Logiweb codex of peanorapport

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peanorapport, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\mathrm{nonfree}}(*,*)$, $\dot{\mathrm{nonfree}}^*(*,*)$, $\dot{\mathrm{free}}\langle *|* := *\rangle$, $\dot{\mathrm{free}}^*\langle *|* := *\rangle$, $*\dot{\equiv}\langle *|* := *\rangle$, $*\dot{\equiv}\langle^* *|* := *\rangle$, S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, L3.2(a)′, L3.2(b)′, L3.2(c)′, L3.2(d)′, L3.2(f)′, L3.2(g)′, L3.2(g)′II, L3.2(h)′basis, L3.2(h)′, Taut1, Taut2, Taut3, Taut4, Weaken, M1.7, MPTwice, $\dot{*}$, $*'$, $*\dot{\cdot}*$, $*\dot{+}*$, $*\stackrel{p}{=}*$, $*^{\mathcal{P}}$, $\dot{\neg}*$, $\dot{\forall}*:*$, $*\dot{\Rightarrow}*$,

peanorapport

[peanorapport $\stackrel{\text{pyk}}{\rightarrow}$ “peanorapport”]

$\dot{0}$

[$\dot{0} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{0}”]

[$\dot{0} \stackrel{\text{pyk}}{\rightarrow}$ “peano zero”]

$\dot{1}$

[$\dot{1} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{1} \ddot{=} \dot{0}']\rceil)$]

[$\dot{1} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{1}”]

[$\dot{1} \stackrel{\text{pyk}}{\rightarrow}$ “peano one”]

$\dot{2}$

[$\dot{2} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{2} \ddot{=} \dot{1}']\rceil)$]

[$\dot{2} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{2}”]

[$\dot{2} \stackrel{\text{pyk}}{\rightarrow}$ “peano two”]

$\dot{a}$

$[\dot{a} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{a} \ddot{=} \dot{\mathsf{a}}] \rceil)]$

$[\dot{a} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{a}}"]

$[\dot{a} \stackrel{\text{pyk}}{\to}$ "peano a"]

$\dot{b}$

$[\dot{b} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{b}}"]

$[\dot{b} \stackrel{\text{pyk}}{\to}$ "peano b"]

$\dot{c}$

$[\dot{c} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{c}}"]

$[\dot{c} \stackrel{\text{pyk}}{\to}$ "peano c"]

$\dot{d}$

$[\dot{d} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{d}}"]

$[\dot{d} \stackrel{\text{pyk}}{\to}$ "peano d"]

$\dot{e}$

$[\dot{e} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{e}}"]

$[\dot{e} \stackrel{\text{pyk}}{\to}$ "peano e"]

$\dot{f}$

$[\dot{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \ddot{=} \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\to}$ "peano f"]

$\dot{g}$

$[\dot{g} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\to}$ "peano g"]

$\dot{h}$

$[\dot{h} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\to}$ "peano h"]

$\dot{i}$

$[\dot{i} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\to}$ "peano i"]

$\dot{j}$

$[\dot{j} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\to}$ "peano j"]

$\dot{k}$

$[\dot{k} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{k} \doteqdot \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{k}}"]

$[\dot{k} \stackrel{\text{pyk}}{\to}$ "peano k"]

$\dot{l}$

$[\dot{l} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{l} \doteqdot \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{l}}"]

$[\dot{l} \stackrel{\text{pyk}}{\to}$ "peano l"]

$\dot{m}$

$[\dot{m} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{m} \doteqdot \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{m}}"]

$[\dot{m} \stackrel{\text{pyk}}{\to}$ "peano m"]

$\dot{n}$

$[\dot{n} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{n} \doteqdot \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{n}}"]

$[\dot{n} \stackrel{\text{pyk}}{\to}$ "peano n"]

$\dot{o}$

$[\dot{o} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{o} \doteqdot \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{o}}"]

$[\dot{o} \stackrel{\text{pyk}}{\to}$ "peano o"]

$\dot{p}$

$[\dot{p} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{p} \mathrel{\ddot{=}} \dot{\mathsf{p}}]\rceil)]$

$[\dot{p} \overset{\text{tex}}{\to}$ "
\dot{\mathit{p}}"]

$[\dot{p} \overset{\text{pyk}}{\to}$ "peano p"]

$\dot{q}$

$[\dot{q} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{q} \mathrel{\ddot{=}} \dot{\mathsf{q}}]\rceil)]$

$[\dot{q} \overset{\text{tex}}{\to}$ "
\dot{\mathit{q}}"]

$[\dot{q} \overset{\text{pyk}}{\to}$ "peano q"]

$\dot{r}$

$[\dot{r} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{r} \mathrel{\ddot{=}} \dot{\mathsf{r}}]\rceil)]$

$[\dot{r} \overset{\text{tex}}{\to}$ "
\dot{\mathit{r}}"]

$[\dot{r} \overset{\text{pyk}}{\to}$ "peano r"]

$\dot{s}$

$[\dot{s} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{s} \mathrel{\ddot{=}} \dot{\mathsf{s}}]\rceil)]$

$[\dot{s} \overset{\text{tex}}{\to}$ "
\dot{\mathit{s}}"]

$[\dot{s} \overset{\text{pyk}}{\to}$ "peano s"]

$\dot{t}$

$[\dot{t} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{t} \mathrel{\ddot{=}} \dot{\mathsf{t}}]\rceil)]$

$[\dot{t} \overset{\text{tex}}{\to}$ "
\dot{\mathit{t}}"]

$[\dot{t} \overset{\text{pyk}}{\to}$ "peano t"]

$\dot{u}$

$[\dot{u} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{u} \ddot{=} \dot{\mathsf{u}}]\rceil)]$

$[\dot{u} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{u}}"]

$[\dot{u} \overset{\text{pyk}}{\rightarrow}$ "peano u"]

$\dot{v}$

$[\dot{v} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{v} \ddot{=} \dot{\mathsf{v}}]\rceil)]$

$[\dot{v} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{v}}"]

$[\dot{v} \overset{\text{pyk}}{\rightarrow}$ "peano v"]

$\dot{w}$

$[\dot{w} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{w} \ddot{=} \dot{\mathsf{w}}]\rceil)]$

$[\dot{w} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{w}}"]

$[\dot{w} \overset{\text{pyk}}{\rightarrow}$ "peano w"]

$\dot{x}$

$[\dot{x} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{x} \ddot{=} \dot{\mathsf{x}}]\rceil)]$

$[\dot{x} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{x}}"]

$[\dot{x} \overset{\text{pyk}}{\rightarrow}$ "peano x"]

$\dot{y}$

$[\dot{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{y} \ddot{=} \dot{\mathsf{y}}]\rceil)]$

$[\dot{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{y}}"]

$[\dot{y} \overset{\text{pyk}}{\rightarrow}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"]

$\dot{\text{nonfree}}(*,*)$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to}$
$\text{If}(\mathsf{y}^{\mathcal{P}},\neg\,[\,\mathsf{x} \stackrel{\text{t}}{=} \mathsf{y}\,]\,,$
$\text{If}(\neg\,[\,\mathsf{y} \stackrel{\text{r}}{=} \lceil\dot{\forall}\mathsf{x}\colon\mathsf{y}\rceil\,]\,,\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=}\ [\,\mathsf{y}^1\,]\,,\mathsf{T},\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^2)))))]$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree $*$ in $*$ end nonfree"]

$\dot{\text{nonfree}}^*(*,*)$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y},\mathsf{T},\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}),\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),\mathsf{F}))]$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree star $*$ in $*$ end nonfree"]

$\dot{\text{free}}\langle *|* := *\rangle$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{val}}{\to} \mathsf{x}!\ [\ \mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}},\mathsf{T},$
$\text{If}(\neg\,[\,\mathsf{a} \stackrel{\text{r}}{=} \lceil\dot{\forall}\mathsf{u}\colon\mathsf{v}\rceil\,]\,,\dot{\text{free}}^*\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$
$\text{If}(\mathsf{a}^1 \stackrel{\text{t}}{=} \mathsf{x},\mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^2),\mathsf{T},$

$\mathrm{If}(\neg\dot{\mathrm{nonfree}}(\mathsf{a}^1,\mathsf{b}),\mathsf{F},$
$\dot{\mathrm{free}}\langle \mathsf{a}^2|\mathsf{x} := \mathsf{b}\rangle)))))$]]

$[\dot{\mathrm{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

$[\dot{\mathrm{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{pyk}}{\to}$ "peano free * set * to * end free"]

$\dot{\mathrm{free}}^*\langle *|* := *\rangle$

$[\dot{\mathrm{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{val}}{\to} \mathsf{x}!$ [$\mathsf{b}!\mathrm{If}(\mathsf{a},\mathsf{T},\mathrm{If}(\dot{\mathrm{free}}\langle \mathsf{a}^{\mathrm{h}}|\mathsf{x} := \mathsf{b}\rangle,\dot{\mathrm{free}}^*\langle \mathsf{a}^{\mathrm{t}}|\mathsf{x} := \mathsf{b}\rangle,\mathsf{F}))$]]

$[\dot{\mathrm{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

$[\dot{\mathrm{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{pyk}}{\to}$ "peano free star * set * to * end free"]

$*\equiv\langle *|* := *\rangle$

$[\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{val}}{\to} \mathsf{a}!$ [$\mathsf{x}!$ [$\mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \overset{\mathrm{r}}{=} \lceil\dot{\forall}\mathsf{u}\colon\mathsf{v}\rceil,\mathsf{b}^1 \overset{\mathrm{t}}{=} \mathsf{x},\mathsf{F}),\mathsf{a} \overset{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge$ [$\mathsf{b} \overset{\mathrm{t}}{=} \mathsf{x}$] $,\mathsf{a} \overset{\mathrm{t}}{=} \mathsf{c},\mathrm{If}($ [
a] $\overset{\mathrm{r}}{=} \mathsf{b},\mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}}|\mathsf{x} := \mathsf{c}\rangle,\mathsf{F})))$]]]

$[\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{tex}}{\to}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

$[\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{pyk}}{\to}$ "peano sub * is * where * is * end sub"]

$*\equiv\langle^* *|* := *\rangle$

$[\mathsf{a}\equiv\langle^* \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{val}}{\to} \mathsf{b}!$ [$\mathsf{x}!$ [$\mathsf{c}!\mathrm{If}(\mathsf{a},\mathsf{T},\mathrm{If}(\mathsf{a}^{\mathrm{h}}\equiv\langle \mathsf{b}^{\mathrm{h}}|\mathsf{x} := \mathsf{c}\rangle,\mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}}|\mathsf{x} := \mathsf{c}\rangle,\mathsf{F}))$]]]

$[\mathsf{a}\equiv\langle^* \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{tex}}{\to}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

$[\mathsf{a}\equiv\langle^*\mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \xrightarrow{\text{pyk}}$ "peano sub star * is * where * is * end sub"]

S′

$$[S' \xrightarrow{\text{stmt}} [\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \dot{+} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}]']]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[[\dot{\neg}\underline{b}] \dot{\Rightarrow} \dot{\neg}\underline{a}] \dot{\Rightarrow} [[[\dot{\neg}\underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [\underline{a}' \stackrel{p}{=} [\underline{b}']]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a}' \stackrel{p}{=} [\underline{b}']] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{b}]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]]]] \oplus [[\forall\underline{x}\colon\forall\underline{a}\colon\forall\underline{b}\colon [\mathrm{nonfree}([\underline{x}], [\underline{a}]) \Vdash [[\dot{\forall}\underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{b}]]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \dot{\cdot} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{\cdot} \underline{b}] \dot{+} \underline{a}]]]] \oplus [[\forall\underline{a}\colon [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [[\underline{a} \stackrel{p}{=} \underline{c}] \dot{\Rightarrow} [\underline{b} \stackrel{p}{=} \underline{c}]]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon\forall\underline{x}\colon [\underline{b}\equiv\langle\underline{a}|\underline{x} := \dot{0}\rangle \Vdash [\underline{c}\equiv\langle\underline{a}|\underline{x} := \underline{x}'\rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall}\underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{a}]]]]]] \oplus [[\forall\underline{a}\colon \neg [\dot{0} \stackrel{p}{=} [\underline{a}']]]] \oplus [[\forall\underline{x}\colon\forall\underline{a}\colon [\underline{a} \vdash \dot{\forall}\underline{x}\colon \underline{a}]]] \oplus [[\forall\underline{c}\colon\forall\underline{a}\colon\forall\underline{x}\colon\forall\underline{b}\colon [\lceil\underline{a}\rceil\equiv\langle\lceil\underline{b}\rceil|\lceil\underline{x}\rceil := \lceil\underline{c}\rceil\rangle \Vdash [[\dot{\forall}\underline{x}\colon \underline{b}] \dot{\Rightarrow} \underline{a}]]]] \oplus \forall\underline{a}\colon [[\underline{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]]]]]]]]]]]]]]]]]$$

[S′ $\xrightarrow{\text{tex}}$ "
S'"]

[S′ $\xrightarrow{\text{pyk}}$ "system prime s"]

A1′

[A1′ $\xrightarrow{\text{proof}}$ Rule tactic]

$[A1' \xrightarrow{\text{stmt}} S' \vdash \forall\underline{a}\colon\forall\underline{b}\colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]]]$

[A1′ $\xrightarrow{\text{tex}}$ "
A1'"]

[A1′ $\xrightarrow{\text{pyk}}$ "axiom prime a one"]

A2′

[A2′ $\xrightarrow{\text{proof}}$ Rule tactic]

$[A2' \xrightarrow{\text{stmt}} S' \vdash \forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]$

$[\mathrm{A2}' \stackrel{\mathrm{tex}}{\rightarrow}$ “
A2”’]

$[\mathrm{A2}' \stackrel{\mathrm{pyk}}{\rightarrow}$ “axiom prime a two”]

A3′

$[\mathrm{A3}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A3}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ [\ \dot{\neg}\, \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow} \dot{\neg}\, \underline{\mathsf{a}}\ ]\ \dot{\Rightarrow}\ [\ [\ [\ \dot{\neg}\, \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]$

$[\mathrm{A3}' \stackrel{\mathrm{tex}}{\rightarrow}$ “
A3”’]

$[\mathrm{A3}' \stackrel{\mathrm{pyk}}{\rightarrow}$ “axiom prime a three”]

A4′

$[\mathrm{A4}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A4}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{b}} \colon [\ \lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil | \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]$

$[\mathrm{A4}' \stackrel{\mathrm{tex}}{\rightarrow}$ “
A4”’]

$[\mathrm{A4}' \stackrel{\mathrm{pyk}}{\rightarrow}$ “axiom prime a four”]

A5′

$[\mathrm{A5}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A5}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \mathrm{non\dot{f}ree}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{a}} \rceil) \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ]\ ]\ ]\ ]$

$[\mathrm{A5}' \stackrel{\mathrm{tex}}{\rightarrow}$ “
A5”’]

$[\mathrm{A5}' \stackrel{\mathrm{pyk}}{\rightarrow}$ “axiom prime a five”]

S1′

$[\mathrm{S1}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S1}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ \dot{\Rightarrow}\ [\ \underline{\mathsf{b}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$

$[\mathrm{S1}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
S1""]

$[\mathrm{S1}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime s one"]

S2′

$[\mathrm{S2}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{S2}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \stackrel{\cdot}{\Rightarrow} [\ \underline{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]]$

$[\mathrm{S2}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
S2""]

$[\mathrm{S2}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime s two"]

S3′

$[\mathrm{S3}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{S3}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \neg [\ \dot{0} \stackrel{\mathrm{p}}{=} [\ \underline{\mathsf{a}}'\ ]\ ]]$

$[\mathrm{S3}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
S3""]

$[\mathrm{S3}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime s three"]

S4′

$[\mathrm{S4}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{S4}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \underline{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ] \stackrel{\cdot}{\Rightarrow} [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}\ ]\ ]]$

$[\mathrm{S4}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
S4""]

$[\mathrm{S4}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime s four"]

S5′

$[\mathrm{S5}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{S5}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon [\ [\ \underline{\mathsf{a}} \stackrel{\cdot}{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]]$

[S5′ $\stackrel{\text{tex}}{\rightarrow}$ "
S5'"]

[S5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s five"]

S6′

[S6′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{+} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{+} \underline{b}\ ]\ '\ ]\ ]$]

[S6′ $\stackrel{\text{tex}}{\rightarrow}$ "
S6'"]

[S6′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s six"]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{\cdot} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]$]

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7'"]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\cdot} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{\cdot} \underline{b}\ ] \dot{+} \underline{a}\ ]\ ]$]

[S8′ $\stackrel{\text{tex}}{\rightarrow}$ "
S8'"]

[S8′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]$]

[S9′ $\stackrel{\text{tex}}{\rightarrow}$ "
S9′"]

[S9′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP′ $\stackrel{\text{tex}}{\rightarrow}$ "
MP′"]

[MP′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen′ $\stackrel{\text{tex}}{\rightarrow}$ "
Gen′"]

[Gen′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime gen"]

L3.2(a)′

[L3.2(a)′ $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon [\ [\ S5' \gg [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ ]\ ; [\ [\ S1' \gg [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \mathrm{MPTwice} \rhd [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ]\ \rhd [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ \rhd [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ ] \gg [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[L3.2(a)′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{t}\colon [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]$]

[L3.2(a)′ $\stackrel{\text{tex}}{\rightarrow}$ "L3.2 (a)′"]

[L3.2(a)′ $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime three two a"]

L3.2(b)′

$$[\mathrm{L3.2(b)}' \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\ [\ \mathrm{S1}' \gg [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Taut1} \rhd [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(a)}' \gg [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]\ ] \rhd [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ] \gg [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{L3.2(b)}' \stackrel{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]$$

$$[\mathrm{L3.2(b)}' \stackrel{\mathrm{tex}}{\to} \text{“L3.2 (b)'”}]$$

$$[\mathrm{L3.2(b)}' \stackrel{\mathrm{pyk}}{\to} \text{“lemma prime three two b”}]$$

L3.2(c)′

$$[\mathrm{L3.2(c)}' \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \mathrm{S1}' \gg [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(b)}' \gg [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Taut2} \rhd [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ]\ ]\ ] \rhd [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{L3.2(c)}' \stackrel{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{r}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]$$

$$[\mathrm{L3.2(c)}' \stackrel{\mathrm{tex}}{\to} \text{“L3.2 (c)'”}]$$

$$[\mathrm{L3.2(c)}' \stackrel{\mathrm{pyk}}{\to} \text{“lemma prime three two c”}]$$

L3.2(d)′

$$[\mathrm{L3.2(d)}' \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \mathrm{L3.2(c)}' \gg [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Taut1} \rhd [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(b)}' \gg [\ [\ \underline{\mathsf{s}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{Taut2} \rhd [\ [\ \underline{\mathsf{s}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ] \rhd [\ [\ \underline{\mathsf{t}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{s}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Taut1} \rhd [\ [\ \underline{\mathsf{s}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{s}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{L3.2(d)}' \stackrel{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{s}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{t}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{r}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{s}}\ ]\ ]\ ]\ ]$$

$$[\mathrm{L3.2(d)}' \stackrel{\mathrm{tex}}{\to} \text{“L3.2 (d)'”}]$$

$$[\mathrm{L3.2(d)}' \stackrel{\mathrm{pyk}}{\to} \text{“lemma prime three two d”}]$$

L3.2(f)′

$[\mathrm{L3.2(f)}' \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [\ [\ \mathrm{S5}' \gg [\ [\ \dot{0}\dot{+}\dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(b)}' \gg [\ [$
$[\ \dot{0}\dot{+}\dot{0}\ ] \stackrel{p}{=} \dot{0}\ ] \dot{\Rightarrow} [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \dot{0}\dot{+}\dot{0}\ ] \stackrel{p}{=} \dot{0}\ ] \dot{\Rightarrow}$
$[\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{0}\dot{+}\dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]\ ] \gg [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{S6}' \gg [$
$[\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+}$
$[\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]$
$]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t} \stackrel{p}{=} [$
$\dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]$
$\stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ] \gg [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [$
$\dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{M1.7} \gg [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]$
$]\ ]\ ]\ ;\ [\ [\ \mathrm{S2}' \gg [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [$
$[\ [\ \mathrm{Taut2} \rhd [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{t} \stackrel{p}{=} [$
$\dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [$
$\dot{t}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(d)}' \gg [\ [\ \dot{t}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [$
$[\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$
$\mathrm{Taut2} \rhd [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{t}' \stackrel{p}{=} [$
$[\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [$
$\dot{t}'\ ]\ ]\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]$
$]'\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{Taut3} \rhd [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]$
$\dot{\Rightarrow} [\ [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ [$
$[\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{t} \stackrel{p}{=}$
$[\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Gen}' \rhd [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]$
$]\ \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{t}\colon [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [$
$\dot{t}'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{S9}' \gg [\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{t}\colon [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]$
$\dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [$
$\mathrm{MPTwice} \rhd [\ [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{t}\colon [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [$
$\dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \dot{0} \stackrel{p}{=} [\ \dot{0}\dot{+}\dot{0}\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{t}\colon$
$[\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ] \dot{\Rightarrow} [\ \dot{t}' \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}'\ ]\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]$
$]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{L3.2(f)}' \stackrel{\text{stmt}}{\to} \mathrm{S}' \vdash \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0}\dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]$

$[\mathrm{L3.2(f)}' \stackrel{\text{tex}}{\to}$ “L3.2 (f)'”]

$[\mathrm{L3.2(f)}' \stackrel{\text{pyk}}{\to}$ “lemma prime three two f”]

L3.2(g)′

$[\mathrm{L3.2(g)}' \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [\ [\ \mathrm{S5}' \gg [\ [\ \dot{t}'\dot{+}\dot{0}\ ] \stackrel{p}{=} [\ \dot{t}'\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{S5}' \gg [\ [$
$\dot{t}\dot{+}\dot{0}\ ] \stackrel{p}{=} [\ \dot{t}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{S2}' \gg [\ [\ [\ \dot{t}\dot{+}\dot{0}\ ] \stackrel{p}{=} [\ \dot{t}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}\dot{+}\dot{0}\ ]' \stackrel{p}{=} [\ \dot{t}'\ ]\ ]$

$]] ; [[[[\mathrm{MP}' \triangleright [[[\dot{t} \dot{+} \dot{0}] \overset{p}{=} [\dot{t}]] \dot{\Rightarrow} [[\dot{t} \dot{+} \dot{0}] ' \overset{p}{=} [\dot{t}']]]] \triangleright [[$
$\dot{t} \dot{+} \dot{0}] \overset{p}{=} [\dot{t}]]] \gg [[\dot{t} \dot{+} \dot{0}] ' \overset{p}{=} [\dot{t}']]] ; [[\mathrm{L3.2(d)}' \gg [[[\dot{t}' \dot{+} \dot{0}]$
$\overset{p}{=} [\dot{t}']] \dot{\Rightarrow} [[[\dot{t} \dot{+} \dot{0}] ' \overset{p}{=} [\dot{t}']] \dot{\Rightarrow} [[\dot{t}' \dot{+} \dot{0}] \overset{p}{=} [[\dot{t} \dot{+} \dot{0}] ']]]]] ;$
$[[[[[\mathrm{MPTwice} \triangleright [[[\dot{t}' \dot{+} \dot{0}] \overset{p}{=} [\dot{t}']] \dot{\Rightarrow} [[[\dot{t} \dot{+} \dot{0}] ' \overset{p}{=} [\dot{t}']] \dot{\Rightarrow} [$
$[\dot{t}' \dot{+} \dot{0}] \overset{p}{=} [[\dot{t} \dot{+} \dot{0}] ']]]]] \triangleright [[\dot{t}' \dot{+} \dot{0}] \overset{p}{=} [\dot{t}']]] \triangleright [[\dot{t} \dot{+} \dot{0}] ' \overset{p}{=} [$
$\dot{t}']]] \gg [[\dot{t}' \dot{+} \dot{0}] \overset{p}{=} [[\dot{t} \dot{+} \dot{0}] ']]] ; [[\mathrm{S6}' \gg [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[$
$\dot{t}' \dot{+} [\dot{r}]] ']]] ; [[[\mathrm{Weaken} \triangleright [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t}' \dot{+} [\dot{r}]] ']]]$
$\gg [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t}' \dot{+} [\dot{r}]$
$] ']]]] ; [[\mathrm{S2}' \gg [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}]$
$] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]]] ; [[\mathrm{L3.2(c)}' \gg [[[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t}' \dot{+} [\dot{r}]$
$] ']] \dot{\Rightarrow} [[[\dot{t}' \dot{+} [\dot{r}]] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[$
$\dot{t} \dot{+} [\dot{r}]] '']]]]] ; [[[[\mathrm{MP}' \triangleright [[[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t}' \dot{+} [\dot{r}]] ']]$
$\dot{\Rightarrow} [[[\dot{t}' \dot{+} [\dot{r}]] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]$
$] '']]]]] \triangleright [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t}' \dot{+} [\dot{r}]] ']]] \gg [[[\dot{t}' \dot{+} [\dot{r}]]$
$' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]]] ; [[[[$
$\mathrm{Taut2} \triangleright [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}]] ' \overset{p}{=} [[\dot{t} \dot{+}$
$[\dot{r}]] '']]]] \triangleright [[[\dot{t}' \dot{+} [\dot{r}]] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']]$
$\overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]]] \gg [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[$
$\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]]] ; [[\mathrm{S6}' \gg [[\dot{t} \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [$
$\dot{r}]] ']]] ; [[\mathrm{S2}' \gg [[[\dot{t} \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}'$
$]] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]]] ; [[[[\mathrm{MP}' \triangleright [[[\dot{t} \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}$
$]] ']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]]]] \triangleright [[\dot{t} \dot{+} [\dot{r}']] \overset{p}{=} [$
$[\dot{t} \dot{+} [\dot{r}]] ']]] \gg [[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]] ; [[$
$\mathrm{L3.2(d)}' \gg [[[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [$
$[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']] ']]]]]] ; [[[[$
$\mathrm{Taut2} \triangleright [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+}$
$[\dot{r}]] '']]]] \triangleright [[[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[[\dot{t} \dot{+} [\dot{r}']$
$] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']] ']]]]]] \gg [$
$[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']$
$] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']] ']]]]] ; [[[\mathrm{Taut1} \triangleright [[[\dot{t}' \dot{+} [\dot{r}$
$]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']] \dot{\Rightarrow} [[$
$\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']] ']]]]]] \gg [[[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]$
$] '']] \dot{\Rightarrow} [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[$
$\dot{t} \dot{+} [\dot{r}']] ']]]]]] ; [[[[\mathrm{MP}' \triangleright [[[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]$
$\dot{\Rightarrow} [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']$
$] ']]]]]] \triangleright [[\dot{t} \dot{+} [\dot{r}']] ' \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] '']]] \gg [[[\dot{t}' \dot{+} [\dot{r}]]$
$\overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']] ']]]] ; [[[$
$\mathrm{Gen}' \triangleright [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']] \overset{p}{=} [[\dot{t} \dot{+} [$
$\dot{r}']] ']]]] \gg \dot{\forall} \dot{r} \colon [[[\dot{t}' \dot{+} [\dot{r}]] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}]] ']] \dot{\Rightarrow} [[\dot{t}' \dot{+} [\dot{r}']$
$] \overset{p}{=} [[\dot{t} \dot{+} [\dot{r}']] ']]]] ; [[\mathrm{S9}' \gg [[[\dot{t}' \dot{+} \dot{0}] \overset{p}{=} [[\dot{t} \dot{+} \dot{0}] ']] \dot{\Rightarrow} [$

$[\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}' \dotplus [\ \dot{r}'\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}'$ $]\ ]'\ ]\ ]\ ]\ ] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ [$ $\mathrm{MPTwice} \rhd [\ [\ [\ \dot{t}' \dotplus \dot{0}\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus \dot{0}\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [$ $\dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}' \dotplus [\ \dot{r}'\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}'\ ]\ ]'\ ]\ ]\ ]\ ] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [$ $\dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ]\ ] \rhd [\ [\ \dot{t}' \dotplus \dot{0}\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus \dot{0}\ ]'\ ]\ ]\ ] \rhd \dot{\forall}\dot{r}\colon [\ [$ $[\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}' \dotplus [\ \dot{r}'\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}'\ ]\ ]'\ ]\ ]\ ]$ $]\ \gg \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Gen}' \rhd \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ]$ $\overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ] \gg \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ]$ $]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{L3.2(g)}' \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]]$

$[\mathrm{L3.2(g)}' \overset{\mathrm{tex}}{\rightarrow}$ "L3.2 (g)'"]

$[\mathrm{L3.2(g)}' \overset{\mathrm{pyk}}{\rightarrow}$ "lemma prime three two g"]

L3.2(g)'II

$[\mathrm{L3.2(g)'II} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ \mathrm{L3.2(g)}' \gg \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus$ $[\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ;\ [\ \lceil\dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\rceil \equiv \langle \lceil\dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]$ $]\ \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\rceil | \lceil\dot{t}\rceil := \lceil\dot{s}\rceil\rangle \Vdash [\ [\ [\ \mathrm{A4}' \unrhd \lceil\dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [$ $\dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\rceil \equiv \langle \lceil\dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\rceil | \lceil\dot{t}\rceil := \lceil\dot{s}\rceil\rangle\ ] \gg$ $[\ [\ \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [$ $\dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]$ $'\ ]\ ]\ ] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dotplus [\ \dot{r}\ ]\ ]$ $\overset{p}{=} [\ [\ \dot{t} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ] \gg \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ;\ [\ \lceil\ [$ $\dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\rceil \equiv \langle \lceil\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]$ $\rceil | \lceil\dot{r}\rceil := \lceil\dot{t}\rceil\rangle \Vdash [\ [\ [\ \mathrm{A4}' \unrhd \lceil\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\rceil \equiv \langle \lceil\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]$ $]\ \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\rceil | \lceil\dot{r}\rceil := \lceil\dot{t}\rceil\rangle\ ] \gg [\ [\ \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]$ $'\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\dot{r}\colon [$ $[\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]$ $]\ ]\ \rhd \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\ ] \gg [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus$ $[\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Gen}' \rhd [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ] \gg \dot{\forall}\dot{s}\colon [\ [$ $\dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ;\ [\ \lceil\ [\ \dot{r}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{r} \dotplus [\ \dot{t}\ ]\ ]'\ ]\rceil \equiv \langle \lceil$ $[\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\rceil | \lceil\dot{s}\rceil := \lceil\dot{r}\rceil\rangle \Vdash [\ [\ [\ \mathrm{A4}' \unrhd \lceil\ [\ \dot{r}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=}$ $[\ [\ \dot{r} \dotplus [\ \dot{t}\ ]\ ]'\ ]\rceil \equiv \langle \lceil\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\rceil | \lceil\dot{s}\rceil := \lceil\dot{r}\rceil\rangle\ ] \gg [\ [$ $\dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{r}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{r} \dotplus [\ \dot{t}\ ]\ ]$ $'\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ] \dot{\Rightarrow} [\ [$ $\dot{r}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{r} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{t}\ ]\ ]'$ $]\ ]\ ] \gg [\ [\ \dot{r}' \dotplus [\ \dot{t}\ ]\ ] \overset{p}{=} [\ [\ \dot{r} \dotplus [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{L3.2(g)'II} \overset{\mathrm{stmt}}{\rightarrow} S' \vdash [\ \lceil\dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dotplus [\ \dot{r}\ ]\ ] \overset{p}{=} [\ [\ \dot{s} \dotplus [\ \dot{r}\ ]\ ]'\ ]\ ]\rceil \equiv \langle \lceil\dot{\forall}\dot{r}\colon [\ [$

$\dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]'\ ]\ ]\ \rceil || \lceil \dot{t} \rceil := \lceil \dot{s} \rceil \rangle \Vdash [\ \lceil [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]'\ ]\ \rceil \equiv \langle \lceil [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]'\ ]\ \rceil || \lceil \dot{r} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [\ \lceil [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]'\ ]\ \rceil \equiv \langle \lceil [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]'\ ]\ \rceil || \lceil \dot{s} \rceil := \lceil \dot{r} \rceil \rangle \Vdash [\ [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ]\ ]]$

[L3.2(g)′II $\stackrel{\text{tex}}{\rightarrow}$ "L3.2 (g)' II"]

[L3.2(g)′II $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime three two g rev"]

L3.2(h)′basis

[L3.2(h)′basis $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ S5' \gg [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{t}\ ]\ ]\ ]\ ; [\ [\ L3.2(f)' \gg \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ; [\ [\lceil \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ \rceil \equiv \langle \lceil \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ \rceil || \lceil \dot{t} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [\ [\ [\ A4' \rhd\!\!\rhd \lceil \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ \rceil \equiv \langle \lceil \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ \rceil || \lceil \dot{t} \rceil := \lceil \dot{t} \rceil \rangle\ ] \gg [\ [\ \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ MP' \rhd [\ [\ \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{t}\colon [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ] \gg [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ; [\ [\ L3.2(c)' \gg [\ [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{t}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ [\ MPTwice \rhd [\ [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{t}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{t}\ ]\ ]\ ]\ \rhd [\ \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ \gg [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ; [\ [\ S9' \gg [\ [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ MP' \rhd [\ [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ \gg [\ [\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[L3.2(h)′basis $\stackrel{\text{stmt}}{\rightarrow} S' \vdash [\ \lceil \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ \rceil \equiv \langle \lceil \dot{t} \stackrel{p}{=} [\ \dot{0} \dot{+} [\ \dot{t}\ ]\ ]\ \rceil || \lceil \dot{t} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [\ [\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ \dot{\Rightarrow} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ ]\ ]\ ]\ ]]$

[L3.2(h)′basis $\stackrel{\text{tex}}{\rightarrow}$ "L3.2 (h)' basis"]

[L3.2(h)′basis $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime three two h base"]

L3.2(h)′

[L3.2(h)′ $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\ [\ S6' \gg [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]'\ ]\ ]\ ]\ ; [\ [\ L3.2(g)'II \gg [\ [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]'\ ]\ ]\ ]\ ; [\ [\ S2' \gg [\ [\ [$

$\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}]]' \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]] ; [[$
$\mathrm{L3.2(c)}' \gg [[[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{t} \dot{+} [\dot{r}]]']] \dot{\Rightarrow} [[[\dot{t} \dot{+} [\dot{r}]]' \stackrel{p}{=} [[$
$\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]]]] ; [[[[\mathrm{MP}' \rhd$
$[[[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{t} \dot{+} [\dot{r}]]']] \dot{\Rightarrow} [[[\dot{t} \dot{+} [\dot{r}]]' \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]'$
$]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]]] \rhd [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{t} \dot{+}$
$[\dot{r}]]']]] \gg [[[\dot{t} \dot{+} [\dot{r}]]' \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=}$
$[[\dot{r} \dot{+} [\dot{t}]]']]]] ; [[[[\mathrm{Taut2} \rhd [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow}$
$[[\dot{t} \dot{+} [\dot{r}]]' \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]] \rhd [[[\dot{t} \dot{+} [\dot{r}]]' \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]$
$']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]]] \gg [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+}$
$[\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]] ; [[\mathrm{L3.2(d)}' \gg [[[$
$\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[[\dot{r}' \dot{+} [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]$
$\dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]]] ; [[[[\mathrm{Taut2} \rhd [[[\dot{t} \dot{+} [\dot{r}]]$
$\stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]]] \rhd [[[\dot{t} \dot{+} [\dot{r}'$
$]] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[[\dot{r}' \dot{+} [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[$
$\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]]] \gg [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [$
$[[\dot{r}' \dot{+} [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]]$
$] ; [[[[\mathrm{Taut4} \rhd [[\dot{r}' \dot{+} [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']]] \rhd [[[\dot{t} \dot{+} [\dot{r}]]$
$\stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[[\dot{r}' \dot{+} [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dot{+} [\dot{t}]]']] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']]$
$\stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]]] \gg [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']$
$] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]] ; [[[\mathrm{Gen}' \rhd [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [$
$[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]] \gg \dot{\forall} \dot{r}\colon [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow}$
$[[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]] ; [[\mathrm{L3.2(h)}'\mathrm{basis} \gg [[\dot{\forall} \dot{r}\colon [[[\dot{t} \dot{+} [\dot{r}$
$]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]] \dot{\Rightarrow} \dot{\forall} \dot{r}\colon [[\dot{t} \dot{+} [$
$\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]]]]] ; [[[[\mathrm{MP}' \rhd [[\dot{\forall} \dot{r}\colon [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}$
$]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+} [\dot{t}]]]]]] \dot{\Rightarrow} \dot{\forall} \dot{r}\colon [[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [$
$\dot{t}]]]]]] \rhd \dot{\forall} \dot{r}\colon [[[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dot{+} [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dot{+}$
$[\dot{t}]]]]]] \gg \dot{\forall} \dot{r}\colon [[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]]]] ; [[\mathrm{Gen}' \rhd \dot{\forall} \dot{r}\colon [[\dot{t} \dot{+} [$
$\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]]]] \gg \dot{\forall} \dot{t}\colon \dot{\forall} \dot{r}\colon [[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]]]]]]]]]]$
$]]]]]]] \rceil, p_0, c)]$

$[\mathrm{L3.2(h)}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall} \dot{t}\colon \dot{\forall} \dot{r}\colon [[\dot{t} \dot{+} [\dot{r}]] \stackrel{p}{=} [\dot{r} \dot{+} [\dot{t}]]]]$

$[\mathrm{L3.2(h)}' \stackrel{\mathrm{tex}}{\rightarrow}$ "L3.2 (h)'"]

$[\mathrm{L3.2(h)}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma prime three two h"]

Taut1

$[\mathrm{Taut1} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \vdash [[\mathrm{A2}' \gg [[\underline{a} \dot{\Rightarrow}$
$[\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] ; [[[[\mathrm{MP}' \rhd [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}$
$]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \rhd [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]]] \gg [[\underline{a} \dot{\Rightarrow} \underline{b}]$
$\dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] ; [[[\mathrm{Weaken} \rhd [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] \gg [\underline{b} \dot{\Rightarrow} [[$

$\underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MPTwice} \rhd [\ [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{b} \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ \rhd [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ \gg [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{Taut1} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \vdash\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]$

$[\mathrm{Taut1} \stackrel{\text{tex}}{\rightarrow}$ "Taut 1"$]$

$[\mathrm{Taut1} \stackrel{\text{pyk}}{\rightarrow}$ "lemma tautology one"$]$

Taut2

$[\mathrm{Taut2} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{d}\colon \forall \underline{e}\colon \forall \underline{f}\colon [\ [\ \underline{d} \dot{\Rightarrow} \underline{e}\ ]\ \vdash\ [\ [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ \vdash\ [\ [\ \mathrm{A1}' \gg [\ [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ \dot{\Rightarrow}\ [\ \underline{d} \dot{\Rightarrow} [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ \dot{\Rightarrow}\ [\ \underline{d} \dot{\Rightarrow} [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ \gg [\ \underline{d} \dot{\Rightarrow} [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{d} \dot{\Rightarrow} [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{d} \dot{\Rightarrow} \underline{e}\ ]\ \dot{\Rightarrow}\ [\ \underline{d} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MPTwice} \rhd [\ [\ \underline{d} \dot{\Rightarrow} [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{d} \dot{\Rightarrow} \underline{e}\ ]\ \dot{\Rightarrow}\ [\ \underline{d} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{d} \dot{\Rightarrow} [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ \rhd [\ \underline{d} \dot{\Rightarrow} \underline{e}\ ]\ ]\ \gg [\ \underline{d} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{Taut2} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{d}\colon \forall \underline{e}\colon \forall \underline{f}\colon [\ [\ \underline{d} \dot{\Rightarrow} \underline{e}\ ]\ \vdash\ [\ [\ \underline{e} \dot{\Rightarrow} \underline{f}\ ]\ \vdash\ [\ \underline{d} \dot{\Rightarrow} \underline{f}\ ]\ ]\ ]\ ]$

$[\mathrm{Taut2} \stackrel{\text{tex}}{\rightarrow}$ "Taut 2"$]$

$[\mathrm{Taut2} \stackrel{\text{pyk}}{\rightarrow}$ "lemma tautology two"$]$

Taut3

$[\mathrm{Taut3} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \vdash\ [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \vdash\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MPTwice} \rhd [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ \rhd [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ]\ \gg [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{Taut3} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \vdash\ [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \vdash\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]$

$[\mathrm{Taut3} \stackrel{\text{tex}}{\rightarrow}$ "Taut 3"$]$

$[\mathrm{Taut3} \stackrel{\text{pyk}}{\rightarrow}$ "lemma tautology three"$]$

Taut4

$[\mathrm{Taut4} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ \underline{a} \vdash [\ [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \vdash\ [\ [\ [\ \mathrm{Taut1} \rhd [\ \underline{b} \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ \gg [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ \rhd \underline{a}\ ]\ \gg [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{Taut4} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ \underline{\mathsf{a}} \vdash [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]]$

$[\mathrm{Taut4} \overset{\mathrm{tex}}{\rightarrow}$ “Taut 4”$]$

$[\mathrm{Taut4} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma tautology four”$]$

Weaken

$[\mathrm{Weaken} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \underline{\mathsf{a}} \vdash [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{MP}' \rhd [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ] \rhd \underline{\mathsf{a}}\ ] \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ ] \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Weaken} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \underline{\mathsf{a}} \vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]]$

$[\mathrm{Weaken} \overset{\mathrm{tex}}{\rightarrow}$ “Weaken”$]$

$[\mathrm{Weaken} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma weaken”$]$

M1.7

$[\mathrm{M1.7} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{b}} \colon [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ; [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ] \rhd [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ; [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ] \rhd [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ] \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{M1.7} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{b}} \colon [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]]$

$[\mathrm{M1.7} \overset{\mathrm{tex}}{\rightarrow}$ “
M1.7”$]$

$[\mathrm{M1.7} \overset{\mathrm{pyk}}{\rightarrow}$ “mendelson one seven”$]$

MPTwice

$[\mathrm{MPTwice} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \vdash [\ \underline{\mathsf{a}} \vdash [\ \underline{\mathsf{b}} \vdash [\ [\ [\ [\ \mathrm{MP}' \rhd [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ] \rhd \underline{\mathsf{a}}\ ] \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ; [\ [\ [\ \mathrm{MP}' \rhd [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \rhd \underline{\mathsf{b}}\ ] \gg \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ] \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MPTwice} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \vdash [\ \underline{\mathsf{a}} \vdash [\ \underline{\mathsf{b}} \vdash \underline{\mathsf{c}}\ ]\ ]\ ]]$

$[\mathrm{MPTwice} \overset{\mathrm{tex}}{\rightarrow}$ “MPTwice”$]$

$[\mathrm{MPTwice} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma mp twice”$]$

$\dot{*}$

$[\dot{\mathsf{x}} \stackrel{\text{tex}}{\to}$ "
\dot{#1.
}"]

$[\dot{\mathsf{x}} \stackrel{\text{pyk}}{\to}$ "$*$ peano var"]

$*'$

$[\mathsf{x}' \stackrel{\text{tex}}{\to}$ "#1.'"]

$[\mathsf{x}' \stackrel{\text{pyk}}{\to}$ "$*$ peano succ"]

$* \mathop{\dot{\cdot}} *$

$[\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathop{\dot{\cdot}} #2."]

$[\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "$*$ peano times $*$"]

$* \mathop{\dot{+}} *$

$[\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathop{\dot{+}} #2."]

$[\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "$*$ peano plus $*$"]

$* \stackrel{p}{=} *$

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\stackrel{p}{=} #2."]

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "$*$ peano is $*$"]

$*^{\mathcal{P}}$

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{val}}{\to} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil]$

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{tex}}{\to}$ "#1.
{} ^ {\cal P}"]

$[\mathsf{x}^{\mathcal{P}} \overset{\text{pyk}}{\to}$ "$*$ is peano var"$]$

$\dot{\neg}\, *$

$[\dot{\neg}\, \mathsf{x} \overset{\text{tex}}{\to}$ "
\dot{\neg}\, #1."]

$[\dot{\neg}\, \mathsf{x} \overset{\text{pyk}}{\to}$ "peano not $*$"$]$

$\dot{\forall} *\colon *$

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\to}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\to}$ "peano all $*$ indeed $*$"$]$

$* \mathrel{\dot{\Rightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \overset{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \overset{\text{pyk}}{\to}$ "$*$ peano imply $*$"$]$

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-30.UTC:07:14:31.615667 = MJD-53551.TAI:07:15:03.615667 = LGT-4626832503615667e-6