Peano arithmetic

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Contents

1 Peano arithmetic

This Logiweb page [1] defines Peano arithmetic.

1.1 The constructs of Peano arithmetic

Terms of Peano arithmetic are constructed from zero $[\dot{0} \stackrel{\text{pyk}}{\rightarrow}$ “peano zero”]$[\dot{0} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{0}”], successor $[\mathsf{x}' \stackrel{\text{pyk}}{\rightarrow}$ “∗ peano succ”]$[\mathsf{x}' \stackrel{\text{tex}}{\rightarrow}$ “#1.’”], plus $[\mathsf{x} \dot{+} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ “∗ peano plus ∗”]$[\mathsf{x} \dot{+} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ “#1.
\mathop{\dot{+}} #2.”], and times $[\mathsf{x} \dot{\cdot} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ “∗ peano times ∗”]$[\mathsf{x} \dot{\cdot} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ “#1.
\mathop{\dot{\cdot}} #2.”].

Formulas of Peano arithmeric are constructed from equality $[\mathsf{x} \stackrel{\text{p}}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ “∗ peano is ∗”]$[\mathsf{x} \stackrel{\text{p}}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ “#1.

\stackrel{p}{=} #2."], negation $[\dot{\neg}\mathsf{x} \stackrel{\text{pyk}}{\to}$ "peano not *"]$[\dot{\neg}\mathsf{x} \stackrel{\text{tex}}{\to}$ "
\dot{\neg}\, {#1.}"], implication $[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano imply
*"]$[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."], and universal quantification
$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\to}$ "peano all * indeed *"]$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\forall} #1.
\colon #2."].

From these constructs we macro define one $[\dot{1} \stackrel{\text{pyk}}{\to}$ "peano one"]$[\dot{1} \stackrel{\text{tex}}{\to}$ "
\dot{1}"], two $[\dot{2} \stackrel{\text{pyk}}{\to}$ "peano two"]$[\dot{2} \stackrel{\text{tex}}{\to}$ "
\dot{2}"], conjunction $[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano and *"]$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\wedge}} #2."], disjunction $[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano or
*"]$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\vee}} #2."], biimplication $[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano iff
*"]$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."], and existential quantification
$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\to}$ "peano exist * indeed *"]$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\exists} #1.
\colon #2."]:

$$[\dot{1} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{1} \ddot{=} \dot{0}']\rceil)]$$

$$[\dot{2} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{2} \ddot{=} \dot{1}']\rceil)]$$

$$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\wedge} \mathsf{y} \ddot{=} \dot{\neg}(\mathsf{x} \dot{\Rightarrow} \dot{\neg}\mathsf{y})]\rceil)]$$

$$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\vee} \mathsf{y} \ddot{=} \dot{\neg}\mathsf{x} \dot{\Rightarrow} \mathsf{y}]\rceil)]$$

$$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \ddot{=} (\mathsf{x} \dot{\Rightarrow} \mathsf{y}) \dot{\wedge} (\mathsf{y} \dot{\Rightarrow} \mathsf{x})]\rceil)]$$

$$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{\exists}\mathsf{x}\colon \mathsf{y} \ddot{=} \dot{\neg}\dot{\forall}\mathsf{x}\colon \dot{\neg}\mathsf{y}]\rceil)]$$

1.2 Variables

We now introduce the unary operator $[\dot{\mathsf{x}} \stackrel{\text{pyk}}{\to}$ "* peano var"]$[\dot{\mathsf{x}} \stackrel{\text{tex}}{\to}$ "
\dot{#1.
}"] and define that a term is a *Peano variable* (i.e. a variable of Peano arithmetic) if it has the $[\dot{\mathsf{x}}]$ operator in its root. $[\mathsf{x}^{\mathcal{P}} \stackrel{\text{pyk}}{\to}$ "* is peano
var"]$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{tex}}{\to}$ "#1.
{} ^ {\cal P}"] is true if $[\mathsf{x}]$ is a Peano variable:

$$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{val}}{\to} \mathsf{x} \stackrel{\text{r}}{=} \lceil\dot{\mathsf{x}}\rceil]$$

We macro define $[\dot{a} \stackrel{\text{pyk}}{\to}$ "peano a"]$[\dot{a} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{a}}"] to be a Peano variable:

$[\dot{a} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{a} \ddot{=} \dot{\mathsf{a}}]\rceil)]$

Appendix A.2 defines Peano variables for the other letters of the English alphabet.

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \overset{\text{pyk}}{\to}$ "peano nonfree * in * end nonfree"$][\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \overset{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"] is true if the Peano variable [x] does not occur free in the Peano term/formula [y]. $[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \overset{\text{pyk}}{\to}$ "peano nonfree star * in * end nonfree"$][\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \overset{\text{tex}}{\to}$ "
\dot{nonfree}ˆ*(#1.
, #2.
)"] is true if the Peano variable [x] does not occur free in the list [y] of Peano terms/formulas.

$$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \overset{\text{val}}{\to}$$
$$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg \mathsf{x} \overset{\text{t}}{=} \mathsf{y},$$
$$\text{If}(\neg \mathsf{y} \overset{\text{r}}{=} \lceil \dot{\forall} \mathsf{x}\colon \mathsf{y}\rceil, \dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),$$
$$\text{If}(\mathsf{x} \overset{\text{t}}{=} \mathsf{y}^1, \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^2)))))]$$

$$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \overset{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y},\mathsf{T},\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}), \dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),\mathsf{F}))]$$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{pyk}}{\to}$ "peano free * set * to * end free"$][\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"] is true if the substitution $[\langle \mathsf{a}\,|\mathsf{x}{:=}\,\mathsf{b}\rangle]$ is free.
$[\dot{\text{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{pyk}}{\to}$ "peano free star * set * to * end free"$][\dot{\text{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{tex}}{\to}$ "
\dot{free}{}ˆ*\langle #1.
| #2.
:= #3.
\rangle"] is the version where [a] is a list of terms.

$$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{val}}{\to} \mathsf{x}!\mathsf{b}!$$
$$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$$
$$\text{If}(\neg \mathsf{a} \overset{\text{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v}\rceil, \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$$
$$\text{If}(\mathsf{a}^1 \overset{\text{t}}{=} \mathsf{x}, \mathsf{T},$$
$$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^2), \mathsf{T},$$
$$\text{If}(\neg \dot{\text{nonfree}}(\mathsf{a}^1,\mathsf{b}), \mathsf{F},$$
$$\dot{\text{free}}\langle \mathsf{a}^2|\mathsf{x} := \mathsf{b}\rangle))))))]$$

$$[\dot{\text{free}}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{val}}{\to} \mathsf{x}!\mathsf{b}!\text{If}(\mathsf{a},\mathsf{T},\text{If}(\dot{\text{free}}\langle \mathsf{a}^{\text{h}}|\mathsf{x} := \mathsf{b}\rangle, \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,\mathsf{F}))]$$

$[\mathsf{a}{\equiv}\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\text{pyk}}{\to}$ "peano sub * is * where * is * end sub"$][\mathsf{a}{\equiv}\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\text{tex}}{\to}$ "#1.
{\equiv}\langle #2.

|#3.
:=#4.
\rangle"] is true if [a] equals [⟨b |x:= c⟩]. [$a \equiv \langle^* b | x := c \rangle \overset{\text{pyk}}{\to}$ "peano sub star * is * where * is * end sub"][$a \equiv \langle^* b | x := c \rangle \overset{\text{tex}}{\to}$ "#1.
{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"] is the version where [a] and [b] are lists.

$$[a \equiv \langle b | x := c \rangle \overset{\text{val}}{\to} a!x!c! \\ \text{If}(\text{If}(b \overset{r}{=} \lceil \dot{\forall} u \colon v \rceil, b^1 \overset{t}{=} x, F), a \overset{t}{=} b, \\ \text{If}(b^{\mathcal{P}} \wedge b \overset{t}{=} x, a \overset{t}{=} c, \text{If}(\\ a \overset{r}{=} b, a^t \equiv \langle^* b^t | x := c \rangle, F)))]$$

$$[a \equiv \langle^* b | x := c \rangle \overset{\text{val}}{\to} b!x!c!\text{If}(a, T, \text{If}(a^h \equiv \langle b^h | x := c \rangle, a^t \equiv \langle^* b^t | x := c \rangle, F))]$$

1.3 Mendelsons system S

System [$S \overset{\text{pyk}}{\to}$ "system s"][$S \overset{\text{tex}}{\to}$ "
S"] of Mendelson [2] expresses Peano arithmetic. It comprises the axioms [$A1 \overset{\text{pyk}}{\to}$ "axiom a one"][$A1 \overset{\text{tex}}{\to}$ "
A1"], [$A2 \overset{\text{pyk}}{\to}$ "axiom a two"][$A2 \overset{\text{tex}}{\to}$ "
A2"], [$A3 \overset{\text{pyk}}{\to}$ "axiom a three"][$A3 \overset{\text{tex}}{\to}$ "
A3"], [$A4 \overset{\text{pyk}}{\to}$ "axiom a four"][$A4 \overset{\text{tex}}{\to}$ "
A4"], and [$A5 \overset{\text{pyk}}{\to}$ "axiom a five"][$A5 \overset{\text{tex}}{\to}$ "
A5"] and inference rules [$MP \overset{\text{pyk}}{\to}$ "rule mp"][$MP \overset{\text{tex}}{\to}$ "
MP"] and [$Gen \overset{\text{pyk}}{\to}$ "rule gen"][$Gen \overset{\text{tex}}{\to}$ "
Gen"] of first order predicate calculus. Furthermore, it comprises the proper axioms [$S1 \overset{\text{pyk}}{\to}$ "axiom s one"][$S1 \overset{\text{tex}}{\to}$ "
S1"], [$S2 \overset{\text{pyk}}{\to}$ "axiom s two"][$S2 \overset{\text{tex}}{\to}$ "
S2"], [$S3 \overset{\text{pyk}}{\to}$ "axiom s three"][$S3 \overset{\text{tex}}{\to}$ "
S3"], [$S4 \overset{\text{pyk}}{\to}$ "axiom s four"][$S4 \overset{\text{tex}}{\to}$ "
S4"], [$S5 \overset{\text{pyk}}{\to}$ "axiom s five"][$S5 \overset{\text{tex}}{\to}$ "
S5"], [$S6 \overset{\text{pyk}}{\to}$ "axiom s six"][$S6 \overset{\text{tex}}{\to}$ "
S6"], [$S7 \overset{\text{pyk}}{\to}$ "axiom s seven"][$S7 \overset{\text{tex}}{\to}$ "
S7"], [$S8 \overset{\text{pyk}}{\to}$ "axiom s eight"][$S8 \overset{\text{tex}}{\to}$ "
S8"], and [$S9 \overset{\text{pyk}}{\to}$ "axiom s nine"][$S9 \overset{\text{tex}}{\to}$ "
S9"]. System [S] is defined thus:

$$[S \overset{\text{stmt}}{\to} \dot{a} \dotplus \dot{b}' \overset{p}{=} \dot{a} \dotplus \dot{b}' \oplus \forall \underline{a} \colon \forall \underline{b} \colon \dot{\neg} \underline{b} \dot{\Rightarrow} \dot{\neg} \underline{a} \dot{\Rightarrow} \dot{\neg} \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b} \oplus \dot{a} \overset{p}{=} \dot{b} \dot{\Rightarrow} \dot{a}' \overset{p}{=} \\ \dot{b}' \oplus \forall \underline{a} \colon \forall \underline{b} \colon \underline{a} \dot{\Rightarrow} \underline{b} \vdash \underline{a} \vdash \underline{b} \oplus \dot{a}' \overset{p}{=} \dot{b}' \dot{\Rightarrow} \dot{a} \overset{p}{=} \dot{b} \oplus \forall \underline{a} \colon \forall \underline{b} \colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \oplus$$

$\forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon \mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash \dot{\forall} \underline{x}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b} \oplus \dot{a} \mathbin{\dot{\cdot}} \dot{b}' \stackrel{p}{=} \dot{a} \mathbin{\dot{\cdot}} \dot{b} \dot{+} \dot{a} \oplus \dot{a} \dot{+} \dot{0} \stackrel{p}{=} \dot{a} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{c} \oplus \dot{a} \stackrel{p}{=} \dot{b} \dot{\Rightarrow} \dot{a} \stackrel{p}{=} \dot{c} \dot{\Rightarrow} \dot{b} \stackrel{p}{=} \dot{c} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash \underline{b} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a} \oplus \dot{\neg} \dot{0} \stackrel{p}{=} \dot{a}' \oplus \forall \underline{x}\colon \forall \underline{a}\colon \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a} \oplus \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash \dot{\forall} \underline{x}\colon \underline{b} \dot{\Rightarrow} \underline{a} \oplus \dot{a} \mathbin{\dot{\cdot}} \dot{0} \stackrel{p}{=} \dot{0}]$

$[\mathrm{A1} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a}][\mathrm{A1} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A2} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{c}][\mathrm{A2} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A3} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg} \underline{b} \dot{\Rightarrow} \dot{\neg} \underline{a} \dot{\Rightarrow} \dot{\neg} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{b}][\mathrm{A3} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

The order of quantifiers in the following axiom is such that $[\underline{c}]$ which the current conclusion tactic cannot guess comes first. This allows to supply a value for $[\underline{c}]$ without having to supply values for the other meta-variables.

$[\mathrm{A4} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash \dot{\forall} \underline{x}\colon \underline{b} \dot{\Rightarrow} \underline{a}][\mathrm{A4} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{A5} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon \mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash \dot{\forall} \underline{x}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}][\mathrm{A5} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{MP} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{\Rightarrow} \underline{b} \vdash \underline{a} \vdash \underline{b}][\mathrm{MP} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{Gen} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{x}\colon \forall \underline{a}\colon \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}][\mathrm{Gen} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

Axiom [S1] to [S8] are stated as Mendelson [2] does. This is done here to test certain parts of the Logiweb system. Serious users of Peano arithmetic are advised to take Mendelson's Lemma 3.1 as axioms instead.

$[\mathrm{S1} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a} \stackrel{p}{=} \dot{b} \dot{\Rightarrow} \dot{a} \stackrel{p}{=} \dot{c} \dot{\Rightarrow} \dot{b} \stackrel{p}{=} \dot{c}][\mathrm{S1} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S2} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a} \stackrel{p}{=} \dot{b} \dot{\Rightarrow} \dot{a}' \stackrel{p}{=} \dot{b}'][\mathrm{S2} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S3} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{\neg} \dot{0} \stackrel{p}{=} \dot{a}'][\mathrm{S3} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S4} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a}' \stackrel{p}{=} \dot{b}' \dot{\Rightarrow} \dot{a} \stackrel{p}{=} \dot{b}][\mathrm{S4} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S5} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a} \dot{+} \dot{0} \stackrel{p}{=} \dot{a}][\mathrm{S5} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S6} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a} \dot{+} \dot{b}' \stackrel{p}{=} \dot{a} \dot{+} \dot{b}'][\mathrm{S6} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S7} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a} \mathbin{\dot{\cdot}} \dot{0} \stackrel{p}{=} \dot{0}][\mathrm{S7} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S8} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \dot{a} \mathbin{\dot{\cdot}} \dot{b}' \stackrel{p}{=} \dot{a} \mathbin{\dot{\cdot}} \dot{b} \dot{+} \dot{a}][\mathrm{S8} \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[S9 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash \underline{b} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}][S9 \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

1.4 An alternative axiomatic system

System [S′ $\stackrel{\text{pyk}}{\rightarrow}$ "system prime s"][S′ $\stackrel{\text{tex}}{\rightarrow}$ "
S′"] is system [S] in which the proper axioms are taken from Lemma 3.1 in Mendelson [2]. It comprises the axioms [A1′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a one"][A1′ $\stackrel{\text{tex}}{\rightarrow}$ "
A1′"], [A2′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a two"][A2′ $\stackrel{\text{tex}}{\rightarrow}$ "
A2′"], [A3′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a three"][A3′ $\stackrel{\text{tex}}{\rightarrow}$ "
A3′"], [A4′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a four"][A4′ $\stackrel{\text{tex}}{\rightarrow}$ "
A4′"], and [A5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a five"][A5′ $\stackrel{\text{tex}}{\rightarrow}$ "
A5′"] and inference rules [MP′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime mp"][MP′ $\stackrel{\text{tex}}{\rightarrow}$ "
MP′"] and [Gen′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime gen"][Gen′ $\stackrel{\text{tex}}{\rightarrow}$ "
Gen′"] of first order predicate calculus. Furthermore, it comprises the proper axioms [S1′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s one"][S1′ $\stackrel{\text{tex}}{\rightarrow}$ "
S1′"], [S2′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s two"][S2′ $\stackrel{\text{tex}}{\rightarrow}$ "
S2′"], [S3′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s three"][S3′ $\stackrel{\text{tex}}{\rightarrow}$ "
S3′"], [S4′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s four"][S4′ $\stackrel{\text{tex}}{\rightarrow}$ "
S4′"], [S5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s five"][S5′ $\stackrel{\text{tex}}{\rightarrow}$ "
S5′"], [S6′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s six"][S6′ $\stackrel{\text{tex}}{\rightarrow}$ "
S6′"], [S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"][S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7′"], [S8′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s eight"][S8′ $\stackrel{\text{tex}}{\rightarrow}$ "
S8′"], and [S9′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s nine"][S9′ $\stackrel{\text{tex}}{\rightarrow}$ "
S9′"].
System [S′] is defined thus:

$$[S' \stackrel{\text{stmt}}{\rightarrow} \forall \underline{a}\colon \underline{a} \dot{+} \dot{0} \stackrel{p}{=} \underline{a} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{c} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \stackrel{p}{=} \underline{b} \dot{\Rightarrow} \underline{a} \stackrel{p}{=} \underline{c} \dot{\Rightarrow} \underline{b} \stackrel{p}{=} \underline{c} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash \underline{b} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a} \oplus \forall \underline{a}\colon \dot{\neg} \dot{0} \stackrel{p}{=} \underline{a}' \oplus \forall \underline{x}\colon \forall \underline{a}\colon \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a} \oplus \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash \dot{\forall} \underline{x}\colon \underline{b} \dot{\Rightarrow} \underline{a} \oplus \forall \underline{a}\colon \underline{a} \dot{\cdot} \dot{0} \stackrel{p}{=} \dot{0} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a}' \stackrel{p}{=} \underline{b}' \dot{\Rightarrow} \underline{a} \stackrel{p}{=} \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \oplus \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon \text{nonfree}(\lceil \underline{x} \rceil, \lceil \underline{a} \rceil) \Vdash \dot{\forall} \underline{x}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{\cdot} \underline{b}' \stackrel{p}{=} \underline{a} \dot{\cdot} \underline{b} \dot{+} \underline{a} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \stackrel{p}{=} \underline{b} \dot{\Rightarrow} \underline{a}' \stackrel{p}{=} \underline{b}' \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{\Rightarrow} \underline{b} \vdash \underline{a} \vdash \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg} \underline{b} \dot{\Rightarrow} \dot{\neg} \underline{a} \dot{\Rightarrow} \dot{\neg} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{+} \underline{b}' \stackrel{p}{=} \underline{a} \dot{+} \underline{b}']$$

$[A1' \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a}][A1' \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$[A2' \stackrel{\text{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{c} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{b} \dot{\Rightarrow} \underline{a} \dot{\Rightarrow} \underline{c}][A2' \stackrel{\text{proof}}{\rightarrow} \text{Rule tactic}]$

$$[\mathrm{A3}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}\, \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}][\mathrm{A3}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{A4}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{b}}\colon \lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil | \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}][\mathrm{A4}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{A5}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{non}\dot{\text{f}}\text{ree}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{a}} \rceil) \Vdash \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{b}}][\mathrm{A5}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{MP}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}][\mathrm{MP}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{Gen}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \vdash \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}][\mathrm{Gen}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S1}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{c}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{c}}][\mathrm{S1}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S2}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}' \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}'][\mathrm{S2}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S3}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \neg \dot{0} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}'][\mathrm{S3}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S4}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}}' \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}' \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}][\mathrm{S4}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S5}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \mathbin{\dot{+}} \dot{0} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}][\mathrm{S5}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S6}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \mathbin{\dot{+}} \underline{\mathsf{b}}' \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}} \mathbin{\dot{+}} \underline{\mathsf{b}}'][\mathrm{S6}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S7}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \mathbin{\dot{\cdot}} \dot{0} \stackrel{\mathrm{p}}{=} \dot{0}][\mathrm{S7}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S8}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \mathbin{\dot{\cdot}} \underline{\mathsf{b}}' \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}} \mathbin{\dot{\cdot}} \underline{\mathsf{b}} \mathbin{\dot{+}} \underline{\mathsf{a}}][\mathrm{S8}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

$$[\mathrm{S9}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{x}}\colon \underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash \underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}][\mathrm{S9}' \stackrel{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$$

Note that [A1] and [A1′] are distinct. The former says $[\mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}}]$ and the latter says $[\mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}]$.

A Chores

A.1 The name of the page

This defines the name of the page:

$$[\mathrm{peano} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“peano”}]$$

A.2 Variables of Peano arithmetic

We use [$\dot{b} \stackrel{\text{pyk}}{\to}$ "peano b"][$\dot{b} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{b}}"], [$\dot{c} \stackrel{\text{pyk}}{\to}$ "peano c"][$\dot{c} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{c}}"], [$\dot{d} \stackrel{\text{pyk}}{\to}$ "peano d"][$\dot{d} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{d}}"], [$\dot{e} \stackrel{\text{pyk}}{\to}$ "peano e"][$\dot{e} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{e}}"], [$\dot{f} \stackrel{\text{pyk}}{\to}$ "peano f"][$\dot{f} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{f}}"], [$\dot{g} \stackrel{\text{pyk}}{\to}$ "peano g"][$\dot{g} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{g}}"], [$\dot{h} \stackrel{\text{pyk}}{\to}$ "peano h"][$\dot{h} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{h}}"], [$\dot{i} \stackrel{\text{pyk}}{\to}$ "peano i"][$\dot{i} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{i}}"], [$\dot{j} \stackrel{\text{pyk}}{\to}$ "peano j"][$\dot{j} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{j}}"], [$\dot{k} \stackrel{\text{pyk}}{\to}$ "peano k"][$\dot{k} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{k}}"], [$\dot{l} \stackrel{\text{pyk}}{\to}$ "peano l"][$\dot{l} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{l}}"], [$\dot{m} \stackrel{\text{pyk}}{\to}$ "peano m"][$\dot{m} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{m}}"], [$\dot{n} \stackrel{\text{pyk}}{\to}$ "peano n"][$\dot{n} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{n}}"], [$\dot{o} \stackrel{\text{pyk}}{\to}$ "peano o"][$\dot{o} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{o}}"], [$\dot{p} \stackrel{\text{pyk}}{\to}$ "peano p"][$\dot{p} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{p}}"], [$\dot{q} \stackrel{\text{pyk}}{\to}$ "peano q"][$\dot{q} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{q}}"], [$\dot{r} \stackrel{\text{pyk}}{\to}$ "peano r"][$\dot{r} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{r}}"], [$\dot{s} \stackrel{\text{pyk}}{\to}$ "peano s"][$\dot{s} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{s}}"], [$\dot{t} \stackrel{\text{pyk}}{\to}$ "peano t"][$\dot{t} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{t}}"], [$\dot{u} \stackrel{\text{pyk}}{\to}$ "peano u"][$\dot{u} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{u}}"], [$\dot{v} \stackrel{\text{pyk}}{\to}$ "peano v"][$\dot{v} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{v}}"], [$\dot{w} \stackrel{\text{pyk}}{\to}$ "peano w"][$\dot{w} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{w}}"], [$\dot{x} \stackrel{\text{pyk}}{\to}$ "peano x"][$\dot{x} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{x}}"], [$\dot{y} \stackrel{\text{pyk}}{\to}$ "peano y"][$\dot{y} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{y}}"], and [$\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"][$\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"] to denote variables of Peano arithmetic:
[$\dot{b} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{b} \ddot{=} \dot{\mathsf{b}}]\rceil)$], [$\dot{c} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{c} \ddot{=} \dot{\mathsf{c}}]\rceil)$],
[$\dot{d} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{d} \ddot{=} \dot{\mathsf{d}}]\rceil)$], [$\dot{e} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{e} \ddot{=} \dot{\mathsf{e}}]\rceil)$],
[$\dot{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{f} \ddot{=} \dot{\mathsf{f}}]\rceil)$], [$\dot{g} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)$],
[$\dot{h} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)$], [$\dot{i} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)$],
[$\dot{j} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)$], [$\dot{k} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{k} \ddot{=} \dot{\mathsf{k}}]\rceil)$],
[$\dot{l} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{l} \ddot{=} \dot{\mathsf{l}}]\rceil)$], [$\dot{m} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{m} \ddot{=} \dot{\mathsf{m}}]\rceil)$],
[$\dot{n} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{n} \ddot{=} \dot{\mathsf{n}}]\rceil)$], [$\dot{o} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{o} \ddot{=} \dot{\mathsf{o}}]\rceil)$],
[$\dot{p} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{p} \ddot{=} \dot{\mathsf{p}}]\rceil)$], [$\dot{q} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{q} \ddot{=} \dot{\mathsf{q}}]\rceil)$],
[$\dot{r} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{r} \ddot{=} \dot{\mathsf{r}}]\rceil)$], [$\dot{s} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{s} \ddot{=} \dot{\mathsf{s}}]\rceil)$],
[$\dot{t} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{t} \ddot{=} \dot{\mathsf{t}}]\rceil)$], [$\dot{u} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{u} \ddot{=} \dot{\mathsf{u}}]\rceil)$],

$[\dot{v} \stackrel{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{v} \ddot{=} \dot{\mathsf{v}}]\rceil)]$, $[\dot{w} \stackrel{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{w} \ddot{=} \dot{\mathsf{w}}]\rceil)]$, $[\dot{x} \stackrel{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{x} \ddot{=} \dot{\mathsf{x}}]\rceil)]$, $[\dot{y} \stackrel{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{y} \ddot{=} \dot{\mathsf{y}}]\rceil)]$, and $[\dot{z} \stackrel{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$.

A.3 TEX definitions

A.4 Test

$[\lceil\dot{\mathsf{a}}\rceil^{\mathcal{P}}]^{\cdot}$

$[\lceil\mathsf{a}\rceil^{\mathcal{P}}]^{-}$

$[\dot{\mathrm{nonfree}}(\lceil\dot{\mathsf{x}}\rceil, \lceil\dot{\mathsf{y}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{z}} \dot{\Rightarrow} \dot{\forall}\dot{\mathsf{x}}\colon \dot{\mathsf{x}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{y}}\rceil)]^{\cdot}$

$[\dot{\mathrm{nonfree}}(\lceil\dot{\mathsf{x}}\rceil, \lceil\dot{\mathsf{x}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{z}} \dot{\Rightarrow} \dot{\forall}\dot{\mathsf{x}}\colon \dot{\mathsf{x}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{y}}\rceil)]^{-}$

$[\dot{\mathrm{nonfree}}(\lceil\dot{\mathsf{x}}\rceil, \lceil\dot{\mathsf{y}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{x}} \dot{\Rightarrow} \dot{\forall}\dot{\mathsf{x}}\colon \dot{\mathsf{x}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{y}}\rceil)]^{-}$

$[\dot{\mathrm{nonfree}}(\lceil\dot{\mathsf{x}}\rceil, \lceil\dot{\mathsf{y}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{z}} \dot{\Rightarrow} \dot{\forall}\dot{\mathsf{y}}\colon \dot{\mathsf{x}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{y}}\rceil)]^{-}$

$[\dot{\mathrm{free}}\langle\lceil\dot{\forall}\dot{\mathsf{x}}\colon \mathsf{b} :: \dot{\mathsf{x}} :: \mathsf{c}\rceil | \lceil\dot{\mathsf{x}}\rceil := \lceil\mathsf{x} :: \dot{\mathsf{y}} :: \mathsf{z}\rceil\rangle]^{\cdot}$

$[\dot{\mathrm{free}}\langle\lceil\dot{\forall}\dot{\mathsf{y}}\colon \mathsf{b} :: \dot{\mathsf{x}} :: \mathsf{c}\rceil | \lceil\dot{\mathsf{x}}\rceil := \lceil\mathsf{x} :: \dot{\mathsf{y}} :: \mathsf{z}\rceil\rangle]^{-}$

$[\dot{\mathrm{free}}\langle\lceil\dot{\forall}\dot{\mathsf{x}}\colon \mathsf{b} :: \dot{\mathsf{x}} :: \mathsf{c}\rceil | \lceil\dot{\mathsf{y}}\rceil := \lceil\mathsf{x} :: \dot{\mathsf{y}} :: \mathsf{z}\rceil\rangle]^{\cdot}$

$[\dot{\mathrm{free}}\langle\lceil\dot{\forall}\dot{\mathsf{y}}\colon \mathsf{b} :: \dot{\mathsf{x}} :: \mathsf{c}\rceil | \lceil\dot{\mathsf{y}}\rceil := \lceil\mathsf{x} :: \dot{\mathsf{y}} :: \mathsf{z}\rceil\rangle]^{\cdot}$

$[\dot{\mathsf{a}} \equiv \langle\dot{\mathsf{a}} | \dot{\mathsf{b}} := \dot{\mathsf{c}}\rangle]^{\cdot}$

$[\dot{\mathsf{c}} \equiv \langle\dot{\mathsf{b}} | \dot{\mathsf{b}} := \dot{\mathsf{c}}\rangle]^{\cdot}$

$[\forall\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{b}} \equiv \langle\forall\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{b}} | \dot{\mathsf{a}} := \dot{\mathsf{c}}\rangle]^{\cdot}$

$[\forall\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{c}} \equiv \langle\forall\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{\mathsf{b}} | \dot{\mathsf{b}} := \dot{\mathsf{c}}\rangle]^{\cdot}$

$[\dot{\forall}\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{a}} \dot{\Rightarrow} \dot{\mathsf{c}} \dot{\cdot} \dot{\mathsf{d}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{c}} \dot{\cdot} \dot{\mathsf{d}} \equiv \langle\dot{\forall}\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{a}} \dot{\Rightarrow} \dot{\mathsf{b}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{b}} | \dot{\mathsf{b}} := \dot{\mathsf{c}} \dot{\cdot} \dot{\mathsf{d}}\rangle]^{\cdot}$

$[\dot{\forall}\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{a}} \dot{\Rightarrow} \dot{\mathsf{b}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{b}} \equiv \langle\dot{\forall}\dot{\mathsf{a}}\colon \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{a}} \dot{\Rightarrow} \dot{\mathsf{b}} \stackrel{\mathrm{p}}{=} \dot{0} \dot{+} \dot{\mathsf{b}} | \dot{\mathsf{a}} := \dot{\mathsf{c}}\rangle]^{\cdot}$

A.5 Priority table

[peano $\stackrel{\rm prio}{\to}$

Preassociative

[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\to} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if($*,*,*$)], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\bot$], [f($*$)], [$(*)^{\rm I}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\rm M}$], [If($*,*,*$)], [array{$*$} $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\rm M}(*)$], [**apply**($*,*$)], [$\mathbf{apply}_1(*,*)$], [identifier($*$)], [identifier$_1(*,*)$], [array-plus($*,*$)], [array-remove($*,*,*$)], [array-put($*,*,*,*$)], [array-add($*,*,*,*,*$)], [bit($*,*$)], [bit$_1(*,*)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*,*,*)$], [$\mathcal{E}_2(*,*,*,*,*)$], [$\mathcal{E}_3(*,*,*,*)$], [$\mathcal{E}_4(*,*,*,*)$], [**lookup**($*,*,*$)], [**abstract**($*,*,*,*$)], [$\lceil * \rceil$], [$\mathcal{M}(*,*,*)$], [$\mathcal{M}_2(*,*,*,*)$], [$\mathcal{M}^*(*,*,*)$], [macro], [s_0], [**zip**($*,*$)], [$\mathbf{assoc}_1(*,*,*)$], [$(*)^{\sf P}$], [self], [[$* \stackrel{..}{=} *$]], [[$* \doteq *$]], [[$* \stackrel{\prime}{=} *$]], [[$* \stackrel{\rm pyk}{=} *$]], [[$* \stackrel{\rm tex}{=} *$]], [[$* \stackrel{\rm name}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*,*,*,*)$], [$\tilde{\mathcal{M}}(*,*,*)$], [$\tilde{\mathcal{Q}}(*,*,*)$], [$\tilde{\mathcal{Q}}_2(*,*,*)$], [$\tilde{\mathcal{Q}}_3(*,*,*,*)$], [$\tilde{\mathcal{Q}}^*(*,*,*)$], [$(*)$], [**aspect**($*,*$)], [**aspect**($*,*,*$)], [$\langle * \rangle$], [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], [let$_2(*,*)$], [let$_1(*,*)$], [[$* \stackrel{\rm claim}{=} *$]], [checker], [**check**($*,*$)], [$\mathbf{check}_2(*,*,*)$], [$\mathbf{check}_3(*,*,*)$], [$\mathbf{check}^*(*,*)$], [$\mathbf{check}^*_2(*,*,*)$], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \stackrel{\rm msg}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\rm stmt}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\bot\!\!\!\bot$], [Contra′], [$T'_{\rm E}$], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\sf V}$], [intro($*,*,*,*$)], [intro($*,*,*$)], [error($*,*$)], [error$_2(*,*)$], [proof($*,*,*$)], [proof$_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\rm I}(*,*)$], [$\mathcal{S}^{\rhd}(*,*)$], [$\mathcal{S}^{\rhd}_1(*,*,*)$], [$\mathcal{S}^{\rm E}(*,*)$], [$\mathcal{S}^{\rm E}_1(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}^{+}_1(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}^{-}_1(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}^{*}_1(*,*,*)$], [$\mathcal{S}^{*}_2(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}^{@}_1(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}^{\vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}^{\Vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\rm i.e.}(*,*)$], [$\mathcal{S}^{\rm i.e.}_1(*,*,*,*)$], [$\mathcal{S}^{\rm i.e.}_2(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}^{\forall}_1(*,*,*,*)$], [$\mathcal{S}^{;}(*,*)$], [$\mathcal{S}^{;}_1(*,*,*)$], [$\mathcal{S}^{;}_2(*,*,*,*)$], [$\mathcal{T}(*)$], [claims($*,*,*$)], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut($*,*$)], [Head$_{\oplus}(*)$], [Tail$_{\oplus}(*)$], [rule$_1(*,*)$], [rule($*,*$)], [Rule tactic], [Plus($*,*$)], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil″], [HeadPair″], [Transitivity″], [Contra″], [HeadNil], [HeadPair], [Transitivity], [Contra], [$T_{\rm E}$], [ragged right], [ragged right expansion], [parm($*,*,*$)], [parm$^*(*,*,*)$], [inst($*,*$)], [inst$^*(*,*)$], [occur($*,*,*$)], [occur$^*(*,*,*)$], [unify($* = *,*$)], [unify$^*(* = *,*)$], [unify$_2(* = *,*)$], [L_a], [L_b], [L_c], [L_d], [L_e], [L_f], [L_g], [L_h], [L_i], [L_j], [L_k], [L_l], [L_m], [L_n], [L_o], [L_p], [L_q], [L_r], [L_s], [L_t], [L_u], [L_v], [L_w], [L_x], [L_y], [L_z], [L_A], [L_B], [L_C],

[L_D], [L_E], [L_F], [L_G], [L_H], [L_I], [L_J], [L_K], [L_L], [L_M], [L_N], [L_O], [L_P], [L_Q], [L_R], [L_S], [L_T], [L_U], [L_V], [L_W], [L_X], [L_Y], [L_Z], [$L_?$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$p_0$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [$\dot{\text{nonfree}}(*,*)$], [$\dot{\text{nonfree}}^*(*,*)$], [free$\langle *|* := *\rangle$], [free$^*\langle *|* := *\rangle$], [$*\equiv\langle *|* := *\rangle$], [$*\equiv\langle^* *|* := *\rangle$], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [S′], [A1′], [A2′], [A3′], [A4′], [A5′], [S1′], [S2′], [S3′], [S4′], [S5′], [S6′], [S7′], [S8′], [S9′], [MP′], [Gen′];

Preassociative

[$*_{\{*\}}$], [$*'$], [$*[*]$], [$*[*\rightarrow*]$], [$*[*\Rightarrow*]$], [$\dot{*}$];

Preassociative

[“ * ”], [], [$(*)^{\mathbf{t}}$], [string$(*) + *$], [string$(*) ++ *$], [*], [*], [!*], [”*], [#*], [$*], [%*], [&*], [’*], [(*], [)*], [**], [+*], [,*], [-*], [.*], [/*], [0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [:*], [;*], [<*], [=*], [>*], [?*], [@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*], [O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [*], []*], [^*], [_*], [‘*], [a*], [b*], [c*], [d*], [e*], [f*], [g*], [h*], [i*], [j*], [k*], [l*], [m*], [n*], [o*], [p*], [q*], [r*], [s*], [t*], [u*], [v*], [w*], [x*], [y*], [z*], [{*], [|*], [}*], [~*], [**Preassociative** *; *], [**Postassociative** *; *], [[*], *], [priority * end], [newline *], [macro newline *];

Preassociative

[*0], [*1], [0b], [*-color(*)], [*-color*(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^0$], [$*^1$], [$*^2$], [$*^3$], [$*^4$], [$*^5$], [$*^6$], [$*^7$], [$*^8$], [$*^9$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*'$];

Preassociative

[$* \cdot *$], [$* \cdot_0 *$], [$* \dot{:} *$];

Preassociative

[$* + *$], [$* +_0 *$], [$* +_1 *$], [$* - *$], [$* -_0 *$], [$* -_1 *$], [$* \dot{+} *$];

Preassociative

[$* \cup \{*\}$], [$* \cup *$], [$* \backslash \{*\}$];

Postassociative

[$* \therefore *$], [$* \underline{\therefore} *$], [$* \underline{::} *$], [$* \underline{+2*} *$], [$* :: *$], [$* +2* *$];

Postassociative

[$*, *$];

Preassociative

[$* \overset{\mathrm{B}}{\approx} *$], [$* \overset{\mathrm{D}}{\approx} *$], [$* \overset{\mathrm{C}}{\approx} *$], [$* \overset{\mathrm{P}}{\approx} *$], [$* \approx *$], [$* = *$], [$* \overset{+}{\rightarrow} *$], [$* \overset{\mathrm{t}}{=} *$], [$* \overset{\mathrm{t}^*}{=} *$], [$* \overset{\mathrm{r}}{=} *$], [$* \in_{\mathrm{t}} *$], [$* \subseteq_{\mathrm{T}} *$], [$* \overset{\mathrm{T}}{=} *$], [$* \overset{\mathrm{s}}{=} *$], [* free in *], [* free in* *], [* free for * in *], [* free for* * in *], [$* \in_{\mathrm{c}} *$], [$* < *$], [$* <' *$], [$* \leq' *$], [$* \overset{\mathrm{p}}{=} *$], [$*^{\mathcal{P}}$];

Preassociative

[$\neg *$], [$\dot{\neg} *$];

Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *];$
Preassociative
$[\dot{\forall} *\colon *], [\dot{\exists} *\colon *];$
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *];$
Postassociative
$[* : *], [*!*];$
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$
Preassociative
$[\lambda * .*], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *];$
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}];$
Preassociative
$[* @ *], [* \rhd *], [* \unrhd *], [* \gg *];$
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
$[\forall *\colon *];$
Postassociative
$[* \oplus *];$
Postassociative
$[*; *];$
Preassociative
$[* \text{ proves } *];$
Preassociative
$[* \textbf{ proof of } * : *], [\text{Line} * : * \gg *; *], [\text{Last line} * \gg * \square],$
$[\text{Line} * : \text{Premise} \gg *; *], [\text{Line} * : \text{Side-condition} \gg *; *], [\text{Arbitrary} \gg *; *],$
$[\text{Local} \gg * = *; *];$
Postassociative
$[* \text{ then } *], [*[*]*];$
Preassociative
$[*\&*];$
Preassociative
$[*\backslash\backslash*];]$

B Index

C Bibliography

[1] K. Grue. Logiweb. In Fairouz Kamareddine, editor, *Mathematical Knowledge Management Symposium 2003*, volume 93 of *Electronic Notes in Theoretical Computer Science*, pages 70–101. Elsevier, 2004.

[2] E. Mendelson. *Introduction to Mathematical Logic*. Wadsworth and Brooks, 3. edition, 1987.