

projekt i logik

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Resumé

1 Introduction

In this paper we will prove the propositions regarding S starting with proposition Prop 3.2i. continuing to 3.4, bevise 3.5, aksiomer, der definerer $x \rightarrow y$, bevise 3.10, og bevise 3.11.

We chose to prove chapter 3 from mendelson, starting with Lemma 3.2, since the first ten wfs' are proved in Mendelson, we will start with Lemma 3.2.i.

2

A modified version of Mendelsons system S (Peano arithmetic) [Men97] may be formulated thus:

$$\begin{aligned} &[S \xrightarrow{\text{stmt}} \Pi \underline{a}: \underline{a} + 0 = \underline{a} \oplus \Pi \underline{a}, \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash [\text{MP} \xrightarrow{\text{stmt}} \Pi \underline{a}, \underline{b}: \underline{a} = \underline{b} \vdash \underline{a} = \underline{c} \vdash \underline{b} = \underline{c} \oplus \Pi \underline{a}, \\ &\quad \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b}][\text{MP} \xrightarrow{\text{proof}} \text{Rule tactic}] \\ &[\text{Gen} \xrightarrow{\text{stmt}} S \vdash \Pi \underline{x}, \underline{a}: \underline{a} \vdash \forall \underline{x}: \underline{a}][\text{Gen} \xrightarrow{\text{proof}} \Pi \underline{x}: \underline{a} \vdash \forall \underline{x}: \underline{a}][\text{Ded} \xrightarrow{\text{stmt}} \Pi \underline{a}, \\ &\quad \underline{b}: \text{Ded}(\underline{a}, \underline{b}) \vdash \underline{a} \vdash \underline{b}][\text{Ded} \xrightarrow{\text{proof}} \text{Rule tactic}] \\ &\quad [S2 \xrightarrow{\text{stmt}} S \vdash \Pi \underline{a}, \\ &\quad \underline{b}: \underline{a} = \underline{b} \vdash \underline{a}' = \underline{b}'] [S2 \xrightarrow{\text{proof}} \text{Rule tactic}] \end{aligned}$$

$[S3 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}: \neg 0 = \underline{a}'] [S3 \xrightarrow{\text{proof}} \text{Rule tactic}] [S4 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}},$
 $\underline{b}: \underline{a}' = \underline{b}' \vdash \underline{a} = \underline{b}] [S4 \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[S5 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}: \underline{a} + 0 = \underline{a}] [S5 \xrightarrow{\text{proof}} \text{Rule tactic}] [S6 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}},$
 $\underline{b}: \underline{a} + \underline{b}' = \underline{a} + \underline{b}'] [S6 \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[S7 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}: \underline{a} \cdot 0 = 0] [S7 \xrightarrow{\text{proof}} \text{Rule tactic}] [S8 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}},$
 $\underline{b}: \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}] [S8 \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[Neg \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}: \Pi_{\underline{b}}: \neg \underline{b} \Rightarrow \neg \underline{a} \vdash \neg \underline{b} \Rightarrow \underline{a} \vdash \underline{b}] [Neg \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[S1 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}, \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} = \underline{c} \vdash \underline{b} = \underline{c}] [S1 \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[S9 \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{x}, \underline{a}, \underline{b}},$
 $\underline{c}: \langle \underline{b} \equiv \underline{a} | \underline{x} := 0 \rangle \Vdash \langle \underline{c} \equiv \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash \underline{b} \vdash \underline{a} \Rightarrow \underline{c} \vdash \underline{a}] [S9 \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2a} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}: \underline{a} = \underline{a}] [\text{Prop 3.2a} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2b} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{a}] [\text{Prop 3.2b} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2c} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}, \underline{c}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{c} \vdash \underline{a} = \underline{c}] [\text{Prop 3.2c} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2d} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}, \underline{c}: \underline{a} = \underline{c} \vdash \underline{b} = \underline{c} \vdash \underline{a} = \underline{b}] [\text{Prop 3.2d} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2e} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}, \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} + \underline{c} = \underline{b} + \underline{c}] [\text{Prop 3.2e} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2f} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}: \underline{a} = 0 + \underline{a}] [\text{Prop 3.2f} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2g} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}: \underline{a}' + \underline{b} = \underline{a} + \underline{b}'] [\text{Prop 3.2g} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2h} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}: \underline{a} + \underline{b} = \underline{b} + \underline{a}] [\text{Prop 3.2h} \xrightarrow{\text{proof}} \text{Rule tactic}]$
 $[\text{Prop 3.2i} \xrightarrow{\text{stmt}} S \vdash \Pi_{\underline{a}}, \underline{b}, \underline{c}: \underline{a} = \underline{b} \vdash \underline{c} + \underline{a} = \underline{c} + \underline{b}]$

we will start by doing Prop 3.2i

$[\text{Prop 3.2i} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S \vdash$
 $L_{\text{?}}: \quad \text{Arbitrary} \gg \quad \underline{a}, \underline{b}, \underline{c} \quad ;$
 $L_{\text{?}}: \quad \text{Block} \gg \quad \text{Begin} \quad ;$
 $L_{\text{?}}: \quad \text{Arbitrary} \gg \quad \underline{a}, \underline{b}, \underline{c} \quad \underline{a} \neq$
 $\underline{b} \vdash \text{Prop 3.2e} \triangleright \underline{a} = \underline{b} \gg \underline{a} + \underline{c} = \underline{b} + \underline{c}; \text{Prop 3.2h} \gg \underline{a} + \underline{c} =$
 $\underline{c} + \underline{a}; \text{Prop 3.2h} \gg \underline{b} + \underline{c} = \underline{c} + \underline{b}; S1 \triangleright \underline{a} + \underline{c} = \underline{b} + \underline{c} \triangleright \underline{a} + \underline{c} = \underline{c} + \underline{a} \gg \underline{b} + \underline{c} =$
 $\underline{c} + \underline{a}; \text{Prop 3.2b} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{b} \gg \underline{c} + \underline{a} = \underline{b} + \underline{c};$
 $L_{\text{?}}: \quad \text{Prop 3.2c} \triangleright \underline{c} + \underline{a} =$
 $\underline{b} + \underline{c} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{b} \gg \quad \underline{c} + \underline{a} = \underline{c} + \underline{b} \quad ;$
 $L_{\text{g}}: \quad \text{Block} \gg \quad \text{End} \quad \text{Ded} \triangleright$
 $L_{\text{g}} \gg \underline{a} = \underline{b} \vdash \underline{c} + \underline{a} = \underline{c} + \underline{b}], p_0, c)]$

Litteratur

- [Men97] E. Mendelson. *Introduction to Mathematical Logic*. Chapman & Hall, 4. edition, 1997.