

Logiweb codex of peano commutativity

Up Help

S', A1', A2', A3', A4', A5', S1', S2', S3', S4', S5', S6', S7', S8', S9', MP', Gen',
peano commutativity, M Lemma 1.8, M Proposition 3.2(a),
M Proposition 3.2(b), M Proposition 3.2(c), M Proposition 3.2(d),
M Proposition 3.2(f) (i), M Proposition 3.2(f) (ii), M Proposition 3.2(f),
M Proposition 3.2(g) (i), M Proposition 3.2(g) (ii), M Proposition 3.2(g),
M Proposition 3.2(h)(i), M Proposition 3.2(h)_{(g)(ii)}, M Proposition 3.2(h)(ii),
M Proposition 3.2(h), M Tautology A, M Tautology B,

S'

[S' $\xrightarrow{\text{stmt}}$ x]

A1'

[A1' $\xrightarrow{\text{proof}}$ Rule tactic]

[A1' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{a}: \forall \underline{b}: [\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{a}]]$]

A2'

[A2' $\xrightarrow{\text{proof}}$ Rule tactic]

[A2' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [[\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{c}]] \Rightarrow [[\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]]$]

A3'

[A3' $\xrightarrow{\text{proof}}$ Rule tactic]

[A3' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{a}: \forall \underline{b}: [[[\underline{a} \dot{\neg} \underline{b}] \Rightarrow \dot{\neg} \underline{a}] \Rightarrow [[[\underline{a} \dot{\neg} \underline{b}] \Rightarrow \underline{a}] \Rightarrow \underline{b}]]$]

A4'

[A4' $\xrightarrow{\text{proof}}$ Rule tactic]

[A4' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: [[\underline{a}] \equiv \langle [\underline{b}] | [\underline{x}] := [\underline{c}] \rangle \Vdash [[\dot{\forall} \underline{x}: \underline{b}] \Rightarrow \underline{a}]]$]

A5'

[A5' $\xrightarrow{\text{proof}}$ Rule tactic]

[A5' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\text{nonfree}(\underline{x}, [\underline{a}])] \Vdash [[\forall \underline{x}: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \forall \underline{x}: \underline{b}]]]$]

S1'

[S1' $\xrightarrow{\text{proof}}$ Rule tactic]

[S1' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [[\underline{a} \stackrel{p}{=} \underline{b}] \Rightarrow [[\underline{a} \stackrel{p}{=} \underline{c}] \Rightarrow [\underline{b} \stackrel{p}{=} \underline{c}]]]$]

S2'

[S2' $\xrightarrow{\text{proof}}$ Rule tactic]

[S2' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \stackrel{p}{=} \underline{b}] \Rightarrow [\underline{a}' \stackrel{p}{=} [\underline{b}']]]$]

S3'

[S3' $\xrightarrow{\text{proof}}$ Rule tactic]

[S3' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \neg [\dot{0} \stackrel{p}{=} [\underline{a}']]]$

S4'

[S4' $\xrightarrow{\text{proof}}$ Rule tactic]

[S4' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a}' \stackrel{p}{=} [\underline{b}']] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{b}]]$]

S5'

[S5' $\xrightarrow{\text{proof}}$ Rule tactic]

[S5' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]]$

S6'

[S6' $\xrightarrow{\text{proof}}$ Rule tactic]

$$[S6' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \dot{+} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}]']]]$$

S7'

$$[S7' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S7' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: [[\underline{a} \dot{:} \dot{0}] \stackrel{p}{=} \dot{0}]]$$

S8'

$$[S8' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S8' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \dot{:} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{:} \underline{b}] \dot{+} \underline{a}]]]$$

S9'

$$[S9' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S9' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \Rightarrow [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \forall \underline{x}: \underline{a}]]]]]]$$

MP'

$$[MP' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[MP' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]]$$

Gen'

$$[Gen' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[Gen' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{x}: \forall \underline{a}: [\underline{a} \vdash \dot{\forall} \underline{x}: \underline{a}]]$$

peano commutativity

$$[\text{peano commutativity} \xrightarrow{\text{prio}}$$

Preassociative

[peano commutativity], [base], [bracket * end bracket],
[big bracket * end bracket], [math * end math], [**flush left** [*]], [x], [y], [z],

$[[* \bowtie *]], [[* \xrightarrow{*} *]], [\text{pyk}], [\text{tex}], [\text{name}], [\text{prio}], [*, [\text{T}], [\text{if}(*, *, *)], [[* \Rightarrow *]], [\text{val}],$
 $[\text{claim}], [\perp], [\text{f}(*)], [(*)^I], [\text{F}], [\text{Q}], [1], [2], [3], [4], [5], [6], [7], [8], [9], [\text{a}], [\text{b}], [\text{c}], [\text{d}], [\text{e}], [\text{f}], [\text{g}], [\text{h}], [\text{i}], [\text{j}], [\text{k}], [\text{l}], [\text{m}], [\text{n}], [\text{o}], [\text{p}], [\text{q}], [\text{r}],$
 $[\text{s}], [\text{t}], [\text{u}], [\text{v}], [\text{w}], [(*)^M], [\text{If}(*, *, *)], [\text{array}\{*\} * \text{end array}], [\text{l}], [\text{c}], [\text{r}], [\text{empty}],$
 $[(*) | := *], [\mathcal{M}(*)], [\mathcal{U}(*)], [\mathcal{U}(*)], [\mathcal{U}^M(*)], [\text{apply}(*, *)], [\text{apply}_1(*, *)],$
 $[\text{identifier}(*)], [\text{identifier}_1(*, *)], [\text{array-plus}(*, *)], [\text{array-remove}(*, *, *)], [\text{array-}$
 $\text{put}(*, *, *, *)], [\text{array-add}(*, *, *, *, *)], [\text{bit}(*, *)], [\text{bit}_1(*, *)], [\text{rack}], [\"vector\"],$
 $[\"bibliography\"], [\"dictionary\"], [\"body\"], [\"codex\"], [\"expansion\"], [\"code\"],$
 $[\"cache\"], [\"diagnose\"], [\"pyk\"], [\"tex\"], [\"texname\"], [\"value\"], [\"message\"],$
 $[\"macro\"], [\"definition\"], [\"unpack\"], [\"claim\"], [\"priority\"], [\"lambda\"],$
 $[\"apply\"], [\"true\"], [\"if\"], [\"quote\"], [\"proclaim\"], [\"define\"], [\"introduce\"],$
 $[\"hide\"], [\"pre\"], [\"post\"], [\mathcal{E}(*, *, *)], [\mathcal{E}_2(*, *, *, *, *)], [\mathcal{E}_3(*, *, *, *)],$
 $[\mathcal{E}_4(*, *, *, *)], [\text{lookup}(*, *, *)], [\text{abstract}(*, *, *, *)], [\"*\"], [\mathcal{M}(*, *, *)],$
 $[\mathcal{M}_2(*, *, *, *)], [\mathcal{M}^*(*, *, *)], [\text{macro}], [\text{s}_0], [\text{zip}(*, *)], [\text{assoc}_1(*, *, *)], [(*)^P],$
 $[\text{self}], [[* \doteq *]], [[* \dot{=} *]], [[* \leq *]], [[* \stackrel{\text{pyk}}{=} *]], [[* \stackrel{\text{tex}}{=} *]], [[* \stackrel{\text{name}}{=} *]],$
 $[\text{Priority table} *], [\mathcal{M}_1], [\mathcal{M}_2(*)], [\mathcal{M}_3(*)], [\mathcal{M}_4(*, *, *, *)], [\mathcal{M}(*, *, *)],$
 $[\hat{Q}(*, *, *)], [\hat{Q}_2(*, *, *)], [\hat{Q}_3(*, *, *, *)], [\hat{Q}^*(*, *, *)], [(*)], [\text{aspect}(*, *)],$
 $[\text{aspect}(*, *, *)], [\langle * \rangle], [\text{tuple}_1(*)], [\text{tuple}_2(*)], [\text{let}_2(*, *)], [\text{let}_1(*, *)],$
 $[[* \stackrel{\text{claim}}{=} *]], [\text{checker}], [\text{check}(*, *)], [\text{check}_2(*, *, *)], [\text{check}_3(*, *, *)],$
 $[\text{check}^*(*, *)], [\text{check}_2^*(*, *, *)], [\"*\"], [\"*\"]^-, [\"*\"]^\circ, [\text{msg}], [[* \stackrel{\text{msg}}{=} *]], [<\text{stmt}>],$
 $[\text{stmt}], [[* \stackrel{\text{stmt}}{=} *]], [\text{HeadNil}'], [\text{HeadPair}'], [\text{Transitivity}'], [\perp], [\text{Contra}'], [\text{T}'_E],$
 $[\text{L}_1], [\text{L}_*], [\mathcal{A}], [\mathcal{B}], [\mathcal{C}], [\mathcal{D}], [\mathcal{E}], [\mathcal{F}], [\mathcal{G}], [\mathcal{H}], [\mathcal{I}], [\mathcal{J}], [\mathcal{K}], [\mathcal{L}], [\mathcal{M}], [\mathcal{N}], [\mathcal{O}], [\mathcal{P}], [\mathcal{Q}],$
 $[\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [(*) | := *], [(*)^* | := *], [\emptyset], [\text{Remainder}],$
 $[(*)^\vee], [\text{intro}(*, *, *, *)], [\text{intro}(*, *, *)], [\text{error}(*, *)], [\text{error}_2(*, *)], [\text{proof}(*, *, *)],$
 $[\text{proof}_2(*, *)], [\mathcal{S}(*, *)], [\mathcal{S}^I(*, *)], [\mathcal{S}^\triangleright(*, *)], [\mathcal{S}_1^\triangleright(*, *, *)], [\mathcal{S}^E(*, *)], [\mathcal{S}_1^E(*, *, *)],$
 $[\mathcal{S}^+(*, *)], [\mathcal{S}_1^+(*, *, *)], [\mathcal{S}^-(*, *)], [\mathcal{S}_1^-(*, *, *)], [\mathcal{S}^*(*, *)], [\mathcal{S}_1^*(*, *, *)],$
 $[\mathcal{S}_2^*(*, *, *, *)], [\mathcal{S}^\oplus(*, *)], [\mathcal{S}_1^\oplus(*, *, *)], [\mathcal{S}^\lrcorner(*, *)], [\mathcal{S}_1^\lrcorner(*, *, *, *)], [\mathcal{S}^\#(*, *)],$
 $[\mathcal{S}_1^\#(*, *, *, *)], [\mathcal{S}^{\text{i.e.}}(*, *)], [\mathcal{S}_1^{\text{i.e.}}(*, *, *, *)], [\mathcal{S}_2^{\text{e.}}(*, *, *, *, *)], [\mathcal{S}^\vee(*, *)],$
 $[\mathcal{S}_1^\vee(*, *, *, *)], [\mathcal{S}^\cdot(*, *)], [\mathcal{S}_1^\cdot(*, *, *, *)], [\mathcal{S}_2^\cdot(*, *, *, *, *)], [\mathcal{T}(*)], [\text{claims}(*, *, *)],$
 $[\text{claims}_2(*, *, *)], [<\text{proof}>], [\text{proof}], [\text{Lemma} * : *], [\text{Proof of} * : *],$
 $[* \text{ lemma} * : *], [* \text{ antilemma} * : *], [* \text{ rule} * : *], [* \text{ antirule} * : *],$
 $[\text{verifier}], [\mathcal{V}_1(*)], [\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)],$
 $[\mathcal{V}_7(*, *, *, *)], [\text{Cut}(*, *)], [\text{Head}_\oplus(*)], [\text{Tail}_\oplus(*)], [\text{rule}_1(*, *)], [\text{rule}(*, *)],$
 $[\text{Rule tactic}], [\text{Plus}(*, *)], [\text{Theory} *], [\text{theory}_2(*, *)], [\text{theory}_3(*, *)],$
 $[\text{theory}_4(*, *, *)], [\text{HeadNil}''], [\text{HeadPair}''], [\text{Transitivity}''], [\text{Contra}''], [\text{HeadNil}],$
 $[\text{HeadPair}], [\text{Transitivity}], [\text{Contra}], [\text{T}_E], [\text{ragged right}],$
 $[\text{ragged right expansion}], [\text{parm}(*, *, *)], [\text{parm}^*(*, *, *)], [\text{inst}(*, *)],$
 $[\text{inst}^*(*, *)], [\text{occur}(*, *, *)], [\text{occur}^*(*, *, *)], [\text{unify}(* = *, *)], [\text{unify}^*(* = *, *)],$
 $[\text{unify}_2(* = *, *)], [\text{L}_a], [\text{L}_b], [\text{L}_c], [\text{L}_d], [\text{L}_e], [\text{L}_f], [\text{L}_g], [\text{L}_h], [\text{L}_i], [\text{L}_j], [\text{L}_k], [\text{L}_l], [\text{L}_m],$
 $[\text{L}_n], [\text{L}_o], [\text{L}_p], [\text{L}_q], [\text{L}_r], [\text{L}_s], [\text{L}_t], [\text{L}_u], [\text{L}_v], [\text{L}_w], [\text{L}_x], [\text{L}_y], [\text{L}_z], [\text{L}_A], [\text{L}_B], [\text{L}_C],$
 $[\text{L}_D], [\text{L}_E], [\text{L}_F], [\text{L}_G], [\text{L}_H], [\text{L}_I], [\text{L}_J], [\text{L}_K], [\text{L}_L], [\text{L}_M], [\text{L}_N], [\text{L}_O], [\text{L}_P], [\text{L}_Q], [\text{L}_R],$
 $[\text{L}_S], [\text{L}_T], [\text{L}_U], [\text{L}_V], [\text{L}_W], [\text{L}_X], [\text{L}_Y], [\text{L}_Z], [\text{L}_?], [\text{Reflexivity}], [\text{Reflexivity}_1],$
 $[\text{Commutativity}], [\text{Commutativity}_1], [<\text{tactic}>], [\text{tactic}], [[* \stackrel{\text{tactic}}{=} *]], [\mathcal{P}(*, *, *)],$
 $[\mathcal{P}^*(*, *, *)], [\text{p}_0], [\text{conclude}_1(*, *)], [\text{conclude}_2(*, *, *)], [\text{conclude}_3(*, *, *, *)],$

[conclude₄(*,*)], [peano], [0], [1], [2], [*a*], [*b*], [*c*], [*d*], [*e*], [*f*], [*g*], [*h*], [*i*], [*j*], [*k*], [*l*], [*m*], [*n*], [*o*], [*p*], [*q*], [*r*], [*s*], [*t*], [*u*], [*v*], [*w*], [*x*], [*y*], [*z*], [nonfree(*,*)],
[nonfree (*,*)], [free(*|* := *)], [free*(** := *)], [*≡(*|* := *)], [*≡(** := *)],
[S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP],
[Gen], [L3.2(a)], [S'], [A1'], [A2'], [A3'], [A4'], [A5'], [S1'], [S2'], [S3'], [S4'], [S5'],
[S6'], [S7'], [S8'], [S9'], [MP'], [Gen'], [L3.2(a)'], [M1.7], [MP'_h], [Hypothesize],
[Gen'_h], [M3.2(a)], [M3.2(a)_h], [M3.2(b)_h], [M3.1(S1')_h], [M3.2(c)_h], [M3.1(S2')_h],
[M3.1(S5')_h], [M3.1(S6')_h], [M3.2(f)], [M Lemma 1.8], [M Proposition 3.2(a)],
[M Proposition 3.2(b)], [M Proposition 3.2(c)], [M Proposition 3.2(d)],
[M Proposition 3.2(f) (i)], [M Proposition 3.2(f) (ii)], [M Proposition 3.2(f)],
[M Proposition 3.2(g) (i)], [M Proposition 3.2(g) (ii)], [M Proposition 3.2(g)],
[M Proposition 3.2(h)(i)], [M Proposition 3.2(h)_{(g)(ii)}],
[M Proposition 3.2(h)(ii)], [M Proposition 3.2(h)], [M Tautology A],
[M Tautology B];

Preassociative

[*_{*}], [*'], [*[*]], [*[*→*]], [*[*⇒*]], [*];

Preassociative

[“*”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [
*, [*], [! *], [” *], [# *], [\$ *], [% *], [& *], [’ *], [(*) , () *], [**], [+ *], [*], [- *], [. *], [/ *],
[0 *], [1 *], [2 *], [3 *], [4 *], [5 *], [6 *], [7 *], [8 *], [9 *], [: *], [; *], [< *], [= *], [> *], [? *],
[@ *], [A *], [B *], [C *], [D *], [E *], [F *], [G *], [H *], [I *], [J *], [K *], [L *], [M *], [N *],
[O *], [P *], [Q *], [R *], [S *], [T *], [U *], [V *], [W *], [X *], [Y *], [Z *], [[*], [\ *], [] *], [^ *],
[_ *], [‘ *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *],
[p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [{ *}, [| *], [} *], [~ *],
[Preassociative *; *], [Postassociative *; *], [[*], *], [priority * end],
[newline *], [macro newline *];

Preassociative

[*0], [*1], [0b], [*-color(*)], [*-color*(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*ⁱ], [*^d], [*^R], [*⁰],
[*¹], [*²], [*³], [*⁴], [*⁵], [*⁶], [*⁷], [*⁸], [*⁹], [*^E], [*^V], [*^C], [*^{C*}], [*^l];

Preassociative

[* · *], [* ·₀ *], [* · : *];

Preassociative

[* + *], [* +₀ *], [* +₁ *], [* − *], [* −₀ *], [* −₁ *], [* ÷ *];

Preassociative

[* ∪ { * }], [* ∪ *], [* \ { * }];

Postassociative

[* ∴ *], [* ∴ *], [* ∴ : *], [* +₂ *], [* ∴ : *], [* +₂ *];

Postassociative

[*, *];

Preassociative

[* ^B ≈ *], [* ^D ≈ *], [* ^C ≈ *], [* ^P ≈ *], [* ≈ *], [* = *], [* ⁺ → *], [* ^t = *], [* ^{t*} = *], [* ^r = *],

$[* \in_t *], [* \subseteq_T *], [* \stackrel{T}{=} *], [* \stackrel{s}{=} *], [* \text{ free in } *], [* \text{ free in }^* *], [* \text{ free for } * \text{ in } *],$
 $[* \text{ free for }^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* \stackrel{P}{=} *], [*^{\mathcal{P}}];$
Preassociative
 $[\neg *], [\dot{\neg} *];$
Preassociative
 $[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$
Preassociative
 $[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$
Preassociative
 $[\dot{\forall} *: *], [\dot{\exists} *: *];$
Postassociative
 $[* \rightrightarrows *], [* \Rightarrow *], [* \Leftrightarrow *];$
Postassociative
 $[* : *], [*! *];$
Preassociative
 $[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$
Preassociative
 $[\lambda *. *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \ddot{=} * \text{ in } *];$
Preassociative
 $[*^!], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$
Preassociative
 $[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *];$
Postassociative
 $[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
 $[\forall *: *];$
Postassociative
 $[* \oplus *];$
Postassociative
 $[*, *];$
Preassociative
 $[* \text{ proves } *];$
Preassociative
 $[* \text{ proof of } *: *], [\text{Line } *: * \gg *: *], [\text{Last line } * \gg * \square],$
 $[\text{Line } *: \text{Premise } \gg *: *], [\text{Line } *: \text{Side-condition } \gg *: *], [\text{Arbitrary } \gg *: *],$
 $[\text{Local } \gg * = *: *];$
Postassociative
 $[* \text{ then } *], [*[*] *];$
Preassociative
 $[* \& *];$
Preassociative
 $[* \setminus \setminus *];$

$[\text{peano commutativity} \xrightarrow{\text{pyk}} \text{“peano commutativity”}]$

M Lemma 1.8

[M Lemma 1.8 $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{b}: [[A2' \gg [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]]]] \Rightarrow [[\underline{b} \Rightarrow [\underline{b} \Rightarrow \underline{b}]] \Rightarrow [\underline{b} \Rightarrow \underline{b}]]]] ; [[A1' \gg [\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]]]] ; [[[[MP' \triangleright [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \Rightarrow [[\underline{b} \Rightarrow [\underline{b} \Rightarrow \underline{b}]] \Rightarrow [\underline{b} \Rightarrow \underline{b}]]]] \triangleright [\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \gg [[\underline{b} \Rightarrow [\underline{b} \Rightarrow \underline{b}]] \Rightarrow [\underline{b} \Rightarrow \underline{b}]]] ; [[A1' \gg [\underline{b} \Rightarrow [\underline{b} \Rightarrow \underline{b}]]] ; [[[MP' \triangleright [[\underline{b} \Rightarrow [\underline{b} \Rightarrow \underline{b}]] \Rightarrow [\underline{b} \Rightarrow \underline{b}]]] \triangleright [\underline{b} \Rightarrow [\underline{b} \Rightarrow \underline{b}]] \gg [\underline{b} \Rightarrow \underline{b}]]]]], p_0, c)]$

[M Lemma 1.8 $\xrightarrow{\text{stmt}} S' \vdash \forall \underline{b}: [\underline{b} \Rightarrow \underline{b}]$

[M Lemma 1.8 $\xrightarrow{\text{tex}} \text{"M \ Lemma \ 1.8"}]$

[M Lemma 1.8 $\xrightarrow{\text{pyk}} \text{"mendelson lemma one eight"}]$

M Proposition 3.2(a)

[M Proposition 3.2(a) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{t}: [[S5' \gg [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]] ; [[S1' \gg [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \Rightarrow [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{t}]]]] ; [[[MP' \triangleright [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \Rightarrow [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{t}]]] \triangleright [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]] \gg [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{t}]]] ; [[[MP' \triangleright [[[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{t}]] \triangleright [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]] \gg [\underline{t} \stackrel{p}{=} \underline{t}]]]]], p_0, c)]$

[M Proposition 3.2(a) $\xrightarrow{\text{stmt}} S' \vdash \forall \underline{t}: [\underline{t} \stackrel{p}{=} \underline{t}]$

[M Proposition 3.2(a) $\xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(a)"}]$

[M Proposition 3.2(a) $\xrightarrow{\text{pyk}} \text{"mendelson proposition three two a"}]$

M Proposition 3.2(b)

[M Proposition 3.2(b) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{t}: \forall \underline{r}: [[S1' \gg [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [[\underline{t} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]]] ; [[[M \text{ Tautology A} \triangleright [\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [[\underline{t} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]]] \gg [[\underline{t} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]]] ; [[M \text{ Proposition 3.2(a)} \gg [\underline{t} \stackrel{p}{=} \underline{t}]] ; [[[MP' \triangleright [[\underline{t} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]] \triangleright [\underline{t} \stackrel{p}{=} \underline{t}]] \gg [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]]]], p_0, c)]$

[M Proposition 3.2(b) $\xrightarrow{\text{stmt}} S' \vdash \forall \underline{t}: \forall \underline{r}: [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]$

[M Proposition 3.2(b) $\xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(b)"}]$

[M Proposition 3.2(b) $\xrightarrow{\text{pyk}} \text{"mendelson proposition three two b"}]$

M Proposition 3.2(c)

[M Proposition 3.2(c) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{t}: \forall \underline{r}: \forall \underline{s}: [[S1' \gg [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{s}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]]] ; [[\text{M Proposition 3.2(b)} \gg [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]]] ; [[[\text{M Tautology B} \triangleright [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]]] \triangleright [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{s}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]]]]] \gg [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{s}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]]]]] , p_0, c)]$

[M Proposition 3.2(c) $\xrightarrow{\text{stmt}} S' \vdash \forall \underline{t}: \forall \underline{r}: \forall \underline{s}: [[\underline{t} \stackrel{p}{=} \underline{r}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{s}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]]]$

[M Proposition 3.2(c) $\xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(c)"}]$

[M Proposition 3.2(c) $\xrightarrow{\text{pyk}} \text{"mendelson proposition three two c"}]$

M Proposition 3.2(d)

[M Proposition 3.2(d) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{r}: \forall \underline{t}: \forall \underline{s}: [[\text{M Proposition 3.2(c)} \gg [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{t} \stackrel{p}{=} \underline{s}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]] ; [[[\text{M Tautology A} \triangleright [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{t} \stackrel{p}{=} \underline{s}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]] \triangleright [[\underline{t} \stackrel{p}{=} \underline{s}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]]] \gg [[\underline{t} \stackrel{p}{=} \underline{s}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]] ; [[\text{M Proposition 3.2(b)} \gg [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]]] ; [[[[\text{M Tautology B} \triangleright [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]] \triangleright [[\underline{t} \stackrel{p}{=} \underline{s}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]] \gg [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]] ; [[[[\text{M Tautology A} \triangleright [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]] \triangleright [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]]] \gg [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]]]] , p_0, c)]$

[M Proposition 3.2(d) $\xrightarrow{\text{stmt}} S' \vdash \forall \underline{r}: \forall \underline{t}: \forall \underline{s}: [[\underline{r} \stackrel{p}{=} \underline{t}] \Rightarrow [[\underline{s} \stackrel{p}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]$

[M Proposition 3.2(d) $\xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(d)"}]$

[M Proposition 3.2(d) $\xrightarrow{\text{pyk}} \text{"mendelson proposition three two d"}]$

M Proposition 3.2(f) (i)

[M Proposition 3.2(f) (i) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash [[S5' \gg [[\underline{\dot{0}} \dot{+} \underline{\dot{0}}] \stackrel{p}{=} \underline{\dot{0}}]] ; [[\text{M Proposition 3.2(b)} \gg [[[\underline{\dot{0}} \dot{+} \underline{\dot{0}}] \stackrel{p}{=} \underline{\dot{0}}] \Rightarrow [\underline{\dot{0}} \stackrel{p}{=} [\underline{\dot{0}} \dot{+} \underline{\dot{0}}]]]] ; [[[\text{MP}' \triangleright [[[\underline{\dot{0}} \dot{+} \underline{\dot{0}}] \stackrel{p}{=} \underline{\dot{0}}] \Rightarrow [\underline{\dot{0}} \stackrel{p}{=} [\underline{\dot{0}} \dot{+} \underline{\dot{0}}]]]] \triangleright [[\underline{\dot{0}} \dot{+} \underline{\dot{0}}] \stackrel{p}{=} \underline{\dot{0}}]] \gg [[\underline{\dot{0}} \stackrel{p}{=} [\underline{\dot{0}} \dot{+} \underline{\dot{0}}]]]]]] , p_0, c)]$

[M Proposition 3.2(f) (i) $\xrightarrow{\text{stmt}} S' \vdash [\underline{\dot{0}} \stackrel{p}{=} [\underline{\dot{0}} \dot{+} \underline{\dot{0}}]]]$

[M Proposition 3.2(f) (i) $\xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(f) \ (i)"}]$

[M Proposition 3.2(f) (i) $\xrightarrow{\text{pyk}}$ “mendelson proposition three two f i”]

M Proposition 3.2(f) (ii)

[M Proposition 3.2(f) (ii) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil S' \vdash [[S6' \gg [[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] ; [[S2' \gg [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']]] ; [[\text{M Proposition 3.2(d)} \gg [[\dot{x}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]]] ; [[[\text{M Tautology B} \triangleright [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']]] \triangleright [[\dot{x}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]] \gg [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]]] ; [[[\text{M Tautology A} \triangleright [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]] \gg [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]]] ; [[[[\text{MP}' \triangleright [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \Rightarrow [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]] \triangleright [[[\dot{0} \dot{+} [\dot{x}']]] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{x}]]']] \gg [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]]] ; [[[[\text{Gen}' \triangleright [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]] \gg \dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]]]] \rceil, p_0, c)]$

[M Proposition 3.2(f) (ii) $\xrightarrow{\text{stmt}} S' \vdash \dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]]]$

[M Proposition 3.2(f) (ii) $\xrightarrow{\text{tex}}$ “M\ Proposition\ 3.2(f)\ (ii)”]

[M Proposition 3.2(f) (ii) $\xrightarrow{\text{pyk}}$ “mendelson proposition three two f ii”]

M Proposition 3.2(f)

[M Proposition 3.2(f) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil S' \vdash [[S9' \gg [[\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]] \Rightarrow [[\dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]] \Rightarrow \dot{V}\dot{x}: [\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]]]] ; [[\text{M Proposition 3.2(f) (i)} \gg [[\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]]] ; [[[[\text{MP}' \triangleright [[\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]] \Rightarrow [[\dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]] \Rightarrow \dot{V}\dot{x}: [\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]]] \triangleright [[\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]]] \gg [[\dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]] \Rightarrow \dot{V}\dot{x}: [\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]]]] ; [[\text{M Proposition 3.2(f) (ii)} \gg \dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]] ; [[[[\text{MP}' \triangleright [[\dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]] \Rightarrow \dot{V}\dot{x}: [\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]]] \triangleright \dot{V}\dot{x}: [[\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}']]]] \gg \dot{V}\dot{x}: [\dot{x} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{x}]]]]] \rceil, p_0, c)]$

$$\begin{aligned}
& \dot{x}' \dot{+} [\dot{y}] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'] \Rightarrow [[[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x}' \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]; [[[M \text{ Tautology A} \triangleright [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]] \gg [[[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x}' \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]] \triangleright [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x}' \dot{+} [\dot{y}]]']] \gg [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]]; [[[S6' \gg [[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]; [[[S2' \gg [[[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]]; [[[M \text{ Proposition 3.2(d)} \gg [[[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \Rightarrow [[[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \\
& \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]]; [[[[M \text{ Tautology B} \triangleright [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]] \\
& \triangleright [[[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]] \Rightarrow [[[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \\
& \gg [[[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]]; [[[M \text{ Tautology A} \triangleright [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \gg [[[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \Rightarrow [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]]; [[[[MP' \triangleright [[[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]] \\
& \triangleright [[[\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \gg [[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]]; [[[[MP' \triangleright [[[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']] \Rightarrow [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \\
& \triangleright [[\dot{x} \dot{+} [\dot{y}']]' \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]'']]] \gg [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]]; [[[Gen' \triangleright [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]] \\
& \gg \dot{y}: [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \Rightarrow [[[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]], p_0, c)]
\end{aligned}$$

$$\begin{aligned}
& [M \text{ Proposition 3.2(g)} \text{ (ii)} \xrightarrow{\text{stmt}} S' \vdash \dot{y}: [[[\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']] \\
& \Rightarrow [[\dot{x}' \dot{+} [\dot{y}']] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]']]]
\end{aligned}$$

$$[M \text{ Proposition 3.2(g)} \text{ (ii)} \xrightarrow{\text{tex}} \text{"M \setminus Proposition \setminus 3.2(g) \setminus (ii)"}]$$

$$[M \text{ Proposition 3.2(g)} \text{ (ii)} \xrightarrow{\text{pyk}} \text{"mendelson proposition three two g ii"}]$$

M Proposition 3.2(g)

$$\begin{aligned}
 & [\text{M Proposition 3.2(g)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash [[S9' \gg [[\dot{V}\dot{x}: [[\dot{x}' \dot{+} \dot{0}] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{0}]']]] \Rightarrow [[\dot{V}\dot{y}: [[[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']] \Rightarrow [[\dot{x}' \dot{+} \dot{y}']] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}']']]] \Rightarrow \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] ; [[\text{M Proposition 3.2(g)} \text{ (i)} \gg \dot{V}\dot{x}: [[\dot{x}' \dot{+} \dot{0}] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{0}]']]] ; [[[\text{MP}' \triangleright [[\dot{V}\dot{x}: [[\dot{x}' \dot{+} \dot{0}] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{0}]']]] \Rightarrow [[\dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] \Rightarrow [[\dot{x}' \dot{+} \dot{y}']] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}']']]] \triangleright \dot{V}\dot{x}: [[\dot{x}' \dot{+} \dot{0}] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{0}]']]] \gg [[\dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']] \Rightarrow [[\dot{x}' \dot{+} \dot{y}']] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}']']]] \Rightarrow \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] \triangleright \dot{V}\dot{x}: [[\dot{x}' \dot{+} \dot{0}] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{0}]']]] \gg [[\dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']] \Rightarrow [[\dot{x}' \dot{+} \dot{y}']] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}']']]] \Rightarrow \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] ; [[\text{M Proposition 3.2(g)} \text{ (ii)} \gg \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']] \Rightarrow [[\dot{x}' \dot{+} \dot{y}']] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}']']]] ; [[[\text{MP}' \triangleright [[\dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] \Rightarrow [[\dot{x}' \dot{+} \dot{y}']] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}']']]] \Rightarrow \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] \triangleright \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]] \gg \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]]] , p_0, c)]
 \end{aligned}$$

$$[\text{M Proposition 3.2(g)} \xrightarrow{\text{stmt}} S' \vdash \dot{V}\dot{y}: [[\dot{x}' \dot{+} \dot{y}]] \stackrel{P}{=} [[\dot{x} \dot{+} \dot{y}]']]]$$

$$[\text{M Proposition 3.2(g)} \xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(g)"}]$$

$$[\text{M Proposition 3.2(g)} \xrightarrow{\text{pyk}} \text{"mendelson proposition three two g"}]$$

M Proposition 3.2(h)(i)

$$\begin{aligned}
 & [\text{M Proposition 3.2(h)(i)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash [[S5' \gg [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]]]] ; [[\text{M Proposition 3.2(f)} \gg \dot{V}\dot{x}: [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] ; [[[A4' @ [\dot{x}]] \gg [[\dot{V}\dot{x}: [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \Rightarrow [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] ; [[[\text{MP}' \triangleright [[\dot{V}\dot{x}: [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \Rightarrow [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] \triangleright \dot{V}\dot{x}: [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] \gg [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] ; [[\text{M Proposition 3.2(c)} \gg [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]] \Rightarrow [[\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \Rightarrow [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] ; [[[\text{MP}' \triangleright [[[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]] \Rightarrow [[\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \Rightarrow [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] \triangleright [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]]] \gg [[\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \Rightarrow [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] ; [[[\text{MP}' \triangleright [[\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \Rightarrow [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] \triangleright [\dot{x} \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \gg [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]] ; [[\text{Gen}' \triangleright [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]] \gg \dot{V}\dot{x}: [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]]] , p_0, c)]
 \end{aligned}$$

$$[\text{M Proposition 3.2(h)(i)} \xrightarrow{\text{stmt}} S' \vdash \dot{V}\dot{x}: [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{0} \dot{+} \dot{x}]]]$$

$$[\text{M Proposition 3.2(h)(i)} \xrightarrow{\text{tex}} \text{"M \ Proposition \ 3.2(h)(i)"}]$$

[M Proposition 3.2(h)(i) $\xrightarrow{\text{pyk}}$ “mendelson proposition three two h i”]

M Proposition 3.2(h)_(g)(ii)
$$\begin{aligned}
& [M \text{ Proposition 3.2(h)}]_{(g)(ii)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash [[\dot{V}\dot{y}: [\dot{x}' + \dot{y}]] \stackrel{P}{=} [[\dot{x} + \dot{y}]]']]] \vdash [[[\dot{A}4' @ [\dot{z}]] \gg [[\dot{V}\dot{y}: [\dot{x}' + \dot{y}]] \stackrel{P}{=} [[\dot{x} + \dot{y}]]']]] \Rightarrow [[[\dot{x}' + \dot{z}]] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']]]; [[[\dot{M}P' \triangleright [\dot{V}\dot{y}: [\dot{x}' + \dot{y}]] \stackrel{P}{=} [[\dot{x} + \dot{y}]]']]] \Rightarrow [[[\dot{x}' + \dot{z}]] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']]] \triangleright \dot{V}\dot{y}: [\dot{x}' + \dot{y}] \stackrel{P}{=} [[\dot{x} + \dot{y}]]'] \gg [[[\dot{x}' + \dot{z}]] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']]; [[[\dot{G}en' \triangleright [[\dot{x}' + \dot{z}]] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']] \gg \dot{V}\dot{x}: [\dot{x}' + \dot{z}] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']]; [[[\dot{A}4' @ [\dot{y}]] \gg [[\dot{V}\dot{x}: [\dot{x}' + \dot{z}]] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']]] \Rightarrow [[[\dot{y}' + \dot{z}]] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']]]; [[[\dot{M}P' \triangleright [[\dot{V}\dot{x}: [\dot{x}' + \dot{z}]] \stackrel{P}{=} [[\dot{x} + \dot{z}]]']]] \Rightarrow [[[\dot{y}' + \dot{z}]] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']]] \triangleright \dot{V}\dot{x}: [\dot{x}' + \dot{z}] \stackrel{P}{=} [[\dot{x} + \dot{z}]]'] \gg [[[\dot{y}' + \dot{z}]] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']]; [[[\dot{G}en' \triangleright [[\dot{y}' + \dot{z}]] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']] \gg \dot{V}\dot{z}: [\dot{y}' + \dot{z}] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']]; [[[\dot{A}4' @ [\dot{x}]] \gg [[\dot{V}\dot{z}: [\dot{y}' + \dot{z}]] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']]] \Rightarrow [[[\dot{y}' + \dot{x}]] \stackrel{P}{=} [[\dot{y} + \dot{x}]]']]]; [[[\dot{M}P' \triangleright [[\dot{V}\dot{z}: [\dot{y}' + \dot{z}]] \stackrel{P}{=} [[\dot{y} + \dot{z}]]']]] \Rightarrow [[[\dot{y}' + \dot{x}]] \stackrel{P}{=} [[\dot{y} + \dot{x}]]']]] \triangleright \dot{V}\dot{z}: [\dot{y}' + \dot{z}] \stackrel{P}{=} [[\dot{y} + \dot{z}]]'] \gg [[[\dot{y}' + \dot{x}]] \stackrel{P}{=} [[\dot{y} + \dot{x}]]']]]], p_0, c)]
\end{aligned}$$
$$[\text{M Proposition 3.2(h)}]_{(g)(ii)} \xrightarrow{\text{stmt}} S' \vdash [[\ddot{y} \dot{y} : [[\dot{x}' \dot{+} [\dot{y}]]]] \stackrel{p}{=} [[\dot{x} \dot{+} [\dot{y}]]]']]] \vdash [[\dot{y}' \dot{+} [\dot{x}]]] \stackrel{p}{=} [[\dot{y} \dot{+} [\dot{x}]]]']]]$$
$$[\text{M Proposition 3.2(h)}]_{(g)(ii)} \xrightarrow{\text{tex}} [\text{M Proposition 3.2(h)}]_{\{(g)(ii)\}}$$

[M Proposition 3.2(h)_{(g)(ii)} $\xrightarrow{\text{pyk}}$ “mendelson proposition three two h g ii”]

M Proposition 3.2(h)(ii)

$$\begin{aligned} & [\text{M Proposition 3.2(h)(ii)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash [\text{S6}' \gg [\dot{x} \dot{+} [\dot{y}]]] \stackrel{p}{=} [\dot{x} \dot{+} [\dot{y}]]')])]; [\text{M Proposition 3.2(g)} \gg \dot{y} \dot{y}: [\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [\dot{x} \dot{+} [\dot{y}]]')]); [\text{M Proposition 3.2(h)}_{(g)(ii)} \triangleright \dot{y} \dot{y}: [\dot{x}' \dot{+} [\dot{y}]] \stackrel{p}{=} [\dot{x} \dot{+} [\dot{y}]]')]) \gg [\dot{y}' \dot{+} [\dot{x}]] \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]')]); [\text{S2}' \gg [\dot{x} \dot{+} [\dot{y}]] \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]] \Rightarrow [\dot{x} \dot{+} [\dot{y}]]' \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]')]); [\text{M Proposition 3.2(c)} \gg [\dot{x} \dot{+} [\dot{y}]] \stackrel{p}{=} [\dot{x} \dot{+} [\dot{y}]]') \Rightarrow [\dot{x} \dot{+} [\dot{y}]]' \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]') \Rightarrow [\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]')]); [\text{M Tautology A} \triangleright [\dot{x} \dot{+} [\dot{y}]] \stackrel{p}{=} [\dot{x} \dot{+} [\dot{y}]]') \Rightarrow [\dot{x} \dot{+} [\dot{y}]]' \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]') \Rightarrow [\dot{x} \dot{+} [\dot{y}']] \stackrel{p}{=} [\dot{y} \dot{+} [\dot{x}]]')]); \end{aligned}$$

M Proposition 3.2(h)

$$\begin{aligned} & [\text{M Proposition 3.2(h)}] \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{S9'} \gg [\dot{\vee}x: [\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{0} + \dot{x}]]] \Rightarrow [\dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]] \Rightarrow [\dot{x} + \dot{y'}] \stackrel{P}{=} [\dot{y'} + \dot{x}]]] \Rightarrow \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]]; \\ & \text{M Proposition 3.2(h)(i)} \gg \dot{\vee}x: [\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{0} + \dot{x}]]]; \\ & [\dot{\vee}x: [\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{0} + \dot{x}]]] \Rightarrow [\dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \Rightarrow [\dot{x} + \dot{y'}] \stackrel{P}{=} [\dot{y'} + \dot{x}]]] \Rightarrow \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \triangleright \dot{\vee}x: [\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{0} + \dot{x}]]] \gg [\dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \Rightarrow [\dot{x} + \dot{y'}] \stackrel{P}{=} [\dot{y'} + \dot{x}]]] \Rightarrow \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]]; \\ & [\text{M Proposition 3.2(h)(ii)} \gg \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \Rightarrow [\dot{x} + \dot{y'}] \stackrel{P}{=} [\dot{y'} + \dot{x}]]]]; \\ & [\dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \Rightarrow [\dot{x} + \dot{y'}] \stackrel{P}{=} [\dot{y'} + \dot{x}]]] \triangleright \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \Rightarrow [\dot{x} + \dot{y'}] \stackrel{P}{=} [\dot{y'} + \dot{x}]]] \gg \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]]; \\ & [\text{Gen'} \triangleright \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]] \gg \dot{\vee}x: \dot{\vee}y: [\dot{x} + \dot{y}] \stackrel{P}{=} [\dot{y} + \dot{x}]]], p_0, c)] \end{aligned}$$
$$[M \text{ Proposition 3.2(h)}] \xrightarrow{\text{stnt}} S' \vdash \dot{x}:\dot{x}.\dot{y}:\dot{y} : \left[\left[\dot{x} \dot{+} \left[\dot{y} \right] \right] \stackrel{p}{=} \left[\dot{y} \dot{+} \left[\dot{x} \right] \right] \right]$$

[M Proposition 3.2(h) $\xrightarrow{\text{tex}}$ “M\ Proposition\ 3.2(h)”]

[M Proposition 3.2(h) $\xrightarrow{\text{pyk}}$ “mendelson proposition three two h”]

M Tautology A

$$\begin{aligned}
& [M \text{ Tautology } A \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil S' \vdash \forall a: \forall \mathbf{b}: \forall \mathbf{c}: [\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c}] \rceil) \vdash [\underline{A2'} \gg [\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c}] \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]] ; [\underline{[\underline{[\underline{MP'} \triangleright [\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c}] \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]} \triangleright [\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c}]}] \gg [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]] ; [\underline{A1'} \gg [\underline{[\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]}] \Rightarrow [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]] ; [\underline{[\underline{MP'} \triangleright [\underline{[\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]}] \Rightarrow [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}] }] } \triangleright [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]] \gg [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]] ; [\underline{A2'} \gg [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]] \Rightarrow [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]] \triangleright [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]] \gg [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]] ; [\underline{A1'} \gg [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] }]] ; [\underline{[\underline{MP'} \triangleright [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] }] \Rightarrow [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{c}] }] } \triangleright [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{b}] }]] \gg [\underline{b} \Rightarrow [\underline{a} \Rightarrow \underline{c}] }]]] , p_0, c)]
\end{aligned}$$
$$[\text{M Tautology } A \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [\textcolor{blue}{[\underline{a} \Rightarrow \textcolor{blue}{[\underline{b} \Rightarrow \underline{c}]}]} \vdash \textcolor{blue}{[\underline{b} \Rightarrow \textcolor{blue}{[\underline{a} \Rightarrow \underline{c}]}]}]]$$

[M Tautology A $\xrightarrow{\text{tex}}$ “M\ Tautology\ A”]

[M Tautology A $\xrightarrow{\text{pyk}}$ “mendelson tautology a”]

M Tautology B

[M Tautology B $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil S' \vdash \forall d: \forall e: \forall f: [[d \Rightarrow e] \vdash [[e \Rightarrow f] \vdash [[A1' \gg [[e \Rightarrow f] \Rightarrow [d \Rightarrow [e \Rightarrow f]]]] ; [[[[MP' \triangleright [[e \Rightarrow f] \Rightarrow [d \Rightarrow [e \Rightarrow f]]]] \triangleright [e \Rightarrow f]] \gg [d \Rightarrow [e \Rightarrow f]]] ; [[A2' \gg [[d \Rightarrow [e \Rightarrow f]] \Rightarrow [[d \Rightarrow e] \Rightarrow [d \Rightarrow f]]]] ; [[[[MP' \triangleright [[d \Rightarrow [e \Rightarrow f]] \Rightarrow [[d \Rightarrow e] \Rightarrow [d \Rightarrow f]]]] \triangleright [d \Rightarrow [e \Rightarrow f]]] \gg [[d \Rightarrow e] \Rightarrow [d \Rightarrow f]]] ; [[[MP' \triangleright [[d \Rightarrow e] \Rightarrow [d \Rightarrow f]]] \triangleright [d \Rightarrow e]] \gg [d \Rightarrow f]]]]] \rceil, p0, c)$]

[M Tautology B $\xrightarrow{\text{stmt}}$ $S' \vdash \forall d: \forall e: \forall f: [[d \Rightarrow e] \vdash [[e \Rightarrow f] \vdash [d \Rightarrow f]]]$]

[M Tautology B $\xrightarrow{\text{tex}}$ “M\ Tautology\ B”]

[M Tautology B $\xrightarrow{\text{pyk}}$ “mendelson tautology b”]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-07-03.UTC:17:02:22.108370 = MJD-53554.TAI:17:02:54.108370 =
LGT-4627126974108370e-6