

Up Help

peano, intro(*, *, *, *), intro(*, *, *, $\dot{0}$, $\dot{1}$, $\dot{2}$, \dot{a} , \dot{b} , \dot{c} , \dot{d} , \dot{e} , \dot{f} , \dot{g} , \dot{h} , \dot{i} , \dot{j} , \dot{k} , \dot{l} , \dot{m} , \dot{n} , \dot{o} , \dot{p} , \dot{q} , \dot{r} , \dot{s} , \dot{t} , \dot{u} , \dot{v} , \dot{w} , \dot{x} , \dot{y} , \dot{z} , nonfree(*, *), nonfree*(*, *), free(*|* := *), free*($*|*$:= *), $\star\equiv$ (*|* := *), $\star\equiv$ (*|* := *), S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), \star , \star' , $\star\ddot{*}$, $\star\dot{+}\star$, $\star\stackrel{P}{=}\star$, $\star^{\mathcal{P}}$, $\dot{\div}\star$, $\star\dot{\wedge}\star$, $\star\dot{\vee}\star$, $\forall\star:\star$, $\exists\star:\star$, $\star\Rightarrow\star$, $\star\Leftrightarrow\star$,

peano

[peano →

Preassociative

[peano], [base], [bracket * end bracket], [big bracket * end bracket],
 [math * end math], [**flush left** [*]], [x], [y], [z], [[* \bowtie *]], [[* \rightarrow^* *]], [pyk], [tex],
 [name], [prio], [*], [T], [if(*, *, *)], [[* \Rightarrow^* *]], [val], [claim], [\perp], [f(*)], [(*)^I], [F], [0],
 [1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d],
 [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(*)^M], [If(*, *, *),
 [*]], [array{*} * end array], [l], [c], [r], [empty], [[* | * := *]], [M(*)], $\mathcal{U}(*)$, $\mathcal{U}(*)$,
 [$\mathcal{U}^M(*)$], [**apply**(*, *)], [**apply**₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-
 plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)],
 [bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"],
 ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
 ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
 ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
 ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
 [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**(*, *, *)],
 [**abstract**(*, *, *, *)], [[*]], [M(*, *, *)], [M₂(*, *, *, *)], [M^{*}(*, *, *)], [macro],
 [s₀], [**zip**(*, *)], [**assoc**₁(*, *, *)], [(*)^P], [self], [[* \equiv *]], [[* \doteq *]], [[* \doteqdot *]],
 [[* $\stackrel{\text{pyk}}{=}$ *]], [[* $\stackrel{\text{tex}}{=}$ *]], [[* $\stackrel{\text{name}}{=}$ *]], [**Priority table***], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$],
 [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$],
 [(*)], [**aspect**(*, *)], [**aspect**(*, *, *)], [[*]], [**tuple**₁(*)], [**tuple**₂(*)], [let₂(*, *)],
 [let₁(*, *)], [[* $\stackrel{\text{claim}}{=}$ *]], [checker], [**check**(*, *)], [**check**₂(*, *, *)], [**check**₃(*, *, *)],
 [**check**^{*}(*, *)], [**check**₂^{*}(*, *, *)], [[* ·]], [[* −]], [[* °]], [msg], [[* $\stackrel{\text{msg}}{=}$ *]], [<stmt>],
 [stmt], [[* $\stackrel{\text{stmt}}{=}$ *]], [HeadNil'], [HeadPair'], [Transitivity'], [\perp], [Contra'], [T'_E],
 [L₁], [*], [\mathcal{A}], [\mathcal{B}], [\mathcal{C}], [\mathcal{D}], [\mathcal{E}], [\mathcal{F}], [\mathcal{G}], [\mathcal{H}], [\mathcal{I}], [\mathcal{J}], [\mathcal{K}], [\mathcal{L}], [\mathcal{M}], [\mathcal{N}], [\mathcal{O}], [\mathcal{P}], [\mathcal{Q}],
 [\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [[* | * := *]], [[* * | * := *]], [∅], [Remainder],
 [(*)^V], [error(*, *)], [error₂(*, *)], [proof(*, *, *)], [proof₂(*, *)], [$\mathcal{S}(*, *)$], [$\mathcal{S}^I(*, *)$],
 [$\mathcal{S}^>(*, *)$], [$\mathcal{S}^>_1(*, *, *)$], [$\mathcal{S}^E(*, *)$], [$\mathcal{S}^E_1(*, *, *)$], [$\mathcal{S}^+(*, *)$], [$\mathcal{S}^+_1(*, *, *)$],

$[S^{\perp}(*, *), [S_1^{\perp}(*, *, *), [S^*(*, *, *)], [S_1^*(*, *, *)], [S_2^*(*, *, *, *)], [S^{\circledast}(*, *)],$
 $[S_1^{\circledast}(*, *, *)], [S^{\vdash}(*, *)], [S_1^{\vdash}(*, *, *, *)], [S^{\dashv}(*, *)], [S_1^{\dashv}(*, *, *, *)], [S^{i.e.}(*, *)],$
 $[S_1^{i.e.}(*, *, *, *)], [S_2^{i.e.}(*, *, *, *, *)], [S^{\vee}(*, *)], [S_1^{\vee}(*, *, *, *)], [S^{\cdot}(*, *)],$
 $[S_1^{\cdot}(*, *, *)], [S_2^{\cdot}(*, *, *, *)], [\mathcal{T}(*)], [\text{claims}(*, *, *)], [\text{claims}_2(*, *, *)], <\text{proof}>,$
 $\text{[proof]}, [[\text{Lemma } * : *]], [[\text{Proof of } * : *]], [[\text{lemma } * : *]],$
 $[[* \text{ antilemma } * : *]], [[* \text{ rule } * : *]], [[* \text{ antirule } * : *]], [\text{verifier}], [\mathcal{V}_1(*)],$
 $[\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)], [\mathcal{V}_7(*, *, *, *)],$
 $[\text{Cut}(*, *)], [\text{Head}_{\oplus}(*)], [\text{Tail}_{\oplus}(*)], [\text{rule}_1(*, *)], [\text{rule}(*, *)], [\text{Rule tactic}],$
 $[\text{Plus}(*, *)], [[\text{Theory } *]], [\text{theory}_2(*, *)], [\text{theory}_3(*, *)], [\text{theory}_4(*, *, *)],$
 $[\text{HeadNil}''], [\text{HeadPair}''], [\text{Transitivity}''], [\text{Contra}''], [\text{HeadNil}], [\text{HeadPair}],$
 $[\text{Transitivity}], [\text{Contra}], [\text{T}_E], [\text{ragged right}], [\text{ragged right expansion}],$
 $[\text{parm}(*, *, *)], [\text{parm}^*(*, *, *)], [\text{inst}(*, *)], [\text{inst}^*(*, *)], [\text{occur}(*, *, *)],$
 $[\text{occur}^*(*, *, *)], [\text{unify}(* = *, *)], [\text{unify}^*(* = *, *)], [\text{unify}_2(* = *, *)], [\text{L}_a], [\text{L}_b],$
 $[\text{L}_c], [\text{L}_d], [\text{L}_e], [\text{L}_f], [\text{L}_g], [\text{L}_h], [\text{L}_i], [\text{L}_j], [\text{L}_k], [\text{L}_l], [\text{L}_m], [\text{L}_n], [\text{L}_o], [\text{L}_p], [\text{L}_q], [\text{L}_r],$
 $[\text{L}_s], [\text{L}_t], [\text{L}_u], [\text{L}_v], [\text{L}_w], [\text{L}_x], [\text{L}_y], [\text{L}_z], [\text{L}_A], [\text{L}_B], [\text{L}_C], [\text{L}_D], [\text{L}_E], [\text{L}_F], [\text{L}_G],$
 $[\text{L}_H], [\text{L}_I], [\text{L}_J], [\text{L}_K], [\text{L}_L], [\text{L}_M], [\text{L}_N], [\text{L}_O], [\text{L}_P], [\text{L}_Q], [\text{L}_R], [\text{L}_S], [\text{L}_T], [\text{L}_U], [\text{L}_V],$
 $[\text{L}_W], [\text{L}_X], [\text{L}_Y], [\text{L}_Z], [\text{L}_?], [\text{Reflexivity}], [\text{Reflexivity}_1], [\text{Commutativity}],$
 $[\text{Commutativity}_1], <\text{tactic}>, [\text{tactic}], [[* \stackrel{\text{tactic}}{=} *]], [\mathcal{P}(*, *, *)], [\mathcal{P}^*(*, *, *)], [\text{p}_0],$
 $[\text{conclude}_1(*, *)], [\text{conclude}_2(*, *, *)], [\text{conclude}_3(*, *, *, *)], [\text{intro}(*, *, *, *)],$
 $[\text{intro}(*, *, *)], [\dot{0}], [\dot{1}], [\dot{2}], [\dot{a}], [\dot{b}], [\dot{c}], [\dot{d}], [\dot{e}], [\dot{f}], [\dot{g}], [\dot{h}], [\dot{i}], [\dot{j}], [\dot{k}], [\dot{l}], [\dot{m}], [\dot{n}], [\dot{o}],$
 $[\dot{p}], [\dot{q}], [\dot{r}], [\dot{s}], [\dot{t}], [\dot{u}], [\dot{v}], [\dot{w}], [\dot{x}], [\dot{y}], [\dot{z}], [\text{nonfree}(*, *)], [\text{nonfree}^*(*, *)],$
 $[\text{free}^*(*|* := *)], [\text{free}^*(*|* := *)], [* \equiv (*|* := *)], [* \equiv (*|* := *)], [\text{S}], [\text{A1}], [\text{A2}],$
 $[\text{A3}], [\text{A4}], [\text{A5}], [\text{S1}], [\text{S2}], [\text{S3}], [\text{S4}], [\text{S5}], [\text{S6}], [\text{S7}], [\text{S8}], [\text{S9}], [\text{MP}], [\text{Gen}],$
 $[\text{L3.2(a)}];$

Preassociative

$[* _ \{ * \}], [*'], [* [*]], [* [* \rightarrow *]], [* [* \Rightarrow *]], [*]:$

Preassociative

[Preassociative *; *], [Postassociative *; *], [[*], *], [priority * end],

[newline *], [macro newline *];

Preassociative

$[*0], [*1], [0b], [-color(*)], [-color^*(*)]$

Preassociative

$$[*, *], [*, *];$$

Preassociative

$[\ast H], [\ast T], [\ast U], [\ast h], [\ast t], [\ast s], [\ast c], [\ast d], [\ast a], [\ast C], [\ast M], [\ast B], [\ast r], [\ast i], [\ast d], [\ast R], [\ast 0],$
 $[\ast 1], [\ast 2], [\ast 3], [\ast 4], [\ast 5], [\ast 6], [\ast 7], [\ast 8], [\ast 9], [\ast E], [\ast V], [\ast C], [\ast C^*], [\ast '];$

Preassociative

$[* \cdot *]$, $[* \cdot_0 *$], $[* : *]$:

Preassociative[* + *], [* +₀ *], [* +₁ *], [* - *], [* -₀ *], [* -₁ *], [* ⋆ *];**Preassociative**

[* ∪ { * }], [* ∪ *], [* \{ * \}];

Postassociative[* .. *], [* .._. *], [* .._. *], [* +₂* *], [* :: *], [* +₂* *];**Postassociative**

[*, *];

Preassociative[* ≈^B *], [* ≈^D *], [* ≈^C *], [* ≈^P *], [* ≈ *], [* = *], [* → *], [* ^t= *], [* ^{t*}= *], [* ^r= *],
[* ∈_t *], [* ⊆_T *], [* ^T= *], [* ^s= *], [* free in *], [* free in^{*} *], [* free for * in *],
[* free for^{*} * in *], [* ∈_c *], [* < *], [* <['] *], [* ≤['] *], [* ^P= *], [* ^P];**Preassociative**

[¬*], [¬..*];

Preassociative[* ∧ *], [* ḥ ∧ *], [* ḥ ∧ *], [* ∧_c *], [* ḥ ∧ *];**Preassociative**

[* ∨ *], [* ∥ *], [* ḥ ∨ *], [* ḥ ∨ *];

Preassociative

[∀*: *], [∃*: *];

Postassociative

[* ⇒ *], [* ⇒ *], [* ⇔ *];

Postassociative

[* : *], [* !*];

Preassociative

[* { * } *];

Preassociative

[λ * . *], [Λ*], [if * then * else *], [let * = * in *], [let * ≡ * in *];

Preassociative[*^I], [*[▷]], [*^V], [*⁺], [*⁻], [*^{*}];**Preassociative**

[* @ *], [* ▷ *], [* ▷ *], [* ≫ *];

Postassociative

[* ⊢ *], [* ⊦ *], [* i.e. *];

Preassociative

[∀*: *];

Postassociative

[* ⊕ *];

Postassociative

[* ; *];

Preassociative

[* proves *];

Preassociative

[* proof of * : *], [Line * : * ≫ *; *], [Last line * ≫ * □],

[Line * : Premise $\gg *; *$], [Line * : Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
 [Local $\gg * = *; *$];
Postassociative
 [* then *], [*[*]*];
Preassociative
 [*&*];
Preassociative
 [**];
 [peano $\xrightarrow{\text{pyk}}$ “peano”]

intro(*, *, *, *)

[intro(x, i, p, t) $\xrightarrow{\text{name}}$ “
 intro(#1.
 ,#2.
 ,#3.
 ,#4.
)”]
 [intro(x, i, p, t) $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{intro}(x, i, p, t) \doteq [x \stackrel{\text{pyk}}{=} p][x \stackrel{\text{tex}}{=} t]]])$]
 [intro(x, i, p, t) $\xrightarrow{\text{tex}}$ “\index{\#2.: #3. @\#2.: \$[\#1/\text{tex name/tex.}]\$ #3.%
 \index{\#3. \$[\#1/\text{tex name/tex.}]\$}%
 \tex{
 \$[\#1/\text{tex name/tex.}
 \stackrel{}{\backslash \text{mathrm}}{\{ \text{tex} \}}\{=\} \#4/\text{tex name.}
]\\$[\#1/\text{tex name/tex.}%
]\\$\\footnote{\$[\#1/\text{tex name/tex.}
 \stackrel{}{\backslash \text{mathrm}}{\{ \text{pyk} \}}\{=\} \#3/\text{tex name.}
]\\$}”]
 [intro(x, i, p, t) $\xrightarrow{\text{pyk}}$ “intro * index * pyk * tex * end intro”]

intro(*, *, *)

[intro(x, p, t) $\xrightarrow{\text{name}}$ “
 intro(#1.
 ,#2.
 ,#3.
)”]
 [intro(x, p, t) $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{intro}(x, p, t) \doteq [x \stackrel{\text{pyk}}{=} p][x \stackrel{\text{tex}}{=} t]]])$]
 [intro(x, p, t) $\xrightarrow{\text{tex}}$ “\index{\alpha \#2. @\back \makebox[20mm][l]{\$[\#1/\text{tex}”]}

```

name/tex.]$}#2.}%
\index{#2. $[#1/tex name/tex.]$}%
\tex{%
$[#1/tex name/tex.%
\stackrel{\mathrm{tex}}{\{=\}} \#3/tex name.%
]$}[ #1/tex name/tex.%%
]$%footnote{$[#1/tex name/tex.%
\stackrel{\mathrm{tex}}{\{=\}} \#2/tex name.%
]$}"]

```

$[introduction(x, p, t) \xrightarrow{pyk} "introduction * pyk * tex * end introduction"]$

$\dot{0}$

$[\dot{0} \xrightarrow{\mathrm{tex}} "\dot{0}"$
 $\backslash dot\{0\}"]$

$[\dot{0} \xrightarrow{pyk} "peano zero"]$

$\dot{1}$

$[\dot{1} \xrightarrow{\mathrm{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{1} \equiv \dot{0}']])]$

$[\dot{1} \xrightarrow{\mathrm{tex}} "\dot{1}"$
 $\backslash dot\{1\}"]$

$[\dot{1} \xrightarrow{pyk} "peano one"]$

$\dot{2}$

$[\dot{2} \xrightarrow{\mathrm{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{2} \equiv \dot{1}']])]$

$[\dot{2} \xrightarrow{\mathrm{tex}} "\dot{2}"$
 $\backslash dot\{2\}"]$

$[\dot{2} \xrightarrow{pyk} "peano two"]$

\dot{a}

$[\dot{a} \xrightarrow{\mathrm{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{a} \equiv \dot{a}]])]$

$[\dot{a} \xrightarrow{\mathrm{tex}} "\dot{a}"$
 $\backslash dot\{\backslash mathit\{a\}\}"]$

$\dot{a} \xrightarrow{\text{pyk}} \text{“peano a”}$

\dot{b}

$\dot{b} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{b} \doteqdot \dot{b}] \rceil)$

$\dot{b} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\backslash\text{mathit}\{b\}\}\text{”}$

$\dot{b} \xrightarrow{\text{pyk}} \text{“peano b”}$

\dot{c}

$\dot{c} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{c} \doteqdot \dot{c}] \rceil)$

$\dot{c} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\backslash\text{mathit}\{c\}\}\text{”}$

$\dot{c} \xrightarrow{\text{pyk}} \text{“peano c”}$

\dot{d}

$\dot{d} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{d} \doteqdot \dot{d}] \rceil)$

$\dot{d} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\backslash\text{mathit}\{d\}\}\text{”}$

$\dot{d} \xrightarrow{\text{pyk}} \text{“peano d”}$

\dot{e}

$\dot{e} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{e} \doteqdot \dot{e}] \rceil)$

$\dot{e} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\backslash\text{mathit}\{e\}\}\text{”}$

$\dot{e} \xrightarrow{\text{pyk}} \text{“peano e”}$

\dot{f}

$\dot{f} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{f} \doteqdot \dot{f}] \rceil)$

$\dot{f} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\mathit{f}}\}\text{''}$

$\dot{f} \xrightarrow{\text{pyk}} \text{“peano f”}$

\dot{g}

$[\dot{g} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{g} \equiv g] \rceil)]$

$[\dot{g} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\text{\mathit{g}}\}\text{”}]$

$[\dot{g} \xrightarrow{\text{pyk}} \text{“peano g”}]$

\dot{h}

$[\dot{h} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{h} \equiv h] \rceil)]$

$[\dot{h} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\text{\mathit{h}}\}\text{”}]$

$[\dot{h} \xrightarrow{\text{pyk}} \text{“peano h”}]$

\dot{i}

$[\dot{i} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{i} \equiv i] \rceil)]$

$[\dot{i} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\text{\mathit{i}}\}\text{”}]$

$[\dot{i} \xrightarrow{\text{pyk}} \text{“peano i”}]$

\dot{j}

$[\dot{j} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{j} \equiv j] \rceil)]$

$[\dot{j} \xrightarrow{\text{tex}} \text{“}\backslash\text{dot}\{\text{\mathit{j}}\}\text{”}]$

$[\dot{j} \xrightarrow{\text{pyk}} \text{“peano j”}]$

\dot{k}

$[\dot{k} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{k} \equiv \dot{k}]])]$
 $[\dot{k} \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{mathit}\{k\}\}\”]$
 $[\dot{k} \xrightarrow{\text{pyk}} “\text{peano k}”]$

\dot{l}

$[\dot{l} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{l} \equiv \dot{l}]])]$
 $[\dot{l} \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{mathit}\{l\}\}\”]$
 $[\dot{l} \xrightarrow{\text{pyk}} “\text{peano l}”]$

\dot{m}

$[\dot{m} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{m} \equiv \dot{m}]])]$
 $[\dot{m} \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{mathit}\{m\}\}\”]$
 $[\dot{m} \xrightarrow{\text{pyk}} “\text{peano m}”]$

\dot{n}

$[\dot{n} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{n} \equiv \dot{n}]])]$
 $[\dot{n} \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{mathit}\{n\}\}\”]$
 $[\dot{n} \xrightarrow{\text{pyk}} “\text{peano n}”]$

\dot{o}

$[\dot{o} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{o} \equiv \dot{o}]])]$
 $[\dot{o} \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{mathit}\{o\}\}\”]$
 $[\dot{o} \xrightarrow{\text{pyk}} “\text{peano o}”]$

\dot{p}

$[\dot{p} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{p} \equiv \dot{p}]])]$

$[\dot{p} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\textit{p}}\}\text{''}]$

$[\dot{p} \xrightarrow{\text{pyk}} \text{``peano p''}]$

\dot{q}

$[\dot{q} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{q} \equiv \dot{q}]])]$

$[\dot{q} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\textit{q}}\}\text{''}]$

$[\dot{q} \xrightarrow{\text{pyk}} \text{``peano q''}]$

\dot{r}

$[\dot{r} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{r} \equiv \dot{r}]])]$

$[\dot{r} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\textit{r}}\}\text{''}]$

$[\dot{r} \xrightarrow{\text{pyk}} \text{``peano r''}]$

\dot{s}

$[\dot{s} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{s} \equiv \dot{s}]])]$

$[\dot{s} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\textit{s}}\}\text{''}]$

$[\dot{s} \xrightarrow{\text{pyk}} \text{``peano s''}]$

\dot{t}

$[\dot{t} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{t} \equiv \dot{t}]])]$

$[\dot{t} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\textit{t}}\}\text{''}]$

$[\dot{t} \xrightarrow{\text{pyk}} \text{``peano t''}]$

\dot{u}

$[\dot{u} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{u} \doteqdot \dot{u}]])]$
 $[\dot{u} \xrightarrow{\text{tex}} "$
 $\backslash dot\{\backslash mathit\{u\}\}"]$
 $[\dot{u} \xrightarrow{\text{pyk}} \text{"peano u"}]$

\dot{v}

$[\dot{v} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{v} \doteqdot \dot{v}]])]$
 $[\dot{v} \xrightarrow{\text{tex}} "$
 $\backslash dot\{\backslash mathit\{v\}\}"]$
 $[\dot{v} \xrightarrow{\text{pyk}} \text{"peano v"}]$

\dot{w}

$[\dot{w} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{w} \doteqdot \dot{w}]])]$
 $[\dot{w} \xrightarrow{\text{tex}} "$
 $\backslash dot\{\backslash mathit\{w\}\}"]$
 $[\dot{w} \xrightarrow{\text{pyk}} \text{"peano w"}]$

\dot{x}

$[\dot{x} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{x} \doteqdot \dot{x}]])]$
 $[\dot{x} \xrightarrow{\text{tex}} "$
 $\backslash dot\{\backslash mathit\{x\}\}"]$
 $[\dot{x} \xrightarrow{\text{pyk}} \text{"peano x"}]$

\dot{y}

$[\dot{y} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{y} \doteqdot \dot{y}]])]$
 $[\dot{y} \xrightarrow{\text{tex}} "$
 $\backslash dot\{\backslash mathit\{y\}\}"]$
 $[\dot{y} \xrightarrow{\text{pyk}} \text{"peano y"}]$

\dot{z}

$[\dot{z} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{M}_4(t, s, c, [[\dot{z} \doteqdot \dot{z}]])]$

$[\dot{z} \xrightarrow{\text{tex}} \text{``}\backslash\text{dot}\{\text{\textit{z}}\}\text{''}]$

$[\dot{z} \xrightarrow{\text{pyk}} \text{``peano z''}]$

$\dot{\text{nonfree}}(*, *)$

$[\dot{\text{nonfree}}(x, y) \xrightarrow{\text{val}}$

$\text{If}(y^P, \neg [x \stackrel{t}{=} y],$

$\text{If}(\neg [y \stackrel{r}{=} [\forall x: y]], \dot{\text{nonfree}}^*(x, y^t),$

$\text{If}(x \stackrel{t}{=} [y^1], T, \dot{\text{nonfree}}(x, y^2))))]$

$[\dot{\text{nonfree}}(x, y) \xrightarrow{\text{tex}} \text{``}$

$\backslash\text{dot}\{\text{nonfree}\}(\#1.$

$, \#2.$

$)'']$

$[\dot{\text{nonfree}}(x, y) \xrightarrow{\text{pyk}} \text{``peano nonfree * in * end nonfree''}]$

$\dot{\text{nonfree}}^*(*, *)$

$[\dot{\text{nonfree}}^*(x, y) \xrightarrow{\text{val}} x! \text{If}(y, T, \dot{\text{nonfree}}(x, y^h), \dot{\text{nonfree}}^*(x, y^t), F))]$

$[\dot{\text{nonfree}}^*(x, y) \xrightarrow{\text{tex}} \text{``}$

$\backslash\text{dot}\{\text{nonfree}\}^*(\#1.$

$, \#2.$

$)'']$

$[\dot{\text{nonfree}}^*(x, y) \xrightarrow{\text{pyk}} \text{``peano nonfree star * in * end nonfree''}]$

$\dot{\text{free}}(*|* := *)$

$[\dot{\text{free}}(a|x := b) \xrightarrow{\text{val}} x! [b!]$

$\text{If}(a^P, T,$

$\text{If}(\neg [a \stackrel{r}{=} [\forall u: v]], \dot{\text{free}}^*(a^t|x := b),$

$\text{If}(a^1 \stackrel{t}{=} x, T,$

$\text{If}(\dot{\text{nonfree}}(x, a^2), T,$

If(¬nonfree(a¹, b), F,
free⟨a²|x := b⟩))))]]

[free⟨a|x := b⟩ $\xrightarrow{\text{tex}}$ “
\dot{free}\{free\}\langle#1.
| #2.
:= #3.
\rangle”]

[free⟨a|x := b⟩ $\xrightarrow{\text{pyk}}$ “peano free * set * to * end free”]

free*⟨*|* := *⟩

[free*⟨a|x := b⟩ $\xrightarrow{\text{val}}$ x! [b!If(a, T, If(free⟨a^h|x := b⟩, free*⟨a^t|x := b⟩, F))]]
[free*⟨a|x := b⟩ $\xrightarrow{\text{tex}}$ “
\dot{free}\{free\}\{}^*\langle#1.
| #2.
:= #3.
\rangle”]

[free*⟨a|x := b⟩ $\xrightarrow{\text{pyk}}$ “peano free star * set * to * end free”]

≡⟨|* := *⟩

[a≡⟨b|x := c⟩ $\xrightarrow{\text{val}}$ a! [x! [c!
If(If(b $\stackrel{r}{=}$ [$\forall u: v$], b¹ $\stackrel{t}{=}$ x, F), a $\stackrel{t}{=}$ b,
If(b^P \wedge [b $\stackrel{t}{=}$ x], a $\stackrel{t}{=}$ c, If([
a] $\stackrel{r}{=}$ b, a^t≡⟨*b^t|x := c⟩, F)))]]]

[a≡⟨b|x := c⟩ $\xrightarrow{\text{tex}}$ “#1.
\{equiv\}\langle#2.
| #3.
:= #4.
\rangle”]

[a≡⟨b|x := c⟩ $\xrightarrow{\text{pyk}}$ “peano sub * is * where * is * end sub”]

≡⟨|* := *⟩

[a≡⟨*b|x := c⟩ $\xrightarrow{\text{val}}$ b! [x! [c!If(a, T, If(a^h≡⟨b^h|x := c⟩, a^t≡⟨*b^t|x := c⟩, F))]]]
[a≡⟨*b|x := c⟩ $\xrightarrow{\text{tex}}$ “#1.

{\equiv}\langle^*\#2.

| #3.

:=#4.

\rangle

$[a \equiv (* b | x := c) \xrightarrow{\text{pyk}} \text{"peano sub star * is * where * is * end sub"}]$

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$\begin{aligned} & [S \xrightarrow{\text{stmt}} [[\dot{a}' \stackrel{P}{=} [b']]] \Rightarrow [\dot{a} \stackrel{P}{=} [b]]] \oplus [[\forall a: \forall b: [a \Rightarrow [b \Rightarrow a]]] \\ & \oplus [[\forall x: \forall a: \forall b: [\text{nonfree}(x, a) \Vdash [\forall x: [a \Rightarrow b]] \Rightarrow [a \Rightarrow \forall x: b]]]] \\ & \oplus [[[\dot{a}: [b']] \stackrel{P}{=} [[\dot{a}: [b]] + [\dot{a}]]] \oplus [[[\dot{a} \stackrel{P}{=} [b]] \Rightarrow [\dot{a}' \stackrel{P}{=} [b']]] \\ & \oplus [[\forall a: \forall b: [a \Rightarrow b] \vdash [a \vdash b]]] \oplus [[[[\neg b] \Rightarrow \neg a] \\ & \Rightarrow [[[\neg b] \Rightarrow a] \Rightarrow b]] \oplus [[[\dot{a} + [b']] \stackrel{P}{=} [[\dot{a} + [b]]']] \oplus [[\neg [\dot{0} \stackrel{P}{=} [\dot{a}']]] \oplus [[\forall x: \forall a: [a \vdash \forall x: a]] \oplus [[\forall c: \forall x: \forall b: \forall a: [a \equiv \langle b | x := c \rangle \Vdash [\forall x: b \Rightarrow a]]] \oplus [[[\dot{a}: \dot{0}] \stackrel{P}{=} \dot{0}] \oplus [[[\dot{a} \stackrel{P}{=} [b]] \\ & \Rightarrow [[\dot{a} \stackrel{P}{=} [c]] \Rightarrow [[b \stackrel{P}{=} [c]]]] \oplus [[\forall a: \forall b: \forall c: \forall x: [b \equiv \langle a | x := \dot{0} \rangle \Vdash [c \equiv \langle a | x := x' \rangle \Vdash [b \Rightarrow [[\forall x: [a \Rightarrow c]] \Rightarrow \forall x: a]]]] \oplus [[\forall a: \forall b: \forall c: [[a \Rightarrow [b \Rightarrow c]] \Rightarrow [[a \Rightarrow b] \Rightarrow [a \Rightarrow c]]]] \oplus [[[\dot{a} + \dot{0}] \stackrel{P}{=} [\dot{a}]] \end{aligned}$

[S $\xrightarrow{\text{tex}}$ “

S”]

[S $\xrightarrow{\text{pyk}}$ “system s”]

A1

[A1 $\xrightarrow{\text{proof}}$ Rule tactic]

[A1 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a} : \forall \underline{b} : [\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{a}]]$]

[A1 $\xrightarrow{\text{tex}}$ “

A1”]

[A1 $\xrightarrow{\text{pyk}}$ “axiom a one”]

A2

[A2 $\xrightarrow{\text{proof}}$ Rule tactic]

$$[\text{A2} \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [[\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{c}]] \Rightarrow [[\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]]]$$

[A2 $\xrightarrow{\text{tex}}$ “

A2”]

[A2 $\xrightarrow{\text{pyk}}$ “axiom a two”]

A3

[A3 $\xrightarrow{\text{proof}}$ Rule tactic]

[A3 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{\neg} \underline{b}] \Rightarrow \dot{\neg} \underline{a}] \Rightarrow [[[\dot{\neg} \underline{b}] \Rightarrow \underline{a}] \Rightarrow \underline{b}]]$

[A3 $\xrightarrow{\text{tex}}$ “

A3”]

[A3 $\xrightarrow{\text{pyk}}$ “axiom a three”]

A4

[A4 $\xrightarrow{\text{proof}}$ Rule tactic]

[A4 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{c}: \forall \underline{x}: \forall \underline{b}: \forall \underline{a}: [\underline{a} \equiv \langle \underline{b} | \underline{x} := \underline{c} \rangle \Vdash [[\dot{\forall} \underline{x}: \underline{b}] \Rightarrow \underline{a}]]$

[A4 $\xrightarrow{\text{tex}}$ “

A4”]

[A4 $\xrightarrow{\text{pyk}}$ “axiom a four”]

A5

[A5 $\xrightarrow{\text{proof}}$ Rule tactic]

[A5 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\text{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \dot{\forall} \underline{x}: \underline{b}]]]$

[A5 $\xrightarrow{\text{tex}}$ “

A5”]

[A5 $\xrightarrow{\text{pyk}}$ “axiom a five”]

S1

[S1 $\xrightarrow{\text{proof}}$ Rule tactic]

[S1 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [[\dot{b} \stackrel{P}{=} [\dot{c}]]]]]$

[S1 $\xrightarrow{\text{tex}}$ “
S1”]

[S1 $\xrightarrow{\text{pyk}}$ “axiom s one”]

S2

[S2 $\xrightarrow{\text{proof}}$ Rule tactic]

[S2 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a} \stackrel{p}{=} [\dot{b}]] \Rightarrow [\dot{a}' \stackrel{p}{=} [\dot{b}']]]$]

[S2 $\xrightarrow{\text{tex}}$ “
S2”]

[S2 $\xrightarrow{\text{pyk}}$ “axiom s two”]

S3

[S3 $\xrightarrow{\text{proof}}$ Rule tactic]

[S3 $\xrightarrow{\text{stmt}}$ S $\vdash \neg [\dot{0} \stackrel{p}{=} [\dot{a}']]$]

[S3 $\xrightarrow{\text{tex}}$ “
S3”]

[S3 $\xrightarrow{\text{pyk}}$ “axiom s three”]

S4

[S4 $\xrightarrow{\text{proof}}$ Rule tactic]

[S4 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a}' \stackrel{p}{=} [\dot{b}']] \Rightarrow [\dot{a} \stackrel{p}{=} [\dot{b}]]]$]

[S4 $\xrightarrow{\text{tex}}$ “
S4”]

[S4 $\xrightarrow{\text{pyk}}$ “axiom s four”]

S5

[S5 $\xrightarrow{\text{proof}}$ Rule tactic]

[S5 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]$]

[S5 $\xrightarrow{\text{tex}}$ “
S5”]

[S5 $\xrightarrow{\text{pyk}}$ “axiom s five”]

S6

[S6 $\xrightarrow{\text{proof}}$ Rule tactic]

[S6 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a} + [\dot{b'}]] \stackrel{P}{=} [[\dot{a} + [\dot{b}]]']]$]

[S6 $\xrightarrow{\text{tex}}$ “
S6”]

[S6 $\xrightarrow{\text{pyk}}$ “axiom s six”]

S7

[S7 $\xrightarrow{\text{proof}}$ Rule tactic]

[S7 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a} : \dot{0}] \stackrel{P}{=} \dot{0}]]$

[S7 $\xrightarrow{\text{tex}}$ “
S7”]

[S7 $\xrightarrow{\text{pyk}}$ “axiom s seven”]

S8

[S8 $\xrightarrow{\text{proof}}$ Rule tactic]

[S8 $\xrightarrow{\text{stmt}}$ S $\vdash [[\dot{a} : [\dot{b'}]] \stackrel{P}{=} [[\dot{a} : [\dot{b}]] + [\dot{a}]]]$]

[S8 $\xrightarrow{\text{tex}}$ “
S8”]

[S8 $\xrightarrow{\text{pyk}}$ “axiom s eight”]

S9

[S9 $\xrightarrow{\text{proof}}$ Rule tactic]

[S9 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a} : \forall \underline{b} : \forall \underline{c} : \forall \underline{x} : [b \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [c \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [b \Rightarrow [[\forall \dot{x} : [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \forall \underline{x} : \underline{a}]]]]$]

[S9 $\xrightarrow{\text{tex}}$ “
S9”]

[S9 $\xrightarrow{\text{pyk}}$ “axiom s nine”]

MP

[MP $\xrightarrow{\text{proof}}$ Rule tactic]

[MP $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a} : \forall \underline{b} : [[\underline{a} \Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]$]

[MP $\xrightarrow{\text{tex}}$ “
MP”]

[MP $\xrightarrow{\text{pyk}}$ “rule mp”]

Gen

[Gen $\xrightarrow{\text{proof}}$ Rule tactic]

[Gen $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{x} : \forall \underline{a} : [\underline{a} \vdash \dot{\forall} \underline{x} : \underline{a}]$]

[Gen $\xrightarrow{\text{tex}}$ “
Gen”]

[Gen $\xrightarrow{\text{pyk}}$ “rule gen”]

L3.2(a)

[L3.2(a) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash [[S5 \gg [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]] ; [[[[Gen \triangleright [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]] \gg \dot{\forall} \dot{a} : [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]] ; [[[[[A4 @ [\dot{x}]] @ [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]] @ [[\dot{x} + \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [[\dot{\forall} \dot{a} : [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]] \Rightarrow [[\dot{x} + \dot{0}] \stackrel{p}{=} [\dot{x}]]] ; [[[[MP \triangleright [[\dot{\forall} \dot{a} : [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]] \Rightarrow [[\dot{x} + \dot{0}] \stackrel{p}{=} [\dot{x}]]]] \triangleright \dot{\forall} \dot{a} : [[\dot{a} + \dot{0}] \stackrel{p}{=} [\dot{a}]]] \gg [[\dot{x} + \dot{0}] \stackrel{p}{=} [\dot{x}]]]]] , p_0, c)]$]

[L3.2(a) $\xrightarrow{\text{stmt}}$ S $\vdash [\dot{x} \stackrel{p}{=} [\dot{x}]]$]

[L3.2(a) $\xrightarrow{\text{tex}}$ “
L3.2(a)”]

[L3.2(a) $\xrightarrow{\text{pyk}}$ “lemma 1 three two a”]

*

[$\dot{x} \xrightarrow{\text{tex}} \text{"}$
 $\backslash\text{dot}\{\#1.$
 $\}\text{"}$]
[$\dot{x} \xrightarrow{\text{pyk}} \text{"* peano var"}$]

*'

[$x' \xrightarrow{\text{tex}} \text{"}\#1.$
 $\text{"}}$]
[$x' \xrightarrow{\text{pyk}} \text{"* peano succ"}$]

* : *

[$x:y \xrightarrow{\text{tex}} \text{"}\#1.$
 $\backslash\text{mathop}\{\backslash\text{dot}\{\backslash\text{cdot}\}\} \#2.\text{"}$]
[$x:y \xrightarrow{\text{pyk}} \text{"* peano times *"}$]

* . *

[$x+y \xrightarrow{\text{tex}} \text{"}\#1.$
 $\backslash\text{mathop}\{\backslash\text{dot}\{+\}\} \#2.\text{"}$]
[$x+y \xrightarrow{\text{pyk}} \text{"* peano plus *"}$]

* $\stackrel{\text{P}}{=}$ *

[$x \stackrel{\text{P}}{=} y \xrightarrow{\text{tex}} \text{"}\#1.$
 $\backslash\text{stackrel}\{p\}\{=\} \#2.\text{"}$]
[$x \stackrel{\text{P}}{=} y \xrightarrow{\text{pyk}} \text{"* peano is *"}$]

*^P

[$x^P \xrightarrow{\text{val}} x \stackrel{r}{=} [\dot{x}]$]

$[x^P \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\{\} \wedge \{\text{\textbackslash cal P}\}]$

$[x^P \xrightarrow{\text{pyk}} \text{"* is peano var"}]$

$\dot{\neg} *$

$[\dot{\neg} x \xrightarrow{\text{tex}} \text{"}$
 $\text{\textbackslash dot\{\text{\textbackslash neg}\}\,}, \#\mathbf{1}.\text{"}]$

$[\dot{\neg} x \xrightarrow{\text{pyk}} \text{"peano not *"}]$

$* \dot{\wedge} *$

$[x \dot{\wedge} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [x \dot{\wedge} y \stackrel{?}{=} \dot{\neg}(x \Rightarrow \dot{\neg} y)] \rceil)]$

$[x \dot{\wedge} y \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\text{\textbackslash mathrel\{\text{\textbackslash dot\{\text{\textbackslash wedge}\}\}} \#\mathbf{2}.}]$

$[x \dot{\wedge} y \xrightarrow{\text{pyk}} \text{"* peano and *"}]$

$* \dot{\vee} *$

$[x \dot{\vee} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [x \dot{\vee} y \stackrel{?}{=} [\dot{\neg} x] \Rightarrow y] \rceil)]$

$[x \dot{\vee} y \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\text{\textbackslash mathrel\{\text{\textbackslash dot\{\text{\textbackslash vee}\}\}} \#\mathbf{2}.}]$

$[x \dot{\vee} y \xrightarrow{\text{pyk}} \text{"* peano or *"}]$

$\dot{\forall}*:*$

$[\dot{\forall} x: y \xrightarrow{\text{tex}} \text{"}$
 $\text{\textbackslash dot\{\text{\textbackslash forall}\}\#1.}$
 $\text{\textbackslash colon\#\mathbf{2}.}]$

$[\dot{\forall} x: y \xrightarrow{\text{pyk}} \text{"peano all * indeed *"}]$

$\dot{\exists}*:*$

$[\dot{\exists} x: y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{\exists} x: y \stackrel{?}{=} \dot{\neg} \dot{\forall} x: \dot{\neg} y] \rceil)]$

$\dot{[\exists x: y \xrightarrow{\text{tex}} ``}$
 $\backslash\text{dot}\{\backslash\text{exists}\} \#1.$
 $\backslash\text{colon} \#2."]$
 $[\exists x: y \xrightarrow{\text{pyk}} \text{"peano exist * indeed *"}]$

$* \xrightarrow{*}$

$[x \dot{\Rightarrow} y \xrightarrow{\text{tex}} "\#1.$
 $\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{Rightarrow}\} \#2."]$
 $[x \dot{\Rightarrow} y \xrightarrow{\text{pyk}} "* \text{ peano imply *"]}$

$* \xleftarrow{*}$

$[x \dot{\Leftrightarrow} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\Leftrightarrow} y \doteq (x \dot{\Rightarrow} y) \wedge (y \dot{\Rightarrow} x)]])]$
 $[x \dot{\Leftrightarrow} y \xrightarrow{\text{tex}} "\#1.$
 $\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{Leftrightarrow}\} \#2."]$
 $[x \dot{\Leftrightarrow} y \xrightarrow{\text{pyk}} "* \text{ peano iff *"]}$

The pyk compiler, version 0.grue.20050502+ by Klaus Grue
 $GRD-2005-06-02.UTC:10:16:52.703168 = MJD-53523.TAI:10:17:24.703168 =$
 $LGT-4624424244703168e-6$