Logiweb codex of peano commutativity

Up Help

S′

$[S' \stackrel{\text{stmt}}{\rightarrow} \mathsf{x}]$

A1′

$[A1' \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[A1' \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,] \,]$

A2′

$[A2' \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[A2' \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\, [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \dot{\Rightarrow} [\, [\, \underline{a} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} [\, \underline{a} \dot{\Rightarrow} \underline{c} \,] \,] \,] \,]$

A3′

$[A3' \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[A3' \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, [\, \dot{\neg} \underline{b} \,] \dot{\Rightarrow} \dot{\neg} \underline{a} \,] \dot{\Rightarrow} [\, [\, [\, \dot{\neg} \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \dot{\Rightarrow} \underline{b} \,] \,] \,]$

A4′

$[A4' \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[A4' \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\, [\, \dot{\forall} \underline{x}\colon \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \,] \,]$

A5′

$[\mathrm{A5'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A5'} \overset{\text{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \dot{\mathrm{nonfree}}(\lceil \underline{x} \rceil, \lceil \underline{a} \rceil)\ ]\ \Vdash\ [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]\ ]$

S1′

$[\mathrm{S1'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S1'} \overset{\text{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \overset{\mathrm{p}}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \overset{\mathrm{p}}{=} \underline{c}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \overset{\mathrm{p}}{=} \underline{c}\ ]\ ]\ ]\ ]$

S2′

$[\mathrm{S2'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S2'} \overset{\text{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \overset{\mathrm{p}}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a}' \overset{\mathrm{p}}{=}\ [\ \underline{b}'\ ]\ ]\ ]\ ]$

S3′

$[\mathrm{S3'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S3'} \overset{\text{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon \neg\ [\ \dot{0} \overset{\mathrm{p}}{=}\ [\ \underline{a}'\ ]\ ]\ ]$

S4′

$[\mathrm{S4'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S4'} \overset{\text{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' \overset{\mathrm{p}}{=}\ [\ \underline{b}'\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \overset{\mathrm{p}}{=} \underline{b}\ ]\ ]\ ]$

S5′

$[\mathrm{S5'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S5'} \overset{\text{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{+} \dot{0}\ ]\ \overset{\mathrm{p}}{=} \underline{a}\ ]\ ]$

S6′

$[\mathrm{S6'} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{+} [\ \underline{b}'\ ]\ ]\ \stackrel{p}{=}\ [\ [\ \underline{a} \dot{+} \underline{b}\ ]\ '\ ]\ ]$]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{\cdot} \dot{0}\ ]\ \stackrel{p}{=} \dot{0}\ ]$]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\cdot} [\ \underline{b}'\ ]\ ]\ \stackrel{p}{=}\ [\ [\ \underline{a} \dot{\cdot} \underline{b}\ ]\ \dot{+} \underline{a}\ ]\ ]$]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]$]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \vdash\ [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

peano commutativity

[peano commutativity $\stackrel{\text{prio}}{\rightarrow}$
Preassociative
[peano commutativity], [base], [bracket $*$ end bracket],
[big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z],

$[[* \bowtie *]]$, $[[* \stackrel{*}{\rightarrow} *]]$, [pyk], [tex], [name], [prio], [*], [T], [if(*, *, *)], $[[* \stackrel{*}{\Rightarrow} *]]$, [val], [claim], $[\perp]$, [f(*)], $[(*)^{\mathrm{I}}]$, [F], $[\underline{0}]$, $[\underline{1}]$, $[\underline{2}]$, $[\underline{3}]$, $[\underline{4}]$, $[\underline{5}]$, $[\underline{6}]$, $[\underline{7}]$, $[\underline{8}]$, $[\underline{9}]$, [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\mathrm{M}}]$, [If(*, *, *)], $[\mathrm{array}\{*\} * \mathrm{end\ array}]$, [l], [c], [r], [empty], $[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$, $[\mathcal{U}^{\mathrm{M}}(*)]$, $[\mathbf{apply}(*, *)]$, $[\mathbf{apply}_1(*, *)]$, [identifier(*)], $[\mathrm{identifier}_1(*, *)]$, [array-plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)], [bit(*, *)], $[\mathrm{bit}_1(*, *)]$, [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], $[\mathcal{E}(*, *, *)]$, $[\mathcal{E}_2(*, *, *, *, *)]$, $[\mathcal{E}_3(*, *, *, *)]$, $[\mathcal{E}_4(*, *, *, *)]$, $[\mathbf{lookup}(*, *, *)]$, $[\mathbf{abstract}(*, *, *, *)]$, $[\lceil * \rceil]$, $[\mathcal{M}(*, *, *)]$, $[\mathcal{M}_2(*, *, *, *)]$, $[\mathcal{M}^*(*, *, *)]$, [macro], $[\mathrm{s}_0]$, $[\mathbf{zip}(*, *)]$, $[\mathbf{assoc}_1(*, *, *)]$, $[(*)^{\mathbf{p}}]$, [self], $[[* \mathrel{\ddot{=}} *]]$, $[[* \doteq *]]$, $[[* \mathrel{\acute{=}} *]]$, $[[* \stackrel{\mathrm{pyk}}{=} *]]$, $[[* \stackrel{\mathrm{tex}}{=} *]]$, $[[* \stackrel{\mathrm{name}}{=} *]]$, [**Priority table**[*]], $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$, $[\tilde{\mathcal{M}}_4(*, *, *, *)]$, $[\tilde{\mathcal{M}}(*, *, *)]$, $[\tilde{\mathcal{Q}}(*, *, *)]$, $[\tilde{\mathcal{Q}}_2(*, *, *)]$, $[\tilde{\mathcal{Q}}_3(*, *, *, *)]$, $[\tilde{\mathcal{Q}}^*(*, *, *)]$, [(*)], $[\mathbf{aspect}(*, *)]$, $[\mathbf{aspect}(*, *, *)]$, $[\langle * \rangle]$, $[\mathbf{tuple}_1(*)]$, $[\mathbf{tuple}_2(*)]$, $[\mathrm{let}_2(*, *)]$, $[\mathrm{let}_1(*, *)]$, $[[* \stackrel{\mathrm{claim}}{=} *]]$, [checker], $[\mathbf{check}(*, *)]$, $[\mathbf{check}_2(*, *, *)]$, $[\mathbf{check}_3(*, *, *)]$, $[\mathbf{check}^*(*, *)]$, $[\mathbf{check}^*_2(*, *, *)]$, $[[*]^{\cdot}]$, $[[*]^{-}]$, $[[*]^{\circ}]$, [msg], $[[* \stackrel{\mathrm{msg}}{=} *]]$, [<stmt>], [stmt], $[[* \stackrel{\mathrm{stmt}}{=} *]]$, [HeadNil′], [HeadPair′], [Transitivity′], $[\perp\!\!\!\perp]$, [Contra′], $[\mathrm{T}'_{\mathrm{E}}]$, $[\mathrm{L}_1]$, $[\underline{*}]$, $[\mathcal{A}]$, $[\mathcal{B}]$, $[\mathcal{C}]$, $[\mathcal{D}]$, $[\mathcal{E}]$, $[\mathcal{F}]$, $[\mathcal{G}]$, $[\mathcal{H}]$, $[\mathcal{I}]$, $[\mathcal{J}]$, $[\mathcal{K}]$, $[\mathcal{L}]$, $[\mathcal{M}]$, $[\mathcal{N}]$, $[\mathcal{O}]$, $[\mathcal{P}]$, $[\mathcal{Q}]$, $[\mathcal{R}]$, $[\mathcal{S}]$, $[\mathcal{T}]$, $[\mathcal{U}]$, $[\mathcal{V}]$, $[\mathcal{W}]$, $[\mathcal{X}]$, $[\mathcal{Y}]$, $[\mathcal{Z}]$, $[\langle * \mid * := * \rangle]$, $[\langle^* * \mid * := * \rangle]$, $[\emptyset]$, [Remainder], $[(*)^{\mathbf{v}}]$, [intro(*, *, *, *)], [intro(*, *, *)], [error(*, *)], $[\mathrm{error}_2(*, *)]$, [proof(*, *, *)], $[\mathrm{proof}_2(*, *)]$, $[\mathcal{S}(*, *)]$, $[\mathcal{S}^{\mathrm{I}}(*, *)]$, $[\mathcal{S}^{\rhd}(*, *)]$, $[\mathcal{S}^{\rhd}_1(*, *, *)]$, $[\mathcal{S}^{\mathrm{E}}(*, *)]$, $[\mathcal{S}^{\mathrm{E}}_1(*, *, *)]$, $[\mathcal{S}^{+}(*, *)]$, $[\mathcal{S}^{+}_1(*, *, *)]$, $[\mathcal{S}^{-}(*, *)]$, $[\mathcal{S}^{-}_1(*, *, *)]$, $[\mathcal{S}^{*}(*, *)]$, $[\mathcal{S}^{*}_1(*, *, *)]$, $[\mathcal{S}^{*}_2(*, *, *, *)]$, $[\mathcal{S}^{@}(*, *)]$, $[\mathcal{S}^{@}_1(*, *, *)]$, $[\mathcal{S}^{\vdash}(*, *)]$, $[\mathcal{S}^{\vdash}_1(*, *, *, *)]$, $[\mathcal{S}^{\Vdash}(*, *)]$, $[\mathcal{S}^{\Vdash}_1(*, *, *, *)]$, $[\mathcal{S}^{\mathrm{i.e.}}(*, *)]$, $[\mathcal{S}^{\mathrm{i.e.}}_1(*, *, *, *)]$, $[\mathcal{S}^{\mathrm{i.e.}}_2(*, *, *, *, *)]$, $[\mathcal{S}^{\forall}(*, *)]$, $[\mathcal{S}^{\forall}_1(*, *, *, *)]$, $[\mathcal{S}^{;}(*, *)]$, $[\mathcal{S}^{;}_1(*, *, *)]$, $[\mathcal{S}^{;}_2(*, *, *, *)]$, $[\mathcal{T}(*)]$, [claims(*, *, *)], $[\mathrm{claims}_2(*, *, *)]$, [<proof>], [proof], [[**Lemma** *: *]], [[**Proof of** *: *]], [[* **lemma** *: *]], [[* **antilemma** *: *]], [[* **rule** *: *]], [[* **antirule** *: *]], [verifier], $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*, *)]$, $[\mathcal{V}_3(*, *, *, *)]$, $[\mathcal{V}_4(*, *)]$, $[\mathcal{V}_5(*, *, *, *)]$, $[\mathcal{V}_6(*, *, *, *)]$, $[\mathcal{V}_7(*, *, *, *)]$, [Cut(*, *)], $[\mathrm{Head}_{\oplus}(*)]$, $[\mathrm{Tail}_{\oplus}(*)]$, $[\mathrm{rule}_1(*, *)]$, [rule(*, *)], [Rule tactic], [Plus(*, *)], [[**Theory** *]], $[\mathrm{theory}_2(*, *)]$, $[\mathrm{theory}_3(*, *)]$, $[\mathrm{theory}_4(*, *, *)]$, [HeadNil″], [HeadPair″], [Transitivity″], [Contra″], [HeadNil], [HeadPair], [Transitivity], [Contra], $[\mathrm{T}_{\mathrm{E}}]$, [ragged right], [ragged right expansion], [parm(*, *, *)], $[\mathrm{parm}^*(*, *, *)]$, [inst(*, *)], $[\mathrm{inst}^*(*, *)]$, [occur(*, *, *)], $[\mathrm{occur}^*(*, *, *)]$, [unify(* = *, *)], $[\mathrm{unify}^*(* = *, *)]$, $[\mathrm{unify}_2(* = *, *)]$, $[\mathrm{L}_{\mathrm{a}}]$, $[\mathrm{L}_{\mathrm{b}}]$, $[\mathrm{L}_{\mathrm{c}}]$, $[\mathrm{L}_{\mathrm{d}}]$, $[\mathrm{L}_{\mathrm{e}}]$, $[\mathrm{L}_{\mathrm{f}}]$, $[\mathrm{L}_{\mathrm{g}}]$, $[\mathrm{L}_{\mathrm{h}}]$, $[\mathrm{L}_{\mathrm{i}}]$, $[\mathrm{L}_{\mathrm{j}}]$, $[\mathrm{L}_{\mathrm{k}}]$, $[\mathrm{L}_{\mathrm{l}}]$, $[\mathrm{L}_{\mathrm{m}}]$, $[\mathrm{L}_{\mathrm{n}}]$, $[\mathrm{L}_{\mathrm{o}}]$, $[\mathrm{L}_{\mathrm{p}}]$, $[\mathrm{L}_{\mathrm{q}}]$, $[\mathrm{L}_{\mathrm{r}}]$, $[\mathrm{L}_{\mathrm{s}}]$, $[\mathrm{L}_{\mathrm{t}}]$, $[\mathrm{L}_{\mathrm{u}}]$, $[\mathrm{L}_{\mathrm{v}}]$, $[\mathrm{L}_{\mathrm{w}}]$, $[\mathrm{L}_{\mathrm{x}}]$, $[\mathrm{L}_{\mathrm{y}}]$, $[\mathrm{L}_{\mathrm{z}}]$, $[\mathrm{L}_{\mathrm{A}}]$, $[\mathrm{L}_{\mathrm{B}}]$, $[\mathrm{L}_{\mathrm{C}}]$, $[\mathrm{L}_{\mathrm{D}}]$, $[\mathrm{L}_{\mathrm{E}}]$, $[\mathrm{L}_{\mathrm{F}}]$, $[\mathrm{L}_{\mathrm{G}}]$, $[\mathrm{L}_{\mathrm{H}}]$, $[\mathrm{L}_{\mathrm{I}}]$, $[\mathrm{L}_{\mathrm{J}}]$, $[\mathrm{L}_{\mathrm{K}}]$, $[\mathrm{L}_{\mathrm{L}}]$, $[\mathrm{L}_{\mathrm{M}}]$, $[\mathrm{L}_{\mathrm{N}}]$, $[\mathrm{L}_{\mathrm{O}}]$, $[\mathrm{L}_{\mathrm{P}}]$, $[\mathrm{L}_{\mathrm{Q}}]$, $[\mathrm{L}_{\mathrm{R}}]$, $[\mathrm{L}_{\mathrm{S}}]$, $[\mathrm{L}_{\mathrm{T}}]$, $[\mathrm{L}_{\mathrm{U}}]$, $[\mathrm{L}_{\mathrm{V}}]$, $[\mathrm{L}_{\mathrm{W}}]$, $[\mathrm{L}_{\mathrm{X}}]$, $[\mathrm{L}_{\mathrm{Y}}]$, $[\mathrm{L}_{\mathrm{Z}}]$, $[\mathrm{L}_{?}]$, [Reflexivity], $[\mathrm{Reflexivity}_1]$, [Commutativity], $[\mathrm{Commutativity}_1]$, [<tactic>], [tactic], $[[* \stackrel{\mathrm{tactic}}{=} *]]$, $[\mathcal{P}(*, *, *)]$, $[\mathcal{P}^*(*, *, *)]$, $[\mathrm{p}_0]$, $[\mathrm{conclude}_1(*, *)]$, $[\mathrm{conclude}_2(*, *, *)]$, $[\mathrm{conclude}_3(*, *, *, *)]$, [conclude$_4$(∗, ∗)], [peano], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [nonfree(∗, ∗)], [$\dot{\text{nonfree}}^*$(∗, ∗)], [$\dot{\text{free}}$⟨∗|∗ := ∗⟩], [$\dot{\text{free}}^*$⟨∗|∗ := ∗⟩], [∗≡⟨∗|∗ := ∗⟩], [∗≡⟨*∗|∗ := ∗⟩], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S′], [A1′], [A2′], [A3′], [A4′], [A5′], [S1′], [S2′], [S3′], [S4′], [S5′], [S6′], [S7′], [S8′], [S9′], [MP′], [Gen′], [L3.2(a)′], [M1.7], [MP'_{h}], [Hypothesize], [Gen'_{h}], [M3.2(a)], [M3.2(a)$_{\text{h}}$], [M3.2(b)$_{\text{h}}$], [M3.1(S1′)$_{\text{h}}$], [M3.2(c)$_{\text{h}}$], [M3.1(S2′)$_{\text{h}}$], [M3.1(S5′)$_{\text{h}}$], [M3.1(S6′)$_{\text{h}}$], [M3.2(f)], [M Lemma 1.8], [M Proposition 3.2(a)], [M Proposition 3.2(b)], [M Proposition 3.2(c)], [M Proposition 3.2(d)], [M Proposition 3.2(f) (i)], [M Proposition 3.2(f) (ii)], [M Proposition 3.2(f)], [M Proposition 3.2(g) (i)], [M Proposition 3.2(g) (ii)], [M Proposition 3.2(g)], [M Proposition 3.2(h)(i)], [M Proposition 3.2(h)$_{\text{(g)(ii)}}$], [M Proposition 3.2(h)(ii)], [M Proposition 3.2(h)], [M Tautology A], [M Tautology B];

Preassociative

[∗_{∗}], [∗′], [∗[∗]], [∗[∗→∗]], [∗[∗⇒∗]], [$\dot{*}$];

Preassociative

[“ ∗ ”], [], [(∗)$^{\mathbf{t}}$], [string(∗) + ∗], [string(∗) ++ ∗], [∗], [∗], [!∗], [”∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗], [0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗], [@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗], [O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [ˆ∗], [_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗], [p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [˜∗], [**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end], [newline ∗], [macro newline ∗];

Preassociative

[∗0], [∗1], [0b], [∗-color(∗)], [∗-color*(∗)];

Preassociative

[∗ ’ ∗], [∗ ‘ ∗];

Preassociative

[$*^{\text{H}}$], [$*^{\text{T}}$], [$*^{\text{U}}$], [$*^{\text{h}}$], [$*^{\text{t}}$], [$*^{\text{s}}$], [$*^{\text{c}}$], [$*^{\text{d}}$], [$*^{\text{a}}$], [$*^{\text{C}}$], [$*^{\text{M}}$], [$*^{\text{B}}$], [$*^{\text{r}}$], [$*^{\text{i}}$], [$*^{\text{d}}$], [$*^{\text{R}}$], [$*^{0}$], [$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\text{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [∗′];

Preassociative

[$* \cdot *$], [$* \cdot_0 *$], [$* \mathrel{\dot{:}} *$];

Preassociative

[$* + *$], [$* +_0 *$], [$* +_1 *$], [$* - *$], [$* -_0 *$], [$* -_1 *$], [$* \dot{+} *$];

Preassociative

[$* \cup \{*\}$], [$* \cup *$], [$*\backslash\{*\}$];

Postassociative

[$* \therefore *$], [$* \underline{\therefore} *$], [$* \underline{::} *$], [$* \underline{+2*} *$], [$* :: *$], [$* +2* *$];

Postassociative

[∗, ∗];

Preassociative

[$* \stackrel{\text{B}}{\approx} *$], [$* \stackrel{\text{D}}{\approx} *$], [$* \stackrel{\text{C}}{\approx} *$], [$* \stackrel{\text{P}}{\approx} *$], [$* \approx *$], [$* = *$], [$* \stackrel{+}{\to} *$], [$* \stackrel{\text{t}}{=} *$], [$* \stackrel{\text{t}^*}{=} *$], [$* \stackrel{\text{r}}{=} *$],

$[* \in_t *], [* \subseteq_T *], [* \stackrel{T}{=} *], [* \stackrel{s}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *],$
$[* \text{ free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* \stackrel{p}{=} *], [*^{\mathcal{P}}];$
Preassociative
$[\neg *], [\dot{\neg} *];$
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$
Preassociative
$[\dot{\forall} *\colon *], [\dot{\exists} *\colon *];$
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *];$
Postassociative
$[* : *], [*!*];$
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right];$
Preassociative
$[\lambda * .*], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *];$
Preassociative
$[*^{\mathrm{I}}], [*^{\rhd}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}];$
Preassociative
$[* @ *], [* \rhd *], [* \unrhd *], [* \gg *];$
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
$[\forall *\colon *];$
Postassociative
$[* \oplus *];$
Postassociative
$[*; *];$
Preassociative
$[* \text{ proves } *];$
Preassociative
$[* \textbf{ proof of } * : *], [\text{Line} * : * \gg *; *], [\text{Last line} * \gg * \square],$
$[\text{Line} * : \text{Premise} \gg *; *], [\text{Line} * : \text{Side-condition} \gg *; *], [\text{Arbitrary} \gg *; *],$
$[\text{Local} \gg * = *; *];$
Postassociative
$[* \text{ then } *], [*[*]*];$
Preassociative
$[*\&*];$
Preassociative
$[*\backslash\backslash *];]$

$[\text{peano commutativity} \stackrel{\text{pyk}}{\rightarrow} \text{“peano commutativity”}]$

M Lemma 1.8

$[\text{M Lemma 1.8} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{b}\colon [[\text{A2}' \gg [[\underline{b} \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]]]] ; [[\text{A1}' \gg [\underline{b} \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} \underline{b}]]] ; [[[[\text{MP}' \rhd [[\underline{b} \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]]]] \rhd [\underline{b} \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} \underline{b}]]] \gg [[\underline{b} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]]] ; [[\text{A1}' \gg [\underline{b} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]]] ; [[[\text{MP}' \rhd [[\underline{b} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]]] \rhd [\underline{b} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{b}]]] \gg [\underline{b} \dot{\Rightarrow} \underline{b}]]]]]] \rceil, p_0, c)]$

$[\text{M Lemma 1.8} \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{b}\colon [\underline{b} \dot{\Rightarrow} \underline{b}]]$

[M Lemma 1.8 $\stackrel{\text{tex}}{\rightarrow}$ "M\ Lemma\ 1.8"]

[M Lemma 1.8 $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson lemma one eight"]

M Proposition 3.2(a)

$[\text{M Proposition 3.2(a)} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon [[\text{S5}' \gg [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]] ; [[\text{S1}' \gg [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{t}]]]] ; [[[[\text{MP}' \rhd [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{t}]]]] \rhd [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]] \gg [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{t}]]] ; [[[\text{MP}' \rhd [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{t}]]] \rhd [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]] \gg [\underline{t} \stackrel{p}{=} \underline{t}]]]]] \rceil, p_0, c)]$

$[\text{M Proposition 3.2(a)} \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{t}\colon [\underline{t} \stackrel{p}{=} \underline{t}]]$

[M Proposition 3.2(a) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(a)"]

[M Proposition 3.2(a) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two a"]

M Proposition 3.2(b)

$[\text{M Proposition 3.2(b)} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon \forall \underline{r}\colon [[\text{S1}' \gg [[\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [[\underline{t} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]]]] ; [[[\text{M Tautology A} \rhd [[\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [[\underline{t} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]]]] \gg [[\underline{t} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [[\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]]]] ; [[\text{M Proposition 3.2(a)} \gg [\underline{t} \stackrel{p}{=} \underline{t}]] ; [[[\text{MP}' \rhd [[\underline{t} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [[\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]]]] \rhd [\underline{t} \stackrel{p}{=} \underline{t}]] \gg [[\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]]]]]] \rceil, p_0, c)]$

$[\text{M Proposition 3.2(b)} \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{t}\colon \forall \underline{r}\colon [[\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]]]$

[M Proposition 3.2(b) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(b)"]

[M Proposition 3.2(b) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two b"]

M Proposition 3.2(c)

[M Proposition 3.2(c) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\ \mathrm{S1}' \gg [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ;\ [\ [\ \text{M Proposition 3.2(b)} \gg [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{M Tautology B} \rhd [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{t}]\ ]\ ] \rhd [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ \gg [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[M Proposition 3.2(c) $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]$]

[M Proposition 3.2(c) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(c)"]

[M Proposition 3.2(c) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two c"]

M Proposition 3.2(d)

[M Proposition 3.2(d) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{r}\colon \forall \underline{t}\colon \forall \underline{s}\colon [\ [\ \text{M Proposition 3.2(c)} \gg [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{t} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{M Tautology A} \rhd [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{t} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ] \gg [\ [\underline{t} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ;\ [\ [\ \text{M Proposition 3.2(b)} \gg [\ [\underline{s} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ;\ [\ [\ [\ [\ \text{M Tautology B} \rhd [\ [\underline{s} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ] \rhd [\ [\underline{t} \stackrel{p}{=} \underline{s}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ \gg [\ [\underline{s} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ;\ [\ [\ \text{M Tautology A} \rhd [\ [\underline{s} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ] \gg [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{s} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[M Proposition 3.2(d) $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{r}\colon \forall \underline{t}\colon \forall \underline{s}\colon [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\ [\underline{s} \stackrel{p}{=} \underline{t}] \dot{\Rightarrow} [\underline{r} \stackrel{p}{=} \underline{s}]\ ]\ ]$]

[M Proposition 3.2(d) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(d)"]

[M Proposition 3.2(d) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two d"]

M Proposition 3.2(f) (i)

[M Proposition 3.2(f) (i) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ \mathrm{S5}' \gg [\ [\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}\ ]\ ]\ ;\ [\ [\ \text{M Proposition 3.2(b)} \gg [\ [\ [\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}\ ] \dot{\Rightarrow} [\ \dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}\ ] \dot{\Rightarrow} [\ \dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ ]\ ]\ ] \rhd [\ [\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}\ ]\ ] \gg [\ \dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[M Proposition 3.2(f) (i) $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash [\ \dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]\ ]$]]

[M Proposition 3.2(f) (i) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(f)\ (i)"]

[M Proposition 3.2(f) (i) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two f i"]

M Proposition 3.2(f) (ii)

[M Proposition 3.2(f) (ii) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ S6' \gg [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ]\ ]\ ;\ [\ [\ S2' \gg [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{M Proposition 3.2(d)} \gg [\ [\ \dot{x}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \text{M Tautology B} \rhd [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{x}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{M Tautology A} \rhd [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \text{MP}' \rhd [\ [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ '\ ]\ ]\ ]\ \gg [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{Gen}' \rhd [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rceil, p_0, c)$]

[M Proposition 3.2(f) (ii) $\stackrel{\text{stmt}}{\rightarrow} S' \vdash \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]$]

[M Proposition 3.2(f) (ii) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(f)\ (ii)"]

[M Proposition 3.2(f) (ii) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two f ii"]

M Proposition 3.2(f)

[M Proposition 3.2(f) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ S9' \gg [\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{M Proposition 3.2(f) (i)} \gg [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \text{MP}' \rhd [\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ]\ ]\ \gg [\ [\ \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{M Proposition 3.2(f) (ii)} \gg \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{MP}' \rhd [\ [\ \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{x}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}'\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[M Proposition 3.2(f) $\stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall}\dot{x}\colon [\, \dot{x} \stackrel{p}{=} [\, \dot{0} \dot{+} [\, \dot{x} \,] \,] \,]$]

[M Proposition 3.2(f) $\stackrel{\mathrm{tex}}{\rightarrow}$ “M\ Proposition\ 3.2(f)”]

[M Proposition 3.2(f) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “mendelson proposition three two f”]

M Proposition 3.2(g) (i)

[M Proposition 3.2(g) (i) $\stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [\, [\, \mathrm{S5}' \gg [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x}' \,] \,] \,] ; [\, [\, \mathrm{S5}' \gg [\, [\, \dot{x} \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \,] ; [\, [\, \mathrm{S2}' \gg [\, [\, [\, \dot{x} \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \dot{\Rightarrow} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \,] \,] \,] ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{x} \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \dot{\Rightarrow} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \,] \,] \rhd [\, [\, \dot{x} \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \,] \gg [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \,] ; [\, [\,$ M Proposition 3.2(d) $\gg [\, [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x}' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] \,] \,] \,] ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x}' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] \,] \,] \,] \rhd [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{x}' \,] \,] \,] \,] \gg [\, [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] \,] ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] \,] \,] \rhd [\, [\, \dot{x} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{x}' \,] \,] \,] \gg [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] ; [\, [\, \mathrm{Gen}' \rhd [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] \gg \dot{\forall}\dot{x}\colon [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,] \,] \,] \,] \,] \,] \,] \,] \,] \rceil, p_0, c)$]

[M Proposition 3.2(g) (i) $\stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall}\dot{x}\colon [\, [\, \dot{x}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} \dot{0} \,]' \,] \,]$]

[M Proposition 3.2(g) (i) $\stackrel{\mathrm{tex}}{\rightarrow}$ “M\ Proposition\ 3.2(g)\ (i)”]

[M Proposition 3.2(g) (i) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “mendelson proposition three two g i”]

M Proposition 3.2(g) (ii)

[M Proposition 3.2(g) (ii) $\stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [\, [\, \mathrm{S6}' \gg [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \,] \,] \,] ; [\, [\, \mathrm{S2}' \gg [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \,] \,] ; [\, [\,$ M Proposition 3.2(c) $\gg [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \,] \,] \,] \,] ; [\, [\, [\,$ M Tautology A $\rhd [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \,] \,] \,] \,] \gg [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \,] \,] \,] ; [\, [\, [\, [\,$ M Tautology B $\rhd [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \,] \,] \rhd [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x}' \dot{+} [\, \dot{y} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{x}' \dot{+} [\, \dot{y}' \,] \,] \stackrel{p}{=} [\, [\, \dot{x} \dot{+} [\, \dot{y} \,] \,]'' \,] \,] \,] \,] \,] \,] \gg [\, [\, [$

$\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x}' \dotplus [\dot{y}]] '$
$] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]]]] ; [[[\text{M Tautology A} \rhd [$
$[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x}' \dotplus [\dot{y}]]$
$']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]]]]] \gg [[[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=}$
$[[\dot{x}' \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [$
$\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]]]]] ; [[[[\text{MP}' \rhd [[[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[$
$\dot{x}' \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']$
$] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]]]]] \rhd [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x}' \dotplus [\dot{y}]] ']]] \gg$
$[[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]]$
$'']]]] ; [[\text{S6}' \gg [[\dot{x} \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']]] ; [[\text{S2}' \gg [[$
$[\dot{x} \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]$
$]] ; [[\text{M Proposition 3.2(d)} \gg [[[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow}$
$[[[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']$
$] ']]]]]] ; [[[[\text{M Tautology B} \rhd [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']$
$] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]]]] \rhd [[[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[$
$\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow} [[[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']$
$] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']] ']]]]]] \gg [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow}$
$[[[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']$
$] ']]]]]] ; [[[\text{M Tautology A} \rhd [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']]$
$\dot{\Rightarrow} [[[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [$
$\dot{y}']] ']]]]]] \gg [[[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']] \dot{\Rightarrow} [[[\dot{x}' \dotplus [$
$\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']] ']]]]] ;$
$[[[[\text{MP}' \rhd [[[\dot{x} \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=}$
$[[\dot{x} \dotplus [\dot{y}]] '']]]]] \rhd [[\dot{x} \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']]] \gg [[\dot{x} \dotplus$
$[\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]] ; [[[[\text{MP}' \rhd [[[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus$
$[\dot{y}]] '']] \dot{\Rightarrow} [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=}$
$[[\dot{x} \dotplus [\dot{y}']] ']]]]]] \rhd [[\dot{x} \dotplus [\dot{y}']] ' \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] '']]] \gg [[$
$[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']] ']]$
$]] ; [[\text{Gen}' \rhd [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=}$
$[[\dot{x} \dotplus [\dot{y}']] ']]]] \gg \dot{\forall} \dot{y}\colon [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']] \dot{\Rightarrow} [[$
$\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']] ']]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$

[M Proposition 3.2(g) (ii) $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \dot{\forall} \dot{y}\colon [[[\dot{x}' \dotplus [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}]] ']]$
$\dot{\Rightarrow} [[\dot{x}' \dotplus [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dotplus [\dot{y}']] ']]]]$]

[M Proposition 3.2(g) (ii) $\stackrel{\text{tex}}{\rightarrow}$ "M\ Proposition\ 3.2(g)\ (ii)"]

[M Proposition 3.2(g) (ii) $\stackrel{\text{pyk}}{\rightarrow}$ "mendelson proposition three two g ii"]

M Proposition 3.2(g)

[M Proposition 3.2(g) $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ \text{S9}' \gg [\ [\ \dot{\forall}\dot{x}\colon [\ [\ \dot{x}' \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} \dot{0}\ ]\ '\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{y}\colon [\ [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x}' \dot{+} [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ \text{M Proposition 3.2(g) (i)} \gg \dot{\forall}\dot{x}\colon [\ [\ \dot{x}' \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} \dot{0}\ ]\ '\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{MP}' \rhd [\ [\ \dot{\forall}\dot{x}\colon [\ [\ \dot{x}' \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} \dot{0}\ ]\ '\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{y}\colon [\ [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x}' \dot{+} [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{x}\colon [\ [\ \dot{x}' \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} \dot{0}\ ]\ '\ ]\ ]\ ]\ \gg [\ [\ \dot{\forall}\dot{y}\colon [\ [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x}' \dot{+} [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ \text{M Proposition 3.2(g) (ii)} \gg \dot{\forall}\dot{y}\colon [\ [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x}' \dot{+} [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \text{MP}' \rhd [\ [\ \dot{\forall}\dot{y}\colon [\ [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x}' \dot{+} [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{y}\colon [\ [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x}' \dot{+} [\ \dot{y}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{y}\colon [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[M Proposition 3.2(g) $\stackrel{\text{stmt}}{\to}$ $S' \vdash \dot{\forall}\dot{y}\colon [\ [\ \dot{x}' \dot{+} [\ \dot{y}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{x} \dot{+} [\ \dot{y}\ ]\ ]\ '\ ]\ ]$]

[M Proposition 3.2(g) $\stackrel{\text{tex}}{\to}$ “M\ Proposition\ 3.2(g)”]

[M Proposition 3.2(g) $\stackrel{\text{pyk}}{\to}$ “mendelson proposition three two g”]

M Proposition 3.2(h)(i)

[M Proposition 3.2(h)(i) $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ \text{S5}' \gg [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ; [\ [\ \text{M Proposition 3.2(f)} \gg \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \text{A4}' @ [\ \dot{x}\ ]\ ]\ \gg [\ [\ \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{MP}' \rhd [\ [\ \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{x}\colon [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ \gg [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ; [\ [\ \text{M Proposition 3.2(c)} \gg [\ [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{MP}' \rhd [\ [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ \gg [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{MP}' \rhd [\ [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \dot{x} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ; [\ [\ \text{Gen}' \rhd [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[M Proposition 3.2(h)(i) $\stackrel{\text{stmt}}{\to}$ $S' \vdash \dot{\forall}\dot{x}\colon [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{x}\ ]\ ]\ ]$]

[M Proposition 3.2(h)(i) $\stackrel{\text{tex}}{\to}$ “M\ Proposition\ 3.2(h)(i)”]

[M Proposition 3.2(h)(i) $\stackrel{pyk}{\rightarrow}$ "mendelson proposition three two h i"]

M Proposition 3.2(h)$_{(g)(ii)}$

[M Proposition 3.2(h)$_{(g)(ii)}$ $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [[\dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \vdash [[[A4' @ [\dot{z}]] \gg [[\dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \Rightarrow [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]]]] ; [[[[MP' \rhd [[\dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \Rightarrow [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]]] \rhd \dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \gg [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]]] ; [[[Gen' \rhd [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]] \gg \dot{\forall}\dot{x}\colon [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]]] ; [[[A4' @ [\dot{y}]] \gg [[\dot{\forall}\dot{x}\colon [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]] \Rightarrow [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]]]] ; [[[MP' \rhd [[\dot{\forall}\dot{x}\colon [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]] \Rightarrow [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]]] \rhd \dot{\forall}\dot{x}\colon [[\dot{x}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{z}]]']]] \gg [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]]] ; [[[Gen' \rhd [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]] \gg \dot{\forall}\dot{z}\colon [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]]] ; [[[A4' @ [\dot{x}]] \gg [[\dot{\forall}\dot{z}\colon [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]] \Rightarrow [[\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]] ; [[[MP' \rhd [[\dot{\forall}\dot{z}\colon [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]] \Rightarrow [[\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]] \rhd \dot{\forall}\dot{z}\colon [[\dot{y}' \dot{+} [\dot{z}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{z}]]']]] \gg [[\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]]]]]]]]] \rceil, p_0, c)$]

[M Proposition 3.2(h)$_{(g)(ii)}$ $\stackrel{stmt}{\rightarrow}$ $S' \vdash [[\dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \vdash [[\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]$]

[M Proposition 3.2(h)$_{(g)(ii)}$ $\stackrel{tex}{\rightarrow}$ "M\ Proposition\ 3.2(h)_{(g)(ii)}"]

[M Proposition 3.2(h)$_{(g)(ii)}$ $\stackrel{pyk}{\rightarrow}$ "mendelson proposition three two h g ii"]

M Proposition 3.2(h)(ii)

[M Proposition 3.2(h)(ii) $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [[S6' \gg [[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]] ; [[\text{M Proposition 3.2(g)} \gg \dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]] ; [[[\text{M Proposition 3.2(h)}_{(g)(ii)} \rhd \dot{\forall}\dot{y}\colon [[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \gg [[\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]] ; [[S2' \gg [[[\dot{x} \dot{+} [\dot{y}]] \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]]] \Rightarrow [[\dot{x} \dot{+} [\dot{y}]]' \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]] ; [[\text{M Proposition 3.2(c)} \gg [[[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[[\dot{x} \dot{+} [\dot{y}]]' \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']] \Rightarrow [[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]]] ; [[[\text{M Tautology A} \rhd [[[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[[\dot{x} \dot{+} [\dot{y}]]' \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']] \Rightarrow [[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]']]]]]$

$\gg [[[\dot{x}\dotplus[\dot{y}]]' \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}$
$]]']] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]]] ; [[[[$
$\text{M Tautology B} \rhd [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}]]' \stackrel{p}{=} [[$
$\dot{y}\dotplus[\dot{x}]]']]]] \rhd [[[\dot{x}\dotplus[\dot{y}]]' \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[[\dot{x}\dotplus[$
$\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}]]']] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]]]]$
$\gg [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}]]']$
$] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]]] ; [[[\text{M Tautology A} \rhd [[$
$[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}]]']] \dot{\Rightarrow} [$
$[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]]]] \gg [[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}]$
$]']] \dot{\Rightarrow} [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}$
$]]']]]]] ; [[[[\text{MP}' \rhd [[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}]]']] \dot{\Rightarrow} [[$
$[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]]]$
$\rhd [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{x}\dotplus[\dot{y}]]']]] \gg [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]$
$] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]] ; [[\text{M Proposition 3.2(d)} \gg [$
$[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]'$
$]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]]]]] ; [[[[\text{M Tautology B} \rhd [[$
$[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']]]] \rhd$
$[[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]$
$']] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]]]]] \gg [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus$
$[\dot{x}]]] \dot{\Rightarrow} [[[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [$
$\dot{y}'\dotplus[\dot{x}]]]]]] ; [[[\text{M Tautology A} \rhd [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]$
$] \dot{\Rightarrow} [[[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]$
$]]]]] \gg [[[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=}$
$[\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]]]]] ; [[[[\text{MP}' \rhd [[$
$[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']] \dot{\Rightarrow} [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow}$
$[[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]]]]]] \rhd [[\dot{y}'\dotplus[\dot{x}]] \stackrel{p}{=} [[\dot{y}\dotplus[\dot{x}]]']$
$]] \gg [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]$
$]]] ; [[\text{Gen}' \rhd [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [$
$\dot{y}'\dotplus[\dot{x}]]]]]] \gg \dot{\forall}\dot{y}\colon [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']]$
$\stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$

$[\text{M Proposition 3.2(h)(ii)} \stackrel{\text{stmt}}{\to} S' \vdash \dot{\forall}\dot{y}\colon [[[\dot{x}\dotplus[\dot{y}]] \stackrel{p}{=} [\dot{y}\dotplus[\dot{x}]]] \dot{\Rightarrow} [[\dot{x}\dotplus[\dot{y}']] \stackrel{p}{=} [\dot{y}'\dotplus[\dot{x}]]]]]$

[M Proposition 3.2(h)(ii) $\stackrel{\text{tex}}{\to}$ "M\ Proposition\ 3.2(h)(ii)"]

[M Proposition 3.2(h)(ii) $\stackrel{\text{pyk}}{\to}$ "mendelson proposition three two h ii"]

M Proposition 3.2(h)

[M Proposition 3.2(h) $\overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [[S9' \gg [[\dot{\forall}\dot{x}\colon [[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [$
$\dot{0} \dot{+} [\dot{x}]]]] \dot{\Rightarrow} [[\dot{\forall}\dot{y}\colon [[[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]] \dot{\Rightarrow} [[\dot{x} \dot{+} [\dot{y}']$
$] \overset{p}{=} [\dot{y}' \dot{+} [\dot{x}]]]]] \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]]]]] ; [[$
M Proposition 3.2(h)(i) $\gg \dot{\forall}\dot{x}\colon [[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{0} \dot{+} [\dot{x}]]]] ; [[[[MP' \triangleright$
$[[\dot{\forall}\dot{x}\colon [[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{0} \dot{+} [\dot{x}]]]] \dot{\Rightarrow} [[\dot{\forall}\dot{y}\colon [[[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}$
$]]] \dot{\Rightarrow} [[\dot{x} \dot{+} [\dot{y}']] \overset{p}{=} [\dot{y}' \dot{+} [\dot{x}]]]]] \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+}$
$[\dot{x}]]]]]] \triangleright \dot{\forall}\dot{x}\colon [[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{0} \dot{+} [\dot{x}]]]] \gg [[\dot{\forall}\dot{y}\colon [[[\dot{x} \dot{+} [\dot{y}]$
$] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]] \dot{\Rightarrow} [[\dot{x} \dot{+} [\dot{y}']] \overset{p}{=} [\dot{y}' \dot{+} [\dot{x}]]]]] \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [$
$\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]]] ; [[$ M Proposition 3.2(h)(ii) $\gg \dot{\forall}\dot{y}\colon [[[\dot{x} \dot{+} [\dot{y}]$
$] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]] \dot{\Rightarrow} [[\dot{x} \dot{+} [\dot{y}']] \overset{p}{=} [\dot{y}' \dot{+} [\dot{x}]]]]] ; [[[[MP' \triangleright$
$[[\dot{\forall}\dot{y}\colon [[[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]] \dot{\Rightarrow} [[\dot{x} \dot{+} [\dot{y}']] \overset{p}{=} [\dot{y}' \dot{+} [\dot{x}]]$
$]]] \dot{\Rightarrow} \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]]] \triangleright \dot{\forall}\dot{y}\colon [[[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [$
$\dot{y} \dot{+} [\dot{x}]]] \dot{\Rightarrow} [[\dot{x} \dot{+} [\dot{y}']] \overset{p}{=} [\dot{y}' \dot{+} [\dot{x}]]]]] \gg \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]]$
$\overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]] ; [[Gen' \triangleright \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]]$
$\gg \dot{\forall}\dot{x}\colon \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]]]]]]]] \rceil, p_0, c)$]

[M Proposition 3.2(h) $\overset{\text{stmt}}{\rightarrow} S' \vdash \dot{\forall}\dot{x}\colon \dot{\forall}\dot{y}\colon [[\dot{x} \dot{+} [\dot{y}]] \overset{p}{=} [\dot{y} \dot{+} [\dot{x}]]]$]

[M Proposition 3.2(h) $\overset{\text{tex}}{\rightarrow}$ “M\ Proposition\ 3.2(h)”]

[M Proposition 3.2(h) $\overset{\text{pyk}}{\rightarrow}$ “mendelson proposition three two h”]

M Tautology A

[M Tautology A $\overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \vdash [[$
$A2' \gg [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] ; [[[[MP' \triangleright$
$[[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \triangleright [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]]]$
$\gg [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] ; [[A1' \gg [[[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow}$
$[\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] ; [[[[MP' \triangleright [[[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [$
$\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] \triangleright [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [$
$\underline{a} \dot{\Rightarrow} \underline{c}]]] \gg [\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] ; [[A2' \gg [[\underline{b} \dot{\Rightarrow} [[$
$\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]]$
$; [[[[MP' \triangleright [[\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]]$
$\dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] \triangleright [\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \gg [[$
$\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] ; [[A1' \gg [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]]] ; [$
$[[MP' \triangleright [[\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \triangleright [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]]$
$] \gg [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]]]]]]]] \rceil, p_0, c)$]

[M Tautology A $\overset{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \vdash [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]$
$]$]

[M Tautology A $\stackrel{\text{tex}}{\to}$ "M\ Tautology\ A"]

[M Tautology A $\stackrel{\text{pyk}}{\to}$ "mendelson tautology a"]

M Tautology B

[M Tautology B $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{d}\colon \forall \underline{e}\colon \forall \underline{f}\colon [\ [\underline{d} \dot{\Rightarrow} \underline{e}] \vdash [\ [\underline{e} \dot{\Rightarrow} \underline{f}] \vdash [\ [A1' \gg [\ [\underline{e} \dot{\Rightarrow} \underline{f}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]\ ]\ ]\ ]; [\ [\ [\ [MP' \rhd [\ [\underline{e} \dot{\Rightarrow} \underline{f}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]\ ]\ ]\ ] \rhd [\underline{e} \dot{\Rightarrow} \underline{f}]\ ] \gg [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]\ ]\ ]; [\ [A2' \gg [\ [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]\ ] \dot{\Rightarrow} [\ [\underline{d} \dot{\Rightarrow} \underline{e}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} \underline{f}]\ ]\ ]\ ]; [\ [\ [\ [MP' \rhd [\ [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]\ ] \dot{\Rightarrow} [\ [\underline{d} \dot{\Rightarrow} \underline{e}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} \underline{f}]\ ]\ ]\ ] \rhd [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]\ ]\ ] \gg [\ [\underline{d} \dot{\Rightarrow} \underline{e}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} \underline{f}]\ ]\ ]; [\ [\ [MP' \rhd [\ [\underline{d} \dot{\Rightarrow} \underline{e}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} \underline{f}]\ ]\ ] \rhd [\underline{d} \dot{\Rightarrow} \underline{e}]\ ] \gg [\underline{d} \dot{\Rightarrow} \underline{f}]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\rceil, p_0, c)$]

[M Tautology B $\stackrel{\text{stmt}}{\to}$ $S' \vdash \forall \underline{d}\colon \forall \underline{e}\colon \forall \underline{f}\colon [\ [\underline{d} \dot{\Rightarrow} \underline{e}] \vdash [\ [\underline{e} \dot{\Rightarrow} \underline{f}] \vdash [\underline{d} \dot{\Rightarrow} \underline{f}]\ ]\ ]$]

[M Tautology B $\stackrel{\text{tex}}{\to}$ "M\ Tautology\ B"]

[M Tautology B $\stackrel{\text{pyk}}{\to}$ "mendelson tautology b"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-07-03.UTC:14:46:55.537781 = MJD-53554.TAI:14:47:27.537781 = LGT-4627118847537781e-6