Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, $\dot{\text{nonfree}}(*, *)$, $\dot{\text{nonfree}}^*(*, *)$, $\dot{\text{free}}\langle *|* := *\rangle$, $\dot{\text{free}}^*\langle *|* := *\rangle$, $*\dot{\equiv}\langle *|* := *\rangle$, $*\dot{\equiv}\langle^* *|* := *\rangle$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, L3.2(a)′, $\dot{*}$, $*'$, $* \dot{\cdot} *$, $* \dot{+} *$, $* \stackrel{\text{p}}{=} *$, $*^{\mathcal{P}}$, $\dot{\neg} *$, $* \dot{\wedge} *$, $* \dot{\vee} *$, $\dot{\forall} *\colon *$, $\dot{\exists} *\colon *$, $* \dot{\Rightarrow} *$, $* \dot{\Leftrightarrow} *$,

peano

[peano $\stackrel{\text{prio}}{\rightarrow}$
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\rightarrow} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if($*, *, *$)], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\perp$], [f($*$)], [$(*)^{\text{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\text{M}}$], [If($*, *, *$)], [array{$*$} $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\text{M}}(*)$], [**apply**($*, *$)], [**apply**$_1$($*, *$)], [identifier($*$)], [identifier$_1$($*, *$)], [array-plus($*, *$)], [array-remove($*, *, *$)], [array-put($*, *, *, *$)], [array-add($*, *, *, *, *$)], [bit($*, *$)], [bit$_1$($*, *$)], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**($*, *, *$)], [**abstract**($*, *, *, *$)], [[$\lceil * \rceil$]], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [$\text{s}_0$], [**zip**($*, *$)], [**assoc**$_1$($*, *, *$)], [$(*)^{\mathbf{P}}$], [self], [[$* \stackrel{..}{=} *$]], [[$* \doteq *$]], [[$* \stackrel{\prime}{=} *$]], [[$* \stackrel{\text{pyk}}{=} *$]], [[$* \stackrel{\text{tex}}{=} *$]], [[$* \stackrel{\text{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [$(*)$], [**aspect**($*, *$)], [**aspect**($*, *, *$)], [$\langle * \rangle$], [**tuple**$_1$($*$)], [**tuple**$_2$($*$)], [let$_2$($*, *$)], [let$_1$($*, *$)], [[$* \stackrel{\text{claim}}{=} *$]], [checker], [**check**($*, *$)], [**check**$_2$($*, *, *$)], [**check**$_3$($*, *, *$)], [**check***($*, *$)], [**check**$^*_2$($*, *, *$)], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \stackrel{\text{msg}}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\text{stmt}}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\perp\!\!\!\perp$], [Contra′], [T'_{E}], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathbf{V}}$], [intro($*, *, *, *$)], [intro($*, *, *$)], [error($*, *$)], [error$_2$($*, *$)], [proof($*, *, *$)],

[proof$_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\mathrm{I}}(*,*)$], [$\mathcal{S}^{\rhd}(*,*)$], [$\mathcal{S}_1^{\rhd}(*,*,*)$], [$\mathcal{S}^{\mathrm{E}}(*,*)$], [$\mathcal{S}_1^{\mathrm{E}}(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}_1^{+}(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}_1^{-}(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}_1^{*}(*,*,*)$], [$\mathcal{S}_2^{*}(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}_1^{@}(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}_1^{\vdash}(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}_1^{\Vdash}(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}(*,*)$], [$\mathcal{S}_1^{\mathrm{i.e.}}(*,*,*,*)$], [$\mathcal{S}_2^{\mathrm{i.e.}}(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}_1^{\forall}(*,*,*,*)$], [$\mathcal{S}^{;}(*,*)$], [$\mathcal{S}_1^{;}(*,*,*)$], [$\mathcal{S}_2^{;}(*,*,*,*)$], [$\mathcal{T}(*)$], [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** ∗: ∗]], [[**Proof of** ∗: ∗]], [[∗ **lemma** ∗: ∗]], [[∗ **antilemma** ∗: ∗]], [[∗ **rule** ∗: ∗]], [[∗ **antirule** ∗: ∗]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut$(*,*)$], [Head$_\oplus(*)$], [Tail$_\oplus(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** ∗]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [$\mathrm{T_E}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(*=*,*)$], [unify$^*(*=*,*)$], [unify$_2(*=*,*)$], [$\mathrm{L_a}$], [$\mathrm{L_b}$], [$\mathrm{L_c}$], [$\mathrm{L_d}$], [$\mathrm{L_e}$], [$\mathrm{L_f}$], [$\mathrm{L_g}$], [$\mathrm{L_h}$], [$\mathrm{L_i}$], [$\mathrm{L_j}$], [$\mathrm{L_k}$], [$\mathrm{L_l}$], [$\mathrm{L_m}$], [$\mathrm{L_n}$], [$\mathrm{L_o}$], [$\mathrm{L_p}$], [$\mathrm{L_q}$], [$\mathrm{L_r}$], [$\mathrm{L_s}$], [$\mathrm{L_t}$], [$\mathrm{L_u}$], [$\mathrm{L_v}$], [$\mathrm{L_w}$], [$\mathrm{L_x}$], [$\mathrm{L_y}$], [$\mathrm{L_z}$], [$\mathrm{L_A}$], [$\mathrm{L_B}$], [$\mathrm{L_C}$], [$\mathrm{L_D}$], [$\mathrm{L_E}$], [$\mathrm{L_F}$], [$\mathrm{L_G}$], [$\mathrm{L_H}$], [$\mathrm{L_I}$], [$\mathrm{L_J}$], [$\mathrm{L_K}$], [$\mathrm{L_L}$], [$\mathrm{L_M}$], [$\mathrm{L_N}$], [$\mathrm{L_O}$], [$\mathrm{L_P}$], [$\mathrm{L_Q}$], [$\mathrm{L_R}$], [$\mathrm{L_S}$], [$\mathrm{L_T}$], [$\mathrm{L_U}$], [$\mathrm{L_V}$], [$\mathrm{L_W}$], [$\mathrm{L_X}$], [$\mathrm{L_Y}$], [$\mathrm{L_Z}$], [$\mathrm{L_?}$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \stackrel{\text{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$\mathrm{p_0}$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [$\dot{\text{nonfree}}(*,*)$], [$\dot{\text{nonfree}}^*(*,*)$], [$\dot{\text{free}}\langle *|* := *\rangle$], [free$^*\langle *|* := *\rangle$], [$*\equiv\langle *|* := *\rangle$], [$*\equiv\langle^* *|* := *\rangle$], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$], [L3.2(a)$'$];

Preassociative

[∗_{∗}], [∗′], [∗[∗]], [∗[∗→∗]], [∗[∗⇒∗]], [$\dot{*}$];

Preassociative

[“ ∗ ”], [], [$(*)^{\mathbf{t}}$], [string(∗) + ∗], [string(∗) ++ ∗], [
∗], [∗], [!∗], [”∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗], [0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗], [@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗], [O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [^∗], [_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗], [p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [~∗], [**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end], [newline ∗], [macro newline ∗];

Preassociative

[∗0], [∗1], [0b], [∗-color(∗)], [∗-color*(∗)];

Preassociative

[∗ ’ ∗], [∗ ‘ ∗];

Preassociative

[$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^{0}$], [$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*'$];

Preassociative
$[* \cdot *], [* \cdot_0 *], [* \mathrel{\dot{:}} *]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*, *]$;
Preassociative
$[* \stackrel{\mathrm{B}}{\approx} *], [* \stackrel{\mathrm{D}}{\approx} *], [* \stackrel{\mathrm{C}}{\approx} *], [* \stackrel{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\rightarrow} *], [* \stackrel{\mathrm{t}}{=} *], [* \stackrel{\mathrm{t}^*}{=} *], [* \stackrel{\mathrm{r}}{=} *]$,
$[* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \stackrel{\mathrm{T}}{=} *], [* \stackrel{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *]$,
$[* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* \stackrel{\mathrm{p}}{=} *], [*^{\mathcal{P}}]$;
Preassociative
$[\neg *], [\dot{\neg} *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\mathrm{c}} *], [* \dot{\wedge} *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *]$;
Preassociative
$[\dot{\forall} * \colon *], [\dot{\exists} * \colon *]$;
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *]$;
Postassociative
$[* : *], [* ! *]$;
Preassociative
$[* \begin{cases} * \\ * \end{cases}]$;
Preassociative
$[\lambda * . *], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *]$;
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}]$;
Preassociative
$[* @ *], [* \triangleright *], [* \mathrel{\triangleright\!\!\triangleright} *], [* \gg *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall * \colon *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
$[* \text{ proves } *]$;

Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \square$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$];
Postassociative
[$*$ then $*$], [$*[*]*$];
Preassociative
[$*\&*$];
Preassociative
[$*\backslash\backslash*$];]

[peano $\stackrel{\text{pyk}}{\rightarrow}$ "peano"]

$\dot{0}$

[$\dot{0} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{0}"]

[$\dot{0} \stackrel{\text{pyk}}{\rightarrow}$ "peano zero"]

$\dot{1}$

[$\dot{1} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{1} \ddot{=} \dot{0}']\rceil)$]

[$\dot{1} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{1}"]

[$\dot{1} \stackrel{\text{pyk}}{\rightarrow}$ "peano one"]

$\dot{2}$

[$\dot{2} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{2} \ddot{=} \dot{1}']\rceil)$]

[$\dot{2} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{2}"]

[$\dot{2} \stackrel{\text{pyk}}{\rightarrow}$ "peano two"]

$\dot{a}$

[$\dot{a} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{a} \ddot{=} \dot{\mathsf{a}}]\rceil)$]

$[\dot{a} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{a}}"]

$[\dot{a} \stackrel{\text{pyk}}{\rightarrow}$ "peano a"]

$\dot{b}$

$[\dot{b} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{b}}"]

$[\dot{b} \stackrel{\text{pyk}}{\rightarrow}$ "peano b"]

$\dot{c}$

$[\dot{c} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{c}}"]

$[\dot{c} \stackrel{\text{pyk}}{\rightarrow}$ "peano c"]

$\dot{d}$

$[\dot{d} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{d}}"]

$[\dot{d} \stackrel{\text{pyk}}{\rightarrow}$ "peano d"]

$\dot{e}$

$[\dot{e} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{e}}"]

$[\dot{e} \stackrel{\text{pyk}}{\rightarrow}$ "peano e"]

$\dot{f}$

$[\dot{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \ddot{=} \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\to}$ "peano f"]

$\dot{g}$

$[\dot{g} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\to}$ "peano g"]

$\dot{h}$

$[\dot{h} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\to}$ "peano h"]

$\dot{i}$

$[\dot{i} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\to}$ "peano i"]

$\dot{j}$

$[\dot{j} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\to}$ "peano j"]

$\dot{k}$

$[\dot{k} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{k} \ddot{=} \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{k}}"]

$[\dot{k} \stackrel{\text{pyk}}{\rightarrow}$ "peano k"]

$\dot{l}$

$[\dot{l} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{l} \ddot{=} \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{l}}"]

$[\dot{l} \stackrel{\text{pyk}}{\rightarrow}$ "peano l"]

$\dot{m}$

$[\dot{m} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{m} \ddot{=} \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{m}}"]

$[\dot{m} \stackrel{\text{pyk}}{\rightarrow}$ "peano m"]

$\dot{n}$

$[\dot{n} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{n} \ddot{=} \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{n}}"]

$[\dot{n} \stackrel{\text{pyk}}{\rightarrow}$ "peano n"]

$\dot{o}$

$[\dot{o} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{o} \ddot{=} \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{o}}"]

$[\dot{o} \stackrel{\text{pyk}}{\rightarrow}$ "peano o"]

$\dot{p}$

$[\dot{p} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{p} \doteq \dot{\mathsf{p}}]\rceil)]$

$[\dot{p} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{p}}"]

$[\dot{p} \stackrel{\text{pyk}}{\to}$ "peano p"]

$\dot{q}$

$[\dot{q} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{q} \doteq \dot{\mathsf{q}}]\rceil)]$

$[\dot{q} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{q}}"]

$[\dot{q} \stackrel{\text{pyk}}{\to}$ "peano q"]

$\dot{r}$

$[\dot{r} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{r} \doteq \dot{\mathsf{r}}]\rceil)]$

$[\dot{r} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{r}}"]

$[\dot{r} \stackrel{\text{pyk}}{\to}$ "peano r"]

$\dot{s}$

$[\dot{s} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{s} \doteq \dot{\mathsf{s}}]\rceil)]$

$[\dot{s} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{s}}"]

$[\dot{s} \stackrel{\text{pyk}}{\to}$ "peano s"]

$\dot{t}$

$[\dot{t} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{t} \doteq \dot{\mathsf{t}}]\rceil)]$

$[\dot{t} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{t}}"]

$[\dot{t} \stackrel{\text{pyk}}{\to}$ "peano t"]

$\dot{u}$

$[\dot{u} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{u} \ddot{=} \dot{\mathsf{u}}]\rceil)]$

$[\dot{u} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{u}}"]

$[\dot{u} \overset{\text{pyk}}{\rightarrow}$ "peano u"]

$\dot{v}$

$[\dot{v} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{v} \ddot{=} \dot{\mathsf{v}}]\rceil)]$

$[\dot{v} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{v}}"]

$[\dot{v} \overset{\text{pyk}}{\rightarrow}$ "peano v"]

$\dot{w}$

$[\dot{w} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{w} \ddot{=} \dot{\mathsf{w}}]\rceil)]$

$[\dot{w} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{w}}"]

$[\dot{w} \overset{\text{pyk}}{\rightarrow}$ "peano w"]

$\dot{x}$

$[\dot{x} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{x} \ddot{=} \dot{\mathsf{x}}]\rceil)]$

$[\dot{x} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{x}}"]

$[\dot{x} \overset{\text{pyk}}{\rightarrow}$ "peano x"]

$\dot{y}$

$[\dot{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{y} \ddot{=} \dot{\mathsf{y}}]\rceil)]$

$[\dot{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{y}}"]

$[\dot{y} \overset{\text{pyk}}{\rightarrow}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"]

$\dot{\text{nonfree}}(*,*)$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg [\mathsf{x} \stackrel{\text{t}}{=} \mathsf{y}],$
$\text{If}(\neg [\mathsf{y} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{x}\colon \mathsf{y} \rceil], \dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=} [\mathsf{y}^1], \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^2))))]$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree * in * end nonfree"]

$\dot{\text{nonfree}}^*(*,*)$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y},\mathsf{T},\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}),\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),\mathsf{F}))]$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree star * in * end nonfree"]

$\dot{\text{free}}\langle *|* := *\rangle$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{val}}{\to} \mathsf{x}! [\mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg [\mathsf{a} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil], \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$
$\text{If}(\mathsf{a}^1 \stackrel{\text{t}}{=} \mathsf{x}, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^2), \mathsf{T},$

$\mathrm{If}(\neg \dot{\mathrm{nonfree}}(\mathsf{a}^1, \mathsf{b}), \mathsf{F},$
$\dot{\mathrm{free}}\langle \mathsf{a}^2 | \mathsf{x} := \mathsf{b} \rangle)))))$]]

[$\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{tex}}{\rightarrow}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{pyk}}{\rightarrow}$ "peano free * set * to * end free"]

$\dot{\mathrm{free}}^*\langle * | * := * \rangle$

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{val}}{\rightarrow} \mathsf{x}!$ [$\mathsf{b}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\dot{\mathrm{free}}\langle \mathsf{a}^{\mathrm{h}} | \mathsf{x} := \mathsf{b} \rangle, \dot{\mathrm{free}}^*\langle \mathsf{a}^{\mathrm{t}} | \mathsf{x} := \mathsf{b} \rangle, \mathsf{F}))$]]

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{tex}}{\rightarrow}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{pyk}}{\rightarrow}$ "peano free star * set * to * end free"]

$* \equiv \langle * | * := * \rangle$

[$\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{val}}{\rightarrow} \mathsf{a}!$ [$\mathsf{x}!$ [$\mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \stackrel{\mathrm{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil, \mathsf{b}^1 \stackrel{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge$ [$\mathsf{b} \stackrel{\mathrm{t}}{=} \mathsf{x}$] $, \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}($ [
a] $\stackrel{\mathrm{r}}{=} \mathsf{b}, \mathsf{a}^{\mathrm{t}} \equiv \langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F})))$]]]

[$\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

[$\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{pyk}}{\rightarrow}$ "peano sub * is * where * is * end sub"]

$* \equiv \langle^* * | * := * \rangle$

[$\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{val}}{\rightarrow} \mathsf{b}!$ [$\mathsf{x}!$ [$\mathsf{c}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\mathsf{a}^{\mathrm{h}} \equiv \langle \mathsf{b}^{\mathrm{h}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{a}^{\mathrm{t}} \equiv \langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F}))$]]]

[$\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

[a≡⟨*b|x := c⟩ $\xrightarrow{\text{pyk}}$ "peano sub star * is * where * is * end sub"]

S

[S $\xrightarrow{\text{stmt}}$ [[ȧ +̇ [ḃ′]] $\stackrel{p}{=}$ [[ȧ +̇ [ḃ]] ′]] ⊕ [[∀a: ∀b: [[[¬̇ b] ⇒̇ ¬̇ a] ⇒̇ [[[¬̇ b] ⇒̇ a] ⇒̇ b]]]] ⊕ [[[ȧ $\stackrel{p}{=}$ [ḃ]] ⇒̇ [ȧ′ $\stackrel{p}{=}$ [ḃ′]]]] ⊕ [[∀a: ∀b: [[a ⇒̇ b] ⊢ [a ⊢ b]]]] ⊕ [[[ȧ′ $\stackrel{p}{=}$ [ḃ′]] ⇒̇ [ȧ $\stackrel{p}{=}$ [ḃ]]]] ⊕ [[∀a: ∀b: [a ⇒̇ [b ⇒̇ a]]]] ⊕ [[∀x: ∀a: ∀b: [nonfree(x, a) ⊩ [[∀̇x: [a ⇒̇ b]] ⇒̇ [a ⇒̇ ∀̇x: b]]]]] ⊕ [[[ȧ ·̇ [ḃ′]] $\stackrel{p}{=}$ [[ȧ ·̇ [ḃ]] +̇ [ȧ]]]] ⊕ [[[ȧ +̇ 0̇] $\stackrel{p}{=}$ [ȧ]]] ⊕ [[∀a: ∀b: ∀c: [[a ⇒̇ [b ⇒̇ c]] ⇒̇ [[a ⇒̇ b] ⇒̇ [a ⇒̇ c]]]]] ⊕ [[[ȧ $\stackrel{p}{=}$ [ḃ]] ⇒̇ [[ȧ $\stackrel{p}{=}$ [ċ]] ⇒̇ [ḃ $\stackrel{p}{=}$ [ċ]]]]] ⊕ [[∀a: ∀b: ∀c: ∀x: [b≡⟨a|x := 0̇⟩ ⊩ [c≡⟨a|x := x′⟩ ⊩ [b ⇒̇ [[∀̇x: [a ⇒̇ c]] ⇒̇ ∀̇x: a]]]]]] ⊕ [[¬ [0̇ $\stackrel{p}{=}$ [ȧ′]]]] ⊕ [[∀x: ∀a: [a ⊢ ∀̇x: a]]] ⊕ [[∀c: ∀a: ∀x: ∀b: [⌈a⌉≡⟨⌈b⌉|⌈x⌉ := ⌈c⌉⟩ ⊩ [[∀̇x: b] ⇒̇ a]]]] ⊕ [[ȧ ·̇ 0̇] $\stackrel{p}{=}$ 0̇]]]]]]]]]]]]]]]]

[S $\xrightarrow{\text{tex}}$ "
S"]

[S $\xrightarrow{\text{pyk}}$ "system s"]

A1

[A1 $\xrightarrow{\text{proof}}$ Rule tactic]

[A1 $\xrightarrow{\text{stmt}}$ S ⊢ ∀a: ∀b: [a ⇒̇ [b ⇒̇ a]]]

[A1 $\xrightarrow{\text{tex}}$ "
A1"]

[A1 $\xrightarrow{\text{pyk}}$ "axiom a one"]

A2

[A2 $\xrightarrow{\text{proof}}$ Rule tactic]

[A2 $\xrightarrow{\text{stmt}}$ S ⊢ ∀a: ∀b: ∀c: [[a ⇒̇ [b ⇒̇ c]] ⇒̇ [[a ⇒̇ b] ⇒̇ [a ⇒̇ c]]]]

[A2 $\stackrel{\text{tex}}{\rightarrow}$ "
A2"]

[A2 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom a two"]

A3

[A3 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A3 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{a} \colon \forall \underline{b} \colon [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \dot{\neg} \underline{a}] \dot{\Rightarrow} [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]]$]

[A3 $\stackrel{\text{tex}}{\rightarrow}$ "
A3"]

[A3 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom a three"]

A4

[A4 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A4 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{c} \colon \forall \underline{a} \colon \forall \underline{x} \colon \forall \underline{b} \colon [\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [[\dot{\forall} \underline{x} \colon \underline{b}] \dot{\Rightarrow} \underline{a}]]$]

[A4 $\stackrel{\text{tex}}{\rightarrow}$ "
A4"]

[A4 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom a four"]

A5

[A5 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{x} \colon \forall \underline{a} \colon \forall \underline{b} \colon [\text{non}\dot{\text{f}}\text{ree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x} \colon [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x} \colon \underline{b}]]]$]

[A5 $\stackrel{\text{tex}}{\rightarrow}$ "
A5"]

[A5 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom a five"]

S1

[S1 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{c}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{c}]]]]$]

[S1 $\stackrel{\text{tex}}{\rightarrow}$ "
S1"]

[S1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s one"]

S2

[S2 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [\dot{a}' \stackrel{p}{=} [\dot{b}']]]$]

[S2 $\stackrel{\text{tex}}{\rightarrow}$ "
S2"]

[S2 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s two"]

S3

[S3 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \neg [\dot{0} \stackrel{p}{=} [\dot{a}']]$]

[S3 $\stackrel{\text{tex}}{\rightarrow}$ "
S3"]

[S3 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s three"]

S4

[S4 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash [[\dot{a}' \stackrel{p}{=} [\dot{b}']] \dot{\Rightarrow} [\dot{a} \stackrel{p}{=} [\dot{b}]]]$]

[S4 $\stackrel{\text{tex}}{\rightarrow}$ "
S4"]

[S4 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s four"]

S5

[S5 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5 $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash [[\dot{a} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{a}]]$]

[S5 $\stackrel{\text{tex}}{\to}$ "
S5"]

[S5 $\stackrel{\text{pyk}}{\to}$ "axiom s five"]

S6

[S6 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S6 $\stackrel{\text{stmt}}{\to}$ $S \vdash [[\dot{a} \dot{+} [\dot{b}']] \stackrel{p}{=} [[\dot{a} \dot{+} [\dot{b}]]']]$]

[S6 $\stackrel{\text{tex}}{\to}$ "
S6"]

[S6 $\stackrel{\text{pyk}}{\to}$ "axiom s six"]

S7

[S7 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S7 $\stackrel{\text{stmt}}{\to}$ $S \vdash [[\dot{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]$]

[S7 $\stackrel{\text{tex}}{\to}$ "
S7"]

[S7 $\stackrel{\text{pyk}}{\to}$ "axiom s seven"]

S8

[S8 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S8 $\stackrel{\text{stmt}}{\to}$ $S \vdash [[\dot{a} \dot{\cdot} [\dot{b}']] \stackrel{p}{=} [[\dot{a} \dot{\cdot} [\dot{b}]] \dot{+} [\dot{a}]]]$]

[S8 $\stackrel{\text{tex}}{\to}$ "
S8"]

[S8 $\stackrel{\text{pyk}}{\to}$ "axiom s eight"]

S9

[S9 $\stackrel{\text{proof}}{\to}$ Rule tactic]

[S9 $\stackrel{\text{stmt}}{\to}$ $S \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall} \underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]]]$]

[S9 $\stackrel{\text{tex}}{\rightarrow}$ “
S9”]

[S9 $\stackrel{\text{pyk}}{\rightarrow}$ “axiom s nine”]

MP

[MP $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP $\stackrel{\text{tex}}{\rightarrow}$ “
MP”]

[MP $\stackrel{\text{pyk}}{\rightarrow}$ “rule mp”]

Gen

[Gen $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen $\stackrel{\text{tex}}{\rightarrow}$ “
Gen”]

[Gen $\stackrel{\text{pyk}}{\rightarrow}$ “rule gen”]

L3.2(a)

[L3.2(a) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash [\ [\ S5 \gg [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ; [\ [\ [\ \text{Gen} \rhd [\ [$
$\dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{a}\colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ]\ ; [\ [\ [\ \text{A4} @ [\ \dot{x}\ ]\ ] \gg [\ [$
$\dot{\forall}\dot{a}\colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{MP} \rhd [\ [\ \dot{\forall}\dot{a}\colon$
$[\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{a}\colon [\ [\ \dot{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{a}\ ]\ ]$
$] \gg [\ [\ \dot{x} \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ; [\ [\ S1 \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}}$
$[\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \text{Gen} \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=}$
$[\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ] \gg \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ]\ ;$
$[\ [\ [\ \text{A4} @ [\ \dot{x}\ ]\ ] \gg [\ [\ \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]$
$]\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [$
$[\ \text{MP} \rhd [\ [\ \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ] \mathrel{\dot{\Rightarrow}} [$
$[\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ] \rhd \dot{\forall}\dot{c}\colon [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ]$
$\mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]\ ] \gg [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]$
$] \mathrel{\dot{\Rightarrow}} [\ \dot{b} \stackrel{p}{=} [\ \dot{x}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ \text{Gen} \rhd [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{x}\ ]\ ] \mathrel{\dot{\Rightarrow}} [$

$\dot{b} \overset{p}{=} [\dot{x}]]]]] \gg \dot{\forall}\dot{b}\colon [[\dot{a} \overset{p}{=} [\dot{b}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{b} \overset{p}{=} [\dot{x}]]]$
$]] ; [[[\mathrm{A4} @ [\dot{x}]] \gg [[\dot{\forall}\dot{b}\colon [[\dot{a} \overset{p}{=} [\dot{b}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{b} \overset{p}{=}$
$[\dot{x}]]]]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]]] ; [$
$[[[\mathrm{MP} \rhd [[\dot{\forall}\dot{b}\colon [[\dot{a} \overset{p}{=} [\dot{b}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{b} \overset{p}{=} [\dot{x}]]]]]$
$\mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]]] \rhd \dot{\forall}\dot{b}\colon [[\dot{a} \overset{p}{=} [$
$\dot{b}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{b} \overset{p}{=} [\dot{x}]]]]]] \gg [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=}$
$[\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]] ; [[[\mathrm{Gen} \rhd [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]]$
$\mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]] \gg \dot{\forall}\dot{a}\colon [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]$
$]]]] ; [[[\mathrm{A4} @ [\dot{x} \dot{+} \dot{0}]] \gg [[\dot{\forall}\dot{a}\colon [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]]$
$\mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]] \mathrel{\dot{\Rightarrow}} [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]]$
$\mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]]] ; [[[[\mathrm{MP} \rhd [[\dot{\forall}\dot{a}\colon [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}$
$]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]] \mathrel{\dot{\Rightarrow}} [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]$
$] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]]]] \rhd \dot{\forall}\dot{a}\colon [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[\dot{a} \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [$
$\dot{x}]]]]] \gg [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}$
$]]]]] ; [[[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}}$
$[\dot{x} \overset{p}{=} [\dot{x}]]]]] \rhd [[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]]] \gg [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [$
$\dot{x} \overset{p}{=} [\dot{x}]]]] ; [[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]] \mathrel{\dot{\Rightarrow}} [\dot{x} \overset{p}{=} [\dot{x}]]]] \rhd [$
$[\dot{x} \dot{+} \dot{0}] \overset{p}{=} [\dot{x}]]] \gg [\dot{x} \overset{p}{=} [\dot{x}]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$

$[\mathrm{L3.2(a)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash [\dot{x} \overset{p}{=} [\dot{x}]]]$

[L3.2(a) $\overset{\mathrm{tex}}{\rightarrow}$ "
L3.2(a)"]

[L3.2(a) $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma l three two a"]

S′

$[\mathrm{S}' \overset{\mathrm{stmt}}{\rightarrow} [\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a}' \overset{p}{=} [\underline{b}']] \mathrel{\dot{\Rightarrow}} [\underline{a} \overset{p}{=} \underline{b}]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [\underline{a} \mathrel{\dot{\Rightarrow}} [$
$\underline{b} \mathrel{\dot{\Rightarrow}} \underline{a}]]] \oplus [[\forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}\colon [\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}]] \mathrel{\dot{\Rightarrow}} [$
$\underline{a} \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{b}]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \mathbin{\dot{\cdot}} [\underline{b}']] \overset{p}{=} [[\underline{a} \mathbin{\dot{\cdot}} \underline{b}] \dot{+} \underline{a}]]] \oplus [[$
$\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \overset{p}{=} \underline{b}] \mathrel{\dot{\Rightarrow}} [\underline{a}' \overset{p}{=} [\underline{b}']]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]$
$]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[[\dot{\neg} \underline{b}] \mathrel{\dot{\Rightarrow}} \dot{\neg} \underline{a}] \mathrel{\dot{\Rightarrow}} [[[\dot{\neg} \underline{b}] \mathrel{\dot{\Rightarrow}} \underline{a}] \mathrel{\dot{\Rightarrow}} \underline{b}]]] \oplus [[$
$\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{+} [\underline{b}']] \overset{p}{=} [[\underline{a} \dot{+} \underline{b}] ']]] \oplus [[\forall \underline{a}\colon \neg [\dot{0} \overset{p}{=} [\underline{a}']]]] \oplus [[$
$\forall \underline{x}\colon \forall \underline{a}\colon [\underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}]] \oplus [[\forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [[\dot{\forall} \underline{x}\colon \underline{b}]$
$\mathrel{\dot{\Rightarrow}} \underline{a}]]] \oplus [[\forall \underline{a}\colon [\underline{a} \mathbin{\dot{\cdot}} \dot{0}] \overset{p}{=} \dot{0}]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \overset{p}{=} \underline{b}] \mathrel{\dot{\Rightarrow}} [[$
$\underline{a} \overset{p}{=} \underline{c}] \mathrel{\dot{\Rightarrow}} [\underline{b} \overset{p}{=} \underline{c}]]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [$
$\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \mathrel{\dot{\Rightarrow}} [[\dot{\forall} \underline{x}\colon [\underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}]] \mathrel{\dot{\Rightarrow}} \forall \underline{x}\colon \underline{a}]]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$
$[[\underline{a} \mathrel{\dot{\Rightarrow}} [\underline{b} \mathrel{\dot{\Rightarrow}} \underline{c}]] \mathrel{\dot{\Rightarrow}} [[\underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}] \mathrel{\dot{\Rightarrow}} [\underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}]]]] \oplus \forall \underline{a}\colon [[\underline{a} \dot{+} \dot{0}] \overset{p}{=} \underline{a}]$
$]]]]]]]]]]]]]]]]$

[S′ $\overset{\mathrm{tex}}{\rightarrow}$ "
S′"]

[S′ $\stackrel{\text{pyk}}{\rightarrow}$ “system prime s”]

A1′

[A1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A1′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,]$]

[A1′ $\stackrel{\text{tex}}{\rightarrow}$ “
A1′”]

[A1′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a one”]

A2′

[A2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A2′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\, [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \dot{\Rightarrow} [\, [\, \underline{a} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} [\, \underline{a} \dot{\Rightarrow} \underline{c} \,] \,] \,]$]

[A2′ $\stackrel{\text{tex}}{\rightarrow}$ “
A2′”]

[A2′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a two”]

A3′

[A3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A3′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\, [\, [\, \dot{\neg}\, \underline{b} \,] \dot{\Rightarrow} \dot{\neg}\, \underline{a} \,] \dot{\Rightarrow} [\, [\, [\, \dot{\neg}\, \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \dot{\Rightarrow} \underline{b} \,] \,]$]

[A3′ $\stackrel{\text{tex}}{\rightarrow}$ “
A3′”]

[A3′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a three”]

A4′

[A4′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A4′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\, [\, \dot{\forall} \underline{x}\colon \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \,]$]

[A4′ $\stackrel{\text{tex}}{\rightarrow}$ “
A4′”]

[A4′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a four”]

A5′

[A5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \text{non}\dot{\text{f}}\text{ree}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall}\underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{b}\ ]\ ]\ ]$]

[A5′ $\stackrel{\text{tex}}{\rightarrow}$ "
A5′"]

[A5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a five"]

S1′

[S1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \dot{\Rightarrow} [\ [\ \underline{a} \stackrel{p}{=} \underline{c}\ ] \dot{\Rightarrow} [\ \underline{b} \stackrel{p}{=} \underline{c}\ ]\ ]\ ]$]

[S1′ $\stackrel{\text{tex}}{\rightarrow}$ "
S1′"]

[S1′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s one"]

S2′

[S2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ ]$]

[S2′ $\stackrel{\text{tex}}{\rightarrow}$ "
S2′"]

[S2′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s two"]

S3′

[S3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \neg [\ \dot{0} \stackrel{p}{=} [\ \underline{a}'\ ]\ ]$]

[S3′ $\stackrel{\text{tex}}{\rightarrow}$ "
S3′"]

[S3′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s three"]

S4′

[S4′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ ]$]

[S4′ $\stackrel{\text{tex}}{\rightarrow}$ "
S4'"]

[S4′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s four"]

S5′

[S5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]$]

[S5′ $\stackrel{\text{tex}}{\rightarrow}$ "
S5'"]

[S5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s five"]

S6′

[S6′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{+} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{+} \underline{b}\ ]'\ ]\ ]$]

[S6′ $\stackrel{\text{tex}}{\rightarrow}$ "
S6'"]

[S6′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s six"]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{\cdot} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]$]

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7'"]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"]

S8′

[S8′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathbin{\dot{:}} [\ \underline{b}'\ ]\ ] \overset{p}{=} [\ [\ \underline{a} \mathbin{\dot{:}} \underline{b}\ ] \dot{+} \underline{a}\ ]\ ]$]

[S8′ $\overset{\text{tex}}{\rightarrow}$ "
S8′"]

[S8′ $\overset{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \mathrel{\dot{\Rightarrow}} [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]$]

[S9′ $\overset{\text{tex}}{\rightarrow}$ "
S9′"]

[S9′ $\overset{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP′ $\overset{\text{tex}}{\rightarrow}$ "
MP′"]

[MP′ $\overset{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\overset{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen′ $\overset{\text{tex}}{\rightarrow}$ "
Gen′"]

[Gen′ $\overset{\text{pyk}}{\rightarrow}$ "rule prime gen"]

L3.2(a)′

[L3.2(a)′ $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon [[S5' \gg [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] ; [[S1' \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]]] ; [[[[MP' \rhd [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]]] \rhd [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]] ; [[[MP' \rhd [[[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{a}]]] \rhd [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]] \gg [\underline{a} \stackrel{p}{=} \underline{a}]]]]] \rceil, p_0, c)$]

[L3.2(a)′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\underline{a} \stackrel{p}{=} \underline{a}]$]

[L3.2(a)′ $\stackrel{\text{tex}}{\rightarrow}$ "
L3.2(a)′"]

[L3.2(a)′ $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime l three two a"]

$\dot{*}$

[$\dot{x}$ $\stackrel{\text{tex}}{\rightarrow}$ "
\dot{#1.
}"]

[$\dot{x}$ $\stackrel{\text{pyk}}{\rightarrow}$ "* peano var"]

$*'$

[x' $\stackrel{\text{tex}}{\rightarrow}$ "#1.'"]

[x' $\stackrel{\text{pyk}}{\rightarrow}$ "* peano succ"]

$* \dot{\cdot} *$

[$x \dot{\cdot} y$ $\stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{\cdot}} #2."]

[$x \dot{\cdot} y$ $\stackrel{\text{pyk}}{\rightarrow}$ "* peano times *"]

$* \dot{+} *$

[$x \dot{+} y$ $\stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{+}} #2."]

[$x \dot{+} y$ $\stackrel{\text{pyk}}{\rightarrow}$ "* peano plus *"]

$* \stackrel{p}{=} *$

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

[$\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano is *"]

$*^{\mathcal{P}}$

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil$]

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

[$\mathsf{x}^{\mathcal{P}} \stackrel{\text{pyk}}{\rightarrow}$ "* is peano var"]

$\dot{\neg} *$

[$\dot{\neg}\, \mathsf{x} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\neg}\, {#1.}"]

[$\dot{\neg}\, \mathsf{x} \stackrel{\text{pyk}}{\rightarrow}$ "peano not *"]

$* \mathrel{\dot{\wedge}} *$

[$\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{..}{=} \dot{\neg}\, (\mathsf{x} \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \mathsf{y})] \rceil)$]

[$\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\wedge}} #2."]

[$\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano and *"]

$* \mathrel{\dot{\vee}} *$

[$\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{..}{=} [\, \dot{\neg}\, \mathsf{x}\,] \mathrel{\dot{\Rightarrow}} \mathsf{y}] \rceil)$]

[$\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\vee}} #2."]

[$\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano or *"]

$\dot{\forall} * \colon *$

$[\dot{\forall} \mathsf{x} \colon \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall} \mathsf{x} \colon \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "peano all * indeed *"]

$\dot{\exists} * \colon *$

$[\dot{\exists} \mathsf{x} \colon \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{\exists} \mathsf{x} \colon \mathsf{y} \ddot{=} \dot{\neg} \dot{\forall} \mathsf{x} \colon \dot{\neg} \mathsf{y}] \rceil)]$

$[\dot{\exists} \mathsf{x} \colon \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\exists} #1.
\colon #2."]

$[\dot{\exists} \mathsf{x} \colon \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "peano exist * indeed *"]

$* \dot{\Rightarrow} *$

$[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \dot{\Rightarrow} \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "* peano imply *"]

$* \dot{\Leftrightarrow} *$

$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \ddot{=} (\mathsf{x} \dot{\Rightarrow} \mathsf{y}) \dot{\wedge} (\mathsf{y} \dot{\Rightarrow} \mathsf{x})] \rceil)]$

$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \overset{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

$[\mathsf{x} \dot{\Leftrightarrow} \mathsf{y} \overset{\text{pyk}}{\rightarrow}$ "* peano iff *"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-21.UTC:18:10:30.616926 = MJD-53542.TAI:18:11:02.616926 = LGT-4626094262616926e-6