Logiweb codex of ijcar

Up Help

$\mathcal{X}$, ijcar, $[* \stackrel{\circ}{=} *]$, RootVisible$(*)$, A, R, C, T, L, $\{*\}$, $\overline{*}$, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, $\langle * \equiv * \mid * := * \rangle$, $\langle * \equiv^0 * \mid * := * \rangle$, $\langle * \equiv^1 * \mid * := * \rangle$, $\langle * \equiv^* * \mid * := * \rangle$, Ded$(*,*)$, Ded$_0(*,*)$, Ded$_1(*,*,*)$, Ded$_2(*,*,*)$, Ded$_3(*,*,*,*)$, Ded$_4(*,*,*,*)$, Ded$_4^*(*,*,*,*)$, Ded$_5(*,*,*)$, Ded$_6(*,*,*,*)$, Ded$_6^*(*,*,*,*)$, Ded$_7(*)$, Ded$_8(*,*)$, Ded$_8^*(*,*)$, S, Neg, MP, Gen, Ded, S1, S2, S3, S4, S5, S6, S7, S8, S9, Repetition, A1′, A2′, A4′, A5′, Prop 3.2a, Prop 3.2b, Prop 3.2c, Prop 3.2d, Prop 3.2e$_1$, Prop 3.2e$_2$, Prop 3.2e, Prop 3.2f$_1$, Prop 3.2f$_2$, Prop 3.2f, Prop 3.2g$_1$, Prop 3.2g$_2$, Prop 3.2g, Prop 3.2h$_1$, Prop 3.2h$_2$, Prop 3.2h, Block$_1(*,*,*)$, Block$_2(*)$, $*^{\text{hide}}$, MacroIndent$(*)$, $*'$, $* = *$, $* \neq *$, $*^{\text{var}}$, $*\#^0*$, $*\#^1*$, $*\#^**$, $\exists *: *$, $\forall *: *$, $\forall_{\text{obj}} *: *$, $* \Rightarrow *$, $* \Leftrightarrow *$, $*\#*$, $* \trianglerighteq *$, $\Pi *: *$, Begin $*$; $*$: End; $*$, Last block line $* \gg *$;, Arbitrary $\gg *; *$, $* \mid *$, $\rightarrow$, $*\backslash\backslash *$,

$\mathcal{X}$

$[\mathcal{X} \stackrel{\text{stmt}}{\rightarrow} \text{x}]$

ijcar

[ijcar $\stackrel{\text{prio}}{\rightarrow}$
Preassociative
[ijcar], [ijcar base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [\$ $*$ \$], [**flush left** $(*)$], [x], [y], [z], $[[* \bowtie *]]$, $[[* \stackrel{*}{\rightarrow} *]]$, [pyk], [tex], [name], [prio], $[*]$, [T], [if$(*,*,*)$], $[[* \stackrel{*}{\Rightarrow} *]]$, [val], [claim], $[\bot]$, [f$(*)$], $[(*)^{\text{I}}]$, [F], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\text{M}}]$, [If$(*,*,*)$], [array$\{*\}$ $*$ end array], [l], [c], [r], [empty], $[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$, $[\mathcal{U}^{\text{M}}(*)]$, [**apply**$(*,*)$], [**apply**$_1(*,*)$], [identifier$(*)$], [identifier$_1(*,*)$], [array-plus$(*,*)$], [array-remove$(*,*,*)$], [array-put$(*,*,*,*)$], [array-add$(*,*,*,*,*)$], [bit$(*,*)$], [bit$_1(*,*)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], $[\mathcal{E}(*,*,*)]$, $[\mathcal{E}_2(*,*,*,*,*)]$, $[\mathcal{E}_3(*,*,*,*)]$, $[\mathcal{E}_4(*,*,*,*)]$, [**lookup**$(*,*,*)$], [**abstract**$(*,*,*,*)$], $[\lceil * \rceil]$, $[\mathcal{M}(*,*,*)]$, $[\mathcal{M}_2(*,*,*,*)]$, $[\mathcal{M}^*(*,*,*)]$, [macro],

$[\mathrm{s}_0]$, $[\mathbf{zip}(*,*)]$, $[\mathbf{assoc}_1(*,*,*)]$, $[(*)^{\mathbf{p}}]$, [self], $[[* \doteq *]]$, $[[* \doteq *]]$, $[[* \doteq *]]$,
$[[* \overset{\text{pyk}}{=} *]]$, $[[* \overset{\text{tex}}{=} *]]$, $[[* \overset{\text{name}}{=} *]]$, [**Priority table**$[*]$], $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$,
$[\tilde{\mathcal{M}}_4(*,*,*,*)]$, $[\tilde{\mathcal{M}}(*,*,*)]$, $[\tilde{\mathcal{Q}}(*,*,*)]$, $[\tilde{\mathcal{Q}}_2(*,*,*)]$, $[\tilde{\mathcal{Q}}_3(*,*,*,*)]$, $[\tilde{\mathcal{Q}}^*(*,*,*)]$,
$[(*)]$, $[(*)]$, [display$(*)$], [statement$(*)$], $[[*]^{\cdot}]$, $[[*]^{-}]$, [**aspect**$(*,*)$],
[**aspect**$(*,*,*)$], $[\langle * \rangle]$, [**tuple**$_1(*)$], [**tuple**$_2(*)$], [let$_2(*,*)$], [let$_1(*,*)$],
$[[* \overset{\text{claim}}{=} *]]$, [checker], [**check**$(*,*)$], [**check**$_2(*,*,*)$], [**check**$_3(*,*,*)$],
[**check**$^*(*,*)$], [**check**$^*_2(*,*,*)$], $[[*]^{\cdot}]$, $[[*]^{-}]$, $[[*]^{\circ}]$, [msg], $[[* \overset{\text{msg}}{=} *]]$, [<stmt>],
[stmt], $[[* \overset{\text{stmt}}{=} *]]$, [HeadNil$'$], [HeadPair$'$], [Transitivity$'$], $[\perp\!\!\!\perp]$, [Contra$'$], $[\mathrm{T}'_{\mathrm{E}}]$,
$[\mathrm{L}_1]$, $[\underline{*}]$, $[\mathcal{A}]$, $[\mathcal{B}]$, $[\mathcal{C}]$, $[\mathcal{D}]$, $[\mathcal{E}]$, $[\mathcal{F}]$, $[\mathcal{G}]$, $[\mathcal{H}]$, $[\mathcal{I}]$, $[\mathcal{J}]$, $[\mathcal{K}]$, $[\mathcal{L}]$, $[\mathcal{M}]$, $[\mathcal{N}]$, $[\mathcal{O}]$, $[\mathcal{P}]$, $[\mathcal{Q}]$,
$[\mathcal{R}]$, $[\mathcal{S}]$, $[\mathcal{T}]$, $[\mathcal{U}]$, $[\mathcal{V}]$, $[\mathcal{W}]$, $[\mathcal{X}]$, $[\mathcal{Y}]$, $[\mathcal{Z}]$, $[\langle * \mid * := * \rangle]$, $[\langle {}^{*} * \mid * := * \rangle]$, $[\emptyset]$, [Remainder],
$[(*)^{\mathbf{v}}]$, [intro$(*,*,*,*)$], [intro$(*,*,*)$], [error$(*,*)$], [error$_2(*,*)$], [proof$(*,*,*)$],
[proof$_2(*,*)$], $[\mathcal{S}(*,*)]$, $[\mathcal{S}^{\mathrm{I}}(*,*)]$, $[\mathcal{S}^{\triangleright}(*,*)]$, $[\mathcal{S}^{\triangleright}_1(*,*,*)]$, $[\mathcal{S}^{\mathrm{E}}(*,*)]$, $[\mathcal{S}^{\mathrm{E}}_1(*,*,*)]$,
$[\mathcal{S}^{+}(*,*)]$, $[\mathcal{S}^{+}_1(*,*,*)]$, $[\mathcal{S}^{-}(*,*)]$, $[\mathcal{S}^{-}_1(*,*,*)]$, $[\mathcal{S}^{*}(*,*)]$, $[\mathcal{S}^{*}_1(*,*,*)]$,
$[\mathcal{S}^{*}_2(*,*,*,*)]$, $[\mathcal{S}^{@}(*,*)]$, $[\mathcal{S}^{@}_1(*,*,*)]$, $[\mathcal{S}^{\vdash}(*,*)]$, $[\mathcal{S}^{\vdash}_1(*,*,*,*)]$, $[\mathcal{S}^{\Vdash}(*,*)]$,
$[\mathcal{S}^{\Vdash}_1(*,*,*,*)]$, $[\mathcal{S}^{\text{i.e.}}(*,*)]$, $[\mathcal{S}^{\text{i.e.}}_1(*,*,*,*)]$, $[\mathcal{S}^{\text{i.e.}}_2(*,*,*,*,*)]$, $[\mathcal{S}^{\forall}(*,*)]$,
$[\mathcal{S}^{\forall}_1(*,*,*,*)]$, $[\mathcal{S}^{:}(*,*)]$, $[\mathcal{S}^{:}_1(*,*,*)]$, $[\mathcal{S}^{:}_2(*,*,*,*)]$, $[\mathcal{T}(*)]$, [claims$(*,*,*)$],
[claims$_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]],
[[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]],
[verifier], $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*,*)]$, $[\mathcal{V}_3(*,*,*,*)]$, $[\mathcal{V}_4(*,*)]$, $[\mathcal{V}_5(*,*,*,*)]$, $[\mathcal{V}_6(*,*,*,*)]$,
$[\mathcal{V}_7(*,*,*,*)]$, [Cut$(*,*)$], [Head$_{\oplus}(*)$], [Tail$_{\oplus}(*)$], [rule$_1(*,*)$], [rule$(*,*)$],
[Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$],
[theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil],
[HeadPair], [Transitivity], [Contra], $[\mathrm{T}_{\mathrm{E}}]$, [ragged right],
[ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$],
[inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(* = *,*)$], [unify$^*(* = *,*)$],
[unify$_2(* = *,*)$], $[\mathrm{L_a}]$, $[\mathrm{L_b}]$, $[\mathrm{L_c}]$, $[\mathrm{L_d}]$, $[\mathrm{L_e}]$, $[\mathrm{L_f}]$, $[\mathrm{L_g}]$, $[\mathrm{L_h}]$, $[\mathrm{L_i}]$, $[\mathrm{L_j}]$, $[\mathrm{L_k}]$, $[\mathrm{L_l}]$, $[\mathrm{L_m}]$,
$[\mathrm{L_n}]$, $[\mathrm{L_o}]$, $[\mathrm{L_p}]$, $[\mathrm{L_q}]$, $[\mathrm{L_r}]$, $[\mathrm{L_s}]$, $[\mathrm{L_t}]$, $[\mathrm{L_u}]$, $[\mathrm{L_v}]$, $[\mathrm{L_w}]$, $[\mathrm{L_x}]$, $[\mathrm{L_y}]$, $[\mathrm{L_z}]$, $[\mathrm{L_A}]$, $[\mathrm{L_B}]$, $[\mathrm{L_C}]$,
$[\mathrm{L_D}]$, $[\mathrm{L_E}]$, $[\mathrm{L_F}]$, $[\mathrm{L_G}]$, $[\mathrm{L_H}]$, $[\mathrm{L_I}]$, $[\mathrm{L_J}]$, $[\mathrm{L_K}]$, $[\mathrm{L_L}]$, $[\mathrm{L_M}]$, $[\mathrm{L_N}]$, $[\mathrm{L_O}]$, $[\mathrm{L_P}]$, $[\mathrm{L_Q}]$, $[\mathrm{L_R}]$,
$[\mathrm{L_S}]$, $[\mathrm{L_T}]$, $[\mathrm{L_U}]$, $[\mathrm{L_V}]$, $[\mathrm{L_W}]$, $[\mathrm{L_X}]$, $[\mathrm{L_Y}]$, $[\mathrm{L_Z}]$, $[\mathrm{L_?}]$, [Reflexivity], [Reflexivity$_1$],
[Commutativity], [Commutativity$_1$], [<tactic>], [tactic], $[[* \overset{\text{tactic}}{=} *]]$, $[\mathcal{P}(*,*,*)]$,
$[\mathcal{P}^*(*,*,*)]$, $[\mathrm{p}_0]$, [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$],
[conclude$_4(*,*)$], $[[* \overset{\circ}{=} *]]$, [RootVisible$(*)$], [A], [R], [C], [T], [L], $[\{*\}]$, $[\overline{*}]$, $[a]$, $[b]$,
$[c]$, $[d]$, $[e]$, $[f]$, $[g]$, $[h]$, $[i]$, $[j]$, $[k]$, $[l]$, $[m]$, $[n]$, $[o]$, $[p]$, $[q]$, $[r]$, $[s]$, $[t]$, $[u]$, $[v]$, $[w]$, $[x]$,
$[y]$, $[z]$, $[\langle * \equiv * \mid * := * \rangle]$, $[\langle * \equiv^0 * \mid * := * \rangle]$, $[\langle * \equiv^1 * \mid * := * \rangle]$, $[\langle * \equiv^* * \mid * := * \rangle]$,
[Ded$(*,*)$], [Ded$_0(*,*)$], [Ded$_1(*,*,*)$], [Ded$_2(*,*,*)$], [Ded$_3(*,*,*,*)$],
[Ded$_4(*,*,*,*)$], [Ded$^*_4(*,*,*,*)$], [Ded$_5(*,*,*)$], [Ded$_6(*,*,*,*)$],
[Ded$^*_6(*,*,*,*)$], [Ded$_7(*)$], [Ded$_8(*,*)$], [Ded$^*_8(*,*)$], [S], [Neg], [MP], [Gen],
[Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1$'$], [A2$'$], [A4$'$],
[A5$'$], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop 3.2e$_1$], [Prop 3.2e$_2$],
[Prop 3.2e], [Prop 3.2f$_1$], [Prop 3.2f$_2$], [Prop 3.2f], [Prop 3.2g$_1$], [Prop 3.2g$_2$],
[Prop 3.2g], [Prop 3.2h$_1$], [Prop 3.2h$_2$], [Prop 3.2h], [Block$_1(*,*,*)$], [Block$_2(*)$];
Preassociative
$[*\text{-}\{*\}]$, [*/indexintro$(*,*,*,*)$], [*/intro$(*,*,*)$], [*/bothintro$(*,*,*,*,*)$],
[*/nameintro$(*,*,*,*)$], $[*']$, $[*[\,*\,]]$, $[*[*\rightarrow *]]$, $[*[*\Rightarrow *]]$, [*0], [*1], [0b], [*-color$(*)$],

$[*\text{-color}^*(*)]$, $[*^{\mathrm{H}}]$, $[*^{\mathrm{T}}]$, $[*^{\mathrm{U}}]$, $[*^{\mathrm{h}}]$, $[*^{\mathrm{t}}]$, $[*^{\mathrm{s}}]$, $[*^{\mathrm{c}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{a}}]$, $[*^{\mathrm{C}}]$, $[*^{\mathrm{M}}]$, $[*^{\mathrm{B}}]$, $[*^{\mathrm{r}}]$, $[*^{\mathrm{i}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{R}}]$, $[*^0]$, $[*^1]$, $[*^2]$, $[*^3]$, $[*^4]$, $[*^5]$, $[*^6]$, $[*^7]$, $[*^8]$, $[*^9]$, $[*^{\mathrm{E}}]$, $[*^{\mathcal{V}}]$, $[*^{\mathcal{C}}]$, $[*^{\mathcal{C}^*}]$, $[*^{\mathrm{hide}}]$;

Preassociative

$[\text{“}*\text{”}]$, $[]$, $[(*)^{\mathbf{t}}]$, $[\mathrm{string}(*)+*]$, $[\mathrm{string}(*)\mathbin{+\!\!+}*]$, $[*]$, $[*]$, $[!*]$, $[\text{”}*]$, $[\#*]$, $[\$*]$, $[\%*]$, $[\&*]$, $[\text{’}*]$, $[(*]$, $[)*]$, $[**]$, $[+*]$, $[,*]$, $[-*]$, $[.*]$, $[/*]$, $[0*]$, $[1*]$, $[2*]$, $[3*]$, $[4*]$, $[5*]$, $[6*]$, $[7*]$, $[8*]$, $[9*]$, $[:*]$, $[;*]$, $[<*]$, $[=*]$, $[>*]$, $[?*]$, $[@*]$, $[A*]$, $[B*]$, $[C*]$, $[D*]$, $[E*]$, $[F*]$, $[G*]$, $[H*]$, $[I*]$, $[J*]$, $[K*]$, $[L*]$, $[M*]$, $[N*]$, $[O*]$, $[P*]$, $[Q*]$, $[R*]$, $[S*]$, $[T*]$, $[U*]$, $[V*]$, $[W*]$, $[X*]$, $[Y*]$, $[Z*]$, $[[*]$, $[\backslash*]$, $[]*]$, $[\hat{}*]$, $[\_*]$, $[\text{‘}*]$, $[a*]$, $[b*]$, $[c*]$, $[d*]$, $[e*]$, $[f*]$, $[g*]$, $[h*]$, $[i*]$, $[j*]$, $[k*]$, $[l*]$, $[m*]$, $[n*]$, $[o*]$, $[p*]$, $[q*]$, $[r*]$, $[s*]$, $[t*]$, $[u*]$, $[v*]$, $[w*]$, $[x*]$, $[y*]$, $[z*]$, $[\{*]$, $[|*]$, $[\}*]$, $[\tilde{}*]$, [**Preassociative** $*;*$], [**Postassociative** $*;*$], $[[*],*]$, [priority $*$ end], [newline $*$], [macro newline $*$], [MacroIndent$(*)$];

Preassociative

$[*\text{ ’ }*]$, $[*\text{ ‘ }*]$;

Preassociative

$[*']$;

Preassociative

$[*\cdot*]$, $[*\cdot_0*]$;

Preassociative

$[*+*]$, $[*+_0*]$, $[*+_1*]$, $[*-*]$, $[*-_0*]$, $[*-_1*]$;

Preassociative

$[*\cup\{*\}]$, $[*\cup*]$, $[*\backslash\{*\}]$;

Postassociative

$[*\mathbin{\therefore}*]$, $[*\mathbin{\underline{\therefore}}*]$, $[*\mathbin{::}*]$, $[*\mathbin{\underline{::}}*]$, $[*\mathbin{\underline{+2*}}*]$, $[*::*]$, $[*+2**]$;

Postassociative

$[*,*]$;

Preassociative

$[*\overset{\mathrm{B}}{\approx}*]$, $[*\overset{\mathrm{D}}{\approx}*]$, $[*\overset{\mathrm{C}}{\approx}*]$, $[*\overset{\mathrm{P}}{\approx}*]$, $[*\approx*]$, $[*=*]$, $[*\overset{+}{\to}*]$, $[*\overset{\mathrm{t}}{=}*]$, $[*\overset{\mathrm{t}^*}{=}*]$, $[*\overset{\mathrm{r}}{=}*]$, $[*\in_{\mathrm{t}}*]$, $[*\subseteq_{\mathrm{T}}*]$, $[*\overset{\mathrm{T}}{=}*]$, $[*\overset{\mathrm{s}}{=}*]$, $[*\text{ free in }*]$, $[*\text{ free in}^**]$, $[*\text{ free for }*\text{ in }*]$, $[*\text{ free for}^**\text{ in }*]$, $[*\in_{\mathrm{c}}*]$, $[*<*]$, $[*<'*]$, $[*\leq'*]$, $[*=*]$, $[*\neq*]$, $[*^{\mathrm{var}}]$, $[*\#^0*]$, $[*\#^1*]$, $[*\#^**]$;

Preassociative

$[\neg*]$;

Preassociative

$[*\wedge*]$, $[*\mathbin{\ddot{\wedge}}*]$, $[*\mathbin{\tilde{\wedge}}*]$, $[*\wedge_{\mathrm{c}}*]$;

Preassociative

$[*\vee*]$, $[*\parallel*]$, $[*\mathbin{\ddot{\vee}}*]$;

Preassociative

$[\exists*\colon*]$, $[\forall*\colon*]$, $[\forall_{\mathrm{obj}}*\colon*]$;

Postassociative

$[*\mathbin{\ddot{\Rightarrow}}*]$, $[*\Rightarrow*]$, $[*\Leftrightarrow*]$;

Postassociative

$[*:*]$, [$*$ spy $*$], $[*!*]$;

Preassociative

$[* \left\{ \begin{array}{l} * \\ * \end{array} \right.]$;
Preassociative
$[\lambda * .*]$, $[\Lambda * .*]$, $[\Lambda *]$, [**if** $*$ **then** $*$ **else** $*$], [**let** $* = *$ **in** $*$], [**let** $* \ddot{=} *$ in $*$];
Preassociative
$[* \# *]$;
Preassociative
$[*^{\mathrm{I}}]$, $[*^{\triangleright}]$, $[*^{\mathrm{V}}]$, $[*^{+}]$, $[*^{-}]$, $[*^{*}]$;
Preassociative
$[* @ *]$, $[* \rhd *]$, $[* \rhd\!\!\!\rhd *]$, $[* \gg *]$, $[* \unrhd *]$;
Postassociative
$[* \vdash *]$, $[* \Vdash *]$, [$*$ i.e. $*$];
Preassociative
$[\forall * \colon *]$, $[\Pi * \colon *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
[$*$ proves $*$];
Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \square$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$], [Begin $*; *$: End; $*$], [Last block line $* \gg *;$],
[Arbitrary $\gg *; *$];
Postassociative
$[* \mid *]$;
Postassociative
[$*$ then $*$], $[*[\,*\,]*]$;
Preassociative
$[* \& *]$, $[\rightarrow]$;
Preassociative
$[* \backslash\backslash *]$, $[* \backslash\backslash *]$;]

[ijcar $\stackrel{\text{pyk}}{\rightarrow}$ "ijcar"]

$[* \stackrel{\circ}{=} *]$

$[[\mathsf{x} \stackrel{\circ}{=} \mathsf{y}] \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[[\mathsf{x} \stackrel{\circ}{=} \mathsf{y}] \ddot{=} [(\mathsf{x})^{\mathbf{p}} \stackrel{\text{macro}}{\rightarrow} \mathsf{y}]]\rceil)]$

$[[\mathsf{x} \stackrel{\circ}{=} \mathsf{y}] \stackrel{\text{tex}}{\rightarrow}$ "
[#1/tex name/tex.
\stackrel {\circ}{=}#2.
]"]

[[$\mathsf{x} \overset{\circ}{=} \mathsf{y}$] $\overset{\text{pyk}}{\to}$ “general macro define $*$ as $*$ end define”]

RootVisible($*$)

[RootVisible(x) $\overset{\text{name}}{\to}$ “
RootVisible(#1.
)”]

[RootVisible(x) $\overset{\text{tex}}{\to}$ “#1/tex name/tex.”]

[RootVisible(x) $\overset{\text{pyk}}{\to}$ “make root visible $*$ end visible”]

A

[A $\overset{\text{tex}}{\to}$ “
A”]

[A $\overset{\text{pyk}}{\to}$ “ijcar example axiom”]

R

[R $\overset{\text{tex}}{\to}$ “
R”]

[R $\overset{\text{pyk}}{\to}$ “ijcar example rule”]

C

[C $\overset{\text{tex}}{\to}$ “
C”]

[C $\overset{\text{pyk}}{\to}$ “ijcar example contradiction”]

T

[T $\overset{\text{tex}}{\to}$ “
T”]

[T $\overset{\text{pyk}}{\to}$ “ijcar example theory”]

L

$[\mathsf{L} \overset{\text{tex}}{\rightarrow} \text{“}$
$\mathsf{L}\text{”}]$

$[\mathsf{L} \overset{\text{pyk}}{\rightarrow} \text{“ijcar example lemma”}]$

$\{*\}$

$[\{\mathsf{x}\} \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\{\#1.$
$\backslash\}\text{”}]$

$[\{\mathsf{x}\} \overset{\text{pyk}}{\rightarrow} \text{“set } * \text{ end set”}]$

$\overline{*}$

$[\overline{\mathsf{x}} \overset{\text{tex}}{\rightarrow} \text{“}\backslash\text{overline}\{\#1.\}\text{”}]$

$[\overline{\mathsf{x}} \overset{\text{pyk}}{\rightarrow} \text{“object var } * \text{ end var”}]$

a

$[a \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[a \doteq \overline{\mathsf{a}}]\rceil)]$

$[a \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{mathit}\{\text{a}\}\text{”}]$

$[a \overset{\text{pyk}}{\rightarrow} \text{“object a”}]$

b

$[b \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[b \doteq \overline{\mathsf{b}}]\rceil)]$

$[b \overset{\text{tex}}{\rightarrow} \text{“}$
$\backslash\text{mathit}\{\text{b}\}\text{”}]$

$[b \overset{\text{pyk}}{\rightarrow} \text{“object b”}]$

c

$[c \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[c \doteq \overline{\mathsf{c}}]\rceil)]$

$[c \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{mathit\{c\}”}]$

$[c \overset{\text{pyk}}{\to} \text{“object c”}]$

d

$[d \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[d \doteqdot \overline{\mathsf{d}}]\urcorner)]$

$[d \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{mathit\{d\}”}]$

$[d \overset{\text{pyk}}{\to} \text{“object d”}]$

e

$[e \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[e \doteqdot \overline{\mathsf{e}}]\urcorner)]$

$[e \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{mathit\{e\}”}]$

$[e \overset{\text{pyk}}{\to} \text{“object e”}]$

f

$[f \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[f \doteqdot \overline{\mathsf{f}}]\urcorner)]$

$[f \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{mathit\{f\}”}]$

$[f \overset{\text{pyk}}{\to} \text{“object f”}]$

g

$[g \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[g \doteqdot \overline{\mathsf{g}}]\urcorner)]$

$[g \overset{\text{tex}}{\to} \text{“}$
$\backslash\text{mathit\{g\}”}]$

$[g \overset{\text{pyk}}{\to} \text{“object g”}]$

h

$[h \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[h \doteq \overline{\mathsf{h}}]\urcorner)]$

$[h \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{mathit}\{\text{h}\}\text{”}]$

$[h \overset{\text{pyk}}{\longrightarrow} \text{“object h”}]$

i

$[i \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[i \doteq \overline{\mathsf{i}}]\urcorner)]$

$[i \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{mathit}\{\text{i}\}\text{”}]$

$[i \overset{\text{pyk}}{\longrightarrow} \text{“object i”}]$

j

$[j \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[j \doteq \overline{\mathsf{j}}]\urcorner)]$

$[j \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{mathit}\{\text{j}\}\text{”}]$

$[j \overset{\text{pyk}}{\longrightarrow} \text{“object j”}]$

k

$[k \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[k \doteq \overline{\mathsf{k}}]\urcorner)]$

$[k \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{mathit}\{\text{k}\}\text{”}]$

$[k \overset{\text{pyk}}{\longrightarrow} \text{“object k”}]$

l

$[l \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[l \doteq \overline{\mathsf{l}}]\urcorner)]$

$[l \overset{\text{tex}}{\longrightarrow} \text{“}$
$\backslash\text{mathit}\{\text{l}\}\text{”}]$

$[l \overset{\text{pyk}}{\longrightarrow} \text{“object l”}]$

m

$[m \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[m \doteq \overline{\mathsf{m}}]\urcorner)]$

$[m \overset{\text{tex}}{\rightarrow}$ “
\mathit{m}”]

$[m \overset{\text{pyk}}{\rightarrow}$ “object m”]

n

$[n \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[n \doteq \overline{\mathsf{n}}]\urcorner)]$

$[n \overset{\text{tex}}{\rightarrow}$ “
\mathit{n}”]

$[n \overset{\text{pyk}}{\rightarrow}$ “object n”]

o

$[o \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[o \doteq \overline{\mathsf{o}}]\urcorner)]$

$[o \overset{\text{tex}}{\rightarrow}$ “
\mathit{o}”]

$[o \overset{\text{pyk}}{\rightarrow}$ “object o”]

p

$[p \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[p \doteq \overline{\mathsf{p}}]\urcorner)]$

$[p \overset{\text{tex}}{\rightarrow}$ “
\mathit{p}”]

$[p \overset{\text{pyk}}{\rightarrow}$ “object p”]

q

$[q \overset{\text{macro}}{\rightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[q \doteq \overline{\mathsf{q}}]\urcorner)]$

$[q \overset{\text{tex}}{\rightarrow}$ “
\mathit{q}”]

$[q \overset{\text{pyk}}{\rightarrow}$ “object q”]

r

$[r \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[r \doteqdot \bar{\mathsf{r}}]\urcorner)]$

$[r \overset{\text{tex}}{\longrightarrow}$ “
\mathit{r}”]

$[r \overset{\text{pyk}}{\longrightarrow}$ “object r”]

s

$[s \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[s \doteqdot \bar{\mathsf{s}}]\urcorner)]$

$[s \overset{\text{tex}}{\longrightarrow}$ “
\mathit{s}”]

$[s \overset{\text{pyk}}{\longrightarrow}$ “object s”]

t

$[t \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[t \doteqdot \bar{\mathsf{t}}]\urcorner)]$

$[t \overset{\text{tex}}{\longrightarrow}$ “
\mathit{t}”]

$[t \overset{\text{pyk}}{\longrightarrow}$ “object t”]

u

$[u \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[u \doteqdot \bar{\mathsf{u}}]\urcorner)]$

$[u \overset{\text{tex}}{\longrightarrow}$ “
\mathit{u}”]

$[u \overset{\text{pyk}}{\longrightarrow}$ “object u”]

v

$[v \overset{\text{macro}}{\longrightarrow} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[v \doteqdot \bar{\mathsf{v}}]\urcorner)]$

$[v \overset{\text{tex}}{\longrightarrow}$ “
\mathit{v}”]

$[v \overset{\text{pyk}}{\longrightarrow}$ “object v”]

w

$[w \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[w \doteq \overline{\mathsf{w}}]\urcorner)]$

$[w \overset{\text{tex}}{\to}$ “
\mathit{w}”]

$[w \overset{\text{pyk}}{\to}$ “object w”]

x

$[x \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[x \doteq \overline{\mathsf{x}}]\urcorner)]$

$[x \overset{\text{tex}}{\to}$ “
\mathit{x}”]

$[x \overset{\text{pyk}}{\to}$ “object x”]

y

$[y \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[y \doteq \overline{\mathsf{y}}]\urcorner)]$

$[y \overset{\text{tex}}{\to}$ “
\mathit{y}”]

$[y \overset{\text{pyk}}{\to}$ “object y”]

z

$[z \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[z \doteq \overline{\mathsf{z}}]\urcorner)]$

$[z \overset{\text{tex}}{\to}$ “
\mathit{z}”]

$[z \overset{\text{pyk}}{\to}$ “object z”]

$\langle * \equiv * \mid * :=* \rangle$

$[\langle \mathsf{a} \equiv \mathsf{b} | \mathsf{x} :=\mathsf{t} \rangle \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\ulcorner[\langle \mathsf{a} \equiv \mathsf{b} | \mathsf{x} :=\mathsf{t} \rangle \doteq \langle \ulcorner\mathsf{a}\urcorner \equiv^0 \ulcorner\mathsf{b}\urcorner | \ulcorner\mathsf{x}\urcorner :=\ulcorner\mathsf{t}\urcorner \rangle]\urcorner)]$

$[\langle \mathsf{x} \equiv \mathsf{y} | \mathsf{z} :=\mathsf{u} \rangle \overset{\text{tex}}{\to}$ “
\langle #1.
{\equiv} #2.
| #3.

{:=} #4.
\rangle "]

$[\langle \mathsf{x} \equiv \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{pyk}}{\to}$ "sub $*$ is $*$ where $*$ is $*$ end sub"$]$

$\langle * \equiv^{0} * \mid * := * \rangle$

$[\langle \mathsf{a} \equiv^{0} \mathsf{b} | \mathsf{x} := \mathsf{t} \rangle \overset{\text{val}}{\to} \lambda \mathsf{c}.\ [\ \mathsf{x}^{\text{var}} \wedge \langle \mathsf{a} \equiv^{1} \mathsf{b} | \mathsf{x} := \mathsf{t} \rangle\ ]\ ]$

$[\langle \mathsf{x} \equiv^{0} \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{tex}}{\to}$ "
\langle #1.
{\equiv}^0 #2.
| #3.
{:=} #4.
\rangle "]

$[\langle \mathsf{x} \equiv^{0} \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{pyk}}{\to}$ "sub zero $*$ is $*$ where $*$ is $*$ end sub"$]$

$\langle * \equiv^{1} * \mid * := * \rangle$

$[\langle \mathsf{a} \equiv^{1} \mathsf{b} | \mathsf{x} := \mathsf{t} \rangle \overset{\text{val}}{\to} \mathsf{a}!\ [\ \mathsf{x}!\ [\ \mathsf{t}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \overset{\text{r}}{=} \lceil \forall_{\text{obj}} \mathsf{u} \colon \mathsf{v} \rceil, \mathsf{b}^{1} \overset{\text{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \overset{\text{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\text{var}} \wedge\ [\ \mathsf{b} \overset{\text{t}}{=} \mathsf{x}\ ]\ , \mathsf{a} \overset{\text{t}}{=} \mathsf{t}, \mathrm{If}(\ [$
$\mathsf{a}\ ]\ \overset{\text{r}}{=} \mathsf{b}, \langle \mathsf{a}^{\mathsf{t}} \equiv^{*} \mathsf{b}^{\mathsf{t}} | \mathsf{x} := \mathsf{t} \rangle, \mathsf{F}))\ ]\ ]\ ]$

$[\langle \mathsf{x} \equiv^{1} \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{tex}}{\to}$ "
\langle #1.
{\equiv}^1 #2.
| #3.
{:=} #4.
\rangle "]

$[\langle \mathsf{x} \equiv^{1} \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{pyk}}{\to}$ "sub one $*$ is $*$ where $*$ is $*$ end sub"$]$

$\langle * \equiv^{*} * \mid * := * \rangle$

$[\langle \mathsf{a} \equiv^{*} \mathsf{b} | \mathsf{x} := \mathsf{t} \rangle \overset{\text{val}}{\to} \mathsf{b}!\ [\ \mathsf{x}!\ [\ \mathsf{t}! \mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\langle \mathsf{a}^{\mathsf{h}} \equiv^{1} \mathsf{b}^{\mathsf{h}} | \mathsf{x} := \mathsf{t} \rangle, \langle \mathsf{a}^{\mathsf{t}} \equiv^{*} \mathsf{b}^{\mathsf{t}} | \mathsf{x} := \mathsf{t} \rangle, \mathsf{F}))\ ]\ ]\ ]$

$[\langle \mathsf{x} \equiv^{*} \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{tex}}{\to}$ "
\langle #1.
{\equiv}^* #2.
| #3.

{:=} #4.
\rangle "]

$[\langle \mathsf{x} \equiv^{*} \mathsf{y} | \mathsf{z} := \mathsf{u} \rangle \overset{\text{pyk}}{\to} \text{“sub star } * \text{ is } * \text{ where } * \text{ is } * \text{ end sub”}]$

$\mathrm{Ded}(*, *)$

$[\mathrm{Ded}(\mathsf{p}, \mathsf{c}) \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{Ded}(\mathsf{p}, \mathsf{c}) \doteq \lambda \mathsf{x}.\mathrm{Ded}_0(\lceil \mathsf{p} \rceil, \lceil \mathsf{c} \rceil)] \rceil)]$

[Ded(x, y) $\overset{\text{tex}}{\to}$ “
Ded(#1.
, #2.
)”]

$[\mathrm{Ded}(\mathsf{x}, \mathsf{y}) \overset{\text{pyk}}{\to} \text{“deduction } * \text{ conclude } * \text{ end deduction”}]$

$\mathrm{Ded}_0(*, *)$

$[\mathrm{Ded}_0(\mathsf{p}, \mathsf{c}) \overset{\text{val}}{\to} \mathsf{c}!\mathrm{If}(\mathrm{Ded}_8(\mathsf{p}, \mathsf{T}), \mathrm{Ded}_1(\mathrm{Ded}_7(\mathsf{p}), \mathsf{c}, \mathsf{T}), \mathsf{F})]$

[$\mathrm{Ded}_0(\mathsf{x}, \mathsf{y}) \overset{\text{tex}}{\to}$ “
Ded_0(#1.
, #2.
)”]

$[\mathrm{Ded}_0(\mathsf{x}, \mathsf{y}) \overset{\text{pyk}}{\to} \text{“deduction zero } * \text{ conclude } * \text{ end deduction”}]$

$\mathrm{Ded}_1(*, *, *)$

$[\mathrm{Ded}_1(\mathsf{p}, \mathsf{c}, \mathsf{s}) \overset{\text{val}}{\to} \mathrm{If}(\mathsf{c} \overset{\text{r}}{=} \lceil \mathsf{x} \Vdash \mathsf{y} \rceil, \mathrm{Ded}_1(\mathsf{p}, \mathsf{c}^2, \mathsf{c}^1 :: \mathsf{s}), \mathrm{Ded}_2(\mathsf{p}, \mathsf{c}, \mathsf{s}))]$

[$\mathrm{Ded}_1(\mathsf{x}, \mathsf{y}, \mathsf{z}) \overset{\text{tex}}{\to}$ “
Ded_1(#1.
, #2.
, #3.
)”]

$[\mathrm{Ded}_1(\mathsf{x}, \mathsf{y}, \mathsf{z}) \overset{\text{pyk}}{\to} \text{“deduction one } * \text{ conclude } * \text{ condition } * \text{ end deduction”}]$

$\mathrm{Ded}_2(*,*,*)$

$[\mathrm{Ded}_2(\mathsf{p},\mathsf{c},\mathsf{s}) \overset{\mathrm{val}}{\rightarrow} [\ \mathsf{s}!\ [\ [\ \mathsf{p} \overset{\mathrm{r}}{=} \lceil \mathsf{x} \vdash \mathsf{y} \rceil\ ]\ \wedge\ [\ \mathsf{c} \overset{\mathrm{r}}{=} \lceil \mathsf{x} \Rightarrow \mathsf{y} \rceil\ ]\ ]\ ]$
$\begin{cases} \mathrm{Ded}_3(\mathsf{p}^1,\mathsf{c}^1,\mathsf{s},\mathsf{T}) \wedge \mathrm{Ded}_2(\mathsf{p}^2,\mathsf{c}^2,\mathsf{s}) \\ \mathrm{Ded}_4(\mathsf{p},\mathsf{c},\mathsf{s},\mathrm{Ded}_6(\mathsf{p},\mathsf{c},\mathsf{T},\mathsf{T})) \end{cases}]$

$[\mathrm{Ded}_2(\mathsf{x},\mathsf{y},\mathsf{z}) \overset{\mathrm{tex}}{\rightarrow}$ “
Ded_2(#1.
, #2.
, #3.
)”]

$[\mathrm{Ded}_2(\mathsf{x},\mathsf{y},\mathsf{z}) \overset{\mathrm{pyk}}{\rightarrow}$ “deduction two $*$ conclude $*$ condition $*$ end deduction”]

$\mathrm{Ded}_3(*,*,*,*)$

$[\mathrm{Ded}_3(\mathsf{p},\mathsf{c},\mathsf{s},\mathsf{b}) \overset{\mathrm{val}}{\rightarrow} \mathrm{If}(\neg\ [\ \mathsf{c} \overset{\mathrm{r}}{=} \lceil \forall_{\mathrm{obj}} \mathsf{x} \colon \mathsf{y} \rceil\ ]\ , \mathrm{Ded}_4(\mathsf{p},\mathsf{c},\mathsf{s},\mathsf{b}), \mathrm{If}(\ [\ \mathsf{p} \overset{\mathrm{r}}{=} \lceil \forall_{\mathrm{obj}} \mathsf{x} \colon \mathsf{y} \rceil\ ]\ \wedge\ [\ \mathsf{p}^1 \overset{\mathrm{t}}{=} [\ \mathsf{c}^1\ ]\ ]\ , \mathrm{Ded}_4(\mathsf{p},\mathsf{c},\mathsf{s},\mathsf{b}), \mathrm{Ded}_3(\mathsf{p},\mathsf{c}^2,\mathsf{s},\ [\ \mathsf{c}^1 :: [\ \mathsf{c}^1\ ]\ ] :: \mathsf{b})))]$

$[\mathrm{Ded}_3(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \overset{\mathrm{tex}}{\rightarrow}$ “
Ded_3(#1.
, #2.
, #3.
, #4.
)”]

$[\mathrm{Ded}_3(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \overset{\mathrm{pyk}}{\rightarrow}$ “deduction three $*$ conclude $*$ condition $*$ bound $*$ end deduction”]

$\mathrm{Ded}_4(*,*,*,*)$

$[\mathrm{Ded}_4(\mathsf{p},\mathsf{c},\mathsf{s},\mathsf{b}) \overset{\mathrm{val}}{\rightarrow} \mathsf{s}!\ [\ \mathsf{b}!\mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil \overline{\mathsf{x}} \rceil, \mathbf{lookup}(\mathsf{p},\mathsf{b},\mathsf{T}) \overset{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}(\neg\ [\ \mathsf{p} \overset{\mathrm{r}}{=} \mathsf{c}\ ]\ , \mathsf{F},$
$\mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil \forall_{\mathrm{obj}} \mathsf{x} \colon \mathsf{y} \rceil, [\ \mathsf{p}^1 \overset{\mathrm{t}}{=} [\ \mathsf{c}^1\ ]\ ] \wedge \mathrm{Ded}_4(\mathsf{p}^2,\mathsf{c}^2,\mathsf{s},\ [\ \mathsf{p}^1 :: [\ \mathsf{p}^1\ ]\ ] :: \mathsf{b}), \mathrm{If}(\neg\ [\ \mathsf{p} \overset{\mathrm{r}}{=} \lceil \underline{\mathsf{x}} \rceil\ ]\ , \mathrm{Ded}_4^*(\mathsf{p}^{\mathsf{t}},\mathsf{c}^{\mathsf{t}},\mathsf{s},\mathsf{b}),\ [\ \mathsf{p}^1 \overset{\mathrm{t}}{=} [\ \mathsf{c}^1\ ]\ ] \wedge \mathrm{Ded}_5(\mathsf{p},\mathsf{s},\mathsf{b}))))\ ]\ ]$

$[\mathrm{Ded}_4(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \overset{\mathrm{tex}}{\rightarrow}$ “
Ded_4(#1.
, #2.
, #3.
, #4.
)”]

$[\mathrm{Ded}_4(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \overset{\mathrm{pyk}}{\rightarrow}$ “deduction four $*$ conclude $*$ condition $*$ bound $*$ end

deduction”]

$\mathrm{Ded}_4^*(*,*,*,*)$

$[\mathrm{Ded}_4^*(\mathsf{p},\mathsf{c},\mathsf{s},\mathsf{b}) \overset{\mathrm{val}}{\to} \mathsf{c}!\ [\ \mathsf{s}!\ [\ \mathsf{b}!\mathrm{If}(\mathsf{p},\mathsf{T},\mathrm{Ded}_4(\mathsf{p}^{\mathrm{h}},\mathsf{c}^{\mathrm{h}},\mathsf{s},\mathsf{b}) \wedge \mathrm{Ded}_4^*(\mathsf{p}^{\mathrm{t}},\mathsf{c}^{\mathrm{t}},\mathsf{s},\mathsf{b}))\ ]\ ]\ ]$

$[\mathrm{Ded}_4^*(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \overset{\mathrm{tex}}{\to}$ “
Ded_4^*(#1.
,#2.
,#3.
,#4.
)”]

$[\mathrm{Ded}_4^*(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \overset{\mathrm{pyk}}{\to}$ “deduction four star $*$ conclude $*$ condition $*$ bound $*$ end deduction”]

$\mathrm{Ded}_5(*,*,*)$

$[\mathrm{Ded}_5(\mathsf{p},\mathsf{s},\mathsf{b}) \overset{\mathrm{val}}{\to} \mathsf{p}!\ [\ \mathsf{s}!\mathrm{If}(\mathsf{b},\mathsf{T},\ [\ [\ \lceil\lceil\mathsf{x}\rceil\#^0\lceil\mathsf{y}\rceil\rceil^{\mathrm{h}} :: [\ [\ \lceil\lceil *\rceil\rceil^{\mathrm{h}} :: [\ \mathsf{b}^{\mathrm{hh}} :: \mathsf{T}\ ]\ ] :: [\ [\ \lceil\lceil\mathsf{x}\rceil\rceil^{\mathrm{h}} :: [\ \mathsf{p} :: \mathsf{T}\ ]\ ]\ :: \mathsf{T}\ ]\ ]\ ] \in_{\mathrm{t}} \mathsf{s}\ ] \wedge \mathrm{Ded}_5(\mathsf{p},\mathsf{s},\mathsf{b}^{\mathrm{t}}))\ ]\ ]$

$[\mathrm{Ded}_5(\mathsf{x},\mathsf{y},\mathsf{z}) \overset{\mathrm{tex}}{\to}$ “
Ded_5(#1.
,#2.
,#3.
)”]

$[\mathrm{Ded}_5(\mathsf{x},\mathsf{y},\mathsf{z}) \overset{\mathrm{pyk}}{\to}$ “deduction five $*$ condition $*$ bound $*$ end deduction”]

$\mathrm{Ded}_6(*,*,*,*)$

$[\mathrm{Ded}_6(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \overset{\mathrm{val}}{\to} \mathsf{p}!\ [\ \mathsf{c}!\ [\ \mathsf{b}!\ [\ \mathsf{e}!\mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil\bar{\mathsf{x}}\rceil,\ [\ \mathsf{p} \in_{\mathrm{t}} \mathsf{e}\ ]\ \begin{cases} \mathsf{b} \\ [\ \mathsf{p} :: \mathsf{c}\ ]\ :: \mathsf{b} \end{cases}, \mathrm{If}(\neg\ [\ \mathsf{p} \overset{\mathrm{r}}{=} \mathsf{c}\ ], \mathsf{T}, \mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil\underline{\mathsf{a}}\rceil, \mathsf{b}, \mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil\forall_{\mathrm{obj}}\mathsf{x}\colon\mathsf{y}\rceil, \mathrm{Ded}_6(\mathsf{p}^2,\mathsf{c}^2,\mathsf{c}^1 :: \mathsf{e},\mathsf{b}),$
$\mathrm{Ded}_6^*(\mathsf{p}^{\mathrm{t}},\mathsf{c}^{\mathrm{t}},\mathsf{e},\mathsf{b})))))\ ]\ ]\ ]\ ]$

$[\mathrm{Ded}_6(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \overset{\mathrm{tex}}{\to}$ “
Ded_6(#1.
,#2.
,#3.
,#4.
)”]

$[\mathrm{Ded}_6(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \overset{\mathrm{pyk}}{\to} \text{“deduction six} * \text{conclude} * \text{exception} * \text{bound} * \text{end deduction”}]$

$\mathrm{Ded}_6^*(*,*,*,*)$

$[\mathrm{Ded}_6^*(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \overset{\mathrm{val}}{\to} \mathsf{p}!\ [\ \mathsf{c}!\ [\ \mathsf{b}!\ [\ \mathsf{e}!\mathrm{If}(\mathsf{p},\mathsf{b},\mathrm{Ded}_6^*(\mathsf{p}^{\mathrm{t}},\mathsf{c}^{\mathrm{t}},\mathsf{e},\mathrm{Ded}_6(\mathsf{p}^{\mathrm{h}},\mathsf{c}^{\mathrm{h}},\mathsf{e},\mathsf{b})))\ ]\ ]\ ]\ ]$

$[\mathrm{Ded}_6^*(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \overset{\mathrm{tex}}{\to} \text{“}$
Ded_6^*(#1.
,#2.
,#3.
,#4.
)”]

$[\mathrm{Ded}_6^*(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \overset{\mathrm{pyk}}{\to} \text{“deduction six star} * \text{conclude} * \text{exception} * \text{bound} * \text{end deduction”}]$

$\mathrm{Ded}_7(*)$

$[\mathrm{Ded}_7(\mathsf{p}) \overset{\mathrm{val}}{\to}\ [\ \mathsf{p} \overset{\mathrm{r}}{=} \lceil \forall \mathsf{x}\colon \mathsf{y} \rceil\ ]\ \begin{cases} \mathrm{Ded}_7(\mathsf{p}^2) \\ \mathsf{p} \end{cases}\ ]$

$[\mathrm{Ded}_7(\mathsf{p}) \overset{\mathrm{tex}}{\to} \text{“}$
Ded_7(#1.
)”]

$[\mathrm{Ded}_7(\mathsf{p}) \overset{\mathrm{pyk}}{\to} \text{“deduction seven} * \text{end deduction”}]$

$\mathrm{Ded}_8(*,*)$

$[\mathrm{Ded}_8(\mathsf{p},\mathsf{b}) \overset{\mathrm{val}}{\to} \mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil \forall \mathsf{x}\colon \mathsf{y} \rceil, \mathrm{Ded}_8(\mathsf{p}^2, \mathsf{p}^1 :: \mathsf{b}), \mathrm{If}(\mathsf{p} \overset{\mathrm{r}}{=} \lceil \underline{\mathsf{a}} \rceil, \mathsf{p} \in_{\mathrm{t}} \mathsf{b}, \mathrm{Ded}_8^*(\mathsf{p}^{\mathrm{t}}, \mathsf{b})))]$

$[\mathrm{Ded}_8(\mathsf{p},\mathsf{b}) \overset{\mathrm{tex}}{\to} \text{“}$
Ded_8(#1.
,#2.
)”]

$[\mathrm{Ded}_8(\mathsf{p},\mathsf{b}) \overset{\mathrm{pyk}}{\to} \text{“deduction eight} * \text{bound} * \text{end deduction”}]$

$\mathrm{Ded}_8^*(*,*)$

$[\mathrm{Ded}_8^*(\mathsf{p},\mathsf{b}) \overset{\mathrm{val}}{\to} \mathsf{b}!\mathrm{If}(\mathsf{p},\mathsf{T},\mathrm{If}(\mathrm{Ded}_8(\mathsf{p}^{\mathrm{h}},\mathsf{b}),\mathrm{Ded}_8^*(\mathsf{p}^{\mathrm{t}},\mathsf{b}),\mathsf{F}))]$

$[\mathrm{Ded}_8^*(\mathsf{p},\mathsf{b}) \overset{\mathrm{tex}}{\to} \text{“}$
Ded_8^*(#1.
,#2.
)”]

$[\mathrm{Ded}_8^*(\mathsf{p},\mathsf{b}) \overset{\mathrm{pyk}}{\to} \text{“deduction eight } * \text{ bound } * \text{ end deduction”}]$

S

$[\mathrm{S} \overset{\mathrm{stmt}}{\to} [\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} + [\,\underline{\mathsf{b}}'\,]\,] = [\,[\,\underline{\mathsf{a}} + \underline{\mathsf{b}}\,]'\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\,] \vdash [\,\underline{\mathsf{a}} \vdash \underline{\mathsf{b}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} = \underline{\mathsf{b}}\,] \vdash [\,\underline{\mathsf{a}}' = [\,\underline{\mathsf{b}}'\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}}' = [\,\underline{\mathsf{b}}'\,]\,] \vdash [\,\underline{\mathsf{a}} = \underline{\mathsf{b}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\lambda \mathsf{x}.\mathrm{Ded}_0(\lceil \underline{\mathsf{a}} \rceil, \lceil \underline{\mathsf{b}} \rceil)\,] \Vdash [\,\underline{\mathsf{a}} \vdash \underline{\mathsf{b}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} \cdot [\,\underline{\mathsf{b}}'\,]\,] = [\,[\,\underline{\mathsf{a}} \cdot \underline{\mathsf{b}}\,] + \underline{\mathsf{a}}\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon [\,[\,\underline{\mathsf{a}} + 0\,] = \underline{\mathsf{a}}\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon [\,[\,\neg\neg \underline{\mathsf{a}}\,] \vdash \underline{\mathsf{a}}\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\,[\,\underline{\mathsf{a}} = \underline{\mathsf{b}}\,] \vdash [\,[\,\underline{\mathsf{a}} = \underline{\mathsf{c}}\,] \vdash [\,\underline{\mathsf{b}} = \underline{\mathsf{c}}\,]\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon [\,\langle \lceil \underline{\mathsf{b}} \rceil \equiv^0 \lceil \underline{\mathsf{a}} \rceil \,|\, \lceil \underline{\mathsf{x}} \rceil := \lceil 0 \rceil \rangle \Vdash [\,\langle \lceil \underline{\mathsf{c}} \rceil \equiv^0 \lceil \underline{\mathsf{a}} \rceil \,|\, \lceil \underline{\mathsf{x}} \rceil := \lceil \underline{\mathsf{x}}' \rceil \rangle \Vdash [\,\underline{\mathsf{b}} \vdash [\,[\,\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\,] \vdash \underline{\mathsf{a}}\,]\,]\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{a}}\colon \neg [\,0 = [\,\underline{\mathsf{a}}'\,]\,]\,] \oplus [\,[\,\forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{a}}\colon [\,\underline{\mathsf{a}} \vdash \forall_{\mathrm{obj}} \underline{\mathsf{x}}\colon \underline{\mathsf{a}}\,]\,] \oplus \forall \underline{\mathsf{a}}\colon [\,[\,\underline{\mathsf{a}} \cdot 0\,] = 0\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]$

$[\mathrm{S} \overset{\mathrm{tex}}{\to} \text{“}$
S”]

$[\mathrm{S} \overset{\mathrm{pyk}}{\to} \text{“system s”}]$

Neg

$[\mathrm{Neg} \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{Neg} \overset{\mathrm{stmt}}{\to} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon [\,[\,\neg\neg \underline{\mathsf{a}}\,] \vdash \underline{\mathsf{a}}\,]\,]$

$[\mathrm{Neg} \overset{\mathrm{tex}}{\to} \text{“}$
Neg”]

$[\mathrm{Neg} \overset{\mathrm{pyk}}{\to} \text{“double negation”}]$

MP

$[\mathrm{MP} \overset{\mathrm{proof}}{\to} \text{Rule tactic}]$

$[\mathrm{MP} \overset{\mathrm{stmt}}{\to} \mathrm{S} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon [\,[\,\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}\,] \vdash [\,\underline{\mathsf{a}} \vdash \underline{\mathsf{b}}\,]\,]\,]$

$[\mathrm{MP} \overset{\mathrm{tex}}{\to} \text{“}$
MP”]

$[\mathrm{MP} \overset{\mathrm{pyk}}{\to} \text{“rule mp”}]$

Gen

[Gen $\overset{\text{proof}}{\to}$ Rule tactic]

[Gen $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{x}\colon \forall \underline{a}\colon\ [\ \underline{a} \vdash \forall_{\mathrm{obj}} \underline{x}\colon \underline{a}\ ]\ ]$

[Gen $\overset{\text{tex}}{\to}$ “
Gen”]

[Gen $\overset{\text{pyk}}{\to}$ “rule gen”]

Ded

[Ded $\overset{\text{proof}}{\to}$ Rule tactic]

[Ded $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon\ [\ [\ \lambda x.\mathrm{Ded}_0(\lceil \underline{a} \rceil, \lceil \underline{b} \rceil)\ ]\ \Vdash\ [\ \underline{a} \vdash \underline{b}\ ]\ ]\ ]$

[Ded $\overset{\text{tex}}{\to}$ “
Ded”]

[Ded $\overset{\text{pyk}}{\to}$ “deduction”]

S1

[S1 $\overset{\text{proof}}{\to}$ Rule tactic]

[S1 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon\ [\ [\ \underline{a} = \underline{b}\ ]\ \vdash\ [\ [\ \underline{a} = \underline{c}\ ]\ \vdash\ [\ \underline{b} = \underline{c}\ ]\ ]\ ]\ ]$

[S1 $\overset{\text{tex}}{\to}$ “
S1”]

[S1 $\overset{\text{pyk}}{\to}$ “axiom s one”]

S2

[S2 $\overset{\text{proof}}{\to}$ Rule tactic]

[S2 $\overset{\text{stmt}}{\to}$ $\mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon\ [\ [\ \underline{a} = \underline{b}\ ]\ \vdash\ [\ \underline{a}' =\ [\ \underline{b}'\ ]\ ]\ ]\ ]$

[S2 $\overset{\text{tex}}{\to}$ “
S2”]

[S2 $\overset{\text{pyk}}{\to}$ “axiom s two”]

S3

[S3 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S3 $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \neg\, [\, 0 = \, [\, \underline{a}' \,]\,]\,]$

[S3 $\overset{\text{tex}}{\rightarrow}$ “
S3”]

[S3 $\overset{\text{pyk}}{\rightarrow}$ “axiom s three”]

S4

[S4 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S4 $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \, [\, [\, [\, \underline{a}' = \, [\, \underline{b}' \,]\,]\,]\, \vdash \, [\, \underline{a} = \underline{b} \,]\,]\,]$

[S4 $\overset{\text{tex}}{\rightarrow}$ “
S4”]

[S4 $\overset{\text{pyk}}{\rightarrow}$ “axiom s four”]

S5

[S5 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S5 $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \, [\, [\, \underline{a} + 0 \,]\, = \underline{a} \,]\,]$

[S5 $\overset{\text{tex}}{\rightarrow}$ “
S5”]

[S5 $\overset{\text{pyk}}{\rightarrow}$ “axiom s five”]

S6

[S6 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S6 $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \, [\, [\, \underline{a} + \, [\, \underline{b}' \,]\,]\, = \, [\, [\, \underline{a} + \underline{b} \,]\, ' \,]\,]\,]$

[S6 $\overset{\text{tex}}{\rightarrow}$ “
S6”]

[S6 $\overset{\text{pyk}}{\rightarrow}$ “axiom s six”]

S7

[S7 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S7 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{\mathrm{a}}\colon\ [\ [\ \underline{\mathrm{a}} \cdot 0\ ]\ = 0\ ]$]

[S7 $\overset{\text{tex}}{\rightarrow}$ “
S7”]

[S7 $\overset{\text{pyk}}{\rightarrow}$ “axiom s seven”]

S8

[S8 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S8 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{\mathrm{a}}\colon \forall \underline{\mathrm{b}}\colon\ [\ [\ \underline{\mathrm{a}} \cdot\ [\ \underline{\mathrm{b}}'\ ]\ ]\ =\ [\ [\ \underline{\mathrm{a}} \cdot \underline{\mathrm{b}}\ ]\ + \underline{\mathrm{a}}\ ]\ ]$]

[S8 $\overset{\text{tex}}{\rightarrow}$ “
S8”]

[S8 $\overset{\text{pyk}}{\rightarrow}$ “axiom s eight”]

S9

[S9 $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[S9 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{\mathrm{x}}\colon \forall \underline{\mathrm{a}}\colon \forall \underline{\mathrm{b}}\colon \forall \underline{\mathrm{c}}\colon\ [\ \langle \lceil \underline{\mathrm{b}} \rceil \equiv^0 \lceil \underline{\mathrm{a}} \rceil \| \lceil \underline{\mathrm{x}} \rceil := \lceil 0 \rceil \rangle \Vdash\ [\ \langle \lceil \underline{\mathrm{c}} \rceil \equiv^0 \lceil \underline{\mathrm{a}} \rceil \| \lceil \underline{\mathrm{x}} \rceil := \lceil \underline{\mathrm{x}}' \rceil \rangle \Vdash\ [\ \underline{\mathrm{b}} \vdash\ [\ [\ \underline{\mathrm{a}} \Rightarrow \underline{\mathrm{c}}\ ] \vdash \underline{\mathrm{a}}\ ]\ ]\ ]\ ]$]

[S9 $\overset{\text{tex}}{\rightarrow}$ “
S9”]

[S9 $\overset{\text{pyk}}{\rightarrow}$ “axiom s nine”]

Repetition

[Repetition $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathrm{c}.\lambda \mathrm{x}.\lceil \mathrm{S} \vdash \forall \underline{\mathrm{a}}\colon\ [\ \underline{\mathrm{a}}^{\mathrm{I}}\ ]\ \rceil$]

[Repetition $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{S} \vdash \forall \underline{\mathrm{a}}\colon\ [\ \underline{\mathrm{a}} \vdash \underline{\mathrm{a}}\ ]$]

[Repetition $\overset{\text{tex}}{\rightarrow}$ “
Repetition”]

[Repetition $\overset{\text{pyk}}{\rightarrow}$ “repetition”]

A1′

$[\mathrm{A1}' \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \vdash [\ \underline{b} \vdash [\ [\ \mathrm{Repetition} \rhd \underline{a}\ ]\ \gg \underline{a}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \vdash [\ \underline{b} \vdash \underline{a}\ ]\ ]\ ]\ \gg [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A1}' \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{a}\ ]\ ]\ ]$

$[\mathrm{A1}' \overset{\mathrm{tex}}{\rightarrow} \text{“}$
A1”’]

$[\mathrm{A1}' \overset{\mathrm{pyk}}{\rightarrow} \text{“lemma a one”}]$

A2′

$[\mathrm{A2}' \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \vdash [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \vdash [\ \underline{a} \vdash [\ [\ [\ [\ \mathrm{MP} \rhd [\ \underline{a} \Rightarrow \underline{b}\ ]\ ]\ \rhd \underline{a}\ ]\ \gg \underline{b}\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP} \rhd [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ]\ \rhd \underline{a}\ ]\ \gg [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP} \rhd [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \rhd \underline{b}\ ]\ \gg \underline{c}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \vdash [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \vdash [\ \underline{a} \vdash \underline{c}\ ]\ ]\ ]\ ]\ \gg [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \Rightarrow [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A2}' \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \Rightarrow [\ \underline{b} \Rightarrow \underline{c}\ ]\ ]\ \Rightarrow [\ [\ \underline{a} \Rightarrow \underline{b}\ ]\ \Rightarrow [\ \underline{a} \Rightarrow \underline{c}\ ]\ ]\ ]\ ]$

$[\mathrm{A2}' \overset{\mathrm{tex}}{\rightarrow} \text{“}$
A2”’]

$[\mathrm{A2}' \overset{\mathrm{pyk}}{\rightarrow} \text{“lemma a two”}]$

A4′

$[\mathrm{A4}' \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S} \vdash [\ [\ [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ \vdash [\ [\ \mathrm{Repetition} \rhd [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ ]\ \gg [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Ded} \rhd [\ [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ \vdash [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ ]\ ]\ \gg [\ [\ \forall_{\mathrm{obj}} \overline{x}\colon \forall_{\mathrm{obj}} \overline{y}\colon [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ ]\ \Rightarrow [\ [\ 2 + 3\ ]\ =\ [\ 3 + 2\ ]\ ]\ ]\ ]\ \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{A4}' \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S} \vdash [\ [\ \forall_{\mathrm{obj}} \overline{x}\colon \forall_{\mathrm{obj}} \overline{y}\colon [\ [\ \overline{x} + \overline{y}\ ]\ =\ [\ \overline{y} + \overline{x}\ ]\ ]\ ]\ \Rightarrow [\ [\ 2 + 3\ ]\ =\ [\ 3 + 2\ ]\ ]\ ]\ ]$

$[\mathrm{A4}' \overset{\mathrm{tex}}{\rightarrow} \text{“}$
A4”’]

$[\mathrm{A4}' \overset{\mathrm{pyk}}{\rightarrow} \text{“lemma a four”}]$

A5′

[A5′ $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash [\,[\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,\vdash\,[\,[\,[\,2+3\,]\,=5\,]\,\vdash\,[\,[\,[\,[\,\text{MP}\,\triangleright\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,\triangleright\,[\,[\,2+3\,]\,=5\,]\,]\,\gg\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,;\,[\,[\,\text{Gen}\,\triangleright\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,\gg\,\forall_{\text{obj}}\bar{x}\colon\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,]\,]\,;\,[\,[\,\text{Ded}\,\triangleright\,[\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,\vdash\,[\,[\,[\,2+3\,]\,=5\,]\,\vdash\,\forall_{\text{obj}}\bar{x}\colon\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,]\,]\,]\,\gg\,[\,[\,\forall_{\text{obj}}\bar{x}\colon\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,\forall_{\text{obj}}\bar{x}\colon\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,]\,]\,]\,]\rceil, p_0, c)]$

[A5′ $\overset{\text{stmt}}{\to}$ $S \vdash [\,[\,\forall_{\text{obj}}\bar{x}\colon\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,\Rightarrow\,[\,[\,[\,2+3\,]\,=5\,]\,\Rightarrow\,\forall_{\text{obj}}\bar{x}\colon\,[\,[\,[\,2+3\,]\,+\bar{x}\,]\,=\,[\,5+\bar{x}\,]\,]\,]\,]\,]$

[A5′ $\overset{\text{tex}}{\to}$ “
A5′”]

[A5′ $\overset{\text{pyk}}{\to}$ “lemma a five”]

Prop 3.2a

[Prop 3.2a $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash \forall \underline{a}\colon\,[\,[\,S5 \gg\,[\,[\,\underline{a}+0\,]\,=\underline{a}\,]\,]\,;\,[\,[\,[\,S1\,\triangleright\,[\,[\,\underline{a}+0\,]\,=\underline{a}\,]\,]\,\triangleright\,[\,[\,\underline{a}+0\,]\,=\underline{a}\,]\,]\,\gg\,[\,\underline{a}=\underline{a}\,]\,]\,]\,]\rceil, p_0, c)]$

[Prop 3.2a $\overset{\text{stmt}}{\to}$ $S \vdash \forall \underline{a}\colon\,[\,\underline{a}=\underline{a}\,]\,]$

[Prop 3.2a $\overset{\text{tex}}{\to}$ “
Prop\ 3.2a”]

[Prop 3.2a $\overset{\text{pyk}}{\to}$ “prop three two a”]

Prop 3.2b

[Prop 3.2b $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash \forall \underline{a}\colon\forall \underline{b}\colon\,[\,[\,\underline{a}=\underline{b}\,]\,\vdash\,[\,[\,\text{Prop 3.2a} \gg\,[\,\underline{a}=\underline{a}\,]\,]\,;\,[\,[\,[\,S1\,\triangleright\,[\,\underline{a}=\underline{b}\,]\,]\,\triangleright\,[\,\underline{a}=\underline{a}\,]\,]\,\gg\,[\,\underline{b}=\underline{a}\,]\,]\,]\,]\rceil, p_0, c)]$

[Prop 3.2b $\overset{\text{stmt}}{\to}$ $S \vdash \forall \underline{a}\colon\forall \underline{b}\colon\,[\,[\,\underline{a}=\underline{b}\,]\,\vdash\,[\,\underline{b}=\underline{a}\,]\,]\,]$

[Prop 3.2b $\overset{\text{tex}}{\to}$ “
Prop\ 3.2b”]

[Prop 3.2b $\overset{\text{pyk}}{\to}$ “prop three two b”]

Prop 3.2c

[Prop 3.2c $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ] \vdash [\ [\ \underline{\mathsf{b}} = \underline{\mathsf{c}}\ ] \vdash [\ [\ [$ Prop 3.2b $\rhd [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ]\ ] \gg [\ \underline{\mathsf{b}} = \underline{\mathsf{a}}\ ]\ ] ; [\ [\ [\ \mathrm{S1} \rhd [\ \underline{\mathsf{b}} = \underline{\mathsf{a}}\ ]\ ] \rhd [\ \underline{\mathsf{b}} = \underline{\mathsf{c}}\ ]\ ] \gg [\ \underline{\mathsf{a}} = \underline{\mathsf{c}}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, \mathsf{c})$]

[Prop 3.2c $\stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ] \vdash [\ [\ \underline{\mathsf{b}} = \underline{\mathsf{c}}\ ] \vdash [\ \underline{\mathsf{a}} = \underline{\mathsf{c}}\ ]\ ]\ ]$]

[Prop 3.2c $\stackrel{\text{tex}}{\rightarrow}$ "
Prop\ 3.2c"]

[Prop 3.2c $\stackrel{\text{pyk}}{\rightarrow}$ "prop three two c"]

Prop 3.2d

[Prop 3.2d $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{c}}\ ] \vdash [\ [\ \underline{\mathsf{b}} = \underline{\mathsf{c}}\ ] \vdash [\ [\ [$ Prop 3.2b $\rhd [\ \underline{\mathsf{b}} = \underline{\mathsf{c}}\ ]\ ] \gg [\ \underline{\mathsf{c}} = \underline{\mathsf{b}}\ ]\ ] ; [\ [\ [$ Prop 3.2c $\rhd [\ \underline{\mathsf{a}} = \underline{\mathsf{c}}\ ]\ ] \rhd [\ \underline{\mathsf{c}} = \underline{\mathsf{b}}\ ]\ ] \gg [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, \mathsf{c})$]

[Prop 3.2d $\stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{c}}\ ] \vdash [\ [\ \underline{\mathsf{b}} = \underline{\mathsf{c}}\ ] \vdash [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ]\ ]\ ]$]

[Prop 3.2d $\stackrel{\text{tex}}{\rightarrow}$ "
Prop\ 3.2d"]

[Prop 3.2d $\stackrel{\text{pyk}}{\rightarrow}$ "prop three two d"]

Prop 3.2e$_1$

[Prop 3.2e$_1$ $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ] \vdash [\ [\ \mathrm{S5} \gg [\ [\ \underline{\mathsf{a}} + 0\ ] = \underline{\mathsf{a}}\ ]\ ] ; [\ [\ [\ [$ Prop 3.2c $\rhd [\ [\ \underline{\mathsf{a}} + 0\ ] = \underline{\mathsf{a}}\ ]\ ] \rhd [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ]\ ] \gg [\ [\ \underline{\mathsf{a}} + 0\ ] = \underline{\mathsf{b}}\ ]\ ] ; [\ [\ \mathrm{S5} \gg [\ [\ \underline{\mathsf{b}} + 0\ ] = \underline{\mathsf{b}}\ ]\ ] ; [\ [\ [$ Prop 3.2d $\rhd [\ [\ \underline{\mathsf{a}} + 0\ ] = \underline{\mathsf{b}}\ ]\ ] \rhd [\ [\ \underline{\mathsf{b}} + 0\ ] = \underline{\mathsf{b}}\ ]\ ] \gg [\ [\ \underline{\mathsf{a}} + 0\ ] = [\ \underline{\mathsf{b}} + 0\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ] ; [\ [\ \mathrm{Ded} \rhd \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ] \vdash [\ [\ \underline{\mathsf{a}} + 0\ ] = [\ \underline{\mathsf{b}} + 0\ ]\ ]\ ]\ ] \gg [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ] \Rightarrow [\ [\ \underline{\mathsf{a}} + 0\ ] = [\ \underline{\mathsf{b}} + 0\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, \mathsf{c})$]

[Prop 3.2e$_1$ $\stackrel{\text{stmt}}{\rightarrow} \mathrm{S} \vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon [\ [\ \underline{\mathsf{a}} = \underline{\mathsf{b}}\ ] \Rightarrow [\ [\ \underline{\mathsf{a}} + 0\ ] = [\ \underline{\mathsf{b}} + 0\ ]\ ]\ ]$]

[Prop 3.2e$_1$ $\stackrel{\text{tex}}{\rightarrow}$ "
Prop\ 3.2e_1"]

[Prop 3.2e$_1$ $\stackrel{\text{pyk}}{\rightarrow}$ "prop three two e one"]

Prop 3.2e$_2$

[Prop 3.2e$_2$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[$\forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]]] $\vdash$ [[$\underline{a} = \underline{b}$] $\vdash$ [[[[MP $\triangleright$ [[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]]] $\triangleright$ [$\underline{a} = \underline{b}$]] $\gg$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]] ; [[[S2 $\triangleright$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]] $\gg$ [[$\underline{a} + \underline{c}$] $'=$ [[$\underline{b} + \underline{c}$] $'$]]] ; [[S6 $\gg$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [[$\underline{a} + \underline{c}$] $'$]]] ; [[[[Prop 3.2c $\triangleright$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [[$\underline{a} + \underline{c}$] $'$]]] $\triangleright$ [[$\underline{a} + \underline{c}$] $'=$ [[$\underline{b} + \underline{c}$] $'$]]] $\gg$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [[$\underline{b} + \underline{c}$] $'$]]] ; [[S6 $\gg$ [[$\underline{b} +$ [$\underline{c}'$]] $=$ [[$\underline{b} + \underline{c}$] $'$]]] ; [[[Prop 3.2d $\triangleright$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [[$\underline{b} + \underline{c}$] $'$]]] $\triangleright$ [[$\underline{b} +$ [$\underline{c}'$]] $=$ [[$\underline{b} + \underline{c}$] $'$]]] $\gg$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [$\underline{b} +$ [$\underline{c}'$]]]]]]]]]]] ; [[Ded $\triangleright \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]] $\vdash$ [[$\underline{a} = \underline{b}$] $\vdash$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [$\underline{b} +$ [$\underline{c}'$]]]]]] $\gg$ [[[[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]] $\Rightarrow$ [[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [$\underline{b} +$ [$\underline{c}'$]]]]]]]]] $\rfloor$, p$_0$, c)]

[Prop 3.2e$_2$ $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[[[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]] $\Rightarrow$ [[[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} +$ [$\underline{c}'$]] $=$ [$\underline{b} +$ [$\underline{c}'$]]]]]]]

[Prop 3.2e$_2$ $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2e_2”]

[Prop 3.2e$_2$ $\overset{\text{pyk}}{\rightarrow}$ “prop three two e two”]

Prop 3.2e

[Prop 3.2e $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[$\underline{a} = \underline{b}$] $\vdash$ [[[Prop 3.2e$_1$ $\gg$ [[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} + 0$] $=$ [$\overline{y} + 0$]]]] ; [[Prop 3.2e$_2$ $\gg$ [[[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} + \overline{z}$] $=$ [$\overline{y} + \overline{z}$]]] $\Rightarrow$ [[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} +$ [$\overline{z}'$]] $=$ [$\overline{y} +$ [$\overline{z}'$]]]]]] ; [[[[S9 @ $\overline{z}$] $\triangleright$ [[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} + 0$] $=$ [$\overline{y} + 0$]]]] $\triangleright$ [[[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} + \overline{z}$] $=$ [$\overline{y} + \overline{z}$]]] $\Rightarrow$ [[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} +$ [$\overline{z}'$]] $=$ [$\overline{y} +$ [$\overline{z}'$]]]]]]] $\gg$ [[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} + \overline{z}$] $=$ [$\overline{y} + \overline{z}$]]]]] ; [[[Ded $\triangleright$ [[$\overline{x} = \overline{y}$] $\Rightarrow$ [[$\overline{x} + \overline{z}$] $=$ [$\overline{y} + \overline{z}$]]]] $\gg$ [[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]]] ; [[[MP $\triangleright$ [[$\underline{a} = \underline{b}$] $\Rightarrow$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]]] $\triangleright$ [$\underline{a} = \underline{b}$]] $\gg$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]]]] $\rfloor$, p$_0$, c)]

[Prop 3.2e $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon$ [[$\underline{a} = \underline{b}$] $\vdash$ [[$\underline{a} + \underline{c}$] $=$ [$\underline{b} + \underline{c}$]]]]

[Prop 3.2e $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2e”]

[Prop 3.2e $\overset{\text{pyk}}{\rightarrow}$ “prop three two e”]

Prop 3.2f$_1$

[Prop 3.2f$_1$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash [\ [\ S5 \gg [\ [\ 0+0\ ]\ = 0\ ]\ ]\ ;\ [\ [\ \text{Prop 3.2b} \rhd [\ [\ 0+0\ ]\ = 0\ ]\ ]\ \gg [\ 0 = [\ [\ 0+0\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[Prop 3.2f$_1$ $\overset{\text{stmt}}{\rightarrow}$ $S \vdash [\ 0 = [\ 0+0\ ]\ ]\ ]$

[Prop 3.2f$_1$ $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2f_1”]

[Prop 3.2f$_1$ $\overset{\text{pyk}}{\rightarrow}$ “prop three two f one”]

Prop 3.2f$_2$

[Prop 3.2f$_2$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash \forall \underline{a}\text{:}\ [\ [\ \forall \underline{a}\text{:}\ [\ [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ \vdash [\ [\ [\ S2 \rhd [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ ]\ ]\ \gg [\ \underline{a}' = [\ [\ 0+\underline{a}\ ]'\ ]\ ]\ ]\ ;\ [\ [\ S6 \gg [\ [\ 0+[\ \underline{a}'\ ]\ ]\ = [\ [\ 0+\underline{a}\ ]'\ ]\ ]\ ]\ ;\ [\ [\ [\ \text{Prop 3.2d} \rhd [\ \underline{a}' = [\ [\ 0+\underline{a}\ ]'\ ]\ ]\ ]\ \rhd [\ [\ 0+[\ \underline{a}'\ ]\ ]\ = [\ [\ 0+\underline{a}\ ]'\ ]\ ]\ ]\ \gg [\ \underline{a}' = [\ 0+[\ \underline{a}'\ ]\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{Ded} \rhd \forall \underline{a}\text{:}\ [\ [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ \vdash [\ \underline{a}' = [\ 0+[\ \underline{a}'\ ]\ ]\ ]\ ]\ ]\ \gg [\ [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ \Rightarrow [\ \underline{a}' = [\ 0+[\ \underline{a}'\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[Prop 3.2f$_2$ $\overset{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{a}\text{:}\ [\ [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ \Rightarrow [\ \underline{a}' = [\ 0+[\ \underline{a}'\ ]\ ]\ ]\ ]\ ]$

[Prop 3.2f$_2$ $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2f_2”]

[Prop 3.2f$_2$ $\overset{\text{pyk}}{\rightarrow}$ “prop three two f two”]

Prop 3.2f

[Prop 3.2f $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S \vdash \forall \underline{a}\text{:}\ [\ [\ [\ \text{Prop 3.2f}_1 \gg [\ 0 = [\ 0+0\ ]\ ]\ ]\ ;\ [\ [\ \text{Prop 3.2f}_2 \gg [\ [\ \overline{x} = [\ 0+\overline{x}\ ]\ ]\ \Rightarrow [\ \overline{x}' = [\ 0+[\ \overline{x}'\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ S9 @ \overline{x}\ ]\ \rhd [\ 0 = [\ 0+0\ ]\ ]\ ]\ \rhd [\ [\ \overline{x} = [\ 0+\overline{x}\ ]\ ]\ \Rightarrow [\ \overline{x}' = [\ 0+[\ \overline{x}'\ ]\ ]\ ]\ ]\ ]\ \gg [\ \overline{x} = [\ 0+\overline{x}\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ \text{Ded} \rhd [\ \overline{x} = [\ 0+\overline{x}\ ]\ ]\ ]\ \gg [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)$]

[Prop 3.2f $\overset{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{a}\text{:}\ [\ \underline{a} = [\ 0+\underline{a}\ ]\ ]\ ]$

[Prop 3.2f $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2f”]

[Prop 3.2f $\overset{\text{pyk}}{\rightarrow}$ “prop three two f”]

Prop 3.2g$_1$

[Prop 3.2g$_1$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}$: [[S5 $\gg$ [[$\underline{a}' + 0$] = [$\underline{a}'$]]] ; [[S5 $\gg$ [[$\underline{a} + 0$] = $\underline{a}$]] ; [[[S2 $\triangleright$ [[$\underline{a} + 0$] = $\underline{a}$]] $\gg$ [[$\underline{a} + 0$] $' =$ [$\underline{a}'$]]] ; [[[Prop 3.2d $\triangleright$ [[$\underline{a}' + 0$] = [$\underline{a}'$]]] $\triangleright$ [[$\underline{a} + 0$] $' =$ [$\underline{a}'$]]] $\gg$ [[$\underline{a}' + 0$] = [[$\underline{a} + 0$] $'$]]]]]] $\rceil$, p_0, c)]

[Prop 3.2g$_1$ $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}$: [[$\underline{a}' + 0$] = [[$\underline{a} + 0$] $'$]]]

[Prop 3.2g$_1$ $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2g_1”]

[Prop 3.2g$_1$ $\overset{\text{pyk}}{\rightarrow}$ “prop three two g one”]

Prop 3.2g$_2$

[Prop 3.2g$_2$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}: \forall \underline{b}$: [[$\forall \underline{a}: \forall \underline{b}$: [[[$\underline{a}' + \underline{b}$] = [[$\underline{a} + \underline{b}$] $'$]] $\vdash$ [[[S2 $\triangleright$ [[$\underline{a}' + \underline{b}$] = [[$\underline{a} + \underline{b}$] $'$]]] $\gg$ [[$\underline{a}' + \underline{b}$] $' =$ [[$\underline{a} + \underline{b}$] $''$]]] ; [[S6 $\gg$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a}' + \underline{b}$] $'$]]] ; [[[[Prop 3.2c $\triangleright$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a}' + \underline{b}$] $'$]]] $\triangleright$ [[$\underline{a}' + \underline{b}$] $' =$ [[$\underline{a} + \underline{b}$] $''$]]] $\gg$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a} + \underline{b}$] $'$]]] ; [[S6 $\gg$ [[$\underline{a} +$ [$\underline{b}'$]] = [[$\underline{a} + \underline{b}$] $'$]]] ; [[[S2 $\triangleright$ [[$\underline{a} +$ [$\underline{b}'$]] = [[$\underline{a} + \underline{b}$] $'$]]] $\gg$ [[$\underline{a} +$ [$\underline{b}'$]] $' =$ [[$\underline{a} + \underline{b}$] $''$]]] ; [[[Prop 3.2d $\triangleright$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a} + \underline{b}$] $''$]]] $\triangleright$ [[$\underline{a} +$ [$\underline{b}'$]] $' =$ [[$\underline{a} + \underline{b}$] $''$]]] $\gg$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a} +$ [$\underline{b}'$]] $'$]]]]]]]] ; [[Ded $\triangleright \forall \underline{a}: \forall \underline{b}$: [[[$\underline{a}' + \underline{b}$] = [[$\underline{a} + \underline{b}$] $'$]] $\vdash$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a} +$ [$\underline{b}'$]] $'$]]]] $\gg$ [[[$\underline{a}' + \underline{b}$] = [[$\underline{a} + \underline{b}$] $'$]] $\Rightarrow$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a} +$ [$\underline{b}'$]] $'$]]]]] $\rceil$, p_0, c)]

[Prop 3.2g$_2$ $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}: \forall \underline{b}$: [[[$\underline{a}' + \underline{b}$] = [[$\underline{a} + \underline{b}$] $'$]] $\Rightarrow$ [[$\underline{a}' +$ [$\underline{b}'$]] = [[$\underline{a} +$ [$\underline{b}'$]] $'$]]]]

[Prop 3.2g$_2$ $\overset{\text{tex}}{\rightarrow}$ “
Prop\ 3.2g_2”]

[Prop 3.2g$_2$ $\overset{\text{pyk}}{\rightarrow}$ “prop three two g two”]

Prop 3.2g

[Prop 3.2g $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}: \forall \underline{b}$: [[[Prop 3.2g$_1$ $\gg$ [[$\bar{x}' + 0$] = [[$\bar{x} + 0$] $'$]]] ; [[Prop 3.2g$_2$ $\gg$ [[[$\bar{x}' + \bar{y}$] = [[$\bar{x} + \bar{y}$] $'$]] $\Rightarrow$ [[$\bar{x}' +$ [$\bar{y}'$]] = [[$\bar{x} +$ [$\bar{y}'$]] $'$]]]] ; [[[[S9 @ $\bar{y}$] $\triangleright$ [[$\bar{x}' + 0$] = [[$\bar{x} + 0$] $'$]]] $\triangleright$ [[[$\bar{x}' + \bar{y}$] = [[$\bar{x} + \bar{y}$] $'$]] $\Rightarrow$ [[$\bar{x}' +$ [$\bar{y}'$]] = [[$\bar{x} +$ [$\bar{y}'$]] $'$]]]] $\gg$ [[$\bar{x}' + \bar{y}$] = [[$\bar{x} + \bar{y}$] $'$]]]] ; [[Ded $\triangleright$ [[$\bar{x}' + \bar{y}$] = [[$\bar{x} + \bar{y}$] $'$]] $\gg$ [[$\underline{a}' + \underline{b}$] = [[$\underline{a} + \underline{b}$] $'$]]]] $\rceil$, p_0, c)]

[Prop 3.2g $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a}' + \underline{b}\ ] = [\ [\ \underline{a} + \underline{b}\ ]'\ ]\ ]$]

[Prop 3.2g $\overset{\text{tex}}{\rightarrow}$ "
Prop\ 3.2g"]

[Prop 3.2g $\overset{\text{pyk}}{\rightarrow}$ "prop three two g"]

Prop 3.2h$_1$

[Prop 3.2h$_1$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}\colon [\ [$ S5 $\gg [\ [\ \underline{a} + 0\ ] = \underline{a}\ ]\ ]$; $[\ [$ Prop 3.2f $\gg [\ \underline{a} = [\ 0 + \underline{a}\ ]\ ]\ ]$; $[\ [\ [$ Prop 3.2c $\triangleright [\ [\ \underline{a} + 0\ ] = \underline{a}\ ]\ ] \triangleright [\ \underline{a} = [\ 0 + \underline{a}\ ]\ ]\ ] \gg [\ [\ \underline{a} + 0\ ] = [\ 0 + \underline{a}\ ]\ ]\ ]\ ]\ \rfloor, p_0, c)$]

[Prop 3.2h$_1$ $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon [\ [\ \underline{a} + 0\ ] = [\ 0 + \underline{a}\ ]\ ]$]

[Prop 3.2h$_1$ $\overset{\text{tex}}{\rightarrow}$ "
Prop\ 3.2h_1"]

[Prop 3.2h$_1$ $\overset{\text{pyk}}{\rightarrow}$ "prop three two h one"]

Prop 3.2h$_2$

[Prop 3.2h$_2$ $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \underline{a} + \underline{b}\ ] = [\ \underline{b} + \underline{a}\ ]\ ] \vdash [\ [\ [$ S2 $\triangleright [\ [\ \underline{a} + \underline{b}\ ] = [\ \underline{b} + \underline{a}\ ]\ ]\ ] \gg [\ [\ \underline{a} + \underline{b}\ ]' = [\ [\ \underline{b} + \underline{a}\ ]'\ ]\ ]\ ]$; $[\ [$ S6 $\gg [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ [\ \underline{a} + \underline{b}\ ]'\ ]\ ]\ ]$; $[\ [\ [\ [$ Prop 3.2c $\triangleright [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ [\ \underline{a} + \underline{b}\ ]'\ ]\ ]\ ] \triangleright [\ [\ \underline{a} + \underline{b}\ ]' = [\ [\ \underline{b} + \underline{a}\ ]'\ ]\ ]\ ] \gg [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ [\ \underline{b} + \underline{a}\ ]'\ ]\ ]\ ]$; $[\ [\ [$ Prop 3.2g $\gg [\ [\ \underline{b}' + \underline{a}\ ] = [\ [\ \underline{b} + \underline{a}\ ]'\ ]\ ]\ ]$; $[\ [\ [$ Prop 3.2d $\triangleright [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ [\ \underline{b} + \underline{a}\ ]'\ ]\ ]\ ] \triangleright [\ [\ \underline{b}' + \underline{a}\ ] = [\ [\ \underline{b} + \underline{a}\ ]'\ ]\ ]\ ] \gg [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ \underline{b}' + \underline{a}\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]$; $[\ [$ Ded $\triangleright \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \underline{a} + \underline{b}\ ] = [\ \underline{b} + \underline{a}\ ]\ ] \vdash [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ \underline{b}' + \underline{a}\ ]\ ]\ ]\ ] \gg [\ [\ [\ \underline{a} + \underline{b}\ ] = [\ \underline{b} + \underline{a}\ ]\ ] \Rightarrow [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ \underline{b}' + \underline{a}\ ]\ ]\ ]\ ]\ ]\ \rfloor, p_0, c)$]

[Prop 3.2h$_2$ $\overset{\text{stmt}}{\rightarrow}$ S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \underline{a} + \underline{b}\ ] = [\ \underline{b} + \underline{a}\ ]\ ] \Rightarrow [\ [\ \underline{a} + [\ \underline{b}'\ ]\ ] = [\ \underline{b}' + \underline{a}\ ]\ ]\ ]$]

[Prop 3.2h$_2$ $\overset{\text{tex}}{\rightarrow}$ "
Prop\ 3.2h_2"]

[Prop 3.2h$_2$ $\overset{\text{pyk}}{\rightarrow}$ "prop three two h two"]

Prop 3.2h

[Prop 3.2h $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$S $\vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [$ Prop 3.2h$_1 \gg [\ [\ \overline{x} + 0\ ] = [\ 0 + \overline{x}\ ]\ ]\ ]$; $[\ [$ Prop 3.2h$_2 \gg [\ [\ [\ \overline{x} + \overline{y}\ ] = [\ \overline{y} + \overline{x}\ ]\ ] \Rightarrow [\ [\ \overline{x} + [\ \overline{y}'\ ]\ ] = [\ \overline{y}' + \overline{x}$

$] \;] \;] \;] ; [\; [\; [\; [\mathrm{S9} @ \bar{y}] \rhd [\; [\bar{x} + 0] = [0 + \bar{x}] \;] \;] \rhd [\; [\; [\bar{x} + \bar{y}] = [\bar{y} + \bar{x}] \;] \Rightarrow [\; [\bar{x} + [\bar{y}'] \;] = [\bar{y}' + \bar{x}] \;] \;] \;] \gg [\; [\bar{x} + \bar{y}] = [\bar{y} + \bar{x}] \;] \;] \;] \;] ; [\; [\mathrm{Ded} \rhd [\; [\bar{x} + \bar{y}] = [\bar{y} + \bar{x}] \;] \;] \gg [\; [\underline{a} + \underline{b}] = [\underline{b} + \underline{a}] \;] \;] \;] \;] \rceil, p_0, c)]$

[Prop 3.2h $\stackrel{\text{stmt}}{\rightarrow}$ $S \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\; [\underline{a} + \underline{b}] = [\underline{b} + \underline{a}] \;] \;]$

[Prop 3.2h $\stackrel{\text{tex}}{\rightarrow}$ "
Prop\ 3.2h"]

[Prop 3.2h $\stackrel{\text{pyk}}{\rightarrow}$ "prop three two h"]

$\mathrm{Block}_1(*, *, *)$

$[\mathrm{Block}_1(t, s, c) \stackrel{\text{val}}{\rightarrow} t! \; [\; s! \; [\; c!\mathrm{let}_1(\lambda b.\mathrm{let}_1(\lambda x.\mathrm{let}_1(\lambda q.\mathrm{let}_1(\lambda q.\tilde{\mathcal{Q}}(t, \lceil b; q \rceil, [\; \lceil b \rceil :: b \;] :: [\; [\; \lceil q \rceil :: q \;] :: \mathsf{T} \;] \;), \tilde{\mathcal{M}}(q, s, c)), \tilde{\mathcal{Q}}(t, \lceil \mathbf{let} \; l \doteq x \; \text{ in } \; p \rceil, [\; \lceil l \rceil :: [\; t^2 \;] \;] :: [\; [\; \lceil p \rceil :: [\; t^3 \;] \;] :: [\; [\; \lceil x \rceil :: x \;] :: \mathsf{T} \;] \;] \;)), \mathrm{Block}_2(b)), \tilde{\mathcal{M}}(t^1, s, c)) \;] \;] \;]$

[$\mathrm{Block}_1(t, s, c)$ $\stackrel{\text{tex}}{\rightarrow}$ "
Block_1(#1.
, #2.
, #3.
)"]

[$\mathrm{Block}_1(t, s, c)$ $\stackrel{\text{pyk}}{\rightarrow}$ "block one * state * cache * end block"]

$\mathrm{Block}_2(*)$

$[\mathrm{Block}_2(b) \stackrel{\text{val}}{\rightarrow} \mathrm{If}(b \stackrel{r}{=} \lceil x \vdash y \rceil, \tilde{\mathcal{Q}}(b, \lceil x \vdash y \rceil, [\; \lceil x \rceil :: [\; b^1 \;] \;] :: [\; [\; \lceil y \rceil :: \mathrm{Block}_2(b^2) \;] :: \mathsf{T} \;] \;), \mathrm{If}(b \stackrel{r}{=} \lceil x \Vdash y \rceil, \tilde{\mathcal{Q}}(b, \lceil x \Vdash y \rceil, [\; \lceil x \rceil :: [\; b^1 \;] \;] :: [\; [\; \lceil y \rceil :: \mathrm{Block}_2(b^2) \;] :: \mathsf{T} \;] \;), \mathrm{If}(b \stackrel{r}{=} \lceil \forall x \colon y \rceil, \tilde{\mathcal{Q}}(b, \lceil \forall x \colon y \rceil, [\; \lceil x \rceil :: [\; b^1 \;] \;] :: [\; [\; \lceil y \rceil :: \mathrm{Block}_2(b^2) \;] :: \mathsf{T} \;] \;), \mathrm{If}(b \stackrel{r}{=} \lceil x; y \rceil, \mathrm{Block}_2(b^2), \mathrm{If}(b \stackrel{r}{=} \lceil x \gg y \rceil, b^2, \perp\!\!\!\perp)))))]$

[$\mathrm{Block}_2(b)$ $\stackrel{\text{tex}}{\rightarrow}$ "
Block_2(#1.
)"]

[$\mathrm{Block}_2(b)$ $\stackrel{\text{pyk}}{\rightarrow}$ "block two * end block"]

$*^{\text{hide}}$

Predef: hide

[x^{hide} $\stackrel{\text{tex}}{\rightarrow}$ "#1.
{}^{hide}"]

$[\mathsf{x}^{\mathrm{hide}} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ hide”}]$

MacroIndent(∗)

$[\mathrm{MacroIndent}(\mathsf{x}) \overset{\mathrm{name}}{\to} \text{“}$
MacroIndent(#1.
)”]

$[\mathrm{MacroIndent}(\mathsf{x}) \overset{\mathrm{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{MacroIndent}(\mathsf{x}) \doteq \mathsf{x}]\rceil)]$

$[\mathrm{MacroIndent}(\mathsf{x}) \overset{\mathrm{tex}}{\to} \text{“}$
$\%$
\leftskip=1em%
$#1.”]

$[\mathrm{MacroIndent}(\mathsf{x}) \overset{\mathrm{pyk}}{\to} \text{“macro indent } * \text{”}]$

$*'$

$[\mathsf{x}' \overset{\mathrm{tex}}{\to} \text{“}\#1.$
$\{\}\text{”}']$

$[\mathsf{x}' \overset{\mathrm{pyk}}{\to} \text{“}* \text{ suc”}]$

$* = *$

$[\mathsf{x} = \mathsf{y} \overset{\mathrm{tex}}{\to} \text{“}\#1.$
$= \#2.\text{”}]$

$[\mathsf{x} = \mathsf{y} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ equal } * \text{”}]$

$* \neq *$

$[\mathsf{x} \neq \mathsf{y} \overset{\mathrm{tex}}{\to} \text{“}\#1.$
$\backslash\mathrm{neq}\ \#2.\text{”}]$

$[\mathsf{x} \neq \mathsf{y} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ unequal } * \text{”}]$

$*^{\mathrm{var}}$

$[\mathsf{x}^{\mathrm{var}} \overset{\mathrm{val}}{\to} \mathsf{x} \overset{\mathrm{r}}{=} \lceil\bar{\mathsf{x}}\rceil]$

$[\mathsf{x}^{\mathrm{var}} \overset{\mathrm{tex}}{\to} \text{“\#1.}$
$\{\}\hat{}\{\mathrm{var}\}\text{”}]$

$[\mathsf{x}^{\mathrm{var}} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ is object var”}]$

$*\#^0*$

$[\mathsf{x}\#^0\mathsf{y} \overset{\mathrm{val}}{\to} \lambda \mathsf{c}.\ [\ [\ \mathsf{x}^{\mathrm{var}} \wedge\ [\ \mathsf{y}^{\mathcal{C}}\ ]\ ]\ \wedge\ [\ \mathsf{x}\#^1\mathsf{y}\ ]\ ]\ ]$

$[\mathsf{x}\#^0\mathsf{y} \overset{\mathrm{tex}}{\to} \text{“\#1.}$
$\backslash\#.\hat{}0\#2.\text{”}]$

$[\mathsf{x}\#^0\mathsf{y} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ avoid zero } *\text{”}]$

$*\#^1*$

$[\mathsf{x}\#^1\mathsf{y} \overset{\mathrm{val}}{\to} \mathrm{If}(\mathsf{y}^{\mathrm{var}}, \neg\ [\ \mathsf{x} \overset{\mathrm{t}}{=} \mathsf{y}\ ]\ ,$
$\mathrm{If}(\neg\ [\ \mathsf{y} \overset{\mathrm{r}}{=} \lceil \forall_{\mathrm{obj}} \mathsf{x}\colon \mathsf{y} \rceil\ ]\ , \mathsf{x}\#^*\ [\ \mathsf{y}^{\mathrm{t}}\ ]\ ,$
$\mathrm{If}(\mathsf{x} \overset{\mathrm{t}}{=}\ [\ \mathsf{y}^1\ ]\ , \mathsf{T}, \mathsf{x}\#^1\ [\ \mathsf{y}^2\ ]\ )))]$

$[\mathsf{x}\#^1\mathsf{y} \overset{\mathrm{tex}}{\to} \text{“\#1.}$
$\backslash\#.\hat{}1\#2.\text{”}]$

$[\mathsf{x}\#^1\mathsf{y} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ avoid one } *\text{”}]$

$*\#^**$

$[\mathsf{x}\#^*\mathsf{y} \overset{\mathrm{val}}{\to} \mathsf{x}!\mathrm{If}(\mathsf{y}, \mathsf{T}, \mathrm{If}(\mathsf{x}\#^1\ [\ \mathsf{y}^{\mathrm{h}}\ ]\ , \mathsf{x}\#^*\ [\ \mathsf{y}^{\mathrm{t}}\ ]\ , \mathsf{F}))]$

$[\mathsf{x}\#^*\mathsf{y} \overset{\mathrm{tex}}{\to} \text{“\#1.}$
$\backslash\#.\hat{}*\#2.\text{”}]$

$[\mathsf{x}\#^*\mathsf{y} \overset{\mathrm{pyk}}{\to} \text{“}* \text{ avoid star } *\text{”}]$

$\exists *\colon *$

$[\exists \mathsf{x}\colon \mathsf{y} \overset{\mathrm{tex}}{\to} \text{“}$
$\backslash\mathrm{exists}\ \#1.$
$\backslash\mathrm{colon}\ \#2.\text{”}]$

$[\exists \mathsf{x}\colon \mathsf{y} \overset{\mathrm{pyk}}{\to} \text{“exist } * \text{ indeed } *\text{”}]$

$\forall *\colon *$

$[\forall \mathsf{x}\colon \mathsf{y} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}(\mathrm{If}(\neg\ [\ \mathsf{t}^1 \overset{\mathrm{r}}{=} \lceil \mathsf{x}, \mathsf{y} \rceil\ ]\ , \tilde{\mathcal{Q}}(\mathsf{t}, \lceil \forall_{\mathrm{obj}} \mathsf{x}\colon \mathsf{y} \rceil,\ [\ \lceil \mathsf{x} \rceil :: [\ \mathsf{t}^1\ ]\ ] :: [\ [\ \lceil \mathsf{y} \rceil :: [\ \mathsf{t}^2\ ]\ ] :: \mathsf{T}\ ]\ ), \tilde{\mathcal{Q}}(\mathsf{t}, \lceil \forall_{\mathrm{obj}} \mathsf{x}\colon \forall \mathsf{y}\colon \mathsf{z} \rceil,\ [\ \lceil \mathsf{x} \rceil :: [\ \mathsf{t}^{11}\ ]\ ] :: [\ [\ \lceil \mathsf{y} \rceil :: [\ \mathsf{t}^{12}\ ]\ ] :: [\ [\ \lceil \mathsf{z} \rceil :: [\ \mathsf{t}^2\ ]\ ] :: \mathsf{T}\ ]\ ]\ )), \mathsf{s}, \mathsf{c})]$

$[\forall \mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\to}$ “
\forall #1.
\colon #2.”]

$[\forall \mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\to}$ “for all $*$ indeed $*$”]

$\forall_{\mathrm{obj}} *\colon *$

$[\forall_{\mathrm{obj}} \mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\to}$ “
\forall_{obj} #1.
\colon #2.”]

$[\forall_{\mathrm{obj}} \mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\to}$ “for all objects $*$ indeed $*$”]

$* \Rightarrow *$

$[\mathsf{x} \Rightarrow \mathsf{y} \overset{\text{tex}}{\to}$ “#1.
\Rightarrow #2.”]

$[\mathsf{x} \Rightarrow \mathsf{y} \overset{\text{pyk}}{\to}$ “$*$ imply $*$”]

$* \Leftrightarrow *$

$[\mathsf{x} \Leftrightarrow \mathsf{y} \overset{\text{tex}}{\to}$ “#1.
\Leftrightarrow #2.”]

$[\mathsf{x} \Leftrightarrow \mathsf{y} \overset{\text{pyk}}{\to}$ “$*$ if and only if $*$”]

$*\#*$

$[\mathsf{x}\#\mathsf{y} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x}\#\mathsf{y} \doteq \lceil \mathsf{x} \rceil \#^0 \lceil \mathsf{y} \rceil] \rceil)]$

$[\mathsf{x}\#\mathsf{y} \overset{\text{tex}}{\to}$ “#1.
\#.#2.”]

$[\mathsf{x}\#\mathsf{y} \overset{\text{pyk}}{\to}$ “$*$ avoid $*$”]

$* \trianglerighteq *$

$[\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \trianglerighteq \mathsf{y} \doteq [\,\mathrm{MP} \triangleright \mathsf{x}\,] \triangleright \mathsf{y}]\rceil)]$

$[\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{tex}}{\to} \text{“\#1.}$
$\text{\unrhd \#2.”}]$

$[\mathsf{x} \trianglerighteq \mathsf{y} \overset{\text{pyk}}{\to} \text{“}* \text{ object modus ponens } *\text{”}]$

$\Pi *: *$

$[\Pi \mathsf{x}\colon \mathsf{y} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}(\mathrm{If}(\neg\,[\,\mathsf{t}^1 \overset{\text{r}}{=} \lceil\mathsf{x},\mathsf{y}\rceil\,]\,, \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\forall \mathsf{x}\colon \mathsf{y}\rceil, [\,\lceil\mathsf{x}\rceil :: [\,\mathsf{t}^1\,]\,] :: [\,[\,\lceil\mathsf{y}\rceil :: [\,\mathsf{t}^2\,]\,] :: \mathsf{T}\,]\,), \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\forall \mathsf{x}\colon \Pi \mathsf{y}\colon \mathsf{z}\rceil, [\,\lceil\mathsf{x}\rceil :: [\,\mathsf{t}^{11}\,]\,] :: [\,[\,\lceil\mathsf{y}\rceil :: [\,\mathsf{t}^{12}\,]\,] :: [\,[\,\lceil\mathsf{z}\rceil :: [\,\mathsf{t}^2\,]\,] :: \mathsf{T}\,]\,]\,)), \mathsf{s}, \mathsf{c})]$

$[\Pi \mathsf{x}\colon \mathsf{y} \overset{\text{tex}}{\to} \text{“}$
$\text{\Pi \#1.}$
$\text{\colon \#2.”}]$

$[\Pi \mathsf{x}\colon \mathsf{y} \overset{\text{pyk}}{\to} \text{“for all terms } * \text{ indeed } *\text{”}]$

Begin $*; * :$ End; $*$

$[\mathrm{Begin}\,\mathsf{b}; \mathsf{l} : \mathrm{End}; \mathsf{p} \overset{\text{name}}{\to} \text{“}$
$\text{Begin \textbackslash, \#1.}$
$\text{; \#2.}$
$\text{: End ; \#3.”}]$

$[\mathrm{Begin}\,\mathsf{b}; \mathsf{l} : \mathrm{End}; \mathsf{p} \overset{\text{macro}}{\to} \lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\mathrm{Block}_1(\mathsf{t},\mathsf{s},\mathsf{c})]$

$[\mathrm{Begin}\,\mathsf{b}; \mathsf{l} : \mathrm{End}; \mathsf{p} \overset{\text{tex}}{\to} \text{“}$
$\text{\newline \makebox [0.1\textwidth]\{\}\%}$
$\text{\parbox [b]\{0.4\textwidth \}\{\raggedright}$
$\text{\setlength \{\parindent \}\{-0.1\textwidth \}\%}$
$\text{\makebox [0.1\textwidth][l]\{\$}$
$\text{\if \relax \csname lgwprooflinep\endcsname L\_? \else}$
$\text{\global \advance \lgwproofline by 1}$
$\text{L\ifnum \lgwproofline <10 0\fi \number \lgwproofline}$
$\text{\fi}$
$\text{\$:\}\$Block \{\}\gg \{\}\$\}\quad}$
$\text{\parbox [t]\{0.4\textwidth \}\{\$Begin}$
$\text{\$\hfill \makebox [0mm][l]\{\quad ;\}\}\#1.}$
$\text{\newline \makebox [0.1\textwidth]\{\}\%}$
$\text{\parbox [b]\{0.4\textwidth \}\{\raggedright}$
$\text{\setlength \{\parindent \}\{-0.1\textwidth \}\%}$

\makebox [0.1\textwidth][l]{$#2.
$:}$Block {}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$End
$\hfill \makebox [0mm][l]{\quad ;}}#3."]

[Begin $\mathsf{b}; \mathsf{l} : \mathrm{End}; \mathsf{p} \overset{\mathrm{pyk}}{\rightarrow}$ "block $*$ line $*$ end block $*$"]

Last block line $* \gg *$;

[Last block line $\mathsf{a} \gg \mathsf{i}\,; \overset{\mathrm{name}}{\rightarrow}$ "
Last\ block\ line \, #1.
\gg #2.
\, ;"]

[Last block line $\mathsf{a} \gg \mathsf{i}\,; \overset{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\text{Last block line } \mathsf{a} \gg \mathsf{i}\,; \doteqdot (\mathsf{a} \gg \mathsf{i})]\rceil)$]

[Last block line $\mathsf{a} \gg \mathsf{i}\,; \overset{\mathrm{tex}}{\rightarrow}$ "
\newline \makebox [0.1\textwidth]{}%
\parbox [b]{0.4\textwidth }{\raggedright
\setlength {\parindent }{-0.1\textwidth }%
\makebox [0.1\textwidth][l]{$
\if \relax \csname lgwprooflinep\endcsname L_? \else
\global \advance \lgwproofline by 1
L\ifnum \lgwproofline <10 0\fi \number \lgwproofline
\fi
$:}$#1.
{}\gg {}$}\quad
\parbox [t]{0.4\textwidth }{$#2.
$\hfill \makebox [0mm][l]{\quad ;}}"]

[Last block line $\mathsf{l} \gg \mathsf{i}\,; \overset{\mathrm{pyk}}{\rightarrow}$ "because $*$ indeed $*$ end line"]

Arbitrary $\gg *; *$

[Arbitrary $\gg \mathsf{i}; \mathsf{p} \overset{\mathrm{name}}{\rightarrow}$ "
Arbitrary \gg #1.
; #2."]

[Arbitrary $\gg \mathsf{i}; \mathsf{p} \overset{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\text{Arbitrary} \gg \mathsf{i}; \mathsf{p} \doteqdot \Pi \mathsf{i}\colon \mathsf{p}]\rceil)$]

[Arbitrary $\gg \mathsf{i}; \mathsf{p} \overset{\mathrm{tex}}{\rightarrow}$ "
\newline \makebox [0.1\textwidth][l]{$
\if \relax \csname lgwprooflinep\endcsname L_? \else
\global \advance \lgwproofline by 1

L\ifnum \lgwproofline <10 0\fi \number \lgwproofline
\fi
$:}\makebox [0.4\textwidth][l]{$Arbitrary{}\gg{}$}\quad
\parbox [t]{0.4\textwidth }{$#1.
$\hfill \makebox [0mm][l]{\quad ; }}#2."]

[Arbitrary $\gg$ i; p $\overset{\text{pyk}}{\to}$ "any term $*$ end line $*$"]

$* \mid *$

[x | y $\overset{\text{tex}}{\to}$ "#1.
\mathrel{|} #2."]

[x | y $\overset{\text{pyk}}{\to}$ "$*$ alternative $*$"]

$\longrightarrow$

[$\to\overset{\text{tex}}{\to}$ "
\rightarrow "]

[$\to\overset{\text{pyk}}{\to}$ "evaluates to"]

$*\backslash\backslash *$

[x\\y $\overset{\text{name}}{\to}$ "#1.
\backslash \backslash #2."]

[x\\y $\overset{\text{tex}}{\to}$ "#1.
\\{}#2."]

[x\\y $\overset{\text{pyk}}{\to}$ "$*$ safe row $*$"]

The pyk compiler, version 0.grue.20060417 by Klaus Grue
GRD-2006-03-06.UTC:13:37:57.803308 = MJD-53800.TAI:13:38:30.803308 = LGT-4648369110803308e-6