

A Logiweb Page for a Propositional Calculus

Martin Røpcke

GRD-2005-06-05.UTC:08:46:27.673984

Contents

1	Introduction	1
2	Propositional Theory	2
3	Implication	2
4	Axioms	2
A	The Name of the Page	3
B	T_EX Definitions	3
C	Test	3
D	Priority Table	3
E	Bibliography	6

1 Introduction

The exercise for this page is to define a theory and prove an implication:

$$\forall \mathcal{A} : \mathcal{A} \Rightarrow \mathcal{A}.$$

We must thus define a formal axiomatic theory for the propositional calculus, which includes the definition of three axioms and two rules of inference, namely modus ponens and generalization. To begin with we only define the first of these rules.

2 Propositional Theory

The page defines and test an axiomatic theory for the propositional calculus, which we shall name **prop calc**. This is also the name of the page:

$$[\text{prop calc} \stackrel{\text{pyk}}{=} \text{“prop calc”}]$$

The T_EX appearance of the propositional calculus when used on this and other pages will be as follows:

$$[\textbf{Theory T}_P] \textcolor{red}{^1}$$

3 Implication

Contrary to [1] we make the implication postassociative. In accordance with [1] p. 45 it is defined as:

$$[b \Rightarrow c \doteq \neg b \vee c]. \textcolor{red}{^2}$$

$$b \Rightarrow c \Rightarrow b$$

That is, the implication is an abbriviation of the or operator with the relevant operands as shown.

4 Axioms

In this page we define a theory as it appears in [1], pp. 35ff. We therefore define a number of axioms and a rule of inference. The first axiom we name A1:

$$[T_P \textbf{ rule A1: } \mathcal{B} \Rightarrow (\mathcal{C} \Rightarrow \mathcal{B})]. \textcolor{red}{^3}$$

And the second, A2:

$$[T_P \textbf{ rule A2: } ((\mathcal{B} \Rightarrow \mathcal{C}) \Rightarrow \mathcal{D}) \Rightarrow ((\mathcal{B} \Rightarrow \mathcal{C}) \Rightarrow (\mathcal{B} \Rightarrow \mathcal{D}))]. \textcolor{red}{^4}$$

And the third, A3:

$$[T_P \textbf{ rule A3: } ((\neg \mathcal{C} \Rightarrow \neg \mathcal{B}) \Rightarrow (\neg \mathcal{C} \Rightarrow \mathcal{B}) \Rightarrow \mathcal{C})]. \textcolor{red}{^5}$$

¹[T_P $\stackrel{\text{pyk}}{=}$ “prop theory”]

²[x $\Rightarrow$ y $\stackrel{\text{pyk}}{=}$ “* imply *”]

³[A1 $\stackrel{\text{pyk}}{=}$ “axiom one”]

⁴[A2 $\stackrel{\text{pyk}}{=}$ “axiom two”]

⁵[A3 $\stackrel{\text{pyk}}{=}$ “axiom three”]

A The Name of the Page

This defines the name of the page:

[prop calc $\stackrel{\text{pyk}}{=}$ “prop calc”]

B $\text{\TeX}$ Definitions

[T_P $\stackrel{\text{tex}}{=}$ “ $\text{T}_{\{P\}}$ ”]

[$\text{b} \Rightarrow \text{c}$ $\stackrel{\text{tex}}{=}$ “ $\#1. \setminus\text{Rightarrow}\{\}$ $\#2.$ ”]

[A1 $\stackrel{\text{tex}}{=}$ “ A1 ”]

[A2 $\stackrel{\text{tex}}{=}$ “ A2 ”]

[A3 $\stackrel{\text{tex}}{=}$ “ A3 ”]

C Test

Test cases are listed here. To avoid $\text{\TeX}$ errors about missing items, a trivial test has been included.

[T]

D Priority Table

Priority table

Preassociative

[prop calc], [base], [bracket * end bracket], [big bracket * end bracket],
[math * end math], [**flush left** *], [x], [y], [z], [$[* \bowtie *]$], [$[* \stackrel{*}{\rightarrow} *]$], [pyk], [tex],
[name], [prio], [*], [T], [if(*, *, *)], [$[* \stackrel{*}{\Rightarrow} *]$], [val], [claim], [$\perp$], [$f(*)$], [$(*)^I$], [F], [$\underline{0}$],
[$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [a], [b], [c], [d],
[e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^M$], [If(*, *,
), [array{} * end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}(*)$],
[$\mathcal{U}^M(*)$], [**apply**(*, *)], [**apply**₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-
plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)],
[bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"],
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
[$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**(*, *, *)],
[**abstract**(*, *, *, *)], [$[*]$], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro],
[s_0], [**zip**(*, *)], [**assoc**₁(*, *, *)], [$(*)^P$], [self], [$[* \dot{=}]$], [$[* \dot{=}]$], [$[* \dot{=}]$],

$[[* \stackrel{\text{pyk}}{=} *]], [[* \stackrel{\text{tex}}{=} *]], [[* \stackrel{\text{name}}{=} *]], [\text{Priority table}[*]], [\tilde{\mathcal{M}}_1], [\tilde{\mathcal{M}}_2(*)], [\tilde{\mathcal{M}}_3(*)],$
 $[\tilde{\mathcal{M}}_4(*, *, *, *)], [\mathcal{M}(*, *, *)], [\mathcal{Q}(*, *, *)], [\mathcal{Q}_2(*, *, *)], [\mathcal{Q}_3(*, *, *, *)], [\mathcal{Q}^*(*, *, *)],$
 $[(*)], [\text{aspect}(*, *)], [\text{aspect}(*, *, *)], [(\langle * \rangle)], [\text{tuple}_1(*)], [\text{tuple}_2(*)], [\text{let}_2(*, *)],$
 $[\text{let}_1(*, *)], [[* \stackrel{\text{claim}}{=} *]], [\text{checker}], [\text{check}(*, *)], [\text{check}_2(*, *, *)], [\text{check}_3(*, *, *)],$
 $[\text{check}^*(*, *)], [\text{check}_2^*(*, *, *)], [[*]'], [[*]^-], [[*]^\circ], [\text{msg}], [[* \stackrel{\text{msg}}{=} *]], [<\text{stmt}>],$
 $[\text{stmt}], [[* \stackrel{\text{stmt}}{=} *]], [\text{HeadNil}'], [\text{HeadPair}'], [\text{Transitivity}'], [\perp], [\text{Contra}'], [\text{T}_E],$
 $[\text{L}_1], [\text{L}_2], [\mathcal{A}], [\mathcal{B}], [\mathcal{C}], [\mathcal{D}], [\mathcal{E}], [\mathcal{F}], [\mathcal{G}], [\mathcal{H}], [\mathcal{I}], [\mathcal{J}], [\mathcal{K}], [\mathcal{L}], [\mathcal{M}], [\mathcal{N}], [\mathcal{O}], [\mathcal{P}], [\mathcal{Q}],$
 $[\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [(* \mid * := *)], [(* * \mid * := *)], [\emptyset], [\text{Remainder}],$
 $[(*)^\vee], [\text{error}(*, *)], [\text{error}_2(*, *)], [\text{proof}(*, *, *)], [\text{proof}_2(*, *)], [\mathcal{S}(*, *)], [\mathcal{S}^I(*, *)],$
 $[\mathcal{S}^\triangleright(*, *)], [\mathcal{S}_1^\triangleright(*, *, *)], [\mathcal{S}^E(*, *)], [\mathcal{S}_1^E(*, *, *)], [\mathcal{S}^+(*, *)], [\mathcal{S}_1^+(*, *, *)],$
 $[\mathcal{S}^-(*, *)], [\mathcal{S}_1^-(*, *, *)], [\mathcal{S}^*(*, *)], [\mathcal{S}_1^*(*, *, *)], [\mathcal{S}_2^*(*, *, *, *)], [\mathcal{S}^\textcircled{*}(*, *)],$
 $[\mathcal{S}_1^\textcircled{*}(*, *, *)], [\mathcal{S}^\vdash(*, *)], [\mathcal{S}_1^\vdash(*, *, *, *)], [\mathcal{S}^\sharp(*, *)], [\mathcal{S}_1^\sharp(*, *, *, *)], [\mathcal{S}^{\text{i.e.}}(*, *)],$
 $[\mathcal{S}_1^{\text{i.e.}}(*, *, *, *)], [\mathcal{S}_2^{\text{i.e.}}(*, *, *, *, *)], [\mathcal{S}^\vee(*, *)], [\mathcal{S}_1^\vee(*, *, *, *)], [\mathcal{S}^i(*, *)],$
 $[\mathcal{S}_1^i(*, *, *)], [\mathcal{S}_2^i(*, *, *, *)], [\mathcal{T}(*)], [\text{claims}(*, *, *)], [\text{claims}_2(*, *, *)], [<\text{proof}>],$
 $[\text{proof}], [[\text{Lemma } *: *]], [[\text{Proof of } *: *]], [[* \text{ lemma } *: *]],$
 $[[* \text{ antilemma } *: *]], [[* \text{ rule } *: *]], [[* \text{ antirule } *: *]], [\text{verifier}], [\mathcal{V}_1(*)],$
 $[\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)], [\mathcal{V}_7(*, *, *, *)],$
 $[\text{Cut}(*, *)], [\text{Head}_\oplus(*)], [\text{Tail}_\oplus(*)], [\text{rule}_1(*, *)], [\text{rule}(*, *)], [\text{Rule tactic}],$
 $[\text{Plus}(*, *)], [[\text{Theory } *]], [\text{theory}_2(*, *)], [\text{theory}_3(*, *)], [\text{theory}_4(*, *, *)],$
 $[\text{HeadNil}''], [\text{HeadPair}''], [\text{Transitivity}''], [\text{Contra}''], [\text{HeadNil}], [\text{HeadPair}],$
 $[\text{Transitivity}], [\text{Contra}], [\text{T}_E], [\text{ragged right}], [\text{ragged right expansion}],$
 $[\text{parm}(*, *, *)], [\text{parm}^*(*, *, *)], [\text{inst}(*, *)], [\text{inst}^*(*, *)], [\text{occur}(*, *, *)],$
 $[\text{occur}^*(*, *, *)], [\text{unify}(* = *, *)], [\text{unify}^*(* = *, *)], [\text{unify}_2(* = *, *)], [\text{L}_a], [\text{L}_b],$
 $[\text{L}_c], [\text{L}_d], [\text{L}_e], [\text{L}_f], [\text{L}_g], [\text{L}_h], [\text{L}_i], [\text{L}_j], [\text{L}_k], [\text{L}_l], [\text{L}_m], [\text{L}_n], [\text{L}_o], [\text{L}_p], [\text{L}_q], [\text{L}_r],$
 $[\text{L}_s], [\text{L}_t], [\text{L}_u], [\text{L}_v], [\text{L}_w], [\text{L}_x], [\text{L}_y], [\text{L}_z], [\text{L}_A], [\text{L}_B], [\text{L}_C], [\text{L}_D], [\text{L}_E], [\text{L}_F], [\text{L}_G],$
 $[\text{L}_H], [\text{L}_I], [\text{L}_J], [\text{L}_K], [\text{L}_L], [\text{L}_M], [\text{L}_N], [\text{L}_O], [\text{L}_P], [\text{L}_Q], [\text{L}_R], [\text{L}_S], [\text{L}_T], [\text{L}_U], [\text{L}_V],$
 $[\text{L}_W], [\text{L}_X], [\text{L}_Y], [\text{L}_Z], [\text{L}_7], [\text{Reflexivity}], [\text{Reflexivity}_1], [\text{Commutativity}],$
 $[\text{Commutativity}_1], [<\text{tactic}>], [\text{tactic}], [[* \stackrel{\text{tactic}}{=} *]], [\mathcal{P}(*, *, *)], [\mathcal{P}^*(*, *, *)], [\text{p}_0],$
 $[\text{conclude}_1(*, *)], [\text{conclude}_2(*, *, *)], [\text{conclude}_3(*, *, *, *)];$

Preassociative

$[\text{T}_P], [\text{A1}], [\text{A2}], [\text{A3}];$

Preassociative

$[*_{-}\{*\}], [*'], [*[\text{ } *]], [*[\text{ } \rightarrow *]], [*[\text{ } \Rightarrow *]];$

Preassociative

$["*"], [], [(*)^\dagger], [\text{string}(*) + *], [\text{string}(*) ++ *], [$
 $*, [\text{ } *], [! *], [\' *], [\# *], [\$ *], [\% *], [\& *], [\' *], [(*)], [() *], [**], [+ *], [*], [- *], [*], [/ *],$
 $[0 *], [1 *], [2 *], [3 *], [4 *], [5 *], [6 *], [7 *], [8 *], [9 *], [: *], [; *], [< *], [= *], [> *], [? *],$
 $[@ *], [A *], [B *], [C *], [D *], [E *], [F *], [G *], [H *], [I *], [J *], [K *], [L *], [M *], [N *],$
 $[O *], [P *], [Q *], [R *], [S *], [T *], [U *], [V *], [W *], [X *], [Y *], [Z *], [[*], [\setminus *], [\text{ } *], [\text{ } ^ *],$
 $[_ *], [\' *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *],$
 $[p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [\{ *], [|\text{ } *], [\} *], [\sim *],$
 $[\text{Preassociative } *; *], [\text{Postassociative } *; *], [[*], *], [\text{priority } * \text{ end}],$
 $[\text{newline } *], [\text{macro newline } *];$

Preassociative

$[*0], [*1], [0b], [*-color(*)], [*-color^*(*)];$

Preassociative

$[* ' *], [* ' *];$

Preassociative

$[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*^i], [*^d], [*^R], [*^0],$
 $[*^1], [*^2], [*^3], [*^4], [*^5], [*^6], [*^7], [*^8], [*^9], [*^E], [*^V], [*^C], [*^{C^*}];$

Preassociative

$[* \cdot *], [* \cdot_0 *];$

Preassociative

$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *];$

Preassociative

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$

Postassociative

$[* \dot{:} *], [* \ddot{:} *], [* \ddot{:} : *], [* \underline{+2*} *], [* :: *], [* +2* *];$

Postassociative

$[*, *];$

Preassociative

$[* \overset{B}{\sim} *], [* \overset{D}{\sim} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \overset{+}{\rightarrow} *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{free in } *], [* \text{free in}^* *], [* \text{free for } * \text{ in } *],$
 $[* \text{free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *];$

Preassociative

$[\neg *];$

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *];$

Postassociative

$[* \ddot{\Rightarrow} *];$

Postassociative

$[* \Rightarrow *];$

Postassociative

$[* : *], [* ! *];$

Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right\};$

Preassociative

$[\lambda * . *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \ddot{=} * \text{ in } *];$

Preassociative

$[*^I], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$

Preassociative

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *];$

Preassociative

$[* \triangleright *], [* \blacktriangleright *], [* \gg *];$

Postassociative

$[* \vdash *], [* \Vdash *], [* \text{i.e. } *];$

Preassociative

$[\forall *: *];$

Postassociative

$[* \oplus *];$

Postassociative

$[*, *];$

Preassociative

$[* \text{ proves } *];$

Preassociative

$[* \text{ proof of } * : *], [\text{Line } * : * \gg *; *], [\text{Last line } * \gg *], [\text{Line } * : \text{Premise } \gg *; *],$
 $[\text{Line } * : \text{Side-condition } \gg *; *], [\text{Arbitrary } \gg *; *], [\text{Local } \gg * = *; *];$

Postassociative

$[* \text{ then } *], [* [*] *];$

Preassociative

$[* \& *];$

Preassociative

$[* \setminus *]; \text{End table}$

E Bibliography

[1] E. Mendelson. *Introduction to Mathematical Logic*. Wadsworth and Brooks, 3. edition, 1987.