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Space-efficient vectors and deques

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Background: the Copenhagen STL

Project start: September 2000

Goal: alternative/enhanced versions of individual STL components

Contributors: ca. 20 students have written parts of the library

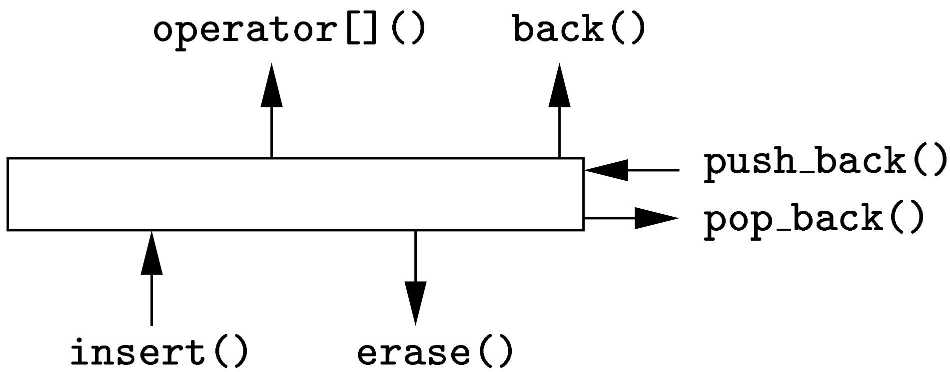
Status: first implementations for the most interesting modules exist

Emphasis: performance engineering, software engineering, algorithmics

Availability: `http://cphstl.dk`

Current problem: How to transfer the existing prototypes to a product?

std::vector in the C++ library



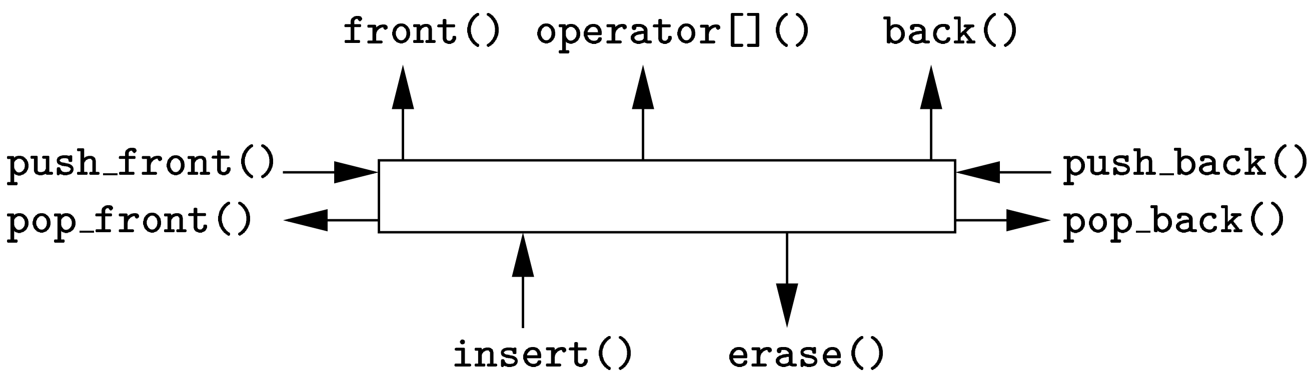
Required by the C++ standard

- sequence operations in $O(1)$ amortized time
- modifying operations in linear time
- according to a technical correction elements must be stored contiguously

SGI STL implementation

- standard doubling technique
- unbounded extra space
- n push_backs require $\Theta(n)$ element moves

`std::deque` in the C++ library



Required by the C++ standard

- sequence operations in $O(1)$ worst-case time
- modifying operations in $O(\min\{i, n-i\})$ time, where i is the insertion/erasure point

SGI STL implementation

- two levels: index blocks and data blocks; data blocks are of a fixed size; only the two extreme data blocks can be non-full
- unbounded extra space
- $O(1)$ amortized time push operations

Earlier results

Vectors and deques [Brodnik et al., 1999]

- sequence operations in $O(1)$ worst-case time
- $O(\sqrt{n})$ extra space (measured in elements and in objects of the built-in types)
- $\Omega(\sqrt{n})$ is a lower bound for the amount of extra space needed

Vectors [Goodrich and Kloss II, 1999]

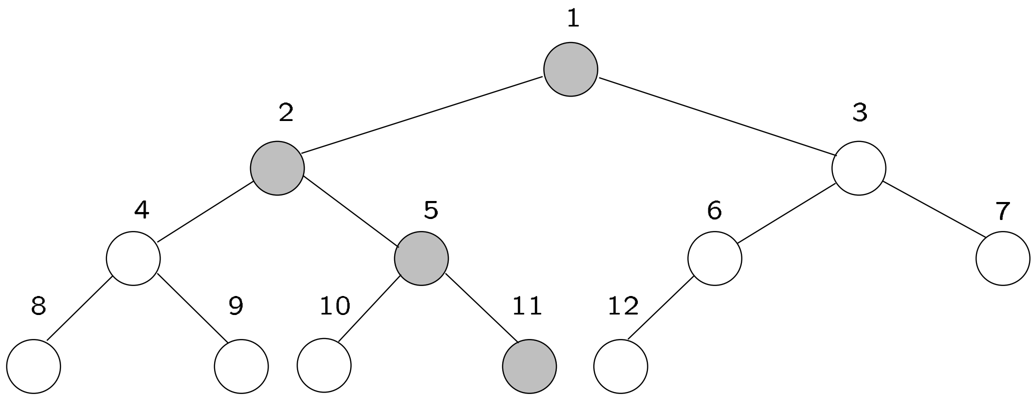
- modifying operations in $O(n^{\varepsilon})$ amortized time for any fixed constant $\varepsilon > 0$

Deques [Mortensen, 2001]

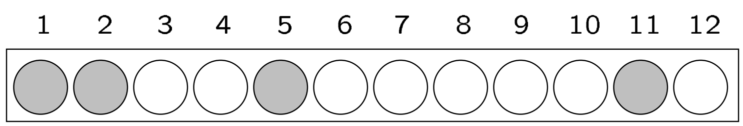
- some implementation details were missing in [Brodnik et al., 1999]
- after filling in these details the implementation got complicated

Piles (and heaps)

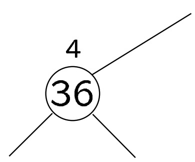
Shape property:



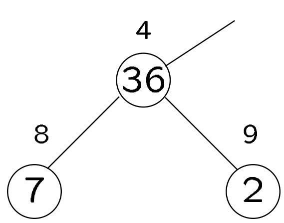
Representation property:



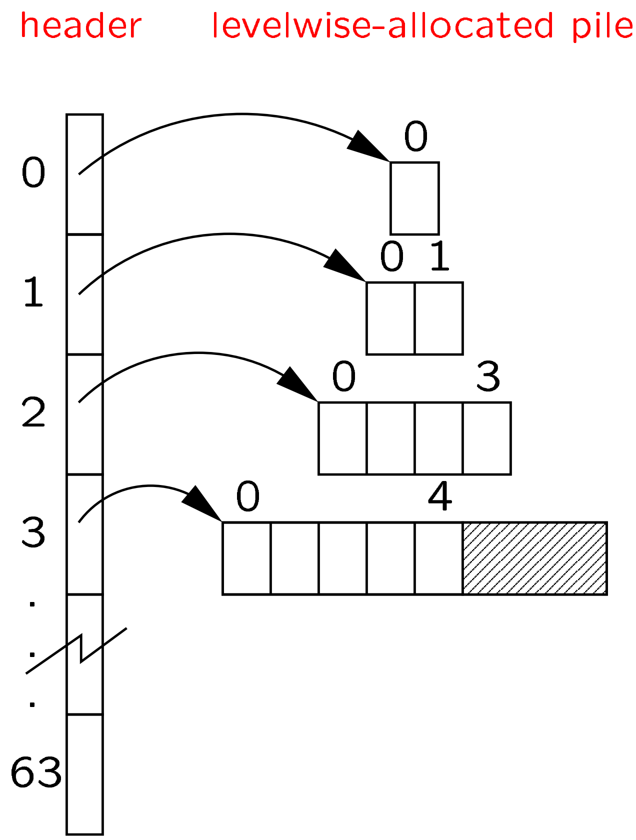
Capacity property:



Order property:

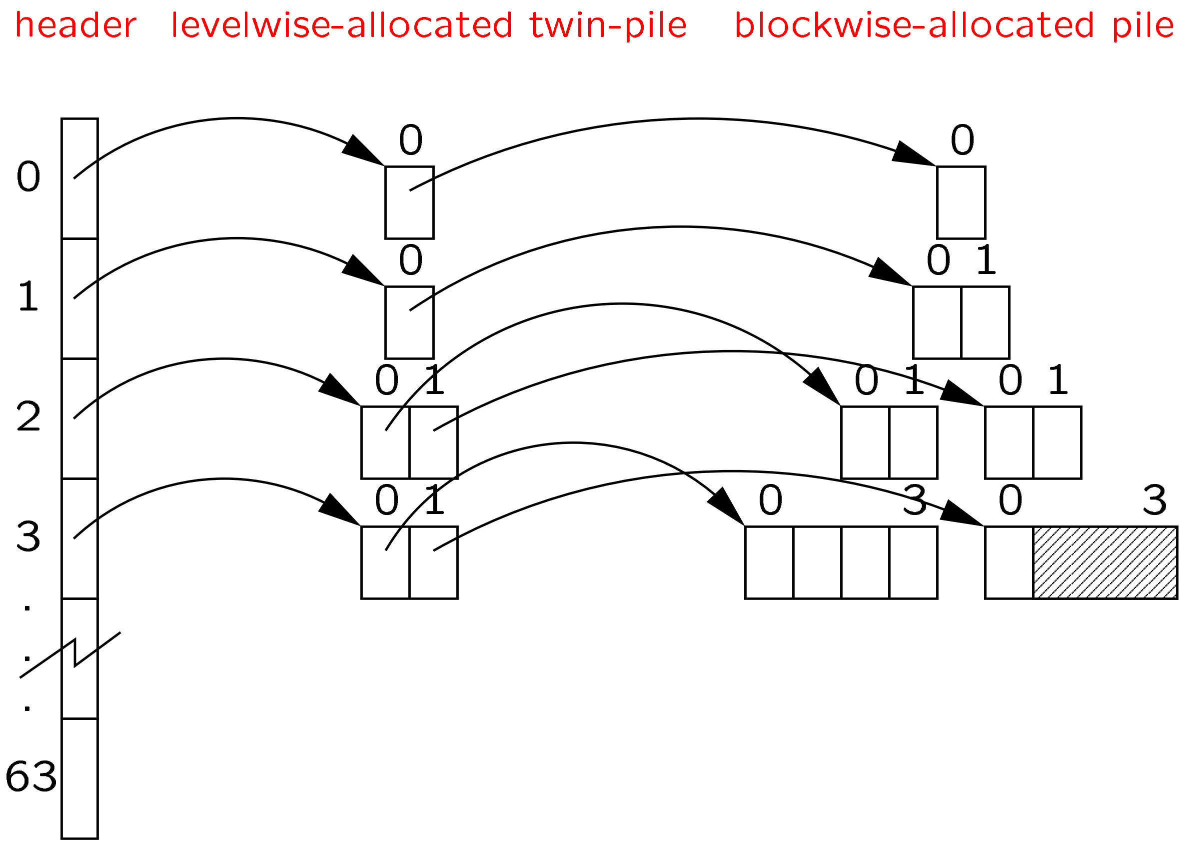


Levelwise-allocated piles



- sequence operations in $O(1)$ worst-case time
- element with index $k \in [0..n-1]$ has index $k - 2^{\lfloor \log_2(k+1) \rfloor} + 1$ at level $\lfloor \log_2(k+1) \rfloor$
- $O(n)$ extra space
- elements are never moved by `push_back` or `pop_back`

Blockwise-allocated piles

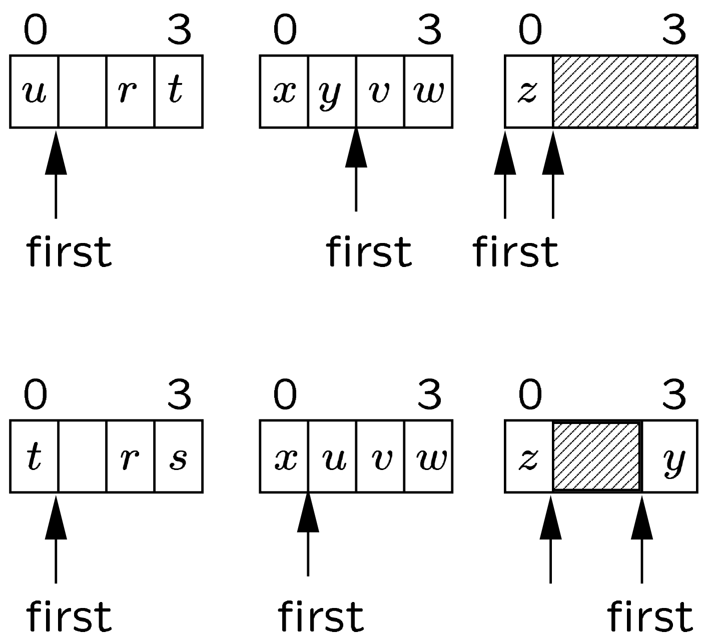


- sequence operations in $O(1)$ worst-case time
- $O(\sqrt{n})$ extra space
- elements are never moved by `push_back` or `pop_back`

Faster modifying operations

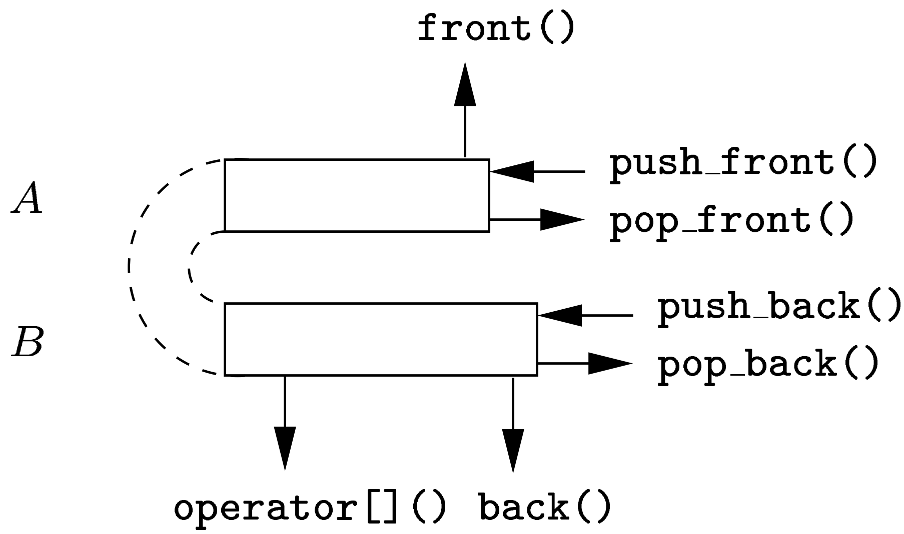
[Goodrich and Kloss II, 1999]

insert element s between r and t



- modifying operations in $O(\sqrt{n})$ worst-case time
- in the twin-pile we have to store double as many pointers

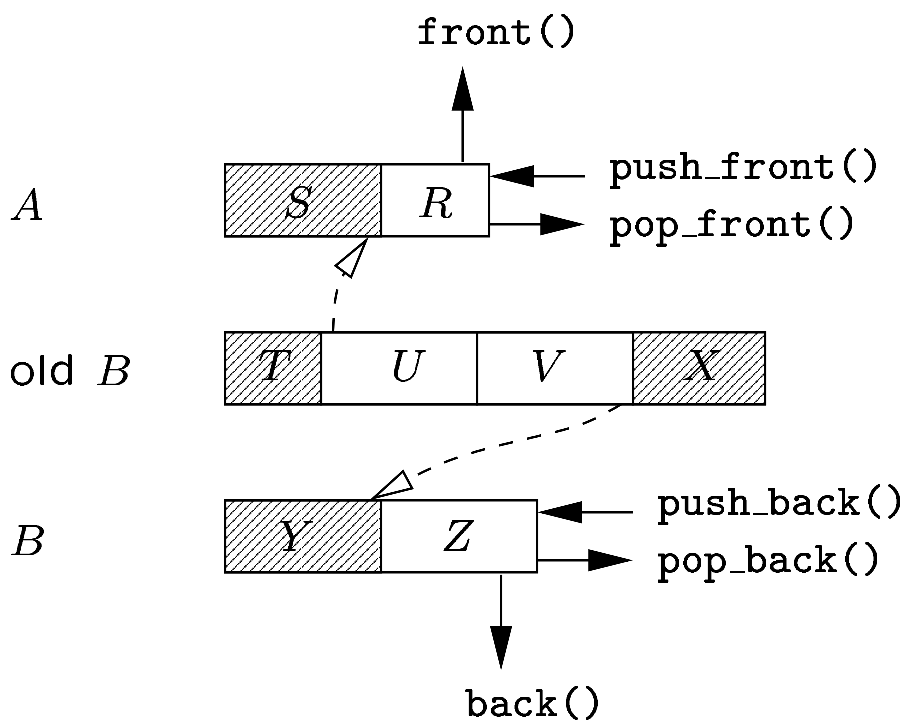
Space-efficient deques



Everything is easy until A or B gets empty.

What if A gets empty?

Observation: A space-efficient vector can be constructed backwards, this can be done piecewise, and the structure can be used simultaneously during such a construction.



- sequence operations in $O(1)$ worst-case time
- modifying operations in $O(\sqrt{n})$ time
- $O(\sqrt{n})$ extra space

Some experimental results

container	push_back (ns)	pop_back (ns)
std::deque	85	11
std::vector	115	2
our deque	113	35
our deque (with reorganization)	113	375

container	sequential access (ns)	random access (ns)
std::deque	117	210
std::vector	2	60
our deque	56	160
our deque (with reorganization)	58	162

container	1 000 inserts (s) initial size 10 000	1 000 inserts (s) initial size 100 000	1 000 inserts (s) initial size 1 000 000
std::deque	0.07	1.00	17.5
std::vector	0.015	0.61	12.9
our deque	0.003	0.01	0.04

Future plans

```
template <
  typename element,
  typename allocator = std::allocator<element>,
  typename implementation =
    bounds_checked_vector<element, allocator>
>
class cphstl::vector {
  ...
}
```

Possible `std::vector` implementations

- `bounds_checked_vector`
- `contiguous_vector`
- `iterator_safe_vector`
- `space_efficient_vector`

Possible `std::deque` implementations

- `bounds_checked_deque`
- `two_level_deque`
- `space_efficient_deque`