Troll, A Language for Specifying Dice-Rolls

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ACM SAC 2009

Why have a DSL for dice-rolls?

- Concise and unambiguous descriptions for communicating between people.
- Internet dice servers.
- Probability calculations for
  - Figuring your chances (player).
  - Deciding difficulty level (GM).
  - Design-space exploration (game designer).

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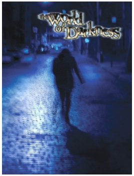
- **Roll** was used in the design of the latest version of the game "World of Darkness" from 2004.



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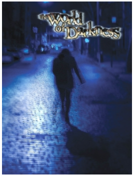

- Some dice-rolls were not easy to describe in **Roll**, so in 2006 I made the successor **Troll**.

Elements of **Troll**

- A roll is a *collection* (multiset) of numbers:
  - Order is irrelevant
  - Number of occurences is significant.
- A collection with one element can be used as a number. Some operations require this.
- Collections can be combined, filtered, counted, summed and in other ways manipulated to find a final result.
- Two different semantics:
  - Random rolling
  - Calculation of probability distribution

Basic **Troll** operations

- `d`N rolls a single N-sided die.
- M`d`N rolls M N-sided dice and makes a collection of the results.
- `sum` C adds the elements in the collection C.
- `count` C counts the elements in the collection C.
- `+, -, *, /` do arithmetic on numbers.
- `@` finds the union of two collections.
- M `<` C returns the elements of C that are greater than M. Also for `=, >, <=, >=, =/=`.
- `min` and `max` find the smallest or largest element in a collection, respectively.
- `least` N and `largest` N find the least or largest N elements of a collection.

Simple `Troll` definitions

- `sum 2d10 + 3`
  Adds two ten-sided dice and adds 3 to the result.
- `sum largest 3 4d6`
  adds the largest 3 of 4 six-sided dice.
- `count 7 < 6d10`
  counts how many out of six d10s are greater than 7.
- `max 3d20`
  finds the largest of three d20.

Advanced features

- M `#` e makes M *independent* samples of expression e and combines the results using `@`.
- `if` C `then` e_1 `else` e_2 If C is non-empty, do e_2, otherwise do e_3.
- x `:=` e_1`;` e_2 defines x to be the value of e_1 inside e_2. x is sampled once and this value used for every occurrence of x inside e_2.
- `repeat` x `:=` e_1 `until` e_2 repeats rolling e_1 until the expression e_2 evaluates to non-empty, then returns last value of e_1.
- `accumulate` x `:=` e_1 `until` e_2 repeats rolling e_1 until the expression e_2 evaluates to non-empty, then returns the union of all values of e_1.
- `foreach` x `in` e_1 `do` e_2 calculates e_1, and for each number n in the result evaluates e_2 with x bound to n, then unions the results of e_2.

Advanced examples

- `b := 2d6; if (min b) = (max b) then b@b else b`
  Backgammon dice.
- `count 7< N#(accumulate x:=d10 while x=10)`
  Die roll for *World of Darkness*.
- `repeat x := 2d6 until (min x) < (max x)`
  Roll two d6 until you don't have a double.
- `x := 7d10; max foreach i in 1..10 do sum i= x`
  Largest sum of identical dice.

Implementation

- The two semantics:

  Random rolls is implemented fairly straightforwardly using a PRNG.

  Probability distribution implemented by enumerating all possible rolls and counting results.

Time or Space

Enumerating all possible rolls can be done in several ways:

- **In time:** Backtrack over all possible rolls, counting at top-level. Advantage: Low space use (only top-level distribution is stored).
- **In space:** Find distributions for subexpressions and combine these to find distribution for complete expression. Advantage: Can combine identical subresults and exploit certain properties of functions.

It turns out that the latter far outweighs the former (details in paper).

Representation of probability distributions

Simple representation: Set of (value, probability) pairs:

$$\{(2, 0.25),\ (3, 0.5),\ (4, 0.25)\}$$

Unnormalised representation to exploit algebraic properties of functions:

$$D \equiv M! \quad + \quad D \cup D \quad + \quad D\,|_p\,D \quad + \quad 2{\times}D$$

- $M!$ means "M with probability 1" where M is a multiset of numbers.
- $d_1 \cup d_2$ combines all outcomes of d_1 and d_2 by union.
- $d_1\,|_p\,d_2$ chooses between the outcomes of d_1 and d_2 with probability p of choosing from d_1.
- $2{\times}d$ is an abbreviation of $d \cup d$.

Main idea: Avoid combinatorial explosion of unioning two distributions.

Linear functions

$$f(M_1 \cup M_2) = f(M_1) \cup f(M_2)$$

Examples: `7<`, `6=`, `foreach`
Can be lifted to unnormalised distributions:

$$\begin{array}{lcl} f(M!) & = & f(M)! \\ f(d_1 \cup d_2) & = & f(d_1) \cup f(d_2) \\ f(d_1 \,|_p\, d_2) & = & f(d_1) \,|_p\, f(d_2) \\ f(2 \times d) & = & 2 \times f(d) \end{array}$$

Homomorphic functions

$$\exists \oplus : \; f(M_1 \cup M_2) = f(M_1) \oplus f(M_2)$$

Examples: `sum, count, min, least` N`, if, different`

Can be lifted to unnormalised distributions:

$$\begin{array}{lcl}
f(M!) & = & f(M)! \\
f(d_1 \cup d_2) & = & f(d_1) \hat{\oplus} f(d_2) \\
f(d_1 \,|_p\, d_2) & = & f(d_1) \,|_p\, f(d_2) \\
f(2 \times d) & = & \oplus^2 f(d) \\
 & & \\
M! \hat{\oplus} N! & = & (M \oplus N)! \\
(d_1 \,|_p\, d_2) \hat{\oplus} d_3 & = & (d_1 \hat{\oplus} d_3) \,|_p\, (d_2 \hat{\oplus} d_3) \\
d_1 \hat{\oplus} (d_2 \,|_p\, d_3) & = & (d_1 \hat{\oplus} d_2) \,|_p\, (d_1 \hat{\oplus} d_3) \\
 & & \\
\oplus^2 M! & = & (M \oplus M)! \\
\oplus^2 (d_1 \,|_p\, d_2) & = & (\oplus^2 d_1) \,|_{p^2}\, ((\oplus^2 d_2) \,|_{\frac{(1-p)^2}{(1-p^2)}}\, (d_1 \hat{\oplus} d_2))
\end{array}$$

Loop optimizations

Exploit that `repeat` and `accumulate` have unchanged conditions in all iterations:

- Distribution of body calculated once, then rewritten into the form

$$d_1 \,|_p\, d_2$$

  where the values in d_1 fulfil the condition and values in d_2 don't.
- For `repeat-until`, the resulting distribution is d_1.
- For `accumulate-until`, the resulting distribution d' is given by the equation

$$d' = d_1 \,|_p\, (d_2 \cup d')$$

  Solution is infinite, but cut off after specified limit.

Experiences with **Troll**

- Non-programmers can write simple definitions.
- While optimisations help a lot, sometimes **Troll** needs to enumerate all combinations, which may be slow.
- New features added occasionally by request from users (latest: text and recursive function definitions).
- Download from `www.diku.dk/~torbenm/Troll` (Requires Moscow ML).