Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\mathrm{nonfree}(*, *)$, $\mathrm{nonfree}^*(*, *)$, $\mathrm{free}\langle *|* := *\rangle$, $\mathrm{free}^*\langle *|* := *\rangle$, $*\equiv\langle *|* := *\rangle$, $*\equiv\langle^* *|* := *\rangle$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S$'$, A1$'$, A2$'$, A3$'$, A4$'$, A5$'$, S1$'$, S2$'$, S3$'$, S4$'$, S5$'$, S6$'$, S7$'$, S8$'$, S9$'$, MP$'$, Gen$'$, L3.2(a)$'$, $\dot{*}$, $*'$, $* \dot{\cdot} *$, $* \dot{+} *$, $* \stackrel{\mathrm{p}}{=} *$, $*^{\mathcal{P}}$, $\dot{\neg} *$, $* \dot{\wedge} *$, $* \dot{\vee} *$, $\dot{\forall} *\colon *$, $\dot{\exists} *\colon *$, $* \dot{\Rightarrow} *$, $* \dot{\Leftrightarrow} *$,

peano

[peano $\stackrel{\mathrm{prio}}{\rightarrow}$
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\rightarrow} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if$(*, *, *)$], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\perp$], [f$(*)$], [$(*)^{\mathrm{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\mathrm{M}}$], [If$(*, *, *)$], [array$\{*\}$ $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\mathrm{M}}(*)$], [**apply**$(*, *)$], [**apply**$_1(*, *)$], [identifier$(*)$], [identifier$_1(*, *)$], [array-plus$(*, *)$], [array-remove$(*, *, *)$], [array-put$(*, *, *, *)$], [array-add$(*, *, *, *, *)$], [bit$(*, *)$], [bit$_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**$(*, *, *)$], [**abstract**$(*, *, *, *)$], [$\lceil * \rceil$], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [$\mathrm{s}_0$], [**zip**$(*, *)$], [**assoc**$_1(*, *, *)$], [$(*)^{\mathbf{P}}$], [self], [[$* \ddot{=} *$]], [[$* \dot{=} *$]], [[$* \acute{=} *$]], [[$* \stackrel{\mathrm{pyk}}{=} *$]], [[$* \stackrel{\mathrm{tex}}{=} *$]], [[$* \stackrel{\mathrm{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [$(*)$], [**aspect**$(*, *)$], [**aspect**$(*, *, *)$], [$\langle * \rangle$], [**tuple**$_1(*)$], [**tuple**$_2(*)$], [let$_2(*, *)$], [let$_1(*, *)$], [[$* \stackrel{\mathrm{claim}}{=} *$]], [checker], [**check**$(*, *)$], [**check**$_2(*, *, *)$], [**check**$_3(*, *, *)$], [**check**$^*(*, *)$], [**check**$^*_2(*, *, *)$], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \stackrel{\mathrm{msg}}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\mathrm{stmt}}{=} *$]], [HeadNil$'$], [HeadPair$'$], [Transitivity$'$], [$\perp\!\!\perp$], [Contra$'$], [$\mathrm{T}'_{\mathrm{E}}$], [$\mathrm{L}_1$], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathbf{V}}$], [intro$(*, *, *, *)$], [intro$(*, *, *)$], [error$(*, *)$], [error$_2(*, *)$], [proof$(*, *, *)$],

[proof$_2$(∗, ∗)], [$\mathcal{S}$(∗, ∗)], [$\mathcal{S}^{\mathrm{I}}$(∗, ∗)], [$\mathcal{S}^{\rhd}$(∗, ∗)], [$\mathcal{S}_1^{\rhd}$(∗, ∗, ∗)], [$\mathcal{S}^{\mathrm{E}}$(∗, ∗)], [$\mathcal{S}_1^{\mathrm{E}}$(∗, ∗, ∗)], [$\mathcal{S}^{+}$(∗, ∗)], [$\mathcal{S}_1^{+}$(∗, ∗, ∗)], [$\mathcal{S}^{-}$(∗, ∗)], [$\mathcal{S}_1^{-}$(∗, ∗, ∗)], [$\mathcal{S}^{*}$(∗, ∗)], [$\mathcal{S}_1^{*}$(∗, ∗, ∗)], [$\mathcal{S}_2^{*}$(∗, ∗, ∗, ∗)], [$\mathcal{S}^{@}$(∗, ∗)], [$\mathcal{S}_1^{@}$(∗, ∗, ∗)], [$\mathcal{S}^{\vdash}$(∗, ∗)], [$\mathcal{S}_1^{\vdash}$(∗, ∗, ∗, ∗)], [$\mathcal{S}^{\Vdash}$(∗, ∗)], [$\mathcal{S}_1^{\Vdash}$(∗, ∗, ∗, ∗)], [$\mathcal{S}^{\mathrm{i.e.}}$(∗, ∗)], [$\mathcal{S}_1^{\mathrm{i.e.}}$(∗, ∗, ∗, ∗)], [$\mathcal{S}_2^{\mathrm{i.e.}}$(∗, ∗, ∗, ∗, ∗)], [$\mathcal{S}^{\forall}$(∗, ∗)], [$\mathcal{S}_1^{\forall}$(∗, ∗, ∗, ∗)], [$\mathcal{S}^{;}$(∗, ∗)], [$\mathcal{S}_1^{;}$(∗, ∗, ∗)], [$\mathcal{S}_2^{;}$(∗, ∗, ∗, ∗)], [$\mathcal{T}$(∗)], [claims(∗, ∗, ∗)], [claims$_2$(∗, ∗, ∗)], [<proof>], [proof], [[**Lemma** ∗: ∗]], [[**Proof of** ∗: ∗]], [[∗ **lemma** ∗: ∗]], [[∗ **antilemma** ∗: ∗]], [[∗ **rule** ∗: ∗]], [[∗ **antirule** ∗: ∗]], [verifier], [$\mathcal{V}_1$(∗)], [$\mathcal{V}_2$(∗, ∗)], [$\mathcal{V}_3$(∗, ∗, ∗, ∗)], [$\mathcal{V}_4$(∗, ∗)], [$\mathcal{V}_5$(∗, ∗, ∗, ∗)], [$\mathcal{V}_6$(∗, ∗, ∗, ∗)], [$\mathcal{V}_7$(∗, ∗, ∗, ∗)], [Cut(∗, ∗)], [Head$_\oplus$(∗)], [Tail$_\oplus$(∗)], [rule$_1$(∗, ∗)], [rule(∗, ∗)], [Rule tactic], [Plus(∗, ∗)], [[**Theory** ∗]], [theory$_2$(∗, ∗)], [theory$_3$(∗, ∗)], [theory$_4$(∗, ∗, ∗)], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [T$_{\mathrm{E}}$], [ragged right], [ragged right expansion], [parm(∗, ∗, ∗)], [parm*(∗, ∗, ∗)], [inst(∗, ∗)], [inst*(∗, ∗)], [occur(∗, ∗, ∗)], [occur*(∗, ∗, ∗)], [unify(∗ = ∗, ∗)], [unify*(∗ = ∗, ∗)], [unify$_2$(∗ = ∗, ∗)], [L$_{\mathrm{a}}$], [L$_{\mathrm{b}}$], [L$_{\mathrm{c}}$], [L$_{\mathrm{d}}$], [L$_{\mathrm{e}}$], [L$_{\mathrm{f}}$], [L$_{\mathrm{g}}$], [L$_{\mathrm{h}}$], [L$_{\mathrm{i}}$], [L$_{\mathrm{j}}$], [L$_{\mathrm{k}}$], [L$_{\mathrm{l}}$], [L$_{\mathrm{m}}$], [L$_{\mathrm{n}}$], [L$_{\mathrm{o}}$], [L$_{\mathrm{p}}$], [L$_{\mathrm{q}}$], [L$_{\mathrm{r}}$], [L$_{\mathrm{s}}$], [L$_{\mathrm{t}}$], [L$_{\mathrm{u}}$], [L$_{\mathrm{v}}$], [L$_{\mathrm{w}}$], [L$_{\mathrm{x}}$], [L$_{\mathrm{y}}$], [L$_{\mathrm{z}}$], [L$_{\mathrm{A}}$], [L$_{\mathrm{B}}$], [L$_{\mathrm{C}}$], [L$_{\mathrm{D}}$], [L$_{\mathrm{E}}$], [L$_{\mathrm{F}}$], [L$_{\mathrm{G}}$], [L$_{\mathrm{H}}$], [L$_{\mathrm{I}}$], [L$_{\mathrm{J}}$], [L$_{\mathrm{K}}$], [L$_{\mathrm{L}}$], [L$_{\mathrm{M}}$], [L$_{\mathrm{N}}$], [L$_{\mathrm{O}}$], [L$_{\mathrm{P}}$], [L$_{\mathrm{Q}}$], [L$_{\mathrm{R}}$], [L$_{\mathrm{S}}$], [L$_{\mathrm{T}}$], [L$_{\mathrm{U}}$], [L$_{\mathrm{V}}$], [L$_{\mathrm{W}}$], [L$_{\mathrm{X}}$], [L$_{\mathrm{Y}}$], [L$_{\mathrm{Z}}$], [L$_{?}$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[∗ $\overset{\text{tactic}}{=}$ ∗]], [$\mathcal{P}$(∗, ∗, ∗)], [$\mathcal{P}^*$(∗, ∗, ∗)], [p$_0$], [conclude$_1$(∗, ∗)], [conclude$_2$(∗, ∗, ∗)], [conclude$_3$(∗, ∗, ∗, ∗)], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [$\dot{\mathrm{nonfree}}$(∗, ∗)], [$\dot{\mathrm{nonfree}}^*$(∗, ∗)], [free⟨∗|∗ := ∗⟩], [free*⟨∗|∗ := ∗⟩], [∗≡⟨∗|∗ := ∗⟩], [∗≡⟨*∗|∗ := ∗⟩], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$], [L3.2(a)$'$];

Preassociative

[∗_{∗}], [∗$'$], [∗[∗]], [∗[∗→∗]], [∗[∗⇒∗]], [$\dot{*}$];

Preassociative

[“ ∗ ”], [], [(∗)$^{\mathbf{t}}$], [string(∗) + ∗], [string(∗) ++ ∗], [
∗], [∗], [!∗], [”∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗], [0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗], [@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗], [O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [^∗], [_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗], [p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [~∗], [**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end], [newline ∗], [macro newline ∗];

Preassociative

[∗0], [∗1], [0b], [∗-color(∗)], [∗-color*(∗)];

Preassociative

[∗ ’ ∗], [∗ ‘ ∗];

Preassociative

[$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^{0}$], [$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*'$];

Preassociative
$[* \cdot *], [* \cdot_0 *], [* \stackrel{\cdot}{:} *]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*, *]$;
Preassociative
$[* \stackrel{B}{\approx} *], [* \stackrel{D}{\approx} *], [* \stackrel{C}{\approx} *], [* \stackrel{P}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\to} *], [* \stackrel{t}{=} *], [* \stackrel{t^*}{=} *], [* \stackrel{r}{=} *]$,
$[* \in_t *], [* \subseteq_T *], [* \stackrel{T}{=} *], [* \stackrel{s}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *]$,
$[* \text{ free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* \stackrel{p}{=} *], [*^{\mathcal{P}}]$;
Preassociative
$[\neg *], [\dot{\neg} *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *]$;
Preassociative
$[\dot{\forall} *: *], [\dot{\exists} *: *]$;
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *]$;
Postassociative
$[* : *], [*!*]$;
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.]$;
Preassociative
$[\lambda * .*], [\Lambda *]$, [**if** $*$ **then** $*$ **else** $*$], [**let** $* = *$ **in** $*$], [**let** $* \ddot{=} *$ in $*$];
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}]$;
Preassociative
$[* @ *], [* \triangleright *], [* \,\triangleright\!\!\!\triangleright\, *], [* \gg *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall *: *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
$[* \text{ proves } *]$;

Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \square$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$];
Postassociative
[$*$ then $*$], [$*[*]*$];
Preassociative
[$*\&*$];
Preassociative
[$*\backslash\backslash*$];]

[peano $\overset{\text{pyk}}{\to}$ "peano"]

$\dot{0}$

[$\dot{0} \overset{\text{tex}}{\to}$ "
\dot{0}"]

[$\dot{0} \overset{\text{pyk}}{\to}$ "peano zero"]

$\dot{1}$

[$\dot{1} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{1} \ddot{=} \dot{0}']\rceil)$]

[$\dot{1} \overset{\text{tex}}{\to}$ "
\dot{1}"]

[$\dot{1} \overset{\text{pyk}}{\to}$ "peano one"]

$\dot{2}$

[$\dot{2} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{2} \ddot{=} \dot{1}']\rceil)$]

[$\dot{2} \overset{\text{tex}}{\to}$ "
\dot{2}"]

[$\dot{2} \overset{\text{pyk}}{\to}$ "peano two"]

$\dot{a}$

[$\dot{a} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{a} \ddot{=} \dot{\mathsf{a}}]\rceil)$]

$[\dot{a} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{a}}"]

$[\dot{a} \overset{\mathrm{pyk}}{\to}$ "peano a"]

$\dot{b}$

$[\dot{b} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{b}}"]

$[\dot{b} \overset{\mathrm{pyk}}{\to}$ "peano b"]

$\dot{c}$

$[\dot{c} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{c}}"]

$[\dot{c} \overset{\mathrm{pyk}}{\to}$ "peano c"]

$\dot{d}$

$[\dot{d} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{d}}"]

$[\dot{d} \overset{\mathrm{pyk}}{\to}$ "peano d"]

$\dot{e}$

$[\dot{e} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{e}}"]

$[\dot{e} \overset{\mathrm{pyk}}{\to}$ "peano e"]

$\dot{f}$

$[\dot{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \ddot{=} \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\to}$ "peano f"]

$\dot{g}$

$[\dot{g} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\to}$ "peano g"]

$\dot{h}$

$[\dot{h} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\to}$ "peano h"]

$\dot{i}$

$[\dot{i} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\to}$ "peano i"]

$\dot{j}$

$[\dot{j} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\to}$ "peano j"]

$\dot{k}$

$[\dot{k} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{k} \ddot{=} \dot{\mathsf{k}}] \rceil)]$

$[\dot{k} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{k}}"]

$[\dot{k} \stackrel{\text{pyk}}{\to}$ "peano k"]

$\dot{l}$

$[\dot{l} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{l} \ddot{=} \dot{\mathsf{l}}] \rceil)]$

$[\dot{l} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{l}}"]

$[\dot{l} \stackrel{\text{pyk}}{\to}$ "peano l"]

$\dot{m}$

$[\dot{m} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{m} \ddot{=} \dot{\mathsf{m}}] \rceil)]$

$[\dot{m} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{m}}"]

$[\dot{m} \stackrel{\text{pyk}}{\to}$ "peano m"]

$\dot{n}$

$[\dot{n} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{n} \ddot{=} \dot{\mathsf{n}}] \rceil)]$

$[\dot{n} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{n}}"]

$[\dot{n} \stackrel{\text{pyk}}{\to}$ "peano n"]

$\dot{o}$

$[\dot{o} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{o} \ddot{=} \dot{\mathsf{o}}] \rceil)]$

$[\dot{o} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{o}}"]

$[\dot{o} \stackrel{\text{pyk}}{\to}$ "peano o"]

$\dot{p}$

$[\dot{p} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{p} \ddot{=} \dot{\mathsf{p}}] \rceil)]$

$[\dot{p} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{p}}"]

$[\dot{p} \stackrel{\text{pyk}}{\to}$ "peano p"]

$\dot{q}$

$[\dot{q} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{q} \ddot{=} \dot{\mathsf{q}}] \rceil)]$

$[\dot{q} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{q}}"]

$[\dot{q} \stackrel{\text{pyk}}{\to}$ "peano q"]

$\dot{r}$

$[\dot{r} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{r} \ddot{=} \dot{\mathsf{r}}] \rceil)]$

$[\dot{r} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{r}}"]

$[\dot{r} \stackrel{\text{pyk}}{\to}$ "peano r"]

$\dot{s}$

$[\dot{s} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{s} \ddot{=} \dot{\mathsf{s}}] \rceil)]$

$[\dot{s} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{s}}"]

$[\dot{s} \stackrel{\text{pyk}}{\to}$ "peano s"]

$\dot{t}$

$[\dot{t} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{t} \ddot{=} \dot{\mathsf{t}}] \rceil)]$

$[\dot{t} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{t}}"]

$[\dot{t} \stackrel{\text{pyk}}{\to}$ "peano t"]

$\dot{u}$

$[\dot{u} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{u} \ddot{=} \dot{\mathsf{u}}] \rceil)]$

$[\dot{u} \overset{\text{tex}}{\to}$ "
\dot{\mathit{u}}"]

$[\dot{u} \overset{\text{pyk}}{\to}$ "peano u"]

$\dot{v}$

$[\dot{v} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{v} \ddot{=} \dot{\mathsf{v}}] \rceil)]$

$[\dot{v} \overset{\text{tex}}{\to}$ "
\dot{\mathit{v}}"]

$[\dot{v} \overset{\text{pyk}}{\to}$ "peano v"]

$\dot{w}$

$[\dot{w} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{w} \ddot{=} \dot{\mathsf{w}}] \rceil)]$

$[\dot{w} \overset{\text{tex}}{\to}$ "
\dot{\mathit{w}}"]

$[\dot{w} \overset{\text{pyk}}{\to}$ "peano w"]

$\dot{x}$

$[\dot{x} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{x} \ddot{=} \dot{\mathsf{x}}] \rceil)]$

$[\dot{x} \overset{\text{tex}}{\to}$ "
\dot{\mathit{x}}"]

$[\dot{x} \overset{\text{pyk}}{\to}$ "peano x"]

$\dot{y}$

$[\dot{y} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{y} \ddot{=} \dot{\mathsf{y}}] \rceil)]$

$[\dot{y} \overset{\text{tex}}{\to}$ "
\dot{\mathit{y}}"]

$[\dot{y} \overset{\text{pyk}}{\to}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{z} \ddot{=} \dot{\mathsf{z}}] \rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\rightarrow}$ "peano z"]

$\dot{\text{nonfree}}(*, *)$

$[\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}) \stackrel{\text{val}}{\rightarrow}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg\, [\, \mathsf{x} \stackrel{\text{t}}{=} \mathsf{y}\,],$
$\text{If}(\neg\, [\, \mathsf{y} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{x}\colon \mathsf{y} \rceil\,], \dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=} [\, \mathsf{y}^1\,], \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}^2))))]$

$[\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}) \stackrel{\text{tex}}{\rightarrow}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}) \stackrel{\text{pyk}}{\rightarrow}$ "peano nonfree $*$ in $*$ end nonfree"]

$\dot{\text{nonfree}}^*(*, *)$

$[\dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}) \stackrel{\text{val}}{\rightarrow} \mathsf{x}!\text{If}(\mathsf{y}, \mathsf{T}, \text{If}(\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}^{\text{h}}), \dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}^{\text{t}}), \mathsf{F}))]$

$[\dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}) \stackrel{\text{tex}}{\rightarrow}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}) \stackrel{\text{pyk}}{\rightarrow}$ "peano nonfree star $*$ in $*$ end nonfree"]

$\dot{\text{free}}\langle * | * := * \rangle$

$[\dot{\text{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\text{val}}{\rightarrow} \mathsf{x}!\ [\ \mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg\, [\, \mathsf{a} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil\,], \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}} | \mathsf{x} := \mathsf{b} \rangle,$
$\text{If}(\mathsf{a}^1 \stackrel{\text{t}}{=} \mathsf{x}, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x}, \mathsf{a}^2), \mathsf{T},$

$\mathrm{If}(\neg\dot{\mathrm{nonfree}}(\mathsf{a}^1, \mathsf{b}), \mathsf{F},$
$\dot{\mathrm{free}}\langle \mathsf{a}^2 | \mathsf{x} := \mathsf{b} \rangle)))))$]]

$[\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

$[\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{pyk}}{\to}$ "peano free $*$ set $*$ to $*$ end free"]

$\dot{\mathrm{free}}^*\langle * | * := * \rangle$

$[\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{val}}{\to} \mathsf{x}!$ [$\mathsf{b}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\dot{\mathrm{free}}\langle \mathsf{a}^{\mathrm{h}} | \mathsf{x} := \mathsf{b} \rangle, \dot{\mathrm{free}}^*\langle \mathsf{a}^{\mathrm{t}} | \mathsf{x} := \mathsf{b} \rangle, \mathsf{F}))$]]

$[\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

$[\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{pyk}}{\to}$ "peano free star $*$ set $*$ to $*$ end free"]

$*\equiv\langle * | * := * \rangle$

$[\mathsf{a}\equiv\langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{val}}{\to} \mathsf{a}!$ [$\mathsf{x}!$ [$\mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \overset{\mathrm{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil, \mathsf{b}^1 \overset{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge$ [$\mathsf{b} \overset{\mathrm{t}}{=} \mathsf{x}$] $, \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}($ [
a] $\overset{\mathrm{r}}{=} \mathsf{b}, \mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F})))$]]]

$[\mathsf{a}\equiv\langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{tex}}{\to}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

$[\mathsf{a}\equiv\langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{pyk}}{\to}$ "peano sub $*$ is $*$ where $*$ is $*$ end sub"]

$*\equiv\langle^* * | * := * \rangle$

$[\mathsf{a}\equiv\langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{val}}{\to} \mathsf{b}!$ [$\mathsf{x}!$ [$\mathsf{c}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\mathsf{a}^{\mathrm{h}}\equiv\langle \mathsf{b}^{\mathrm{h}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F}))$]]]

$[\mathsf{a}\equiv\langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{tex}}{\to}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

$[\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\text{pyk}}{\rightarrow}$ "peano sub star * is * where * is * end sub"]

S

$$[S \stackrel{\text{stmt}}{\rightarrow} [[\dot{a} \dot{+} [\dot{b}']] \stackrel{p}{=} [[\dot{a} \dot{+} [\dot{b}]] ']] \oplus [[[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \dot{\neg} \underline{a}] \dot{\Rightarrow} [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]] \oplus [[[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [\dot{a}' \stackrel{p}{=} [\dot{b}']]] \oplus [[\forall \underline{a} \colon \forall \underline{b} \colon [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]] \oplus [[[\dot{a}' \stackrel{p}{=} [\dot{b}']] \dot{\Rightarrow} [\dot{a} \stackrel{p}{=} [\dot{b}]]] \oplus [[\forall \underline{a} \colon \forall \underline{b} \colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]]] \oplus [[\forall \underline{x} \colon \forall \underline{a} \colon \forall \underline{b} \colon [\mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x} \colon [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x} \colon \underline{b}]]]] \oplus [[[\dot{a} \dot{\cdot} [\dot{b}']] \stackrel{p}{=} [[\dot{a} \dot{\cdot} [\dot{b}]] \dot{+} [\dot{a}]]] \oplus [[[\dot{a} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{a}]] \oplus [[\forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \oplus [[[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{c}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{c}]]]] \oplus [[\forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon \forall \underline{x} \colon [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall} \underline{x} \colon [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall} \underline{x} \colon \underline{a}]]]]]] \oplus [[\neg [\dot{0} \stackrel{p}{=} [\dot{a}']]] \oplus [[\forall \underline{x} \colon \forall \underline{a} \colon [\underline{a} \vdash \dot{\forall} \underline{x} \colon \underline{a}]] \oplus [[\forall \underline{c} \colon \forall \underline{a} \colon \forall \underline{x} \colon \forall \underline{b} \colon [\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [[\dot{\forall} \underline{x} \colon \underline{b}] \dot{\Rightarrow} \underline{a}]]] \oplus [[\dot{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]]]]]]]]]]]]]]]]]$$

$[S \stackrel{\text{tex}}{\rightarrow}$ "
S"]

$[S \stackrel{\text{pyk}}{\rightarrow}$ "system s"]

A1

$[A1 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A1 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]]]$

$[A1 \stackrel{\text{tex}}{\rightarrow}$ "
A1"]

$[A1 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a one"]

A2

$[A2 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A2 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]$

$[A2 \stackrel{\text{tex}}{\rightarrow}$ "
A2"]

$[A2 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a two"]

A3

$[A3 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A3 \stackrel{\text{stmt}}{\rightarrow} S \vdash [[[\dot{\neg}\underline{b}] \dot{\Rightarrow} \dot{\neg}\underline{a}] \dot{\Rightarrow} [[[\dot{\neg}\underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]]]$

$[A3 \stackrel{\text{tex}}{\rightarrow}$ "
A3"]

$[A3 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a three"]

A4

$[A4 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A4 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall\underline{c}\colon \forall\underline{a}\colon \forall\underline{x}\colon \forall\underline{b}\colon [\lceil\underline{a}\rceil \equiv \langle\lceil\underline{b}\rceil | \lceil\underline{x}\rceil := \lceil\underline{c}\rceil\rangle \Vdash [[\dot{\forall}\underline{x}\colon \underline{b}] \dot{\Rightarrow} \underline{a}]]]$

$[A4 \stackrel{\text{tex}}{\rightarrow}$ "
A4"]

$[A4 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a four"]

A5

$[A5 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A5 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall\underline{x}\colon \forall\underline{a}\colon \forall\underline{b}\colon [\text{non}\dot{\text{f}}\text{ree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall}\underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{b}]]]]$

$[A5 \stackrel{\text{tex}}{\rightarrow}$ "
A5"]

$[A5 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a five"]

S1

$[S1 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S1 \stackrel{\text{stmt}}{\rightarrow} S \vdash [[\dot{a} \stackrel{\text{p}}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{\text{p}}{=} [\dot{c}]] \dot{\Rightarrow} [\dot{b} \stackrel{\text{p}}{=} [\dot{c}]]]]]$

[S1 $\stackrel{\text{tex}}{\rightarrow}$ "
S1"]

[S1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s one"]

S2

[S2 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\, [\, \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} [\, \dot{\mathsf{b}} \,] \,] \dot{\Rightarrow} [\, \dot{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\, \dot{\mathsf{b}}' \,] \,] \,]$]

[S2 $\stackrel{\text{tex}}{\rightarrow}$ "
S2"]

[S2 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s two"]

S3

[S3 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \neg [\, \dot{0} \stackrel{\mathrm{p}}{=} [\, \dot{\mathsf{a}}' \,] \,]$]

[S3 $\stackrel{\text{tex}}{\rightarrow}$ "
S3"]

[S3 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s three"]

S4

[S4 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\, [\, \dot{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\, \dot{\mathsf{b}}' \,] \,] \dot{\Rightarrow} [\, \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} [\, \dot{\mathsf{b}} \,] \,] \,]$]

[S4 $\stackrel{\text{tex}}{\rightarrow}$ "
S4"]

[S4 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s four"]

S5

[S5 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\, [\, \dot{\mathsf{a}} \dot{+} \dot{0} \,] \stackrel{\mathrm{p}}{=} [\, \dot{\mathsf{a}} \,] \,]$]

[S5 $\stackrel{\text{tex}}{\rightarrow}$ "
S5"]

[S5 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s five"]

S6

[S6 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{+} [\dot{\mathsf{b}}'\ ]\ ] \stackrel{\mathrm{p}}{=} [\ [\dot{\mathsf{a}} \dot{+} [\dot{\mathsf{b}}\ ]\ ]'\ ]\ ]$]

[S6 $\stackrel{\text{tex}}{\rightarrow}$ "
S6"]

[S6 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s six"]

S7

[S7 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{\cdot} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \dot{0}\ ]$]

[S7 $\stackrel{\text{tex}}{\rightarrow}$ "
S7"]

[S7 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s seven"]

S8

[S8 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{\cdot} [\dot{\mathsf{b}}'\ ]\ ] \stackrel{\mathrm{p}}{=} [\ [\dot{\mathsf{a}} \dot{\cdot} [\dot{\mathsf{b}}\ ]\ ] \dot{+} [\dot{\mathsf{a}}\ ]\ ]\ ]$]

[S8 $\stackrel{\text{tex}}{\rightarrow}$ "
S8"]

[S8 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s eight"]

S9

[S9 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{x}}\colon [\ \underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash [\ \underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon [$
$\underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]$]

[S9 $\stackrel{\text{tex}}{\rightarrow}$ “
S9”]

[S9 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom s nine”]

MP

[MP $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP $\stackrel{\text{tex}}{\rightarrow}$ “
MP”]

[MP $\stackrel{\text{pyk}}{\rightarrow}$ “rule mp”]

Gen

[Gen $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{x} \colon \forall \underline{a} \colon [\ \underline{a} \vdash \dot{\forall} \underline{x} \colon \underline{a}\ ]$]

[Gen $\stackrel{\text{tex}}{\rightarrow}$ “
Gen”]

[Gen $\stackrel{\text{pyk}}{\rightarrow}$ “rule gen”]

L3.2(a)

[L3.2(a) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash [\ [\ \mathrm{S5} \gg [\ [\ \dot{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{Gen} \rhd [\ [\ \dot{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ \gg \dot{\forall}\dot{a} \colon [\ [\ \dot{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{A4} @ [\ \dot{x}\ ]\ ] \gg [\ [\ \dot{\forall}\dot{a} \colon [\ [\ \dot{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{x} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \mathrm{MP} \rhd [\ [\ \dot{\forall}\dot{a} \colon [\ [\ \dot{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{x} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{a} \colon [\ [\ \dot{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \gg [\ [\ \dot{x} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ; [\ [\ \mathrm{S1} \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{Gen} \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{A4} @ [\ \dot{x}\ ]\ ] \gg [\ [\ \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \mathrm{MP} \rhd [\ [\ \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{c} \colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \mathrm{Gen} \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [$

$$\dot{b} \stackrel{p}{=} [\dot{x}]]]]] \gg \dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]]] ; [[[\mathrm{A4} \mathbin{@} [\dot{x}]] \gg [[\dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]]]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]] ; [[[[\mathrm{MP} \rhd [[\dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]]]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]] \rhd \dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]]]] \gg [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] ; [[[\mathrm{Gen} \rhd [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \gg \dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] ; [[[\mathrm{A4} \mathbin{@} [\dot{x} \dot{+} \dot{0}]] \gg [[\dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]]] ; [[[[\mathrm{MP} \rhd [[\dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]] \rhd \dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \gg [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] ; [[[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \rhd [[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] ; [[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \rhd [[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$$

$$[\mathrm{L3.2(a)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash [\dot{x} \stackrel{p}{=} [\dot{x}]]]$$

[L3.2(a) $\stackrel{\mathrm{tex}}{\rightarrow}$ "
L3.2(a)"]

[L3.2(a) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma l three two a"]

S′

$$[\mathrm{S}' \stackrel{\mathrm{stmt}}{\rightarrow} [\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a}' \stackrel{p}{=} [\underline{b}']] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{b}]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]]] \oplus [[\forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{\cdot} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{\cdot} \underline{b}] \dot{+} \underline{a}]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [\underline{a}' \stackrel{p}{=} [\underline{b}']]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]]] \oplus [[[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \dot{\neg} \underline{a}] \dot{\Rightarrow} [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{+} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}]']]]] \oplus [[\forall \underline{a}\colon \neg [\dot{0} \stackrel{p}{=} [\underline{a}']]]] \oplus [[\forall \underline{x}\colon \forall \underline{a}\colon [\underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}]] \oplus [[\forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [[\dot{\forall} \underline{x}\colon \underline{b}] \dot{\Rightarrow} \underline{a}]]]] \oplus [[\forall \underline{a}\colon [[\underline{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [[\underline{a} \stackrel{p}{=} \underline{c}] \dot{\Rightarrow} [\underline{b} \stackrel{p}{=} \underline{c}]]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall} \underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] \oplus \forall \underline{a}\colon [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]]]]]]]]]]]]]]]]$$

[S′ $\stackrel{\mathrm{tex}}{\rightarrow}$ "
S'"]

[$S' \xrightarrow{\text{pyk}}$ "system prime s"]

A1′

[$A1' \xrightarrow{\text{proof}}$ Rule tactic]

[$A1' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,]$]

[$A1' \xrightarrow{\text{tex}}$ "
A1′"]

[$A1' \xrightarrow{\text{pyk}}$ "axiom prime a one"]

A2′

[$A2' \xrightarrow{\text{proof}}$ Rule tactic]

[$A2' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\, [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \dot{\Rightarrow} [\, [\, \underline{a} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} [\, \underline{a} \dot{\Rightarrow} \underline{c} \,] \,] \,]$]

[$A2' \xrightarrow{\text{tex}}$ "
A2′"]

[$A2' \xrightarrow{\text{pyk}}$ "axiom prime a two"]

A3′

[$A3' \xrightarrow{\text{proof}}$ Rule tactic]

[$A3' \xrightarrow{\text{stmt}} S' \vdash [\, [\, [\, \dot{\neg} \underline{b} \,] \dot{\Rightarrow} \dot{\neg} \underline{a} \,] \dot{\Rightarrow} [\, [\, [\, \dot{\neg} \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \dot{\Rightarrow} \underline{b} \,] \,]$]

[$A3' \xrightarrow{\text{tex}}$ "
A3′"]

[$A3' \xrightarrow{\text{pyk}}$ "axiom prime a three"]

A4′

[$A4' \xrightarrow{\text{proof}}$ Rule tactic]

[$A4' \xrightarrow{\text{stmt}} S' \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\, [\, \dot{\forall} \underline{x}\colon \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \,]$]

[$A4' \xrightarrow{\text{tex}}$ "
A4′"]

[$A4' \xrightarrow{\text{pyk}}$ "axiom prime a four"]

A5′

[A5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \dot{\text{nonfree}}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]$]

[A5′ $\stackrel{\text{tex}}{\rightarrow}$ “
A5’”]

[A5′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a five”]

S1′

[S1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \underline{a} \stackrel{p}{=} \underline{c}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{b} \stackrel{p}{=} \underline{c}\ ]\ ]\ ]$]

[S1′ $\stackrel{\text{tex}}{\rightarrow}$ “
S1’”]

[S1′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s one”]

S2′

[S2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ ]$]

[S2′ $\stackrel{\text{tex}}{\rightarrow}$ “
S2’”]

[S2′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s two”]

S3′

[S3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \neg [\ \dot{0} \stackrel{p}{=} [\ \underline{a}'\ ]\ ]$]

[S3′ $\stackrel{\text{tex}}{\rightarrow}$ “
S3’”]

[S3′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s three”]

S4′

[S4′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ ]$]

[S4′ $\stackrel{\text{tex}}{\rightarrow}$ "
S4'"]

[S4′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s four"]

S5′

[S5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]$]

[S5′ $\stackrel{\text{tex}}{\rightarrow}$ "
S5'"]

[S5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s five"]

S6′

[S6′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\ \underline{a} \dot{+} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{+} \underline{b}\ ]'\ ]\ ]$]

[S6′ $\stackrel{\text{tex}}{\rightarrow}$ "
S6'"]

[S6′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s six"]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon [\ [\ \underline{a} \dot{\cdot} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]$]

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7'"]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon$ [[$\underline{a} \mathbin{\dot{:}}$ [$\underline{b}'$]] $\stackrel{p}{=}$ [[$\underline{a} \mathbin{\dot{:}} \underline{b}$] $\dot{+}\, \underline{a}$]]]

[S8′ $\stackrel{\text{tex}}{\rightarrow}$ "
S8′"]

[S8′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon$ [$\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash$ [$\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash$ [$\underline{b} \mathrel{\dot{\Rightarrow}}$ [[$\dot{\forall} \underline{x}\colon$ [$\underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}$]] $\mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{a}$]]]]]

[S9′ $\stackrel{\text{tex}}{\rightarrow}$ "
S9′"]

[S9′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon$ [[$\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}$] $\vdash$ [$\underline{a} \vdash \underline{b}$]]]

[MP′ $\stackrel{\text{tex}}{\rightarrow}$ "
MP′"]

[MP′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{x}\colon \forall \underline{a}\colon$ [$\underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}$]]

[Gen′ $\stackrel{\text{tex}}{\rightarrow}$ "
Gen′"]

[Gen′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime gen"]

L3.2(a)′

[L3.2(a)′ $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon [[S5' \gg [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] ; [[S1' \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]]] ; [[[[MP' \rhd [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]]] \rhd [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]] ; [[[MP' \rhd [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]] \rhd [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [\underline{a} \stackrel{p}{=} \underline{a}]]]]] \rceil, p_0, c)$]

[L3.2(a)′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\underline{a} \stackrel{p}{=} \underline{a}]$]

[L3.2(a)′ $\stackrel{\text{tex}}{\rightarrow}$ “
L3.2(a)’”]

[L3.2(a)′ $\stackrel{\text{pyk}}{\rightarrow}$ “lemma prime l three two a”]

$\dot{*}$

[$\dot{x}$ $\stackrel{\text{tex}}{\rightarrow}$ “
\dot{#1.
}”]

[$\dot{x}$ $\stackrel{\text{pyk}}{\rightarrow}$ “* peano var”]

$*'$

[x' $\stackrel{\text{tex}}{\rightarrow}$ “#1.
’”]

[x' $\stackrel{\text{pyk}}{\rightarrow}$ “* peano succ”]

$* \mathop{\dot{\cdot}} *$

[$x \mathop{\dot{\cdot}} y$ $\stackrel{\text{tex}}{\rightarrow}$ “#1.
\mathop{\dot{\cdot}} #2.”]

[$x \mathop{\dot{\cdot}} y$ $\stackrel{\text{pyk}}{\rightarrow}$ “* peano times *”]

$* \mathop{\dot{+}} *$

[$x \mathop{\dot{+}} y$ $\stackrel{\text{tex}}{\rightarrow}$ “#1.
\mathop{\dot{+}} #2.”]

$[\mathsf{x} \dot{+} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano plus $*$"]

$* \stackrel{p}{=} *$

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano is $*$"]

$*^{\mathcal{P}}$

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil]$

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ is peano var"]

$\dot{\neg} *$

$[\dot{\neg} \mathsf{x} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\neg}\, #1."]

$[\dot{\neg} \mathsf{x} \stackrel{\text{pyk}}{\rightarrow}$ "peano not $*$"]

$* \dot{\wedge} *$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \dot{\wedge} \mathsf{y} \ddot{=} \dot{\neg} (\mathsf{x} \dot{\Rightarrow} \dot{\neg} \mathsf{y})] \rceil)]$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\wedge}} #2."]

$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano and $*$"]

$* \dot{\vee} *$

$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \dot{\vee} \mathsf{y} \ddot{=} \; [\, \dot{\neg} \mathsf{x} \,] \; \dot{\Rightarrow} \mathsf{y}] \rceil)]$

$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\vee}} #2."]

$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano or $*$"]

$\dot{\forall} {*}\colon {*}$

$[\dot{\forall} \mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall} \mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "peano all * indeed *"]

$\dot{\exists} {*}\colon {*}$

$[\dot{\exists} \mathsf{x}\colon \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{\exists} \mathsf{x}\colon \mathsf{y} \ddot{=} \dot{\neg} \dot{\forall} \mathsf{x}\colon \dot{\neg} \mathsf{y}] \rceil)]$

$[\dot{\exists} \mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\exists} #1.
\colon #2."]

$[\dot{\exists} \mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "peano exist * indeed *"]

$* \mathrel{\dot{\Rightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano imply *"]

$* \mathrel{\dot{\Leftrightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \ddot{=} (\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y}) \mathrel{\dot{\wedge}} (\mathsf{y} \mathrel{\dot{\Rightarrow}} \mathsf{x})] \rceil)]$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano iff *"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-20.UTC:11:17:32.910911 = MJD-53541.TAI:11:18:04.910911 = LGT-4625983084910911e-6