Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, nonfree(∗, ∗), nonfree*(∗, ∗), free⟨∗|∗ := ∗⟩, free*⟨∗|∗ := ∗⟩, ∗≡⟨∗|∗ := ∗⟩, ∗≡⟨*∗|∗ := ∗⟩, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, L3.2(a)′, M1.7, MP$'_{\mathrm{h}}$, Hypothesize, Gen$'_{\mathrm{h}}$, M3.2(a), M3.2(a)$_{\mathrm{h}}$, M3.2(b)$_{\mathrm{h}}$, M3.1(S1′)$_{\mathrm{h}}$, M3.2(c)$_{\mathrm{h}}$, M3.1(S2′)$_{\mathrm{h}}$, M3.1(S5′)$_{\mathrm{h}}$, M3.1(S6′)$_{\mathrm{h}}$, M3.2(f), $\dot{*}$, $*'$, $* \dot{\cdot} *$, $* \dot{+} *$, $* \stackrel{\mathrm{p}}{=} *$, $*^{\mathcal{P}}$, $\dot{\neg} *$, $* \dot{\wedge} *$, $* \dot{\vee} *$, $\dot{\forall} *: *$, $\dot{\exists} *: *$, $* \dot{\Rightarrow} *$, $* \dot{\Leftrightarrow} *$, $* \trianglerighteq *$, $* \trianglerighteq_{\mathrm{h}} *$,

peano

[peano $\stackrel{\text{prio}}{\rightarrow}$
Preassociative
[peano], [base], [bracket ∗ end bracket], [big bracket ∗ end bracket], [math ∗ end math], [**flush left** [∗]], [x], [y], [z], [[∗ ⋈ ∗]], [[∗ $\stackrel{*}{\rightarrow}$ ∗]], [pyk], [tex], [name], [prio], [∗], [T], [if(∗, ∗, ∗)], [[∗ $\stackrel{*}{\Rightarrow}$ ∗]], [val], [claim], [⊥], [f(∗)], [(∗)$^{\mathrm{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(∗)$^{\mathrm{M}}$], [If(∗, ∗, ∗)], [array{∗} ∗ end array], [l], [c], [r], [empty], [⟨∗ | ∗ := ∗⟩], [$\mathcal{M}$(∗)], [$\tilde{\mathcal{U}}$(∗)], [$\mathcal{U}$(∗)], [$\mathcal{U}^{\mathrm{M}}$(∗)], [**apply**(∗, ∗)], [**apply**$_1$(∗, ∗)], [identifier(∗)], [identifier$_1$(∗, ∗)], [array-plus(∗, ∗)], [array-remove(∗, ∗, ∗)], [array-put(∗, ∗, ∗, ∗)], [array-add(∗, ∗, ∗, ∗, ∗)], [bit(∗, ∗)], [bit$_1$(∗, ∗)], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}$(∗, ∗, ∗)], [$\mathcal{E}_2$(∗, ∗, ∗, ∗, ∗)], [$\mathcal{E}_3$(∗, ∗, ∗, ∗)], [$\mathcal{E}_4$(∗, ∗, ∗, ∗)], [**lookup**(∗, ∗, ∗)], [**abstract**(∗, ∗, ∗, ∗)], [[⌈∗]], [$\mathcal{M}$(∗, ∗, ∗)], [$\mathcal{M}_2$(∗, ∗, ∗, ∗)], [$\mathcal{M}^*$(∗, ∗, ∗)], [macro], [s$_0$], [**zip**(∗, ∗)], [**assoc**$_1$(∗, ∗, ∗)], [(∗)$^{\mathbf{P}}$], [self], [[∗ $\ddot{=}$ ∗]], [[∗ $\dot{=}$ ∗]], [[∗ $\acute{=}$ ∗]], [[∗ $\stackrel{\text{pyk}}{=}$ ∗]], [[∗ $\stackrel{\text{tex}}{=}$ ∗]], [[∗ $\stackrel{\text{name}}{=}$ ∗]], [**Priority table**[∗]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2$(∗)], [$\tilde{\mathcal{M}}_3$(∗)], [$\tilde{\mathcal{M}}_4$(∗, ∗, ∗, ∗)], [$\tilde{\mathcal{M}}$(∗, ∗, ∗)], [$\tilde{\mathcal{Q}}$(∗, ∗, ∗)], [$\tilde{\mathcal{Q}}_2$(∗, ∗, ∗)], [$\tilde{\mathcal{Q}}_3$(∗, ∗, ∗, ∗)], [$\tilde{\mathcal{Q}}^*$(∗, ∗, ∗)], [(∗)], [**aspect**(∗, ∗)], [**aspect**(∗, ∗, ∗)], [⟨∗⟩], [**tuple**$_1$(∗)], [**tuple**$_2$(∗)], [let$_2$(∗, ∗)], [let$_1$(∗, ∗)], [[∗ $\stackrel{\text{claim}}{=}$ ∗]], [checker], [**check**(∗, ∗)], [**check**$_2$(∗, ∗, ∗)], [**check**$_3$(∗, ∗, ∗)], [**check***(∗, ∗)], [**check**$^*_2$(∗, ∗, ∗)], [[∗]$^{\cdot}$], [[∗]$^-$], [[∗]$^{\circ}$], [msg], [[∗ $\stackrel{\text{msg}}{=}$ ∗]], [<stmt>], [stmt], [[∗ $\stackrel{\text{stmt}}{=}$ ∗]], [HeadNil′], [HeadPair′], [Transitivity′], [⊥⊥], [Contra′], [T$'_{\mathrm{E}}$], [L$_1$], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$],

$[\mathcal{R}]$, $[\mathcal{S}]$, $[\mathcal{T}]$, $[\mathcal{U}]$, $[\mathcal{V}]$, $[\mathcal{W}]$, $[\mathcal{X}]$, $[\mathcal{Y}]$, $[\mathcal{Z}]$, $[\langle * \mid * := * \rangle]$, $[\langle^* * \mid * := * \rangle]$, $[\emptyset]$, [Remainder],
$[(*)^{\mathbf{v}}]$, [intro(∗, ∗, ∗, ∗)], [intro(∗, ∗, ∗)], [error(∗, ∗)], $[\mathrm{error}_2(*, *)]$, [proof(∗, ∗, ∗)],
$[\mathrm{proof}_2(*, *)]$, $[\mathcal{S}(*, *)]$, $[\mathcal{S}^{\mathrm{I}}(*, *)]$, $[\mathcal{S}^{\rhd}(*, *)]$, $[\mathcal{S}_1^{\rhd}(*, *, *)]$, $[\mathcal{S}^{\mathrm{E}}(*, *)]$, $[\mathcal{S}_1^{\mathrm{E}}(*, *, *)]$,
$[\mathcal{S}^{+}(*, *)]$, $[\mathcal{S}_1^{+}(*, *, *)]$, $[\mathcal{S}^{-}(*, *)]$, $[\mathcal{S}_1^{-}(*, *, *)]$, $[\mathcal{S}^{*}(*, *)]$, $[\mathcal{S}_1^{*}(*, *, *)]$,
$[\mathcal{S}_2^{*}(*, *, *, *)]$, $[\mathcal{S}^{@}(*, *)]$, $[\mathcal{S}_1^{@}(*, *, *)]$, $[\mathcal{S}^{\vdash}(*, *)]$, $[\mathcal{S}_1^{\vdash}(*, *, *, *)]$, $[\mathcal{S}^{\Vdash}(*, *)]$,
$[\mathcal{S}_1^{\Vdash}(*, *, *, *)]$, $[\mathcal{S}^{\mathrm{i.e.}}(*, *)]$, $[\mathcal{S}_1^{\mathrm{i.e.}}(*, *, *, *)]$, $[\mathcal{S}_2^{\mathrm{i.e.}}(*, *, *, *, *)]$, $[\mathcal{S}^{\forall}(*, *)]$,
$[\mathcal{S}_1^{\forall}(*, *, *, *)]$, $[\mathcal{S}^{;}(*, *)]$, $[\mathcal{S}_1^{;}(*, *, *)]$, $[\mathcal{S}_2^{;}(*, *, *, *)]$, $[\mathcal{T}(*)]$, [claims(∗, ∗, ∗)],
$[\mathrm{claims}_2(*, *, *)]$, [<proof>], [proof], [[**Lemma** ∗: ∗]], [[**Proof of** ∗: ∗]],
[[∗ **lemma** ∗: ∗]], [[∗ **antilemma** ∗: ∗]], [[∗ **rule** ∗: ∗]], [[∗ **antirule** ∗: ∗]],
[verifier], $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*, *)]$, $[\mathcal{V}_3(*, *, *, *)]$, $[\mathcal{V}_4(*, *)]$, $[\mathcal{V}_5(*, *, *, *)]$, $[\mathcal{V}_6(*, *, *, *)]$,
$[\mathcal{V}_7(*, *, *, *)]$, [Cut(∗, ∗)], $[\mathrm{Head}_{\oplus}(*)]$, $[\mathrm{Tail}_{\oplus}(*)]$, $[\mathrm{rule}_1(*, *)]$, [rule(∗, ∗)],
[Rule tactic], [Plus(∗, ∗)], [[**Theory** ∗]], $[\mathrm{theory}_2(*, *)]$, $[\mathrm{theory}_3(*, *)]$,
$[\mathrm{theory}_4(*, *, *)]$, [HeadNil″], [HeadPair″], [Transitivity″], [Contra″], [HeadNil],
[HeadPair], [Transitivity], [Contra], $[\mathrm{T}_{\mathrm{E}}]$, [ragged right],
[ragged right expansion], [parm(∗, ∗, ∗)], $[\mathrm{parm}^*(*, *, *)]$, [inst(∗, ∗)],
$[\mathrm{inst}^*(*, *)]$, [occur(∗, ∗, ∗)], $[\mathrm{occur}^*(*, *, *)]$, [unify(∗ = ∗, ∗)], $[\mathrm{unify}^*(* = *, *)]$,
$[\mathrm{unify}_2(* = *, *)]$, $[\mathrm{L_a}]$, $[\mathrm{L_b}]$, $[\mathrm{L_c}]$, $[\mathrm{L_d}]$, $[\mathrm{L_e}]$, $[\mathrm{L_f}]$, $[\mathrm{L_g}]$, $[\mathrm{L_h}]$, $[\mathrm{L_i}]$, $[\mathrm{L_j}]$, $[\mathrm{L_k}]$, $[\mathrm{L_l}]$, $[\mathrm{L_m}]$,
$[\mathrm{L_n}]$, $[\mathrm{L_o}]$, $[\mathrm{L_p}]$, $[\mathrm{L_q}]$, $[\mathrm{L_r}]$, $[\mathrm{L_s}]$, $[\mathrm{L_t}]$, $[\mathrm{L_u}]$, $[\mathrm{L_v}]$, $[\mathrm{L_w}]$, $[\mathrm{L_x}]$, $[\mathrm{L_y}]$, $[\mathrm{L_z}]$, $[\mathrm{L_A}]$, $[\mathrm{L_B}]$, $[\mathrm{L_C}]$,
$[\mathrm{L_D}]$, $[\mathrm{L_E}]$, $[\mathrm{L_F}]$, $[\mathrm{L_G}]$, $[\mathrm{L_H}]$, $[\mathrm{L_I}]$, $[\mathrm{L_J}]$, $[\mathrm{L_K}]$, $[\mathrm{L_L}]$, $[\mathrm{L_M}]$, $[\mathrm{L_N}]$, $[\mathrm{L_O}]$, $[\mathrm{L_P}]$, $[\mathrm{L_Q}]$, $[\mathrm{L_R}]$,
$[\mathrm{L_S}]$, $[\mathrm{L_T}]$, $[\mathrm{L_U}]$, $[\mathrm{L_V}]$, $[\mathrm{L_W}]$, $[\mathrm{L_X}]$, $[\mathrm{L_Y}]$, $[\mathrm{L_Z}]$, $[\mathrm{L_?}]$, [Reflexivity], $[\mathrm{Reflexivity}_1]$,
[Commutativity], $[\mathrm{Commutativity}_1]$, [<tactic>], [tactic], $[[* \overset{\mathrm{tactic}}{=} *]]$, $[\mathcal{P}(*, *, *)]$,
$[\mathcal{P}^*(*, *, *)]$, $[\mathrm{p}_0]$, $[\mathrm{conclude}_1(*, *)]$, $[\mathrm{conclude}_2(*, *, *)]$, $[\mathrm{conclude}_3(*, *, *, *)]$,
$[\mathrm{conclude}_4(*, *)]$, $[\dot{0}]$, $[\dot{1}]$, $[\dot{2}]$, $[\dot{a}]$, $[\dot{b}]$, $[\dot{c}]$, $[\dot{d}]$, $[\dot{e}]$, $[\dot{f}]$, $[\dot{g}]$, $[\dot{h}]$, $[\dot{i}]$, $[\dot{j}]$, $[\dot{k}]$, $[\dot{l}]$, $[\dot{m}]$, $[\dot{n}]$,
$[\dot{o}]$, $[\dot{p}]$, $[\dot{q}]$, $[\dot{r}]$, $[\dot{s}]$, $[\dot{t}]$, $[\dot{u}]$, $[\dot{v}]$, $[\dot{w}]$, $[\dot{x}]$, $[\dot{y}]$, $[\dot{z}]$, $[\dot{\mathrm{nonfree}}(*, *)]$, $[\dot{\mathrm{nonfree}}^*(*, *)]$,
$[\mathrm{free}\langle * \mid * := * \rangle]$, $[\mathrm{free}^*\langle * \mid * := * \rangle]$, $[* \equiv \langle * \mid * := * \rangle]$, $[* \equiv \langle^* * \mid * := * \rangle]$, [S], [A1], [A2],
[A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen],
[L3.2(a)], [S′], [A1′], [A2′], [A3′], [A4′], [A5′], [S1′], [S2′], [S3′], [S4′], [S5′], [S6′],
[S7′], [S8′], [S9′], [MP′], [Gen′], [L3.2(a)′], [M1.7], $[\mathrm{MP}'_{\mathrm{h}}]$, [Hypothesize], $[\mathrm{Gen}'_{\mathrm{h}}]$,
[M3.2(a)], $[\mathrm{M3.2(a)_h}]$, $[\mathrm{M3.2(b)_h}]$, $[\mathrm{M3.1(S1')_h}]$, $[\mathrm{M3.2(c)_h}]$, $[\mathrm{M3.1(S2')_h}]$,
$[\mathrm{M3.1(S5')_h}]$, $[\mathrm{M3.1(S6')_h}]$, [M3.2(f)];
Preassociative
$[*_{\{*\}}]$, $[*']$, $[*[*]]$, $[*[* \rightarrow *]]$, $[*[* \Rightarrow *]]$, $[\dot{*}]$;
Preassociative
[“ ∗ ”], [], $[(*)^{\mathbf{t}}]$, [string(∗) + ∗], [string(∗) ++ ∗], [
∗], [∗], [!∗], [”∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗],
[0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗],
[@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗],
[O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [^∗],
[_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗],
[p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [~∗],
[**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end],
[newline ∗], [macro newline ∗];
Preassociative
[∗0], [∗1], [0b], [∗-color(∗)], $[*\text{-color}^*(*)]$;
Preassociative

$[* \text{'} *], [* \text{`} *]$;
Preassociative
$[*^{\mathrm{H}}], [*^{\mathrm{T}}], [*^{\mathrm{U}}], [*^{\mathrm{h}}], [*^{\mathrm{t}}], [*^{\mathrm{s}}], [*^{\mathrm{c}}], [*^{\mathrm{d}}], [*^{\mathrm{a}}], [*^{\mathrm{C}}], [*^{\mathrm{M}}], [*^{\mathrm{B}}], [*^{\mathrm{r}}], [*^{\mathrm{i}}], [*^{\mathrm{d}}], [*^{\mathrm{R}}], [*^{0}], [*^{1}], [*^{2}], [*^{3}], [*^{4}], [*^{5}], [*^{6}], [*^{7}], [*^{8}], [*^{9}], [*^{\mathrm{E}}], [*^{\mathcal{V}}], [*^{\mathcal{C}}], [*^{\mathcal{C}^*}], [*']$;
Preassociative
$[* \cdot *], [* \cdot_0 *], [* \colon *]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*, *]$;
Preassociative
$[* \stackrel{\mathrm{B}}{\approx} *], [* \stackrel{\mathrm{D}}{\approx} *], [* \stackrel{\mathrm{C}}{\approx} *], [* \stackrel{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\to} *], [* \stackrel{\mathrm{t}}{=} *], [* \stackrel{\mathrm{t}^*}{=} *], [* \stackrel{\mathrm{r}}{=} *], [* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \stackrel{\mathrm{T}}{=} *], [* \stackrel{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *], [* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* \stackrel{\mathrm{p}}{=} *], [*^{\mathcal{P}}]$;
Preassociative
$[\neg *], [\dot{\neg} *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\mathrm{c}} *], [* \dot{\wedge} *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *]$;
Preassociative
$[\dot{\forall} *\colon *], [\dot{\exists} *\colon *]$;
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *]$;
Postassociative
$[* : *], [*!*]$;
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.]$;
Preassociative
$[\lambda * .*], [\Lambda *], [\mathbf{if}\ *\ \mathbf{then}\ *\ \mathbf{else}\ *], [\mathbf{let}\ * = *\ \mathbf{in}\ *], [\mathbf{let}\ * \stackrel{..}{=} *\ \text{ in }\ *]$;
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}]$;
Preassociative
$[* @ *], [* \rhd *], [* \gg\!\!\rhd *], [* \gg *], [* \trianglerighteq *], [* \trianglerighteq_{\mathrm{h}} *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall *\colon *]$;
Postassociative
$[* \oplus *]$;

Postassociative
$[*;*]$;
Preassociative
$[*\text{ proves }*]$;
Preassociative
$[*\ \textbf{proof of}\ *:*]$, $[\text{Line}\,*:*\gg *;*]$, $[\text{Last line}\,*\gg *\,\Box]$,
$[\text{Line}\,*:\text{Premise}\gg *;*]$, $[\text{Line}\,*:\text{Side-condition}\gg *;*]$, $[\text{Arbitrary}\gg *;*]$,
$[\text{Local}\gg *=*;*]$;
Postassociative
$[*\text{ then }*]$, $[*[*]*]$;
Preassociative
$[*\&*]$;
Preassociative
$[*\backslash\backslash*]$;]

$[\text{peano} \stackrel{\text{pyk}}{\rightarrow} \text{“peano”}]$

$\dot{0}$

$[\dot{0} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{0}”]

$[\dot{0} \stackrel{\text{pyk}}{\rightarrow} \text{“peano zero”}]$

$\dot{1}$

$[\dot{1} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{1} \ddot{=} \dot{0}']\rceil)]$

$[\dot{1} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{1}”]

$[\dot{1} \stackrel{\text{pyk}}{\rightarrow} \text{“peano one”}]$

$\dot{2}$

$[\dot{2} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{2} \ddot{=} \dot{1}']\rceil)]$

$[\dot{2} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{2}”]

$[\dot{2} \stackrel{\text{pyk}}{\rightarrow} \text{“peano two”}]$

$\dot{a}$

$[\dot{a} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{a} \ddot{=} \dot{\mathsf{a}}] \rceil)]$

$[\dot{a} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{a}}"]

$[\dot{a} \overset{\text{pyk}}{\rightarrow}$ "peano a"]

$\dot{b}$

$[\dot{b} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{b}}"]

$[\dot{b} \overset{\text{pyk}}{\rightarrow}$ "peano b"]

$\dot{c}$

$[\dot{c} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{c}}"]

$[\dot{c} \overset{\text{pyk}}{\rightarrow}$ "peano c"]

$\dot{d}$

$[\dot{d} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{d}}"]

$[\dot{d} \overset{\text{pyk}}{\rightarrow}$ "peano d"]

$\dot{e}$

$[\dot{e} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{e}}"]

$[\dot{e} \overset{\text{pyk}}{\rightarrow}$ "peano e"]

$$\dot{f}$$

$[\dot{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \ddot{=} \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\to}$ "peano f"]

$$\dot{g}$$

$[\dot{g} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\to}$ "peano g"]

$$\dot{h}$$

$[\dot{h} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\to}$ "peano h"]

$$\dot{i}$$

$[\dot{i} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\to}$ "peano i"]

$$\dot{j}$$

$[\dot{j} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\to}$ "peano j"]

$\dot{k}$

$[\dot{k} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{k} \ddot{=} \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{k}}"]

$[\dot{k} \overset{\mathrm{pyk}}{\to}$ "peano k"]

$\dot{l}$

$[\dot{l} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{l} \ddot{=} \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{l}}"]

$[\dot{l} \overset{\mathrm{pyk}}{\to}$ "peano l"]

$\dot{m}$

$[\dot{m} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{m} \ddot{=} \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{m}}"]

$[\dot{m} \overset{\mathrm{pyk}}{\to}$ "peano m"]

$\dot{n}$

$[\dot{n} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{n} \ddot{=} \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{n}}"]

$[\dot{n} \overset{\mathrm{pyk}}{\to}$ "peano n"]

$\dot{o}$

$[\dot{o} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{o} \ddot{=} \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \overset{\mathrm{tex}}{\to}$ "
\dot{\mathit{o}}"]

$[\dot{o} \overset{\mathrm{pyk}}{\to}$ "peano o"]

$\dot{p}$

$[\dot{p} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{p} \ddot{=} \dot{\mathsf{p}}]\rceil)]$

$[\dot{p} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{p}}"]

$[\dot{p} \stackrel{\text{pyk}}{\to}$ "peano p"]

$\dot{q}$

$[\dot{q} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{q} \ddot{=} \dot{\mathsf{q}}]\rceil)]$

$[\dot{q} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{q}}"]

$[\dot{q} \stackrel{\text{pyk}}{\to}$ "peano q"]

$\dot{r}$

$[\dot{r} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{r} \ddot{=} \dot{\mathsf{r}}]\rceil)]$

$[\dot{r} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{r}}"]

$[\dot{r} \stackrel{\text{pyk}}{\to}$ "peano r"]

$\dot{s}$

$[\dot{s} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{s} \ddot{=} \dot{\mathsf{s}}]\rceil)]$

$[\dot{s} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{s}}"]

$[\dot{s} \stackrel{\text{pyk}}{\to}$ "peano s"]

$\dot{t}$

$[\dot{t} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{t} \ddot{=} \dot{\mathsf{t}}]\rceil)]$

$[\dot{t} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{t}}"]

$[\dot{t} \stackrel{\text{pyk}}{\to}$ "peano t"]

$\dot{u}$

$[\dot{u} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{u} \ddot{=} \dot{\mathsf{u}}] \rceil)]$

$[\dot{u} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{u}}"]

$[\dot{u} \stackrel{\text{pyk}}{\to}$ "peano u"]

$\dot{v}$

$[\dot{v} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{v} \ddot{=} \dot{\mathsf{v}}] \rceil)]$

$[\dot{v} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{v}}"]

$[\dot{v} \stackrel{\text{pyk}}{\to}$ "peano v"]

$\dot{w}$

$[\dot{w} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{w} \ddot{=} \dot{\mathsf{w}}] \rceil)]$

$[\dot{w} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{w}}"]

$[\dot{w} \stackrel{\text{pyk}}{\to}$ "peano w"]

$\dot{x}$

$[\dot{x} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{x} \ddot{=} \dot{\mathsf{x}}] \rceil)]$

$[\dot{x} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{x}}"]

$[\dot{x} \stackrel{\text{pyk}}{\to}$ "peano x"]

$\dot{y}$

$[\dot{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{y} \ddot{=} \dot{\mathsf{y}}] \rceil)]$

$[\dot{y} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{y}}"]

$[\dot{y} \stackrel{\text{pyk}}{\to}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{z} \mathrel{\ddot{=}} \dot{\mathsf{z}}] \rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"]

$\dot{\text{nonfree}}(*, *)$

$[\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}) \stackrel{\text{val}}{\to}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg \, [\, \mathsf{x} \stackrel{\text{t}}{=} \mathsf{y} \,] \,,$
$\text{If}(\neg \, [\, \mathsf{y} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{x} \colon \mathsf{y} \rceil \,] \,, \dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=} \, [\, \mathsf{y}^1 \,] \,, \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}^2))))]$

$[\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree ∗ in ∗ end nonfree"]

$\dot{\text{nonfree}}^*(*, *)$

$[\dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}) \stackrel{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y}, \mathsf{T}, \text{If}(\dot{\text{nonfree}}(\mathsf{x}, \mathsf{y}^{\text{h}}), \dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}^{\text{t}}), \mathsf{F}))]$

$[\dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^*(\mathsf{x}, \mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree star ∗ in ∗ end nonfree"]

$\dot{\text{free}}\langle * | * := * \rangle$

$[\dot{\text{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\text{val}}{\to} \mathsf{x}! \, [\, \mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg \, [\, \mathsf{a} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{u} \colon \mathsf{v} \rceil \,] \,, \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}} | \mathsf{x} := \mathsf{b} \rangle,$
$\text{If}(\mathsf{a}^1 \stackrel{\text{t}}{=} \mathsf{x}, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x}, \mathsf{a}^2), \mathsf{T},$

$\mathrm{If}(\neg\dot{\mathrm{nonfree}}(\mathsf{a}^1,\mathsf{b}),\mathsf{F},$
$\dot{\mathrm{free}}\langle\mathsf{a}^2|\mathsf{x}:=\mathsf{b}\rangle)))))$]]

[$\dot{\mathrm{free}}\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{free}\langle #1.
| #2.
:= #3.
\rangle”]

[$\dot{\mathrm{free}}\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano free * set * to * end free”]

$\dot{\mathrm{free}}^*\langle*|*:=*\rangle$

[$\dot{\mathrm{free}}^*\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \stackrel{\mathrm{val}}{\rightarrow} \mathsf{x}!$ [$\mathsf{b}!\mathrm{If}(\mathsf{a},\mathsf{T},\mathrm{If}(\dot{\mathrm{free}}\langle\mathsf{a}^{\mathrm{h}}|\mathsf{x}:=\mathsf{b}\rangle,\dot{\mathrm{free}}^*\langle\mathsf{a}^{\mathrm{t}}|\mathsf{x}:=\mathsf{b}\rangle,\mathsf{F}))$]]

[$\dot{\mathrm{free}}^*\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \stackrel{\mathrm{tex}}{\rightarrow}$ “
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle”]

[$\dot{\mathrm{free}}^*\langle\mathsf{a}|\mathsf{x}:=\mathsf{b}\rangle \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano free star * set * to * end free”]

$*\equiv\langle*|*:=*\rangle$

[$\mathsf{a}\equiv\langle\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \stackrel{\mathrm{val}}{\rightarrow} \mathsf{a}!$ [$\mathsf{x}!$ [$\mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \stackrel{\mathrm{r}}{=} \lceil\dot{\forall}\mathsf{u}\colon\mathsf{v}\rceil,\mathsf{b}^1 \stackrel{\mathrm{t}}{=} \mathsf{x},\mathsf{F}),\mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge$ [$\mathsf{b} \stackrel{\mathrm{t}}{=} \mathsf{x}$] $,\mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{c},\mathrm{If}($ [
a] $\stackrel{\mathrm{r}}{=} \mathsf{b},\mathsf{a}^{\mathrm{t}}\equiv\langle^*\mathsf{b}^{\mathrm{t}}|\mathsf{x}:=\mathsf{c}\rangle,\mathsf{F})))$]]]

[$\mathsf{a}\equiv\langle\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \stackrel{\mathrm{tex}}{\rightarrow}$ “#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle”]

[$\mathsf{a}\equiv\langle\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \stackrel{\mathrm{pyk}}{\rightarrow}$ “peano sub * is * where * is * end sub”]

$*\equiv\langle^**|*:=*\rangle$

[$\mathsf{a}\equiv\langle^*\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \stackrel{\mathrm{val}}{\rightarrow} \mathsf{b}!$ [$\mathsf{x}!$ [$\mathsf{c}!\mathrm{If}(\mathsf{a},\mathsf{T},\mathrm{If}(\mathsf{a}^{\mathrm{h}}\equiv\langle\mathsf{b}^{\mathrm{h}}|\mathsf{x}:=\mathsf{c}\rangle,\mathsf{a}^{\mathrm{t}}\equiv\langle^*\mathsf{b}^{\mathrm{t}}|\mathsf{x}:=\mathsf{c}\rangle,\mathsf{F}))$]]]

[$\mathsf{a}\equiv\langle^*\mathsf{b}|\mathsf{x}:=\mathsf{c}\rangle \stackrel{\mathrm{tex}}{\rightarrow}$ “#1.

{\equiv}\langleˆ∗ #2.
|#3.
:=#4.
\rangle”]

$[\mathsf{a} \equiv \langle^{*} \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\text{pyk}}{\rightarrow}$ “peano sub star ∗ is ∗ where ∗ is ∗ end sub”]

S

$[\mathrm{S} \stackrel{\text{stmt}}{\rightarrow} [\ [\dot{\mathsf{a}} \mathbin{\dot{+}} [\dot{\mathsf{b}}']\ ] \stackrel{p}{=} [\ [\dot{\mathsf{a}} \mathbin{\dot{+}} [\dot{\mathsf{b}}]\ ]'\ ]\ ] \oplus [\ [\forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ [\dot{\neg} \underline{\mathsf{b}}] \mathrel{\dot{\Rightarrow}} \dot{\neg} \underline{\mathsf{a}}\ ] \mathrel{\dot{\Rightarrow}} [\ [\ [\dot{\neg} \underline{\mathsf{b}}] \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}\ ] \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}\ ]\ ]\ ]\ ] \oplus [\ [\ [\dot{\mathsf{a}} \stackrel{p}{=} [\dot{\mathsf{b}}]\ ] \mathrel{\dot{\Rightarrow}} [\dot{\mathsf{a}}' \stackrel{p}{=} [\dot{\mathsf{b}}']\ ]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}] \vdash [\underline{\mathsf{a}} \vdash \underline{\mathsf{b}}]\ ]\ ]\ ] \oplus [\ [\ [\dot{\mathsf{a}}' \stackrel{p}{=} [\dot{\mathsf{b}}']\ ] \mathrel{\dot{\Rightarrow}} [\dot{\mathsf{a}} \stackrel{p}{=} [\dot{\mathsf{b}}]\ ]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}]\ ]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\text{nonfree}(\underline{\mathsf{x}}, \underline{\mathsf{a}}) \Vdash [\ [\dot{\forall} \underline{\mathsf{x}} \colon [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}]\ ] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}]\ ]\ ]\ ]\ ] \oplus [\ [\ [\dot{\mathsf{a}} \mathbin{\dot{\cdot}} [\dot{\mathsf{b}}']\ ] \stackrel{p}{=} [\ [\dot{\mathsf{a}} \mathbin{\dot{\cdot}} [\dot{\mathsf{b}}]\ ] \mathbin{\dot{+}} [\dot{\mathsf{a}}]\ ]\ ]\ ] \oplus [\ [\ [\dot{\mathsf{a}} \mathbin{\dot{+}} \dot{0}] \stackrel{p}{=} [\dot{\mathsf{a}}]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}}]\ ] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}}]\ ]\ ]\ ]\ ] \oplus [\ [\ [\dot{\mathsf{a}} \stackrel{p}{=} [\dot{\mathsf{b}}]\ ] \mathrel{\dot{\Rightarrow}} [\ [\dot{\mathsf{a}} \stackrel{p}{=} [\dot{\mathsf{c}}]\ ] \mathrel{\dot{\Rightarrow}} [\dot{\mathsf{b}} \stackrel{p}{=} [\dot{\mathsf{c}}]\ ]\ ]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{x}} \colon [\underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash [\underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash [\underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} [\ [\dot{\forall} \underline{\mathsf{x}} \colon [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}}]\ ] \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ]\ ] \oplus [\ [\neg [\dot{0} \stackrel{p}{=} [\dot{\mathsf{a}}']\ ]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon [\underline{\mathsf{a}} \vdash \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{a}}]\ ]\ ] \oplus [\ [\forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{b}} \colon [\lceil \underline{\mathsf{a}} \rceil \equiv \langle \lceil \underline{\mathsf{b}} \rceil | \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil \rangle \Vdash [\ [\dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}] \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}\ ]\ ]\ ]\ ] \oplus [\ [\dot{\mathsf{a}} \mathbin{\dot{\cdot}} \dot{0}] \stackrel{p}{=} \dot{0}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]]$

[S $\stackrel{\text{tex}}{\rightarrow}$ “
S”]

[S $\stackrel{\text{pyk}}{\rightarrow}$ “system s”]

A1

[A1 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{A1} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{a}}]\ ]\ ]$

[A1 $\stackrel{\text{tex}}{\rightarrow}$ “
A1”]

[A1 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom a one”]

A2

[A2 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{A2} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}}]\ ] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{b}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}}]\ ]\ ]\ ]$

$[A2 \stackrel{tex}{\rightarrow}$ "
A2"]

$[A2 \stackrel{pyk}{\rightarrow}$ "axiom a two"]

A3

$[A3 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[A3 \stackrel{stmt}{\rightarrow} S \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\,[\,[\,\dot{\neg}\,\underline{b}\,] \mathrel{\dot{\Rightarrow}} \dot{\neg}\,\underline{a}\,] \mathrel{\dot{\Rightarrow}} [\,[\,[\,\dot{\neg}\,\underline{b}\,] \mathrel{\dot{\Rightarrow}} \underline{a}\,] \mathrel{\dot{\Rightarrow}} \underline{b}\,]\,]\,]$

$[A3 \stackrel{tex}{\rightarrow}$ "
A3"]

$[A3 \stackrel{pyk}{\rightarrow}$ "axiom a three"]

A4

$[A4 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[A4 \stackrel{stmt}{\rightarrow} S \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\,\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\,[\,\dot{\forall} \underline{x}\colon \underline{b}\,] \mathrel{\dot{\Rightarrow}} \underline{a}\,]\,]\,]$

$[A4 \stackrel{tex}{\rightarrow}$ "
A4"]

$[A4 \stackrel{pyk}{\rightarrow}$ "axiom a four"]

A5

$[A5 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[A5 \stackrel{stmt}{\rightarrow} S \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\,\mathrm{non\dot{f}ree}(\underline{x}, \underline{a}) \Vdash [\,[\,\dot{\forall} \underline{x}\colon [\,\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\,]\,] \mathrel{\dot{\Rightarrow}} [\,\underline{a} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{b}\,]\,]\,]\,]$

$[A5 \stackrel{tex}{\rightarrow}$ "
A5"]

$[A5 \stackrel{pyk}{\rightarrow}$ "axiom a five"]

S1

$[S1 \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S1 \stackrel{stmt}{\rightarrow} S \vdash [\,[\,\dot{a} \stackrel{p}{=} [\,\dot{b}\,]\,] \mathrel{\dot{\Rightarrow}} [\,[\,\dot{a} \stackrel{p}{=} [\,\dot{c}\,]\,] \mathrel{\dot{\Rightarrow}} [\,\dot{b} \stackrel{p}{=} [\,\dot{c}\,]\,]\,]\,]\,]$

[S1 $\stackrel{\text{tex}}{\rightarrow}$ "
S1"]

[S1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s one"]

S2

[S2 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{a} \stackrel{p}{=} [\dot{b}]\ ] \dot{\Rightarrow} [\dot{a}' \stackrel{p}{=} [\dot{b}']\ ]\ ]$]

[S2 $\stackrel{\text{tex}}{\rightarrow}$ "
S2"]

[S2 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s two"]

S3

[S3 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \neg [\dot{0} \stackrel{p}{=} [\dot{a}']\ ]$]

[S3 $\stackrel{\text{tex}}{\rightarrow}$ "
S3"]

[S3 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s three"]

S4

[S4 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{a}' \stackrel{p}{=} [\dot{b}']\ ] \dot{\Rightarrow} [\dot{a} \stackrel{p}{=} [\dot{b}]\ ]\ ]$]

[S4 $\stackrel{\text{tex}}{\rightarrow}$ "
S4"]

[S4 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s four"]

S5

[S5 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{a} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{a}]\ ]$]

[S5 $\stackrel{\text{tex}}{\rightarrow}$ "
S5"]

[S5 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s five"]

S6

[S6 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{+} [\dot{\mathsf{b}}'\ ]\ ] \stackrel{\mathrm{p}}{=} [\ [\dot{\mathsf{a}} \dot{+} [\dot{\mathsf{b}}\ ]\ ]'\ ]\ ]$]

[S6 $\stackrel{\text{tex}}{\rightarrow}$ "
S6"]

[S6 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s six"]

S7

[S7 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{\cdot} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \dot{0}\ ]$]

[S7 $\stackrel{\text{tex}}{\rightarrow}$ "
S7"]

[S7 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s seven"]

S8

[S8 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\dot{\mathsf{a}} \dot{\cdot} [\dot{\mathsf{b}}'\ ]\ ] \stackrel{\mathrm{p}}{=} [\ [\dot{\mathsf{a}} \dot{\cdot} [\dot{\mathsf{b}}\ ]\ ] \dot{+} [\dot{\mathsf{a}}\ ]\ ]\ ]$]

[S8 $\stackrel{\text{tex}}{\rightarrow}$ "
S8"]

[S8 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s eight"]

S9

[S9 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{x}} \colon [\ \underline{\mathsf{b}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \dot{0} \rangle \Vdash [\ \underline{\mathsf{c}} \equiv \langle \underline{\mathsf{a}} | \underline{\mathsf{x}} := \underline{\mathsf{x}}' \rangle \Vdash [\ \underline{\mathsf{b}} \mathrel{\dot{\Rightarrow}} [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon [\ \underline{\mathsf{a}} \mathrel{\dot{\Rightarrow}} \underline{\mathsf{c}}\ ]\ ] \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]$]

[S9 $\stackrel{\text{tex}}{\rightarrow}$ "
S9"]

[S9 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s nine"]

MP

[MP $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP $\stackrel{\text{tex}}{\rightarrow}$ "
MP"]

[MP $\stackrel{\text{pyk}}{\rightarrow}$ "rule mp"]

Gen

[Gen $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen $\stackrel{\text{tex}}{\rightarrow}$ "
Gen"]

[Gen $\stackrel{\text{pyk}}{\rightarrow}$ "rule gen"]

L3.2(a)

[L3.2(a) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash [\ [\ \mathrm{S5} \gg [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ]; [\ [\ [\ \mathrm{Gen} \rhd [\ [$
$\dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{a}\colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ]; [\ [\ [\ \mathrm{A4} @ [\ \dot{x}\ ]\ ] \gg [\ [$
$\dot{\forall}\dot{a}\colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]; [\ [\ [\ [\ \mathrm{MP} \rhd [\ [\ \dot{\forall}\dot{a}\colon$
$[\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{a}\colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]$
$] \gg [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]; [\ [\ \mathrm{S1} \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow}$
$[\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ]; [\ [\ [\ \mathrm{Gen} \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=}$
$[\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ];$
$[\ [\ [\ \mathrm{A4} @ [\ \dot{x}\ ]\ ] \gg [\ [\ \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}$
$]\ ]\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ]; [\ [\ [$
$[\ \mathrm{MP} \rhd [\ [\ \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \dot{\Rightarrow} [$
$[\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ]$
$\dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]$
$] \dot{\Rightarrow} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]; [\ [\ [\ \mathrm{Gen} \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \dot{\Rightarrow} [$

$\dot{b} \stackrel{p}{=} [\dot{x}]]]]] \gg \dot\forall\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{b} \stackrel{p}{=} [\dot{x}]]]$
$]]; [[[\mathrm{A4} @ [\dot{x}]] \gg [[\dot\forall\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{b} \stackrel{p}{=}$
$[\dot{x}]]]]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]; [$
$[[[\mathrm{MP} \rhd [[\dot\forall\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{b} \stackrel{p}{=} [\dot{x}]]]]]$
$\dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \rhd \dot\forall\dot{b}\colon [[\dot{a} \stackrel{p}{=} [$
$\dot{b}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{b} \stackrel{p}{=} [\dot{x}]]]]] \gg [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=}$
$[\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]; [[[\mathrm{Gen} \rhd [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]]$
$\dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \gg \dot\forall\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]$
$]]]]; [[[\mathrm{A4} @ [\dot{x} \dot{+} \dot{0}]] \gg [[\dot\forall\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]]$
$\dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \dot\Rightarrow [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]$
$\dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]; [[[[\mathrm{MP} \rhd [[\dot\forall\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}$
$]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \dot\Rightarrow [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]$
$] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \rhd \dot\forall\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [$
$\dot{x}]]]]] \gg [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}$
$]]]]]; [[[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow$
$[\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \rhd [[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [$
$\dot{x} \stackrel{p}{=} [\dot{x}]]]]; [[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot\Rightarrow [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \rhd [$
$[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$

$[\mathrm{L3.2(a)} \stackrel{\mathrm{stmt}}{\rightarrow} S \vdash [\dot{x} \stackrel{p}{=} [\dot{x}]]]$

[L3.2(a) $\stackrel{\mathrm{tex}}{\rightarrow}$ “
L3.2(a)”]

[L3.2(a) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma l three two a”]

S′

$[S' \stackrel{\mathrm{stmt}}{\rightarrow} [\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a}' \stackrel{p}{=} [\underline{b}']] \dot\Rightarrow [\underline{a} \stackrel{p}{=} \underline{b}]]] \oplus [[\forall\underline{h}\colon\forall\underline{t}\colon\forall\underline{r}\colon\forall\underline{s}\colon [[\underline{h} \Rightarrow$
$[\underline{t} \stackrel{p}{=} \underline{r}]] \vdash [[\underline{h} \Rightarrow [\underline{t} \stackrel{p}{=} \underline{s}]] \vdash [\underline{h} \Rightarrow [\underline{r} \stackrel{p}{=} \underline{s}]]]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [$
$\underline{a} \dot\Rightarrow [\underline{b} \dot\Rightarrow \underline{a}]]] \oplus [[\forall\underline{x}\colon\forall\underline{a}\colon\forall\underline{b}\colon [\mathrm{nonfree}(\lceil\underline{x}\rceil, \lceil\underline{a}\rceil) \Vdash [[\dot\forall\underline{x}\colon [\underline{a} \dot\Rightarrow \underline{b}]]$
$\dot\Rightarrow [\underline{a} \dot\Rightarrow \dot\forall\underline{x}\colon \underline{b}]]]]] \oplus [[\forall\underline{h}\colon\forall\underline{t}\colon\forall\underline{r}\colon [\underline{h} \Rightarrow [[\underline{t} \dot{+} [\underline{r}']] \stackrel{p}{=} [[\underline{t} \dot{+} \underline{r}]']]$
$]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \dot{\cdot} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{\cdot} \underline{b}] \dot{+} \underline{a}]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \stackrel{p}{=} \underline{b}$
$] \dot\Rightarrow [\underline{a}' \stackrel{p}{=} [\underline{b}']]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \dot\Rightarrow \underline{b}] \vdash [\underline{a} \vdash \underline{b}]]]] \oplus [[$
$\forall\underline{a}\colon\forall\underline{b}\colon [[[\dot\neg\underline{b}] \dot\Rightarrow \dot\neg\underline{a}] \dot\Rightarrow [[[\dot\neg\underline{b}] \dot\Rightarrow \underline{a}] \dot\Rightarrow \underline{b}]]]] \oplus [[\forall\underline{h}\colon\forall\underline{t}\colon\forall\underline{r}\colon [$
$[\underline{h} \Rightarrow [\underline{t} \stackrel{p}{=} \underline{r}]] \vdash [\underline{h} \Rightarrow [\underline{t}' \stackrel{p}{=} [\underline{r}']]]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon [[\underline{a} \dot{+} [\underline{b}']]$
$\stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}]']]]] \oplus [[\forall\underline{a}\colon \neg[\dot{0} \stackrel{p}{=} [\underline{a}']]]] \oplus [[\forall\underline{x}\colon\forall\underline{a}\colon [\underline{a} \vdash \dot\forall\underline{x}\colon \underline{a}]]$
$\oplus [[\forall\underline{c}\colon\forall\underline{a}\colon\forall\underline{x}\colon\forall\underline{b}\colon [\lceil\underline{a}\rceil \equiv \langle\lceil\underline{b}\rceil | \lceil\underline{x}\rceil := \lceil\underline{c}\rceil\rangle \Vdash [[\dot\forall\underline{x}\colon \underline{b}] \dot\Rightarrow \underline{a}]]]] \oplus [[$
$\forall\underline{h}\colon\forall\underline{t}\colon [\underline{h} \Rightarrow [[\underline{t} \dot{+} \dot{0}] \stackrel{p}{=} \underline{t}]]]] \oplus [[\forall\underline{a}\colon [[\underline{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]]] \oplus [[$
$\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot\Rightarrow [[\underline{a} \stackrel{p}{=} \underline{c}] \dot\Rightarrow [\underline{b} \stackrel{p}{=} \underline{c}]]]]] \oplus [[\forall\underline{t}\colon [\underline{t} \stackrel{p}{=} \underline{t}]]$
$\oplus [[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon\forall\underline{x}\colon [\underline{b} \equiv \langle\underline{a}|\underline{x} := \dot{0}\rangle \Vdash [\underline{c} \equiv \langle\underline{a}|\underline{x} := \underline{x}'\rangle \Vdash [\underline{b} \dot\Rightarrow [[\dot\forall\underline{x}\colon [\underline{a} \dot\Rightarrow \underline{c}]$
$] \dot\Rightarrow \dot\forall\underline{x}\colon \underline{a}]]]]] \oplus [[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [[\underline{a} \dot\Rightarrow [\underline{b} \dot\Rightarrow \underline{c}]] \dot\Rightarrow [[\underline{a} \dot\Rightarrow \underline{b}] \dot\Rightarrow [$

$\underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]\ ]\ \oplus\ [\ [\ \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon\ [\ [\ \underline{h} \dot{\Rightarrow}\ [\ \underline{t} \stackrel{p}{=} \underline{r}\ ]\ ]\ \vdash\ [\ [\ \underline{h} \dot{\Rightarrow}\ [\ \underline{r} \stackrel{p}{=} \underline{s}\ ]\ ]\ \vdash$
$[\ \underline{h} \dot{\Rightarrow}\ [\ \underline{t} \stackrel{p}{=} \underline{s}\ ]\ ]\ ]\ ]\ ]\ \oplus \forall \underline{a}\colon\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]$
$]\ ]\ ]\ ]\ ]\ ]$

[S$'$ $\stackrel{\text{tex}}{\to}$ "
S'"]

[S$'$ $\stackrel{\text{pyk}}{\to}$ "system prime s"]

A1$'$

[A1$'$ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A1$'$ $\stackrel{\text{stmt}}{\to}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon\ [\ \underline{a} \dot{\Rightarrow}\ [\ \underline{b} \dot{\Rightarrow} \underline{a}\ ]\ ]$]

[A1$'$ $\stackrel{\text{tex}}{\to}$ "
A1'"]

[A1$'$ $\stackrel{\text{pyk}}{\to}$ "axiom prime a one"]

A2$'$

[A2$'$ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A2$'$ $\stackrel{\text{stmt}}{\to}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon\ [\ [\ \underline{a} \dot{\Rightarrow}\ [\ \underline{b} \dot{\Rightarrow} \underline{c}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ]\ ]$]

[A2$'$ $\stackrel{\text{tex}}{\to}$ "
A2'"]

[A2$'$ $\stackrel{\text{pyk}}{\to}$ "axiom prime a two"]

A3$'$

[A3$'$ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A3$'$ $\stackrel{\text{stmt}}{\to}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon\ [\ [\ [\ \dot{\neg} \underline{b}\ ]\ \dot{\Rightarrow} \dot{\neg} \underline{a}\ ]\ \dot{\Rightarrow}\ [\ [\ [\ \dot{\neg} \underline{b}\ ]\ \dot{\Rightarrow} \underline{a}\ ]\ \dot{\Rightarrow} \underline{b}\ ]\ ]$]

[A3$'$ $\stackrel{\text{tex}}{\to}$ "
A3'"]

[A3$'$ $\stackrel{\text{pyk}}{\to}$ "axiom prime a three"]

A4$'$

[A4$'$ $\stackrel{\text{proof}}{\to}$ Rule tactic]

[A4′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon$ [$\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash$ [[$\dot{\forall} \underline{x}\colon \underline{b}$] $\dot{\Rightarrow} \underline{a}$]]]

[A4′ $\stackrel{\text{tex}}{\rightarrow}$ "
A4′"]

[A4′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a four"]

A5′

[A5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon$ [$\dot{\text{nonfree}}(\lceil \underline{x} \rceil, \lceil \underline{a} \rceil) \Vdash$ [[$\dot{\forall} \underline{x}\colon$ [$\underline{a} \dot{\Rightarrow} \underline{b}$]] $\dot{\Rightarrow}$ [
$\underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}$]]]]

[A5′ $\stackrel{\text{tex}}{\rightarrow}$ "
A5′"]

[A5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a five"]

S1′

[S1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[$\underline{a} \stackrel{\text{p}}{=} \underline{b}$] $\dot{\Rightarrow}$ [[$\underline{a} \stackrel{\text{p}}{=} \underline{c}$] $\dot{\Rightarrow}$ [$\underline{b} \stackrel{\text{p}}{=} \underline{c}$]]]]

[S1′ $\stackrel{\text{tex}}{\rightarrow}$ "
S1′"]

[S1′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s one"]

S2′

[S2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon$ [[$\underline{a} \stackrel{\text{p}}{=} \underline{b}$] $\dot{\Rightarrow}$ [$\underline{a}' \stackrel{\text{p}}{=}$ [$\underline{b}'$]]]]

[S2′ $\stackrel{\text{tex}}{\rightarrow}$ "
S2′"]

[S2′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s two"]

S3′

[S3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S3' \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \neg\, [\, \dot{0} \overset{p}{=} [\, \underline{a}' \,]\,]\,]$

[S3′ $\overset{\mathrm{tex}}{\rightarrow}$ "
S3′"]

[S3′ $\overset{\mathrm{pyk}}{\rightarrow}$ "axiom prime s three"]

S4′

[S4′ $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[S4' \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, \underline{a}' \overset{p}{=} [\, \underline{b}' \,]\,] \dot{\Rightarrow} [\, \underline{a} \overset{p}{=} \underline{b} \,]\,]\,]$

[S4′ $\overset{\mathrm{tex}}{\rightarrow}$ "
S4′"]

[S4′ $\overset{\mathrm{pyk}}{\rightarrow}$ "axiom prime s four"]

S5′

[S5′ $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[S5' \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon [\, [\, \underline{a} \dot{+} \dot{0} \,] \overset{p}{=} \underline{a} \,]\,]$

[S5′ $\overset{\mathrm{tex}}{\rightarrow}$ "
S5′"]

[S5′ $\overset{\mathrm{pyk}}{\rightarrow}$ "axiom prime s five"]

S6′

[S6′ $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[S6' \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, \underline{a} \dot{+} [\, \underline{b}' \,]\,] \overset{p}{=} [\, [\, \underline{a} \dot{+} \underline{b} \,]\, ' \,]\,]\,]$

[S6′ $\overset{\mathrm{tex}}{\rightarrow}$ "
S6′"]

[S6′ $\overset{\mathrm{pyk}}{\rightarrow}$ "axiom prime s six"]

S7′

[S7′ $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[S7' \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a}\colon [\, [\, \underline{a} \,\dot{\cdot}\, \dot{0} \,] \overset{p}{=} \dot{0} \,]\,]$

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7'"]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\ \underline{a} \mathrel{\dot{\cdot}} [\ \underline{b}'\ ]\ ]\ \stackrel{p}{=}\ [\ [\ \underline{a} \mathrel{\dot{\cdot}} \underline{b}\ ] \mathrel{\dot{+}} \underline{a}\ ]\ ]$]

[S8′ $\stackrel{\text{tex}}{\rightarrow}$ "
S8'"]

[S8′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon \forall \underline{x} \colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \mathrel{\dot{\Rightarrow}} [\ [\ \dot{\forall} \underline{x} \colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ]\ \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x} \colon \underline{a}\ ]\ ]\ ]\ ]$]

[S9′ $\stackrel{\text{tex}}{\rightarrow}$ "
S9'"]

[S9′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP′ $\stackrel{\text{tex}}{\rightarrow}$ "
MP'"]

[MP′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x} \colon \forall \underline{a} \colon [\ \underline{a} \vdash \dot{\forall} \underline{x} \colon \underline{a}\ ]$]

$[\mathrm{Gen}' \stackrel{\mathrm{tex}}{\to}$ “
Gen’”]

$[\mathrm{Gen}' \stackrel{\mathrm{pyk}}{\to}$ “rule prime gen”]

L3.2(a)′

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon [\ [\ \mathrm{S5}' \gg [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ;\ [\ [\ \mathrm{S1}' \gg [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ \rhd [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ] \gg [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ \rhd [\ [\ \underline{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ] \gg [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{a}}\colon [\ \underline{\mathsf{a}} \stackrel{\mathrm{p}}{=} \underline{\mathsf{a}}\ ]]$

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{tex}}{\to}$ “
L3.2(a)’”]

$[\mathrm{L3.2(a)}' \stackrel{\mathrm{pyk}}{\to}$ “lemma prime l three two a”]

M1.7

$[\mathrm{M1.7} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{b}}\colon [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{A1}' \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ] \gg [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{M1.7} \stackrel{\mathrm{stmt}}{\to} \mathrm{S}' \vdash \forall \underline{\mathsf{b}}\colon [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]]$

$[\mathrm{M1.7} \stackrel{\mathrm{tex}}{\to}$ “
M1.7”]

$[\mathrm{M1.7} \stackrel{\mathrm{pyk}}{\to}$ “mendelson one seven”]

$\mathrm{MP}'_{\mathrm{h}}$

$[\mathrm{MP}'_{\mathrm{h}} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{\mathsf{h}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \vdash [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ] \vdash [\ [\ \mathrm{A2}' \gg [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ \rhd [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ] \gg [\ \underline{\mathsf{h}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MP}'_h \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ [\underline{h} \Rightarrow [\underline{a} \Rightarrow \underline{b}]\ ] \vdash [\ [\underline{h} \Rightarrow \underline{a}] \vdash [\underline{h} \Rightarrow \underline{b}]\ ]\ ]]$

$[\mathrm{MP}'_h \overset{\mathrm{tex}}{\rightarrow}$ "
MP'_h"]

$[\mathrm{MP}'_h \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical rule prime mp"]

Hypothesize

$[\mathrm{Hypothesize} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon [\underline{a} \vdash [\ [\mathrm{A1}' \gg [\underline{a} \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \underline{a}]\ ]\ ]; [\ [\ [\mathrm{MP}' \rhd [\underline{a} \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \underline{a}]\ ]\ ] \rhd \underline{a}] \gg [\underline{h} \dot{\Rightarrow} \underline{a}]\ ]\ ]\ ]\ ]\rceil, p_0, c)]$

$[\mathrm{Hypothesize} \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{h}\colon \forall \underline{a}\colon [\underline{a} \vdash [\underline{h} \dot{\Rightarrow} \underline{a}]\ ]]$

$[\mathrm{Hypothesize} \overset{\mathrm{tex}}{\rightarrow}$ "
Hypothesize"]

$[\mathrm{Hypothesize} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothesize"]

Gen'_h

$[\mathrm{Gen}'_h \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{h}\colon \forall \underline{x}\colon \forall \underline{a}\colon [\dot{\mathrm{nonfree}}(\lceil \underline{x}\rceil, \lceil \underline{h}\rceil) \Vdash [\ [\underline{h} \dot{\Rightarrow} \underline{a}] \vdash [\ [\ [\mathrm{A5}' \unrhd \dot{\mathrm{nonfree}}(\lceil \underline{x}\rceil, \lceil \underline{h}\rceil)] \gg [\ [\dot{\forall}\underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]\ ] \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{a}]\ ]\ ]; [\ [\ [\mathrm{Gen}' \rhd [\underline{h} \dot{\Rightarrow} \underline{a}]\ ] \gg \dot{\forall}\underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]\ ]; [\ [\ [\mathrm{MP}' \rhd [\ [\dot{\forall}\underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]\ ] \dot{\Rightarrow} [\underline{h} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{a}]\ ]\ ] \rhd \dot{\forall}\underline{x}\colon [\underline{h} \dot{\Rightarrow} \underline{a}]\ ] \gg [\underline{h} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{a}]\ ]\ ]\ ]\ ]\ ]\ ]\rceil, p_0, c)]$

$[\mathrm{Gen}'_h \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{h}\colon \forall \underline{x}\colon \forall \underline{a}\colon [\dot{\mathrm{nonfree}}(\lceil \underline{x}\rceil, \lceil \underline{h}\rceil) \Vdash [\ [\underline{h} \dot{\Rightarrow} \underline{a}] \vdash [\underline{h} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{a}]\ ]\ ]\ ]$

$[\mathrm{Gen}'_h \overset{\mathrm{tex}}{\rightarrow}$ "
Gen'_h"]

$[\mathrm{Gen}'_h \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical rule prime gen"]

M3.2(a)

$[\mathrm{M3.2(a)} \overset{\mathrm{proof}}{\rightarrow} \mathrm{Rule\ tactic}]$

$[\mathrm{M3.2(a)} \overset{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{t}\colon [\underline{t} \overset{p}{=} \underline{t}]\ ]$

$[\mathrm{M3.2(a)} \overset{\mathrm{tex}}{\rightarrow}$ "
M3.2(a)"]

$[\mathrm{M3.2(a)} \overset{\mathrm{pyk}}{\rightarrow}$ "mendelson three two a"]

$\mathrm{M3.2(a)_h}$

$[\mathrm{M3.2(a)_h} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ [\ \mathrm{M3.2(a)} \gg [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ;\ [\ [\ \mathrm{Hypothesize} \rhd [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, c)]$

$[\mathrm{M3.2(a)_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]$

$[\mathrm{M3.2(a)_h} \overset{\mathrm{tex}}{\rightarrow}$ “
M3.2(a)_h”]

$[\mathrm{M3.2(a)_h} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical three two a”]

$\mathrm{M3.2(b)_h}$

$[\mathrm{M3.2(b)_h} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ] \vdash [\ [\ \mathrm{S1}' \gg [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Hypothesize} \rhd [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}'_{\mathrm{h}} \rhd [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ] \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{M3.2(a)_h} \gg [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}'_{\mathrm{h}} \rhd [\ \underline{h} \dot{\Rightarrow} [\ [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ] \rhd [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ] \gg [\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, c)]$

$[\mathrm{M3.2(b)_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ] \vdash [\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{t}\ ]\ ]\ ]\ ]$

$[\mathrm{M3.2(b)_h} \overset{\mathrm{tex}}{\rightarrow}$ “
M3.2(b)_h”]

$[\mathrm{M3.2(b)_h} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical three two b”]

$\mathrm{M3.1(S1')_h}$

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{r}\ ]\ ] \vdash [\ [\ \underline{h} \dot{\Rightarrow} [\ \underline{t} \overset{\mathrm{p}}{=} \underline{s}\ ]\ ] \vdash [\ \underline{h} \dot{\Rightarrow} [\ \underline{r} \overset{\mathrm{p}}{=} \underline{s}\ ]\ ]\ ]\ ]\ ]$

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{tex}}{\rightarrow}$ “
M3.1(S1’)_h”]

$[\mathrm{M3.1(S1')_h} \overset{\mathrm{pyk}}{\rightarrow}$ “hypothetical three one s one”]

$\mathrm{M3.2(c)_h}$

[$\mathrm{M3.2(c)_h} \overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.2(c)_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon \forall \underline{s}\colon [\ [\underline{h} \dot{\Rightarrow} [\underline{t} \overset{p}{=} \underline{r}]\ ] \vdash [\ [\underline{h} \dot{\Rightarrow} [\underline{r} \overset{p}{=} \underline{s}]\ ] \vdash [\underline{h} \dot{\Rightarrow} [\underline{t} \overset{p}{=} \underline{s}]\ ]\ ]\ ]$]

[$\mathrm{M3.2(c)_h} \overset{\mathrm{tex}}{\rightarrow}$ "
M3.2(c)_h"]

[$\mathrm{M3.2(c)_h} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical three two c"]

$\mathrm{M3.1(S2')_h}$

[$\mathrm{M3.1(S2')_h} \overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.1(S2')_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\ [\underline{h} \dot{\Rightarrow} [\underline{t} \overset{p}{=} \underline{r}]\ ] \vdash [\underline{h} \dot{\Rightarrow} [\underline{t}' \overset{p}{=} [\underline{r}']\ ]\ ]\ ]$]

[$\mathrm{M3.1(S2')_h} \overset{\mathrm{tex}}{\rightarrow}$ "
M3.1(S2')_h"]

[$\mathrm{M3.1(S2')_h} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical three one s two"]

$\mathrm{M3.1(S5')_h}$

[$\mathrm{M3.1(S5')_h} \overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.1(S5')_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon [\underline{h} \dot{\Rightarrow} [\ [\underline{t} \dot{+} \dot{0}] \overset{p}{=} \underline{t}]\ ]$]

[$\mathrm{M3.1(S5')_h} \overset{\mathrm{tex}}{\rightarrow}$ "
M3.1(S5')_h"]

[$\mathrm{M3.1(S5')_h} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical three one s five"]

$\mathrm{M3.1(S6')_h}$

[$\mathrm{M3.1(S6')_h} \overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{M3.1(S6')_h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{h}\colon \forall \underline{t}\colon \forall \underline{r}\colon [\underline{h} \dot{\Rightarrow} [\ [\underline{t} \dot{+} [\underline{r}']\ ] \overset{p}{=} [\ [\underline{t} \dot{+} \underline{r}]'\ ]\ ]\ ]$]

[$\mathrm{M3.1(S6')_h} \overset{\mathrm{tex}}{\rightarrow}$ "
M3.1(S6')_h"]

[$\mathrm{M3.1(S6')_h} \overset{\mathrm{pyk}}{\rightarrow}$ "hypothetical three one s six"]

M3.2(f)

[M3.2(f) $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [[A1' \gg [x \dot{\Rightarrow} [x \dot{\Rightarrow} x]]] ; [[M3.1(S5')_h \gg$
$[[x \dot{\Rightarrow} [x \dot{\Rightarrow} x]] \dot{\Rightarrow} [[\dot{0} \dot{+} \dot{0}] \overset{p}{=} \dot{0}]]] ; [[[M3.2(b)_h \rhd [[x \dot{\Rightarrow} [$
$x \dot{\Rightarrow} x]] \dot{\Rightarrow} [[\dot{0} \dot{+} \dot{0}] \overset{p}{=} \dot{0}]]] \gg [[x \dot{\Rightarrow} [x \dot{\Rightarrow} x]] \dot{\Rightarrow} [\dot{0} \overset{p}{=} [\dot{0} \dot{+} \dot{0}]$
$]]] ; [[[[MP' \rhd [[x \dot{\Rightarrow} [x \dot{\Rightarrow} x]] \dot{\Rightarrow} [\dot{0} \overset{p}{=} [\dot{0} \dot{+} \dot{0}]]]]] \rhd [x \dot{\Rightarrow} [$
$x \dot{\Rightarrow} x]]] \gg [\dot{0} \overset{p}{=} [\dot{0} \dot{+} \dot{0}]]] ; [[M1.7 \gg [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [$
$\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]] ; [[[M3.1(S2')_h \rhd [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t} \overset{p}{=} [$
$\dot{0} \dot{+} [\dot{t}]]]]]] \gg [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]] ;$
$[[M3.1(S6')_h \gg [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}']] \overset{p}{=} [[\dot{0} \dot{+} [\dot{t}]] '$
$]]]] ; [[[M3.2(b)_h \rhd [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}']] \overset{p}{=} [[$
$\dot{0} \dot{+} [\dot{t}]] ']]]] \gg [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}]] ' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']$
$]]]] ; [[[[M3.2(c)_h \rhd [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']$
$]]] \rhd [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}]] ' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] \gg [$
$[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]] ; [[[Gen' \rhd [[\dot{t} \overset{p}{=} [\dot{0} \dot{+}$
$[\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] \gg \dot{\forall} t\colon [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=}$
$[\dot{0} \dot{+} [\dot{t}']]]]]] ; [[S9' \gg [[\dot{0} \overset{p}{=} [\dot{0} \dot{+} \dot{0}]] \dot{\Rightarrow} [[\dot{\forall} t\colon [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}$
$]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]] \dot{\Rightarrow} \dot{\forall} t\colon [\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]]] ; [[[[$
$MP' \rhd [[\dot{0} \overset{p}{=} [\dot{0} \dot{+} \dot{0}]] \dot{\Rightarrow} [[\dot{\forall} t\colon [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [$
$\dot{t}']]]]] \dot{\Rightarrow} \dot{\forall} t\colon [\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]]] \rhd [\dot{0} \overset{p}{=} [\dot{0} \dot{+} \dot{0}]]] \gg [[\dot{\forall} t\colon$
$[[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]] \dot{\Rightarrow} \dot{\forall} t\colon [\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]$
$]] ; [[[MP' \rhd [[\dot{\forall} t\colon [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]]$
$\dot{\Rightarrow} \dot{\forall} t\colon [\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]] \rhd \dot{\forall} t\colon [[\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \overset{p}{=} [\dot{0} \dot{+} [$
$\dot{t}']]]]]] \gg \dot{\forall} t\colon [\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$

[M3.2(f) $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \dot{\forall} t\colon [\dot{t} \overset{p}{=} [\dot{0} \dot{+} [\dot{t}]]]$]

[M3.2(f) $\overset{\text{tex}}{\rightarrow}$ "
M3.2(f)"]

[M3.2(f) $\overset{\text{pyk}}{\rightarrow}$ "mendelson three two f"]

$\dot{*}$

[$\dot{x}$ $\overset{\text{tex}}{\rightarrow}$ "
\dot{#1.
}"]

[$\dot{x}$ $\overset{\text{pyk}}{\rightarrow}$ "* peano var"]

$*'$

[$\mathsf{x}' \stackrel{\text{tex}}{\rightarrow}$ "#1.'"]

[$\mathsf{x}' \stackrel{\text{pyk}}{\rightarrow}$ "* peano succ"]

$* \mathop{\dot{\cdot}} *$

[$\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{\cdot}} #2."]

[$\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano times *"]

$* \mathop{\dot{+}} *$

[$\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{+}} #2."]

[$\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano plus *"]

$* \stackrel{p}{=} *$

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano is *"]

$*^{\mathcal{P}}$

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil$]

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{pyk}}{\rightarrow}$ "* is peano var"]

$\dot{\neg} *$

[$\dot{\neg} \mathsf{x} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\neg}\, {#1.}"]

[$\dot{\neg} \mathsf{x} \stackrel{\text{pyk}}{\rightarrow}$ "peano not *"]

$* \mathrel{\dot{\wedge}} *$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \doteq \dot{\neg}(\mathsf{x} \mathrel{\dot{\Rightarrow}} \dot{\neg}\mathsf{y})]\rceil)]$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\wedge}} #2."]

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "* peano and *"]

$* \mathrel{\dot{\vee}} *$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \doteq [\dot{\neg}\mathsf{x}] \mathrel{\dot{\Rightarrow}} \mathsf{y}]\rceil)]$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\vee}} #2."]

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "* peano or *"]

$\dot{\forall} *\colon *$

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "peano all * indeed *"]

$\dot{\exists} *\colon *$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{\exists}\mathsf{x}\colon \mathsf{y} \doteq \dot{\neg}\dot{\forall}\mathsf{x}\colon \dot{\neg}\mathsf{y}]\rceil)]$

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\exists} #1.
\colon #2."]

$[\dot{\exists}\mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "peano exist * indeed *"]

$* \mathrel{\dot{\Rightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "* peano imply *"]

$* \dot{\Leftrightarrow} *$

$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \ddot{=} (\mathsf{x} \dot{\Rightarrow} \mathsf{y}) \dot{\wedge} (\mathsf{y} \dot{\Rightarrow} \mathsf{x})] \rceil)]$

$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* peano iff *"]

$* \unrhd *$

$[\mathsf{x} \unrhd \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \unrhd \mathsf{y} \ddot{=} [\ \mathrm{MP}' \rhd \mathsf{x}\ ] \rhd \mathsf{y}] \rceil)]$

$[\mathsf{x} \unrhd \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\unrhd #2."]

$[\mathsf{x} \unrhd \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* macro modus ponens *"]

$* \unrhd_{\mathrm{h}} *$

$[\mathsf{x} \unrhd_{\mathrm{h}} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \unrhd_{\mathrm{h}} \mathsf{y} \ddot{=} [\ \mathrm{MP}'_{\mathrm{h}} \rhd \mathsf{x}\ ] \rhd \mathsf{y}] \rceil)]$

$[\mathsf{x} \unrhd_{\mathrm{h}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\unrhd_h #2."]

$[\mathsf{x} \unrhd_{\mathrm{h}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "* hypothetical modus ponens *"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-22.UTC:11:16:56.809226 = MJD-53543.TAI:11:17:28.809226 = LGT-4626155848809226e-6