

projekt i logik

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Resumé

1 Introduction

In this paper we will prove the propositions regarding S starting with proposition Prop 3.2i. continuing to 3.4, bevise 3.5, aksiomer, der definerer $x-y$, bevise 3.10, og bevise 3.11.

We chose to prove chapter 3 from mendelson, starting with Lemma 3.2, since the first ten wfs' are proved in Mendelson, we will start with Lemma 3.2.i.

2

A modified version of Mendelsons system S (Peano arithmetic) [Men97] may be formulated thus:

[Theory S]	[S rule MP: $\Pi A, B: A \Rightarrow B \vdash A \vdash B$]
[S rule Gen: $\Pi X, A: A \vdash \forall X: A$]	[S rule Ded: $\Pi A, B: \text{Ded}(A, B) \Vdash A \vdash B$]
	[S rule S2: $\Pi A, B: A = B \vdash A' = B'$]
[S rule S3: $\Pi A: \neg 0 = A'$]	[S rule S4: $\Pi A, B: A' = B' \vdash A = B$]
[S rule S5: $\Pi A: A + 0 = A$]	[S rule S6: $\Pi A, B: A + B' = (A + B)'$]
[S rule S7: $\Pi A: A \cdot 0 = 0$]	[S rule S8: $\Pi A, B: A \cdot (B') = (A \cdot B) + A$]
[S rule Neg: $\Pi A, B: \neg B \Rightarrow \neg A \vdash \neg B \Rightarrow A \vdash B$]	

[S rule S1: $\Pi A, B, C: A = B \vdash A = C \vdash B = C$]

[S rule S9: $\Pi \mathcal{X}, A, B, C: \langle B \equiv A | \mathcal{X}: = 0 \rangle \vdash \langle C \equiv A | \mathcal{X}: = \mathcal{X}' \rangle \vdash B \vdash A \Rightarrow C \vdash A$]

[S rule Prop 3.2a: $\Pi A: A = A$]

[S rule Prop 3.2b: $\Pi A, B: A = B \vdash B = A$]

[S rule Prop 3.2c: $\Pi A, B, C: A = B \vdash B = C \vdash A = C$]

[S rule Prop 3.2d: $\Pi A, B, C: A = C \vdash B = C \vdash A = B$]

[S rule Prop 3.2e: $\Pi A, B, C: A = B \vdash A + C = B + C$]

[S rule Prop 3.2f: $\Pi A: A = 0 + A$]

[S rule Prop 3.2g: $\Pi A, B: A' + B = (A + B)'$]

[S rule Prop 3.2h: $\Pi A, B: A + B = B + A$]

[S lemma Prop 3.2i: $\Pi A, B, C: A = B \vdash C + A = C + B$]

we will start by doing Prop 3.2i

S proof of Prop 3.2i:

L01:	Arbitrary \gg	A, B, C	;
L02:	Block \gg	Begin	;
L03:	Arbitrary \gg	A, B, C	;
L04:	Premise \gg	$A = B$;
L05:	Prop 3.2e \triangleright L04 \gg	$A + C = B + C$;
L06:	Prop 3.2h \gg	$A + C = C + A$;
L07:	Prop 3.2h \gg	$B + C = C + B$;
L08:	S1 \triangleright L05 \triangleright L06 \gg	$B + C = C + A$;
L09:	Prop 3.2b \triangleright L08 \gg	$C + A = B + C$;
L10:	Prop 3.2c \triangleright L09 \triangleright L07 \gg	$C + A = C + B$;
L11:	Block \gg	End	;
L12:	Ded \triangleright L11 \gg	$A = B \vdash C + A = C + B$	□

Litteratur

[Men97] E. Mendelson. *Introduction to Mathematical Logic*. Chapman & Hall, 4. edition, 1997.