

# A Logiweb Page for a Propositional Calculus

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## 1 Introduction

The exercise for this page is to define a theory and prove an implication:

$$\forall \mathcal{A} : \mathcal{A} \Rightarrow \mathcal{A}.$$

We must thus define a formal axiomatic theory for the propositional calculus, which includes the definition of three axioms and two rules of inference, namely modus ponens and generalization. To begin with we only define the first of these rules.

## 2 Propositional Theory

The page defines and test an axiomatic theory for the propositional calculus, which we shall name `prop calc`. This is also the name of the page:

$$[\text{prop calc} \xrightarrow{\text{pyk}} \text{“prop calc”}]$$

The T<sub>E</sub>X appearance of the propositional calculus when used on this and other pages will be as follows:

$$[\text{T}_P \xrightarrow{\text{stmt}} \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{d} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{d} \oplus \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{b} \oplus \neg \underline{c} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{c}]^1$$

## 3 Implication

Contrary to [1] we make the implication postassociative. In accordance with [1] p. 45 it is defined as:

$$[b \Rightarrow c \xrightarrow{\text{val}} \neg b \vee c].^2$$

$$b \Rightarrow c \Rightarrow b$$

That is, the implication is an abbreviation of the or operator with the relevant operands as shown.

## 4 Axioms

In this page we define a theory as it appears in [1], pp. 35ff. We therefore define a number of axioms and a rule of inference. The first axiom we name A1:

$$[A1 \xrightarrow{\text{stmt}} \text{T}_P \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{b}][A1 \xrightarrow{\text{proof}} \text{Rule tactic}].^3$$

And the second, A2:

$$[A2 \xrightarrow{\text{stmt}} \text{T}_P \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{d} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{d}][A2 \xrightarrow{\text{proof}} \text{Rule tactic}].^4$$

And the third, A3:

$$[A3 \xrightarrow{\text{stmt}} \text{T}_P \vdash \neg \underline{c} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{c}][A3 \xrightarrow{\text{proof}} \text{Rule tactic}].^5$$

---

<sup>1</sup>[T<sub>P</sub>  $\xrightarrow{\text{pyk}}$  “prop theory”]

<sup>2</sup>[x  $\Rightarrow$  y  $\xrightarrow{\text{pyk}}$  “\* imply \*”]

<sup>3</sup>[A1  $\xrightarrow{\text{pyk}}$  “axiom one”]

<sup>4</sup>[A2  $\xrightarrow{\text{pyk}}$  “axiom two”]

<sup>5</sup>[A3  $\xrightarrow{\text{pyk}}$  “axiom three”]

# A The Name of the Page

This defines the name of the page:

$$[\text{prop calc} \xrightarrow{\text{pyk}} \text{“prop calc”}]$$

## B T<sub>E</sub>X Definitions

$$[\text{T}_P \xrightarrow{\text{tex}} \text{“T-}\{P\}\text{”}]$$

$$[\text{b} \Rightarrow \text{c} \xrightarrow{\text{tex}} \text{“}\#1. \setrightarrow\{ \} \#2.\text{”}]$$

$$[\text{A1} \xrightarrow{\text{tex}} \text{“A1”}]$$

$$[\text{A2} \xrightarrow{\text{tex}} \text{“A2”}]$$

$$[\text{A3} \xrightarrow{\text{tex}} \text{“A3”}]$$

## C Test

Test cases are listed here. To avoid T<sub>E</sub>X errors about missing items, a trivial test has been included.

$$[\text{T}]$$

## D Priority Table

$$[\text{prop calc} \xrightarrow{\text{prio}}$$

**Preassociative**

[prop calc], [base], [bracket \* end bracket], [big bracket \* end bracket],  
[math \* end math], [**flush left** [\*]], [x], [y], [z], [[\*  $\bowtie$  \*]], [[\*  $\xrightarrow{*}$  \*]], [pyk], [tex],  
[name], [prio], [\*,], [T], [if(\*,\*,\*)], [[\*  $\xrightarrow{*}$  \*]], [val], [claim], [ $\perp$ ], [f(\*)], [(\*)<sup>I</sup>], [F], [0],  
[1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d],  
[e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(\*)<sup>M</sup>], [If(\*,\*,  
\*)], [array{\*} \* end array], [l], [c], [r], [empty], [( \* | \* := \*)], [ $\mathcal{M}(*)$ ], [ $\mathcal{U}(*)$ ], [ $\mathcal{U}(*)$ ],  
[ $\mathcal{U}^M(*)$ ], [**apply**(\*,\*)], [**apply**<sub>1</sub>(\*,\*)], [identifier(\*)], [identifier<sub>1</sub>(\*,\*)], [array-  
plus(\*,\*)], [array-remove(\*,\*,\*)], [array-put(\*,\*,\*,\*)], [array-add(\*,\*,\*,\*,\*)],  
[bit(\*,\*)], [bit<sub>1</sub>(\*,\*)], [rack], ["vector"], ["bibliography"], ["dictionary"],  
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],  
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],  
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],  
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],  
[ $\mathcal{E}(*,*,*)$ ], [ $\mathcal{E}_2(*,*,*,*,*)$ ], [ $\mathcal{E}_3(*,*,*,*,*)$ ], [ $\mathcal{E}_4(*,*,*,*,*)$ ], [**lookup**(\*,\*,\*)],  
[**abstract**(\*,\*,\*,\*,\*)], [[\*]], [ $\mathcal{M}(*,*,*)$ ], [ $\mathcal{M}_2(*,*,*,*,*)$ ], [ $\mathcal{M}^*(*,*,*,*)$ ], [macro],

$[so], [zip(*, *)], [assoc_1(*, *, *)], [(*)^P], [self], [[* \doteq *]], [[* \dot{=} *]], [[* \dot{=} *]],$   
 $[[* \stackrel{pyk}{=} *]], [[* \stackrel{tex}{=} *]], [[* \stackrel{name}{=} *]], [Priority\ table[*]], [\tilde{\mathcal{M}}_1], [\tilde{\mathcal{M}}_2(*)], [\tilde{\mathcal{M}}_3(*)],$   
 $[\tilde{\mathcal{M}}_4(*, *, *, *)], [\mathcal{M}(*, *, *)], [\tilde{\mathcal{Q}}(*, *, *)], [\tilde{\mathcal{Q}}_2(*, *, *)], [\tilde{\mathcal{Q}}_3(*, *, *, *)], [\tilde{\mathcal{Q}}^*(*, *, *)],$   
 $[(*)], [aspect(*, *)], [aspect(*, *, *)], [(*)], [tuple_1(*)], [tuple_2(*)], [let_2(*, *)],$   
 $[let_1(*, *)], [[* \stackrel{claim}{=} *]], [checker], [check(*, *)], [check_2(*, *, *)], [check_3(*, *, *)],$   
 $[check^*(*, *)], [check^*_2(*, *, *)], [[*]'], [[*]^-], [[*]^\circ], [msg], [[* \stackrel{msg}{=} *]], [<stmt>],$   
 $[stmt], [[* \stackrel{stmt}{=} *]], [HeadNil'], [HeadPair'], [Transitivity'], [\perp], [Contra'], [T'_E],$   
 $[L_1], [\underline{*}], [\mathcal{A}], [\mathcal{B}], [\mathcal{C}], [\mathcal{D}], [\mathcal{E}], [\mathcal{F}], [\mathcal{G}], [\mathcal{H}], [\mathcal{I}], [\mathcal{J}], [\mathcal{K}], [\mathcal{L}], [\mathcal{M}], [\mathcal{N}], [\mathcal{O}], [\mathcal{P}], [\mathcal{Q}],$   
 $[\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [[* \mid * := *]], [(*)^* \mid * := *]], [\emptyset], [Remainder],$   
 $[(*)^\vee], [error(*, *)], [error_2(*, *)], [proof(*, *, *)], [proof_2(*, *)], [\mathcal{S}(*, *)], [\mathcal{S}^I(*, *)],$   
 $[\mathcal{S}^\triangleright(*, *)], [\mathcal{S}_1^\triangleright(*, *, *)], [\mathcal{S}^E(*, *)], [\mathcal{S}_1^E(*, *, *)], [\mathcal{S}^+(*, *)], [\mathcal{S}_1^+(*, *, *)],$   
 $[\mathcal{S}^-(*, *)], [\mathcal{S}_1^-(*, *, *)], [\mathcal{S}^*(*, *)], [\mathcal{S}_1^*(*, *, *)], [\mathcal{S}_2^*(*, *, *, *)], [\mathcal{S}^\oplus(*, *)],$   
 $[\mathcal{S}_1^\oplus(*, *, *, *)], [\mathcal{S}^\mp(*, *)], [\mathcal{S}_1^\mp(*, *, *, *)], [\mathcal{S}^{\#}(*, *)], [\mathcal{S}_1^{\#}(*, *, *, *)], [\mathcal{S}^{i.e.}(*, *)],$   
 $[\mathcal{S}_1^{i.e.}(*, *, *, *, *)], [\mathcal{S}_2^{i.e.}(*, *, *, *, *)], [\mathcal{S}^\vee(*, *)], [\mathcal{S}_1^\vee(*, *, *, *)], [\mathcal{S}^i(*, *)],$   
 $[\mathcal{S}_1^i(*, *, *, *)], [\mathcal{S}_2^i(*, *, *, *, *)], [\mathcal{T}(*)], [claims(*, *, *)], [claims_2(*, *, *)], [<proof>],$   
 $[proof], [[Lemma\ * : *]], [[Proof\ of\ * : *]], [[* lemma\ * : *]],$   
 $[[* antilemma\ * : *]], [[* rule\ * : *]], [[* antirule\ * : *]], [verifier], [\mathcal{V}_1(*)],$   
 $[\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)], [\mathcal{V}_7(*, *, *, *)],$   
 $[Cut(*, *)], [Head_\oplus(*)], [Tail_\oplus(*)], [rule_1(*, *)], [rule(*, *)], [Rule\ tactic],$   
 $[Plus(*, *)], [[Theory\ *]], [theory_2(*, *)], [theory_3(*, *)], [theory_4(*, *, *)],$   
 $[HeadNil''], [HeadPair''], [Transitivity''], [Contra''], [HeadNil], [HeadPair],$   
 $[Transitivity], [Contra], [T_E], [ragged\ right], [ragged\ right\ expansion],$   
 $[parm(*, *, *)], [parm^*(*, *, *)], [inst(*, *)], [inst^*(*, *)], [occur(*, *, *)],$   
 $[occur^*(*, *, *)], [unify(* = *, *)], [unify^*(* = *, *)], [unify_2(* = *, *)], [L_a], [L_b],$   
 $[L_c], [L_d], [L_e], [L_f], [L_g], [L_h], [L_i], [L_j], [L_k], [L_l], [L_m], [L_n], [L_o], [L_p], [L_q], [L_r],$   
 $[L_s], [L_t], [L_u], [L_v], [L_w], [L_x], [L_y], [L_z], [L_A], [L_B], [L_C], [L_D], [L_E], [L_F], [L_G],$   
 $[L_H], [L_I], [L_J], [L_K], [L_L], [L_M], [L_N], [L_O], [L_P], [L_Q], [L_R], [L_S], [L_T], [L_U], [L_V],$   
 $[L_W], [L_X], [L_Y], [L_Z], [L_?], [Reflexivity], [Reflexivity_1], [Commutativity],$   
 $[Commutativity_1], [<tactic>], [tactic], [[* \stackrel{tactic}{=} *]], [\mathcal{P}(*, *, *)], [\mathcal{P}^*(*, *, *)], [p_0],$   
 $[conclude_1(*, *)], [conclude_2(*, *, *)], [conclude_3(*, *, *, *)];$

**Preassociative**

$[T_P], [A1], [A2], [A3];$

**Preassociative**

$[* \{ * \}], [*'], [* \mid *], [* \multimap *], [* \Rightarrow *];$

**Preassociative**

$[“\ *”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [$   
 $*, [ \ ], [ * ], [ ! * ], [ ? * ], [ \# * ], [ \$ * ], [ \% * ], [ \& * ], [ ' * ], [ ( * ), ( ) * ], [ ** ], [ + * ], [ , * ], [ - * ], [ . * ], [ / * ],$   
 $[ 0 * ], [ 1 * ], [ 2 * ], [ 3 * ], [ 4 * ], [ 5 * ], [ 6 * ], [ 7 * ], [ 8 * ], [ 9 * ], [ : * ], [ ; * ], [ < * ], [ = * ], [ > * ], [ ? * ],$   
 $[ @ * ], [ A * ], [ B * ], [ C * ], [ D * ], [ E * ], [ F * ], [ G * ], [ H * ], [ I * ], [ J * ], [ K * ], [ L * ], [ M * ], [ N * ],$   
 $[ O * ], [ P * ], [ Q * ], [ R * ], [ S * ], [ T * ], [ U * ], [ V * ], [ W * ], [ X * ], [ Y * ], [ Z * ], [ [ * ], [ \setminus * ], [ ] * ], [ ^ * ],$   
 $[ * ], [ * ], [ a * ], [ b * ], [ c * ], [ d * ], [ e * ], [ f * ], [ g * ], [ h * ], [ i * ], [ j * ], [ k * ], [ l * ], [ m * ], [ n * ], [ o * ],$   
 $[ p * ], [ q * ], [ r * ], [ s * ], [ t * ], [ u * ], [ v * ], [ w * ], [ x * ], [ y * ], [ z * ], [ \{ * ], [ [ * ], [ \} * ], [ \sim * ],$   
 $[Preassociative\ *; *], [Postassociative\ *; *], [ [ * ], * ], [priority\ * end],$   
 $[newline\ *], [macro\ newline\ *];$

**Preassociative**
 $[*0], [*1], [0b], [*-color(*)], [*-color^*(*)];$ 
**Preassociative**
 $[* ' *], [* ^ *];$ 
**Preassociative**
 $[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*^i], [*^d], [*^R], [*^0],$   
 $[*^1], [*^2], [*^3], [*^4], [*^5], [*^6], [*^7], [*^8], [*^9], [*^E], [*^V], [*^C], [*^{C^*}];$ 
**Preassociative**
 $[* \cdot *], [* \cdot_0 *];$ 
**Preassociative**
 $[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *];$ 
**Preassociative**
 $[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$ 
**Postassociative**
 $[* \dot{.} *], [* \dot{.}^* *], [* \dot{:} *], [* \dot{+}2* *], [* \dot{:}^* *], [* \dot{+}2* *];$ 
**Postassociative**
 $[*, *];$ 
**Preassociative**
 $[* \overset{B}{\approx} *], [* \overset{D}{\approx} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \overset{+}{\rightarrow} *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$   
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{ free in } *], [* \text{ free in }^* *], [* \text{ free for } * \text{ in } *],$   
 $[* \text{ free for }^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *];$ 
**Preassociative**
 $[\neg *];$ 
**Preassociative**
 $[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *];$ 
**Preassociative**
 $[* \vee *], [* \parallel *], [* \ddot{\vee} *];$ 
**Postassociative**
 $[* \ddot{\Rightarrow} *];$ 
**Postassociative**
 $[* \Rightarrow *];$ 
**Postassociative**
 $[* : *], [* ! *];$ 
**Preassociative**
 $[* \begin{Bmatrix} * \\ * \end{Bmatrix}];$ 
**Preassociative**
 $[\lambda * . *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \ddot{=} * \text{ in } *];$ 
**Preassociative**
 $[*^I], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$ 
**Preassociative**
 $[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *];$ 
**Preassociative**
 $[* \triangleright *], [* \blacktriangleright *], [* \gg *];$ 
**Postassociative**

[\*  $\vdash$  \*], [\*  $\Vdash$  \*], [\* i.e. \*];

**Preassociative**

[ $\forall$ \*: \*];

**Postassociative**

[\*  $\oplus$  \*];

**Postassociative**

[\*, \*];

**Preassociative**

[\* proves \*];

**Preassociative**

[\* **proof of** \* : \*], [Line \* : \*  $\gg$  \*; \*], [Last line \*  $\gg$  \*], [Line \* : Premise  $\gg$  \*; \*],

[Line \* : Side-condition  $\gg$  \*; \*], [Arbitrary  $\gg$  \*; \*], [Local  $\gg$  \* = \*; \*];

**Postassociative**

[\* then \*], [\* [\* ]\*];

**Preassociative**

[\*&\*];

**Preassociative**

[\*\\[\*];]

## E Bibliography

- [1] E. Mendelson. *Introduction to Mathematical Logic*. Wadsworth and Brooks, 3. edition, 1987.