Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\mathrm{nonfree}}(*, *)$, $\dot{\mathrm{nonfree}}^*(*, *)$, $\dot{\mathrm{free}}\langle *|* := *\rangle$, $\dot{\mathrm{free}}^*\langle *|* := *\rangle$, $*\dot{\equiv}\langle *|* := *\rangle$, $*\dot{\equiv}\langle^* *|* := *\rangle$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, $\dot{*}$, $*'$, $* \dot{\cdot} *$, $* \dot{+} *$, $* \stackrel{\mathrm{p}}{=} *$, $*^{\mathcal{P}}$, $\dot{\neg} *$, $* \dot{\wedge} *$, $* \dot{\vee} *$, $\dot{\forall} *: *$, $\dot{\exists} *: *$, $* \dot{\Rightarrow} *$, $* \dot{\Leftrightarrow} *$,

peano

[peano $\stackrel{\mathrm{prio}}{\to}$
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\to} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if($*, *, *$)], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\perp$], [f($*$)], [$(*)^{\mathrm{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\mathrm{M}}$], [If($*, *, *$)], [array{$*$} $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\mathrm{M}}(*)$], [**apply**($*, *$)], [$\mathbf{apply}_1(*, *)$], [identifier($*$)], [$\mathrm{identifier}_1(*, *)$], [array-plus($*, *$)], [array-remove($*, *, *$)], [array-put($*, *, *, *$)], [array-add($*, *, *, *, *$)], [bit($*, *$)], [$\mathrm{bit}_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**($*, *, *$)], [**abstract**($*, *, *, *$)], [[$\lceil *$]], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [$\mathrm{s}_0$], [**zip**($*, *$)], [$\mathbf{assoc}_1(*, *, *)$], [$(*)^{\mathbf{P}}$], [self], [[$* \ddot{=} *$]], [[$* \dot{=} *$]], [[$* \acute{=} *$]], [[$* \stackrel{\mathrm{pyk}}{=} *$]], [[$* \stackrel{\mathrm{tex}}{=} *$]], [[$* \stackrel{\mathrm{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [$(*)$], [**aspect**($*, *$)], [**aspect**($*, *, *$)], [$\langle * \rangle$], [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], [$\mathrm{let}_2(*, *)$], [$\mathrm{let}_1(*, *)$], [[$* \stackrel{\mathrm{claim}}{=} *$]], [checker], [**check**($*, *$)], [$\mathbf{check}_2(*, *, *)$], [$\mathbf{check}_3(*, *, *)$], [$\mathbf{check}^*(*, *)$], [$\mathbf{check}^*_2(*, *, *)$], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \stackrel{\mathrm{msg}}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\mathrm{stmt}}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\perp\!\!\!\perp$], [Contra′], [T'_{E}], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathbf{v}}$], [intro($*, *, *, *$)], [intro($*, *, *$)], [error($*, *$)], [$\mathrm{error}_2(*, *)$], [proof($*, *, *$)], [$\mathrm{proof}_2(*, *)$], [$\mathcal{S}(*, *)$], [$\mathcal{S}^{\mathrm{I}}(*, *)$], [$\mathcal{S}^{\rhd}(*, *)$], [$\mathcal{S}^{\rhd}_1(*, *, *)$], [$\mathcal{S}^{\mathrm{E}}(*, *)$], [$\mathcal{S}^{\mathrm{E}}_1(*, *, *)$],

$[\mathcal{S}^{+}(*,*)]$, $[\mathcal{S}^{+}_{1}(*,*,*)]$, $[\mathcal{S}^{-}(*,*)]$, $[\mathcal{S}^{-}_{1}(*,*,*)]$, $[\mathcal{S}^{*}(*,*)]$, $[\mathcal{S}^{*}_{1}(*,*,*)]$, $[\mathcal{S}^{*}_{2}(*,*,*,*)]$, $[\mathcal{S}^{@}(*,*)]$, $[\mathcal{S}^{@}_{1}(*,*,*)]$, $[\mathcal{S}^{\vdash}(*,*)]$, $[\mathcal{S}^{\vdash}_{1}(*,*,*,*)]$, $[\mathcal{S}^{\Vdash}(*,*)]$, $[\mathcal{S}^{\Vdash}_{1}(*,*,*,*)]$, $[\mathcal{S}^{\text{i.e.}}(*,*)]$, $[\mathcal{S}^{\text{i.e.}}_{1}(*,*,*,*)]$, $[\mathcal{S}^{\text{i.e.}}_{2}(*,*,*,*,*)]$, $[\mathcal{S}^{\forall}(*,*)]$, $[\mathcal{S}^{\forall}_{1}(*,*,*,*)]$, $[\mathcal{S}^{;}(*,*)]$, $[\mathcal{S}^{;}_{1}(*,*,*)]$, $[\mathcal{S}^{;}_{2}(*,*,*,*)]$, $[\mathcal{T}(*)]$, [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*,*)]$, $[\mathcal{V}_3(*,*,*,*)]$, $[\mathcal{V}_4(*,*)]$, $[\mathcal{V}_5(*,*,*,*)]$, $[\mathcal{V}_6(*,*,*,*)]$, $[\mathcal{V}_7(*,*,*,*)]$, [Cut$(*,*)$], [Head$_\oplus(*)$], [Tail$_\oplus(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [T$_\mathrm{E}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(*=*,*)$], [unify$^*(*=*,*)$], [unify$_2(*=*,*)$], [L$_\mathrm{a}$], [L$_\mathrm{b}$], [L$_\mathrm{c}$], [L$_\mathrm{d}$], [L$_\mathrm{e}$], [L$_\mathrm{f}$], [L$_\mathrm{g}$], [L$_\mathrm{h}$], [L$_\mathrm{i}$], [L$_\mathrm{j}$], [L$_\mathrm{k}$], [L$_\mathrm{l}$], [L$_\mathrm{m}$], [L$_\mathrm{n}$], [L$_\mathrm{o}$], [L$_\mathrm{p}$], [L$_\mathrm{q}$], [L$_\mathrm{r}$], [L$_\mathrm{s}$], [L$_\mathrm{t}$], [L$_\mathrm{u}$], [L$_\mathrm{v}$], [L$_\mathrm{w}$], [L$_\mathrm{x}$], [L$_\mathrm{y}$], [L$_\mathrm{z}$], [L$_\mathrm{A}$], [L$_\mathrm{B}$], [L$_\mathrm{C}$], [L$_\mathrm{D}$], [L$_\mathrm{E}$], [L$_\mathrm{F}$], [L$_\mathrm{G}$], [L$_\mathrm{H}$], [L$_\mathrm{I}$], [L$_\mathrm{J}$], [L$_\mathrm{K}$], [L$_\mathrm{L}$], [L$_\mathrm{M}$], [L$_\mathrm{N}$], [L$_\mathrm{O}$], [L$_\mathrm{P}$], [L$_\mathrm{Q}$], [L$_\mathrm{R}$], [L$_\mathrm{S}$], [L$_\mathrm{T}$], [L$_\mathrm{U}$], [L$_\mathrm{V}$], [L$_\mathrm{W}$], [L$_\mathrm{X}$], [L$_\mathrm{Y}$], [L$_\mathrm{Z}$], [L$_?$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]], $[\mathcal{P}(*,*,*)]$, $[\mathcal{P}^*(*,*,*)]$, [p$_0$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], $[\dot{0}]$, $[\dot{1}]$, $[\dot{2}]$, $[\dot{a}]$, $[\dot{b}]$, $[\dot{c}]$, $[\dot{d}]$, $[\dot{e}]$, $[\dot{f}]$, $[\dot{g}]$, $[\dot{h}]$, $[\dot{i}]$, $[\dot{j}]$, $[\dot{k}]$, $[\dot{l}]$, $[\dot{m}]$, $[\dot{n}]$, $[\dot{o}]$, $[\dot{p}]$, $[\dot{q}]$, $[\dot{r}]$, $[\dot{s}]$, $[\dot{t}]$, $[\dot{u}]$, $[\dot{v}]$, $[\dot{w}]$, $[\dot{x}]$, $[\dot{y}]$, $[\dot{z}]$, [nonfree$(*,*)$], [nonfree$^*(*,*)$], [free$\langle *|*:=*\rangle$], [free$^*\langle *|*:=*\rangle$], $[*{\equiv}\langle *|*:=*\rangle]$, $[*{\equiv}\langle^* *|*:=*\rangle]$, [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$];

Preassociative

$[*_\{*\}]$, $[*']$, $[*[\,*\,]]$, $[*[*{\to}*]]$, $[*[*{\Rightarrow}*]]$, $[\dot{*}]$;

Preassociative

[“ $*$ ”], [], [$(*)^\mathbf{t}$], [string$(*)+*$], [string$(*)$ ++ $*$], [
$*$], [$*$], [!$*$], ["$*$], [#$*$], [\$$*$], [%$*$], [&$*$], ['$*$], [($*$], [)$*$], [$**$], [+$*$], [,$*$], [-$*$], [.$*$], [/$*$], [0$*$], [1$*$], [2$*$], [3$*$], [4$*$], [5$*$], [6$*$], [7$*$], [8$*$], [9$*$], [:$*$], [;$*$], [$<*$], [$=*$], [$>*$], [?$*$], [@$*$], [A$*$], [B$*$], [C$*$], [D$*$], [E$*$], [F$*$], [G$*$], [H$*$], [I$*$], [J$*$], [K$*$], [L$*$], [M$*$], [N$*$], [O$*$], [P$*$], [Q$*$], [R$*$], [S$*$], [T$*$], [U$*$], [V$*$], [W$*$], [X$*$], [Y$*$], [Z$*$], [[$*$], [\$*$], []$*$], [^$*$], [_$*$], [‘$*$], [a$*$], [b$*$], [c$*$], [d$*$], [e$*$], [f$*$], [g$*$], [h$*$], [i$*$], [j$*$], [k$*$], [l$*$], [m$*$], [n$*$], [o$*$], [p$*$], [q$*$], [r$*$], [s$*$], [t$*$], [u$*$], [v$*$], [w$*$], [x$*$], [y$*$], [z$*$], [{$*$], [|$*$], [}$*$], [~$*$], [**Preassociative** $*$; $*$], [**Postassociative** $*$; $*$], [[$*$], $*$], [priority $*$ end], [newline $*$], [macro newline $*$];

Preassociative

[$*0$], [$*1$], [0b], [$*$-color$(*)$], [$*$-color$^*(*)$];

Preassociative

[$*$ ’ $*$], [$*$ ‘ $*$];

Preassociative

$[*^\mathrm{H}]$, $[*^\mathrm{T}]$, $[*^\mathrm{U}]$, $[*^\mathrm{h}]$, $[*^\mathrm{t}]$, $[*^\mathrm{s}]$, $[*^\mathrm{c}]$, $[*^\mathrm{d}]$, $[*^\mathrm{a}]$, $[*^\mathrm{C}]$, $[*^\mathrm{M}]$, $[*^\mathrm{B}]$, $[*^\mathrm{r}]$, $[*^\mathrm{i}]$, $[*^\mathrm{d}]$, $[*^\mathrm{R}]$, $[*^0]$, $[*^1]$, $[*^2]$, $[*^3]$, $[*^4]$, $[*^5]$, $[*^6]$, $[*^7]$, $[*^8]$, $[*^9]$, $[*^\mathrm{E}]$, $[*^\mathcal{V}]$, $[*^\mathcal{C}]$, $[*^{\mathcal{C}^*}]$, $[*']$;

Preassociative

$[* \cdot *], [* \cdot_0 *], [* \mathrel{\dot{:}} *]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*, *]$;
Preassociative
$[* \stackrel{\mathrm{B}}{\approx} *], [* \stackrel{\mathrm{D}}{\approx} *], [* \stackrel{\mathrm{C}}{\approx} *], [* \stackrel{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\rightarrow} *], [* \stackrel{\mathrm{t}}{=} *], [* \stackrel{\mathrm{t}^*}{=} *], [* \stackrel{\mathrm{r}}{=} *]$,
$[* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \stackrel{\mathrm{T}}{=} *], [* \stackrel{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *]$,
$[* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* \stackrel{\mathrm{p}}{=} *], [*^{\mathcal{P}}]$;
Preassociative
$[\neg *], [\dot{\neg} *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\mathrm{c}} *], [* \dot{\wedge} *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *]$;
Preassociative
$[\dot{\forall} * \colon *], [\dot{\exists} * \colon *]$;
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *]$;
Postassociative
$[* : *], [* ! *]$;
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.]$;
Preassociative
$[\lambda * . *], [\Lambda *], [\mathbf{if}\ *\ \mathbf{then}\ *\ \mathbf{else}\ *], [\mathbf{let}\ * = *\ \mathbf{in}\ *], [\mathbf{let}\ * \ddot{=} *\ \text{ in }\ *]$;
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}]$;
Preassociative
$[* @ *], [* \triangleright *], [* \Vvdash *], [* \gg *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall * \colon *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
$[* \text{ proves } *]$;
Preassociative

[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \Box$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$];
Postassociative
[$*$ then $*$], [$*[\,*\,]*$];
Preassociative
[$*\&*$];
Preassociative
[$*\backslash\backslash*$];]

[peano $\overset{\text{pyk}}{\to}$ "peano"]

$\dot{0}$

[$\dot{0} \overset{\text{tex}}{\to}$ "
\dot{0}"]

[$\dot{0} \overset{\text{pyk}}{\to}$ "peano zero"]

$\dot{1}$

[$\dot{1} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{1} \ddot{=} \dot{0}']\rceil)$]

[$\dot{1} \overset{\text{tex}}{\to}$ "
\dot{1}"]

[$\dot{1} \overset{\text{pyk}}{\to}$ "peano one"]

$\dot{2}$

[$\dot{2} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{2} \ddot{=} \dot{1}']\rceil)$]

[$\dot{2} \overset{\text{tex}}{\to}$ "
\dot{2}"]

[$\dot{2} \overset{\text{pyk}}{\to}$ "peano two"]

$\dot{a}$

[$\dot{a} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{a} \ddot{=} \dot{\mathsf{a}}]\rceil)$]

[$\dot{a} \overset{\text{tex}}{\to}$ "
\dot{\mathit{a}}"]

$[\dot{a} \stackrel{\mathrm{pyk}}{\to}$ "peano a"$]$

$\dot{b}$

$[\dot{b} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{b} \stackrel{..}{=} \dot{\mathsf{b}}]\rceil)]$

$[\dot{b} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{b}}"$]$

$[\dot{b} \stackrel{\mathrm{pyk}}{\to}$ "peano b"$]$

$\dot{c}$

$[\dot{c} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{c} \stackrel{..}{=} \dot{\mathsf{c}}]\rceil)]$

$[\dot{c} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{c}}"$]$

$[\dot{c} \stackrel{\mathrm{pyk}}{\to}$ "peano c"$]$

$\dot{d}$

$[\dot{d} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{d} \stackrel{..}{=} \dot{\mathsf{d}}]\rceil)]$

$[\dot{d} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{d}}"$]$

$[\dot{d} \stackrel{\mathrm{pyk}}{\to}$ "peano d"$]$

$\dot{e}$

$[\dot{e} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{e} \stackrel{..}{=} \dot{\mathsf{e}}]\rceil)]$

$[\dot{e} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{e}}"$]$

$[\dot{e} \stackrel{\mathrm{pyk}}{\to}$ "peano e"$]$

$\dot{f}$

$[\dot{f} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \stackrel{..}{=} \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{\mathit{f}}”]

$[\dot{f} \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano f”]

$\dot{g}$

$[\dot{g} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{\mathit{g}}”]

$[\dot{g} \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano g”]

$\dot{h}$

$[\dot{h} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{\mathit{h}}”]

$[\dot{h} \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano h”]

$\dot{i}$

$[\dot{i} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{\mathit{i}}”]

$[\dot{i} \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano i”]

$\dot{j}$

$[\dot{j} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{\mathit{j}}”]

$[\dot{j} \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano j”]

$\dot{k}$

$[\dot{k} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{k} \doteq \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{k}}"]

$[\dot{k} \stackrel{\text{pyk}}{\to}$ "peano k"]

$\dot{l}$

$[\dot{l} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{l} \doteq \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{l}}"]

$[\dot{l} \stackrel{\text{pyk}}{\to}$ "peano l"]

$\dot{m}$

$[\dot{m} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{m} \doteq \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{m}}"]

$[\dot{m} \stackrel{\text{pyk}}{\to}$ "peano m"]

$\dot{n}$

$[\dot{n} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{n} \doteq \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{n}}"]

$[\dot{n} \stackrel{\text{pyk}}{\to}$ "peano n"]

$\dot{o}$

$[\dot{o} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{o} \doteq \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{o}}"]

$[\dot{o} \stackrel{\text{pyk}}{\to}$ "peano o"]

$\dot{p}$

$[\dot{p} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{p} \ddot{=} \dot{\mathsf{p}}]\rceil)]$

$[\dot{p} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{p}}"]

$[\dot{p} \overset{\text{pyk}}{\rightarrow}$ "peano p"]

$\dot{q}$

$[\dot{q} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{q} \ddot{=} \dot{\mathsf{q}}]\rceil)]$

$[\dot{q} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{q}}"]

$[\dot{q} \overset{\text{pyk}}{\rightarrow}$ "peano q"]

$\dot{r}$

$[\dot{r} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{r} \ddot{=} \dot{\mathsf{r}}]\rceil)]$

$[\dot{r} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{r}}"]

$[\dot{r} \overset{\text{pyk}}{\rightarrow}$ "peano r"]

$\dot{s}$

$[\dot{s} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{s} \ddot{=} \dot{\mathsf{s}}]\rceil)]$

$[\dot{s} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{s}}"]

$[\dot{s} \overset{\text{pyk}}{\rightarrow}$ "peano s"]

$\dot{t}$

$[\dot{t} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{t} \ddot{=} \dot{\mathsf{t}}]\rceil)]$

$[\dot{t} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{t}}"]

$[\dot{t} \overset{\text{pyk}}{\rightarrow}$ "peano t"]

$\dot{u}$

$[\dot{u} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{u} \ddot{=} \dot{\mathsf{u}}]\rceil)]$

$[\dot{u} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{u}}"]

$[\dot{u} \stackrel{\text{pyk}}{\to}$ "peano u"]

$\dot{v}$

$[\dot{v} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{v} \ddot{=} \dot{\mathsf{v}}]\rceil)]$

$[\dot{v} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{v}}"]

$[\dot{v} \stackrel{\text{pyk}}{\to}$ "peano v"]

$\dot{w}$

$[\dot{w} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{w} \ddot{=} \dot{\mathsf{w}}]\rceil)]$

$[\dot{w} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{w}}"]

$[\dot{w} \stackrel{\text{pyk}}{\to}$ "peano w"]

$\dot{x}$

$[\dot{x} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{x} \ddot{=} \dot{\mathsf{x}}]\rceil)]$

$[\dot{x} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{x}}"]

$[\dot{x} \stackrel{\text{pyk}}{\to}$ "peano x"]

$\dot{y}$

$[\dot{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{y} \ddot{=} \dot{\mathsf{y}}]\rceil)]$

$[\dot{y} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{y}}"]

$[\dot{y} \stackrel{\text{pyk}}{\to}$ "peano y"]

$\dot{z}$

$[\dot{z} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil [\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$

$[\dot{z} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{z}}"]

$[\dot{z} \overset{\text{pyk}}{\rightarrow}$ "peano z"]

$\dot{\text{nonfree}}(*,*)$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \overset{\text{val}}{\rightarrow}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg\ [\ \mathsf{x} \overset{\text{t}}{=} \mathsf{y}\ ]\ ,$
$\text{If}(\neg\ [\ \mathsf{y} \overset{\text{r}}{=} \lceil \dot{\forall} \mathsf{x}\colon \mathsf{y} \rceil\ ]\ , \dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \overset{\text{t}}{=}\ [\ \mathsf{y}^{1}\ ]\ , \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{2}))))]$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \overset{\text{tex}}{\rightarrow}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \overset{\text{pyk}}{\rightarrow}$ "peano nonfree $*$ in $*$ end nonfree"]

$\dot{\text{nonfree}}^{*}(*,*)$

$[\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}) \overset{\text{val}}{\rightarrow} \mathsf{x}!\text{If}(\mathsf{y},\mathsf{T},\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}),\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}^{\text{t}}),\mathsf{F}))]$

$[\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}) \overset{\text{tex}}{\rightarrow}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}) \overset{\text{pyk}}{\rightarrow}$ "peano nonfree star $*$ in $*$ end nonfree"]

$\dot{\text{free}}\langle *|* := *\rangle$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\text{val}}{\rightarrow} \mathsf{x}!\ [\ \mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg\ [\ \mathsf{a} \overset{\text{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil\ ]\ , \dot{\text{free}}^{*}\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$
$\text{If}(\mathsf{a}^{1} \overset{\text{t}}{=} \mathsf{x}, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^{2}), \mathsf{T},$

$\mathrm{If}(\neg \dot{\mathrm{nonfree}}(\mathsf{a}^1, \mathsf{b}), \mathsf{F},$
$\dot{\mathrm{free}}\langle \mathsf{a}^2 | \mathsf{x} := \mathsf{b} \rangle))))) \;]\;]$

$[\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

$[\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{pyk}}{\to}$ "peano free * set * to * end free"]

$\dot{\mathrm{free}}^*\langle * | * := * \rangle$

$[\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{val}}{\to} \mathsf{x}! \;[\; \mathsf{b}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\dot{\mathrm{free}}\langle \mathsf{a}^{\mathrm{h}} | \mathsf{x} := \mathsf{b} \rangle, \dot{\mathrm{free}}^*\langle \mathsf{a}^{\mathrm{t}} | \mathsf{x} := \mathsf{b} \rangle, \mathsf{F})) \;]\;]$

$[\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

$[\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \overset{\mathrm{pyk}}{\to}$ "peano free star * set * to * end free"]

$* \equiv \langle * | * := * \rangle$

$[\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{val}}{\to} \mathsf{a}! \;[\; \mathsf{x}! \;[\; \mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \overset{\mathrm{r}}{=} \lceil \dot{\forall} \mathsf{u} \colon \mathsf{v} \rceil, \mathsf{b}^1 \overset{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge \;[\; \mathsf{b} \overset{\mathrm{t}}{=} \mathsf{x} \;]\;, \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}(\;[$
$\mathsf{a} \;]\; \overset{\mathrm{r}}{=} \mathsf{b}, \mathsf{a}^{\mathrm{t}} \equiv \langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F}))) \;]\;\;]\;]$

$[\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{tex}}{\to}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

$[\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{pyk}}{\to}$ "peano sub * is * where * is * end sub"]

$* \equiv \langle^* * | * := * \rangle$

$[\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{val}}{\to} \mathsf{b}! \;[\; \mathsf{x}! \;[\; \mathsf{c}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\mathsf{a}^{\mathrm{h}} \equiv \langle \mathsf{b}^{\mathrm{h}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{a}^{\mathrm{t}} \equiv \langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F})) \;]\;\;]\;]$

$[\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \overset{\mathrm{tex}}{\to}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

[$a\equiv\langle^* b|x := c\rangle \overset{\text{pyk}}{\rightarrow}$ "peano sub star * is * where * is * end sub"]

S

$$[S \overset{\text{stmt}}{\rightarrow} [\ [\ \dot{a} \dot{+} [\ \dot{b}'\ ]\ ] \overset{p}{=} [\ [\ \dot{a} \dot{+} [\ \dot{b}\ ]\ ]'\ ]\ ] \oplus [\ [\ \forall \underline{a}: \forall \underline{b}: [\ [\ [\ \dot{\neg} \underline{b}\ ] \dot{\Rightarrow} \dot{\neg} \underline{a}\ ] \dot{\Rightarrow} [\ [\ [\ \dot{\neg} \underline{b}\ ] \dot{\Rightarrow} \underline{a}\ ] \dot{\Rightarrow} \underline{b}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \overset{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ \dot{a}' \overset{p}{=} [\ \dot{b}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}: \forall \underline{b}: [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a}' \overset{p}{=} [\ \dot{b}'\ ]\ ] \dot{\Rightarrow} [\ \dot{a} \overset{p}{=} [\ \dot{b}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}: \forall \underline{b}: [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{a}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: [\ \mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall} \underline{x}: [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}: \underline{b}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \dot{\cdot} [\ \dot{b}'\ ]\ ] \overset{p}{=} [\ [\ \dot{a} \dot{\cdot} [\ \dot{b}\ ]\ ] \dot{+} [\ \dot{a}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \dot{+} \dot{0}\ ] \overset{p}{=} [\ \dot{a}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{a} \overset{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \overset{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \overset{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{x}: [\ \underline{b}\equiv\langle \underline{a}|\underline{x} := \dot{0}\rangle \Vdash [\ \underline{c}\equiv\langle \underline{a}|\underline{x} := \underline{x}'\rangle \Vdash [\ \underline{b} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{x}: [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{x}: \underline{a}\ ]\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \neg [\ \dot{0} \overset{p}{=} [\ \dot{a}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{x}: \forall \underline{a}: [\ \underline{a} \vdash \dot{\forall} \underline{x}: \underline{a}\ ]\ ] \oplus [\ [\ \forall \underline{c}: \forall \underline{a}: \forall \underline{x}: \forall \underline{b}: [\ \lceil \underline{a} \rceil\equiv\langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil\rangle \Vdash [\ [\ \dot{\forall} \underline{x}: \underline{b}\ ] \dot{\Rightarrow} \underline{a}\ ]\ ]\ ] \oplus [\ [\ \dot{a} \dot{\cdot} \dot{0}\ ] \overset{p}{=} \dot{0}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]]$$

[S $\overset{\text{tex}}{\rightarrow}$ "
S"]

[S $\overset{\text{pyk}}{\rightarrow}$ "system s"]

A1

[A1 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[A1 $\overset{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a}: \forall \underline{b}: [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{a}\ ]\ ]$]

[A1 $\overset{\text{tex}}{\rightarrow}$ "
A1"]

[A1 $\overset{\text{pyk}}{\rightarrow}$ "axiom a one"]

A2

[A2 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[A2 $\overset{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: [\ [\ \underline{a} \dot{\Rightarrow} [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]$]

$[A2 \stackrel{\text{tex}}{\rightarrow}$ "
A2"]

$[A2 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a two"]

A3

$[A3 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A3 \stackrel{\text{stmt}}{\rightarrow} \mathsf{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ [\ \dot{\neg}\, \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \underline{\mathsf{a}}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ [\ \dot{\neg}\, \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}\ ] \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}\ ]\ ]\ ]$

$[A3 \stackrel{\text{tex}}{\rightarrow}$ "
A3"]

$[A3 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a three"]

A4

$[A4 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A4 \stackrel{\text{stmt}}{\rightarrow} \mathsf{S} \vdash \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{b}} \colon [\ \lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil \mid \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}\ ]\ ]\ ]$

$[A4 \stackrel{\text{tex}}{\rightarrow}$ "
A4"]

$[A4 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a four"]

A5

$[A5 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[A5 \stackrel{\text{stmt}}{\rightarrow} \mathsf{S} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \mathrm{non\dot{f}ree}(\underline{\mathsf{x}}, \underline{\mathsf{a}}) \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon [\ \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ]\ ]\ ]\ ]$

$[A5 \stackrel{\text{tex}}{\rightarrow}$ "
A5"]

$[A5 \stackrel{\text{pyk}}{\rightarrow}$ "axiom a five"]

S1

$[S1 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S1 \stackrel{\text{stmt}}{\rightarrow} \mathsf{S} \vdash [\ [\ \dot{\mathsf{a}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{\mathsf{a}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{c}}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{\mathsf{b}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{c}}\ ]\ ]\ ]\ ]\ ]$

$[S1 \stackrel{\text{tex}}{\rightarrow}$ "
S1"]

$[S1 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s one"]

S2

$[S2 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S2 \stackrel{\text{stmt}}{\rightarrow} S \vdash [\ [\dot{a} \stackrel{p}{=} [\dot{b}]\ ] \dot{\Rightarrow} [\dot{a}' \stackrel{p}{=} [\dot{b}']\ ]\ ]\ ]$

$[S2 \stackrel{\text{tex}}{\rightarrow}$ "
S2"]

$[S2 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s two"]

S3

$[S3 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S3 \stackrel{\text{stmt}}{\rightarrow} S \vdash \neg [\dot{0} \stackrel{p}{=} [\dot{a}']\ ]\ ]$

$[S3 \stackrel{\text{tex}}{\rightarrow}$ "
S3"]

$[S3 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s three"]

S4

$[S4 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S4 \stackrel{\text{stmt}}{\rightarrow} S \vdash [\ [\dot{a}' \stackrel{p}{=} [\dot{b}']\ ] \dot{\Rightarrow} [\dot{a} \stackrel{p}{=} [\dot{b}]\ ]\ ]\ ]$

$[S4 \stackrel{\text{tex}}{\rightarrow}$ "
S4"]

$[S4 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s four"]

S5

$[S5 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S5 \stackrel{\text{stmt}}{\rightarrow} S \vdash [\ [\dot{a} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{a}]\ ]\ ]$

[S5 $\stackrel{\text{tex}}{\to}$ "
S5"]

[S5 $\stackrel{\text{pyk}}{\to}$ "axiom s five"]

S6

[S6 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S6 $\stackrel{\text{stmt}}{\to}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{+} [\dot{\mathsf{b}}'\ ]\ ] \stackrel{\mathrm{p}}{=} [\ [\dot{\mathsf{a}} \dot{+} [\dot{\mathsf{b}}\ ]\ ]'\ ]\ ]$]

[S6 $\stackrel{\text{tex}}{\to}$ "
S6"]

[S6 $\stackrel{\text{pyk}}{\to}$ "axiom s six"]

S7

[S7 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S7 $\stackrel{\text{stmt}}{\to}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{\cdot} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \dot{0}\ ]$]

[S7 $\stackrel{\text{tex}}{\to}$ "
S7"]

[S7 $\stackrel{\text{pyk}}{\to}$ "axiom s seven"]

S8

[S8 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S8 $\stackrel{\text{stmt}}{\to}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{\cdot} [\dot{\mathsf{b}}'\ ]\ ] \stackrel{\mathrm{p}}{=} [\ [\dot{\mathsf{a}} \dot{\cdot} [\dot{\mathsf{b}}\ ]\ ] \dot{+} [\dot{\mathsf{a}}\ ]\ ]\ ]$]

[S8 $\stackrel{\text{tex}}{\to}$ "
S8"]

[S8 $\stackrel{\text{pyk}}{\to}$ "axiom s eight"]

S9

[S9 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S9 $\stackrel{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{x}}\colon [\ \underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash [\ \underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{\mathsf{x}}\colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]$]

[S9 $\stackrel{\text{tex}}{\to}$ "
S9"]

[S9 $\stackrel{\text{pyk}}{\to}$ "axiom s nine"]

MP

[MP $\stackrel{\text{proof}}{\to}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\to}$ $S \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP $\stackrel{\text{tex}}{\to}$ "
MP"]

[MP $\stackrel{\text{pyk}}{\to}$ "rule mp"]

Gen

[Gen $\stackrel{\text{proof}}{\to}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\to}$ $S \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen $\stackrel{\text{tex}}{\to}$ "
Gen"]

[Gen $\stackrel{\text{pyk}}{\to}$ "rule gen"]

S′

$$
\begin{array}{l}
[S' \stackrel{\text{stmt}}{\to} [\ \forall \underline{a}\colon [\ [\ \underline{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} [\ \underline{b} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\\
\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \underline{a} \stackrel{p}{=} \underline{c}\ ] \mathrel{\dot{\Rightarrow}} [\\
\underline{b} \stackrel{p}{=} \underline{c}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \mathrel{\dot{\Rightarrow}} [\\
[\ \dot{\forall} \underline{x}\colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \neg [\ \dot{0} \stackrel{p}{=} [\ \underline{a}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{x}\colon \forall \underline{a}\colon [\\
\underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{x}\colon \underline{b}\ ] \mathrel{\dot{\Rightarrow}} \underline{a}\ ]\ ]\ ] \\
\oplus [\ [\ \forall \underline{a}\colon [\ [\ \underline{a} \mathbin{\dot{\cdot}} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ \mathrel{\dot{\Rightarrow}} [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ ]\ ]\ ] \oplus \\
[\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} [\ \underline{b} \mathrel{\dot{\Rightarrow}} \underline{a}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \mathrm{nonfree}(\lceil \underline{x} \rceil, \lceil \underline{a} \rceil) \Vdash [\ [\ \dot{\forall} \underline{x}\colon [\\
\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathbin{\dot{\cdot}} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \mathbin{\dot{\cdot}} \underline{b}\ ] \mathbin{\dot{+}} \underline{a} \\
]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \\
\vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \dot{\neg} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} \dot{\neg} \underline{a}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ [\ \dot{\neg} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} \underline{a}\ ] \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \\
]\ ]\ ] \oplus \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathbin{\dot{+}} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \mathbin{\dot{+}} \underline{b}\ ]'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]]
\end{array}
$$

[S′ $\stackrel{\text{tex}}{\to}$ "
S'"]

[S′ $\overset{\text{pyk}}{\rightarrow}$ “system prime s”]

A1′

[A1′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[A1′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} [\ \underline{b} \mathrel{\dot{\Rightarrow}} \underline{a}\ ]\ ]$]

[A1′ $\overset{\text{tex}}{\rightarrow}$ “
A1’”]

[A1′ $\overset{\text{pyk}}{\rightarrow}$ “axiom prime a one”]

A2′

[A2′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[A2′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} [\ \underline{b} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ]\ ]$]

[A2′ $\overset{\text{tex}}{\rightarrow}$ “
A2’”]

[A2′ $\overset{\text{pyk}}{\rightarrow}$ “axiom prime a two”]

A3′

[A3′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[A3′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \dot{\neg}\, \underline{b}\ ] \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \underline{a}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ [\ \dot{\neg}\, \underline{b}\ ] \mathrel{\dot{\Rightarrow}} \underline{a}\ ] \mathrel{\dot{\Rightarrow}} \underline{b}\ ]\ ]$]

[A3′ $\overset{\text{tex}}{\rightarrow}$ “
A3’”]

[A3′ $\overset{\text{pyk}}{\rightarrow}$ “axiom prime a three”]

A4′

[A4′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[A4′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{x}\colon \underline{b}\ ] \mathrel{\dot{\Rightarrow}} \underline{a}\ ]\ ]$]

[A4′ $\overset{\text{tex}}{\rightarrow}$ “
A4’”]

[A4′ $\overset{\text{pyk}}{\rightarrow}$ “axiom prime a four”]

A5′

[A5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5′ $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \mathrm{non\dot{f}ree}(\lceil \underline{\mathsf{x}} \rceil, \lceil \underline{\mathsf{a}} \rceil) \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon [\ \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ]\ ]\ ]$]

[A5′ $\stackrel{\text{tex}}{\rightarrow}$ “
A5’”]

[A5′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a five”]

S1′

[S1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1′ $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{c}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{b}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{c}}\ ]\ ]\ ]$]

[S1′ $\stackrel{\text{tex}}{\rightarrow}$ “
S1’”]

[S1′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s one”]

S2′

[S2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2′ $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{b}}\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\ \underline{\mathsf{b}}'\ ]\ ]\ ]$]

[S2′ $\stackrel{\text{tex}}{\rightarrow}$ “
S2’”]

[S2′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s two”]

S3′

[S3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3′ $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{\mathsf{a}} \colon \neg [\ \dot{0} \stackrel{\mathrm{p}}{=} [\ \underline{\mathsf{a}}'\ ]\ ]$]

[S3′ $\stackrel{\text{tex}}{\rightarrow}$ “
S3’”]

[S3′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s three”]

S4′

[S4′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ ]$]

[S4′ $\stackrel{\text{tex}}{\rightarrow}$ “
S4’”]

[S4′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s four”]

S5′

[S5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]$]

[S5′ $\stackrel{\text{tex}}{\rightarrow}$ “
S5’”]

[S5′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s five”]

S6′

[S6′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathbin{\dot{+}} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \mathbin{\dot{+}} \underline{b}\ ]'\ ]\ ]$]

[S6′ $\stackrel{\text{tex}}{\rightarrow}$ “
S6’”]

[S6′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s six”]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \mathbin{\dot{\cdot}} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]$]

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ “
S7’”]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s seven”]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: [[a ∶ [b′]] $\stackrel{\text{p}}{=}$ [[a ∶ b] $\dot{+}$ a]]]

[S8′ $\stackrel{\text{tex}}{\rightarrow}$ "
S8′"]

[S8′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: ∀c: ∀x: [b≡⟨a|x := $\dot{0}$⟩ ⊩ [c≡⟨a|x := x′⟩ ⊩ [b $\dot{\Rightarrow}$ [[$\dot{\forall}$x: [a $\dot{\Rightarrow}$ c]] $\dot{\Rightarrow}$ $\dot{\forall}$x: a]]]]]

[S9′ $\stackrel{\text{tex}}{\rightarrow}$ "
S9′"]

[S9′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀a: ∀b: [[a $\dot{\Rightarrow}$ b] ⊢ [a ⊢ b]]]

[MP′ $\stackrel{\text{tex}}{\rightarrow}$ "
MP′"]

[MP′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀x: ∀a: [a ⊢ $\dot{\forall}$x: a]]

[Gen′ $\stackrel{\text{tex}}{\rightarrow}$ "
Gen′"]

[Gen′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime gen"]

$\dot{*}$

[$\dot{\mathsf{x}} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{#1.
}"]

[$\dot{\mathsf{x}} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano var"]

$*'$

[$\mathsf{x}' \stackrel{\text{tex}}{\rightarrow}$ "#1.'"]

[$\mathsf{x}' \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano succ"]

$* \mathop{\dot{\cdot}} *$

[$\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{\cdot}} #2."]

[$\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano times $*$"]

$* \mathop{\dot{+}} *$

[$\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{+}} #2."]

[$\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano plus $*$"]

$* \stackrel{p}{=} *$

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "$*$ peano is $*$"]

$*^{\mathcal{P}}$

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil$]

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{pyk}}{\to}$ "∗ is peano var"]

$\dot{\neg} *$

$[\dot{\neg}\, \mathsf{x} \stackrel{\text{tex}}{\to}$ "
\dot{\neg}\, {#1.}"]

$[\dot{\neg}\, \mathsf{x} \stackrel{\text{pyk}}{\to}$ "peano not ∗"]

$* \mathrel{\dot{\wedge}} *$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \mathrel{\ddot{=}} \dot{\neg}\, (\mathsf{x} \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \mathsf{y})] \rceil)]$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\wedge}} #2."]

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "∗ peano and ∗"]

$* \mathrel{\dot{\vee}} *$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \mathrel{\ddot{=}} [\, \dot{\neg}\, \mathsf{x}\,] \mathrel{\dot{\Rightarrow}} \mathsf{y}] \rceil)]$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\vee}} #2."]

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "∗ peano or ∗"]

$\dot{\forall} * \colon *$

$[\dot{\forall} \mathsf{x} \colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall} \mathsf{x} \colon \mathsf{y} \stackrel{\text{pyk}}{\to}$ "peano all ∗ indeed ∗"]

$\dot{\exists} * \colon *$

$[\dot{\exists} \mathsf{x} \colon \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{\exists} \mathsf{x} \colon \mathsf{y} \mathrel{\ddot{=}} \dot{\neg}\, \dot{\forall} \mathsf{x} \colon \dot{\neg}\, \mathsf{y}] \rceil)]$

$[\dot{\exists} \mathsf{x} \colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\exists} #1.
\colon #2."]

$[\dot{\exists} \mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\to}$ "peano exist * indeed *"]

$* \mathrel{\dot{\Rightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \overset{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \overset{\text{pyk}}{\to}$ "* peano imply *"]

$* \mathrel{\dot{\Leftrightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \mathrel{\ddot{=}} (\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y}) \mathrel{\dot{\wedge}} (\mathsf{y} \mathrel{\dot{\Rightarrow}} \mathsf{x})]\rceil)]$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \overset{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \overset{\text{pyk}}{\to}$ "* peano iff *"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-22.UTC:13:28:24.620658 = MJD-53543.TAI:13:28:56.620658 = LGT-4626163736620658e-6