Logiweb codex of peano commutativity

Up Help

A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, peano commutativity, Weakening, ConditionedMP′, PermuteAntecedents, ImplyTransitivity, EqualReflexivity, EqualSymmetry, EqualTransitivity, Mendelson3.2(d), SubstitutionMacro, Mendelson3.2(f)Base, Mendelson3.2(f)Indu, Mendelson3.2(f), Mendelson3.2(g)Base, Mendelson3.2(g)Indu, Mendelson3.2(g), Mendelson3.2(g)Switch, AddOne, Mendelson3.2(d)Rule, Mendelson3.2(d)CondRule, EqualTransitivityRule, EqualTransitivityCondRule, PlusCommutativityBase, PlusCommutativityIndu, PlusCommutativity, DoubleMP′, InductionMacro, EqualSymmetryRule, DoubleConditionedMP′,

A1′

[A1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A1′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon$ [$\underline{a} \mathrel{\dot\Rightarrow}$ [$\underline{b} \mathrel{\dot\Rightarrow} \underline{a}$]]]

A2′

[A2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A2′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[$\underline{a} \mathrel{\dot\Rightarrow}$ [$\underline{b} \mathrel{\dot\Rightarrow} \underline{c}$]] $\mathrel{\dot\Rightarrow}$ [[$\underline{a} \mathrel{\dot\Rightarrow} \underline{b}$] $\mathrel{\dot\Rightarrow}$ [$\underline{a} \mathrel{\dot\Rightarrow} \underline{c}$]]]]

A3′

[A3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A3′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash$ [[[$\dot\neg \underline{b}$] $\mathrel{\dot\Rightarrow} \dot\neg \underline{a}$] $\mathrel{\dot\Rightarrow}$ [[[$\dot\neg \underline{b}$] $\mathrel{\dot\Rightarrow} \underline{a}$] $\mathrel{\dot\Rightarrow} \underline{b}$]]]

A4′

[A4′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A4′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon$ [$\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash$ [[$\dot\forall \underline{x}\colon \underline{b}$] $\mathrel{\dot\Rightarrow} \underline{a}$]]]

A5′

[A5′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

[A5′ $\stackrel{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \text{nonfr}\dot{\text{e}}\text{e}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]$]

S1′

[S1′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

[S1′ $\stackrel{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \dot{\Rightarrow} [\ [\ \underline{a} \stackrel{p}{=} \underline{c}\ ] \dot{\Rightarrow} [\ \underline{b} \stackrel{p}{=} \underline{c}\ ]\ ]\ ]$]

S2′

[S2′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

[S2′ $\stackrel{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ ]$]

S3′

[S3′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

[S3′ $\stackrel{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \neg [\ \dot{0} \stackrel{p}{=} [\ \underline{a}'\ ]\ ]$]

S4′

[S4′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

[S4′ $\stackrel{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ ]$]

S5′

[S5′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

[S5′ $\stackrel{\text{stmt}}{\longrightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]$]

S6′

[S6′ $\stackrel{\text{proof}}{\longrightarrow}$ Rule tactic]

$[\mathrm{S6}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\underline{a} \dot{+} [\underline{b}'\ ]\ ] \overset{\mathrm{p}}{=} [\ [\underline{a} \dot{+} \underline{b}\ ]'\ ]\ ]]$

S7′

$[\mathrm{S7}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{S7}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon [\ [\underline{a} \dot{\cdot} \dot{0}\ ] \overset{\mathrm{p}}{=} \dot{0}\ ]]$

S8′

$[\mathrm{S8}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{S8}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\underline{a} \dot{\cdot} [\underline{b}'\ ]\ ] \overset{\mathrm{p}}{=} [\ [\underline{a} \dot{\cdot} \underline{b}\ ] \dot{+} \underline{a}\ ]\ ]]$

S9′

$[\mathrm{S9}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{S9}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \dot{\Rightarrow} [\ [\dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]]$

MP′

$[\mathrm{MP}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{MP}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash [\underline{a} \vdash \underline{b}\ ]\ ]]$

Gen′

$[\mathrm{Gen}' \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{Gen}' \overset{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]]$

peano commutativity

$[\text{peano commutativity} \overset{\mathrm{pyk}}{\to}$ “peano commutativity”$]$

Weakening

$[\mathrm{Weakening} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\ \underline{\mathsf{a}} \vdash [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ \rhd \underline{\mathsf{a}}\ ]\ \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Weakening} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\ \underline{\mathsf{a}} \vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]$

$[\mathrm{Weakening} \overset{\mathrm{tex}}{\rightarrow}$ "Weakening"$]$

$[\mathrm{Weakening} \overset{\mathrm{pyk}}{\rightarrow}$ "weakening"$]$

ConditionedMP′

$[\mathrm{ConditionedMP}' \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ \vdash [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ \vdash [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{DoubleMP}' \rhd [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ \gg [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ConditionedMP}' \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ \vdash [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ \vdash [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$

$[\mathrm{ConditionedMP}' \overset{\mathrm{tex}}{\rightarrow}$ "ConditionedMP'"$]$

$[\mathrm{ConditionedMP}' \overset{\mathrm{pyk}}{\rightarrow}$ "conditioned mp prime"$]$

PermuteAntecedents

$[\mathrm{PermuteAntecedents} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ \vdash [\ [\ [\ \mathrm{Weakening} \rhd [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Weakening} \rhd [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{ConditionedMP}' \rhd [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{ConditionedMP}' \rhd [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}\ ]\ ]\ ]\ \gg [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PermuteAntecedents} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon [\ [\ \underline{\mathsf{d}} \dot{\Rightarrow} [\ \underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ \vdash [\ \underline{\mathsf{e}} \dot{\Rightarrow} [\ \underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}\ ]\ ]\ ]\ ]$

$[\mathrm{PermuteAntecedents} \overset{\mathrm{tex}}{\rightarrow}$ "PermuteAntecedents"$]$

$[\mathrm{PermuteAntecedents} \overset{\mathrm{pyk}}{\rightarrow}$ "permute antecedents"$]$

ImplyTransitivity

[ImplyTransitivity $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon [[\underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}] \vdash [[\underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}] \vdash [[[\mathrm{Weakening} \rhd [\underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}]] \gg [\underline{\mathsf{d}} \dot{\Rightarrow} [\underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}]]]; [[[\mathrm{ConditionedMP}' \rhd [\underline{\mathsf{d}} \dot{\Rightarrow} [\underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}]]] \rhd [\underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}]] \gg [\underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}]]]]] \rceil, \mathrm{p}_0, \mathsf{c})$]

[ImplyTransitivity $\stackrel{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon [[\underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{e}}] \vdash [[\underline{\mathsf{e}} \dot{\Rightarrow} \underline{\mathsf{f}}] \vdash [\underline{\mathsf{d}} \dot{\Rightarrow} \underline{\mathsf{f}}]]]$]

[ImplyTransitivity $\stackrel{\text{tex}}{\to}$ "ImplyTransitivity"]

[ImplyTransitivity $\stackrel{\text{pyk}}{\to}$ "imply transitivity"]

EqualReflexivity

[EqualReflexivity $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon [[\mathrm{S5}' \gg [[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}]]; [[\mathrm{S1}' \gg [[[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [[[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}]]]]; [[[[\mathrm{DoubleMP}' \rhd [[[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [[[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}]]]] \rhd [[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}]] \rhd [[\underline{\mathsf{t}} \dot{+} \dot{0}] \stackrel{p}{=} \underline{\mathsf{t}}]] \gg [\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}]]]] \rceil, \mathrm{p}_0, \mathsf{c})$]

[EqualReflexivity $\stackrel{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon [\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}]$]

[EqualReflexivity $\stackrel{\text{tex}}{\to}$ "EqualReflexivity"]

[EqualReflexivity $\stackrel{\text{pyk}}{\to}$ "equal reflexivity"]

EqualSymmetry

[EqualSymmetry $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [[\mathrm{S1}' \gg [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{r}}] \dot{\Rightarrow} [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [\underline{\mathsf{r}} \stackrel{p}{=} \underline{\mathsf{t}}]]]]; [[[\mathrm{PermuteAntecedents} \rhd [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{r}}] \dot{\Rightarrow} [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [\underline{\mathsf{r}} \stackrel{p}{=} \underline{\mathsf{t}}]]]] \gg [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{r}}] \dot{\Rightarrow} [\underline{\mathsf{r}} \stackrel{p}{=} \underline{\mathsf{t}}]]]]; [[\mathrm{EqualReflexivity} \gg [\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}]]; [[[\mathrm{MP}' \rhd [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}] \dot{\Rightarrow} [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{r}}] \dot{\Rightarrow} [\underline{\mathsf{r}} \stackrel{p}{=} \underline{\mathsf{t}}]]]] \rhd [\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{t}}]] \gg [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{r}}] \dot{\Rightarrow} [\underline{\mathsf{r}} \stackrel{p}{=} \underline{\mathsf{t}}]]]]]] \rceil, \mathrm{p}_0, \mathsf{c})$]

[EqualSymmetry $\stackrel{\text{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [[\underline{\mathsf{t}} \stackrel{p}{=} \underline{\mathsf{r}}] \dot{\Rightarrow} [\underline{\mathsf{r}} \stackrel{p}{=} \underline{\mathsf{t}}]]$]

[EqualSymmetry $\stackrel{\text{tex}}{\to}$ "EqualSymmetry"]

[EqualSymmetry $\stackrel{\text{pyk}}{\to}$ "equal symmetry"]

EqualTransitivity

$[\mathrm{EqualTransitivity} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\ \mathrm{S1}' \gg [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{Weakening} \rhd [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ] \gg [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ]\ ; [\ [\ \mathrm{EqualSymmetry} \gg [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot\Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]\ ]\ ]\ ; [\ [\ [\ \mathrm{ConditionedMP}' \rhd [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{t}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ] \rhd [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot\Rightarrow [\underline{r} \stackrel{p}{=} \underline{t}]\ ]\ ] \gg [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\rceil, p_0, c)]$

$[\mathrm{EqualTransitivity} \stackrel{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\underline{t} \stackrel{p}{=} \underline{r}] \dot\Rightarrow [\ [\underline{r} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]]$

$[\mathrm{EqualTransitivity} \stackrel{\mathrm{tex}}{\rightarrow}$ "EqualTransitivity"$]$

$[\mathrm{EqualTransitivity} \stackrel{\mathrm{pyk}}{\rightarrow}$ "equal transitivity"$]$

Mendelson3.2(d)

$[\mathrm{Mendelson3.2(d)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon \forall \underline{u}\colon \forall \underline{s}\colon [\ [\ \mathrm{EqualTransitivity} \gg [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\ [\underline{u} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{PermuteAntecedents} \rhd [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\ [\underline{u} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ] \gg [\ [\underline{u} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ; [\ [\ \mathrm{EqualSymmetry} \gg [\ [\underline{s} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{u} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ; [\ [\ [\ [\ \mathrm{ImplyTransitivity} \rhd [\ [\underline{s} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{u} \stackrel{p}{=} \underline{s}]\ ]\ ] \rhd [\ [\underline{u} \stackrel{p}{=} \underline{s}] \dot\Rightarrow [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ] \gg [\ [\underline{s} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ; [\ [\ \mathrm{PermuteAntecedents} \rhd [\ [\underline{s} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ] \gg [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\ [\underline{s} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\rceil, p_0, c)]$

$[\mathrm{Mendelson3.2(d)} \stackrel{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{t}\colon \forall \underline{u}\colon \forall \underline{s}\colon [\ [\underline{t} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\ [\underline{s} \stackrel{p}{=} \underline{u}] \dot\Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]\ ]\ ]]$

$[\mathrm{Mendelson3.2(d)} \stackrel{\mathrm{tex}}{\rightarrow}$ "Mendelson3.2(d)"$]$

$[\mathrm{Mendelson3.2(d)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "mendelson three two d"$]$

SubstitutionMacro

$[\mathrm{SubstitutionMacro} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{z}\colon \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{d}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{d} \rceil | \lceil \underline{z} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ \underline{d} \vdash [\ [\ [\ \mathrm{Gen}' \rhd \underline{d}\ ] \gg \dot\forall \underline{z}\colon \underline{d}\ ]\ ; [\ [\ [\ \mathrm{A4}' \rhd\!\!\rhd \lceil \underline{a} \rceil \equiv \langle \lceil \underline{d} \rceil | \lceil \underline{z} \rceil := \lceil \underline{c} \rceil \rangle\ ] \gg [\ [\ \dot\forall \underline{z}\colon \underline{d}\ ] \dot\Rightarrow \underline{a}\ ]\ ]\ ; [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot\forall \underline{z}\colon \underline{d}\ ] \dot\Rightarrow \underline{a}\ ]\ ] \rhd \dot\forall \underline{z}\colon \underline{d}\ ] \gg \underline{a}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{SubstitutionMacro} \stackrel{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{z}\colon \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{d}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{d} \rceil | \lceil \underline{z} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ \underline{d} \vdash \underline{a}\ ]\ ]\ ]$

$[\mathrm{SubstitutionMacro} \stackrel{\mathrm{tex}}{\rightarrow}$ "SubstitutionMacro"$]$

[SubstitutionMacro $\overset{\text{pyk}}{\to}$ "substitution macro"]

Mendelson3.2(f)Base

[Mendelson3.2(f)Base $\overset{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\,[\,S5' \gg [\,[\,\dot{t} \dot{+} \dot{0}\,] \overset{p}{=} [\,\dot{t}\,]\,]\,]\,; [\,[\,[\,\mathrm{EqualSymmetryRule} \rhd [\,[\,\dot{t} \dot{+} \dot{0}\,] \overset{p}{=} [\,\dot{t}\,]\,]\,] \gg [\,\dot{t} \overset{p}{=} [\,\dot{t} \dot{+} \dot{0}\,]\,]\,]\,; [\,[\,[\,[\,\mathrm{SubstitutionMacro} @ [\,\dot{t}\,]\,] @ \dot{0}\,] \rhd [\,\dot{t} \overset{p}{=} [\,\dot{t} \dot{+} \dot{0}\,]\,]\,]\,] \gg [\,\dot{0} \overset{p}{=} [\,\dot{0} \dot{+} \dot{0}\,]\,]\,]\,]\,]\,] \rceil, p_0, c)$]

[Mendelson3.2(f)Base $\overset{\text{stmt}}{\to} S' \vdash [\,\dot{0} \overset{p}{=} [\,\dot{0} \dot{+} \dot{0}\,]\,]$]

[Mendelson3.2(f)Base $\overset{\text{tex}}{\to}$ "Mendelson3.2(f)Base"]

[Mendelson3.2(f)Base $\overset{\text{pyk}}{\to}$ "mendelson three two f base"]

Mendelson3.2(f)Indu

[Mendelson3.2(f)Indu $\overset{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\,[\,S2' \gg [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,\dot{t}' \overset{p}{=} [\,[\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,'\,]\,]\,]\,]\,]\,; [\,[\,S6' \gg [\,[\,\dot{0} \dot{+} [\,\dot{t}'\,]\,] \overset{p}{=} [\,[\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,'\,]\,]\,]\,; [\,[\,[\,\mathrm{Weakening} \rhd [\,[\,\dot{0} \dot{+} [\,\dot{t}'\,]\,] \overset{p}{=} [\,[\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,'\,]\,]\,] \gg [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,[\,\dot{0} \dot{+} [\,\dot{t}'\,]\,] \overset{p}{=} [\,[\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,'\,]\,]\,]\,]\,]\,; [\,[\,[\,\mathrm{Mendelson3.2(d)CondRule} \rhd [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,\dot{t}' \overset{p}{=} [\,[\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,'\,]\,]\,]\,]\,] \rhd [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,[\,\dot{0} \dot{+} [\,\dot{t}'\,]\,] \overset{p}{=} [\,[\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,'\,]\,]\,]\,]\,] \gg [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,\dot{t}' \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}'\,]\,]\,]\,]\,]\,]\,]\,]\,]\,] \rceil, p_0, c)$]

[Mendelson3.2(f)Indu $\overset{\text{stmt}}{\to} S' \vdash [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,\dot{t}' \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}'\,]\,]\,]\,]\,]$]

[Mendelson3.2(f)Indu $\overset{\text{tex}}{\to}$ "Mendelson3.2(f)Indu"]

[Mendelson3.2(f)Indu $\overset{\text{pyk}}{\to}$ "mendelson three two f induction"]

Mendelson3.2(f)

[Mendelson3.2(f) $\overset{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [\,[\,\mathrm{Mendelson3.2(f)Base} \gg [\,\dot{0} \overset{p}{=} [\,\dot{0} \dot{+} \dot{0}\,]\,]\,]\,; [\,[\,\mathrm{Mendelson3.2(f)Indu} \gg [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,\dot{t}' \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}'\,]\,]\,]\,]\,]\,]\,; [\,[\,[\,\mathrm{InductionMacro} \rhd [\,\dot{0} \overset{p}{=} [\,\dot{0} \dot{+} \dot{0}\,]\,]\,]\,] \rhd [\,[\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,] \dot{\Rightarrow} [\,\dot{t}' \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}'\,]\,]\,]\,]\,]\,]\,] \gg [\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,]\,]\,]\,]\,] \rceil, p_0, c)$]

[Mendelson3.2(f) $\overset{\text{stmt}}{\to} S' \vdash [\,\dot{t} \overset{p}{=} [\,\dot{0} \dot{+} [\,\dot{t}\,]\,]\,]\,]$]

[Mendelson3.2(f) $\overset{\text{tex}}{\to}$ "Mendelson3.2(f)"]