

Introduction to Logiweb

Klaus Grue

GRD-2005-06-02.UTC:10:16:52.703168

Contents

1	Peano arithmetic	1
1.1	Introduction of new constructs	1
1.2	The constructs of Peano arithmetic	2
1.3	Variables	2
1.4	Axiomatic system	4
1.5	A lemma and a proof	5
A	Chores	6
A.1	The name of the page	6
A.2	Variables of Peano arithmetic	6
A.3	T _E X definitions	7
A.4	Test	12
A.5	Priority table	13
B	Index	15
C	Bibliography	19

1 Peano arithmetic

This Logiweb page [1] defines Peano arithmetic.

1.1 Introduction of new constructs

The following constructs allow to introduce a new construct (“introduce” in the normal sense of the word, not in the sense of a Logiweb revelation). The constructs allow to define the pyk aspect [p] and tex aspect [t] of a new construct [x] and also makes two entries in the index. The first of the constructs below adds the text [i] in front of one of the index entries.

$$[\text{intro}(x, i, p, t) \doteq [x \stackrel{\text{pyk}}{=} p][x \stackrel{\text{tex}}{=} t]]^1$$

¹ $[\text{intro}(x, i, p, t) \stackrel{\text{pyk}}{=} \text{“intro * index * pyk * tex * end intro”}]$

$$[\text{intro}(x, p, t) \doteq [x \stackrel{\text{pyk}}{=} p][x \stackrel{\text{tex}}{=} t]]^2$$

1.2 The constructs of Peano arithmetic

Terms of Peano arithmetic are constructed from zero $[0]^3$, successor $[x']^4$, plus $[x + y]^5$, and times $[x \cdot y]^6$.

Formulas of Peano arithmetic are constructed from equality $[x \stackrel{p}{=} y]^7$, negation $[\neg x]^8$, implication $[x \Rightarrow y]^9$, and universal quantification $[\forall x: y]^10$.

From these constructs we macro define one $[1]^11$, two $[2]^12$, conjunction $[x \wedge y]^13$, disjunction $[x \vee y]^14$, biimplication $[x \Leftrightarrow y]^15$, and existential quantification $[\exists x: y]^16$:

$$[1 \doteq 0']$$

$$[2 \doteq 1']$$

$$[x \wedge y \doteq \neg (x \Rightarrow \neg y)]$$

$$[x \vee y \doteq \neg x \Rightarrow y]$$

$$[x \Leftrightarrow y \doteq (x \Rightarrow y) \wedge (y \Rightarrow x)]$$

$$[\exists x: y \doteq \neg \forall x: \neg y]$$

1.3 Variables

We now introduce the unary operator $[\dot{x}]^{17}$ and define that a term is a *Peano variable* (i.e. a variable of Peano arithmetic) if it has the $[\dot{x}]$ operator in its root. $[x^{\mathcal{P}}]^{18}$ is true if $[x]$ is a Peano variable:

$$^2[\text{intro}(x, p, t) \stackrel{\text{pyk}}{=} \text{"intro * pyk * tex * end intro"}]$$

$$^3[0 \stackrel{\text{pyk}}{=} \text{"peano zero"}]$$

$$^4[x' \stackrel{\text{pyk}}{=} \text{"* peano succ"}]$$

$$^5[x + y \stackrel{\text{pyk}}{=} \text{"* peano plus *"}]$$

$$^6[x \cdot y \stackrel{\text{pyk}}{=} \text{"* peano times *"}]$$

$$^7[x \stackrel{p}{=} y \stackrel{\text{pyk}}{=} \text{"* peano is *"}]$$

$$^8[\neg x \stackrel{\text{pyk}}{=} \text{"peano not *"}]$$

$$^9[x \Rightarrow y \stackrel{\text{pyk}}{=} \text{"* peano imply *"}]$$

$$^{10}[\forall x: y \stackrel{\text{pyk}}{=} \text{"peano all * indeed *"}]$$

$$^{11}[1 \stackrel{\text{pyk}}{=} \text{"peano one"}]$$

$$^{12}[2 \stackrel{\text{pyk}}{=} \text{"peano two"}]$$

$$^{13}[x \wedge y \stackrel{\text{pyk}}{=} \text{"* peano and *"}]$$

$$^{14}[x \vee y \stackrel{\text{pyk}}{=} \text{"* peano or *"}]$$

$$^{15}[x \Leftrightarrow y \stackrel{\text{pyk}}{=} \text{"* peano iff *"}]$$

$$^{16}[\exists x: y \stackrel{\text{pyk}}{=} \text{"peano exist * indeed *"}]$$

$$^{17}[\dot{x} \stackrel{\text{pyk}}{=} \text{"* peano var"}]$$

$$^{18}[x^{\mathcal{P}} \stackrel{\text{pyk}}{=} \text{"* is peano var"}]$$

$$[x^{\mathcal{P}} \doteq x \doteq [\dot{x}]]$$

We macro define $[\dot{a}]$ ¹⁹ to be a Peano variable:

$$[\dot{a} \doteq \dot{a}]$$

Appendix A.2 defines Peano variables for the other letters of the English alphabet.

$[\text{nonfree}(x, y)]$ ²⁰ is true if the Peano variable $[x]$ does not occur free in the Peano term/formula $[y]$. $[\text{nonfree}^*(x, y)]$ ²¹ is true if the Peano variable $[x]$ does not occur free in the list $[y]$ of Peano terms/formulas.

$$\begin{aligned} &[\text{nonfree}(x, y) \doteq \\ &\text{if } y^{\mathcal{P}} \text{ then } \neg x \doteq y \text{ else} \\ &\text{if } \neg y \doteq [\dot{\forall}x: y] \text{ then nonfree}^*(x, y^t) \text{ else} \\ &\text{if } x \doteq y^1 \text{ then } \top \text{ else nonfree}(x, y^2)] \end{aligned}$$

$$[\text{nonfree}^*(x, y) \doteq x! \text{If}(y, \top, \text{nonfree}(x, y^h) \wedge \text{nonfree}^*(x, y^t))]$$

$[\text{free}\langle a|x := b \rangle]$ ²² is true if the substitution $[\langle a|x := b \rangle]$ is free. $[\text{free}^*\langle a|x := b \rangle]$ ²³ is the version where $[a]$ is a list of terms.

$$\begin{aligned} &[\text{free}\langle a|x := b \rangle \doteq x!b! \\ &\text{if } a^{\mathcal{P}} \text{ then } \top \text{ else} \\ &\text{if } \neg a \doteq [\dot{\forall}u: v] \text{ then free}^*\langle a^t|x := b \rangle \text{ else} \\ &\text{if } a^1 \doteq x \text{ then } \top \text{ else} \\ &\text{if nonfree}(x, a^2) \text{ then } \top \text{ else} \\ &\text{if } \neg \text{nonfree}(a^1, b) \text{ then } \top \text{ else} \\ &\text{free}\langle a^2|x := b \rangle] \end{aligned}$$

$$[\text{free}^*\langle a|x := b \rangle \doteq x!b! \text{If}(a, \top, \text{free}\langle a^h|x := b \rangle \wedge \text{free}^*\langle a^t|x := b \rangle)]$$

$[a \equiv \langle b|x := c \rangle]$ ²⁴ is true if $[a]$ equals $[\langle b|x := c \rangle]$. $[a \equiv \langle^*b|x := c \rangle]$ ²⁵ is the version where $[a]$ and $[b]$ are lists.

$$\begin{aligned} &[a \equiv \langle b|x := c \rangle \doteq a!x!c! \\ &\text{if } b \doteq [\dot{\forall}u: v] \wedge b^1 \doteq x \text{ then } a \doteq b \text{ else} \\ &\text{if } b^{\mathcal{P}} \wedge b \doteq x \text{ then } a \doteq c \text{ else} \\ &a \doteq b \wedge a^t \equiv \langle^*b^t|x := c \rangle] \end{aligned}$$

¹⁹ $[\dot{a} \doteq^{\text{pyk}} \text{“peano } a\text{”}]$

²⁰ $[\text{nonfree}(x, y) \doteq^{\text{pyk}} \text{“peano nonfree * in * end nonfree”}]$

²¹ $[\text{nonfree}^*(x, y) \doteq^{\text{pyk}} \text{“peano nonfree star * in * end nonfree”}]$

²² $[\text{free}\langle a|x := b \rangle \doteq^{\text{pyk}} \text{“peano free * set * to * end free”}]$

²³ $[\text{free}^*\langle a|x := b \rangle \doteq^{\text{pyk}} \text{“peano free star * set * to * end free”}]$

²⁴ $[a \equiv \langle b|x := c \rangle \doteq^{\text{pyk}} \text{“peano sub * is * where * is * end sub”}]$

²⁵ $[a \equiv \langle^*b|x := c \rangle \doteq^{\text{pyk}} \text{“peano sub star * is * where * is * end sub”}]$

$$[a \equiv \langle *b | x := c \rangle \doteq b!x!c! \text{If}(a, T, a^h \equiv \langle b^h | x := c \rangle) \wedge a^t \equiv \langle *b^t | x := c \rangle)]$$

1.4 Axiomatic system

System [S]²⁶ of Mendelson [2] expresses Peano arithmetic. It comprises the axioms [A1]²⁷, [A2]²⁸, [A3]²⁹, [A4]³⁰, and [A5]³¹ and inference rules [MP]³² and [Gen]³³ of first order predicate calculus. Furthermore, it comprises the proper axioms [S1]³⁴, [S2]³⁵, [S3]³⁶, [S4]³⁷, [S5]³⁸, [S6]³⁹, [S7]⁴⁰, [S8]⁴¹, and [S9]⁴². System [S] is defined thus:

[Theory S]

[S rule A1: $\forall A: \forall B: A \Rightarrow B \Rightarrow A$]

[S rule A2: $\forall A: \forall B: \forall C: (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$]

[S rule A3: $(\neg B \Rightarrow \neg A) \Rightarrow (\neg B \Rightarrow A) \Rightarrow B$]

The order of quantifiers in the following axiom is such that [A] which the system can currently guess comes first. At a later stage, when a planned generalisation of the conclusion tactic has been implemented, the system will be able to guess [X] and [B] also. Guessing [C] would require a separate tactic.

[S rule A4: $\forall C: \forall X: \forall B: \forall A: A \equiv \langle B | X := C \rangle \Vdash \forall X: B \Rightarrow A$]

[S rule A5: $\forall X: \forall A: \forall B: \text{nonfree}(X, A) \Vdash \forall X: (A \Rightarrow B) \Rightarrow A \Rightarrow \forall X: B$]

[S rule MP: $\forall A: \forall B: A \Rightarrow B \vdash A \vdash B$]

²⁶[S $\stackrel{\text{pyk}}{=}$ “system s”]

²⁷[A1 $\stackrel{\text{pyk}}{=}$ “axiom a one”]

²⁸[A2 $\stackrel{\text{pyk}}{=}$ “axiom a two”]

²⁹[A3 $\stackrel{\text{pyk}}{=}$ “axiom a three”]

³⁰[A4 $\stackrel{\text{pyk}}{=}$ “axiom a four”]

³¹[A5 $\stackrel{\text{pyk}}{=}$ “axiom a five”]

³²[MP $\stackrel{\text{pyk}}{=}$ “rule mp”]

³³[Gen $\stackrel{\text{pyk}}{=}$ “rule gen”]

³⁴[S1 $\stackrel{\text{pyk}}{=}$ “axiom s one”]

³⁵[S2 $\stackrel{\text{pyk}}{=}$ “axiom s two”]

³⁶[S3 $\stackrel{\text{pyk}}{=}$ “axiom s three”]

³⁷[S4 $\stackrel{\text{pyk}}{=}$ “axiom s four”]

³⁸[S5 $\stackrel{\text{pyk}}{=}$ “axiom s five”]

³⁹[S6 $\stackrel{\text{pyk}}{=}$ “axiom s six”]

⁴⁰[S7 $\stackrel{\text{pyk}}{=}$ “axiom s seven”]

⁴¹[S8 $\stackrel{\text{pyk}}{=}$ “axiom s eight”]

⁴²[S9 $\stackrel{\text{pyk}}{=}$ “axiom s nine”]

[S **rule** Gen: $\forall \mathcal{X}: \forall \mathcal{A}: \mathcal{A} \vdash \dot{\forall} \mathcal{X}: \mathcal{A}$]

Axiom [S1] to [S8] are stated as Mendelson [2] does. This is done here to test certain parts of the Logiweb system. Serious users of Peano arithmetic are advised to take Mendelson's Lemma 3.1 as axioms instead.

[S **rule** S1: $\dot{a} \stackrel{P}{=} \dot{b} \Rightarrow \dot{a} \stackrel{P}{=} \dot{c} \Rightarrow \dot{b} \stackrel{P}{=} \dot{c}$]

[S **rule** S2: $\dot{a} \stackrel{P}{=} \dot{b} \Rightarrow \dot{a}' \stackrel{P}{=} \dot{b}'$]

[S **rule** S3: $\neg \dot{0} \stackrel{P}{=} \dot{a}'$]

[S **rule** S4: $\dot{a}' \stackrel{P}{=} \dot{b}' \Rightarrow \dot{a} \stackrel{P}{=} \dot{b}$]

[S **rule** S5: $\dot{a} \dot{+} \dot{0} \stackrel{P}{=} \dot{a}$]

[S **rule** S6: $\dot{a} \dot{+} \dot{b}' \stackrel{P}{=} (\dot{a} \dot{+} \dot{b})'$]

[S **rule** S7: $\dot{a} : \dot{0} \stackrel{P}{=} \dot{0}$]

[S **rule** S8: $\dot{a} : (\dot{b}') \stackrel{P}{=} (\dot{a} : \dot{b}) \dot{+} \dot{a}$]

[S **rule** S9: $\forall \mathcal{A}: \forall \mathcal{B}: \forall \mathcal{C}: \forall \mathcal{X}: \mathcal{B} \equiv \langle \mathcal{A} | \mathcal{X} := \dot{0} \rangle \Vdash \mathcal{C} \equiv \langle \mathcal{A} | \mathcal{X} := \mathcal{X}' \rangle \Vdash \mathcal{B} \Rightarrow \dot{\forall} \mathcal{X}: (\mathcal{A} \Rightarrow \mathcal{C}) \Rightarrow \dot{\forall} \mathcal{X}: \mathcal{A}$]

1.5 A lemma and a proof

We now prove Lemma [L3.2(a)]⁴³ which is an instance of the corresponding proposition in Mendelson [2]:

[S **lemma** L3.2(a): $\dot{x} \stackrel{P}{=} \dot{x}$]

The first four lines of the proof has revealed an error in the conclusion tactic which has to be corrected before the proof will be accepted:

S **proof** of L3.2(a):

L01:	S5 $\gg$	$\dot{a} \dot{+} \dot{0} \stackrel{P}{=} \dot{a}$	;
L02:	Gen $\triangleright$ L01 $\gg$	$\dot{\forall} \dot{a}: \dot{a} \dot{+} \dot{0} \stackrel{P}{=} \dot{a}$	;
L03:	A4 @ $\dot{x}$ @ $\dot{a}$ @ $\dot{a} \dot{+} \dot{0} \stackrel{P}{=} \dot{a}$ @ $\dot{x} \dot{+} \dot{0} \stackrel{P}{=} \dot{x} \gg$	$\dot{\forall} \dot{a}: \dot{a} \dot{+} \dot{0} \stackrel{P}{=} \dot{a} \Rightarrow \dot{x} \dot{+} \dot{0} \stackrel{P}{=} \dot{x}$	;
L04:	MP $\triangleright$ L03 $\triangleright$ L02 $\gg$	$\dot{x} \dot{+} \dot{0} \stackrel{P}{=} \dot{x}$	□

⁴³[L3.2(a)]^{pyk} $\equiv$ "lemma l three two a"]

A Chores

A.1 The name of the page

This defines the name of the page:

$$[\text{peano} \stackrel{\text{pyk}}{=} \text{“peano”}]$$

A.2 Variables of Peano arithmetic

We use $[\dot{b}]^{44}$, $[\dot{c}]^{45}$, $[\dot{d}]^{46}$, $[\dot{e}]^{47}$, $[\dot{f}]^{48}$, $[\dot{g}]^{49}$, $[\dot{h}]^{50}$, $[\dot{i}]^{51}$, $[\dot{j}]^{52}$, $[\dot{k}]^{53}$, $[\dot{l}]^{54}$, $[\dot{m}]^{55}$, $[\dot{n}]^{56}$, $[\dot{o}]^{57}$, $[\dot{p}]^{58}$, $[\dot{q}]^{59}$, $[\dot{r}]^{60}$, $[\dot{s}]^{61}$, $[\dot{t}]^{62}$, $[\dot{u}]^{63}$, $[\dot{v}]^{64}$, $[\dot{w}]^{65}$, $[\dot{x}]^{66}$, $[\dot{y}]^{67}$, and $[\dot{z}]^{68}$ to denote variables of Peano arithmetic:

$[\dot{b} \doteq \dot{b}]$, $[\dot{c} \doteq \dot{c}]$, $[\dot{d} \doteq \dot{d}]$, $[\dot{e} \doteq \dot{e}]$, $[\dot{f} \doteq \dot{f}]$, $[\dot{g} \doteq \dot{g}]$, $[\dot{h} \doteq \dot{h}]$, $[\dot{i} \doteq \dot{i}]$, $[\dot{j} \doteq \dot{j}]$, $[\dot{k} \doteq \dot{k}]$, $[\dot{l} \doteq \dot{l}]$, $[\dot{m} \doteq \dot{m}]$, $[\dot{n} \doteq \dot{n}]$, $[\dot{o} \doteq \dot{o}]$, $[\dot{p} \doteq \dot{p}]$, $[\dot{q} \doteq \dot{q}]$, $[\dot{r} \doteq \dot{r}]$, $[\dot{s} \doteq \dot{s}]$, $[\dot{t} \doteq \dot{t}]$, $[\dot{u} \doteq \dot{u}]$, $[\dot{v} \doteq \dot{v}]$, $[\dot{w} \doteq \dot{w}]$, $[\dot{x} \doteq \dot{x}]$, $[\dot{y} \doteq \dot{y}]$, and $[\dot{z} \doteq \dot{z}]$.

-
- 44 $[\dot{b} \stackrel{\text{pyk}}{=} \text{“peano b”}]$
 - 45 $[\dot{c} \stackrel{\text{pyk}}{=} \text{“peano c”}]$
 - 46 $[\dot{d} \stackrel{\text{pyk}}{=} \text{“peano d”}]$
 - 47 $[\dot{e} \stackrel{\text{pyk}}{=} \text{“peano e”}]$
 - 48 $[\dot{f} \stackrel{\text{pyk}}{=} \text{“peano f”}]$
 - 49 $[\dot{g} \stackrel{\text{pyk}}{=} \text{“peano g”}]$
 - 50 $[\dot{h} \stackrel{\text{pyk}}{=} \text{“peano h”}]$
 - 51 $[\dot{i} \stackrel{\text{pyk}}{=} \text{“peano i”}]$
 - 52 $[\dot{j} \stackrel{\text{pyk}}{=} \text{“peano j”}]$
 - 53 $[\dot{k} \stackrel{\text{pyk}}{=} \text{“peano k”}]$
 - 54 $[\dot{l} \stackrel{\text{pyk}}{=} \text{“peano l”}]$
 - 55 $[\dot{m} \stackrel{\text{pyk}}{=} \text{“peano m”}]$
 - 56 $[\dot{n} \stackrel{\text{pyk}}{=} \text{“peano n”}]$
 - 57 $[\dot{o} \stackrel{\text{pyk}}{=} \text{“peano o”}]$
 - 58 $[\dot{p} \stackrel{\text{pyk}}{=} \text{“peano p”}]$
 - 59 $[\dot{q} \stackrel{\text{pyk}}{=} \text{“peano q”}]$
 - 60 $[\dot{r} \stackrel{\text{pyk}}{=} \text{“peano r”}]$
 - 61 $[\dot{s} \stackrel{\text{pyk}}{=} \text{“peano s”}]$
 - 62 $[\dot{t} \stackrel{\text{pyk}}{=} \text{“peano t”}]$
 - 63 $[\dot{u} \stackrel{\text{pyk}}{=} \text{“peano u”}]$
 - 64 $[\dot{v} \stackrel{\text{pyk}}{=} \text{“peano v”}]$
 - 65 $[\dot{w} \stackrel{\text{pyk}}{=} \text{“peano w”}]$
 - 66 $[\dot{x} \stackrel{\text{pyk}}{=} \text{“peano x”}]$
 - 67 $[\dot{y} \stackrel{\text{pyk}}{=} \text{“peano y”}]$
 - 68 $[\dot{z} \stackrel{\text{pyk}}{=} \text{“peano z”}]$

A.3 T_EX definitions

```
[intro(x, i, p, t) tex ≡ “\index{#2.: #3. @#2.: $[#1/tex name/tex.]$ #3.}%
\index{#3. $[#1/tex name/tex.]$}%
\tex{
$[#1/tex name/tex.
\stackrel{\mathrm{tex}}{=} #4/tex name.
}]$ $[ #1/tex name/tex.%
]$ \footnote{$[#1/tex name/tex.
\stackrel{\mathrm{pyk}}{=} #3/tex name.
]$}”]
```

```
[intro(x, i, p, t) name ≡ “
intro(#1.
, #2.
, #3.
, #4.
)”]
```

```
[intro(x, p, t) tex ≡ “\index{\alpha #2. @\back \makebox[20mm][l]{$[#1/tex
name/tex.]$}#2.}%
\index{#2. $[#1/tex name/tex.]$}%
\tex{
$[#1/tex name/tex.
\stackrel{\mathrm{tex}}{=} #3/tex name.
}]$ $[ #1/tex name/tex.%
]$ \footnote{$[#1/tex name/tex.
\stackrel{\mathrm{pyk}}{=} #2/tex name.
]$}”]
```

```
[intro(x, p, t) name ≡ “
intro(#1.
, #2.
, #3.
)”]
```

```
[0̇ tex ≡ “
\dot{0}”]
```

```
[x' tex ≡ “#1.
””]
```

```
[x + y tex ≡ “#1.
\mathop{\dot{+}} #2.”]
```

```
[x : y tex ≡ “#1.
\mathop{\dot{\cdot}} #2.”]
```

$$\begin{aligned}
&[x \stackrel{\mathrm{p}}{=} y \stackrel{\mathrm{tex}}{=} \text{“}\#1. \\
&\quad \backslash\mathrm{stackrel}\{\mathrm{p}\}\{=\} \text{ }\#2.\text{”}] \\
&[\dot{-}x \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{\backslash\mathrm{neg}\}\backslash, \text{ }\#1.\text{”}] \\
&[x \Rightarrow y \stackrel{\mathrm{tex}}{=} \text{“}\#1. \\
&\quad \backslash\mathrm{mathrel}\{\backslash\mathrm{dot}\{\backslash\mathrm{Rrightarrow}\}\} \text{ }\#2.\text{”}] \\
&[\dot{\forall}x:y \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{\backslash\mathrm{forall}\} \text{ }\#1. \\
&\quad \backslash\mathrm{colon} \text{ }\#2.\text{”}] \\
&[\dot{1} \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{1\}\text{”}] \\
&[\dot{2} \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{2\}\text{”}] \\
&[x \dot{\wedge} y \stackrel{\mathrm{tex}}{=} \text{“}\#1. \\
&\quad \backslash\mathrm{mathrel}\{\backslash\mathrm{dot}\{\backslash\mathrm{wedge}\}\} \text{ }\#2.\text{”}] \\
&[x \dot{\vee} y \stackrel{\mathrm{tex}}{=} \text{“}\#1. \\
&\quad \backslash\mathrm{mathrel}\{\backslash\mathrm{dot}\{\backslash\mathrm{vee}\}\} \text{ }\#2.\text{”}] \\
&[x \Leftrightarrow y \stackrel{\mathrm{tex}}{=} \text{“}\#1. \\
&\quad \backslash\mathrm{mathrel}\{\backslash\mathrm{dot}\{\backslash\mathrm{Leftrightarrow}\}\} \text{ }\#2.\text{”}] \\
&[\dot{\exists}x:y \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{\backslash\mathrm{exists}\} \text{ }\#1. \\
&\quad \backslash\mathrm{colon} \text{ }\#2.\text{”}] \\
&[\dot{x} \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{\#1. \\
&\quad \}\text{”}] \\
&[x^{\mathcal{P}} \stackrel{\mathrm{tex}}{=} \text{“}\#1. \\
&\quad \{\} \wedge \{\backslash\mathrm{cal} \text{ P}\}\text{”}] \\
&[\dot{a} \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{\backslash\mathrm{mathit}\{\mathrm{a}\}\}\text{”}] \\
&[\mathrm{nonfree}(x,y) \stackrel{\mathrm{tex}}{=} \text{“} \\
&\quad \backslash\mathrm{dot}\{\mathrm{nonfree}\}(\#1. \\
&\quad , \text{ }\#2. \\
&\quad)\text{”}]
\end{aligned}$$

$$[\text{nonfree}^*(x, y) \stackrel{\text{tex}}{=} \text{"} \backslash \text{dot}\{\text{nonfree}\}^*(\#1. \\ , \#2. \\)"]$$
$$\begin{aligned} [\text{free}\langle a|x := b \rangle] &\stackrel{\text{tex}}{=} \text{“} \\ &\quad \backslash \text{dot}\{\text{free}\} \backslash \text{angle} \#1. \\ &\quad | \#2. \\ &\quad := \#3. \\ &\quad \backslash \text{rangle}\text{”} \end{aligned}$$
$$[\text{free}^* \langle a | x := b \rangle \stackrel{\text{tex}}{=} \langle \dot{\text{free}} \rangle^* \rangle \# 1. \\ | \# 2. \\ := \# 3. \\ \rangle \text{rangle}"]$$
$$[a \equiv b | x := c]^{\text{tex}} \equiv \begin{array}{l} \text{"\#1.} \\ \{\backslash\text{equiv}\} \backslash\text{rangle \#2.} \\ \backslash\text{\#3.} \\ \text{:}=\text{\#4.} \\ \backslash\text{rangle"} \end{array}$$
$$\begin{aligned} [a \equiv \langle * b | x := c \rangle] &\stackrel{\text{tex}}{=} \text{"\#1.} \\ &\{\backslash \text{equiv}\} \backslash \text{angle}^* \text{\#2.} \\ &|\text{\#3.} \\ &:= \text{\#4.} \\ &\backslash \text{angle}"] \end{aligned}$$

[S tex “ S”]

[A1 ^{tex} “ A1”]

[A2^{tex} “ A2”]

[A3^{tex} “ A3”]

[A4^{tex} “ A4”]

[A5^{tex} “ A5”]

$$[\text{MP} \stackrel{\text{tex}}{=} \text{“MP”}]$$

$$[\text{Gen} \stackrel{\text{tex}}{=} \text{“Gen”}]$$

$$[\text{S1} \stackrel{\text{tex}}{=} \text{“S1”}]$$

$$[\text{S2} \stackrel{\text{tex}}{=} \text{“S2”}]$$

$$[\text{S3} \stackrel{\text{tex}}{=} \text{“S3”}]$$

$$[\text{S4} \stackrel{\text{tex}}{=} \text{“S4”}]$$

$$[\text{S5} \stackrel{\text{tex}}{=} \text{“S5”}]$$

$$[\text{S6} \stackrel{\text{tex}}{=} \text{“S6”}]$$

$$[\text{S7} \stackrel{\text{tex}}{=} \text{“S7”}]$$

$$[\text{S8} \stackrel{\text{tex}}{=} \text{“S8”}]$$

$$[\text{S9} \stackrel{\text{tex}}{=} \text{“S9”}]$$

$$[\text{L3.2(a)} \stackrel{\text{tex}}{=} \text{“L3.2(a)”}]$$

$$[\dot{b} \stackrel{\text{tex}}{=} \text{“}\dot{\mathit{b}}\text{”}]$$

$$[\dot{c} \stackrel{\text{tex}}{=} \text{“}\dot{\mathit{c}}\text{”}]$$

$$[\dot{d} \stackrel{\text{tex}}{=} \text{“}\dot{\mathit{d}}\text{”}]$$

$$[\dot{e} \stackrel{\text{tex}}{=} \text{“}\dot{\mathit{e}}\text{”}]$$

$\dot{f}^{\text{tex}}$ “	$\dot{\mathit{f}}$ ”]
$\dot{g}^{\text{tex}}$ “	$\dot{\mathit{g}}$ ”]
$\dot{h}^{\text{tex}}$ “	$\dot{\mathit{h}}$ ”]
$\dot{i}^{\text{tex}}$ “	$\dot{\mathit{i}}$ ”]
$\dot{j}^{\text{tex}}$ “	$\dot{\mathit{j}}$ ”]
$\dot{k}^{\text{tex}}$ “	$\dot{\mathit{k}}$ ”]
$\dot{l}^{\text{tex}}$ “	$\dot{\mathit{l}}$ ”]
$\dot{m}^{\text{tex}}$ “	$\dot{\mathit{m}}$ ”]
$\dot{n}^{\text{tex}}$ “	$\dot{\mathit{n}}$ ”]
$\dot{o}^{\text{tex}}$ “	$\dot{\mathit{o}}$ ”]
$\dot{p}^{\text{tex}}$ “	$\dot{\mathit{p}}$ ”]
$\dot{q}^{\text{tex}}$ “	$\dot{\mathit{q}}$ ”]
$\dot{r}^{\text{tex}}$ “	$\dot{\mathit{r}}$ ”]
$\dot{s}^{\text{tex}}$ “	$\dot{\mathit{s}}$ ”]
$\dot{t}^{\text{tex}}$ “	$\dot{\mathit{t}}$ ”]
$\dot{u}^{\text{tex}}$ “	$\dot{\mathit{u}}$ ”]

$$[\dot{v} \stackrel{\text{tex}}{=} \text{“} \quad \backslash\text{dot}\{\backslash\text{mathit}\{\text{v}\}\}\text{”}]$$

$$[\dot{w} \stackrel{\text{tex}}{=} \text{“} \quad \backslash\text{dot}\{\backslash\text{mathit}\{\text{w}\}\}\text{”}]$$

$$[\dot{x} \stackrel{\text{tex}}{=} \text{“} \quad \backslash\text{dot}\{\backslash\text{mathit}\{\text{x}\}\}\text{”}]$$

$$[\dot{y} \stackrel{\text{tex}}{=} \text{“} \quad \backslash\text{dot}\{\backslash\text{mathit}\{\text{y}\}\}\text{”}]$$

$$[\dot{z} \stackrel{\text{tex}}{=} \text{“} \quad \backslash\text{dot}\{\backslash\text{mathit}\{\text{z}\}\}\text{”}]$$

A.4 Test

$$[[a]^{\mathcal{P}}].$$

$$[[a]^{\mathcal{P}}]^{-}$$

$$[\text{nonfree}(\lceil \dot{x} \rceil, [\dot{y} \stackrel{\text{P}}{=} z \Rightarrow \dot{\forall} \dot{x}: \dot{x} \stackrel{\text{P}}{=} \dot{y}])].$$

$$[\text{nonfree}(\lceil \dot{x} \rceil, [\dot{x} \stackrel{\text{P}}{=} z \Rightarrow \dot{\forall} \dot{x}: \dot{x} \stackrel{\text{P}}{=} \dot{y}])]^{-}$$

$$[\text{nonfree}(\lceil \dot{x} \rceil, [\dot{y} \stackrel{\text{P}}{=} \dot{x} \Rightarrow \dot{\forall} \dot{x}: \dot{x} \stackrel{\text{P}}{=} \dot{y}])]^{-}$$

$$[\text{nonfree}(\lceil \dot{x} \rceil, [\dot{y} \stackrel{\text{P}}{=} z \Rightarrow \dot{\forall} \dot{y}: \dot{x} \stackrel{\text{P}}{=} \dot{y}])]^{-}$$

$$[\text{free}(\langle \lceil \dot{\forall} \dot{x}: \mathbf{b} :: \dot{x} :: \mathbf{c} \rceil \mid \lceil \dot{x} \rceil := \lceil \mathbf{x} :: \dot{y} :: \mathbf{z} \rceil \rangle)].$$

$$[\text{free}(\langle \lceil \dot{\forall} \dot{y}: \mathbf{b} :: \dot{x} :: \mathbf{c} \rceil \mid \lceil \dot{x} \rceil := \lceil \mathbf{x} :: \dot{y} :: \mathbf{z} \rceil \rangle)]^{-}$$

$$[\text{free}(\langle \lceil \dot{\forall} \dot{x}: \mathbf{b} :: \dot{x} :: \mathbf{c} \rceil \mid \lceil \dot{y} \rceil := \lceil \mathbf{x} :: \dot{y} :: \mathbf{z} \rceil \rangle)].$$

$$[\text{free}(\langle \lceil \dot{\forall} \dot{y}: \mathbf{b} :: \dot{x} :: \mathbf{c} \rceil \mid \lceil \dot{y} \rceil := \lceil \mathbf{x} :: \dot{y} :: \mathbf{z} \rceil \rangle)].$$

$$[\dot{a} \equiv \langle \dot{a} \mid \dot{b} := \dot{c} \rangle].$$

$$[\dot{c} \equiv \langle \dot{b} \mid \dot{b} := \dot{c} \rangle].$$

$$[\forall \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{b} \equiv \langle \forall \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{b} \mid \dot{a} := \dot{c} \rangle].$$

$$[\forall \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{c} \equiv \langle \forall \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{b} \mid \dot{b} := \dot{c} \rangle].$$

$$[\dot{\forall} \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{a} \Rightarrow \dot{c}: \dot{d} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{c}: \dot{d} \equiv \langle \dot{\forall} \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{a} \Rightarrow \dot{b} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{b} \mid \dot{b} := \dot{c}: \dot{d} \rangle].$$

$$[\dot{\forall} \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{a} \Rightarrow \dot{b} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{b} \equiv \langle \dot{\forall} \dot{a}: \dot{a} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{a} \Rightarrow \dot{b} \stackrel{\text{P}}{=} \dot{0} \dot{+} \dot{b} \mid \dot{a} := \dot{c} \rangle].$$

A.5 Priority table

Priority table

Preassociative

[peano], [base], [bracket * end bracket], [big bracket * end bracket],
[math * end math], [**flush left** *], [x], [y], [z], [$[* \bowtie *]$], [$[* \rightarrow *]$], [pyk], [tex],
[name], [prio], [*,], [T], [if(*, *, *)], [$[* \xrightarrow{*} *]$], [val], [claim], [$\perp$], [f(*)], [$(*)^I$], [F], [0],
[1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d],
[e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^M$], [If(*, *,
)], [array{} * end array], [l], [c], [r], [empty], [$[* | * := *]$], [$\mathcal{M}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}(*)$],
 $\mathcal{U}^M(*)$], [**apply**(*, *)], [**apply**₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-
plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)],
[bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"],
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
 $\mathcal{E}(*, *, *)$, [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *, *)$], [$\mathcal{E}_4(*, *, *, *, *)$], [**lookup**(*, *, *)],
[**abstract**(*, *, *, *)], [$[*]$], [$\mathcal{M}(*, *, *, *)$], [$\mathcal{M}_2(*, *, *, *, *)$], [$\mathcal{M}^*(*, *, *, *)$], [macro],
[s₀], [**zip**(*, *)], [**assoc**₁(*, *, *)], [$(*)^P$], [self], [$[* \doteq *]$], [$[* \doteq *]$], [$[* \doteq *]$],
 $[* \stackrel{\text{pyk}}{=} *]$, [$[* \stackrel{\text{tex}}{=} *]$], [$[* \stackrel{\text{name}}{=} *]$], [**Priority table**[*]], [$\hat{\mathcal{M}}_1$], [$\hat{\mathcal{M}}_2(*)$], [$\hat{\mathcal{M}}_3(*)$],
 $\hat{\mathcal{M}}_4(*, *, *, *, *)$], [$\mathcal{M}(*, *, *, *)$], [$\hat{\mathcal{Q}}(*, *, *, *)$], [$\hat{\mathcal{Q}}_2(*, *, *, *)$], [$\hat{\mathcal{Q}}_3(*, *, *, *, *)$], [$\hat{\mathcal{Q}}^*(*, *, *, *)$],
 $(*)$], [**aspect**(*, *)], [**aspect**(*, *, *)], [$\langle * \rangle$], [**tuple**₁(*)], [**tuple**₂(*)], [let₂(*, *)],
[let₁(*, *)], [$[* \stackrel{\text{claim}}{=} *]$], [checker], [**check**(*, *)], [**check**₂(*, *, *, *)], [**check**₃(*, *, *, *)],
[**check**^{*}(*, *)], [**check**₂^{*}(*, *, *, *)], [$[* \cdot]$], [$[* \cdot]^-$], [$[* \cdot]^\circ$], [msg], [$[* \stackrel{\text{msg}}{=} *]$], [<stmt>],
[stmt], [$[* \stackrel{\text{stmt}}{=} *]$], [HeadNil'], [HeadPair'], [Transitivity'], [$\perp$], [Contra'], [T_E],
[L₁], [$\underline{*}$], [A], [B], [C], [D], [E], [F], [G], [H], [I], [J], [K], [L], [M], [N], [O], [P], [Q],
[R], [S], [T], [U], [V], [W], [X], [Y], [Z], [$[* | * := *]$], [$[*^* | * := *]$], [$\emptyset$], [Remainder],
 $(*)^\vee$], [error(*, *)], [error₂(*, *)], [proof(*, *, *)], [proof₂(*, *, *)], [S(*, *)], [S^I(*, *, *)],
 $S^\triangleright(*, *)$], [S₁[▷](*, *, *, *)], [S^E(*, *)], [S₁^E(*, *, *, *)], [S⁺(*, *)], [S₁⁺(*, *, *, *)],
 $S^-(*, *)$], [S₁⁻(*, *, *, *)], [S^{*}(*, *)], [S₁^{*}(*, *, *, *)], [S₂^{*}(*, *, *, *, *)], [S[@](*, *)],
[S₁[@](*, *, *, *)], [S[†](*, *)], [S₁[†](*, *, *, *, *)], [S[‡](*, *)], [S₁[‡](*, *, *, *, *)], [S^{i.e.}(*, *, *)],
[S₁^{i.e.}(*, *, *, *, *)], [S₂^{i.e.}(*, *, *, *, *)], [S[∇](*, *)], [S₁[∇](*, *, *, *, *)], [Sⁱ(*, *)],
[S₁ⁱ(*, *, *, *)], [S₂ⁱ(*, *, *, *, *)], [T(*)], [claims(*, *, *)], [claims₂(*, *, *)], [<proof>],
[proof], [**Lemma** *: *], [**Proof of** *: *], [*** lemma** *: *],
 $[* \text{ antilemma } *: *]$, [$[* \text{ rule } *: *]$], [$[* \text{ antirule } *: *]$], [verifier], [V₁(*)],
V₂(*, *)], [V₃(*, *, *, *)], [V₄(*, *)], [V₅(*, *, *, *)], [V₆(*, *, *, *, *)], [V₇(*, *, *, *, *)],
[Cut(*, *)], [Head_⊕(*)], [Tail_⊕(*)], [rule₁(*, *)], [rule(*, *)], [Rule tactic],
[Plus(*, *)], [**Theory** *], [theory₂(*, *)], [theory₃(*, *)], [theory₄(*, *, *)],
[HeadNil''], [HeadPair''], [Transitivity''], [Contra''], [HeadNil], [HeadPair],
[Transitivity], [Contra], [T_E], [ragged right], [ragged right expansion],
[parm(*, *, *)], [parm^{*}(*, *, *)], [inst(*, *)], [inst^{*}(*, *)], [occur(*, *, *)],
occur^{*}(*, *, *)], [unify(* = *, *)], [unify^{*}(* = *, *)], [unify₂(* = *, *)], [L_a], [L_b],
[L_c], [L_d], [L_e], [L_f], [L_g], [L_h], [L_i], [L_j], [L_k], [L_l], [L_m], [L_n], [L_o], [L_p], [L_q], [L_r],
[L_s], [L_t], [L_u], [L_v], [L_w], [L_x], [L_y], [L_z], [L_A], [L_B], [L_C], [L_D], [L_E], [L_F], [L_G],

[L_H], [L_I], [L_J], [L_K], [L_L], [L_M], [L_N], [L_O], [L_P], [L_Q], [L_R], [L_S], [L_T], [L_U], [L_V],
[L_W], [L_X], [L_Y], [L_Z], [L_?], [Reflexivity], [Reflexivity₁], [Commutativity],
[Commutativity₁], [<tactic>], [tactic], [[* $\stackrel{\text{tactic}}{=}$ *]], [$\mathcal{P}^*(*, *, *)$], [$\mathcal{P}^*(*, *, *)$], [p₀],
[conclude₁(*, *)], [conclude₂(*, *, *)], [conclude₃(*, *, *, *)], [intro(*, *, *, *)],
[intro(*, *, *)], [0̇], [1̇], [2̇], [ā], [ḃ], [ċ], [ḋ], [ē], [ḟ], [ġ], [ḣ], [i̇], [j̇], [k̇], [l̇], [ṁ], [ṅ], [ō],
[ṗ], [q̇], [ṙ], [ṡ], [ṫ], [ū], [v̇], [ẇ], [ẋ], [ẏ], [ż], [nonfree(*, *)], [nonfree^{*}(*, *)],
[free(*|* := *)], [free^{*}(*|* := *)], [*≡(*|* := *)], [*≡^{*}(*|* := *)], [S], [A1], [A2],
[A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen],
[L3.2(a)];

Preassociative

[*_{*}], [*'], [*|*], [*|* → *], [*|* ⇒ *], [*];

Preassociative

[“ * ”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [
*, [*], [! *], [” *], [# *], [\$ *], [% *], [& *], [’ *], [(*)], [*)], [**], [+ *], [*], [- *], [· *], [/ *],
[0 *], [1 *], [2 *], [3 *], [4 *], [5 *], [6 *], [7 *], [8 *], [9 *], [: *], [; *], [< *], [= *], [> *], [? *],
[@ *], [A *], [B *], [C *], [D *], [E *], [F *], [G *], [H *], [I *], [J *], [K *], [L *], [M *], [N *],
[O *], [P *], [Q *], [R *], [S *], [T *], [U *], [V *], [W *], [X *], [Y *], [Z *], [[*], [\ *], [] *], [^ *],
[_ *], [‘ *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *],
[p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [{ * }, [| *], { } *], [~ *],
[Preassociative *; *], [Postassociative *; *], [[*], *], [priority * end],
[newline *], [macro newline *];

Preassociative

[*0], [*1], [0b], [*-color(*)], [*-color^{*}(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*ⁱ], [*^d], [*^R], [*⁰],
[*¹], [*²], [*³], [*⁴], [*⁵], [*⁶], [*⁷], [*⁸], [*⁹], [*^E], [*^V], [*^C], [*^{C*}], [*[′]];

Preassociative

[* · *], [* ·₀ *], [* · *];

Preassociative

[* + *], [* +₀ *], [* +₁ *], [* − *], [* −₀ *], [* −₁ *], [* ÷ *];

Preassociative

[* ∪ { * }], [* ∪ *], [* \ { * }];

Postassociative

[* ∴ *], [* ∴̇ *], [* ∴̈ *], [* +₂ *], [* ∴ ∴ *], [* +₂ * *];

Postassociative

[*, *];

Preassociative

[* $\stackrel{B}{\approx}$ *], [* $\stackrel{D}{\approx}$ *], [* $\stackrel{C}{\approx}$ *], [* $\stackrel{P}{\approx}$ *], [* $\approx$ *], [* = *], [* $\stackrel{+}{\rightarrow}$ *], [* $\stackrel{t}{=}$ *], [* $\stackrel{t^*}{=}$ *], [* $\stackrel{r}{=}$ *],
[* $\in_t$ *], [* $\subseteq_T$ *], [* $\stackrel{T}{=}$ *], [* $\stackrel{s}{=}$ *], [* free in *], [* free in^{*} *], [* free for * in *],
[* free for^{*} * in *], [* $\in_c$ *], [* < *], [* <′ *], [* ≤′ *], [* $\stackrel{P}{=}$ *], [*^P];

Preassociative

[¬ *], [¬̇ *];

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$

Preassociative

$[\dot{\forall}*: *], [\dot{\exists}*: *];$

Postassociative

$[* \rightrightarrows *], [* \Rightarrow *], [* \Leftrightarrow *];$

Postassociative

$[* : *], [*! *];$

Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right\}];$

Preassociative

$[\lambda *. *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \ddot{=} * \text{ in } *];$

Preassociative

$[*!], [* \triangleright], [*^V], [*^+], [*^-], [*^*];$

Preassociative

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *];$

Postassociative

$[* \vdash *], [* \Vdash *], [* \text{i.e. } *];$

Preassociative

$[\forall *: *];$

Postassociative

$[* \oplus *];$

Postassociative

$[*, *];$

Preassociative

$[* \text{ proves } *];$

Preassociative

$[* \text{ proof of } *: *], [\text{Line } *: * \gg *], [\text{Last line } * \gg * \square],$
 $[\text{Line } *: \text{Premise } \gg *: *], [\text{Line } *: \text{Side-condition } \gg *: *], [\text{Arbitrary } \gg *: *],$
 $[\text{Local } \gg * = *: *];$

Postassociative

$[* \text{ then } *], [* [*] *];$

Preassociative

$[* \& *];$

Preassociative

$[* \setminus *]; \text{End table}$

B Index

- * is peano var $[x^{\mathcal{P}}]$, 2
- * peano and $* [x \wedge y]$, 2
- * peano iff $* [x \Leftrightarrow y]$, 2
- * peano imply $* [x \Rightarrow y]$, 2

$* \text{ peano is } *$	$[x \stackrel{p}{=} y], 2$
$* \text{ peano or } *$	$[x \dot{\vee} y], 2$
$* \text{ peano plus } *$	$[x \dot{+} y], 2$
$* \text{ peano succ }$	$[x'], 2$
$* \text{ peano times } *$	$[x \dot{:} y], 2$
$* \text{ peano var }$	$[\dot{x}], 2$
$[x^p]$	$* \text{ is peano var}, 2$
$[x \wedge y]$	$* \text{ peano and } *, 2$
$[x \Leftrightarrow y]$	$* \text{ peano iff } *, 2$
$[x \Rightarrow y]$	$* \text{ peano imply } *, 2$
$[x \stackrel{p}{=} y]$	$* \text{ peano is } *, 2$
$[x \dot{\vee} y]$	$* \text{ peano or } *, 2$
$[x \dot{+} y]$	$* \text{ peano plus } *, 2$
$[x']$	$* \text{ peano succ}, 2$
$[x \dot{:} y]$	$* \text{ peano times } *, 2$
$[\dot{x}]$	$* \text{ peano var}, 2$
$[A5]$	axiom a five, 4
$[A4]$	axiom a four, 4
$[A1]$	axiom a one, 4
$[A3]$	axiom a three, 4
$[A2]$	axiom a two, 4
$[S8]$	axiom s eight, 4
$[S5]$	axiom s five, 4
$[S4]$	axiom s four, 4
$[S9]$	axiom s nine, 4
$[S1]$	axiom s one, 4
$[S7]$	axiom s seven, 4
$[S6]$	axiom s six, 4
$[S3]$	axiom s three, 4
$[S2]$	axiom s two, 4
$[L3.2(a)]$	lemma l three two a, 5
$[\dot{a}]$	peano a, 3
$[\dot{\forall}x: y]$	peano all $*$ indeed $*$, 2
$[\dot{b}]$	peano b, 6
$[\dot{c}]$	peano c, 6
$[\dot{d}]$	peano d, 6
$[\dot{e}]$	peano e, 6
$[\dot{\exists}x: y]$	peano exist $*$ indeed $*$, 2
$[\dot{f}]$	peano f, 6
$[\dot{g}]$	peano g, 6
$[\dot{h}]$	peano h, 6
$[\dot{i}]$	peano i, 6
$[\dot{j}]$	peano j, 6
$[\dot{k}]$	peano k, 6
$[\dot{l}]$	peano l, 6

$[\dot{m}]$	peano m, 6
$[\dot{n}]$	peano n, 6
$[\text{nonfree}(x, y)]$	peano nonfree * in * end nonfree, 3
$[\text{nonfree}^*(x, y)]$	peano nonfree star * in * end nonfree, 3
$[\neg x]$	peano not *, 2
$[\dot{o}]$	peano o, 6
$[\dot{1}]$	peano one, 2
$[\dot{p}]$	peano p, 6
$[\dot{q}]$	peano q, 6
$[\dot{r}]$	peano r, 6
$[\dot{s}]$	peano s, 6
$[\dot{t}]$	peano t, 6
$[\dot{2}]$	peano two, 2
$[\dot{u}]$	peano u, 6
$[\dot{v}]$	peano v, 6
$[\dot{w}]$	peano w, 6
$[\dot{x}]$	peano x, 6
$[\dot{y}]$	peano y, 6
$[\dot{z}]$	peano z, 6
$[\dot{0}]$	peano zero, 2
[Gen]	rule gen, 4
[MP]	rule mp, 4
[S]	system s, 4

axiom a five [A5], **4**
 axiom a four [A4], **4**
 axiom a one [A1], **4**
 axiom a three [A3], **4**
 axiom a two [A2], **4**
 axiom s eight [S8], **4**
 axiom s five [S5], **4**
 axiom s four [S4], **4**
 axiom s nine [S9], **4**
 axiom s one [S1], **4**
 axiom s seven [S7], **4**
 axiom s six [S6], **4**
 axiom s three [S3], **4**
 axiom s two [S2], **4**

intro * index * pyk * tex * end intro [intro(x, i, p, t)], **1**
 intro * pyk * tex * end intro [intro(x, p, t)], **2**
 intro: [intro(x, i, p, t)] intro * index * pyk * tex * end intro, **1**
 intro: [intro(x, p, t)] intro * pyk * tex * end intro, **2**

lemma l three two a [L3.2(a)], **5**

peano a $[\dot{a}]$, 3
 peano all * indeed * $[\dot{\forall}x: y]$, 2
 peano b $[\dot{b}]$, 6
 peano c $[\dot{c}]$, 6
 peano d $[\dot{d}]$, 6
 peano e $[\dot{e}]$, 6
 peano exist * indeed * $[\dot{\exists}x: y]$, 2
 peano f $[\dot{f}]$, 6
 peano free * set * to * end free $[\text{free}\langle a, x := b \rangle]3$
 peano free star * set * to * end free $[\text{free}^*\langle a, x := b \rangle]3$
 peano g $[\dot{g}]$, 6
 peano h $[\dot{h}]$, 6
 peano i $[\dot{i}]$, 6
 peano j $[\dot{j}]$, 6
 peano k $[\dot{k}]$, 6
 peano l $[\dot{l}]$, 6
 peano m $[\dot{m}]$, 6
 peano n $[\dot{n}]$, 6
 peano nonfree * in * end nonfree $[\text{nonfree}(x, y)]$, 3
 peano nonfree star * in * end nonfree $[\text{nonfree}^*(x, y)]$, 3
 peano not * $[\dot{\neg}x]$, 2
 peano o $[\dot{o}]$, 6
 peano one $[\dot{1}]$, 2
 peano p $[\dot{p}]$, 6
 peano q $[\dot{q}]$, 6
 peano r $[\dot{r}]$, 6
 peano s $[\dot{s}]$, 6
 peano sub * is * where * is * end sub $[a \equiv \langle b, x := c \rangle]3$
 peano sub star * is * where * is * end sub $[a \equiv \langle ^*b, x := c \rangle]3$
 peano t $[\dot{t}]$, 6
 peano two $[\dot{2}]$, 2
 peano u $[\dot{u}]$, 6
 peano v $[\dot{v}]$, 6
 Peano variable, 2
 peano w $[\dot{w}]$, 6
 peano x $[\dot{x}]$, 6
 peano y $[\dot{y}]$, 6
 peano z $[\dot{z}]$, 6
 peano zero $[\dot{0}]$, 2

 rule gen $[\text{Gen}]$, 4
 rule mp $[\text{MP}]$, 4

 system s $[\text{S}]$, 4

 variable, Peano, 2

C Bibliography

- [1] K. Grue. Logiweb. In Fairouz Kamareddine, editor, *Mathematical Knowledge Management Symposium 2003*, volume 93 of *Electronic Notes in Theoretical Computer Science*, pages 70–101. Elsevier, 2004.
- [2] E. Mendelson. *Introduction to Mathematical Logic*. Wadsworth and Brooks, 3. edition, 1987.