Logiweb codex of peano

Up Help

peano, $\dot{0}$, $\dot{1}$, $\dot{2}$, $\dot{a}$, $\dot{b}$, $\dot{c}$, $\dot{d}$, $\dot{e}$, $\dot{f}$, $\dot{g}$, $\dot{h}$, $\dot{i}$, $\dot{j}$, $\dot{k}$, $\dot{l}$, $\dot{m}$, $\dot{n}$, $\dot{o}$, $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{s}$, $\dot{t}$, $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{x}$, $\dot{y}$, $\dot{z}$, nonfree$(*,*)$, $\text{nonfree}^{*}(*,*)$, free$\langle *|* := *\rangle$, $\text{free}^{*}\langle *|* := *\rangle$, $*\equiv\langle *|* := *\rangle$, $*\equiv\langle^{*} *|* := *\rangle$, S, A1, A2, A3, A4, A5, S1, S2, S3, S4, S5, S6, S7, S8, S9, MP, Gen, L3.2(a), S′, A1′, A2′, A3′, A4′, A5′, S1′, S2′, S3′, S4′, S5′, S6′, S7′, S8′, S9′, MP′, Gen′, L3.2(a)′, $\dot{*}$, $*'$, $* \mathbin{\dot{:}} *$, $* \dot{+} *$, $* \overset{\mathrm{p}}{=} *$, $*^{\mathcal{P}}$, $\dot{\neg} *$, $* \dot{\wedge} *$, $* \dot{\vee} *$, $\dot{\forall} *: *$, $\dot{\exists} *: *$, $* \dot{\Rightarrow} *$, $* \dot{\Leftrightarrow} *$,

peano

[peano $\overset{\text{prio}}{\rightarrow}$
Preassociative
[peano], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [math $*$ end math], [**flush left** $[*]$], [x], [y], [z], $[[* \bowtie *]]$, $[[* \overset{*}{\rightarrow} *]]$, [pyk], [tex], [name], [prio], $[*]$, [T], [if$(*, *, *)$], $[[* \overset{*}{\Rightarrow} *]]$, [val], [claim], $[\bot]$, [f$(*)$], $[(*)^{\mathrm{I}}]$, [F], $[\underline{0}]$, $[\underline{1}]$, $[\underline{2}]$, $[\underline{3}]$, $[\underline{4}]$, $[\underline{5}]$, $[\underline{6}]$, $[\underline{7}]$, $[\underline{8}]$, $[\underline{9}]$, [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\mathrm{M}}]$, [If$(*, *, *)$], [array$\{*\}$ $*$ end array], [l], [c], [r], [empty], $[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$, $[\mathcal{U}^{\mathrm{M}}(*)]$, [**apply**$(*, *)$], [$\mathbf{apply}_1(*, *)$], [identifier$(*)$], [$\text{identifier}_1(*, *)$], [array-plus$(*, *)$], [array-remove$(*, *, *)$], [array-put$(*, *, *, *)$], [array-add$(*, *, *, *, *)$], [bit$(*, *)$], [$\text{bit}_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], $[\mathcal{E}(*, *, *)]$, $[\mathcal{E}_2(*, *, *, *, *)]$, $[\mathcal{E}_3(*, *, *, *)]$, $[\mathcal{E}_4(*, *, *, *)]$, [**lookup**$(*, *, *)$], [**abstract**$(*, *, *, *)$], $[\lceil * \rceil]$, $[\mathcal{M}(*, *, *)]$, $[\mathcal{M}_2(*, *, *, *)]$, $[\mathcal{M}^{*}(*, *, *)]$, [macro], $[\mathrm{s}_0]$, [**zip**$(*, *)$], [$\mathbf{assoc}_1(*, *, *)$], $[(*)^{\mathbf{P}}]$, [self], $[[* \ddot{=} *]]$, $[[* \dot{=} *]]$, $[[* \acute{=} *]]$, $[[* \overset{\text{pyk}}{=} *]]$, $[[* \overset{\text{tex}}{=} *]]$, $[[* \overset{\text{name}}{=} *]]$, [**Priority table**$[*]$], $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$, $[\tilde{\mathcal{M}}_4(*, *, *, *)]$, $[\tilde{\mathcal{M}}(*, *, *)]$, $[\tilde{\mathcal{Q}}(*, *, *)]$, $[\tilde{\mathcal{Q}}_2(*, *, *)]$, $[\tilde{\mathcal{Q}}_3(*, *, *, *)]$, $[\tilde{\mathcal{Q}}^{*}(*, *, *)]$, $[(*)]$, [**aspect**$(*, *)$], [**aspect**$(*, *, *)$], $[\langle * \rangle]$, [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], [$\text{let}_2(*, *)$], [$\text{let}_1(*, *)$], $[[* \overset{\text{claim}}{=} *]]$, [checker], [**check**$(*, *)$], [$\mathbf{check}_2(*, *, *)$], [$\mathbf{check}_3(*, *, *)$], [$\mathbf{check}^{*}(*, *)$], [$\mathbf{check}^{*}_2(*, *, *)$], $[[*]^{\cdot}]$, $[[*]^{-}]$, $[[*]^{\circ}]$, [msg], $[[* \overset{\text{msg}}{=} *]]$, [<stmt>], [stmt], $[[* \overset{\text{stmt}}{=} *]]$, [HeadNil′], [HeadPair′], [Transitivity′], $[\perp\!\!\!\perp]$, [Contra′], $[\mathrm{T}'_{\mathrm{E}}]$, $[\mathrm{L}_1]$, $[\underline{*}]$, $[\mathcal{A}]$, $[\mathcal{B}]$, $[\mathcal{C}]$, $[\mathcal{D}]$, $[\mathcal{E}]$, $[\mathcal{F}]$, $[\mathcal{G}]$, $[\mathcal{H}]$, $[\mathcal{I}]$, $[\mathcal{J}]$, $[\mathcal{K}]$, $[\mathcal{L}]$, $[\mathcal{M}]$, $[\mathcal{N}]$, $[\mathcal{O}]$, $[\mathcal{P}]$, $[\mathcal{Q}]$, $[\mathcal{R}]$, $[\mathcal{S}]$, $[\mathcal{T}]$, $[\mathcal{U}]$, $[\mathcal{V}]$, $[\mathcal{W}]$, $[\mathcal{X}]$, $[\mathcal{Y}]$, $[\mathcal{Z}]$, $[\langle * \mid * := * \rangle]$, $[\langle^{*} * \mid * := * \rangle]$, $[\emptyset]$, [Remainder], $[(*)^{\mathbf{v}}]$, [intro$(*, *, *, *)$], [intro$(*, *, *)$], [error$(*, *)$], [$\text{error}_2(*, *)$], [proof$(*, *, *)$],

[$\mathrm{proof}_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\mathrm{I}}(*,*)$], [$\mathcal{S}^{\rhd}(*,*)$], [$\mathcal{S}^{\rhd}_1(*,*,*)$], [$\mathcal{S}^{\mathrm{E}}(*,*)$], [$\mathcal{S}^{\mathrm{E}}_1(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}^{+}_1(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}^{-}_1(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}^{*}_1(*,*,*)$], [$\mathcal{S}^{*}_2(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}^{@}_1(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}^{\vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}^{\Vdash}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}(*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}_2(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}^{\forall}_1(*,*,*,*)$], [$\mathcal{S}^{;}(*,*)$], [$\mathcal{S}^{;}_1(*,*,*)$], [$\mathcal{S}^{;}_2(*,*,*,*)$], [$\mathcal{T}(*)$], [claims$(*,*,*)$], [$\mathrm{claims}_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut$(*,*)$], [$\mathrm{Head}_{\oplus}(*)$], [$\mathrm{Tail}_{\oplus}(*)$], [$\mathrm{rule}_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [$\mathrm{theory}_2(*,*)$], [$\mathrm{theory}_3(*,*)$], [$\mathrm{theory}_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [$\mathrm{T_E}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(*=*,*)$], [unify$^*(*=*,*)$], [$\mathrm{unify}_2(*=*,*)$], [$\mathrm{L_a}$], [$\mathrm{L_b}$], [$\mathrm{L_c}$], [$\mathrm{L_d}$], [$\mathrm{L_e}$], [$\mathrm{L_f}$], [$\mathrm{L_g}$], [$\mathrm{L_h}$], [$\mathrm{L_i}$], [$\mathrm{L_j}$], [$\mathrm{L_k}$], [$\mathrm{L_l}$], [$\mathrm{L_m}$], [$\mathrm{L_n}$], [$\mathrm{L_o}$], [$\mathrm{L_p}$], [$\mathrm{L_q}$], [$\mathrm{L_r}$], [$\mathrm{L_s}$], [$\mathrm{L_t}$], [$\mathrm{L_u}$], [$\mathrm{L_v}$], [$\mathrm{L_w}$], [$\mathrm{L_x}$], [$\mathrm{L_y}$], [$\mathrm{L_z}$], [$\mathrm{L_A}$], [$\mathrm{L_B}$], [$\mathrm{L_C}$], [$\mathrm{L_D}$], [$\mathrm{L_E}$], [$\mathrm{L_F}$], [$\mathrm{L_G}$], [$\mathrm{L_H}$], [$\mathrm{L_I}$], [$\mathrm{L_J}$], [$\mathrm{L_K}$], [$\mathrm{L_L}$], [$\mathrm{L_M}$], [$\mathrm{L_N}$], [$\mathrm{L_O}$], [$\mathrm{L_P}$], [$\mathrm{L_Q}$], [$\mathrm{L_R}$], [$\mathrm{L_S}$], [$\mathrm{L_T}$], [$\mathrm{L_U}$], [$\mathrm{L_V}$], [$\mathrm{L_W}$], [$\mathrm{L_X}$], [$\mathrm{L_Y}$], [$\mathrm{L_Z}$], [$\mathrm{L_?}$], [Reflexivity], [$\mathrm{Reflexivity}_1$], [Commutativity], [$\mathrm{Commutativity}_1$], [<tactic>], [tactic], [[$* \overset{\mathrm{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$\mathrm{p_0}$], [$\mathrm{conclude}_1(*,*)$], [$\mathrm{conclude}_2(*,*,*)$], [$\mathrm{conclude}_3(*,*,*,*)$], [$\dot{0}$], [$\dot{1}$], [$\dot{2}$], [$\dot{a}$], [$\dot{b}$], [$\dot{c}$], [$\dot{d}$], [$\dot{e}$], [$\dot{f}$], [$\dot{g}$], [$\dot{h}$], [$\dot{i}$], [$\dot{j}$], [$\dot{k}$], [$\dot{l}$], [$\dot{m}$], [$\dot{n}$], [$\dot{o}$], [$\dot{p}$], [$\dot{q}$], [$\dot{r}$], [$\dot{s}$], [$\dot{t}$], [$\dot{u}$], [$\dot{v}$], [$\dot{w}$], [$\dot{x}$], [$\dot{y}$], [$\dot{z}$], [nonfree$(*,*)$], [nonfree$^*(*,*)$], [free$\langle *|* := *\rangle$], [free$^*\langle *|* := *\rangle$], [$*\equiv\langle *|* := *\rangle$], [$*\equiv\langle^* *|* := *\rangle$], [S], [A1], [A2], [A3], [A4], [A5], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [MP], [Gen], [L3.2(a)], [S$'$], [A1$'$], [A2$'$], [A3$'$], [A4$'$], [A5$'$], [S1$'$], [S2$'$], [S3$'$], [S4$'$], [S5$'$], [S6$'$], [S7$'$], [S8$'$], [S9$'$], [MP$'$], [Gen$'$], [L3.2(a)$'$];

Preassociative

[$*_{\{*\}}$], [$*'$], [$*[*]$], [$*[*\rightarrow*]$], [$*[*\Rightarrow*]$], [$\dot{*}$];

Preassociative

[“ $*$ ”], [], [$(*)^{\mathbf{t}}$], [string$(*)+*$], [string$(*)$ ++ $*$], [
$*$], [$*$], [!$*$], [”$*$], [#$*$], [$\$*$], [%$*$], [&$*$], [’$*$], [($*$], [)$*$], [$**$], [+$*$], [,$*$], [-$*$], [.$*$], [/$*$], [0$*$], [1$*$], [2$*$], [3$*$], [4$*$], [5$*$], [6$*$], [7$*$], [8$*$], [9$*$], [:$*$], [;$*$], [<$*$], [=$*$], [>$*$], [?$*$], [@$*$], [A$*$], [B$*$], [C$*$], [D$*$], [E$*$], [F$*$], [G$*$], [H$*$], [I$*$], [J$*$], [K$*$], [L$*$], [M$*$], [N$*$], [O$*$], [P$*$], [Q$*$], [R$*$], [S$*$], [T$*$], [U$*$], [V$*$], [W$*$], [X$*$], [Y$*$], [Z$*$], [[$*$], [\$*$], []$*$], [^$*$], [_$*$], [‘$*$], [a$*$], [b$*$], [c$*$], [d$*$], [e$*$], [f$*$], [g$*$], [h$*$], [i$*$], [j$*$], [k$*$], [l$*$], [m$*$], [n$*$], [o$*$], [p$*$], [q$*$], [r$*$], [s$*$], [t$*$], [u$*$], [v$*$], [w$*$], [x$*$], [y$*$], [z$*$], [{$*$], [|$*$], [}$*$], [~$*$], [**Preassociative** $*$; $*$], [**Postassociative** $*$; $*$], [[$*$], $*$], [priority $*$ end], [newline $*$], [macro newline $*$];

Preassociative

[$*0$], [$*1$], [0b], [$*$-color$(*)$], [$*$-color$^*(*)$];

Preassociative

[$*$ ’ $*$], [$*$ ‘ $*$];

Preassociative

[$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^{0}$], [$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*'$];

Preassociative
$[* \cdot *], [* \cdot_0 *], [* \vdots *]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [* \dot{+} *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*, *]$;
Preassociative
$[* \stackrel{\mathrm{B}}{\approx} *], [* \stackrel{\mathrm{D}}{\approx} *], [* \stackrel{\mathrm{C}}{\approx} *], [* \stackrel{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\to} *], [* \stackrel{\mathrm{t}}{=} *], [* \stackrel{\mathrm{t}^*}{=} *], [* \stackrel{\mathrm{r}}{=} *]$, $[* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \stackrel{\mathrm{T}}{=} *], [* \stackrel{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *]$, $[* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* \stackrel{\mathrm{p}}{=} *], [*^{\mathcal{P}}]$;
Preassociative
$[\neg *], [\dot{\neg} *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\mathrm{c}} *], [* \dot{\wedge} *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \dot{\vee} *]$;
Preassociative
$[\dot{\forall} *: *], [\dot{\exists} *: *]$;
Postassociative
$[* \ddot{\Rightarrow} *], [* \dot{\Rightarrow} *], [* \dot{\Leftrightarrow} *]$;
Postassociative
$[* : *], [*!*]$;
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.]$;
Preassociative
$[\lambda * .*], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *]$;
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}]$;
Preassociative
$[* @ *], [* \rhd *], [* \Vrhd *], [* \gg *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall *: *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
$[* \text{ proves } *]$;

Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \Box$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$];
Postassociative
[$*$ then $*$], [$*[*]*$];
Preassociative
[$*\&*$];
Preassociative
[$*\backslash\backslash*$];]

[peano $\overset{\text{pyk}}{\rightarrow}$ "peano"]

$\dot{0}$

[$\dot{0} \overset{\text{tex}}{\rightarrow}$ "
\dot{0}"]

[$\dot{0} \overset{\text{pyk}}{\rightarrow}$ "peano zero"]

$\dot{1}$

[$\dot{1} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{1} \ddot{=} \dot{0}'] \rceil)$]

[$\dot{1} \overset{\text{tex}}{\rightarrow}$ "
\dot{1}"]

[$\dot{1} \overset{\text{pyk}}{\rightarrow}$ "peano one"]

$\dot{2}$

[$\dot{2} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{2} \ddot{=} \dot{1}'] \rceil)$]

[$\dot{2} \overset{\text{tex}}{\rightarrow}$ "
\dot{2}"]

[$\dot{2} \overset{\text{pyk}}{\rightarrow}$ "peano two"]

$\dot{a}$

[$\dot{a} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{a} \ddot{=} \dot{\mathsf{a}}] \rceil)$]

$[\dot{a} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{\mathit{a}}”]

$[\dot{a} \stackrel{\text{pyk}}{\rightarrow}$ “peano a”]

$\dot{b}$

$[\dot{b} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{\mathit{b}}”]

$[\dot{b} \stackrel{\text{pyk}}{\rightarrow}$ “peano b”]

$\dot{c}$

$[\dot{c} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{\mathit{c}}”]

$[\dot{c} \stackrel{\text{pyk}}{\rightarrow}$ “peano c”]

$\dot{d}$

$[\dot{d} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{\mathit{d}}”]

$[\dot{d} \stackrel{\text{pyk}}{\rightarrow}$ “peano d”]

$\dot{e}$

$[\dot{e} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \stackrel{\text{tex}}{\rightarrow}$ “
\dot{\mathit{e}}”]

$[\dot{e} \stackrel{\text{pyk}}{\rightarrow}$ “peano e”]

$\dot{f}$

$[\dot{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{f} \doteq \dot{\mathsf{f}}] \rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\to}$ "peano f"]

$\dot{g}$

$[\dot{g} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{g} \doteq \dot{\mathsf{g}}] \rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\to}$ "peano g"]

$\dot{h}$

$[\dot{h} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{h} \doteq \dot{\mathsf{h}}] \rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\to}$ "peano h"]

$\dot{i}$

$[\dot{i} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{i} \doteq \dot{\mathsf{i}}] \rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\to}$ "peano i"]

$\dot{j}$

$[\dot{j} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{j} \doteq \dot{\mathsf{j}}] \rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\to}$ "peano j"]

$\dot{k}$

$[\dot{k} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{k} \doteqdot \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{k}}"]

$[\dot{k} \stackrel{\text{pyk}}{\to}$ "peano k"]

$\dot{l}$

$[\dot{l} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{l} \doteqdot \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{l}}"]

$[\dot{l} \stackrel{\text{pyk}}{\to}$ "peano l"]

$\dot{m}$

$[\dot{m} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{m} \doteqdot \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{m}}"]

$[\dot{m} \stackrel{\text{pyk}}{\to}$ "peano m"]

$\dot{n}$

$[\dot{n} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{n} \doteqdot \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{n}}"]

$[\dot{n} \stackrel{\text{pyk}}{\to}$ "peano n"]

$\dot{o}$

$[\dot{o} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{o} \doteqdot \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{o}}"]

$[\dot{o} \stackrel{\text{pyk}}{\to}$ "peano o"]

$\dot{p}$

$[\dot{p} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{p} \mathrel{\ddot{=}} \dot{\mathsf{p}}]\rceil)]$

$[\dot{p} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{p}}"]

$[\dot{p} \stackrel{\mathrm{pyk}}{\to}$ "peano p"]

$\dot{q}$

$[\dot{q} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{q} \mathrel{\ddot{=}} \dot{\mathsf{q}}]\rceil)]$

$[\dot{q} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{q}}"]

$[\dot{q} \stackrel{\mathrm{pyk}}{\to}$ "peano q"]

$\dot{r}$

$[\dot{r} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{r} \mathrel{\ddot{=}} \dot{\mathsf{r}}]\rceil)]$

$[\dot{r} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{r}}"]

$[\dot{r} \stackrel{\mathrm{pyk}}{\to}$ "peano r"]

$\dot{s}$

$[\dot{s} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{s} \mathrel{\ddot{=}} \dot{\mathsf{s}}]\rceil)]$

$[\dot{s} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{s}}"]

$[\dot{s} \stackrel{\mathrm{pyk}}{\to}$ "peano s"]

$\dot{t}$

$[\dot{t} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{t} \mathrel{\ddot{=}} \dot{\mathsf{t}}]\rceil)]$

$[\dot{t} \stackrel{\mathrm{tex}}{\to}$ "
\dot{\mathit{t}}"]

$[\dot{t} \stackrel{\mathrm{pyk}}{\to}$ "peano t"]

$\dot{u}$

$[\dot{u} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{u} \ddot{=} \dot{\mathsf{u}}]\rceil)]$

$[\dot{u} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{u}}"]

$[\dot{u} \overset{\text{pyk}}{\rightarrow}$ "peano u"]

$\dot{v}$

$[\dot{v} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{v} \ddot{=} \dot{\mathsf{v}}]\rceil)]$

$[\dot{v} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{v}}"]

$[\dot{v} \overset{\text{pyk}}{\rightarrow}$ "peano v"]

$\dot{w}$

$[\dot{w} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{w} \ddot{=} \dot{\mathsf{w}}]\rceil)]$

$[\dot{w} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{w}}"]

$[\dot{w} \overset{\text{pyk}}{\rightarrow}$ "peano w"]

$\dot{x}$

$[\dot{x} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{x} \ddot{=} \dot{\mathsf{x}}]\rceil)]$

$[\dot{x} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{x}}"]

$[\dot{x} \overset{\text{pyk}}{\rightarrow}$ "peano x"]

$\dot{y}$

$[\dot{y} \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{y} \ddot{=} \dot{\mathsf{y}}]\rceil)]$

$[\dot{y} \overset{\text{tex}}{\rightarrow}$ "
\dot{\mathit{y}}"]

$[\dot{y} \overset{\text{pyk}}{\rightarrow}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"]

$\dot{\text{nonfree}}(*,*)$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg\, [\, \mathsf{x} \stackrel{\text{t}}{=} \mathsf{y}\,]\,,$
$\text{If}(\neg\, [\, \mathsf{y} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{x} \colon \mathsf{y} \rceil\,]\,, \dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=}\ [\, \mathsf{y}^{1}\,]\,, \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{2}))))]$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree $*$ in $*$ end nonfree"]

$\dot{\text{nonfree}}^{*}(*,*)$

$[\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y},\mathsf{T},\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}), \dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}^{\text{t}}),\mathsf{F}))]$

$[\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^{*}(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree star $*$ in $*$ end nonfree"]

$\dot{\text{free}}\langle *|* := *\rangle$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{val}}{\to} \mathsf{x}!\ [\ \mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}},\mathsf{T},$
$\text{If}(\neg\, [\, \mathsf{a} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{u} \colon \mathsf{v} \rceil\,]\,, \dot{\text{free}}^{*}\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$
$\text{If}(\mathsf{a}^{1} \stackrel{\text{t}}{=} \mathsf{x},\mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^{2}),\mathsf{T},$

$\mathrm{If}(\neg\mathrm{no\dot{n}free}(\mathsf{a}^1,\mathsf{b}),\mathsf{F},$
$\mathrm{\dot{f}ree}\langle \mathsf{a}^2|\mathsf{x} := \mathsf{b}\rangle)))))$]]

[$\mathrm{fr\dot{e}e}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

[$\mathrm{fr\dot{e}e}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{pyk}}{\to}$ "peano free * set * to * end free"]

$\mathrm{fr\dot{e}e}^*\langle *|* := *\rangle$

[$\mathrm{fr\dot{e}e}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{val}}{\to} \mathsf{x}!$ [$\mathsf{b}!\mathrm{If}(\mathsf{a},\mathsf{T},\mathrm{If}(\mathrm{fr\dot{e}e}\langle \mathsf{a}^{\mathrm{h}}|\mathsf{x} := \mathsf{b}\rangle, \mathrm{fr\dot{e}e}^*\langle \mathsf{a}^{\mathrm{t}}|\mathsf{x} := \mathsf{b}\rangle,\mathsf{F}))$]]

[$\mathrm{fr\dot{e}e}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{tex}}{\to}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

[$\mathrm{fr\dot{e}e}^*\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \overset{\mathrm{pyk}}{\to}$ "peano free star * set * to * end free"]

$*\equiv\langle *|* := *\rangle$

[$\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{val}}{\to} \mathsf{a}!$ [$\mathsf{x}!$ [$\mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \overset{\mathrm{r}}{=} \lceil \dot{\forall}\mathsf{u}\colon \mathsf{v}\rceil, \mathsf{b}^1 \overset{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge$ [$\mathsf{b} \overset{\mathrm{t}}{=} \mathsf{x}$] $, \mathsf{a} \overset{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}($ [
a] $\overset{\mathrm{r}}{=} \mathsf{b}, \mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}}|\mathsf{x} := \mathsf{c}\rangle, \mathsf{F})))$]]]

[$\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{tex}}{\to}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

[$\mathsf{a}\equiv\langle \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{pyk}}{\to}$ "peano sub * is * where * is * end sub"]

$*\equiv\langle^* *|* := *\rangle$

[$\mathsf{a}\equiv\langle^* \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{val}}{\to} \mathsf{b}!$ [$\mathsf{x}!$ [$\mathsf{c}!\mathrm{If}(\mathsf{a},\mathsf{T},\mathrm{If}(\mathsf{a}^{\mathrm{h}}\equiv\langle \mathsf{b}^{\mathrm{h}}|\mathsf{x} := \mathsf{c}\rangle, \mathsf{a}^{\mathrm{t}}\equiv\langle^* \mathsf{b}^{\mathrm{t}}|\mathsf{x} := \mathsf{c}\rangle, \mathsf{F}))$]]]

[$\mathsf{a}\equiv\langle^* \mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \overset{\mathrm{tex}}{\to}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

$[\mathsf{a}{\equiv}\langle^*\mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \stackrel{\text{pyk}}{\rightarrow}$ "peano sub star $*$ is $*$ where $*$ is $*$ end sub"$]$

S

$[\mathrm{S} \stackrel{\text{stmt}}{\rightarrow} [\ [\ \dot{\mathsf{a}} \dot{+} [\ \dot{\mathsf{b}}'\ ]\ ] \stackrel{\text{p}}{=} [\ [\ \dot{\mathsf{a}} \dot{+} [\ \dot{\mathsf{b}}\ ]\ ]\ '\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ [\ \dot{\neg} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \dot{\neg} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} [\ [\ [\ \dot{\neg} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{a}}\ ] \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{\mathsf{a}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \dot{\mathsf{a}}' \stackrel{\text{p}}{=} [\ \dot{\mathsf{b}}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \vdash [\ \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{\mathsf{a}}' \stackrel{\text{p}}{=} [\ \dot{\mathsf{b}}'\ ]\ ] \dot{\Rightarrow} [\ \dot{\mathsf{a}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \mathrm{nonfree}(\underline{\mathsf{x}}, \underline{\mathsf{a}}) \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{\mathsf{a}} \dot{\cdot} [\ \dot{\mathsf{b}}'\ ]\ ] \stackrel{\text{p}}{=} [\ [\ \dot{\mathsf{a}} \dot{\cdot} [\ \dot{\mathsf{b}}\ ]\ ] \dot{+} [\ \dot{\mathsf{a}}\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{\mathsf{a}} \dot{+} \dot{0}\ ] \stackrel{\text{p}}{=} [\ \dot{\mathsf{a}}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ [\ \dot{\mathsf{a}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\mathsf{a}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} [\ \dot{\mathsf{b}} \stackrel{\text{p}}{=} [\ \dot{\mathsf{c}}\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{x}} \colon [\ \underline{\mathsf{b}}{\equiv}\langle \underline{\mathsf{a}}|\underline{\mathsf{x}} := \dot{0}\rangle \Vdash [\ \underline{\mathsf{c}}{\equiv}\langle \underline{\mathsf{a}}|\underline{\mathsf{x}} := \underline{\mathsf{x}}'\rangle \Vdash [\ \underline{\mathsf{b}} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{a}}\ ]\ ]\ ]\ ]\ ]\ ] \oplus [\ [\ \neg [\ \dot{0} \stackrel{\text{p}}{=} [\ \dot{\mathsf{a}}'\ ]\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon [\ \underline{\mathsf{a}} \vdash \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{a}}\ ]\ ]\ ] \oplus [\ [\ \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{b}} \colon [\ \lceil \underline{\mathsf{a}} \rceil{\equiv}\langle \lceil \underline{\mathsf{b}} \rceil | \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{c}} \rceil\rangle \Vdash [\ [\ \dot{\forall} \underline{\mathsf{x}} \colon \underline{\mathsf{b}}\ ] \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]\ ] \oplus [\ [\ \dot{\mathsf{a}} \dot{\cdot} \dot{0}\ ] \stackrel{\text{p}}{=} \dot{0}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]$

[S $\stackrel{\text{tex}}{\rightarrow}$ "
S"]

[S $\stackrel{\text{pyk}}{\rightarrow}$ "system s"]

A1

[A1 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{A1} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{a}}\ ]\ ]\ ]$

[A1 $\stackrel{\text{tex}}{\rightarrow}$ "
A1"]

[A1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom a one"]

A2

[A2 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{A2} \stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} [\ \underline{\mathsf{b}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ] \dot{\Rightarrow} [\ [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{b}}\ ] \dot{\Rightarrow} [\ \underline{\mathsf{a}} \dot{\Rightarrow} \underline{\mathsf{c}}\ ]\ ]\ ]\ ]$

$[A2 \xrightarrow{\text{tex}}$ "
A2"]

[A2 $\xrightarrow{\text{pyk}}$ "axiom a two"]

A3

[A3 $\xrightarrow{\text{proof}}$ Rule tactic]

[A3 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \dot{\neg}\, \underline{b}\ ]\ \dot{\Rightarrow}\ \dot{\neg}\, \underline{a}\ ]\ \dot{\Rightarrow}\ [\ [\ [\ \dot{\neg}\, \underline{b}\ ]\ \dot{\Rightarrow}\ \underline{a}\ ]\ \dot{\Rightarrow}\ \underline{b}\ ]\ ]$]

[A3 $\xrightarrow{\text{tex}}$ "
A3"]

[A3 $\xrightarrow{\text{pyk}}$ "axiom a three"]

A4

[A4 $\xrightarrow{\text{proof}}$ Rule tactic]

[A4 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{x}\colon \underline{b}\ ]\ \dot{\Rightarrow}\ \underline{a}\ ]\ ]$]

[A4 $\xrightarrow{\text{tex}}$ "
A4"]

[A4 $\xrightarrow{\text{pyk}}$ "axiom a four"]

A5

[A5 $\xrightarrow{\text{proof}}$ Rule tactic]

[A5 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \text{non}\dot{\text{f}}\text{ree}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a}\ \dot{\Rightarrow}\ \underline{b}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{a}\ \dot{\Rightarrow}\ \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]$]

[A5 $\xrightarrow{\text{tex}}$ "
A5"]

[A5 $\xrightarrow{\text{pyk}}$ "axiom a five"]

S1

[S1 $\xrightarrow{\text{proof}}$ Rule tactic]

[S1 $\xrightarrow{\text{stmt}}$ $S \vdash [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{b}\ ]\ ]\ \dot{\Rightarrow}\ [\ [\ \dot{a} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ \dot{\Rightarrow}\ [\ \dot{b} \stackrel{p}{=} [\ \dot{c}\ ]\ ]\ ]\ ]$]

[S1 $\stackrel{\text{tex}}{\rightarrow}$ "
S1"]

[S1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s one"]

S2

[S2 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\ \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\ \dot{\mathsf{b}}'\ ]\ ]\ ]$]

[S2 $\stackrel{\text{tex}}{\rightarrow}$ "
S2"]

[S2 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s two"]

S3

[S3 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \neg [\ \dot{0} \stackrel{\mathrm{p}}{=} [\ \dot{\mathsf{a}}'\ ]\ ]$]

[S3 $\stackrel{\text{tex}}{\rightarrow}$ "
S3"]

[S3 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s three"]

S4

[S4 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\ \dot{\mathsf{a}}' \stackrel{\mathrm{p}}{=} [\ \dot{\mathsf{b}}'\ ]\ ] \mathrel{\dot{\Rightarrow}} [\ \dot{\mathsf{a}} \stackrel{\mathrm{p}}{=} [\ \dot{\mathsf{b}}\ ]\ ]\ ]$]

[S4 $\stackrel{\text{tex}}{\rightarrow}$ "
S4"]

[S4 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s four"]

S5

[S5 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash [\ [\ \dot{\mathsf{a}} \mathbin{\dot{+}} \dot{0}\ ] \stackrel{\mathrm{p}}{=} [\ \dot{\mathsf{a}}\ ]\ ]$]

$[S5 \stackrel{\text{tex}}{\rightarrow}$ "
S5"]

$[S5 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s five"]

S6

$[S6 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S6 \stackrel{\text{stmt}}{\rightarrow} S \vdash [[\dot{a} \dot{+} [\dot{b}']]] \stackrel{p}{=} [[\dot{a} \dot{+} [\dot{b}]]']]]$

$[S6 \stackrel{\text{tex}}{\rightarrow}$ "
S6"]

$[S6 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s six"]

S7

$[S7 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S7 \stackrel{\text{stmt}}{\rightarrow} S \vdash [[\dot{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]]$

$[S7 \stackrel{\text{tex}}{\rightarrow}$ "
S7"]

$[S7 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s seven"]

S8

$[S8 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S8 \stackrel{\text{stmt}}{\rightarrow} S \vdash [[\dot{a} \dot{\cdot} [\dot{b}']]] \stackrel{p}{=} [[\dot{a} \dot{\cdot} [\dot{b}]] \dot{+} [\dot{a}]]]]$

$[S8 \stackrel{\text{tex}}{\rightarrow}$ "
S8"]

$[S8 \stackrel{\text{pyk}}{\rightarrow}$ "axiom s eight"]

S9

$[S9 \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$[S9 \stackrel{\text{stmt}}{\rightarrow} S \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall} \underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]]]]$

[S9 $\stackrel{\text{tex}}{\rightarrow}$ "
S9"]

[S9 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom s nine"]

MP

[MP $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\, [\, \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b} \,] \vdash [\, \underline{a} \vdash \underline{b} \,] \,]$]

[MP $\stackrel{\text{tex}}{\rightarrow}$ "
MP"]

[MP $\stackrel{\text{pyk}}{\rightarrow}$ "rule mp"]

Gen

[Gen $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{x} \colon \forall \underline{a} \colon [\, \underline{a} \vdash \dot{\forall} \underline{x} \colon \underline{a} \,]$]

[Gen $\stackrel{\text{tex}}{\rightarrow}$ "
Gen"]

[Gen $\stackrel{\text{pyk}}{\rightarrow}$ "rule gen"]

L3.2(a)

[L3.2(a) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash [\, [\, S5 \gg [\, [\, \dot{a} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{a} \,] \,] \,] ; [\, [\, [\, Gen \rhd [\, [$
$\dot{a} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{a} \,] \,] \,] \gg \dot{\forall}\dot{a} \colon [\, [\, \dot{a} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{a} \,] \,] \,] ; [\, [\, [\, A4 @ [\, \dot{x} \,] \,] \gg [\, [$
$\dot{\forall}\dot{a} \colon [\, [\, \dot{a} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{a} \,] \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{x} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \,] \,] ; [\, [\, [\, [\, MP \rhd [\, [\, \dot{\forall}\dot{a} \colon$
$[\, [\, \dot{a} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{a} \,] \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{x} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \,] \,] \rhd \dot{\forall}\dot{a} \colon [\, [\, \dot{a} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{a} \,] \,]$
$] \gg [\, [\, \dot{x} \mathbin{\dot{+}} \dot{0} \,] \stackrel{p}{=} [\, \dot{x} \,] \,] \,] ; [\, [\, S1 \gg [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{c} \,] \,] \mathrel{\dot{\Rightarrow}}$
$[\, \dot{b} \stackrel{p}{=} [\, \dot{c} \,] \,] \,] \,] \,] ; [\, [\, [\, Gen \rhd [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{c} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=}$
$[\, \dot{c} \,] \,] \,] \,] \,] \gg \dot{\forall}\dot{c} \colon [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{c} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{c} \,] \,] \,] \,] \,] ;$
$[\, [\, [\, A4 @ [\, \dot{x} \,] \,] \gg [\, [\, \dot{\forall}\dot{c} \colon [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{c} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{c} \,]$
$] \,] \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{x} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{x} \,] \,] \,] \,] \,] \,] ; [\, [\, [$
$[\, MP \rhd [\, [\, \dot{\forall}\dot{c} \colon [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{c} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{c} \,] \,] \,] \,] \,] \mathrel{\dot{\Rightarrow}} [$
$[\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{x} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{x} \,] \,] \,] \,] \,] \,] \rhd \dot{\forall}\dot{c} \colon [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,]$
$\mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{c} \,] \,] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{c} \,] \,] \,] \,] \,] \gg [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{x} \,]$
$] \mathrel{\dot{\Rightarrow}} [\, \dot{b} \stackrel{p}{=} [\, \dot{x} \,] \,] \,] \,] \,] ; [\, [\, [\, Gen \rhd [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{b} \,] \,] \mathrel{\dot{\Rightarrow}} [\, [\, \dot{a} \stackrel{p}{=} [\, \dot{x} \,] \,] \mathrel{\dot{\Rightarrow}} [$

$\dot{b} \stackrel{p}{=} [\dot{x}]]]]] \gg \dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]$
$]] ; [[[\mathrm{A4} @ [\dot{x}]] \gg [[\dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=}$
$[\dot{x}]]]]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] ; [$
$[[[\mathrm{MP} \rhd [[\dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]]]$
$\dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \rhd \dot{\forall}\dot{b}\colon [[\dot{a} \stackrel{p}{=} [$
$\dot{b}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{b} \stackrel{p}{=} [\dot{x}]]]]] \gg [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=}$
$[\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] ; [[[\mathrm{Gen} \rhd [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]]$
$\dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \gg \dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]$
$]]]] ; [[[\mathrm{A4} @ [\dot{x} \dot{+} \dot{0}]] \gg [[\dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]]$
$\dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]$
$\dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] ; [[[[\mathrm{MP} \rhd [[\dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}$
$]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]$
$] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]] \rhd \dot{\forall}\dot{a}\colon [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[\dot{a} \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [$
$\dot{x}]]]]] \gg [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}$
$]]]]] ; [[[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow}$
$[\dot{x} \stackrel{p}{=} [\dot{x}]]]]] \rhd [[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [$
$\dot{x} \stackrel{p}{=} [\dot{x}]]]] ; [[[\mathrm{MP} \rhd [[[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]] \dot{\Rightarrow} [\dot{x} \stackrel{p}{=} [\dot{x}]]]] \rhd [$
$[\dot{x} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{x}]]] \gg [\dot{x} \stackrel{p}{=} [\dot{x}]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$

$[\mathrm{L3.2(a)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash [\dot{x} \stackrel{p}{=} [\dot{x}]]]$

$[\mathrm{L3.2(a)} \stackrel{\mathrm{tex}}{\rightarrow}$ “
L3.2(a)”]

$[\mathrm{L3.2(a)} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma l three two a”]

S′

$[\mathrm{S}' \stackrel{\mathrm{stmt}}{\rightarrow} [\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a}' \stackrel{p}{=} [\underline{b}']] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{b}]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [\underline{a} \dot{\Rightarrow} [$
$\underline{b} \dot{\Rightarrow} \underline{a}]]] \oplus [[\forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\mathrm{nonfree}(\underline{x}, \underline{a}) \Vdash [[\dot{\forall} \underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [$
$\underline{a} \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{b}]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{\cdot} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{\cdot} \underline{b}] \dot{+} \underline{a}]]] \oplus [[$
$\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [\underline{a}' \stackrel{p}{=} [\underline{b}']]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]$
$]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \dot{\neg} \underline{a}] \dot{\Rightarrow} [[[\dot{\neg} \underline{b}] \dot{\Rightarrow} \underline{a}] \dot{\Rightarrow} \underline{b}]]] \oplus [[$
$\forall \underline{a}\colon \forall \underline{b}\colon [[\underline{a} \dot{+} [\underline{b}']] \stackrel{p}{=} [[\underline{a} \dot{+} \underline{b}] ']]] \oplus [[\forall \underline{a}\colon \neg [\dot{0} \stackrel{p}{=} [\underline{a}']]]] \oplus [[$
$\forall \underline{x}\colon \forall \underline{a}\colon [\underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}]] \oplus [[\forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [[\dot{\forall} \underline{x}\colon \underline{b}]$
$\dot{\Rightarrow} \underline{a}]]] \oplus [[\forall \underline{a}\colon [\underline{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [[$
$\underline{a} \stackrel{p}{=} \underline{c}] \dot{\Rightarrow} [\underline{b} \stackrel{p}{=} \underline{c}]]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [$
$\underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\underline{b} \dot{\Rightarrow} [[\dot{\forall} \underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}]]]]] \oplus [[\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$
$[[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \oplus \forall \underline{a}\colon [[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}]$
$]]]]]]]]]]]]]]]$

$[\mathrm{S}' \stackrel{\mathrm{tex}}{\rightarrow}$ “
S′”]

[$\mathrm{S}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "system prime s"]

$\mathrm{A1}'$

[$\mathrm{A1}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{A1}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,]$]

[$\mathrm{A1}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
A1"]

[$\mathrm{A1}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime a one"]

$\mathrm{A2}'$

[$\mathrm{A2}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{A2}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [\, [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \dot{\Rightarrow} [\, [\, \underline{a} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} [\, \underline{a} \dot{\Rightarrow} \underline{c} \,] \,] \,]$]

[$\mathrm{A2}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
A2"]

[$\mathrm{A2}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime a two"]

$\mathrm{A3}'$

[$\mathrm{A3}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{A3}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\, [\, [\, \dot{\neg} \underline{b} \,] \dot{\Rightarrow} \dot{\neg} \underline{a} \,] \dot{\Rightarrow} [\, [\, [\, \dot{\neg} \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \dot{\Rightarrow} \underline{b} \,] \,]$]

[$\mathrm{A3}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
A3"]

[$\mathrm{A3}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime a three"]

$\mathrm{A4}'$

[$\mathrm{A4}' \stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{A4}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{c} \colon \forall \underline{a} \colon \forall \underline{x} \colon \forall \underline{b} \colon [\, \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\, [\, \dot{\forall} \underline{x} \colon \underline{b} \,] \dot{\Rightarrow} \underline{a} \,] \,]$]

[$\mathrm{A4}' \stackrel{\mathrm{tex}}{\rightarrow}$ "
A4"]

[$\mathrm{A4}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "axiom prime a four"]

A5′

[A5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[A5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \dot{\text{nonfree}}(\underline{x}, \underline{a}) \Vdash [\ [\ \dot{\forall}\underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ]\ ] \dot{\Rightarrow} [\ \underline{a} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{b}\ ]\ ]\ ]$]

[A5′ $\stackrel{\text{tex}}{\rightarrow}$ “
A5’”]

[A5′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime a five”]

S1′

[S1′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S1′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \stackrel{\text{p}}{=} \underline{b}\ ] \dot{\Rightarrow} [\ [\ \underline{a} \stackrel{\text{p}}{=} \underline{c}\ ] \dot{\Rightarrow} [\ \underline{b} \stackrel{\text{p}}{=} \underline{c}\ ]\ ]\ ]$]

[S1′ $\stackrel{\text{tex}}{\rightarrow}$ “
S1’”]

[S1′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s one”]

S2′

[S2′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S2′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \stackrel{\text{p}}{=} \underline{b}\ ] \dot{\Rightarrow} [\ \underline{a}' \stackrel{\text{p}}{=} [\ \underline{b}'\ ]\ ]\ ]$]

[S2′ $\stackrel{\text{tex}}{\rightarrow}$ “
S2’”]

[S2′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s two”]

S3′

[S3′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S3′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \neg [\ \dot{0} \stackrel{\text{p}}{=} [\ \underline{a}'\ ]\ ]$]

[S3′ $\stackrel{\text{tex}}{\rightarrow}$ “
S3’”]

[S3′ $\stackrel{\text{pyk}}{\rightarrow}$ “axiom prime s three”]

S4′

[S4′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S4′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' \stackrel{p}{=} [\ \underline{b}'\ ]\ ]\ \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ ]$]

[S4′ $\stackrel{\text{tex}}{\rightarrow}$ "
S4'"]

[S4′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s four"]

S5′

[S5′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S5′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]$]

[S5′ $\stackrel{\text{tex}}{\rightarrow}$ "
S5'"]

[S5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s five"]

S6′

[S6′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{+} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{+} \underline{b}\ ]'\ ]\ ]$]

[S6′ $\stackrel{\text{tex}}{\rightarrow}$ "
S6'"]

[S6′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s six"]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{\cdot} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]$]

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7'"]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"]

S8′

[S8′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{:}} [\ \underline{b}'\ ]\ ] \overset{\mathrm{p}}{=} [\ [\ \underline{a} \mathrel{\dot{:}} \underline{b}\ ] \dot{+} \underline{a}\ ]\ ]$]

[S8′ $\overset{\text{tex}}{\rightarrow}$ "
S8′"]

[S8′ $\overset{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \mathrel{\dot{\Rightarrow}} [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{c}\ ]\ ] \mathrel{\dot{\Rightarrow}} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]$]

[S9′ $\overset{\text{tex}}{\rightarrow}$ "
S9′"]

[S9′ $\overset{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \mathrel{\dot{\Rightarrow}} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP′ $\overset{\text{tex}}{\rightarrow}$ "
MP′"]

[MP′ $\overset{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S}' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen′ $\overset{\text{tex}}{\rightarrow}$ "
Gen′"]

[Gen′ $\overset{\text{pyk}}{\rightarrow}$ "rule prime gen"]

L3.2(a)′

[L3.2(a)′ $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a}\colon [\ [\ S5' \gg [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ]\ ; [\ [\ S1' \gg [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ MP' \rhd [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ] \rhd [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ] \gg [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ; [\ [\ [\ MP' \rhd [\ [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ] \dot{\Rightarrow} [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ] \rhd [\ [\ \underline{a} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{a}\ ]\ ] \gg [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]\ ]\ ]\ ]\ ]\ ]\rceil, p_0, c)$]

[L3.2(a)′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ \underline{a} \stackrel{p}{=} \underline{a}\ ]$]

[L3.2(a)′ $\stackrel{\text{tex}}{\rightarrow}$ "
L3.2(a)′"]

[L3.2(a)′ $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime l three two a"]

$\dot{*}$

[$\dot{x}$ $\stackrel{\text{tex}}{\rightarrow}$ "
\dot{#1.
}"]

[$\dot{x}$ $\stackrel{\text{pyk}}{\rightarrow}$ "* peano var"]

$*'$

[x' $\stackrel{\text{tex}}{\rightarrow}$ "#1.′"]

[x' $\stackrel{\text{pyk}}{\rightarrow}$ "* peano succ"]

$* \mathop{\dot{\cdot}} *$

[$x \mathop{\dot{\cdot}} y$ $\stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{\cdot}} #2."]

[$x \mathop{\dot{\cdot}} y$ $\stackrel{\text{pyk}}{\rightarrow}$ "* peano times *"]

$* \mathop{\dot{+}} *$

[$x \mathop{\dot{+}} y$ $\stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{+}} #2."]

[$x \mathop{\dot{+}} y$ $\stackrel{\text{pyk}}{\rightarrow}$ "* peano plus *"]

$* \stackrel{p}{=} *$

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\mathrm{pyk}}{\rightarrow}$ "* peano is *"]

$*^{\mathcal{P}}$

$[\mathsf{x}^{\mathcal{P}} \stackrel{\mathrm{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil]$

$[\mathsf{x}^{\mathcal{P}} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

$[\mathsf{x}^{\mathcal{P}} \stackrel{\mathrm{pyk}}{\rightarrow}$ "* is peano var"]

$\dot{\neg}\, *$

$[\dot{\neg}\, \mathsf{x} \stackrel{\mathrm{tex}}{\rightarrow}$ "
\dot{\neg}\, #1."]

$[\dot{\neg}\, \mathsf{x} \stackrel{\mathrm{pyk}}{\rightarrow}$ "peano not *"]

$* \mathrel{\dot{\wedge}} *$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \mathrel{\ddot{=}} \dot{\neg}\, (\mathsf{x} \mathrel{\dot{\Rightarrow}} \dot{\neg}\, \mathsf{y})] \rceil)]$

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\wedge}} #2."]

$[\mathsf{x} \mathrel{\dot{\wedge}} \mathsf{y} \stackrel{\mathrm{pyk}}{\rightarrow}$ "* peano and *"]

$* \mathrel{\dot{\vee}} *$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \mathrel{\ddot{=}} [\, \dot{\neg}\, \mathsf{x}\,] \mathrel{\dot{\Rightarrow}} \mathsf{y}] \rceil)]$

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\vee}} #2."]

$[\mathsf{x} \mathrel{\dot{\vee}} \mathsf{y} \stackrel{\mathrm{pyk}}{\rightarrow}$ "* peano or *"]

$\dot{\forall}*\colon *$

[$\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\forall} #1.
\colon #2."]

[$\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "peano all * indeed *"]

$\dot{\exists}*\colon *$

[$\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{\exists}\mathsf{x}\colon \mathsf{y} \ddot{=} \dot{\neg}\dot{\forall}\mathsf{x}\colon \dot{\neg}\,\mathsf{y}]\rceil)$]

[$\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\exists} #1.
\colon #2."]

[$\dot{\exists}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "peano exist * indeed *"]

$* \mathrel{\dot{\Rightarrow}} *$

[$\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

[$\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano imply *"]

$* \mathrel{\dot{\Leftrightarrow}} *$

[$\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \ddot{=} (\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y}) \mathrel{\dot{\wedge}} (\mathsf{y} \mathrel{\dot{\Rightarrow}} \mathsf{x})]\rceil)$]

[$\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

[$\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano iff *"]

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-20.UTC:11:21:10.729935 = MJD-53541.TAI:11:21:42.729935 = LGT-4625983302729935e-6