projekt i logik

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Indhold

1 Introduction **1**

2 **1**

Resumé

1 Introduction

In this paper we will prove the propositions regarding S starting with proposition Prop 3.2i. continuing to 3.4, bevise 3.5, aksiomer, der definerer x—y, bevise 3.10, og bevise 3.11.
We chose to prove chapter 3 from mendelson, starting with Lemma 3.2, since the first ten wfs' are proved in Mendelson, we will start with Lemma 3.2.i.

2

A modified version of Mendelsons system S (Peano arithmetic) [Men97] may be formulated thus:

$[\mathrm{S} \overset{\text{stmt}}{\longrightarrow} \mathsf{x}]$

$[\mathrm{MP} \overset{\text{stmt}}{\longrightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}][\mathrm{MP} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{Gen} \overset{\text{stmt}}{\longrightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \vdash \forall_{\mathrm{obj}} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}][\mathrm{Gen} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{Ded} \overset{\text{stmt}}{\longrightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \lambda \mathrm{x}.\mathrm{Ded}_0(\lceil \underline{\mathsf{a}} \rceil, \lceil \underline{\mathsf{b}} \rceil) \Vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}][\mathrm{Ded} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{S2} \overset{\text{stmt}}{\longrightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{a}}' = \underline{\mathsf{b}}'][\mathrm{S2} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{S3} \overset{\text{stmt}}{\longrightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \neg 0 = \underline{\mathsf{a}}'][\mathrm{S3} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{S4} \overset{\text{stmt}}{\longrightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}}' = \underline{\mathsf{b}}' \vdash \underline{\mathsf{a}} = \underline{\mathsf{b}}][\mathrm{S4} \overset{\text{proof}}{\longrightarrow} \text{Rule tactic}]$

$[\mathrm{S5} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \underline{\mathsf{a}} + 0 = \underline{\mathsf{a}}][\mathrm{S5} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}][\mathrm{S6} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} + \underline{\mathsf{b}}' = \underline{\mathsf{a}} + \underline{\mathsf{b}}'][\mathrm{S6} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S7} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \underline{\mathsf{a}} \cdot 0 = 0][\mathrm{S7} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}][\mathrm{S8} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} \cdot \underline{\mathsf{b}}' = \underline{\mathsf{a}} \cdot \underline{\mathsf{b}} + \underline{\mathsf{a}}][\mathrm{S8} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{Neg} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \neg \underline{\mathsf{b}} \Rightarrow \neg \underline{\mathsf{a}} \vdash \neg \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}][\mathrm{Neg} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S1} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} = \underline{\mathsf{c}} \vdash \underline{\mathsf{b}} = \underline{\mathsf{c}}][\mathrm{S1} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\mathrm{S9} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \langle \lceil \underline{\mathsf{b}} \rceil {\equiv}^0 \lceil \underline{\mathsf{a}} \rceil | \lceil \underline{\mathsf{x}} \rceil {:=} \lceil 0 \rceil \rangle \Vdash \langle \lceil \underline{\mathsf{c}} \rceil {\equiv}^0 \lceil \underline{\mathsf{a}} \rceil | \lceil \underline{\mathsf{x}} \rceil {:=} \lceil \underline{\mathsf{x}}' \rceil \rangle \Vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}}][\mathrm{S9} \overset{\mathrm{proof}}{\rightarrow} \text{Rule tactic}]$

$[\text{Prop 3.2a} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \underline{\mathsf{a}} = \underline{\mathsf{a}}]$

$[\text{Prop 3.2b} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{b}} = \underline{\mathsf{a}}]$

$[\text{Prop 3.2c} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{b}} = \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} = \underline{\mathsf{c}}]$

$[\text{Prop 3.2d} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{c}} \vdash \underline{\mathsf{b}} = \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} = \underline{\mathsf{b}}]$

$[\text{Prop 3.2e} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} + \underline{\mathsf{c}} = \underline{\mathsf{b}} + \underline{\mathsf{c}}]$

$[\text{Prop 3.2f} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \underline{\mathsf{a}} = 0 + \underline{\mathsf{a}}]$

$[\text{Prop 3.2g} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}}' + \underline{\mathsf{b}} = \underline{\mathsf{a}} + \underline{\mathsf{b}}']$

$[\text{Prop 3.2h} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} + \underline{\mathsf{b}} = \underline{\mathsf{b}} + \underline{\mathsf{a}}]$

$[\text{Prop 3.2i} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} + \underline{\mathsf{a}} = \underline{\mathsf{c}} + \underline{\mathsf{b}}]$

$[\text{Prop 3.2j} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} + \underline{\mathsf{b}} + \underline{\mathsf{c}} = \underline{\mathsf{a}} + \underline{\mathsf{b}} + \underline{\mathsf{c}}]$

$[\text{Prop 3.2k} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \cdot \underline{\mathsf{c}} = \underline{\mathsf{b}} \cdot \underline{\mathsf{c}}]$

$[\text{Prop 3.2l} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon 0 \cdot \underline{\mathsf{a}} = 0]$

$[\text{Prop 3.2m} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}}' \cdot \underline{\mathsf{b}} = \underline{\mathsf{a}} \cdot \underline{\mathsf{b}} + \underline{\mathsf{b}}]$

$[\text{Prop 3.2n} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} \cdot \underline{\mathsf{b}} = \underline{\mathsf{b}} \cdot \underline{\mathsf{a}}]$

$[\text{Prop 3.2o} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \cdot \underline{\mathsf{a}} = \underline{\mathsf{c}} \cdot \underline{\mathsf{b}}]$

we will start by doing Prop 3.2i

$[\text{Prop 3.2i} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \underline{\mathsf{a}} = \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} + \underline{\mathsf{a}} = \underline{\mathsf{c}} + \underline{\mathsf{b}}]$

$[\text{Prop 3.2i} \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \text{Prop 3.2e} \gg \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} =$
$\underline{b} + \underline{c}; \text{Prop 3.2h} \gg \underline{a} + \underline{c} = \underline{c} + \underline{a}; \text{Prop 3.2h} \gg \underline{b} + \underline{c} = \underline{c} + \underline{b}; \underline{a} = \underline{b} \vdash \text{MP} \triangleright \underline{a} =$
$\underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \triangleright \underline{a} = \underline{b} \gg \underline{a} + \underline{c} = \underline{b} + \underline{c}; \text{S1} \gg \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{c} = \underline{c} + \underline{a} \Rightarrow$
$\underline{b} + \underline{c} = \underline{c} + \underline{a}; \text{MP} \triangleright \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{c} = \underline{c} + \underline{a} \Rightarrow \underline{b} + \underline{c} = \underline{c} + \underline{a} \triangleright \underline{a} + \underline{c} =$
$\underline{b} + \underline{c} \gg \underline{a} + \underline{c} = \underline{c} + \underline{a} \Rightarrow \underline{b} + \underline{c} = \underline{c} + \underline{a}; \text{MP} \triangleright \underline{a} + \underline{c} = \underline{c} + \underline{a} \Rightarrow \underline{b} + \underline{c} =$
$\underline{c} + \underline{a} \triangleright \underline{a} + \underline{c} = \underline{c} + \underline{a} \gg \underline{b} + \underline{c} = \underline{c} + \underline{a}; \text{Prop 3.2b} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{a} \gg \underline{c} + \underline{a} =$
$\underline{b} + \underline{c}; \text{Prop 3.2e} \gg \underline{c} + \underline{a} = \underline{b} + \underline{c} \Rightarrow \underline{b} + \underline{c} = \underline{c} + \underline{b} \Rightarrow \underline{c} + \underline{a} = \underline{c} + \underline{b}; \text{MP} \triangleright \underline{c} + \underline{a} =$
$\underline{b} + \underline{c} \Rightarrow \underline{b} + \underline{c} = \underline{c} + \underline{b} \Rightarrow \underline{c} + \underline{a} = \underline{c} + \underline{b} \triangleright \underline{c} + \underline{a} = \underline{b} + \underline{c} \gg \underline{b} + \underline{c} = \underline{c} + \underline{b} \Rightarrow$
$\underline{c} + \underline{a} = \underline{c} + \underline{b}; \text{MP} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{b} \Rightarrow \underline{c} + \underline{a} = \underline{c} + \underline{b} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{b} \gg \underline{c} + \underline{a} =$
$\underline{c} + \underline{b}; \text{Ded} \triangleright \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} = \underline{b} \vdash \underline{c} + \underline{a} = \underline{c} + \underline{b} \gg \underline{a} = \underline{b} \Rightarrow \underline{c} + \underline{a} = \underline{c} + \underline{b}\rceil, \mathrm{p}_0, c)]$

$[\text{Prop 3.2j}_1 \stackrel{\text{stmt}}{\to} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} + 0]$

$[\text{Prop 3.2j}_1 \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \text{S5} \gg \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b}; \text{S5} \gg \underline{b} + 0 =$
$\underline{b}; \text{Prop 3.2i} \triangleright \underline{b} + 0 = \underline{b} \gg \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b}; \text{Prop 3.2d} \triangleright \underline{a} + \underline{b} + 0 =$
$\underline{a} + \underline{b} \triangleright \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} \gg \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} + 0\rceil, \mathrm{p}_0, c)]$

$[\text{Prop 3.2j}_2 \stackrel{\text{stmt}}{\to} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}']$

$[\text{Prop 3.2j}_2 \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \vdash \text{S6} \gg$
$\underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{S2} \triangleright \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \gg \underline{a} + \underline{b} + \underline{c}' =$
$\underline{a} + \underline{b} + \underline{c}'; \text{Prop 3.2c} \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' =$
$\underline{a} + \underline{b} + \underline{c}'; \text{S6} \gg \underline{b} + \underline{c}' = \underline{b} + \underline{c}'; \text{Prop 3.2i} \triangleright \underline{b} + \underline{c}' = \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' =$
$\underline{a} + \underline{b} + \underline{c}'; \text{S6} \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Prop 3.2c} \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \triangleright \underline{a} + \underline{b} + \underline{c}' =$
$\underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Prop 3.2d} \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \triangleright \underline{a} + \underline{b} + \underline{c}' =$
$\underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Ded} \triangleright \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \vdash$
$\underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'\rceil, \mathrm{p}_0, c)]$

$[\text{Prop 3.2j} \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \text{Prop 3.2j}_1 \gg \bar{x} + \bar{y} + 0 =$
$\bar{x} + \bar{y} + 0; \text{Prop 3.2j}_2 \gg \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z} \Rightarrow \bar{x} + \bar{y} + \bar{z}' =$
$\bar{x} + \bar{y} + \bar{z}'; \text{S9} \mathbin{@} \bar{z} \triangleright \bar{x} + \bar{y} + 0 = \bar{x} + \bar{y} + 0 \triangleright \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z} \Rightarrow \bar{x} + \bar{y} + \bar{z}' = \bar{x} + \bar{y} + \bar{z}' \gg$
$\bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z}; \text{Ded} \triangleright \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z} \gg \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c}\rceil, \mathrm{p}_0, c)]$

$[\text{Prop 3.2k}_1 \stackrel{\text{stmt}}{\to} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} = \underline{b} \vdash \underline{a} \cdot 0 = \underline{b} \cdot 0]$

$[\text{Prop 3.2k}_1 \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \text{S7} \gg \underline{a} \cdot 0 = 0; \text{S7} \gg \underline{b} \cdot 0 = 0; \text{Prop 3.2a} \gg$
$0 = 0; \text{Prop 3.2b} \triangleright \underline{b} \cdot 0 = 0 \gg 0 = \underline{b} \cdot 0; \text{Prop 3.2c} \gg \underline{a} \cdot 0 = 0 \Rightarrow 0 = \underline{b} \cdot 0 \Rightarrow \underline{a} \cdot 0 =$
$\underline{b} \cdot 0; \underline{a} \cdot 0 = 0 \Rightarrow 0 = \underline{b} \cdot 0 \Rightarrow \underline{a} \cdot 0 = \underline{b} \cdot 0 \triangleright \underline{a} \cdot 0 = 0 \triangleright 0 = \underline{b} \cdot 0 \gg \underline{a} \cdot 0 = \underline{b} \cdot 0\rceil, \mathrm{p}_0, c)]$

$[\text{Prop 3.2k}_2 \stackrel{\text{stmt}}{\to} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \vdash \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c}']$

$[\text{Prop 3.2k}_2 \stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \vdash \underline{a} =$
$\underline{b} \vdash \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \triangleright \underline{a} = \underline{b} \gg \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}; \text{S8} \gg \underline{a} \cdot \underline{c}' = \underline{a} \cdot \underline{c} + \underline{a}; \text{S8} \gg \underline{b} \cdot \underline{c}' =$
$\underline{b} \cdot \underline{c} + \underline{b}; \text{Prop 3.2e} \gg \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{a} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{a}; \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{a} \cdot \underline{c} + \underline{a} =$
$\underline{b} \cdot \underline{c} + \underline{a} \triangleright \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \gg \underline{a} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{a}; \text{Prop 3.2i} \gg \underline{a} = \underline{b} \Rightarrow \underline{b} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b}; \underline{a} =$
$\underline{b} \Rightarrow \underline{b} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b} \triangleright \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \gg \underline{b} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b}; \text{Prop 3.2c} \gg \underline{a} \cdot \underline{c} + \underline{a} =$

$\underline{b}\cdot\underline{c}+\underline{a} \Rightarrow \underline{b}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b} \Rightarrow \underline{a}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b}; \underline{a}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{a} \Rightarrow \underline{b}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b} \Rightarrow \underline{a}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b} \rhd \underline{a}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{a} \rhd \underline{b}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b} \gg \underline{a}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b}; \underline{a}\cdot\underline{c}+\underline{a} = \underline{b}\cdot\underline{c}+\underline{b} \rhd \underline{a}\cdot\underline{c}' = \underline{a}\cdot\underline{c}+\underline{a} \rhd \underline{b}\cdot\underline{c}' = \underline{b}\cdot\underline{c}+\underline{b} \gg \underline{a}\cdot\underline{c}' = \underline{b}\cdot\underline{c}'; \mathrm{Ded} \rhd \forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon \underline{a} = \underline{b} \Rightarrow \underline{a}\cdot\underline{c} = \underline{b}\cdot\underline{c} \vdash \underline{a} = \underline{b} \vdash \underline{a}\cdot\underline{c}' = \underline{b}\cdot\underline{c}' \gg \underline{a} = \underline{b} \Rightarrow \underline{a}\cdot\underline{c} = \underline{b}\cdot\underline{c} \vdash \underline{a} = \underline{b} \Rightarrow \underline{a}\cdot\underline{c}' = \underline{b}\cdot\underline{c}'\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{Prop 3.2k} \overset{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon \text{Prop 3.2k}_1 \gg \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot 0 = \bar{y}\cdot 0; \text{Prop 3.2k}_2 \gg \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot\bar{z} = \bar{y}\cdot\bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot\bar{z}' = \bar{y}\cdot\bar{z}'; \mathrm{S9} \mathbin{@} \bar{z} \rhd \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot 0 = \bar{y}\cdot 0 \rhd \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot\bar{z} = \bar{y}\cdot\bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot\bar{z}' = \bar{y}\cdot\bar{z}' \gg \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot\bar{z} = \bar{y}\cdot\bar{z}; \mathrm{Ded} \rhd \bar{x} = \bar{y} \Rightarrow \bar{x}\cdot\bar{z} = \bar{y}\cdot\bar{z} \gg \underline{a} = \underline{b} \vdash \underline{a}\cdot\underline{c} = \underline{b}\cdot\underline{c}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{Prop 3.2l}_1 \overset{\text{stmt}}{\to} \mathrm{S} \vdash \forall\underline{a}\colon 0\cdot 0 = 0]$

$[\text{Prop 3.2l}_1 \overset{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \mathrm{S7} \gg 0\cdot 0 = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{Prop 3.2l}_2 \overset{\text{stmt}}{\to} \mathrm{S} \vdash \forall\underline{a}\colon 0\cdot\underline{a} = 0 \Rightarrow 0\cdot\underline{a}' = 0]$

$[\text{Prop 3.2l}_2 \overset{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall\underline{a}\colon\forall\underline{a}\colon 0\cdot\underline{a} = 0 \vdash \mathrm{S8} \gg 0\cdot\underline{a}' = 0\cdot\underline{a} + 0; \mathrm{S5} \gg 0\cdot\underline{a} + 0 = 0\cdot\underline{a}; \text{Prop 3.2c} \rhd 0\cdot\underline{a} + 0 = 0\cdot\underline{a} \rhd 0\cdot\underline{a}' = 0\cdot\underline{a} + 0 \gg 0\cdot\underline{a}' = 0\cdot\underline{a}; \text{Prop 3.2c} \rhd 0\cdot\underline{a}' = 0\cdot\underline{a} \rhd 0\cdot\underline{a} = 0 \gg 0\cdot\underline{a}' = 0; \mathrm{Ded} \rhd \forall\underline{a}\colon 0\cdot\underline{a} = 0 \vdash 0\cdot\underline{a}' = 0 \gg 0\cdot\underline{a} = 0 \Rightarrow 0\cdot\underline{a}' = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{Prop 3.2l} \overset{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall\underline{a}\colon \text{Prop 3.2l}_1 \gg 0\cdot 0 = 0; \text{Prop 3.2l}_2 \gg 0\cdot\bar{x} = 0 \Rightarrow 0\cdot\bar{x}' = 0; \mathrm{S9} \mathbin{@} \bar{x} \rhd 0\cdot 0 = 0 \rhd 0\cdot\bar{x} = 0 \Rightarrow 0\cdot\bar{x}' = 0 \gg 0\cdot\bar{x} = 0; \mathrm{Ded} \rhd 0\cdot\bar{x} = 0 \gg 0\cdot\underline{a} = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{Prop 3.2m}_1 \overset{\text{stmt}}{\to} \mathrm{S} \vdash \forall\underline{a}\colon \underline{a}'\cdot 0 = \underline{t}\cdot 0 + 0]$

$[\text{Prop 3.2m}_1 \overset{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall\underline{a}\colon \mathrm{S7} \gg \underline{a}'\cdot 0 = 0; \text{Prop 3.2f} \gg 0 = 0 + 0; \mathrm{S7} \gg 0 = \underline{a}\cdot 0; \text{Prop 3.2e} \gg 0 = \underline{a}\cdot 0 \Rightarrow 0 + 0 = \underline{a}\cdot 0 + 0; 0 = \underline{a}\cdot 0 \Rightarrow 0 + 0 = \underline{a}\cdot 0 + 0 \rhd 0 = \underline{a}\cdot 0 \gg 0 + 0 = \underline{a}\cdot 0 + 0; \text{Prop 3.2c} \gg 0 = 0 + 0 \Rightarrow 0 + 0 = \underline{a}\cdot 0 + 0 \Rightarrow 0 = \underline{a}\cdot 0 + 0; 0 = 0 + 0 \Rightarrow 0 + 0 = \underline{a}\cdot 0 + 0 \Rightarrow 0 = \underline{a}\cdot 0 + 0 \rhd 0 = 0 + 0 \rhd 0 + 0 = \underline{a}\cdot 0 + 0 \gg 0 = \underline{a}\cdot 0 + 0; \text{Prop 3.2c} \gg \underline{a}'\cdot 0 = 0 \Rightarrow 0 = \underline{a}\cdot 0 + 0 \Rightarrow \underline{a}'\cdot 0 = \underline{a}\cdot 0 + 0; \underline{a}'\cdot 0 = 0 \Rightarrow 0 = \underline{a}\cdot 0 + 0 \Rightarrow \underline{a}'\cdot 0 = \underline{a}\cdot 0 + 0 \rhd \underline{a}'\cdot 0 = 0 \rhd \underline{a}'\cdot 0 = 0 \Rightarrow 0 = \underline{a}\cdot 0 + 0 \Rightarrow \underline{a}'\cdot 0 = \underline{a}\cdot 0 + 0 \gg \underline{a}'\cdot 0 = \underline{a}\cdot 0 + 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{Prop 3.2m}_2 \overset{\text{stmt}}{\to} \mathrm{S} \vdash \forall\underline{a}\colon\forall\underline{b}\colon \underline{a}'\cdot\underline{b} = \underline{a}\cdot\underline{b} + \underline{b} \Rightarrow \underline{a}'\cdot\underline{b}' = \underline{a}\cdot\underline{b}' + \underline{b}']$

$[\text{Prop 3.2m}_2 \overset{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{a}\colon\forall\underline{b}\colon \underline{a}'\cdot\underline{b} = \underline{a}\cdot\underline{b} + \underline{b} \vdash \mathrm{S8} \gg \underline{a}'\cdot\underline{b}' = \underline{a}'\cdot\underline{b}' + \underline{a}'; \text{Prop 3.2e} \gg \underline{a}'\cdot\underline{b} = \underline{a}\cdot\underline{b} + \underline{b} \Rightarrow \underline{a}'\cdot\underline{b} + \underline{a}' = \underline{a}\cdot\underline{b} + \underline{b} + \underline{a}'; \underline{a}'\cdot\underline{b} = \underline{a}\cdot\underline{b} + \underline{b} \Rightarrow \underline{a}'\cdot\underline{b} + \underline{a}' = \underline{a}\cdot\underline{b} + \underline{b} + \underline{a}' \rhd \underline{a}'\cdot\underline{b} = \underline{a}\cdot\underline{b} + \underline{b} \gg \underline{a}'\cdot\underline{b} + \underline{a}' = \underline{a}\cdot\underline{b} + \underline{b} + \underline{a}'; \mathrm{S6} \gg \underline{b} + \underline{a}' = \underline{b} + \underline{a}'; \text{Prop 3.2g} \gg \underline{b}' + \underline{a} = \underline{b} + \underline{a}'; \text{Prop 3.2d} \rhd \underline{b} + \underline{a}' = \underline{b} + \underline{a}' \rhd \underline{b}' + \underline{a} = \underline{b} + \underline{a}' \gg \underline{b} + \underline{a}' = \underline{b}' + \underline{a}; \text{Prop 3.2h} \gg \underline{b}' + \underline{a} = \underline{a} + \underline{b}'; \text{Prop 3.2c} \rhd \underline{b} + \underline{a}' = \underline{b}' + \underline{a} \rhd \underline{b}' + \underline{a} = \underline{a} + \underline{b}' \gg \underline{b} + \underline{a}' = \underline{a} + \underline{b}'; \text{Prop 3.2i} \rhd \underline{b} + \underline{a}' = \underline{a} + \underline{b}' \gg \underline{a}\cdot\underline{b} + \underline{b} + \underline{a}' = \underline{a}\cdot\underline{b} + \underline{a} + \underline{b}'; \mathrm{S8} \gg \underline{a}\cdot\underline{b}' = \underline{a}\cdot\underline{b} + \underline{a}; \text{Prop 3.2e} \rhd \underline{a}\cdot\underline{b}' = \underline{a}\cdot\underline{b} + \underline{a} \gg \underline{a}\cdot\underline{b}' + \underline{b}' = \underline{a}\cdot\underline{b} + \underline{a} + \underline{b}'; \text{Prop 3.2d} \rhd \underline{a}\cdot\underline{b}' + \underline{b}' = \underline{a}\cdot\underline{b} + \underline{a} + \underline{b}' \rhd \underline{a}\cdot\underline{b} + \underline{b} + \underline{a}' = \underline{a}\cdot\underline{b} + \underline{a} + \underline{b}' \gg \underline{a}\cdot\underline{b} + \underline{b} + \underline{a}' = \underline{a}\cdot\underline{b}' + \underline{b}'; \text{Prop 3.2c} \rhd \underline{a}'\cdot\underline{b} + \underline{a}' =$

$\underline{a} \cdot \underline{b} + \underline{b} + \underline{a}' \rhd \underline{a} \cdot \underline{b} + \underline{b} + \underline{a}' = \underline{a} \cdot \underline{b}' + \underline{b}' \gg \underline{a}' \cdot \underline{b} + \underline{a}' = \underline{a} \cdot \underline{b}' + \underline{b}'; \text{Prop 3.2c} \rhd \underline{a}' \cdot \underline{b}' = \underline{a}' \cdot \underline{b}' + \underline{a}' \rhd \underline{a}' \cdot \underline{b} + \underline{a}' = \underline{a} \cdot \underline{b}' + \underline{b}' \gg \underline{a}' \cdot \underline{b}' = \underline{a} \cdot \underline{b}' + \underline{b}'; \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \underline{a}' \cdot \underline{b} = \underline{a} \cdot \underline{b} + \underline{b} \vdash \underline{a}' \cdot \underline{b}' = \underline{a} \cdot \underline{b}' + \underline{b}' \gg \underline{a}' \cdot \underline{b} = \underline{a} \cdot \underline{b} + \underline{b} \Rightarrow \underline{a}' \cdot \underline{b}' = \underline{a} \cdot \underline{b}' + \underline{b}' \rceil, p_0, \mathsf{c})]$

$[\text{Prop 3.2m} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \text{Prop 3.2m}_1 \gg \overline{x}' \cdot 0 = \overline{x} \cdot 0 + 0; \text{Prop 3.2m}_2 \gg \overline{x}' \cdot \overline{y} = \overline{x} \cdot \overline{y} + \overline{y} \Rightarrow \overline{x}' \cdot \overline{y}' = \overline{x} \cdot \overline{y}' + \overline{y}; \text{S9} \mathbin{@} \overline{y} \rhd \overline{x}' \cdot 0 = \overline{x} \cdot 0 + 0 \rhd \overline{x}' \cdot \overline{y} = \overline{x} \cdot \overline{y} + \overline{y} \Rightarrow \overline{x}' \cdot \overline{y}' = \overline{x} \cdot \overline{y}' + \overline{y} \gg \overline{x}' \cdot \overline{y} = \overline{x} \cdot \overline{y} + \overline{y}; \text{Ded} \rhd \overline{x}' \cdot \overline{y} = \overline{x} \cdot \overline{y} + \overline{y} \gg \underline{a}' \cdot \underline{b} = \underline{a} \cdot \underline{b} + \underline{b} \rceil, p_0, \mathsf{c})]$

$[\text{Prop 3.2n}_1 \overset{\text{stmt}}{\to} \text{S} \vdash \forall \underline{a}\colon \underline{a} \cdot 0 = 0 \cdot \underline{a}]$

$[\text{Prop 3.2n}_1 \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{S} \vdash \forall \underline{a}\colon \text{S7} \gg \underline{a} \cdot 0 = 0; \text{Prop 3.2l} \gg 0 \cdot \underline{a} = 0; \text{Prop 3.2c} \rhd \underline{a} \cdot 0 = 0 \rhd 0 \cdot \underline{a} = 0 \gg \underline{a} \cdot 0 = 0 \cdot \underline{a} \rceil, p_0, \mathsf{c})]$

$[\text{Prop 3.2n}_2 \overset{\text{stmt}}{\to} \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \Rightarrow \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a}]$

$[\text{Prop 3.2n}_2 \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \vdash \text{S8} \gg \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}; \text{Prop 3.2e} \rhd \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \gg \underline{a} \cdot \underline{b} + \underline{a} = \underline{b} \cdot \underline{a} + \underline{a}; \text{Prop 3.2b} \rhd \text{Prop 3.2m} \gg \underline{b} \cdot \underline{a} + \underline{a} = \underline{b}' \cdot \underline{a}; \text{Prop 3.2c} \rhd \text{L}_\text{e} \rhd \underline{b} \cdot \underline{a} + \underline{a} = \underline{b}' \cdot \underline{a} \gg \underline{a} \cdot \underline{b} + \underline{a} = \underline{b}' \cdot \underline{a}; \text{Prop 3.2c} \rhd \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a} \rhd \underline{a} \cdot \underline{b} + \underline{a} = \underline{b}' \cdot \underline{a} \gg \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a}; \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \vdash \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a} \gg \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \Rightarrow \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{s} \rceil, p_0, \mathsf{c})]$

$[\text{Prop 3.2n} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \text{Prop 3.2n}_1 \gg \overline{x} \cdot 0 = 0 \cdot \overline{x}; \text{Prop 3.2n}_2 \gg \overline{x} \cdot \overline{y} = \overline{y} \cdot \overline{x} \Rightarrow \overline{x} \cdot \overline{y}' = \overline{y}' \cdot \overline{x}; \text{S9} \mathbin{@} \overline{y} \rhd \overline{x} \cdot 0 = 0 \cdot \overline{x} \rhd \overline{x} \cdot \overline{y} = \overline{y} \cdot \overline{x} \Rightarrow \overline{x} \cdot \overline{y}' = \overline{y}' \cdot \overline{x} \gg \overline{x} \cdot \overline{y} = \overline{y} = \overline{x}; \text{Ded} \rhd \overline{x} \cdot \overline{y} = \overline{y} = \overline{x} \gg \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \rceil, p_0, \mathsf{c})]$

$[\text{Prop 3.2o} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} = \underline{b} \vdash \text{Prop 3.2k} \gg \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}; \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \rhd \underline{a} = \underline{b} \gg \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}; \text{Prop 3.2n} \gg \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{a}; \text{Prop 3.2n} \gg \underline{b} \cdot \underline{c} = \underline{c} \cdot \underline{b}; \text{Prop 3.2c} \gg \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{b} \cdot \underline{c} = \underline{c} \cdot \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{b}; \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{b} \cdot \underline{c} = \underline{c} \cdot \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{b} \rhd \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \rhd \underline{b} \cdot \underline{c} = \underline{c} \cdot \underline{b} \gg \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{b}; \text{S1} \gg \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{a} \Rightarrow \underline{c} \cdot \underline{a} = \underline{c} \cdot \underline{b}; \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{a} \Rightarrow \underline{c} \cdot \underline{a} = \underline{c} \cdot \underline{b} \rhd \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{b} \rhd \underline{a} \cdot \underline{c} = \underline{c} \cdot \underline{a} \gg \underline{c} \cdot \underline{a} = \underline{c} \cdot \underline{b}; \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} = \underline{b} \vdash \underline{c} \cdot \underline{a} = \underline{c} \cdot \underline{b} \gg \underline{a} = \underline{b} \Rightarrow \underline{c} \cdot \underline{a} = \underline{c} \cdot \underline{b} \rceil, p_0, \mathsf{c})]$

$[\text{Prop 3.4a} \overset{\text{stmt}}{\to} \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \cdot \underline{b} + \underline{c} = \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}]$

$[\text{Prop 3.4b} \overset{\text{stmt}}{\to} \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{b} + \underline{c} \cdot \underline{a} = \underline{b} \cdot \underline{a} + \underline{c} \cdot \underline{b}]$

$[\text{Prop 3.4c} \overset{\text{stmt}}{\to} \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \cdot \underline{b} \cdot \underline{c} = \underline{a} \cdot \underline{b} \cdot \underline{c}]$

$[\text{Prop 3.4d} \overset{\text{stmt}}{\to} \text{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} = \underline{b}]$

Litteratur

[Men97] E. Mendelson. *Introduction to Mathematical Logic.* Chapmann & Hall, 4. edition, 1997.