# Introduction to Logiweb

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## Contents

## 1 Introduction

Logiweb [**?**] is a system for distribution of mathematical definitions, lemmas, and proofs. More precisely, the Logiweb standard comprises a format, a semantics, and a protocol for storage, interpretation, and transmission of formal mathematics.

A "clean" implementation of Logiweb comprises a Logiweb server, a Logiweb client, and an authoring tool. The Logiweb standard governs the server, the client, and the backend of the authoring tool.

Logiweb allows a user to submit Logiweb pages that contain formal formal mathematics. To submit a page, the user must first produce the page using the authoring tool and then make the page available on the ordinary World Wide Web (WWW) under some Hyper Text Transport Protocal Uniform Resource Locator (http URL). Then the authoring tool must notify the nearest Logiweb server about the submission.

Each Logiweb page has a Logiweb reference which essentially is a global hash key computed on the basis of the contents of the page. Logiweb pages are addressed using Logiweb references but retrieved from WWW using their http URL. The main task of the mesh of Logiweb servers is to translate Logiweb references to http URLs.

A single Logiweb server cannot do the translation from Logiweb reference to http URL alone. Rather, the Logiweb servers form a mesh that cooperate on indexing all Logiweb pages in the world.

### 1.1 References

The reference of the present page is

```
01
   53 DA 8F 72 F5  AE 93 67 4B 8C
   6C 75 F2 D4 EA  19 36 BD D1 B7
   82 9F 8F D7 93  A0 A1 08 06
```

when expressed in *mixed endian hexadecimal*. "Mixed endian" indicates that the reference is written in network byte order but each byte is written with the most significant digit first as in dot-notation for ip-addresses. When written in mixed endian Kana, the reference of the present page reads

```
nani
   tine kisu sake tenu keti  suku sine tute tase saka
   tuka teti kenu kita kusu  nisi netu seki kini sete
   sanu sike sake kite sine  suna suni nasa natu
```

In the Kana notation, "n", "t", "s", "k" represent 00, 01, 10, and 11, respectively. Also, "a", "i", "u", "e" represent 00, 01, 10, and 11, respectively. So "11000100 11000000" translates to "kata kana" and "00 01 10 11 11 10 01 00" translates to "nise kuta". The Kana format is useful if one has to pronounce a reference.

## 1.2 Bibliographies

Each Logiweb page contains a bibliography which is a list of references to other Logiweb pages. As an example the present page references a page named base.

As mentioned, the reference of a page depends on the contents of the page and one cannot change a page without affecting its reference. For that reason, an author of a Logiweb page can safely refer to another Logiweb page without fearing that that other page will change. Furthermore, if an author references a Logiweb page, then the author may copy and resubmit that page and in that way ensure that the referenced Logiweb page will not disappear from Logiweb.

The base page referenced by the present page contains loads of definitions of computing machinery, macro expansion machinery, proof checking machinery, and many other things. The present page states a proof that has been machine checked by Logiweb plus a some other examples of the merits of Logiweb.

## 1.3 Programming

Logiweb supports programming. As an example,

$$[\mathsf{n}! \overset{\text{val}}{\to} \text{If}(\mathsf{n} \approx 0, 1, \mathsf{n} \cdot \mathsf{n} - 1!)][\mathsf{n}! \overset{\text{pyk}}{\to} \text{“}* \text{ factorial”}][\mathsf{n}! \overset{\text{tex}}{\to} \text{“\#1. \char33”}]$$

The footnote of the definition above states what one should type on a keyboard or say in a microphone to enter the factorial function. Appendix **??** states how Logiweb should render the factorial function using TEX.

Enclosing formulas in brackets is a stylistic choice of the author, not something enforced by Logiweb.

## 1.4 Evaluation

Logiweb supports evaluation, compilation, and test of programs. As an example,

$$[3! \approx 6]^{\cdot}$$

tests that [3!] evaluates to [6].

Logiweb has checked the test case above as can be seen from the "After verifying: The page is correct" in the Logiweb main menu of the present page (The name of the present page is "check" for historical reasons).

For further test cases see Section **??**.

## 1.5 Theories

Logiweb supports formal mathematics. As an example, propositional calculus [L] as defined in [**?**] reads:

$$[\mathrm{L} \overset{\text{stmt}}{\to} \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \underline{a} \Rightarrow \underline{b} \vdash \underline{b} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b}][\mathrm{L} \overset{\text{pyk}}{\to} \text{“propositional calculus”}][\mathrm{L} \overset{\text{tex}}{\to} \text{“}\\ \mathrm{L}\text{”}]$$

$$[\mathrm{A1} \overset{\text{stmt}}{\to} \mathrm{L} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a}][\mathrm{A1} \overset{\text{proof}}{\to} \text{Rule tactic}][\mathrm{A1} \overset{\text{pyk}}{\to} \text{“propositional a one”}][\mathrm{A1} \overset{\text{tex}}{\to} \text{“}\\ \mathrm{A1}\text{”}]$$

$$[\mathrm{A2} \overset{\text{stmt}}{\to} \mathrm{L} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c}][\mathrm{A2} \overset{\text{proof}}{\to}\\ \text{Rule tactic}][\mathrm{A2} \overset{\text{pyk}}{\to} \text{“propositional a two”}][\mathrm{A2} \overset{\text{tex}}{\to} \text{“}\\ \mathrm{A2}\text{”}]$$

$$[\mathrm{A3} \overset{\text{stmt}}{\to} \mathrm{L} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b}][\mathrm{A3} \overset{\text{proof}}{\to} \text{Rule tactic}][\mathrm{A3} \overset{\text{pyk}}{\to}\\ \text{“propositional a three”}][\mathrm{A3} \overset{\text{tex}}{\to} \text{“}\\ \mathrm{A3}\text{”}]$$

$$[\mathrm{MP} \overset{\text{stmt}}{\to} \mathrm{L} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \underline{a} \Rightarrow \underline{b} \vdash \underline{b}][\mathrm{MP} \overset{\text{proof}}{\to} \text{Rule tactic}][\mathrm{MP} \overset{\text{pyk}}{\to}\\ \text{“propositional modus ponens”}][\mathrm{MP} \overset{\text{tex}}{\to} \text{“}\\ \mathrm{MP}\text{”}]$$

The construct $[x \Rightarrow y][x \Rightarrow y \overset{\text{pyk}}{\to} \text{“}* \text{ imply } *\text{”}][x \Rightarrow y \overset{\text{tex}}{\to} \text{“\#1. \textbackslash Rightarrow \#2.”}]$ is post-associative (i.e. right associative in text that runs left to right, c.f. Appendix **??**).

The construct $[\neg x]$ is introduced on the base page and has higher associativity than $[x \Rightarrow y]$.

Parentheses $[x]$ are macro defined on the base page. Parentheses affect the structure of parse trees but are macro expanded away before e.g. the proof checker sees the terms they occur in.

## 1.6 Proof checking

Logiweb supports proof checking. As an example, Lemma [I] below says that $[\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}]$ holds for all terms $[\underline{\mathsf{a}}]$ in propositional calculus:

$$[\mathrm{I} \overset{\text{stmt}}{\rightarrow} \mathrm{L} \vdash \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}][\mathrm{I} \overset{\text{pyk}}{\rightarrow} \text{“propositional identity”}][\mathrm{I} \overset{\text{tex}}{\rightarrow} \text{“I”}]$$

We have chosen to render meta-quantification as $[\forall \underline{\mathsf{a}}\colon \underline{\mathsf{b}}]$. When formalizing first order predicate calculus which has an object-quantifier, it may be more convenient to pick another name for the meta-quantifier. That is possible to do since Logiweb offers complete notational freedom.

A proof of the statement above could read:

$$[\mathrm{I} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{L} \vdash \forall \underline{\mathsf{a}}\colon \mathrm{A1} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{A1} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{A2} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rceil, \mathrm{p}_0, \mathsf{c})]$$

Logiweb has checked the proof above as can be seen from the “After verifying: The page is correct” in the Logiweb main menu of the present page.

## 1.7 Logiweb and copyright

Once a Logiweb page is submitted to Logiweb it is understood that anybody is allowed to make verbatim copies of the page and resubmit them to Logiweb. Resubmitted copies have the same Logiweb reference as the original but a different http URL. For this reason, each Logiweb page may have many http URLs but only one Logiweb reference.

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