Propositional Calculus

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[propositional calculus $\overset{\text{pyk}}{\rightarrow}$ "propositional calculus"]
Define theory: [$\mathrm{T}_{\mathrm{Prop}} \overset{\text{stmt}}{\rightarrow} \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \underline{a} \Rightarrow \underline{b} \vdash \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{c}$ free for $\underline{b}$ in $\underline{a} \Vdash \forall \underline{b}\colon \underline{a} \Rightarrow \langle \underline{a} \,|\underline{b}{:=}\, \underline{c} \rangle \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \neg \underline{c}$ free in $\underline{a} \Vdash \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \forall \underline{c}\colon \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \forall \underline{b}\colon \underline{a}$][1]

Implication

[$\mathsf{x} \Rightarrow \mathsf{y} \overset{\text{val}}{\rightarrow} \neg \mathsf{x} \vee \mathsf{y}$] [2]

Axioms

[A1 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a}$][A1 $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [3]

[A2 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c}$][A2 $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [4]

[A3 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a}$][A3 $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [5]

[A4 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{c}$ free for $\underline{b}$ in $\underline{a} \Vdash \forall \underline{b}\colon \underline{a} \Rightarrow \langle \underline{a} \,|\underline{b}{:=}\, \underline{c} \rangle$][A4 $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [6]

[A5 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \neg \underline{c}$ free in $\underline{a} \Vdash \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \forall \underline{c}\colon \underline{b}$][A5 $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [7]

[MP $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \underline{a} \Rightarrow \underline{b} \vdash \underline{b}$][MP $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [8]

[Gen $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \forall \underline{b}\colon \underline{a}$][Gen $\overset{\text{proof}}{\rightarrow}$ Rule tactic] [9]

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[1] [$\mathrm{T}_{\mathrm{Prop}} \overset{\text{pyk}}{\rightarrow}$ "propositional theory"]
[2] [$\mathsf{x} \Rightarrow \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "$*$ imply $*$"]
[3] [A1 $\overset{\text{pyk}}{\rightarrow}$ "axiom one"]
[4] [A2 $\overset{\text{pyk}}{\rightarrow}$ "axiom two"]
[5] [A3 $\overset{\text{pyk}}{\rightarrow}$ "axiom three"]
[6] [A4 $\overset{\text{pyk}}{\rightarrow}$ "axiom four"]
[7] [A5 $\overset{\text{pyk}}{\rightarrow}$ "axiom five"]
[8] [MP $\overset{\text{pyk}}{\rightarrow}$ "axiom mp"]
[9] [Gen $\overset{\text{pyk}}{\rightarrow}$ "axiom gen"]

Proof

[Mendelson **1.8** $\overset{\text{stmt}}{\to}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}$] [10]

[Mendelson **1.8** $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{a}}\colon \mathrm{A2} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{A1} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{A1} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rceil, \mathrm{p}_0, \mathsf{c})$]

Experimental

[Mendelson **1.10**a $\overset{\text{stmt}}{\to}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{f}}\colon \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \vdash \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \vdash \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}}$] [11]

[Mendelson **1.10**a $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{f}}\colon \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \vdash \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \vdash \mathrm{A2} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}}; \mathrm{A1} \gg \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}}; \mathrm{MP} \rhd \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \rhd \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}}; \mathrm{MP} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}}; \mathrm{MP} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})$]

[Mendelson **1.10**b $\overset{\text{stmt}}{\to}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \vdash \underline{\mathsf{e}} \vdash \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}}$] [12]

[Mendelson **1.10**b $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \vdash \underline{\mathsf{e}} \vdash \mathrm{A2} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}}; \mathrm{A1} \gg \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}}; \mathrm{MP} \rhd \underline{\mathsf{e}} \rhd \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}}; \mathrm{MP} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}}; \mathrm{MP} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})$]

[Mendelson **1.11**a $\overset{\text{stmt}}{\to}$ $\mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{g}}\colon \neg\neg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{g}}$] [13]

[Mendelson **1.11**a $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{T}_{\mathrm{Prop}} \vdash \forall \underline{\mathsf{g}}\colon \mathrm{A3} \gg \neg \underline{\mathsf{g}} \Rightarrow \neg\neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{g}}; \text{Mendelson } \mathbf{1.8} \gg \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}}; \mathrm{A1} \gg \neg\neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg\neg \underline{\mathsf{g}}; \text{Mendelson } \mathbf{1.10}\text{b} \rhd \neg \underline{\mathsf{g}} \Rightarrow \neg\neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{g}} \rhd \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \gg \neg \underline{\mathsf{g}} \Rightarrow \neg\neg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{g}}; \text{Mendelson } \mathbf{1.10}\text{a} \rhd \neg\neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg\neg \underline{\mathsf{g}} \rhd \neg \underline{\mathsf{g}} \Rightarrow \neg\neg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{g}} \gg \neg\neg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{g}} \rceil, \mathrm{p}_0, \mathsf{c})$]

A TEX definitions

[$\mathrm{T}_{\mathrm{Prop}} \overset{\text{tex}}{\to}$ "
T_{Prop}"]

[$\mathsf{x} \Rightarrow \mathsf{y} \overset{\text{tex}}{\to}$ "#1.
\Rightarrow{}#2."]

[A1 $\overset{\text{tex}}{\to}$ "A1"]

[A2 $\overset{\text{tex}}{\to}$ "A2"]

[A3 $\overset{\text{tex}}{\to}$ "A3"]

[10] [Mendelson **1.8** $\overset{\text{pyk}}{\to}$ "mendelson lemma one eight"]

[11] [Mendelson **1.10**a $\overset{\text{pyk}}{\to}$ "mendelson corollary one ten a"]

[12] [Mendelson **1.10**b $\overset{\text{pyk}}{\to}$ "mendelson corollary one ten b"]

[13] [Mendelson **1.11**a $\overset{\text{pyk}}{\to}$ "mendelson lemma one eleven a"]

[A4 $\overset{\text{tex}}{\to}$ "A4"]

[A5 $\overset{\text{tex}}{\to}$ "A5"]

[MP $\overset{\text{tex}}{\to}$ "MP"]

[Gen $\overset{\text{tex}}{\to}$ "Gen"]

[Mendelson **1.8** $\overset{\text{tex}}{\to}$ "Mendelson\ \textbf{1.8}"]

[Mendelson **1.10**a $\overset{\text{tex}}{\to}$ "Mendelson\ \textbf{1.10} a"]

[Mendelson **1.10**b $\overset{\text{tex}}{\to}$ "Mendelson\ \textbf{1.10} b"]

[Mendelson **1.11**a $\overset{\text{tex}}{\to}$ "Mendelson\ \textbf{1.11} a"]

B Test

$[\mathsf{T} \Rightarrow \mathsf{T}]^{\cdot}$

$[\mathsf{T} \Rightarrow \mathsf{F}]^{-}$

$[\mathsf{F} \Rightarrow \mathsf{T}]^{\cdot}$

$[\mathsf{F} \Rightarrow \mathsf{F}]^{\cdot}$

C Priority table

[propositional calculus $\overset{\text{prio}}{\to}$
Preassociative
[propositional calculus], [base], [bracket $*$ end bracket],
[big bracket $*$ end bracket], [math $*$ end math], [**flush left** $[*]$], [x], [y], [z],
$[[* \bowtie *]]$, $[[* \overset{*}{\to} *]]$, [pyk], [tex], [name], [prio], $[*]$, $[\mathsf{T}]$, $[\text{if}(*,*,*)]$, $[[* \overset{*}{\Rightarrow} *]]$, [val],
[claim], $[\bot]$, $[\text{f}(*)]$, $[(*)^{\text{I}}]$, $[\mathsf{F}]$, $[\underline{0}]$, $[\underline{1}]$, $[\underline{2}]$, $[\underline{3}]$, $[\underline{4}]$, $[\underline{5}]$, $[\underline{6}]$, $[\underline{7}]$, $[\underline{8}]$, $[\underline{9}]$, [0], [1], [2], [3], [4],
[5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r],
[s], [t], [u], [v], [w], $[(*)^{\text{M}}]$, $[\text{If}(*,*,*)]$, [array$\{*\}$ $*$ end array], [l], [c], [r], [empty],
$[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$, $[\mathcal{U}^{\text{M}}(*)]$, $[\mathbf{apply}(*,*)]$, $[\mathbf{apply}_1(*,*)]$,
[identifier$(*)$], [identifier$_1(*,*)$], [array-plus$(*,*)$], [array-remove$(*,*,*)$], [array-
put$(*,*,*,*)$], [array-add$(*,*,*,*,*)$], [bit$(*,*)$], [bit$_1(*,*)$], [rack], ["vector"],
["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"],
["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"],
["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"],
["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"],
["hide"], ["pre"], ["post"], $[\mathcal{E}(*,*,*)]$, $[\mathcal{E}_2(*,*,*,*,*)]$, $[\mathcal{E}_3(*,*,*,*)]$,
$[\mathcal{E}_4(*,*,*,*)]$, $[\mathbf{lookup}(*,*,*)]$, $[\mathbf{abstract}(*,*,*,*)]$, $[\lceil * \rceil]$, $[\mathcal{M}(*,*,*)]$,
$[\mathcal{M}_2(*,*,*,*)]$, $[\mathcal{M}^*(*,*,*)]$, [macro], $[\text{s}_0]$, $[\mathbf{zip}(*,*)]$, $[\mathbf{assoc}_1(*,*,*)]$, $[(*)^{\mathbf{p}}]$,

[self], [[$* \doteqdot *$]], [[$* \doteq *$]], [[$* \doteq *$]], [[$* \overset{\text{pyk}}{=} *$]], [[$* \overset{\text{tex}}{=} *$]], [[$* \overset{\text{name}}{=} *$]],
[**Priority table**$(*)$], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*,*,*,*)$], [$\tilde{\mathcal{M}}(*,*,*)$],
[$\tilde{\mathcal{Q}}(*,*,*)$], [$\tilde{\mathcal{Q}}_2(*,*,*)$], [$\tilde{\mathcal{Q}}_3(*,*,*,*)$], [$\tilde{\mathcal{Q}}^*(*,*,*)$], [$(*)$], [**aspect**$(*,*)$],
[**aspect**$(*,*,*)$], [$\langle * \rangle$], [**tuple**$_1(*)$], [**tuple**$_2(*)$], [let$_2(*,*)$], [let$_1(*,*)$],
[[$* \overset{\text{claim}}{=} *$]], [checker], [**check**$(*,*)$], [**check**$_2(*,*,*)$], [**check**$_3(*,*,*)$],
[**check**$^*(*,*)$], [**check**$^*_2(*,*,*)$], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \overset{\text{msg}}{=} *$]], [<stmt>],
[stmt], [[$* \overset{\text{stmt}}{=} *$]], [HeadNil$'$], [HeadPair$'$], [Transitivity$'$], [$\perp\!\!\!\perp$], [Contra$'$], [T$'_{\text{E}}$],
[L$_1$], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$],
[$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle {}^{*} * \mid * := * \rangle$], [$\emptyset$], [Remainder],
[$(*)^{\mathbf{v}}$], [error$(*,*)$], [error$_2(*,*)$], [proof$(*,*,*)$], [proof$_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\text{I}}(*,*)$],
[$\mathcal{S}^{\triangleright}(*,*)$], [$\mathcal{S}^{\triangleright}_1(*,*,*)$], [$\mathcal{S}^{\text{E}}(*,*)$], [$\mathcal{S}^{\text{E}}_1(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}^{+}_1(*,*,*)$],
[$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}^{-}_1(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}^{*}_1(*,*,*)$], [$\mathcal{S}^{*}_2(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$],
[$\mathcal{S}^{@}_1(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}^{\vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}^{\Vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\text{i.e.}}(*,*)$],
[$\mathcal{S}^{\text{i.e.}}_1(*,*,*,*)$], [$\mathcal{S}^{\text{i.e.}}_2(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}^{\forall}_1(*,*,*,*)$], [$\mathcal{S}^{\text{i}}(*,*)$],
[$\mathcal{S}^{\text{i}}_1(*,*,*)$], [$\mathcal{S}^{\text{i}}_2(*,*,*,*)$], [$\mathcal{T}(*)$], [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>],
[proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]],
[[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$],
[$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$],
[Cut$(*,*)$], [Head$_{\oplus}(*)$], [Tail$_{\oplus}(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic],
[Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$],
[HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair],
[Transitivity], [Contra], [T$_{\text{E}}$], [ragged right], [ragged right expansion],
[color$(* : *)$], [color$^*(* : *)$], [vars$(*)$], [vars$^*(*)$], [instantiate$(* + * : *)$],
[instantiate$^*(* + * : *)$], [unify$(* : * = * : * + *)$], [unify$^*(* : * = * : * + *)$],
[unify$_1(* : * = * : * + *)$], [unify$_2(* + *)$], [unify$_3(* = * + *)$], [check],
[parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*)$],
[occur$^*(*,*)$], [circular$(* = *,*)$], [circular$^*(* = *,*)$], [unify$(* = *,*)$],
[unify$^*(* = *,*)$], [unify$_2(* = *,*)$], [L$_{\text{a}}$], [L$_{\text{b}}$], [L$_{\text{c}}$], [L$_{\text{d}}$], [L$_{\text{e}}$], [L$_{\text{f}}$], [L$_{\text{g}}$], [L$_{\text{h}}$], [L$_{\text{i}}$],
[L$_{\text{j}}$], [L$_{\text{k}}$], [L$_{\text{l}}$], [L$_{\text{m}}$], [L$_{\text{n}}$], [L$_{\text{o}}$], [L$_{\text{p}}$], [L$_{\text{q}}$], [L$_{\text{r}}$], [L$_{\text{s}}$], [L$_{\text{t}}$], [L$_{\text{u}}$], [L$_{\text{v}}$], [L$_{\text{w}}$], [L$_{\text{x}}$], [L$_{\text{y}}$],
[L$_{\text{z}}$], [L$_{\text{A}}$], [L$_{\text{B}}$], [L$_{\text{C}}$], [L$_{\text{D}}$], [L$_{\text{E}}$], [L$_{\text{F}}$], [L$_{\text{G}}$], [L$_{\text{H}}$], [L$_{\text{I}}$], [L$_{\text{J}}$], [L$_{\text{K}}$], [L$_{\text{L}}$], [L$_{\text{M}}$], [L$_{\text{N}}$],
[L$_{\text{O}}$], [L$_{\text{P}}$], [L$_{\text{Q}}$], [L$_{\text{R}}$], [L$_{\text{S}}$], [L$_{\text{T}}$], [L$_{\text{U}}$], [L$_{\text{V}}$], [L$_{\text{W}}$], [L$_{\text{X}}$], [L$_{\text{Y}}$], [L$_{\text{Z}}$], [L$_{?}$],
[Reflexivity], [Reflexivity$_1$], [Commutativity], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]],
[$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [p$_0$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$],
[conclude$_3(*,*,*,*)$], [T$_{\text{Prop}}$], [A1], [A2], [A3], [A4], [A5], [MP], [Gen],
[Mendelson **1.8**], [Mendelson **1.10**a], [Mendelson **1.10**b], [Mendelson **1.11**a],
[nani teta saka...:13];

Preassociative

[$*$-{$*$}], [$*'$], [$*[\ *\ ]$], [$*[*\rightarrow*]$], [$*[*\Rightarrow*]$];

Preassociative

[“ $*$ ”], [], [$(*)^{\mathbf{t}}$], [string$(*) + *$], [string$(*)$ ++ $*$], [
$*$], [$*$], [!$*$], [”$*$], [#$*$], [$$$*], [%$*], [&$*], [’$*], [($*$), ()$*$], [**$*$], [+$*$], [,$*$], [-$*$], [.$*$], [/$*$],
[0$*$], [1$*$], [2$*$], [3$*$], [4$*$], [5$*$], [6$*$], [7$*$], [8$*$], [9$*$], [:$*$], [;$*$], [<$*$], [=$*$], [>$*$], [?$*$],
[@$*$], [A$*$], [B$*$], [C$*$], [D$*$], [E$*$], [F$*$], [G$*$], [H$*$], [I$*$], [J$*$], [K$*$], [L$*$], [M$*$], [N$*$],
[O$*$], [P$*$], [Q$*$], [R$*$], [S$*$], [T$*$], [U$*$], [V$*$], [W$*$], [X$*$], [Y$*$], [Z$*$], [[$*$], [\$*$], []$*$], [^$*$],

$[\_*]$, $[`*]$, $[a*]$, $[b*]$, $[c*]$, $[d*]$, $[e*]$, $[f*]$, $[g*]$, $[h*]$, $[i*]$, $[j*]$, $[k*]$, $[l*]$, $[m*]$, $[n*]$, $[o*]$, $[p*]$, $[q*]$, $[r*]$, $[s*]$, $[t*]$, $[u*]$, $[v*]$, $[w*]$, $[x*]$, $[y*]$, $[z*]$, $[\{*]$, $[|*]$, $[\}*]$, $[\tilde{}*]$,
$[\textbf{Preassociative}\ *;\ *]$, $[\textbf{Postassociative}\ *;\ *]$, $[[*],\ *]$, [priority $*$ end],
[newline $*$], [macro newline $*$];

Preassociative

$[*0]$, $[*1]$, [0b], $[*\text{-color}(*)]$, $[*\text{-color}^{*}(*)]$;

Preassociative

$[*\ '\ *]$, $[*\ `\ *]$;

Preassociative

$[*^{\mathrm{H}}]$, $[*^{\mathrm{T}}]$, $[*^{\mathrm{U}}]$, $[*^{\mathrm{h}}]$, $[*^{\mathrm{t}}]$, $[*^{\mathrm{s}}]$, $[*^{\mathrm{c}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{a}}]$, $[*^{\mathrm{C}}]$, $[*^{\mathrm{M}}]$, $[*^{\mathrm{B}}]$, $[*^{\mathrm{r}}]$, $[*^{\mathrm{i}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{R}}]$, $[*^{0}]$, $[*^{1}]$, $[*^{2}]$, $[*^{3}]$, $[*^{4}]$, $[*^{5}]$, $[*^{6}]$, $[*^{7}]$, $[*^{8}]$, $[*^{9}]$, $[*^{\mathrm{E}}]$, $[*^{\mathcal{V}}]$, $[*^{\mathcal{C}}]$, $[*^{\mathcal{C}^{*}}]$;

Preassociative

$[*\cdot *]$, $[*\cdot_0 *]$;

Preassociative

$[*+*]$, $[*+_0 *]$, $[*+_1 *]$, $[*-*]$, $[*-_0 *]$, $[*-_1 *]$;

Preassociative

$[*\cup\{*\}]$, $[*\cup *]$, $[*\backslash\{*\}]$;

Postassociative

$[*\mathbin{\therefore} *]$, $[*\mathbin{\underline{\therefore}} *]$, $[*\mathbin{\underline{::}} *]$, $[*\mathbin{\underline{+2*}} *]$, $[*::*]$, $[*+2*\ *]$;

Postassociative

$[*, *]$;

Preassociative

$[*\overset{\mathrm{B}}{\approx} *]$, $[*\overset{\mathrm{D}}{\approx} *]$, $[*\overset{\mathrm{C}}{\approx} *]$, $[*\overset{\mathrm{P}}{\approx} *]$, $[*\approx *]$, $[*=*]$, $[*\overset{+}{\to} *]$, $[*\overset{\mathrm{t}}{=} *]$, $[*\overset{\mathrm{t}^{*}}{=} *]$, $[*\overset{\mathrm{r}}{=} *]$, $[*\in_{\mathrm{t}} *]$, $[*\subseteq_{\mathrm{T}} *]$, $[*\overset{\mathrm{T}}{=} *]$, $[*\overset{\mathrm{s}}{=} *]$, [$*$ free in $*$], [$*$ free in* $*$], [$*$ free for $*$ in $*$], [$*$ free for* $*$ in $*$], $[*\in_{\mathrm{c}} *]$, $[*<*]$, $[*<'*]$, $[*\leq'*]$;

Preassociative

$[\neg *]$;

Preassociative

$[*\wedge *]$, $[*\mathbin{\ddot{\wedge}} *]$, $[*\mathbin{\tilde{\wedge}} *]$, $[*\wedge_{\mathrm{c}} *]$;

Preassociative

$[*\vee *]$, $[*\parallel *]$, $[*\mathbin{\ddot{\vee}} *]$;

Postassociative

$[*\mathbin{\ddot{\Rightarrow}} *]$;

Postassociative

$[*:*]$, $[*!*]$;

Postassociative

$[*\Rightarrow *]$;

Preassociative

$[*\begin{cases} * \\ * \end{cases}]$;

Preassociative

$[\lambda * .*]$, $[\Lambda *]$, [**if** $*$ **then** $*$ **else** $*$], [**let** $*=*$ **in** $*$], [**let** $*\mathbin{\ddot{=}}*$ in $*$];

Preassociative

$[*^{\mathrm{I}}]$, $[*^{\triangleright}]$, $[*^{\mathrm{V}}]$, $[*^{+}]$, $[*^{-}]$, $[*^{*}]$;

Preassociative

$[* \mathbin{@} *], [* \rhd *], [* \mathbin{|\!\!>} *], [* \gg *]$;
Postassociative
$[* \vdash *], [* \Vdash *],$ [* i.e. *];
Preassociative
$[\forall *\colon *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
[* proves *];
Preassociative
[* **proof of** * : *], [Line * : * $\gg$ *; *], [Last line * $\gg$ *], [Line * : Premise $\gg$ *; *], [Line * : Side-condition $\gg$ *; *], [Arbitrary $\gg$ *; *], [Local $\gg$ * = *; *];
Postassociative
[* then *], [*[*]*];
Preassociative
[*&*];
Preassociative
[**];]