

Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\text{nonfree}(*, *)$, $\text{nonfree}^*(*, *)$, $\text{free}(*|* := *)$, $\text{free}^*(** := *)$, $*\equiv(*|* := *)$, $*\equiv^*(** := *)$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S', A1', A2', A3', A4', A5', S1', S2', S3', S4', S5', S6', S7', S8', S9', MP', Gen', L3.2(a)', M1.7, MP'_h, Hypothesize, Gen'_h, M3.2(a), M3.2(a)_h, M3.2(b)_h, M3.1(S1')_h, M3.2(c)_h, M3.1(S2')_h, M3.1(S5')_h, M3.1(S6')_h, M3.2(f), $\dot{*}$, $\dot{*}'$, $\dot{*} : *$, $\dot{*} \dot{+} *$, $\dot{*} \dot{=} *$, $\dot{*} \dot{P}$, $\dot{*} \dot{\wedge} *$, $\dot{*} \dot{\vee} *$, $\dot{\vee} \dot{*} : *$, $\dot{\exists} \dot{*} : *$, $\dot{*} \dot{\Rightarrow} *$, $\dot{*} \dot{\Leftrightarrow} *$, $\dot{*} \dot{\supseteq} *$, $\dot{*} \dot{\supseteq}_h *$,

peano

[peano $\xrightarrow{\text{prio}}$

Preassociative

[peano], [base], [bracket * end bracket], [big bracket * end bracket], [math * end math], [**flush left** *], [x], [y], [z], [$[* \bowtie *]$], [$[* \rightarrow *]$], [pyk], [tex], [name], [prio], [*], [T], [if(*, *, *)], [$[* \xRightarrow{*} *]$], [val], [claim], [$\perp$], [f(*)], [$(*)^I$], [F], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^M$], [If(*, *, *)], [array{*} * end array], [l], [c], [r], [empty], [$\langle * | * := * \rangle$], [$\mathcal{M}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^M(*)$], [**apply**(*, *)], [**apply**₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)], [bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *, *)$], [$\mathcal{E}_4(*, *, *, *, *)$], [**lookup**(*, *, *)], [**abstract**(*, *, *, *)], [$[*]$], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [s₀], [**zip**(*, *)], [**assoc**₁(*, *, *)], [$(*)^P$], [self], [$[* \ddot{=}]$], [$[* \dot{=}]$], [$[* \dot{=}^*]$], [$[* \stackrel{\text{pyk}}{=}]$], [$[* \stackrel{\text{tex}}{=}]$], [$[* \stackrel{\text{name}}{=}]$], [**Priority table**[*]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\mathcal{M}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [$\langle * \rangle$], [**aspect**(*, *)], [**aspect**(*, *, *)], [$\langle * \rangle$], [**tuple**₁(*)], [**tuple**₂(*)], [let₂(*, *)], [let₁(*, *)], [$[* \stackrel{\text{claim}}{=}]$], [checker], [**check**(*, *)], [**check**₂(*, *, *)], [**check**₃(*, *, *)], [**check**^{*}(*, *)], [**check**₂^{*}(*, *, *)], [$[*]'$], [$[*]^-$], [$[*]^\circ$], [msg], [$[* \stackrel{\text{msg}}{=}]$], [$\langle \text{stmt} \rangle$], [stmt], [$[* \stackrel{\text{stmt}}{=}]$], [HeadNil'], [HeadPair'], [Transitivity'], [$\perp$], [Contra'], [T_E], [L₁], [$\underline{x}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$],

$[\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [(* \mid * := *)], [(* \mid * := *)], [\emptyset], [\text{Remainder}],$
 $[(*)^\vee], [\text{intro}(*, *, *, *)], [\text{intro}(*, *, *)], [\text{error}(*, *)], [\text{error}_2(*, *)], [\text{proof}(*, *, *)],$
 $[\text{proof}_2(*, *)], [\mathcal{S}(*, *)], [\mathcal{S}^I(*, *)], [\mathcal{S}^\triangleright(*, *)], [\mathcal{S}_1^\triangleright(*, *, *)], [\mathcal{S}^E(*, *)], [\mathcal{S}_1^E(*, *, *)],$
 $[\mathcal{S}^+(*, *)], [\mathcal{S}_1^+(*, *, *)], [\mathcal{S}^-(*, *)], [\mathcal{S}_1^-(*, *, *)], [\mathcal{S}^*(*, *)], [\mathcal{S}_1^*(*, *, *)],$
 $[\mathcal{S}_2^*(*, *, *, *)], [\mathcal{S}^\oplus(*, *)], [\mathcal{S}_1^\oplus(*, *, *, *)], [\mathcal{S}^\perp(*, *)], [\mathcal{S}_1^\perp(*, *, *, *)], [\mathcal{S}^\oplus(*, *)],$
 $[\mathcal{S}_1^\oplus(*, *, *, *)], [\mathcal{S}^{\text{i.e.}}(*, *)], [\mathcal{S}_1^{\text{i.e.}}(*, *, *, *)], [\mathcal{S}_2^{\text{i.e.}}(*, *, *, *, *)], [\mathcal{S}^\forall(*, *)],$
 $[\mathcal{S}_1^\forall(*, *, *, *, *)], [\mathcal{S}^{\text{i}}(*, *)], [\mathcal{S}_1^{\text{i}}(*, *, *, *)], [\mathcal{S}_2^{\text{i}}(*, *, *, *, *)], [\mathcal{T}(*)], [\text{claims}(*, *, *)],$
 $[\text{claims}_2(*, *, *)], [<\text{proof}>], [\text{proof}], [[\text{Lemma } *: *]], [[\text{Proof of } *: *]],$
 $[[* \text{ lemma } *: *]], [[* \text{ antilemma } *: *]], [[* \text{ rule } *: *]], [[* \text{ antirule } *: *]],$
 $[\text{verifier}], [\mathcal{V}_1(*)], [\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)],$
 $[\mathcal{V}_7(*, *, *, *)], [\text{Cut}(*, *)], [\text{Head}_\oplus(*)], [\text{Tail}_\oplus(*)], [\text{rule}_1(*, *)], [\text{rule}(*, *)],$
 $[\text{Rule tactic}], [\text{Plus}(*, *)], [[\text{Theory } *]], [\text{theory}_2(*, *)], [\text{theory}_3(*, *)],$
 $[\text{theory}_4(*, *, *)], [\text{HeadNil}'], [\text{HeadPair}'], [\text{Transitivity}'], [\text{Contra}'], [\text{HeadNil}],$
 $[\text{HeadPair}], [\text{Transitivity}], [\text{Contra}], [\text{T}_E], [\text{ragged right}],$
 $[\text{ragged right expansion }], [\text{parm}(*, *, *)], [\text{parm}^*(*, *, *)], [\text{inst}(*, *)],$
 $[\text{inst}^*(*, *)], [\text{occur}(*, *, *)], [\text{occur}^*(*, *, *)], [\text{unify}(* = *, *)], [\text{unify}^*(* = *, *)],$
 $[\text{unify}_2(* = *, *)], [\text{L}_a], [\text{L}_b], [\text{L}_c], [\text{L}_d], [\text{L}_e], [\text{L}_f], [\text{L}_g], [\text{L}_h], [\text{L}_i], [\text{L}_j], [\text{L}_k], [\text{L}_l], [\text{L}_m],$
 $[\text{L}_n], [\text{L}_o], [\text{L}_p], [\text{L}_q], [\text{L}_r], [\text{L}_s], [\text{L}_t], [\text{L}_u], [\text{L}_v], [\text{L}_w], [\text{L}_x], [\text{L}_y], [\text{L}_z], [\text{L}_A], [\text{L}_B], [\text{L}_C],$
 $[\text{L}_D], [\text{L}_E], [\text{L}_F], [\text{L}_G], [\text{L}_H], [\text{L}_I], [\text{L}_J], [\text{L}_K], [\text{L}_L], [\text{L}_M], [\text{L}_N], [\text{L}_O], [\text{L}_P], [\text{L}_Q], [\text{L}_R],$
 $[\text{L}_S], [\text{L}_T], [\text{L}_U], [\text{L}_V], [\text{L}_W], [\text{L}_X], [\text{L}_Y], [\text{L}_Z], [\text{L}_?], [\text{Reflexivity}], [\text{Reflexivity}_1],$
 $[\text{Commutativity}], [\text{Commutativity}_1], [<\text{tactic}>], [\text{tactic}], [[* \stackrel{\text{tactic}}{=} *]], [\mathcal{P}(*, *, *)],$
 $[\mathcal{P}^*(*, *, *)], [\text{p}_0], [\text{conclude}_1(*, *)], [\text{conclude}_2(*, *, *)], [\text{conclude}_3(*, *, *, *)],$
 $[\text{conclude}_4(*, *)], [\hat{0}], [\hat{1}], [\hat{2}], [\hat{a}], [\hat{b}], [\hat{c}], [\hat{d}], [\hat{e}], [f], [\hat{g}], [\hat{h}], [\hat{i}], [\hat{j}], [\hat{k}], [\hat{l}], [\hat{m}], [\hat{n}],$
 $[\hat{o}], [\hat{p}], [\hat{q}], [\hat{r}], [\hat{s}], [\hat{t}], [\hat{u}], [\hat{v}], [\hat{w}], [\hat{x}], [\hat{y}], [\hat{z}], [\text{nonfree}(*, *)], [\text{nonfree}^*(*, *)],$
 $[\text{free}(* \mid * := *)], [\text{free}^*(* \mid * := *)], [* \equiv (* \mid * := *)], [* \equiv^*(* \mid * := *)], [\text{S}], [\text{A1}], [\text{A2}],$
 $[\text{A3}], [\text{A4}], [\text{A5}], [\text{S1}], [\text{S2}], [\text{S3}], [\text{S4}], [\text{S5}], [\text{S6}], [\text{S7}], [\text{S8}], [\text{S9}], [\text{MP}], [\text{Gen}],$
 $[\text{L3.2(a)}], [\text{S}'], [\text{A1}'], [\text{A2}'], [\text{A3}'], [\text{A4}'], [\text{A5}'], [\text{S1}'], [\text{S2}'], [\text{S3}'], [\text{S4}'], [\text{S5}'], [\text{S6}'],$
 $[\text{S7}'], [\text{S8}'], [\text{S9}'], [\text{MP}'], [\text{Gen}'], [\text{L3.2(a)}'], [\text{M1.7}], [\text{MP}'_h], [\text{Hypothesize}], [\text{Gen}'_h],$
 $[\text{M3.2(a)}], [\text{M3.2(a)}_h], [\text{M3.2(b)}_h], [\text{M3.1(S1')}_h], [\text{M3.2(c)}_h], [\text{M3.1(S2')}_h],$
 $[\text{M3.1(S5')}_h], [\text{M3.1(S6')}_h], [\text{M3.2(f)}];$

Preassociative

$[_*\{*\}], [*'], [* [*]], [* \multimap *], [* \Rightarrow *], [*];$

Preassociative

$[" * "], [], [(*)^\dagger], [\text{string}(* + *)], [\text{string}(* ++ *)], [$
 $*, [*], [! *], [" *], [\# *], [\$ *], [\% *], [\& *], [' *], [(*), () *], [* *], [+ *], [, *], [- *], [. *], [/ *],$
 $[0 *], [1 *], [2 *], [3 *], [4 *], [5 *], [6 *], [7 *], [8 *], [9 *], [: *], [; *], [< *], [= *], [> *], [? *],$
 $[@ *], [A *], [B *], [C *], [D *], [E *], [F *], [G *], [H *], [I *], [J *], [K *], [L *], [M *], [N *],$
 $[O *], [P *], [Q *], [R *], [S *], [T *], [U *], [V *], [W *], [X *], [Y *], [Z *], [[*], [\setminus *], [] *], [^ *],$
 $[_ *], [` *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *],$
 $[p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [\{ *], [| *], [} *], [~ *],$
 $[\text{Preassociative } *; *], [\text{Postassociative } *; *], [[*], *], [\text{priority } * \text{ end}],$
 $[\text{newline } *], [\text{macro newline } *];$

Preassociative

$[*0], [*1], [0b], [* \text{-color}(*)], [* \text{-color}^*(*)];$

Preassociative

$[* ' *], [* ' *];$

Preassociative

$[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*^i], [*^d], [*^R], [*^0],$
 $[*^1], [*^2], [*^3], [*^4], [*^5], [*^6], [*^7], [*^8], [*^9], [*^E], [*^V], [*^C], [*^{C*}], [*^'];$

Preassociative

$[* \cdot *], [* \cdot_0 *], [* \dot{\cdot} *];$

Preassociative

$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *];$

Preassociative

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$

Postassociative

$[* \dot{:} *], [* \dot{:} *], [* \ddot{:} *], [* \underline{+2*}], [* \ddot{:} *], [* +2* *];$

Postassociative

$[*, *];$

Preassociative

$[* \overset{B}{\sim} *], [* \overset{D}{\sim} *], [* \overset{C}{\sim} *], [* \overset{P}{\sim} *], [* \approx *], [* = *], [* \overset{+}{\rightarrow} *], [* \overset{t}{\leftarrow} *], [* \overset{t}{=} *], [* \overset{r}{=} *],$
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{free in } *], [* \text{free in}^* *], [* \text{free for } * \text{ in } *],$
 $[* \text{free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* \overset{P}{=} *], [*^{\mathcal{P}}];$

Preassociative

$[\neg *], [\dot{\neg} *];$

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$

Preassociative

$[\dot{\forall} *: *], [\dot{\exists} *: *];$

Postassociative

$[* \rightrightarrows *], [* \Rightarrow *], [* \Leftrightarrow *];$

Postassociative

$[* : *], [* ! *];$

Preassociative

$[* \begin{Bmatrix} * \\ * \end{Bmatrix}];$

Preassociative

$[\lambda *. *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \doteq * \text{ in } *];$

Preassociative

$[*^I], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$

Preassociative

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *], [* \supseteq *], [* \supseteq_h *];$

Postassociative

$[* \vdash *], [* \Vdash *], [* \text{i.e. } *];$

Preassociative

$[\forall *: *];$

Postassociative

$[* \oplus *];$

Postassociative

[*,*];

Preassociative

[* proves *];

Preassociative

[* **proof of** * : *], [Line * : * $\gg$ *; *], [Last line * $\gg$ * $\square$],
[Line * : Premise $\gg$ *; *], [Line * : Side-condition $\gg$ *; *], [Arbitrary $\gg$ *; *],
[Local $\gg$ * = *; *];

Postassociative

[* then *], [* [*] *];

Preassociative

[*&*];

Preassociative

[**];]

[peano $\xrightarrow{\text{pyk}}$ “peano”]

$\dot{0}$

[$\dot{0} \xrightarrow{\text{tex}}$ “
\dot{0}”]

[$\dot{0} \xrightarrow{\text{pyk}}$ “peano zero”]

$\dot{1}$

[$\dot{1} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{1} \doteq \dot{0}'] \rrbracket)]$

[$\dot{1} \xrightarrow{\text{tex}}$ “
\dot{1}”]

[$\dot{1} \xrightarrow{\text{pyk}}$ “peano one”]

$\dot{2}$

[$\dot{2} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{2} \doteq \dot{1}'] \rrbracket)]$

[$\dot{2} \xrightarrow{\text{tex}}$ “
\dot{2}”]

[$\dot{2} \xrightarrow{\text{pyk}}$ “peano two”]

$\dot{a}$

$[\dot{a} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{a} \doteq \dot{a}]])]$
 $[\dot{a} \xrightarrow{\text{tex}} “\dot{\mathit{a}}”]$
 $[\dot{a} \xrightarrow{\text{pyk}} “\text{peano a}”]$

$\dot{b}$

$[\dot{b} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{b} \doteq \dot{b}]])]$
 $[\dot{b} \xrightarrow{\text{tex}} “\dot{\mathit{b}}”]$
 $[\dot{b} \xrightarrow{\text{pyk}} “\text{peano b}”]$

$\dot{c}$

$[\dot{c} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{c} \doteq \dot{c}]])]$
 $[\dot{c} \xrightarrow{\text{tex}} “\dot{\mathit{c}}”]$
 $[\dot{c} \xrightarrow{\text{pyk}} “\text{peano c}”]$

$\dot{d}$

$[\dot{d} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{d} \doteq \dot{d}]])]$
 $[\dot{d} \xrightarrow{\text{tex}} “\dot{\mathit{d}}”]$
 $[\dot{d} \xrightarrow{\text{pyk}} “\text{peano d}”]$

$\dot{e}$

$[\dot{e} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{e} \doteq \dot{e}]])]$
 $[\dot{e} \xrightarrow{\text{tex}} “\dot{\mathit{e}}”]$
 $[\dot{e} \xrightarrow{\text{pyk}} “\text{peano e}”]$

$\dot{f}$

$[\dot{f} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{f} \doteq \mathring{f}] \rrbracket)]$

$[\dot{f} \xrightarrow{\text{tex}} “\dot{\mathit{f}}”]$

$[\dot{f} \xrightarrow{\text{pyk}} “\text{peano f}”]$

$\dot{g}$

$[\dot{g} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{g} \doteq \mathring{g}] \rrbracket)]$

$[\dot{g} \xrightarrow{\text{tex}} “\dot{\mathit{g}}”]$

$[\dot{g} \xrightarrow{\text{pyk}} “\text{peano g}”]$

$\dot{h}$

$[\dot{h} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{h} \doteq \mathring{h}] \rrbracket)]$

$[\dot{h} \xrightarrow{\text{tex}} “\dot{\mathit{h}}”]$

$[\dot{h} \xrightarrow{\text{pyk}} “\text{peano h}”]$

$\dot{i}$

$[\dot{i} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{i} \doteq \mathring{i}] \rrbracket)]$

$[\dot{i} \xrightarrow{\text{tex}} “\dot{\mathit{i}}”]$

$[\dot{i} \xrightarrow{\text{pyk}} “\text{peano i}”]$

$\dot{j}$

$[\dot{j} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{j} \doteq \mathring{j}] \rrbracket)]$

$[\dot{j} \xrightarrow{\text{tex}} “\dot{\mathit{j}}”]$

$[\dot{j} \xrightarrow{\text{pyk}} “\text{peano j}”]$

$\dot{k}$

$[\dot{k} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{k} \doteq \dot{k}]])]$

$[\dot{k} \xrightarrow{\text{tex}} “\dot{\mathit{k}}”]$

$[\dot{k} \xrightarrow{\text{pyk}} “\text{peano k}”]$

$\dot{l}$

$[\dot{l} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{l} \doteq \dot{l}]])]$

$[\dot{l} \xrightarrow{\text{tex}} “\dot{\mathit{l}}”]$

$[\dot{l} \xrightarrow{\text{pyk}} “\text{peano l}”]$

$\dot{m}$

$[\dot{m} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{m} \doteq \dot{m}]])]$

$[\dot{m} \xrightarrow{\text{tex}} “\dot{\mathit{m}}”]$

$[\dot{m} \xrightarrow{\text{pyk}} “\text{peano m}”]$

$\dot{n}$

$[\dot{n} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{n} \doteq \dot{n}]])]$

$[\dot{n} \xrightarrow{\text{tex}} “\dot{\mathit{n}}”]$

$[\dot{n} \xrightarrow{\text{pyk}} “\text{peano n}”]$

$\dot{o}$

$[\dot{o} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{o} \doteq \dot{o}]])]$

$[\dot{o} \xrightarrow{\text{tex}} “\dot{\mathit{o}}”]$

$[\dot{o} \xrightarrow{\text{pyk}} “\text{peano o}”]$

$\dot{p}$

$[\dot{p} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{p} \doteq \dot{p}] \rrbracket)]$

$[\dot{p} \xrightarrow{\text{tex}} “\dot{\mathit{p}}”]$

$[\dot{p} \xrightarrow{\text{pyk}} “\text{peano } p”]$

$\dot{q}$

$[\dot{q} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{q} \doteq \dot{q}] \rrbracket)]$

$[\dot{q} \xrightarrow{\text{tex}} “\dot{\mathit{q}}”]$

$[\dot{q} \xrightarrow{\text{pyk}} “\text{peano } q”]$

$\dot{r}$

$[\dot{r} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{r} \doteq \dot{r}] \rrbracket)]$

$[\dot{r} \xrightarrow{\text{tex}} “\dot{\mathit{r}}”]$

$[\dot{r} \xrightarrow{\text{pyk}} “\text{peano } r”]$

$\dot{s}$

$[\dot{s} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{s} \doteq \dot{s}] \rrbracket)]$

$[\dot{s} \xrightarrow{\text{tex}} “\dot{\mathit{s}}”]$

$[\dot{s} \xrightarrow{\text{pyk}} “\text{peano } s”]$

$\dot{t}$

$[\dot{t} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{t} \doteq \dot{t}] \rrbracket)]$

$[\dot{t} \xrightarrow{\text{tex}} “\dot{\mathit{t}}”]$

$[\dot{t} \xrightarrow{\text{pyk}} “\text{peano } t”]$

$\dot{\mathcal{U}}$
$$[\dot{u} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{u} \doteq \dot{u}]])]$$
$$[\dot{u}^{\text{tex}} \rightarrow \text{“}\dot{\mathit{u}}\text{”}]$$
$$[i \xrightarrow{\text{pyk}} \text{“peano u”}]$$
 $\dot{\nu}$
$$[\dot{v} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{v} \doteq \dot{v}]])]$$
$$[\dot{v}^{\text{tex}} \rightarrow “\dot{\mathit{v}}”]$$
$$[\dot{v} \xrightarrow{\text{pyk}} \text{“peano v”}]$$
 $\dot{w}$
$$[\dot{w} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{w} \doteq \dot{w}]])]$$
$$\left[\dot{w}^{\text{tex}} \rightarrow "\dot{\mathit{w}}"\right]$$
$$[\dot{w} \xrightarrow{\text{pyk}} \text{“peano w”}]$$
 $\dot{x}$
$$[\dot{x} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{x} \doteq \dot{x}]])]$$
$$\left[\dot{x} \stackrel{\text{tex}}{\rightarrow} “\dot{\mathit{x}}”\right]$$
$$[\dot{x} \xrightarrow{\text{pyk}} \text{“peano x”}]$$
 $\dot{y}$
$$[\dot{y} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{y} \doteq \dot{y}]])]$$
$$[\dot{y} \overset{\text{tex}}{\rightarrow} “\dot{\mathit{y}}”]$$
$$[\dot{y} \xrightarrow{\text{pyk}} \text{“peano } y”]$$

$\dot{z}$

$[\dot{z} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{z} \doteq \dot{z}] \rrbracket)]$

$[\dot{z} \xrightarrow{\text{tex}} “$
 $\backslash \text{dot}\{\backslash \text{mathit}\{z\}\}”]$

$[\dot{z} \xrightarrow{\text{pyk}} “\text{peano } z”]$

$\text{nonfree}(*, *)$

$[\text{nonfree}(x, y) \xrightarrow{\text{val}}$
 $\text{If}(y^{\mathcal{P}}, \neg \llbracket x \stackrel{t}{=} y \rrbracket ,$
 $\text{If}(\neg \llbracket y \stackrel{r}{=} \llbracket \forall x: y \rrbracket \rrbracket , \text{nonfree}^*(x, y^t),$
 $\text{If}(x \stackrel{t}{=} \llbracket y^1 \rrbracket , \top, \text{nonfree}(x, y^2)))]]$

$[\text{nonfree}(x, y) \xrightarrow{\text{tex}} “$
 $\backslash \text{dot}\{\text{nonfree}\}(\#1.$
 $, \#2.$
 $)”]$

$[\text{nonfree}(x, y) \xrightarrow{\text{pyk}} “\text{peano nonfree } * \text{ in } * \text{ end nonfree}”]$

$\text{nonfree}^*(*, *)$

$[\text{nonfree}^*(x, y) \xrightarrow{\text{val}} x! \text{If}(y, \top, \text{If}(\text{nonfree}(x, y^h), \text{nonfree}^*(x, y^t), F))]$

$[\text{nonfree}^*(x, y) \xrightarrow{\text{tex}} “$
 $\backslash \text{dot}\{\text{nonfree}\}^*(\#1.$
 $, \#2.$
 $)”]$

$[\text{nonfree}^*(x, y) \xrightarrow{\text{pyk}} “\text{peano nonfree star } * \text{ in } * \text{ end nonfree}”]$

$\text{free}\langle * | * := * \rangle$

$[\text{free}\langle a | x := b \rangle \xrightarrow{\text{val}} x! \llbracket b!$
 $\text{If}(a^{\mathcal{P}}, \top,$
 $\text{If}(\neg \llbracket a \stackrel{r}{=} \llbracket \forall u: v \rrbracket \rrbracket , \text{free}^*\langle a^t | x := b \rangle,$
 $\text{If}(a^1 \stackrel{t}{=} x, \top,$
 $\text{If}(\text{nonfree}(x, a^2), \top,$

If($\neg$ nonfree(a^1, b), F,
free($a^2|x := b$)))))]]

[free($a|x := b$) $\xrightarrow{\text{tex}}$ “
 $\backslash\dot{\text{free}}\backslash\langle \rangle \#1$.
| $\#2$.
:= $\#3$.
 $\backslash\langle \rangle$ ”]

[free($a|x := b$) $\xrightarrow{\text{pyk}}$ “peano free * set * to * end free”]

free* $\langle *|* := * \rangle$

[free* $\langle a|x := b \rangle \xrightarrow{\text{val}} x!$ [b!If($a, T, \text{If}(\text{free}\langle a^h|x := b \rangle, \text{free}^*\langle a^t|x := b \rangle, F)$))]]

[free* $\langle a|x := b \rangle \xrightarrow{\text{tex}}$ “
 $\backslash\dot{\text{free}}\{\}^*\backslash\langle \rangle \#1$.
| $\#2$.
:= $\#3$.
 $\backslash\langle \rangle$ ”]

[free* $\langle a|x := b \rangle \xrightarrow{\text{pyk}}$ “peano free star * set * to * end free”]

* $\equiv \langle *|* := * \rangle$

[$a \equiv \langle b|x := c \rangle \xrightarrow{\text{val}} a!$ [$x!$ [$c!$
If($\text{If}(b \stackrel{r}{=} \backslash\forall u: v, b^1 \stackrel{t}{=} x, F), a \stackrel{t}{=} b,$
If($b^P \wedge [b \stackrel{t}{=} x], a \stackrel{t}{=} c, \text{If}([$
 $a] \stackrel{r}{=} b, a^t \equiv \langle *b^t|x := c \rangle, F))$)]]]

[$a \equiv \langle b|x := c \rangle \xrightarrow{\text{tex}}$ “ $\#1$.
 $\{\backslash\text{equiv}\}\backslash\langle \rangle \#2$.
| $\#3$.
:= $\#4$.
 $\backslash\langle \rangle$ ”]

[$a \equiv \langle b|x := c \rangle \xrightarrow{\text{pyk}}$ “peano sub * is * where * is * end sub”]

* $\equiv \langle *|* := * \rangle$

[$a \equiv \langle *b|x := c \rangle \xrightarrow{\text{val}} b!$ [$x!$ [$c!\text{If}(a, T, \text{If}(a^h \equiv \langle b^h|x := c \rangle, a^t \equiv \langle *b^t|x := c \rangle, F))$)]]]
[$a \equiv \langle *b|x := c \rangle \xrightarrow{\text{tex}}$ “ $\#1$.

$$\{\equiv\}\langle^* \#2.$$

|#3.

$$:= \#4.$$

\range"]

$$[\mathbf{a} \equiv \langle^* \mathbf{b} | \mathbf{x} := \mathbf{c} \rangle \xrightarrow{\text{pyk}} \text{“peano sub star } * \text{ is } * \text{ where } * \text{ is } * \text{ end sub”}]$$

S

$$[S \xrightarrow{\text{stnt}} [[\dot{a} + [\dot{b}']] \stackrel{P}{=} [[\dot{a} + [\dot{b}]]'] \oplus [[\forall a: \forall b: [[\neg \underline{b}] \Rightarrow \neg \underline{a}] \Rightarrow [[\neg \underline{b}] \Rightarrow \underline{a}] \Rightarrow \underline{b}]] \oplus [[[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [\dot{a}' \stackrel{P}{=} [\dot{b}']]] \oplus [[[\forall a: \forall b: [[\underline{a} \Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]] \oplus [[[\dot{a}' \stackrel{P}{=} [\dot{b}']] \Rightarrow [\dot{a} \stackrel{P}{=} [\dot{b}]]]] \oplus [[[\forall a: \forall b: [\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{a}]]] \oplus [[[\forall x: \forall a: \forall b: \text{nonfree}(x, a) \Vdash [[\forall x: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \dot{x}: \underline{b}]]]] \oplus [[[\dot{a}: [\dot{b}']] \stackrel{P}{=} [\dot{a}: [\dot{b}]] + [\dot{a}]]] \oplus [[[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]] \oplus [[\forall a: \forall b: \forall c: [[\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{c}]] \Rightarrow [[\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]] \oplus [[[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]] \oplus [[[\forall a: \forall b: \forall c: \forall x: [\underline{b} \equiv \langle \underline{a}|x := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a}|x := x' \rangle] \Vdash [\underline{b} \Rightarrow [[\dot{x}: [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \dot{x}: \underline{a}]]] \oplus [[[\neg \dot{0} \stackrel{P}{=} [\dot{a}']]] \oplus [[\forall x: \forall a: [\underline{a} \vdash \dot{x}: \underline{a}]]] \oplus [[[\forall c: \forall a: \forall x: \forall b: [\underline{a} \equiv \langle \underline{b}||\underline{x} := \underline{c} \rangle] \Vdash [[\forall x: \underline{b}] \Rightarrow \underline{a}]]] \oplus [[[\dot{a}: \dot{0}] \stackrel{P}{=} \dot{0}]]]]]]]]$$
$$[S \xrightarrow{\text{tex}} S'']$$
$$[S \xrightarrow{\text{pyk}} \text{“system s”}]$$

A1

[A1 $\xrightarrow{\text{proof}}$ Rule tactic]

$$[A1 \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}. \forall \underline{b}. [\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{a}]]]$$
$$[A1 \xrightarrow{\text{tex}} \text{“} A1\text{”}]$$
$$[A1 \xrightarrow{\text{pyk}} \text{“axiom a one”}]$$

A2

[A2 $\xrightarrow{\text{proof}}$ Rule tactic]

$$[A2 \xrightarrow{\text{stmt}} S \vdash \forall \mathbf{a}:\forall \mathbf{b}:\forall \mathbf{c}: [[\mathbf{a} \Rightarrow [\mathbf{b} \Rightarrow \mathbf{c}]] \Rightarrow [[\mathbf{a} \Rightarrow \mathbf{b}] \Rightarrow [\mathbf{a} \Rightarrow \mathbf{c}]]]]$$

[A2 $\xrightarrow{\text{tex}}$ “
A2”]

[A2 $\xrightarrow{\text{pyk}}$ “axiom a two”]

A3

[A3 $\xrightarrow{\text{proof}}$ Rule tactic]

[A3 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: [[[\dot{\neg} \underline{b}] \Rightarrow \dot{\neg} \underline{a}] \Rightarrow [[[\dot{\neg} \underline{b}] \Rightarrow \underline{a}] \Rightarrow \underline{b}]]]$]

[A3 $\xrightarrow{\text{tex}}$ “
A3”]

[A3 $\xrightarrow{\text{pyk}}$ “axiom a three”]

A4

[A4 $\xrightarrow{\text{proof}}$ Rule tactic]

[A4 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: [[\underline{a}] \equiv \langle [\underline{b}] | [\underline{x}] \rangle := [\underline{c}] \rangle \Vdash [[\dot{\forall} \underline{x}: \underline{b}] \Rightarrow \underline{a}]]]$]

[A4 $\xrightarrow{\text{tex}}$ “
A4”]

[A4 $\xrightarrow{\text{pyk}}$ “axiom a four”]

A5

[A5 $\xrightarrow{\text{proof}}$ Rule tactic]

[A5 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\text{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \dot{\forall} \underline{x}: \underline{b}]]]]$]

[A5 $\xrightarrow{\text{tex}}$ “
A5”]

[A5 $\xrightarrow{\text{pyk}}$ “axiom a five”]

S1

[S1 $\xrightarrow{\text{proof}}$ Rule tactic]

[S1 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{\underline{a}} \stackrel{P}{=} [\dot{\underline{b}}]] \Rightarrow [[\dot{\underline{a}} \stackrel{P}{=} [\dot{\underline{c}}]] \Rightarrow [\dot{\underline{b}} \stackrel{P}{=} [\dot{\underline{c}}]]]]]$]

[S1 $\xrightarrow{\text{tex}}$ “
S1”]

[S1 $\xrightarrow{\text{pyk}}$ “axiom s one”]

S2

[S2 $\xrightarrow{\text{proof}}$ Rule tactic]

[S2 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \stackrel{p}{=} [\dot{b}]] \Rightarrow [\dot{a}' \stackrel{p}{=} [\dot{b}']]]$]

[S2 $\xrightarrow{\text{tex}}$ “
S2”]

[S2 $\xrightarrow{\text{pyk}}$ “axiom s two”]

S3

[S3 $\xrightarrow{\text{proof}}$ Rule tactic]

[S3 $\xrightarrow{\text{stmt}}$ $S \vdash \neg [\dot{0} \stackrel{p}{=} [\dot{a}']]$]

[S3 $\xrightarrow{\text{tex}}$ “
S3”]

[S3 $\xrightarrow{\text{pyk}}$ “axiom s three”]

S4

[S4 $\xrightarrow{\text{proof}}$ Rule tactic]

[S4 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a}' \stackrel{p}{=} [\dot{b}']] \Rightarrow [\dot{a} \stackrel{p}{=} [\dot{b}]]]$]

[S4 $\xrightarrow{\text{tex}}$ “
S4”]

[S4 $\xrightarrow{\text{pyk}}$ “axiom s four”]

S5

[S5 $\xrightarrow{\text{proof}}$ Rule tactic]

[S5 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{a}]]$]

[S5 $\xrightarrow{\text{tex}}$ “
S5”]

[S5 $\xrightarrow{\text{pyk}}$ “axiom s five”]

S6

[S6 $\xrightarrow{\text{proof}}$ Rule tactic]

[S6 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \dot{+} [\dot{b}']] \stackrel{p}{=} [[\dot{a} \dot{+} [\dot{b}]]']]$]

[S6 $\xrightarrow{\text{tex}}$ “
S6”]

[S6 $\xrightarrow{\text{pyk}}$ “axiom s six”]

S7

[S7 $\xrightarrow{\text{proof}}$ Rule tactic]

[S7 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \dot{:} \dot{0}] \stackrel{p}{=} \dot{0}]$]

[S7 $\xrightarrow{\text{tex}}$ “
S7”]

[S7 $\xrightarrow{\text{pyk}}$ “axiom s seven”]

S8

[S8 $\xrightarrow{\text{proof}}$ Rule tactic]

[S8 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \dot{:} [\dot{b}']] \stackrel{p}{=} [[\dot{a} \dot{:} [\dot{b}]] \dot{+} [\dot{a}]]]$]

[S8 $\xrightarrow{\text{tex}}$ “
S8”]

[S8 $\xrightarrow{\text{pyk}}$ “axiom s eight”]

S9

[S9 $\xrightarrow{\text{proof}}$ Rule tactic]

[S9 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \Rightarrow [[\forall \underline{x}: [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \forall \underline{x}: \underline{a}]]]]$]

$$[S9 \xrightarrow{\text{pyk}} \text{“axiom s nine”}]$$
$$[\text{MP} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{MP} \xrightarrow{\text{stmt}} S \vdash \forall \mathbf{a}: \forall \mathbf{b}: [\ [\ \underline{\mathbf{a} \Rightarrow \mathbf{b}} \] \vdash [\ \underline{\mathbf{a} \vdash \mathbf{b}} \] \]]$$
$$[\text{MP} \xrightarrow{\text{pyk}} \text{“rule mp”}]$$
$$\begin{aligned} & [\text{Gen} \xrightarrow{\text{proof}} \text{Rule tactic}] \\ & [\text{Gen} \xrightarrow{\text{stmt}} S \vdash \forall x: \forall a: [a \vdash \dot{\forall} x: a]] \end{aligned}$$
$$[\text{Gen} \xrightarrow{\text{pyk}} \text{“rule gen”}]$$
$$\begin{aligned} & [\text{L3.2(a)}] \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S \vdash [[S5 \gg [[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]]]; [[\text{Gen} \triangleright [[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]]]] \gg \check{V}\dot{a}: [[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]]]; [[A4@[\dot{x}]]] \gg [\\ & \check{V}\dot{a}: [[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]]] \Rightarrow [[\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{x}]]]; [[[MP \triangleright [[\check{V}\dot{a}: [[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]]]] \Rightarrow [[\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{x}]]]] \triangleright \check{V}\dot{a}: [[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]]] \\ & \gg [[\dot{x} + \dot{0}] \stackrel{P}{=} [\dot{x}]]]; [[S1 \gg [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]]]; [[[\text{Gen} \triangleright [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]]] \gg \check{V}\dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]]]; \\ & [[[A4@[\dot{x}]]] \gg [[\check{V}\dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{x}]]]]; [[[MP \triangleright [[\check{V}\dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{x}]]]]] \triangleright \check{V}\dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \\ & \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]] \gg [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{x}]]]]; [[[\text{Gen} \triangleright [[\dot{a} \stackrel{P}{=} [\dot{b}]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{x}]]]]] \end{aligned}$$

$$[A4' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: [[\underline{a}] \equiv \langle [\underline{b}] | [\underline{x}] \rangle := [\underline{c}]] \Vdash [[\dot{\forall} \underline{x}: \underline{b}] \Rightarrow \underline{a}]]]$$

$$[A4' \xrightarrow{\text{tex}} \text{“} \\ A4' \text{”}]$$

$$[A4' \xrightarrow{\text{pyk}} \text{“axiom prime a four”}]$$

A5'

$$[A5' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[A5' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\text{nonfree}([\underline{x}], [\underline{a}]) \Vdash [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \dot{\forall} \underline{x}: \underline{b}]]]]$$

$$[A5' \xrightarrow{\text{tex}} \text{“} \\ A5' \text{”}]$$

$$[A5' \xrightarrow{\text{pyk}} \text{“axiom prime a five”}]$$

S1'

$$[S1' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S1' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [[\underline{a} \stackrel{p}{=} \underline{b}] \Rightarrow [[\underline{a} \stackrel{p}{=} \underline{c}] \Rightarrow [\underline{b} \stackrel{p}{=} \underline{c}]]]]$$

$$[S1' \xrightarrow{\text{tex}} \text{“} \\ S1' \text{”}]$$

$$[S1' \xrightarrow{\text{pyk}} \text{“axiom prime s one”}]$$

S2'

$$[S2' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S2' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \stackrel{p}{=} \underline{b}] \Rightarrow [\underline{a}' \stackrel{p}{=} [\underline{b}']]]]]$$

$$[S2' \xrightarrow{\text{tex}} \text{“} \\ S2' \text{”}]$$

$$[S2' \xrightarrow{\text{pyk}} \text{“axiom prime s two”}]$$

S3'

$$[S3' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S3' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \neg [\dot{0} \stackrel{p}{=} [\underline{a'}]]]$$

$$[S3' \xrightarrow{\text{tex}} \text{“} \\ S3' \text{”}]$$

$$[S3' \xrightarrow{\text{pyk}} \text{“axiom prime s three”}]$$

S4'

$$[S4' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S4' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a'} \stackrel{p}{=} [\underline{b'}]] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{b}]]]$$

$$[S4' \xrightarrow{\text{tex}} \text{“} \\ S4' \text{”}]$$

$$[S4' \xrightarrow{\text{pyk}} \text{“axiom prime s four”}]$$

S5'

$$[S5' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S5' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]]$$

$$[S5' \xrightarrow{\text{tex}} \text{“} \\ S5' \text{”}]$$

$$[S5' \xrightarrow{\text{pyk}} \text{“axiom prime s five”}]$$

S6'

$$[S6' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S6' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \dot{+} [\underline{b'}]] \stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}] ']]]$$

$$[S6' \xrightarrow{\text{tex}} \text{“} \\ S6' \text{”}]$$

$$[S6' \xrightarrow{\text{pyk}} \text{“axiom prime s six”}]$$

S7'

$$[S7' \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[S7' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: [[\underline{a} \dot{:} \dot{0}] \stackrel{p}{=} \dot{0}]]$$

[S7' $\xrightarrow{\text{tex}}$ “
S7”]

[S7' $\xrightarrow{\text{pyk}}$ “axiom prime s seven”]

S8'

[S8' $\xrightarrow{\text{proof}}$ Rule tactic]

[S8' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a}: [\underline{b}']] \stackrel{p}{=} [[\underline{a}: \underline{b}] \dot{+} \underline{a}]]$]

[S8' $\xrightarrow{\text{tex}}$ “
S8”]

[S8' $\xrightarrow{\text{pyk}}$ “axiom prime s eight”]

S9'

[S9' $\xrightarrow{\text{proof}}$ Rule tactic]

[S9' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \Rightarrow [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \dot{\forall} \underline{x}: \underline{a}]]]]$]

[S9' $\xrightarrow{\text{tex}}$ “
S9”]

[S9' $\xrightarrow{\text{pyk}}$ “axiom prime s nine”]

MP'

[MP' $\xrightarrow{\text{proof}}$ Rule tactic]

[MP' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]$]

[MP' $\xrightarrow{\text{tex}}$ “
MP”]

[MP' $\xrightarrow{\text{pyk}}$ “rule prime mp”]

Gen'

[Gen' $\xrightarrow{\text{proof}}$ Rule tactic]

[Gen' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{x}: \forall \underline{a}: [\underline{a} \vdash \dot{\forall} \underline{x}: \underline{a}]]$

[Gen' $\xrightarrow{\text{tex}}$ “
Gen'”]

[Gen' $\xrightarrow{\text{pyk}}$ “rule prime gen”]

L3.2(a)'

[L3.2(a)' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{a}: [[S5' \gg [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] ; [[S1' \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]] ; [[[[MP' \triangleright [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]] \triangleright [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]] ; [[[MP' \triangleright [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]] \triangleright [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [\underline{a} \stackrel{p}{=} \underline{a}]]]]] , p_0, c)]$

[L3.2(a)' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: [\underline{a} \stackrel{p}{=} \underline{a}]$]

[L3.2(a)' $\xrightarrow{\text{tex}}$ “
L3.2(a)”]

[L3.2(a)' $\xrightarrow{\text{pyk}}$ “lemma prime l three two a”]

M1.7

[M1.7 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{b}: [[A1' \gg [\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]]] ; [[A2' \gg [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \Rightarrow [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]]]] ; [[[[MP' \triangleright [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \Rightarrow [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]]] \triangleright [\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \gg [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \Rightarrow [\underline{b} \Rightarrow \underline{b}]]] ; [[A1' \gg [\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]]] ; [[[MP' \triangleright [[\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \Rightarrow [\underline{b} \Rightarrow \underline{b}]]] \triangleright [\underline{b} \Rightarrow [[\underline{b} \Rightarrow \underline{b}] \Rightarrow \underline{b}]] \gg [\underline{b} \Rightarrow \underline{b}]]]]] , p_0, c)]$

[M1.7 $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{b}: [\underline{b} \Rightarrow \underline{b}]$]

[M1.7 $\xrightarrow{\text{tex}}$ “
M1.7”]

[M1.7 $\xrightarrow{\text{pyk}}$ “mendelson one seven”]

MP'_h

[MP'_h $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{h}: \forall \underline{a}: \forall \underline{b}: [[\underline{h} \Rightarrow [\underline{a} \Rightarrow \underline{b}]] \vdash [[\underline{h} \Rightarrow \underline{a}] \vdash [[A2' \gg [[\underline{h} \Rightarrow [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [[\underline{h} \Rightarrow \underline{a}] \Rightarrow [\underline{h} \Rightarrow \underline{b}]]]] ; [[[[MP' \triangleright [[\underline{h} \Rightarrow [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [[\underline{h} \Rightarrow \underline{a}] \Rightarrow [\underline{h} \Rightarrow \underline{b}]]] \triangleright [\underline{h} \Rightarrow [\underline{a} \Rightarrow \underline{b}]]] \gg [[\underline{h} \Rightarrow \underline{a}] \Rightarrow [\underline{h} \Rightarrow \underline{b}]]] ; [[[MP' \triangleright [[\underline{h} \Rightarrow \underline{a}] \Rightarrow [\underline{h} \Rightarrow \underline{b}]]] \triangleright [\underline{h} \Rightarrow \underline{a}]] \gg [\underline{h} \Rightarrow \underline{b}]]]]] , p_0, c)]$

$[MP'_h \xrightarrow{\text{stmt}} S' \vdash \forall \underline{h}: \forall \underline{a}: \forall \underline{b}: [[\underline{h} \Rightarrow [\underline{a} \Rightarrow \underline{b}]]] \vdash [[\underline{h} \Rightarrow \underline{a}] \vdash [\underline{h} \Rightarrow \underline{b}]]]]$

$[MP'_h \xrightarrow{\text{tex}} \text{“}$
 $MP'_h \text{”}]$

$[MP'_h \xrightarrow{\text{pyk}} \text{“hypothetical rule prime mp”}]$

Hypothesize

$[Hypothesize \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{h}: \forall \underline{a}: [\underline{a} \vdash [[A1' \gg [\underline{a} \Rightarrow [\underline{h} \Rightarrow \underline{a}]]]] ;$
 $[[[MP' \triangleright [\underline{a} \Rightarrow [\underline{h} \Rightarrow \underline{a}]]]] \triangleright \underline{a}] \gg [\underline{h} \Rightarrow \underline{a}]]]], p_0, c)]$

$[Hypothesize \xrightarrow{\text{stmt}} S' \vdash \forall \underline{h}: \forall \underline{a}: [\underline{a} \vdash [\underline{h} \Rightarrow \underline{a}]]]$

$[Hypothesize \xrightarrow{\text{tex}} \text{“}$
 $Hypothesize \text{”}]$

$[Hypothesize \xrightarrow{\text{pyk}} \text{“hypothesize”}]$

Gen_h'

$[Gen'_h \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{h}: \forall \underline{x}: \forall \underline{a}: [\text{nonfree}(\underline{x}, [\underline{h}]) \Vdash [[\underline{h} \Rightarrow \underline{a}] \vdash [[$
 $A5' \triangleright \text{nonfree}(\underline{x}, [\underline{h}])]] \gg [[\forall \underline{x}: [\underline{h} \Rightarrow \underline{a}]]] \Rightarrow [\underline{h} \Rightarrow \forall \underline{x}: \underline{a}]]] ; [[[$
 $Gen' \triangleright [\underline{h} \Rightarrow \underline{a}]]] \gg \forall \underline{x}: [\underline{h} \Rightarrow \underline{a}]]] ; [[[MP' \triangleright [[\forall \underline{x}: [\underline{h} \Rightarrow \underline{a}]]] \Rightarrow [$
 $\underline{h} \Rightarrow \forall \underline{x}: \underline{a}]]]] \triangleright \forall \underline{x}: [\underline{h} \Rightarrow \underline{a}]] \gg [\underline{h} \Rightarrow \forall \underline{x}: \underline{a}]]]], p_0, c)]$

$[Gen'_h \xrightarrow{\text{stmt}} S' \vdash \forall \underline{h}: \forall \underline{x}: \forall \underline{a}: [\text{nonfree}(\underline{x}, [\underline{h}]) \Vdash [[\underline{h} \Rightarrow \underline{a}] \vdash [\underline{h} \Rightarrow \forall \underline{x}: \underline{a}]]]]$
 $]$

$[Gen'_h \xrightarrow{\text{tex}} \text{“}$
 $Gen'_h \text{”}]$

$[Gen'_h \xrightarrow{\text{pyk}} \text{“hypothetical rule prime gen”}]$

M3.2(a)

$[M3.2(a) \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[M3.2(a) \xrightarrow{\text{stmt}} S' \vdash \forall \underline{t}: [\underline{t} \stackrel{p}{=} \underline{t}]]$

$[M3.2(a) \xrightarrow{\text{tex}} \text{“}$
 $M3.2(a) \text{”}]$

$[M3.2(a) \xrightarrow{\text{pyk}} \text{“mendelson three two a”}]$

M3.2(a)_h

$[M3.2(a)_h \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{h}: \forall \underline{t}: [[M3.2(a) \gg [\underline{t} \stackrel{P}{=} \underline{t}]] ; [[$
Hypothesize $\triangleright [\underline{t} \stackrel{P}{=} \underline{t}]] \gg [\underline{h} \Rightarrow [\underline{t} \stackrel{P}{=} \underline{t}]]]], p_0, c)]$

$[M3.2(a)_h \xrightarrow{\text{stmt}} S' \vdash \forall \underline{h}: \forall \underline{t}: [\underline{h} \Rightarrow [\underline{t} \stackrel{P}{=} \underline{t}]]]$

$[M3.2(a)_h \xrightarrow{\text{tex}} \text{“}$
M3.2(a)_h”]

$[M3.2(a)_h \xrightarrow{\text{pyk}} \text{“hypothetical three two a”}]$

M3.2(b)_h

$[M3.2(b)_h \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{h}: \forall \underline{t}: \forall \underline{r}: [[\underline{h} \Rightarrow [\underline{t} \stackrel{P}{=} \underline{r}]] \vdash [[S1' \gg [[$
 $\underline{t} \stackrel{P}{=} \underline{r}] \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]] ; [[[\text{Hypothesize} \triangleright [[\underline{t} \stackrel{P}{=} \underline{r}] \Rightarrow [$
 $[\underline{t} \stackrel{P}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]] \gg [\underline{h} \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{r}] \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]]$
 $]] ; [[[[\text{MP}'_h \triangleright [\underline{h} \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{r}] \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]]] \triangleright [$
 $\underline{h} \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{r}]]] \gg [\underline{h} \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]] ; [[M3.2(a)_h \gg [$
 $\underline{h} \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{t}]]] ; [[[\text{MP}'_h \triangleright [\underline{h} \Rightarrow [[\underline{t} \stackrel{P}{=} \underline{t}] \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]] \triangleright [\underline{h} \Rightarrow$
 $[\underline{t} \stackrel{P}{=} \underline{t}]]] \gg [\underline{h} \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]]]], p_0, c)]$

$[M3.2(b)_h \xrightarrow{\text{stmt}} S' \vdash \forall \underline{h}: \forall \underline{t}: \forall \underline{r}: [[\underline{h} \Rightarrow [\underline{t} \stackrel{P}{=} \underline{r}]] \vdash [\underline{h} \Rightarrow [\underline{r} \stackrel{P}{=} \underline{t}]]]]$

$[M3.2(b)_h \xrightarrow{\text{tex}} \text{“}$
M3.2(b)_h”]

$[M3.2(b)_h \xrightarrow{\text{pyk}} \text{“hypothetical three two b”}]$

M3.1(S1')_h

$[M3.1(S1')_h \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[M3.1(S1')_h \xrightarrow{\text{stmt}} S' \vdash \forall \underline{h}: \forall \underline{t}: \forall \underline{r}: \forall \underline{s}: [[\underline{h} \Rightarrow [\underline{t} \stackrel{P}{=} \underline{r}]] \vdash [[\underline{h} \Rightarrow [\underline{t} \stackrel{P}{=} \underline{s}]] \vdash$
 $[\underline{h} \Rightarrow [\underline{r} \stackrel{P}{=} \underline{s}]]]]]$

$[M3.1(S1')_h \xrightarrow{\text{tex}} \text{“}$
M3.1(S1')_h”]

$[M3.1(S1')_h \xrightarrow{\text{pyk}} \text{“hypothetical three one s one”}]$

M3.2(c)_h

[M3.2(c)_h $\xrightarrow{\text{proof}}$ Rule tactic]

[M3.2(c)_h $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{h}: \forall \underline{t}: \forall \underline{r}: \forall \underline{s}: [[\underline{h} \Rightarrow [\underline{t} \stackrel{p}{=} \underline{r}]] \vdash [[\underline{h} \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]] \vdash [\underline{h} \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]]]]$]

[M3.2(c)_h $\xrightarrow{\text{tex}}$ “
M3.2(c)_h”]

[M3.2(c)_h $\xrightarrow{\text{pyk}}$ “hypothetical three two c”]

M3.1(S2')_h

[M3.1(S2')_h $\xrightarrow{\text{proof}}$ Rule tactic]

[M3.1(S2')_h $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{h}: \forall \underline{t}: \forall \underline{r}: [[\underline{h} \Rightarrow [\underline{t} \stackrel{p}{=} \underline{r}]] \vdash [\underline{h} \Rightarrow [\underline{t}' \stackrel{p}{=} [\underline{r}']]]]]$

[M3.1(S2')_h $\xrightarrow{\text{tex}}$ “
M3.1(S2')_h”]

[M3.1(S2')_h $\xrightarrow{\text{pyk}}$ “hypothetical three one s two”]

M3.1(S5')_h

[M3.1(S5')_h $\xrightarrow{\text{proof}}$ Rule tactic]

[M3.1(S5')_h $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{h}: \forall \underline{t}: [\underline{h} \Rightarrow [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]]]$

[M3.1(S5')_h $\xrightarrow{\text{tex}}$ “
M3.1(S5')_h”]

[M3.1(S5')_h $\xrightarrow{\text{pyk}}$ “hypothetical three one s five”]

M3.1(S6')_h

[M3.1(S6')_h $\xrightarrow{\text{proof}}$ Rule tactic]

[M3.1(S6')_h $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{h}: \forall \underline{t}: \forall \underline{r}: [\underline{h} \Rightarrow [[\underline{t} \dot{+} [\underline{r}']] \stackrel{p}{=} [[\underline{t} \dot{+} \underline{r}] ']]]]$

[M3.1(S6')_h $\xrightarrow{\text{tex}}$ “
M3.1(S6')_h”]

[M3.1(S6')_h $\xrightarrow{\text{pyk}}$ “hypothetical three one s six”]

M3.2(f)

[illegible]
$$[\text{M3.2(f)} \xrightarrow{\text{stmt}} S' \vdash \forall \dot{t}: [\dot{t} \stackrel{\text{p}}{=} [\dot{0} + [\dot{t}]]]]$$

[M3.2(f) $\xrightarrow{\text{tex}}$ “M3.2(f)”]

[M3.2(f) $\xrightarrow{\text{pyk}}$ “mendelson three two f”]

•
*

$$[\dot{x} \stackrel{\text{tex}}{\rightarrow} “\dot{\{ \#1. \}}”]$$
$$[\dot{x} \xrightarrow{\text{pyk}} \text{“} * \text{ peano var”}]$$

$\ast'$

$[x' \overset{\text{tex}}{\rightarrow} \text{“}\#1.$
”]

$[x' \overset{\text{pyk}}{\rightarrow} \text{“}\ast \text{ peano succ”}]$

$\ast \dot{\cdot} \ast$

$[x \dot{\cdot} y \overset{\text{tex}}{\rightarrow} \text{“}\#1.$
 $\backslash\mathop{\mathrm{dot}}\{\backslash\mathrm{cdot}\}\} \#2.”]$

$[x \dot{\cdot} y \overset{\text{pyk}}{\rightarrow} \text{“}\ast \text{ peano times } \ast”]$

$\ast \dot{+} \ast$

$[x \dot{+} y \overset{\text{tex}}{\rightarrow} \text{“}\#1.$
 $\backslash\mathop{\mathrm{dot}}\{\dot{+}\}\} \#2.”]$

$[x \dot{+} y \overset{\text{pyk}}{\rightarrow} \text{“}\ast \text{ peano plus } \ast”]$

$\ast \overset{\text{p}}{=} \ast$

$[x \overset{\text{p}}{=} y \overset{\text{tex}}{\rightarrow} \text{“}\#1.$
 $\backslash\mathrm{stackrel{p}{=}}\{=\} \#2.”]$

$[x \overset{\text{p}}{=} y \overset{\text{pyk}}{\rightarrow} \text{“}\ast \text{ peano is } \ast”]$

$\ast \mathcal{P}$

$[x^{\mathcal{P}} \overset{\text{val}}{\rightarrow} x \overset{\text{r}}{=} [\dot{x}]]$

$[x^{\mathcal{P}} \overset{\text{tex}}{\rightarrow} \text{“}\#1.$
 $\{\}^{\wedge} \{\backslash\mathrm{cal} P\}”]$

$[x^{\mathcal{P}} \overset{\text{pyk}}{\rightarrow} \text{“}\ast \text{ is peano var”}]$

$\dot{-} \ast$

$[\dot{-} x \overset{\text{tex}}{\rightarrow} \text{“}$
 $\backslash\mathrm{dot}\{\backslash\mathrm{neg}\}\backslash, \#1.”]$

$[\dot{\neg} x \xrightarrow{\text{pyk}} \text{“peano not } * \text{”}]$

$* \dot{\wedge} *$

$[x \dot{\wedge} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\wedge} y \dot{=} \dot{\neg} (x \dot{\Rightarrow} \dot{\neg} y)])]]$

$[x \dot{\wedge} y \xrightarrow{\text{tex}} \text{“}\#1.$
 $\backslash\mathrel{\dot{\wedge}}\{ \backslash\dot{\wedge} \} \} \#2.\text{”}]$

$[x \dot{\wedge} y \xrightarrow{\text{pyk}} \text{“} * \text{ peano and } * \text{”}]$

$* \dot{\vee} *$

$[x \dot{\vee} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\vee} y \dot{=} [\dot{\neg} x] \dot{\Rightarrow} y]])]$

$[x \dot{\vee} y \xrightarrow{\text{tex}} \text{“}\#1.$
 $\backslash\mathrel{\dot{\vee}}\{ \backslash\dot{\vee} \} \} \#2.\text{”}]$

$[x \dot{\vee} y \xrightarrow{\text{pyk}} \text{“} * \text{ peano or } * \text{”}]$

$\dot{\forall} *: *$

$[\dot{\forall} x. y \xrightarrow{\text{tex}} \text{“}$
 $\backslash\dot{\forall}\{ \backslash\text{forall} \} \#1.$
 $\backslash\text{colon} \#2.\text{”}]$

$[\dot{\forall} x. y \xrightarrow{\text{pyk}} \text{“peano all } * \text{ indeed } * \text{”}]$

$\dot{\exists} *: *$

$[\dot{\exists} x. y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{\exists} x. y \dot{=} \dot{\neg} \dot{\forall} x. \dot{\neg} y]])]$

$[\dot{\exists} x. y \xrightarrow{\text{tex}} \text{“}$
 $\backslash\dot{\exists}\{ \backslash\text{exists} \} \#1.$
 $\backslash\text{colon} \#2.\text{”}]$

$[\dot{\exists} x. y \xrightarrow{\text{pyk}} \text{“peano exist } * \text{ indeed } * \text{”}]$

$* \dot{\Rightarrow} *$

$[x \dot{\Rightarrow} y \xrightarrow{\text{tex}} \text{“}\#1.$
 $\backslash\mathrel{\dot{\Rightarrow}}\{ \backslash\dot{\Rightarrow} \} \} \#2.\text{”}]$

$[x \Rightarrow y \xrightarrow{\text{pyk}} \text{“} * \text{ peano imply } * \text{”}]$

$* \Leftrightarrow *$

$[x \Leftrightarrow y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \Leftrightarrow y \doteq (x \Rightarrow y) \wedge (y \Rightarrow x)]])]$

$[x \Leftrightarrow y \xrightarrow{\text{tex}} \text{“}\#1.$
 $\backslash\mathrel{\dot{\rightarrow}} \text{“}\#2.\text{”}]$

$[x \Leftrightarrow y \xrightarrow{\text{pyk}} \text{“} * \text{ peano iff } * \text{”}]$

$* \supseteq *$

$[x \supseteq y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \supseteq y \doteq [\text{MP}' \supset x] \supset y]])]$

$[x \supseteq y \xrightarrow{\text{tex}} \text{“}\#1.$
 $\backslash\text{unrhd} \text{“}\#2.\text{”}]$

$[x \supseteq y \xrightarrow{\text{pyk}} \text{“} * \text{ macro modus ponens } * \text{”}]$

$* \supseteq_h *$

$[x \supseteq_h y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \supseteq_h y \doteq [\text{MP}'_h \supset x] \supset y]])]$

$[x \supseteq_h y \xrightarrow{\text{tex}} \text{“}\#1.$
 $\backslash\text{unrhd}_h \text{“}\#2.\text{”}]$

$[x \supseteq_h y \xrightarrow{\text{pyk}} \text{“} * \text{ hypothetical modus ponens } * \text{”}]$

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-22.UTC:07:23:31.271829 = MJD-53543.TAI:07:24:03.271829 =
LGT-4626141843271829e-6