Introduction to Logiweb

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Contents

1 Peano arithmetic

This Logiweb page [1] defines Peano arithmetic.

1.1 Introduction of new constructs

The following constructs allow to introduce a new construct ("introduce" in the normal sense of the word, not in the sense of a Logiweb revelation). The constructs allow to define the pyk aspect [p] and tex aspect [t] of a new construct [x] and also makes two entries in the index. The first of the constructs below adds the text [i] in front of one of the index entries.

$$[\mathrm{intro}(\mathsf{x},\mathsf{i},\mathsf{p},\mathsf{t}) \ddot{=} [\mathsf{x} \stackrel{\text{pyk}}{=} \mathsf{p}][\mathsf{x} \stackrel{\text{tex}}{=} \mathsf{t}]]$$ [1]

[1] $[\mathrm{intro}(\mathsf{x},\mathsf{i},\mathsf{p},\mathsf{t}) \stackrel{\text{pyk}}{=}$ "intro ∗ index ∗ pyk ∗ tex ∗ end intro"]

$[\mathrm{intro}(\mathsf{x},\mathsf{p},\mathsf{t}) \ddot{=} [\mathsf{x} \stackrel{\mathrm{pyk}}{=} \mathsf{p}][\mathsf{x} \stackrel{\mathrm{tex}}{=} \mathsf{t}]]$[2]

1.2 The constructs of Peano arithmetic

Terms of Peano arithmetic are constructed from zero $[\dot{0}]$[3], successor $[\mathsf{x}']$[4], plus $[\mathsf{x} \dot{+} \mathsf{y}]$[5], and times $[\mathsf{x} \dot{\cdot} \mathsf{y}]$[6].

Formulas of Peano arithmeric are constructed from equality $[\mathsf{x} \stackrel{\mathrm{p}}{=} \mathsf{y}]$[7], negation $[\dot{\neg}\, \mathsf{x}]$[8], implication $[\mathsf{x} \dot{\Rightarrow} \mathsf{y}]$[9], and universal quantification $[\dot{\forall} \mathsf{x} \colon \mathsf{y}]$[10].

From these constructs we macro define one $[\dot{1}]$[11], two $[\dot{2}]$[12], conjunction $[\mathsf{x} \dot{\wedge} \mathsf{y}]$[13], disjunction $[\mathsf{x} \dot{\vee} \mathsf{y}]$[14], biimplication $[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y}]$[15], and existential quantification $[\dot{\exists} \mathsf{x} \colon \mathsf{y}]$[16]:

$$[\dot{1} \ddot{=} \dot{0}']$$

$$[\dot{2} \ddot{=} \dot{1}']$$

$$[\mathsf{x} \dot{\wedge} \mathsf{y} \ddot{=} \dot{\neg}(\mathsf{x} \dot{\Rightarrow} \dot{\neg}\, \mathsf{y})]$$

$$[\mathsf{x} \dot{\vee} \mathsf{y} \ddot{=} \dot{\neg}\, \mathsf{x} \dot{\Rightarrow} \mathsf{y}]$$

$$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \ddot{=} (\mathsf{x} \dot{\Rightarrow} \mathsf{y}) \dot{\wedge} (\mathsf{y} \dot{\Rightarrow} \mathsf{x})]$$

$$[\dot{\exists} \mathsf{x} \colon \mathsf{y} \ddot{=} \dot{\neg} \dot{\forall} \mathsf{x} \colon \dot{\neg}\, \mathsf{y}]$$

1.3 Variables

We now introduce the unary operator $[\dot{\mathsf{x}}]$[17] and define that a term is a *Peano variable* (i.e. a variable of Peano arithmetic) if it has the $[\dot{\mathsf{x}}]$ operator in its root. $[\mathsf{x}^{\mathcal{P}}]$[18] is true if $[\mathsf{x}]$ is a Peano variable:

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[2] $[\mathrm{intro}(\mathsf{x},\mathsf{p},\mathsf{t}) \stackrel{\mathrm{pyk}}{=}$ "intro * pyk * tex * end intro"]

[3] $[\dot{0} \stackrel{\mathrm{pyk}}{=}$ "peano zero"]

[4] $[\mathsf{x}' \stackrel{\mathrm{pyk}}{=}$ "* peano succ"]

[5] $[\mathsf{x} \dot{+} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano plus *"]

[6] $[\mathsf{x} \dot{\cdot} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano times *"]

[7] $[\mathsf{x} \stackrel{\mathrm{p}}{=} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano is *"]

[8] $[\dot{\neg}\, \mathsf{x} \stackrel{\mathrm{pyk}}{=}$ "peano not *"]

[9] $[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano imply *"]

[10] $[\dot{\forall} \mathsf{x} \colon \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "peano all * indeed *"]

[11] $[\dot{1} \stackrel{\mathrm{pyk}}{=}$ "peano one"]

[12] $[\dot{2} \stackrel{\mathrm{pyk}}{=}$ "peano two"]

[13] $[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano and *"]

[14] $[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano or *"]

[15] $[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "* peano iff *"]

[16] $[\dot{\exists} \mathsf{x} \colon \mathsf{y} \stackrel{\mathrm{pyk}}{=}$ "peano exist * indeed *"]

[17] $[\dot{\mathsf{x}} \stackrel{\mathrm{pyk}}{=}$ "* peano var"]

[18] $[\mathsf{x}^{\mathcal{P}} \stackrel{\mathrm{pyk}}{=}$ "* is peano var"]

$[x^{\mathcal{P}} \doteq x \stackrel{r}{=} \lceil \dot{x} \rceil]$

We macro define $[\dot{a}]$[19] to be a Peano variable:

$$[\dot{a} \ddot{=} \dot{\mathsf{a}}]$$

Appendix A.2 defines Peano variables for the other letters of the English alphabet.

$[\dot{\text{nonfree}}(x, y)]$[20] is true if the Peano variable $[x]$ does not occur free in the Peano term/formula $[y]$. $[\dot{\text{nonfree}}^*(x, y)]$[21] is true if the Peano variable $[x]$ does not occur free in the list $[y]$ of Peano terms/formulas.

$$[\dot{\text{nonfree}}(x, y) \doteq$$
$$\textbf{if } y^{\mathcal{P}} \textbf{ then } \neg x \stackrel{t}{=} y \textbf{ else}$$
$$\textbf{if } \neg y \stackrel{r}{=} \lceil \dot{\forall} x \colon y \rceil \textbf{ then } \dot{\text{nonfree}}^*(x, y^t) \textbf{ else}$$
$$\textbf{if } x \stackrel{t}{=} y^1 \textbf{ then } \mathsf{T} \textbf{ else } \dot{\text{nonfree}}(x, y^2)]$$

$$[\dot{\text{nonfree}}^*(x, y) \doteq x!\text{If}(y, \mathsf{T}, \dot{\text{nonfree}}(x, y^h) \ddot{\wedge} \dot{\text{nonfree}}^*(x, y^t))]$$

$[\dot{\text{free}}\langle a|x := b\rangle]$[22] is true if the substitution $[\langle a\,|x{:=}\,b\rangle]$ is free. $[\dot{\text{free}}^*\langle a|x := b\rangle]$[23] is the version where $[a]$ is a list of terms.

$$[\dot{\text{free}}\langle a|x := b\rangle \doteq x!b!$$
$$\textbf{if } a^{\mathcal{P}} \textbf{ then } \mathsf{T} \textbf{ else}$$
$$\textbf{if } \neg a \stackrel{r}{=} \lceil \dot{\forall} u \colon v \rceil \textbf{ then } \dot{\text{free}}^*\langle a^t|x := b\rangle \textbf{ else}$$
$$\textbf{if } a^1 \stackrel{t}{=} x \textbf{ then } \mathsf{T} \textbf{ else}$$
$$\textbf{if } \dot{\text{nonfree}}(x, a^2) \textbf{ then } \mathsf{T} \textbf{ else}$$
$$\textbf{if } \neg \dot{\text{nonfree}}(a^1, b) \textbf{ then } \mathsf{F} \textbf{ else}$$
$$\dot{\text{free}}\langle a^2|x := b\rangle]$$

$$[\dot{\text{free}}^*\langle a|x := b\rangle \doteq x!b!\text{If}(a, \mathsf{T}, \dot{\text{free}}\langle a^h|x := b\rangle \ddot{\wedge} \dot{\text{free}}^*\langle a^t|x := b\rangle)]$$

$[a \equiv \langle b|x := c\rangle]$[24] is true if $[a]$ equals $[\langle b\,|x{:=}\,c\rangle]$. $[a \equiv \langle^* b|x := c\rangle]$[25] is the version where $[a]$ and $[b]$ are lists.

$$[a \equiv \langle b|x := c\rangle \doteq a!x!c!$$
$$\textbf{if } b \stackrel{r}{=} \lceil \dot{\forall} u \colon v \rceil \ddot{\wedge} b^1 \stackrel{t}{=} x \textbf{ then } a \stackrel{t}{=} b \textbf{ else}$$
$$\textbf{if } b^{\mathcal{P}} \wedge b \stackrel{t}{=} x \textbf{ then } a \stackrel{t}{=} c \textbf{ else}$$
$$a \stackrel{r}{=} b \ddot{\wedge} a^t \equiv \langle^* b^t|x := c\rangle]$$

[19] $[\dot{a} \stackrel{\text{pyk}}{=}$ "peano a"$]$

[20] $[\dot{\text{nonfree}}(x, y) \stackrel{\text{pyk}}{=}$ "peano nonfree * in * end nonfree"$]$

[21] $[\dot{\text{nonfree}}^*(x, y) \stackrel{\text{pyk}}{=}$ "peano nonfree star * in * end nonfree"$]$

[22] $[\dot{\text{free}}\langle a|x := b\rangle \stackrel{\text{pyk}}{=}$ "peano free * set * to * end free"$]$

[23] $[\dot{\text{free}}^*\langle a|x := b\rangle \stackrel{\text{pyk}}{=}$ "peano free star * set * to * end free"$]$

[24] $[a \equiv \langle b|x := c\rangle \stackrel{\text{pyk}}{=}$ "peano sub * is * where * is * end sub"$]$

[25] $[a \equiv \langle^* b|x := c\rangle \stackrel{\text{pyk}}{=}$ "peano sub star * is * where * is * end sub"$]$

$[\mathsf{a}\equiv\langle{}^*\mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \doteq \mathsf{b}!\mathsf{x}!\mathsf{c}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathsf{a}^{\mathrm{h}}\equiv\langle\mathsf{b}^{\mathrm{h}}|\mathsf{x} := \mathsf{c}\rangle \ddot{\wedge} \mathsf{a}^{\mathrm{t}}\equiv\langle{}^*\mathsf{b}^{\mathrm{t}}|\mathsf{x} := \mathsf{c}\rangle)]$

1.4 Axiomatic system

System [S][26] of Mendelson [2] expresses Peano arithmetic. It comprises the axioms [A1][27], [A2][28], [A3][29], [A4][30], and [A5][31] and inference rules [MP][32] and [Gen][33] of first order predicate calculus. Furthermore, it comprises the proper axioms [S1][34], [S2][35], [S3][36], [S4][37], [S5][38], [S6][39], [S7][40], [S8][41], and [S9][42]. System [S] is defined thus:

[**Theory** S]

$[\mathrm{S}\ \mathbf{rule}\ \mathrm{A1}\colon \forall\mathcal{A}\colon \forall\mathcal{B}\colon \mathcal{A} \dot{\Rightarrow} \mathcal{B} \dot{\Rightarrow} \mathcal{A}]$

$[\mathrm{S}\ \mathbf{rule}\ \mathrm{A2}\colon \forall\mathcal{A}\colon \forall\mathcal{B}\colon \forall\mathcal{C}\colon (\mathcal{A} \dot{\Rightarrow} \mathcal{B} \dot{\Rightarrow} \mathcal{C}) \dot{\Rightarrow} (\mathcal{A} \dot{\Rightarrow} \mathcal{B}) \dot{\Rightarrow} \mathcal{A} \dot{\Rightarrow} \mathcal{C}]$

$[\mathrm{S}\ \mathbf{rule}\ \mathrm{A3}\colon (\dot{\neg}\mathcal{B} \dot{\Rightarrow} \dot{\neg}\mathcal{A}) \dot{\Rightarrow} (\dot{\neg}\mathcal{B} \dot{\Rightarrow} \mathcal{A}) \dot{\Rightarrow} \mathcal{B}]$

The order of quantifiers in the following axiom is such that [$\mathcal{A}$] which the system can currently guess comes first. At a later stage, when a planned generalisation of the conclusion tactic has been implemented, the system will be able to guess [$\mathcal{X}$] and [$\mathcal{B}$] also. Guessing [$\mathcal{C}$] would require a separate tactic.

$[\mathrm{S}\ \mathbf{rule}\ \mathrm{A4}\colon \forall\mathcal{C}\colon \forall\mathcal{X}\colon \forall\mathcal{B}\colon \forall\mathcal{A}\colon \mathcal{A}\equiv\langle\mathcal{B}|\mathcal{X} := \mathcal{C}\rangle \Vdash \dot{\forall}\mathcal{X}\colon \mathcal{B} \dot{\Rightarrow} \mathcal{A}]$

$[\mathrm{S}\ \mathbf{rule}\ \mathrm{A5}\colon \forall\mathcal{X}\colon \forall\mathcal{A}\colon \forall\mathcal{B}\colon \mathrm{non\dot{f}ree}(\mathcal{X}, \mathcal{A}) \Vdash \dot{\forall}\mathcal{X}\colon (\mathcal{A} \dot{\Rightarrow} \mathcal{B}) \dot{\Rightarrow} \mathcal{A} \dot{\Rightarrow} \dot{\forall}\mathcal{X}\colon \mathcal{B}]$

$[\mathrm{S}\ \mathbf{rule}\ \mathrm{MP}\colon \forall\mathcal{A}\colon \forall\mathcal{B}\colon \mathcal{A} \dot{\Rightarrow} \mathcal{B} \vdash \mathcal{A} \vdash \mathcal{B}]$

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26 [S $\stackrel{\text{pyk}}{=}$ "system s"]

27 [A1 $\stackrel{\text{pyk}}{=}$ "axiom a one"]

28 [A2 $\stackrel{\text{pyk}}{=}$ "axiom a two"]

29 [A3 $\stackrel{\text{pyk}}{=}$ "axiom a three"]

30 [A4 $\stackrel{\text{pyk}}{=}$ "axiom a four"]

31 [A5 $\stackrel{\text{pyk}}{=}$ "axiom a five"]

32 [MP $\stackrel{\text{pyk}}{=}$ "rule mp"]

33 [Gen $\stackrel{\text{pyk}}{=}$ "rule gen"]

34 [S1 $\stackrel{\text{pyk}}{=}$ "axiom s one"]

35 [S2 $\stackrel{\text{pyk}}{=}$ "axiom s two"]

36 [S3 $\stackrel{\text{pyk}}{=}$ "axiom s three"]

37 [S4 $\stackrel{\text{pyk}}{=}$ "axiom s four"]

38 [S5 $\stackrel{\text{pyk}}{=}$ "axiom s five"]

39 [S6 $\stackrel{\text{pyk}}{=}$ "axiom s six"]

40 [S7 $\stackrel{\text{pyk}}{=}$ "axiom s seven"]

41 [S8 $\stackrel{\text{pyk}}{=}$ "axiom s eight"]

42 [S9 $\stackrel{\text{pyk}}{=}$ "axiom s nine"]

[S **rule** Gen: $\forall \mathcal{X}\colon \forall \mathcal{A}\colon \mathcal{A} \vdash \dot{\forall} \mathcal{X}\colon \mathcal{A}$]

Axiom [S1] to [S8] are stated as Mendelson [2] does. This is done here to test certain parts of the Logiweb system. Serious users of Peano arithmetic are advised to take Mendelson's Lemma 3.1 as axioms instead.

[S **rule** S1: $\dot{a} \overset{\mathrm{p}}{=} \dot{b} \mathrel{\dot{\Rightarrow}} \dot{a} \overset{\mathrm{p}}{=} \dot{c} \mathrel{\dot{\Rightarrow}} \dot{b} \overset{\mathrm{p}}{=} \dot{c}$]

[S **rule** S2: $\dot{a} \overset{\mathrm{p}}{=} \dot{b} \mathrel{\dot{\Rightarrow}} \dot{a}' \overset{\mathrm{p}}{=} \dot{b}'$]

[S **rule** S3: $\dot{\neg} \dot{0} \overset{\mathrm{p}}{=} \dot{a}'$]

[S **rule** S4: $\dot{a}' \overset{\mathrm{p}}{=} \dot{b}' \mathrel{\dot{\Rightarrow}} \dot{a} \overset{\mathrm{p}}{=} \dot{b}$]

[S **rule** S5: $\dot{a} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{a}$]

[S **rule** S6: $\dot{a} \mathbin{\dot{+}} \dot{b}' \overset{\mathrm{p}}{=} (\dot{a} \mathbin{\dot{+}} \dot{b})'$]

[S **rule** S7: $\dot{a} \mathbin{\dot{\cdot}} \dot{0} \overset{\mathrm{p}}{=} \dot{0}$]

[S **rule** S8: $\dot{a} \mathbin{\dot{\cdot}} (\dot{b}') \overset{\mathrm{p}}{=} (\dot{a} \mathbin{\dot{\cdot}} \dot{b}) \mathbin{\dot{+}} \dot{a}$]

[S **rule** S9: $\forall \mathcal{A}\colon \forall \mathcal{B}\colon \forall \mathcal{C}\colon \forall \mathcal{X}\colon$
$\mathcal{B} \equiv \langle \mathcal{A} | \mathcal{X} := \dot{0} \rangle \Vdash \mathcal{C} \equiv \langle \mathcal{A} | \mathcal{X} := \mathcal{X}' \rangle \Vdash$
$\mathcal{B} \mathrel{\dot{\Rightarrow}} \dot{\forall} \mathcal{X}\colon (\mathcal{A} \mathrel{\dot{\Rightarrow}} \mathcal{C}) \mathrel{\dot{\Rightarrow}} \dot{\forall} \mathcal{X}\colon \mathcal{A}$]

1.5 A lemma and a proof

We now prove Lemma [L3.2(a)][43] which is an instance of the corresponding proposition in Mendelson [2]:

[S **lemma** L3.2(a): $\dot{x} \overset{\mathrm{p}}{=} \dot{x}$]

The first four lines of the proof has revealed an error in the conclusion tactic which has to be corrected before the proof will be accepted:

S **proof of** L3.2(a):

L01:	S5 $\gg$	$\dot{a} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{a}$	;
L02:	Gen $\rhd$ L01 $\gg$	$\dot{\forall} \dot{a}\colon \dot{a} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{a}$	;
L03:	A4 @ $\dot{x}$ @ $\dot{a}$ @ $\dot{a} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{a}$ @ $\dot{x} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{x}$ $\gg$	$\dot{\forall} \dot{a}\colon \dot{a} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{a} \mathrel{\dot{\Rightarrow}} \dot{x} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{x}$	;
L04:	MP $\rhd$ L03 $\rhd$ L02 $\gg$	$\dot{x} \mathbin{\dot{+}} \dot{0} \overset{\mathrm{p}}{=} \dot{x}$	□

[43] [L3.2(a) $\overset{\mathrm{pyk}}{=}$ "lemma 1 three two a"]

A Chores

A.1 The name of the page

This defines the name of the page:

$$[\text{peano} \stackrel{\text{pyk}}{=} \text{“peano”}]$$

A.2 Variables of Peano arithmetic

We use $[\dot{b}]$[44], $[\dot{c}]$[45], $[\dot{d}]$[46], $[\dot{e}]$[47], $[\dot{f}]$[48], $[\dot{g}]$[49], $[\dot{h}]$[50], $[\dot{i}]$[51], $[\dot{j}]$[52], $[\dot{k}]$[53], $[\dot{l}]$[54], $[\dot{m}]$[55], $[\dot{n}]$[56], $[\dot{o}]$[57], $[\dot{p}]$[58], $[\dot{q}]$[59], $[\dot{r}]$[60], $[\dot{s}]$[61], $[\dot{t}]$[62], $[\dot{u}]$[63], $[\dot{v}]$[64], $[\dot{w}]$[65], $[\dot{x}]$[66], $[\dot{y}]$[67], and $[\dot{z}]$[68] to denote variables of Peano arithmetic:

$[\dot{b} \ddot{=} \dot{\mathsf{b}}]$, $[\dot{c} \ddot{=} \dot{\mathsf{c}}]$, $[\dot{d} \ddot{=} \dot{\mathsf{d}}]$, $[\dot{e} \ddot{=} \dot{\mathsf{e}}]$, $[\dot{f} \ddot{=} \dot{\mathsf{f}}]$, $[\dot{g} \ddot{=} \dot{\mathsf{g}}]$, $[\dot{h} \ddot{=} \dot{\mathsf{h}}]$, $[\dot{i} \ddot{=} \dot{\mathsf{i}}]$, $[\dot{j} \ddot{=} \dot{\mathsf{j}}]$, $[\dot{k} \ddot{=} \dot{\mathsf{k}}]$, $[\dot{l} \ddot{=} \dot{\mathsf{l}}]$, $[\dot{m} \ddot{=} \dot{\mathsf{m}}]$, $[\dot{n} \ddot{=} \dot{\mathsf{n}}]$, $[\dot{o} \ddot{=} \dot{\mathsf{o}}]$, $[\dot{p} \ddot{=} \dot{\mathsf{p}}]$, $[\dot{q} \ddot{=} \dot{\mathsf{q}}]$, $[\dot{r} \ddot{=} \dot{\mathsf{r}}]$, $[\dot{s} \ddot{=} \dot{\mathsf{s}}]$, $[\dot{t} \ddot{=} \dot{\mathsf{t}}]$, $[\dot{u} \ddot{=} \dot{\mathsf{u}}]$, $[\dot{v} \ddot{=} \dot{\mathsf{v}}]$, $[\dot{w} \ddot{=} \dot{\mathsf{w}}]$, $[\dot{x} \ddot{=} \dot{\mathsf{x}}]$, $[\dot{y} \ddot{=} \dot{\mathsf{y}}]$, and $[\dot{z} \ddot{=} \dot{\mathsf{z}}]$.

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[44] $[\dot{b} \stackrel{\text{pyk}}{=} \text{“peano b”}]$

[45] $[\dot{c} \stackrel{\text{pyk}}{=} \text{“peano c”}]$

[46] $[\dot{d} \stackrel{\text{pyk}}{=} \text{“peano d”}]$

[47] $[\dot{e} \stackrel{\text{pyk}}{=} \text{“peano e”}]$

[48] $[\dot{f} \stackrel{\text{pyk}}{=} \text{“peano f”}]$

[49] $[\dot{g} \stackrel{\text{pyk}}{=} \text{“peano g”}]$

[50] $[\dot{h} \stackrel{\text{pyk}}{=} \text{“peano h”}]$

[51] $[\dot{i} \stackrel{\text{pyk}}{=} \text{“peano i”}]$

[52] $[\dot{j} \stackrel{\text{pyk}}{=} \text{“peano j”}]$

[53] $[\dot{k} \stackrel{\text{pyk}}{=} \text{“peano k”}]$

[54] $[\dot{l} \stackrel{\text{pyk}}{=} \text{“peano l”}]$

[55] $[\dot{m} \stackrel{\text{pyk}}{=} \text{“peano m”}]$

[56] $[\dot{n} \stackrel{\text{pyk}}{=} \text{“peano n”}]$

[57] $[\dot{o} \stackrel{\text{pyk}}{=} \text{“peano o”}]$

[58] $[\dot{p} \stackrel{\text{pyk}}{=} \text{“peano p”}]$

[59] $[\dot{q} \stackrel{\text{pyk}}{=} \text{“peano q”}]$

[60] $[\dot{r} \stackrel{\text{pyk}}{=} \text{“peano r”}]$

[61] $[\dot{s} \stackrel{\text{pyk}}{=} \text{“peano s”}]$

[62] $[\dot{t} \stackrel{\text{pyk}}{=} \text{“peano t”}]$

[63] $[\dot{u} \stackrel{\text{pyk}}{=} \text{“peano u”}]$

[64] $[\dot{v} \stackrel{\text{pyk}}{=} \text{“peano v”}]$

[65] $[\dot{w} \stackrel{\text{pyk}}{=} \text{“peano w”}]$

[66] $[\dot{x} \stackrel{\text{pyk}}{=} \text{“peano x”}]$

[67] $[\dot{y} \stackrel{\text{pyk}}{=} \text{“peano y”}]$

[68] $[\dot{z} \stackrel{\text{pyk}}{=} \text{“peano z”}]$

A.3 TEX definitions

[intro(x, i, p, t) $\stackrel{\text{tex}}{=}$ “\index{#2.: #3. @#2.: $[#1/tex name/tex.]$ #3.}%
\index{#3. $[#1/tex name/tex.]$}%
\tex{
$[#1/tex name/tex.
\stackrel {\mathrm {tex}}{=} #4/tex name.
]$}$[#1/tex name/tex.%
]$\footnote{$[#1/tex name/tex.
\stackrel {\mathrm {pyk}}{=} #3/tex name.
]$}”]

[intro(x, i, p, t) $\stackrel{\text{name}}{=}$ “
intro(#1.
, #2.
, #3.
, #4.
)”]

[intro(x, p, t) $\stackrel{\text{tex}}{=}$ “\index{\alpha #2. @\back \makebox[20mm][l]{$[#1/tex name/tex.]$}#2.}%
\index{#2. $[#1/tex name/tex.]$}%
\tex{
$[#1/tex name/tex.
\stackrel {\mathrm {tex}}{=} #3/tex name.
]$}$[#1/tex name/tex.%
]$\footnote{$[#1/tex name/tex.
\stackrel {\mathrm {pyk}}{=} #2/tex name.
]$}”]

[intro(x, p, t) $\stackrel{\text{name}}{=}$ “
intro(#1.
, #2.
, #3.
)”]

[$\dot{0}$ $\stackrel{\text{tex}}{=}$ “
\dot{0}”]

[x' $\stackrel{\text{tex}}{=}$ “#1.
'”]

[$x \mathop{\dot{+}} y$ $\stackrel{\text{tex}}{=}$ “#1.
\mathop{\dot{+}} #2.”]

[$x \mathop{\dot{\cdot}} y$ $\stackrel{\text{tex}}{=}$ “#1.
\mathop{\dot{\cdot}} #2.”]

[$\mathsf{x} \stackrel{\mathrm{p}}{=} \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “#1.
\stackrel{p}{=} #2.”]

[$\dot{\neg}\, \mathsf{x}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{\neg}\, #1.”]

[$\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “#1.
\mathrel{\dot{\Rightarrow}} #2.”]

[$\dot{\forall} \mathsf{x} \colon \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{\forall} #1.
\colon #2.”]

[$\dot{1}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{1}”]

[$\dot{2}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{2}”]

[$\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “#1.
\mathrel{\dot{\wedge}} #2.”]

[$\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “#1.
\mathrel{\dot{\vee}} #2.”]

[$\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “#1.
\mathrel{\dot{\Leftrightarrow}} #2.”]

[$\dot{\exists} \mathsf{x} \colon \mathsf{y}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{\exists} #1.
\colon #2.”]

[$\dot{\mathsf{x}}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{#1.
}”]

[$\mathsf{x}^{\cal P}$ $\stackrel{\mathrm{tex}}{=}$ “#1.
{} ^ {\cal P}”]

[$\dot{a}$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{\mathit{a}}”]

[$\dot{\mathrm{nonfree}}(\mathsf{x}, \mathsf{y})$ $\stackrel{\mathrm{tex}}{=}$ “
\dot{nonfree}(#1.
, #2.
)”]

[$\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{=}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

[$\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{tex}}{=}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\text{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{tex}}{=}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

[$\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \stackrel{\text{tex}}{=}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

[$\mathsf{a}\equiv\langle^* \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \stackrel{\text{tex}}{=}$ "#1.
{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

[S $\stackrel{\text{tex}}{=}$ "
S"]

[A1 $\stackrel{\text{tex}}{=}$ "
A1"]

[A2 $\stackrel{\text{tex}}{=}$ "
A2"]

[A3 $\stackrel{\text{tex}}{=}$ "
A3"]

[A4 $\stackrel{\text{tex}}{=}$ "
A4"]

[A5 $\stackrel{\text{tex}}{=}$ "
A5"]

[MP $\stackrel{\text{tex}}{=}$ "
MP"]

[Gen $\stackrel{\text{tex}}{=}$ "
Gen"]

[S1 $\stackrel{\text{tex}}{=}$ "
S1"]

[S2 $\stackrel{\text{tex}}{=}$ "
S2"]

[S3 $\stackrel{\text{tex}}{=}$ "
S3"]

[S4 $\stackrel{\text{tex}}{=}$ "
S4"]

[S5 $\stackrel{\text{tex}}{=}$ "
S5"]

[S6 $\stackrel{\text{tex}}{=}$ "
S6"]

[S7 $\stackrel{\text{tex}}{=}$ "
S7"]

[S8 $\stackrel{\text{tex}}{=}$ "
S8"]

[S9 $\stackrel{\text{tex}}{=}$ "
S9"]

[L3.2(a) $\stackrel{\text{tex}}{=}$ "
L3.2(a)"]

[$\dot{b}$ $\stackrel{\text{tex}}{=}$ "
\dot{\mathit{b}}"]

[$\dot{c}$ $\stackrel{\text{tex}}{=}$ "
\dot{\mathit{c}}"]

[$\dot{d}$ $\stackrel{\text{tex}}{=}$ "
\dot{\mathit{d}}"]

[$\dot{e}$ $\stackrel{\text{tex}}{=}$ "
\dot{\mathit{e}}"]

[$\dot{f} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{f}}”]

[$\dot{g} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{g}}”]

[$\dot{h} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{h}}”]

[$\dot{i} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{i}}”]

[$\dot{j} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{j}}”]

[$\dot{k} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{k}}”]

[$\dot{l} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{l}}”]

[$\dot{m} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{m}}”]

[$\dot{n} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{n}}”]

[$\dot{o} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{o}}”]

[$\dot{p} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{p}}”]

[$\dot{q} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{q}}”]

[$\dot{r} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{r}}”]

[$\dot{s} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{s}}”]

[$\dot{t} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{t}}”]

[$\dot{u} \stackrel{\text{tex}}{=}$ “
\dot{\mathit{u}}”]

$[\dot{v} \stackrel{\text{tex}}{=}$ “

\dot{\mathit{v}}”]

$[\dot{w} \stackrel{\text{tex}}{=}$ “

\dot{\mathit{w}}”]

$[\dot{x} \stackrel{\text{tex}}{=}$ “

\dot{\mathit{x}}”]

$[\dot{y} \stackrel{\text{tex}}{=}$ “

\dot{\mathit{y}}”]

$[\dot{z} \stackrel{\text{tex}}{=}$ “

\dot{\mathit{z}}”]

A.4 Test

$[\lceil \dot{a} \rceil^{\mathcal{P}}]^{\cdot}$

$[\lceil \mathsf{a} \rceil^{\mathcal{P}}]^{-}$

$[\dot{\text{nonfree}}(\lceil \dot{x} \rceil, \lceil \dot{y} \stackrel{\text{p}}{=} \dot{z} \mathrel{\dot{\Rightarrow}} \dot{\forall} \dot{x}\colon \dot{x} \stackrel{\text{p}}{=} \dot{y} \rceil)]^{\cdot}$

$[\dot{\text{nonfree}}(\lceil \dot{x} \rceil, \lceil \dot{x} \stackrel{\text{p}}{=} \dot{z} \mathrel{\dot{\Rightarrow}} \dot{\forall} \dot{x}\colon \dot{x} \stackrel{\text{p}}{=} \dot{y} \rceil)]^{-}$

$[\dot{\text{nonfree}}(\lceil \dot{x} \rceil, \lceil \dot{y} \stackrel{\text{p}}{=} \dot{x} \mathrel{\dot{\Rightarrow}} \dot{\forall} \dot{x}\colon \dot{x} \stackrel{\text{p}}{=} \dot{y} \rceil)]^{-}$

$[\dot{\text{nonfree}}(\lceil \dot{x} \rceil, \lceil \dot{y} \stackrel{\text{p}}{=} \dot{z} \mathrel{\dot{\Rightarrow}} \dot{\forall} \dot{y}\colon \dot{x} \stackrel{\text{p}}{=} \dot{y} \rceil)]^{-}$

$[\dot{\text{free}}\langle \lceil \dot{\forall} \dot{x}\colon \mathsf{b} :: \dot{x} :: \mathsf{c} \rceil | \lceil \dot{x} \rceil := \lceil \mathsf{x} :: \dot{y} :: \mathsf{z} \rceil \rangle]^{\cdot}$

$[\dot{\text{free}}\langle \lceil \dot{\forall} \dot{y}\colon \mathsf{b} :: \dot{x} :: \mathsf{c} \rceil | \lceil \dot{x} \rceil := \lceil \mathsf{x} :: \dot{y} :: \mathsf{z} \rceil \rangle]^{-}$

$[\dot{\text{free}}\langle \lceil \dot{\forall} \dot{x}\colon \mathsf{b} :: \dot{x} :: \mathsf{c} \rceil | \lceil \dot{y} \rceil := \lceil \mathsf{x} :: \dot{y} :: \mathsf{z} \rceil \rangle]^{\cdot}$

$[\dot{\text{free}}\langle \lceil \dot{\forall} \dot{y}\colon \mathsf{b} :: \dot{x} :: \mathsf{c} \rceil | \lceil \dot{y} \rceil := \lceil \mathsf{x} :: \dot{y} :: \mathsf{z} \rceil \rangle]^{\cdot}$

$[\dot{a} \equiv \langle \dot{a} | \dot{b} := \dot{c} \rangle]^{\cdot}$

$[\dot{c} \equiv \langle \dot{b} | \dot{b} := \dot{c} \rangle]^{\cdot}$

$[\forall \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{b} \equiv \langle \forall \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{b} | \dot{a} := \dot{c} \rangle]^{\cdot}$

$[\forall \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{c} \equiv \langle \forall \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{b} | \dot{b} := \dot{c} \rangle]^{\cdot}$

$[\dot{\forall} \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{a} \Rightarrow \dot{c} \mathbin{\dot{\cdot}} \dot{d} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{c} \mathbin{\dot{\cdot}} \dot{d} \equiv \langle \dot{\forall} \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{a} \Rightarrow \dot{b} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{b} | \dot{b} := \dot{c} \mathbin{\dot{\cdot}} \dot{d} \rangle]^{\cdot}$

$[\dot{\forall} \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{a} \Rightarrow \dot{b} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{b} \equiv \langle \dot{\forall} \dot{a}\colon \dot{a} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{a} \Rightarrow \dot{b} \stackrel{\text{p}}{=} \dot{0} \mathbin{\dot{+}} \dot{b} | \dot{a} := \dot{c} \rangle]^{\cdot}$

A.5 Priority table

Priority table
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\rightarrow} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if($*, *, *$)], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\perp$], [f($*$)], [$(*)^{\mathrm{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\mathrm{M}}$], [If($*, *, *$)], [array{$*$} $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\mathrm{M}}(*)$], [**apply**($*, *$)], [$\mathbf{apply}_1(*, *)$], [identifier($*$)], [$\mathrm{identifier}_1(*, *)$], [array-plus($*, *$)], [array-remove($*, *, *$)], [array-put($*, *, *, *$)], [array-add($*, *, *, *, *$)], [bit($*, *$)], [$\mathrm{bit}_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**($*, *, *$)], [**abstract**($*, *, *, *$)], [[$\lceil *$]], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [$s_0$], [**zip**($*, *$)], [$\mathbf{assoc}_1(*, *, *)$], [$(*)^{\mathbf{p}}$], [self], [[$* \ddot{=} *$]], [[$* \doteq *$]], [[$* \acute{=} *$]], [[$* \stackrel{\mathrm{pyk}}{=} *$]], [[$* \stackrel{\mathrm{tex}}{=} *$]], [[$* \stackrel{\mathrm{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [($*$)], [**aspect**($*, *$)], [**aspect**($*, *, *$)], [$\langle * \rangle$], [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], [$\mathrm{let}_2(*, *)$], [$\mathrm{let}_1(*, *)$], [[$* \stackrel{\mathrm{claim}}{=} *$]], [checker], [**check**($*, *$)], [$\mathbf{check}_2(*, *, *)$], [$\mathbf{check}_3(*, *, *)$], [$\mathbf{check}^*(*, *)$], [$\mathbf{check}^*_2(*, *, *)$], [[$*]^{\cdot}$], [[$*]^{-}$], [[$*]^{\circ}$], [msg], [[$* \stackrel{\mathrm{msg}}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\mathrm{stmt}}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\perp\!\!\!\perp$], [Contra′], [T'_{E}], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathbf{v}}$], [error($*, *$)], [$\mathrm{error}_2(*, *)$], [proof($*, *, *$)], [$\mathrm{proof}_2(*, *)$], [$\mathcal{S}(*, *)$], [$\mathcal{S}^{\mathrm{I}}(*, *)$], [$\mathcal{S}^{\rhd}(*, *)$], [$\mathcal{S}^{\rhd}_1(*, *, *)$], [$\mathcal{S}^{\mathrm{E}}(*, *)$], [$\mathcal{S}^{\mathrm{E}}_1(*, *, *)$], [$\mathcal{S}^{+}(*, *)$], [$\mathcal{S}^{+}_1(*, *, *)$], [$\mathcal{S}^{-}(*, *)$], [$\mathcal{S}^{-}_1(*, *, *)$], [$\mathcal{S}^{*}(*, *)$], [$\mathcal{S}^{*}_1(*, *, *)$], [$\mathcal{S}^{*}_2(*, *, *, *)$], [$\mathcal{S}^{@}(*, *)$], [$\mathcal{S}^{@}_1(*, *, *)$], [$\mathcal{S}^{\vdash}(*, *)$], [$\mathcal{S}^{\vdash}_1(*, *, *, *)$], [$\mathcal{S}^{\Vdash}(*, *)$], [$\mathcal{S}^{\Vdash}_1(*, *, *, *)$], [$\mathcal{S}^{\mathrm{i.e.}}(*, *)$], [$\mathcal{S}^{\mathrm{i.e.}}_1(*, *, *, *)$], [$\mathcal{S}^{\mathrm{i.e.}}_2(*, *, *, *, *)$], [$\mathcal{S}^{\forall}(*, *)$], [$\mathcal{S}^{\forall}_1(*, *, *, *)$], [$\mathcal{S}^{;}(*, *)$], [$\mathcal{S}^{;}_1(*, *, *)$], [$\mathcal{S}^{;}_2(*, *, *, *)$], [$\mathcal{T}(*)$], [claims($*, *, *$)], [$\mathrm{claims}_2(*, *, *)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*, *)$], [$\mathcal{V}_3(*, *, *, *)$], [$\mathcal{V}_4(*, *)$], [$\mathcal{V}_5(*, *, *, *)$], [$\mathcal{V}_6(*, *, *, *)$], [$\mathcal{V}_7(*, *, *, *)$], [Cut($*, *$)], [$\mathrm{Head}_{\oplus}(*)$], [$\mathrm{Tail}_{\oplus}(*)$], [$\mathrm{rule}_1(*, *)$], [rule($*, *$)], [Rule tactic], [Plus($*, *$)], [[**Theory** $*$]], [$\mathrm{theory}_2(*, *)$], [$\mathrm{theory}_3(*, *)$], [$\mathrm{theory}_4(*, *, *)$], [HeadNil″], [HeadPair″], [Transitivity″], [Contra″], [HeadNil], [HeadPair], [Transitivity], [Contra], [T_{E}], [ragged right], [ragged right expansion], [parm($*, *, *$)], [$\mathrm{parm}^*(*, *, *)$], [inst($*, *$)], [$\mathrm{inst}^*(*, *)$], [occur($*, *, *$)], [$\mathrm{occur}^*(*, *, *)$], [unify($* = *, *$)], [$\mathrm{unify}^*(* = *, *)$], [$\mathrm{unify}_2(* = *, *)$], [$\mathrm{L}_{\mathrm{a}}$], [$\mathrm{L}_{\mathrm{b}}$], [$\mathrm{L}_{\mathrm{c}}$], [$\mathrm{L}_{\mathrm{d}}$], [$\mathrm{L}_{\mathrm{e}}$], [$\mathrm{L}_{\mathrm{f}}$], [$\mathrm{L}_{\mathrm{g}}$], [$\mathrm{L}_{\mathrm{h}}$], [$\mathrm{L}_{\mathrm{i}}$], [$\mathrm{L}_{\mathrm{j}}$], [$\mathrm{L}_{\mathrm{k}}$], [$\mathrm{L}_{\mathrm{l}}$], [$\mathrm{L}_{\mathrm{m}}$], [$\mathrm{L}_{\mathrm{n}}$], [$\mathrm{L}_{\mathrm{o}}$], [$\mathrm{L}_{\mathrm{p}}$], [$\mathrm{L}_{\mathrm{q}}$], [$\mathrm{L}_{\mathrm{r}}$], [$\mathrm{L}_{\mathrm{s}}$], [$\mathrm{L}_{\mathrm{t}}$], [$\mathrm{L}_{\mathrm{u}}$], [$\mathrm{L}_{\mathrm{v}}$], [$\mathrm{L}_{\mathrm{w}}$], [$\mathrm{L}_{\mathrm{x}}$], [$\mathrm{L}_{\mathrm{y}}$], [$\mathrm{L}_{\mathrm{z}}$], [$\mathrm{L}_{\mathrm{A}}$], [$\mathrm{L}_{\mathrm{B}}$], [$\mathrm{L}_{\mathrm{C}}$], [$\mathrm{L}_{\mathrm{D}}$], [$\mathrm{L}_{\mathrm{E}}$], [$\mathrm{L}_{\mathrm{F}}$], [$\mathrm{L}_{\mathrm{G}}$],

$[\mathrm{L_H}], [\mathrm{L_I}], [\mathrm{L_J}], [\mathrm{L_K}], [\mathrm{L_L}], [\mathrm{L_M}], [\mathrm{L_N}], [\mathrm{L_O}], [\mathrm{L_P}], [\mathrm{L_Q}], [\mathrm{L_R}], [\mathrm{L_S}], [\mathrm{L_T}], [\mathrm{L_U}], [\mathrm{L_V}]$,
$[\mathrm{L_W}], [\mathrm{L_X}], [\mathrm{L_Y}], [\mathrm{L_Z}], [\mathrm{L_?}]$, [Reflexivity], $[\mathrm{Reflexivity_1}]$, [Commutativity],
$[\mathrm{Commutativity_1}]$, [<tactic>], [tactic], $[[* \overset{\mathrm{tactic}}{=} *]], [\mathcal{P}(*,*,*)], [\mathcal{P}^*(*,*,*)], [\mathrm{p_0}]$,
$[\mathrm{conclude_1}(*,*)], [\mathrm{conclude_2}(*,*,*)], [\mathrm{conclude_3}(*,*,*,*)], [\mathrm{intro}(*,*,*,*)]$,
$[\mathrm{intro}(*,*,*)], [\dot{0}], [\dot{1}], [\dot{2}], [\dot{a}], [\dot{b}], [\dot{c}], [\dot{d}], [\dot{e}], [\dot{f}], [\dot{g}], [\dot{h}], [\dot{i}], [\dot{j}], [\dot{k}], [\dot{l}], [\dot{m}], [\dot{n}], [\dot{o}]$,
$[\dot{p}], [\dot{q}], [\dot{r}], [\dot{s}], [\dot{t}], [\dot{u}], [\dot{v}], [\dot{w}], [\dot{x}], [\dot{y}], [\dot{z}], [\mathrm{nonfree}(*,*)], [\mathrm{nonfree}^*(*,*)]$,
$[\mathrm{free}\langle *|* := *\rangle], [\mathrm{free}^*\langle *|* := *\rangle], [*{\equiv}\langle *|* := *\rangle], [*{\equiv}\langle^* *|* := *\rangle]$, [S], [A1], [A2],
[A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen],
[L3.2(a)];

Preassociative

$[*\_\{*\}], [*'], [*[\,*\,]], [*[*{\rightarrow}*]], [*[*{\Rightarrow}*]], [\dot{*}]$;

Preassociative

[“ * ”], [], $[(*)^{\mathbf{t}}]$, [string(*) + *], [string(*) ++ *], [
*], [*], [!*], [”*], [#*], [$*], [%*], [&*], [’*], [(*], [)*], [**], [+*], [, *], [-*], [.*], [/*],
[0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [:*], [; *], [<*], [=*], [>*], [?*],
[@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*],
[O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [*], []*], [ˆ*],
[_*], [‘*], [a*], [b*], [c*], [d*], [e*], [f*], [g*], [h*], [i*], [j*], [k*], [l*], [m*], [n*], [o*],
[p*], [q*], [r*], [s*], [t*], [u*], [v*], [w*], [x*], [y*], [z*], [{*], [|*], [}*], [˜*],
[**Preassociative** *; *], [**Postassociative** *; *], [[*], *], [priority * end],
[newline *], [macro newline *];

Preassociative

[*0], [*1], [0b], [*-color(*)], $[*\text{-color}^*(*)]$;

Preassociative

[* ’ *], [* ‘ *];

Preassociative

$[*^{\mathrm{H}}], [*^{\mathrm{T}}], [*^{\mathrm{U}}], [*^{\mathrm{h}}], [*^{\mathrm{t}}], [*^{\mathrm{s}}], [*^{\mathrm{c}}], [*^{\mathrm{d}}], [*^{\mathrm{a}}], [*^{\mathrm{C}}], [*^{\mathrm{M}}], [*^{\mathrm{B}}], [*^{\mathrm{r}}], [*^{\mathrm{i}}], [*^{\mathrm{d}}], [*^{\mathrm{R}}], [*^{0}]$,
$[*^{1}], [*^{2}], [*^{3}], [*^{4}], [*^{5}], [*^{6}], [*^{7}], [*^{8}], [*^{9}], [*^{\mathrm{E}}], [*^{\mathcal{V}}], [*^{\mathcal{C}}], [*^{\mathcal{C}^*}], [*']$;

Preassociative

$[* \cdot *], [* \cdot_0 *], [* \mathrel{\dot{:}} *]$;

Preassociative

$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *]$;

Preassociative

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}]$;

Postassociative

$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;

Postassociative

$[*, *]$;

Preassociative

$[* \overset{\mathrm{B}}{\approx} *], [* \overset{\mathrm{D}}{\approx} *], [* \overset{\mathrm{C}}{\approx} *], [* \overset{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \xrightarrow{+} *], [* \overset{\mathrm{t}}{=} *], [* \overset{\mathrm{t}^*}{=} *], [* \overset{\mathrm{r}}{=} *]$,
$[* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \overset{\mathrm{T}}{=} *], [* \overset{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *]$,
$[* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* \overset{\mathrm{p}}{=} *], [*^{\mathcal{P}}]$;

Preassociative

$[\neg *], [\dot{\neg} *]$;

Preassociative

[$* \wedge *$], [$* \ddot{\wedge} *$], [$* \tilde{\wedge} *$], [$* \wedge_{\mathrm{c}} *$], [$* \dot{\wedge} *$];
Preassociative
[$* \vee *$], [$* \parallel *$], [$* \ddot{\vee} *$], [$* \dot{\vee} *$];
Preassociative
[$\dot{\forall} * \colon *$], [$\dot{\exists} * \colon *$];
Postassociative
[$* \ddot{\Rightarrow} *$], [$* \dot{\Rightarrow} *$], [$* \dot{\Leftrightarrow} *$];
Postassociative
[$* : *$], [$* ! *$];
Preassociative
[$* \left\{ \begin{array}{c} * \\ * \end{array} \right.$];
Preassociative
[$\lambda * . *$], [$\Lambda *$], [**if** $*$ **then** $*$ **else** $*$], [**let** $* = *$ **in** $*$], [**let** $* \ddot{=} *$ in $*$];
Preassociative
[$*^{\mathrm{I}}$], [$*^{\rhd}$], [$*^{\mathrm{V}}$], [$*^{+}$], [$*^{-}$], [$*^{*}$];
Preassociative
[$* @ *$], [$* \rhd *$], [$* \rhd\!\!\rhd *$], [$* \gg *$];
Postassociative
[$* \vdash *$], [$* \Vdash *$], [$*$ i.e. $*$];
Preassociative
[$\forall * \colon *$];
Postassociative
[$* \oplus *$];
Postassociative
[$*; *$];
Preassociative
[$*$ proves $*$];
Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \square$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$];
Postassociative
[$*$ then $*$], [$*[*]*$];
Preassociative
[$* \& *$];
Preassociative
[$* \backslash\backslash *$]; **End table**

B Index

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