# Understanding the Hough transform: Hough cell support and its utilisation.

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*ABSTRACT*

The Standard Hough Transform (SHT) is used to find lines, circles and other image features in edge maps containing edge points. Edge points which potentially can 'vote' for a given line (called its 'line support') are highly dependent of line position and orientation due to the boundedness of images. If accumulator array peaks are used to find lines then peripherally located lines are missed. Uniform feature detection probability in all parts of the image requires normalization of the accumulator array by the support. The paper describes some further applications of the concept of support when finding lines, e.g. determining dominant directions, and gives some guidelines on the choice of the coordinate system origin.

**Keywords:**

Hough transform, Curve detection, Hough space cell support, Bias, Parameter quantisation.

## 1. Introduction

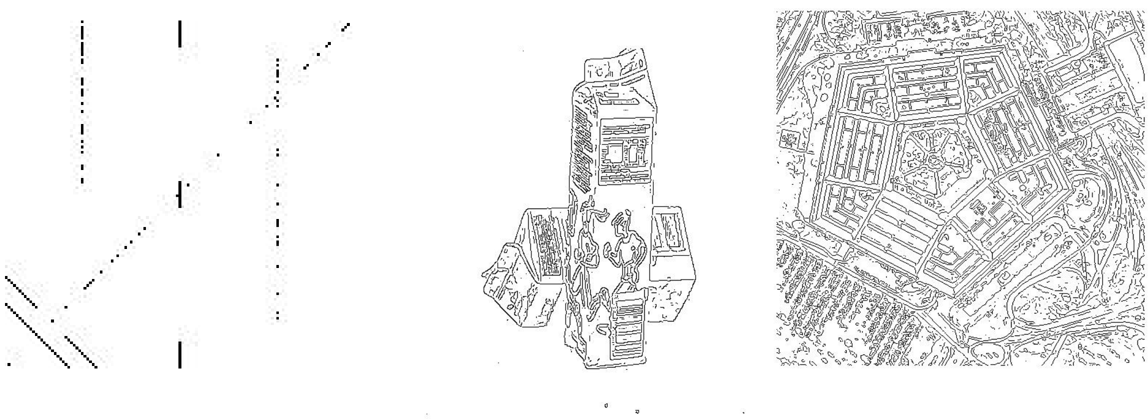
The Standard Hough Transform (SHT) is used to find groups of pixels forming geometrical elements from an edge point image which typically is the result of using an edge detector on a grey-level image. Examples of edge point images are shown in figure 1. In this paper we restrict ourselves to elements being straight line segments of width one to two pixels.

As the Hough transform may be used as well in a bootstrapping procedure (i.e. extracting data from a digital image of which nothing is known a priori) as in later processing of it, we avoid arbitrary assumptions about it, like " ... a short line, which is near the edge of the image space. In general an image will not be composed of such lines." (Leavers [9], p. 76).

For each edge point votes are cast in the Hough parameter space for those parameter combinations which result in a line through the point (see section 2 for a brief description). By associating an integer value (or Hough cell) to each parameter combination, this may be done by counting in the cell. A parameter combination having a significant number of votes is a candidate for a line. The support for a line or more precisely a Hough cell may be defined as the set of pixels which potentially counts in it. If the size of the support for a given cell is the number of pixels in the set, it is easily seen that because the edge image is bounded, the size of the support as a function of the parameters will not be constant. Cohen and Toussaint

[3] showed this for a circular retina. For rectangular images the support will be largest for diagonal lines through the center, and be near zero for lines in corners. The support is given approximately by the area covered by the the inverse Hough transform of a cell representing a given parameter combination; it has the general shape of a bow tie (see Princen et al. [10], where its is called the template shape, Ratley & Lundgren [12] and figure 3).

If line candidates are selected simply by choosing those cells which have the largest counts, disregarding the support, the line finding process may have a bias against lines in the corners, favouring the diagonals and lines parallel to the image boundaries. For noisy images, noise in the central part may actually overshadow lines near the boundary. This bias is relatively insignificant for sparse images or images with edge points mainly in the central part, but for dense images with a lot of texture (like figure 1c) it is quite pronounced and the votes have to be normalised in some way before the selection process starts (this is discussed in section 4). Filtering the Hough space to enhance the peak counts (Leavers [9], Princen et al. [10], van Veen and Groen [13]) will not affect the bias.



(a) (b) (c)

*Figure 1: Edge images. In (a) is shown a simple test image ("test") featuring six lines (128x128, 161 edge points (1% black)). In (b) is a fairly typical test image ("milk3") with many lines in two directions (512x512, 12206 edge points (4.7% black)). The original image is quite noisy. In (c) is shown a dense edge image ("pentagon"), with lines in five dominant directions (512x512, $M(E) =$ 40481 edge points (15.4% black)).*

The support may be used for other purposes than normalisation of the counts. For a completely filled edge image (completely "black"), the count in a cell is a close approximation to the size of the support for it. If an edge image has no structure, edge points will be distributed uniformly, and the number of votes cast for a given parameter combination would be expected to be a constant fraction of the support. The presense of structure in form of lines will be indicated by a vote significantly different.

This may be used to find dominant line directions (Andersen & Hansen [1]), as discussed briefly below.

## 2. The Standard Hough Transform (SHT), and notation used

The Standard Hough Transform (SHT) is used to find points forming lines in an edge (point) image called $E$ (dimensions $N_r$ rows and $N_c$ columns), which typically is the result of using an edge detector on a grey-level image $I$. Edge points are marked e.g. as black on a white background. The number of edge points is called $M(E)$. Examples of edge images are shown in figure 1. The three examples are chosen because they illustrate how different edge images may be. The test image (figure 1a) is generated by a program and has few edge points forming six clearly visible groups each corresponding to a line. Figure 1b is a typical scene image with a medium number of edge points and a mixture of lines and texture. Figure 1c is a difficult image with a high density of edge points and much local texture, and a small number of dominant line directions.

The Hough transform is based on lines being parametrisised by two parameters. A Hough transform commonly used (Duda & Hart [4]) assumes polar representation of a line, a line being the points

$$L_{\theta,\rho} = \{(x, y) | \rho = x\cos(\theta) + y\sin(\theta)\} \tag{1}$$

$\theta$ being in $[0; \pi[$. The coordinates $(x, y)$ are in a coordinate system with origin $O$; here center $(\left\lfloor \frac{N_r}{2} \right\rfloor, \left\lfloor \frac{N_c - 1}{2} \right\rfloor)$ of the image is chosen as $O$. $\theta$ and $\rho$ are both limited to a number of discrete values $N_\theta$ and $N_\rho$. In this paper nearly all examples use the values $N_\theta = 45$ (corresponding to a resolution of 4°) and $N_\rho = 256$ (corresponding to a resolution of 2.8 pixels in a 512 by 512 image). These values have been chosen to obtain clear plots illustrating the principles rather than useful results, but they are still in the range of useful values according to Lam et.al. [8]. The effects of peak spreading and peak extension can be observed, but are not significant for this paper.

An image $E$ is transformed by the SHT by taking each edge pixel $P = (x, y)$ in $E$ and incrementing a counter (also called a cell in the Hough space) for each possible coordinate pair $(\theta, \rho)$. If the polar coordinates for $P$ are $(v_p, R_p)$, $P$ will be represented by the Hough transform as a sinusoid

$$\rho = R_p \cos(\theta - v_p) \tag{2}$$

The maximum value for $R_p$ is $R_E = \frac{1}{2}\sqrt{N_r^2 + N_c^2}$.

The family of sinusoids resulting from a Hough transform of a line will form a butterfly (Leavers [9], Lam et al. [8], and figure 3b). Typically $\theta$ will be chosen from the set $\theta_i = \frac{(i-1)\pi}{N_\theta}$ ( $i$ in $[1; N_\theta]$), and $\rho$ is restricted to a member of $\rho_j = (\frac{N_\rho}{2} - j + 1)R_E/N_\rho$ ($j$ in $[1; N_\rho]$). Figure 2 shows plots of the results of two Hough transforms. The letters $i$ and $j$ are in the following used for the indices (called theta and rho indices) corresponding to $\theta_i$ and $\rho_j$ values. A cell $H_{ij}$ in the Hough space with a high count indicates a candidate for a line. This is clearly illustrated in figure 2a.

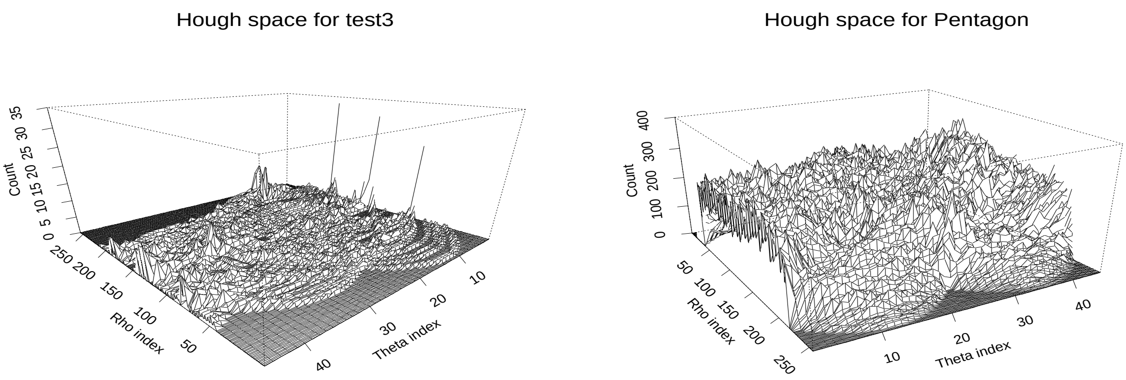
The Hough cells forms a 2D histogram. For certain purposes it may be viewed alternatively as a collection of 1D histograms; an example is the directional analysis desribed later. This idea is also found in Cohen and Toussaint [3]. For a given $\theta$ the counter values $H_{ij}$ for all $\rho$ values form a histogram. Two histograms for $\theta = 25°$ and 36° are shown in figure 4. Lines corresponding to the maxima are plotted in figure 5. The two histograms have a common $\cap$-shaped form. A closer examination shows that of the 20 maxima in figure 4a, only the leftmost 10 correspond to actual lines. The 11 maxima in figure 4b do not correspond to lines at all. The form of the histogram is discussed further in section 3.

The effects of the choice of values for $N_\theta$ and $N_\rho$ and the effects of over- or undersampling (respectively under- and overquantisation) are discussed in Lam et al. [8], Kiryati & Bruckstein [6] and and van Veen and Groen [13]. The effects of choice of origin for the image $E$, or of the image $E$ being bounded by a rectangle have however not been investigated, and will be topics covered by this paper.

## 3. Support and bias

In this section we will define the concept of support for a Hough cell, and show how the traditional method of selecting candidates by using maxima in the Hough space will result in a bias favouring diagonal lines in the middle of the image, if the edge image is sufficiently dense and uniform.

The *voting support* $V(C)$ for a cell $C_{\theta,\rho}$ is defined as the set of edge points in $E$ that causes that cell to be incremented. The *support* $S(C)$ for a cell $C_{\theta,\rho}$ is defined as the set of points in $E$ which potentially is in $V(C)$. $S(C)$ has the general shape of a bow tie, the borders of which are formed by the lines corresponding to $(\theta, \rho)$ plus/minus half the cell size. This is illustrated in figure 3. Princen et al. [10] calls this bow tie shape a template shape, in this case corresponding to an rectangular cell. The support for a cell is a particular instance an template. We distinguish between the support for a cell and the support for the line $L_{\theta,\rho}$, because the latter is not well defined.




(a) (b)

*Figure 2: Hough transforms of two of the images in figure 1: (a) test image (1a). Note the six peaks corresponding to three vertical lines ($\theta$ index 1), one diagonal ($\theta$ index 34) through the origin O and two diagonals in the left lower corner ($\theta$ index 12). In (b) is shown the Hough transform for pentagon (1c). The peaks corresponding to approximately 100 lines in the five dominant directions ($\theta$ indices 7, 16, 25, 34 and 43) are not clearly distinguishable.*

The following observations hold for the support: (1) if $\theta$ is undersampled, the support for neighbour cells in the $\theta$ direction may overlap; (2) a line may have only a few pixels in common with the nearest cell; (3) if $\theta$ is limited to values $\theta_i$, the cell shrinks to width zero, and the points of of $S(C)$ will fall between parallel lines in a band of width $\Delta\rho = R_E/N_\rho$ (Princen et al. [10]); (4) if $\theta$ is undersampled (or $\rho$ overquantisised), some points may be excluded from the support although they actually belong to it.

Point (4) needs an explanation. If voting is done by incrementing counters in cells indicated by the sinusoid $R_p \cos(\theta_i - v_p)$, cells corresponding to some $\rho_j$ may be skipped where the sinusoid declines or inclines steeply, unless $\theta$ is subsampled, or the sinusoid is traversed using interpolation. A pixel may in this way not vote for all possible lines through it, and may consequently be missing from the support for these lines. This will happen whenever $R_p$ is larger than a critical radius $R_{crit}$ defined in section 6. This radius is closely linked to the equivalent line length $L_q$ found in Lam et al. [8], which depends on the dimension of the image and the quantisation scheme.

As the edge image $E$ is bounded (typically rectangular), the size of the support $S(C)$ will not be constant. Lines near the corners of an image will have insignificant support, while the diagonals have large support. Lines parallel to the borders have all equal support which falls in between. This is illustrated as follows. A line between (0,9) and (9,0) has the Euclidean length $10\sqrt{2}$ or 14, while the line between (0,99) and (99,0) has the length $100\sqrt{2}$ or 141. Let us assume that the former is solid, and the latter is missing 85% of the points. The SHT will then select the latter as a better candidate, unless the counts are normalised by the size of the support (giving normalised counts 1.00 and 0.85 respectively). Similar results were demonstrated for a circular retina in Cohen and Toussaint [3], who gives the probability density function for noise as $f(y) = \frac{2}{\pi}\sqrt{1-y^2}$ ($y$ being the distance from the center of the image). The effect is found for any bounded retina independent of its shape, but the density function depends on the shape.

A complication may be that the number of counts coming from the points of a certain line depends on the definition of a digital line. If the points of a line have been generated by a scan-conversion algorithm for drawing 1-pixel lines (see e.g. Foley et. al. [5]), the number of points forming it will be equal to the difference in coordinates; as an example, a line between (0,9) and (9,0) will only have 10 points, one for each first coordinate. van Veen and Groen [13] contains a detailed discussion of this. It is however only relevant if the edge image $E$ is generated by a line drawing program and will not be taken into consideration

(a) (b)

*Figure 3: The inverse Hough transform for an 128 * 128 image. In (a) is shown the inverse Hough transform. Light grey and black indicates which pixels will count in the cell for $\theta = 60°$ and $\rho = +24$ ( $H_{6,25}$); these pixels form the support for that cell. Black indicate the line corresponding to the $\theta_i$ and $\rho_j$ of the cell. Coordinate axes are dark grey. The "bow tie" form is exaggerated by using a very coarse sampling of $\theta$ ($N_\theta = 15$) and $\rho$ ( $N_\rho = 64$), and to allow $\theta$ to have 20 intermediate values between given $\theta_i$ and $\theta_{i+1}$. If this subsampling was not used, the "bow tie" would be a band bounded by two parallel lines with distance 2.8 pixels. In (b) is shown why the support for a cell may miss pixels, using $N_\theta = 36$ and $N_\rho = 128$. The two sinusoids forming the boundary of a "butterfly" form are the Hough transforms for the points (9,3) and (63,123), which are end points for a line segment with $\theta = 65.8°$ and $\rho = 24$ as in (a). The nearest cell has indices $(i, j) = (14, 47)$, corresponding to $\theta = 60°$. Note that as $\rho$ is overquantisised, not all cells having the points in their support will have their counter incremented. The sinusoids are more dense when $N_\theta$ is increased, and for points near the origin (because the amplitude is equal to the distance $R_p$).*
here.

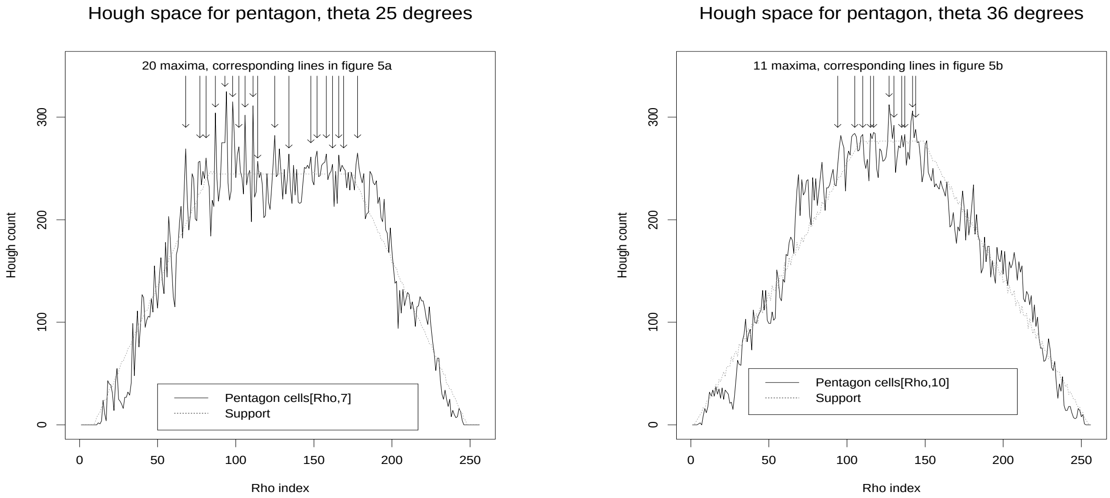
The number of points in $S(C)$ may be computed or it may be measured by doing a SHL on a " black" image consisting entirely of edge points. The resulting Hough space $B$ has the form shown in figure 7, having maxima at the image diagonals. It is important to use the same Hough transform algorithm on the black image to obtain a $B$ usable as described below, as the effects of e.g. aliasing should be identical for both the edge image and the corresponding support image $B$.

If the 40481 edge points in figure 1c were uniformly distributed, it could be expected that the resulting Hough transform would have approximately the same shape as figure 7, all values being scaled down by a factor $\frac{M(E)}{M(allblack)} = \frac{40481}{262144}$; this scaled-down parameter space is called $B'$. In figure 4 two sets of the values of $B'$ have been plotted with the signature "support".

We assume in this paper that the scale has been chosen that the width of lines is one or two pixels.

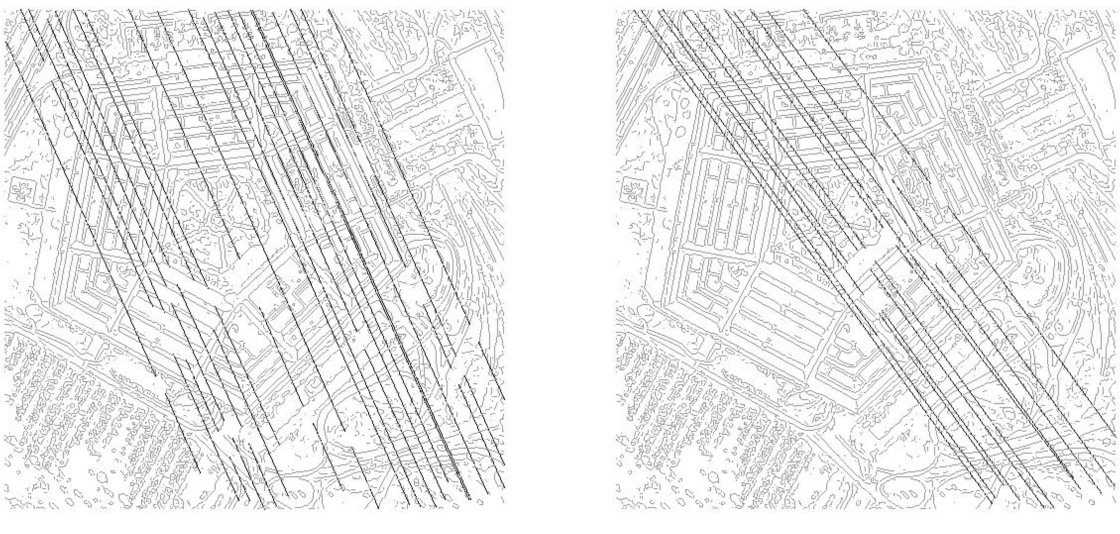
A reasonable assumption about the general form of a Hough space histogram $H$ is that it is the scaled support $B'$ plus or minus an amount due to local variations in the distribution of edge points due to e.g. random variations, texture or lines.

A further assumption based on preliminary analysis and evidence about edge detectors is that we will expect that lines will have a voting support $V(C)$ along the line containing at least an average number of pixels (giving counts above average), surrounded by areas having pixel counts below average. This is because edge detectors suppress other edge points in a small neighbourhood; it can be seen in the edge image as a "white" band on both sides of a line, as illustrated in figure 6. A line will thus appear in the histogram as a peak, as may be seen in figure 4. Such peaks may of course be caused by other features in the image. Princen et al. [11] and Cohen and Toussaint [3] do not consider multiple edge point suppression




(a) (b)

*Figure 4: Hough counts corresponding to one $\theta$. In (a) $\theta$ is 25°, or cell index 7 (of 45). This is one of the dominating directions of 115°. The 20 largest values are marked (see plot in figure 5a). The 10 peaks in the $\rho$ index interval 116 to 178 does not correspond to lines in the image. The image however contains lines corresponding to the peaks corresponding to low $\rho$ values. In (b) $\theta$ is 36°, or cell index 10. It has been chosen as an example of a direction with none or very few lines. The 11 largest values are marked (see plot in figure 5b). They do not correspond to any lines. The signature "support" is the scaled support size as described in the text.*



(a) (b)

*Figure 5: Lines corresponding to one $\theta$. The lines have been plotted with a program which produces line segments instead of lines going between images borders. In (a) $\theta$ is 25°. Lines corresponding to the 20 largest values are plotted. The lowest $\rho$ index 68 corresponds to the upper left line. The lines have all approximately the same support. In (b) $\theta$ is 36°, and the 11 largest values plotted. The lowest $\rho$ index 94 is in the upper left.*

done by some edge detectors, but make the assumption that the distribution is a combination of uniformly distributed additive noise and "real" linepoints.

(a) (b)

*Figure 6: Edge detector behaviour. (a) is a part of figure 1b, and (b) is taken from figure 1c. Edge points forming lines are surrounded by non-edge ("white") pixels. Even in textured regions with no obvious lines, clusters of edge points tend to be surrounded by non-edge bands.*

The concept of support is not limited to lines. Any geometrical element like a circle, an ellipsis, or a parabola will for certain parameter sets have points outside the image boundary, thus decreasing the support. The discussion above will consequently be valid in the general case.

## 4. Avoiding the bias

Using the maxima without taking the support into consideration may introduce a bias favouring lines which are in the middle of an image and at oblique angles, because only a small fraction of a line with large support is needed to outvote a line with small support, even if all points in the latter are present. Another effect is that votes coming from texture in regions with large support may "drown" lines with small support. Because $B' = B * M(E)/(N_r * N_c)$, the significance of $B'$ increases with the total number of edge points. The bias is not a problem for images with a low number of edge points, or when the edge points are in the middle of the image, or if the dominant directions are parallel with the axes, but is quite pronounced in images like "pentagon". This may be observed in figure 2a (a sparse image), and in figure 2b (a dense image).

The problem is to a certain extent avoided using the Random Hough Transform (Kälviäinen et al. [7]). Any random sampling of edge points will reduce the number of edge points contributing votes in the Hough space, thus effectively reducing $M(E)$. This is equivalent to making the original edge image less dense.

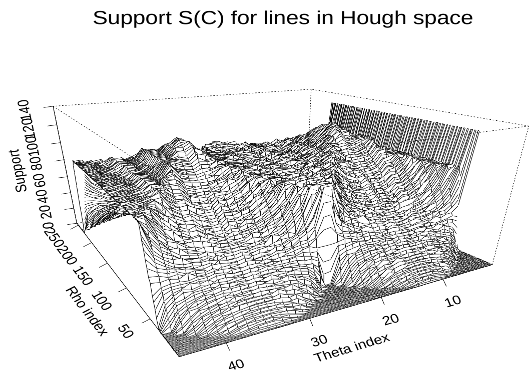



*Figure 7: The support $S(C)$ for all possible cells $C_{\theta,\rho}$ in an 512x512 image, plotted as a function of $\theta$ and $\rho$ indices. The peaks at $\theta$ index 1 is due to undersampling in $\rho$. The two maxima are at the diagonals (($\theta$, $\rho$) indices (12,128) and (34,128)). The zero counts in the border area are for lines outside the $E$ image boundaries.*

The selection of candidates has to be based on some normalisation to avoid the bias. It is clear that the histogram $H$ should be normalised in some way by either the size of the support $B$ or the scaled value $B'$ before the selection process. From the assumptions about $H$ in section 3 it is however not clear how this may be done in the best way. The method used should preserve the peaks corresponding to lines, and scale that part of a peak not due to noise or texture in a way to make selection of a maximum independent of the size of the support. There are several ways to normalise $H$ by either $B$ or $B'$. Normalisation of a quantity $Q$ is done by dividing it by a number $Q_N$. If the normalisation factor is an expected value as well, it may be subtracted before the division $(Q - Q_N)/Q_N$; the squared difference is often used to eliminate the sign and emphasise the difference. Taking combinations, five possibilities could be:

| Variant | New histogram $H'$ computed as |
|---|---|
| 1 | $H'_{ij} = \frac{(H_{ij} - B'_{ij})^2}{B'_{ij}}$ |
| 2 | $H'_{ij} = \frac{H_{ij} - B'_{ij}}{B'_{ij}}$ |
| 3 | $H'_{ij} = \frac{H_{ij} - B_{ij}}{B_{ij}}$ |
| 4 | $H'_{ij} = \frac{H_{ij}}{B_{ij}}$ |
| 5 | $H'_{ij} = H_{ij} - B'_{ij}$ |

When $B_{ij}$ or $B'_{ij}$ is zero in a denominator, $H'_{ij}$ is arbitrarily set to zero.

Variant 1 has the familiar $\chi^2$ form, but this does not indicate whether the count is higher or lower than expected. Furthermore a count which accidentally is equal to the average and has low-count neighbours will give a local minimum rather than a maximum. Variant 2 preserves the sign, but will also produce zero if the count is average. Variant 3 compares the count to the maximum possible, and will give negative values, but with maxima at interesting counters. Variant 4 scales the peaks, but otherwise preserves the shape of the histogram. These four variants all scales the height of the peaks by a number which is small for lines near the image boundary; this has the unfortunate effect to reverse the bias to some extent. Variant 5 avoids this reversal, but does on the other hand not fully remove the bias. The conclusion of this is that to normalise properly, we need amore precise model for $H$.

Two plots showing variant 3 are shown in figures 8 and 9. Compared to figures 4 and 5, this is a clear improvement even when using a method found purely by heuristics.

## 5. Utilising the support

As the scaled support expresses the expected number of votes for evenly distributed edge points, deviations from it (e.g. table 1, variant 1) is a measure of uneveness, caused e.g. by texture or lines. The $\chi^2$-like sum

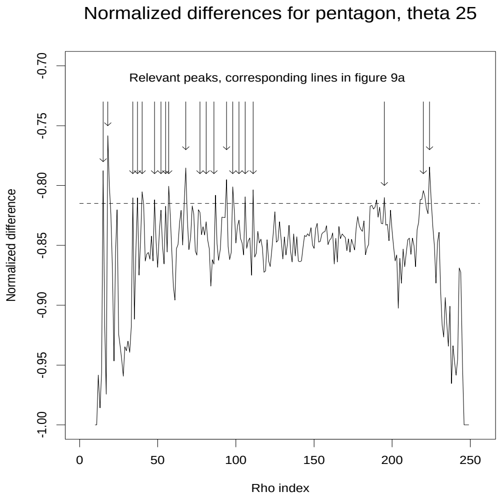
$$S_i = \sum_{j=1}^{N_\theta} \frac{(H_{ij} - B'_{ij})^2}{B'_{ij}} \tag{3}$$

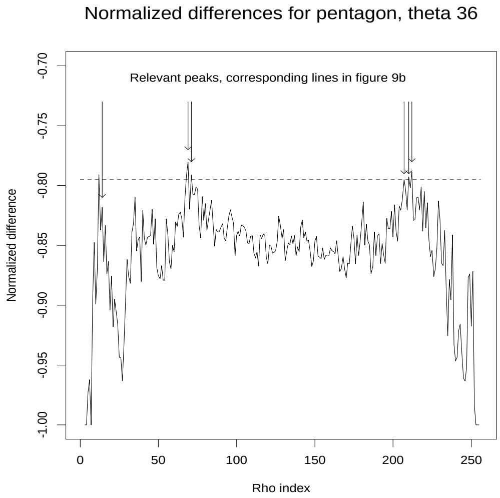
can be used to select $\theta$ values most likely to correspond to dominant line directions (see Andersen & Hansen [1]). This is illustrated in figure 10, which shows that the main directions in the images from figure 1 are seen as peaks in the plot of $S_i$.

The variant 3 in table 1 can be used to select line candidates for a given $\theta$. The method of selecting candidates by looking at peaks is not completely satisfactory. In figure 9a some lines are missing, as they have not corresponding peaks in the histogram. This may be explained by peak spreading and extension. In figure 9b it can on the other hand be seen that peaks in the normalised histogram does not always correspond to actual lines, but is a response to other image features. The selection of candidates have thus to be followed by an evaluation of the candidates to decide if the lines are present or not. This evaluation process is the topic of e.g. Andersen & Hansen [1,2].

## 6. Choosing the origin and quantisation parameters

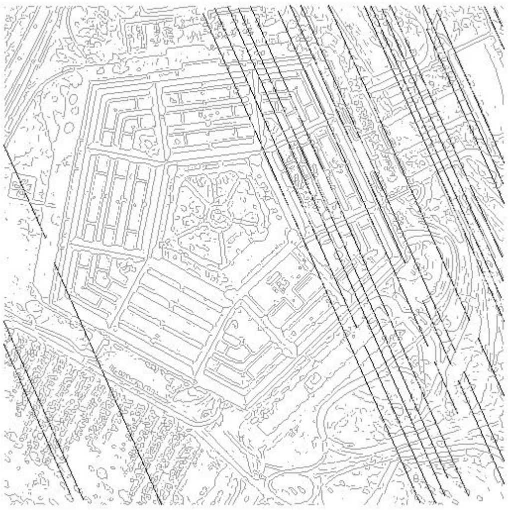
As illustrated in figure 3b, the tangent to the sinusoid may have an slope exceeding 45° for certain points and certain quantisation values $N_\theta$ and $N_\rho$, with the result that a pixel will not cast a vote in all relevant cells. The derivative of equation (2) is $d\rho/d\theta = -R_p \sin(\theta - v_p)$. Transforming this back to image coordinates, and observing that $|-\sin(\theta - v_p)|$ is less or equal to 1, this happens when



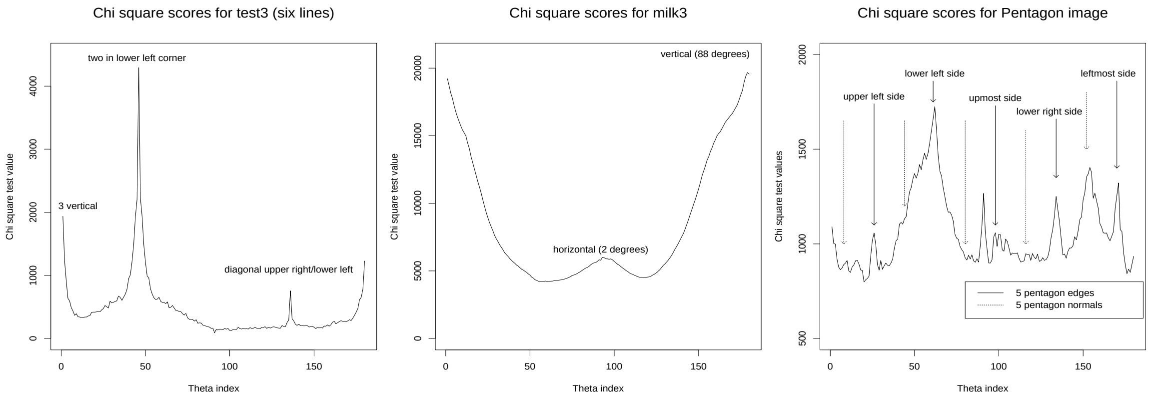

(a) (b)

*Figure 8: Normalised Hough counts, computed as* $(H_{ij} - B_{ij})/B_{ij}$. *In (a)* $\theta$ *is 25°, or cell index* $i = 7$ *(of 45). The 18 most significant peaks are marked (see plot in figure 9a). Note that some of the peaks are common with figure 4a. Most marked values correspond to actual lines. In (b)* $\theta$ *is 36°, or cell index* $i = 10$. *The most significant 6 values are marked (see plot in figure 9b). None of the values correspond to actual lines. The stippled lines (at -0.82 and -0.79) is a visual aid only.*



(a) (b)

*Figure 9: Lines corresponding to one* $\theta$. *In (a)* $\theta$ *is 25°. Lines corresponding to the 18 values in figure 8a are plotted. The lowest* $\rho$ *index 15 corresponds to the upper left line. The lines drawn for* $\rho$ *indices 195, 220 and 224 (lower left three lines) does not correspond to actual lines. Generally the lines plotted have quite different support, the least support for the lines in the upper left and lower right corners. In (b)* $\theta$ *is 36°. Lines corresponding to the 6 values in figure 8b are plotted. The lowest* $\rho$ *index 14 is the upper left line. None of these lines correspond to actual lines.*




(a) (b) (c)

*Figure 10: Finding dominant line directions. The values of $S_i$ from equation (3) are plotted for (a) "test3" (figure 1a, (b) "milk3" (figure 1b) and (c) "pentagon" (figure 1c). The texture in the lower left corner of "pentagon" broadens the peak for theta index 62. In this figure $N_\rho = 256$ and $N_\theta = 180$.*

$$R_p \geq R_{crit} = \frac{N_\theta R_E}{\pi N_\rho} \tag{4}$$

This effect is most pronounced for lines with $\rho$ near zero.

In Lam et al. [8] is defined an equivalent line length $L_q = \dfrac{\Delta_q \rho}{\sin(\Delta_q \theta/2)}$, $\Delta_q \rho$ being the $\rho$ quantisation interval (here $R_E/N_\rho$) and $\Delta_q \theta$ being the $\theta$ interval (here $\pi/N_\theta$). If a line has length less than $L_q$, we will have peak extension, and if the length is greater, we will have peak spreading.

If we approximate $\sin(x)$ by $x$ for small values, we get an approximation

$$L_q = \frac{\Delta_q \rho}{\Delta_q \theta/2} = 2R_{crit}$$

From this it clearly follows that the origin $O$ and maximal radius $R_{\max}$ should be chosen so the circle defined by these two parameters covers the area of interest, $O$ being placed at the barycenter for the area. If nothing is known about an image, the natural choice is to place $O$ at the center of the image itself, and let $R_{\max} = R_E$. This is different from e.g. Leavers [9], who chooses $R_{\max} = N_r/2 - 1$.

The effects of choosing a small value for $R_{crit}$ is that long lines near the diagonal may get fewer votes, and that lines near the corners may not count in the relevant Hough cell.

## 7. Discussion and conclusion

The notion of support has surprisingly enough not found its way to the literature about the SHT. Cohen and Toussaint[3] suggest maximum entropy quantization to avoid the bias problem, but this requires pooling of Hough cells corresponding to regions having less support. This reduces the detection and localization accuracy in low support areas, e.g. near the image corners.

The effects of support for dense edge images has been demonstrated in this paper, and two applications have been mentioned. Support and bias are general concepts, which may be used for the application of the Hough transform on other geometrical elements as well. An explanation of why the problem with bias has only been sparsely mentioned in the literature until now may be that the edge images used have not been very dense or had lines near the corners. Using the Random Hough Transform seems to be another way of avoiding the bias.

The idea of considering one row (or column) in the Hough space as a histogram and combine it with normalisation seems to be a very promising technique which can be used as one step in a multi-step selection-evaluation cycle. In such a process the computation of a $\chi^2$ value for each direction may be used to find dominant line directions. Princen et al. [11] use hypothesis testing and normalize cell counts by dividing by an average of neighbouring cell counts. They use a specific model of a line profile on which they suggest a test statistic. This test statistic corresponds to variant 4 of the methods discussed in section 4 for avoiding bias. Further research is however needed to find a suitable normalisation method for use in the selection process, and to verify that the edge image line model is valid for commonly used edge detectors.

## 8. Acknowledgements

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## 9. Literature

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