

Logiweb codex of opgave

Up Help

S, Neg, MP, Gen, Ded, S1, S2, S3, S4, S5, S6, S7, S8, S9, Prop 3.2a, Prop 3.2b, Prop 3.2c, Prop 3.2d, Prop 3.2e, Prop 3.2f, Prop 3.2g, Prop 3.2h, opgave, $\bar{0}$, $\bar{1}$, $\bar{2}$, $\bar{3}$, $\bar{4}$, $\bar{5}$, $\bar{6}$, $\bar{7}$, $\bar{8}$, $\bar{9}$, \bar{n} , rule div, R, R1, R2, R3, R4, R5, R6, Con1, Con2, Con, Dis1, Dis2, Lem1.11c, Cor1.10a, Cor1.10b, Lem1.11a, Lem1.11b, H3, Prop3.2c', S1'', Neg', Repetition', Lem1.11e, Lem1.11d, Prop3.2b', H10, H11, Lem1.11g, MT, S10, Prop 3.2, Prop 3.2i, Prop 3.2j₁, Prop 3.2j₂, Prop 3.2j, Prop 3.2k₁, Prop 3.2k₂, Prop 3.2k, Prop 3.2l₁, Prop 3.2l₂, Prop 3.2l, Prop 3.2m₁, Prop 3.2m₂, Prop 3.2m, Prop 3.2n₁, Prop 3.2n₂, Prop 3.2n, Prop 3.2o, Prop 3.4, Prop 3.4a₁, Prop 3.4a₂, Prop 3.4a, Prop 3.4b, Prop 3.4c₁, Prop 3.4c₂, Prop 3.4c, Prop 3.4d₁, Prop 3.4d₂, Prop 3.4d, Prop 3.5, Prop 3.5a, Prop 3.5b, Prop 3.5c, Prop 3.5d₁, Prop 3.5d₂, Prop 3.5d, Prop 3.5e₁, Prop 3.5e₂, Prop 3.5e, Prop 3.5f₁, Prop 3.5f₂, Prop 3.5f, Prop 3.5g₁, Prop 3.5g₄, Prop 3.5g₂, Prop 3.5g₃, Prop 3.5g, Prop 3.5h₁, Prop 3.5h₂, Prop 3.5h, Prop 3.5i₁, Prop 3.5i₂, Prop 3.5i, Prop 3.5j₁, Prop 3.5j₂, Prop 3.5j, Prop 3.7, Prop 3.7a, Prop 3.7b, Prop 3.7c, Prop 3.7d, Prop 3.7e, Prop 3.7f, Prop 3.7g, Prop 3.7g', Prop 3.7h, Prop 3.7i, Prop 3.7j, Prop 3.7k, Prop 3.7k', Prop 3.7l, Prop 3.7l', Prop 3.7m, Prop 3.7n, Prop 3.7o, Prop 3.7p, Prop 3.7q, Prop 3.7r, Prop 3.7s, Prop 3.7t, Prop 3.7u, Prop 3.7u', Prop 3.7v, Prop 3.7w, Prop 3.7x, Prop 3.7x', Prop 3.7y, Prop 3.7y', Prop 3.7z, Prop 3.7z', Prop 3.10, Prop 3.10a, Prop 3.10b, Prop 3.10c, Prop 3.10d, Prop 3.10e, Prop 3.10f, Prop 3.10g, Prop 3.10h, Prop 3.11, * < *, * ≤ *, * \neq^*, * ≠ *, * ∧ *, * ∨ *, ∃*:*, * | *, * ...,

S

[S $\xrightarrow{\text{stmt}}$ x]

Neg

[Neg $\xrightarrow{\text{proof}}$ Rule tactic]

[Neg $\xrightarrow{\text{stmt}}$ S ⊢ ∀a:∀b:¬b ⇒ ¬a ⊢ ¬b ⇒ a ⊢ b]

MP

[MP $\xrightarrow{\text{proof}}$ Rule tactic]

[MP $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b}$]

Gen

[Gen $\xrightarrow{\text{proof}}$ Rule tactic]

[Gen $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{x}: \forall \underline{a}: \underline{a} \vdash \forall_{\text{obj}} \underline{x}: \underline{a}$]

Ded

[Ded $\xrightarrow{\text{proof}}$ Rule tactic]

[Ded $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \lambda x. \text{Ded}_0([\underline{a}], [\underline{b}]) \Vdash \underline{a} \vdash \underline{b}$]

S1

[S1 $\xrightarrow{\text{proof}}$ Rule tactic]

[S1 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} = \underline{c} \vdash \underline{b} = \underline{c}$]

S2

[S2 $\xrightarrow{\text{proof}}$ Rule tactic]

[S2 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{a}' = \underline{b}'$]

S3

[S3 $\xrightarrow{\text{proof}}$ Rule tactic]

[S3 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: -0 = \underline{a}'$]

S4

[S4 $\xrightarrow{\text{proof}}$ Rule tactic]

[S4 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a}' = \underline{b}' \vdash \underline{a} = \underline{b}$]

S5

[S5 $\xrightarrow{\text{proof}}$ Rule tactic]

[S5 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} + 0 = \underline{a}$]

S6

[S6 $\xrightarrow{\text{proof}}$ Rule tactic]

[S6 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b}' = \underline{a} + \underline{b}$]

S7

[S7 $\xrightarrow{\text{proof}}$ Rule tactic]

[S7 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \cdot 0 = 0$]

S8

[S8 $\xrightarrow{\text{proof}}$ Rule tactic]

[S8 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}$]

S9

[S9 $\xrightarrow{\text{proof}}$ Rule tactic]

[S9 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \langle [\underline{b}] \equiv^0 [\underline{a}] \mid [\underline{x}] := [0] \rangle \Vdash \langle [\underline{c}] \equiv^0 [\underline{a}] \mid [\underline{x}] := [\underline{x}'] \rangle \Vdash \underline{b} \vdash \underline{a} \Rightarrow \underline{c} \vdash \underline{a}$]

Prop 3.2a

[Prop 3.2a $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} = \underline{a}$]

Prop 3.2b

[Prop 3.2b $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{a}$]

Prop 3.2c

[Prop 3.2c $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{c} \vdash \underline{a} = \underline{c}$]

Prop 3.2d

[Prop 3.2d $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{c} \vdash \underline{b} = \underline{c} \vdash \underline{a} = \underline{b}$]

Prop 3.2e

[Prop 3.2e $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} + \underline{c} = \underline{b} + \underline{c}$]

Prop 3.2f

[Prop 3.2f $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} = 0 + \underline{a}$]

Prop 3.2g

[Prop 3.2g $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a}' + \underline{b} = \underline{a} + \underline{b}'$]

Prop 3.2h

[Prop 3.2h $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = \underline{b} + \underline{a}$]

opgave

[opgave $\xrightarrow{\text{prio}}$

Preassociative

[opgave], [base], [bracket * end bracket], [big bracket * end bracket], [\$ * \$],
[flush left [*]], [x], [y], [z], [[* \bowtie *]], [[* \rightarrow *]], [pyk], [tex], [name], [prio], [*], [T],
[if(*, *, *)], [[* \Rightarrow *]], [val], [claim], [\perp], [f(*)], [(*)^I], [F], [0], [1], [2], [3], [4], [5], [6],
[7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],
[k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(*)^M], [If(*, *, *)],
[array{*} * end array], [l], [c], [r], [empty], [(* | * := *)], [\mathcal{M} (*)], [$\tilde{\mathcal{U}}$ (*)], [\mathcal{U} (*)],
[\mathcal{U}^M (*)], [apply(*, *)], [apply₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-
plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)],

[bit(*, *), [bit₁(*, *), [rack], ["vector"], ["bibliography"], ["dictionary"],
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
 $\mathcal{E}(*, *, *)$, $\mathcal{E}_2(*, *, *, *, *)$, $\mathcal{E}_3(*, *, *, *, *)$, $\mathcal{E}_4(*, *, *, *, *)$, **lookup**(*, *, *),
abstract(*, *, *, *), [[*]], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro],
[s₀], [**zip**(*, *)], [**assoc**₁(*, *, *)], [(*)^P], [self], [[* \doteq *]], [[* \doteq *]], [[* \doteq *]],
[[* $\stackrel{\text{pyk}}{=}$ *]], [[* $\stackrel{\text{tex}}{=}$ *]], [[* $\stackrel{\text{name}}{=}$ *]], [**Priority table**[*]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*, *)$], [$\tilde{\mathcal{M}}_3(*, *)$],
 $\tilde{\mathcal{M}}_4(*, *, *, *, *)$], [$\mathcal{M}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *, *)$],
[[*]], [[*]], [display(*)], [statement(*)], [[*]'], [[*]'], [**aspect**(*, *)],
aspect(*, *, *), [$\langle * \rangle$], [**tuple**₁(*), [**tuple**₂(*), [let₂(*, *)], [let₁(*, *)],
[[* $\stackrel{\text{claim}}{=}$ *]], [checker], [**check**(*, *)], [**check**₂(*, *, *)], [**check**₃(*, *, *, *)],
check^{*}(*, *), [**check**₂^{*}(*, *, *)], [[*]'], [[*]'], [[*]'], [msg], [[* $\stackrel{\text{msg}}{=}$ *]], [$\langle \text{stmt} \rangle$],
[stmt], [[* $\stackrel{\text{stmt}}{=}$ *]], [HeadNil'], [HeadPair'], [Transitivity'], [\perp], [Contra'], [T_E'],
[L₁], [$\underline{*}$], [A], [B], [C], [D], [E], [F], [G], [H], [I], [J], [K], [L], [M], [N], [O], [P], [Q],
[R], [S], [T], [U], [V], [W], [X], [Y], [Z], [($* | * := *$)], [($* * | * := *$)], [\emptyset], [Remainder],
[($*$)^v], [intro(*, *, *, *)], [intro(*, *, *)], [error(*, *)], [error₂(*, *)], [proof(*, *, *)],
[proof₂(*, *)], [S(*, *)], [S^I(*, *)], [S^D(*, *)], [S^D₁(*, *, *)], [S^E(*, *)], [S^E₁(*, *, *)],
[S⁺(*, *)], [S⁺₁(*, *, *)], [S⁻(*, *)], [S⁻₁(*, *, *)], [S^{*}(*, *)], [S^{*}₁(*, *, *)],
[S²₁(*, *, *, *)], [S[@](*, *)], [S[@]₁(*, *, *, *)], [S⁺₁(*, *)], [S⁺₁(*, *, *, *)], [S⁺₁(*, *)],
[S⁺₁(*, *, *, *)], [S^{i.e.}(*, *)], [S^{i.e.}₁(*, *, *, *)], [S^{i.e.}₂(*, *, *, *, *)], [S^v(*, *)],
[S^v₁(*, *, *, *)], [Sⁱ(*, *)], [Sⁱ₁(*, *, *, *)], [Sⁱ₂(*, *, *, *, *)], [T(*)], [claims(*, *, *)],
[claims₂(*, *, *)], [$\langle \text{proof} \rangle$], [proof], [[**Lemma** *: *]], [[**Proof of** *: *]],
[[* **lemma** *: *]], [[* **antilemma** *: *]], [[* **rule** *: *]], [[* **antirule** *: *]],
[verifier], [V₁(*), [V₂(*, *)], [V₃(*, *, *, *)], [V₄(*, *)], [V₅(*, *, *, *)], [V₆(*, *, *, *)],
[V₇(*, *, *, *)], [Cut(*, *)], [Head \oplus (*), [Tail \oplus (*), [rule₁(*, *)], [rule(*, *)],
[Rule tactic], [Plus(*, *)], [[**Theory** *]], [theory₂(*, *)], [theory₃(*, *)],
[theory₄(*, *, *, *)], [HeadNil''], [HeadPair''], [Transitivity''], [Contra''], [HeadNil],
[HeadPair], [Transitivity], [Contra], [T_E], [ragged right],
[ragged right expansion], [parm(*, *, *)], [parm^{*}(*, *, *)], [inst(*, *)],
[inst^{*}(*, *)], [occur(*, *, *)], [occur^{*}(*, *, *)], [unify(* = *, *)], [unify^{*}(* = *, *)],
[unify₂(* = *, *)], [L_a], [L_b], [L_c], [L_d], [L_e], [L_f], [L_g], [L_h], [L_i], [L_j], [L_k], [L_l], [L_m],
[L_n], [L_o], [L_p], [L_q], [L_r], [L_s], [L_t], [L_u], [L_v], [L_w], [L_x], [L_y], [L_z], [L_A], [L_B], [L_C],
[L_D], [L_E], [L_F], [L_G], [L_H], [L_I], [L_J], [L_K], [L_L], [L_M], [L_N], [L_O], [L_P], [L_Q], [L_R],
[L_S], [L_T], [L_U], [L_V], [L_W], [L_X], [L_Y], [L_Z], [L_?], [Reflexivity], [Reflexivity₁],
[Commutativity], [Commutativity₁], [$\langle \text{tactic} \rangle$], [tactic], [[* $\stackrel{\text{tactic}}{=}$ *]], [$\mathcal{P}(*, *, *)$],
 $\mathcal{P}^*(*, *, *)$], [p₀], [conclude₁(*, *)], [conclude₂(*, *, *)], [conclude₃(*, *, *, *)],
[conclude₄(*, *)], [check], [[* $\stackrel{=}{=}$ *]], [RootVisible(*), [A], [R], [C], [T], [L], [{*}], [$\bar{*}$],
[a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v],
[w], [x], [y], [z], [($* \equiv * | * := *$)], [($* \equiv^0 * | * := *$)], [($* \equiv^1 * | * := *$)], [($* \equiv^* * | * := *$)],
[Ded(*, *)], [Ded₀(*, *)], [Ded₁(*, *, *)], [Ded₂(*, *, *)], [Ded₃(*, *, *, *)],
[Ded₄(*, *, *, *)], [Ded₄^{*}(*, *, *, *)], [Ded₅(*, *, *)], [Ded₆(*, *, *, *)],
[Ded₆^{*}(*, *, *, *)], [Ded₇(*), [Ded₈(*, *)], [Ded₈^{*}(*, *)], [S], [Neg], [MP], [Gen],

[Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1'], [A2'], [A4'], [A5'], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop 3.2e₁], [Prop 3.2e₂], [Prop 3.2e], [Prop 3.2f₁], [Prop 3.2f₂], [Prop 3.2f], [Prop 3.2g₁], [Prop 3.2g₂], [Prop 3.2g], [Prop 3.2h₁], [Prop 3.2h₂], [Prop 3.2h], [Block₁(*, *, *)], [Block₂(*)], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [n], [rule div], [R], [R1], [R2], [R3], [R4], [R5], [R6], [Con1], [Con2], [Con], [Dis1], [Dis2], [Lem1.11c], [Cor1.10a], [Cor1.10b], [Lem1.11a], [Lem1.11b], [H3], [Prop3.2c'], [S1''], [Neg'], [Repetition'], [Lem1.11e], [Lem1.11d], [Prop3.2b'], [H10], [H11], [Lem1.11g], [MT], [S10], [Prop 3.2], [Prop 3.2i], [Prop 3.2j₁], [Prop 3.2j₂], [Prop 3.2j], [Prop 3.2k₁], [Prop 3.2k₂], [Prop 3.2k], [Prop 3.2l₁], [Prop 3.2l₂], [Prop 3.2l], [Prop 3.2m₁], [Prop 3.2m₂], [Prop 3.2m], [Prop 3.2n₁], [Prop 3.2n₂], [Prop 3.2n], [Prop 3.2o], [Prop 3.4], [Prop 3.4a₁], [Prop 3.4a₂], [Prop 3.4a], [Prop 3.4b], [Prop 3.4c₁], [Prop 3.4c₂], [Prop 3.4c], [Prop 3.4d₁], [Prop 3.4d₂], [Prop 3.4d], [Prop 3.5], [Prop 3.5a], [Prop 3.5b], [Prop 3.5c], [Prop 3.5d₁], [Prop 3.5d₂], [Prop 3.5d], [Prop 3.5e₁], [Prop 3.5e₂], [Prop 3.5e], [Prop 3.5f₁], [Prop 3.5f₂], [Prop 3.5f], [Prop 3.5g₁], [Prop 3.5g₄], [Prop 3.5g₂], [Prop 3.5g₃], [Prop 3.5g], [Prop 3.5h₁], [Prop 3.5h₂], [Prop 3.5h], [Prop 3.5i₁], [Prop 3.5i₂], [Prop 3.5i], [Prop 3.5j₁], [Prop 3.5j₂], [Prop 3.5j], [Prop 3.7], [Prop 3.7a], [Prop 3.7b], [Prop 3.7c], [Prop 3.7d], [Prop 3.7e], [Prop 3.7f], [Prop 3.7g], [Prop 3.7g'], [Prop 3.7h], [Prop 3.7i], [Prop 3.7j], [Prop 3.7k], [Prop 3.7k'], [Prop 3.7l], [Prop 3.7l'], [Prop 3.7m], [Prop 3.7n], [Prop 3.7o], [Prop 3.7p], [Prop 3.7q], [Prop 3.7r], [Prop 3.7s], [Prop 3.7t], [Prop 3.7u], [Prop 3.7u'], [Prop 3.7v], [Prop 3.7w], [Prop 3.7x], [Prop 3.7x'], [Prop 3.7y], [Prop 3.7y'], [Prop 3.7z], [Prop 3.7z'], [Prop 3.10], [Prop 3.10a], [Prop 3.10b], [Prop 3.10c], [Prop 3.10d], [Prop 3.10e], [Prop 3.10f], [Prop 3.10g], [Prop 3.10h], [Prop 3.11];

Preassociative

[*-{*}], [* /indexintro(*, *, *, *)], [* /intro(*, *, *)], [* /bothintro(*, *, *, *, *)], [* /nameintro(*, *, *, *)], [*'], [* [*]], [* [* → *]], [* [* ⇒ *]], [* 0], [* 1], [0b], [* -color (*)], [* -color* (*)], [*^H], [*^T], [*^U], [*^h], [*^t], [*^s], [*^c], [*^d], [*^a], [*^C], [*^M], [*^B], [*^r], [*ⁱ], [*^d], [*^R], [*⁰], [*¹], [*²], [*³], [*⁴], [*⁵], [*⁶], [*⁷], [*⁸], [*⁹], [*^E], [*^V], [*^C], [*^{C*}], [*hide];

Preassociative

[“ * ”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [*], [*], [! *], [“ * ”], [# *], [\$ *], [% *], [& *], [’ *], [(*)], [()*], [**], [+ *], [*], [- *], [. *], [/ *], [0 *], [1 *], [2 *], [3 *], [4 *], [5 *], [6 *], [7 *], [8 *], [9 *], [: *], [; *], [< *], [= *], [> *], [? *], [@ *], [A *], [B *], [C *], [D *], [E *], [F *], [G *], [H *], [I *], [J *], [K *], [L *], [M *], [N *], [O *], [P *], [Q *], [R *], [S *], [T *], [U *], [V *], [W *], [X *], [Y *], [Z *], [[*], [\ *], [] *], [^ *], [_ *], [‘ *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *], [p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [{ * }, [| * }, [} * }, [~ *], [Preassociative *; *], [Postassociative *; *], [[*], [*], [priority * end], [newline *], [macro newline *], [MacroIndent(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*’];

Preassociative

$[\cdot \cdot *], [* \cdot_0 *];$

Preassociative

$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *];$

Preassociative

$[* \cup \{ * \}], [* \cup *], [* \setminus \{ * \}];$

Postassociative

$[* \dot{\cdot} *], [* \dot{\cdot} *], [* \dot{\cdot} : *], [* \underline{+2} *], [* \dot{\cdot} : *], [* +2 * *];$

Postassociative

$[* , *];$

Preassociative

$[* \overset{B}{\approx} *], [* \overset{D}{\approx} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \overset{+}{\dashv} *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$

$[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{ free in } *], [* \text{ free in }^* *], [* \text{ free for } * \text{ in } *],$

$[* \text{ free for }^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\text{var}}],$

$[* \#^0 *], [* \#^1 *], [* \#^* *], [* < *], [* \leq *], [* \not< *], [* > *], [* \geq *], [* \not> *], [* \neq *];$

Preassociative

$[\neg *];$

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \wedge *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *], [* \vee *];$

Preassociative

$[\exists * : *], [\forall * : *], [\forall_{\text{obj}} * : *], [\exists * : *];$

Postassociative

$[* \overset{\cdot}{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *];$

Postassociative

$[* : *], [* \text{ spy } *], [* ! *];$

Preassociative

$[* \left\{ \begin{array}{l} * \\ * \end{array} \right.];$

Preassociative

$[\lambda * . *], [\Lambda * . *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \dot{=} * \text{ in } *];$

Preassociative

$[* \# *];$

Preassociative

$[*^I], [* \triangleright], [* \vee], [* ^+], [* ^-], [* ^*];$

Preassociative

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *], [* \triangleright *];$

Postassociative

$[* \vdash *], [* \dashv *], [* \text{ i.e. } *];$

Preassociative

$[\forall * : *], [\Pi * : *];$

Postassociative

$[* \oplus *];$

Postassociative

$[* , *];$

Preassociative

[* proves *];

Preassociative

[* **proof of** * : *], [Line * : * \gg *; *], [Last line * \gg * \square],
[Line * : Premise \gg *; *], [Line * : Side-condition \gg *; *], [Arbitrary \gg *; *],
[Local \gg * = *; *], [Begin *; * : End; *], [Last block line * \gg *; *],
[Arbitrary \gg *; *];

Postassociative

[* | *];

Postassociative

[* , *], [* [*]*];

Preassociative

[*&*], [\rightarrow];

Preassociative

[* \\ *], [* linebreak[4] *], [* \\ *], [* | *], [* ...];]

[opgave $\xrightarrow{\text{pyk}}$ “opgave”]

$\overline{0}$

[$\overline{0}$ $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\overline{0} \doteq 0]])$]

[$\overline{0}$ $\xrightarrow{\text{tex}}$ “
 $\overline{\{0\}}$ ”]

[$\overline{0}$ $\xrightarrow{\text{pyk}}$ “numeral zero”]

$\overline{1}$

[$\overline{1}$ $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\overline{1} \doteq 0'])$]

[$\overline{1}$ $\xrightarrow{\text{tex}}$ “
 $\overline{\{1\}}$ ”]

[$\overline{1}$ $\xrightarrow{\text{pyk}}$ “numeral one”]

$\overline{2}$

[$\overline{2}$ $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\overline{2} \doteq 0'']$)]

[$\overline{2}$ $\xrightarrow{\text{tex}}$ “
 $\overline{\{2\}}$ ”]

[$\overline{2}$ $\xrightarrow{\text{pyk}}$ “numeral two”]

3

$\overline{3} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\overline{3} \doteq 0'''])]$

$\overline{3} \xrightarrow{\text{tex}} \text{“}\overline{3}\text{”}$
`\overline{3}`]

$\overline{3} \xrightarrow{\text{pyk}} \text{“numeral three”}$

4

$\overline{4} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\overline{4} \doteq 0'''''])]$

$\overline{4} \xrightarrow{\text{tex}} \text{“}\overline{4}\text{”}$
`\overline{4}`]

$\overline{4} \xrightarrow{\text{pyk}} \text{“numeral four”}$

5

$\overline{5} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\overline{5} \doteq 0'''''''])]$

$\overline{5} \xrightarrow{\text{tex}} \text{“}\overline{5}\text{”}$
`\overline{5}`]

$\overline{5} \xrightarrow{\text{pyk}} \text{“numeral five”}$

6

$\overline{6} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\overline{6} \doteq 0'''''''''])]$

$\overline{6} \xrightarrow{\text{tex}} \text{“}\overline{6}\text{”}$
`\overline{6}`]

$\overline{6} \xrightarrow{\text{pyk}} \text{“numeral six”}$

7

$\overline{7} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [\overline{7} \doteq 0'''''''''''])]$

$\overline{7} \xrightarrow{\text{tex}} \text{“}\overline{7}\text{”}$
`\overline{7}`]

$\overline{7} \xrightarrow{\text{pyk}} \text{“numeral seven”}$

$\overline{8}$

$[\overline{8} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\overline{8} \doteq 0'''''''']])]$

$[\overline{8} \xrightarrow{\text{tex}} “\backslash\overline{8}”]$

$[\overline{8} \xrightarrow{\text{pyk}} “\text{numeral eight}”]$

$\overline{9}$

$[\overline{9} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\overline{9} \doteq 0'''''''']])]$

$[\overline{9} \xrightarrow{\text{tex}} “\backslash\overline{9}”]$

$[\overline{9} \xrightarrow{\text{pyk}} “\text{numeral nine}”]$

\overline{n}

$[\overline{n} \xrightarrow{\text{tex}} “\backslash\overline{n}”]$

$[\overline{n} \xrightarrow{\text{pyk}} “\text{numeral n}”]$

rule div

$[\text{rule div} \xrightarrow{\text{pyk}} “\text{rule div}”]$

\mathbb{R}

$[\mathbb{R} \xrightarrow{\text{tex}} “\mathbb{R}”]$

$[\mathbb{R} \xrightarrow{\text{pyk}} “\text{rule r}”]$

$\mathbb{R}1$

$[\mathbb{R}1 \xrightarrow{\text{tex}} “\mathbb{R}1”]$

$[\mathbb{R}1 \xrightarrow{\text{pyk}} “\text{rule r one}”]$

R2

[R2 $\xrightarrow{\text{tex}}$ “
R2”]

[R2 $\xrightarrow{\text{pyk}}$ “rule r two”]

R3

[R3 $\xrightarrow{\text{tex}}$ “
R3”]

[R3 $\xrightarrow{\text{pyk}}$ “rule r three”]

R4

[R4 $\xrightarrow{\text{tex}}$ “
R4”]

[R4 $\xrightarrow{\text{pyk}}$ “rule r four”]

R5

[R5 $\xrightarrow{\text{tex}}$ “
R5”]

[R5 $\xrightarrow{\text{pyk}}$ “rule r five”]

R6

[R6 $\xrightarrow{\text{tex}}$ “
R6”]

[R6 $\xrightarrow{\text{pyk}}$ “rule r six”]

Con1

[Con1 $\xrightarrow{\text{proof}}$ $\lambda c.\lambda x.\mathcal{P}([\text{S} \vdash \forall \underline{a}:\forall \underline{b}:\neg \underline{a} \Rightarrow \neg \underline{b} \vdash \forall \underline{a}:\forall \underline{b}:\neg \underline{a} \vdash A1' \gg \neg \underline{a} \Rightarrow \neg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \underline{a} \Rightarrow \neg \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{a} \gg \neg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{Lem1.11d} \gg \neg \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \underline{b}; \text{MP} \triangleright \neg \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \underline{b} \triangleright \neg \neg \underline{b} \Rightarrow \neg \underline{a} \gg \underline{a} \Rightarrow \neg \underline{b}; \text{Ded} \triangleright \forall \underline{a}:\forall \underline{b}:\neg \underline{a} \vdash$

$\underline{a} \Rightarrow \neg \underline{b} \gg \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \underline{b}$; MT $\triangleright \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \underline{b} \triangleright \neg \underline{a} \Rightarrow \neg \underline{b} \gg \neg \neg \underline{a}$; Lem1.11a $\gg \neg \neg \underline{a} \Rightarrow \underline{a}$; MP $\triangleright \neg \neg \underline{a} \Rightarrow \underline{a} \triangleright \neg \neg \underline{a} \gg \underline{a}$], p_0, c)]

[Con1 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \Rightarrow \neg \underline{b} \vdash \underline{a}$]

[Con1 $\xrightarrow{\text{tex}}$ “
Con1”]

[Con1 $\xrightarrow{\text{pyk}}$ “conjel1”]

Con2

[Con2 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \Rightarrow \neg \underline{b} \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \vdash A1' \gg \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \neg \underline{b}$; MP $\triangleright \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \neg \underline{b} \triangleright \neg \underline{b} \gg \underline{a} \Rightarrow \neg \underline{b}$; Ded $\triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \vdash \underline{a} \Rightarrow \neg \underline{b} \gg \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \neg \underline{b}$; MT $\triangleright \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \neg \underline{b} \triangleright \neg \underline{a} \Rightarrow \neg \underline{b} \gg \neg \neg \underline{b}$; Lem1.11a $\gg \neg \neg \underline{b} \Rightarrow \underline{b}$; MP $\triangleright \neg \neg \underline{b} \Rightarrow \underline{b} \triangleright \neg \neg \underline{b} \gg \underline{b}$], p_0, c)]

[Con2 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \Rightarrow \neg \underline{b} \vdash \underline{b}$]

[Con2 $\xrightarrow{\text{tex}}$ “
Con2”]

[Con2 $\xrightarrow{\text{pyk}}$ “conjel2”]

Con

[Con $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \neg \underline{b} \vdash \text{Repetition} \triangleright \underline{a} \gg \underline{a}$; MP $\triangleright \underline{a} \Rightarrow \neg \underline{b} \triangleright \underline{a} \gg \neg \underline{b}$; Ded $\triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \neg \underline{b} \vdash \neg \underline{b} \gg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{b}$; Lem1.11b $\gg \underline{b} \Rightarrow \neg \neg \underline{b}$; MP $\triangleright \underline{b} \Rightarrow \neg \neg \underline{b} \triangleright \underline{b} \gg \neg \neg \underline{b}$; MT $\triangleright \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{b} \triangleright \neg \neg \underline{b} \gg \neg \underline{a} \Rightarrow \neg \underline{b}$], p_0, c)]

[Con $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b} \vdash \neg \underline{a} \Rightarrow \neg \underline{b}$]

[Con $\xrightarrow{\text{tex}}$ “
Con”]

[Con $\xrightarrow{\text{pyk}}$ “conjin”]

Dis1

[Dis1 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \text{Lem1.11c} \gg \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \underline{b}$; Lem1.11b $\gg \underline{a} \Rightarrow \neg \neg \underline{a}$; MP $\triangleright \underline{a} \Rightarrow \neg \neg \underline{a} \triangleright \underline{a} \gg \neg \neg \underline{a}$; MP $\triangleright \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \underline{b} \triangleright \neg \neg \underline{a} \gg \neg \underline{a} \Rightarrow \underline{b}$], p_0, c)]

[Dis1 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \neg \underline{a} \Rightarrow \underline{b}$]

[Dis1 $\xrightarrow{\text{tex}}$ “
Dis1”]

[Dis1 $\xrightarrow{\text{pyk}}$ “disjin1”]

Dis2

[Dis2 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \underline{b} \vdash A1' \gg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{b}; \text{MP} \triangleright \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{b} \triangleright \underline{b} \gg \neg \underline{a} \Rightarrow \underline{b}], p_0, c)$]

[Dis2 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{b} \vdash \neg \underline{a} \Rightarrow \underline{b}$]

[Dis2 $\xrightarrow{\text{tex}}$ “
Dis2”]

[Dis2 $\xrightarrow{\text{pyk}}$ “disjin2”]

Lem1.11c

[Lem1.11c $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \text{Repetition} \triangleright \neg \underline{a} \gg \neg \underline{a}; A1' \gg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a}; \text{MP} \triangleright \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a} \triangleright \underline{a} \gg \neg \underline{b} \Rightarrow \underline{a}; A1' \gg \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{a} \gg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{Neg} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{b} \Rightarrow \underline{a} \gg \underline{b}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b} \gg \underline{a} \Rightarrow \underline{b}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \vdash \underline{a} \Rightarrow \underline{b} \gg \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \underline{b}], p_0, c)$]

[Lem1.11c $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \underline{b}$]

[Lem1.11c $\xrightarrow{\text{tex}}$ “
Lem 1.11c”]

[Lem1.11c $\xrightarrow{\text{pyk}}$ “t one”]

Cor1.10a

[Cor1.10a $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \vdash \underline{b} \Rightarrow \underline{c} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \vdash \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \vdash \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \triangleright \underline{a} \gg \underline{b}; \text{MP} \triangleright \underline{b} \Rightarrow \underline{c} \triangleright \underline{b} \gg \underline{c}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \vdash \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \vdash \underline{c} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{c}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{c} \triangleright \underline{a} \Rightarrow \underline{b} \gg \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{c}; \text{MP} \triangleright \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{c} \triangleright \underline{b} \Rightarrow \underline{c} \gg \underline{a} \Rightarrow \underline{c}], p_0, c)$]

[Cor1.10a $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \vdash \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{c}$]

[Cor1.10a $\xrightarrow{\text{tex}}$ “
Cor 1.10a”]

[Cor1.10a $\xrightarrow{\text{pyk}}$ “h zero a”]

Cor1.10b

[Cor1.10b $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{b} \vdash A2' \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c}; A1' \gg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b}; \text{MP} \triangleright \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b} \triangleright \underline{b} \gg \underline{a} \Rightarrow \underline{b}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c} \triangleright \underline{a} \Rightarrow \underline{b} \gg \underline{a} \Rightarrow \underline{c}], p_0, c)$

[Cor1.10b $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{b} \vdash \underline{a} \Rightarrow \underline{c}$]

[Cor1.10b $\xrightarrow{\text{tex}}$ “
Cor 1.10b”]

[Cor1.10b $\xrightarrow{\text{pyk}}$ “h zero b”]

Lem1.11a

[Lem1.11a $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \text{Neg}' \gg \neg \underline{a} \Rightarrow \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \underline{a}; \text{Repetition}' \gg \neg \underline{a} \Rightarrow \neg \underline{a}; \text{Cor1.10b} \triangleright \neg \underline{a} \Rightarrow \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \underline{a} \triangleright \neg \underline{a} \Rightarrow \neg \underline{a} \gg \neg \underline{a} \Rightarrow \neg \neg \underline{a} \Rightarrow \underline{a}; A1' \gg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \neg \neg \underline{a}; \text{Cor1.10a} \triangleright \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \neg \neg \underline{a} \triangleright \neg \underline{a} \Rightarrow \neg \neg \underline{a} \Rightarrow \underline{a} \gg \neg \neg \underline{a} \Rightarrow \underline{a}], p_0, c)$

[Lem1.11a $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \neg \neg \underline{a} \Rightarrow \underline{a}$]

[Lem1.11a $\xrightarrow{\text{tex}}$ “
Lem 1.11a”]

[Lem1.11a $\xrightarrow{\text{pyk}}$ “h one”]

Lem1.11b

[Lem1.11b $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \text{Neg}' \gg \neg \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \neg \neg \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \neg \underline{a}; \text{Lem1.11a} \gg \neg \neg \neg \underline{a} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \neg \neg \underline{a} \Rightarrow \neg \underline{a} \Rightarrow \neg \neg \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \neg \underline{a} \triangleright \neg \neg \neg \underline{a} \Rightarrow \neg \underline{a} \gg \neg \neg \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \neg \underline{a}; A1' \gg \underline{a} \Rightarrow \neg \neg \neg \underline{a} \Rightarrow \underline{a}; \text{Cor1.10a} \triangleright \underline{a} \Rightarrow \neg \neg \neg \underline{a} \Rightarrow \underline{a} \triangleright \neg \neg \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \neg \neg \underline{a} \gg \underline{a} \Rightarrow \neg \neg \underline{a}], p_0, c)$

[Lem1.11b $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \underline{a} \Rightarrow \neg \neg \underline{a}$]

[Lem1.11b $\xrightarrow{\text{tex}}$ “
Lem 1.11b”]

[Lem1.11b $\xrightarrow{\text{pyk}}$ “h two”]

H3

[H3 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: a \Rightarrow b \Rightarrow c \vdash a \vdash \neg c \vdash \text{MP} \triangleright a \Rightarrow b \Rightarrow c \triangleright a \gg b \Rightarrow c; \text{MT} \triangleright b \Rightarrow c \triangleright \neg c \gg \neg b], p_0, c)$]

[H3 $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: a \Rightarrow b \Rightarrow c \vdash a \vdash \neg c \vdash \neg b$]

[H3 $\xrightarrow{\text{tex}}$ “
H3”]

[H3 $\xrightarrow{\text{pyk}}$ “h three”]

Prop3.2c'

[Prop3.2c' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: \forall a: \forall b: \forall c: a = b \vdash \forall a: \forall b: \forall c: b = c \vdash \text{Prop 3.2c} \triangleright a = b \triangleright b = c \gg a = c; \text{Ded} \triangleright \forall a: \forall b: \forall c: b = c \vdash a = c \gg b = c \Rightarrow a = c; \text{Ded} \triangleright \forall a: \forall b: \forall c: a = b \vdash b = c \Rightarrow a = c \gg a = b \Rightarrow b = c \Rightarrow a = c], p_0, c)$]

[Prop3.2c' $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: a = b \Rightarrow b = c \Rightarrow a = c$]

[Prop3.2c' $\xrightarrow{\text{tex}}$ “
Prop 3.2c'”]

[Prop3.2c' $\xrightarrow{\text{pyk}}$ “h four”]

S1''

[S1'' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: \forall a: \forall b: \forall c: a = b \vdash \forall a: \forall b: \forall c: a = c \vdash S1 \triangleright a = b \triangleright a = c \gg b = c; \text{Ded} \triangleright \forall a: \forall b: \forall c: a = c \vdash b = c \gg a = c \Rightarrow b = c; \text{Ded} \triangleright \forall a: \forall b: \forall c: a = b \vdash a = c \Rightarrow b = c \gg a = b \Rightarrow a = c \Rightarrow b = c], p_0, c)$]

[S1'' $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: a = b \Rightarrow a = c \Rightarrow b = c$]

[S1'' $\xrightarrow{\text{tex}}$ “
S1''”]

[S1'' $\xrightarrow{\text{pyk}}$ “h four mark”]

Neg'

[Neg' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall a: \forall b: \neg b \Rightarrow \neg a \vdash \forall a: \forall b: \neg b \Rightarrow a \vdash \text{Neg} \triangleright \neg b \Rightarrow \neg a \triangleright \neg b \Rightarrow a \gg b; \text{Ded} \triangleright \forall a: \forall b: \neg b \Rightarrow a \vdash b \gg \neg b \Rightarrow a \Rightarrow b; \text{Ded} \triangleright \forall a: \forall b: \neg b \Rightarrow \neg a \vdash \neg b \Rightarrow a \Rightarrow b \gg \neg b \Rightarrow \neg a \Rightarrow \neg b \Rightarrow a \Rightarrow b], p_0, c)$]

[Neg' $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \neg b \Rightarrow \neg a \Rightarrow \neg b \Rightarrow a \Rightarrow b$]

[Neg' $\xrightarrow{\text{tex}}$ “
Neg”]

[Neg' $\xrightarrow{\text{pyk}}$ “h five”]

Repetition'

[Repetition' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([\text{S} \vdash \forall \underline{a}: \forall \underline{a}: \underline{a} \vdash \text{Repetition} \triangleright \underline{a} \gg \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \underline{a} \vdash \underline{a} \gg \underline{a} \Rightarrow \underline{a}], p_0, c)$]

[Repetition' $\xrightarrow{\text{stmt}}$ $\text{S} \vdash \forall \underline{a}: \underline{a} \Rightarrow \underline{a}$]

[Repetition' $\xrightarrow{\text{tex}}$ “
Repetition”]

[Repetition' $\xrightarrow{\text{pyk}}$ “h six”]

Lem1.11e

[Lem1.11e $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([\text{S} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \text{Lem1.11a} \gg \neg \underline{a} \Rightarrow \underline{a}; \text{Cor1.10a} \triangleright \neg \underline{a} \Rightarrow \underline{a} \triangleright \underline{a} \Rightarrow \underline{b} \gg \neg \underline{a} \Rightarrow \underline{b}; \text{Lem1.11b} \gg \underline{b} \Rightarrow \neg \underline{b}; \text{Cor1.10a} \triangleright \neg \underline{a} \Rightarrow \underline{b} \triangleright \underline{b} \Rightarrow \neg \underline{b} \gg \neg \underline{a} \Rightarrow \neg \underline{b}; \text{Lem1.11d} \gg \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{a} \Rightarrow \neg \underline{b} \gg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{b} \Rightarrow \neg \underline{a} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}], p_0, c)$]

[Lem1.11e $\xrightarrow{\text{stmt}}$ $\text{S} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}$]

[Lem1.11e $\xrightarrow{\text{tex}}$ “
Lem 1.11e”]

[Lem1.11e $\xrightarrow{\text{pyk}}$ “h seven”]

Lem1.11d

[Lem1.11d $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([\text{S} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \Rightarrow \neg \underline{a} \vdash \text{Neg}' \gg \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b}; \text{A1}' \gg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a}; \text{MP} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \gg \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b}; \text{Cor1.10a} \triangleright \underline{a} \Rightarrow \neg \underline{b} \Rightarrow \underline{a} \triangleright \neg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b} \gg \underline{a} \Rightarrow \underline{b}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \Rightarrow \neg \underline{a} \vdash \underline{a} \Rightarrow \underline{b} \gg \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \underline{b}], p_0, c)$]

[Lem1.11d $\xrightarrow{\text{stmt}}$ $\text{S} \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \underline{a} \Rightarrow \underline{b}$]

[Lem1.11d $\xrightarrow{\text{tex}}$ “
Lem 1.11d”]

[Lem1.11d $\xrightarrow{\text{pyk}}$ “h eight”]

Prop3.2b'

[Prop3.2b' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \text{Prop 3.2b} \triangleright \underline{a} = \underline{b} \gg \underline{b} = \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{a} \gg \underline{a} = \underline{b} \Rightarrow \underline{b} = \underline{a}]$, p_0, c)]

[Prop3.2b' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \Rightarrow \underline{b} = \underline{a}$]

[Prop3.2b' $\xrightarrow{\text{tex}}$ “
Prop 3.2b”]

[Prop3.2b' $\xrightarrow{\text{pyk}}$ “h nine”]

H10

[H10 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = \underline{a} \vdash \text{Prop3.2b}' \gg \underline{a} = \underline{b} \Rightarrow \underline{b} = \underline{a}; \text{Lem1.11e} \gg \underline{a} = \underline{b} \Rightarrow \underline{b} = \underline{a} \Rightarrow \neg \underline{b} = \underline{a} \Rightarrow \neg \underline{a} = \underline{b}; \text{MP} \triangleright \underline{a} = \underline{b} \Rightarrow \underline{b} = \underline{a} \Rightarrow \neg \underline{b} = \underline{a} \Rightarrow \neg \underline{a} = \underline{b} \triangleright \underline{a} = \underline{b} \Rightarrow \underline{b} = \underline{a} \gg \neg \underline{b} = \underline{a} \Rightarrow \neg \underline{a} = \underline{b}; \text{MP} \triangleright \neg \underline{b} = \underline{a} \Rightarrow \neg \underline{a} = \underline{b} \triangleright \underline{b} \triangleright \neg \underline{b} = \underline{a} \gg \neg \underline{a} = \underline{b}]$, p_0, c)]

[H10 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = \underline{a} \vdash \neg \underline{a} = \underline{b}$]

[H10 $\xrightarrow{\text{tex}}$ “
H10”]

[H10 $\xrightarrow{\text{pyk}}$ “h ten”]

H11

[H11 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \Rightarrow \underline{c} \vdash \underline{b} \Rightarrow \underline{c} \vdash \text{Cor1.10a} \triangleright \neg \underline{a} \Rightarrow \underline{b} \triangleright \underline{b} \Rightarrow \underline{c} \gg \neg \underline{a} \Rightarrow \underline{c}; \text{Lem1.11g} \triangleright \underline{a} \Rightarrow \underline{c} \triangleright \neg \underline{a} \Rightarrow \underline{c} \gg \underline{c}]$, p_0, c)]

[H11 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \Rightarrow \underline{c} \vdash \underline{b} \Rightarrow \underline{c} \vdash \underline{c}$]

[H11 $\xrightarrow{\text{tex}}$ “
H11”]

[H11 $\xrightarrow{\text{pyk}}$ “h eleven”]

Lem1.11g

[Lem1.11g $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{a} \Rightarrow \underline{b} \vdash \text{Lem1.11e} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \underline{a} \Rightarrow \underline{b} \gg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{Lem1.11e} \gg \neg \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{a} \Rightarrow \underline{b} \gg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{Neg} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \gg \underline{b}]$, p_0, c)]

[Lem1.11g $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{a} \Rightarrow \underline{b} \vdash \underline{b}$]

[Lem1.11g $\xrightarrow{\text{tex}}$ “
Lem 1.11g”]

[Lem1.11g $\xrightarrow{\text{pyk}}$ “h twelwe”]

MT

[MT $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([\text{S} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{b} \vdash \text{Lem1.11a} \gg \neg \underline{a} \Rightarrow \underline{a}; \text{Cor1.10a} \triangleright \neg \underline{a} \Rightarrow \underline{a} \triangleright \underline{a} \Rightarrow \underline{b} \gg \neg \underline{a} \Rightarrow \underline{b}; \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{b} \vdash \text{A1}' \gg \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{b}; \text{MP} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \Rightarrow \neg \underline{b} \triangleright \neg \underline{b} \gg \neg \underline{a} \Rightarrow \neg \underline{b}; \text{Neg} \triangleright \neg \underline{a} \Rightarrow \neg \underline{b} \triangleright \neg \underline{a} \Rightarrow \underline{b} \gg \neg \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{b} \vdash \neg \underline{a} \gg \neg \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \underline{a} \Rightarrow \underline{b} \Rightarrow \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{a} \Rightarrow \underline{b} \gg \neg \underline{b} \Rightarrow \neg \underline{a}; \text{MP} \triangleright \neg \underline{b} \Rightarrow \neg \underline{a} \triangleright \neg \underline{b} \gg \neg \underline{a}], p_0, c)]$

[MT $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \neg \underline{b} \vdash \neg \underline{a}$]

[MT $\xrightarrow{\text{tex}}$ “
MT”]

[MT $\xrightarrow{\text{pyk}}$ “modus tollens”]

S10

[S10 $\xrightarrow{\text{tex}}$ “
S10”]

[S10 $\xrightarrow{\text{pyk}}$ “axiom s ten”]

Prop 3.2

[Prop 3.2 $\xrightarrow{\text{tex}}$ “
Prop\ 3.2”]

[Prop 3.2 $\xrightarrow{\text{pyk}}$ “prop three two”]

Prop 3.2i

[Prop 3.2i $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([\text{S} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \text{Prop 3.2e} \triangleright \underline{a} = \underline{b} \gg \underline{a} + \underline{c} = \underline{b} + \underline{c}; \text{Prop 3.2h} \gg \underline{a} + \underline{c} = \underline{c} + \underline{a}; \text{Prop 3.2h} \gg \underline{b} + \underline{c} = \underline{c} + \underline{b}; \text{S1} \triangleright \underline{a} + \underline{c} = \underline{b} + \underline{c} \triangleright \underline{a} + \underline{c} = \underline{c} + \underline{a} \gg \underline{b} + \underline{c} = \underline{c} + \underline{a}; \text{Prop 3.2b} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{a} \gg \underline{c} + \underline{a} = \underline{b} + \underline{c}; \text{Prop 3.2c} \triangleright \underline{c} + \underline{a} = \underline{b} + \underline{c} \triangleright \underline{b} + \underline{c} = \underline{c} + \underline{b} \gg \underline{c} + \underline{a} = \underline{c} + \underline{b}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{c} + \underline{a} = \underline{c} + \underline{b} \gg \underline{a} = \underline{b} \vdash \underline{c} + \underline{a} = \underline{c} + \underline{b}], p_0, c)]$

[Prop 3.2i $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{c} + \underline{a} = \underline{c} + \underline{b}$]

[Prop 3.2i $\xrightarrow{\text{tex}}$ “
Prop\ 3.2i”]

[Prop 3.2i $\xrightarrow{\text{pyk}}$ “prop three two i”]

Prop 3.2j₁

[Prop 3.2j₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \forall \underline{a}: \forall \underline{b}: S5 \gg \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b}; S5 \gg \underline{b} + 0 = \underline{b}; \text{Prop 3.2i} \triangleright \underline{b} + 0 = \underline{b} \gg \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b}; \text{Prop 3.2d} \triangleright \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} \triangleright \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} \gg \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} + 0 \rceil, p_0, c)$]

[Prop 3.2j₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} + 0 = \underline{a} + \underline{b} + 0$]

[Prop 3.2j₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2j₁”]

[Prop 3.2j₁ $\xrightarrow{\text{pyk}}$ “prop three two j one”]

Prop 3.2j₂

[Prop 3.2j₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{a}': \forall \underline{b}': \forall \underline{c}': \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \vdash S6 \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; S2 \triangleright \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Prop 3.2c} \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; S6 \gg \underline{b} + \underline{c}' = \underline{b} + \underline{c}'; \text{Prop 3.2i} \triangleright \underline{b} + \underline{c}' = \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; S6 \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Prop 3.2c} \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Prop 3.2d} \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \triangleright \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \vdash \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \gg \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}' \rceil, p_0, c)$]

[Prop 3.2j₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{b} + \underline{c}' = \underline{a} + \underline{b} + \underline{c}'$]

[Prop 3.2j₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2j₂”]

[Prop 3.2j₂ $\xrightarrow{\text{pyk}}$ “prop three two j two”]

Prop 3.2j

[Prop 3.2j $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \text{Prop 3.2j}_1 \gg \bar{x} + \bar{y} + 0 = \bar{x} + \bar{y} + 0; \text{Prop 3.2j}_2 \gg \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z} \Rightarrow \bar{x} + \bar{y} + \bar{z}' = \bar{x} + \bar{y} + \bar{z}'; S9 @ \bar{z} \triangleright \bar{x} + \bar{y} + 0 = \bar{x} + \bar{y} + 0 \triangleright \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z} \Rightarrow \bar{x} + \bar{y} + \bar{z}' = \bar{x} + \bar{y} + \bar{z}' \gg \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z}; \text{Ded} \triangleright \bar{x} + \bar{y} + \bar{z} = \bar{x} + \bar{y} + \bar{z} \gg \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c} \rceil, p_0, c)$]

[Prop 3.2j $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} + \underline{b} + \underline{c} = \underline{a} + \underline{b} + \underline{c}$]

[Prop 3.2j $\xrightarrow{\text{tex}}$ “
Prop\ 3.2j”]

[Prop 3.2j $\xrightarrow{\text{pyk}}$ “prop three two j”]

Prop 3.2k₁

[Prop 3.2k₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash S7 \gg \underline{a} \cdot 0 = 0; S7 \gg \underline{b} \cdot 0 = 0; \text{Prop } 3.2a \gg 0 = 0; \text{Prop } 3.2b \triangleright \underline{b} \cdot 0 = 0 \gg 0 = \underline{b} \cdot 0; \text{Prop } 3.2c \triangleright \underline{a} \cdot 0 = 0 \triangleright 0 = \underline{b} \cdot 0 \gg \underline{a} \cdot 0 = \underline{b} \cdot 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{a} \cdot 0 = \underline{b} \cdot 0 \gg \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot 0 = \underline{b} \cdot 0], p_0, c)$]

[Prop 3.2k₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot 0 = \underline{b} \cdot 0$]

[Prop 3.2k₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2k.1”]

[Prop 3.2k₁ $\xrightarrow{\text{pyk}}$ “prop three two k one”]

Prop 3.2k₂

[Prop 3.2k₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \vdash \underline{a} = \underline{b} \vdash \text{MP} \triangleright \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \triangleright \underline{a} = \underline{b} \gg \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}; S8 \gg \underline{a} \cdot \underline{c}' = \underline{a} \cdot \underline{c} + \underline{a}; S8 \gg \underline{b} \cdot \underline{c}' = \underline{b} \cdot \underline{c} + \underline{b}; \text{Prop } 3.2e \triangleright \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \gg \underline{a} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{a}; \text{Prop } 3.2i \triangleright \underline{a} = \underline{b} \gg \underline{b} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b}; \text{Prop } 3.2c \triangleright \underline{a} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{a} \triangleright \underline{b} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b} \gg \underline{a} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b}; \text{Prop } 3.2c \triangleright \underline{a} \cdot \underline{c}' = \underline{a} \cdot \underline{c} + \underline{a} \triangleright \underline{a} \cdot \underline{c} + \underline{a} = \underline{b} \cdot \underline{c} + \underline{b} \gg \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c} + \underline{b}; \text{Prop } 3.2d \triangleright \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c} + \underline{b} \triangleright \underline{b} \cdot \underline{c}' = \underline{b} \cdot \underline{c} + \underline{b} \gg \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c}'; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \vdash \underline{a} = \underline{b} \vdash \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c}' \gg \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c}'], p_0, c)$]

[Prop 3.2k₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c}' = \underline{b} \cdot \underline{c}'$]

[Prop 3.2k₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2k.2”]

[Prop 3.2k₂ $\xrightarrow{\text{pyk}}$ “prop three two k two”]

Prop 3.2k

[Prop 3.2k $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \text{Prop } 3.2k_1 \gg \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot 0 = \bar{y} \cdot 0; \text{Prop } 3.2k_2 \gg \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot \bar{z} = \bar{y} \cdot \bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot \bar{z}' = \bar{y} \cdot \bar{z}'; S9 @ \bar{z} \triangleright \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot 0 = \bar{y} \cdot 0 \triangleright \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot \bar{z} = \bar{y} \cdot \bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot \bar{z}' = \bar{y} \cdot \bar{z}' \gg \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot \bar{z} = \bar{y} \cdot \bar{z}; \text{Ded} \triangleright \bar{x} = \bar{y} \Rightarrow \bar{x} \cdot \bar{z} = \bar{y} \cdot \bar{z} \gg \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}; \text{MP} \triangleright \underline{a} = \underline{b} \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \triangleright \underline{a} = \underline{b} \gg \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}], p_0, c)$]

[Prop 3.2k $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c}$]

[Prop 3.2k $\xrightarrow{\text{tex}}$ “
Prop\ 3.2k”]

[Prop 3.2k $\xrightarrow{\text{pyk}}$ “prop three two k”]

Prop 3.2l₁

[Prop 3.2l₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2l₁”]

[Prop 3.2l₁ $\xrightarrow{\text{pyk}}$ “prop three two l one”]

Prop 3.2l₂

[Prop 3.2l₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{a}: 0 \cdot \underline{a} = 0 \vdash S8 \gg 0 \cdot \underline{a}' = 0 \cdot \underline{a} + 0; S5 \gg 0 \cdot \underline{a} + 0 = 0 \cdot \underline{a}; \text{Prop 3.2c} \triangleright 0 \cdot \underline{a}' = 0 \cdot \underline{a} + 0 \triangleright 0 \cdot \underline{a} + 0 = 0 \cdot \underline{a} \gg 0 \cdot \underline{a}' = 0 \cdot \underline{a}; \text{Prop 3.2c} \triangleright 0 \cdot \underline{a}' = 0 \cdot \underline{a} \triangleright 0 \cdot \underline{a} = 0 \gg 0 \cdot \underline{a}' = 0; \text{Ded} \triangleright \forall \underline{a}: 0 \cdot \underline{a} = 0 \vdash 0 \cdot \underline{a}' = 0 \gg 0 \cdot \underline{a} = 0 \Rightarrow 0 \cdot \underline{a}' = 0], p_0, c)$]

[Prop 3.2l₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: 0 \cdot \underline{a} = 0 \Rightarrow 0 \cdot \underline{a}' = 0$]

[Prop 3.2l₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2l₂”]

[Prop 3.2l₂ $\xrightarrow{\text{pyk}}$ “prop three two l two”]

Prop 3.2l

[Prop 3.2l $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S7 \gg 0 \cdot 0 = 0; \text{Prop 3.2l}_2 \gg 0 \cdot \bar{x} = 0 \Rightarrow 0 \cdot \bar{x}' = 0; S9 \text{ @ } \bar{x} \triangleright 0 \cdot 0 = 0 \triangleright 0 \cdot \bar{x} = 0 \Rightarrow 0 \cdot \bar{x}' = 0 \gg 0 \cdot \bar{x} = 0; \text{Ded} \triangleright 0 \cdot \bar{x} = 0 \vdash 0 \cdot \underline{a} = 0], p_0, c)$]

[Prop 3.2l $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: 0 \cdot \underline{a} = 0$]

[Prop 3.2l $\xrightarrow{\text{tex}}$ “
Prop\ 3.2l”]

[Prop 3.2l $\xrightarrow{\text{pyk}}$ “prop three two l”]

[Prop 3.2m $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a}' \cdot \underline{b} = \underline{a} \cdot \underline{b} + \underline{b}$]

[Prop 3.2m $\xrightarrow{\text{tex}}$ “
Prop\ 3.2m”]

[Prop 3.2m $\xrightarrow{\text{pyk}}$ “prop three two m”]

Prop 3.2n₁

[Prop 3.2n₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S7 \gg \underline{a} \cdot 0 = 0; \text{Prop 3.2l} \gg 0 \cdot \underline{a} = 0; \text{Prop 3.2d} \triangleright \underline{a} \cdot 0 = 0 \triangleright 0 \cdot \underline{a} = 0 \gg \underline{a} \cdot 0 = 0 \cdot \underline{a}], p_0, c)$]

[Prop 3.2n₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \cdot 0 = 0 \cdot \underline{a}$]

[Prop 3.2n₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2n.1”]

[Prop 3.2n₁ $\xrightarrow{\text{pyk}}$ “prop three two n one”]

Prop 3.2n₂

[Prop 3.2n₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}': \forall \underline{b}': \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \vdash S8 \gg \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}; \text{Prop 3.2e} \triangleright \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \gg \underline{a} \cdot \underline{b} + \underline{a} = \underline{b} \cdot \underline{a} + \underline{a}; \text{Prop 3.2m} \gg \underline{b}' \cdot \underline{a} = \underline{b} \cdot \underline{a} + \underline{a}; \text{Prop 3.2b} \triangleright \underline{b}' \cdot \underline{a} = \underline{b} \cdot \underline{a} + \underline{a} \gg \underline{b} \cdot \underline{a} + \underline{a} = \underline{b}' \cdot \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} \cdot \underline{b} + \underline{a} = \underline{b} \cdot \underline{a} + \underline{a} \triangleright \underline{b} \cdot \underline{a} + \underline{a} = \underline{b}' \cdot \underline{a} \gg \underline{a} \cdot \underline{b} + \underline{a} = \underline{b}' \cdot \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a} \triangleright \underline{a} \cdot \underline{b} + \underline{a} = \underline{b}' \cdot \underline{a} \gg \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \vdash \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a} \gg \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \Rightarrow \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a}], p_0, c)$]

[Prop 3.2n₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \Rightarrow \underline{a} \cdot \underline{b}' = \underline{b}' \cdot \underline{a}$]

[Prop 3.2n₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2n.2”]

[Prop 3.2n₂ $\xrightarrow{\text{pyk}}$ “prop three two n two”]

Prop 3.2n

[Prop 3.2n $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop 3.2n}_1 \gg \bar{x} \cdot 0 = 0 \cdot \bar{x}; \text{Prop 3.2n}_2 \gg \bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x} \Rightarrow \bar{x} \cdot \bar{y}' = \bar{y}' \cdot \bar{x}; S9 @ \bar{y} \triangleright \bar{x} \cdot 0 = 0 \cdot \bar{x} \triangleright \bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x} \Rightarrow \bar{x} \cdot \bar{y}' = \bar{y}' \cdot \bar{x} \gg \bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x}; \text{Ded} \triangleright \bar{x} \cdot \bar{y} = \bar{y} \cdot \bar{x} \gg \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}], p_0, c)$]

[Prop 3.2n $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$]

[Prop 3.2n $\xrightarrow{\text{tex}}$ “
Prop\ 3.2n”]

[Prop 3.2n $\xrightarrow{\text{pyk}}$ “prop three two n”]

Prop 3.2o

[Prop 3.2o $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: a = \underline{b} \vdash \forall a: \forall b: \forall c: a = \underline{b} \vdash$
Prop 3.2k $\triangleright a = \underline{b} \gg a \cdot c = \underline{b} \cdot c$; Prop 3.2n $\gg a \cdot c = \underline{c} \cdot a$; Prop 3.2n $\gg b \cdot c =$
 $\underline{c} \cdot b$; Prop 3.2c $\triangleright a \cdot c = \underline{b} \cdot c \triangleright b \cdot c = \underline{c} \cdot b \gg a \cdot c = \underline{c} \cdot b$; S1 $\triangleright a \cdot c =$
 $\underline{c} \cdot a \triangleright a \cdot c = \underline{c} \cdot b \gg \underline{c} \cdot a = \underline{c} \cdot b$; Ded $\triangleright \forall a: \forall b: \forall c: a = \underline{b} \vdash \underline{c} \cdot a = \underline{c} \cdot b \gg a = \underline{b} \Rightarrow$
 $\underline{c} \cdot a = \underline{c} \cdot b$; MP $\triangleright a = \underline{b} \Rightarrow \underline{c} \cdot a = \underline{c} \cdot b \triangleright a = \underline{b} \gg \underline{c} \cdot a = \underline{c} \cdot b$], p0, c)]

[Prop 3.2o $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: a = \underline{b} \vdash \underline{c} \cdot a = \underline{c} \cdot b$]

[Prop 3.2o $\xrightarrow{\text{tex}}$ “
Prop\ 3.2o”]

[Prop 3.2o $\xrightarrow{\text{pyk}}$ “prop three two o”]

Prop 3.4

[Prop 3.4 $\xrightarrow{\text{tex}}$ “
Prop\ 3.4”]

[Prop 3.4 $\xrightarrow{\text{pyk}}$ “prop three four”]

Prop 3.4a₁

[Prop 3.4a₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: S5 \gg b + 0 = \underline{b}$; Prop 3.2o $\triangleright b + 0 = \underline{b} \gg$
 $a \cdot b + 0 = \underline{a} \cdot b$; S5 $\gg a \cdot b + 0 = \underline{a} \cdot b$; Prop 3.2d $\triangleright a \cdot b + 0 = \underline{a} \cdot b \triangleright a \cdot b + 0 = \underline{a} \cdot b \gg$
 $a \cdot b + 0 = \underline{a} \cdot b + 0$; S7 $\gg a \cdot 0 = 0$; Prop 3.2i $\triangleright a \cdot 0 = 0 \gg a \cdot b + a \cdot 0 = \underline{a} \cdot b +$
 0 ; Prop 3.2d $\triangleright a \cdot b + 0 = \underline{a} \cdot b + 0 \triangleright a \cdot b + a \cdot 0 = \underline{a} \cdot b + 0 \gg a \cdot b + 0 = \underline{a} \cdot b + a \cdot 0$], p0, c)]

[Prop 3.4a₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: a \cdot b + 0 = \underline{a} \cdot b + a \cdot 0$]

[Prop 3.4a₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.4a.1”]

[Prop 3.4a₁ $\xrightarrow{\text{pyk}}$ “prop three four a one”]

Prop 3.4a₂

[Prop 3.4a₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: \forall a': \forall b': \forall c': a \cdot b + c = \underline{a} \cdot b + a \cdot c \vdash S6 \gg$
 $\underline{b} + c' = \underline{b} + c'$; Prop 3.2o $\triangleright \underline{b} + c' = \underline{b} + c' \gg a \cdot b + c' = \underline{a} \cdot b + c'$; S8 $\gg a \cdot b + c' =$

Prop 3.4d₁

[Prop 3.4d₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall a': \forall b': a + 0 = b + 0 \vdash S5 \gg a + 0 = a; S5 \gg b + 0 = b; S1 \triangleright a + 0 = a \triangleright a + 0 = b + 0 \gg a = b + 0; \text{Prop 3.2c} \triangleright a = b + 0 \triangleright b + 0 = b \gg a = b; \text{Ded} \triangleright \forall a: \forall b: a + 0 = b + 0 \vdash a = b \gg a + 0 = b + 0 \Rightarrow a = b], p_0, c)]$

[Prop 3.4d₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: a + 0 = b + 0 \Rightarrow a = b]$

[Prop 3.4d₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.4d.1”]

[Prop 3.4d₁ $\xrightarrow{\text{pyk}}$ “prop three four d one”]

Prop 3.4d₂

[Prop 3.4d₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: \forall a': \forall b': \forall c': a + c = b + c \Rightarrow a = b \vdash a + c' = b + c' \vdash S6 \gg a + c' = a + c'; S6 \gg b + c' = b + c'; S1 \triangleright a + c' = a + c' \triangleright a + c' = b + c' \gg a + c' = b + c'; \text{Prop 3.2c} \triangleright a + c' = b + c' \triangleright b + c' = b + c' \gg a + c' = b + c'; S4 \triangleright a + c' = b + c' \gg a + c = b + c; \text{MP} \triangleright a + c = b + c \Rightarrow a = b \triangleright a + c = b + c \gg a = b; \text{Ded} \triangleright \forall a: \forall b: \forall c: a + c = b + c \Rightarrow a = b \vdash a + c' = b + c' \vdash a = b \gg a + c = b + c \Rightarrow a = b \Rightarrow a + c' = b + c' \Rightarrow a = b], p_0, c)]$

[Prop 3.4d₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: a + c = b + c \Rightarrow a = b \Rightarrow a + c' = b + c' \Rightarrow a = b]$

[Prop 3.4d₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.4d.2”]

[Prop 3.4d₂ $\xrightarrow{\text{pyk}}$ “prop three four d two”]

Prop 3.4d

[Prop 3.4d $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall a: \forall b: \forall c: \text{Prop 3.4d}_1 \gg \bar{x} + 0 = \bar{y} + 0 \Rightarrow \bar{x} = \bar{y}; \text{Prop 3.4d}_2 \gg \bar{x} + \bar{z} = \bar{y} + \bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z}' = \bar{y} + \bar{z}' \Rightarrow \bar{x} = \bar{y}; S9 @ \bar{z} \triangleright \bar{x} + 0 = \bar{y} + 0 \Rightarrow \bar{x} = \bar{y} \triangleright \bar{x} + \bar{z} = \bar{y} + \bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z}' = \bar{y} + \bar{z}' \Rightarrow \bar{x} = \bar{y} \gg \bar{x} + \bar{z} = \bar{y} + \bar{z} \Rightarrow \bar{x} = \bar{y}; \text{Ded} \triangleright \bar{x} + \bar{z} = \bar{y} + \bar{z} \Rightarrow \bar{x} = \bar{y} \gg a + c = b + c \Rightarrow a = b], p_0, c)]$

[Prop 3.4d $\xrightarrow{\text{stmt}}$ $S \vdash \forall a: \forall b: \forall c: a + c = b + c \Rightarrow a = b]$

[Prop 3.4d $\xrightarrow{\text{tex}}$ “
Prop\ 3.4d”]

[Prop 3.4d $\xrightarrow{\text{pyk}}$ “prop three four d”]

Prop 3.5

[Prop 3.5 $\xrightarrow{\text{tex}}$ “
Prop\ 3.5”]

[Prop 3.5 $\xrightarrow{\text{pyk}}$ “prop three five”]

Prop 3.5a

[Prop 3.5a $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S6 \gg \underline{a} + 0' = \underline{a} + 0'; S5 \gg \underline{a} + 0 = \underline{a}; S2 \triangleright \underline{a} + 0 = \underline{a} \gg \underline{a} + 0' = \underline{a}'; \text{Prop 3.2c} \triangleright \underline{a} + 0' = \underline{a} + 0' \triangleright \underline{a} + 0' = \underline{a}' \gg \underline{a} + 0' = \underline{a}'; \text{Prop 3.2a} \gg \underline{a} + 0' = \underline{a} + 0'; S1 \triangleright \underline{a} + 0' = \underline{a} + 0' \triangleright \underline{a} + 0' = \underline{a}' \gg \underline{a} + 0' = \underline{a}']$, p_0, c)]

[Prop 3.5a $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} + 0' = \underline{a}'$]

[Prop 3.5a $\xrightarrow{\text{tex}}$ “
Prop\ 3.5a”]

[Prop 3.5a $\xrightarrow{\text{pyk}}$ “prop three five a”]

Prop 3.5b

[Prop 3.5b $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S8 \gg \underline{a} \cdot 0' = \underline{a} \cdot 0 + \underline{a}; S7 \gg \underline{a} \cdot 0 = 0; \text{Prop 3.2e} \triangleright \underline{a} \cdot 0 = 0 \gg \underline{a} \cdot 0 + \underline{a} = 0 + \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} \cdot 0' = \underline{a} \cdot 0 + \underline{a} \triangleright \underline{a} \cdot 0 + \underline{a} = 0 + \underline{a} \gg \underline{a} \cdot 0' = 0 + \underline{a}; \text{Prop 3.2f} \gg \underline{a} = 0 + \underline{a}; \text{Prop 3.2b} \triangleright \underline{a} = 0 + \underline{a} \gg 0 + \underline{a} = \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} \cdot 0' = 0 + \underline{a} \triangleright 0 + \underline{a} = \underline{a} \gg \underline{a} \cdot 0' = \underline{a}; \text{Prop 3.2a} \gg \underline{a} \cdot 0' = \underline{a} \cdot 0'; S1 \triangleright \underline{a} \cdot 0' = \underline{a} \cdot 0' \triangleright \underline{a} \cdot 0' = \underline{a} \gg \underline{a} \cdot 0' = \underline{a}']$, p_0, c)]

[Prop 3.5b $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \cdot 0' = \underline{a}$]

[Prop 3.5b $\xrightarrow{\text{tex}}$ “
Prop\ 3.5b”]

[Prop 3.5b $\xrightarrow{\text{pyk}}$ “prop three five b”]

Prop 3.5c

[Prop 3.5c $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S8 \gg \underline{a} \cdot 0'' = \underline{a} \cdot 0' + \underline{a}; \text{Prop 3.5b} \gg \underline{a} \cdot 0' = \underline{a}; \text{Prop 3.2e} \triangleright \underline{a} \cdot 0' = \underline{a} \gg \underline{a} \cdot 0' + \underline{a} = \underline{a} + \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} \cdot 0'' = \underline{a} \cdot 0' + \underline{a} \triangleright \underline{a} \cdot 0' + \underline{a} = \underline{a} + \underline{a} \gg \underline{a} \cdot 0'' = \underline{a} + \underline{a}; \text{Prop 3.2a} \gg \underline{a} \cdot 0'' = \underline{a} \cdot 0''; S1 \triangleright \underline{a} \cdot 0'' = \underline{a} \cdot 0'' \triangleright \underline{a} \cdot 0'' = \underline{a} + \underline{a} \gg \underline{a} \cdot 0'' = \underline{a} + \underline{a}']$, p_0, c)]

[Prop 3.5c $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \cdot 0'' = \underline{a} + \underline{a}$]

[Prop 3.5c $\xrightarrow{\text{tex}}$ “
Prop\ 3.5c”]

[Prop 3.5c $\xrightarrow{\text{pyk}}$ “prop three five c”]

Prop 3.5d₁

[Prop 3.5d₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{a}: \underline{a} + 0 = 0 \vdash S5 \gg \underline{a} + 0 = \underline{a}; S1 \triangleright \underline{a} + 0 = \underline{a} \triangleright \underline{a} + 0 = 0 \gg \underline{a} = 0; \text{Prop } 3.2a \gg 0 = 0; \text{Con} \triangleright \underline{a} = 0 \triangleright 0 = 0 \gg \neg \underline{a} = 0 \Rightarrow \neg 0 = 0; \text{Ded} \triangleright \forall \underline{a}: \underline{a} + 0 = 0 \vdash \neg \underline{a} = 0 \Rightarrow \neg 0 = 0 \gg \underline{a} + 0 = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg 0 = 0], p_0, c)$]

[Prop 3.5d₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} + 0 = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg 0 = 0$]

[Prop 3.5d₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5d.1”]

[Prop 3.5d₁ $\xrightarrow{\text{pyk}}$ “prop three five d one”]

Prop 3.5d₂

[Prop 3.5d₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b}' = 0 \vdash S3 \gg \neg 0 = \underline{a} + \underline{b}'; \text{Prop } 3.2b' \gg \underline{a} + \underline{b}' = 0 \Rightarrow 0 = \underline{a} + \underline{b}'; \text{Lem } 1.11e \gg \underline{a} + \underline{b}' = 0 \Rightarrow 0 = \underline{a} + \underline{b}' \Rightarrow \neg 0 = \underline{a} + \underline{b}' \Rightarrow \neg \underline{a} + \underline{b}' = 0; \text{MP} \triangleright \underline{a} + \underline{b}' = 0 \Rightarrow 0 = \underline{a} + \underline{b}' \Rightarrow \neg 0 = \underline{a} + \underline{b}' \Rightarrow \neg \underline{a} + \underline{b}' = 0 \triangleright \underline{a} + \underline{b}' = 0 \Rightarrow 0 = \underline{a} + \underline{b}' \gg \neg 0 = \underline{a} + \underline{b}' \Rightarrow \neg \underline{a} + \underline{b}' = 0; \text{MP} \triangleright \neg 0 = \underline{a} + \underline{b}' \Rightarrow \neg \underline{a} + \underline{b}' = 0 \triangleright \neg 0 = \underline{a} + \underline{b}' \gg \neg \underline{a} + \underline{b}' = 0; S6 \gg \underline{a} + \underline{b}' = \underline{a} + \underline{b}'; S1'' \gg \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \Rightarrow \underline{a} + \underline{b}' = 0 \Rightarrow \underline{a} + \underline{b}' = 0; H3 \triangleright \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \Rightarrow \underline{a} + \underline{b}' = 0 \Rightarrow \underline{a} + \underline{b}' = 0 \triangleright \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \triangleright \neg \underline{a} + \underline{b}' = 0 \gg \neg \underline{a} + \underline{b}' = 0; \text{Lem } 1.11c \gg \neg \underline{a} + \underline{b}' = 0 \Rightarrow \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0; \text{MP} \triangleright \neg \underline{a} + \underline{b}' = 0 \Rightarrow \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0 \triangleright \neg \underline{a} + \underline{b}' = 0 \gg \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0; \text{MP} \triangleright \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0 \triangleright \underline{a} + \underline{b}' = 0 \gg \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b}' = 0 \vdash \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0 \gg \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \vdash \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0 \gg \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \Rightarrow \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0], p_0, c)$]

[Prop 3.5d₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \Rightarrow \underline{a} + \underline{b}' = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0$]

[Prop 3.5d₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5d.2”]

[Prop 3.5d₂ $\xrightarrow{\text{pyk}}$ “prop three five d two”]

Prop 3.5d

[Prop 3.5d $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop 3.5d}_1 \gg \bar{x} + 0 = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg 0 = 0; \text{Prop 3.5d}_2 \gg \bar{x} + \bar{y} = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0 \Rightarrow \bar{x} + \bar{y}' = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y}' = 0; \text{S9} @ \bar{y} \triangleright \bar{x} + 0 = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg 0 = 0 \triangleright \bar{x} + \bar{y} = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0 \Rightarrow \bar{x} + \bar{y}' = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y}' = 0 \gg \bar{x} + \bar{y} = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0; \text{Ded} \triangleright \bar{x} + \bar{y} = 0 \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0 \gg \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0], p_0, c)]$

[Prop 3.5d $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0]$

[Prop 3.5d $\xrightarrow{\text{tex}}$ “
Prop\ 3.5d”]

[Prop 3.5d $\xrightarrow{\text{pyk}}$ “prop three five d”]

Prop 3.5e₁

[Prop 3.5e₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{a}: \neg 0 = 0 \vdash \text{Prop 3.2a} \gg 0 = 0; \text{Lem1.11c} \gg \neg 0 = 0 \Rightarrow 0 = 0 \Rightarrow \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0; \text{MP} \triangleright \neg 0 = 0 \Rightarrow 0 = 0 \Rightarrow \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0 \triangleright \neg 0 = 0 \gg 0 = 0 \Rightarrow \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0; \text{MP} \triangleright 0 = 0 \Rightarrow \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0 \triangleright 0 = 0 \gg \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0; \text{Ded} \triangleright \forall \underline{a}: \neg 0 = 0 \vdash \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0 \gg \neg 0 = 0 \Rightarrow \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0], p_0, c)]$

[Prop 3.5e₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg 0 = 0 \Rightarrow \underline{a} \cdot 0 = 0 \Rightarrow \underline{a} = 0]$

[Prop 3.5e₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5e.1”]

[Prop 3.5e₁ $\xrightarrow{\text{pyk}}$ “prop three five e one”]

Prop 3.5e₂

[Prop 3.5e₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = 0 \Rightarrow \underline{a} \cdot \underline{b} = 0 \Rightarrow \underline{a} = 0 \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b}' = 0 \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b}' = 0 \vdash \text{S8} \gg \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}; \text{S1} \triangleright \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a} \triangleright \underline{a} \cdot \underline{b}' = 0 \gg \underline{a} \cdot \underline{b} + \underline{a} = 0; \text{Prop 3.5d} \gg \underline{a} \cdot \underline{b} + \underline{a} = 0 \Rightarrow \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0; \text{MP} \triangleright \underline{a} \cdot \underline{b} + \underline{a} = 0 \Rightarrow \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \triangleright \underline{a} \cdot \underline{b} + \underline{a} = 0 \gg \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0; \text{Con2} \triangleright \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \gg \underline{a} = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b}' = 0 \vdash \underline{a} = 0 \gg \underline{a} \cdot \underline{b}' = 0 \Rightarrow \underline{a} = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{b}' = 0 \vdash \underline{a} \cdot \underline{b}' = 0 \Rightarrow \underline{a} = 0 \gg \neg \underline{b}' = 0 \Rightarrow \underline{a} \cdot \underline{b}' = 0 \Rightarrow \underline{a} = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = 0 \Rightarrow \underline{a} \cdot \underline{b} = 0 \Rightarrow \underline{a} = 0 \vdash \neg \underline{b}' = 0 \Rightarrow \underline{a} \cdot \underline{b}' = 0 \Rightarrow \underline{a} = 0 \gg \neg \underline{b} = 0 \Rightarrow \underline{a} \cdot \underline{b} = 0 \Rightarrow \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0 \Rightarrow \underline{a} \cdot \underline{b}' = 0 \Rightarrow \underline{a} = 0], p_0, c)]$

[Prop 3.5e₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = 0 \Rightarrow \underline{a} \cdot \underline{b} = 0 \Rightarrow \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0 \Rightarrow \underline{a} \cdot \underline{b}' = 0 \Rightarrow \underline{a} = 0]$

[Prop 3.5e₂ $\xrightarrow{\text{tex}}$ “

Prop\ 3.5e_2"]

[Prop 3.5e₂ $\xrightarrow{\text{pyk}}$ "prop three five e two"]

Prop 3.5e

[Prop 3.5e $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop } 3.5e_1 \gg \neg 0 = 0 \Rightarrow \bar{x} \cdot 0 = 0 \Rightarrow \bar{x} = 0; \text{Prop } 3.5e_2 \gg \neg \bar{y} = 0 \Rightarrow \bar{x} \cdot \bar{y} = 0 \Rightarrow \bar{x} = 0 \Rightarrow \neg \bar{y}' = 0 \Rightarrow \bar{x} \cdot \bar{y}' = 0 \Rightarrow \bar{x} = 0; S9 @ \bar{y} \triangleright \neg 0 = 0 \Rightarrow \bar{x} \cdot 0 = 0 \Rightarrow \bar{x} = 0 \triangleright \neg \bar{y} = 0 \Rightarrow \bar{x} \cdot \bar{y} = 0 \Rightarrow \bar{x} = 0 \Rightarrow \neg \bar{y}' = 0 \Rightarrow \bar{x} \cdot \bar{y}' = 0 \Rightarrow \bar{x} = 0 \gg \neg \bar{y} = 0 \Rightarrow \bar{x} \cdot \bar{y} = 0 \Rightarrow \bar{x} = 0; \text{Ded} \triangleright \neg \bar{y} = 0 \Rightarrow \bar{x} \cdot \bar{y} = 0 \Rightarrow \bar{x} = 0 \gg \neg \underline{b} = 0 \Rightarrow \underline{a} \cdot \underline{b} = 0 \Rightarrow \underline{a} = 0], p_0, c)$]

[Prop 3.5e $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = 0 \Rightarrow \underline{a} \cdot \underline{b} = 0 \Rightarrow \underline{a} = 0$]

[Prop 3.5e $\xrightarrow{\text{tex}}$ "
Prop\ 3.5e"]

[Prop 3.5e $\xrightarrow{\text{pyk}}$ "prop three five e"]

Prop 3.5f₁

[Prop 3.5f₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{a}': \underline{a} + 0 = 0' \vdash S5 \gg \underline{a} + 0 = \underline{a}; S1 \triangleright \underline{a} + 0 = \underline{a} \triangleright \underline{a} + 0 = 0' \gg \underline{a} = 0'; \text{Prop } 3.2a \gg 0 = 0; \text{Con} \triangleright \underline{a} = 0' \triangleright 0 = 0 \gg \neg \underline{a} = 0' \Rightarrow \neg 0 = 0; \text{Dis2} \triangleright \neg \underline{a} = 0' \Rightarrow \neg 0 = 0 \gg \neg \neg \underline{a} = 0 \Rightarrow \neg 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0; \text{Ded} \triangleright \forall \underline{a}: \underline{a} + 0 = 0' \vdash \neg \neg \underline{a} = 0 \Rightarrow \neg 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0 \gg \underline{a} + 0 = 0' \Rightarrow \neg \neg \underline{a} = 0 \Rightarrow \neg 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0], p_0, c)$]

[Prop 3.5f₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} + 0 = 0' \Rightarrow \neg \neg \underline{a} = 0 \Rightarrow \neg 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0$]

[Prop 3.5f₁ $\xrightarrow{\text{tex}}$ "
Prop\ 3.5f_1"]

[Prop 3.5f₁ $\xrightarrow{\text{pyk}}$ "prop three five f one"]

Prop 3.5f₂

[Prop 3.5f₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}': \forall \underline{b}': \underline{a} + \underline{b} = 0' \Rightarrow \neg \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0 \vdash \forall \underline{a}': \forall \underline{b}': \underline{a} + \underline{b}' = 0' \vdash S6 \gg \underline{a} + \underline{b}' = \underline{a} + \underline{b}'; S1 \triangleright \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \triangleright \underline{a} + \underline{b}' = 0' \gg \underline{a} + \underline{b}' = 0'; S4 \triangleright \underline{a} + \underline{b}' = 0' \gg \underline{a} + \underline{b} = 0; \text{Prop } 3.5d \gg \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0; \text{MP} \triangleright \underline{a} + \underline{b} = 0 \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \triangleright \underline{a} + \underline{b} = 0 \gg \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0; \text{Con1} \triangleright \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \gg \underline{a} = 0; \text{Con2} \triangleright \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0 \gg \underline{b} = 0; S2 \triangleright \underline{b} = 0 \gg \underline{b}' = 0'; \text{Con} \triangleright \underline{a} = 0 \triangleright \underline{b}' = 0' \gg \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0'; \text{Dis1} \triangleright \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \gg \neg \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b}' = 0' \vdash \neg \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0 \gg \underline{a} +$

$\underline{b}' = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0 \vdash \underline{a} + \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0 \gg \underline{a} + \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0 \Rightarrow \underline{a} + \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0], p_0, c]$

[Prop 3.5f₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0 \Rightarrow \underline{a} + \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0]$

[Prop 3.5f₂ $\xrightarrow{\text{tex}}$ “
Prop \ 3.5f_2”]

[Prop 3.5f₂ $\xrightarrow{\text{pyk}}$ “prop three five f two”]

Prop 3.5f

[Prop 3.5f $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop 3.5f}_1 \gg \bar{x} + 0 = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg 0 = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg 0 = 0; \text{Prop 3.5f}_2 \gg \bar{x} + \bar{y} = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0 \Rightarrow \bar{x} + \bar{y}' = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y}' = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y}' = 0; S9 @ \bar{y} \triangleright \bar{x} + 0 = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg 0 = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg 0 = 0 \triangleright \bar{x} + \bar{y} = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0 \Rightarrow \bar{x} + \bar{y}' = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y}' = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y}' = 0 \gg \bar{x} + \bar{y} = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0; \text{Ded} \triangleright \bar{x} + \bar{y} = 0' \Rightarrow \neg \bar{x} = 0 \Rightarrow \neg \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0 \gg \underline{a} + \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0], p_0, c)]$

[Prop 3.5f $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0]$

[Prop 3.5f $\xrightarrow{\text{tex}}$ “
Prop \ 3.5f”]

[Prop 3.5f $\xrightarrow{\text{pyk}}$ “prop three five f”]

Prop 3.5g₁

[Prop 3.5g₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{a}: \underline{a} \cdot 0 = 0' \vdash S7 \gg \underline{a} \cdot 0 = 0; S1 \triangleright \underline{a} \cdot 0 = 0 \triangleright \underline{a} \cdot 0 = 0' \gg 0 = 0'; S3 \gg \neg 0 = 0'; \text{Lem1.11c} \gg \neg 0 = 0' \Rightarrow 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0'; \text{MP} \triangleright \neg 0 = 0' \Rightarrow 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0' \triangleright \neg 0 = 0' \gg 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0'; \text{MP} \triangleright 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0' \triangleright 0 = 0' \gg \neg \underline{a} = 0' \Rightarrow \neg 0 = 0'; \text{Ded} \triangleright \forall \underline{a}: \underline{a} \cdot 0 = 0' \vdash \neg \underline{a} = 0' \Rightarrow \neg 0 = 0' \gg \underline{a} \cdot 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0'], p_0, c)]$

[Prop 3.5g₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \cdot 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg 0 = 0'$]

[Prop 3.5g₁ $\xrightarrow{\text{tex}}$ “
Prop \ 3.5g_1”]

[Prop 3.5g₁ $\xrightarrow{\text{pyk}}$ “prop three five g one”]

Prop 3.5g₄

[Prop 3.5g₄ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}': \forall \underline{b}': \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \vdash \underline{a} \cdot \underline{b}' = 0' \vdash S8 \gg \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}; S1 \triangleright \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a} \triangleright \underline{a} \cdot \underline{b}' = 0' \gg \underline{a} \cdot \underline{b} + \underline{a} = 0'; \text{Prop 3.5f} \gg \underline{a} \cdot \underline{b} + \underline{a} = 0' \Rightarrow \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0; \text{MP} \triangleright \underline{a} \cdot \underline{b} + \underline{a} = 0' \Rightarrow \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \triangleright \underline{a} \cdot \underline{b} + \underline{a} = 0' \gg \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0; \text{Prop 3.5g}_2 \gg \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{Prop 3.5g}_3 \triangleright \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \gg \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{H11} \triangleright \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \triangleright \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \triangleright \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \gg \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \vdash \underline{a} \cdot \underline{b}' = 0' \vdash \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \gg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \Rightarrow \underline{a} \cdot \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0']]$

[Prop 3.5g₄ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \Rightarrow \underline{a} \cdot \underline{b}' = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'$]

[Prop 3.5g₄ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5g.4”]

[Prop 3.5g₄ $\xrightarrow{\text{pyk}}$ “prop three five g two”]

Prop 3.5g₂

[Prop 3.5g₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}': \forall \underline{b}': \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \vdash \text{Con2} \triangleright \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \gg \underline{a} = 0'; \text{Con1} \triangleright \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \gg \underline{a} \cdot \underline{b} = 0; \text{S3} \gg \neg 0 = 0'; \text{H10} \triangleright \neg 0 = 0' \gg \neg 0' = 0; \text{S1}'' \gg \underline{a} = 0' \Rightarrow \underline{a} = 0 \Rightarrow 0' = 0; \text{H3} \triangleright \underline{a} = 0' \Rightarrow \underline{a} = 0 \Rightarrow 0' = 0 \triangleright \underline{a} = 0' \triangleright \neg 0' = 0 \gg \neg \underline{a} = 0; \text{Prop 3.2n} \gg \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}; \text{S1} \triangleright \underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a} \triangleright \underline{a} \cdot \underline{b} = 0 \gg \underline{b} \cdot \underline{a} = 0; \text{Prop 3.5e} \gg \neg \underline{a} = 0 \Rightarrow \underline{b} \cdot \underline{a} = 0 \Rightarrow \underline{b} = 0; \text{MP} \triangleright \neg \underline{a} = 0 \Rightarrow \underline{b} \cdot \underline{a} = 0 \Rightarrow \underline{b} = 0 \triangleright \neg \underline{a} = 0 \gg \underline{b} \cdot \underline{a} = 0 \Rightarrow \underline{b} = 0; \text{MP} \triangleright \underline{b} \cdot \underline{a} = 0 \Rightarrow \underline{b} = 0 \triangleright \underline{b} \cdot \underline{a} = 0 \gg \underline{b} = 0; \text{S2} \triangleright \underline{b} = 0 \gg \underline{b}' = 0'; \text{Con} \triangleright \underline{a} = 0' \triangleright \underline{b}' = 0' \gg \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \vdash \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \gg \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0']]$

[Prop 3.5g₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \cdot \underline{b} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'$]

[Prop 3.5g₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5g.2”]

[Prop 3.5g₂ $\xrightarrow{\text{pyk}}$ “prop three five g three”]

Prop 3.5g₃

[Prop 3.5g₃ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \vdash \text{Con1} \triangleright \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \gg \underline{a} \cdot \underline{b} = 0'; \text{MP} \triangleright \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \triangleright \underline{a} \cdot \underline{b} = 0' \gg \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0'; \text{Con1} \triangleright \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \gg \underline{a} = 0'; \text{Con2} \triangleright \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \gg \underline{a} = 0; \text{S1} \triangleright \underline{a} = 0 \triangleright \underline{a} = 0' \gg 0 = 0'; \text{S3} \gg \neg 0 = 0'; \text{Lem1.11c} \gg \neg 0 = 0' \Rightarrow 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{MP} \triangleright \neg 0 = 0' \Rightarrow 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \triangleright \neg 0 = 0' \gg 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{MP} \triangleright 0 = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \triangleright 0 = 0' \gg \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \vdash \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0' \gg \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0']$, p₀, c)]

[Prop 3.5g₃ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0' \vdash \neg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b}' = 0'$]

[Prop 3.5g₃ $\xrightarrow{\text{tex}}$ “
Prop \ 3.5g.3”]

[Prop 3.5g₃ $\xrightarrow{\text{pyk}}$ “prop three five g four”]

Prop 3.5g

[Prop 3.5g $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop 3.5g}_1 \gg \bar{x} \cdot 0 = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg 0 = 0'; \text{Prop 3.5g}_4 \gg \bar{x} \cdot \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0' \Rightarrow \bar{x} \cdot \bar{y}' = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y}' = 0'; \text{S9} @ \bar{y} \triangleright \bar{x} \cdot 0 = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg 0 = 0' \triangleright \bar{x} \cdot \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0' \Rightarrow \bar{x} \cdot \bar{y}' = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y}' = 0' \gg \bar{x} \cdot \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0'; \text{Ded} \triangleright \bar{x} \cdot \bar{y} = 0' \Rightarrow \neg \bar{x} = 0' \Rightarrow \neg \bar{y} = 0' \gg \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0']$, p₀, c)]

[Prop 3.5g $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b} = 0' \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \underline{b} = 0'$]

[Prop 3.5g $\xrightarrow{\text{tex}}$ “
Prop \ 3.5g”]

[Prop 3.5g $\xrightarrow{\text{pyk}}$ “prop three five g”]

Prop 3.5h₁

[Prop 3.5h₁ $\xrightarrow{\text{tex}}$ “
Prop \ 3.5h.1”]

[Prop 3.5h₁ $\xrightarrow{\text{pyk}}$ “prop three five h one”]

Prop 3.5h₂

[Prop 3.5h₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5h.2”]

[Prop 3.5h₂ $\xrightarrow{\text{pyk}}$ “prop three five h two”]

Prop 3.5h

[Prop 3.5h $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}} \underline{b}: \neg \underline{a} = \underline{b}'$]

[Prop 3.5h $\xrightarrow{\text{tex}}$ “
Prop\ 3.5h”]

[Prop 3.5h $\xrightarrow{\text{pyk}}$ “prop three five h”]

Prop 3.5i₁

[Prop 3.5i₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5i.1”]

[Prop 3.5i₁ $\xrightarrow{\text{pyk}}$ “prop three five i one”]

Prop 3.5i₂

[Prop 3.5i₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5i.2”]

[Prop 3.5i₂ $\xrightarrow{\text{pyk}}$ “prop three five i two”]

Prop 3.5i

[Prop 3.5i $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{c} = 0 \Rightarrow \underline{a} \cdot \underline{c} = \underline{b} \cdot \underline{c} \Rightarrow \underline{a} = \underline{b}$]

[Prop 3.5i $\xrightarrow{\text{tex}}$ “
Prop\ 3.5i”]

[Prop 3.5i $\xrightarrow{\text{pyk}}$ “prop three five i”]

Prop 3.5j₁

[Prop 3.5j₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5j-1”]

[Prop 3.5j₁ $\xrightarrow{\text{pyk}}$ “prop three five j one”]

Prop 3.5j₂

[Prop 3.5j₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.5j-2”]

[Prop 3.5j₂ $\xrightarrow{\text{pyk}}$ “prop three five j two”]

Prop 3.5j

[Prop 3.5j $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \underline{a} = 0 \Rightarrow \neg \underline{a} = 0' \Rightarrow \neg \forall_{\text{obj}} \underline{b}: \neg \underline{a} = \underline{b}''$]

[Prop 3.5j $\xrightarrow{\text{tex}}$ “
Prop\ 3.5j”]

[Prop 3.5j $\xrightarrow{\text{pyk}}$ “prop three five j”]

Prop 3.7

[Prop 3.7 $\xrightarrow{\text{tex}}$ “
Prop\ 3.7”]

[Prop 3.7 $\xrightarrow{\text{pyk}}$ “prop three seven”]

Prop 3.7a

[Prop 3.7a $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \neg \forall_{\text{obj}} \underline{z}: \neg \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{a} = \underline{a}$]

[Prop 3.7a $\xrightarrow{\text{tex}}$ “
Prop\ 3.7a”]

[Prop 3.7a $\xrightarrow{\text{pyk}}$ “prop three seven a”]

Prop 3.7b

[Prop 3.7b $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{c}$]

[Prop 3.7b $\xrightarrow{\text{tex}}$ “
Prop\ 3.7b”]

[Prop 3.7b $\xrightarrow{\text{pyk}}$ “prop three seven b”]

Prop 3.7c

[Prop 3.7c $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{a}$]

[Prop 3.7c $\xrightarrow{\text{tex}}$ “
Prop\ 3.7c”]

[Prop 3.7c $\xrightarrow{\text{pyk}}$ “prop three seven c”]

Prop 3.7d

[Prop 3.7d $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} + \underline{c} = \underline{b} + \underline{c}$]

[Prop 3.7d $\xrightarrow{\text{tex}}$ “
Prop\ 3.7d”]

[Prop 3.7d $\xrightarrow{\text{pyk}}$ “prop three seven d”]

Prop 3.7e

[Prop 3.7e $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \forall_{\text{obj}Z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{a} \Rightarrow \underline{a} = \underline{a}$]

[Prop 3.7e $\xrightarrow{\text{tex}}$ “
Prop\ 3.7e”]

[Prop 3.7e $\xrightarrow{\text{pyk}}$ “prop three seven e”]

Prop 3.7f

[Prop 3.7f $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c} \Rightarrow \underline{b} = \underline{c} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{c} \Rightarrow \underline{a} = \underline{c}$]

[Prop 3.7f $\xrightarrow{\text{tex}}$ “
Prop\ 3.7f”]

[Prop 3.7f $\xrightarrow{\text{pyk}}$ “prop three seven f”]

Prop 3.7g

[Prop 3.7g $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c}$]

[Prop 3.7g $\xrightarrow{\text{tex}}$ “
Prop\ 3.7g”]

[Prop 3.7g $\xrightarrow{\text{pyk}}$ “prop three seven g”]

Prop 3.7g'

[Prop 3.7g' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b}$]

[Prop 3.7g' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7g'”]

[Prop 3.7g' $\xrightarrow{\text{pyk}}$ “prop three seven g mark”]

Prop 3.7h

[Prop 3.7h $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{c}$]

[Prop 3.7h $\xrightarrow{\text{tex}}$ “
Prop\ 3.7h”]

[Prop 3.7h $\xrightarrow{\text{pyk}}$ “prop three seven h”]

Prop 3.7i

[Prop 3.7i $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{a} \Rightarrow 0 = \underline{a}$]

[Prop 3.7i $\xrightarrow{\text{tex}}$ “
Prop\ 3.7i”]

[Prop 3.7i $\xrightarrow{\text{pyk}}$ “prop three seven i”]

Prop 3.7j

[Prop 3.7j $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{a}'$]

[Prop 3.7j $\xrightarrow{\text{tex}}$ “
Prop\ 3.7j”]

[Prop 3.7j $\xrightarrow{\text{pyk}}$ “prop three seven j”]

Prop 3.7k

[Prop 3.7k $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a}' = \underline{b} \Rightarrow \underline{a}' = \underline{b}$]

[Prop 3.7k $\xrightarrow{\text{tex}}$ “
Prop\ 3.7k”]

[Prop 3.7k $\xrightarrow{\text{pyk}}$ “prop three seven k”]

Prop 3.7k'

[Prop 3.7k' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a}' = \underline{b} \Rightarrow \underline{a}' = \underline{b} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b}$]

[Prop 3.7k' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7k'”]

[Prop 3.7k' $\xrightarrow{\text{pyk}}$ “prop three seven k mark”]

Prop 3.7l

[Prop 3.7l $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b}'$]

[Prop 3.7l $\xrightarrow{\text{tex}}$ “
Prop\ 3.7l”]

[Prop 3.7l $\xrightarrow{\text{pyk}}$ “prop three seven l”]

Prop 3.7l'

[Prop 3.7l' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b}' \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b}$]

[Prop 3.7l' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7l'”]

[Prop 3.7l' $\xrightarrow{\text{pyk}}$ “prop three seven l mark”]

Prop 3.7m

[Prop 3.7m $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{a}'$]

[Prop 3.7m $\xrightarrow{\text{tex}}$ “
Prop\ 3.7m”]

[Prop 3.7m $\xrightarrow{\text{pyk}}$ “prop three seven m”]

Prop 3.7n

[Prop 3.7n $\xrightarrow{\text{tex}}$ “
Prop\ 3.7n”]

[Prop 3.7n $\xrightarrow{\text{pyk}}$ “prop three seven n”]

Prop 3.7o

[Prop 3.7o $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{a}$]

[Prop 3.7o $\xrightarrow{\text{tex}}$ “
Prop\ 3.7o”]

[Prop 3.7o $\xrightarrow{\text{pyk}}$ “prop three seven o”]

Prop 3.7p

[Prop 3.7p $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{b} = \underline{a}$]

[Prop 3.7p $\xrightarrow{\text{tex}}$ “
Prop\ 3.7p”]

[Prop 3.7p $\xrightarrow{\text{pyk}}$ “prop three seven p”]

Prop 3.7q

[Prop 3.7q $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b} \Rightarrow \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{b} = \underline{a} \Rightarrow \underline{b} = \underline{a}$]

[Prop 3.7q $\xrightarrow{\text{tex}}$ “
Prop\ 3.7q”]

[Prop 3.7q $\xrightarrow{\text{pyk}}$ “prop three seven q”]

Prop 3.7r

[Prop 3.7r $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{a} = \underline{a} + \underline{b} \Rightarrow \underline{a} = \underline{a} + \underline{b}$]

[Prop 3.7r $\xrightarrow{\text{tex}}$ “
Prop\ 3.7r”]

[Prop 3.7r $\xrightarrow{\text{pyk}}$ “prop three seven r”]

Prop 3.7s

[Prop 3.7s $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = 0 \Rightarrow \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{a} = \underline{a} + \underline{b}$]

[Prop 3.7s $\xrightarrow{\text{tex}}$ “
Prop\ 3.7s”]

[Prop 3.7s $\xrightarrow{\text{pyk}}$ “prop three seven s”]

Prop 3.7t

[Prop 3.7t $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} = 0 \Rightarrow \neg \forall_{\text{obj}} \underline{z}: \neg \neg \underline{z} = 0 \Rightarrow \neg \underline{z} + \underline{a} = \underline{a} \cdot \underline{b} \Rightarrow \underline{a} = \underline{a} \cdot \underline{b}$]

[Prop 3.7t $\xrightarrow{\text{tex}}$ “
Prop\ 3.7t”]

[Prop 3.7t $\xrightarrow{\text{pyk}}$ “prop three seven t”]

Prop 3.7u

[Prop 3.7u $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{a}$]

[Prop 3.7u $\xrightarrow{\text{tex}}$ “
Prop\ 3.7u”]

[Prop 3.7u $\xrightarrow{\text{pyk}}$ “prop three seven u”]

Prop 3.7u'

[Prop 3.7u' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{a} \Rightarrow \neg \underline{a} = 0$]

[Prop 3.7u' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7u'”]

[Prop 3.7u' $\xrightarrow{\text{pyk}}$ “prop three seven u mark”]

Prop 3.7v

[Prop 3.7v $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{a} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{b} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0 = \underline{a} \cdot \underline{b}$]

[Prop 3.7v $\xrightarrow{\text{tex}}$ “
Prop\ 3.7v”]

[Prop 3.7v $\xrightarrow{\text{pyk}}$ “prop three seven v”]

Prop 3.7w

[Prop 3.7w $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + 0' = \underline{b} \Rightarrow \neg \forall_{\text{obj}Z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \cdot \underline{a}$]

[Prop 3.7w $\xrightarrow{\text{tex}}$ “
Prop\ 3.7w”]

[Prop 3.7w $\xrightarrow{\text{pyk}}$ “prop three seven w”]

Prop 3.7x

[Prop 3.7x $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} \cdot \underline{a} = \underline{c} \cdot \underline{a}$]

[Prop 3.7x $\xrightarrow{\text{tex}}$ “
Prop\ 3.7x”]

[Prop 3.7x $\xrightarrow{\text{pyk}}$ “prop three seven x”]

Prop 3.7x'

[Prop 3.7x' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} \cdot \underline{a} = \underline{c} \cdot \underline{a} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c}$]

[Prop 3.7x' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7x'”]

[Prop 3.7x' $\xrightarrow{\text{pyk}}$ “prop three seven x mark”]

Prop 3.7y

[Prop 3.7y $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c} \Rightarrow \underline{b} = \underline{c} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} \cdot \underline{a} = \underline{c} \cdot \underline{a} \Rightarrow \underline{b} \cdot \underline{a} = \underline{c} \cdot \underline{a}$]

[Prop 3.7y $\xrightarrow{\text{tex}}$ “
Prop\ 3.7y”]

[Prop 3.7y $\xrightarrow{\text{pyk}}$ “prop three seven y”]

Prop 3.7y'

[Prop 3.7y' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} \cdot \underline{a} = \underline{c} \cdot \underline{a} \Rightarrow \underline{b} \cdot \underline{a} = \underline{c} \cdot \underline{a} \Rightarrow \neg \forall_{\text{obj}z}: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{c} \Rightarrow \underline{b} = \underline{c}$]

[Prop 3.7y' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7y'”]

[Prop 3.7y' $\xrightarrow{\text{pyk}}$ “prop three seven y mark”]

Prop 3.7z

[Prop 3.7z $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \neg \neg \forall_{\text{obj}} z: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = 0$]

[Prop 3.7z $\xrightarrow{\text{tex}}$ “
Prop\ 3.7z”]

[Prop 3.7z $\xrightarrow{\text{pyk}}$ “prop three seven z”]

Prop 3.7z'

[Prop 3.7z' $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \neg \neg \forall_{\text{obj}} z: \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{b} \Rightarrow \underline{a} = \underline{b} \Rightarrow \neg \neg \neg \forall_{\text{obj}} z: \neg \neg z = 0 \Rightarrow \neg z + \underline{b} = \underline{a} \Rightarrow \underline{b} = \underline{a} \Rightarrow \underline{a} = \underline{b}$]

[Prop 3.7z' $\xrightarrow{\text{tex}}$ “
Prop\ 3.7z'”]

[Prop 3.7z' $\xrightarrow{\text{pyk}}$ “prop three seven z mark”]

Prop 3.10

[Prop 3.10 $\xrightarrow{\text{tex}}$ “
Prop\ 3.10”]

[Prop 3.10 $\xrightarrow{\text{pyk}}$ “prop three ten”]

Prop 3.10a

[Prop 3.10a $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}} z: \neg \underline{a} = \forall \underline{a}: \underline{a} \cdot z$]

[Prop 3.10a $\xrightarrow{\text{tex}}$ “
Prop\ 3.10a”]

[Prop 3.10a $\xrightarrow{\text{pyk}}$ “prop three ten a”]

Prop 3.10b

[Prop 3.10b $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}} z: \neg \underline{a} = \forall \underline{a}: 0' \cdot z$]

[Prop 3.10b $\xrightarrow{\text{tex}}$ “
Prop\ 3.10b”]

[Prop 3.10b $\xrightarrow{\text{pyk}}$ “prop three ten b”]

Prop 3.10c

[Prop 3.10c $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}z}: \neg 0 = \forall \underline{a}: \underline{a} \cdot z$]

[Prop 3.10c $\xrightarrow{\text{tex}}$ “
Prop\ 3.10c”]

[Prop 3.10c $\xrightarrow{\text{pyk}}$ “prop three ten c”]

Prop 3.10d

[Prop 3.10d $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}z}: \neg \underline{c} = \neg \forall_{\text{obj}z}: \neg \underline{c} \Rightarrow \underline{a} = \neg \forall_{\text{obj}z}: \neg \underline{b} \wedge \underline{b} = \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \cdot z \cdot z \cdot z$]

[Prop 3.10d $\xrightarrow{\text{tex}}$ “
Prop\ 3.10d”]

[Prop 3.10d $\xrightarrow{\text{pyk}}$ “prop three ten d”]

Prop 3.10e

[Prop 3.10e $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}z}: \neg \underline{b} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{a} = \underline{a} \Rightarrow \underline{a} = \underline{a} = \forall \underline{a}: \forall \underline{b}: \neg \neg \underline{a} = 0 \Rightarrow \neg \underline{b} \cdot z$]

[Prop 3.10e $\xrightarrow{\text{tex}}$ “
Prop\ 3.10e”]

[Prop 3.10e $\xrightarrow{\text{pyk}}$ “prop three ten e”]

Prop 3.10f

[Prop 3.10f $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}z}: \neg \underline{a} \Rightarrow \underline{a} = \underline{b} = \neg \forall_{\text{obj}z}: \neg \neg \underline{b} \Rightarrow \neg \underline{b} = \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot z \cdot z$]

[Prop 3.10f $\xrightarrow{\text{tex}}$ “
Prop\ 3.10f”]

[Prop 3.10f $\xrightarrow{\text{pyk}}$ “prop three ten f”]

Prop 3.10g

[Prop 3.10g $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}z}: \neg \underline{b} \cdot \underline{c} = \neg \forall_{\text{obj}z}: \neg \underline{b} \Rightarrow \underline{a} = \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \cdot z \cdot z$]

[Prop 3.10g $\xrightarrow{\text{tex}}$ “
Prop\ 3.10g”]

[Prop 3.10g $\xrightarrow{\text{pyk}}$ “prop three ten g”]

Prop 3.10h

[Prop 3.10h $\xrightarrow{\text{stmt}}$ $S \vdash \neg \forall_{\text{obj}z}: \neg \underline{b} + \underline{c} = \neg \forall_{\text{obj}z}: \neg \underline{c} \Rightarrow \underline{a} = \neg \forall_{\text{obj}z}: \neg \underline{b} \wedge \underline{a} = \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \cdot z \cdot z \cdot z$]

[Prop 3.10h $\xrightarrow{\text{tex}}$ “
Prop\ 3.10h”]

[Prop 3.10h $\xrightarrow{\text{pyk}}$ “prop three ten h”]

Prop 3.11

[Prop 3.11 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{a} = 0 \Rightarrow \neg \forall_{\text{obj}c}: \forall_{\text{obj}d}: \neg \neg \neg \underline{b} = \underline{a} \cdot \underline{c} + \underline{d} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{d} = \underline{a} \Rightarrow \neg \forall_{\text{obj}e}: \forall_{\text{obj}f}: \neg \underline{b} = \underline{a} \cdot \underline{e} + \underline{f} \Rightarrow \neg \neg \forall_{\text{obj}z}: \neg \neg \neg z = 0 \Rightarrow \neg z + \underline{f} = \underline{a} \Rightarrow \neg \underline{c} = \underline{e} \Rightarrow \neg \underline{d} = \underline{f}$]

[Prop 3.11 $\xrightarrow{\text{tex}}$ “
Prop\ 3.11”]

[Prop 3.11 $\xrightarrow{\text{pyk}}$ “prop three eleven”]

* < *

[$x < y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x < y \doteq \exists z: (z \neq 0 \wedge z + x = y)])]]$]

[$x < y \xrightarrow{\text{tex}}$ “#1.
< #2.”]

[* < * $\xrightarrow{\text{pyk}}$ “" ist "”]

* \leq *

[$x \leq y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \leq y \doteq x < y \vee x = y]])$]

[$x \leq y \xrightarrow{\text{tex}}$ “#1.
\leq #2.”]

[* \leq * $\xrightarrow{\text{pyk}}$ “" istq "”]

* $\not<$ *

[$x \not< y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \not< y \doteq \neg(x < y)])]$]

[$x \not< y \xrightarrow{\text{tex}}$ “#1.
 $\backslash\text{not} < \#2.$ ”]

[$* \not< * \xrightarrow{\text{pyk}}$ “ inst ”]

* $>$ *

[$x > y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x > y \doteq y < x]])]$

[$x > y \xrightarrow{\text{tex}}$ “#1.
 $> \#2.$ ”]

[$* > * \xrightarrow{\text{pyk}}$ “ igt ”]

* \geq *

[$x \geq y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \geq y \doteq y \leq x]])]$

[$x \geq y \xrightarrow{\text{tex}}$ “#1.
 $\backslash\text{geq} \#2.$ ”]

[$* \geq * \xrightarrow{\text{pyk}}$ “ igtq ”]

* $\not>$ *

[$x \not> y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \not> y \doteq y \not< x]])]$

[$x \not> y \xrightarrow{\text{tex}}$ “#1.
 $\backslash\text{not} > \#2.$ ”]

[$* \not> * \xrightarrow{\text{pyk}}$ “ ingt ”]

* \neq *

[$x \neq y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \neq y \doteq \neg(x = y)])]$]

[$x \neq y \xrightarrow{\text{tex}}$ “#1.
 $\backslash\text{neq} \#2.$ ”]

[$* \neq * \xrightarrow{\text{pyk}}$ “ neq ”]

* \wedge *

[$x \wedge y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \wedge y \doteq \neg(x \Rightarrow \neg y)])])$]

[$x \wedge y \xrightarrow{\text{tex}}$ “#1.
 \wedge #2.”]

[$* \wedge * \xrightarrow{\text{pyk}}$ “and1 ”]

* \vee *

[$x \vee y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \vee y \doteq (\neg x) \Rightarrow y]])$]

[$x \vee y \xrightarrow{\text{tex}}$ “#1.
 \vee #2.”]

[$* \vee * \xrightarrow{\text{pyk}}$ “or1 ”]

\exists *: *

[$\exists x: y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\exists x: y \doteq \neg(\forall x: \neg y)])])$]

[$\exists x: y \xrightarrow{\text{tex}}$ “
 \exists #1.
: #2.”]

[$\exists *: * \xrightarrow{\text{pyk}}$ “exists " indeed ”]

* | *

[$x | y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x | y \doteq \exists z: y = x \cdot z]])$]

[$x | y \xrightarrow{\text{tex}}$ “#1.
 $\mathrel{\{}}$ #2.”]

[$* | * \xrightarrow{\text{pyk}}$ “divides ”]

* . . .

[$x \dots \xrightarrow{\text{tex}}$ “#1.
 \ldots ”]

[$* \dots \xrightarrow{\text{pyk}}$ “ldots”]

*The pyk compiler, version 0.grue.20060417+ by Klaus Grue
GRD-2006-06-28.UTC:01:17:25.716427 = MJD-53914.TAI:01:17:58.716427 =
LGT-4658174278716427e-6*