

Logiweb codex of peano axioms

Up Help

peano axioms, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$,
 $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, **nonfree**(*,*), **nonfree**^{*}(*,*), **free**(*|* := *), **free**^{*}(*|* := *),
***≡**(*|* := *), ***≡**^{*}(*|* := *), **S**, **A1**, **A2**, **A3**, **A4**, **A5**, **S1**, **S2**, **S3**, **S4**, **S5**, **S6**, **S7**,
S8, **S9**, **MP**, **Gen**, **L3.2(a)**, **S'**, **A1'**, **A2'**, **A3'**, **A4'**, **A5'**, **S1'**, **S2'**, **S3'**, **S4'**, **S5'**,
S6', **S7'**, **S8'**, **S9'**, **MP'**, **Gen'**, **L3.2(a)'**, $\dot{*}$, $\dot{*}'$, $\dot{*} \dot{:} \dot{*}$, $\dot{*} \dot{+} \dot{*}$, $\dot{*} \dot{=} \dot{*}$, $\dot{*} \dot{\mathcal{P}}$, $\dot{*} \dot{\wedge} \dot{*}$,
 $\dot{*} \dot{\vee} \dot{*}$, $\dot{\vee} \dot{*} \dot{:} \dot{*}$, $\dot{\exists} \dot{*} \dot{:} \dot{*}$, $\dot{*} \dot{\Rightarrow} \dot{*}$, $\dot{*} \dot{\Leftrightarrow} \dot{*}$,

peano axioms

[peano axioms] $\xrightarrow{\text{prio}}$

Preassociative

[peano axioms], [base], [bracket * end bracket], [big bracket * end bracket],
[math * end math], [**flush left** *], [x], [y], [z], [$* \bowtie *$], [$* \rightarrow *$], [pyk], [tex],
[name], [prio], [*], [T], [if(*,*,*)], [$* \xRightarrow{*} *$], [val], [claim], [$\perp$], [f(*)], [$(*)^I$], [F], [Q],
[1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d],
[e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^M$], [If(*,*,
)], [array{}*end array], [l], [c], [r], [empty], [$\langle * | * := * \rangle$], [$\mathcal{M}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}(*)$],
[$\mathcal{U}^M(*)$], [**apply**(*,*)], [**apply**₁(*,*)], [identifier(*)], [identifier₁(*,*)], [array-
plus(*,*)], [array-remove(*,*,*)], [array-put(*,*,*,*)], [array-add(*,*,*,*,*)],
[bit(*,*)], [bit₁(*,*)], [rack], ["vector"], ["bibliography"], ["dictionary"],
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
[$\mathcal{E}(*,*,*)$], [$\mathcal{E}_2(*,*,*,*,*)$], [$\mathcal{E}_3(*,*,*,*,*)$], [$\mathcal{E}_4(*,*,*,*,*)$], [**lookup**(*,*,*)],
[**abstract**(*,*,*,*)], [$\lceil * \rceil$], [$\mathcal{M}(*,*,*)$], [$\mathcal{M}_2(*,*,*,*)$], [$\mathcal{M}^*(*,*,*)$], [macro],
[s₀], [**zip**(*,*)], [**assoc**₁(*,*,*)], [$(*)^P$], [self], [$* \ddot{=}$ *], [$* \dot{=}$ *], [$* \dot{=} *$],
[$* \stackrel{\text{pyk}}{=} *$], [$* \stackrel{\text{tex}}{=} *$], [$* \stackrel{\text{name}}{=} *$], [**Priority table**[*]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$],
[$\tilde{\mathcal{M}}_4(*,*,*,*)$], [$\mathcal{M}(*,*,*)$], [$\mathcal{Q}(*,*,*)$], [$\tilde{\mathcal{Q}}_2(*,*,*)$], [$\tilde{\mathcal{Q}}_3(*,*,*,*)$], [$\tilde{\mathcal{Q}}^*(*,*,*)$],
[(*)], [**aspect**(*,*)], [**aspect**(*,*,*)], [$\langle * \rangle$], [**tuple**₁(*)], [**tuple**₂(*)], [let₂(*,*)],
[let₁(*,*)], [$* \stackrel{\text{claim}}{=} *$], [checker], [**check**(*,*)], [**check**₂(*,*,*)], [**check**₃(*,*,*)],
[**check**^{*}(*,*)], [**check**₂^{*}(*,*,*)], [$\lceil * \rceil$], [$*^-$], [$*^\circ$], [msg], [$* \stackrel{\text{msg}}{=}$ *], [$\langle \text{stmt} \rangle$],
[stmt], [$* \stackrel{\text{stmt}}{=} *$], [HeadNil'], [HeadPair'], [Transitivity'], [$\perp$], [Contra'], [T_E],
[L₁], [$\llbracket$], [**A**], [**B**], [**C**], [**D**], [**E**], [**F**], [**G**], [**H**], [**I**], [**J**], [**K**], [**L**], [**M**], [**N**], [**O**], [**P**], [**Q**],
[**R**], [**S**], [**T**], [**U**], [**V**], [**W**], [**X**], [**Y**], [**Z**], [$\langle * | * := * \rangle$], [$\langle * * | * := * \rangle$], [$\emptyset$], [Remainder],
[$(*)^\vee$], [intro(*,*,*,*)], [intro(*,*,*)], [error(*,*)], [error₂(*,*)], [proof(*,*,*)],

[proof₂(*,*), [S(*,*), [S^I(*,*), [S[▷](*,*), [S[▷]₁(*,*,*), [S^E(*,*), [S^F₁(*,*,*), [S⁺(*,*), [S⁺₁(*,*,*), [S⁻(*,*), [S⁻₁(*,*,*), [S^{*}(*,*), [S^{*}₁(*,*,*), [S^{*}₂(*,*,*,*), [S[@](*,*), [S[@]₁(*,*,*), [S⁺(*,*), [S⁺₁(*,*,*,*), [S⁺₁(*,*,*,*), [S^{i.e.}(*,*), [S^{i.e.}₁(*,*,*,*), [S^{i.e.}₂(*,*,*,*), [S[∇](*,*), [S[∇]₁(*,*,*,*), [Sⁱ(*,*), [Sⁱ₁(*,*,*,*), [Sⁱ₂(*,*,*,*), [T(*), [claims(*,*,*), [claims₂(*,*,*), [<proof>], [proof], [[**Lemma** *: *]], [[**Proof of** *: *]], [[* **lemma** *: *]], [[* **antilemma** *: *]], [[* **rule** *: *]], [[* **antirule** *: *]], [verifier], [V₁(*)], [V₂(*,*)], [V₃(*,*,*,*)], [V₄(*,*)], [V₅(*,*,*,*)], [V₆(*,*,*,*)], [V₇(*,*,*,*)], [Cut(*,*)], [Head_⊕(*)], [Tail_⊕(*)], [rule₁(*,*)], [rule(*,*)], [Rule tactic], [Plus(*,*)], [[**Theory** *]], [theory₂(*,*)], [theory₃(*,*)], [theory₄(*,*,*)], [HeadNil''], [HeadPair''], [Transitivity''], [Contra''], [HeadNil], [HeadPair], [Transitivity], [Contra], [T_E], [ragged right], [ragged right expansion], [parm(*,*,*)], [parm^{*}(*,*,*)], [inst(*,*)], [inst^{*}(*,*)], [occur(*,*,*)], [occur^{*}(*,*,*)], [unify(* = *,*)], [unify^{*}(* = *,*)], [unify₂(* = *,*)], [L_a], [L_b], [L_c], [L_d], [L_e], [L_f], [L_g], [L_h], [L_i], [L_j], [L_k], [L_l], [L_m], [L_n], [L_o], [L_p], [L_q], [L_r], [L_s], [L_t], [L_u], [L_v], [L_w], [L_x], [L_y], [L_z], [L_A], [L_B], [L_C], [L_D], [L_E], [L_F], [L_G], [L_H], [L_I], [L_J], [L_K], [L_L], [L_M], [L_N], [L_O], [L_P], [L_Q], [L_R], [L_S], [L_T], [L_U], [L_V], [L_W], [L_X], [L_Y], [L_Z], [L_?], [Reflexivity], [Reflexivity₁], [Commutativity], [Commutativity₁], [<tactic>], [tactic], [[* ^{tactic} *]], [P(*,*,*)], [P^{*}(*,*,*)], [p₀], [conclude₁(*,*)], [conclude₂(*,*,*)], [conclude₃(*,*,*,*)], [0], [1], [2], [ā], [b̄], [c̄], [d̄], [ē], [f̄], [ḡ], [h̄], [ī], [j̄], [k̄], [l̄], [m̄], [n̄], [ō], [p̄], [q̄], [r̄], [s̄], [t̄], [ū], [v̄], [w̄], [x̄], [ȳ], [z̄], [nonfree(*,*)], [nonfree^{*}(*,*)], [free(* := *)], [free^{*}(* := *)], [* ≡ (* := *)], [* ≡^{*}(* := *)], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S'], [A1'], [A2'], [A3'], [A4'], [A5'], [S1'], [S2'], [S3'], [S4'], [S5'], [S6'], [S7'], [S8'], [S9'], [MP'], [Gen'], [L3.2(a)']];

Preassociative

[*-{*}], [*'], [* *], [* *→*], [* *⇒*], [*];

Preassociative

[“ * ”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [*, [*, [*, [’*], [#*], [\$*], [%*], [&*], [’*], [(*)], [()*], [**], [+*], [*, [-*], [.*], [/ *], [0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [:*], [*;], [< *], [= *], [> *], [?*], [@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*], [O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [\ *], [*], [^ *], [*], [*], [a*], [b*], [c*], [d*], [e*], [f*], [g*], [h*], [i*], [j*], [k*], [l*], [m*], [n*], [o*], [p*], [q*], [r*], [s*], [t*], [u*], [v*], [w*], [x*], [y*], [z*], [{*}, [*], [}*], [~ *], [**Preassociative** *; *], [**Postassociative** *; *], [*], [*], [priority * end], [newline *], [macro newline *];

Preassociative

[*0], [*1], [0b], [*-color(*)], [*-color^{*}(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*ⁱ], [*^d], [*^R], [*⁰], [*¹], [*²], [*³], [*⁴], [*⁵], [*⁶], [*⁷], [*⁸], [*⁹], [*^E], [*^V], [*^C], [*^{C*}], [*'];

Preassociative

$[* \cdot *], [* \cdot_0 *], [* \dot{\cdot} *];$

Preassociative

$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *];$

Preassociative

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$

Postassociative

$[* \dot{\cdot} *], [* \dot{\cdot} *], [* \dot{\cdot} :: *], [* \underline{+2} *], [* :: *], [* +2 * *];$

Postassociative

$[*, *];$

Preassociative

$[* \overset{B}{\approx} *], [* \overset{D}{\approx} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \overset{+}{\rightarrow} *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{free in } *], [* \text{free in}^* *], [* \text{free for } * \text{ in } *],$
 $[* \text{free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* \overset{P}{=} *], [* \mathcal{P}];$

Preassociative

$[\neg *], [\dot{\neg} *];$

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$

Preassociative

$[\dot{\forall} *: *], [\dot{\exists} *: *];$

Postassociative

$[* \rightrightarrows *], [* \Rightarrow *], [* \Leftrightarrow *];$

Postassociative

$[* : *], [* ! *];$

Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right\}];$

Preassociative

$[\lambda *. *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \doteq * \text{ in } *];$

Preassociative

$[*^I], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$

Preassociative

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *];$

Postassociative

$[* \vdash *], [* \Vdash *], [* \text{i.e. } *];$

Preassociative

$[\forall *: *];$

Postassociative

$[* \oplus *];$

Postassociative

$[*, *];$

Preassociative

$[* \text{ proves } *];$

Preassociative

[* **proof of** * : *], [Line * : * $\gg$ *; *], [Last line * $\gg$ * $\square$],
[Line * : Premise $\gg$ *; *], [Line * : Side-condition $\gg$ *; *], [Arbitrary $\gg$ *; *],
[Local $\gg$ * = *; *];

Postassociative

[* then *], [* [*]*];

Preassociative

[*&*];

Preassociative

[*\\[*];]

[peano axioms $\xrightarrow{\text{pyk}}$ “peano axioms”]

$\dot{0}$

[$\dot{0} \xrightarrow{\text{tex}}$ “
 $\dot{\{0\}}$ ”]

[$\dot{0} \xrightarrow{\text{pyk}}$ “peano zero”]

$\dot{1}$

[$\dot{1} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{1} \doteq \dot{0}'] \rrbracket)$]

[$\dot{1} \xrightarrow{\text{tex}}$ “
 $\dot{\{1\}}$ ”]

[$\dot{1} \xrightarrow{\text{pyk}}$ “peano one”]

$\dot{2}$

[$\dot{2} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{2} \doteq \dot{1}'] \rrbracket)$]

[$\dot{2} \xrightarrow{\text{tex}}$ “
 $\dot{\{2\}}$ ”]

[$\dot{2} \xrightarrow{\text{pyk}}$ “peano two”]

$\dot{a}$

[$\dot{a} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{a} \doteq \dot{a}'] \rrbracket)$]

$\dot{a} \xrightarrow{\text{tex}} \text{\dot{\mathit{a}}}$

$\dot{a} \xrightarrow{\text{pyk}} \text{“peano a”}$

$\dot{b}$

$\dot{b} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{b} \doteq \dot{b}]])$

$\dot{b} \xrightarrow{\text{tex}} \text{\dot{\mathit{b}}}$

$\dot{b} \xrightarrow{\text{pyk}} \text{“peano b”}$

$\dot{c}$

$\dot{c} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{c} \doteq \dot{c}]])$

$\dot{c} \xrightarrow{\text{tex}} \text{\dot{\mathit{c}}}$

$\dot{c} \xrightarrow{\text{pyk}} \text{“peano c”}$

$\dot{d}$

$\dot{d} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{d} \doteq \dot{d}]])$

$\dot{d} \xrightarrow{\text{tex}} \text{\dot{\mathit{d}}}$

$\dot{d} \xrightarrow{\text{pyk}} \text{“peano d”}$

$\dot{e}$

$\dot{e} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[\dot{e} \doteq \dot{e}]])$

$\dot{e} \xrightarrow{\text{tex}} \text{\dot{\mathit{e}}}$

$\dot{e} \xrightarrow{\text{pyk}} \text{“peano e”}$

$\dot{f}$

$[\dot{f} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{f} \doteq \mathring{f}] \rrbracket)]$

$[\dot{f} \xrightarrow{\text{tex}} \text{“}\dot{\mathit{f}}\text{”}]$

$[\dot{f} \xrightarrow{\text{pyk}} \text{“peano f”}]$

$\dot{g}$

$[\dot{g} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{g} \doteq \mathring{g}] \rrbracket)]$

$[\dot{g} \xrightarrow{\text{tex}} \text{“}\dot{\mathit{g}}\text{”}]$

$[\dot{g} \xrightarrow{\text{pyk}} \text{“peano g”}]$

$\dot{h}$

$[\dot{h} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{h} \doteq \mathring{h}] \rrbracket)]$

$[\dot{h} \xrightarrow{\text{tex}} \text{“}\dot{\mathit{h}}\text{”}]$

$[\dot{h} \xrightarrow{\text{pyk}} \text{“peano h”}]$

$\dot{i}$

$[\dot{i} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{i} \doteq \mathring{i}] \rrbracket)]$

$[\dot{i} \xrightarrow{\text{tex}} \text{“}\dot{\mathit{i}}\text{”}]$

$[\dot{i} \xrightarrow{\text{pyk}} \text{“peano i”}]$

$\dot{j}$

$[\dot{j} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, \llbracket [\dot{j} \doteq \mathring{j}] \rrbracket)]$

$[\dot{j} \xrightarrow{\text{tex}} \text{“}\dot{\mathit{j}}\text{”}]$

$[\dot{j} \xrightarrow{\text{pyk}} \text{“peano j”}]$

$\dot{k}$

$[\dot{k} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{k} \doteq \dot{k}]])]$

$[\dot{k} \xrightarrow{\text{tex}} “\dot{\mathit{k}}”]$

$[\dot{k} \xrightarrow{\text{pyk}} “\text{peano k}”]$

$\dot{l}$

$[\dot{l} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{l} \doteq \dot{l}]])]$

$[\dot{l} \xrightarrow{\text{tex}} “\dot{\mathit{l}}”]$

$[\dot{l} \xrightarrow{\text{pyk}} “\text{peano l}”]$

$\dot{m}$

$[\dot{m} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{m} \doteq \dot{m}]])]$

$[\dot{m} \xrightarrow{\text{tex}} “\dot{\mathit{m}}”]$

$[\dot{m} \xrightarrow{\text{pyk}} “\text{peano m}”]$

$\dot{n}$

$[\dot{n} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{n} \doteq \dot{n}]])]$

$[\dot{n} \xrightarrow{\text{tex}} “\dot{\mathit{n}}”]$

$[\dot{n} \xrightarrow{\text{pyk}} “\text{peano n}”]$

$\dot{o}$

$[\dot{o} \xrightarrow{\text{macro}} \lambda \mathbf{t}.\lambda \mathbf{s}.\lambda \mathbf{c}.\tilde{\mathcal{M}}_4(\mathbf{t}, \mathbf{s}, \mathbf{c}, [[\dot{o} \doteq \dot{o}]])]$

$[\dot{o} \xrightarrow{\text{tex}} “\dot{\mathit{o}}”]$

$[\dot{o} \xrightarrow{\text{pyk}} “\text{peano o}”]$

$\dot{p}$

$[\dot{p} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{p} \doteq \dot{p}] \rrbracket)]$

$[\dot{p} \xrightarrow{\text{tex}} “\dot{\mathit{p}}”]$

$[\dot{p} \xrightarrow{\text{pyk}} “\text{peano } p”]$

$\dot{q}$

$[\dot{q} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{q} \doteq \dot{q}] \rrbracket)]$

$[\dot{q} \xrightarrow{\text{tex}} “\dot{\mathit{q}}”]$

$[\dot{q} \xrightarrow{\text{pyk}} “\text{peano } q”]$

$\dot{r}$

$[\dot{r} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{r} \doteq \dot{r}] \rrbracket)]$

$[\dot{r} \xrightarrow{\text{tex}} “\dot{\mathit{r}}”]$

$[\dot{r} \xrightarrow{\text{pyk}} “\text{peano } r”]$

$\dot{s}$

$[\dot{s} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{s} \doteq \dot{s}] \rrbracket)]$

$[\dot{s} \xrightarrow{\text{tex}} “\dot{\mathit{s}}”]$

$[\dot{s} \xrightarrow{\text{pyk}} “\text{peano } s”]$

$\dot{t}$

$[\dot{t} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \llbracket [\dot{t} \doteq \dot{t}] \rrbracket)]$

$[\dot{t} \xrightarrow{\text{tex}} “\dot{\mathit{t}}”]$

$[\dot{t} \xrightarrow{\text{pyk}} “\text{peano } t”]$

$\dot{u}$

$[\dot{u} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\dot{u} \doteq \dot{u}]])]$

$[\dot{u} \xrightarrow{\text{tex}} “\dot{\mathit{u}}”]$

$[\dot{u} \xrightarrow{\text{pyk}} “\text{peano } u”]$

$\dot{v}$

$[\dot{v} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\dot{v} \doteq \dot{v}]])]$

$[\dot{v} \xrightarrow{\text{tex}} “\dot{\mathit{v}}”]$

$[\dot{v} \xrightarrow{\text{pyk}} “\text{peano } v”]$

$\dot{w}$

$[\dot{w} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\dot{w} \doteq \dot{w}]])]$

$[\dot{w} \xrightarrow{\text{tex}} “\dot{\mathit{w}}”]$

$[\dot{w} \xrightarrow{\text{pyk}} “\text{peano } w”]$

$\dot{x}$

$[\dot{x} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\dot{x} \doteq \dot{x}]])]$

$[\dot{x} \xrightarrow{\text{tex}} “\dot{\mathit{x}}”]$

$[\dot{x} \xrightarrow{\text{pyk}} “\text{peano } x”]$

$\dot{y}$

$[\dot{y} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\dot{y} \doteq \dot{y}]])]$

$[\dot{y} \xrightarrow{\text{tex}} “\dot{\mathit{y}}”]$

$[\dot{y} \xrightarrow{\text{pyk}} “\text{peano } y”]$

$\dot{z}$

$[\dot{z} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\dot{z} \doteq \dot{z}] \rceil)]$

$[\dot{z} \xrightarrow{\text{tex}} “$
 $\backslash \text{dot}\{\backslash \text{mathit}\{z\}\}”]$

$[\dot{z} \xrightarrow{\text{pyk}} “\text{peano } z”]$

$\text{nonfree}(*, *)$

$[\text{nonfree}(x, y) \xrightarrow{\text{val}}$
 $\text{If}(y^{\mathcal{P}}, \neg [\ x \stackrel{t}{=} y \] ,$
 $\text{If}(\neg [\ y \stackrel{r}{=} [\forall x: y \] \] , \text{nonfree}^*(x, y^t),$
 $\text{If}(x \stackrel{t}{=} [\ y^1 \] , \top, \text{nonfree}(x, y^2)))]]$

$[\text{nonfree}(x, y) \xrightarrow{\text{tex}} “$
 $\backslash \text{dot}\{\text{nonfree}\}(\#1.$
 $, \#2.$
 $)”]$

$[\text{nonfree}(x, y) \xrightarrow{\text{pyk}} “\text{peano nonfree } * \text{ in } * \text{ end nonfree}”]$

$\text{nonfree}^*(*, *)$

$[\text{nonfree}^*(x, y) \xrightarrow{\text{val}} x! \text{If}(y, \top, \text{If}(\text{nonfree}(x, y^h), \text{nonfree}^*(x, y^t), F))]$

$[\text{nonfree}^*(x, y) \xrightarrow{\text{tex}} “$
 $\backslash \text{dot}\{\text{nonfree}\}^*(\#1.$
 $, \#2.$
 $)”]$

$[\text{nonfree}^*(x, y) \xrightarrow{\text{pyk}} “\text{peano nonfree star } * \text{ in } * \text{ end nonfree}”]$

$\text{free}\langle * | * := * \rangle$

$[\text{free}\langle a | x := b \rangle \xrightarrow{\text{val}} x! [\ b!$
 $\text{If}(a^{\mathcal{P}}, \top,$
 $\text{If}(\neg [\ a \stackrel{r}{=} [\forall u: v \] \] , \text{free}^*\langle a^t | x := b \rangle,$
 $\text{If}(a^1 \stackrel{t}{=} x, \top,$
 $\text{If}(\text{nonfree}(x, a^2), \top,$

If($\neg$ nonfree(a^1, b), F,
free($a^2|x := b$)))))]]

[free($a|x := b$) $\xrightarrow{\text{tex}}$ “
\dot{free}\langle #1.
| #2.
:= #3.
\rangle”]

[free($a|x := b$) $\xrightarrow{\text{pyk}}$ “peano free * set * to * end free”]

free* $\langle *|* := * \rangle$

[free* $\langle a|x := b \rangle \xrightarrow{\text{val}} x!$ [b!If($a, T, \text{If}(\text{free}\langle a^h|x := b \rangle, \text{free}^*\langle a^t|x := b \rangle, F)$))]]

[free* $\langle a|x := b \rangle \xrightarrow{\text{tex}}$ “
\dot{free}\{\}^*\langle #1.
| #2.
:= #3.
\rangle”]

[free* $\langle a|x := b \rangle \xrightarrow{\text{pyk}}$ “peano free star * set * to * end free”]

* $\equiv \langle *|* := * \rangle$

[$a \equiv \langle b|x := c \rangle \xrightarrow{\text{val}} a!$ [$x!$ [$c!$
If($\text{If}(b \stackrel{r}{=} \forall u: v, b^1 \stackrel{t}{=} x, F), a \stackrel{t}{=} b,$
If($b^p \wedge [b \stackrel{t}{=} x], a \stackrel{t}{=} c, \text{If}([$
 $a] \stackrel{r}{=} b, a^t \equiv \langle *b^t|x := c \rangle, F))$)]]]

[$a \equiv \langle b|x := c \rangle \xrightarrow{\text{tex}}$ “#1.
\equiv\langle #2.
| #3.
:= #4.
\rangle”]

[$a \equiv \langle b|x := c \rangle \xrightarrow{\text{pyk}}$ “peano sub * is * where * is * end sub”]

* $\equiv \langle *|* := * \rangle$

[$a \equiv \langle *b|x := c \rangle \xrightarrow{\text{val}} b!$ [$x!$ [$c!$ If($a, T, \text{If}(a^h \equiv \langle b^h|x := c \rangle, a^t \equiv \langle *b^t|x := c \rangle, F)$))]]]

[$a \equiv \langle *b|x := c \rangle \xrightarrow{\text{tex}}$ “#1.

$$\{\equiv\}\langle^* \#2.$$

|#3.

$$:= \#4.$$

\range"]

$$[a \equiv \langle *b | x := c \rangle \xrightarrow{\text{pyk}} \text{“peano sub star } * \text{ is } * \text{ where } * \text{ is } * \text{ end sub”}]$$

S

$$\begin{aligned}
[S \xrightarrow{\text{stnt}} & \left[\left[\dot{a} \dot{+} \dot{b}' \right] \stackrel{P}{=} \left[\dot{a} \dot{+} \left[\dot{b} \right]' \right] \right] \oplus \left[\left[\left[\dot{+} \underline{b} \right] \Rightarrow \dot{+} \underline{a} \right] \Rightarrow \left[\left[\dot{+} \underline{b} \right] \Rightarrow \underline{a} \right] \Rightarrow \underline{b} \right] \oplus \left[\left[\left[\dot{a} \stackrel{P}{=} \dot{b} \right] \right] \Rightarrow \left[\dot{a}' \stackrel{P}{=} \left[\dot{b}' \right] \right] \right] \oplus \left[\left[\forall \underline{a}: \forall \underline{b}: \left[\underline{a} \Rightarrow \underline{b} \right] \vdash \left[\underline{a} \vdash \underline{b} \right] \right] \right] \oplus \left[\left[\left[\dot{a}' \stackrel{P}{=} \left[\dot{b}' \right] \right] \right] \Rightarrow \left[\dot{a} \stackrel{P}{=} \left[\dot{b} \right] \right] \right] \oplus \left[\left[\forall \underline{a}: \forall \underline{b}: \left[\underline{a} \Rightarrow \left[\underline{b} \Rightarrow \underline{a} \right] \right] \right] \right] \oplus \left[\left[\forall \underline{x}: \forall \underline{a}: \forall \underline{b}: \left[\text{nonfree}(\underline{x}, \underline{a}) \Vdash \left[\dot{\forall} \underline{x}: \left[\underline{a} \Rightarrow \underline{b} \right] \right] \right] \Rightarrow \left[\underline{a} \Rightarrow \dot{\forall} \underline{x}: \underline{b} \right] \right] \right] \oplus \left[\left[\left[\dot{a}: \left[\dot{b}' \right] \right] \stackrel{P}{=} \left[\dot{a}: \left[\dot{b} \right] \right] \dot{+} \left[\dot{a} \right] \right] \oplus \left[\left[\left[\dot{a} \dot{+} \dot{0} \right] \stackrel{P}{=} \left[\dot{a} \right] \right] \oplus \left[\left[\forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \left[\underline{a} \Rightarrow \left[\underline{b} \Rightarrow \underline{c} \right] \right] \Rightarrow \left[\underline{a} \Rightarrow \underline{b} \right] \right] \right] \right] \right] \oplus \left[\left[\left[\dot{a} \stackrel{P}{=} \left[\dot{b} \right] \right] \right] \Rightarrow \left[\left[\dot{a} \stackrel{P}{=} \left[\dot{c} \right] \right] \right] \Rightarrow \left[\dot{b} \stackrel{P}{=} \left[\dot{c} \right] \right] \right] \oplus \left[\left[\forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: \left[\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash \left[\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash \left[\underline{b} \Rightarrow \left[\dot{\forall} \underline{x}: \left[\underline{a} \Rightarrow \underline{c} \right] \right] \Rightarrow \dot{\forall} \underline{x}: \underline{a} \right] \right] \right] \right] \oplus \left[\left[\neg \left[\dot{0} \stackrel{P}{=} \left[\dot{a}' \right] \right] \right] \right] \oplus \left[\left[\forall \underline{x}: \forall \underline{a}: \left[\underline{a} \vdash \dot{\forall} \underline{x}: \underline{a} \right] \right] \right] \oplus \left[\left[\forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: \left[\left[\underline{a} \right] \equiv \langle \left[\underline{b} \right] | \left[\underline{x} \right] := \left[\underline{c} \right] \rangle \Vdash \left[\left[\dot{\forall} \underline{x}: \underline{b} \right] \Rightarrow \underline{a} \right] \right] \right] \oplus \left[\left[\left[\dot{a}: \dot{0} \right] \stackrel{P}{=} \dot{0} \right] \right] \right] \right]
\end{aligned}$$
$$[S \xrightarrow{\text{tex}} S'']$$
$$[S \xrightarrow{\text{pyk}} \text{“system s”}]$$

A1

[A1 $\xrightarrow{\text{proof}}$ Rule tactic]

$$[A1 \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}. \forall \underline{b}. [\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{a}]]]$$
$$[A1 \xrightarrow{\text{tex}} \text{“} A1\text{”}]$$
$$[A1 \xrightarrow{\text{pyk}} \text{“axiom a one”}]$$

A2

[A2 $\xrightarrow{\text{proof}}$ Rule tactic]

$$[A2 \xrightarrow{\text{stmt}} S \vdash \forall \mathbf{a}:\forall \mathbf{b}:\forall \mathbf{c}: [[\mathbf{a} \Rightarrow [\mathbf{b} \Rightarrow \mathbf{c}]] \Rightarrow [[\mathbf{a} \Rightarrow \mathbf{b}] \Rightarrow [\mathbf{a} \Rightarrow \mathbf{c}]]]]$$

[A2 $\xrightarrow{\text{tex}}$ “
A2”]

[A2 $\xrightarrow{\text{pyk}}$ “axiom a two”]

A3

[A3 $\xrightarrow{\text{proof}}$ Rule tactic]

[A3 $\xrightarrow{\text{stmt}}$ S $\vdash$ [[[$\dot{\neg} \underline{b}$] $\Rightarrow \dot{\neg} \underline{a}$] $\Rightarrow$ [[[$\dot{\neg} \underline{b}$] $\Rightarrow \underline{a}$] $\Rightarrow \underline{b}$]]]

[A3 $\xrightarrow{\text{tex}}$ “
A3”]

[A3 $\xrightarrow{\text{pyk}}$ “axiom a three”]

A4

[A4 $\xrightarrow{\text{proof}}$ Rule tactic]

[A4 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: [[\underline{a}] \equiv \langle [\underline{b}] | [\underline{x}] := [\underline{c}] \rangle \Vdash [[\dot{\forall} \underline{x}: \underline{b}] \Rightarrow \underline{a}]]]$

[A4 $\xrightarrow{\text{tex}}$ “
A4”]

[A4 $\xrightarrow{\text{pyk}}$ “axiom a four”]

A5

[A5 $\xrightarrow{\text{proof}}$ Rule tactic]

[A5 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\text{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \dot{\forall} \underline{x}: \underline{b}]]]$

[A5 $\xrightarrow{\text{tex}}$ “
A5”]

[A5 $\xrightarrow{\text{pyk}}$ “axiom a five”]

S1

[S1 $\xrightarrow{\text{proof}}$ Rule tactic]

[S1 $\xrightarrow{\text{stmt}}$ S $\vdash [[\underline{a} \stackrel{P}{=} [\underline{b}]] \Rightarrow [[\underline{a} \stackrel{P}{=} [\underline{c}]] \Rightarrow [\underline{b} \stackrel{P}{=} [\underline{c}]]]]]$

[S1 $\xrightarrow{\text{tex}}$ “
S1”]

[S1 $\xrightarrow{\text{pyk}}$ “axiom s one”]

S2

[S2 $\xrightarrow{\text{proof}}$ Rule tactic]

[S2 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \stackrel{p}{=} [\dot{b}]] \Rightarrow [\dot{a}' \stackrel{p}{=} [\dot{b}']]]$]

[S2 $\xrightarrow{\text{tex}}$ “
S2”]

[S2 $\xrightarrow{\text{pyk}}$ “axiom s two”]

S3

[S3 $\xrightarrow{\text{proof}}$ Rule tactic]

[S3 $\xrightarrow{\text{stmt}}$ $S \vdash \neg [\dot{0} \stackrel{p}{=} [\dot{a}']]$]

[S3 $\xrightarrow{\text{tex}}$ “
S3”]

[S3 $\xrightarrow{\text{pyk}}$ “axiom s three”]

S4

[S4 $\xrightarrow{\text{proof}}$ Rule tactic]

[S4 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a}' \stackrel{p}{=} [\dot{b}']] \Rightarrow [\dot{a} \stackrel{p}{=} [\dot{b}]]]$]

[S4 $\xrightarrow{\text{tex}}$ “
S4”]

[S4 $\xrightarrow{\text{pyk}}$ “axiom s four”]

S5

[S5 $\xrightarrow{\text{proof}}$ Rule tactic]

[S5 $\xrightarrow{\text{stmt}}$ $S \vdash [[\dot{a} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{a}]]$]

[S5 $\xrightarrow{\text{tex}}$ “
S5”]

[S5 $\xrightarrow{\text{pyk}}$ “axiom s five”]

S6

[S6 $\xrightarrow{\text{proof}}$ Rule tactic]

[S6 $\xrightarrow{\text{stmt}}$ S $\vdash$ [[$\dot{a} \dot{+}$ [$\dot{b}'$]] $\stackrel{p}{=}$ [[$\dot{a} \dot{+}$ [$\dot{b}$]] ']]]

[S6 $\xrightarrow{\text{tex}}$ “
S6”]

[S6 $\xrightarrow{\text{pyk}}$ “axiom s six”]

S7

[S7 $\xrightarrow{\text{proof}}$ Rule tactic]

[S7 $\xrightarrow{\text{stmt}}$ S $\vdash$ [[$\dot{a} \dot{:}$ $\dot{0}$]] $\stackrel{p}{=}$ $\dot{0}$]]

[S7 $\xrightarrow{\text{tex}}$ “
S7”]

[S7 $\xrightarrow{\text{pyk}}$ “axiom s seven”]

S8

[S8 $\xrightarrow{\text{proof}}$ Rule tactic]

[S8 $\xrightarrow{\text{stmt}}$ S $\vdash$ [[$\dot{a} \dot{:}$ [$\dot{b}'$]]] $\stackrel{p}{=}$ [[$\dot{a} \dot{:}$ [$\dot{b}$]] $\dot{+}$ [$\dot{a}$]]]]

[S8 $\xrightarrow{\text{tex}}$ “
S8”]

[S8 $\xrightarrow{\text{pyk}}$ “axiom s eight”]

S9

[S9 $\xrightarrow{\text{proof}}$ Rule tactic]

[S9 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \Rightarrow [[\forall \underline{x}: [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \forall \underline{x}: \underline{a}]]]]]]]$

[S9 $\xrightarrow{\text{tex}}$ “
S9”]

[S9 $\xrightarrow{\text{pyk}}$ “axiom s nine”]

MP

[MP $\xrightarrow{\text{proof}}$ Rule tactic]

[MP $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]$]

[MP $\xrightarrow{\text{tex}}$ “
MP”]

[MP $\xrightarrow{\text{pyk}}$ “rule mp”]

Gen

[Gen $\xrightarrow{\text{proof}}$ Rule tactic]

[Gen $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{x}: \forall \underline{a}: [\underline{a} \vdash \dot{\forall} \underline{x}: \underline{a}]$]

[Gen $\xrightarrow{\text{tex}}$ “
Gen”]

[Gen $\xrightarrow{\text{pyk}}$ “rule gen”]

L3.2(a)

[L3.2(a) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash [[S5 \gg [[\dot{a} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{a}]]] ; [[[\text{Gen} \triangleright [[\dot{a} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{a}]]] \gg \dot{\forall} \dot{a}: [[\dot{a} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{a}]]] ; [[[A4@ [\dot{x}]] \gg [[\dot{\forall} \dot{a}: [[\dot{a} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{a}]]] \Rightarrow [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]]] ; [[[MP \triangleright [[\dot{\forall} \dot{a}: [[\dot{a} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{a}]]] \Rightarrow [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]]]] \triangleright \dot{\forall} \dot{a}: [[\dot{a} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{a}]]] \gg [[\dot{x} \dot{+} \dot{0}] \stackrel{P}{=} [\dot{x}]]] ; [[S1 \gg [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]]]] ; [[[\text{Gen} \triangleright [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]]] \gg \dot{\forall} \dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]]]] ; [[[A4@ [\dot{x}]] \gg [[\dot{\forall} \dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{x}]]]]] ; [[[MP \triangleright [[\dot{\forall} \dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]]]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{x}]]]]] \triangleright \dot{\forall} \dot{c}: [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{c}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{c}]]]] \gg [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{x}]]]]] ; [[[\text{Gen} \triangleright [[\dot{a} \stackrel{P}{=} [\dot{b}]] \Rightarrow [[\dot{a} \stackrel{P}{=} [\dot{x}]] \Rightarrow [\dot{b} \stackrel{P}{=} [\dot{x}]]]]]$]

[S' $\xrightarrow{\text{pyk}}$ “system prime s”]

A1'

[A1' $\xrightarrow{\text{proof}}$ Rule tactic]

[A1' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{a}: \forall \underline{b}: [\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{a}]]$]

[A1' $\xrightarrow{\text{tex}}$ “
A1”]

[A1' $\xrightarrow{\text{pyk}}$ “axiom prime a one”]

A2'

[A2' $\xrightarrow{\text{proof}}$ Rule tactic]

[A2' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [[\underline{a} \Rightarrow [\underline{b} \Rightarrow \underline{c}]] \Rightarrow [[\underline{a} \Rightarrow \underline{b}] \Rightarrow [\underline{a} \Rightarrow \underline{c}]]]$]

[A2' $\xrightarrow{\text{tex}}$ “
A2”]

[A2' $\xrightarrow{\text{pyk}}$ “axiom prime a two”]

A3'

[A3' $\xrightarrow{\text{proof}}$ Rule tactic]

[A3' $\xrightarrow{\text{stmt}}$ S' $\vdash [[[\neg \underline{b}] \Rightarrow \neg \underline{a}] \Rightarrow [[[\neg \underline{b}] \Rightarrow \underline{a}] \Rightarrow \underline{b}]]]$]

[A3' $\xrightarrow{\text{tex}}$ “
A3”]

[A3' $\xrightarrow{\text{pyk}}$ “axiom prime a three”]

A4'

[A4' $\xrightarrow{\text{proof}}$ Rule tactic]

[A4' $\xrightarrow{\text{stmt}}$ S' $\vdash \forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: [[\underline{a}] \equiv \langle [\underline{b}] | [\underline{x}] := [\underline{c}] \rangle \Vdash [[\forall \underline{x}: \underline{b}] \Rightarrow \underline{a}]]]$]

[A4' $\xrightarrow{\text{tex}}$ “
A4”]

[A4' $\xrightarrow{\text{pyk}}$ “axiom prime a four”]

A5'

[A5' $\xrightarrow{\text{proof}}$ Rule tactic]
 [A5' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\text{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{b}]] \Rightarrow [\underline{a} \Rightarrow \dot{\forall} \underline{x}: \underline{b}]]]$]
 [A5' $\xrightarrow{\text{tex}}$ “
 A5'”]
 [A5' $\xrightarrow{\text{pyk}}$ “axiom prime a five”]

S1'

[S1' $\xrightarrow{\text{proof}}$ Rule tactic]
 [S1' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [[\underline{a} \stackrel{P}{=} \underline{b}] \Rightarrow [[\underline{a} \stackrel{P}{=} \underline{c}] \Rightarrow [\underline{b} \stackrel{P}{=} \underline{c}]]]$]
 [S1' $\xrightarrow{\text{tex}}$ “
 S1'”]
 [S1' $\xrightarrow{\text{pyk}}$ “axiom prime s one”]

S2'

[S2' $\xrightarrow{\text{proof}}$ Rule tactic]
 [S2' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \stackrel{P}{=} \underline{b}] \Rightarrow [\underline{a}' \stackrel{P}{=} [\underline{b}']]]$]
 [S2' $\xrightarrow{\text{tex}}$ “
 S2'”]
 [S2' $\xrightarrow{\text{pyk}}$ “axiom prime s two”]

S3'

[S3' $\xrightarrow{\text{proof}}$ Rule tactic]
 [S3' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \neg [\dot{0} \stackrel{P}{=} [\underline{a}']]$]
 [S3' $\xrightarrow{\text{tex}}$ “
 S3'”]
 [S3' $\xrightarrow{\text{pyk}}$ “axiom prime s three”]

S4'

[S4' $\xrightarrow{\text{proof}}$ Rule tactic]

[S4' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a}' \stackrel{p}{=} [\underline{b}']] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{b}]]$]

[S4' $\xrightarrow{\text{tex}}$ “
S4'”]

[S4' $\xrightarrow{\text{pyk}}$ “axiom prime s four”]

S5'

[S5' $\xrightarrow{\text{proof}}$ Rule tactic]

[S5' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]$]

[S5' $\xrightarrow{\text{tex}}$ “
S5'”]

[S5' $\xrightarrow{\text{pyk}}$ “axiom prime s five”]

S6'

[S6' $\xrightarrow{\text{proof}}$ Rule tactic]

[S6' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \dot{+} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}] ']]$]

[S6' $\xrightarrow{\text{tex}}$ “
S6'”]

[S6' $\xrightarrow{\text{pyk}}$ “axiom prime s six”]

S7'

[S7' $\xrightarrow{\text{proof}}$ Rule tactic]

[S7' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: [[\underline{a} \dot{:} \dot{0}] \stackrel{p}{=} \dot{0}]$]

[S7' $\xrightarrow{\text{tex}}$ “
S7'”]

[S7' $\xrightarrow{\text{pyk}}$ “axiom prime s seven”]

S8'

[S8' $\xrightarrow{\text{proof}}$ Rule tactic]

[S8' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} : [\underline{b}']] \stackrel{p}{=} [[\underline{a} : \underline{b}] \dot{+} \underline{a}]]$]

[S8' $\xrightarrow{\text{tex}}$ “
S8'''”]

[S8' $\xrightarrow{\text{pyk}}$ “axiom prime s eight”]

S9'

[S9' $\xrightarrow{\text{proof}}$ Rule tactic]

[S9' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \Rightarrow [[\dot{\forall} \underline{x}: [\underline{a} \Rightarrow \underline{c}]] \Rightarrow \dot{\forall} \underline{x}: \underline{a}]]]]$]

[S9' $\xrightarrow{\text{tex}}$ “
S9'''”]

[S9' $\xrightarrow{\text{pyk}}$ “axiom prime s nine”]

MP'

[MP' $\xrightarrow{\text{proof}}$ Rule tactic]

[MP' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{a}: \forall \underline{b}: [[\underline{a} \Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]$]

[MP' $\xrightarrow{\text{tex}}$ “
MP'''”]

[MP' $\xrightarrow{\text{pyk}}$ “rule prime mp”]

Gen'

[Gen' $\xrightarrow{\text{proof}}$ Rule tactic]

[Gen' $\xrightarrow{\text{stmt}}$ $S' \vdash \forall \underline{x}: \forall \underline{a}: [\underline{a} \vdash \dot{\forall} \underline{x}: \underline{a}]$]

[Gen' $\xrightarrow{\text{tex}}$ “
Gen'''”]

[Gen' $\xrightarrow{\text{pyk}}$ “rule prime gen”]

L3.2(a)'

$$[\text{L3.2(a)}' \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S' \vdash \forall \underline{a}: [[S5' \gg [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]]; [[S1' \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]]; [[[[MP' \triangleright [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]] \triangleright [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]]; [[[MP' \triangleright [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \Rightarrow [\underline{a} \stackrel{p}{=} \underline{a}]]] \triangleright [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [\underline{a} \stackrel{p}{=} \underline{a}]]]], p_0, c)]$$

$$[\text{L3.2(a)}' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}: [\underline{a} \stackrel{p}{=} \underline{a}]]$$

$$[\text{L3.2(a)}' \xrightarrow{\text{tex}} \text{“} \\ \text{L3.2(a)”}]$$

$$[\text{L3.2(a)}' \xrightarrow{\text{pyk}} \text{“lemma prime l three two a”}]$$

$$\dot{*}$$

$$[\dot{x} \xrightarrow{\text{tex}} \text{“} \\ \backslash \text{dot}\{\#1. \\ \}\text{”}]$$

$$[\dot{x} \xrightarrow{\text{pyk}} \text{“} * \text{ peano var”}]$$

$$*'$$

$$[x' \xrightarrow{\text{tex}} \text{“}\#1. \\ \text{”}]$$

$$[x' \xrightarrow{\text{pyk}} \text{“} * \text{ peano succ”}]$$

$$* \dot{+} *$$

$$[x \dot{+} y \xrightarrow{\text{tex}} \text{“}\#1. \\ \backslash \text{mathop}\{\backslash \text{dot}\{\backslash \text{cdot}\}\} \#2.\text{”}]$$

$$[x \dot{+} y \xrightarrow{\text{pyk}} \text{“} * \text{ peano times } * \text{”}]$$

$$* \dot{+} *$$

$$[x \dot{+} y \xrightarrow{\text{tex}} \text{“}\#1. \\ \backslash \text{mathop}\{\backslash \text{dot}\{+\}\} \#2.\text{”}]$$

$$[\mathop{\dot{+}}\nolimits x\,y\stackrel{\text{pyk}}{\rightarrow} “* \text{ peano plus }*”]$$

$$*\stackrel{\text{p}}{=}*$$

$$[x\stackrel{\text{p}}{=}y\stackrel{\text{tex}}{\rightarrow} “\#1.\backslash\stackrel{\text{rel}}{\text{p}}\{=\}\{=\}\ \#2.”]$$

$$[x\stackrel{\text{p}}{=}y\stackrel{\text{pyk}}{\rightarrow} “* \text{ peano is }*”]$$

$$*\mathcal{P}$$

$$[x^{\mathcal{P}}\stackrel{\text{val}}{\rightarrow}x\stackrel{\text{r}}{=}\lceil\mathop{\dot{x}}\nolimits\rceil]$$

$$[x^{\mathcal{P}}\stackrel{\text{tex}}{\rightarrow} “\#1.\{\}\ \wedge\ \{\backslash\text{cal P}\}”]$$

$$[x^{\mathcal{P}}\stackrel{\text{pyk}}{\rightarrow} “* \text{ is peano var}”]$$

$$\mathop{\dot{\neg}}\nolimits *$$

$$[\mathop{\dot{\neg}}\nolimits x\stackrel{\text{tex}}{\rightarrow} “\backslash\dot{\{\neg\}}\backslash,\ \#1.”]$$

$$[\mathop{\dot{\neg}}\nolimits x\stackrel{\text{pyk}}{\rightarrow} “\text{peano not }*”]$$

$$*\mathop{\dot{\wedge}}\nolimits *$$

$$[x\mathop{\dot{\wedge}}\nolimits y\stackrel{\text{macro}}{\rightarrow}\lambda\text{t}.\lambda\text{s}.\lambda\text{c}.\tilde{\mathcal{M}}_4(\text{t},\text{s},\text{c},\lceil[x\mathop{\dot{\wedge}}\nolimits y\mathop{\dot{=}}\nolimits \mathop{\dot{\neg}}\nolimits (x\mathop{\dot{\Rightarrow}}\nolimits \mathop{\dot{\neg}}\nolimits y)]\rceil)]$$

$$[x\mathop{\dot{\wedge}}\nolimits y\stackrel{\text{tex}}{\rightarrow} “\#1.\backslash\mathrel{\dot{\wedge}}\{\backslash\text{wedge}\}\ \#2.”]$$

$$[x\mathop{\dot{\wedge}}\nolimits y\stackrel{\text{pyk}}{\rightarrow} “* \text{ peano and }*”]$$

$$*\mathop{\dot{\vee}}\nolimits *$$

$$[x\mathop{\dot{\vee}}\nolimits y\stackrel{\text{macro}}{\rightarrow}\lambda\text{t}.\lambda\text{s}.\lambda\text{c}.\tilde{\mathcal{M}}_4(\text{t},\text{s},\text{c},\lceil[x\mathop{\dot{\vee}}\nolimits y\mathop{\dot{=}}\nolimits \text{ [} \mathop{\dot{\neg}}\nolimits x \text{] } \mathop{\dot{\Rightarrow}}\nolimits y]\rceil)]$$

$$[x\mathop{\dot{\vee}}\nolimits y\stackrel{\text{tex}}{\rightarrow} “\#1.\backslash\mathrel{\dot{\vee}}\{\backslash\text{vee}\}\ \#2.”]$$

$$[x\mathop{\dot{\vee}}\nolimits y\stackrel{\text{pyk}}{\rightarrow} “* \text{ peano or }*”]$$

$\dot{\forall}*: *$

$[\dot{\forall}x: y \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{forall}\} \#1.$
 $\backslash\text{colon} \#2.”]$

$[\dot{\forall}x: y \xrightarrow{\text{pyk}} “\text{peano all } * \text{ indeed } *”]$

$\dot{\exists}*: *$

$[\dot{\exists}x: y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\dot{\exists}x: y \dot{=} \dot{\neg} \dot{\forall}x: \dot{\neg} y]])]$

$[\dot{\exists}x: y \xrightarrow{\text{tex}} “$
 $\backslash\text{dot}\{\backslash\text{exists}\} \#1.$
 $\backslash\text{colon} \#2.”]$

$[\dot{\exists}x: y \xrightarrow{\text{pyk}} “\text{peano exist } * \text{ indeed } *”]$

$* \Rightarrow *$

$[x \Rightarrow y \xrightarrow{\text{tex}} “\#1.$
 $\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{Rrightarrow}\}\} \#2.”]$

$[x \Rightarrow y \xrightarrow{\text{pyk}} “* \text{ peano imply } *”]$

$* \Leftrightarrow *$

$[x \Leftrightarrow y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \Leftrightarrow y \dot{=} (x \Rightarrow y) \dot{\wedge} (y \Rightarrow x)])])]$

$[x \Leftrightarrow y \xrightarrow{\text{tex}} “\#1.$
 $\backslash\text{mathrel}\{\backslash\text{dot}\{\backslash\text{Leftrightarrow}\}\} \#2.”]$

$[x \Leftrightarrow y \xrightarrow{\text{pyk}} “* \text{ peano iff } *”]$

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-03.UTC:15:58:02.061312 = MJD-53524.TAI:15:58:34.061312 =
LGT-4624531114061312e-6