Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\text{nonfree}}(*,*)$, $\dot{\text{nonfree}}^{*}(*,*)$, $\dot{\text{free}}\langle *|* := *\rangle$, $\dot{\text{free}}^{*}\langle *|* := *\rangle$, $*\dot{\equiv}\langle *|* := *\rangle$, $*\dot{\equiv}\langle^{*} *|* := *\rangle$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, L3.2(a)′, M1.7, MP'_{h}, Hypothesize, Gen'_{h}, M3.2(a), $\text{M3.2(a)}_{\text{h}}$, $\text{M3.2(b)}_{\text{h}}$, $\text{M3.1(S1}')_{\text{h}}$, $\text{M3.2(c)}_{\text{h}}$, $\text{M3.1(S2}')_{\text{h}}$, $\text{M3.1(S5}')_{\text{h}}$, $\text{M3.1(S6}')_{\text{h}}$, M3.2(f), $\dot{*}$, $*'$, $* \dot{\cdot} *$, $* \dot{+} *$, $* \overset{\text{p}}{=} *$, $*^{\mathcal{P}}$, $\dot{\neg} *$, $* \dot{\wedge} *$, $* \dot{\vee} *$, $\dot{\forall} *\colon *$, $\dot{\exists} *\colon *$, $* \dot{\Rightarrow} *$, $* \dot{\Leftrightarrow} *$, $* \trianglerighteq *$, $* \trianglerighteq_{\text{h}} *$,

peano

[peano $\overset{\text{prio}}{\rightarrow}$
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \overset{*}{\rightarrow} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if$(*,*,*)$], [[$* \overset{*}{\Rightarrow} *$]], [val], [claim], [$\perp$], [f$(*)$], [$(*)^{\text{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\text{M}}$], [If$(*,*,*)$], [array$\{*\}$ $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\text{M}}(*)$], [**apply**$(*,*)$], [$\textbf{apply}_1(*,*)$], [identifier$(*)$], [$\text{identifier}_1(*,*)$], [array-plus$(*,*)$], [array-remove$(*,*,*)$], [array-put$(*,*,*,*)$], [array-add$(*,*,*,*,*)$], [bit$(*,*)$], [$\text{bit}_1(*,*)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*,*,*)$], [$\mathcal{E}_2(*,*,*,*,*)$], [$\mathcal{E}_3(*,*,*,*)$], [$\mathcal{E}_4(*,*,*,*)$], [**lookup**$(*,*,*)$], [**abstract**$(*,*,*,*)$], [[$\lceil * \rceil$]], [$\mathcal{M}(*,*,*)$], [$\mathcal{M}_2(*,*,*,*)$], [$\mathcal{M}^{*}(*,*,*)$], [macro], [s_0], [**zip**$(*,*)$], [$\textbf{assoc}_1(*,*,*)$], [$(*)^{\text{P}}$], [self], [[$* \ddot{=} *$]], [[$* \dot{=} *$]], [[$* \acute{=} *$]], [[$* \overset{\text{pyk}}{=} *$]], [[$* \overset{\text{tex}}{=} *$]], [[$* \overset{\text{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*,*,*,*)$], [$\tilde{\mathcal{M}}(*,*,*)$], [$\tilde{\mathcal{Q}}(*,*,*)$], [$\tilde{\mathcal{Q}}_2(*,*,*)$], [$\tilde{\mathcal{Q}}_3(*,*,*,*)$], [$\tilde{\mathcal{Q}}^{*}(*,*,*)$], [$(*)$], [**aspect**$(*,*)$], [**aspect**$(*,*,*)$], [$\langle * \rangle$], [$\textbf{tuple}_1(*)$], [$\textbf{tuple}_2(*)$], [$\text{let}_2(*,*)$], [$\text{let}_1(*,*)$], [[$* \overset{\text{claim}}{=} *$]], [checker], [**check**$(*,*)$], [$\textbf{check}_2(*,*,*)$], [$\textbf{check}_3(*,*,*)$], [$\textbf{check}^{*}(*,*)$], [$\textbf{check}^{*}_2(*,*,*)$], [[$*]^{\cdot}$], [[$*]^{-}$], [[$*]^{\circ}$], [msg], [[$* \overset{\text{msg}}{=} *$]], [<stmt>], [stmt], [[$* \overset{\text{stmt}}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\perp\!\!\!\perp$], [Contra′], [T'_{E}], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$],

[$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathbf{v}}$], [intro$(*,*,*,*)$], [intro$(*,*,*)$], [error$(*,*)$], [error$_2(*,*)$], [proof$(*,*,*)$], [proof$_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\mathrm{I}}(*,*)$], [$\mathcal{S}^{\rhd}(*,*)$], [$\mathcal{S}^{\rhd}_1(*,*,*)$], [$\mathcal{S}^{\mathrm{E}}(*,*)$], [$\mathcal{S}^{\mathrm{E}}_1(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}^{+}_1(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}^{-}_1(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}^{*}_1(*,*,*)$], [$\mathcal{S}^{*}_2(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}^{@}_1(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}^{\vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}^{\Vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}(*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}_2(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}^{\forall}_1(*,*,*,*)$], [$\mathcal{S}^{;}(*,*)$], [$\mathcal{S}^{;}_1(*,*,*)$], [$\mathcal{S}^{;}_2(*,*,*,*)$], [$\mathcal{T}(*)$], [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut$(*,*)$], [Head$_\oplus(*)$], [Tail$_\oplus(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [$\mathrm{T_E}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(* = *, *)$], [unify$^*(* = *, *)$], [unify$_2(* = *, *)$], [$\mathrm{L_a}$], [$\mathrm{L_b}$], [$\mathrm{L_c}$], [$\mathrm{L_d}$], [$\mathrm{L_e}$], [$\mathrm{L_f}$], [$\mathrm{L_g}$], [$\mathrm{L_h}$], [$\mathrm{L_i}$], [$\mathrm{L_j}$], [$\mathrm{L_k}$], [$\mathrm{L_l}$], [$\mathrm{L_m}$], [$\mathrm{L_n}$], [$\mathrm{L_o}$], [$\mathrm{L_p}$], [$\mathrm{L_q}$], [$\mathrm{L_r}$], [$\mathrm{L_s}$], [$\mathrm{L_t}$], [$\mathrm{L_u}$], [$\mathrm{L_v}$], [$\mathrm{L_w}$], [$\mathrm{L_x}$], [$\mathrm{L_y}$], [$\mathrm{L_z}$], [$\mathrm{L_A}$], [$\mathrm{L_B}$], [$\mathrm{L_C}$], [$\mathrm{L_D}$], [$\mathrm{L_E}$], [$\mathrm{L_F}$], [$\mathrm{L_G}$], [$\mathrm{L_H}$], [$\mathrm{L_I}$], [$\mathrm{L_J}$], [$\mathrm{L_K}$], [$\mathrm{L_L}$], [$\mathrm{L_M}$], [$\mathrm{L_N}$], [$\mathrm{L_O}$], [$\mathrm{L_P}$], [$\mathrm{L_Q}$], [$\mathrm{L_R}$], [$\mathrm{L_S}$], [$\mathrm{L_T}$], [$\mathrm{L_U}$], [$\mathrm{L_V}$], [$\mathrm{L_W}$], [$\mathrm{L_X}$], [$\mathrm{L_Y}$], [$\mathrm{L_Z}$], [$\mathrm{L_?}$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \overset{\mathrm{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$\mathrm{p_0}$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [nonfree$(*,*)$], [$\dot{\text{nonfree}}^*(*,*)$], [free$\langle *|* := *\rangle$], [free$^*\langle *|* := *\rangle$], [$*\equiv\langle *|* := *\rangle$], [$*\equiv\langle^* *|* := *\rangle$], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$], [L3.2(a)$'$], [M1.7], [$\mathrm{MP'_h}$], [Hypothesize], [$\mathrm{Gen'_h}$], [M3.2(a)], [M3.2(a)$_\mathrm{h}$], [M3.2(b)$_\mathrm{h}$], [M3.1(S1$'$)$_\mathrm{h}$], [M3.2(c)$_\mathrm{h}$], [M3.1(S2$'$)$_\mathrm{h}$], [M3.1(S5$'$)$_\mathrm{h}$], [M3.1(S6$'$)$_\mathrm{h}$], [M3.2(f)];

Preassociative

[$*_{\{*\}}$], [$*'$], [$*[*]$], [$*[*\to*]$], [$*[*\Rightarrow*]$], [$\dot{*}$];

Preassociative

[“ * ”], [], [$(*)^{\mathbf{t}}$], [string$(*)$ + $*$], [string$(*)$ ++ $*$], [
*], [*], [!*], [”*], [#*], [$*], [%*], [&*], [’*], [(*], [)*], [**], [+*], [,*], [-*], [.*], [/*], [0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [:*], [;*], [<*], [=*], [>*], [?*], [@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*], [O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [*], []*], [^*], [_*], [‘*], [a*], [b*], [c*], [d*], [e*], [f*], [g*], [h*], [i*], [j*], [k*], [l*], [m*], [n*], [o*], [p*], [q*], [r*], [s*], [t*], [u*], [v*], [w*], [x*], [y*], [z*], [{*], [|*], [}*], [~*], [**Preassociative** *; *], [**Postassociative** *; *], [[*], *], [priority * end], [newline *], [macro newline *];

Preassociative

[*0], [*1], [0b], [*-color(*)], [*-color*(*)];

Preassociative

$[* \text{'} *], [* \text{`} *];$

Preassociative

$[*^{\mathrm{H}}], [*^{\mathrm{T}}], [*^{\mathrm{U}}], [*^{\mathrm{h}}], [*^{\mathrm{t}}], [*^{\mathrm{s}}], [*^{\mathrm{c}}], [*^{\mathrm{d}}], [*^{\mathrm{a}}], [*^{\mathrm{C}}], [*^{\mathrm{M}}], [*^{\mathrm{B}}], [*^{\mathrm{r}}], [*^{\mathrm{i}}], [*^{\mathrm{d}}], [*^{\mathrm{R}}], [*^{0}], [*^{1}], [*^{2}], [*^{3}], [*^{4}], [*^{5}], [*^{6}], [*^{7}], [*^{8}], [*^{9}], [*^{\mathrm{E}}], [*^{\mathcal{V}}], [*^{\mathcal{C}}], [*^{\mathcal{C}^*}], [*']; $

Preassociative

$[* \cdot *], [* \cdot_0 *], [* \stackrel{\cdot}{\cdot} *];$

Preassociative

$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *];$

Preassociative

$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}];$

Postassociative

$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *];$

Postassociative

$[*, *];$

Preassociative

$[* \stackrel{\mathrm{B}}{\approx} *], [* \stackrel{\mathrm{D}}{\approx} *], [* \stackrel{\mathrm{C}}{\approx} *], [* \stackrel{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\rightarrow} *], [* \stackrel{\mathrm{t}}{=} *], [* \stackrel{\mathrm{t}^*}{=} *], [* \stackrel{\mathrm{r}}{=} *], [* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \stackrel{\mathrm{T}}{=} *], [* \stackrel{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *], [* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* \stackrel{\mathrm{p}}{=} *], [*^{\mathcal{P}}];$

Preassociative

$[\neg *], [\dot{\neg} *];$

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\mathrm{c}} *], [* \dot{\wedge} *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$

Preassociative

$[\dot{\forall} *: *], [\dot{\exists} *: *];$

Postassociative

$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *];$

Postassociative

$[* : *], [* ! *];$

Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$

Preassociative

$[\lambda * . *], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *];$

Preassociative

$[*^{\mathrm{I}}], [*^{\rhd}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}];$

Preassociative

$[* @ *], [* \rhd *], [* \Rrightarrow *], [* \gg *], [* \trianglerighteq *], [* \trianglerighteq_{\mathrm{h}} *];$

Postassociative

$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$

Preassociative

$[\forall *: *];$

Postassociative

$[* \oplus *];$

Postassociative
[$*;*$];
Preassociative
[$*$ proves $*$];
Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *;*$], [Last line $* \gg * \square$],
[Line $*$: Premise $\gg *;*$], [Line $*$: Side-condition $\gg *;*$], [Arbitrary $\gg *;*$],
[Local $\gg * = *;*$];
Postassociative
[$*$ then $*$], [$*[*]*$];
Preassociative
[$*\&*$];
Preassociative
[$*\backslash\backslash*$];]

[peano $\stackrel{\text{pyk}}{\rightarrow}$ "peano"]

$\dot{0}$

[$\dot{0}$ $\stackrel{\text{tex}}{\rightarrow}$ "
\dot{0}"]

[$\dot{0}$ $\stackrel{\text{pyk}}{\rightarrow}$ "peano zero"]

$\dot{1}$

[$\dot{1} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{1} \ddot{=} \dot{0}']\rceil)$]

[$\dot{1}$ $\stackrel{\text{tex}}{\rightarrow}$ "
\dot{1}"]

[$\dot{1}$ $\stackrel{\text{pyk}}{\rightarrow}$ "peano one"]

$\dot{2}$

[$\dot{2} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{2} \ddot{=} \dot{1}']\rceil)$]

[$\dot{2}$ $\stackrel{\text{tex}}{\rightarrow}$ "
\dot{2}"]

[$\dot{2}$ $\stackrel{\text{pyk}}{\rightarrow}$ "peano two"]

$\dot{a}$

$[\dot{a} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{a} \ddot{=} \dot{\mathsf{a}}] \rceil)]$

$[\dot{a} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{a}}"]

$[\dot{a} \stackrel{\text{pyk}}{\to}$ "peano a"]

$\dot{b}$

$[\dot{b} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{b}}"]

$[\dot{b} \stackrel{\text{pyk}}{\to}$ "peano b"]

$\dot{c}$

$[\dot{c} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{c}}"]

$[\dot{c} \stackrel{\text{pyk}}{\to}$ "peano c"]

$\dot{d}$

$[\dot{d} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{d}}"]

$[\dot{d} \stackrel{\text{pyk}}{\to}$ "peano d"]

$\dot{e}$

$[\dot{e} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{e}}"]

$[\dot{e} \stackrel{\text{pyk}}{\to}$ "peano e"]

$\dot{f}$

$[\dot{f} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil [\dot{f} \doteq \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\rightarrow}$ "peano f"]

$\dot{g}$

$[\dot{g} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil [\dot{g} \doteq \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\rightarrow}$ "peano g"]

$\dot{h}$

$[\dot{h} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil [\dot{h} \doteq \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\rightarrow}$ "peano h"]

$\dot{i}$

$[\dot{i} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil [\dot{i} \doteq \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\rightarrow}$ "peano i"]

$\dot{j}$

$[\dot{j} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil [\dot{j} \doteq \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\rightarrow}$ "peano j"]

$\dot{k}$

[$\dot{k} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{k} \ddot{=} \dot{\mathsf{k}}] \rceil)$]

[$\dot{k} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{k}}"]

[$\dot{k} \stackrel{\text{pyk}}{\rightarrow}$ "peano k"]

$\dot{l}$

[$\dot{l} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{l} \ddot{=} \dot{\mathsf{l}}] \rceil)$]

[$\dot{l} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{l}}"]

[$\dot{l} \stackrel{\text{pyk}}{\rightarrow}$ "peano l"]

$\dot{m}$

[$\dot{m} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{m} \ddot{=} \dot{\mathsf{m}}] \rceil)$]

[$\dot{m} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{m}}"]

[$\dot{m} \stackrel{\text{pyk}}{\rightarrow}$ "peano m"]

$\dot{n}$

[$\dot{n} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{n} \ddot{=} \dot{\mathsf{n}}] \rceil)$]

[$\dot{n} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{n}}"]

[$\dot{n} \stackrel{\text{pyk}}{\rightarrow}$ "peano n"]

$\dot{o}$

[$\dot{o} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{o} \ddot{=} \dot{\mathsf{o}}] \rceil)$]

[$\dot{o} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{o}}"]

[$\dot{o} \stackrel{\text{pyk}}{\rightarrow}$ "peano o"]

$\dot{p}$

$[\dot{p} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{p} \mathrel{\ddot{=}} \dot{\mathsf{p}}] \rceil)]$

$[\dot{p} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{p}}"]

$[\dot{p} \stackrel{\text{pyk}}{\to}$ "peano p"]

$\dot{q}$

$[\dot{q} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{q} \mathrel{\ddot{=}} \dot{\mathsf{q}}] \rceil)]$

$[\dot{q} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{q}}"]

$[\dot{q} \stackrel{\text{pyk}}{\to}$ "peano q"]

$\dot{r}$

$[\dot{r} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{r} \mathrel{\ddot{=}} \dot{\mathsf{r}}] \rceil)]$

$[\dot{r} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{r}}"]

$[\dot{r} \stackrel{\text{pyk}}{\to}$ "peano r"]

$\dot{s}$

$[\dot{s} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{s} \mathrel{\ddot{=}} \dot{\mathsf{s}}] \rceil)]$

$[\dot{s} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{s}}"]

$[\dot{s} \stackrel{\text{pyk}}{\to}$ "peano s"]

$\dot{t}$

$[\dot{t} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{t} \mathrel{\ddot{=}} \dot{\mathsf{t}}] \rceil)]$

$[\dot{t} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{t}}"]

$[\dot{t} \stackrel{\text{pyk}}{\to}$ "peano t"]

$\dot{u}$

$[\dot{u} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{u} \doteq \dot{\mathsf{u}}]\rceil)]$

$[\dot{u} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{u}}"]

$[\dot{u} \stackrel{\text{pyk}}{\to}$ "peano u"]

$\dot{v}$

$[\dot{v} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{v} \doteq \dot{\mathsf{v}}]\rceil)]$

$[\dot{v} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{v}}"]

$[\dot{v} \stackrel{\text{pyk}}{\to}$ "peano v"]

$\dot{w}$

$[\dot{w} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{w} \doteq \dot{\mathsf{w}}]\rceil)]$

$[\dot{w} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{w}}"]

$[\dot{w} \stackrel{\text{pyk}}{\to}$ "peano w"]

$\dot{x}$

$[\dot{x} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{x} \doteq \dot{\mathsf{x}}]\rceil)]$

$[\dot{x} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{x}}"]

$[\dot{x} \stackrel{\text{pyk}}{\to}$ "peano x"]

$\dot{y}$

$[\dot{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{y} \doteq \dot{\mathsf{y}}]\rceil)]$

$[\dot{y} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{y}}"]

$[\dot{y} \stackrel{\text{pyk}}{\to}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"]

$\dot{\text{nonfree}}(*, *)$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg [\mathsf{x} \stackrel{\text{t}}{=} \mathsf{y}],$
$\text{If}(\neg [\mathsf{y} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{x}\colon \mathsf{y}\rceil], \dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=} [\mathsf{y}^1], \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^2)))))]$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree $*$ in $*$ end nonfree"]

$\dot{\text{nonfree}}^*(*, *)$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y},\mathsf{T},\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}),\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),\mathsf{F}))]$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree star $*$ in $*$ end nonfree"]

$\dot{\text{free}}\langle * | * := * \rangle$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{val}}{\to} \mathsf{x}! [\mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg [\mathsf{a} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v}\rceil], \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$
$\text{If}(\mathsf{a}^1 \stackrel{\text{t}}{=} \mathsf{x}, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^2), \mathsf{T},$

$\mathrm{If}(\neg \dot{\mathrm{nonfree}}(\mathsf{a}^1, \mathsf{b}), \mathsf{F},$
$\dot{\mathrm{free}}\langle \mathsf{a}^2 | \mathsf{x} := \mathsf{b}\rangle)))))$]]

[$\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{pyk}}{\to}$ "peano free * set * to * end free"]

$\dot{\mathrm{free}}^*\langle * | * := *\rangle$

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{val}}{\to} \mathsf{x}!$ [$\mathsf{b}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\dot{\mathrm{free}}\langle \mathsf{a}^{\mathrm{h}} | \mathsf{x} := \mathsf{b}\rangle, \dot{\mathrm{free}}^*\langle \mathsf{a}^{\mathrm{t}} | \mathsf{x} := \mathsf{b}\rangle, \mathsf{F}))$]]

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{pyk}}{\to}$ "peano free star * set * to * end free"]

$*\equiv\langle * | * := *\rangle$

[$\mathsf{a}\equiv\langle \mathsf{b} | \mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{val}}{\to} \mathsf{a}!$ [$\mathsf{x}!$ [$\mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \overset{\mathrm{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v}\rceil, \mathsf{b}^1 \overset{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge$ [$\mathsf{b} \overset{\mathrm{t}}{=} \mathsf{x}$] $, \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}($ [
a] $\overset{\mathrm{r}}{=} \mathsf{b}, \mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c}\rangle, \mathsf{F})))$]]]

[$\mathsf{a}\equiv\langle \mathsf{b} | \mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{tex}}{\to}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

[$\mathsf{a}\equiv\langle \mathsf{b} | \mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{pyk}}{\to}$ "peano sub * is * where * is * end sub"]

$*\equiv\langle^* * | * := *\rangle$

[$\mathsf{a}\equiv\langle^* \mathsf{b} | \mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{val}}{\to} \mathsf{b}!$ [$\mathsf{x}!$ [$\mathsf{c}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\mathsf{a}^{\mathrm{h}}\equiv\langle \mathsf{b}^{\mathrm{h}} | \mathsf{x} := \mathsf{c}\rangle, \mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c}\rangle, \mathsf{F}))$]]]

[$\mathsf{a}\equiv\langle^* \mathsf{b} | \mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{tex}}{\to}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle”]

[$a \equiv \langle^* b | x := c \rangle \stackrel{\text{pyk}}{\rightarrow}$ “peano sub star ∗ is ∗ where ∗ is ∗ end sub”]

S

[$S \stackrel{\text{stmt}}{\rightarrow}$ $[\ [\ \dot{a} \dot{+} [\ \dot{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{a} \dot{+} [\ \dot{b}\ ]\ ]'\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \dot{\neg} \underline{b}\ ] \dot{\Rightarrow} \dot{\neg} \underline{a}\ ] \dot{\Rightarrow} [\ [\ [\ \dot{\neg} \underline{b}\ ] \dot{\Rightarrow} \underline{a}\ ] \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ \dot{a}' \stackrel{p}{=} [\ \dot{b}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a}' \stackrel{p}{=} [\ \dot{b}'\ ]\ ] \dot{\Rightarrow} [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{a}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \dot{\cdot} [\ \dot{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{a} \dot{\cdot} [\ \dot{b}\ ]\ ] \dot{+} [\ \dot{a}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \neg [\ \dot{0} \stackrel{p}{=} [\ \dot{a}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \forall \underline{x}\colon \underline{a}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{x}\colon \underline{b}\ ] \dot{\Rightarrow} \underline{a}\ ]\ ]\ ]\ ] \oplus [\ [\ \dot{a} \dot{\cdot} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]$]

[$S \stackrel{\text{tex}}{\rightarrow}$ “
S”]

[$S \stackrel{\text{pyk}}{\rightarrow}$ “system s”]

A1

[$A1 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$A1 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{a}\ ]\ ]$]

[$A1 \stackrel{\text{tex}}{\rightarrow}$ “
A1”]

[$A1 \stackrel{\text{pyk}}{\rightarrow}$ “axiom a one”]

A2

[$A2 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$A2 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]$]

[A2 $\stackrel{\text{tex}}{\rightarrow}$ “
A2”]

[A2 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom a two”]

A3

[A3 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A3 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\,[\,[\,\dot{\neg}\,\underline{b}\,] \dot{\Rightarrow} \dot{\neg}\,\underline{a}\,] \dot{\Rightarrow} [\,[\,[\,\dot{\neg}\,\underline{b}\,] \dot{\Rightarrow} \underline{a}\,] \dot{\Rightarrow} \underline{b}\,]\,]$]

[A3 $\stackrel{\text{tex}}{\rightarrow}$ “
A3”]

[A3 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom a three”]

A4

[A4 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A4 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\,\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\,[\,\dot{\forall}\underline{x}\colon \underline{b}\,] \dot{\Rightarrow} \underline{a}\,]\,]$]

[A4 $\stackrel{\text{tex}}{\rightarrow}$ “
A4”]

[A4 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom a four”]

A5

[A5 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\,\mathrm{non}\dot{\mathrm{f}}\mathrm{ree}(\underline{x}, \underline{a}) \Vdash [\,[\,\dot{\forall}\underline{x}\colon [\,\underline{a} \dot{\Rightarrow} \underline{b}\,]\,] \dot{\Rightarrow} [\,\underline{a} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{b}\,]\,]\,]$]

[A5 $\stackrel{\text{tex}}{\rightarrow}$ “
A5”]

[A5 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom a five”]

S1

[S1 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\,[\,\dot{a} \stackrel{p}{=} [\,\dot{b}\,]\,] \dot{\Rightarrow} [\,[\,\dot{a} \stackrel{p}{=} [\,\dot{c}\,]\,] \dot{\Rightarrow} [\,\dot{b} \stackrel{p}{=} [\,\dot{c}\,]\,]\,]\,]$]

$[S1 \stackrel{tex}{\rightarrow}$ "
S1"]

$[S1 \stackrel{pyk}{\rightarrow}$ "axiom s one"]

S2

$[S2 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S2 \stackrel{stmt}{\rightarrow} S \vdash [\,[\,\dot{a} \stackrel{p}{=} [\,\dot{b}\,]\,] \mathrel{\dot{\Rightarrow}} [\,\dot{a}' \stackrel{p}{=} [\,\dot{b}'\,]\,]\,]]$

$[S2 \stackrel{tex}{\rightarrow}$ "
S2"]

$[S2 \stackrel{pyk}{\rightarrow}$ "axiom s two"]

S3

$[S3 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S3 \stackrel{stmt}{\rightarrow} S \vdash \neg [\,\dot{0} \stackrel{p}{=} [\,\dot{a}'\,]\,]]$

$[S3 \stackrel{tex}{\rightarrow}$ "
S3"]

$[S3 \stackrel{pyk}{\rightarrow}$ "axiom s three"]

S4

$[S4 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S4 \stackrel{stmt}{\rightarrow} S \vdash [\,[\,\dot{a}' \stackrel{p}{=} [\,\dot{b}'\,]\,] \mathrel{\dot{\Rightarrow}} [\,\dot{a} \stackrel{p}{=} [\,\dot{b}\,]\,]\,]]$

$[S4 \stackrel{tex}{\rightarrow}$ "
S4"]

$[S4 \stackrel{pyk}{\rightarrow}$ "axiom s four"]

S5

$[S5 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S5 \stackrel{stmt}{\rightarrow} S \vdash [\,[\,\dot{a} \mathbin{\dot{+}} \dot{0}\,] \stackrel{p}{=} [\,\dot{a}\,]\,]]$

[S5 $\stackrel{\text{tex}}{\to}$ “
S5”]

[S5 $\stackrel{\text{pyk}}{\to}$ “axiom s five”]

S6

[S6 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S6 $\stackrel{\text{stmt}}{\to}$ $S \vdash [\, [\, \dot{a} \mathbin{\dot{+}} [\, \dot{b}' \,] \,] \stackrel{p}{=} [\, [\, \dot{a} \mathbin{\dot{+}} [\, \dot{b} \,] \,]' \,] \,]$]

[S6 $\stackrel{\text{tex}}{\to}$ “
S6”]

[S6 $\stackrel{\text{pyk}}{\to}$ “axiom s six”]

S7

[S7 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S7 $\stackrel{\text{stmt}}{\to}$ $S \vdash [\, [\, \dot{a} \mathbin{\dot{\cdot}} \dot{0} \,] \stackrel{p}{=} \dot{0} \,]$]

[S7 $\stackrel{\text{tex}}{\to}$ “
S7”]

[S7 $\stackrel{\text{pyk}}{\to}$ “axiom s seven”]

S8

[S8 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S8 $\stackrel{\text{stmt}}{\to}$ $S \vdash [\, [\, \dot{a} \mathbin{\dot{\cdot}} [\, \dot{b}' \,] \,] \stackrel{p}{=} [\, [\, \dot{a} \mathbin{\dot{\cdot}} [\, \dot{b} \,] \,] \mathbin{\dot{+}} [\, \dot{a} \,] \,] \,]$]

[S8 $\stackrel{\text{tex}}{\to}$ “
S8”]

[S8 $\stackrel{\text{pyk}}{\to}$ “axiom s eight”]

S9

[S9 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S9 $\stackrel{\text{stmt}}{\to}$ $S \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\, \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\, \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\, \underline{b} \mathrel{\dot{\Rightarrow}} [\, [\, \dot{\forall} \underline{x}\colon [\, \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c} \,] \,] \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{a} \,] \,] \,] \,]$]

[S9 $\stackrel{\text{tex}}{\rightarrow}$ “
S9”]

[S9 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom s nine”]

MP

[MP $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP $\stackrel{\text{tex}}{\rightarrow}$ “
MP”]

[MP $\stackrel{\text{pyk}}{\rightarrow}$ “rule mp”]

Gen

[Gen $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{x} \colon \forall \underline{a} \colon [\ \underline{a} \vdash \dot{\forall} \underline{x} \colon \underline{a}\ ]$]

[Gen $\stackrel{\text{tex}}{\rightarrow}$ “
Gen”]

[Gen $\stackrel{\text{pyk}}{\rightarrow}$ “rule gen”]

L3.2(a)

[L3.2(a) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S \vdash [\ [\ S5 \gg [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ; [\ [\ [\ Gen \rhd [\ [$
$\dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{a} \colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ; [\ [\ [\ A4 @ [\ \dot{x}\ ]\ ] \gg [\ [$
$\dot{\forall}\dot{a} \colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ MP \rhd [\ [\ \dot{\forall}\dot{a} \colon$
$[\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{a} \colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]$
$] \gg [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ; [\ [\ S1 \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow}$
$[\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ Gen \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=}$
$[\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ;$
$[\ [\ [\ A4 @ [\ \dot{x}\ ]\ ] \gg [\ [\ \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}$
$]\ ]\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [$
$[\ MP \rhd [\ [\ \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ] \dot{\Rightarrow} [$
$[\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ]$
$\dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]$
$] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ Gen \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \dot{\Rightarrow} [$

ḃ $\stackrel{p}{=}$ [ẋ]]]]] ≫ ∀̇ḃ: [[ȧ $\stackrel{p}{=}$ [ḃ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ḃ $\stackrel{p}{=}$ [ẋ]]]]] ; [[[A4 @ [ẋ]] ≫ [[∀̇ḃ: [[ȧ $\stackrel{p}{=}$ [ḃ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ḃ $\stackrel{p}{=}$ [ẋ]]]]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]]] ; [[[[MP ▷ [[∀̇ḃ: [[ȧ $\stackrel{p}{=}$ [ḃ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ḃ $\stackrel{p}{=}$ [ẋ]]]]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]]] ▷ ∀̇ḃ: [[ȧ $\stackrel{p}{=}$ [ḃ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ḃ $\stackrel{p}{=}$ [ẋ]]]]] ≫ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ; [[[Gen ▷ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ≫ ∀̇ȧ: [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ; [[[A4 @ [ẋ +̇ 0̇]] ≫ [[∀̇ȧ: [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ⇒̇ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]]] ; [[[[MP ▷ [[∀̇ȧ: [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ⇒̇ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]]] ▷ ∀̇ȧ: [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ≫ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]]] ; [[[[MP ▷ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]]] ▷ [[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]]] ≫ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]] ; [[[MP ▷ [[[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]] ⇒̇ [ẋ $\stackrel{p}{=}$ [ẋ]]]] ▷ [[ẋ +̇ 0̇] $\stackrel{p}{=}$ [ẋ]]] ≫ [ẋ $\stackrel{p}{=}$ [ẋ]]]]]]]]]]]]]]]]]]] ⌉, p_0, c)]

[L3.2(a) $\stackrel{\text{stmt}}{\to}$ S ⊢ [ẋ $\stackrel{p}{=}$ [ẋ]]]

[L3.2(a) $\stackrel{\text{tex}}{\to}$ “
L3.2(a)”]

[L3.2(a) $\stackrel{\text{pyk}}{\to}$ “lemma l three two a”]

S′

[S′ $\stackrel{\text{stmt}}{\to}$ [∀a: ∀b: [[a′ $\stackrel{p}{=}$ [b′]] ⇒̇ [a $\stackrel{p}{=}$ b]]] ⊕ [[∀h: ∀t: ∀r: ∀s: [[h ⇒̇ [t $\stackrel{p}{=}$ r]] ⊢ [[h ⇒̇ [t $\stackrel{p}{=}$ s]] ⊢ [h ⇒̇ [r $\stackrel{p}{=}$ s]]]]] ⊕ [[∀a: ∀b: [a ⇒̇ [b ⇒̇ a]]] ⊕ [[∀x: ∀a: ∀b: [nonfree(⌈x⌉, ⌈a⌉) ⊩ [[∀̇x: [a ⇒̇ b]] ⇒̇ [a ⇒̇ ∀̇x: b]]]] ⊕ [[∀h: ∀t: ∀r: [h ⇒̇ [[t +̇ [r′]] $\stackrel{p}{=}$ [[t +̇ r] ′]]]] ⊕ [[∀a: ∀b: [[a ·̇ [b′]] $\stackrel{p}{=}$ [[a ·̇ b] +̇ a]]] ⊕ [[∀a: ∀b: [[a $\stackrel{p}{=}$ b] ⇒̇ [a′ $\stackrel{p}{=}$ [b′]]]] ⊕ [[∀a: ∀b: [[a ⇒̇ b] ⊢ [a ⊢ b]]] ⊕ [[∀a: ∀b: [[[¬̇ b] ⇒̇ ¬̇ a] ⇒̇ [[[¬̇ b] ⇒̇ a] ⇒̇ b]]] ⊕ [[∀h: ∀t: ∀r: [[h ⇒̇ [t $\stackrel{p}{=}$ r]] ⊢ [h ⇒̇ [t′ $\stackrel{p}{=}$ [r′]]]]] ⊕ [[∀a: ∀b: [[a +̇ [b′]] $\stackrel{p}{=}$ [[a +̇ b] ′]]] ⊕ [[∀a: ¬ [0̇ $\stackrel{p}{=}$ [a′]]] ⊕ [[∀x: ∀a: [a ⊢ ∀̇x: a]] ⊕ [[∀c: ∀a: ∀x: ∀b: [⌈a⌉≡⟨⌈b⌉|⌈x⌉ := ⌈c⌉⟩ ⊩ [[∀̇x: b] ⇒̇ a]]] ⊕ [[∀h: ∀t: [h ⇒̇ [[t +̇ 0̇] $\stackrel{p}{=}$ t]]] ⊕ [[∀a: [[a ·̇ 0̇] $\stackrel{p}{=}$ 0̇]] ⊕ [[∀a: ∀b: ∀c: [[a $\stackrel{p}{=}$ b] ⇒̇ [[a $\stackrel{p}{=}$ c] ⇒̇ [b $\stackrel{p}{=}$ c]]]] ⊕ [[∀t: [t $\stackrel{p}{=}$ t]] ⊕ [[∀a: ∀b: ∀c: ∀x: [b≡⟨a|x := 0̇⟩ ⊩ [c≡⟨a|x := x′⟩ ⊩ [b ⇒̇ [[∀̇x: [a ⇒̇ c]] ⇒̇ ∀̇x: a]]]]] ⊕ [[∀a: ∀b: ∀c: [[a ⇒̇ [b ⇒̇ c]] ⇒̇ [[a ⇒̇ b] ⇒̇ [

$\underline{a} \dot{\Rightarrow} \underline{c}]]]] \oplus [[\forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [[\underline{h} \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{r}]] \vdash [[\underline{h} \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]] \vdash [\underline{h} \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]]]]] \oplus \forall \underline{a}\colon [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]$

[S′ $\stackrel{\text{tex}}{\to}$ “
S’”]

[S′ $\stackrel{\text{pyk}}{\to}$ “system prime s”]

A1′

[A1′ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A1′ $\stackrel{\text{stmt}}{\to}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]]$]

[A1′ $\stackrel{\text{tex}}{\to}$ “
A1’”]

[A1′ $\stackrel{\text{pyk}}{\to}$ “axiom prime a one”]

A2′

[A2′ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A2′ $\stackrel{\text{stmt}}{\to}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]$]

[A2′ $\stackrel{\text{tex}}{\to}$ “
A2’”]

[A2′ $\stackrel{\text{pyk}}{\to}$ “axiom prime a two”]

A3′

[A3′ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A3′ $\stackrel{\text{stmt}}{\to}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \dot{\neg} \underline{a}] \dot{\Rightarrow} [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]]$]

[A3′ $\stackrel{\text{tex}}{\to}$ “
A3’”]

[A3′ $\stackrel{\text{pyk}}{\to}$ “axiom prime a three”]

A4′

[A4′ $\stackrel{\text{proof}}{\to}$ Rule tactic]

$[\mathrm{A4'} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\, [\, \dot{\forall} \underline{x}\colon \underline{b} \,] \mathrel{\dot{\Rightarrow}} \underline{a} \,] \,]]$

[A4′ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
A4'"]

[A4′ $\stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime a four"]

A5′

[A5′ $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{A5'} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\, \mathrm{non\dot{f}ree}(\lceil \underline{x} \rceil, \lceil \underline{a} \rceil) \Vdash [\, [\, \dot{\forall} \underline{x}\colon [\, \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \underline{a} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{b} \,] \,] \,]]$

[A5′ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
A5'"]

[A5′ $\stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime a five"]

S1′

[S1′ $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{S1'} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\, [\, \underline{a} \stackrel{\mathrm{p}}{=} \underline{b} \,] \mathrel{\dot{\Rightarrow}} [\, [\, \underline{a} \stackrel{\mathrm{p}}{=} \underline{c} \,] \mathrel{\dot{\Rightarrow}} [\, \underline{b} \stackrel{\mathrm{p}}{=} \underline{c} \,] \,] \,]]$

[S1′ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
S1'"]

[S1′ $\stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime s one"]

S2′

[S2′ $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{S2'} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, \underline{a} \stackrel{\mathrm{p}}{=} \underline{b} \,] \mathrel{\dot{\Rightarrow}} [\, \underline{a}' \stackrel{\mathrm{p}}{=} [\, \underline{b}' \,] \,] \,]]$

[S2′ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
S2'"]

[S2′ $\stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime s two"]

S3′

[S3′ $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[S3' \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \neg [\, \dot{0} \overset{p}{=} [\, \underline{a}' \,] \,]]$

$[S3' \overset{\text{tex}}{\rightarrow}$ “
S3”’]

$[S3' \overset{\text{pyk}}{\rightarrow}$ “axiom prime s three”]

S4′

$[S4' \overset{\text{proof}}{\rightarrow}$ Rule tactic]

$[S4' \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, \underline{a}' \overset{p}{=} [\, \underline{b}' \,] \,] \dot{\Rightarrow} [\, \underline{a} \overset{p}{=} \underline{b} \,] \,]]$

$[S4' \overset{\text{tex}}{\rightarrow}$ “
S4”’]

$[S4' \overset{\text{pyk}}{\rightarrow}$ “axiom prime s four”]

S5′

$[S5' \overset{\text{proof}}{\rightarrow}$ Rule tactic]

$[S5' \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon [\, [\, \underline{a} \dot{+} \dot{0} \,] \overset{p}{=} \underline{a} \,]]$

$[S5' \overset{\text{tex}}{\rightarrow}$ “
S5”’]

$[S5' \overset{\text{pyk}}{\rightarrow}$ “axiom prime s five”]

S6′

$[S6' \overset{\text{proof}}{\rightarrow}$ Rule tactic]

$[S6' \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, \underline{a} \dot{+} [\, \underline{b}' \,] \,] \overset{p}{=} [\, [\, \underline{a} \dot{+} \underline{b} \,]' \,] \,]]$

$[S6' \overset{\text{tex}}{\rightarrow}$ “
S6”’]

$[S6' \overset{\text{pyk}}{\rightarrow}$ “axiom prime s six”]

S7′

$[S7' \overset{\text{proof}}{\rightarrow}$ Rule tactic]

$[S7' \overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon [\, [\, \underline{a} \dot{\cdot} \dot{0} \,] \overset{p}{=} \dot{0} \,]]$

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ “
S7’”]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s seven”]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \dot{:} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{:} \underline{b}] \dot{+} \underline{a}]]$]

[S8′ $\stackrel{\text{tex}}{\rightarrow}$ “
S8’”]

[S8′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s eight”]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall} \underline{x}: [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall} \underline{x}: \underline{a}]]]]$]

[S9′ $\stackrel{\text{tex}}{\rightarrow}$ “
S9’”]

[S9′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s nine”]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]$]

[MP′ $\stackrel{\text{tex}}{\rightarrow}$ “
MP’”]

[MP′ $\stackrel{\text{pyk}}{\rightarrow}$ “rule prime mp”]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}: \forall \underline{a}: [\underline{a} \vdash \dot{\forall} \underline{x}: \underline{a}]$]

$[\mathrm{Gen}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
Gen′"]

$[\mathrm{Gen}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "rule prime gen"]

L3.2(a)′

$$[\mathrm{L3.2(a)}' \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon [\ [\ \mathrm{S5}' \gg [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ]\ ;\ [\ [\ \mathrm{S1}' \gg [\ [$$
$$[\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [$$
$$\underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ] \rhd [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ]$$
$$\gg [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [$$
$$\underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ] \rhd [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ] \gg [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ] \rceil, \mathrm{p}_0, c)]$$

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a}\colon [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]$

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
L3.2(a)′"]

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma prime l three two a"]

M1.7

$$[\mathrm{M1.7} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{b}\colon [\ [\ \mathrm{A1}' \gg [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ;\ [\ [$$
$$\mathrm{A2}' \gg [\ [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]$$
$$]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [$$
$$\underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ] \rhd [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} \underline{b}\ ]\ ]\ ] \gg [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [$$
$$\underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ \underline{b} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]$$
$$] \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ] \rhd [\ \underline{b} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ] \gg [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ]\ ]\ ] \rceil, \mathrm{p}_0, c)]$$

$[\mathrm{M1.7} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{b}\colon [\ \underline{b} \dot{\Rightarrow} \underline{b}\ ]\ ]$

$[\mathrm{M1.7} \stackrel{\mathrm{tex}}{\rightarrow}$ "
M1.7"]

$[\mathrm{M1.7} \stackrel{\mathrm{pyk}}{\rightarrow}$ "mendelson one seven"]

$\mathrm{MP}'_{\mathrm{h}}$

$$[\mathrm{MP}'_{\mathrm{h}} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \vdash [\ [\ \underline{h} \dot{\Rightarrow} \underline{a}\ ] \vdash [\ [$$
$$\mathrm{A2}' \gg [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{h} \dot{\Rightarrow} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{h} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$$
$$\mathrm{MP}' \rhd [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{h} \dot{\Rightarrow} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{h} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \dot{\Rightarrow} [$$
$$\underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ] \gg [\ [\ \underline{h} \dot{\Rightarrow} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{h} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{h} \dot{\Rightarrow} \underline{a}\ ] \dot{\Rightarrow} [$$
$$\underline{h} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ] \rhd [\ \underline{h} \dot{\Rightarrow} \underline{a}\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ]\ ]\ ] \rceil, \mathrm{p}_0, c)]$$

[$MP'_h \overset{stmt}{\rightarrow} S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon [[\underline{h} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]] \vdash [[\underline{h} \dot{\Rightarrow} \underline{a}] \vdash [\underline{h} \dot{\Rightarrow} \underline{b}]]]$]

[$MP'_h \overset{tex}{\rightarrow}$ "
MP'_h"]

[$MP'_h \overset{pyk}{\rightarrow}$ "hypothetical rule prime mp"]

Hypothesize

[Hypothesize $\overset{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon [\underline{a} \vdash [[A1' \gg [\underline{a} \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \underline{a}]]] ; [[[MP' \rhd [\underline{a} \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \underline{a}]]] \rhd \underline{a}] \gg [\underline{h} \dot{\Rightarrow} \underline{a}]]]] \rceil, p_0, c)$]

[Hypothesize $\overset{stmt}{\rightarrow} S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon [\underline{a} \vdash [\underline{h} \dot{\Rightarrow} \underline{a}]]$]

[Hypothesize $\overset{tex}{\rightarrow}$ "
Hypothesize"]

[Hypothesize $\overset{pyk}{\rightarrow}$ "hypothesize"]

Gen'_h

[$Gen'_h \overset{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{x}\colon \forall \underline{a}\colon [\dot{nonfree}(\lceil \underline{x} \rceil, \lceil \underline{h} \rceil) \Vdash [[\underline{h} \dot{\Rightarrow} \underline{a}] \vdash [[[A5' \unrhd \dot{nonfree}(\lceil \underline{x} \rceil, \lceil \underline{h} \rceil)] \gg [[\dot{\forall} \underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]] \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]] ; [[[Gen' \rhd [\underline{h} \dot{\Rightarrow} \underline{a}]] \gg \dot{\forall} \underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]] ; [[[MP' \rhd [[\dot{\forall} \underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]] \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]] \rhd \dot{\forall} \underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]] \gg [\underline{h} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]]]]] \rceil, p_0, c)$]

[$Gen'_h \overset{stmt}{\rightarrow} S' \vdash \forall \underline{h}\colon \forall \underline{x}\colon \forall \underline{a}\colon [\dot{nonfree}(\lceil \underline{x} \rceil, \lceil \underline{h} \rceil) \Vdash [[\underline{h} \dot{\Rightarrow} \underline{a}] \vdash [\underline{h} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]]$
]

[$Gen'_h \overset{tex}{\rightarrow}$ "
Gen'_h"]

[$Gen'_h \overset{pyk}{\rightarrow}$ "hypothetical rule prime gen"]

M3.2(a)

[M3.2(a) $\overset{proof}{\rightarrow}$ Rule tactic]

[M3.2(a) $\overset{stmt}{\rightarrow} S' \vdash \forall \underline{t}\colon [\underline{t} \overset{p}{=} \underline{t}]$]

[M3.2(a) $\overset{tex}{\rightarrow}$ "
M3.2(a)"]

[M3.2(a) $\overset{pyk}{\rightarrow}$ "mendelson three two a"]

$\mathrm{M3.2(a)_h}$

$[\mathrm{M3.2(a)_h} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ [\ \mathrm{M3.2(a)} \gg [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ;\ [\ [$ $\mathrm{Hypothesize} \rhd [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, c)]$

$[\mathrm{M3.2(a)_h} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]]$

$[\mathrm{M3.2(a)_h} \stackrel{\mathrm{tex}}{\rightarrow}$ "
M3.2(a)_h"]

$[\mathrm{M3.2(a)_h} \stackrel{\mathrm{pyk}}{\rightarrow}$ "hypothetical three two a"]

$\mathrm{M3.2(b)_h}$

$[\mathrm{M3.2(b)_h} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ]\ ] \vdash [\ [\ \mathrm{S1}' \gg [\ [$ $\underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Hypothesize} \rhd [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [$ $[\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]$ $]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}'_{\mathrm{h}} \rhd [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ] \rhd [$ $\underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ]\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{M3.2(a)_h} \gg [$ $\underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}'_{\mathrm{h}} \rhd [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \dot{\Rightarrow}$ $[\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, c)]$

$[\mathrm{M3.2(b)_h} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ]\ ] \vdash [\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]]$

$[\mathrm{M3.2(b)_h} \stackrel{\mathrm{tex}}{\rightarrow}$ "
M3.2(b)_h"]

$[\mathrm{M3.2(b)_h} \stackrel{\mathrm{pyk}}{\rightarrow}$ "hypothetical three two b"]

$\mathrm{M3.1(S1')_h}$

$[\mathrm{M3.1(S1')_h} \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{M3.1(S1')_h} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{r}\ ]\ ] \vdash [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{\mathrm{p}}{=} \underline{s}\ ]\ ] \vdash$ $[\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \stackrel{\mathrm{p}}{=} \underline{s}\ ]\ ]\ ]\ ]]$

$[\mathrm{M3.1(S1')_h} \stackrel{\mathrm{tex}}{\rightarrow}$ "
M3.1(S1')_h"]

$[\mathrm{M3.1(S1')_h} \stackrel{\mathrm{pyk}}{\rightarrow}$ "hypothetical three one s one"]

$\mathrm{M3.2(c)_h}$

[$\mathrm{M3.2(c)_h} \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.2(c)_h} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{p}{=} \underline{r}\ ]\ ] \vdash [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \stackrel{p}{=} \underline{s}\ ]\ ] \vdash [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{p}{=} \underline{s}\ ]\ ]\ ]\ ]$]

[$\mathrm{M3.2(c)_h} \stackrel{\text{tex}}{\rightarrow}$ "
M3.2(c)_h"]

[$\mathrm{M3.2(c)_h} \stackrel{\text{pyk}}{\rightarrow}$ "hypothetical three two c"]

$\mathrm{M3.1(S2')_h}$

[$\mathrm{M3.1(S2')_h} \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.1(S2')_h} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \stackrel{p}{=} \underline{r}\ ]\ ] \vdash [\ \underline{h} \dot{\Rightarrow} [\ \underline{t}' \stackrel{p}{=} [\ \underline{r}'\ ]\ ]\ ]\ ]$]

[$\mathrm{M3.1(S2')_h} \stackrel{\text{tex}}{\rightarrow}$ "
M3.1(S2')_h"]

[$\mathrm{M3.1(S2')_h} \stackrel{\text{pyk}}{\rightarrow}$ "hypothetical three one s two"]

$\mathrm{M3.1(S5')_h}$

[$\mathrm{M3.1(S5')_h} \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.1(S5')_h} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ ]$]

[$\mathrm{M3.1(S5')_h} \stackrel{\text{tex}}{\rightarrow}$ "
M3.1(S5')_h"]

[$\mathrm{M3.1(S5')_h} \stackrel{\text{pyk}}{\rightarrow}$ "hypothetical three one s five"]

$\mathrm{M3.1(S6')_h}$

[$\mathrm{M3.1(S6')_h} \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.1(S6')_h} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \dot{+} [\ \underline{r}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{t} \dot{+} \underline{r}\ ]'\ ]\ ]\ ]$]

[$\mathrm{M3.1(S6')_h} \stackrel{\text{tex}}{\rightarrow}$ "
M3.1(S6')_h"]

[$\mathrm{M3.1(S6')_h} \stackrel{\text{pyk}}{\rightarrow}$ "hypothetical three one s six"]

M3.2(f)

$$[\mathrm{M3.2(f)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash [\ [\ \mathrm{A1}' \gg [\ \mathsf{x} \dot{\Rightarrow} [\ \mathsf{x} \dot{\Rightarrow} \mathsf{x}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{M3.1(S5')_h} \gg$$
$$[\ [\ \mathsf{x} \dot{\Rightarrow} [\ \mathsf{x} \dot{\Rightarrow} \mathsf{x}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{0} \dot{+} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{M3.2(b)_h} \rhd [\ [\ \mathsf{x} \dot{\Rightarrow} [$$
$$\mathsf{x} \dot{\Rightarrow} \mathsf{x}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{0} \dot{+} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]\ ]\ ]\ \gg [\ [\ \mathsf{x} \dot{\Rightarrow} [\ \mathsf{x} \dot{\Rightarrow} \mathsf{x}\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0} \dot{+} \dot{0}\ ]$$
$$]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \mathsf{x} \dot{\Rightarrow} [\ \mathsf{x} \dot{\Rightarrow} \mathsf{x}\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0} \dot{+} \dot{0}\ ]\ ]\ ]\ ]\ \rhd [\ \mathsf{x} \dot{\Rightarrow} [$$
$$\mathsf{x} \dot{\Rightarrow} \mathsf{x}\ ]\ ]\ ]\ \gg [\ \dot{0} \stackrel{p}{=} [\ \dot{0} \dot{+} \dot{0}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{M1.7} \gg [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [$$
$$\dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{M3.1(S2')_h} \rhd [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [$$
$$\dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]'\ ]\ ]\ ]\ ]\ ;$$
$$[\ [\ \mathrm{M3.1(S6')_h} \gg [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]'$$
$$]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{M3.2(b)_h} \rhd [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ] \stackrel{p}{=} [\ [$$
$$\dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]'\ ]\ ]\ ]\ ]\ \gg [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]$$
$$]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{M3.2(c)_h} \rhd [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]'\ ]$$
$$]\ ]\ ]\ \rhd [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ ]\ \gg [$$
$$[\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Gen}' \rhd [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+}$$
$$[\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\mathsf{t}\colon [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=}$$
$$[\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{S9}' \gg [\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0} \dot{+} \dot{0}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{\forall}\mathsf{t}\colon [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}$$
$$]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$$
$$\mathrm{MP}' \rhd [\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0} \dot{+} \dot{0}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{\forall}\mathsf{t}\colon [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [$$
$$\dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \dot{0} \stackrel{p}{=} [\ \dot{0} \dot{+} \dot{0}\ ]\ ]\ ]\ \gg [\ [\ \dot{\forall}\mathsf{t}\colon$$
$$[\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]$$
$$]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\mathsf{t}\colon [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]$$
$$\dot{\Rightarrow} \dot{\forall}\mathsf{t}\colon [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\mathsf{t}\colon [\ [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{\mathsf{t}}' \stackrel{p}{=} [\ \dot{0} \dot{+} [$$
$$\dot{\mathsf{t}}'\ ]\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\mathsf{t}\colon [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{M3.2(f)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall}\mathsf{t}\colon [\ \dot{\mathsf{t}} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{\mathsf{t}}\ ]\ ]\ ]\ ]$$

[M3.2(f) $\stackrel{\mathrm{tex}}{\rightarrow}$ "
M3.2(f)"]

[M3.2(f) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "mendelson three two f"]

$\dot{*}$

[$\dot{\mathsf{x}}$ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
\dot{#1.
}"]

[$\dot{\mathsf{x}}$ $\stackrel{\mathrm{pyk}}{\rightarrow}$ "* peano var"]

$*'$

[$\mathsf{x}' \stackrel{\text{tex}}{\rightarrow}$ "#1.'"]

[$\mathsf{x}' \stackrel{\text{pyk}}{\rightarrow}$ "* peano succ"]

$* \mathop{\dot{\cdot}} *$

[$\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{\cdot}} #2."]

[$\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano times *"]

$* \mathop{\dot{+}} *$

[$\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{+}} #2."]

[$\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano plus *"]

$* \stackrel{p}{=} *$

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano is *"]

$*^{\mathcal P}$

[$\mathsf{x}^{\mathcal P} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil$]

[$\mathsf{x}^{\mathcal P} \stackrel{\text{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

[$\mathsf{x}^{\mathcal P} \stackrel{\text{pyk}}{\rightarrow}$ "* is peano var"]

$\dot{\neg} *$

[$\dot{\neg} \mathsf{x} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\neg}\, #1."]

[$\dot{\neg} \mathsf{x} \stackrel{\text{pyk}}{\rightarrow}$ "peano not *"]

$* \dot{\wedge} *$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \doteqdot \dot{\neg}(\mathsf{x} \mathrel{\dot{\Rightarrow}} \dot{\neg}\mathsf{y})]\rceil)]$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\wedge}} #2."]

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano and *"]

$* \dot{\vee} *$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \doteqdot [\dot{\neg}\mathsf{x}] \mathrel{\dot{\Rightarrow}} \mathsf{y}]\rceil)]$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\vee}} #2."]

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano or *"]

$\dot{\forall} *\colon *$

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\to}$ "peano all * indeed *"]

$\dot{\exists} *\colon *$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{\exists}\mathsf{x}\colon \mathsf{y} \doteqdot \dot{\neg}\dot{\forall}\mathsf{x}\colon \dot{\neg}\mathsf{y}]\rceil)]$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\exists} #1.
\colon #2."]

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\to}$ "peano exist * indeed *"]

$* \dot{\Rightarrow} *$

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano imply *"]

$* \dot{\Leftrightarrow} *$

$[x \dot{\Leftrightarrow} y \stackrel{\text{macro}}{\to} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil [x \dot{\Leftrightarrow} y \ddot{=} (x \dot{\Rightarrow} y) \dot{\wedge} (y \dot{\Rightarrow} x)] \rceil)]$

$[x \dot{\Leftrightarrow} y \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

$[x \dot{\Leftrightarrow} y \stackrel{\text{pyk}}{\to}$ "* peano iff *"]

$* \unrhd *$

$[x \unrhd y \stackrel{\text{macro}}{\to} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil [x \unrhd y \ddot{=} [\, \mathrm{MP}' \rhd x \,] \rhd y] \rceil)]$

$[x \unrhd y \stackrel{\text{tex}}{\to}$ "#1.
\unrhd #2."]

$[x \unrhd y \stackrel{\text{pyk}}{\to}$ "* macro modus ponens *"]

$* \unrhd_{\mathrm{h}} *$

$[x \unrhd_{\mathrm{h}} y \stackrel{\text{macro}}{\to} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil [x \unrhd_{\mathrm{h}} y \ddot{=} [\, \mathrm{MP}'_{\mathrm{h}} \rhd x \,] \rhd y] \rceil)]$

$[x \unrhd_{\mathrm{h}} y \stackrel{\text{tex}}{\to}$ "#1.
\unrhd_h #2."]

$[x \unrhd_{\mathrm{h}} y \stackrel{\text{pyk}}{\to}$ "* hypothetical modus ponens *"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-29.UTC:12:32:14.418881 = MJD-53550.TAI:12:32:46.418881 = LGT-4626765166418881e-6