Formal Logic

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1 Initial remarks

Initially we planned to show a simple result of Group Theory namely the uniqueness of the neutral element. Our idea was to develop propositional logic and predicate calculus first. Based on these we planned to develop the axiomatic set theory ZFC and finally when we had sets we could define groups. Unfortunately it turned out that this was much more cumbersome than we thought both because we are newcommers to Logiweb^{TM}and also because core Logiweb^{TM}is very low level. Being newcommers to Logiweb^{TM}we have used a lot of time trying to find out how to use the system. This hasn't been easy due to the total absence of a hands on users manual. Thus we wasted a lot of time early on trying to parse other peoples code from earlier years in order to understand how to use pyk (the language used to construct proofs ect. in Logiweb^{TM}). This was a very frustrating and non-trivial task since this years pyk syntax is different from earlier years! A lot of emailing back and forth with Klaus Grue helped us, but progress was slow. Very late in the course we had the oportunity to sit down with Klaus in a kind of assisted programming session, where Klaus helped us with our problems as they occured - this was very rewarding. After that we revised our goals with respect to this project and we found that even though we were now able to prove things in Logiweb^{TM}our initial goal was out of range because of the assembler like nature of our predicate calculus. Instead we decided to take the first step towards a more high level interface to our predicate calculus.

2 Conclusion

In this report we define .... todo

Mainly we have experienced that Logiweb^{TM}is very

Soon it turned that it wasn't as easy as we thought to master Logiweb^{TM}.

3 Introduction

In this report we set out to formally prove a simple result of Group Theory namely:

Theorem 3.1 *Let* e *be a neutral element of a Group* G *then* e *is unique. Thus we can talk about* the neutral element *of a Group* G.

The theorem above is very loosely formulated. In this report we set out to formalize the theorem and give a formal proof of it's correctness. In order to do this we need to do a lot of other work. First in XXTODO we define *Propositional Calculus* and then in MMTODO we define *First order Predicate Calculus*. Then using this we define ZFC set theory in YYTODO and finally having set theory available we can define a Group in ZZTODO. Then in WWTODO we restate the above theorem in a formal setting and we give a formal proof of it's correctness.

TODO mere jalla.

4 First order predicate calculus

Based on mathworld[1] and thus on Kleene (2002) we define first-order predicate calculus below. We note that the axioms 1 through 10 together with the inference rule modus ponens constitutes the propositional calculus.

Our definitions are not excately like those found on Mathworld. The reason is that we have made $\Rightarrow$ right associative this means that $\underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f}$ really means $\underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f}$ below.

The $[\text{pred calc} \overset{\text{stmt}}{\rightarrow} \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \Rightarrow \underline{f} \oplus \forall \underline{f}\colon \neg\neg \underline{f} \Rightarrow \underline{f} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{h}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{h} \Rightarrow \underline{f} \Rightarrow \underline{h} \oplus \forall \underline{x}\colon \forall \underline{r}\colon \forall \underline{g}\colon \forall \underline{f}\colon \langle \lceil \underline{h} \rceil \equiv^0 \lceil \underline{f} \rceil \| \lceil \underline{x} \rceil := \lceil \underline{r} \rceil \rangle \Vdash \underline{h} \Rightarrow \exists \underline{x}.\, (\underline{f}) \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{f} \vee \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{h}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{h} \Rightarrow \underline{g} \Rightarrow \underline{f} \vee \underline{h} \Rightarrow \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{x}\colon \lceil \underline{x} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{f} \Rightarrow \underline{g} \vdash \exists \underline{x}.\, (\underline{f}) \Rightarrow \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{f} \Rightarrow \underline{g} \vdash \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \vee \underline{f} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \Rightarrow \neg \underline{g} \Rightarrow \neg \underline{f} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \lambda x.\mathrm{Ded}_0(\lceil \underline{a} \rceil, \lceil \underline{b} \rceil) \Vdash \underline{a} \vdash \underline{b} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \oplus \forall \underline{x}\colon \forall \underline{r}\colon \forall \underline{g}\colon \forall \underline{f}\colon \langle \lceil \underline{h} \rceil \equiv^0 \lceil \underline{f} \rceil \| \lceil \underline{x} \rceil := \lceil \underline{r} \rceil \rangle \Vdash \forall \underline{x}.\, (\underline{f}) \Rightarrow \underline{h} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \wedge \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \Rightarrow \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{x}\colon \lceil \underline{x} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{g} \Rightarrow \underline{f} \vdash \underline{g} \Rightarrow \forall \underline{x}.\, (\underline{f})]$ contains the following axioms

1. $[\text{pc1} \overset{\text{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f}][\text{pc1} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$
2. $[\text{pc2} \overset{\text{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{h}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{h} \Rightarrow \underline{f} \Rightarrow \underline{h}][\text{pc2} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$
3. $[\text{pc3} \overset{\text{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \wedge \underline{g}][\text{pc3} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$
4. $[\text{pc4} \overset{\text{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{f} \vee \underline{g}][\text{pc4} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$
5. $[\text{pc5} \overset{\text{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \vee \underline{f}][\text{pc5} \overset{\text{proof}}{\rightarrow} \text{Rule tactic}]$

[1] http://mathworld.wolfram.com/First-OrderLogic.html.

6. $[\mathrm{pc6} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \Rightarrow \underline{f}][\mathrm{pc6} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

7. $[\mathrm{pc7} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \Rightarrow \underline{g}][\mathrm{pc7} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

8. $[\mathrm{pc8} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{h}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{h} \Rightarrow \underline{g} \Rightarrow \underline{f} \vee \underline{h} \Rightarrow \underline{g}][\mathrm{pc8} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

9. $[\mathrm{pc9} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \Rightarrow \neg \underline{g} \Rightarrow \neg \underline{f}][\mathrm{pc9} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

10. $[\mathrm{pc10} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \neg\neg \underline{f} \Rightarrow \underline{f}][\mathrm{pc10} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

11. $[\mathrm{pc11} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{x}\colon \forall \underline{r}\colon \forall \underline{g}\colon \forall \underline{f}\colon \langle \lceil \underline{h} \rceil \equiv^0 \lceil \underline{f} \rceil \| \lceil \underline{x} \rceil := \lceil \underline{r} \rceil \rangle \Vdash \forall \underline{x}.\, (\underline{f}) \Rightarrow \underline{h}][\mathrm{pc11} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

12. $[\mathrm{pc12} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{x}\colon \forall \underline{r}\colon \forall \underline{g}\colon \forall \underline{f}\colon \langle \lceil \underline{h} \rceil \equiv^0 \lceil \underline{f} \rceil \| \lceil \underline{x} \rceil := \lceil \underline{r} \rceil \rangle \Vdash \underline{h} \Rightarrow \exists \underline{x}.\, (\underline{f})][\mathrm{pc12} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

We note that in first order predicate calculus metavariables used in functions F and predicates P are *object metavariables*.

The only proof rule in $[\text{pred calc} \overset{\mathrm{stmt}}{\rightarrow} \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \Rightarrow \underline{f} \oplus \forall \underline{f}\colon \neg\neg \underline{f} \Rightarrow \underline{f} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{h}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{h} \Rightarrow \underline{f} \Rightarrow \underline{h} \oplus \forall \underline{x}\colon \forall \underline{r}\colon \forall \underline{g}\colon \forall \underline{f}\colon \langle \lceil \underline{h} \rceil \equiv^0 \lceil \underline{f} \rceil \| \lceil \underline{x} \rceil := \lceil \underline{r} \rceil \rangle \Vdash \underline{h} \Rightarrow \exists \underline{x}.\, (\underline{f}) \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{f} \vee \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{h}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{h} \Rightarrow \underline{g} \Rightarrow \underline{f} \vee \underline{h} \Rightarrow \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{x}\colon \lceil \underline{x} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{f} \Rightarrow \underline{g} \vdash \exists \underline{x}.\, (\underline{f}) \Rightarrow \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{f} \Rightarrow \underline{g} \vdash \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \vee \underline{f} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \Rightarrow \neg \underline{g} \Rightarrow \neg \underline{f} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \lambda \mathrm{x}.\mathrm{Ded}_0(\lceil \underline{a} \rceil, \lceil \underline{b} \rceil) \Vdash \underline{a} \vdash \underline{b} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \oplus \forall \underline{x}\colon \forall \underline{r}\colon \forall \underline{g}\colon \forall \underline{f}\colon \langle \lceil \underline{h} \rceil \equiv^0 \lceil \underline{f} \rceil \| \lceil \underline{x} \rceil := \lceil \underline{r} \rceil \rangle \Vdash \forall \underline{x}.\, (\underline{f}) \Rightarrow \underline{h} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \wedge \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \Rightarrow \underline{g} \oplus \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{x}\colon \lceil \underline{x} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{g} \Rightarrow \underline{f} \vdash \underline{g} \Rightarrow \forall \underline{x}.\, (\underline{f})]$ is Modus Ponens which says

- $[\mathrm{pcmp} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{f} \Rightarrow \underline{g} \vdash \underline{g}][\mathrm{pcmp} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$
- $[\mathrm{pcia} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{x}\colon \lceil \underline{x} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{g} \Rightarrow \underline{f} \vdash \underline{g} \Rightarrow \forall \underline{x}.\, (\underline{f})][\mathrm{pcia} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$
- $[\mathrm{pcie} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \forall \underline{x}\colon \lceil \underline{x} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{f} \Rightarrow \underline{g} \vdash \exists \underline{x}.\, (\underline{f}) \Rightarrow \underline{g}][\mathrm{pcie} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$
- $[\mathrm{pcdeduction} \overset{\mathrm{stmt}}{\rightarrow} \text{pred calc} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \lambda \mathrm{x}.\mathrm{Ded}_0(\lceil \underline{a} \rceil, \lceil \underline{b} \rceil) \Vdash \underline{a} \vdash \underline{b}][\mathrm{pcdeduction} \text{ Rule tactic}]$

todo hvorfor tilfoejer vi deduction. bemaerk pcmp er imply elim mens pcded er imply intro

4.1 Deduction lemma

Lemma 4.1 $[\textit{pcded} \overset{\mathrm{stmt}}{\rightarrow} \textit{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{g} \vdash \underline{f} \Rightarrow \underline{g}]$

$[\mathrm{pcded} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathrm{c}.\lambda \mathrm{x}.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{g} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{f} \vdash \underline{g} \rhd \underline{f} \gg \underline{g}; \text{pcdeduction} \rhd \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{g} \gg \underline{f} \Rightarrow \underline{g} \rceil, \mathrm{p}_0, \mathrm{c})]$

4.2 A little proof

Below we make some small proofs just to demonstrate how to do it but also in order to test our definitions above.

Lemma 4.2 $[\mathit{trivia} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{f}\colon \underline{f} \Rightarrow \underline{f}]$

$[\text{trivia} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f}\colon \text{pc2} \gg \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f}; \text{pc1} \gg \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f}; \text{pcmp} \rhd \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \rhd \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \gg \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f}; \text{pc1} \gg \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f}; \text{pcmp} \rhd \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \rhd \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \Rightarrow \underline{f} \gg \underline{f} \Rightarrow \underline{f} \rceil, \mathrm{p}_0, \mathsf{c})]$

Lemma 4.3 (Repetition) $[\mathit{repeat} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{f}\colon \underline{f} \vdash \underline{f}]$

$[\text{repeat} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f}\colon \underline{f} \vdash \text{trivia} \gg \underline{f} \Rightarrow \underline{f}; \text{pcmp} \rhd \underline{f} \rhd \underline{f} \Rightarrow \underline{f} \gg \underline{f} \rceil, \mathrm{p}_0, \mathsf{c})]$

Lemma 4.4 $[\mathit{iatest} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{g}\colon \forall \underline{y}\colon \lceil \underline{y} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \underline{g} \Rightarrow \forall \underline{y}.\ (\underline{y} \Rightarrow \underline{g})]$

$[\text{iatest} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{g}\colon \forall \underline{y}\colon \lceil \underline{y} \rceil \#^0 \lceil \underline{g} \rceil \Vdash \text{pc1} \gg \underline{g} \Rightarrow \underline{y} \Rightarrow \underline{g}; \text{pcia} \Rrightarrow \lceil \underline{y} \rceil \#^0 \lceil \underline{g} \rceil \rhd \underline{g} \Rightarrow \underline{y} \Rightarrow \underline{g} \gg \underline{g} \Rightarrow \forall \underline{y}.\ (\underline{y} \Rightarrow \underline{g}) \rceil, \mathrm{p}_0, \mathsf{c})]$

5 Natural deduction

TODO skriv noget tekst

Lemma 5.1 $[\mathit{andintro} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{g} \vdash \underline{f} \wedge \underline{g}]$

$[\text{andintro} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{g} \vdash \text{pc3} \gg \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \wedge \underline{g}; \text{pcmp} \rhd \underline{f} \rhd \underline{f} \Rightarrow \underline{g} \Rightarrow \underline{f} \wedge \underline{g} \gg \underline{g} \Rightarrow \underline{f} \wedge \underline{g}; \text{pcmp} \rhd \underline{g} \rhd \underline{g} \Rightarrow \underline{f} \wedge \underline{g} \gg \underline{f} \wedge \underline{g} \rceil, \mathrm{p}_0, \mathsf{c})]$

Lemma 5.2 $[\mathit{andelim1} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \vdash \underline{f}]$

$[\text{andelim1} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \vdash \text{pc6} \gg \underline{f} \wedge \underline{g} \Rightarrow \underline{f}; \text{pcmp} \rhd \underline{f} \wedge \underline{g} \rhd \underline{f} \wedge \underline{g} \Rightarrow \underline{f} \gg \underline{f} \rceil, \mathrm{p}_0, \mathsf{c})]$

Lemma 5.3 $[\mathit{andelim2} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \vdash \underline{g}]$

$[\text{andelim2} \overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \wedge \underline{g} \vdash \text{pc7} \gg \underline{f} \wedge \underline{g} \Rightarrow \underline{g}; \text{pcmp} \rhd \underline{f} \wedge \underline{g} \rhd \underline{f} \wedge \underline{g} \Rightarrow \underline{g} \gg \underline{g} \rceil, \mathrm{p}_0, \mathsf{c})]$

Lemma 5.4 $[\mathit{orintro1} \overset{\text{stmt}}{\rightarrow} \mathit{pred\ calc} \vdash \forall \underline{f}\colon \forall \underline{g}\colon \underline{f} \vdash \underline{f} \vee \underline{g}]$

[orintro1 $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash$ pc4 $\gg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}}$; pcmp $\rhd \underline{\mathsf{f}} \rhd \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \gg \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \rceil, \mathrm{p}_0, \mathsf{c})$]

Lemma 5.5 [*orintro2* $\overset{\text{stmt}}{\rightarrow}$ *pred calc* $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{g}} \vdash \underline{\mathsf{f}} \vee \underline{\mathsf{g}}$]

[orintro2 $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{g}} \vdash$ pc5 $\gg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}}$; pcmp $\rhd \underline{\mathsf{g}} \rhd \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \gg \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \rceil, \mathrm{p}_0, \mathsf{c})$]

Lemma 5.6 [*orelim* $\overset{\text{stmt}}{\rightarrow}$ *pred calc* $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \forall \underline{\mathsf{h}}\colon \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \vdash \underline{\mathsf{f}} \vdash \underline{\mathsf{h}} \vdash \underline{\mathsf{g}} \vdash \underline{\mathsf{h}} \vdash \underline{\mathsf{h}}$]

[orelim $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \forall \underline{\mathsf{h}}\colon \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \vdash \underline{\mathsf{f}} \vdash \underline{\mathsf{h}} \vdash \underline{\mathsf{g}} \vdash \underline{\mathsf{h}} \vdash$ pcded$\rhd \underline{\mathsf{f}} \vdash \underline{\mathsf{h}} \gg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{h}}$; pcded$\rhd \underline{\mathsf{g}} \vdash \underline{\mathsf{h}} \gg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}}$; pc8 $\gg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}}$; pcmp$\rhd \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{h}} \rhd \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \gg \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}}$; pcmp$\rhd \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \rhd \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \Rightarrow \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \gg \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}}$; pcmp $\rhd \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \rhd \underline{\mathsf{f}} \vee \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{h}} \gg \underline{\mathsf{h}} \rceil, \mathrm{p}_0, \mathsf{c})$]

TODO lemma changed from natural deduction!!! skriv afsnit om det.

Lemma 5.7 [*notintro* $\overset{\text{stmt}}{\rightarrow}$ *pred calc* $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash \underline{\mathsf{g}} \vdash \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{g}} \vdash \neg \underline{\mathsf{f}}$]

[notintro $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash \underline{\mathsf{g}} \vdash \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{g}} \vdash$ pcded$\rhd \underline{\mathsf{f}} \vdash \underline{\mathsf{g}} \gg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}}$; pcded $\rhd \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{g}} \gg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{g}}$; pc9 $\gg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{f}}$; pcmp $\rhd \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}} \rhd \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{f}} \gg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{f}}$; pcmp $\rhd \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{g}} \rhd \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{g}} \Rightarrow \neg \underline{\mathsf{f}} \gg \neg \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})$]

Lemma 5.8 [*notnotelim* $\overset{\text{stmt}}{\rightarrow}$ *pred calc* $\vdash \forall \underline{\mathsf{f}}\colon \neg\neg \underline{\mathsf{f}} \vdash \underline{\mathsf{f}}$]

[notnotelim $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \neg\neg \underline{\mathsf{f}} \vdash$ pc10 $\gg \neg\neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}}$; pcmp $\rhd \neg\neg \underline{\mathsf{f}} \rhd \neg\neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \gg \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})$]

5.1 Derived theorems

Below we apply the theorems above to prove some other fairly standard rules.

Lemma 5.9 [*mt* $\overset{\text{stmt}}{\rightarrow}$ *pred calc* $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}} \vdash \neg \underline{\mathsf{g}} \vdash \neg \underline{\mathsf{f}}$]

[mt $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}} \vdash \neg \underline{\mathsf{g}} \vdash \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash$ pcmp $\rhd \underline{\mathsf{f}} \rhd \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{g}} \gg \underline{\mathsf{g}}$; pcdeduction $\rhd \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash \underline{\mathsf{g}} \gg \underline{\mathsf{f}} \vdash \underline{\mathsf{g}}; \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash$ repeat $\rhd \neg \underline{\mathsf{g}} \gg \neg \underline{\mathsf{g}}$; pcdeduction $\rhd \forall \underline{\mathsf{f}}\colon \forall \underline{\mathsf{g}}\colon \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{g}} \gg \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{g}}$; notintro $\rhd \underline{\mathsf{f}} \vdash \underline{\mathsf{g}} \rhd \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{g}} \gg \neg \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})$]

Lemma 5.10 [*notnotintro* $\overset{\text{stmt}}{\rightarrow}$ *pred calc* $\vdash \forall \underline{\mathsf{f}}\colon \underline{\mathsf{f}} \vdash \neg\neg \underline{\mathsf{f}}$]

[notnotintro $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$pred calc $\vdash \forall \underline{\mathsf{f}}\colon \underline{\mathsf{f}} \vdash \forall \underline{\mathsf{f}}\colon \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{f}} \vdash$ repeat $\rhd \underline{\mathsf{f}} \gg \underline{\mathsf{f}}$; pcdeduction $\rhd \forall \underline{\mathsf{f}}\colon \underline{\mathsf{f}} \vdash \neg \underline{\mathsf{f}} \vdash \underline{\mathsf{f}} \gg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}}$; pcmp $\rhd \underline{\mathsf{f}} \rhd \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \gg \neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}}$; trivia $\gg \neg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}}$; pc9 $\gg \neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \neg\neg \underline{\mathsf{f}}$; pcmp $\rhd \neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \rhd \neg \underline{\mathsf{f}} \Rightarrow \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \neg\neg \underline{\mathsf{f}} \gg \neg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \neg\neg \underline{\mathsf{f}}$; pcmp $\rhd \neg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \rhd \neg \underline{\mathsf{f}} \Rightarrow \neg \underline{\mathsf{f}} \Rightarrow \neg\neg \underline{\mathsf{f}} \gg \neg\neg \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})$]

Lemma 5.11 $[pbc \overset{\text{stmt}}{\to} pred\ calc \vdash \forall \underline{f} \colon \forall \underline{g} \colon \neg \underline{f} \vdash \underline{g} \vdash \neg \underline{f} \vdash \neg \underline{g} \vdash \underline{f}]$

$[\text{pbc} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f} \colon \forall \underline{g} \colon \neg \underline{f} \vdash \underline{g} \vdash \neg \underline{f} \vdash \neg \underline{g} \vdash \text{notintro} \rhd \neg \underline{f} \vdash \underline{g} \rhd \neg \underline{f} \vdash \neg \underline{g} \gg \neg\neg \underline{f}; \text{notnotelim} \rhd \neg\neg \underline{f} \gg \underline{f} \rceil, \mathrm{p}_0, \mathsf{c})]$

5.2 Law of the Excluded Middle

In this section we prove the *Law of the Excluded Middle*.

Theorem 5.12 $[lem \overset{\text{stmt}}{\to} pred\ calc \vdash \forall \underline{f} \colon \underline{f} \vee \neg \underline{f}]$

$[\text{lem} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{pred calc} \vdash \forall \underline{f} \colon \forall \underline{f} \colon \neg \underline{f} \vee \neg \underline{f} \vdash \forall \underline{f} \colon \underline{f} \vdash \text{orintro1} \rhd \underline{f} \gg \underline{f} \vee \neg \underline{f}; \text{pcdeduction} \rhd \forall \underline{f} \colon \underline{f} \vdash \underline{f} \vee \neg \underline{f} \gg \underline{f} \vdash \underline{f} \vee \neg \underline{f}; \forall \underline{f} \colon \underline{f} \vdash \text{repeat} \rhd \neg \underline{f} \vee \neg \underline{f} \gg \neg \underline{f} \vee \neg \underline{f}; \text{pcdeduction} \rhd \forall \underline{f} \colon \underline{f} \vdash \neg \underline{f} \vee \neg \underline{f} \gg \underline{f} \vdash \neg \underline{f} \vee \neg \underline{f}; \text{notintro} \rhd \underline{f} \vdash \underline{f} \vee \neg \underline{f} \rhd \underline{f} \vdash \neg \underline{f} \vee \neg \underline{f} \gg \neg \underline{f}; \text{orintro2} \rhd \neg \underline{f} \gg \underline{f} \vee \neg \underline{f}; \text{pcdeduction} \rhd \forall \underline{f} \colon \neg \underline{f} \vee \neg \underline{f} \vdash \underline{f} \vee \neg \underline{f} \gg \neg \underline{f} \vee \neg \underline{f} \vdash \underline{f} \vee \neg \underline{f}; \forall \underline{f} \colon \neg \underline{f} \vee \neg \underline{f} \vdash \text{repeat} \rhd \neg \underline{f} \vee \neg \underline{f} \gg \neg \underline{f} \vee \neg \underline{f}; \text{pcdeduction} \rhd \forall \underline{f} \colon \neg \underline{f} \vee \neg \underline{f} \vdash \neg \underline{f} \vee \neg \underline{f} \gg \neg \underline{f} \vee \neg \underline{f} \vdash \neg \underline{f} \vee \neg \underline{f}; \text{notintro} \rhd \neg \underline{f} \vee \neg \underline{f} \vdash \underline{f} \vee \neg \underline{f} \rhd \neg \underline{f} \vee \neg \underline{f} \vdash \neg \underline{f} \vee \neg \underline{f} \gg \neg\neg \underline{f} \vee \neg \underline{f}; \text{notnotelim} \rhd \neg\neg \underline{f} \vee \neg \underline{f} \gg \underline{f} \vee \neg \underline{f} \rceil, \mathrm{p}_0, \mathsf{c})]$

A Pyk definitions

[pred calc $\overset{\text{pyk}}{\to}$ "pred calc"]
[pc1 $\overset{\text{pyk}}{\to}$ "pc1"]
[pc2 $\overset{\text{pyk}}{\to}$ "pc2"]
[pc3 $\overset{\text{pyk}}{\to}$ "pc3"]
[pc4 $\overset{\text{pyk}}{\to}$ "pc4"]
[pc5 $\overset{\text{pyk}}{\to}$ "pc5"]
[pc6 $\overset{\text{pyk}}{\to}$ "pc6"]
[pc7 $\overset{\text{pyk}}{\to}$ "pc7"]
[pc8 $\overset{\text{pyk}}{\to}$ "pc8"]
[pc9 $\overset{\text{pyk}}{\to}$ "pc9"]
[pc10 $\overset{\text{pyk}}{\to}$ "pc10"]
[pc11 $\overset{\text{pyk}}{\to}$ "pc11"]
[pc12 $\overset{\text{pyk}}{\to}$ "pc12"]
[pcmp $\overset{\text{pyk}}{\to}$ "pcmp"]
[pcded $\overset{\text{pyk}}{\to}$ "pcded"]
[pcia $\overset{\text{pyk}}{\to}$ "pcia"]
[pcie $\overset{\text{pyk}}{\to}$ "pcie"]
[pcdeduction $\overset{\text{pyk}}{\to}$ "pcdeduction"]

[trivia $\overset{\text{pyk}}{\to}$ “trivia”]
[iatest $\overset{\text{pyk}}{\to}$ “iatest”]
[andintro $\overset{\text{pyk}}{\to}$ “andintro”]
[andelim1 $\overset{\text{pyk}}{\to}$ “andelim1”]
[andelim2 $\overset{\text{pyk}}{\to}$ “andelim2”]
[orintro1 $\overset{\text{pyk}}{\to}$ “orintro1”]
[orintro2 $\overset{\text{pyk}}{\to}$ “orintro2”]
[orelim $\overset{\text{pyk}}{\to}$ “orelim”]
[notintro $\overset{\text{pyk}}{\to}$ “notintro”]
[notnotintro $\overset{\text{pyk}}{\to}$ “notnotintro”]
[notnotelim $\overset{\text{pyk}}{\to}$ “notnotelim”]
[mt $\overset{\text{pyk}}{\to}$ “mt”]
[pbc $\overset{\text{pyk}}{\to}$ “pbc”]
[repeat $\overset{\text{pyk}}{\to}$ “repeat”]
[lem $\overset{\text{pyk}}{\to}$ “lem”]
[$* \equiv *$ $\overset{\text{pyk}}{\to}$ “" setequiv "”]
[$* = *$ $\overset{\text{pyk}}{\to}$ “" setequals "”]
[$\neg *$ $\overset{\text{pyk}}{\to}$ “lnot "”]
[$* \wedge *$ $\overset{\text{pyk}}{\to}$ “" land "”]
[$* \vee *$ $\overset{\text{pyk}}{\to}$ “" lor "”]
[$\forall * . (*)$ $\overset{\text{pyk}}{\to}$ “forall " dot " end forall”]
[$\exists * . (*)$ $\overset{\text{pyk}}{\to}$ “exists " dot " end exists”]
[$* \in *$ $\overset{\text{pyk}}{\to}$ “" setin "”]
[problemone $\overset{\text{pyk}}{\to}$ “problemone”]

B Tex definitions

- [$\neg \mathsf{x}$ $\overset{\text{tex}}{\to}$ “\neg #1.”]
- [$\mathsf{x} \wedge \mathsf{y}$ $\overset{\text{tex}}{\to}$ “#1. \wedge #2.”]
- [$\mathsf{x} \vee \mathsf{y}$ $\overset{\text{tex}}{\to}$ “#1. \vee #2.”]
- [$\mathsf{x} \Rightarrow \mathsf{y}$ $\overset{\text{tex}}{\to}$ “#1. \Rightarrow #2.”]
- [$\forall \mathsf{y}. (\mathsf{b})$ $\overset{\text{tex}}{\to}$ “\forall #1. . \left(#2.\right)”]
- [$\exists \mathsf{y}. (\mathsf{b})$ $\overset{\text{tex}}{\to}$ “\exists #1. . \left(#2.\right)”]

- $[\mathsf{y} \in \mathsf{b} \overset{\text{tex}}{\to}$ "#1. \in #2."]
- $[\mathsf{y} \equiv \mathsf{b} \overset{\text{tex}}{\to}$ "#1. \equiv #2."]
- $[\mathsf{y} = \mathsf{b} \overset{\text{tex}}{\to}$ "#1. = #2."]

C Priority table

[problemone $\overset{\text{prio}}{\to}$
Preassociative
[problemone], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [$\$ * \$$], [**flush left** $*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \overset{*}{\to} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if$(*,*,*)$], [[$* \overset{*}{\Rightarrow} *$]], [val], [claim], [$\bot$], [f$(*)$], [$(*)^{\mathrm{I}}$], [$\mathsf{F}$], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\mathrm{M}}]$, [If$(*,*,*)$], [array$\{*\} * $ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\mathrm{M}}(*)$], [**apply**$(*,*)$], [**apply**$_1(*,*)$], [identifier$(*)$], [identifier$_1(*,*)$], [array-plus$(*,*)$], [array-remove$(*,*,*)$], [array-put$(*,*,*,*)$], [array-add$(*,*,*,*,*)$], [bit$(*,*)$], [bit$_1(*,*)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*,*,*)$], [$\mathcal{E}_2(*,*,*,*,*)$], [$\mathcal{E}_3(*,*,*,*)$], [$\mathcal{E}_4(*,*,*,*,*)$], [**lookup**$(*,*,*)$], [**abstract**$(*,*,*,*)$], [$\lceil * \rceil$], [$\mathcal{M}(*,*,*)$], [$\mathcal{M}_2(*,*,*,*)$], [$\mathcal{M}^*(*,*,*)$], [macro], [s_0], [**zip**$(*,*)$], [**assoc**$_1(*,*,*)$], $[(*)^{\mathbf{P}}]$, [self], [[$* \doteq *$]], [[$* \dot{=} *$]], [[$* \dot{\leftarrow} *$]], [[$* \overset{\text{pyk}}{=} *$]], [[$* \overset{\text{tex}}{=} *$]], [[$* \overset{\text{name}}{=} *$]], [**Priority table**$[*]$], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*,*,*,*)$], [$\tilde{\mathcal{M}}(*,*,*)$], [$\tilde{\mathcal{Q}}(*,*,*)$], [$\tilde{\mathcal{Q}}_2(*,*,*)$], [$\tilde{\mathcal{Q}}_3(*,*,*,*)$], [$\tilde{\mathcal{Q}}^*(*,*,*)$], [$(*)$], [$(*)$], [display$(*)$], [statement$(*)$], [$[*]^{\cdot}$], [$[*]^{-}$], [**aspect**$(*,*)$], [**aspect**$(*,*,*)$], [$\langle * \rangle$], [**tuple**$_1(*)$], [**tuple**$_2(*)$], [let$_2(*,*)$], [let$_1(*,*)$], [[$* \overset{\text{claim}}{=} *$]], [checker], [**check**$(*,*)$], [**check**$_2(*,*,*)$], [**check**$_3(*,*,*)$], [**check**$^*(*,*)$], [**check**$^*_2(*,*,*)$], $[[*]^{\cdot}]$, $[[*]^{-}]$, $[[*]^{\circ}]$, [msg], [[$* \overset{\text{msg}}{=} *$]], [<stmt>], [stmt], [[$* \overset{\text{stmt}}{=} *$]], [HeadNil$'$], [HeadPair$'$], [Transitivity$'$], [$\perp\!\!\!\perp$], [Contra$'$], [$\mathrm{T}'_{\mathrm{E}}$], [$\mathrm{L}_1$], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle {}^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], $[(*)^{\mathbf{v}}]$, [intro$(*,*,*,*)$], [intro$(*,*,*)$], [error$(*,*)$], [error$_2(*,*)$], [proof$(*,*,*)$], [proof$_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\mathrm{I}}(*,*)$], [$\mathcal{S}^{\triangleright}(*,*)$], [$\mathcal{S}^{\triangleright}_1(*,*,*)$], [$\mathcal{S}^{\mathrm{E}}(*,*)$], [$\mathcal{S}^{\mathrm{E}}_1(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}^{+}_1(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}^{-}_1(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}^{*}_1(*,*,*)$], [$\mathcal{S}^{*}_2(*,*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}^{@}_1(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}^{\vdash}_1(*,*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}^{\Vdash}_1(*,*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}(*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}_1(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}_2(*,*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}^{\forall}_1(*,*,*,*)$], [$\mathcal{S}^{\cdot}(*,*)$], [$\mathcal{S}^{\cdot}_1(*,*,*)$], [$\mathcal{S}^{\cdot}_2(*,*,*,*)$], [$\mathcal{T}(*)$], [claims$(*,*,*)$], [claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$],

$[\mathcal{V}_7(*,*,*,*)]$, [Cut$(*,*)$], [Head$_{\oplus}(*)$], [Tail$_{\oplus}(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [T$_{\mathrm{E}}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(*=*,*)$], [unify$^*(*=*,*)$], [unify$_2(*=*,*)$], [L$_{\mathrm{a}}$], [L$_{\mathrm{b}}$], [L$_{\mathrm{c}}$], [L$_{\mathrm{d}}$], [L$_{\mathrm{e}}$], [L$_{\mathrm{f}}$], [L$_{\mathrm{g}}$], [L$_{\mathrm{h}}$], [L$_{\mathrm{i}}$], [L$_{\mathrm{j}}$], [L$_{\mathrm{k}}$], [L$_{\mathrm{l}}$], [L$_{\mathrm{m}}$], [L$_{\mathrm{n}}$], [L$_{\mathrm{o}}$], [L$_{\mathrm{p}}$], [L$_{\mathrm{q}}$], [L$_{\mathrm{r}}$], [L$_{\mathrm{s}}$], [L$_{\mathrm{t}}$], [L$_{\mathrm{u}}$], [L$_{\mathrm{v}}$], [L$_{\mathrm{w}}$], [L$_{\mathrm{x}}$], [L$_{\mathrm{y}}$], [L$_{\mathrm{z}}$], [L$_{\mathrm{A}}$], [L$_{\mathrm{B}}$], [L$_{\mathrm{C}}$], [L$_{\mathrm{D}}$], [L$_{\mathrm{E}}$], [L$_{\mathrm{F}}$], [L$_{\mathrm{G}}$], [L$_{\mathrm{H}}$], [L$_{\mathrm{I}}$], [L$_{\mathrm{J}}$], [L$_{\mathrm{K}}$], [L$_{\mathrm{L}}$], [L$_{\mathrm{M}}$], [L$_{\mathrm{N}}$], [L$_{\mathrm{O}}$], [L$_{\mathrm{P}}$], [L$_{\mathrm{Q}}$], [L$_{\mathrm{R}}$], [L$_{\mathrm{S}}$], [L$_{\mathrm{T}}$], [L$_{\mathrm{U}}$], [L$_{\mathrm{V}}$], [L$_{\mathrm{W}}$], [L$_{\mathrm{X}}$], [L$_{\mathrm{Y}}$], [L$_{\mathrm{Z}}$], [L$_?$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], $[[* \overset{\text{tactic}}{=} *]]$, $[\mathcal{P}(*,*,*)]$, $[\mathcal{P}^*(*,*,*)]$, [p$_0$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], [check], $[[* \overset{\circ}{=} *]]$, [RootVisible$(*)$], [A], [R], [C], [T], [L], [{$*$}], $[\overline{*}]$, [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [x], [y], [z], $[\langle * \equiv * \mid * :=* \rangle]$, $[\langle * \equiv^0 * \mid * :=* \rangle]$, $[\langle * \equiv^1 * \mid * :=* \rangle]$, $[\langle * \equiv^* * \mid * :=* \rangle]$, [Ded$(*,*)$], [Ded$_0(*,*)$], [Ded$_1(*,*,*)$], [Ded$_2(*,*,*)$], [Ded$_3(*,*,*,*)$], [Ded$_4(*,*,*,*)$], [Ded$_4^*(*,*,*,*)$], [Ded$_5(*,*,*)$], [Ded$_6(*,*,*,*)$], [Ded$_6^*(*,*,*,*)$], [Ded$_7(*)$], [Ded$_8(*,*)$], [Ded$_8^*(*,*)$], [S], [Neg], [MP], [Gen], [Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1$'$], [A2$'$], [A4$'$], [A5$'$], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop 3.2e$_1$], [Prop 3.2e$_2$], [Prop 3.2e], [Prop 3.2f$_1$], [Prop 3.2f$_2$], [Prop 3.2f], [Prop 3.2g$_1$], [Prop 3.2g$_2$], [Prop 3.2g], [Prop 3.2h$_1$], [Prop 3.2h$_2$], [Prop 3.2h], [Block$_1(*,*,*)$], [Block$_2(*)$], [pred calc], [pc1], [pc2], [pc3], [pc4], [pc5], [pc6], [pc7], [pc8], [pc9], [pc10], [pc11], [pc12], [pcmp], [pcded], [pcia], [pcie], [pcdeduction], [trivia], [iatest], [andintro], [andelim1], [andelim2], [orintro1], [orintro2], [orelim], [notintro], [notnotintro], [notnotelim], [mt], [pbc], [repeat], [lem];

Preassociative

[$*$_{$*$}], [$*$/indexintro$(*,*,*,*)$], [$*$/intro$(*,*,*)$], [$*$/bothintro$(*,*,*,*,*)$], [$*$/nameintro$(*,*,*,*)$], [$*'$], [$*$[$*$]], [$*$[$*\rightarrow*$]], [$*$[$*\Rightarrow*$]], [$*0$], [$*1$], [0b], [$*$-color$(*)$], [$*$-color$^*(*)$], [$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^0$], [$*^1$], [$*^2$], [$*^3$], [$*^4$], [$*^5$], [$*^6$], [$*^7$], [$*^8$], [$*^9$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*^{\mathrm{hide}}$];

Preassociative

[" $*$ "], [], [$(*)^{\mathbf{t}}$], [string$(*)$ + $*$], [string$(*)$ ++ $*$], [$*$], [$*$], [!$*$], ["$*$], [#$*$], [$$$*$], [%$*$], [&$*$], ['$*$], [($*$], [)$*$], [**], [+$*$], [, $*$], [-$*$], [.$*$], [/$*$], [0$*$], [1$*$], [2$*$], [3$*$], [4$*$], [5$*$], [6$*$], [7$*$], [8$*$], [9$*$], [:$*$], [; $*$], [<$*$], [=$*$], [>$*$], [?$*$], [@$*$], [A$*$], [B$*$], [C$*$], [D$*$], [E$*$], [F$*$], [G$*$], [H$*$], [I$*$], [J$*$], [K$*$], [L$*$], [M$*$], [N$*$], [O$*$], [P$*$], [Q$*$], [R$*$], [S$*$], [T$*$], [U$*$], [V$*$], [W$*$], [X$*$], [Y$*$], [Z$*$], [[$*$], [\$*$], []$*$], [^ $*$], [_$*$], [`$*$], [a$*$], [b$*$], [c$*$], [d$*$], [e$*$], [f$*$], [g$*$], [h$*$], [i$*$], [j$*$], [k$*$], [l$*$], [m$*$], [n$*$], [o$*$], [p$*$], [q$*$], [r$*$], [s$*$], [t$*$], [u$*$], [v$*$], [w$*$], [x$*$], [y$*$], [z$*$], [{$*$], [|$*$], [}$*$], [~$*$], [**Preassociative** $*;*$], [**Postassociative** $*;*$], [[$*$], $*$], [priority $*$ end], [newline $*$], [macro newline $*$], [MacroIndent$(*)$];

Preassociative

[$*$ ' $*$], [$*$ ` $*$];

Preassociative

$[*']$;
Preassociative
$[* \text{ ' } *], [* \text{ ` } *]$;
Preassociative
$[* \cdot *], [* \cdot_0 *]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* :: *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*, *]$;
Preassociative
$[* \overset{\text{B}}{\approx} *], [* \overset{\text{D}}{\approx} *], [* \overset{\text{C}}{\approx} *], [* \overset{\text{P}}{\approx} *], [* \approx *], [* = *], [* \overset{+}{\to} *], [* \overset{\text{t}}{=} *], [* \overset{\text{t}^*}{=} *], [* \overset{\text{r}}{=} *],$
$[* \in_{\text{t}} *], [* \subseteq_{\text{T}} *], [* \overset{\text{T}}{=} *], [* \overset{\text{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* * *], [* \text{ free for } * \text{ in } *],$
$[* \text{ free for}^* * \text{ in } *], [* \in_{\text{c}} *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\text{var}}],$
$[*\#^0*], [*\#^1*], [*\#^**], [* \equiv *], [* = *]$;
Preassociative
$[\neg *], [\neg *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\text{c}} *], [* \wedge *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \vee *]$;
Preassociative
$[\exists * : *], [\forall * : *], [\forall_{\text{obj}} * : *], [\forall * . (*)], [\exists * . (*)]$;
Postassociative
$[* \ddot{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *]$;
Postassociative
$[* : *], [* \text{ spy } *], [*!*]$;
Preassociative
$[* \begin{cases} * \\ * \end{cases}]$;
Preassociative
$[\lambda * . *], [\Lambda * . *], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *]$;
Preassociative
$[*\#*]$;
Preassociative
$[*^{\text{I}}], [*^{\triangleright}], [*^{\text{V}}], [*^+], [*^-], [*^*]$;
Preassociative
$[* @ *], [* \triangleright *], [* \Rrightarrow *], [* \gg *], [* \trianglerighteq *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall * : *], [\Pi * : *]$;

Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
$[*$ proves $*]$;
Preassociative
$[*$ **proof of** $* : *]$, $[\text{Line}\, * : * \gg *; *]$, $[\text{Last line}\, * \gg * \,\square]$,
$[\text{Line}\, * : \text{Premise} \gg *; *]$, $[\text{Line}\, * : \text{Side-condition} \gg *; *]$, $[\text{Arbitrary} \gg *; *]$,
$[\text{Local} \gg * = *; *]$, $[\text{Begin}\, *; * : \text{End}; *]$, $[\text{Last block line}\, * \gg * ;]$,
$[\text{Arbitrary} \gg *; *]$;
Postassociative
$[* \mid *]$;
Postassociative
$[* , *]$, $[*[\, * \,]*]$;
Preassociative
$[*\&*]$, $[\rightarrow]$;
Preassociative
$[*\backslash\backslash *]$, $[*$ linebreak[4] $*]$, $[*\backslash\backslash *]$;
Preassociative
$[* \in *];]$