

Logiweb codex of base

Up Help

base, $[* \bowtie *]$, “ $*$ ”, , $*$ then $*$, $*[*]*$, Preassociative $*;*$,
Postassociative $*;*$, $[*]_*$, $*$, priority $*$ end,
 $*, *, (*^t)$, string($*$) + $*$, string($*$) ++ $*$, bracket $*$ end bracket,
big bracket $*$ end bracket, math $*$ end math, flush left $[*]$, $x, y, z, [* \xrightarrow{*} *]$,
pyk, tex, name, prio, T, if($*, *, *$), $[* \Rightarrow *]$, val, claim, $*, !*, ''*, \#*, \$*, \%*,$
 $\&*, ', (*, *)$, $**$, $+*$, $, *$, $-*$, $.*$, $/*$, $0*$, $1*$, $2*$, $3*$, $4*$, $5*$, $6*$, $7*$, $8*$, $9*$, $::$, $;*$,
 $<*$, $=*$, $>*$, $?*$, $@*$, A*, B*, C*, D*, E*, F*, G*, H*, I*, J*, K*, L*, M*, N*,
O*, P*, Q*, R*, S*, T*, U*, V*, W*, X*, Y*, Z*, $[*, \backslash*,]*$, * , $_*$, $'*$, a*, b*,
c*, d*, e*, f*, g*, h*, i*, j*, k*, l*, m*, n*, o*, p*, q*, r*, s*, t*, u*, v*, w*,
x*, y*, z*, $\{*, |*, *\}$, $\sim*$, \perp , f($*$), $(*)^I$, F, 0, 1, 2, 3, 4,
5, 6, 7, 8, 9, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, $(*)^M$, If($*, *, *$),
array{ $*$ } * end array, l, c, r, empty, $\langle * | * := * \rangle$, $\mathcal{M}(*)$, $\tilde{\mathcal{U}}(*)$, $\mathcal{U}(*)$, $\mathcal{U}^M(*)$,
apply($*, *$), apply₁($*, *$), identifier($*$), identifier₁($*, *$), array-plus($*, *$), array-
remove($*, *, *$), array-put($*, *, *, *$), array-add($*, *, *, *, *$), bit($*, *$), bit₁($*, *$),
rack, "vector", "bibliography", "dictionary", "body", "codex", "expansion",
"code", "cache", "diagnose", "pyk", "tex", "texname", "value", "message",
"macro", "definition", "unpack", "claim", "priority", "lambda", "apply",
"true", "if", "quote", "proclaim", "define", "introduce", "hide", "pre",
"post", $\mathcal{E}(*, *, *)$, $\mathcal{E}_2(*, *, *, *, *)$, $\mathcal{E}_3(*, *, *, *)$, $\mathcal{E}_4(*, *, *, *)$, lookup($*, *, *$),
abstract($*, *, *, *$), $[\ast]$, $\mathcal{M}(*, *, *)$, $\mathcal{M}_2(*, *, *, *)$, $\mathcal{M}^*(*, *, *)$, macro, s₀,
zip($*, *$), assoc₁($*, *, *$), $(*)^P$, self, $[* \equiv *]$, $[* \doteq *]$, $[* \stackrel{\text{pyk}}{\equiv} *]$, $[* \stackrel{\text{tex}}{\equiv} *]$,
 $[* \stackrel{\text{name}}{\equiv} *]$, Priority table[*], $\tilde{\mathcal{M}}_1$, $\tilde{\mathcal{M}}_2(*)$, $\tilde{\mathcal{M}}_3(*)$, $\tilde{\mathcal{M}}_4(*, *, *, *)$, $\tilde{\mathcal{M}}(*, *, *)$,
 $\tilde{\mathcal{Q}}(*, *, *)$, $\tilde{\mathcal{Q}}_2(*, *, *)$, $\tilde{\mathcal{Q}}_3(*, *, *, *)$, $\tilde{\mathcal{Q}}^*(*, *, *)$, $(*)$, aspect($*, *$),
aspect($*, *, *$), $\langle *$, tuple₁($*$), tuple₂($*$), let₂($*, *$), let₁($*, *$), $[* \stackrel{\text{claim}}{\equiv} *]$,
checker, check($*, *$), check₂($*, *, *$), check₃($*, *, *$), check^{*}($*, *$),
check₂^{*}($*, *, *$), $[\ast]', [\ast]^-, [\ast]^0$, msg, $[* \stackrel{\text{msg}}{\equiv} *]$, <stmt>, stmt, $[* \stackrel{\text{stmt}}{\equiv} *]$,
HeadNil', HeadPair', Transitivity', \perp , Contra', T'_E, L₁, \perp , A, B, C, D, E, F, G,
H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, $\langle * | * := * \rangle$,
 $\langle * | * := * \rangle$, \emptyset , Remainder, $(*)^v$, intro($*, *, *, *$), intro($*, *, *$), error($*, *$),
error₂($*, *$), proof($*, *, *$), proof₂($*, *$), $\mathcal{S}(*, *)$, $\mathcal{S}^I(*, *)$, $\mathcal{S}^D(*, *)$, $\mathcal{S}_1^D(*, *, *)$,
 $\mathcal{S}^E(*, *)$, $\mathcal{S}_1^E(*, *, *)$, $\mathcal{S}^+(*, *)$, $\mathcal{S}_1^+(*, *, *)$, $\mathcal{S}^-(*, *)$, $\mathcal{S}_1^-(*, *, *)$, $\mathcal{S}^*(*, *)$,
 $\mathcal{S}_1^*(*, *, *)$, $\mathcal{S}_2^*(*, *, *, *)$, $\mathcal{S}^@(*, *)$, $\mathcal{S}_1^@(*, *, *)$, $\mathcal{S}^\vdash(*, *)$, $\mathcal{S}_1^\vdash(*, *, *, *)$, $\mathcal{S}^\Vdash(*, *)$,
 $\mathcal{S}_1^\Vdash(*, *, *, *)$, $\mathcal{S}^{i.e.}(*, *)$, $\mathcal{S}_1^{i.e.}(*, *, *, *)$, $\mathcal{S}^{i.e.}(*, *, *, *, *)$, $\mathcal{S}^\forall(*, *)$, $\mathcal{S}_1^\forall(*, *, *, *)$,
 $\mathcal{S}^i(*, *)$, $\mathcal{S}_1^i(*, *, *)$, $\mathcal{S}_2^i(*, *, *, *)$, T(*), claims($*, *, *$), claims₂($*, *, *$),
<proof>, proof, [Lemma $*:*$], [Proof of $*:*$], [$* \text{ lemma } * :*$],
[$* \text{ antilemma } * :*$], [$* \text{ rule } * :*$], [$* \text{ antirule } * :*$], verifier, $\mathcal{V}_1(*)$, $\mathcal{V}_2(*)$,
 $\mathcal{V}_3(*, *, *, *)$, $\mathcal{V}_4(*, *)$, $\mathcal{V}_5(*, *, *, *)$, $\mathcal{V}_6(*, *, *, *)$, $\mathcal{V}_7(*, *, *, *)$, Cut($*, *$),
Head_⊕($*$), Tail_⊕($*$), rule₁($*, *$), rule($*, *$), Rule tactic, Plus($*, *$), [Theory $*$],

theory₂(*, *), theory₃(*, *), theory₄(*, *, *), HeadNil'', HeadPair'',
 Transitivity'', Contra'', HeadNil, HeadPair, Transitivity, Contra, T_E,
 ragged right, ragged right expansion , parm(*, *, *), parm*(*, *, *), inst(*, *),
 inst*(*, *), occur(*, *, *), occur*(*, *, *), unify(* = *, *), unify*(* = *, *),
 unify₂(* = *, *), L_a, L_b, L_c, L_d, L_e, L_f, L_g, L_h, L_i, L_j, L_k, L_l, L_m, L_n, L_o, L_p,
 L_q, L_r, L_s, L_t, L_u, L_v, L_w, L_x, L_y, L_z, L_A, L_B, L_C, L_D, L_E, L_F, L_G, L_H, L_I, L_J,
 L_K, L_L, L_M, L_N, L_O, L_P, L_Q, L_R, L_S, L_T, L_U, L_V, L_W, L_X, L_Y, L_Z, L_?,
 Reflexivity, Reflexivity₁, Commutativity, Commutativity₁, <tactic>, tactic,
 [* ^{tactic} = *], P(*, *, *), P*(*, *, *), p₀, conclude₁(*, *), conclude₂(*, *, *),
 conclude₃(*, *, *, *), *-{*}, *', *[*], *[*→*], *[*⇒*], newline *,
 macro newline *, *0, *1, 0b, --color(*), --color*(*), *' *, *' *, *^H, *^T, *^U, *^h,
 *^t, *^s, *^c, *^d, *^a, *^C, *^B, *^r, *ⁱ, *^d, *^R, *⁰, *¹, *², *³, *⁴, *⁵, *⁶, *⁷, *⁸, *⁹,
 *^E, *^V, *^C, *^C, *·*, *+*, *+0*, *+1*, *-* , *-0*, *-1*, * ∪ {*},
 * ∪ *, *＼{*}, * . *, * . *, * :: *, * +2* *, * :: *, * +2* *, *, *, * ≈ *, * ≈ *,
 * ≈ *, * ≈ *, * ≈ *, * = *, * → *, * = *, * = *, * = *, * = *, * ∈ t *, * ⊆ T *, * = *,
 * = *, * free in *, * free in * *, * free for * in *, * free for * in *, * ∈ c *, * < *,
 * < ', * ≤ ', * → *, * ∧ *, * Ā *, * Ā *, * ∧c *, * ∨ *, * || *, * Ÿ *, * ⇒ *, * : *,
 !, * { * , λ * . *, Λ*, if * then * else *, let * = * in *, let * ≡ * in *, *^I,
 *▷, *^V, *+, *-, **, *@*, *▷*, *▷*, * ≫ *, * ⊢ *, * ⊦ *, * i.e. *, ∀*:*,
 * ⊕ *, *; *, * proves *, * proof of * : *, Line * : * ≫ *; *, Last line * ≫ * □,
 Line * : Premise ≫ *; *, Line * : Side-condition ≫ *; *, Arbitrary ≫ *; *,
 Local ≫ * = *; *, *&*, *＼*,

base

[base ^{prio} →

Preassociative

[base], [bracket * end bracket], [big bracket * end bracket], [math * end math],
 [flush left [*]], [x], [y], [z], [[* ⋮ *]], [[* → *]], [pyk], [tex], [name], [prio], [*], [T],
 [if(*, *, *)], [[* ⇒ *]], [val], [claim], [⊥], [f(*)], [(*)^I], [F], [0], [1], [2], [3], [4], [5], [6],
 [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],
 [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(*)^M], [If(*, *, *)],
 [array {* } * end array], [l], [c], [r], [empty], [(* | * := *)], [M(*)], [U(*)], [U(*)],
 [U^M(*)], [apply(*, *)], [apply₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-
 plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)],
 [bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"],
 ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
 ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
 ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
 ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
 [E(*, *, *)], [E₂(*, *, *, *, *)], [E₃(*, *, *, *)], [E₄(*, *, *, *)], [lookup(*, *, *)],
 [abstract(*, *, *, *)], [[*]], [M(*, *, *)], [M₂(*, *, *, *)], [M^{*}(*, *, *)], [macro],

Preassociative

$$[*-\{*\}], [*'], [*[*]], [*[* \rightarrow *]], [*[* \Rightarrow *]];$$

Preassociative

```
[["*"],[],[(*tPreassociative
[*0], [*1], [0b], [*~color(*)], [*~color*(*)];
```

Preassociative

[* ' *], [* ' *];

Preassociative

[*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*ⁱ], [*^d], [*^R], [*⁰], [*¹], [*²], [*³], [*⁴], [*⁵], [*⁶], [*⁷], [*⁸], [*⁹], [*^E], [*^V], [*^C], [*^{C*}];

Preassociative

[* · *], [* ·₀ *];

Preassociative

[* + *], [* +₀ *], [* +₁ *], [* - *], [* -₀ *], [* -₁ *];

Preassociative

[* ∪ { * }], [* ∪ *], [* \ { * }];

Postassociative

[* . . *], [* . . *], [* :: *], [* +2* *], [* :: *], [* +2* *];

Postassociative

[*, *];

Preassociative

[* ≈^B *], [* ≈^D *], [* ≈^C *], [* ≈^P *], [* ≈ *], [* = *], [* → *], [* =^t *], [* =^r *], [* =^{t*} *],

[* ∈_t *], [* ⊆_T *], [* =^T *], [* =^s *], [* free in *], [* free in^{*} *], [* free for * in *],

[* free for^{*} * in *], [* ∈_c *], [* < *], [* <' *], [* ≤' *];

Preassociative

[¬*];

Preassociative

[* ∧ *], [* ḥ *], [* ḥ *], [* ∧_c *];

Preassociative

[* ∨ *], [* || *], [* ḕ *];

Postassociative

[* ⇒ *];

Postassociative

[*: *], [*!*];

Preassociative

[* { * }];

Preassociative

[λ * . *], [Λ*], [if * then * else *], [let * = * in *], [let * ≡ * in *];

Preassociative

[*^I], [*▷], [*^V], [*⁺], [*⁻], [*^{*}];

Preassociative

[* @ *], [* ▷ *], [* ▷ *], [* ≫ *];

Postassociative

[* ⊢ *], [* ⊦ *], [* i.e. *];

Preassociative

[∀*: *];

Postassociative

[* ⊕ *];

Postassociative

```

[*; *];
Preassociative
[* proves *];
Preassociative
[* proof of * : *], [Line * : *  $\gg$  *; *], [Last line *  $\gg$  *  $\square$ ],
[Line * : Premise  $\gg$  *; *], [Line * : Side-condition  $\gg$  *; *], [Arbitrary  $\gg$  *; *],
[Local  $\gg$  * = *; *];
Postassociative
[* then *], [*[*]*];
Preassociative
[*&*];
Preassociative
[*\/*];
[base  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda c. M(t, s_0, c)$ ]
[base  $\xrightarrow{\text{claim}}$  checker  $\wedge_c$  verifier]
[base  $\xrightarrow{\text{pyk}}$  “base”]

```

[* \bowtie *]

Predef: proclaim

```

[[x  $\bowtie$  y]  $\xrightarrow{\text{tex}}$  “
[#1/tex name/tex.
\bowtie#2.
]”]
[[x  $\bowtie$  y]  $\xrightarrow{\text{pyk}}$  “proclaim * as * end proclaim”]

```

“*”

Predef: hide

```

“x”  $\xrightarrow{\text{name}}$  “
\mbox{“}#1.
\mbox{”}”
[“x”  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. t$ ]
[“x”  $\xrightarrow{\text{tex}}$  “#1.”]
[“x”  $\xrightarrow{\text{pyk}}$  “unicode start of text * end unicode text”]

```

[$\xrightarrow{\text{tex}}$ “”]

[$\xrightarrow{\text{pyk}}$ “unicode end of text”]

* then *

[x then $y \xrightarrow{\text{name}} “\#1.$
 $\backslash\mathrm{mathrel}\{\backslash\mathrm{mathrm}\{then\}\}\#2.”$]

[x then $y \xrightarrow{\text{val}} [**]^R :: [x :: [y :: T]]$]

[x then $y \xrightarrow{\text{tex}} “\#1.\#2.”$]

[x then $y \xrightarrow{\text{pyk}} “*$

then *”]

*[*]*

[$x[y]z \xrightarrow{\text{name}} “\#1.$
 $\{\} \#2.$
 $\{\} \#3.”$]

[$x[y]z \xrightarrow{\text{tex}} “\#1.\#2.\#3.”$]

[$x[y]z \xrightarrow{\text{pyk}} “*$

begin *

end *”]

Preassociative *; *

Predef: pre

[**Preassociative** $x; y \xrightarrow{\text{name}} “$
 $\backslash\mathrm{mathbf}\{\mathrm{Preassociative}\}\backslash, \#1.$
; #2.”]

[**Preassociative** $x; y \xrightarrow{\text{tex}} “$
 $\backslash\mathrm{newline}\backslash\mathrm{mathbf}\{\mathrm{Preassociative}\}\backslash\mathrm{newline} \#1.$
; #2.”]

[**Preassociative** $x; y \xrightarrow{\text{pyk}}$ “preassociative * greater than *”]

Postassociative *; *

Predef: post

[**Postassociative** $x; y \xrightarrow{\text{name}}$ “
\mathbf{Postassociative}\, \#1.
; \#2.”]

[**Postassociative** $x; y \xrightarrow{\text{tex}}$ “
\newline\mathbf{Postassociative} \newline \#1.
; \#2.”]

[**Postassociative** $x; y \xrightarrow{\text{pyk}}$ “postassociative * greater than *”]

[*], *

[[x], y $\xrightarrow{\text{name}}$ “
[#1.
] , \linebreak [0] #2.”]
[[x], y $\xrightarrow{\text{tex}}$ “
[#1/tex name/tex.
] , \linebreak [0] #2.”]
[[x], y $\xrightarrow{\text{pyk}}$ “priority * equal *”]

priority * end

[priority x end $\xrightarrow{\text{name}}$ “
\mathrm{priority} \, \#1.
\, \mathrm{end}”]

[priority x end $\xrightarrow{\text{tex}}$ “
[#1/tex name/tex.
]”]

[priority x end $\xrightarrow{\text{pyk}}$ “priority * end priority”]

*

```
[  
x  $\xrightarrow{\text{name}}$  “  
\newline #1.”]  
[  
x  $\xrightarrow{\text{tex}}$  “  
#1.”]  
[  
x  $\xrightarrow{\text{pyk}}$  “  
unicode newline *”]
```

*

```
[*  $\xrightarrow{\text{tex}}$  “  
\ast ”]  
[*  $\xrightarrow{\text{pyk}}$  “x”]
```

$(*)^t$

```
[(x) $t$   $\xrightarrow{\text{name}}$  “  
(#1.  
)^{\backslash bf t}”]  
[(x) $t$   $\xrightarrow{\text{val}}$  [*] $R$  :: [ x :: T ] ]  
[(x) $t$   $\xrightarrow{\text{tex}}$  “#1/tex name.”]  
[(x) $t$   $\xrightarrow{\text{pyk}}$  “text * end text”]
```

string(*) + *

```
[string(x) + y  $\xrightarrow{\text{name}}$  “  
\mbox{string} (#1.  
) + #2.”]  
[string(x) + y  $\xrightarrow{\text{tex}}$  “  
\mathrm{#1/tex name.  
} + \newline #2.”]
```

```
[string(x) + y  $\xrightarrow{\text{pyk}}$  “text * plus *”]
```

string(*) ++ *

```
[string(x) ++ y  $\xrightarrow{\text{name}}$  “\mbox{string}(\#1.  
)\mathrel{++}\#2.”]
```

```
[string(x) ++ y  $\xrightarrow{\text{tex}}$  “  
\mathrm{\#1/tex name.  
}\;{++}\newline{}\qqquad\#2.”]
```

```
[string(x) ++ y  $\xrightarrow{\text{pyk}}$  “text * plus indent *”]
```

bracket * end bracket

```
[bracket x end bracket  $\xrightarrow{\text{name}}$  “  
\mbox{bracket \$\#1.\$ end bracket}”]
```

```
[bracket x end bracket  $\xrightarrow{\text{tex}}$  “\$[\#1.]\$”]
```

```
[bracket x end bracket  $\xrightarrow{\text{pyk}}$  “bracket * end bracket”]
```

big bracket * end bracket

```
[big bracket x end bracket  $\xrightarrow{\text{name}}$  “  
\mbox{big bracket \$\#1.\$ end bracket}”]
```

```
[big bracket x end bracket  $\xrightarrow{\text{tex}}$  “\$\\left[\#1.\\right]\$”]
```

```
[big bracket x end bracket  $\xrightarrow{\text{pyk}}$  “big bracket * end bracket”]
```

math * end math

```
[math x end math  $\xrightarrow{\text{name}}$  “  
\mbox{math \$\#1.\$ end math}”]
```

```
[math x end math  $\xrightarrow{\text{tex}}$  “\$#\#1.\$”]
```

```
[math x end math  $\xrightarrow{\text{pyk}}$  “math * end math”]
```

flush left [*]

[**flush left** [x] $\xrightarrow{\text{name}}$ “
 \mathbf{flush\ left\ }#[\ #1.
]”]

[**flush left** [x] $\xrightarrow{\text{tex}}$ “
 \begin{flushleft}#1.
 \end{flushleft}”]

[**flush left** [x] $\xrightarrow{\text{pyk}}$ “flush left * end left”]

X

[x $\xrightarrow{\text{tactic}}$ y]

[x $\xrightarrow{\text{stmt}}$ x]

[x $\xrightarrow{\text{msg}}$ y]

[x $\xrightarrow{\text{claim}}$ y]

[x $\xrightarrow{\text{tex}}$ “
 \mathsf{x}”]

[x $\xrightarrow{\text{pyk}}$ “var x”]

Y

[y \xrightarrow{x} z]

[y $\xrightarrow{\text{tex}}$ “
 \mathsf{y}”]

[y $\xrightarrow{\text{pyk}}$ “var y”]

Z

[z $\xrightarrow{\text{tex}}$ “
 \mathsf{z}”]

[z $\xrightarrow{\text{pyk}}$ “var z”]

$[* \xrightarrow{*} *]$

Predef: define

$[[y \xrightarrow{x} z] \xrightarrow{\text{tex}} “\#2/\text{tex name}/\text{tex.}\backslash\text{stackrel}\{\#1.\}\{\backslash\text{rightarrow}\}\#\#3.\”]$

$[y \xrightarrow{x} z] \xrightarrow{\text{pyk}} \text{“define } *\text{ of } * \text{ as } * \text{ end define”}$

pyk

Predef: pyk

$[\text{pyk} \xrightarrow{\text{tex}} “\text{\\mathrm}\{\text{pyk}\}\”]$

$[\text{pyk} \xrightarrow{\text{pyk}} \text{“pyk”}]$

tex

Predef: tex

$[\text{tex} \xrightarrow{\text{tex}} “\text{\\mathrm}\{\text{tex}\}\”]$

$[\text{tex} \xrightarrow{\text{pyk}} \text{“tex”}]$

name

Predef: texname

$[\text{name} \xrightarrow{\text{tex}} “\text{\\mathrm}\{\text{name}\}\”]$

$[\text{name} \xrightarrow{\text{pyk}} \text{“tex name”}]$

prio

Predef: priority

[prio $\xrightarrow{\text{tex}}$ “
\\mathrm{prio}”]
[prio $\xrightarrow{\text{pyk}}$ “priority”]

T

Predef: true
[T $\xrightarrow{\text{tex}}$ “
\\mathsf{T}”]
[T $\xrightarrow{\text{pyk}}$ “true”]

if(*, *, *)

Predef: if
[if(*, *, *) $\xrightarrow{\text{tex}}$ “
\\mathrm{if}{}(\#1.
, \\linebreak [0]\#2.
, \\linebreak [0]\#3.
)”]
[if(*, *, *) $\xrightarrow{\text{pyk}}$ “if * then * else * end if”]

[* $\stackrel{*}{\Rightarrow}$ *]

Predef: introduce
[[* $\stackrel{*}{\Rightarrow}$ *] $\xrightarrow{\text{tex}}$ “
[#2/tex name/tex.
\\stackrel{}{\Rightarrow} \#1.
}\{\Rightarrow \#3.
”]
[* $\stackrel{*}{\Rightarrow}$ *] $\xrightarrow{\text{pyk}}$ “introduce * of * as * end introduce”]

val

Predef: value
[val $\xrightarrow{\text{tex}}$ “
\\mathrm{val}”]

```
[val  $\xrightarrow{\text{pyk}}$  "value"]
```

claim

Predef: claim

```
[claim  $\xrightarrow{\text{tex}}$  "  
\mathrm{claim}"]  
[claim  $\xrightarrow{\text{pyk}}$  "claim"]
```

*

```
[ x  $\xrightarrow{\text{name}}$  "  
\linebreak [0]\hspace{2.0em}\#1."]  
[ x  $\xrightarrow{\text{tex}}$  "\#1."  
[ x  $\xrightarrow{\text{pyk}}$  "unicode space *"]
```

!*

```
[!x  $\xrightarrow{\text{tex}}$  "\#1."  
[!x  $\xrightarrow{\text{pyk}}$  "unicode exclamation mark *"]
```

”*

```
[”x  $\xrightarrow{\text{tex}}$  "\#1."  
[”x  $\xrightarrow{\text{pyk}}$  "unicode quotation mark *"]
```

#*

```
[#x  $\xrightarrow{\text{name}}$  "  
\#\#\#1."  
[#x  $\xrightarrow{\text{tex}}$  "\#\#\#1."  
[#x  $\xrightarrow{\text{pyk}}$  "unicode number sign *"]
```

\$*

```
[$x ^{name} “  
\\$#1.”]  
[$x ^{tex} “$#1.”]  
[$x ^{pyk} “unicode dollar sign *”]
```

%*

```
[%x ^{name} “  
\\%#1.”]  
[%x ^{tex} “%#1.”]  
[%x ^{pyk} “unicode percent *”]
```

&*

```
[&x ^{name} “  
\\&#1.”]  
[&x ^{tex} “&#1.”]  
[&x ^{pyk} “unicode ampersand *”]
```

,

```
[’x ^{name} “  
\\mbox {’}#1.”]  
[’x ^{tex} “’#1.”]  
[’x ^{pyk} “unicode apostrophe *”]
```

(

```
[(x ^{tex} “(#1.”]  
[(x ^{pyk} “unicode left parenthesis *”]
```

)*

[$\text{D}\times \xrightarrow{\text{tex}} \text{"})\#1."]$]

[$\text{D}\times \xrightarrow{\text{pyk}} \text{"unicode right parenthesis *"}]$]

**

[$\text{*}\times \xrightarrow{\text{name}} \text{"}\{\ast\}\#1.\text{"}$]

[$\text{*}\times \xrightarrow{\text{tex}} \text{"*\#1."}$]

[$\text{*}\times \xrightarrow{\text{pyk}} \text{"unicode asterisk *"}]$]

+*

[$\text{+}\times \xrightarrow{\text{name}} \text{"}\{+\}\#1.\text{"}$]

[$\text{+}\times \xrightarrow{\text{tex}} \text{"+\#1."}$]

[$\text{+}\times \xrightarrow{\text{pyk}} \text{"unicode plus sign *"}]$]

, *

[$\text{,}\times \xrightarrow{\text{tex}} \text{"}\#\text{1."}$]

[$\text{,}\times \xrightarrow{\text{pyk}} \text{"unicode comma *"}]$]

-*

[$\text{-}\times \xrightarrow{\text{name}} \text{"}\backslash\text{mbox}\{-\}\#1.\text{"}$]

[$\text{-}\times \xrightarrow{\text{tex}} \text{"-\#1."}$]

[$\text{-}\times \xrightarrow{\text{pyk}} \text{"unicode hyphen *"}]$]

.*

[$\text{.}\times \xrightarrow{\text{tex}} \text{"}\.\#\text{1."}$]

[.x $\xrightarrow{\text{pyk}}$ “unicode period *”]

/*

[/x $\xrightarrow{\text{tex}}$ “/#1.”]

[/x $\xrightarrow{\text{pyk}}$ “unicode slash *”]

0*

[0x $\xrightarrow{\text{tex}}$ “0#1.”]

[0x $\xrightarrow{\text{pyk}}$ “unicode zero *”]

1*

[1x $\xrightarrow{\text{tex}}$ “1#1.”]

[1x $\xrightarrow{\text{pyk}}$ “unicode one *”]

2*

[2x $\xrightarrow{\text{tex}}$ “2#1.”]

[2x $\xrightarrow{\text{pyk}}$ “unicode two *”]

3*

[3x $\xrightarrow{\text{tex}}$ “3#1.”]

[3x $\xrightarrow{\text{pyk}}$ “unicode three *”]

4*

[4x $\xrightarrow{\text{tex}}$ “4#1.”]

[4x $\xrightarrow{\text{pyk}}$ “unicode four *”]

5*

[5x $\xrightarrow{\text{tex}}$ “5#1.”]

[5x $\xrightarrow{\text{pyk}}$ “unicode five *”]

6*

[6x $\xrightarrow{\text{tex}}$ “6#1.”]

[6x $\xrightarrow{\text{pyk}}$ “unicode six *”]

7*

[7x $\xrightarrow{\text{tex}}$ “7#1.”]

[7x $\xrightarrow{\text{pyk}}$ “unicode seven *”]

8*

[8x $\xrightarrow{\text{tex}}$ “8#1.”]

[8x $\xrightarrow{\text{pyk}}$ “unicode eight *”]

9*

[9x $\xrightarrow{\text{tex}}$ “9#1.”]

[9x $\xrightarrow{\text{pyk}}$ “unicode nine *”]

:*

[{:x $\xrightarrow{\text{name}}$ “

{:}#1.”}]

[{:x $\xrightarrow{\text{tex}}$ “:#1.”}]

[{:x $\xrightarrow{\text{pyk}}$ “unicode colon *”}]

; *

[;x $\xrightarrow{\text{tex}}$ “;#1.”]

[;x $\xrightarrow{\text{pyk}}$ “unicode semicolon *”]

<*

[<x $\xrightarrow{\text{name}}$ “
{<}#1.”]

[<x $\xrightarrow{\text{tex}}$ “<#1.”]

[<x $\xrightarrow{\text{pyk}}$ “unicode less than *”]

=*

[=x $\xrightarrow{\text{name}}$ “
{=}#1.”]

[=x $\xrightarrow{\text{tex}}$ “=#1.”]

[=x $\xrightarrow{\text{pyk}}$ “unicode equal sign *”]

>*

[>x $\xrightarrow{\text{name}}$ “
{>}#1.”]

[>x $\xrightarrow{\text{tex}}$ “>#1.”]

[>x $\xrightarrow{\text{pyk}}$ “unicode greater than *”]

?*

[?x $\xrightarrow{\text{tex}}$ “?#1.”]

[?x $\xrightarrow{\text{pyk}}$ “unicode question mark *”]

@*

[@x $\xrightarrow{\text{tex}}$ “@#1.”]

[@x $\xrightarrow{\text{pyk}}$ “unicode commercial at *”]

A*

[Ax $\xrightarrow{\text{tex}}$ “A#1.”]

[Ax $\xrightarrow{\text{pyk}}$ “unicode capital a *”]

B*

[Bx $\xrightarrow{\text{tex}}$ “B#1.”]

[Bx $\xrightarrow{\text{pyk}}$ “unicode capital b *”]

C*

[Cx $\xrightarrow{\text{tex}}$ “C#1.”]

[Cx $\xrightarrow{\text{pyk}}$ “unicode capital c *”]

D*

[Dx $\xrightarrow{\text{tex}}$ “D#1.”]

[Dx $\xrightarrow{\text{pyk}}$ “unicode capital d *”]

E*

[Ex $\xrightarrow{\text{tex}}$ “E#1.”]

[Ex $\xrightarrow{\text{pyk}}$ “unicode capital e *”]

F*

[Fx $\xrightarrow{\text{tex}}$ “F#1.”]

[Fx $\xrightarrow{\text{pyk}}$ “unicode capital f *”]

G*

[Gx $\xrightarrow{\text{tex}}$ “G#1.”]

[Gx $\xrightarrow{\text{pyk}}$ “unicode capital g *”]

H*

[Hx $\xrightarrow{\text{tex}}$ “H#1.”]

[Hx $\xrightarrow{\text{pyk}}$ “unicode capital h *”]

I*

[Ix $\xrightarrow{\text{tex}}$ “I#1.”]

[Ix $\xrightarrow{\text{pyk}}$ “unicode capital i *”]

J*

[Jx $\xrightarrow{\text{tex}}$ “J#1.”]

[Jx $\xrightarrow{\text{pyk}}$ “unicode capital j *”]

K*

[Kx $\xrightarrow{\text{tex}}$ “K#1.”]

[Kx $\xrightarrow{\text{pyk}}$ “unicode capital k *”]

L*

[Lx $\xrightarrow{\text{tex}}$ “L#1.”]

[Lx $\xrightarrow{\text{pyk}}$ “unicode capital l *”]

M*

[Mx $\xrightarrow{\text{tex}}$ “M#1.”]

[Mx $\xrightarrow{\text{pyk}}$ “unicode capital m *”]

N*

[Nx $\xrightarrow{\text{tex}}$ “N#1.”]

[Nx $\xrightarrow{\text{pyk}}$ “unicode capital n *”]

O*

[Ox $\xrightarrow{\text{tex}}$ “O#1.”]

[Ox $\xrightarrow{\text{pyk}}$ “unicode capital o *”]

P*

[Px $\xrightarrow{\text{tex}}$ “P#1.”]

[Px $\xrightarrow{\text{pyk}}$ “unicode capital p *”]

Q*

[Qx $\xrightarrow{\text{tex}}$ “Q#1.”]

[Qx $\xrightarrow{\text{pyk}}$ “unicode capital q *”]

R*

[Rx $\xrightarrow{\text{tex}}$ “R#1.”]

[Rx $\xrightarrow{\text{pyk}}$ “unicode capital r *”]

S*

[Sx $\xrightarrow{\text{tex}}$ “S#1.”]

[Sx $\xrightarrow{\text{pyk}}$ “unicode capital s *”]

T*

[Tx $\xrightarrow{\text{tex}}$ “T#1.”]

[Tx $\xrightarrow{\text{pyk}}$ “unicode capital t *”]

U*

[Ux $\xrightarrow{\text{tex}}$ “U#1.”]

[Ux $\xrightarrow{\text{pyk}}$ “unicode capital u *”]

V*

[Vx $\xrightarrow{\text{tex}}$ “V#1.”]

[Vx $\xrightarrow{\text{pyk}}$ “unicode capital v *”]

W*

[Wx $\xrightarrow{\text{tex}}$ “W#1.”]

[Wx $\xrightarrow{\text{pyk}}$ “unicode capital w *”]

X*

[Xx $\xrightarrow{\text{tex}}$ “X#1.”]

[Xx $\xrightarrow{\text{pyk}}$ “unicode capital x *”]

Y*

[Yx $\xrightarrow{\text{tex}}$ “Y#1.”]

[Yx $\xrightarrow{\text{pyk}}$ “unicode capital y *”]

Z*

[Zx $\xrightarrow{\text{tex}}$ “Z#1.”]

[Zx $\xrightarrow{\text{pyk}}$ “unicode capital z *”]

[*

[[x $\xrightarrow{\text{tex}}$ “[#1.”]

[[x $\xrightarrow{\text{pyk}}$ “unicode left bracket *”]

*

[$\backslash x \xrightarrow{\text{name}}$ “
\\mbox {\$\\backslash\$}#1.”]

[$\backslash x \xrightarrow{\text{tex}}$ “\\#1.”]

[$\backslash x \xrightarrow{\text{pyk}}$ “unicode backslash *”]

]*

[]x $\xrightarrow{\text{tex}}$ “[#1.”]

[]x $\xrightarrow{\text{pyk}}$ “unicode right bracket *”]

^*

[$\wedge x \xrightarrow{\text{name}}$ “
{\\char94}#1.”]

[$\wedge x \xrightarrow{\text{tex}}$ “^#1.”]

[$\wedge x \xrightarrow{\text{pyk}}$ “unicode circumflex *”]

_*

[_x $\xrightarrow{\text{name}}$ “
\\-#1.”]

[_x $\xrightarrow{\text{tex}}$ “-_1.”]

[_x $\xrightarrow{\text{pyk}}$ “unicode underscore *”]

‘*

[‘x ^{name} → “\mbox {\#1.”]
[‘x ^{tex} → “\#1.”]
[‘x ^{pyk} → “unicode grave accent *”]

a*

[ax ^{tex} → “a\#1.”]
[ax ^{pyk} → “unicode small a *”]

b*

[bx ^{tex} → “b\#1.”]
[bx ^{pyk} → “unicode small b *”]

c*

[cx ^{tex} → “c\#1.”]
[cx ^{pyk} → “unicode small c *”]

d*

[dx ^{tex} → “d\#1.”]
[dx ^{pyk} → “unicode small d *”]

e*

[ex ^{tex} → “e\#1.”]
[ex ^{pyk} → “unicode small e *”]

f*

[fx $\xrightarrow{\text{tex}}$ “f#1.”]

[fx $\xrightarrow{\text{pyk}}$ “unicode small f *”]

g*

[gx $\xrightarrow{\text{tex}}$ “g#1.”]

[gx $\xrightarrow{\text{pyk}}$ “unicode small g *”]

h*

[hx $\xrightarrow{\text{tex}}$ “h#1.”]

[hx $\xrightarrow{\text{pyk}}$ “unicode small h *”]

i*

[ix $\xrightarrow{\text{tex}}$ “i#1.”]

[ix $\xrightarrow{\text{pyk}}$ “unicode small i *”]

j*

[jx $\xrightarrow{\text{tex}}$ “j#1.”]

[jx $\xrightarrow{\text{pyk}}$ “unicode small j *”]

k*

[kx $\xrightarrow{\text{tex}}$ “k#1.”]

[kx $\xrightarrow{\text{pyk}}$ “unicode small k *”]

l*

[lx $\xrightarrow{\text{tex}}$ “l#1.”]

[lx $\xrightarrow{\text{pyk}}$ “unicode small l *”]

m*

[mx $\xrightarrow{\text{tex}}$ “m#1.”]

[mx $\xrightarrow{\text{pyk}}$ “unicode small m *”]

n*

[nx $\xrightarrow{\text{tex}}$ “n#1.”]

[nx $\xrightarrow{\text{pyk}}$ “unicode small n *”]

O*

[ox $\xrightarrow{\text{tex}}$ “o#1.”]

[ox $\xrightarrow{\text{pyk}}$ “unicode small o *”]

p*

[px $\xrightarrow{\text{tex}}$ “p#1.”]

[px $\xrightarrow{\text{pyk}}$ “unicode small p *”]

q*

[qx $\xrightarrow{\text{tex}}$ “q#1.”]

[qx $\xrightarrow{\text{pyk}}$ “unicode small q *”]

r*

[rx $\xrightarrow{\text{tex}}$ “r#1.”]

[rx $\xrightarrow{\text{pyk}}$ “unicode small r *”]

S*

[sx $\xrightarrow{\text{tex}}$ “s#1.”]

[sx $\xrightarrow{\text{pyk}}$ “unicode small s *”]

t*

[tx $\xrightarrow{\text{tex}}$ “t#1.”]

[tx $\xrightarrow{\text{pyk}}$ “unicode small t *”]

u*

[ux $\xrightarrow{\text{tex}}$ “u#1.”]

[ux $\xrightarrow{\text{pyk}}$ “unicode small u *”]

v*

[vx $\xrightarrow{\text{tex}}$ “v#1.”]

[vx $\xrightarrow{\text{pyk}}$ “unicode small v *”]

w*

[wx $\xrightarrow{\text{tex}}$ “w#1.”]

[wx $\xrightarrow{\text{pyk}}$ “unicode small w *”]

x*

[xx $\xrightarrow{\text{tex}}$ “x#1.”]

[xx $\xrightarrow{\text{pyk}}$ “unicode small x *”]

y*

[yx $\xrightarrow{\text{tex}}$ “y#1.”]

[yx $\xrightarrow{\text{pyk}}$ “unicode small y *”]

Z*

[zx $\xrightarrow{\text{tex}}$ “z#1.”]

[zx $\xrightarrow{\text{pyk}}$ “unicode small z *”]

{*

[{x $\xrightarrow{\text{name}}$ “
\\{#1.”]

[{x $\xrightarrow{\text{tex}}$ “{#1.”]

[{x $\xrightarrow{\text{pyk}}$ “unicode left brace *”]

|*

[|x $\xrightarrow{\text{tex}}$ “|#1.”]

[|x $\xrightarrow{\text{pyk}}$ “unicode vertical line *”]

}*

[}x $\xrightarrow{\text{name}}$ “
\\}#1.”]

[}x $\xrightarrow{\text{tex}}$ “}#1.”]

[}x $\xrightarrow{\text{pyk}}$ “unicode right brace *”]

~*

[~x $\xrightarrow{\text{name}}$ “
\\char126 #1.”]

[~x $\xrightarrow{\text{tex}}$ “~#1.”]

[~x $\xrightarrow{\text{pyk}}$ “unicode tilde *”]

\perp

$[\perp \xrightarrow{\text{val}} (\lambda x. [x, x])^I, (\lambda x. [x, x])^I]$
 $[\perp \xrightarrow{\text{tex}} “\backslash\text{bot}”]$
 $[\perp \xrightarrow{\text{pyk}} “\text{bottom}”]$

$f(*)$

$[f(x) \xrightarrow{\text{val}} \text{if}(x, T, f(x, T))]$
 $[f(*) \xrightarrow{\text{tex}} “f(\#1.)”]$
 $[f(*) \xrightarrow{\text{pyk}} “\text{function } f \text{ of } * \text{ end function}”]$

$(*)^I$

$[(x)^I \xrightarrow{\text{val}} x]$
 $[(*)^I \xrightarrow{\text{tex}} “(\#1.)\{\}^{\{I\}}”]$
 $[(*)^I \xrightarrow{\text{pyk}} “\text{identity } * \text{ end identity}”]$

F

$[F \xrightarrow{\text{val}} T :: T]$
 $[F \xrightarrow{\text{tex}} “\backslash\text{mathsf}\{F\}”]$
 $[F \xrightarrow{\text{pyk}} “\text{false}”]$

0

$[0 \xrightarrow{\text{val}} T]$

$[0 \xrightarrow{\text{tex}} \text{``}\underline{\text{underline }} \{0\}\text{''}]$

$[0 \xrightarrow{\text{pyk}} \text{``untagged zero''}]$

1

$[1 \xrightarrow{\text{val}} F \underline{+2*} 0]$

$[1 \xrightarrow{\text{tex}} \text{``}\underline{\text{underline }} \{1\}\text{''}]$

$[1 \xrightarrow{\text{pyk}} \text{``untagged one''}]$

2

$[2 \xrightarrow{\text{val}} T \underline{+2*} 1]$

$[2 \xrightarrow{\text{tex}} \text{``}\underline{\text{underline }} \{2\}\text{''}]$

$[2 \xrightarrow{\text{pyk}} \text{``untagged two''}]$

3

$[3 \xrightarrow{\text{val}} F \underline{+2*} 1]$

$[3 \xrightarrow{\text{tex}} \text{``}\underline{\text{underline }} \{3\}\text{''}]$

$[3 \xrightarrow{\text{pyk}} \text{``untagged three''}]$

4

$[4 \xrightarrow{\text{val}} T \underline{+2*} 2]$

$[4 \xrightarrow{\text{tex}} \text{``}\underline{\text{underline }} \{4\}\text{''}]$

$[4 \xrightarrow{\text{pyk}} \text{``untagged four''}]$

5

[$\underline{5} \xrightarrow{\text{val}} F \underline{+2*} \underline{2}$]
[$\underline{5} \xrightarrow{\text{tex}} "$
 $\backslash\text{underline }\{5\}"$]
[$\underline{5} \xrightarrow{\text{pyk}} \text{"untagged five"}$]

6

[$\underline{6} \xrightarrow{\text{val}} T \underline{+2*} \underline{3}$]
[$\underline{6} \xrightarrow{\text{tex}} "$
 $\backslash\text{underline }\{6\}"$]
[$\underline{6} \xrightarrow{\text{pyk}} \text{"untagged six"}$]

7

[$\underline{7} \xrightarrow{\text{val}} F \underline{+2*} \underline{3}$]
[$\underline{7} \xrightarrow{\text{tex}} "$
 $\backslash\text{underline }\{7\}"$]
[$\underline{7} \xrightarrow{\text{pyk}} \text{"untagged seven"}$]

8

[$\underline{8} \xrightarrow{\text{val}} T \underline{+2*} \underline{4}$]
[$\underline{8} \xrightarrow{\text{tex}} "$
 $\backslash\text{underline }\{8\}"$]
[$\underline{8} \xrightarrow{\text{pyk}} \text{"untagged eight"}$]

9

[$\underline{9} \xrightarrow{\text{val}} F \underline{+2*} \underline{4}$]
[$\underline{9} \xrightarrow{\text{tex}} "$
 $\backslash\text{underline }\{9\}"$]

$[9 \xrightarrow{\text{pyk}} \text{“untagged nine”}]$

0

$[0 \xrightarrow{\text{val}} T \therefore T]$

$[0 \xrightarrow{\text{tex}} \text{“} 0 \text{”}]$

$[0 \xrightarrow{\text{pyk}} \text{“zero”}]$

1

$[1 \xrightarrow{\text{val}} F + 2 * 0]$

$[1 \xrightarrow{\text{tex}} \text{“} 1 \text{”}]$

$[1 \xrightarrow{\text{pyk}} \text{“one”}]$

2

$[2 \xrightarrow{\text{val}} T + 2 * 1]$

$[2 \xrightarrow{\text{tex}} \text{“} 2 \text{”}]$

$[2 \xrightarrow{\text{pyk}} \text{“two”}]$

3

$[3 \xrightarrow{\text{val}} F + 2 * 1]$

$[3 \xrightarrow{\text{tex}} \text{“} 3 \text{”}]$

$[3 \xrightarrow{\text{pyk}} \text{“three”}]$

4

$[4 \xrightarrow{\text{val}} T + 2 * 2]$

[4 $\xrightarrow{\text{tex}}$ “
4”]

[4 $\xrightarrow{\text{pyk}}$ “four”]

5

[5 $\xrightarrow{\text{val}}$ F +2* 2]

[5 $\xrightarrow{\text{tex}}$ “
5”]

[5 $\xrightarrow{\text{pyk}}$ “five”]

6

[6 $\xrightarrow{\text{val}}$ T +2* 3]

[6 $\xrightarrow{\text{tex}}$ “
6”]

[6 $\xrightarrow{\text{pyk}}$ “six”]

7

[7 $\xrightarrow{\text{val}}$ F +2* 3]

[7 $\xrightarrow{\text{tex}}$ “
7”]

[7 $\xrightarrow{\text{pyk}}$ “seven”]

8

[8 $\xrightarrow{\text{val}}$ T +2* 4]

[8 $\xrightarrow{\text{tex}}$ “
8”]

[8 $\xrightarrow{\text{pyk}}$ “eight”]

9

[9 $\xrightarrow{\text{val}}$ F +2* 4]

[9 $\xrightarrow{\text{tex}}$ “
9”]

[9 $\xrightarrow{\text{pyk}}$ “nine”]

a

[a $\xrightarrow{\text{tex}}$ “
\mathsf{a}”]

[a $\xrightarrow{\text{pyk}}$ “var a”]

b

[b $\xrightarrow{\text{tex}}$ “
\mathsf{b}”]

[b $\xrightarrow{\text{pyk}}$ “var b”]

c

[c $\xrightarrow{\text{tex}}$ “
\mathsf{c}”]

[c $\xrightarrow{\text{pyk}}$ “var c”]

d

[d $\xrightarrow{\text{tex}}$ “
\mathsf{d}”]

[d $\xrightarrow{\text{pyk}}$ “var d”]

e

[e $\xrightarrow{\text{tex}}$ “
\mathsf{e}”]

[$e \xrightarrow{\text{pyk}}$ “var e”]

f

[$f \xrightarrow{\text{tex}}$ “
\\mathsf {f}”]

[$f \xrightarrow{\text{pyk}}$ “var f”]

g

[$g \xrightarrow{\text{tex}}$ “
\\mathsf {g}”]

[$g \xrightarrow{\text{pyk}}$ “var g”]

h

[$h \xrightarrow{\text{tex}}$ “
\\mathsf {h}”]

[$h \xrightarrow{\text{pyk}}$ “var h”]

i

[$i \xrightarrow{\text{tex}}$ “
\\mathsf {i}”]

[$i \xrightarrow{\text{pyk}}$ “var i”]

j

[$j \xrightarrow{\text{tex}}$ “
\\mathsf {j}”]

[$j \xrightarrow{\text{pyk}}$ “var j”]

k

[$k \xrightarrow{\text{tex}} \text{“}\backslash\text{mathsf }\{k\}\text{”}$]
[$k \xrightarrow{\text{pyk}} \text{“var k”}$]

l

[$l \xrightarrow{\text{tex}} \text{“}\backslash\text{mathsf }\{l\}\text{”}$]
[$l \xrightarrow{\text{pyk}} \text{“var l”}$]

m

[$m \xrightarrow{\text{tex}} \text{“}\backslash\text{mathsf }\{m\}\text{”}$]
[$m \xrightarrow{\text{pyk}} \text{“var m”}$]

n

[$n \xrightarrow{\text{tex}} \text{“}\backslash\text{mathsf }\{n\}\text{”}$]
[$n \xrightarrow{\text{pyk}} \text{“var n”}$]

o

[$o \xrightarrow{\text{tex}} \text{“}\backslash\text{mathsf }\{o\}\text{”}$]
[$o \xrightarrow{\text{pyk}} \text{“var o”}$]

p

[$p \xrightarrow{\text{tex}} \text{“}\backslash\text{mathsf }\{p\}\text{”}$]
[$p \xrightarrow{\text{pyk}} \text{“var p”}$]

q

$[q \xrightarrow{\text{tex}} “\backslashmathsf{q}”]$
 $[q \xrightarrow{\text{pyk}} “\text{var q}”]$

r

$[r \xrightarrow{\text{tex}} “\backslashmathsf{r}”]$
 $[r \xrightarrow{\text{pyk}} “\text{var r}”]$

s

$[s \xrightarrow{\text{tex}} “\backslashmathsf{s}”]$
 $[s \xrightarrow{\text{pyk}} “\text{var s}”]$

t

$[t \xrightarrow{\text{tex}} “\backslashmathsf{t}”]$
 $[t \xrightarrow{\text{pyk}} “\text{var t}”]$

u

$[u \xrightarrow{\text{tex}} “\backslashmathsf{u}”]$
 $[u \xrightarrow{\text{pyk}} “\text{var u}”]$

v

$[v \xrightarrow{\text{tex}} “\backslashmathsf{v}”]$
 $[v \xrightarrow{\text{pyk}} “\text{var v}”]$

w

[w $\xrightarrow{\text{tex}}$ “\\mathsf {w}”]
[w $\xrightarrow{\text{pyk}}$ “var w”]

(*)^M

[(x)^M $\xrightarrow{\text{val}}$ x^M]
[(*)^M $\xrightarrow{\text{tex}}$ “#1.
)^M”]
[(*)^M $\xrightarrow{\text{pyk}}$ “tagged parenthesis * end tagged”]

If(*, *, *)

[If(x, y, z) $\xrightarrow{\text{val}}$ if(x^M, y^M, z^M)
[If(*, *, *) $\xrightarrow{\text{tex}}$ “\\mathrm {If}(\#1.
, \\linebreak [0]\#2.
, \\linebreak [0]\#3.
)”]
[If(*, *, *) $\xrightarrow{\text{pyk}}$ “tagged if * then * else * end if”]

array{*} * end array

[array{*} * end array $\xrightarrow{\text{name}}$ “\\mathrm {array}\{\#1.
\}\#2.
\\mathrm {end}\\ array}”]
[array{*} * end array $\xrightarrow{\text{tex}}$ “
\\begin {array}\{\#1.
\}\#2.
\\end {array}”]
[array{*} * end array $\xrightarrow{\text{pyk}}$ “array * is * end array”]

l

[$l \xrightarrow{\text{tex}} "l"$]
[$l \xrightarrow{\text{pyk}} \text{"left"}$]

c

[$c \xrightarrow{\text{tex}} "c"$]
[$c \xrightarrow{\text{pyk}} \text{"center"}$]

r

[$r \xrightarrow{\text{tex}} "r"$]
[$r \xrightarrow{\text{pyk}} \text{"right"}$]

empty

[$\text{empty} \xrightarrow{\text{name}} "\backslash\mathrm{empty}"$]
[$\text{empty} \xrightarrow{\text{tex}} "$]
[$\text{empty} \xrightarrow{\text{pyk}} \text{"empty"}$]

$\langle * | * := * \rangle$

[$\langle * | * := * \rangle \xrightarrow{\text{tex}} "\backslash\langle \#1.$
 $\backslash, \{ \} \#2.$
 $\{ := \} \backslash, \#3.$
 $\backslash\rangle"$]
[$\langle * | * := * \rangle \xrightarrow{\text{pyk}} \text{"substitute * set * to * end substitute"}$]

$\mathcal{M}(*)$

$[\mathcal{M}(x) \xrightarrow{\text{val}} (0 \mathrel{\dot{::}} [\mathbf{1} \mathrel{\dot{::}} T])^I \mathrel{\dot{::}} x]$

$[\mathcal{M}(*) \xrightarrow{\text{tex}} ``\{\backslash\text{cal M}\}(\#1.)'']$

$[\mathcal{M}(*) \xrightarrow{\text{pyk}} \text{“map tag * end tag”}]$

$\tilde{\mathcal{U}}(*)$

$[\tilde{\mathcal{U}}(x) \xrightarrow{\text{val}} \text{if}(x^d, x, x^T)]$

$[\tilde{\mathcal{U}}(*) \xrightarrow{\text{tex}} ``\backslash\text{tilde }\{\{\backslash\text{cal U}\}\}(\#1.)'']$

$[\tilde{\mathcal{U}}(*) \xrightarrow{\text{pyk}} \text{“raw map untag * end untag”}]$

$\mathcal{U}(*)$

$[\mathcal{U}(x) \xrightarrow{\text{val}} \tilde{\mathcal{U}}(x^M)]$

$[\mathcal{U}(*) \xrightarrow{\text{tex}} ``\{\backslash\text{cal U}\}(\#1.)'']$

$[\mathcal{U}(*) \xrightarrow{\text{pyk}} \text{“map untag * end untag”}]$

$\mathcal{U}^M(*)$

$[\mathcal{U}^M(x) \xrightarrow{\text{val}} \mathcal{U}(x)^M]$

$[\mathcal{U}^M(*) \xrightarrow{\text{tex}} ``\{\backslash\text{cal U}\}^M(\#1.)'']$

$[\mathcal{U}^M(*) \xrightarrow{\text{pyk}} \text{“normalizing untag * end untag”}]$

apply(*, *)

[**apply**(f, x) $\xrightarrow{\text{val}}$ **apply**₁(f^M, x^M)]

[**apply**(* , *) $\xrightarrow{\text{tex}}$ “
\mathbf{apply} \{apply\}(\#1.
,\#2.
)”]

[**apply**(* , *) $\xrightarrow{\text{pyk}}$ “apply * to * end apply”]

apply₁(* , *)

[**apply**₁(f, x) $\xrightarrow{\text{val}}$ f^d { If(x^d, f, f)
If(x^d, M(f^T, x), M(f^T, (x^T)^I)) }]

[**apply**₁(* , *) $\xrightarrow{\text{tex}}$ “
\mathbf{apply}\{apply\}_1(\#1.
,\#2.
)”]

[**apply**₁(* , *) $\xrightarrow{\text{pyk}}$ “apply one * to * end apply”]

identifier(*)

[identifier(x) $\xrightarrow{\text{val}}$ If(x^t, 0, identifier₁(xⁱ, identifier(x¹)))]

[identifier(x) $\xrightarrow{\text{tex}}$ “
identifier(#1.
)”]

[identifier(x) $\xrightarrow{\text{pyk}}$ “identifier * end identifier”]

identifier₁(* , *)

[identifier₁(x, y) $\xrightarrow{\text{val}}$ If(x⁶, y, x⁰ + 2* [x¹ + 2* [x² + 2* [x³ + 2* [x⁴ + 2* [x⁵ + 2* [F + 2* [T + 2* y]]]]]])]

[identifier₁(x, y) $\xrightarrow{\text{tex}}$ “
identifier_{1}(\#1.
,\#2.
)”]

[identifier₁(x, y) $\xrightarrow{\text{pyk}}$ “identifier one * plus id * end identifier”]

array-plus(*, *)

[array-plus(x, y) $\xrightarrow{\text{val}}$ x^a {

y^a	T
y^{hc}	y
x^{hc}	$x :: y$
$x :: y$	

}]

[array-plus(x, y) $\xrightarrow{\text{tex}}$ “
array\mbox{-}\linebreak[0]plus(#1.
, #2.
)”]

[array-plus(x, y) $\xrightarrow{\text{pyk}}$ “array plus * and * end plus”]

array-remove(*, *, *)

[array-remove(i, a, l) $\xrightarrow{\text{val}}$ [[
i] ! [! [a^a]]]

T	{	$a^h \approx i$	T
a^{hc}		a	array-plus(array-remove(i, a ^h , l + 1), a ^t)
bit(l, i)	array-plus(a ^h , array-remove(i, a ^t , l + 1))]	

[array-remove(i, a, l) $\xrightarrow{\text{tex}}$ “
array\mbox{-}\linebreak[0]remove(#1.
, #2.
, #3.
)”]

[array-remove(i, a, l) $\xrightarrow{\text{pyk}}$ “array remove * array * level * end remove”]

array-put(*, *, *, *)

[array-put(i, v, a, l) $\xrightarrow{\text{val}}$ [[
 i :: v
] ! [a^a]]] $\left\{ \begin{array}{l} a^h \approx i \\ a^h c \end{array} \right\} \left\{ \begin{array}{l} i :: v \\ \text{array-add}(i, v, a^h, a^t, l) \\ \text{bit}(l, i) \left\{ \begin{array}{l} \text{array-put}(i, v, a^h, l + 1) :: [a^t] \\ a^n :: \text{array-put}(i, v, a^t, l + 1) \end{array} \right. \end{array} \right.$]
[array-put(i, v, a, l) $\xrightarrow{\text{tex}}$ “
array\mbox{-}\linebreak[0]put(#1.
, #2.
, #3.
, #4.
)”]
[array-put(i, v, a, l) $\xrightarrow{\text{pyk}}$ “array put * value * array * level * end put”]

array-add(*, *, *, *, *)

[array-add(i, v, i', v', l) $\xrightarrow{\text{val}}$ [
 bit(l, i') $\left\{ \begin{array}{l} \text{array-add}(i, v, i', v', l + 1) :: T \\ (i :: v)^M :: (i' :: [v'])^M \end{array} \right. \right.$]
 bit(l, i') $\left\{ \begin{array}{l} (i' :: [v'])^M :: (i :: v)^M \\ T :: \text{array-add}(i, v, i', v', l + 1) \end{array} \right. \right]$

[array-add(i, v, i', v', l) $\xrightarrow{\text{tex}}$ “
array\mbox{-}\linebreak[0]add(#1.
, #2.
, #3.
, #4.
, #5.
)”]
[array-add(i, v, i', v', l) $\xrightarrow{\text{pyk}}$ “array add * value * index * value * level * end add”]

bit(*, *)

[bit(x, y) $\xrightarrow{\text{val}}$ If(x^c \wedge [y^c], bit₁(x, y), T)]
[bit(x, y) $\xrightarrow{\text{tex}}$ “
bit(#1.
, #2.
)”]

[bit(x,y) $\xrightarrow{\text{pyk}}$ “bit * of * end bit”]

bit₁(*，“*)

[bit₁(x,y) $\xrightarrow{\text{val}}$ If(x^s,y^h,bit(x - 1,y^t))]

[bit₁(x,y) $\xrightarrow{\text{tex}}$ “
bit₁(#1.
, #2.
)”]

[bit₁(x,y) $\xrightarrow{\text{pyk}}$ “bit one * of * end bit”]

rack

[rack $\xrightarrow{\text{val}}$ base[base[0]]]

[rack $\xrightarrow{\text{tex}}$ “
rack”]

[rack $\xrightarrow{\text{pyk}}$ “example rack”]

"vector"

["vector" $\xrightarrow{\text{val}}$ identifier(["vector"])]

["vector" $\xrightarrow{\text{tex}}$ “
\mbox {\tt \char34}\mathrm {vector}\mbox {\tt \char34}”]

["vector" $\xrightarrow{\text{pyk}}$ “vector hook”]

"bibliography"

["bibliography" $\xrightarrow{\text{val}}$ identifier(["bibliography"])]

["bibliography" $\xrightarrow{\text{tex}}$ “
\mbox {\tt \char34}\mathrm {bibliography}\mbox {\tt \char34}”]

["bibliography" $\xrightarrow{\text{pyk}}$ “bibliography hook”]

"dictionary"

```
[ "dictionary"  $\xrightarrow{\text{val}}$  identifier( [ "dictionary" ] ) ]  
[ "dictionary"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {dictionary}\mbox {\tt \char34}" ]  
[ "dictionary"  $\xrightarrow{\text{pyk}}$  "dictionary hook" ]
```

"body"

```
[ "body"  $\xrightarrow{\text{val}}$  identifier( [ "body" ] ) ]  
[ "body"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {body}\mbox {\tt \char34}" ]  
[ "body"  $\xrightarrow{\text{pyk}}$  "body hook" ]
```

"codex"

```
[ "codex"  $\xrightarrow{\text{val}}$  identifier( [ "codex" ] ) ]  
[ "codex"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {codex}\mbox {\tt \char34}" ]  
[ "codex"  $\xrightarrow{\text{pyk}}$  "codex hook" ]
```

"expansion"

```
[ "expansion"  $\xrightarrow{\text{val}}$  identifier( [ "expansion" ] ) ]  
[ "expansion"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {expansion}\mbox {\tt \char34}" ]  
[ "expansion"  $\xrightarrow{\text{pyk}}$  "expansion hook" ]
```

"code"

```
[ "code"  $\xrightarrow{\text{val}}$  identifier( [ "code" ] ) ]  
[ "code"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {code}\mbox {\tt \char34}" ]
```

["code" $\xrightarrow{\text{pyk}}$ "code hook"]

"cache"

["cache" $\xrightarrow{\text{val}}$ identifier(["cache"])]

["cache" $\xrightarrow{\text{tex}}$ "
\mbox {\tt \char34}\mathrm {cache}\mbox {\tt \char34}"]

["cache" $\xrightarrow{\text{pyk}}$ "cache hook"]

"diagnose"

["diagnose" $\xrightarrow{\text{val}}$ identifier(["diagnose"])]

["diagnose" $\xrightarrow{\text{tex}}$ "
\mbox {\tt \char34}\mathrm {diagnose}\mbox {\tt \char34}"]

["diagnose" $\xrightarrow{\text{pyk}}$ "diagnose hook"]

"pyk"

["pyk" $\xrightarrow{\text{val}}$ identifier(["pyk"])]

["pyk" $\xrightarrow{\text{tex}}$ "
\mbox {\tt \char34}\mathrm {pyk}\mbox {\tt \char34}"]

["pyk" $\xrightarrow{\text{pyk}}$ "pyk aspect"]

"tex"

["tex" $\xrightarrow{\text{val}}$ identifier(["tex"])]

["tex" $\xrightarrow{\text{tex}}$ "
\mbox {\tt \char34}\mathrm {tex}\mbox {\tt \char34}"]

["tex" $\xrightarrow{\text{pyk}}$ "tex aspect"]

"texname"

["texname" $\xrightarrow{\text{val}}$ identifier(["texname"])]

```
["texname"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {texname}\mbox {\tt \char34}"]  
["texname"  $\xrightarrow{\text{pyk}}$  "texname aspect"]
```

"value"

```
["value"  $\xrightarrow{\text{val}}$  identifier(["value"])]  
["value"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {value}\mbox {\tt \char34}"]  
["value"  $\xrightarrow{\text{pyk}}$  "value aspect"]
```

"message"

```
["message"  $\xrightarrow{\text{val}}$  identifier(["message"])]  
["message"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {message}\mbox {\tt \char34}"]  
["message"  $\xrightarrow{\text{pyk}}$  "message aspect"]
```

"macro"

```
["macro"  $\xrightarrow{\text{val}}$  identifier(["macro"])]  
["macro"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {macro}\mbox {\tt \char34}"]  
["macro"  $\xrightarrow{\text{pyk}}$  "macro aspect"]
```

"definition"

```
["definition"  $\xrightarrow{\text{val}}$  identifier(["definition"])]  
["definition"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {definition}\mbox {\tt \char34}"]  
["definition"  $\xrightarrow{\text{pyk}}$  "definition aspect"]
```

"unpack"

```
[ "unpack"  $\xrightarrow{\text{val}}$  identifier( [ "unpack" ] ) ]  
[ "unpack"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {unpack}\mbox {\tt \char34}" ]  
[ "unpack"  $\xrightarrow{\text{pyk}}$  "unpack aspect" ]
```

"claim"

```
[ "claim"  $\xrightarrow{\text{val}}$  identifier( [ "claim" ] ) ]  
[ "claim"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {claim}\mbox {\tt \char34}" ]  
[ "claim"  $\xrightarrow{\text{pyk}}$  "claim aspect" ]
```

"priority"

```
[ "priority"  $\xrightarrow{\text{val}}$  identifier( [ "priority" ] ) ]  
[ "priority"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {priority}\mbox {\tt \char34}" ]  
[ "priority"  $\xrightarrow{\text{pyk}}$  "priority aspect" ]
```

"lambda"

```
[ "lambda"  $\xrightarrow{\text{val}}$  identifier( [ "lambda" ] ) ]  
[ "lambda"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {\lambda}\mbox {\tt \char34}" ]  
[ "lambda"  $\xrightarrow{\text{pyk}}$  "lambda identifier" ]
```

"apply"

```
[ "apply"  $\xrightarrow{\text{val}}$  identifier( [ "apply" ] ) ]  
[ "apply"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {apply}\mbox {\tt \char34}" ]
```

["apply" $\xrightarrow{\text{pyk}}$ “apply identifier”]

"true"

["true" $\xrightarrow{\text{val}}$ identifier(["“true”"])]

["true" $\xrightarrow{\text{tex}}$ “
\\mbox {\\tt ‘char34}mathrm {true}\\mbox {\\tt ‘char34}”]
["true" $\xrightarrow{\text{pyk}}$ “true identifier”]

"if"

["if" $\xrightarrow{\text{val}}$ identifier(["“if”"])]

["if" $\xrightarrow{\text{tex}}$ “
\\mbox {\\tt ‘char34}mathrm {if}\\mbox {\\tt ‘char34}”]
["if" $\xrightarrow{\text{pyk}}$ “if identifier”]

"quote"

["quote" $\xrightarrow{\text{val}}$ identifier(["“quote”"])]

["quote" $\xrightarrow{\text{tex}}$ “
\\mbox {\\tt ‘char34}mathrm {quote}\\mbox {\\tt ‘char34}”]
["quote" $\xrightarrow{\text{pyk}}$ “quote identifier”]

"proclaim"

["proclaim" $\xrightarrow{\text{val}}$ identifier(["“proclaim”"])]

["proclaim" $\xrightarrow{\text{tex}}$ “
\\mbox {\\tt ‘char34}mathrm {proclaim}\\mbox {\\tt ‘char34}”]
["proclaim" $\xrightarrow{\text{pyk}}$ “proclaim identifier”]

"define"

["define" $\xrightarrow{\text{val}}$ identifier(["“define”"])]

```
["define"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {define}\mbox {\tt \char34}"]  
["define"  $\xrightarrow{\text{pyk}}$  "define identifier"]
```

"introduce"

```
["introduce"  $\xrightarrow{\text{val}}$  identifier([“introduce”])]  
["introduce"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {introduce}\mbox {\tt \char34}"]  
["introduce"  $\xrightarrow{\text{pyk}}$  “introduce identifier”]
```

"hide"

```
["hide"  $\xrightarrow{\text{val}}$  identifier([“hide”])]  
["hide"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {hide}\mbox {\tt \char34}"]  
["hide"  $\xrightarrow{\text{pyk}}$  “hide identifier”]
```

"pre"

```
["pre"  $\xrightarrow{\text{val}}$  identifier([“pre”])]  
["pre"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {pre}\mbox {\tt \char34}"]  
["pre"  $\xrightarrow{\text{pyk}}$  “pre identifier”]
```

"post"

```
["post"  $\xrightarrow{\text{val}}$  identifier([“post”])]  
["post"  $\xrightarrow{\text{tex}}$  "  
\mbox {\tt \char34}\mathrm {post}\mbox {\tt \char34}"]  
["post"  $\xrightarrow{\text{pyk}}$  “post identifier”]
```

$\mathcal{E}(*, *, *)$

$[\mathcal{E}(t, s, c) \xrightarrow{\text{val}} \mathcal{E}_2(t, t^r, t^i, s, c)]$

$[\mathcal{E}(*, *, *) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal E}\}(\#1.$

$, \#2.$

$, \#3.$

$)"]$

$[\mathcal{E}(*, *, *) \xrightarrow{\text{pyk}} \text{"eval * stack * cache * end eval"}]$

$\mathcal{E}_2(*, *, *, *, *)$

$[\mathcal{E}_2(t, r, i, s, c) \xrightarrow{\text{val}} \mathfrak{f}^s \left\{ \begin{array}{l} t! [s!\text{If}(r \approx [c[0]], c, c[c[0]]["cache"][r])] \\ \mathcal{E}_3(t, c[r]["code"][i], s, c) \end{array} \right\}]$

$[\mathcal{E}_2(*, *, *, *, *) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal E}\}_2(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$, \#5.$

$)"]$

$[\mathcal{E}_2(*, *, *, *, *) \xrightarrow{\text{pyk}} \text{"eval two * ref * id * stack * cache * end eval"}]$

$\mathcal{E}_3(*, *, *, *)$

$[\mathcal{E}_3(t, f, s, c) \xrightarrow{\text{val}} \mathfrak{f}^c \left\{ \begin{array}{l} \mathfrak{f}^s \left\{ \begin{array}{l} \mathbf{abstract}(t^1, t^2, s, c) \\ c! [s! [t^1]] \end{array} \right\} \\ f \left\{ \begin{array}{l} \mathbf{cllookup}(t, s, T) \\ \mathcal{E}_4(f, t^t, s, c) \end{array} \right\} \end{array} \right\}]$

$[\mathcal{E}_3(*, *, *, *) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal E}\}_3(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$)"]$

$[\mathcal{E}_3(*, *, *, *) \xrightarrow{\text{pyk}} \text{"eval three * function * stack * cache * end eval"}]$

$\mathcal{E}_4(*, *, *, *)$

$[\mathcal{E}_4(f, a, s, c) \xrightarrow{\text{val}} \text{If}(a, s! [c!f], \mathcal{E}_4(\text{apply}(f, \mathcal{E}(a^h, s, c)), a^t, s, c))]$

$[\mathcal{E}_4(*, *, *, *) \xrightarrow{\text{tex}} "$

$\{\backslash\text{cal E}\}_4(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$)"]$

$[\mathcal{E}_4(*, *, *, *) \xrightarrow{\text{pyk}} \text{"eval four * arguments * stack * cache * end eval"}]$

$\text{lookup}(*, *, *)$

$[\text{lookup}(v, s, d) \xrightarrow{\text{val}} v! [d! \text{If}(s, d, \text{If}(v \stackrel{t}{=} [s^{hh}], s^{ht}, \text{lookup}(v, s^t, d)))]]$

$[\text{lookup}(*, *, *) \xrightarrow{\text{tex}} "$

$\backslash\text{mathbf }\{\text{lookup}\}(\#1.$

$, \#2.$

$, \#3.$

$)"]$

$[\text{lookup}(*, *, *) \xrightarrow{\text{pyk}} \text{"lookup * stack * default * end lookup"}]$

$\text{abstract}(*, *, *, *)$

$[\text{abstract}(v, t, s, c) \xrightarrow{\text{val}} v! [t! [s! [c! \Lambda x. \mathcal{E}(t, (v :: x)^M :: s, c)]]]]$

$[\text{abstract}(*, *, *, *) \xrightarrow{\text{tex}} "$

$\backslash\text{mathbf }\{\text{abstract}\}(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$)"]$

$[\text{abstract}(*, *, *, *) \xrightarrow{\text{pyk}} \text{"abstract * term * stack * cache * end abstract"}]$

$[\ast]$

Predef: quote

$[[\ast] \xrightarrow{\text{tex}} "$

\lceil \text{ceil} \#1.
\rceil \text{ceil }]

[[*] $\xrightarrow{\text{pyk}}$ “quote * end quote”]

$\mathcal{M}(*, *, *)$

$[\mathcal{M}(t, s, c) \xrightarrow{\text{val}} s! [c!\text{If}(t^{\text{is}}, t, \mathcal{M}_2(t, \text{aspect}("macro", t, c), s, c))]]$

$[\mathcal{M}(*, *, *) \xrightarrow{\text{tex}} "$
 $\{\text{\cal M}\}(\#1.$
 $,\#2.$
 $,\#3.$
)"]

$[\mathcal{M}(*, *, *) \xrightarrow{\text{pyk}} "expand * state * cache * end expand"]$

$\mathcal{M}_2(*, *, *, *)$

$[\mathcal{M}_2(t, d, s, c) \xrightarrow{\text{val}} d \left\{ \begin{array}{l} t^h :: \mathcal{M}^*(t^t, s, c) \\ \mathcal{U}^M([[\mathcal{E}(d^3, T, c) ' t] ' s] ' c) \end{array} \right\}]$

$[\mathcal{M}_2(*, *, *, *) \xrightarrow{\text{tex}} "$
 $\{\text{\cal M}\}_2(\#1.$
 $,\#2.$
 $,\#3.$
 $,\#4.$
)"]

$[\mathcal{M}_2(*, *, *, *) \xrightarrow{\text{pyk}} "expand two * definition * state * cache * end expand"]$

$\mathcal{M}^*(*, *, *)$

$[\mathcal{M}^*(a, s, c) \xrightarrow{\text{val}} s! [c!\text{If}(a, T, \mathcal{M}(a^h, s, c) :: \mathcal{M}^*(a^t, s, c))]]$

$[\mathcal{M}^*(*, *, *) \xrightarrow{\text{tex}} "$
 $\{\text{\cal M}\}^*(\#1.$
 $,\#2.$
 $,\#3.$
)"]

$[\mathcal{M}^*(*, *, *) \xrightarrow{\text{pyk}} "expand list * state * cache * end expand"]$

macro

Predef: macro

[macro $\xrightarrow{\text{tex}}$ “
\\mathrm {macro}”]
[macro $\xrightarrow{\text{pyk}}$ “macro”]

S₀

[S₀ $\xrightarrow{\text{val}}$ $\mathcal{M}(\lambda t.\lambda s.\lambda c.\mathcal{M}(t,s,c)) :: T$]

[S₀ $\xrightarrow{\text{tex}}$ “
s_0”]
[S₀ $\xrightarrow{\text{pyk}}$ “macro state”]

zip(* , *)

[zip(p, a) $\xrightarrow{\text{val}}$ a!If(p, T, (p^h :: [a^h])^M :: zip(p^t, a^t))]

[zip(* , *) $\xrightarrow{\text{tex}}$ “
\\mathbf {zip}(\#1.
,\#2.
)”]

[zip(* , *) $\xrightarrow{\text{pyk}}$ “zip * with * end zip”]

assoc₁(* , *, *)

[assoc₁(a, d, i) $\xrightarrow{\text{val}}$ a^a $\left\{ \begin{array}{l} d! [i!T] \\ a^{hc} \left\{ \begin{array}{l} [i \approx [a^h]] \left\{ \begin{array}{l} d! [a^t] \\ d!T \end{array} \right. \\ d^h \left\{ \begin{array}{l} assoc_1(a^h, d^t, i) \\ assoc_1(a^t, d^t, i) \end{array} \right. \end{array} \right. \end{array} \right.]$]

[assoc₁(* , *, *) $\xrightarrow{\text{tex}}$ “
\\mathbf {assoc}_1(\#1.
,\#2.
,\#3.
)”]

[assoc₁(* , *, *) $\xrightarrow{\text{pyk}}$ “assoc one * address * index * end assoc”]

$(*)^{\mathbf{P}}$

$[(x)^{\mathbf{P}} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. [t^1]]$

$[(*)^{\mathbf{P}} \xrightarrow{\text{tex}} “(\#1.\nolimits^{\wedge}\{\\mathbf{p}\})”]$

$[(*)^{\mathbf{P}} \xrightarrow{\text{pyk}} “\text{protect } * \text{ end protect}”]$

self

$[\text{self} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. [(c[0] :: [0 :: [t^d]])^I :: T]]$

$[\text{self} \xrightarrow{\text{pyk}} “\text{self}”]$

$[* \ddot{=} *]$

$[[x \ddot{=} y] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_3(t)]$

$[[* \ddot{=} *] \xrightarrow{\text{tex}} “\nolimits^{\wedge}\{\\mathrel{\\{\ddot{=}}}{=}\} \#2.\nolimits^{\wedge}”]$

$[[* \ddot{=} *] \xrightarrow{\text{pyk}} \text{macrodefine } * \text{ as } * \text{ enddefine}]$

$[* \dot{=} *]$

$[[x \dot{=} y] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \dot{=} y] \ddot{=} [(x)^{\mathbf{P}} \xrightarrow{\text{val}} y]])]$

$[[* \dot{=} *] \xrightarrow{\text{tex}} “\nolimits^{\wedge}\{\\mathrel{\\{\dot{=}}}{=}\} \#2.\nolimits^{\wedge}”]$

$[[* \dot{=} *] \xrightarrow{\text{pyk}} “\text{value define } * \text{ as } * \text{ end define}”]$

$[* \acute{=} *]$

$[[x \acute{=} y] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \acute{=} y] \ddot{=} [(x)^{\mathbf{P}} \xrightarrow{\text{val}} y]])]$

$[[* \acute{=} *] \xrightarrow{\text{tex}} “$

[#1/tex name/tex.

\mathrel {\acute{=}}\#2.

”]

[[* $\stackrel{\text{pyk}}{=}$ *] $\xrightarrow{\text{pyk}}$ “intro define * as * end define”]

[* $\stackrel{\text{pyk}}{=}$ *]

[*[x $\stackrel{\text{pyk}}{=}$ y] $\xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[(x \stackrel{\text{pyk}}{=} y) \stackrel{\text{def}}{=} ((x)^p \stackrel{\text{pyk}}{\rightarrow} y)])$]

[[* $\stackrel{\text{pyk}}{=}$ *] $\xrightarrow{\text{tex}}$ “

[#1/tex name/tex.

\stackrel{\text{tex}}{=}\#2/tex name.

”]

[[* $\stackrel{\text{pyk}}{=}$ *] $\xrightarrow{\text{pyk}}$ “pyk define * as * end define”]

[* $\stackrel{\text{tex}}{=}$ *]

[*[x $\stackrel{\text{tex}}{=}$ y] $\xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[(x \stackrel{\text{tex}}{=} y) \stackrel{\text{def}}{=} ((x)^p \stackrel{\text{tex}}{\rightarrow} y)])$]

[[* $\stackrel{\text{tex}}{=}$ *] $\xrightarrow{\text{tex}}$ “

[#1/tex name/tex.

\stackrel{\text{tex}}{=}\#2/tex name.

”]

[[* $\stackrel{\text{tex}}{=}$ *] $\xrightarrow{\text{pyk}}$ “tex define * as * end define”]

[* $\stackrel{\text{name}}{=}$ *]

[*[x $\stackrel{\text{name}}{=}$ y] $\xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[[(x \stackrel{\text{name}}{=} y) \stackrel{\text{def}}{=} ((x)^p \stackrel{\text{name}}{\rightarrow} y)])$]

[[* $\stackrel{\text{name}}{=}$ *] $\xrightarrow{\text{tex}}$ “

[#1/tex name/tex.

\stackrel{\text{tex}}{=}\#2/tex name.

”]

[[* $\stackrel{\text{name}}{=}$ *] $\xrightarrow{\text{pyk}}$ “tex name define * as * end define”]

Priority table[$*$]

[**Priority table**[x] $\xrightarrow{\text{name}}$ “
 $\backslash\text{mathbf}\{\text{Priority}\backslash\text{ table}\} [\#1.$
 $]\”]$

[**Priority table**[x] $\xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[$ [**Priority table**[x] \doteq [self $\xrightarrow{\text{prio}}$
 $(x^P)]])]$]

[**Priority table**[x] $\xrightarrow{\text{tex}}$ “
 $\backslash\text{mathbf}\{\text{Priority}\backslash\text{ table}\} \#1.$
 $\backslash\text{mathbf}\{\text{End}\backslash\text{ table}\}\”]$

[**Priority table**[x] $\xrightarrow{\text{pyk}}$ “priority table * end table”]

$\tilde{\mathcal{M}}_1$

[$\tilde{\mathcal{M}}_1 \xrightarrow{\text{val}} [[x \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[d])]]$]
 $[\tilde{\mathcal{M}}_1 \xrightarrow{\text{tex}}$ “
 $\backslash\text{tilde }\{\{\backslash\text{cal M}\}\}_1\”]$
 $[\tilde{\mathcal{M}}_1 \xrightarrow{\text{pyk}}$ “macro define one”]

$\tilde{\mathcal{M}}_2(*)$

[$\tilde{\mathcal{M}}_2(t) \xrightarrow{\text{val}} ([x :: [t^1]]^M :: [(d :: t)^M :: T])$]
 $[\tilde{\mathcal{M}}_2(*) \xrightarrow{\text{tex}}$ “
 $\backslash\text{tilde }\{\{\backslash\text{cal M}\}\}_2(\#1.$
 $)\”]$
 $[\tilde{\mathcal{M}}_2(*) \xrightarrow{\text{pyk}}$ “macro define two * end define”]

$\tilde{\mathcal{M}}_3(*)$

[$\tilde{\mathcal{M}}_3(t) \xrightarrow{\text{val}} \tilde{\mathcal{Q}}(t, \tilde{\mathcal{M}}_1, \tilde{\mathcal{M}}_2(t))$]
 $[\tilde{\mathcal{M}}_3(*) \xrightarrow{\text{tex}}$ “
 $\backslash\text{tilde }\{\{\backslash\text{cal M}\}\}_3(\#1.$
 $)\”]$
 $[\tilde{\mathcal{M}}_3(*) \xrightarrow{\text{pyk}}$ “macro define three * end define”]

$\tilde{\mathcal{M}}_4(*, *, *, *)$

$[\tilde{\mathcal{M}}_4(t, s, c, d) \xrightarrow{\text{val}} \tilde{\mathcal{M}}(\tilde{\mathcal{Q}}(t, d^2, \mathbf{zip}(d^{1t}, t^t)), s, c)]$

$[\tilde{\mathcal{M}}_4(*, *, *, *) \xrightarrow{\text{tex}} ``$

$\backslash\text{tilde }\{\{\backslash\text{cal M}\}\}_4(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$)"]$

$[\tilde{\mathcal{M}}_4(*, *, *, *) \xrightarrow{\text{pyk}} \text{"macro define four * state * cache * definition * end define"}]$

$\tilde{\mathcal{M}}(*, *, *)$

$[\tilde{\mathcal{M}}(t, s, c) \xrightarrow{\text{val}} \mathcal{U}([[s^h \cdot t] \cdot s] \cdot c)]$

$[\tilde{\mathcal{M}}(*, *, *) \xrightarrow{\text{tex}} ``$

$\backslash\text{tilde }\{\{\backslash\text{cal M}\}\}(\#1.$

$, \#2.$

$, \#3.$

$)"]$

$[\tilde{\mathcal{M}}(*, *, *) \xrightarrow{\text{pyk}} \text{"state expand * state * cache * end expand"}]$

$\tilde{\mathcal{Q}}(*, *, *)$

$[\tilde{\mathcal{Q}}(r, t, s) \xrightarrow{\text{val}} \tilde{\mathcal{Q}}_2(r^d, t, s)]$

$[\tilde{\mathcal{Q}}(*, *, *) \xrightarrow{\text{tex}} ``$

$\backslash\text{tilde }\{\{\backslash\text{cal Q}\}\}(\#1.$

$, \#2.$

$, \#3.$

$)"]$

$[\tilde{\mathcal{Q}}(*, *, *) \xrightarrow{\text{pyk}} \text{"quote expand * term * stack * end expand"}]$

$\tilde{\mathcal{Q}}_2(*, *, *)$

$[\tilde{\mathcal{Q}}_2(r, t, s) \xrightarrow{\text{val}} \tilde{\mathcal{Q}}_3(r, t, s, \mathbf{lookup}(t, s, T))]$

$[\tilde{\mathcal{Q}}_2(*, *, *) \xrightarrow{\text{tex}} ``$

$\backslash\text{tilde }\{\{\backslash\text{cal Q}\}\}_2(\#1.$

,#2.
,#3.
)”]

$[\tilde{\mathcal{Q}}_2(*, *, *) \xrightarrow{\text{pyk}}$ “quote expand two * term * stack * end expand”]

$\tilde{\mathcal{Q}}_3(*, *, *, *)$

$[\tilde{\mathcal{Q}}_3(r, t, s, v) \xrightarrow{\text{val}} v \left\{ \begin{array}{l} (t^r :: [t^i :: r])^M :: \tilde{\mathcal{Q}}^*(r, t^t, s) \\ r! [t! [s!v]] \end{array} \right]$

$[\tilde{\mathcal{Q}}_3(*, *, *, *) \xrightarrow{\text{tex}}$ “
\tilde {\{{\cal Q}\}}_3(\#1.
,#2.
,#3.
,#4.
)”]

$[\tilde{\mathcal{Q}}_3(*, *, *, *) \xrightarrow{\text{pyk}}$ “quote expand three * term * stack * value * end expand”]

$\tilde{\mathcal{Q}}^*(*, *, *)$

$[\tilde{\mathcal{Q}}^*(r, t, s) \xrightarrow{\text{val}} t \left\{ \begin{array}{l} r! [t! [s!T]] \\ \tilde{\mathcal{Q}}_2(r, t^h, s) :: \tilde{\mathcal{Q}}^*(r, t^t, s) \end{array} \right]$

$[\tilde{\mathcal{Q}}^*(*, *, *) \xrightarrow{\text{tex}}$ “
\tilde {\{{\cal Q}\}}^*(\#1.
,#2.
,#3.
)”]

$[\tilde{\mathcal{Q}}^*(*, *, *) \xrightarrow{\text{pyk}}$ “quote expand star * term * stack * end expand”]

$(*)$

$[(x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [(x) \ddot{=} x] \rceil)]$
 $[(*) \xrightarrow{\text{tex}}$ “
(\#1.
)”]

$[(*) \xrightarrow{\text{pyk}}$ “parenthesis * end parenthesis”]

aspect(* , *)

[**aspect**(a, c) $\xrightarrow{\text{val}}$ a^c { c[0][a] | c[a^r][aⁱ] }]

[**aspect**(a, c) $\xrightarrow{\text{tex}}$ “\mathbf{aspect}(\#1, \#2, \#3)”]

[**aspect**(a, c) $\xrightarrow{\text{pyk}}$ “aspect * subcodex * end aspect”]

aspect(* , * , *)

[**aspect**(a, t, c) $\xrightarrow{\text{val}}$ **aspect**(a, c[t^r]["codex"][t^r][tⁱ])]

[**aspect**(* , * , *) $\xrightarrow{\text{tex}}$ “\mathbf{aspect}(\#1, \#2, \#3)”]

[**aspect**(a, t, c) $\xrightarrow{\text{pyk}}$ “aspect * term * cache * end aspect”]

$\langle *\rangle$

[$\langle x \rangle \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}(\mathbf{tuple}_1(t), s, c)$]

[$\langle *\rangle \xrightarrow{\text{tex}}$ “\langle \#1 \rangle”]

[$\langle x \rangle \xrightarrow{\text{pyk}}$ “tuple * end tuple”]

tuple₁(*)

[**tuple**₁(t) $\xrightarrow{\text{val}}$ [t¹ $\stackrel{r}{=}$ [x, y]] { $\tilde{\mathcal{Q}}(t, [x :: \langle y \rangle], \mathbf{tuple}_2(t^1))$ | $\tilde{\mathcal{Q}}(t, [x :: T], [x :: [t^1]] :: T)$ }]

[**tuple**₁(*) $\xrightarrow{\text{tex}}$ “\mathbf{tuple}_1(\#1)”]

[**tuple**₁(t) $\xrightarrow{\text{pyk}}$ “tuple one * end tuple”]

tuple₂(*)

[tuple₂(t) $\xrightarrow{\text{val}}$ [[x] :: [t¹]] :: [[y] :: [t²]] :: T]
[tuple₂(*) $\xrightarrow{\text{tex}}$ “
\mathbf{tuple} \{tuple\}_2(\#1.
) ”]
]

[tuple₂(t) $\xrightarrow{\text{pyk}}$ “tuple two * end tuple”]

let₂(*, *)

[let₂(f, y) $\xrightarrow{\text{val}}$ (y! [f , y])^I]
[let₂(f, y) $\xrightarrow{\text{tex}}$ “
let_2(#1.
, #2.
)”]
[let₂(f, y) $\xrightarrow{\text{pyk}}$ “let two * apply * end let”]

let₁(*, *)

[let₁(f, y) $\xrightarrow{\text{val}}$ let₂(f, y^M)^M]
[let₁(f, y) $\xrightarrow{\text{tex}}$ “
let_1(#1.
, #2.
)”]
[let₁(f, y) $\xrightarrow{\text{pyk}}$ “let one * apply * end let”]

[* $\stackrel{\text{claim}}{=}$ *]

[[x $\stackrel{\text{claim}}{=}$ y] $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \stackrel{\text{claim}}{=} y] \doteq [x \xrightarrow{\text{claim}} y]])$]
[[* $\stackrel{\text{claim}}{=}$ *] $\xrightarrow{\text{tex}}$ “
[#1/tex name/tex.
\stackrel{\text{claim}}{=} \{=\}\#2.
]”]
[[x $\stackrel{\text{claim}}{=}$ y] $\xrightarrow{\text{pyk}}$ “claim define * as * end define”]

checker

[checker $\xrightarrow{\text{val}} \lambda t. \lambda c. \mathbf{check}(t, c)$]

[checker $\xrightarrow{\text{pyk}}$ “checker”]

check(*, *)

[$\mathbf{check}(t, c) \xrightarrow{\text{val}} \text{If}(t^{\text{is}}, c!T, \mathbf{check}_2(t, c, \mathbf{aspect}("claim", t, c)))$]

[$\mathbf{check}(*, *) \xrightarrow{\text{tex}}$ “
\\mathbf{check}(#1.
, #2.
)”]

[$\mathbf{check}(t, c) \xrightarrow{\text{pyk}}$ “check * cache * end check”]

check₂(* , *, *)

[$\mathbf{check}_2(t, c, d) \xrightarrow{\text{val}} d \left\{ \begin{array}{l} \mathbf{check}_3(t, c, \mathbf{aspect}("definition", t, c)) \\ \mathcal{U}^M(\mathcal{E}(d^3, T, c) ` t) ` c \end{array} \right\}$]

[$\mathbf{check}_2(*, *, *) \xrightarrow{\text{tex}}$ “
\\mathbf{check}_2(#1.
, #2.
, #3.
)”]

[$\mathbf{check}_2(t, c, d) \xrightarrow{\text{pyk}}$ “check two * cache * def * end check”]

check₃(* , *, *)

[$\mathbf{check}_3(t, c, d) \xrightarrow{\text{val}} \text{If}(d, \mathbf{check}^*(t^t, c), t! [c!T])$]

[$\mathbf{check}_3(*, *, *) \xrightarrow{\text{tex}}$ “
\\mathbf{check}_3(#1.
, #2.
, #3.
)”]

[$\mathbf{check}_3(t, c, d) \xrightarrow{\text{pyk}}$ “check three * cache * def * end check”]

check^{*}(*, *)

[**check**^{*}(t, c) $\xrightarrow{\text{val}}$ If(t, c!T, **check**₂^{*}(t^t, c, **check**(t^h, c)))]

[**check**^{*}(*, *) $\xrightarrow{\text{tex}}$ “\mathbf{check}^*(*(\#1.
, \#2.
)”]

[**check**^{*}(t, c) $\xrightarrow{\text{pyk}}$ “check list * cache * end check”]

check₂^{*}(*, *, *)

[**check**₂^{*}(t, c, v) $\xrightarrow{\text{val}}$ If($\neg v$, t! [c!v], **check**^{*}(t, c))]

[**check**₂^{*}(*, *, *) $\xrightarrow{\text{tex}}$ “\mathbf{check}_2^*(\#1.
, \#2.
, \#3.
)”]

[**check**₂^{*}(t, c, v) $\xrightarrow{\text{pyk}}$ “check list two * cache * value * end check”]

[*]·

[|t]· $\xrightarrow{\text{claim}}$ $\lambda t. \lambda c. \text{if}(\mathcal{U}(\mathcal{E}(t^1, T, c)), T, t)]$

[[*]· $\xrightarrow{\text{tex}}$ “\relax [\#1.
\relax]^{\{\cdot\}}”]

[|t]· $\xrightarrow{\text{pyk}}$ “test * end test”]

[*]−

[|x]− $\xrightarrow{\text{claim}}$ $\lambda t. \lambda c. \text{if}(\mathcal{U}(\mathcal{E}(t^1, T, c)), t, T)]$

[|x]− $\xrightarrow{\text{tex}}$ “\relax [\#1.
\relax]^{\{-\}}”]

[|x]− $\xrightarrow{\text{pyk}}$ “false test * end test”]

$[*]^\circ$

$[[t]^\circ \xrightarrow{\text{claim}} \lambda t. \lambda c. \mathcal{U}(\mathcal{E}(t^1, T, c))]$

$[[*]^\circ \xrightarrow{\text{tex}} ``$

$\backslash \text{relax} [\#1.$

$\backslash \text{relax}] ^\{\backslash \text{circ}\} "]$

$[[t]^\circ \xrightarrow{\text{pyk}} \text{``raw test * end test''}]$

msg

Predef: message

$[\text{msg} \xrightarrow{\text{tex}} ``$
msg”]

$[\text{msg} \xrightarrow{\text{pyk}} \text{``message''}]$

$[* \stackrel{\text{msg}}{=} *]$

$[[x \stackrel{\text{msg}}{=} y] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \stackrel{\text{msg}}{=} y] \stackrel{=} [(x)^p \xrightarrow{\text{msg}} y]])]$

$[[x \stackrel{\text{msg}}{=} y] \xrightarrow{\text{tex}} ``$

$\#1/\text{tex name/tex}.$

$\backslash \text{stackrel} \{ \text{msg} \} \{ = \} \#2.$

$]"]$

$[[x \stackrel{\text{msg}}{=} y] \xrightarrow{\text{pyk}} \text{``message define * as * end define''}]$

<stmt>

$[<\text{stmt}> \xrightarrow{\text{val}} [<\text{stmt}>]]$

$[<\text{stmt}> \xrightarrow{\text{tex}} ``$

$\{ < \} \text{stmt} \{ > \} "]$

$[<\text{stmt}> \xrightarrow{\text{pyk}} \text{``the statement aspect''}]$

stmt

$[\text{stmt} \xrightarrow{\text{msg}} <\text{stmt}>]$

[stmt $\xrightarrow{\text{tex}}$ “
stmt”]

[stmt $\xrightarrow{\text{pyk}}$ “statement”]

[* stmt *]

[$[x \stackrel{\text{stmt}}{=} y] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \stackrel{\text{stmt}}{=} y] \doteq [(x)^p \stackrel{\text{stmt}}{\rightarrow} y]])$]

[$[x \stackrel{\text{stmt}}{=} y] \xrightarrow{\text{tex}}$ “
[#1/tex name/tex.
\stackrel{\text{tex}}{\text{stackrel}} {\{ \text{stmt} \}} \{ = \} \#2.
]”]

[$[x \stackrel{\text{stmt}}{=} y] \xrightarrow{\text{pyk}}$ “statement define * as * end define”]

HeadNil'

[HeadNil' $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{HeadNil}' \doteq T^h = T]])$]

[HeadNil' $\xrightarrow{\text{tex}}$ “
HeadNil”]

[HeadNil' $\xrightarrow{\text{pyk}}$ “example axiom”]

HeadPair'

[HeadPair' $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{HeadPair}' \doteq \forall A: \forall B: [(A :: B)^h = A]])$]

[HeadPair' $\xrightarrow{\text{tex}}$ “
HeadPair”]

[HeadPair' $\xrightarrow{\text{pyk}}$ “example scheme”]

Transitivity'

[Transitivity' $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Transitivity}' \doteq \forall A: \forall B: \forall C: [[A = B] \vdash [[A = C] \vdash [[B = C]]]]])$]

[Transitivity' $\xrightarrow{\text{tex}}$ “
Transitivity”]

[Transitivity' $\xrightarrow{\text{pyk}}$ “example rule”]

¶

[$\perp \xrightarrow{\text{val}} [\perp]^R :: T$]

[$\perp \xrightarrow{\text{tex}} "$

{\makebox [0mm][l]{\\$\\bot \\$}\backslash,{\\bot }}"]

[$\perp \xrightarrow{\text{pyk}} \text{"absurdity"}$]

Contra'

[$\text{Contra}' \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{M}_4(t, s, c, [[\text{Contra}' \equiv [[T :: T] = T] \vdash \perp]])$]

[$\text{Contra}' \xrightarrow{\text{tex}} "$

Contra"]

[$\text{Contra}' \xrightarrow{\text{pyk}} \text{"contraexample"}$]

T'_E

[$T'_E \xrightarrow{\text{stmt}} [T^h = T] \oplus [[\forall \underline{a} : \forall \underline{b} : [[\underline{a} :: \underline{b}]^h = \underline{a}]] \oplus [[\forall \underline{a} : \forall \underline{b} : \forall \underline{c} : [[\underline{a} = \underline{b}] \vdash [[\underline{a} = \underline{c}] \vdash [[\underline{b} = \underline{c}]]]]] \oplus [[[T :: T] = T] \vdash \perp]]]$]

[$T'_E \xrightarrow{\text{tex}} "$

$T' - \{E\}$ "

[$T'_E \xrightarrow{\text{pyk}} \text{"example theory primed"}$]

L_1

[$L_1 \xrightarrow{\text{stmt}} T'_E \vdash \forall \underline{a} : \forall \underline{b} : [[\underline{a} = \underline{b}] \vdash [\underline{b} = \underline{a}]]]$]

[$L_1 \xrightarrow{\text{tex}} "$

$L - \{1\}$ "

[$L_1 \xrightarrow{\text{pyk}} \text{"example lemma"}$]

*

[$\underline{x} \xrightarrow{\text{tex}} "\underline{\text{underline}\{\#1.\}}"$]

[$\underline{x} \xrightarrow{\text{pyk}} \text{"metavar * end metavar"}$]

\mathcal{A}

$[\mathcal{A} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{A} \doteq \underline{a}] \rceil)]$

$[\mathcal{A} \xrightarrow{\text{tex}} “\{\backslash\text{cal A}\}”]$

$[\mathcal{A} \xrightarrow{\text{pyk}} “\text{meta a}”]$

\mathcal{B}

$[\mathcal{B} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{B} \doteq \underline{b}] \rceil)]$

$[\mathcal{B} \xrightarrow{\text{tex}} “\{\backslash\text{cal B}\}”]$

$[\mathcal{B} \xrightarrow{\text{pyk}} “\text{meta b}”]$

\mathcal{C}

$[\mathcal{C} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{C} \doteq \underline{c}] \rceil)]$

$[\mathcal{C} \xrightarrow{\text{tex}} “\{\backslash\text{cal C}\}”]$

$[\mathcal{C} \xrightarrow{\text{pyk}} “\text{meta c}”]$

\mathcal{D}

$[\mathcal{D} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{D} \doteq \underline{d}] \rceil)]$

$[\mathcal{D} \xrightarrow{\text{tex}} “\{\backslash\text{cal D}\}”]$

$[\mathcal{D} \xrightarrow{\text{pyk}} “\text{meta d}”]$

\mathcal{E}

$[\mathcal{E} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{E} \doteq \underline{e}] \rceil)]$

$[\mathcal{E} \xrightarrow{\text{tex}} “\{\backslash\text{cal E}\}”]$

$[\mathcal{E} \xrightarrow{\text{pyk}} “\text{meta e}”]$

\mathcal{F}

$[\mathcal{F} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{F} \doteq \underline{f}] \rceil)]$

$\mathcal{F} \xrightarrow{\text{tex}} \{\backslash\text{cal F}\}$

$\mathcal{F} \xrightarrow{\text{pyk}} \text{meta f}$

\mathcal{G}

$\mathcal{G} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{G} \doteq g]\rceil)$

$\mathcal{G} \xrightarrow{\text{tex}} \{\backslash\text{cal G}\}$

$\mathcal{G} \xrightarrow{\text{pyk}} \text{meta g}$

\mathcal{H}

$\mathcal{H} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{H} \doteq h]\rceil)$

$\mathcal{H} \xrightarrow{\text{tex}} \{\backslash\text{cal H}\}$

$\mathcal{H} \xrightarrow{\text{pyk}} \text{meta h}$

\mathcal{I}

$\mathcal{I} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{I} \doteq i]\rceil)$

$\mathcal{I} \xrightarrow{\text{tex}} \{\backslash\text{cal I}\}$

$\mathcal{I} \xrightarrow{\text{pyk}} \text{meta i}$

\mathcal{J}

$\mathcal{J} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{J} \doteq j]\rceil)$

$\mathcal{J} \xrightarrow{\text{tex}} \{\backslash\text{cal J}\}$

$\mathcal{J} \xrightarrow{\text{pyk}} \text{meta j}$

\mathcal{K}

$\mathcal{K} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{K} \doteq k]\rceil)$

$\mathcal{K} \xrightarrow{\text{tex}} \{\backslash\text{cal K}\}$

$\mathcal{K} \xrightarrow{\text{pyk}} \text{meta k}$

\mathcal{L}

$[\mathcal{L} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{L} \doteq l] \rceil)]$
 $[\mathcal{L} \xrightarrow{\text{tex}} “\{\backslash\text{cal L}\}”]$
 $[\mathcal{L} \xrightarrow{\text{pyk}} “\text{meta l}”]$

\mathcal{M}

$[\mathcal{M} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{M} \doteq m] \rceil)]$
 $[\mathcal{M} \xrightarrow{\text{tex}} “\{\backslash\text{cal M}\}”]$
 $[\mathcal{M} \xrightarrow{\text{pyk}} “\text{meta m}”]$

\mathcal{N}

$[\mathcal{N} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{N} \doteq n] \rceil)]$
 $[\mathcal{N} \xrightarrow{\text{tex}} “\{\backslash\text{cal N}\}”]$
 $[\mathcal{N} \xrightarrow{\text{pyk}} “\text{meta n}”]$

\mathcal{O}

$[\mathcal{O} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{O} \doteq o] \rceil)]$
 $[\mathcal{O} \xrightarrow{\text{tex}} “\{\backslash\text{cal O}\}”]$
 $[\mathcal{O} \xrightarrow{\text{pyk}} “\text{meta o}”]$

\mathcal{P}

$[\mathcal{P} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{P} \doteq p] \rceil)]$
 $[\mathcal{P} \xrightarrow{\text{tex}} “\{\backslash\text{cal P}\}”]$
 $[\mathcal{P} \xrightarrow{\text{pyk}} “\text{meta p}”]$

\mathcal{Q}

$[\mathcal{Q} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{Q} \doteq q] \rceil)]$

$\mathcal{Q} \xrightarrow{\text{tex}} \{"\backslash\text{cal Q}"\}$

$\mathcal{Q} \xrightarrow{\text{pyk}} \text{"meta q"}$

\mathcal{R}

$\mathcal{R} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{R} \equiv r]\rceil)$

$\mathcal{R} \xrightarrow{\text{tex}} \{"\backslash\text{cal R}"\}$

$\mathcal{R} \xrightarrow{\text{pyk}} \text{"meta r"}$

\mathcal{S}

$\mathcal{S} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{S} \equiv s]\rceil)$

$\mathcal{S} \xrightarrow{\text{tex}} \{"\backslash\text{cal S}"\}$

$\mathcal{S} \xrightarrow{\text{pyk}} \text{"meta s"}$

\mathcal{T}

$\mathcal{T} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{T} \equiv t]\rceil)$

$\mathcal{T} \xrightarrow{\text{tex}} \{"\backslash\text{cal T}"\}$

$\mathcal{T} \xrightarrow{\text{pyk}} \text{"meta t"}$

\mathcal{U}

$\mathcal{U} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{U} \equiv u]\rceil)$

$\mathcal{U} \xrightarrow{\text{tex}} \{"\backslash\text{cal U}"\}$

$\mathcal{U} \xrightarrow{\text{pyk}} \text{"meta u"}$

\mathcal{V}

$\mathcal{V} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,\lceil[\mathcal{V} \equiv v]\rceil)$

$\mathcal{V} \xrightarrow{\text{tex}} \{"\backslash\text{cal V}"\}$

$\mathcal{V} \xrightarrow{\text{pyk}} \text{"meta v"}$

\mathcal{W}

$[\mathcal{W} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{W} \doteq w] \rceil)]$
 $[\mathcal{W} \xrightarrow{\text{tex}} \{"\backslash\text{cal } W\}"]$
 $[\mathcal{W} \xrightarrow{\text{pyk}} \text{"meta w"}]$

\mathcal{X}

$[\mathcal{X} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{X} \doteq x] \rceil)]$
 $[\mathcal{X} \xrightarrow{\text{tex}} \{"\backslash\text{cal } X\}"]$
 $[\mathcal{X} \xrightarrow{\text{pyk}} \text{"meta x"}]$

\mathcal{Y}

$[\mathcal{Y} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{Y} \doteq y] \rceil)]$
 $[\mathcal{Y} \xrightarrow{\text{tex}} \{"\backslash\text{cal } Y\}"]$
 $[\mathcal{Y} \xrightarrow{\text{pyk}} \text{"meta y"}]$

\mathcal{Z}

$[\mathcal{Z} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\mathcal{Z} \doteq z] \rceil)]$
 $[\mathcal{Z} \xrightarrow{\text{tex}} \{"\backslash\text{cal } Z\}"]$
 $[\mathcal{Z} \xrightarrow{\text{pyk}} \text{"meta z"}]$

$\langle * \mid * := * \rangle$

$[\langle a | x := b \rangle \xrightarrow{\text{val}} x! [b!$
 $\text{If}(a^v, \text{If}(a \stackrel{t}{=} x, b, a),$
 $\text{If}(\neg [a \stackrel{r}{=} \forall * : *] , a^h :: \langle * a^t | x := b \rangle,$
 $\text{If}(a^1 \stackrel{t}{=} x, a,$
 $[a^0 :: [a^1 :: [\langle a^2 | x := b \rangle :: T]]])))]$
 $[\langle a | x := b \rangle \xrightarrow{\text{tex}} \text{"$
 $\backslash\text{lang}\#1.$
 $\backslash, \{\backslash\text{protect}\backslash\text{vert}\}\#2.$

$\{:=\} \backslash, \#3.$
 $\backslash \text{rangle } "]$

$[\langle a | x := b \rangle \xrightarrow{\text{pyk}} \text{"sub * set * to * end sub"}]$

$\langle * * | * := * \rangle$

$[\langle * a | x := b \rangle \xrightarrow{\text{val}} x! [b! \text{If}(a, T, \langle a^h | x := b \rangle :: \langle * a^t | x := b \rangle)]]$

$[\langle * a | x := b \rangle \xrightarrow{\text{tex}} "$
 $\backslash \text{langle } ^\wedge \{ \backslash \text{ast} \} \#1.$
 $\backslash, \{ \backslash \text{protect} \backslash \text{vert} \} \#2.$
 $\{:=\} \backslash, \#3.$
 $\backslash \text{rangle } "]$

$[\langle * a | x := b \rangle \xrightarrow{\text{pyk}} \text{"sub star * set * to * end sub"}]$

\emptyset

$[\emptyset \xrightarrow{\text{val}} T]$

$[\emptyset \xrightarrow{\text{tex}} "$
 $\backslash \text{emptyset } "]$

$[\emptyset \xrightarrow{\text{pyk}} \text{"the empty set"}]$

Remainder

$[\text{Remainder} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Remainder} \doteq \text{HeadPair}' \oplus [\text{Transitivity}' \oplus \text{Contra}']]])]$

$[\text{Remainder} \xrightarrow{\text{tex}} "$
 $\text{Remainder}"]$

$[\text{Remainder} \xrightarrow{\text{pyk}} \text{"example remainder"}]$

$(*)^\mathbf{v}$

$[(x)^\mathbf{v} \xrightarrow{\text{name}} "$
 $(\#1.$
 $)^\wedge \{ \backslash \text{bf v} \}"]$

$[(x)^\mathbf{v} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [(x)^\mathbf{v} \doteq x])]$

$[(x)^v \xrightarrow{\text{tex}} \#\text{1/tex name.}]$

$[(x)^v \xrightarrow{\text{pyk}} \text{"make visible * end visible"}]$

$\text{intro}(*, *, *, *)$

$[\text{intro}(x, i, p, t) \xrightarrow{\text{name}} \text{"}$

$\text{intro}(\#\text{1.}$

$, \#\text{2.}$

$, \#\text{3.}$

$, \#\text{4.}$

$)"]$

$[\text{intro}(x, i, p, t) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\text{intro}(x, i, p, t) \doteq$

$\text{math}[x \xrightarrow{\text{pyk}} p] \text{ end mathmath}[x \xrightarrow{\text{tex}} t] \text{ end math}])]$

$[\text{intro}(x, i, p, t) \xrightarrow{\text{tex}} \text{"}\backslash\text{index}\{\#\text{2.: } \#\text{3. } @\#\text{2.: } \$[\#\text{1/tex name/tex.}]\$ \#\text{3.}\} \%$

$\text{\backslash index}\{\#\text{3. } \$[\#\text{1/tex name/tex.}]\$}\%$

$\text{\backslash tex}\{$

$\$[\#\text{1/tex name/tex.}$

$\text{\backslash stackrel}\{ \text{\backslash mathrm}\{ \text{tex}\}\} \{=\} \#\text{4/tex name.}$

$\} \$\} \$[\#\text{1/tex name/tex.}\%$

$\} \$\text{\footnote}\{ \$[\#\text{1/tex name/tex.}$

$\text{\backslash stackrel}\{ \text{\backslash mathrm}\{ \text{pyk}\}\} \{=\} \#\text{3/tex name.}$

$\} \$\}]$

$[\text{intro}(x, i, p, t) \xrightarrow{\text{pyk}} \text{"intro * index * pyk * tex * end intro"}]$

$\text{intro}(*, *, *, *)$

$[\text{intro}(x, p, t) \xrightarrow{\text{name}} \text{"}$

$\text{intro}(\#\text{1.}$

$, \#\text{2.}$

$, \#\text{3.}$

$)"]$

$[\text{intro}(x, p, t) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\text{intro}(x, p, t) \doteq$

$\text{math}[x \xrightarrow{\text{pyk}} p] \text{ end mathmath}[x \xrightarrow{\text{tex}} t] \text{ end math}])]$

$[\text{intro}(x, p, t) \xrightarrow{\text{tex}} \text{"}\backslash\text{index}\{\backslash\alpha \#\text{2. } @\backslash\text{back }\text{\backslash makebox}[20mm][l]\{ \$[\#\text{1/tex name/tex.}]\$}\#\text{2.}\} \%$

$\text{\backslash index}\{\#\text{2. } \$[\#\text{1/tex name/tex.}]\$}\%$

$\text{\backslash tex}\{$

$\$[\#\text{1/tex name/tex.}$

$\text{\backslash stackrel}\{ \text{\backslash mathrm}\{ \text{tex}\}\} \{=\} \#\text{3/tex name.}$

```
]$}$_[ #1/tex name/tex.%  
]$\footnote{\$_[ #1/tex name/tex.  
\stackrel{\{ \mathsf{pyk} \}}{=} \#2/tex name.  
]}$_]
```

[intro(x, p, t) $\xrightarrow{\text{pyk}}$ “intro * pyk * tex * end intro”]

error(*, *)

```
[error(m, t)  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{error}(m, t) \doteq \text{error}_2([m], t)]])$ ]  
[error(m, t)  $\xrightarrow{\text{tex}}$  “  
error(#1/tex name.  
, #2.  
)”]
```

[error(m, t) $\xrightarrow{\text{pyk}}$ “error * term * end error”]

error₂(* , *)

```
[error2(m, t)  $\xrightarrow{\text{val}}$  t-color(m1 [ [ “  
”] ] )]  
[error2(m, t)  $\xrightarrow{\text{tex}}$  “  
error-{2} (#1/tex name.  
, #2.  
)”]
```

[error₂(m, t) $\xrightarrow{\text{pyk}}$ “error two * term * end error”]

proof(*, *, *)

```
[proof(p, t, c)  $\xrightarrow{\text{val}}$  proof2(S(c, p), t)]  
[proof(p, t, c)  $\xrightarrow{\text{tex}}$  “  
proof( #1.  
, #2.  
, #3.  
)”]
```

[proof(p, t, c) $\xrightarrow{\text{pyk}}$ “proof * term * cache * end proof”]

$\text{proof}_2(*, *)$

[$\text{proof}_2(q, t) \xrightarrow{\text{val}} t!$
If($q^E, q,$
If($\neg [q^0]$, error₂(“Proof has at least one unresolved premise.
Lemma; premise reads:”], t; [q^{0h}]),
If($\neg [q^1]$, error₂(“Proof has at least one unresolved side condition.
Lemma; condition reads:”], t; [q^{1h}]),
If($q^2 \stackrel{t}{=} t, T,$
error₂(“Lemma does not match conclusion. Lemma; conclusion reads:”], t; [q^2])))))]
[$\text{proof}_2(q, t) \xrightarrow{\text{tex}} “\text{proof_}\{2\}(\ #1.$
 $, \ #2.$
)”]
[$\text{proof}_2(q, t) \xrightarrow{\text{pyk}} “\text{proof two * term * end proof}”$]

$\mathcal{S}(*, *)$

[$\mathcal{S}(c, t) \xrightarrow{\text{val}} c!$
If($t^E, t,$
If($t \stackrel{r}{=} [*^I]$, $\mathcal{S}^I(c, t),$
If($t \stackrel{r}{=} [*^\triangleright]$, $\mathcal{S}^\triangleright(c, t),$
If($t \stackrel{r}{=} [*^V]$, $\mathcal{S}^E(c, t),$
If($t \stackrel{r}{=} [*^+]$, $\mathcal{S}^+(c, t),$
If($t \stackrel{r}{=} [*^-]$, $\mathcal{S}^-(c, t),$
If($t \stackrel{r}{=} [*^*]$, $\mathcal{S}^*(c, t),$
If($t \stackrel{r}{=} [* @ *]$, $\mathcal{S}^@ (c, t),$
If($t \stackrel{r}{=} [* \vdash *]$, $\mathcal{S}^\vdash (c, t),$
If($t \stackrel{r}{=} [* \Vdash *]$, $\mathcal{S}^\Vdash (c, t),$
If($t \stackrel{r}{=} [* \text{i.e.} *]$, $\mathcal{S}^{\text{i.e.}}(c, t),$
If($t \stackrel{r}{=} [\forall * : *]$, $\mathcal{S}^\forall(c, t),$
If($t \stackrel{r}{=} [* ; *]$, $\mathcal{S}^:(c, t),$
error₂(“Unknown sequent operator:”], t)))))))))))])
[$\mathcal{S}(x, y) \xrightarrow{\text{tex}} “\{\text{\cal S}\}(\#1.$
 $, \ #2.$
)”]
[$\mathcal{S}(x, y) \xrightarrow{\text{pyk}} “\text{sequent eval * term * end eval}”$]

$\mathcal{S}^I(*, *)$

$[\mathcal{S}^I(c, t) \xrightarrow{\text{val}} c! [\emptyset :: [\emptyset :: [\text{t-color}(t^1 \vdash [t^1]) :: T]]]]$

$[\mathcal{S}^I(x, y) \xrightarrow{\text{tex}} ``$
 $\{\backslash\text{cal S}\}^{\wedge} \{I\}(\#1.$
 $, \#2.$
 $)"]$

$[\mathcal{S}^I(x, y) \xrightarrow{\text{pyk}} ``\text{seqeval init * term * end eval}"]$

$\mathcal{S}^{\triangleright}(*, *)$

$[\mathcal{S}^{\triangleright}(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^{\triangleright}(c, t, \mathcal{S}(c, t^1))]$

$[\mathcal{S}^{\triangleright}(x, y) \xrightarrow{\text{tex}} ``$
 $\{\backslash\text{cal S}\}^{\wedge} \{\backslash\text{rhd}\}(\#1.$
 $, \#2.$
 $)"]$

$[\mathcal{S}^{\triangleright}(x, y) \xrightarrow{\text{pyk}} ``\text{seqeval modus * term * end eval}"]$

$\mathcal{S}_1^{\triangleright}(*, *, *)$

$[\mathcal{S}_1^{\triangleright}(c, t, q) \xrightarrow{\text{val}} c! [t!$

If($q^E, q,$

If($q^2 \stackrel{r}{=} [* \vdash *], q^0 \cup \{q^{21}\} :: [q^1 :: [q^{22} :: T]]$,

If($q^2 \stackrel{r}{=} [* \Vdash *], q^0 :: [q^1 \cup \{q^{21}\} :: [q^{22} :: T]]$,

error₂([“The modus operation requires the conclusion of its argument to be an inference or an endorsement”, t))))]]

$[\mathcal{S}_1^{\triangleright}(x, y, z) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal S}\}_{-1}^{\wedge} \{\backslash\text{rhd}\}(\#1.$
 $, \#2.$
 $, \#3.$
 $)"]$

$[\mathcal{S}_1^{\triangleright}(x, y, z) \xrightarrow{\text{pyk}} ``\text{seqeval modus one * term * sequent * end eval}"]$

$\mathcal{S}^E(*, *)$

$[\mathcal{S}^E(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^E(c, t, \mathcal{S}(c, t^1))]$

$[\mathcal{S}^E(x, y) \xrightarrow{\text{tex}} ``\{\backslash\text{cal } S\}^{\wedge}\{E\}(\#1.$
 $, \#2.$
 $)"']$

$[\mathcal{S}^E(x, y) \xrightarrow{\text{pyk}} ``\text{seqeval verify * term * end eval}"]$

$\mathcal{S}_1^E(*, *, *)$

$[\mathcal{S}_1^E(c, t, q) \xrightarrow{\text{val}} c! [t!$

If($q^E, q,$

If($\neg [q^2 \stackrel{r}{=} [* \vdash *]], \text{error}_2([\text{"The verify operation requires the conclusion of its argument to be an endorsement:"}], t),$

If($\neg [q^{21C}], \text{error}_2([\text{"The verify operation requires the side condition to contain no metavariables. Term; condition:"}], t; [q^{21}]),$

If($(U^M(U^M(E(q^{21}, T, c)) \cdot c), q^0 :: [q^1 :: [q^{22} :: T]]),$

error₂([“False side condition:”], t))))]

$[\mathcal{S}_1^E(x, y, z) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal } S\}_{\{-1\}}^{\wedge}\{E\}(\#1.$

, #2.

, #3.

)"']

$[\mathcal{S}_1^E(x, y, z) \xrightarrow{\text{pyk}} ``\text{seqeval verify one * term * sequent * end eval}"]$

$\mathcal{S}^+(*, *)$

$[\mathcal{S}^+(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^+(c, t, \mathcal{S}(c, t^1))]$

$[\mathcal{S}^+(x, y) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal } S\}^{\wedge}\{+\}(\#1.$

, #2.

)"']

$[\mathcal{S}^+(x, y) \xrightarrow{\text{pyk}} ``\text{sequent eval plus * term * end eval}"]$

$\mathcal{S}_1^+(*, *, *)$

$[\mathcal{S}_1^+(c, t, q) \xrightarrow{\text{val}} c! [t!$

If($q^E, q,$

If($[q^2 \stackrel{r}{=} [* \vdash *]] \wedge [q^{22} \stackrel{r}{=} [* \vdash *]],$

$[q^0 :: [q^1 :: [t\text{-color}([q^{21} \oplus [q^{221}]]] \vdash [q^{222}]) :: T]]],$
 $\text{error}_2(\lceil\text{"Term; conclusion not fit for currying:"}\rceil, t; [q^2]))]$

$[\mathcal{S}_1^+(x, y, z) \xrightarrow{\text{tex}} ``$
 $\{\text{\cal S}\}_{-1}^{\wedge}\{+\}(\#1.$
 $, \#2.$
 $, \#3.$
 $)"]$

$[\mathcal{S}_1^+(x, y, z) \xrightarrow{\text{pyk}} \text{"seqeval plus one * term * sequent * end eval"}]$

$\mathcal{S}^-(*, *)$

$[\mathcal{S}^-(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^-(c, t, \mathcal{S}(c, t^1))]$

$[\mathcal{S}^-(x, y) \xrightarrow{\text{tex}} ``$
 $\{\text{\cal S}\}_{-1}^{\wedge}(\#1.$
 $, \#2.$
 $)"]$

$[\mathcal{S}^-(x, y) \xrightarrow{\text{pyk}} \text{"seqeval minus * term * end eval"}]$

$\mathcal{S}_1^-(*, *, *)$

$[\mathcal{S}_1^-(c, t, q) \xrightarrow{\text{val}} c! [t!]$

$\text{If}(q^E, q,$

$\text{If}([\lceil q^2 \stackrel{r}{=} [* \vdash *] \rceil \wedge [\lceil q^{21} \stackrel{r}{=} [* \oplus *] \rceil],$
 $[\lceil q^0 :: [q^1 :: [t\text{-color}(q^{211} \vdash [q^{212} \vdash [q^{22}]]]) :: T]]],$
 $\text{error}_2(\lceil\text{"Term; conclusion not fit for currying:"}\rceil, t; [q^2]))]$

$[\mathcal{S}_1^-(x, y, z) \xrightarrow{\text{tex}} ``$
 $\{\text{\cal S}\}_{-1}^{\wedge}(\#1.$
 $, \#2.$
 $, \#3.$
 $)"]$

$[\mathcal{S}_1^-(x, y, z) \xrightarrow{\text{pyk}} \text{"seqeval minus one * term * sequent * end eval"}]$

$\mathcal{S}^*(*, *)$

$[\mathcal{S}^*(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^*(c, t, \mathcal{S}(c, t^1))]$

$[\mathcal{S}^*(x, y) \xrightarrow{\text{tex}} ``$

$\{\backslash\text{cal S}\}^{\wedge}\{\backslash\text{ast}\}(\#1.$
 $, \#2.$
 $)”]$

$[\mathcal{S}^*(x, y) \xrightarrow{\text{pyk}} \text{“seqeval deref * term * end eval”}]$

$\mathcal{S}_1^*(*, *, *)$

$[\mathcal{S}_1^*(c, t, q) \xrightarrow{\text{val}} c! [t!$
 $\text{If}(q^E, q,$
 $\mathcal{S}_2^*(c, t, q, \text{aspect}(<\text{stmt}>, q^2, c)))]]$
 $[\mathcal{S}_1^*(x, y, z) \xrightarrow{\text{tex}} \text{“}$
 $\{\backslash\text{cal S}\}_{-}\{1\}^{\wedge}\{\backslash\text{ast}\}(\#1.$
 $, \#2.$
 $, \#3.$
 $)”]$

$[\mathcal{S}_1^*(x, y, z) \xrightarrow{\text{pyk}} \text{“seqeval deref one * term * sequent * end eval”}]$

$\mathcal{S}_2^*(*, *, *, *)$

$[\mathcal{S}_2^*(c, t, q, d) \xrightarrow{\text{val}} c! [t! [q!$
 $\text{If}(d, \text{error}_2([\text{“Dereferencing construct that has no statement def:”}], t),$
 $[q^0 :: [q^1 :: [d^3 :: T]]])]]$
 $[\mathcal{S}_2^*(c, t, q, d) \xrightarrow{\text{tex}} \text{“}$
 $\{\backslash\text{cal S}\}_{-}\{2\}^{\wedge}\{\backslash\text{ast}\}(\#1.$
 $, \#2.$
 $, \#3.$
 $, \#4.$
 $)”]$

$[\mathcal{S}_2^*(c, t, q, d) \xrightarrow{\text{pyk}} \text{“seqeval deref two * term * sequent * def * end eval”}]$

$\mathcal{S}^@(*, *)$

$[\mathcal{S}^@ (c, t) \xrightarrow{\text{val}} \mathcal{S}_1^@ (c, t, \mathcal{S}(c, t^1))]$
 $[\mathcal{S}^@ (x, y) \xrightarrow{\text{tex}} \text{“}$
 $\{\backslash\text{cal S}\}^{\wedge}\{\backslash\text{char64}\}(\#1.$
 $, \#2.$
 $)”]$

$[\mathcal{S}^{\circledast}(x, y) \xrightarrow{\text{pyk}} \text{“seqeval at * term * end eval”}]$

$\mathcal{S}_1^{\circledast}(*, *, *)$

$[\mathcal{S}_1^{\circledast}(c, t, q) \xrightarrow{\text{val}} c! [t!$

If($q^E, q,$

If($\neg [q^2 = \lceil \forall*: * \rceil]$, error₂($\lceil \text{“Quantifier elimination requires the conclusion of its argument to be a quantifier:”} \rceil, t),$

If($\neg [t^2 \text{ free for } q^{21} \text{ in } [q^{22}]]$, error₂($\lceil \text{“Quantifier elimination leads to variable clash:”} \rceil, t),$

$[q^0 :: [q^1 :: [\langle q^{22} | q^{21} := t^2 \rangle :: T]]]))]$)

$[\mathcal{S}_1^{\circledast}(c, t, q) \xrightarrow{\text{tex}} \text{“}$

$\{\backslash\text{cal S}\}_{-1}^{\wedge} \{\backslash\text{char64}\}(\#1.$

, #2.

, #3.

)”]

$[\mathcal{S}_1^{\circledast}(c, t, q) \xrightarrow{\text{pyk}} \text{“seqeval at one * term * sequent * end eval”}]$

$\mathcal{S}^{\vdash}(*, *)$

$[\mathcal{S}^{\vdash}(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^{\vdash}(c, t, t^1, \mathcal{S}(c, t^2))]$

$[\mathcal{S}^{\vdash}(x, y) \xrightarrow{\text{tex}} \text{“}$

$\{\backslash\text{cal S}\}^{\wedge} \{\backslash\text{vdash}\}(\#1.$

, #2.

)”]

$[\mathcal{S}^{\vdash}(x, y) \xrightarrow{\text{pyk}} \text{“seqeval infer * term * end eval”}]$

$\mathcal{S}_1^{\vdash}(*, *, *, *)$

$[\mathcal{S}_1^{\vdash}(c, t, p, q) \xrightarrow{\text{val}} c! [t! [p!$

If($q^E, q,$

$[q^0 \setminus \{p\} :: [q^1 :: [t\text{-color}(p \vdash [q^2]) :: T]]])]$)

$[\mathcal{S}_1^{\vdash}(x, y, z, u) \xrightarrow{\text{tex}} \text{“}$

$\{\backslash\text{cal S}\}_{-1}^{\wedge} \{\backslash\text{vdash}\}(\#1.$

, #2.

, #3.

, #4.

)”]

$[S_1^{\vdash}(x, y, z, u) \xrightarrow{\text{pyk}} \text{“seqeval infer one * term * premise * sequent * end eval”}]$

$\mathcal{S}^{\vdash}(*, *)$

$[S^{\vdash}(c, t) \xrightarrow{\text{val}} S_1^{\vdash}(c, t, t^1, S(c, t^2))]$

$[S^{\vdash}(x, y) \xrightarrow{\text{tex}} \text{“}\{\backslash\text{cal S}\}^{\wedge}\{\backslash\text{makebox } [0mm][l]\{\backslash\text{scriptsize }\$\backslash\text{vdash }\$\}, \{\backslash\text{vdash }\}\}(\#1. , \#2.)”]$

$[S^{\vdash}(x, y) \xrightarrow{\text{pyk}} \text{“seqeval endorse * term * end eval”}]$

$S_1^{\vdash}(*, *, *, *)$

$[S_1^{\vdash}(c, t, p, q) \xrightarrow{\text{val}} c! [t! [p!$

$\text{If}(q^E, q,$

$[q^0 :: [q^1 \setminus \{p\} :: [t\text{-color}(p \Vdash [q^2]) :: T]]]]$

$[S_1^{\vdash}(x, y, z, u) \xrightarrow{\text{tex}} \text{“}$

$\{\backslash\text{cal S}\}_{-1}^{\wedge}\{\backslash\text{makebox } [0mm][l]\{\backslash\text{scriptsize }\$\backslash\text{vdash }\$\}, \{\backslash\text{vdash }\}\}(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$)”]$

$[S_1^{\vdash}(x, y, z, u) \xrightarrow{\text{pyk}} \text{“seqeval endorse one * term * side * sequent * end eval”}]$

$\mathcal{S}^{\text{i.e.}}(*, *)$

$[S^{\text{i.e.}}(c, t) \xrightarrow{\text{val}} S_1^{\text{i.e.}}(c, t, t^2, S(c, t^1))]$

$[S^{\text{i.e.}}(x, y) \xrightarrow{\text{tex}} \text{“}$

$\{\backslash\text{cal S}\}^{\wedge}\{\text{i.e.}\}(\#1.$

$, \#2.$

$)”]$

$[S^{\text{i.e.}}(x, y) \xrightarrow{\text{pyk}} \text{“seqeval est * term * end eval”}]$

$\mathcal{S}_1^{\text{i.e.}}(*, *, *, *)$

$[\mathcal{S}_1^{\text{i.e.}}(c, t, a, q) \xrightarrow{\text{val}} c! [t! [a!$
If($q^E, q, \mathcal{S}_2^{\text{i.e.}}(c, t, a, q, \text{aspect}(<\text{stmt}>, a, c)))]]]$

$[\mathcal{S}_1^{\text{i.e.}}(x, y, z, u) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal S}\}_{-1}^{\text{i.e.}}(\#1.$
, #2.
, #3.
, #4.
)"]

$[\mathcal{S}_1^{\text{i.e.}}(x, y, z, u) \xrightarrow{\text{pyk}} \text{"seqeval est one * term * name * sequent * end eval"}]$

$\mathcal{S}_2^{\text{i.e.}}(*, *, *, *, *)$

$[\mathcal{S}_2^{\text{i.e.}}(c, t, a, q, d) \xrightarrow{\text{val}} c! [t! [a! [q!$
If($d, \text{error}_2([\text{"Referencing construct that has no statement def:"]}, t),$
If($\neg [d^3 \stackrel{t}{=} [q^2]], \text{error}_2([\text{"Reference; conclusion do not match:"]}, a; [q^2]),$
[$q^0 :: [q^1 :: [a :: T]]])]]]$

$[\mathcal{S}_2^{\text{i.e.}}(c, t, a, q, d) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal S}\}_{-2}^{\text{i.e.}}(\#1.$
, #2.
, #3.
, #4.
, #5.
)"]

$[\mathcal{S}_2^{\text{i.e.}}(c, t, a, q, d) \xrightarrow{\text{pyk}} \text{"seqeval est two * term * name * sequent * def * end eval"}]$

$\mathcal{S}^{\forall}(*, *)$

$[\mathcal{S}^{\forall}(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^{\forall}(c, t, t^1, \mathcal{S}(c, t^2))]$

$[\mathcal{S}^{\forall}(x, y) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal S}\}^{\forall}(\#1.$
, #2.
)"]

$[\mathcal{S}^{\forall}(x, y) \xrightarrow{\text{pyk}} \text{"seqeval all * term * end eval"}]$

$\mathcal{S}_1^{\forall}(*, *, *, *)$

$[\mathcal{S}_1^{\forall}(c, t, v, q) \xrightarrow{\text{val}} c! [t! [v!$
 If(q^E, q ,
 If($\neg [v^V]$, error₂(["Metageneralization over non-metavariable:"], t),
 If(v free in $[q^0]$, error₂(["Metageneralization over metavariable that occurs free in some premise:"], t),
 If(v free in $[q^1]$, error₂(["Metageneralization over metavariable that occurs free in some side condition:"], t),
 $[q^0 :: [q^1 :: [t\text{-color}(\forall v: [q^2]) :: T]]])]))]]]$

$[\mathcal{S}_1^{\forall}(c, t, v, q) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal } S\}_{-}\{1\}^{\wedge}\{\backslash\text{forall}\}(\#1.$
 $, \#2.$
 $, \#3.$
 $, \#4.$
 $)"]$

$[\mathcal{S}_1^{\forall}(c, t, v, q) \xrightarrow{\text{pyk}} \text{"seqeval all one * term * variable * sequent * end eval"}]$

$\mathcal{S}^{:}(*, *)$

$[\mathcal{S}^{:}(c, t) \xrightarrow{\text{val}} \mathcal{S}_1^{:}(c, t, \mathcal{S}(c, t^1))]$
 $[\mathcal{S}^{:}(x, y) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal } S\}^{\wedge}\{; \}(\#1.$
 $, \#2.$
 $)"]$

$[\mathcal{S}^{:}(x, y) \xrightarrow{\text{pyk}} \text{"seqeval cut * term * end eval"}]$

$\mathcal{S}_1^{:}(*, *, *)$

$[\mathcal{S}_1^{:}(c, t, p) \xrightarrow{\text{val}} c! [t!$
 If($p^E, p, \mathcal{S}_2^{:}(c, t, p, \mathcal{S}(c, t^2)))]]$
 $[\mathcal{S}_1^{:}(x, y, z) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal } S\}_{-}\{1\}^{\wedge}\{; \}(\#1.$
 $, \#2.$
 $, \#3.$
 $)"]$

$[\mathcal{S}_1^{:}(c, t, p) \xrightarrow{\text{pyk}} \text{"seqeval cut one * term * forerunner * end eval"}]$

$\mathcal{S}_2^{\cdot}(*, *, *, *)$

$[\mathcal{S}_2^{\cdot}(c, t, p, q) \xrightarrow{\text{val}} c! [t! [p!]] :: [[p^1 \cup [q^1]] :: [q^2 :: T]]]]$
If($q^E, q, [p^0 \cup [q^0 \setminus \{p^2\}]] :: [[p^1 \cup [q^1]] :: [q^2 :: T]])$)
 $[\mathcal{S}_2^{\cdot}(c, t, p, q) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal S}\}_{-2}^{\cdot} \{; \}(\#1.$
 $, \#2.$
 $, \#3.$
 $, \#4.$
)"]

$[\mathcal{S}_2^{\cdot}(x, y, z, u) \xrightarrow{\text{pyk}} \text{"seqeval cut two * term * forerunner * sequent * end eval"}]$

$\mathcal{T}(*)$

$[\mathcal{T}(x) \xrightarrow{\text{val}} x! \Lambda \lambda c. x]$
 $[\mathcal{T}(x) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal T}\}(\#1.$
)"]

$[\mathcal{T}(x) \xrightarrow{\text{pyk}} \text{"computably true * end true"}]$

$\text{claims}(*, *, *)$

$[\text{claims}(t, c, r) \xrightarrow{\text{val}}$
If($\text{claims}_2(t, c, r), T, \text{claims}_2(t, c, c[r][\text{"bibliography"}]^1))$]
 $[\text{claims}(t, c, r) \xrightarrow{\text{tex}} "$
 $\text{claims}(\#1.$
 $, \#2.$
 $, \#3.$
)"]

$[\text{claims}(t, c, r) \xrightarrow{\text{pyk}} \text{"claims * cache * ref * end claims"}]$

$\text{claims}_2(*, *, *)$

$[\text{claims}_2(t, c, r) \xrightarrow{\text{val}} \text{If}(\neg [r^c], t! [c!F], t \in_c [c[r][\text{"codex"}][r][0][0][\text{"claim"}]^3])]$
 $[\text{claims}_2(t, c, r) \xrightarrow{\text{tex}} "$
 $\text{claims_2}(\#1.$

, #2.
, #3.
)]”]

[claims₂(t, c, r) $\xrightarrow{\text{pyk}}$ “claims two * cache * ref * end claims”]

<proof>

[<proof> $\xrightarrow{\text{val}}$ [<proof>]]

[<proof> $\xrightarrow{\text{tex}}$ “
{<}proof{>}”]

[<proof> $\xrightarrow{\text{pyk}}$ “the proof aspect”]

proof

[proof $\xrightarrow{\text{msg}}$ <proof>]

[proof $\xrightarrow{\text{tex}}$ “
proof”]

[proof $\xrightarrow{\text{pyk}}$ “proof”]

[**Lemma** *: *]

[[**Lemma** x: y] $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Lemma } x: y] \doteq [x \xrightarrow{\text{stmt}} y]])$]

[[**Lemma** x: y] $\xrightarrow{\text{tex}}$ “
[\mathbf{Lemma} \] \#1.
\colon \#2.
]”]

[[**Lemma** x: y] $\xrightarrow{\text{pyk}}$ “lemma * says * end lemma”]

[**Proof of** *: *]

[[**Proof of** x: y] $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Proof of } x: y] \doteq [x \xrightarrow{\text{proof}} y]])$]

[[**Proof of** x: y] $\xrightarrow{\text{tex}}$ “
[\mathbf{Proof} \ of \] \#1/tex name/tex.
\colon \#2.
]”]

$\llbracket [\text{[Proof of } x:y] \rrbracket \xrightarrow{\text{pyk}} \text{“proof of } * \text{ reads } * \text{ end proof”}]$

$\llbracket [* \text{ lemma } * : *] \rrbracket$

$\llbracket [x \text{ lemma } y:z] \rrbracket \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c, \lceil \llbracket [x \text{ lemma } y:z] \rrbracket \doteq [y \stackrel{\text{stmt}}{=} x \vdash z] \rceil))$

$\llbracket [x \text{ lemma } y:z] \rrbracket \xrightarrow{\text{tex}} \text{“}$

$\lceil \#1.$

$\backslash\text{mathbf}{\{ \backslash \text{ lemma} \}} \#2.$

$\backslash\text{colon } \#3.$

$\rceil ”]$

$\llbracket [x \text{ lemma } y:z] \rrbracket \xrightarrow{\text{pyk}} \text{“in theory } * \text{ lemma } * \text{ says } * \text{ end lemma”}]$

$\llbracket [* \text{ antilemma } * : *] \rrbracket$

$\llbracket [x \text{ antilemma } y:z] \rrbracket \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c, \lceil \llbracket [x \text{ antilemma } y:z] \rrbracket \doteq [x \text{ lemma } y:z \vdash \perp] \rceil))$

$\llbracket [x \text{ antilemma } y:z] \rrbracket \xrightarrow{\text{tex}} \text{“}$

$\lceil \#1.$

$\backslash\text{mathbf}{\{ \backslash \text{ antilemma} \}} \#2.$

$\backslash\text{colon } \#3.$

$\rceil ”]$

$\llbracket [x \text{ antilemma } y:z] \rrbracket \xrightarrow{\text{pyk}} \text{“in theory } * \text{ antilemma } * \text{ says } * \text{ end antilemma”}]$

$\llbracket [* \text{ rule } * : *] \rrbracket$

$\llbracket [x \text{ rule } y:z] \rrbracket \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c, \lceil \llbracket [x \text{ rule } y:z] \rrbracket \doteq [x \text{ lemma } y:z][\text{[Proof of } y:\text{Rule tactic}]] \rceil))$

$\llbracket [x \text{ rule } y:z] \rrbracket \xrightarrow{\text{tex}} \text{“}$

$\lceil \#1.$

$\backslash\text{mathbf}{\{ \backslash \text{ rule} \}} \#2.$

$\backslash\text{colon } \#3.$

$\rceil ”]$

$\llbracket [x \text{ rule } y:z] \rrbracket \xrightarrow{\text{pyk}} \text{“in theory } * \text{ rule } * \text{ says } * \text{ end rule”}]$

[* **antirule** *: *]

$[[x \text{ antirule } y: z]] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \text{ antirule } y: z]] \equiv [x \text{ rule } y: z \vdash \perp])])$

$[[x \text{ antirule } y: z]] \xrightarrow{\text{tex}} ``$

[#1.

\mathbf{\{ \backslash antirule \} } #2.

\colon #3.

]"]

$[[x \text{ antirule } y: z]] \xrightarrow{\text{pyk}} \text{``in theory * antirule * says * end antirule''}$

verifier

[verifier $\xrightarrow{\text{val}} \lambda t. \lambda c. \mathcal{V}_1(c)$]

[verifier $\xrightarrow{\text{pyk}}$ “verifier”]

$\mathcal{V}_1(*)$

$[\mathcal{V}_1(c) \xrightarrow{\text{val}}$

let₁($\lambda r.$

let₁($\lambda x.$

let₁($\lambda p.$

let₁($\lambda d.$

If($\neg d, d,$

let₁($\lambda i.$

If($\neg [i^c], T,$

error₂([“Circular proof. Circle

includes:”], p[i]^{0h})), $\mathcal{V}_5(c, r, p, p))), \mathcal{V}_3(c, r, p, T)), \mathcal{V}_2(c, x)), c[r][\text{"codex"}][r]), c[0]]$

$[\mathcal{V}_1(c) \xrightarrow{\text{tex}} ``$

{\cal V}_1 -1(#1.

)"]

$[\mathcal{V}_1(c) \xrightarrow{\text{pyk}} \text{``verify one * end verify''}]$

$\mathcal{V}_2(*, *)$

$[\mathcal{V}_2(c, p) \xrightarrow{\text{val}} c!$

If($p, T,$

If($\neg [p^{hc}], \mathcal{V}_2(c, p^h) :: \mathcal{V}_2(c, p^t), p^h ::$

```

let1(λd.
If(d, T,
let1(λr.
If(rE, error2([“Error in proof of”], d2 [ [“
”]¹r ] ), r), S(c, UM( [ E(d3, T, c) ‘ c ] ‘ p))), aspect(<proof>, pt))))]
[V2(c, p)  $\xrightarrow{\text{tex}}$  “
{\cal V} -2( #1.
, #2.
)”]
[V2(c, p)  $\xrightarrow{\text{pyk}}$  “verify two * proofs * end verify”]

```

$\mathcal{V}_3(*, *, *, *)$

```

[V3(c, r, p, d)  $\xrightarrow{\text{val}}$  c! [ r! [ p!
If(¬d, d,
If(p, T,
If(¬ [ phc ], V3(c, r, pt, V3(c, r, ph, T)),
let1(λi.
let1(λq.
If(q, T,
If(qE, q,
If(¬ [ q1 ], error2([“Unchecked sidecondition:”], q1h),
let1(λd.
If(d, error2([“Proof of non-existent lemma:”], q2),
If(¬ [ q2  $\stackrel{t}{=}$  [ d3 ] ], error2([“Lemma/proof mismatch:”], d2; [ q2 ] ),
V4(c, q0))), aspect(<stmt>, c[r][“codex”][r][i])))), pt), ph)))) ] ]
[V3(c, r, p, d)  $\xrightarrow{\text{tex}}$  “
{\cal V} -3( #1.
, #2.
, #3.
, #4.
)”]
[V3(c, r, p, d)  $\xrightarrow{\text{pyk}}$  “verify three * ref * sequents * diagnose * end verify”]

```

$\mathcal{V}_4(*, *)$

```

[V4(c, p)  $\xrightarrow{\text{val}}$  c!
If(p, T,
let1(λd.
If(¬d, d,

```

$\text{let}_1(\lambda p.$
 $\text{let}_1(\lambda r.$
 $\text{let}_1(\lambda i.$
 $\text{If}(\neg [c[r]["\text{diagnose"}]],$
 $\text{error}_2([\text{"Reference to erroneous page"}], p),$
 $\text{If}(\neg \text{claims}([\text{verifier}], c, r),$
 $\text{error}_2([\text{"Reference to unchecked lemma"}], p),$
 $\text{If}(\mathbf{aspect}(<\text{proof}>, p, c),$
 $\text{error}_2([\text{"Reference to unproved lemma"}], p), T))), p^i), p^r), p^h)), \mathcal{V}_4(c, p^t)))]$

$[\mathcal{V}_4(c, p) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal V}\}_4(\#1.$
 $, \#2.$
 $)"]$

$[\mathcal{V}_4(c, p) \xrightarrow{\text{pyk}} \text{"verify four * premises * end verify"}]$

$\mathcal{V}_5(*, *, *, *)$

$[\mathcal{V}_5(c, r, a, q) \xrightarrow{\text{val}} c! [r! [a!$
 $\text{If}(q^c, q,$
 $\text{If}(a, q,$
 $\text{If}(\neg [a^{hc}], \mathcal{V}_5(c, r, a^t, \mathcal{V}_5(c, r, a^h, q)),$
 $\mathcal{V}_7(c, r, a^h, q))))]]]$

$[\mathcal{V}_5(c, r, a, q) \xrightarrow{\text{tex}} "$
 $\{\backslash\text{cal V}\}_5(\#1.$
 $, \#2.$
 $, \#3.$
 $, \#4.$
 $)"]$

$[\mathcal{V}_5(c, r, a, q) \xrightarrow{\text{pyk}} \text{"verify five * ref * array * sequents * end verify"}]$

$\mathcal{V}_6(*, *, *, *)$

$[\mathcal{V}_6(c, r, p, q) \xrightarrow{\text{val}} c! [r! [p!$
 $\text{If}(q^c, q,$
 $\text{If}(p, q,$
 $\text{let}_1(\lambda q.$
 $\text{If}(q^c, q,$
 $\text{If}(\neg [r \approx [p^{hr}]], q,$
 $\mathcal{V}_7(c, r, p^{hi}, q))), \mathcal{V}_6(c, r, p^t, q))))]]]$

$[\mathcal{V}_6(c, r, p, q) \xrightarrow{\text{tex}} “\{\backslash\text{cal V}\}_6(\ #1.$
 $, \ #2.$
 $, \ #3.$
 $, \ #4.$
 $)”]$

$[\mathcal{V}_6(c, r, p, q) \xrightarrow{\text{pyk}} “\text{verify six * ref * list * sequents * end verify}”]$

$\mathcal{V}_7(*, *, *, *)$

$[\mathcal{V}_7(c, r, i, q) \xrightarrow{\text{val}} c! \ [r!$
 $\text{let}_1(\lambda v.$
 $\text{If}(v, q,$
 $\text{If}(v \approx 0, i,$
 $\text{If}(v \approx 1, q,$
 $\text{let}_1(\lambda q.$
 $\text{If}(q^c, q, q[i \rightarrow 1]), \mathcal{V}_6(c, r, v^0, q[i \rightarrow 0]))))), q[i]) \]]$
 $[\mathcal{V}_7(c, r, i, q) \xrightarrow{\text{tex}} “\{\backslash\text{cal V}\}_7(\ #1.$
 $, \ #2.$
 $, \ #3.$
 $, \ #4.$
 $)”]$

$[\mathcal{V}_7(c, r, i, q) \xrightarrow{\text{pyk}} “\text{verify seven * ref * id * sequents * end verify}”]$

$\text{Cut}(*, *)$

$[\text{Cut}(a, b) \xrightarrow{\text{val}} \text{If}(b, a, a; b)]$
 $[\text{Cut}(a, b) \xrightarrow{\text{tex}} “\text{Cut}(\ #1.$
 $, \ #2.$
 $)”]$

$[\text{Cut}(a, b) \xrightarrow{\text{pyk}} “\text{cut * and * end cut}”]$

$\text{Head}_{\oplus}(*)$

$[\text{Head}_{\oplus}(s) \xrightarrow{\text{val}} [s^1 \vdash [s^2 \vdash [s^{1I \triangleright}]]]^{+ \triangleright}]$

[Head_⊕(s) $\xrightarrow{\text{tex}}$ “
 Head_{\oplus} (\#1.
)”]

[Head_⊕(s) $\xrightarrow{\text{pyk}}$ “head * end head”]

Tail_⊕(*)

[Tail_⊕(s) $\xrightarrow{\text{val}}$ [s¹ \vdash [s^{2I}]] \triangleright]

[Tail_⊕(s) $\xrightarrow{\text{tex}}$ “
 Tail_{\oplus} (\#1.
)”]

[Tail_⊕(s) $\xrightarrow{\text{pyk}}$ “tail * end tail”]

rule₁(* , *)

[rule₁(s, t) $\xrightarrow{\text{val}}$

If(s $\stackrel{t}{=}$ t, T,

If(\neg [t $\stackrel{r}{=}$ [x \oplus y]], 0,

let₁($\lambda p.$

If(\neg [p^c], Cut(Head_⊕(t), p),

let₁($\lambda p.$

If(\neg [p^c], Cut(Tail_⊕(t), p), 0), rule₁(s, t²)), rule₁(s, t¹))))]

[rule₁(s, t) $\xrightarrow{\text{tex}}$ “

rule_1(#1.

, #2.

)”]

[rule₁(s, t) $\xrightarrow{\text{pyk}}$ “rule one * theory * end rule”]

rule(* , *)

[rule(c, p) $\xrightarrow{\text{val}}$ c!

let₁($\lambda s.$

If(s, [“Rule has no statement aspect”],

If(\neg [s $\stackrel{r}{=}$ [x \vdash y]], error₂([“Rule has invalid statement aspect”], s),

let₁($\lambda t.$

If(t, [“Theory has no statement aspect”],

let₁($\lambda r.$

If(r^c , error₂([“The theory does not assert the given rule”], $s; t$),
 $\boxed{[s^1 \vdash \text{Cut}(s^{1\text{I}\triangleright^*}, r)]}$, rule₁(s^2, t)), aspect($\langle \text{stmt} \rangle, s^1, c)^3$)), aspect($\langle \text{stmt} \rangle, p^t)^3$])
[rule(c, p) $\xrightarrow{\text{tex}}$ “rule(#1.
rule(#2.
)”]
[rule(c, p) $\xrightarrow{\text{pyk}}$ “rule * subcodex * end rule”]

Rule tactic

[Rule tactic $\xrightarrow{\text{val}}$ $\lambda c. \lambda p. \text{rule}(c, p)$]
[Rule tactic $\xrightarrow{\text{tex}}$ “Rule\ tactic”]
[Rule tactic $\xrightarrow{\text{pyk}}$ “rule tactic”]

Plus(*, *)

[Plus(a, b) $\xrightarrow{\text{val}}$ If($b, a, a \oplus b$)]
[Plus(a, b) $\xrightarrow{\text{tex}}$ “Plus(#1.
, #2.
)”]
[Plus(a, b) $\xrightarrow{\text{pyk}}$ “plus * and * end plus”]

[Theory *]

[[Theory n] $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \text{theory}_2(t, c)$]
[[Theory n] $\xrightarrow{\text{tex}}$ “[\mathbf{Theory} \] #1.
”]
[[Theory n] $\xrightarrow{\text{pyk}}$ “theory * end theory”]

theory₂(*, *)

[theory₂(t, c) $\xrightarrow{\text{val}}$
let₁(λn.
let₁(λs.
 $\tilde{Q}(t, [[n \xrightarrow{\text{stmt}} x]], s)$, [[n] :: n] :: [[x] :: theory₃(c, n)] :: T], t¹)]
[theory₂(t, c) $\xrightarrow{\text{tex}}$ “
theory_2(#1.
, #2.
)”]
[theory₂(t, c) $\xrightarrow{\text{pyk}}$ “theory two * cache * end theory”]

theory₃(*, *)

[theory₃(c, n) $\xrightarrow{\text{val}}$ n!
let₁(λr.
theory₄(c[r][“codex”][r], n, T), c[0])]
[theory₃(c, n) $\xrightarrow{\text{tex}}$ “
theory_3(#1.
, #2.
)”]
[theory₃(c, n) $\xrightarrow{\text{pyk}}$ “theory three * name * end theory”]

theory₄(*, *, *)

[theory₄(c, n, s) $\xrightarrow{\text{val}}$ n!
If(c, s,
If(\neg [c^{hc}], theory₄(c^t, n, theory₄(c^h, n, s)),
If(\neg [aspect(<proof>, c^t)³ $\stackrel{t}{=}$ [Rule tactic]], s,
let₁(λd.
If(\neg [d¹ $\stackrel{t}{=}$ n], s,
Plus(d², s)), aspect(<stmt>, c^t)³))))]
[theory₄(c, n, s) $\xrightarrow{\text{tex}}$ “
theory_4(#1.
, #2.
, #3.
)”]

[theory₄(c, n, s) $\xrightarrow{\text{pyk}}$ “theory four * name * sum * end theory”]

HeadNil”

[HeadNil” $\xrightarrow{\text{proof}}$ Rule tactic]

[HeadNil” $\xrightarrow{\text{stmt}}$ T'_E $\vdash [\top^h = \top]$]

[HeadNil” $\xrightarrow{\text{tex}}$ “
HeadNil”””]

[HeadNil” $\xrightarrow{\text{pyk}}$ “example axiom lemma primed”]

HeadPair”

[HeadPair” $\xrightarrow{\text{proof}}$ Rule tactic]

[HeadPair” $\xrightarrow{\text{stmt}}$ T'_E $\vdash \forall \underline{a} : \forall \underline{b} : [[\underline{a} :: \underline{b}]^h = \underline{a}]$]

[HeadPair” $\xrightarrow{\text{tex}}$ “
HeadPair”””]

[HeadPair” $\xrightarrow{\text{pyk}}$ “example scheme lemma primed”]

Transitivity”

[Transitivity” $\xrightarrow{\text{proof}}$ Rule tactic]

[Transitivity” $\xrightarrow{\text{stmt}}$ T'_E $\vdash \forall \underline{a} : \forall \underline{b} : \forall \underline{c} : [[\underline{a} = \underline{b}] \vdash [[\underline{a} = \underline{c}] \vdash [\underline{b} = \underline{c}]]]$]

[Transitivity” $\xrightarrow{\text{tex}}$ “
Transitivity”””]

[Transitivity” $\xrightarrow{\text{pyk}}$ “example rule lemma primed”]

Contra”

[Contra” $\xrightarrow{\text{proof}}$ Rule tactic]

[Contra” $\xrightarrow{\text{stmt}}$ T'_E $\vdash [[[\top :: \top] = \top] \vdash \perp]$]

[Contra” $\xrightarrow{\text{tex}}$ “
Contra”””]

[Contra” $\xrightarrow{\text{pyk}}$ “contraexample lemma primed”]

HeadNil

[HeadNil $\xrightarrow{\text{proof}}$ Rule tactic]

[HeadNil $\xrightarrow{\text{stmt}}$ $T_E \vdash [T^h = T]$]

[HeadNil $\xrightarrow{\text{tex}}$ “
HeadNil”]

[HeadNil $\xrightarrow{\text{pyk}}$ “example axiom lemma”]

HeadPair

[HeadPair $\xrightarrow{\text{proof}}$ Rule tactic]

[HeadPair $\xrightarrow{\text{stmt}}$ $T_E \vdash \forall a: \forall b: [[a :: b]^h = a]$]

[HeadPair $\xrightarrow{\text{tex}}$ “
HeadPair”]

[HeadPair $\xrightarrow{\text{pyk}}$ “example scheme lemma”]

Transitivity

[Transitivity $\xrightarrow{\text{proof}}$ Rule tactic]

[Transitivity $\xrightarrow{\text{stmt}}$ $T_E \vdash \forall a: \forall b: \forall c: [[a = b] \vdash [[a = c] \vdash [b = c]]]$]

[Transitivity $\xrightarrow{\text{tex}}$ “
Transitivity”]

[Transitivity $\xrightarrow{\text{pyk}}$ “example rule lemma”]

Contra

[Contra $\xrightarrow{\text{proof}}$ Rule tactic]

[Contra $\xrightarrow{\text{stmt}}$ $T_E \vdash [[[T :: T] = T] \vdash \perp]$]

[Contra $\xrightarrow{\text{tex}}$ “
Contra”]

[Contra $\xrightarrow{\text{pyk}}$ “contraexample lemma”]

T_E

$[T_E \xrightarrow{\text{stmt}} [[[T :: T] = T] \vdash \perp\perp] \oplus [[\forall \underline{a} : \forall \underline{b} : [[\underline{a} :: \underline{b}]^h = \underline{a}]] \oplus [[\underline{a} : \forall \underline{b} : \forall \underline{c} : [[\underline{a} = \underline{b}] \vdash [[\underline{a} = \underline{c}] \vdash [[\underline{b} = \underline{c}]]]] \oplus [[T^h = T]]]]$

$[T_E \xrightarrow{\text{tex}} "T_E"]$

$[T_E \xrightarrow{\text{pyk}} \text{"example theory"}]$

ragged right

$[\text{ragged right} \xrightarrow{\text{name}} " \text{ragged}\backslash \text{ right}"]$

$[\text{ragged right} \xrightarrow{\text{tex}} "\backslash \text{raggedright}"]$

$[\text{ragged right} \xrightarrow{\text{pyk}} \text{"ragged right"}]$

ragged right expansion

$[\text{ragged right expansion} \xrightarrow{\text{name}} " \text{ragged}\backslash \text{ right}\backslash \text{ expansion}\backslash "]$

$[\text{ragged right expansion} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{M}_4(t, s, c, [[\text{ragged right expansion} \doteq \text{ragged right}]])]$

$[\text{ragged right expansion} \xrightarrow{\text{tex}} ""]$

$[\text{ragged right expansion} \xrightarrow{\text{pyk}} \text{"ragged right expansion"}]$

parm(*, *, *)

$[\text{parm}(t, s, n) \xrightarrow{\text{val}} n! \\ \text{If}(t \stackrel{r}{=} [\forall x: y], \forall n: \text{parm}(t^2, [t^1 :: n] :: s, T + 2 * n), \\ \text{let}_1(\lambda m. \\ \text{If}(\neg m, m, t^R :: \text{parm}^*(t^t, s, n)), \text{lookup}(t, s, T)))]$

$[\text{parm}(t, s, n) \xrightarrow{\text{tex}} " \\ \text{parm}(\#1. \\ , \#2. \\ , \#3. \\)"]$

$\text{parm}(t, s, n) \xrightarrow{\text{pyk}} \text{“parameter term * stack * seed * end parameter”}$

$\text{parm}^*(*, *, *)$

$\text{[parm}^*(t, s, n) \xrightarrow{\text{val}} s! [n!\text{If}(t^a, T, \text{parm}(t^h, s, n) :: \text{parm}^*(t^t, s, n))]]$

$\text{[parm}^*(t, s, n) \xrightarrow{\text{tex}} “}$

$\text{parm}^*(\#1.$

$, \#2.$

$, \#3.$

$)”]$

$\text{[parm}^*(t, s, n) \xrightarrow{\text{pyk}} \text{“parameter term star * stack * seed * end parameter”}]$

$\text{inst}^*(*, *)$

$\text{[inst}(t, s) \xrightarrow{\text{val}} \text{If}(t^c, \text{inst}(s[t], s), t^R :: \text{inst}^*(t^t, s))]$

$\text{[inst}(t, s) \xrightarrow{\text{tex}} “}$

$\text{inst}(\#1.$

$, \#2.$

$)”]$

$\text{[inst}(t, s) \xrightarrow{\text{pyk}} \text{“instantiate * with * end instantiate”}]$

$\text{inst}^*(*, *)$

$\text{[inst}^*(t, s) \xrightarrow{\text{val}} s!\text{If}(t^a, T, \text{inst}(t^h, s) :: \text{inst}^*(t^t, s))]$

$\text{[inst}^*(t, s) \xrightarrow{\text{tex}} “}$

$\text{inst}^*(\#1.$

$, \#2.$

$)”]$

$\text{[inst}^*(t, s) \xrightarrow{\text{pyk}} \text{“instantiate star * with * end instantiate”}]$

$\text{occur}^*(*, *, *)$

$\text{[occur}(t, u, s) \xrightarrow{\text{val}} s!\text{If}(u^c, \text{If}(t \approx u, T, \text{occur}(t, s[u], s)), \text{occur}^*(t, u^t, s))]$

$\text{[occur}(t, u, s) \xrightarrow{\text{tex}} “}$

$\text{occur}(\#1.$

,#2.
,#3.
)]

[occur(t, u, s) $\xrightarrow{\text{pyk}}$ “occur * in * substitution * end occur”]

occur*(*, *, *)

[occur*(t, u, s) $\xrightarrow{\text{val}}$ t! [s!If(u^a, F, If(occur(t, u^h, s), T, occur*(t, u^t, s)))]]

[occur*(t, u, s) $\xrightarrow{\text{tex}}$ “
occur^*(#1.
,#2.
,#3.
)]

[occur*(t, u, s) $\xrightarrow{\text{pyk}}$ “occur star * in * substitution * end occur”]

unify(* = *, *)

[unify(t = u, s) $\xrightarrow{\text{val}}$ t! [u!
If(s^c, s,
If(t^c, unify₂(t = u, s),
If(u^c, unify₂(u = t, s),
If(t $\stackrel{r}{=}$ u, unify*(t^t = u^t, s, 0))))]]

[unify(t = u, s) $\xrightarrow{\text{tex}}$ “
unify(#1.
=#2.
,#3.
)]

[unify(t = u, s) $\xrightarrow{\text{pyk}}$ “unify * with * substitution * end unify”]

unify*(* = *, *)

[unify*(t = u, s) $\xrightarrow{\text{val}}$ u!If(t^a, s, unify*(t^t = u^t, unify(t^h = u^h, s)))]
[unify*(t = u, s) $\xrightarrow{\text{tex}}$ “
unify^*(#1.
=#2.
,#3.
)]

[unify*($t = u, s$) $\xrightarrow{\text{pyk}}$ “unify star * with * substitution * end unify”]

unify₂(* = *, *)

[unify₂($t = u, s$) $\xrightarrow{\text{val}}$
If($t \approx u, s$,
let₁($\lambda t'$.
If($\neg [t']$, unify($t' = u, s$),
If(occur($t, u, s, 0, s[t \rightarrow u]$), $s[t]$)))]
[unify₂($t = u, s$) $\xrightarrow{\text{tex}}$ “
unify_2(#1.
=#2.
,#3.
)”]

[unify₂($t = u, s$) $\xrightarrow{\text{pyk}}$ “unify two * with * substitution * end unify”]

L_a

[L_a $\xrightarrow{\text{name}}$ “L_a”]
[L_a $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_a \else
\if \relax \csname lgwella\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwella {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwella \fi ”]
[L_a $\xrightarrow{\text{pyk}}$ “ell a”]

L_b

[L_b $\xrightarrow{\text{name}}$ “L_b”]
[L_b $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_b \else
\if \relax \csname lgwellb\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellb {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellb \fi ”]
[L_b $\xrightarrow{\text{pyk}}$ “ell b”]

L_c

[L_c $\xrightarrow{\text{name}}$ “L_c”]
[L_c $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_c \else
\if \relax \csname lgwellc\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellc {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellc \fi ”]
[L_c $\xrightarrow{\text{pyk}}$ “ell c”]

L_d

[L_d $\xrightarrow{\text{name}}$ “L_d”]
[L_d $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_d \else
\if \relax \csname lgwelld\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwelld {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwelld \fi ”]
[L_d $\xrightarrow{\text{pyk}}$ “ell d”]

L_e

[L_e $\xrightarrow{\text{name}}$ “L_e”]
[L_e $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_e \else
\if \relax \csname lgwelle\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwelle {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwelle \fi ”]
[L_e $\xrightarrow{\text{pyk}}$ “ell e”]

L_f

[L_f $\xrightarrow{\text{name}}$ “L_f”]
[L_f $\xrightarrow{\text{tex}}$ “

```

\if \relax \csname lgwprooflinep\endcsname L_f \else
\if \relax \csname lgwellf\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellf {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellf \fi "]
[L_f  $\xrightarrow{\text{pyk}}$  "ell f"]

```

L_g

```

[L_g  $\xrightarrow{\text{name}}$  "L_g"]
[L_g  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_g \else
\if \relax \csname lgwellg\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellg {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellg \fi "]
[L_g  $\xrightarrow{\text{pyk}}$  "ell g"]

```

L_h

```

[L_h  $\xrightarrow{\text{name}}$  "L_h"]
[L_h  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_h \else
\if \relax \csname lgwellh\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellh {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellh \fi "]
[L_h  $\xrightarrow{\text{pyk}}$  "ell h"]

```

L_i

```

[L_i  $\xrightarrow{\text{name}}$  "L_i"]
[L_i  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_i \else
\if \relax \csname lgwelli\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwelli {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwelli \fi "]

```

[$L_i \xrightarrow{\text{pyk}} \text{"ell i"}$]

L_j

[$L_j \xrightarrow{\text{name}} \text{"L_j"}$]

[$L_j \xrightarrow{\text{tex}} \text{"}$

```
\if \relax \csname lgwprooflinep\endcsname L_j \else
\if \relax \csname lgwellj\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellj {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellj \fi "]
```

[$L_j \xrightarrow{\text{pyk}} \text{"ell j"}$]

L_k

[$L_k \xrightarrow{\text{name}} \text{"L_k"}$]

[$L_k \xrightarrow{\text{tex}} \text{"}$

```
\if \relax \csname lgwprooflinep\endcsname L_k \else
\if \relax \csname lgwellk\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellk {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellk \fi "]
```

[$L_k \xrightarrow{\text{pyk}} \text{"ell k"}$]

L_l

[$L_l \xrightarrow{\text{name}} \text{"L_l"}$]

[$L_l \xrightarrow{\text{tex}} \text{"}$

```
\if \relax \csname lgwprooflinep\endcsname L_l \else
\if \relax \csname lgwelll\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwelll {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwelll \fi "]
```

[$L_l \xrightarrow{\text{pyk}} \text{"ell l"}$]

L_m

[L_m $\xrightarrow{\text{name}}$ “L_m”]
[L_m $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_m \else
\if \relax \csname lgwellm\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellm {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellm \fi ”]
[L_m $\xrightarrow{\text{pyk}}$ “ell m”]

L_n

[L_n $\xrightarrow{\text{name}}$ “L_n”]
[L_n $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_n \else
\if \relax \csname lgwelln\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwelln {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwelln \fi ”]
[L_n $\xrightarrow{\text{pyk}}$ “ell n”]

L_o

[L_o $\xrightarrow{\text{name}}$ “L_o”]
[L_o $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_o \else
\if \relax \csname lgwello\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwello {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwello \fi ”]
[L_o $\xrightarrow{\text{pyk}}$ “ell o”]

L_p

[L_p $\xrightarrow{\text{name}}$ “L_p”]
[L_p $\xrightarrow{\text{tex}}$ “

```

\if \relax \csname lgwprooflinep\endcsname L_p \else
\if \relax \csname lgwellp\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellp {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellp \fi ”]
[Lp  $\xrightarrow{\text{pyk}}$  “ell p”]

```

L_q

```

[Lq  $\xrightarrow{\text{name}}$  “L_q”]
[Lq  $\xrightarrow{\text{tex}}$  “
\if \relax \csname lgwprooflinep\endcsname L_q \else
\if \relax \csname lgwellq\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellq {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellq \fi ”]
[Lq  $\xrightarrow{\text{pyk}}$  “ell q”]

```

L_r

```

[Lr  $\xrightarrow{\text{name}}$  “L_r”]
[Lr  $\xrightarrow{\text{tex}}$  “
\if \relax \csname lgwprooflinep\endcsname L_r \else
\if \relax \csname lgwellr\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellr {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellr \fi ”]
[Lr  $\xrightarrow{\text{pyk}}$  “ell r”]

```

L_s

```

[Ls  $\xrightarrow{\text{name}}$  “L_s”]
[Ls  $\xrightarrow{\text{tex}}$  “
\if \relax \csname lgwprooflinep\endcsname L_s \else
\if \relax \csname lgwells\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwells {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwells \fi ”]

```

[$L_s \xrightarrow{\text{pyk}} \text{“ell s”}$]

L_t

[$L_t \xrightarrow{\text{name}} \text{“L_t”}$]

[$L_t \xrightarrow{\text{tex}} \text{“}$

```
\if \relax \csname lgwprooflinep\endcsname L_t \else
\if \relax \csname lgwellt\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellt {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellt \fi ”]
```

[$L_t \xrightarrow{\text{pyk}} \text{“ell t”}$]

L_u

[$L_u \xrightarrow{\text{name}} \text{“L_u”}$]

[$L_u \xrightarrow{\text{tex}} \text{“}$

```
\if \relax \csname lgwprooflinep\endcsname L_u \else
\if \relax \csname lgwellu\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellu {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellu \fi ”]
```

[$L_u \xrightarrow{\text{pyk}} \text{“ell u”}$]

L_v

[$L_v \xrightarrow{\text{name}} \text{“L_v”}$]

[$L_v \xrightarrow{\text{tex}} \text{“}$

```
\if \relax \csname lgwprooflinep\endcsname L_v \else
\if \relax \csname lgwellv\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellv {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellv \fi ”]
```

[$L_v \xrightarrow{\text{pyk}} \text{“ell v”}$]

L_w

[L_w $\xrightarrow{\text{name}}$ “L_w”]
[L_w $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_w \else
\if \relax \csname lgwellw\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellw {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellw \fi ”]
[L_w $\xrightarrow{\text{pyk}}$ “ell w”]

L_x

[L_x $\xrightarrow{\text{name}}$ “L_x”]
[L_x $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_x \else
\if \relax \csname lgwellx\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellx {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellx \fi ”]
[L_x $\xrightarrow{\text{pyk}}$ “ell x”]

L_y

[L_y $\xrightarrow{\text{name}}$ “L_y”]
[L_y $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_y \else
\if \relax \csname lgwelly\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwelly {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwelly \fi ”]
[L_y $\xrightarrow{\text{pyk}}$ “ell y”]

L_z

[L_z $\xrightarrow{\text{name}}$ “L_z”]
[L_z $\xrightarrow{\text{tex}}$ “

```

\if \relax \csname lgwprooflinep\endcsname L_z \else
\if \relax \csname lgwellz\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellz {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellz \fi "]
[L_z  $\xrightarrow{\text{pyk}}$  "ell z"]

```

L_A

```

[L_A  $\xrightarrow{\text{name}}$  "L_A"]
[L_A  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_A \else
\if \relax \csname lgwellbiga\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbiga {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbiga \fi "]
[L_A  $\xrightarrow{\text{pyk}}$  "ell big a"]

```

L_B

```

[L_B  $\xrightarrow{\text{name}}$  "L_B"]
[L_B  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_B \else
\if \relax \csname lgwellbigb\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigb {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigb \fi "]
[L_B  $\xrightarrow{\text{pyk}}$  "ell big b"]

```

L_C

```

[L_C  $\xrightarrow{\text{name}}$  "L_C"]
[L_C  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_C \else
\if \relax \csname lgwellbigc\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigc {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigc \fi "]

```

[L_C $\xrightarrow{\text{pyk}}$ “ell big c”]

L_D

[L_D $\xrightarrow{\text{name}}$ “L_D”]

[L_D $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_D \else
\if \relax \csname lgwellbigd\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigd {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigd \fi ”]
```

[L_D $\xrightarrow{\text{pyk}}$ “ell big d”]

L_E

[L_E $\xrightarrow{\text{name}}$ “L_E”]

[L_E $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_E \else
\if \relax \csname lgwellbige\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbige {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbige \fi ”]
```

[L_E $\xrightarrow{\text{pyk}}$ “ell big e”]

L_F

[L_F $\xrightarrow{\text{name}}$ “L_F”]

[L_F $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_F \else
\if \relax \csname lgwellbigf\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigf {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigf \fi ”]
```

[L_F $\xrightarrow{\text{pyk}}$ “ell big f”]

L_G

[L_G $\xrightarrow{\text{name}}$ “L_G”]
[L_G $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_G \else
\if \relax \csname lgwellbigg\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigg {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigg \fi ”]
[L_G $\xrightarrow{\text{pyk}}$ “ell big g”]

L_H

[L_H $\xrightarrow{\text{name}}$ “L_H”]
[L_H $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_H \else
\if \relax \csname lgwellbigh\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigh {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigh \fi ”]
[L_H $\xrightarrow{\text{pyk}}$ “ell big h”]

L_I

[L_I $\xrightarrow{\text{name}}$ “L_I”]
[L_I $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_I \else
\if \relax \csname lgwellbigi\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigi {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigi \fi ”]
[L_I $\xrightarrow{\text{pyk}}$ “ell big i”]

L_J

[L_J $\xrightarrow{\text{name}}$ “L_J”]
[L_J $\xrightarrow{\text{tex}}$ “

```

\if \relax \csname lgwprooflinep\endcsname L_J \else
\if \relax \csname lgwellbigj\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigj {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigj \fi "]
[L_J  $\xrightarrow{\text{pyk}}$  "ell big j"]

```

L_K

```

[L_K  $\xrightarrow{\text{name}}$  "L_K"]
[L_K  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_K \else
\if \relax \csname lgwellbigk\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigk {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigk \fi "]
[L_K  $\xrightarrow{\text{pyk}}$  "ell big k"]

```

L_L

```

[L_L  $\xrightarrow{\text{name}}$  "L_L"]
[L_L  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_L \else
\if \relax \csname lgwellbigl\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigl {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigl \fi "]
[L_L  $\xrightarrow{\text{pyk}}$  "ell big l"]

```

L_M

```

[L_M  $\xrightarrow{\text{name}}$  "L_M"]
[L_M  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_M \else
\if \relax \csname lgwellbigm\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigm {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigm \fi "]

```

[L_M $\xrightarrow{\text{pyk}}$ “ell big m”]

L_N

[L_N $\xrightarrow{\text{name}}$ “L_N”]

[L_N $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_N \else
\if \relax \csname lgwellbign\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbign {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbign \fi ”]
```

[L_N $\xrightarrow{\text{pyk}}$ “ell big n”]

L_O

[L_O $\xrightarrow{\text{name}}$ “L_O”]

[L_O $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_O \else
\if \relax \csname lgwellbigo\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigo {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigo \fi ”]
```

[L_O $\xrightarrow{\text{pyk}}$ “ell big o”]

L_P

[L_P $\xrightarrow{\text{name}}$ “L_P”]

[L_P $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_P \else
\if \relax \csname lgwellbigp\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigp {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigp \fi ”]
```

[L_P $\xrightarrow{\text{pyk}}$ “ell big p”]

L_Q

[L_Q $\xrightarrow{\text{name}}$ “L_Q”]
[L_Q $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_Q \else
\if \relax \csname lgwellbigq\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigq {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigq \fi ”]
[L_Q $\xrightarrow{\text{pyk}}$ “ell big q”]

L_R

[L_R $\xrightarrow{\text{name}}$ “L_R”]
[L_R $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_R \else
\if \relax \csname lgwellbigr\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigr {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigr \fi ”]
[L_R $\xrightarrow{\text{pyk}}$ “ell big r”]

L_S

[L_S $\xrightarrow{\text{name}}$ “L_S”]
[L_S $\xrightarrow{\text{tex}}$ “
\if \relax \csname lgwprooflinep\endcsname L_S \else
\if \relax \csname lgwellbigs\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigs {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigs \fi ”]
[L_S $\xrightarrow{\text{pyk}}$ “ell big s”]

L_T

[L_T $\xrightarrow{\text{name}}$ “L_T”]
[L_T $\xrightarrow{\text{tex}}$ “

```

\if \relax \csname lgwprooflinep\endcsname L_T \else
\if \relax \csname lgwellbigt\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigt {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigt \fi "
[L_T  $\xrightarrow{\text{pyk}}$  "ell big t"]

```

L_U

```

[L_U  $\xrightarrow{\text{name}}$  "L_U"]
[L_U  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_U \else
\if \relax \csname lgwellbigu\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigu {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigu \fi "
[L_U  $\xrightarrow{\text{pyk}}$  "ell big u"]

```

L_V

```

[L_V  $\xrightarrow{\text{name}}$  "L_V"]
[L_V  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_V \else
\if \relax \csname lgwellbigv\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigv {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigv \fi "
[L_V  $\xrightarrow{\text{pyk}}$  "ell big v"]

```

L_W

```

[L_W  $\xrightarrow{\text{name}}$  "L_W"]
[L_W  $\xrightarrow{\text{tex}}$  "
\if \relax \csname lgwprooflinep\endcsname L_W \else
\if \relax \csname lgwellbigw\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigw {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigw \fi "

```

[L_W $\xrightarrow{\text{pyk}}$ “ell big w”]

L_X

[L_X $\xrightarrow{\text{name}}$ “L_X”]

[L_X $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_X \else
\if \relax \csname lgwellbigx\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigx {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigx \fi ”]
```

[L_X $\xrightarrow{\text{pyk}}$ “ell big x”]

L_Y

[L_Y $\xrightarrow{\text{name}}$ “L_Y”]

[L_Y $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_Y \else
\if \relax \csname lgwellbigy\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigy {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigy \fi ”]
```

[L_Y $\xrightarrow{\text{pyk}}$ “ell big y”]

L_Z

[L_Z $\xrightarrow{\text{name}}$ “L_Z”]

[L_Z $\xrightarrow{\text{tex}}$ “

```
\if \relax \csname lgwprooflinep\endcsname L_Z \else
\if \relax \csname lgwellbigz\endcsname
\global \advance \lgwproofline by 1
\xdef \lgwellbigz {L\ifnum \lgwproofline <10 0\fi \number \lgwproofline }
\fi \lgwellbigz \fi ”]
```

[L_Z $\xrightarrow{\text{pyk}}$ “ell big z”]

L?

[L? $\xrightarrow{\text{name}}$ “L_?”]

[L? $\xrightarrow{\text{tex}}$ “

\if \relax \csname lgwprooflinep\endcsname L_? \else
\global \advance \lgwproofline by 1

L\ifnum \lgwproofline <10 0\fi \number \lgwproofline
\fi ”]

[L? $\xrightarrow{\text{pyk}}$ “ell dummy”]

Reflexivity

[Reflexivity $\xrightarrow{\text{proof}}$ $T_E \vdash \forall \underline{a} : [[[\text{HeadPair}^{I\triangleright * \triangleright} @ \underline{a}] @ \underline{a}] ; [[[[$
Transitivity $^{I\triangleright * \triangleright} @ [[\underline{a} :: \underline{a}]^h] @ \underline{a}] @ \underline{a}]^{>\triangleright}]]]$]

[Reflexivity $\xrightarrow{\text{stmt}}$ $T_E \vdash \forall \underline{a} : [\underline{a} = \underline{a}]$]

[Reflexivity $\xrightarrow{\text{tex}}$ “
Reflexivity”]

[Reflexivity $\xrightarrow{\text{pyk}}$ “sequent reflexivity”]

Reflexivity₁

[Reflexivity₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([T_E \vdash \forall \underline{a} : [[[\text{HeadPair} \gg [[\underline{a} :: \underline{a}]^h = \underline{a}]] ; [[$
Transitivity $\triangleright [[\underline{a} :: \underline{a}]^h = \underline{a}] \triangleright [[\underline{a} :: \underline{a}]^h = \underline{a}] \gg [\underline{a} = \underline{a}]]]] , p_0, c)]$]

[Reflexivity₁ $\xrightarrow{\text{stmt}}$ $T_E \vdash \forall \underline{a} : [\underline{a} = \underline{a}]$]

[Reflexivity₁ $\xrightarrow{\text{tex}}$ “
Reflexivity_1”]

[Reflexivity₁ $\xrightarrow{\text{pyk}}$ “tactic reflexivity”]

Commutativity

[Commutativity $\xrightarrow{\text{proof}}$ $T_E \vdash \forall \underline{a} : \forall \underline{b} : [[[\underline{a} = \underline{b}] \vdash [[[\text{Reflexivity}^{I\triangleright * \triangleright} @ \underline{a}] ; [[[[$
Transitivity $^{I\triangleright * \triangleright} @ \underline{a}] @ \underline{b}] @ \underline{a}]^{>\triangleright}]]]]$]

[Commutativity $\xrightarrow{\text{stmt}}$ $T_E \vdash \forall \underline{a} : \forall \underline{b} : [[[\underline{a} = \underline{b}] \vdash [\underline{b} = \underline{a}]]]$]

[Commutativity $\xrightarrow{\text{tex}}$ “
Commutativity”]

[Commutativity $\xrightarrow{\text{pyk}}$ “sequent commutativity”]

Commutativity₁

[Commutativity₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil T_E \vdash \forall \underline{a}: \forall \underline{b}: [\underline{a} = \underline{b}] \vdash [\underline{a} = \underline{a}] \gg [\underline{b} = \underline{a}] \gg [\text{Reflexivity}_1 \gg [\underline{a} = \underline{a}]] ; [\underline{a} = \underline{b}] \triangleright [\underline{a} = \underline{b}] \triangleright [\underline{a} = \underline{a}] \gg [\underline{b} = \underline{a}]]], p_0, c)$]

[Commutativity₁ $\xrightarrow{\text{stmt}}$ $T_E \vdash \forall \underline{a}: \forall \underline{b}: [\underline{a} = \underline{b}] \vdash [\underline{b} = \underline{a}]$]

[Commutativity₁ $\xrightarrow{\text{tex}}$ “
Commutativity₋₁”]

[Commutativity₁ $\xrightarrow{\text{pyk}}$ “tactic commutativity”]

<tactic>

[<tactic> $\xrightarrow{\text{val}}$ [<tactic>]]

[<tactic> $\xrightarrow{\text{tex}}$ “
{<}tactic{>}”]

[<tactic> $\xrightarrow{\text{pyk}}$ “the tactic aspect”]

tactic

[tactic $\xrightarrow{\text{msg}}$ <tactic>]

[tactic $\xrightarrow{\text{tex}}$ “
tactic”]

[tactic $\xrightarrow{\text{pyk}}$ “tactic”]

[* $\stackrel{\text{tactic}}{=}$ *]

$[[x \stackrel{\text{tactic}}{=} y]] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \stackrel{\text{tactic}}{=} y] \doteq [(x) \xrightarrow{\text{pyk}} \text{tactic} y]])$

$[[x \stackrel{\text{tactic}}{=} y]] \xrightarrow{\text{tex}}$ “
[#1/tex name/tex.

\stackrel{\text{tactic}}{\text{stackrel}} \{tactic\}\{=\} \#2.
]”]

[$x^{\text{tactic}} = y$] $\xrightarrow{\text{pyk}}$ “tactic define * as * end define”]

$\mathcal{P}(*, *, *)$

$[\mathcal{P}(t, s, c) \xrightarrow{\text{val}} s!]$
let₁($\lambda d.$

If($d, t^h :: \mathcal{P}^*(t^t, s, c)$,
 $\mathcal{U}^M([[\mathcal{E}(d^3, T, c) ' t] ' s] ' c))$, aspect(<tactic>, t, c)))

$[\mathcal{P}(t, s, c) \xrightarrow{\text{tex}} “$
 $\{\backslash\text{cal } P\}(\ #1.$
, #2.
, #3.
)”]

$[\mathcal{P}(t, s, c) \xrightarrow{\text{pyk}} “\text{proof expand * state * cache * end expand}”]$

$\mathcal{P}^*(*, *, *)$

$[\mathcal{P}^*(t, s, c) \xrightarrow{\text{val}} s! [c!\text{If}(t, T, \mathcal{P}(t^h, s, c) :: \mathcal{P}^*(t^t, s, c))]]$

$[\mathcal{P}^*(t, s, c) \xrightarrow{\text{tex}} “$
 $\{\backslash\text{cal } P\}^*(\ #1.$
, #2.
, #3.
)”]

$[\mathcal{P}^*(t, s, c) \xrightarrow{\text{pyk}} “\text{proof expand list * state * cache * end expand}”]$

p_0

$[p_0 \xrightarrow{\text{val}} \mathcal{M}(\lambda t.\lambda s.\lambda c.\mathcal{P}(t, s, c)) :: T]$

$[p_0 \xrightarrow{\text{tex}} “$
 $p_0”]$

$[p_0 \xrightarrow{\text{pyk}} “\text{proof state}”]$

conclude₁(* , *)

[conclude₁(t, c) $\xrightarrow{\text{val}}$
let₁($\lambda r.$
If(r^c, error₂(["Unification failed"], t), r), conclude₂(t¹, t², c))]
[conclude₁(t, c) $\xrightarrow{\text{tex}}$ "
conclude_1 (#1.
, #2.
)"]
[conclude₁(t, c) $\xrightarrow{\text{pyk}}$ "conclude one * cache * end conclude"]

conclude₂(* , *, *)

[conclude₂(a, t, c) $\xrightarrow{\text{val}}$ t!
If(a $\stackrel{r}{=}$ [x \triangleright y], conclude₂(a¹, a-color(t \triangleright [a²]), c),
If(a $\stackrel{r}{=}$ [x \bowtie y], conclude₂(a¹, a-color(t \bowtie [a²]), c),
If(a $\stackrel{r}{=}$ [x @ y], conclude₂(a¹, a-color(t @ [a²]), c),
If(**aspect**(<proof>, a, c), error₂(["Lemma expected"], a),
let₁($\lambda d.$
conclude₃(a^{I \triangleright * \triangleright , t, $\text{parm}(d^{32}, T, 1)$, T), **aspect**(<stmt>, a, c))))])}

[conclude₂(a, t, c) $\xrightarrow{\text{tex}}$ "
conclude_2 (#1.
, #2.
, #3.
)"]
[conclude₂(a, t, c) $\xrightarrow{\text{pyk}}$ "conclude two * proves * cache * end conclude"]

conclude₃(* , *, *, *)

[conclude₃(a, t, l, s) $\xrightarrow{\text{val}}$ a! [t! [l! [s!
If(l $\stackrel{r}{=}$ [x \vdash y], [[
t] $\stackrel{r}{=}$ [x \triangleright y]] { conclude₃(a \triangleright , t¹, l², unify(l¹ = t², s)) ,
conclude₃(a \triangleright , t, l², s)
}
If(l $\stackrel{r}{=}$ [x \dashv y], [[
t] $\stackrel{r}{=}$ [x \bowtie y]] { conclude₃(a \triangleright , t¹, l², unify(l¹ = t², s)) ,
conclude₃(a \vee , t, l², s)
}
If(l $\stackrel{r}{=}$ [forall x: y], [[
t] $\stackrel{r}{=}$ [x @ y]] { conclude₃(a @ [t²], t¹, l², unify(l¹ = t², s)) ,
conclude₃(a @ [l¹], t, l², s)
}

let₁(λs.
If(s^c, s,
inst(a, s)), unify(l = t, s))))]]]]

[conclude₃(a, t, l, s) $\xrightarrow{\text{tex}}$ “
conclude_3 (#1.
, #2.
, #3.
, #4.
)”]

[conclude₃(a, t, l, s) $\xrightarrow{\text{pyk}}$ “conclude three * proves * lemma * substitution * end
conclude”]

-{}

[*-{*} $\xrightarrow{\text{name}}$ “#1.
\{\#2.
\}”]

[*-{*} $\xrightarrow{\text{tex}}$ “#1.
\{\#2.
\}”]

[*-{*} $\xrightarrow{\text{pyk}}$ “* sub * end sub”]

*

[x' $\xrightarrow{\text{tex}}$ “#1.”]
[x' $\xrightarrow{\text{pyk}}$ “* prime”]

*[*]

[a[k] $\xrightarrow{\text{val}}$ assoc₁(a, k, k)]

[*[*] $\xrightarrow{\text{tex}}$ “#1.
{[]}\#2.
{[]}”]
[*[*] $\xrightarrow{\text{pyk}}$ “* assoc * end assoc”]

$*[* \rightarrow *]$

$[a[i \rightarrow v] \xrightarrow{\text{val}} i^a \left\{ \begin{array}{l} v \left\{ \begin{array}{l} \text{array-remove}(i, a, 0) \\ \text{array-put}(i, v, a, 0) \end{array} \right. \\ v!a \end{array} \right\}]$

$[a[i \rightarrow v] \xrightarrow{\text{tex}} "\#1.$

$[\#2.$

$\{\backslash rightarrow\} \#3.$

$]"]$

$[a[i \rightarrow v] \xrightarrow{\text{pyk}} "* \text{ set } * \text{ to } * \text{ end set"}]$

$*[* \Rightarrow *]$

$[a[i \Rightarrow v] \xrightarrow{\text{val}} i^a \left\{ \begin{array}{l} a!v \\ a[i^h \rightarrow a[i^h][i^t \Rightarrow v]] \end{array} \right\}]$

$[a[i \Rightarrow v] \xrightarrow{\text{tex}} "\#1.$

$[\#2.$

$\{\backslash Rightarrow\} \#3.$

$]"]$

$[a[i \Rightarrow v] \xrightarrow{\text{pyk}} "* \text{ set multi } * \text{ to } * \text{ end set"}]$

newline *

$[\text{newline } x \xrightarrow{\text{name}} "$
 $\text{newline}\backslash \#1."]$

$[\text{newline } x \xrightarrow{\text{val}} x^M]$

$[\text{newline } x \xrightarrow{\text{tex}} "$
 $\backslash \text{newline } \#1."]$

$[\text{newline } x \xrightarrow{\text{pyk}} "\text{newline } *"]$

macro newline *

$[\text{macro newline } x \xrightarrow{\text{name}} "$
 $\text{macro}\backslash \text{ newline}\backslash \#1."]$

$[\text{macro newline } x \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{macro newline } x \doteq x]])]$

[macro newline x $\xrightarrow{\text{tex}}$ “\newline #1.”]

[macro newline x $\xrightarrow{\text{pyk}}$ “macro newline *”]

*0

[x0 $\xrightarrow{\text{val}}$ T +2* x]

[*0 $\xrightarrow{\text{tex}}$ “#1.
0”]

[*0 $\xrightarrow{\text{pyk}}$ “* bit nil”]

*1

[x1 $\xrightarrow{\text{val}}$ F +2* x]

[*1 $\xrightarrow{\text{tex}}$ “#1.
1”]

[*1 $\xrightarrow{\text{pyk}}$ “* bit one”]

0b

[0b $\xrightarrow{\text{val}}$ 0]

[0b $\xrightarrow{\text{tex}}$ “
0 \mathbf{b}”]

[0b $\xrightarrow{\text{pyk}}$ “binary”]

***-color(*)**

[a-color(t) $\xrightarrow{\text{val}}$ t^d { [t^r :: [tⁱ :: [a^d]]] :: [a-color*(t^t)]]

[x-color(y) $\xrightarrow{\text{tex}}$ “#1.
\mbox {-color}(#2.
)”]

[x-color(y) $\xrightarrow{\text{pyk}}$ “* color * end color”]

-color(*)

[$a\text{-color}^*(t) \xrightarrow{\text{val}} t \left\{ \begin{array}{l} a!T \\ a\text{-color}(t^h) :: [a\text{-color}^*(t^t)] \end{array} \right.]$

[$x\text{-color}^*(y) \xrightarrow{\text{tex}} "\#1.\newline \backslash mbox \{ -color \} ^{\backslash \ast}(\#2.\newline)"$]

[$x\text{-color}^*(y) \xrightarrow{\text{pyk}} "* \text{ color star } * \text{ end color}"$]

* , *

Predef: apply

[$*' * \xrightarrow{\text{tex}} "\#1.\newline \backslash mathbin \{ \backslash mbox \{ '\} \} \#2."$]

[$*' * \xrightarrow{\text{pyk}} "* \text{ apply } *"$]

* ‘ *

[$f ' x \xrightarrow{\text{val}} \text{apply}(f, x)$]

[$*' * \xrightarrow{\text{tex}} "\#1.\newline \backslash mathbin \{ \backslash mbox \{ '\} \} \#2."$]

[$*' * \xrightarrow{\text{pyk}} "* \text{ tagged apply } *"$]

$*^H$

[$x^H \xrightarrow{\text{val}} x , T$]

[$*^H \xrightarrow{\text{tex}} "\#1.\newline \{ \} ^H"$]

[$*^H \xrightarrow{\text{pyk}} "* \text{ raw head}"$]

$*^T$

[$x^T \xrightarrow{\text{val}} x , F$]

[$*^T \xrightarrow{\text{tex}} "\#1.\newline \{ \} ^T"$]

$[*^T \xrightarrow{\text{pyk}} \text{"* raw tail"}]$

$*^U$

$[x^U \xrightarrow{\text{val}} \text{if}(x^H, x^T, T)]$

$[*^U \xrightarrow{\text{tex}} \text{"\#1."}$

$\{\}^U"]$

$[*^U \xrightarrow{\text{pyk}} \text{"* cardinal untag"}]$

$*^h$

$[x^h \xrightarrow{\text{val}} x^{MTH}]$

$[*^h \xrightarrow{\text{tex}} \text{"\#1."}$

$\{\}^h"]$

$[*^h \xrightarrow{\text{pyk}} \text{"* head"}]$

$*^t$

$[x^t \xrightarrow{\text{val}} \text{if}(x^d, \text{if}(x^c, T \triangleleft [x^{MTT}], x^{MTT}), T)]$

$[*^t \xrightarrow{\text{tex}} \text{"\#1."}$

$\{\}^t"]$

$[*^t \xrightarrow{\text{pyk}} \text{"* tail"}]$

$*^s$

$[x^s \xrightarrow{\text{val}} x^{MTB}]$

$[*^s \xrightarrow{\text{tex}} \text{"\#1."}$

$\{\}^s"]$

$[*^s \xrightarrow{\text{pyk}} \text{"* is singular"}]$

$*^c$

$[x^c \xrightarrow{\text{val}} \text{if}(x, F, x^{MHB})]$

$[*_c \xrightarrow{\text{tex}} “\#1.”]$

$\{\}^c”]$

$[*_c \xrightarrow{\text{pyk}} “*_c \text{ is cardinal}”]$

$*^d$

$[x^d \xrightarrow{\text{val}} x^{\text{MHTHB}}]$

$[*_d \xrightarrow{\text{tex}} “\#1.”]$

$\{\}^d”]$

$[*_d \xrightarrow{\text{pyk}} “*_d \text{ is data}”]$

$*^a$

$[x^a \xrightarrow{\text{val}} [[\neg [x^d]] \vee [x^c]] \vee [x^s]]$

$[*_a \xrightarrow{\text{tex}} “\#1.”]$

$\{\}^a”]$

$[*_a \xrightarrow{\text{pyk}} “*_a \text{ is atomic}”]$

$*^C$

$[x^C \xrightarrow{\text{val}} \text{if}(x, T, x^{\text{HB}} \underline{+} 2 * [x^{TC}])]$

$[*_C \xrightarrow{\text{tex}} “\#1.”]$

$\{\}^C”]$

$[*_C \xrightarrow{\text{pyk}} “*_C \text{ cardinal retract}”]$

$*^M$

$[x^M \xrightarrow{\text{val}} \text{if}(x, T, \text{if}(x^H, T \dot{\subseteq} [x^{TC}], \text{if}(x^{\text{HTH}}, x^{\text{THM}} \dot{\subseteq} [x^{\text{TTM}}], \mathcal{M}(x^T))))]$

$[*_M \xrightarrow{\text{tex}} “\#1.”]$

$\{\}^M”]$

$[*_M \xrightarrow{\text{pyk}} “*_M \text{ tagged retract}”]$

$*^B$

$[x^B \xrightarrow{\text{val}} \text{if}(x, T, F)]$

$[*^B \xrightarrow{\text{tex}} "\#1.\n{}^B"]$

$[*^B \xrightarrow{\text{pyk}} "* \text{ boolean retract}]$

$*^r$

$[x^r \xrightarrow{\text{val}} x^{hh}]$

$[x^r \xrightarrow{\text{tex}} "\#1.\n{}^r"]$

$[x^r \xrightarrow{\text{pyk}} "* \text{ ref}]$

$*^i$

$[x^i \xrightarrow{\text{val}} x^{hth}]$

$[x^i \xrightarrow{\text{tex}} "\#1.\n{}^i"]$

$[x^i \xrightarrow{\text{pyk}} "* \text{ id}]$

$*^d$

$[x^d \xrightarrow{\text{val}} x^{htt}]$

$[x^d \xrightarrow{\text{tex}} "\#1.\n{}^d"]$

$[x^d \xrightarrow{\text{pyk}} "* \text{ debug}]$

$*^R$

$[x^R \xrightarrow{\text{val}} x^r :: [x^i :: T]]$

$[x^R \xrightarrow{\text{tex}} "\#1.\n{}^R"]$

$[x^R \xrightarrow{\text{pyk}} "* \text{ root}]$

$*^0$

$[x^0 \xrightarrow{\text{val}} x^h]$

$[x^0 \xrightarrow{\text{tex}} "\#1."]$
 $\{\}^{\wedge}\{0\}"]$

$[x^0 \xrightarrow{\text{pyk}} "* \text{zeroth}"]$

$*^1$

$[x^1 \xrightarrow{\text{val}} x^{t0}]$

$[x^1 \xrightarrow{\text{tex}} "\#1."]$
 $\{\}^{\wedge}\{1\}"]$

$[x^1 \xrightarrow{\text{pyk}} "* \text{first}"]$

$*^2$

$[x^2 \xrightarrow{\text{val}} x^{t1}]$

$[x^2 \xrightarrow{\text{tex}} "\#1."]$
 $\{\}^{\wedge}\{2\}"]$

$[x^2 \xrightarrow{\text{pyk}} "* \text{second}"]$

$*^3$

$[x^3 \xrightarrow{\text{val}} x^{t2}]$

$[x^3 \xrightarrow{\text{tex}} "\#1."]$
 $\{\}^{\wedge}\{3\}"]$

$[x^3 \xrightarrow{\text{pyk}} "* \text{third}"]$

$*^4$

$[x^4 \xrightarrow{\text{val}} x^{t3}]$

$[x^4 \xrightarrow{\text{tex}} "\#1."]$
 $\{\}^{\wedge}\{4\}"]$

$[x^4 \xrightarrow{\text{pyk}} "* \text{fourth}"]$

*⁵

[$x^5 \xrightarrow{\text{val}} x^{t4}$]

[$x^5 \xrightarrow{\text{tex}} \#\!1.$

{ }[^]{ 5 } ”]

[$x^5 \xrightarrow{\text{pyk}} \text{* fifth”}$]

*⁶

[$x^6 \xrightarrow{\text{val}} x^{t5}$]

[$x^6 \xrightarrow{\text{tex}} \#\!1.$

{ }[^]{ 6 } ”]

[$x^6 \xrightarrow{\text{pyk}} \text{* sixth”}$]

*⁷

[$x^7 \xrightarrow{\text{val}} x^{t6}$]

[$x^7 \xrightarrow{\text{tex}} \#\!1.$

{ }[^]{ 7 } ”]

[$x^7 \xrightarrow{\text{pyk}} \text{* seventh”}$]

*⁸

[$x^8 \xrightarrow{\text{val}} x^{t7}$]

[$x^8 \xrightarrow{\text{tex}} \#\!1.$

{ }[^]{ 8 } ”]

[$x^8 \xrightarrow{\text{pyk}} \text{* eighth”}$]

*⁹

[$x^9 \xrightarrow{\text{val}} x^{t8}$]

[$x^9 \xrightarrow{\text{tex}} \#\!1.$

{ }[^]{ 9 } ”]

[$x^9 \xrightarrow{\text{pyk}} \text{* ninth”}$]

$*^E$

$[x^E \xrightarrow{\text{val}} x \stackrel{r}{=} [xy]]$

$[x^E \xrightarrow{\text{tex}} "\#1.$

$\{\} \wedge \{ E \}"]$

$[x^E \xrightarrow{\text{pyk}} "* \text{ is error}"]$

$*^V$

$[t^V \xrightarrow{\text{val}} t \stackrel{r}{=} [a]]$

$[t^V \xrightarrow{\text{tex}} "\#1.$

$\{\} \wedge \{\backslash \text{cal V}\}"]$

$[t^V \xrightarrow{\text{pyk}} "* \text{ is metavar}"]$

$*^C$

$[t^C \xrightarrow{\text{val}} \text{If}(t^V, F, t^{t^C})]$

$[t^C \xrightarrow{\text{tex}} "\#1.$

$\{\} \wedge \{\backslash \text{cal C}\}"]$

$[t^C \xrightarrow{\text{pyk}} "* \text{ is metaclosed}"]$

$*^{C^*}$

$[t^{C^*} \xrightarrow{\text{val}} \text{If}(t, T, \text{If}(t^{hC}, t^{t^{C^*}}, F))]$

$[t^{C^*} \xrightarrow{\text{tex}} "\#1.$

$\{\} \wedge \{\{\backslash \text{cal C}\} \wedge \{ \backslash \text{ast}\} \}]$

$[t^{C^*} \xrightarrow{\text{pyk}} "* \text{ is metaclosed star}"]$

$* \cdot *$

$[x \cdot y \xrightarrow{\text{val}} \text{If}(x^c \wedge [y^c], x \cdot_0 y, T)]$

$[x \cdot y \xrightarrow{\text{tex}} "\#1.$

$\backslash \text{cdot} \#2."]$

$[x \cdot y \xrightarrow{\text{pyk}} "* \text{ times } *"]$

* ·₀ *

$$[x \cdot_0 y \xrightarrow{\text{val}} y^s \left\{ \begin{array}{l} x!0 \\ y^h \left\{ \begin{array}{l} T + 2 * [x \cdot_0 [y^t]] \\ (T + 2 * [x \cdot_0 [y^t]])^M +_0 x \end{array} \right\} \end{array} \right\}]$$

[$x \cdot_0 y \xrightarrow{\text{tex}} \#\!\!1.$
 $\backslash\text{cdot}_0 \#\!\!2.\!]$

[$x \cdot_0 y \xrightarrow{\text{pyk}} \text{"* times zero *"}]$

* + *

[$x + y \xrightarrow{\text{val}} \text{If}(x^c \wedge [y^c], x +_0 y, T)$]

[$x + y \xrightarrow{\text{tex}} \#\!\!1.$
 $+ \#\!\!2.\!]$

[$x + y \xrightarrow{\text{pyk}} \text{"* plus *"}]$

* +₀ *

$$[x +_0 y \xrightarrow{\text{val}} x^s \left\{ \begin{array}{l} y \\ y^s \left\{ \begin{array}{l} x \\ x^h \left\{ \begin{array}{l} y^h \left\{ \begin{array}{l} T + 2 * [x^t +_0 [y^t]] \\ F + 2 * [x^t +_0 [y^t]] \\ F + 2 * [x^t +_0 [y^t]] \\ T + 2 * [x^t +_1 [y^t]] \end{array} \right\} \end{array} \right\} \end{array} \right\} \end{array} \right\}]$$

[$x +_0 y \xrightarrow{\text{tex}} \#\!\!1.$
 $\backslash\text{mathop}\{+_0\} \#\!\!2.\!]$

[$x +_0 y \xrightarrow{\text{pyk}} \text{"* plus zero *"}]$

* +₁ *

$$[x +_1 y \xrightarrow{\text{val}} x^s \left\{ \begin{array}{l} y +_0 1 \\ y^s \left\{ \begin{array}{l} x +_0 1 \\ x^h \left\{ \begin{array}{l} y^h \left\{ \begin{array}{l} F + 2 * [x^t +_0 [y^t]] \\ T + 2 * [x^t +_1 [y^t]] \\ T + 2 * [x^t +_1 [y^t]] \\ F + 2 * [x^t +_1 [y^t]] \end{array} \right\} \end{array} \right\} \end{array} \right\} \end{array} \right\}]$$

[$x +_1 y \xrightarrow{\text{tex}} \#\!\!1.$

\mathop{+_1} \#2.]

[x +_1 y $\xrightarrow{\text{pyk}}$ “* plus one *”]

* — *

[x - y $\xrightarrow{\text{val}}$ If(x^c \wedge [y^c], If(x < y, 0, x -₀ y), T)]

[x - y $\xrightarrow{\text{tex}}$ “#1.
- #2.”]

[x - y $\xrightarrow{\text{pyk}}$ “* minus *”]

* -₀ *

[x -₀ y $\xrightarrow{\text{val}}$ y^s {
 x {
 y^h {
 T + 2* [x^t -₀ [y^t]]
 F + 2* [x^t -₁ [y^t]]
 }
 y^h {
 F + 2* [x^t -₀ [y^t]]
 T + 2* [x^t -₀ [y^t]]
 }
 }
}]

[x -₀ y $\xrightarrow{\text{tex}}$ “#1.
\mathop{-_0} \#2.”]

[x -₀ y $\xrightarrow{\text{pyk}}$ “* minus zero *”]

* -₁ *

[x -₁ y $\xrightarrow{\text{val}}$ y^s {
 x -₀ 1 {
 y^h {
 F + 2* [x^t -₁ [y^t]]
 T + 2* [x^t -₁ [y^t]]
 }
 y^h {
 T + 2* [x^t -₀ [y^t]]
 F + 2* [x^t -₁ [y^t]]
 }
 }
}]

[x -₁ y $\xrightarrow{\text{tex}}$ “#1.
\mathop{-_1} \#2.”]

[x -₁ y $\xrightarrow{\text{pyk}}$ “* minus one *”]

* \cup {*}

[x \cup {y} $\xrightarrow{\text{val}}$ If(y \in_t x, x, y :: x)]

$[x \cup \{y\} \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\cup \backslash\{ \#2.$
 $\backslash\}]$

$[x \cup \{y\} \xrightarrow{\text{pyk}} \text{"* term plus * end plus"}]$

$* \cup *$

$[x \cup y \xrightarrow{\text{val}} \text{If}(x^a, y, [x^t \cup y] \cup \{x^h\})]$

$[x \cup y \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\cup \#\mathbf{2}.]$

$[x \cup y \xrightarrow{\text{pyk}} \text{"* term union *"}]$

$* \backslash \{*\}$

$[x \setminus \{y\} \xrightarrow{\text{val}} \text{If}(x^a, y \neq \emptyset, \text{If}(y \stackrel{t}{=} [x^h], x^t, x^h :: [x^t \setminus \{y\}]))]$

$[x \setminus \{y\} \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\backslash \{ \#\mathbf{2}.$
 $\backslash\}]$

$[x \setminus \{y\} \xrightarrow{\text{pyk}} \text{"* term minus * end minus"}]$

$* \cdot *$

$[y \therefore z \xrightarrow{\text{val}} \lambda x. \text{if}(x, y, z)]$

$[* \therefore * \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\mathrel{.} \{ \backslash\dot{.} \backslash, . \} \#\mathbf{2}.]$

$[* \therefore * \xrightarrow{\text{pyk}} \text{"* raw pair *"}]$

$* \underline{\cdot} *$

$[x \underline{\cdot} y \xrightarrow{\text{val}} x : [y : [x \therefore y]]]$

$[* \underline{\cdot} * \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\mathrel{.} \{ \backslash\underline{\dot{.}} \backslash, . \} \#\mathbf{2}.]$

$[* \underline{\cdot} * \xrightarrow{\text{pyk}} \text{"* eager pair *"}]$

* : *

[$x \mathrel{\underline{:}} y \xrightarrow{\text{val}} (0 \mathrel{\underline{:}} [\mathrel{\underline{0}} \mathrel{\underline{:}} T])^I \mathrel{\underline{:}} [x \mathrel{\underline{:}} y]$]

[* : * $\xrightarrow{\text{tex}}$ “#1.

\mathrel{\underline{:}} \mathrel{\underline{:}} \mathrel{\underline{}} \mathrel{\underline{:}} \mathrel{\underline{}}

[* : * $\xrightarrow{\text{pyk}}$ “* tagged pair *”]

* +2* *

[$x +2* y \xrightarrow{\text{val}} \text{if}(x, \text{if}(y, T, x \mathrel{\underline{:}} y), x \mathrel{\underline{:}} y)$]

[* +2* * $\xrightarrow{\text{tex}}$ “#1.

\mathrel{\underline{:}} \mathrel{\underline{+}} \mathrel{\underline{:}} \mathrel{\underline{}} \mathrel{\underline{:}} \mathrel{\underline{}}

[* +2* * $\xrightarrow{\text{pyk}}$ “* untagged double *”]

* : *

[$x :: y \xrightarrow{\text{val}} x^M \mathrel{\underline{:}} [y^M]$]

[$x :: y \xrightarrow{\text{tex}}$ “#1.

\mathrel{\underline{:}} \mathrel{\underline{:}} \mathrel{\underline{}}

[* :: * $\xrightarrow{\text{pyk}}$ “* pair *”]

* +2* *

[$x +2* y \xrightarrow{\text{val}} T \mathrel{\underline{:}} [x^B \mathrel{\underline{:}} [y^{UC}]]$]

[* +2* * $\xrightarrow{\text{tex}}$ “#1.

\mathrel{\underline{:}} \mathrel{\underline{+}} \mathrel{\underline{:}} \mathrel{\underline{}} \mathrel{\underline{:}} \mathrel{\underline{}}

[* +2* * $\xrightarrow{\text{pyk}}$ “* double *”]

*, *

[$x, y \xrightarrow{\text{tex}}$ “#1.

, \linebreak [0] #2.”]

[$x, y \xrightarrow{\text{pyk}}$ “* comma *”]

$* \approx^B *$

$[x \approx^B y \xrightarrow{\text{val}} x \left\{ \begin{array}{l} \text{If}(y, T, F) \\ \text{If}(y, F, T) \end{array} \right\}]$

$[* \approx^B * \xrightarrow{\text{tex}} "\#1.\\" \text{stackrel}\{B\}{\backslash \text{approx}} \#\#2.]$

$[* \approx^B * \xrightarrow{\text{pyk}} "* \text{ boolean equal } *"]$

$* \approx^D *$

$[x \approx^D y \xrightarrow{\text{val}} x^c \left\{ \begin{array}{l} \text{If}(y^c, x \approx^C y, F) \\ \text{If}(y^c, F, x \approx^P y) \end{array} \right\}]$

$[* \approx^D * \xrightarrow{\text{tex}} "\#1.\\" \text{stackrel}\{D\}{\backslash \text{approx}} \#\#2.]$

$[* \approx^D * \xrightarrow{\text{pyk}} "* \text{ data equal } *"]$

$* \approx^C *$

$[x \approx^C y \xrightarrow{\text{val}} x^s \left\{ \begin{array}{l} \text{If}(y^s, T, F) \\ \text{If}(y^s, F, [x^h \approx^B [y^h]] \wedge [x^t \approx^C [y^t]]) \end{array} \right\}]$

$[* \approx^C * \xrightarrow{\text{tex}} "\#1.\\" \text{stackrel}\{C\}{\backslash \text{approx}} \#\#2.]$

$[* \approx^C * \xrightarrow{\text{pyk}} "* \text{ cardinal equal } *"]$

$* \approx^P *$

$[x \approx^P y \xrightarrow{\text{val}} x^s \left\{ \begin{array}{l} \text{If}(y^s, T, F) \\ \text{If}(y^s, F, [x^h \approx [y^h] \wedge [x^t \approx [y^t]]) \end{array} \right\}]$

$[* \approx^P * \xrightarrow{\text{tex}} "\#1.\\" \text{stackrel}\{P\}{\backslash \text{approx}} \#\#2.]$

$[* \approx^P * \xrightarrow{\text{pyk}} "* \text{ peano equal } *"]$

$* \approx *$

$$[x \approx y \xrightarrow{\text{val}} x^d \left\{ \begin{array}{l} \text{If}(y^d, x \overset{D}{\approx} y, F) \\ \text{If}(y^d, F, T) \end{array} \right]$$

$[* \approx * \xrightarrow{\text{tex}} "\#1."]$
 $\backslash \text{approx} \#2."]$

$[* \approx * \xrightarrow{\text{pyk}} "* \text{ tagged equal } *"]$

$* = *$

$[* = * \xrightarrow{\text{tex}} "\#1."]$
 $= \#2."]$

$[* = * \xrightarrow{\text{pyk}} "* \text{ math equal } *"]$

$* \xrightarrow{+} *$

$[* \xrightarrow{+} * \xrightarrow{\text{tex}} "\#1."]$
 $\backslash \text{stackrel} \{+\} \{\rightarrow\} \#2."]$

$[* \xrightarrow{+} * \xrightarrow{\text{pyk}} "* \text{ reduce to } *"]$

$* \xrightarrow{t} *$

$[x \xrightarrow{t} y \xrightarrow{\text{val}} \text{If}(x = y, x^t \xrightarrow{t^*} [y^t], F)]$

$[* \xrightarrow{t} * \xrightarrow{\text{tex}} "\#1."]$
 $\backslash \text{stackrel} \{t\} \{=\} \#2."]$

$[* \xrightarrow{t} * \xrightarrow{\text{pyk}} "* \text{ term equal } *"]$

$* \xrightarrow{t^*} *$

$[x \xrightarrow{t^*} y \xrightarrow{\text{val}} x^a \left\{ \begin{array}{l} \text{If}(y^a, T, F) \\ \text{If}(y^a, F, \text{If}(x^h \xrightarrow{t} [y^h], x^t \xrightarrow{t^*} [y^t], F)) \end{array} \right]$

$[* \xrightarrow{t^*} * \xrightarrow{\text{tex}} "\#1."]$
 $\backslash \text{stackrel} \{t^*\} \{=\} \#2."]$

$[* \xrightarrow{t^*} * \xrightarrow{\text{pyk}} "* \text{ term list equal } *"]$

$* \stackrel{r}{=} *$

$[x \stackrel{r}{=} y \stackrel{\text{val}}{\rightarrow} \text{If}(x^r \approx [y^r], x^i \approx [y^i], F)]$

$[x \stackrel{r}{=} y \stackrel{\text{tex}}{\rightarrow} "\#1."$

$\backslash \text{stackrel}\{r\}\{=\} \#2."$

$[x \stackrel{r}{=} y \stackrel{\text{pyk}}{\rightarrow} "* \text{ term root equal } *"]$

$* \in_t *$

$[x \in_t y \stackrel{\text{val}}{\rightarrow} \text{If}(y^a, x!F, \text{If}(x \stackrel{t}{=} [y^h], T, x \in_t [y^t]))]$

$[x \in_t y \stackrel{\text{tex}}{\rightarrow} "\#1."$

$\backslash \text{in_t} \#2."$

$[x \in_t y \stackrel{\text{pyk}}{\rightarrow} "* \text{ term in } *"]$

$* \subseteq_T *$

$[x \subseteq_T y \stackrel{\text{val}}{\rightarrow} \text{If}(x^a, y!T, \text{If}(x^h \in_t y, x^t \subseteq_T y, F))]$

$[x \subseteq_T y \stackrel{\text{tex}}{\rightarrow} "\#1."$

$\backslash \text{subsequeq_T} \#2."$

$[x \subseteq_T y \stackrel{\text{pyk}}{\rightarrow} "* \text{ term subset } *"]$

$* \stackrel{T}{=} *$

$[x \stackrel{T}{=} y \stackrel{\text{val}}{\rightarrow} \text{If}(x \subseteq_T y, y \subseteq_T x, F)]$

$[x \stackrel{T}{=} y \stackrel{\text{tex}}{\rightarrow} "\#1."$

$\backslash \text{stackrel}\{T\}\{=\} \#2."$

$[x \stackrel{T}{=} y \stackrel{\text{pyk}}{\rightarrow} "* \text{ term set equal } *"]$

$* \stackrel{s}{=} *$

$[x \stackrel{s}{=} y \stackrel{\text{val}}{\rightarrow} \text{If}(\neg [x^2 \stackrel{t}{=} [y^2]], F, \text{If}(x^0 \stackrel{T}{=} [y^0], x^1 \stackrel{T}{=} [y^1], F))]$

$[x \stackrel{s}{=} y \stackrel{\text{tex}}{\rightarrow} "\#1."$

$\backslash \text{stackrel}\{s\}\{=\} \#2."$

$[x \stackrel{s}{=} y \stackrel{\text{pyk}}{\rightarrow} "* \text{ sequent equal } *"]$

* free in *

[v free in t $\xrightarrow{\text{val}}$
 If(v $\stackrel{t}{=}$ t, T,
 If(\neg [t $\stackrel{r}{=}$ $\lceil \forall*: * \rceil$], v free in* [t^t],
 If(v $\stackrel{t}{=}$ [t¹], F, v free in [t²])))]

[x free in y $\xrightarrow{\text{tex}}$ “#1.
 \mathrel{\{free\}} \in \#2.”]
 [v free in t $\xrightarrow{\text{pyk}}$ “* free in *”]

* free in* *

[v free in* t $\xrightarrow{\text{val}}$ If(t, v!F, If(v free in [t^h], T, v free in* [t^t]))]
 [x free in* y $\xrightarrow{\text{tex}}$ “#1.
 \mathrel{\{free\}} \in \{\ast\} \#2.”]
 [v free in* t $\xrightarrow{\text{pyk}}$ “* free in star *”]

* free for * in *

[a free for x in b $\xrightarrow{\text{val}}$ a! [x!
 If(b^v, T,
 If(\neg [b $\stackrel{r}{=}$ $\lceil \forall*: * \rceil$], a free for* x in [b^t],
 If(x $\stackrel{t}{=}$ [b¹], T,
 If(\neg [x free in [b²]], T,
 If(b¹ free in a, F, [
 a] free for x in [b²])))))]]
 [a free for x in b $\xrightarrow{\text{tex}}$ “#1.
 \mathrel{\{free\}} \text{for} \#2.
 \mathrel{\{in\}} \#3.”]
 [a free for x in b $\xrightarrow{\text{pyk}}$ “* free for * in *”]

* free for* * in *

[a free for* x in b $\xrightarrow{\text{val}}$
 If(b, a! [x!T],
 If(a free for x in [b^h], a free for* x in [b^t], F))]

[a free for* x in $b \xrightarrow{\text{tex}} \#1$.

\mathrel{\{free\}} for \} ^{\{ \ast \}} \#2.

\mathrel{\{in\}} \#3.]

[a free for* $*$ in $b \xrightarrow{\text{pyk}} \text{"* free for star * in *"}]$

$* \in_c *$

[$x \in_c y \xrightarrow{\text{val}} [y \stackrel{r}{=} [x \wedge_c y]] \left\{ \begin{array}{l} \text{If}(x \in_c [y^1], T, x \in_c [y^2]) \\ x \stackrel{t}{=} y \end{array} \right.]$

[$x \in_c y \xrightarrow{\text{tex}} \#1$.

\in_c \#2.]

[$x \in_c y \xrightarrow{\text{pyk}} \text{"* claim in *"}]$

$* < *$

[$x < y \xrightarrow{\text{val}} \text{If}(x^c \wedge [y^c], x <^r y, F)$]

[$x < y \xrightarrow{\text{tex}} \#1$.

< \#2.]

[$x < y \xrightarrow{\text{pyk}} \text{"* less *"}]$

$* <^r *$

[$x <^r y \xrightarrow{\text{val}} y^s \left\{ \begin{array}{l} x!F \\ x^s \left\{ \begin{array}{l} T \\ x^h \left\{ \begin{array}{l} y^h \left\{ \begin{array}{l} x^t <^r [y^t] \\ x^t \leq^r [y^t] \end{array} \right. \\ y^h \left\{ \begin{array}{l} x^t <^r [y^t] \\ x^t <^r [y^t] \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right.]$

[$x <^r y \xrightarrow{\text{tex}} \#1$.

<^r \#2.]

[$x <^r y \xrightarrow{\text{pyk}} \text{"* less zero *"}]$

$* \leq' *$

$$[x \leq' y \xrightarrow{\text{val}} x^s \left\{ \begin{array}{l} y^t \\ y^s \left\{ \begin{array}{l} F \\ x^h \left\{ \begin{array}{l} y^h \left\{ \begin{array}{l} x^t \leq' [y^t] \\ x^t \leq' [y^t] \\ x^t < [y^t] \\ x^t \leq' [y^t] \end{array} \right. \end{array} \right. \end{array} \right. \end{array} \right]$$

$[x \leq' y \xrightarrow{\text{tex}} "\#1." \\ \backslash le' \#2.]$

$[x \leq' y \xrightarrow{\text{pyk}} "* \text{ less one } *"]$

$\neg *$

$[\neg x \xrightarrow{\text{val}} \text{If}(x, F, T)]$

$[\neg * \xrightarrow{\text{tex}} " \\ \{\backslash neg \} \#1."]$

$[\neg * \xrightarrow{\text{pyk}} "\text{not } *"]$

$* \wedge *$

$[x \wedge y \xrightarrow{\text{val}} x \left\{ \begin{array}{l} \text{If}(y, T, F) \\ \text{If}(y, F, F) \end{array} \right.]$

$[* \wedge * \xrightarrow{\text{tex}} "\#1." \\ \backslash wedge \#2.]$

$[* \wedge * \xrightarrow{\text{pyk}} "* \text{ and } *"]$

$* \ddot{\wedge} *$

$[x \ddot{\wedge} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\lceil [x \ddot{\wedge} y \doteq \text{If}(x, y, F)] \rceil])]$

$[x \ddot{\wedge} y \xrightarrow{\text{tex}} "\#1." \\ \backslash mathrel{\{\ddot{\wedge}\}} \#2.]$

$[x \ddot{\wedge} y \xrightarrow{\text{pyk}} "* \text{ macro and } *"]$

$* \tilde{\wedge} *$

$[x \tilde{\wedge} y \xrightarrow{\text{val}} \text{if}(x, y, x)]$

$[x \tilde{\wedge} y \xrightarrow{\text{tex}} "\#1.\newline \mathrel{\{\backslash tilde\{\backslash wedge\}} \#2.}]$

$[x \tilde{\wedge} y \xrightarrow{\text{pyk}} "* \text{ simple and } *"]$

$* \wedge_c *$

$[x \wedge_c y \xrightarrow{\text{val}} \lambda t. \lambda c. [[[x't]'c] \tilde{\wedge} [[y't]'c]]]$

$[x \wedge_c y \xrightarrow{\text{tex}} "\#1.\newline \wedge_c \#2.]"$

$[x \wedge_c y \xrightarrow{\text{pyk}} "* \text{ claim and } *"]$

$* \vee *$

$[x \vee y \xrightarrow{\text{val}} x \left\{ \begin{array}{l} \text{If}(y, T, T) \\ \text{If}(y, T, F) \end{array} \right.$

$[* \vee * \xrightarrow{\text{tex}} "\#1.\newline \vee \#2.]"$

$[* \vee * \xrightarrow{\text{pyk}} "* \text{ or } *"]$

$* \parallel *$

$[* \parallel * \xrightarrow{\text{tex}} "\#1.\newline \parallel \#2.]"$

$[* \parallel * \xrightarrow{\text{pyk}} "* \text{ parallel } *"]$

$* \ddot{\vee} *$

$[x \ddot{\vee} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \ddot{\vee} y \doteq \text{If}(x, T, y)]])]$

$[x \ddot{\vee} y \xrightarrow{\text{tex}} "\#1.\newline \mathrel{\{\backslash ddot\{\backslash vee\}} \#2.}"$

$[x \ddot{\vee} y \xrightarrow{\text{pyk}} "* \text{ macro or } *"]$

* → *

```
[x \Rightarrow y \stackrel{\text{macro}}{\rightarrow} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [x \Rightarrow y \stackrel{\text{If}}{=} \text{If}(x, y, T)])]
[x \Rightarrow y \stackrel{\text{tex}}{\rightarrow} "\#1.
\mathrel{\{\ddot{\rangle}\}} \#2."]
[x \Rightarrow y \stackrel{\text{pyk}}{\rightarrow} "* \text{ macro imply } *"]
```

* : *

$[x : y \xrightarrow{\text{val}} \text{if}(x, y, y)]$

$[*: * \xrightarrow{\text{tex}} \text{"#1."} : \#2.]$

$[* : * \xrightarrow{\text{pyk}} “* \text{ guard } *”]$

!

$$[x!y \stackrel{\text{val}}{\Rightarrow} \text{If}(x, y, y)]$$

[*!*^{tex} → “#1.
!#2.”]

$\left[*!*\stackrel{\text{pyk}}{\rightarrow} “\text{* tagged guard *}” \right]$

* *

$$[\times \left\{ \begin{array}{l} y \\ z \end{array} \right. \stackrel{\text{val}}{\Rightarrow} \text{If}(x, y, z)]$$

```
[* { * tex "#1.  
\left{\protect \begin{array}{l}#2.  
\#3.
```

```
\protect \end {array}\right." ]  
[ * { *  $\stackrel{\text{pyk}}{\rightarrow}$  "* select * else * end select" ]
```

$\lambda * .*$

Predef: lambda

$[\lambda * .* \xrightarrow{\text{tex}} “$

$\backslash\text{lambda } \#1.$

$.\#2.”]$

$[\lambda * .* \xrightarrow{\text{pyk}} “\text{lambda * dot *}”]$

$\Lambda *$

$[\Lambda x \xrightarrow{\text{val}} \mathcal{M}(\lambda u.\mathcal{U}(x, \mathcal{M}(u)))]$

$[\Lambda * \xrightarrow{\text{tex}} “$

$\backslash\text{Lambda } \#1.”]$

$[\Lambda * \xrightarrow{\text{pyk}} “\text{tagging *}”]$

if * **then** * **else** *

$[\text{if } x \text{ then } y \text{ else } z \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [\text{if } x \text{ then } y \text{ else } z \doteq \text{If}(x, y, z)])])$

$[\text{if } x \text{ then } y \text{ else } z \xrightarrow{\text{tex}} “$

$\{\backslash\text{bf if}\} \backslash \#1.$

$\backslash \{\backslash\text{bf then}\} \backslash \#2.$

$\backslash \{\backslash\text{bf else}\} \backslash \#3.”]$

$[\text{if } x \text{ then } y \text{ else } z \xrightarrow{\text{pyk}} “\text{open if * then * else *}”]$

let * = * **in** *

$[\text{let } x = y \text{ in } z \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [\text{let } x = y \text{ in } z \doteq \text{let}_1(\lambda x.z, y)])])$

$[\text{let } x = y \text{ in } z \xrightarrow{\text{tex}} “$

$\backslash\text{mathbf{let}}\backslash \} \#1.$

$= \#2.$

$\backslash\text{mathbf{\{} \backslash\text{ in} \backslash\text{ \}}} \#3.”]$

$[\text{let } x = y \text{ in } z \xrightarrow{\text{pyk}} “\text{let * be * in *}”]$

let * $\ddot{=}$ * in *

[**let** x $\ddot{=}$ y in z $\xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.$

$\tilde{M}(t^3, s, c[t^{1r} :: ["codex" :: [t^{1r} :: [t^{1i} :: [0 :: ["macro" :: T]]]]] \Rightarrow \tilde{M}_3(t)])]$

[**let** x $\ddot{=}$ y in z $\xrightarrow{\text{tex}}$ “

$\backslash\text{mathbf}{\{let\}} \#1.$

$\backslash\text{mathrel}{\{\ddot{=}\}} \#2.$

$\backslash\text{mathrel}{\{\text{in}\}} \#3."$]

[**let** x $\ddot{=}$ y in z $\xrightarrow{\text{pyk}}$ “let * abbreviate * in *”]

*^I

[x^I $\xrightarrow{\text{val}}$ $\lceil x^I \rceil^R :: [x :: T]$]

[x^I $\xrightarrow{\text{tex}}$ “#1.

{ } ^ { I }

[x^I $\xrightarrow{\text{pyk}}$ “* init”]

*^D

[x^D $\xrightarrow{\text{val}}$ $\lceil x^D \rceil^R :: [x :: T]$]

[x^D $\xrightarrow{\text{tex}}$ “#1.

{ } ^ { \backslash rhd }

[x^D $\xrightarrow{\text{pyk}}$ “* modus”]

*^V

[x^V $\xrightarrow{\text{val}}$ $\lceil x^V \rceil^R :: [x :: T]$]

[x^V $\xrightarrow{\text{tex}}$ “#1.

{ } ^ { V }

[x^V $\xrightarrow{\text{pyk}}$ “* verify”]

*⁺

[x⁺ $\xrightarrow{\text{val}}$ $\lceil x^+ \rceil^R :: [x :: T]$]

$[x^+ \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\{\} \wedge \{ + \}"]$
 $[x^+ \xrightarrow{\text{pyk}} \text{"* curry plus"}]$

$*^-$

$[x^- \xrightarrow{\text{val}} [x^-]^R :: [x :: T]]$
 $[x^- \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\{\} \wedge \{ - \}"]$
 $[x^- \xrightarrow{\text{pyk}} \text{"* curry minus"}]$

$*^*$

$[x^* \xrightarrow{\text{val}} [x^*]^R :: [x :: T]]$
 $[x^* \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\{\} \wedge \{ \backslash\text{ast} \}"]$
 $[x^* \xrightarrow{\text{pyk}} \text{"* dereference"}]$

$* @ *$

$[x @ y \xrightarrow{\text{val}} [x @ y]^R :: [x :: [y :: T]]]$
 $[x @ y \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\text{mathop} \{ \backslash\text{char64} \} \#2.]$
 $[x @ y \xrightarrow{\text{pyk}} \text{"* at *"}]$

$* \triangleright *$

$[x \triangleright y \xrightarrow{\text{val}} [x \triangleright y]^R :: [x :: [y :: T]]]$
 $[x \triangleright y \xrightarrow{\text{tex}} \#\mathbf{1}.$
 $\backslash\text{rhd} \#2.]$
 $[x \triangleright y \xrightarrow{\text{pyk}} \text{"* modus ponens *"}]$

* \triangleright *

[$x \triangleright y \xrightarrow{\text{val}} [x \triangleright y]^R :: [x :: [y :: T]]$]
[$x \triangleright y \xrightarrow{\text{tex}} "\#1.\mathrel{\backslash\text{mathrel{}}\{\backslash\text{makebox}[0mm][l]\{\$\backslash\text{rhd}\$}\},\{\backslash\text{rhd}\}}\#2."$]
[$x \triangleright y \xrightarrow{\text{pyk}} "* \text{ modus probans } *$]

* \gg *

[$x \gg y \xrightarrow{\text{tactic}} \lambda t. \lambda s. \lambda c. \text{conclude}_1(t, c)$]
[$x \gg y \xrightarrow{\text{tex}} "\#1.\mathrel{\backslash\text{gg}}\#2."$]
[$x \gg y \xrightarrow{\text{pyk}} "* \text{ conclude } *$]

* \vdash *

[$x \vdash y \xrightarrow{\text{val}} [x \vdash y]^R :: [x :: [y :: T]]$]
[$x \vdash y \xrightarrow{\text{tex}} "\#1.\mathrel{\backslash\text{vdash}}\#2."$]
[$x \vdash y \xrightarrow{\text{pyk}} "* \text{ infer } *$]

* \Vdash *

[$x \Vdash y \xrightarrow{\text{val}} [x \Vdash y]^R :: [x :: [y :: T]]$]
[$x \Vdash y \xrightarrow{\text{tex}} "\#1.\mathrel{\backslash\text{mathrel{}}\{\backslash\text{makebox}[0mm][l]\{\$\backslash\text{vdash}\$}\},\{\backslash\text{vdash}\}}\#2."$]
[$x \Vdash y \xrightarrow{\text{pyk}} "* \text{ endorse } *$]

* i.e. *

[$x \text{ i.e. } y \xrightarrow{\text{val}} [x \text{ i.e. } y]^R :: [x :: [y :: T]]$]
[$x \text{ i.e. } y \xrightarrow{\text{tex}} "\#1.\mathrel{\backslash\text{mathrel{}}\{\text{i.e.}\}}\#2."$]
[$x \text{ i.e. } y \xrightarrow{\text{pyk}} "* \text{ id est } *$]

$\forall * : *$

$[\forall x: y \xrightarrow{\text{val}} [\forall x: y]^R :: [x :: [y :: T]]]$

$[\forall x: y \xrightarrow{\text{tex}} ``$

$\backslash \text{forall } \#1.$

$\backslash \text{colon } \#2."]$

$[\forall x: y \xrightarrow{\text{pyk}} ``\text{all } * \text{ indeed } *"]$

$* \oplus *$

$[x \oplus y \xrightarrow{\text{val}} [x \oplus y]^R :: [x :: [y :: T]]]$

$[x \oplus y \xrightarrow{\text{tex}} "\#1.$

$\backslash \text{mathrel } \{\backslash \text{oplus}\} \#2."]$

$[x \oplus y \xrightarrow{\text{pyk}} "* \text{ rule plus } *"]$

$*; *$

$[x; y \xrightarrow{\text{val}} [x; y]^R :: [x :: [y :: T]]]$

$[x; y \xrightarrow{\text{tex}} "\#1.$

$; \#2."]$

$[x; y \xrightarrow{\text{pyk}} "* \text{ cut } *"]$

$* \text{ proves } *$

$[\mathbf{p} \text{ proves } t \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\mathbf{p} \text{ proves } t \doteq \text{proof}([\mathbf{p}], [t], \text{self})]])]$

$[\mathbf{p} \text{ proves } t \xrightarrow{\text{tex}} "\#1.$

$\backslash \text{ proves} \backslash \#2."]$

$[x \text{ proves } y \xrightarrow{\text{pyk}} "* \text{ proves } *"]$

$* \text{ proof of } * : *$

$[t \text{ proof of } s : p \xrightarrow{\text{name}} "\#1.$

$\backslash \text{mathbf{\{}} \backslash \text{ proof} \backslash \text{ of} \backslash \text{ }} \#2.$

$: \#3."]$

[t proof of s : p $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [t \text{ proof of } s : p \ddot{=} [\text{Proof of } s : \lambda c. \lambda x. \mathcal{P}([t \vdash p], p_0, c)]])]$]

[t proof of s : p $\xrightarrow{\text{tex}}$ “
\if\relax\csname lgwprooflinep\endcsname
\def\lgwprooflinep{x}
\newcount\lgwproofline
\fi
\begingroup
\def\insideproof{x}
\lgwproofline=0 #1.
\mathbf{\{ \ proof \ of \ } } #2.
\colon #3.
\gdef\lgwella{\relax}
\gdef\lgwellb{\relax}
\gdef\lgwellc{\relax}
\gdef\lgwelld{\relax}
\gdef\lgwelle{\relax}
\gdef\lgwellf{\relax}
\gdef\lgwellg{\relax}
\gdef\lgwellh{\relax}
\gdef\lgwelli{\relax}
\gdef\lgwellj{\relax}
\gdef\lgwellk{\relax}
\gdef\lgwelll{\relax}
\gdef\lgwellm{\relax}
\gdef\lgwelln{\relax}
\gdef\lgwello{\relax}
\gdef\lgwellp{\relax}
\gdef\lgwellq{\relax}
\gdef\lgwellr{\relax}
\gdef\lgwells{\relax}
\gdef\lgwellt{\relax}
\gdef\lgwellu{\relax}
\gdef\lgwellv{\relax}
\gdef\lgwellw{\relax}
\gdef\lgwellx{\relax}
\gdef\lgwelly{\relax}
\gdef\lgwellz{\relax}
\gdef\lgwellbiga{\relax}
\gdef\lgwellbigb{\relax}
\gdef\lgwellbigc{\relax}
\gdef\lgwellbigd{\relax}
\gdef\lgwellbige{\relax}
\gdef\lgwellbigf{\relax}
\gdef\lgwellbigg{\relax}

```

\gdef\lgwellbigh{\relax}
\gdef\lgwellbigi{\relax}
\gdef\lgwellbigj{\relax}
\gdef\lgwellbigk{\relax}
\gdef\lgwellbigl{\relax}
\gdef\lgwellbigm{\relax}
\gdef\lgwellbign{\relax}
\gdef\lgwellbigo{\relax}
\gdef\lgwellbigp{\relax}
\gdef\lgwellbigq{\relax}
\gdef\lgwellbigr{\relax}
\gdef\lgwellbigs{\relax}
\gdef\lgwellbigt{\relax}
\gdef\lgwellbigu{\relax}
\gdef\lgwellbigv{\relax}
\gdef\lgwellbigw{\relax}
\gdef\lgwellbigx{\relax}
\gdef\lgwellbigy{\relax}
\gdef\lgwellbigz{\relax}
\endgroup "

```

[**t proof of** $\mathbf{l} : \mathbf{p} \xrightarrow{\text{pyk}}$ “* proof of * reads *”]

Line * : * \gg *; *

[Line $\mathbf{l} : \mathbf{a} \gg \mathbf{i}; \mathbf{p} \xrightarrow{\text{name}}$ “

Line \, #1.

: #2.

\gg #3.

; #4.”]

[Line $\mathbf{l} : \mathbf{a} \gg \mathbf{i}; \mathbf{p} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\mathbf{l} : \mathbf{a} \gg \mathbf{i}; \mathbf{p} \doteq ([\mathbf{a} \gg \mathbf{i}] ; \mathbf{let} \mathbf{l} \doteq \mathbf{i} \mathbf{in} \mathbf{p})])])$

[Line $\mathbf{l} : \mathbf{a} \gg \mathbf{i}; \mathbf{p} \xrightarrow{\text{tex}}$ “

\newline \makebox [0.1\textwidth]{}%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{\$\#1\$.
\$\#2\$.}
{} \gg {} \quad
\parbox [t]{0.4\textwidth }{\$\#3\$.
\\$ \hfill \makebox [0mm][l]{\quad ; } \$\#4.”]

[Line $\mathbf{l} : \mathbf{a} \gg \mathbf{i}; \mathbf{p} \xrightarrow{\text{pyk}}$ “line * because * indeed * end line *”]

Last line * \gg *

[Last line a \gg i $\square \xrightarrow{\text{name}}$ “
Last\ line \, #1.
\gg #2.
\, \Box”]
[Last line a \gg i $\square \xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[$ [Last line a \gg i $\square \doteq (a \gg i)])])$
[Last line a \gg i $\square \xrightarrow{\text{tex}}$ “
\newline \makebox [0.1\textwidth]{ }%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][]{\\$}
\if \relax \csname lgwprooflinep\endcsname L_-? \else
\global \advance \lgwproofline by 1
L\ifnum \lgwproofline <10 0\fi \number \lgwproofline
\fi
\\$;}\\$#1.
{ }\gg { }\\$}\quad
\parbox [t]{0.4\textwidth }{\\$#2.
\hfill \makebox [0mm][]{\\$\Box\\$}}”]
[Last line a \gg i $\square \xrightarrow{\text{pyk}}$ “because * indeed * qed”]

Line * : Premise \gg *; *

[Line l : Premise \gg i; p $\xrightarrow{\text{name}}$ “
Line \, #1.
: Premise \gg #2.
;\#3.”]
[Line l : Premise \gg i; p $\xrightarrow{\text{macro}}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t,s,c,[$ [Line l : Premise \gg i; p $\doteq (i \vdash$
let l $\doteq i$ in p)])])
[Line l : Premise \gg i; p $\xrightarrow{\text{tex}}$ “
\newline \makebox [0.1\textwidth][l]{\\$#1.
\\$}\makebox [0.4\textwidth][l]{\\$Premise{}\gg{}\\$}\quad
\parbox [t]{0.4\textwidth }{\\$#2.
\hfill \makebox [0mm][l]{\\$quad ; }#3.”}
[Line l : Premise \gg i; p $\xrightarrow{\text{pyk}}$ “line * premise * end line *”]

Line * : Side-condition $\gg *; *$

```
[Line l : Side-condition  $\gg i; p \xrightarrow{\text{name}} "$ 
Line \, #1.
: \mbox{Side-condition} \gg #2.
; #3."]

[Line l : Side-condition  $\gg i; p \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{M}_4(t, s, c, [[Line l :
Side-condition  $\gg i; p \doteq (i \vdash \text{let } l \doteq i \text{ in } p)])])$$ 
```

```
[Line l : Side-condition  $\gg i; p \xrightarrow{\text{tex}} "$ 
\newline \makebox [0.1\textwidth ][l]{$#1.
$:$}\makebox [0.4\textwidth ][l]{
$\mbox{Side-condition}{}{}\gg{}$\quad
\parbox [t]{0.4\textwidth }{$#2.
\$\\hfill \makebox [0mm][l]{$\quad ; $}$#3."}
```

```
[Line l : Side-condition  $\gg i; p \xrightarrow{\text{pyk}} "\text{line * side condition * end line *}"$ ]
```

Arbitrary $\gg *; *$

```
[Arbitrary  $\gg i; p \xrightarrow{\text{name}} "$ 
Arbitrary \gg #1.
; #2."]

[Arbitrary  $\gg i; p \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{M}_4(t, s, c, [[Arbitrary  $\gg i; p \doteq (\forall i: p)])])$$ 
```

```
[Arbitrary  $\gg i; p \xrightarrow{\text{tex}} "$ 
\newline \makebox [0.1\textwidth ][l]{$
\if \relax \csname lgwprooflinep \endcsname L_? \else
\global \advance \lgwproofline by 1
L\ifnum \lgwproofline < 10 0\fi \number \lgwproofline
\fi
$:$}\makebox [0.4\textwidth ][l]{$\Arbitrary{}{}\gg{}$\quad
\parbox [t]{0.4\textwidth }{$#1.
\$\\hfill \makebox [0mm][l]{$\quad ; $}$#2."}
```

```
[Arbitrary  $\gg i; p \xrightarrow{\text{pyk}} "\text{arbitrary * end line *}"$ ]
```

Local $\gg * = *; *$

```
[Local  $\gg a = i; p \xrightarrow{\text{name}} "$ 
Local \gg #1.
= #2.
; #3."]
```

$$[\text{Local} \gg a = i; p \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\text{Local} \gg a = i; p \ddot{=} (\text{let } a \ddot{=} i \text{ in } p)] \rceil)]$$

```

[Local >> a = i; p  $\xrightarrow{\text{tex}}$  "
\newline\makebox[0.1\textwidth]{$
\if \relax \csname lgwprooflinep\endcsname L_? \else
\global \advance \lgwproofline by 1
\ifnum \lgwproofline <10 0\fi \number \lgwproofline
\fi
$:}%
\makebox[0.4\textwidth]{$Local{}\gg{}$}%
\quad%
\parbox[t]{0.4\textwidth}{$\#1.
=\#2.
\$\\hfill\makebox[0mm][l]{\quad ; }$\}#3."]
[Local >> u = v; p  $\xrightarrow{\text{pyk}}$  "locally define * as * end line *"]

```

* & *

[*&* → “#1.
\\& #2.”]

[*&* $\xrightarrow{\text{tex}}$ “#1.
.”]

[*&*> pyk “* tab *”]

* \ / *

[**^{name} “#1.
\\backslash \\backslash #2.”]

```
[*\\*tex “#1.  
\\ #2.”]
```

$[*\backslash\backslash * \xrightarrow{\text{pyk}} “* \text{ row}$

*”]

The pyk compiler, version 0.grue.20050603 by Klaus Grue

GRD-2005-06-03.UTC:15:49:33.495567 = MJD-53524.TAI:15:50:05.495567 = LGT-4624530605495567e-6