Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\text{nonfree}}(*,*)$, $\dot{\text{nonfree}}^{*}(*,*)$, $\dot{\text{free}}\langle *|* := *\rangle$, $\dot{\text{free}}^{*}\langle *|* := *\rangle$, $*\dot{\equiv}\langle *|* := *\rangle$, $*\dot{\equiv}\langle {}^{*}*|* := *\rangle$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, L3.2(a)′, M1.7, MP'_{h}, Hypothesize, Gen'_{h}, M3.2(a), $\text{M3.2(a)}_{\text{h}}$, $\text{M3.2(b)}_{\text{h}}$, $\text{M3.1(S1}')_{\text{h}}$, $\text{M3.2(c)}_{\text{h}}$, $\text{M3.1(S2}')_{\text{h}}$, $\text{M3.1(S5}')_{\text{h}}$, $\text{M3.1(S6}')_{\text{h}}$, M3.2(f), $\dot{*}$, $*'$, $*\dot{:}*$, $*\dot{+}*$, $*\overset{\text{p}}{=}*$, $*^{\mathcal{P}}$, $\dot{\neg}*$, $*\dot{\wedge}*$, $*\dot{\vee}*$, $\dot{\forall}*:*$, $\dot{\exists}*:*$, $*\dot{\Rightarrow}*$, $*\dot{\Leftrightarrow}*$, $*\dot{\trianglerighteq}*$, $*\dot{\trianglerighteq}_{\text{h}}*$,

peano

[peano $\overset{\text{prio}}{\rightarrow}$
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** $[*]$], [x], [y], [z], $[[* \bowtie *]]$, $[[* \overset{*}{\rightarrow} *]]$, [pyk], [tex], [name], [prio], $[*]$, [T], $[\text{if}(*,*,*)]$, $[[* \overset{*}{\Rightarrow} *]]$, [val], [claim], $[\bot]$, $[\text{f}(*)]$, $[(*)^{\text{I}}]$, [F], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\text{M}}]$, $[\text{If}(*,*,*)]$, [array $\{*\}$ $*$ end array], [l], [c], [r], [empty], $[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$, $[\mathcal{U}^{\text{M}}(*)]$, $[\textbf{apply}(*,*)]$, $[\textbf{apply}_1(*,*)]$, $[\text{identifier}(*)]$, $[\text{identifier}_1(*,*)]$, $[\text{array-plus}(*,*)]$, $[\text{array-remove}(*,*,*)]$, $[\text{array-put}(*,*,*,*)]$, $[\text{array-add}(*,*,*,*,*)]$, $[\text{bit}(*,*)]$, $[\text{bit}_1(*,*)]$, [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], $[\mathcal{E}(*,*,*)]$, $[\mathcal{E}_2(*,*,*,*,*)]$, $[\mathcal{E}_3(*,*,*,*)]$, $[\mathcal{E}_4(*,*,*,*)]$, $[\textbf{lookup}(*,*,*)]$, $[\textbf{abstract}(*,*,*,*)]$, $[\lceil * \rceil]$, $[\mathcal{M}(*,*,*)]$, $[\mathcal{M}_2(*,*,*,*)]$, $[\mathcal{M}^{*}(*,*,*)]$, [macro], $[\text{s}_0]$, $[\textbf{zip}(*,*)]$, $[\textbf{assoc}_1(*,*,*)]$, $[(*)^{\text{P}}]$, [self], $[[* \doteqdot *]]$, $[[* \doteq *]]$, $[[* \doteq *]]$, $[[* \overset{\text{pyk}}{=} *]]$, $[[* \overset{\text{tex}}{=} *]]$, $[[* \overset{\text{name}}{=} *]]$, [**Priority table**$[*]$], $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$, $[\tilde{\mathcal{M}}_4(*,*,*,*)]$, $[\tilde{\mathcal{M}}(*,*,*)]$, $[\tilde{\mathcal{Q}}(*,*,*)]$, $[\tilde{\mathcal{Q}}_2(*,*,*)]$, $[\tilde{\mathcal{Q}}_3(*,*,*,*)]$, $[\tilde{\mathcal{Q}}^{*}(*,*,*)]$, $[(*)]$, $[\textbf{aspect}(*,*)]$, $[\textbf{aspect}(*,*,*)]$, $[\langle * \rangle]$, $[\textbf{tuple}_1(*)]$, $[\textbf{tuple}_2(*)]$, $[\text{let}_2(*,*)]$, $[\text{let}_1(*,*)]$, $[[* \overset{\text{claim}}{=} *]]$, [checker], $[\textbf{check}(*,*)]$, $[\textbf{check}_2(*,*,*)]$, $[\textbf{check}_3(*,*,*)]$, $[\textbf{check}^{*}(*,*)]$, $[\textbf{check}_2^{*}(*,*,*)]$, $[[*]^{\cdot}]$, $[[*]^{-}]$, $[[*]^{\circ}]$, [msg], $[[* \overset{\text{msg}}{=} *]]$, [<stmt>], [stmt], $[[* \overset{\text{stmt}}{=} *]]$, [HeadNil′], [HeadPair′], [Transitivity′], $[\perp\!\!\!\perp]$, [Contra′], $[\text{T}'_{\text{E}}]$, $[\text{L}_1]$, $[\underline{*}]$, $[\mathcal{A}]$, $[\mathcal{B}]$, $[\mathcal{C}]$, $[\mathcal{D}]$, $[\mathcal{E}]$, $[\mathcal{F}]$, $[\mathcal{G}]$, $[\mathcal{H}]$, $[\mathcal{I}]$, $[\mathcal{J}]$, $[\mathcal{K}]$, $[\mathcal{L}]$, $[\mathcal{M}]$, $[\mathcal{N}]$, $[\mathcal{O}]$, $[\mathcal{P}]$, $[\mathcal{Q}]$,

$[\mathcal{R}]$, $[\mathcal{S}]$, $[\mathcal{T}]$, $[\mathcal{U}]$, $[\mathcal{V}]$, $[\mathcal{W}]$, $[\mathcal{X}]$, $[\mathcal{Y}]$, $[\mathcal{Z}]$, $[\langle * \mid * := * \rangle]$, $[\langle {}^{*} * \mid * := * \rangle]$, $[\emptyset]$, [Remainder], $[(*)^{\mathbf{v}}]$, [intro$(*,*,*,*)$], [intro$(*,*,*)$], [error$(*,*)$], [error$_2(*,*)$], [proof$(*,*,*)$], [proof$_2(*,*)$], $[\mathcal{S}(*,*)]$, $[\mathcal{S}^{\mathrm{I}}(*,*)]$, $[\mathcal{S}^{\triangleright}(*,*)]$, $[\mathcal{S}_1^{\triangleright}(*,*,*)]$, $[\mathcal{S}^{\mathrm{E}}(*,*)]$, $[\mathcal{S}_1^{\mathrm{E}}(*,*,*)]$, $[\mathcal{S}^{+}(*,*)]$, $[\mathcal{S}_1^{+}(*,*,*)]$, $[\mathcal{S}^{-}(*,*)]$, $[\mathcal{S}_1^{-}(*,*,*)]$, $[\mathcal{S}^{*}(*,*)]$, $[\mathcal{S}_1^{*}(*,*,*)]$, $[\mathcal{S}_2^{*}(*,*,*,*)]$, $[\mathcal{S}^{@}(*,*)]$, $[\mathcal{S}_1^{@}(*,*,*)]$, $[\mathcal{S}^{\vdash}(*,*)]$, $[\mathcal{S}_1^{\vdash}(*,*,*,*)]$, $[\mathcal{S}^{\Vdash}(*,*)]$, $[\mathcal{S}_1^{\Vdash}(*,*,*,*)]$, $[\mathcal{S}^{\mathrm{i.e.}\cdot}(*,*)]$, $[\mathcal{S}_1^{\mathrm{i.e.}\cdot}(*,*,*,*)]$, $[\mathcal{S}_2^{\mathrm{i.e.}\cdot}(*,*,*,*,*)]$, $[\mathcal{S}^{\forall}(*,*)]$, $[\mathcal{S}_1^{\forall}(*,*,*,*)]$, $[\mathcal{S}^{\mathrm{;}}(*,*)]$, $[\mathcal{S}_1^{\mathrm{;}}(*,*,*)]$, $[\mathcal{S}_2^{\mathrm{;}}(*,*,*,*)]$, $[\mathcal{T}(*)]$, [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*,*)]$, $[\mathcal{V}_3(*,*,*,*)]$, $[\mathcal{V}_4(*,*)]$, $[\mathcal{V}_5(*,*,*,*)]$, $[\mathcal{V}_6(*,*,*,*)]$, $[\mathcal{V}_7(*,*,*,*)]$, [Cut$(*,*)$], [Head$_{\oplus}(*)$], [Tail$_{\oplus}(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [$\mathrm{T_E}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^{*}(*,*,*)$], [inst$(*,*)$], [inst$^{*}(*,*)$], [occur$(*,*,*)$], [occur$^{*}(*,*,*)$], [unify$(*=*,*)$], [unify$^{*}(*=*,*)$], [unify$_2(*=*,*)$], [$\mathrm{L_a}$], [$\mathrm{L_b}$], [$\mathrm{L_c}$], [$\mathrm{L_d}$], [$\mathrm{L_e}$], [$\mathrm{L_f}$], [$\mathrm{L_g}$], [$\mathrm{L_h}$], [$\mathrm{L_i}$], [$\mathrm{L_j}$], [$\mathrm{L_k}$], [$\mathrm{L_l}$], [$\mathrm{L_m}$], [$\mathrm{L_n}$], [$\mathrm{L_o}$], [$\mathrm{L_p}$], [$\mathrm{L_q}$], [$\mathrm{L_r}$], [$\mathrm{L_s}$], [$\mathrm{L_t}$], [$\mathrm{L_u}$], [$\mathrm{L_v}$], [$\mathrm{L_w}$], [$\mathrm{L_x}$], [$\mathrm{L_y}$], [$\mathrm{L_z}$], [$\mathrm{L_A}$], [$\mathrm{L_B}$], [$\mathrm{L_C}$], [$\mathrm{L_D}$], [$\mathrm{L_E}$], [$\mathrm{L_F}$], [$\mathrm{L_G}$], [$\mathrm{L_H}$], [$\mathrm{L_I}$], [$\mathrm{L_J}$], [$\mathrm{L_K}$], [$\mathrm{L_L}$], [$\mathrm{L_M}$], [$\mathrm{L_N}$], [$\mathrm{L_O}$], [$\mathrm{L_P}$], [$\mathrm{L_Q}$], [$\mathrm{L_R}$], [$\mathrm{L_S}$], [$\mathrm{L_T}$], [$\mathrm{L_U}$], [$\mathrm{L_V}$], [$\mathrm{L_W}$], [$\mathrm{L_X}$], [$\mathrm{L_Y}$], [$\mathrm{L_Z}$], [$\mathrm{L_?}$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]], $[\mathcal{P}(*,*,*)]$, $[\mathcal{P}^{*}(*,*,*)]$, [p$_0$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], $[\dot{0}]$, $[\dot{1}]$, $[\dot{2}]$, $[\dot{a}]$, $[\dot{b}]$, $[\dot{c}]$, $[\dot{d}]$, $[\dot{e}]$, $[\dot{f}]$, $[\dot{g}]$, $[\dot{h}]$, $[\dot{i}]$, $[\dot{j}]$, $[\dot{k}]$, $[\dot{l}]$, $[\dot{m}]$, $[\dot{n}]$, $[\dot{o}]$, $[\dot{p}]$, $[\dot{q}]$, $[\dot{r}]$, $[\dot{s}]$, $[\dot{t}]$, $[\dot{u}]$, $[\dot{v}]$, $[\dot{w}]$, $[\dot{x}]$, $[\dot{y}]$, $[\dot{z}]$, [nonfree$(*,*)$], [nonfree$^{*}(*,*)$], [free$\langle * | * := * \rangle$], [free$^{*}\langle * | * := * \rangle$], [$* \equiv \langle * | * := * \rangle$], [$* \equiv \langle {}^{*} * | * := * \rangle$], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$], [L3.2(a)$'$], [M1.7], [MP$'_{\mathrm{h}}$], [Hypothesize], [Gen$'_{\mathrm{h}}$], [M3.2(a)], [M3.2(a)$_{\mathrm{h}}$], [M3.2(b)$_{\mathrm{h}}$], [M3.1(S1$'$)$_{\mathrm{h}}$], [M3.2(c)$_{\mathrm{h}}$], [M3.1(S2$'$)$_{\mathrm{h}}$], [M3.1(S5$'$)$_{\mathrm{h}}$], [M3.1(S6$'$)$_{\mathrm{h}}$], [M3.2(f)];

Preassociative

[$*$_{$*$}], [$*'$], [$*[\,*\,]$], [$*[*\rightarrow*]$], [$*[*\Rightarrow*]$], [$\dot{*}$];

Preassociative

[“ $*$ ”], [], $[(*)^{\mathbf{t}}]$, [string$(*)$ + $*$], [string$(*)$ ++ $*$], [$*$], [$*$], [!$*$], [”$*$], [#$*$], [\$$*$], [%$*$], [&$*$], [’$*$], [($*$], [)$*$], [$**$], [+$*$], [,$*$], [-$*$], [.$*$], [/$*$], [0$*$], [1$*$], [2$*$], [3$*$], [4$*$], [5$*$], [6$*$], [7$*$], [8$*$], [9$*$], [:$*$], [;$*$], [<$*$], [=$*$], [>$*$], [?$*$], [@$*$], [A$*$], [B$*$], [C$*$], [D$*$], [E$*$], [F$*$], [G$*$], [H$*$], [I$*$], [J$*$], [K$*$], [L$*$], [M$*$], [N$*$], [O$*$], [P$*$], [Q$*$], [R$*$], [S$*$], [T$*$], [U$*$], [V$*$], [W$*$], [X$*$], [Y$*$], [Z$*$], [[$*$], [\$*$], []$*$], [ˆ$*$], [_$*$], [‘$*$], [a$*$], [b$*$], [c$*$], [d$*$], [e$*$], [f$*$], [g$*$], [h$*$], [i$*$], [j$*$], [k$*$], [l$*$], [m$*$], [n$*$], [o$*$], [p$*$], [q$*$], [r$*$], [s$*$], [t$*$], [u$*$], [v$*$], [w$*$], [x$*$], [y$*$], [z$*$], [{$*$], [|$*$], [}$*$], [˜$*$], [**Preassociative** $*$; $*$], [**Postassociative** $*$; $*$], [[$*$], $*$], [priority $*$ end], [newline $*$], [macro newline $*$];

Preassociative

[$*$0], [$*$1], [0b], [$*$-color$(*)$], [$*$-color$^{*}(*)$];

Preassociative

$[* \,{}' *]$, $[* \,{}` *]$;

Preassociative

$[*^{\mathrm{H}}]$, $[*^{\mathrm{T}}]$, $[*^{\mathrm{U}}]$, $[*^{\mathrm{h}}]$, $[*^{\mathrm{t}}]$, $[*^{\mathrm{s}}]$, $[*^{\mathrm{c}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{a}}]$, $[*^{\mathrm{C}}]$, $[*^{\mathrm{M}}]$, $[*^{\mathrm{B}}]$, $[*^{\mathrm{r}}]$, $[*^{\mathrm{i}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{R}}]$, $[*^{0}]$, $[*^{1}]$, $[*^{2}]$, $[*^{3}]$, $[*^{4}]$, $[*^{5}]$, $[*^{6}]$, $[*^{7}]$, $[*^{8}]$, $[*^{9}]$, $[*^{\mathrm{E}}]$, $[*^{\mathcal{V}}]$, $[*^{\mathcal{C}}]$, $[*^{\mathcal{C}^*}]$, $[*']$;

Preassociative

$[* \cdot *]$, $[* \cdot_0 *]$, $[* \dot{:} *]$;

Preassociative

$[* + *]$, $[* +_0 *]$, $[* +_1 *]$, $[* - *]$, $[* -_0 *]$, $[* -_1 *]$, $[* \dot{+} *]$;

Preassociative

$[* \cup \{*\}]$, $[* \cup *]$, $[* \backslash \{*\}]$;

Postassociative

$[* \mathbin{\dot{\cdot}} *]$, $[* \mathbin{\underline{\dot{\cdot}}} *]$, $[* \mathbin{\underline{::}} *]$, $[* \mathbin{\underline{+2*}} *]$, $[* :: *]$, $[* +2* *]$;

Postassociative

$[*, *]$;

Preassociative

$[* \overset{\mathrm{B}}{\approx} *]$, $[* \overset{\mathrm{D}}{\approx} *]$, $[* \overset{\mathrm{C}}{\approx} *]$, $[* \overset{\mathrm{P}}{\approx} *]$, $[* \approx *]$, $[* = *]$, $[* \overset{+}{\to} *]$, $[* \overset{\mathrm{t}}{=} *]$, $[* \overset{\mathrm{t}^*}{=} *]$, $[* \overset{\mathrm{r}}{=} *]$, $[* \in_{\mathrm{t}} *]$, $[* \subseteq_{\mathrm{T}} *]$, $[* \overset{\mathrm{T}}{=} *]$, $[* \overset{\mathrm{s}}{=} *]$, $[* \text{ free in } *]$, $[* \text{ free in}^* *]$, $[* \text{ free for } * \text{ in } *]$, $[* \text{ free for}^* * \text{ in } *]$, $[* \in_{\mathrm{c}} *]$, $[* < *]$, $[* <' *]$, $[* \leq' *]$, $[* \overset{\mathrm{p}}{=} *]$, $[*^{\mathcal{P}}]$;

Preassociative

$[\neg *]$, $[\dot{\neg} *]$;

Preassociative

$[* \wedge *]$, $[* \ddot{\wedge} *]$, $[* \tilde{\wedge} *]$, $[* \wedge_{\mathrm{c}} *]$, $[* \dot{\wedge} *]$;

Preassociative

$[* \vee *]$, $[* \parallel *]$, $[* \ddot{\vee} *]$, $[* \dot{\vee} *]$;

Preassociative

$[\dot{\forall} *: *]$, $[\dot{\exists} *: *]$;

Postassociative

$[* \ddot{\Rightarrow} *]$, $[* \dot{\Rightarrow} *]$, $[* \dot{\Leftrightarrow} *]$;

Postassociative

$[* : *]$, $[*!*]$;

Preassociative

$[* \begin{cases} * \\ * \end{cases}]$;

Preassociative

$[\lambda * .*]$, $[\Lambda *]$, $[\textbf{if } * \textbf{ then } * \textbf{ else } *]$, $[\textbf{let } * = * \textbf{ in } *]$, $[\textbf{let } * \ddot{=} * \text{ in } *]$;

Preassociative

$[*^{\mathrm{I}}]$, $[*^{\triangleright}]$, $[*^{\mathrm{V}}]$, $[*^{+}]$, $[*^{-}]$, $[*^*]$;

Preassociative

$[* @ *]$, $[* \triangleright *]$, $[* \mathbb{\triangleright} *]$, $[* \gg *]$, $[* \trianglerighteq *]$, $[* \trianglerighteq_{\mathrm{h}} *]$;

Postassociative

$[* \vdash *]$, $[* \Vdash *]$, $[* \text{ i.e. } *]$;

Preassociative

$[\forall *: *]$;

Postassociative

$[* \oplus *]$;

Postassociative
$[*;*];$
Preassociative
$[*\ \text{proves}\ *];$
Preassociative
$[*\ \textbf{proof of}\ *:*], [\text{Line}\ *:*\gg *;*], [\text{Last line}\ *\gg *\,\square],$
$[\text{Line}\ *:\text{Premise}\gg *;*], [\text{Line}\ *:\text{Side-condition}\gg *;*], [\text{Arbitrary}\gg *;*],$
$[\text{Local}\gg *=*;*];$
Postassociative
$[*\ \text{then}\ *], [*[\ *\ ]*];$
Preassociative
$[*\&*];$
Preassociative
$[*\backslash\backslash *];]$

$[\text{peano} \overset{\text{pyk}}{\rightarrow} \text{“peano”}]$

$\dot{0}$

$[\dot{0} \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{dot}\{0\}\text{”}]$

$[\dot{0} \overset{\text{pyk}}{\rightarrow} \text{“peano zero”}]$

$\dot{1}$

$[\dot{1} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{1} \doteqdot \dot{0}']\urcorner)]$

$[\dot{1} \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{dot}\{1\}\text{”}]$

$[\dot{1} \overset{\text{pyk}}{\rightarrow} \text{“peano one”}]$

$\dot{2}$

$[\dot{2} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{2} \doteqdot \dot{1}']\urcorner)]$

$[\dot{2} \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{dot}\{2\}\text{”}]$

$[\dot{2} \overset{\text{pyk}}{\rightarrow} \text{“peano two”}]$

$\dot{a}$

$[\dot{a} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{a} \doteqdot \dot{\mathsf{a}}]\urcorner)]$

$[\dot{a} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{a}}”]

$[\dot{a} \overset{\text{pyk}}{\rightarrow}$ “peano a”]

$\dot{b}$

$[\dot{b} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{b} \doteqdot \dot{\mathsf{b}}]\urcorner)]$

$[\dot{b} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{b}}”]

$[\dot{b} \overset{\text{pyk}}{\rightarrow}$ “peano b”]

$\dot{c}$

$[\dot{c} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{c} \doteqdot \dot{\mathsf{c}}]\urcorner)]$

$[\dot{c} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{c}}”]

$[\dot{c} \overset{\text{pyk}}{\rightarrow}$ “peano c”]

$\dot{d}$

$[\dot{d} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{d} \doteqdot \dot{\mathsf{d}}]\urcorner)]$

$[\dot{d} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{d}}”]

$[\dot{d} \overset{\text{pyk}}{\rightarrow}$ “peano d”]

$\dot{e}$

$[\dot{e} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{e} \doteqdot \dot{\mathsf{e}}]\urcorner)]$

$[\dot{e} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{e}}”]

$[\dot{e} \overset{\text{pyk}}{\rightarrow}$ “peano e”]

$\dot{f}$

$[\dot{f} \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \doteqdot \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{f}\}\}\text{”}]$

$[\dot{f} \overset{\text{pyk}}{\longrightarrow} \text{“peano f”}]$

$\dot{g}$

$[\dot{g} \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{g} \doteqdot \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{g}\}\}\text{”}]$

$[\dot{g} \overset{\text{pyk}}{\longrightarrow} \text{“peano g”}]$

$\dot{h}$

$[\dot{h} \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{h} \doteqdot \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{h}\}\}\text{”}]$

$[\dot{h} \overset{\text{pyk}}{\longrightarrow} \text{“peano h”}]$

$\dot{i}$

$[\dot{i} \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{i} \doteqdot \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{i}\}\}\text{”}]$

$[\dot{i} \overset{\text{pyk}}{\longrightarrow} \text{“peano i”}]$

$\dot{j}$

$[\dot{j} \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{j} \doteqdot \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{j}\}\}\text{”}]$

$[\dot{j} \overset{\text{pyk}}{\longrightarrow} \text{“peano j”}]$

$\dot{k}$

$[\dot{k} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{k} \doteqdot \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{k}\}\}\text{”}]$

$[\dot{k} \overset{\text{pyk}}{\to} \text{“peano k”}]$

$\dot{l}$

$[\dot{l} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{l} \doteqdot \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{l}\}\}\text{”}]$

$[\dot{l} \overset{\text{pyk}}{\to} \text{“peano l”}]$

$\dot{m}$

$[\dot{m} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{m} \doteqdot \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{m}\}\}\text{”}]$

$[\dot{m} \overset{\text{pyk}}{\to} \text{“peano m”}]$

$\dot{n}$

$[\dot{n} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{n} \doteqdot \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{n}\}\}\text{”}]$

$[\dot{n} \overset{\text{pyk}}{\to} \text{“peano n”}]$

$\dot{o}$

$[\dot{o} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{o} \doteqdot \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{o}\}\}\text{”}]$

$[\dot{o} \overset{\text{pyk}}{\to} \text{“peano o”}]$

$\dot{p}$

$[\dot{p} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{p} \doteqdot \dot{\mathsf{p}}]\rceil)]$

$[\dot{p} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{p}}”]

$[\dot{p} \overset{\text{pyk}}{\rightarrow}$ “peano p”]

$\dot{q}$

$[\dot{q} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{q} \doteqdot \dot{\mathsf{q}}]\rceil)]$

$[\dot{q} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{q}}”]

$[\dot{q} \overset{\text{pyk}}{\rightarrow}$ “peano q”]

$\dot{r}$

$[\dot{r} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{r} \doteqdot \dot{\mathsf{r}}]\rceil)]$

$[\dot{r} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{r}}”]

$[\dot{r} \overset{\text{pyk}}{\rightarrow}$ “peano r”]

$\dot{s}$

$[\dot{s} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{s} \doteqdot \dot{\mathsf{s}}]\rceil)]$

$[\dot{s} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{s}}”]

$[\dot{s} \overset{\text{pyk}}{\rightarrow}$ “peano s”]

$\dot{t}$

$[\dot{t} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{t} \doteqdot \dot{\mathsf{t}}]\rceil)]$

$[\dot{t} \overset{\text{tex}}{\rightarrow}$ “
\dot{\mathit{t}}”]

$[\dot{t} \overset{\text{pyk}}{\rightarrow}$ “peano t”]

$\dot{u}$

$[\dot{u} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{u} \doteqdot \dot{\mathsf{u}}]\urcorner)]$

$[\dot{u} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{u}\}\}\text{”}]$

$[\dot{u} \overset{\text{pyk}}{\to} \text{“peano u”}]$

$\dot{v}$

$[\dot{v} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{v} \doteqdot \dot{\mathsf{v}}]\urcorner)]$

$[\dot{v} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{v}\}\}\text{”}]$

$[\dot{v} \overset{\text{pyk}}{\to} \text{“peano v”}]$

$\dot{w}$

$[\dot{w} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{w} \doteqdot \dot{\mathsf{w}}]\urcorner)]$

$[\dot{w} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{w}\}\}\text{”}]$

$[\dot{w} \overset{\text{pyk}}{\to} \text{“peano w”}]$

$\dot{x}$

$[\dot{x} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{x} \doteqdot \dot{\mathsf{x}}]\urcorner)]$

$[\dot{x} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{x}\}\}\text{”}]$

$[\dot{x} \overset{\text{pyk}}{\to} \text{“peano x”}]$

$\dot{y}$

$[\dot{y} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\dot{y} \doteqdot \dot{\mathsf{y}}]\urcorner)]$

$[\dot{y} \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{dot}\{\backslash\text{mathit}\{\text{y}\}\}\text{”}]$

$[\dot{y} \overset{\text{pyk}}{\to} \text{“peano y”}]$

$\dot{z}$

$[\dot{z} \overset{\text{macro}}{\to} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\dot{z} \doteqdot \ddot{z}]\rceil)]$

$[\dot{z} \overset{\text{tex}}{\to} \text{“}$
$\backslash dot\{\backslash mathit\{z\}\}\text{”}]$

$[\dot{z} \overset{\text{pyk}}{\to} \text{“peano z”}]$

$\dot{\text{nonfree}}(*, *)$

$[\dot{\text{nonfree}}(x, y) \overset{\text{val}}{\to}$
$\text{If}(y^{\mathcal{P}}, \neg\, [\, x \overset{t}{=} y\,]\,,$
$\text{If}(\neg\, [\, y \overset{r}{=} \lceil\dot{\forall} x{:}\, y\rceil\,]\,, \dot{\text{nonfree}}^{*}(x, y^{t}),$
$\text{If}(x \overset{t}{=} [\, y^{1}\,]\,, \mathsf{T}, \dot{\text{nonfree}}(x, y^{2}))))]$

$[\dot{\text{nonfree}}(x, y) \overset{\text{tex}}{\to} \text{“}$
$\backslash dot\{nonfree\}(\#1.$
$,\#2.$
$)\text{”}]$

$[\dot{\text{nonfree}}(x, y) \overset{\text{pyk}}{\to} \text{“peano nonfree } * \text{ in } * \text{ end nonfree”}]$

$\dot{\text{nonfree}}^{*}(*, *)$

$[\dot{\text{nonfree}}^{*}(x, y) \overset{\text{val}}{\to} x!\text{If}(y, \mathsf{T}, \text{If}(\dot{\text{nonfree}}(x, y^{h}), \dot{\text{nonfree}}^{*}(x, y^{t}), \mathsf{F}))]$

$[\dot{\text{nonfree}}^{*}(x, y) \overset{\text{tex}}{\to} \text{“}$
$\backslash dot\{nonfree\}\hat{}*(\#1.$
$,\#2.$
$)\text{”}]$

$[\dot{\text{nonfree}}^{*}(x, y) \overset{\text{pyk}}{\to} \text{“peano nonfree star } * \text{ in } * \text{ end nonfree”}]$

$\dot{\text{free}}\langle *|* := * \rangle$

$[\dot{\text{free}}\langle a|x := b\rangle \overset{\text{val}}{\to} x!\, [\, b!$
$\text{If}(a^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg\, [\, a \overset{r}{=} \lceil\dot{\forall} u{:}\, v\rceil\,]\,, \dot{\text{free}}^{*}\langle a^{t}|x := b\rangle,$
$\text{If}(a^{1} \overset{t}{=} x, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(x, a^{2}), \mathsf{T},$

$\mathsf{If}(\neg\dot{\mathsf{nonfree}}(\mathsf{a}^1,\mathsf{b}),\mathsf{F},$
$\dot{\mathsf{free}}\langle\mathsf{a}^2|\mathsf{x}:=\mathsf{b}\rangle))))))\ ]\ ]$

$[\dot{\mathsf{free}}\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \overset{\text{tex}}{\to}$ “
\dot{free}\langle #1.
| #2.
:= #3.
\rangle”]

$[\dot{\mathsf{free}}\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \overset{\text{pyk}}{\to}$ “peano free ∗ set ∗ to ∗ end free”]

$\dot{\mathsf{free}}^*\langle *|*:=*\rangle$

$[\dot{\mathsf{free}}^*\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \overset{\text{val}}{\to} \mathsf{x}!\ [\ \mathsf{b}!\mathsf{If}(\mathsf{a},\mathsf{T},\mathsf{If}(\dot{\mathsf{free}}\langle\mathsf{a}^{\mathrm{h}}|\mathsf{x}:=\mathsf{b}\rangle,\dot{\mathsf{free}}^*\langle\mathsf{a}^{\mathrm{t}}|\mathsf{x}:=\mathsf{b}\rangle,\mathsf{F}))\ ]\ ]$

$[\dot{\mathsf{free}}^*\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \overset{\text{tex}}{\to}$ “
\dot{free}{}\hat{}*\langle #1.
| #2.
:= #3.
\rangle”]

$[\dot{\mathsf{free}}^*\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \overset{\text{pyk}}{\to}$ “peano free star ∗ set ∗ to ∗ end free”]

$*\equiv\langle *|*:=*\rangle$

$[\mathsf{a}\equiv\langle\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \overset{\text{val}}{\to} \mathsf{a}!\ [\ \mathsf{x}!\ [\ \mathsf{c}!$
$\mathsf{If}(\mathsf{If}(\mathsf{b}\overset{\mathrm{r}}{=}\lceil\dot{\forall}\mathsf{u}\colon\mathsf{v}\rceil,\mathsf{b}^1\overset{\mathrm{t}}{=}\mathsf{x},\mathsf{F}),\mathsf{a}\overset{\mathrm{t}}{=}\mathsf{b},$
$\mathsf{If}(\mathsf{b}^{\mathcal{P}}\wedge\ [\ \mathsf{b}\overset{\mathrm{t}}{=}\mathsf{x}\ ]\ ,\mathsf{a}\overset{\mathrm{t}}{=}\mathsf{c},\mathsf{If}(\ [$
$\mathsf{a}\ ]\ \overset{\mathrm{r}}{=}\mathsf{b},\mathsf{a}^{\mathrm{t}}\equiv\langle{}^*\mathsf{b}^{\mathrm{t}}|\mathsf{x}:=\mathsf{c}\rangle,\mathsf{F})))\ ]\ \ ]\ ]$

$[\mathsf{a}\equiv\langle\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \overset{\text{tex}}{\to}$ “#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle”]

$[\mathsf{a}\equiv\langle\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \overset{\text{pyk}}{\to}$ “peano sub ∗ is ∗ where ∗ is ∗ end sub”]

$*\equiv\langle {}^**|*:=*\rangle$

$[\mathsf{a}\equiv\langle{}^*\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \overset{\text{val}}{\to} \mathsf{b}!\ [\ \mathsf{x}!\ [\ \mathsf{c}!\mathsf{If}(\mathsf{a},\mathsf{T},\mathsf{If}(\mathsf{a}^{\mathrm{h}}\equiv\langle\mathsf{b}^{\mathrm{h}}|\mathsf{x}:=\mathsf{c}\rangle,\mathsf{a}^{\mathrm{t}}\equiv\langle{}^*\mathsf{b}^{\mathrm{t}}|\mathsf{x}:=\mathsf{c}\rangle,\mathsf{F}))\ ]\ \ ]\ ]$

$[\mathsf{a}\equiv\langle{}^*\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \overset{\text{tex}}{\to}$ “#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

[a≡⟨*b|x := c⟩ $\overset{\text{pyk}}{\to}$ "peano sub star * is * where * is * end sub"]

S

[S $\overset{\text{stmt}}{\to}$ [[ȧ ∔ [ḃ′]] $\overset{\text{p}}{=}$ [[ȧ ∔ [ḃ]] ′]] ⊕ [[∀a: ∀b: [[[¬ b] ⇒ ¬ a] ⇒ [[[¬ b] ⇒ a] ⇒ b]]] ⊕ [[[ȧ $\overset{\text{p}}{=}$ [ḃ]] ⇒ [ȧ′ $\overset{\text{p}}{=}$ [ḃ′]]] ⊕ [[∀a: ∀b: [[a ⇒ b] ⊢ [a ⊢ b]]] ⊕ [[[ȧ′ $\overset{\text{p}}{=}$ [ḃ′]] ⇒ [ȧ $\overset{\text{p}}{=}$ [ḃ]]] ⊕ [[∀a: ∀b: [a ⇒ [b ⇒ a]]] ⊕ [[∀x: ∀a: ∀b: [nonfree(x, a) ⊩ [[∀x: [a ⇒ b]] ⇒ [a ⇒ ∀x: b]]]] ⊕ [[[ȧ ∶ [ḃ′]] $\overset{\text{p}}{=}$ [[ȧ ∶ [ḃ]] ∔ [ȧ]]] ⊕ [[[ȧ ∔ 0̇] $\overset{\text{p}}{=}$ [ȧ]] ⊕ [[∀a: ∀b: ∀c: [[a ⇒ [b ⇒ c]] ⇒ [[a ⇒ b] ⇒ [a ⇒ c]]]] ⊕ [[[ȧ $\overset{\text{p}}{=}$ [ḃ]] ⇒ [[ȧ $\overset{\text{p}}{=}$ [ċ]] ⇒ [ḃ $\overset{\text{p}}{=}$ [ċ]]]] ⊕ [[∀a: ∀b: ∀c: ∀x: [b≡⟨a|x := 0̇⟩ ⊩ [c≡⟨a|x := x′⟩ ⊩ [b ⇒ [[∀x: [a ⇒ c]] ⇒ ∀x: a]]]]] ⊕ [[¬ [0̇ $\overset{\text{p}}{=}$ [ȧ′]]] ⊕ [[∀x: ∀a: [a ⊢ ∀x: a]] ⊕ [[∀c: ∀a: ∀x: ∀b: [⌈a⌉≡⟨⌈b⌉||⌈x⌉ := ⌈c⌉⟩ ⊩ [[∀x: b] ⇒ a]]] ⊕ [[ȧ ∶ 0̇] $\overset{\text{p}}{=}$ 0̇]]]]]]]]]]]]]]]]

[S $\overset{\text{tex}}{\to}$ "
S"]

[S $\overset{\text{pyk}}{\to}$ "system s"]

A1

[A1 $\overset{\text{proof}}{\to}$ Rule tactic]

[A1 $\overset{\text{stmt}}{\to}$ S ⊢ ∀a: ∀b: [a ⇒ [b ⇒ a]]]

[A1 $\overset{\text{tex}}{\to}$ "
A1"]

[A1 $\overset{\text{pyk}}{\to}$ "axiom a one"]

A2

[A2 $\overset{\text{proof}}{\to}$ Rule tactic]

[A2 $\overset{\text{stmt}}{\to}$ S ⊢ ∀a: ∀b: ∀c: [[a ⇒ [b ⇒ c]] ⇒ [[a ⇒ b] ⇒ [a ⇒ c]]]]

[A2 $\overset{\text{tex}}{\to}$ “
A2”]

[A2 $\overset{\text{pyk}}{\to}$ “axiom a two”]

A3

[A3 $\overset{\text{proof}}{\to}$ Rule tactic]

[A3 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \; [\; [\; [\; \dot{\neg}\, \underline{\mathsf{b}}\;]\; \dot{\Rightarrow}\; \dot{\neg}\, \underline{\mathsf{a}}\;]\; \dot{\Rightarrow}\; [\; [\; [\; \dot{\neg}\, \underline{\mathsf{b}}\;]\; \dot{\Rightarrow}\; \underline{\mathsf{a}}\;]\; \dot{\Rightarrow}\; \underline{\mathsf{b}}\;]\;]\;]$]

[A3 $\overset{\text{tex}}{\to}$ “
A3”]

[A3 $\overset{\text{pyk}}{\to}$ “axiom a three”]

A4

[A4 $\overset{\text{proof}}{\to}$ Rule tactic]

[A4 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{b}}\colon \; [\; \lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil \,||\, \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash \; [\; [\; \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{b}}\;]\; \dot{\Rightarrow}\; \underline{\mathsf{a}}\;]\;]\;]$]

[A4 $\overset{\text{tex}}{\to}$ “
A4”]

[A4 $\overset{\text{pyk}}{\to}$ “axiom a four”]

A5

[A5 $\overset{\text{proof}}{\to}$ Rule tactic]

[A5 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \; [\; \dot{\text{nonfree}}(\underline{\mathsf{x}}, \underline{\mathsf{a}}) \Vdash \; [\; [\; \dot{\forall} \underline{\mathsf{x}}\colon \; [\; \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\;]\;]\; \dot{\Rightarrow}\; [\; \underline{\mathsf{a}} \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{b}}\;]\;]\;]\;]$]

[A5 $\overset{\text{tex}}{\to}$ “
A5”]

[A5 $\overset{\text{pyk}}{\to}$ “axiom a five”]

S1

[S1 $\overset{\text{proof}}{\to}$ Rule tactic]

[S1 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \; [\; [\; \dot{\mathsf{a}} \overset{\mathrm{p}}{=} \; [\; \dot{\mathsf{b}}\;]\;]\; \dot{\Rightarrow}\; [\; [\; \dot{\mathsf{a}} \overset{\mathrm{p}}{=} \; [\; \dot{\mathsf{c}}\;]\;]\; \dot{\Rightarrow}\; [\; \dot{\mathsf{b}} \overset{\mathrm{p}}{=} \; [\; \dot{\mathsf{c}}\;]\;]\;]\;]\;]$]

[S1 $\overset{\text{tex}}{\to}$ “
S1”]

[S1 $\overset{\text{pyk}}{\to}$ “axiom s one”]

S2

[S2 $\overset{\text{proof}}{\to}$ Rule tactic]

[S2 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash [\ [\ \dot{\mathsf{a}} \overset{\mathrm{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ] \Rightarrow [\ \dot{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \dot{\mathsf{b}}'\ ]\ ]\ ]$]

[S2 $\overset{\text{tex}}{\to}$ “
S2”]

[S2 $\overset{\text{pyk}}{\to}$ “axiom s two”]

S3

[S3 $\overset{\text{proof}}{\to}$ Rule tactic]

[S3 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \neg [\ \dot{0} \overset{\mathrm{p}}{=} [\ \dot{\mathsf{a}}'\ ]\ ]$]

[S3 $\overset{\text{tex}}{\to}$ “
S3”]

[S3 $\overset{\text{pyk}}{\to}$ “axiom s three”]

S4

[S4 $\overset{\text{proof}}{\to}$ Rule tactic]

[S4 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash [\ [\ \dot{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \dot{\mathsf{b}}'\ ]\ ] \Rightarrow [\ \dot{\mathsf{a}} \overset{\mathrm{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ]\ ]$]

[S4 $\overset{\text{tex}}{\to}$ “
S4”]

[S4 $\overset{\text{pyk}}{\to}$ “axiom s four”]

S5

[S5 $\overset{\text{proof}}{\to}$ Rule tactic]

[S5 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash [\ [\ \dot{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} [\ \dot{\mathsf{a}}\ ]\ ]$]

[S5 $\overset{\text{tex}}{\to}$ “
S5”]

[S5 $\overset{\text{pyk}}{\to}$ “axiom s five”]

S6

[S6 $\overset{\text{proof}}{\to}$ Rule tactic]

[S6 $\overset{\text{stmt}}{\to}$ $\mathsf{S} \vdash [\ [\ \dot{\mathsf{a}} \dot{+} [\ \dot{\mathsf{b}}'\ ]\ ] \overset{\text{p}}{=} [\ [\ \dot{\mathsf{a}} \dot{+} [\ \dot{\mathsf{b}}\ ]\ ]'\ ]\ ]$]

[S6 $\overset{\text{tex}}{\to}$ “
S6”]

[S6 $\overset{\text{pyk}}{\to}$ “axiom s six”]

S7

[S7 $\overset{\text{proof}}{\to}$ Rule tactic]

[S7 $\overset{\text{stmt}}{\to}$ $\mathsf{S} \vdash [\ [\ \dot{\mathsf{a}} \dot{:} \dot{0}\ ] \overset{\text{p}}{=} \dot{0}\ ]$]

[S7 $\overset{\text{tex}}{\to}$ “
S7”]

[S7 $\overset{\text{pyk}}{\to}$ “axiom s seven”]

S8

[S8 $\overset{\text{proof}}{\to}$ Rule tactic]

[S8 $\overset{\text{stmt}}{\to}$ $\mathsf{S} \vdash [\ [\ \dot{\mathsf{a}} \dot{:} [\ \dot{\mathsf{b}}'\ ]\ ] \overset{\text{p}}{=} [\ [\ \dot{\mathsf{a}} \dot{:} [\ \dot{\mathsf{b}}\ ]\ ] \dot{+} [\ \dot{\mathsf{a}}\ ]\ ]\ ]$]

[S8 $\overset{\text{tex}}{\to}$ “
S8”]

[S8 $\overset{\text{pyk}}{\to}$ “axiom s eight”]

S9

[S9 $\overset{\text{proof}}{\to}$ Rule tactic]

[S9 $\overset{\text{stmt}}{\to}$ $\mathsf{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{x}}\colon [\ \underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash [\ \underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} \forall \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]$]

[S9 $\overset{\text{tex}}{\to}$ “
S9”]

[S9 $\overset{\text{pyk}}{\to}$ “axiom s nine”]

MP

[MP $\overset{\text{proof}}{\to}$ Rule tactic]

[MP $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{\mathrm{a}} \colon \forall \underline{\mathrm{b}} \colon \; [\; [\; \underline{\mathrm{a}} \Rightarrow \underline{\mathrm{b}} \;] \vdash [\; \underline{\mathrm{a}} \vdash \underline{\mathrm{b}} \;] \;]$]

[MP $\overset{\text{tex}}{\to}$ “
MP”]

[MP $\overset{\text{pyk}}{\to}$ “rule mp”]

Gen

[Gen $\overset{\text{proof}}{\to}$ Rule tactic]

[Gen $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{\mathrm{x}} \colon \forall \underline{\mathrm{a}} \colon \; [\; \underline{\mathrm{a}} \vdash \dot{\forall} \underline{\mathrm{x}} \colon \underline{\mathrm{a}} \;]$]

[Gen $\overset{\text{tex}}{\to}$ “
Gen”]

[Gen $\overset{\text{pyk}}{\to}$ “rule gen”]

L3.2(a)

[L3.2(a) $\overset{\text{proof}}{\to}$ $\lambda \mathrm{c}. \lambda \mathrm{x}. \mathcal{P}(\lceil \mathrm{S} \vdash [\; [\; \mathrm{S5} \gg [\; [\; \dot{\mathrm{a}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{a}} \;] \;] \;] ; [\; [\; [\; \mathrm{Gen} \rhd [\; [\; \dot{\mathrm{a}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{a}} \;] \;] \;] \gg \dot{\forall} \dot{\mathrm{a}} \colon [\; [\; \dot{\mathrm{a}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{a}} \;] \;] \;] ; [\; [\; [\; \mathrm{A4} \mathbin{@} [\; \dot{\mathrm{x}} \;] \;] \gg [\; [\; \dot{\forall} \dot{\mathrm{a}} \colon [\; [\; \dot{\mathrm{a}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{a}} \;] \;] \;] \Rightarrow [\; [\; \dot{\mathrm{x}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \;] \;] ; [\; [\; [\; [\; \mathrm{MP} \rhd [\; [\; \dot{\forall} \dot{\mathrm{a}} \colon [\; [\; \dot{\mathrm{a}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{a}} \;] \;] \;] \Rightarrow [\; [\; \dot{\mathrm{x}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \;] \;] \rhd \dot{\forall} \dot{\mathrm{a}} \colon [\; [\; \dot{\mathrm{a}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{a}} \;] \;] \;] \gg [\; [\; \dot{\mathrm{x}} \dot{+} \dot{0} \;] \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \;] ; [\; [\; \mathrm{S1} \gg [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \;] \;] \;] ; [\; [\; [\; \mathrm{Gen} \rhd [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \;] \;] \;] \gg \dot{\forall} \dot{\mathrm{c}} \colon [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \;] \;] \;] ; [\; [\; [\; \mathrm{A4} \mathbin{@} [\; \dot{\mathrm{x}} \;] \;] \gg [\; [\; \dot{\forall} \dot{\mathrm{c}} \colon [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \;] \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \;] \;] \;] \;] ; [\; [\; [\; [\; \mathrm{MP} \rhd [\; [\; \dot{\forall} \dot{\mathrm{c}} \colon [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \;] \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \;] \;] \;] \rhd \dot{\forall} \dot{\mathrm{c}} \colon [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{c}} \;] \;] \;] \;] \;] \gg [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \Rightarrow [\; \dot{\mathrm{b}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \;] \;] \;] ; [\; [\; [\; \mathrm{Gen} \rhd [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{b}} \;] \;] \Rightarrow [\; [\; \dot{\mathrm{a}} \overset{\mathrm{p}}{=} [\; \dot{\mathrm{x}} \;] \;] \Rightarrow [$

$\dot{\mathsf{b}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,] \gg \dot{\forall}\dot{\mathsf{b}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{b}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{b}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,;\, [\,[\,[\,\mathrm{A4}\,@\,[\,\dot{\mathsf{x}}\,]\,] \gg [\,[\,\dot{\forall}\dot{\mathsf{b}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{b}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{b}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,]\,;\, [\,[\,[\,\mathrm{MP} \rhd [\,[\,\dot{\forall}\dot{\mathsf{b}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{b}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{b}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \rhd \dot{\forall}\dot{\mathsf{b}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{b}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{b}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \gg [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,;\, [\,[\,[\,\mathrm{Gen} \rhd [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \gg \dot{\forall}\dot{\mathsf{a}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,;\, [\,[\,[\,\mathrm{A4}\,@\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,]\,] \gg [\,[\,\dot{\forall}\dot{\mathsf{a}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \Rightarrow [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,;\, [\,[\,[\,[\,\mathrm{MP} \rhd [\,[\,\dot{\forall}\dot{\mathsf{a}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \Rightarrow [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,] \rhd \dot{\forall}\dot{\mathsf{a}}\colon [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,\dot{\mathsf{a}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \gg [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,;\, [\,[\,[\,[\,\mathrm{MP} \rhd [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \rhd [\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \gg [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,;\, [\,[\,[\,[\,\mathrm{MP} \rhd [\,[\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,] \Rightarrow [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,] \rhd [\,[\,\dot{\mathsf{x}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,] \gg [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,], \mathsf{p}_0, \mathsf{c})]$

[L3.2(a) $\overset{\mathsf{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\,\dot{\mathsf{x}} \overset{\mathsf{p}}{=} [\,\dot{\mathsf{x}}\,]\,]$]

[L3.2(a) $\overset{\mathsf{tex}}{\rightarrow}$ “
L3.2(a)”]

[L3.2(a) $\overset{\mathsf{pyk}}{\rightarrow}$ “lemma l three two a”]

S′

[S′ $\overset{\mathsf{stmt}}{\rightarrow}$ $[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}}' \overset{\mathsf{p}}{=} [\,\underline{\mathsf{b}}'\,]\,] \Rightarrow [\,\underline{\mathsf{a}} \overset{\mathsf{p}}{=} \underline{\mathsf{b}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\,[\,\underline{\mathsf{h}} \Rightarrow [\,\underline{\mathsf{t}} \overset{\mathsf{p}}{=} \underline{\mathsf{r}}\,]\,] \vdash [\,[\,\underline{\mathsf{h}} \Rightarrow [\,\underline{\mathsf{t}} \overset{\mathsf{p}}{=} \underline{\mathsf{s}}\,]\,] \vdash [\,\underline{\mathsf{h}} \Rightarrow [\,\underline{\mathsf{r}} \overset{\mathsf{p}}{=} \underline{\mathsf{s}}\,]\,]\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,\underline{\mathsf{a}} \Rightarrow [\,\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,\mathrm{nonfree}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{a}} \rceil)\,] \Vdash [\,[\,\dot{\forall}\underline{\mathsf{x}}\colon [\,\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\,]\,] \Rightarrow [\,\underline{\mathsf{a}} \Rightarrow \dot{\forall}\underline{\mathsf{x}}\colon \underline{\mathsf{b}}\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\,\underline{\mathsf{h}} \Rightarrow [\,[\,\underline{\mathsf{t}} \dot{+} [\,\underline{\mathsf{r}}'\,]\,] \overset{\mathsf{p}}{=} [\,[\,\underline{\mathsf{t}} \dot{+} \underline{\mathsf{r}}\,]'\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} \dot{:} [\,\underline{\mathsf{b}}'\,]\,] \overset{\mathsf{p}}{=} [\,[\,\underline{\mathsf{a}} \dot{:} \underline{\mathsf{b}}\,] \dot{+} \underline{\mathsf{a}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} \overset{\mathsf{p}}{=} \underline{\mathsf{b}}\,] \Rightarrow [\,\underline{\mathsf{a}}' \overset{\mathsf{p}}{=} [\,\underline{\mathsf{b}}'\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,[\,\dot{\neg}\,\underline{\mathsf{b}}\,] \Rightarrow \dot{\neg}\,\underline{\mathsf{a}}\,] \Rightarrow [\,[\,[\,\dot{\neg}\,\underline{\mathsf{b}}\,] \Rightarrow \underline{\mathsf{a}}\,] \Rightarrow \underline{\mathsf{b}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\,[\,\underline{\mathsf{h}} \Rightarrow [\,\underline{\mathsf{t}} \overset{\mathsf{p}}{=} \underline{\mathsf{r}}\,]\,] \vdash [\,\underline{\mathsf{h}} \Rightarrow [\,\underline{\mathsf{t}}' \overset{\mathsf{p}}{=} [\,\underline{\mathsf{r}}'\,]\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} \dot{+} [\,\underline{\mathsf{b}}'\,]\,] \overset{\mathsf{p}}{=} [\,[\,\underline{\mathsf{a}} \dot{+} \underline{\mathsf{b}}\,]'\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \neg\,[\,\dot{0} \overset{\mathsf{p}}{=} [\,\underline{\mathsf{a}}'\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon [\,\underline{\mathsf{a}} \vdash \dot{\forall}\underline{\mathsf{x}}\colon \underline{\mathsf{a}}\,]\,] \oplus [\,[\,\forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{b}}\colon [\,\lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil | \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash [\,[\,\dot{\forall}\underline{\mathsf{x}}\colon \underline{\mathsf{b}}\,] \Rightarrow \underline{\mathsf{a}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon [\,\underline{\mathsf{h}} \Rightarrow [\,[\,\underline{\mathsf{t}} \dot{+} \dot{0}\,] \overset{\mathsf{p}}{=} \underline{\mathsf{t}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon [\,[\,\underline{\mathsf{a}} \dot{:} \dot{0}\,] \overset{\mathsf{p}}{=} \dot{0}\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\,[\,\underline{\mathsf{a}} \overset{\mathsf{p}}{=} \underline{\mathsf{b}}\,] \Rightarrow [\,[\,\underline{\mathsf{a}} \overset{\mathsf{p}}{=} \underline{\mathsf{c}}\,] \Rightarrow [\,\underline{\mathsf{b}} \overset{\mathsf{p}}{=} \underline{\mathsf{c}}\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{t}}\colon [\,\underline{\mathsf{t}} \overset{\mathsf{p}}{=} \underline{\mathsf{t}}\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{x}}\colon [\,\underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash [\,\underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash [\,\underline{\mathsf{b}} \Rightarrow [\,[\,\dot{\forall}\underline{\mathsf{x}}\colon [\,\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\,]\,] \Rightarrow \forall \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\,]\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\,[\,\underline{\mathsf{a}} \Rightarrow [\,\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}\,]\,] \Rightarrow [\,[\,\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\,] \Rightarrow [$

$\underline{a} \dot{\Rightarrow} \underline{c} \,]\,]\,]\,] \oplus [\,[\,\forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\,[\,\underline{h} \dot{\Rightarrow} [\,\underline{t} \overset{p}{=} \underline{r}\,]\,] \vdash [\,[\,\underline{h} \dot{\Rightarrow} [\,\underline{r} \overset{p}{=} \underline{s}\,]\,] \vdash [\,\underline{h} \dot{\Rightarrow} [\,\underline{t} \overset{p}{=} \underline{s}\,]\,]\,]\,]\,]\,] \oplus \forall \underline{a}\colon [\,[\,\underline{a} \dot{+} \dot{0}\,] \overset{p}{=} \underline{a}\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]$

[S′ $\overset{\text{tex}}{\to}$ “
S’”]

[S′ $\overset{\text{pyk}}{\to}$ “system prime s”]

A1′

[A1′ $\overset{\text{proof}}{\to}$ Rule tactic]

[A1′ $\overset{\text{stmt}}{\to}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\,\underline{a} \dot{\Rightarrow} [\,\underline{b} \dot{\Rightarrow} \underline{a}\,]\,]\,]$

[A1′ $\overset{\text{tex}}{\to}$ “
A1’”]

[A1′ $\overset{\text{pyk}}{\to}$ “axiom prime a one”]

A2′

[A2′ $\overset{\text{proof}}{\to}$ Rule tactic]

[A2′ $\overset{\text{stmt}}{\to}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\,[\,\underline{a} \dot{\Rightarrow} [\,\underline{b} \dot{\Rightarrow} \underline{c}\,]\,] \dot{\Rightarrow} [\,[\,\underline{a} \dot{\Rightarrow} \underline{b}\,] \dot{\Rightarrow} [\,\underline{a} \dot{\Rightarrow} \underline{c}\,]\,]\,]$

[A2′ $\overset{\text{tex}}{\to}$ “
A2’”]

[A2′ $\overset{\text{pyk}}{\to}$ “axiom prime a two”]

A3′

[A3′ $\overset{\text{proof}}{\to}$ Rule tactic]

[A3′ $\overset{\text{stmt}}{\to}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\,[\,[\,\dot{\neg} \underline{b}\,] \dot{\Rightarrow} \dot{\neg} \underline{a}\,] \dot{\Rightarrow} [\,[\,[\,\dot{\neg} \underline{b}\,] \dot{\Rightarrow} \underline{a}\,] \dot{\Rightarrow} \underline{b}\,]\,]$

[A3′ $\overset{\text{tex}}{\to}$ “
A3’”]

[A3′ $\overset{\text{pyk}}{\to}$ “axiom prime a three”]

A4′

[A4′ $\overset{\text{proof}}{\to}$ Rule tactic]

$[\mathrm{A4}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{b}}\colon\ [\ \lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash\ [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow}\ \underline{\mathsf{a}}\ ]\ ]\ ]$

$[\mathrm{A4}' \overset{\mathrm{tex}}{\to} \text{“}$
A4”"]

$[\mathrm{A4}' \overset{\mathrm{pyk}}{\to} \text{“axiom prime a four”}]$

A5′

$[\mathrm{A5}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{A5}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon\ [\ \dot{\mathrm{nonfree}}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{a}} \rceil) \Vdash\ [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{b}}\ ]\ ]\ ]\ ]$

$[\mathrm{A5}' \overset{\mathrm{tex}}{\to} \text{“}$
A5”"]

$[\mathrm{A5}' \overset{\mathrm{pyk}}{\to} \text{“axiom prime a five”}]$

S1′

$[\mathrm{S1}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{S1}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ \dot{\Rightarrow}\ [\ \underline{\mathsf{b}} \overset{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$

$[\mathrm{S1}' \overset{\mathrm{tex}}{\to} \text{“}$
S1”"]

$[\mathrm{S1}' \overset{\mathrm{pyk}}{\to} \text{“axiom prime s one”}]$

S2′

$[\mathrm{S2}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{S2}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon\ [\ [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow}\ [\ \underline{\mathsf{a}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]\ ]$

$[\mathrm{S2}' \overset{\mathrm{tex}}{\to} \text{“}$
S2”"]

$[\mathrm{S2}' \overset{\mathrm{pyk}}{\to} \text{“axiom prime s two”}]$

S3′

$[\mathrm{S3}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

[S3′ $\overset{\text{stmt}}{\to}$ S′ ⊢ ∀a: ¬ [$\dot{0} \overset{\text{p}}{=}$ [a′]]]

[S3′ $\overset{\text{tex}}{\to}$ “
S3’”]

[S3′ $\overset{\text{pyk}}{\to}$ “axiom prime s three”]

S4′

[S4′ $\overset{\text{proof}}{\to}$ Rule tactic]

[S4′ $\overset{\text{stmt}}{\to}$ S′ ⊢ ∀a: ∀b: [[a′ $\overset{\text{p}}{=}$ [b′]] $\dot{\Rightarrow}$ [a $\overset{\text{p}}{=}$ b]]]

[S4′ $\overset{\text{tex}}{\to}$ “
S4’”]

[S4′ $\overset{\text{pyk}}{\to}$ “axiom prime s four”]

S5′

[S5′ $\overset{\text{proof}}{\to}$ Rule tactic]

[S5′ $\overset{\text{stmt}}{\to}$ S′ ⊢ ∀a: [[a $\dot{+}$ $\dot{0}$] $\overset{\text{p}}{=}$ a]]

[S5′ $\overset{\text{tex}}{\to}$ “
S5’”]

[S5′ $\overset{\text{pyk}}{\to}$ “axiom prime s five”]

S6′

[S6′ $\overset{\text{proof}}{\to}$ Rule tactic]

[S6′ $\overset{\text{stmt}}{\to}$ S′ ⊢ ∀a: ∀b: [[a $\dot{+}$ [b′]] $\overset{\text{p}}{=}$ [[a $\dot{+}$ b] ′]]]

[S6′ $\overset{\text{tex}}{\to}$ “
S6’”]

[S6′ $\overset{\text{pyk}}{\to}$ “axiom prime s six”]

S7′

[S7′ $\overset{\text{proof}}{\to}$ Rule tactic]

[S7′ $\overset{\text{stmt}}{\to}$ S′ ⊢ ∀a: [[a $\dot{\cdot}$ $\dot{0}$] $\overset{\text{p}}{=}$ $\dot{0}$]]

[$\mathrm{S7}' \overset{\mathrm{tex}}{\to}$ “
S7”’]

[$\mathrm{S7}' \overset{\mathrm{pyk}}{\to}$ “axiom prime s seven”]

$\mathrm{S8}'$

[$\mathrm{S8}' \overset{\mathrm{proof}}{\to}$ Rule tactic]

[$\mathrm{S8}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} : [\ \underline{b}'\ ]\ ]\ \overset{\mathrm{p}}{=}\ [\ [\ \underline{a} : \underline{b}\ ]\ \dot{+}\ \underline{a}\ ]\ ]$]

[$\mathrm{S8}' \overset{\mathrm{tex}}{\to}$ “
S8”’]

[$\mathrm{S8}' \overset{\mathrm{pyk}}{\to}$ “axiom prime s eight”]

$\mathrm{S9}'$

[$\mathrm{S9}' \overset{\mathrm{proof}}{\to}$ Rule tactic]

[$\mathrm{S9}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash\ [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash\ [\ \underline{b} \dot{\Rightarrow}\ [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]$]

[$\mathrm{S9}' \overset{\mathrm{tex}}{\to}$ “
S9”’]

[$\mathrm{S9}' \overset{\mathrm{pyk}}{\to}$ “axiom prime s nine”]

MP'

[$\mathrm{MP}' \overset{\mathrm{proof}}{\to}$ Rule tactic]

[$\mathrm{MP}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash\ [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[$\mathrm{MP}' \overset{\mathrm{tex}}{\to}$ “
MP”’]

[$\mathrm{MP}' \overset{\mathrm{pyk}}{\to}$ “rule prime mp”]

Gen'

[$\mathrm{Gen}' \overset{\mathrm{proof}}{\to}$ Rule tactic]

[$\mathrm{Gen}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]$]

[$\mathrm{Gen}' \overset{\mathrm{tex}}{\to}$ “
Gen’”]

[$\mathrm{Gen}' \overset{\mathrm{pyk}}{\to}$ “rule prime gen”]

L3.2(a)′

[$\mathrm{L3.2(a)}' \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon [\ [\ \mathrm{S5}' \gg [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ;\ [\ [\ \mathrm{S1}' \gg [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \Rightarrow [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \Rightarrow [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ \rhd [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ] \gg [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ \rhd [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ] \gg [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ]\rfloor, \mathrm{p}_0, \mathsf{c})]$

[$\mathrm{L3.2(a)}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon [\ \underline{\mathsf{a}} \overset{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]$

[$\mathrm{L3.2(a)}' \overset{\mathrm{tex}}{\to}$ “
L3.2(a)”]

[$\mathrm{L3.2(a)}' \overset{\mathrm{pyk}}{\to}$ “lemma prime l three two a”]

M1.7

[$\mathrm{M1.7} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{b}}\colon [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ] \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{b}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{b}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{b}} \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{b}} \Rightarrow [\ [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ] \gg [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ ]\rfloor, \mathrm{p}_0, \mathsf{c})]$

[$\mathrm{M1.7} \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{b}}\colon [\ \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]$

[$\mathrm{M1.7} \overset{\mathrm{tex}}{\to}$ “
M1.7”]

[$\mathrm{M1.7} \overset{\mathrm{pyk}}{\to}$ “mendelson one seven”]

$\mathrm{MP}'_{\mathrm{h}}$

[$\mathrm{MP}'_{\mathrm{h}} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \vdash [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \vdash [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \Rightarrow [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \Rightarrow [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ] \gg [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ ]\rfloor, \mathrm{p}_0, \mathsf{c})]$

[$\mathrm{MP}'_{\mathrm{h}} \overset{\mathrm{stmt}}{\to} \mathsf{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow [\ \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\ ]\ ] \vdash [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \vdash [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{b}}\ ]\ ]\ ]$]

[$\mathrm{MP}'_{\mathrm{h}} \overset{\mathrm{tex}}{\to}$ “
MP'_h”]

[$\mathrm{MP}'_{\mathrm{h}} \overset{\mathrm{pyk}}{\to}$ “hypothetical rule prime mp”]

Hypothesize

[Hypothesize $\overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathsf{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon [\ \underline{\mathsf{a}} \vdash [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{MP}' \rhd [\ \underline{\mathsf{a}} \Rightarrow [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ \rhd \underline{\mathsf{a}}\ ] \gg [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ \rfloor, \mathrm{p}_0, \mathsf{c})$]

[Hypothesize $\overset{\mathrm{stmt}}{\to} \mathsf{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon [\ \underline{\mathsf{a}} \vdash [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]$]

[Hypothesize $\overset{\mathrm{tex}}{\to}$ “
Hypothesize”]

[Hypothesize $\overset{\mathrm{pyk}}{\to}$ “hypothesize”]

$\mathrm{Gen}'_{\mathrm{h}}$

[$\mathrm{Gen}'_{\mathrm{h}} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathsf{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon [\ \dot{\mathrm{nonfree}}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{h}} \rceil) \Vdash [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \vdash [\ [\ [\ \mathrm{A5}' \Rrightarrow \dot{\mathrm{nonfree}}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{h}} \rceil)\ ] \gg [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{h}} \Rightarrow \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{Gen}' \rhd [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ \gg \dot{\forall} \underline{\mathsf{x}}\colon [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ ; [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{h}} \Rightarrow \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]\ \rhd \dot{\forall} \underline{\mathsf{x}}\colon [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ]\ ]\ \gg [\ \underline{\mathsf{h}} \Rightarrow \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ]\ \rfloor, \mathrm{p}_0, \mathsf{c})$]

[$\mathrm{Gen}'_{\mathrm{h}} \overset{\mathrm{stmt}}{\to} \mathsf{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon [\ \dot{\mathrm{nonfree}}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{h}} \rceil) \Vdash [\ [\ \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{a}}\ ] \vdash [\ \underline{\mathsf{h}} \Rightarrow \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]$]

[$\mathrm{Gen}'_{\mathrm{h}} \overset{\mathrm{tex}}{\to}$ “
Gen'_h”]

[$\mathrm{Gen}'_{\mathrm{h}} \overset{\mathrm{pyk}}{\to}$ “hypothetical rule prime gen”]

M3.2(a)

[M3.2(a) $\overset{\mathrm{proof}}{\to}$ Rule tactic]

[M3.2(a) $\overset{\mathrm{stmt}}{\to} \mathsf{S}' \vdash \forall \underline{\mathsf{t}}\colon [\ \underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]$]

[M3.2(a) $\overset{\mathrm{tex}}{\to}$ “
M3.2(a)”]

[M3.2(a) $\overset{\mathrm{pyk}}{\to}$ “mendelson three two a”]

$\mathrm{M3.2(a)_h}$

$[\mathrm{M3.2(a)_h} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ [\ \mathrm{M3.2(a)} \gg [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ;\ [\ [\ \mathrm{Hypothesize} \rhd [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ \gg [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, c)]$

$[\mathrm{M3.2(a)_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]$

$[\mathrm{M3.2(a)_h} \overset{\mathrm{tex}}{\rightarrow}$ “
M3.2(a)_h”]

$[\mathrm{M3.2(a)_h} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical three two a”]

$\mathrm{M3.2(b)_h}$

$[\mathrm{M3.2(b)_h} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ]\ \vdash [\ [\ \mathrm{S1}' \gg [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Hypothesize} \rhd [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ] \gg [\ \underline{h} \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}'_{\mathrm{h}} \rhd [\ \underline{h} \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ]\ ]\ \gg [\ \underline{h} \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{M3.2(a)_h} \gg [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}'_{\mathrm{h}} \rhd [\ \underline{h} \Rightarrow [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \Rightarrow [\ [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ \gg [\ \underline{h} \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, c)]$

$[\mathrm{M3.2(b)_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ]\ \vdash [\ \underline{h} \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]$

$[\mathrm{M3.2(b)_h} \overset{\mathrm{tex}}{\rightarrow}$ “
M3.2(b)_h”]

$[\mathrm{M3.2(b)_h} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical three two b”]

$\mathrm{M3.1(S1')_h}$

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ]\ \vdash [\ [\ \underline{h} \Rightarrow [\ \underline{t} \overset{\mathrm{p}}{=} \underline{s}\ ]\ ]\ \vdash [\ \underline{h} \Rightarrow [\ \underline{r} \overset{\mathrm{p}}{=} \underline{s}\ ]\ ]\ ]\ ]\ ]$

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{tex}}{\rightarrow}$ “
M3.1(S1')_h”]

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical three one s one”]

$\mathrm{M3.2(c)_h}$

$[\mathrm{M3.2(c)_h} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{M3.2(c)_h} \overset{\text{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow\ [\ \underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}\ ]\ ] \vdash [\ [\ \underline{\mathsf{h}} \Rightarrow\ [\ \underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ] \vdash [\ \underline{\mathsf{h}} \Rightarrow\ [\ \underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]]$

$[\mathrm{M3.2(c)_h} \overset{\text{tex}}{\longrightarrow}$ “
M3.2(c)_h”]

$[\mathrm{M3.2(c)_h} \overset{\text{pyk}}{\longrightarrow}$ “hypothetical three two c”]

$\mathrm{M3.1(S2')_h}$

$[\mathrm{M3.1(S2')_h} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{M3.1(S2')_h} \overset{\text{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\ [\ \underline{\mathsf{h}} \Rightarrow\ [\ \underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}\ ]\ ] \vdash [\ \underline{\mathsf{h}} \Rightarrow\ [\ \underline{\mathsf{t}}' \overset{\mathrm{p}}{=} [\ \underline{\mathsf{r}}'\ ]\ ]\ ]\ ]]$

$[\mathrm{M3.1(S2')_h} \overset{\text{tex}}{\longrightarrow}$ “
M3.1(S2')_h”]

$[\mathrm{M3.1(S2')_h} \overset{\text{pyk}}{\longrightarrow}$ “hypothetical three one s two”]

$\mathrm{M3.1(S5')_h}$

$[\mathrm{M3.1(S5')_h} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{M3.1(S5')_h} \overset{\text{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon [\ \underline{\mathsf{h}} \Rightarrow\ [\ [\ \underline{\mathsf{t}} \dot{+} \dot{0}\ ] \overset{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]]$

$[\mathrm{M3.1(S5')_h} \overset{\text{tex}}{\longrightarrow}$ “
M3.1(S5')_h”]

$[\mathrm{M3.1(S5')_h} \overset{\text{pyk}}{\longrightarrow}$ “hypothetical three one s five”]

$\mathrm{M3.1(S6')_h}$

$[\mathrm{M3.1(S6')_h} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{M3.1(S6')_h} \overset{\text{stmt}}{\longrightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\ \underline{\mathsf{h}} \Rightarrow\ [\ [\ \underline{\mathsf{t}} \dot{+}\ [\ \underline{\mathsf{r}}'\ ]\ ] \overset{\mathrm{p}}{=} [\ [\ \underline{\mathsf{t}} \dot{+} \underline{\mathsf{r}}\ ]'\ ]\ ]\ ]]$

$[\mathrm{M3.1(S6')_h} \overset{\text{tex}}{\longrightarrow}$ “
M3.1(S6')_h”]

$[\mathrm{M3.1(S6')_h} \overset{\text{pyk}}{\longrightarrow}$ “hypothetical three one s six”]

M3.2(f)

$[\mathrm{M3.2(f)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\ulcorner \mathrm{S}' \vdash [\ [\ \mathrm{A1}' \gg [\mathsf{x} \dot{\Rightarrow} [\mathsf{x} \dot{\Rightarrow} \mathsf{x}]\ ]\ ]\ ;\ [\ [\ \mathrm{M3.1(S5')_h} \gg$
$[\ [\mathsf{x} \dot{\Rightarrow} [\mathsf{x} \dot{\Rightarrow} \mathsf{x}]\ ]\ \dot{\Rightarrow}\ [\ [\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{M3.2(b)_h} \rhd [\ [\mathsf{x} \dot{\Rightarrow} [$
$\mathsf{x} \dot{\Rightarrow} \mathsf{x}]\ ]\ \dot{\Rightarrow}\ [\ [\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}\ ]\ ]\ ]\ \gg [\ [\mathsf{x} \dot{\Rightarrow} [\mathsf{x} \dot{\Rightarrow} \mathsf{x}]\ ]\ \dot{\Rightarrow} [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]$
$]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\mathsf{x} \dot{\Rightarrow} [\mathsf{x} \dot{\Rightarrow} \mathsf{x}]\ ]\ \dot{\Rightarrow} [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ ]\ ]\ ] \rhd [\mathsf{x} \dot{\Rightarrow} [$
$\mathsf{x} \dot{\Rightarrow} \mathsf{x}]\ ]\ ]\ \gg [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ ]\ ;\ [\ [\ \mathrm{M1.7} \gg [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [$
$\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{M3.1(S2')_h} \rhd [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t} \stackrel{p}{=} [$
$\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\ [\dot{0} \dot{+} [\dot{t}]\ ]'\ ]\ ]\ ]\ ]\ ;$
$[\ [\ \mathrm{M3.1(S6')_h} \gg [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\ [\dot{0} \dot{+} [\dot{t}']\ ] \stackrel{p}{=} [\ [\dot{0} \dot{+} [\dot{t}]\ ]'$
$]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{M3.2(b)_h} \rhd [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\ [\dot{0} \dot{+} [\dot{t}']\ ] \stackrel{p}{=} [\ [$
$\dot{0} \dot{+} [\dot{t}]\ ]'\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\ [\dot{0} \dot{+} [\dot{t}]\ ]' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']$
$]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{M3.2(c)_h} \rhd [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\ [\dot{0} \dot{+} [\dot{t}]\ ]'\ ]$
$]\ ]\ ]\ \rhd [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\ [\dot{0} \dot{+} [\dot{t}]\ ]' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]\ ]\ \gg [$
$[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Gen}' \rhd [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+}$
$[\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\mathsf{t}\colon [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=}$
$[\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{S9}' \gg [\ [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ \dot{\Rightarrow} [\ [\dot{\forall}\mathsf{t}\colon [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}$
$]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$
$\mathrm{MP}' \rhd [\ [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ \dot{\Rightarrow} [\ [\dot{\forall}\mathsf{t}\colon [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [$
$\dot{t}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ ]\ \gg [\ [\dot{\forall}\mathsf{t}\colon$
$[\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]$
$]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\dot{\forall}\mathsf{t}\colon [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']\ ]\ ]\ ]\ ]$
$\dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\mathsf{t}\colon [\ [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [$
$\dot{t}'\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\mathsf{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \urcorner, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{M3.2(f)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall}\mathsf{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]\ ]\ ]\ ]$

[M3.2(f) $\stackrel{\mathrm{tex}}{\rightarrow}$ "
M3.2(f)"]

[M3.2(f) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "mendelson three two f"]

$\dot{*}$

[$\dot{\mathsf{x}}$ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
\dot{#1.
}"]

[$\dot{\mathsf{x}}$ $\stackrel{\mathrm{pyk}}{\rightarrow}$ "* peano var"]

$*'$

$[\mathsf{x}' \overset{\text{tex}}{\to} \text{“\#1.'”}]$

$[\mathsf{x}' \overset{\text{pyk}}{\to} \text{“}* \text{ peano succ”}]$

$* \mathbin{\dot{:}} *$

$[\mathsf{x} \mathbin{\dot{:}} \mathsf{y} \overset{\text{tex}}{\to} \text{“\#1.}$
$\text{\mathop\{\dot\{\cdot\}\} \#2.”}]$

$[\mathsf{x} \mathbin{\dot{:}} \mathsf{y} \overset{\text{pyk}}{\to} \text{“}* \text{ peano times }*\text{”}]$

$* \mathbin{\dot{+}} *$

$[\mathsf{x} \mathbin{\dot{+}} \mathsf{y} \overset{\text{tex}}{\to} \text{“\#1.}$
$\text{\mathop\{\dot\{+\}\} \#2.”}]$

$[\mathsf{x} \mathbin{\dot{+}} \mathsf{y} \overset{\text{pyk}}{\to} \text{“}* \text{ peano plus }*\text{”}]$

$* \overset{\mathsf{p}}{=} *$

$[\mathsf{x} \overset{\mathsf{p}}{=} \mathsf{y} \overset{\text{tex}}{\to} \text{“\#1.}$
$\text{\stackrel\{p\}\{=\} \#2.”}]$

$[\mathsf{x} \overset{\mathsf{p}}{=} \mathsf{y} \overset{\text{pyk}}{\to} \text{“}* \text{ peano is }*\text{”}]$

$*^{\mathcal{P}}$

$[\mathsf{x}^{\mathcal{P}} \overset{\text{val}}{\to} \mathsf{x} \overset{\mathsf{r}}{=} [\dot{\mathsf{x}}]]$

$[\mathsf{x}^{\mathcal{P}} \overset{\text{tex}}{\to} \text{“\#1.}$
$\text{\{\} \^{} \{\cal P\}”}]$

$[\mathsf{x}^{\mathcal{P}} \overset{\text{pyk}}{\to} \text{“}* \text{ is peano var”}]$

$\dot{\neg} *$

$[\dot{\neg}\mathsf{x} \overset{\text{tex}}{\to} \text{“}$
$\text{\dot\{\neg\}\textbackslash, \#1.”}]$

$[\dot{\neg}\mathsf{x} \overset{\text{pyk}}{\to} \text{“peano not }*\text{”}]$

$* \dot{\wedge} *$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\wedge} \mathsf{y} \doteqdot \dot{\neg}(\mathsf{x} \dot{\Rightarrow} \dot{\neg}\mathsf{y})]\rceil)]$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“\#1.}$
$\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{wedge}\}\}\ \#2.\text{”}]$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“}* \text{ peano and } *\text{”}]$

$* \dot{\vee} *$

$[\mathsf{x} \dot{\vee} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\vee} \mathsf{y} \doteqdot [\dot{\neg}\mathsf{x}] \dot{\Rightarrow} \mathsf{y}]\rceil)]$

$[\mathsf{x} \dot{\vee} \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“\#1.}$
$\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{vee}\}\}\ \#2.\text{”}]$

$[\mathsf{x} \dot{\vee} \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“}* \text{ peano or } *\text{”}]$

$\dot{\forall} *\colon *$

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{forall}\}\ \#1.$
$\backslash\text{colon}\ \#2.\text{”}]$

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“peano all } * \text{ indeed } *\text{”}]$

$\dot{\exists} *\colon *$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{\exists}\mathsf{x}\colon \mathsf{y} \doteqdot \dot{\neg}\dot{\forall}\mathsf{x}\colon \dot{\neg}\mathsf{y}]\rceil)]$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{dot}\{\backslash\text{exists}\}\ \#1.$
$\backslash\text{colon}\ \#2.\text{”}]$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“peano exist } * \text{ indeed } *\text{”}]$

$* \dot{\Rightarrow} *$

$[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“\#1.}$
$\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{Rightarrow}\}\}\ \#2.\text{”}]$

$[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“}* \text{ peano imply } *\text{”}]$

$* \mathrel{\dot{\Leftrightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \ddot{=} (\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y}) \mathrel{\dot{\wedge}} (\mathsf{y} \mathrel{\dot{\Rightarrow}} \mathsf{x})]\rceil)]$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“\#1.}$
$\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{Leftrightarrow}\}\}\ \#2.\text{”}]$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“}* \text{ peano iff } *\text{”}]$

$* \mathrel{\dot{\trianglerighteq}} *$

$[\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq \mathsf{y} \ddot{=} \ [\ \mathrm{MP}' \rhd \mathsf{x}\ ]\ \rhd \mathsf{y}]\rceil)]$

$[\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“\#1.}$
$\backslash\text{unrhd}\ \#2.\text{”}]$

$[\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“}* \text{ macro modus ponens } *\text{”}]$

$* \trianglerighteq_{\mathrm{h}} *$

$[\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \ddot{=} \ [\ \mathrm{MP}'_{\mathrm{h}} \rhd \mathsf{x}\ ]\ \rhd \mathsf{y}]\rceil)]$

$[\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \overset{\text{tex}}{\rightarrow} \text{“\#1.}$
$\backslash\text{unrhd\_h}\ \#2.\text{”}]$

$[\mathsf{x} \trianglerighteq_{\mathrm{h}} \mathsf{y} \overset{\text{pyk}}{\rightarrow} \text{“}* \text{ hypothetical modus ponens } *\text{”}]$

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-29.UTC:12:28:36.959100 = MJD-53550.TAI:12:29:08.959100 = LGT-4626764948959100e-6