

Logiweb codex of check

Up Help

\mathcal{X} , check, $[* \overset{\circ}{*}]$, RootVisible(*), A, R, C, T, L, {*}, $\bar{*}$, a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, $\langle * \equiv * \mid * := * \rangle$, $\langle * \equiv^0 * \mid * := * \rangle$, $\langle * \equiv^1 * \mid * := * \rangle$, $\langle * \equiv^* * \mid * := * \rangle$, Ded(*, *), Ded₀(*, *), Ded₁(*, *, *), Ded₂(*, *, *), Ded₃(*, *, *, *), Ded₄(*, *, *, *), Ded₄^{*}(*, *, *, *), Ded₅(*, *, *), Ded₆(*, *, *, *), Ded₆^{*}(*, *, *, *), Ded₇(*), Ded₈(*, *), Ded₈^{*}(*, *), S, Neg, MP, Gen, Ded, S1, S2, S3, S4, S5, S6, S7, S8, S9, Repetition, A1', A2', A4', A5', Prop 3.2a, Prop 3.2b, Prop 3.2c, Prop 3.2d, Prop 3.2e₁, Prop 3.2e₂, Prop 3.2e, Prop 3.2f₁, Prop 3.2f₂, Prop 3.2f, Prop 3.2g₁, Prop 3.2g₂, Prop 3.2g, Prop 3.2h₁, Prop 3.2h₂, Prop 3.2h, Block₁(*, *, *), Block₂(*), *^{hide}, MacroIndent(*), *', * = *, * ≠ *, *^{var}, *#⁰*, *#¹*, *#^{*}*, ∃*:*, ∀*:*, ∇_{obj}*:*, * ⇒ *, * ⇔ *, *#*, * ⊇ *, Π*:*, Begin*:*, End*:*, Last block line* ≫ *;*, Arbitrary ≫ *;*, * | *, →, * \ *,

\mathcal{X}

$[\mathcal{X} \xrightarrow{\text{stmt}} \mathcal{X}]$

check

$[\text{check} \xrightarrow{\text{prio}}$

Preassociative

[check], [base], [bracket * end bracket], [big bracket * end bracket], [\$ * \$], [flush left [*], [x], [y], [z], [[* ⋈ *], [[* ^{*} *], [pyk], [tex], [name], [prio], [*], [T], [if(*, *, *)], [[* ^{*} *], [val], [claim], [⊥], [f(*)], [(*)^l], [F], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(*)^M], [If(*, *, *)], [array{*} * end array], [l], [c], [r], [empty], [(* | * := *)], [\mathcal{M} (*)], [$\tilde{\mathcal{U}}$ (*)], [\mathcal{U} (*)], [\mathcal{U}^M (*)], [apply(*, *)], [apply₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)], [bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [\mathcal{E} (*, *, *)], [\mathcal{E}_2 (*, *, *, *)], [\mathcal{E}_3 (*, *, *, *)], [\mathcal{E}_4 (*, *, *, *)], [lookup(*, *, *)], [abstract(*, *, *, *)], [[*], [\mathcal{M} (*, *, *)], [\mathcal{M}_2 (*, *, *, *)], [\mathcal{M}^* (*, *, *)], [macro],

$[so]$, $[zip(*, *)]$, $[assoc_1(*, *, *)]$, $[(*)^P]$, $[self]$, $[[* \doteq *]]$, $[[* \dot{=} *]]$, $[[* \dot{=} *]]$,
 $[[* \stackrel{pyk}{=} *]]$, $[[* \stackrel{tex}{=} *]]$, $[[* \stackrel{name}{=} *]]$, $[Priority\ table[*]]$, $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$,
 $[\tilde{\mathcal{M}}_4(*, *, *, *)]$, $[\mathcal{M}(*, *, *)]$, $[\tilde{\mathcal{Q}}(*, *, *)]$, $[\tilde{\mathcal{Q}}_2(*, *, *)]$, $[\tilde{\mathcal{Q}}_3(*, *, *, *)]$, $[\tilde{\mathcal{Q}}^*(*, *, *)]$,
 $[(*)]$, $[(*)]$, $[display(*)]$, $[statement(*)]$, $[[*]]$, $[[*^-]]$, $[aspect(*, *)]$,
 $[aspect(*, *, *)]$, $[(*)]$, $[tuple_1(*)]$, $[tuple_2(*)]$, $[let_2(*, *)]$, $[let_1(*, *)]$,
 $[[* \stackrel{claim}{=} *]]$, $[checker]$, $[check(*, *)]$, $[check_2(*, *, *)]$, $[check_3(*, *, *)]$,
 $[check^*(*, *)]$, $[check_2^*(*, *, *)]$, $[[*]]$, $[[*^-]]$, $[[*^\circ]]$, $[msg]$, $[[* \stackrel{msg}{=} *]]$, $[<stmt>]$,
 $[stmt]$, $[[* \stackrel{stmt}{=} *]]$, $[HeadNil']$, $[HeadPair']$, $[Transitivity']$, $[⊥]$, $[Contra']$, $[T'_E]$,
 $[L_1]$, $[*]$, $[A]$, $[B]$, $[C]$, $[D]$, $[E]$, $[F]$, $[G]$, $[H]$, $[I]$, $[J]$, $[K]$, $[L]$, $[M]$, $[N]$, $[O]$, $[P]$, $[Q]$,
 $[R]$, $[S]$, $[T]$, $[U]$, $[V]$, $[W]$, $[X]$, $[Y]$, $[Z]$, $[[* \mid * := *]]$, $[[* \mid * := *]]$, $[\emptyset]$, $[Remainder]$,
 $[(*)^\vee]$, $[intro(*, *, *, *)]$, $[intro(*, *, *)]$, $[error(*, *)]$, $[error_2(*, *)]$, $[proof(*, *, *)]$,
 $[proof_2(*, *)]$, $[S(*, *)]$, $[S^1(*, *)]$, $[S^\triangleright(*, *)]$, $[S_1^\triangleright(*, *, *)]$, $[S^E(*, *)]$, $[S_1^E(*, *, *)]$,
 $[S^+(*, *)]$, $[S_1^+(*, *, *)]$, $[S^-(*, *)]$, $[S_1^-(*, *, *)]$, $[S^*(*, *)]$, $[S_1^*(*, *, *)]$,
 $[S_2^*(*, *, *, *)]$, $[S^\circ(*, *)]$, $[S_1^\circ(*, *, *)]$, $[S^+(*, *)]$, $[S_1^+(*, *, *, *)]$, $[S^{\#}(*, *)]$,
 $[S_1^{\#}(*, *, *, *)]$, $[S^{i.e.}(*, *)]$, $[S_1^{i.e.}(*, *, *, *)]$, $[S_2^{i.e.}(*, *, *, *, *)]$, $[S^\vee(*, *)]$,
 $[S_1^\vee(*, *, *, *)]$, $[S^i(*, *)]$, $[S_1^i(*, *, *, *)]$, $[S_2^i(*, *, *, *, *)]$, $[T(*)]$, $[claims(*, *, *)]$,
 $[claims_2(*, *, *)]$, $[<proof>]$, $[proof]$, $[Lemma\ * : *]$, $[Proof\ of\ * : *]$,
 $[* \text{ lemma } * : *]$, $[* \text{ antilemma } * : *]$, $[* \text{ rule } * : *]$, $[* \text{ antirule } * : *]$,
 $[verifier]$, $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*, *)]$, $[\mathcal{V}_3(*, *, *, *)]$, $[\mathcal{V}_4(*, *)]$, $[\mathcal{V}_5(*, *, *, *)]$, $[\mathcal{V}_6(*, *, *, *, *)]$,
 $[\mathcal{V}_7(*, *, *, *)]$, $[Cut(*, *)]$, $[Head_\oplus(*)]$, $[Tail_\oplus(*)]$, $[rule_1(*, *)]$, $[rule(*, *)]$,
 $[Rule\ tactic]$, $[Plus(*, *)]$, $[Theory\ *]$, $[theory_2(*, *)]$, $[theory_3(*, *)]$,
 $[theory_4(*, *, *)]$, $[HeadNil'']$, $[HeadPair'']$, $[Transitivity'']$, $[Contra'']$, $[HeadNil]$,
 $[HeadPair]$, $[Transitivity]$, $[Contra]$, $[T_E]$, $[ragged\ right]$,
 $[ragged\ right\ expansion]$, $[parm(*, *, *)]$, $[parm^*(*, *, *)]$, $[inst(*, *)]$,
 $[inst^*(*, *)]$, $[occur(*, *, *)]$, $[occur^*(*, *, *)]$, $[unify(* = *, *)]$, $[unify^*(* = *, *)]$,
 $[unify_2(* = *, *)]$, $[L_a]$, $[L_b]$, $[L_c]$, $[L_d]$, $[L_e]$, $[L_f]$, $[L_g]$, $[L_h]$, $[L_i]$, $[L_j]$, $[L_k]$, $[L_l]$, $[L_m]$,
 $[L_n]$, $[L_o]$, $[L_p]$, $[L_q]$, $[L_r]$, $[L_s]$, $[L_t]$, $[L_u]$, $[L_v]$, $[L_w]$, $[L_x]$, $[L_y]$, $[L_z]$, $[L_A]$, $[L_B]$, $[L_C]$,
 $[L_D]$, $[L_E]$, $[L_F]$, $[L_G]$, $[L_H]$, $[L_I]$, $[L_J]$, $[L_K]$, $[L_L]$, $[L_M]$, $[L_N]$, $[L_O]$, $[L_P]$, $[L_Q]$, $[L_R]$,
 $[L_S]$, $[L_T]$, $[L_U]$, $[L_V]$, $[L_W]$, $[L_X]$, $[L_Y]$, $[L_Z]$, $[L_?]$, $[Reflexivity]$, $[Reflexivity_1]$,
 $[Commutativity]$, $[Commutativity_1]$, $[<tactic>]$, $[tactic]$, $[[* \stackrel{tactic}{=} *]]$, $[P(*, *, *)]$,
 $[P^*(*, *, *)]$, $[p_0]$, $[conclude_1(*, *)]$, $[conclude_2(*, *, *)]$, $[conclude_3(*, *, *, *)]$,
 $[conclude_4(*, *)]$, $[[* \stackrel{\circ}{=} *]]$, $[RootVisible(*)]$, $[A]$, $[R]$, $[C]$, $[T]$, $[L]$, $[*]$, $[a]$, $[b]$,
 $[c]$, $[d]$, $[e]$, $[f]$, $[g]$, $[h]$, $[i]$, $[j]$, $[k]$, $[l]$, $[m]$, $[n]$, $[o]$, $[p]$, $[q]$, $[r]$, $[s]$, $[t]$, $[u]$, $[v]$, $[w]$, $[x]$,
 $[y]$, $[z]$, $[[* \equiv * \mid * := *]]$, $[[* \equiv 0 \mid * := *]]$, $[[* \equiv 1 \mid * := *]]$, $[[* \equiv * \mid * := *]]$,
 $[Ded(*, *)]$, $[Ded_0(*, *)]$, $[Ded_1(*, *, *)]$, $[Ded_2(*, *, *)]$, $[Ded_3(*, *, *, *)]$,
 $[Ded_4(*, *, *, *)]$, $[Ded_4^*(*, *, *, *)]$, $[Ded_5(*, *, *)]$, $[Ded_6(*, *, *, *)]$,
 $[Ded_6^*(*, *, *, *)]$, $[Ded_7(*)]$, $[Ded_8(*, *)]$, $[Ded_8^*(*, *)]$, $[S]$, $[Neg]$, $[MP]$, $[Gen]$,
 $[Ded]$, $[S1]$, $[S2]$, $[S3]$, $[S4]$, $[S5]$, $[S6]$, $[S7]$, $[S8]$, $[S9]$, $[Repetition]$, $[A1']$, $[A2']$, $[A4']$,
 $[A5']$, $[Prop\ 3.2a]$, $[Prop\ 3.2b]$, $[Prop\ 3.2c]$, $[Prop\ 3.2d]$, $[Prop\ 3.2e_1]$, $[Prop\ 3.2e_2]$,
 $[Prop\ 3.2e]$, $[Prop\ 3.2f_1]$, $[Prop\ 3.2f_2]$, $[Prop\ 3.2f]$, $[Prop\ 3.2g_1]$, $[Prop\ 3.2g_2]$,
 $[Prop\ 3.2g]$, $[Prop\ 3.2h_1]$, $[Prop\ 3.2h_2]$, $[Prop\ 3.2h]$, $[Block_1(*, *, *)]$, $[Block_2(*)]$;
Preassociative
 $[* \{ * \}]$, $[*/indexintro(*, *, *, *)]$, $[*/intro(*, *, *)]$, $[*/bothintro(*, *, *, *, *)]$,
 $[*/nameintro(*, *, *, *)]$, $[*']$, $[[* \ *]]$, $[[* \ * \rightarrow *]]$, $[[* \ * \Rightarrow *]]$, $[*0]$, $[*1]$, $[0b]$, $[* \text{-color} (*)]$,

[*-color* (*), [*H], [*T], [*U], [*h], [*t], [*s], [*c], [*d], [*a], [*C], [*M], [*B], [*F], [*i], [*d], [*R], [*0], [*1], [*2], [*3], [*4], [*5], [*6], [*7], [*8], [*9], [*E], [*V], [*C], [*C[#]], [*hide];

Preassociative

[“ * ”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [*, [*], [!*, [\"*], [#*], [*\$], [%*], [&*], [’*], [(*)], [*], [**], [+*], [*, [*], [-*], [.*], [/ *], [0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [*:], [*;], [*< *], [*= *], [*> *], [*? *], [@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*], [O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [\ *], [] *], [^ *], [_ *], [‘ *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *], [p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [{ *], [| *], [} *], [~ *], [Preassociative *; *], [Postassociative *; *], [[*], [*], [priority * end], [newline *], [macro newline *], [MacroIndent(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*’];

Preassociative

[* · *], [* · 0 *];

Preassociative

[* + *], [* + 0 *], [* + 1 *], [* - *], [* - 0 *], [* - 1 *];

Preassociative

[* ∪ { * }, [* ∪ *], [* \ { * }];

Postassociative

[* . : *], [* . : *], [* : : *], [* + 2 * *], [* : : *], [* + 2 * *];

Postassociative

[* , *];

Preassociative

[* ^B ≈ *], [* ^D ≈ *], [* ^C ≈ *], [* ^P ≈ *], [* ≈ *], [* = *], [* → *], [* ^t = *], [* ^{t*} = *], [* ^r = *], [* ∈_T *], [* ⊆_T *], [* ^T = *], [* ^s = *], [* free in *], [* free in* *], [* free for * in *], [* free for* * in *], [* ∈_C *], [* < *], [* <’ *], [* ≤’ *], [* = *], [* ≠ *], [*^{var}], [*#⁰ *], [*#¹ *], [*#* *];

Preassociative

[¬*];

Preassociative

[* ∧ *], [* ^λ *], [* ^λ *], [* ∧_C *];

Preassociative

[* ∨ *], [* || *], [* [∨] *];

Preassociative

[∃* : *], [∀* : *], [∀_{obj}* : *];

Postassociative

[* [⇒] *], [* ⇒ *], [* ⇔ *];

Postassociative

[* : *], [* spy *], [*! *];

Preassociative

[* $\left\{ \begin{array}{c} * \\ * \end{array} \right.$ *];

Preassociative

[λ * .*], [Λ * .*], [Λ *], [if * then * else *], [let * = * in *], [let * \doteq * in *];

Preassociative

[*#*];

Preassociative

[*^I], [*[▷]], [*^V], [*⁺], [*⁻], [*^{*}];

Preassociative

[* @ *], [* ▷ *], [* ▷ *], [* ≫ *], [* ≳ *];

Postassociative

[* ⊢ *], [* ⊢ *], [* i.e. *];

Preassociative

[\forall *: *], [Π *: *];

Postassociative

[* ⊕ *];

Postassociative

[*; *];

Preassociative

[* proves *];

Preassociative

[* **proof of** * : *], [Line * : * ≫ *; *], [Last line * ≫ * □],
[Line * : Premise ≫ *; *], [Line * : Side-condition ≫ *; *], [Arbitrary ≫ *; *],
[Local ≫ * = *; *], [Begin *; * : End; *], [Last block line * ≫ *; *],
[Arbitrary ≫ *; *];

Postassociative

[* | *];

Postassociative

[* , *], [* [* *]];

Preassociative

[*&*], [→];

Preassociative

[* \\ *], [* linebreak[4] *], [* \\ *];

[check $\xrightarrow{\text{pyk}}$ “check”]

[* $\overset{\circ}{=}$ *]

[$[x \overset{\circ}{=} y] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[[x \overset{\circ}{=} y] \doteq [(x) \overset{\text{macro}}{\rightarrow} y]])]]$]

[$[x \overset{\circ}{=} y] \xrightarrow{\text{tex}}$ “

[#1/tex name/tex.

\stackrel{\circ}{=} {=} #2.

]”]

$[[* \overset{\circ}{=} *] \xrightarrow{\text{pyk}} \text{"general macro define " as " end define"}]$

RootVisible(*)

$[\text{RootVisible}(x) \xrightarrow{\text{name}} \text{"RootVisible(\#1.} \\ \text{)}"]]$

$[\text{RootVisible}(x) \xrightarrow{\text{tex}} \text{"\#1/tex name/tex."}]$

$[\text{RootVisible}(*) \xrightarrow{\text{pyk}} \text{"make root visible " end visible"}]$

A

$[A \xrightarrow{\text{tex}} \text{"A"}]$

$[A \xrightarrow{\text{pyk}} \text{"sequent example axiom"}]$

R

$[R \xrightarrow{\text{tex}} \text{"R"}]$

$[R \xrightarrow{\text{pyk}} \text{"sequent example rule"}]$

C

$[C \xrightarrow{\text{tex}} \text{"C"}]$

$[C \xrightarrow{\text{pyk}} \text{"sequent example contradiction"}]$

T

$[T \xrightarrow{\text{tex}} \text{"T"}]$

$[T \xrightarrow{\text{pyk}} \text{"sequent example theory"}]$

L

[L $\xrightarrow{\text{tex}}$ “
L”]

[L $\xrightarrow{\text{pyk}}$ “sequent example lemma”]

{*}

[{x} $\xrightarrow{\text{tex}}$ “
\{#1.
\}”]

[{*} $\xrightarrow{\text{pyk}}$ “set " end set”]

$\overline{*}$

[\overline{x} $\xrightarrow{\text{tex}}$ “\overline{\#1.}”]

[$\overline{*}$ $\xrightarrow{\text{pyk}}$ “object var " end var”]

a

[*a* $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[a \doteq \bar{a}]])$]

[*a* $\xrightarrow{\text{tex}}$ “
\mathit{a}”]

[*a* $\xrightarrow{\text{pyk}}$ “object a”]

b

[*b* $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[b \doteq \bar{b}]])$]

[*b* $\xrightarrow{\text{tex}}$ “
\mathit{b}”]

[*b* $\xrightarrow{\text{pyk}}$ “object b”]

c

[*c* $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[c \doteq \bar{c}]])$]

[$c \xrightarrow{\text{tex}}$ “
 $\backslash\mathit{c}$ ”]

[$c \xrightarrow{\text{pyk}}$ “object c”]

d

[$d \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[d \doteq \bar{d}]])$]

[$d \xrightarrow{\text{tex}}$ “
 $\backslash\mathit{d}$ ”]

[$d \xrightarrow{\text{pyk}}$ “object d”]

e

[$e \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[e \doteq \bar{e}]])$]

[$e \xrightarrow{\text{tex}}$ “
 $\backslash\mathit{e}$ ”]

[$e \xrightarrow{\text{pyk}}$ “object e”]

f

[$f \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[f \doteq \bar{f}]])$]

[$f \xrightarrow{\text{tex}}$ “
 $\backslash\mathit{f}$ ”]

[$f \xrightarrow{\text{pyk}}$ “object f”]

g

[$g \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[g \doteq \bar{g}]])$]

[$g \xrightarrow{\text{tex}}$ “
 $\backslash\mathit{g}$ ”]

[$g \xrightarrow{\text{pyk}}$ “object g”]

h

[$h \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[h \doteq \bar{h}]])$]

[$h \xrightarrow{\text{tex}}$ “
`\mathit{h}`”]

[$h \xrightarrow{\text{pyk}}$ “object h”]

i

[$i \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[i \doteq \bar{i}]])$]

[$i \xrightarrow{\text{tex}}$ “
`\mathit{i}`”]

[$i \xrightarrow{\text{pyk}}$ “object i”]

j

[$j \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[j \doteq \bar{j}]])$]

[$j \xrightarrow{\text{tex}}$ “
`\mathit{j}`”]

[$j \xrightarrow{\text{pyk}}$ “object j”]

k

[$k \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[k \doteq \bar{k}]])$]

[$k \xrightarrow{\text{tex}}$ “
`\mathit{k}`”]

[$k \xrightarrow{\text{pyk}}$ “object k”]

l

[$l \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[l \doteq \bar{l}]])$]

[$l \xrightarrow{\text{tex}}$ “
`\mathit{l}`”]

[$l \xrightarrow{\text{pyk}}$ “object l”]

m

[$m \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[m \doteq \bar{m}]])$]

[$m \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{m\}\text{”}$]

[$m \xrightarrow{\text{pyk}} \text{“object } m\text{”}$]

n

[$n \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[n \doteq \bar{n}]])$]

[$n \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{n\}\text{”}$]

[$n \xrightarrow{\text{pyk}} \text{“object } n\text{”}$]

o

[$o \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[o \doteq \bar{o}]])$]

[$o \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{o\}\text{”}$]

[$o \xrightarrow{\text{pyk}} \text{“object } o\text{”}$]

p

[$p \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[p \doteq \bar{p}]])$]

[$p \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{p\}\text{”}$]

[$p \xrightarrow{\text{pyk}} \text{“object } p\text{”}$]

q

[$q \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[q \doteq \bar{q}]])$]

[$q \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{q\}\text{”}$]

[$q \xrightarrow{\text{pyk}} \text{“object } q\text{”}$]

r

$[r \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[r \doteq \bar{r}]])]$

$[r \xrightarrow{\text{tex}} \text{"}\backslash\text{mathit}\{r\}\text{"}]$

$[r \xrightarrow{\text{pyk}} \text{"object r"}]$

s

$[s \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[s \doteq \bar{s}]])]$

$[s \xrightarrow{\text{tex}} \text{"}\backslash\text{mathit}\{s\}\text{"}]$

$[s \xrightarrow{\text{pyk}} \text{"object s"}]$

t

$[t \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[t \doteq \bar{t}]])]$

$[t \xrightarrow{\text{tex}} \text{"}\backslash\text{mathit}\{t\}\text{"}]$

$[t \xrightarrow{\text{pyk}} \text{"object t"}]$

u

$[u \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[u \doteq \bar{u}]])]$

$[u \xrightarrow{\text{tex}} \text{"}\backslash\text{mathit}\{u\}\text{"}]$

$[u \xrightarrow{\text{pyk}} \text{"object u"}]$

v

$[v \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[v \doteq \bar{v}]])]$

$[v \xrightarrow{\text{tex}} \text{"}\backslash\text{mathit}\{v\}\text{"}]$

$[v \xrightarrow{\text{pyk}} \text{"object v"}]$

w

$[w \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[w \doteq \bar{w}]])]$

$[w \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{w\}\text{”}]$

$[w \xrightarrow{\text{pyk}} \text{“object } w\text{”}]$

x

$[x \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \doteq \bar{x}]])]$

$[x \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{x\}\text{”}]$

$[x \xrightarrow{\text{pyk}} \text{“object } x\text{”}]$

y

$[y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[y \doteq \bar{y}]])]$

$[y \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{y\}\text{”}]$

$[y \xrightarrow{\text{pyk}} \text{“object } y\text{”}]$

z

$[z \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[z \doteq \bar{z}]])]$

$[z \xrightarrow{\text{tex}} \text{“}\backslash\text{mathit}\{z\}\text{”}]$

$[z \xrightarrow{\text{pyk}} \text{“object } z\text{”}]$

$\langle * \equiv * \mid * := * \rangle$

$[\langle a \equiv b \mid x := t \rangle \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\langle a \equiv b \mid x := t \rangle \doteq \langle [a] \equiv^0 [b] \mid [x] := [t] \rangle]])]$

$[\langle x \equiv y \mid z := u \rangle \xrightarrow{\text{tex}} \text{“}\backslash\text{langle } \#1.$

$\{\backslash\text{equiv}\} \#2.$

$\mid \#3.$

{:=} #4.
 \rangle ”]

[⟨*≡* | * :=*⟩^{pyk} “sub " is " where " is " end sub”]

⟨*≡⁰* | * :=*⟩

[⟨a≡⁰b|x:=t⟩^{val} λc.x^{var} ∧ ⟨a≡¹b|x:=t⟩]

[⟨x≡⁰y|z:=u⟩^{tex} “

\rangle #1.

{\equiv}^0 #2.

| #3.

{:=} #4.

\rangle ”]

[⟨*≡⁰* | * :=*⟩^{pyk} “sub zero " is " where " is " end sub”]

⟨*≡¹* | * :=*⟩

[⟨a≡¹b|x:=t⟩^{val} a!x!t!

If(If(b ^r [∇_{obj}u:v], b¹ ^t x, F), a ^t b,

If(b^{var} ∧ b ^t x, a ^t t, If(

a ^r b, ⟨a^t≡*b^t|x:=t⟩, F)))]

[⟨x≡¹y|z:=u⟩^{tex} “

\rangle #1.

{\equiv}^1 #2.

| #3.

{:=} #4.

\rangle ”]

[⟨*≡¹* | * :=*⟩^{pyk} “sub one " is " where " is " end sub”]

⟨*≡* | * :=*⟩

[⟨a≡*b|x:=t⟩^{val} b!x!t!If(a, T, If(⟨a^h≡¹b^h|x:=t⟩, ⟨a^t≡*b^t|x:=t⟩, F)))]

[⟨x≡*y|z:=u⟩^{tex} “

\rangle #1.

{\equiv}^* #2.

| #3.

{:=} #4.
\rangle ”]

[(<≡* * | * :=*) $\xrightarrow{\text{pyk}}$ “sub star " is " where " is " end sub”]

Ded(*, *)

[Ded(p, c) $\xrightarrow{\text{macro}}$ $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket \text{Ded}(p, c) \doteq \lambda x. \text{Ded}_0(\llbracket p \rrbracket, \llbracket c \rrbracket) \rrbracket \rrbracket)$]

[Ded(x, y) $\xrightarrow{\text{tex}}$ “
Ded(#1.
, #2.
)”]

[Ded(*, *) $\xrightarrow{\text{pyk}}$ “deduction " conclude " end deduction”]

Ded₀(*, *)

[Ded₀(p, c) $\xrightarrow{\text{val}}$ $\text{c!If}(\text{Ded}_8(p, T), \text{Ded}_1(\text{Ded}_7(p), c, T), F)$]

[Ded₀(x, y) $\xrightarrow{\text{tex}}$ “
Ded_0(#1.
, #2.
)”]

[Ded₀(*, *) $\xrightarrow{\text{pyk}}$ “deduction zero " conclude " end deduction”]

Ded₁(*, *, *)

[Ded₁(p, c, s) $\xrightarrow{\text{val}}$ $\text{If}(c \stackrel{r}{=} [x \vdash y], \text{Ded}_1(p, c^2, c^1 :: s), \text{Ded}_2(p, c, s))$]

[Ded₁(x, y, z) $\xrightarrow{\text{tex}}$ “
Ded_1(#1.
, #2.
, #3.
)”]

[Ded₁(*, *, *) $\xrightarrow{\text{pyk}}$ “deduction one " conclude " condition " end deduction”]

Ded₂(* , * , *)

[Ded₂(p, c, s) $\xrightarrow{\text{val}}$ s!p $\stackrel{r}{\equiv}$ [x \vdash y] \wedge c $\stackrel{r}{\equiv}$ [x \Rightarrow
y] { Ded₃(p¹, c¹, s, T) \wedge Ded₂(p², c², s) }
Ded₄(p, c, s, Ded₆(p, c, T, T))]

[Ded₂(x, y, z) $\xrightarrow{\text{tex}}$ "
Ded_2(#1.
, #2.
, #3.
)"]

[Ded₂(* , * , *) $\xrightarrow{\text{pyk}}$ "deduction two " conclude " condition " end deduction"]

Ded₃(* , * , * , *)

[Ded₃(p, c, s, b) $\xrightarrow{\text{val}}$ If(\neg c $\stackrel{r}{\equiv}$ [$\forall_{\text{obj}x}$: y], Ded₄(p, c, s, b), If(p $\stackrel{r}{\equiv}$ [$\forall_{\text{obj}x}$: y] \wedge p¹ $\stackrel{t}{\equiv}$ c¹,
Ded₄(p, c, s, b), Ded₃(p, c², s, c¹ :: c¹ :: b)))]

[Ded₃(x, y, z, u) $\xrightarrow{\text{tex}}$ "
Ded_3(#1.
, #2.
, #3.
, #4.
)"]

[Ded₃(* , * , * , *) $\xrightarrow{\text{pyk}}$ "deduction three " conclude " condition " bound " end
deduction"]

Ded₄(* , * , * , *)

[Ded₄(p, c, s, b) $\xrightarrow{\text{val}}$ s!b!If(p $\stackrel{r}{\equiv}$ [\bar{x}], **lookup**(p, b, T) $\stackrel{t}{\equiv}$ c, If(\neg p $\stackrel{r}{\equiv}$ c, F,
If(p $\stackrel{r}{\equiv}$ [$\forall_{\text{obj}x}$: y], p¹ $\stackrel{t}{\equiv}$ c¹ \wedge Ded₄(p², c², s, p¹ :: p¹ :: b), If(\neg p $\stackrel{r}{\equiv}$ [\underline{x}],
Ded₄^{*}(p^t, c^t, s, b), p¹ $\stackrel{t}{\equiv}$ c¹ \wedge Ded₅(p, s, b)))]

[Ded₄(x, y, z, u) $\xrightarrow{\text{tex}}$ "
Ded_4(#1.
, #2.
, #3.
, #4.
)"]

[Ded₄(* , * , * , *) $\xrightarrow{\text{pyk}}$ "deduction four " conclude " condition " bound " end
deduction"]

$\text{Ded}_4^*(*, *, *, *)$

$[\text{Ded}_4^*(p, c, s, b) \xrightarrow{\text{val}} \text{c!s!b!If}(p, T, \text{Ded}_4(p^h, c^h, s, b) \wedge \text{Ded}_4^*(p^t, c^t, s, b))]$

$[\text{Ded}_4^*(x, y, z, u) \xrightarrow{\text{tex}} \text{“$

$\text{Ded}_4^*(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$\text{”)”]$

$[\text{Ded}_4^*(*, *, *, *) \xrightarrow{\text{pyk}} \text{“deduction four star " conclude " condition " bound " end deduction”}]$

$\text{Ded}_5(*, *, *)$

$[\text{Ded}_5(p, s, b) \xrightarrow{\text{val}} \text{p!s!If}(b, T, \text{[[x]}\#^0\text{[y]}\text{]}^h :: \text{[[*]}\text{]}^h :: b^{hh} :: T :: \text{[[x]}\text{]}^h :: p :: T :: T \in_t s \wedge \text{Ded}_5(p, s, b^t))]$

$[\text{Ded}_5(x, y, z) \xrightarrow{\text{tex}} \text{“$

$\text{Ded}_5(\#1.$

$, \#2.$

$, \#3.$

$\text{”)”]$

$[\text{Ded}_5(*, *, *) \xrightarrow{\text{pyk}} \text{“deduction five " condition " bound " end deduction”}]$

$\text{Ded}_6(*, *, *, *)$

$[\text{Ded}_6(p, c, e, b) \xrightarrow{\text{val}} \text{p!c!b!e!If}(p \stackrel{r}{=} [\bar{x}], p \in_t e \left\{ \begin{array}{l} b \\ p :: c :: b \end{array} \right., \text{If}(\neg p \stackrel{r}{=} c, T, \text{If}(p \stackrel{r}{=} [\underline{a}], b, \text{If}(p \stackrel{r}{=} [\forall_{\text{obj}x}: y], \text{Ded}_6(p^2, c^2, c^1 :: e, b), \text{Ded}_6^*(p^t, c^t, e, b)))))]$

$[\text{Ded}_6(p, c, e, b) \xrightarrow{\text{tex}} \text{“$

$\text{Ded}_6(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$\text{”)”]$

$[\text{Ded}_6(*, *, *, *) \xrightarrow{\text{pyk}} \text{“deduction six " conclude " exception " bound " end deduction”}]$

$\text{Ded}_6^*(*, *, *, *)$

$[\text{Ded}_6^*(p, c, e, b) \xrightarrow{\text{val}} p!c!b!e!If(p, b, \text{Ded}_6^*(p^t, c^t, e, \text{Ded}_6(p^h, c^h, e, b)))]$

$[\text{Ded}_6^*(p, c, e, b) \xrightarrow{\text{tex}} "$

$\text{Ded}_6^*(\#1.$

$, \#2.$

$, \#3.$

$, \#4.$

$)"]$

$[\text{Ded}_6^*(*, *, *, *) \xrightarrow{\text{pyk}} \text{"deduction six star " conclude " exception " bound " end deduction"}]$

$\text{Ded}_7(*)$

$[\text{Ded}_7(p) \xrightarrow{\text{val}} p \stackrel{r}{=} [\forall x: y] \left\{ \begin{array}{l} \text{Ded}_7(p^2) \\ p \end{array} \right\}]$

$[\text{Ded}_7(p) \xrightarrow{\text{tex}} "$

$\text{Ded}_7(\#1.$

$)"]$

$[\text{Ded}_7(*) \xrightarrow{\text{pyk}} \text{"deduction seven " end deduction"}]$

$\text{Ded}_8(*, *)$

$[\text{Ded}_8(p, b) \xrightarrow{\text{val}} If(p \stackrel{r}{=} [\forall x: y], \text{Ded}_8(p^2, p^1 :: b), If(p \stackrel{r}{=} [a], p \in_t b, \text{Ded}_8^*(p^t, b)))]$

$[\text{Ded}_8(p, b) \xrightarrow{\text{tex}} "$

$\text{Ded}_8(\#1.$

$, \#2.$

$)"]$

$[\text{Ded}_8(*, *) \xrightarrow{\text{pyk}} \text{"deduction eight " bound " end deduction"}]$

$\text{Ded}_8^*(*, *)$

$[\text{Ded}_8^*(p, b) \xrightarrow{\text{val}} b!If(p, \top, If(\text{Ded}_8(p^h, b), \text{Ded}_8^*(p^t, b), F))]$

$[\text{Ded}_8^*(p, b) \xrightarrow{\text{tex}} "$

$\text{Ded}_8^*(\#1.$

, #2.
)”]

[Ded₈^{*}(*, *) $\xrightarrow{\text{pyk}}$ “deduction eight star ” bound ” end deduction”]

S

[S $\xrightarrow{\text{stmt}}$ $\forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \oplus \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b} \oplus \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{a}' = \underline{b}' \oplus$
 $\forall \underline{a}: \forall \underline{b}: \underline{a}' = \underline{b}' \vdash \underline{a} = \underline{b} \oplus \forall \underline{a}: \forall \underline{b}: \lambda x. \text{Ded}_0([\underline{a}], [\underline{b}]) \Vdash \underline{a} \vdash \underline{b} \oplus \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b}' =$
 $\underline{a} \cdot \underline{b} + \underline{a} \oplus \forall \underline{a}: \underline{a} + 0 = \underline{a} \oplus \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \Rightarrow \neg \underline{a} \vdash \neg \underline{b} \Rightarrow \underline{a} \vdash \underline{b} \oplus \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash$
 $\underline{a} = \underline{c} \vdash \underline{b} = \underline{c} \oplus \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \langle [\underline{b}] \equiv^0 [\underline{a}] \mid [\underline{x}] := [0] \rangle \Vdash \langle [\underline{c}] \equiv^0 [\underline{a}] \mid [\underline{x}] := [\underline{x}'] \rangle \Vdash$
 $\underline{b} \vdash \underline{a} \Rightarrow \underline{c} \vdash \underline{a} \oplus \forall \underline{a}: \neg 0 = \underline{a}' \oplus \forall \underline{x}: \forall \underline{a}: \underline{a} \vdash \forall_{\text{obj } \underline{x}}: \underline{a} \oplus \forall \underline{a}: \underline{a} \cdot 0 = 0]$

[S $\xrightarrow{\text{tex}}$ “
S”]

[S $\xrightarrow{\text{pyk}}$ “system s”]

Neg

[Neg $\xrightarrow{\text{proof}}$ Rule tactic]

[Neg $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \neg \underline{b} \Rightarrow \neg \underline{a} \vdash \neg \underline{b} \Rightarrow \underline{a} \vdash \underline{b}$]

[Neg $\xrightarrow{\text{tex}}$ “
Neg”]

[Neg $\xrightarrow{\text{pyk}}$ “double negation”]

MP

[MP $\xrightarrow{\text{proof}}$ Rule tactic]

[MP $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b}$]

[MP $\xrightarrow{\text{tex}}$ “
MP”]

[MP $\xrightarrow{\text{pyk}}$ “rule mp”]

Gen

[Gen $\xrightarrow{\text{proof}}$ Rule tactic]

[Gen $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{x}: \forall \underline{a}: \underline{a} \vdash \forall_{\text{obj}} \underline{x}: \underline{a}$]

[Gen $\xrightarrow{\text{tex}}$ “
Gen”]

[Gen $\xrightarrow{\text{pyk}}$ “rule gen”]

Ded

[Ded $\xrightarrow{\text{proof}}$ Rule tactic]

[Ded $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \lambda x. \text{Ded}_0([\underline{a}], [\underline{b}]) \Vdash \underline{a} \vdash \underline{b}$]

[Ded $\xrightarrow{\text{tex}}$ “
Ded”]

[Ded $\xrightarrow{\text{pyk}}$ “deduction”]

S1

[S1 $\xrightarrow{\text{proof}}$ Rule tactic]

[S1 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} = \underline{c} \vdash \underline{b} = \underline{c}$]

[S1 $\xrightarrow{\text{tex}}$ “
S1”]

[S1 $\xrightarrow{\text{pyk}}$ “axiom s one”]

S2

[S2 $\xrightarrow{\text{proof}}$ Rule tactic]

[S2 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{a}' = \underline{b}'$]

[S2 $\xrightarrow{\text{tex}}$ “
S2”]

[S2 $\xrightarrow{\text{pyk}}$ “axiom s two”]

S3

[S3 $\xrightarrow{\text{proof}}$ Rule tactic]

[S3 $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \neg 0 = \underline{a}'$]

[S3 $\xrightarrow{\text{tex}}$ “
S3”]

[S3 $\xrightarrow{\text{pyk}}$ “axiom s three”]

S4

[S4 $\xrightarrow{\text{proof}}$ Rule tactic]

[S4 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a}' = \underline{b}' \vdash \underline{a} = \underline{b}$]

[S4 $\xrightarrow{\text{tex}}$ “
S4”]

[S4 $\xrightarrow{\text{pyk}}$ “axiom s four”]

S5

[S5 $\xrightarrow{\text{proof}}$ Rule tactic]

[S5 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} + 0 = \underline{a}$]

[S5 $\xrightarrow{\text{tex}}$ “
S5”]

[S5 $\xrightarrow{\text{pyk}}$ “axiom s five”]

S6

[S6 $\xrightarrow{\text{proof}}$ Rule tactic]

[S6 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b}' = \underline{a} + \underline{b}'$]

[S6 $\xrightarrow{\text{tex}}$ “
S6”]

[S6 $\xrightarrow{\text{pyk}}$ “axiom s six”]

S7

[S7 $\xrightarrow{\text{proof}}$ Rule tactic]

[S7 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \cdot 0 = 0$]

[S7 $\xrightarrow{\text{tex}}$ “
S7”]

[S7 $\xrightarrow{\text{pyk}}$ “axiom s seven”]

S8

[S8 $\xrightarrow{\text{proof}}$ Rule tactic]

[S8 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \cdot \underline{b}' = \underline{a} \cdot \underline{b} + \underline{a}$]

[S8 $\xrightarrow{\text{tex}}$ “
S8”]

[S8 $\xrightarrow{\text{pyk}}$ “axiom s eight”]

S9

[S9 $\xrightarrow{\text{proof}}$ Rule tactic]

[S9 $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{x}: \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \langle [\underline{b}] \equiv^0 [\underline{a}] \mid [\underline{x}] := [0] \rangle \Vdash \langle [\underline{c}] \equiv^0 [\underline{a}] \mid [\underline{x}] := [\underline{x}'] \rangle \Vdash \underline{b} \vdash \underline{a} \Rightarrow \underline{c} \vdash \underline{a}$]

[S9 $\xrightarrow{\text{tex}}$ “
S9”]

[S9 $\xrightarrow{\text{pyk}}$ “axiom s nine”]

Repetition

[Repetition $\xrightarrow{\text{proof}}$ $\lambda \underline{c}. \lambda \underline{x}. [S \vdash \forall \underline{a}: \underline{a}^I]$]

[Repetition $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} \vdash \underline{a}$]

[Repetition $\xrightarrow{\text{tex}}$ “
Repetition”]

[Repetition $\xrightarrow{\text{pyk}}$ “repetition”]

A1'

[A1' $\xrightarrow{\text{proof}}$ $\lambda \underline{c}. \lambda \underline{x}. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b} \vdash \text{Repetition} \triangleright \underline{a} \gg \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b} \vdash \underline{a} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a}], \rho_0, \underline{c})$]

[A1' $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a}$]

[A1' $\xrightarrow{\text{tex}}$ “
A1”]

[A1' $\xrightarrow{\text{pyk}}$ “lemma a one”]

A2'

[A2' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \triangleright \underline{a} \gg \underline{b}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \triangleright \underline{a} \gg \underline{b} \Rightarrow \underline{c}; \text{MP} \triangleright \underline{b} \Rightarrow \underline{c} \triangleright \underline{b} \gg \underline{c}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{c} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c}]$, p0, c)]

[A2' $\xrightarrow{\text{stmt}}$ S $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{c}$]

[A2' $\xrightarrow{\text{tex}}$ “
A2”]

[A2' $\xrightarrow{\text{pyk}}$ “lemma a two”]

A4'

[A4' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \bar{x} + \bar{y} = \bar{y} + \bar{x} \vdash \text{Repetition} \triangleright \bar{x} + \bar{y} = \bar{y} + \bar{x} \gg \bar{x} + \bar{y} = \bar{y} + \bar{x}; \text{Ded} \triangleright \bar{x} + \bar{y} = \bar{y} + \bar{x} \vdash \bar{x} + \bar{y} = \bar{y} + \bar{x} \gg \forall_{\text{obj}} \bar{x}: \forall_{\text{obj}} \bar{y}: \bar{x} + \bar{y} = \bar{y} + \bar{x} \Rightarrow 2 + 3 = 3 + 2]$, p0, c)]

[A4' $\xrightarrow{\text{stmt}}$ S $\vdash \forall_{\text{obj}} \bar{x}: \forall_{\text{obj}} \bar{y}: \bar{x} + \bar{y} = \bar{y} + \bar{x} \Rightarrow 2 + 3 = 3 + 2$]

[A4' $\xrightarrow{\text{tex}}$ “
A4”]

[A4' $\xrightarrow{\text{pyk}}$ “lemma a four”]

A5'

[A5' $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash 2 + 3 = 5 \Rightarrow 2 + 3 + \bar{x} = 5 + \bar{x} \vdash 2 + 3 = 5 \vdash \text{MP} \triangleright 2 + 3 = 5 \Rightarrow 2 + 3 + \bar{x} = 5 + \bar{x} \triangleright 2 + 3 = 5 \gg 2 + 3 + \bar{x} = 5 + \bar{x}; \text{Gen} \triangleright 2 + 3 + \bar{x} = 5 + \bar{x} \gg \forall_{\text{obj}} \bar{x}: 2 + 3 + \bar{x} = 5 + \bar{x}; \text{Ded} \triangleright 2 + 3 = 5 \Rightarrow 2 + 3 + \bar{x} = 5 + \bar{x} \vdash 2 + 3 = 5 \vdash \forall_{\text{obj}} \bar{x}: 2 + 3 + \bar{x} = 5 + \bar{x} \gg \forall_{\text{obj}} \bar{x}: 2 + 3 = 5 \Rightarrow 2 + 3 + \bar{x} = 5 + \bar{x} \Rightarrow 2 + 3 = 5 \Rightarrow \forall_{\text{obj}} \bar{x}: 2 + 3 + \bar{x} = 5 + \bar{x}]$, p0, c)]

[A5' $\xrightarrow{\text{stmt}}$ S $\vdash \forall_{\text{obj}} \bar{x}: 2 + 3 = 5 \Rightarrow 2 + 3 + \bar{x} = 5 + \bar{x} \Rightarrow 2 + 3 = 5 \Rightarrow \forall_{\text{obj}} \bar{x}: 2 + 3 + \bar{x} = 5 + \bar{x}$]

[A5' $\xrightarrow{\text{tex}}$ “

A5”]

[A5' $\xrightarrow{\text{pyk}}$ “lemma a five”]

Prop 3.2a

[Prop 3.2a $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S5 \gg \underline{a} + 0 = \underline{a}; S1 \triangleright \underline{a} + 0 = \underline{a} \triangleright \underline{a} + 0 = \underline{a} \gg \underline{a} = \underline{a}], p_0, c)$]

[Prop 3.2a $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} = \underline{a}$]

[Prop 3.2a $\xrightarrow{\text{tex}}$ “
Prop\ 3.2a”]

[Prop 3.2a $\xrightarrow{\text{pyk}}$ “prop three two a”]

Prop 3.2b

[Prop 3.2b $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \text{Prop 3.2a} \gg \underline{a} = \underline{a}; S1 \triangleright \underline{a} = \underline{b} \triangleright \underline{a} = \underline{a} \gg \underline{b} = \underline{a}], p_0, c)$]

[Prop 3.2b $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{a}$]

[Prop 3.2b $\xrightarrow{\text{tex}}$ “
Prop\ 3.2b”]

[Prop 3.2b $\xrightarrow{\text{pyk}}$ “prop three two b”]

Prop 3.2c

[Prop 3.2c $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{c} \vdash \text{Prop 3.2b} \triangleright \underline{a} = \underline{b} \gg \underline{b} = \underline{a}; S1 \triangleright \underline{b} = \underline{a} \triangleright \underline{b} = \underline{c} \gg \underline{a} = \underline{c}], p_0, c)$]

[Prop 3.2c $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{b} = \underline{c} \vdash \underline{a} = \underline{c}$]

[Prop 3.2c $\xrightarrow{\text{tex}}$ “
Prop\ 3.2c”]

[Prop 3.2c $\xrightarrow{\text{pyk}}$ “prop three two c”]

Prop 3.2d

[Prop 3.2d $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{c} \vdash \underline{b} = \underline{c} \vdash \text{Prop 3.2b} \triangleright \underline{b} = \underline{c} \gg \underline{c} = \underline{b}; \text{Prop 3.2c} \triangleright \underline{a} = \underline{c} \triangleright \underline{c} = \underline{b} \gg \underline{a} = \underline{b}], p_0, c)$]

[Prop 3.2d $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{c} \vdash \underline{b} = \underline{c} \vdash \underline{a} = \underline{b}$]

[Prop 3.2d $\xrightarrow{\text{tex}}$ “
Prop\ 3.2d”]

[Prop 3.2d $\xrightarrow{\text{pyk}}$ “prop three two d”]

Prop 3.2e₁

[Prop 3.2e₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}': \forall \underline{b}': \underline{a} = \underline{b} \vdash S5 \gg \underline{a} + 0 = \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} + 0 = \underline{a} \triangleright \underline{a} = \underline{b} \gg \underline{a} + 0 = \underline{b}; S5 \gg \underline{b} + 0 = \underline{b}; \text{Prop 3.2d} \triangleright \underline{a} + 0 = \underline{b} \triangleright \underline{b} + 0 = \underline{b} \gg \underline{a} + 0 = \underline{b} + 0; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \vdash \underline{a} + 0 = \underline{b} + 0 \gg \underline{a} = \underline{b} \Rightarrow \underline{a} + 0 = \underline{b} + 0], p_0, c)$]

[Prop 3.2e₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} = \underline{b} \Rightarrow \underline{a} + 0 = \underline{b} + 0$]

[Prop 3.2e₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2e.1”]

[Prop 3.2e₁ $\xrightarrow{\text{pyk}}$ “prop three two e one”]

Prop 3.2e₂

[Prop 3.2e₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{a}': \forall \underline{b}': \forall \underline{c}': \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \vdash \underline{a} = \underline{b} \vdash \text{MP} \triangleright \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \triangleright \underline{a} = \underline{b} \gg \underline{a} + \underline{c} = \underline{b} + \underline{c}; S2 \triangleright \underline{a} + \underline{c} = \underline{b} + \underline{c} \gg \underline{a} + \underline{c}' = \underline{b} + \underline{c}'; S6 \gg \underline{a} + \underline{c}' = \underline{a} + \underline{c}'; \text{Prop 3.2c} \triangleright \underline{a} + \underline{c}' = \underline{a} + \underline{c}' \triangleright \underline{a} + \underline{c}' = \underline{b} + \underline{c}' \gg \underline{a} + \underline{c}' = \underline{b} + \underline{c}'; S6 \gg \underline{b} + \underline{c}' = \underline{b} + \underline{c}'; \text{Prop 3.2d} \triangleright \underline{a} + \underline{c}' = \underline{b} + \underline{c}' \triangleright \underline{b} + \underline{c}' = \underline{b} + \underline{c}' \gg \underline{a} + \underline{c}' = \underline{b} + \underline{c}'; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \vdash \underline{a} = \underline{b} \vdash \underline{a} + \underline{c}' = \underline{b} + \underline{c}' \gg \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c}' = \underline{b} + \underline{c}'], p_0, c)$]

[Prop 3.2e₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \Rightarrow \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c}' = \underline{b} + \underline{c}'$]

[Prop 3.2e₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2e.2”]

[Prop 3.2e₂ $\xrightarrow{\text{pyk}}$ “prop three two e two”]

Prop 3.2e

[Prop 3.2e $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \text{Prop 3.2e}_1 \gg \bar{x} = \bar{y} \Rightarrow \bar{x} + 0 = \bar{y} + 0; \text{Prop 3.2e}_2 \gg \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z} = \bar{y} + \bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z}' = \bar{y} + \bar{z}'; \text{S9} @ \bar{z} \triangleright \bar{x} = \bar{y} \Rightarrow \bar{x} + 0 = \bar{y} + 0 \triangleright \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z} = \bar{y} + \bar{z} \Rightarrow \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z}' = \bar{y} + \bar{z}' \gg \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z} = \bar{y} + \bar{z}; \text{Ded} \triangleright \bar{x} = \bar{y} \Rightarrow \bar{x} + \bar{z} = \bar{y} + \bar{z} \gg \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c}; \text{MP} \triangleright \underline{a} = \underline{b} \Rightarrow \underline{a} + \underline{c} = \underline{b} + \underline{c} \triangleright \underline{a} = \underline{b} \gg \underline{a} + \underline{c} = \underline{b} + \underline{c} \rceil, p_0, c)$]

[Prop 3.2e $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \underline{a} = \underline{b} \vdash \underline{a} + \underline{c} = \underline{b} + \underline{c}$]

[Prop 3.2e $\xrightarrow{\text{tex}}$ “
Prop\ 3.2e”]

[Prop 3.2e $\xrightarrow{\text{pyk}}$ “prop three two e”]

Prop 3.2f₁

[Prop 3.2f₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \text{S5} \gg 0 + 0 = 0; \text{Prop 3.2b} \triangleright 0 + 0 = 0 \gg 0 = 0 + 0 \rceil, p_0, c)$]

[Prop 3.2f₁ $\xrightarrow{\text{stmt}}$ $S \vdash 0 = 0 + 0$]

[Prop 3.2f₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2f_1”]

[Prop 3.2f₁ $\xrightarrow{\text{pyk}}$ “prop three two f one”]

Prop 3.2f₂

[Prop 3.2f₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \forall \underline{a}: \forall \underline{a}': \underline{a} = 0 + \underline{a} \vdash \text{S2} \triangleright \underline{a} = 0 + \underline{a} \gg \underline{a}' = 0 + \underline{a}'; \text{S6} \gg 0 + \underline{a}' = 0 + \underline{a}'; \text{Prop 3.2d} \triangleright \underline{a}' = 0 + \underline{a}' \triangleright 0 + \underline{a}' = 0 + \underline{a}' \gg \underline{a}' = 0 + \underline{a}'; \text{Ded} \triangleright \forall \underline{a}: \underline{a} = 0 + \underline{a} \vdash \underline{a}' = 0 + \underline{a}' \gg \underline{a} = 0 + \underline{a} \Rightarrow \underline{a}' = 0 + \underline{a}' \rceil, p_0, c)$]

[Prop 3.2f₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} = 0 + \underline{a} \Rightarrow \underline{a}' = 0 + \underline{a}'$]

[Prop 3.2f₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2f_2”]

[Prop 3.2f₂ $\xrightarrow{\text{pyk}}$ “prop three two f two”]

Prop 3.2f

[Prop 3.2f $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil S \vdash \forall \underline{a}: \text{Prop 3.2f}_1 \gg 0 = 0 + 0; \text{Prop 3.2f}_2 \gg \bar{x} = 0 + \bar{x} \Rightarrow \bar{x}' = 0 + \bar{x}'; \text{S9} @ \bar{x} \triangleright 0 = 0 + 0 \triangleright \bar{x} = 0 + \bar{x} \Rightarrow \bar{x}' = 0 + \bar{x}' \gg \bar{x} =$

$0 + \bar{x}; \text{Ded} \triangleright \bar{x} = 0 + \bar{x} \gg \underline{a} = 0 + \underline{a}], p_0, c)$

$[\text{Prop } 3.2f \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}: \underline{a} = 0 + \underline{a}]$

$[\text{Prop } 3.2f \xrightarrow{\text{tex}} \text{“}$
 $\text{Prop} \setminus 3.2f\text{”}]$

$[\text{Prop } 3.2f \xrightarrow{\text{pyk}} \text{“prop three two f”}]$

Prop 3.2g₁

$[\text{Prop } 3.2g_1 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S5 \gg \underline{a}' + 0 = \underline{a}'; S5 \gg \underline{a} + 0 = \underline{a}; S2 \triangleright \underline{a} + 0 = \underline{a} \gg \underline{a} + 0' = \underline{a}'; \text{Prop } 3.2d \triangleright \underline{a}' + 0 = \underline{a}' \triangleright \underline{a} + 0' = \underline{a}' \gg \underline{a}' + 0 = \underline{a} + 0'], p_0, c)]$

$[\text{Prop } 3.2g_1 \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}: \underline{a}' + 0 = \underline{a} + 0']$

$[\text{Prop } 3.2g_1 \xrightarrow{\text{tex}} \text{“}$
 $\text{Prop} \setminus 3.2g_1\text{”}]$

$[\text{Prop } 3.2g_1 \xrightarrow{\text{pyk}} \text{“prop three two g one”}]$

Prop 3.2g₂

$[\text{Prop } 3.2g_2 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}': \forall \underline{b}': \underline{a}' + \underline{b} = \underline{a} + \underline{b}' \vdash S2 \triangleright \underline{a}' + \underline{b} = \underline{a} + \underline{b}' \gg \underline{a}' + \underline{b}' = \underline{a} + \underline{b}''; S6 \gg \underline{a}' + \underline{b}' = \underline{a}' + \underline{b}''; \text{Prop } 3.2c \triangleright \underline{a}' + \underline{b}' = \underline{a}' + \underline{b}' \triangleright \underline{a}' + \underline{b}' = \underline{a} + \underline{b}'' \gg \underline{a}' + \underline{b}' = \underline{a} + \underline{b}''; S6 \gg \underline{a} + \underline{b}' = \underline{a} + \underline{b}''; S2 \triangleright \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \gg \underline{a} + \underline{b}'' = \underline{a} + \underline{b}''; \text{Prop } 3.2d \triangleright \underline{a}' + \underline{b}' = \underline{a} + \underline{b}'' \triangleright \underline{a} + \underline{b}'' = \underline{a} + \underline{b}'' \gg \underline{a}' + \underline{b}' = \underline{a} + \underline{b}''; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a}' + \underline{b} = \underline{a} + \underline{b}' \vdash \underline{a}' + \underline{b}' = \underline{a} + \underline{b}'' \gg \underline{a}' + \underline{b} = \underline{a} + \underline{b}' \Rightarrow \underline{a}' + \underline{b}' = \underline{a} + \underline{b}''], p_0, c)]$

$[\text{Prop } 3.2g_2 \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a}' + \underline{b} = \underline{a} + \underline{b}' \Rightarrow \underline{a}' + \underline{b}' = \underline{a} + \underline{b}'']$

$[\text{Prop } 3.2g_2 \xrightarrow{\text{tex}} \text{“}$
 $\text{Prop} \setminus 3.2g_2\text{”}]$

$[\text{Prop } 3.2g_2 \xrightarrow{\text{pyk}} \text{“prop three two g two”}]$

Prop 3.2g

$[\text{Prop } 3.2g \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop } 3.2g_1 \gg \bar{x}' + 0 = \bar{x} + 0'; \text{Prop } 3.2g_2 \gg \bar{x}' + \bar{y} = \bar{x} + \bar{y}' \Rightarrow \bar{x}' + \bar{y}' = \bar{x} + \bar{y}''; S9 @ \bar{y} \triangleright \bar{x}' + 0 = \bar{x} + 0' \triangleright \bar{x}' + \bar{y} = \bar{x} + \bar{y}' \Rightarrow \bar{x}' + \bar{y}' = \bar{x} + \bar{y}'' \gg \bar{x}' + \bar{y} = \bar{x} + \bar{y}''; \text{Ded} \triangleright \bar{x}' + \bar{y} = \bar{x} + \bar{y}' \gg \underline{a}' + \underline{b} = \underline{a} + \underline{b}'], p_0, c)]$

$[\text{Prop } 3.2g \xrightarrow{\text{stmt}} S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a}' + \underline{b} = \underline{a} + \underline{b}']$

[Prop 3.2g $\xrightarrow{\text{tex}}$ “
Prop\ 3.2g”]

[Prop 3.2g $\xrightarrow{\text{pyk}}$ “prop three two g”]

Prop 3.2h₁

[Prop 3.2h₁ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: S5 \gg \underline{a} + 0 = \underline{a}; \text{Prop 3.2f} \gg \underline{a} = 0 + \underline{a}; \text{Prop 3.2c} \triangleright \underline{a} + 0 = \underline{a} \triangleright \underline{a} = 0 + \underline{a} \gg \underline{a} + 0 = 0 + \underline{a}], p_0, c)$]

[Prop 3.2h₁ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \underline{a} + 0 = 0 + \underline{a}$]

[Prop 3.2h₁ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2h.1”]

[Prop 3.2h₁ $\xrightarrow{\text{pyk}}$ “prop three two h one”]

Prop 3.2h₂

[Prop 3.2h₂ $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = \underline{b} + \underline{a} \vdash S2 \triangleright \underline{a} + \underline{b} = \underline{b} + \underline{a} \gg \underline{a} + \underline{b}' = \underline{b} + \underline{a}'; S6 \gg \underline{a} + \underline{b}' = \underline{a} + \underline{b}'; \text{Prop 3.2c} \triangleright \underline{a} + \underline{b}' = \underline{a} + \underline{b}' \triangleright \underline{a} + \underline{b}' = \underline{b} + \underline{a}' \gg \underline{a} + \underline{b}' = \underline{b} + \underline{a}'; \text{Prop 3.2g} \gg \underline{b}' + \underline{a} = \underline{b} + \underline{a}'; \text{Prop 3.2d} \triangleright \underline{a} + \underline{b}' = \underline{b} + \underline{a}' \triangleright \underline{b}' + \underline{a} = \underline{b} + \underline{a}' \gg \underline{a} + \underline{b}' = \underline{b}' + \underline{a}; \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = \underline{b} + \underline{a} \vdash \underline{a} + \underline{b}' = \underline{b}' + \underline{a} \gg \underline{a} + \underline{b} = \underline{b} + \underline{a} \Rightarrow \underline{a} + \underline{b}' = \underline{b}' + \underline{a}], p_0, c)$]

[Prop 3.2h₂ $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = \underline{b} + \underline{a} \Rightarrow \underline{a} + \underline{b}' = \underline{b}' + \underline{a}$]

[Prop 3.2h₂ $\xrightarrow{\text{tex}}$ “
Prop\ 3.2h.2”]

[Prop 3.2h₂ $\xrightarrow{\text{pyk}}$ “prop three two h two”]

Prop 3.2h

[Prop 3.2h $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}([S \vdash \forall \underline{a}: \forall \underline{b}: \text{Prop 3.2h}_1 \gg \bar{x} + 0 = 0 + \bar{x}; \text{Prop 3.2h}_2 \gg \bar{x} + \bar{y} = \bar{y} + \bar{x} \Rightarrow \bar{x} + \bar{y}' = \bar{y}' + \bar{x}; S9 @ \bar{y} \triangleright \bar{x} + 0 = 0 + \bar{x} \triangleright \bar{x} + \bar{y} = \bar{y} + \bar{x} \Rightarrow \bar{x} + \bar{y}' = \bar{y}' + \bar{x} \gg \bar{x} + \bar{y} = \bar{y} + \bar{x}; \text{Ded} \triangleright \bar{x} + \bar{y} = \bar{y} + \bar{x} \gg \underline{a} + \underline{b} = \underline{b} + \underline{a}], p_0, c)$]

[Prop 3.2h $\xrightarrow{\text{stmt}}$ $S \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} + \underline{b} = \underline{b} + \underline{a}$]

[Prop 3.2h $\xrightarrow{\text{tex}}$ “
Prop\ 3.2h”]

[Prop 3.2h $\xrightarrow{\text{pyk}}$ “prop three two h”]

Block₁(* , * , *)

[Block₁(t, s, c) $\xrightarrow{\text{val}}$ t!s!c!let₁(λb.let₁(λx.let₁(λq.let₁(λq. \tilde{Q} (t, [b; q], [b] :: b :: [q] :: q :: T), \tilde{M} (q, s, c)), \tilde{Q} (t, [let l ≐ x in p], [l] :: t² :: [p] :: t³ :: [x] :: x :: T)), Block₂(b)), \mathcal{M} (t¹, s, c)]

[Block₁(t, s, c) $\xrightarrow{\text{tex}}$ “
Block_1(#1.
, #2.
, #3.
)”]

[Block₁(* , * , *) $\xrightarrow{\text{pyk}}$ “block one " state " cache " end block”]

Block₂(*)

[Block₂(b) $\xrightarrow{\text{val}}$ If(b $\stackrel{r}{=} [x \vdash y]$, \tilde{Q} (b, [x ⊢ y], [x] :: b¹ :: [y] :: Block₂(b²) :: T),
If(b $\stackrel{r}{=} [x \Vdash y]$, \tilde{Q} (b, [x ⊢ y], [x] :: b¹ :: [y] :: Block₂(b²) :: T), If(b $\stackrel{r}{=} [\forall x: y]$,
 \tilde{Q} (b, [∀x: y], [x] :: b¹ :: [y] :: Block₂(b²) :: T), If(b $\stackrel{r}{=} [x; y]$, Block₂(b²),
If(b $\stackrel{r}{=} [x \gg y]$, b², ⊥)))]

[Block₂(b) $\xrightarrow{\text{tex}}$ “
Block_2(#1.
)”]

[Block₂(*) $\xrightarrow{\text{pyk}}$ “block two " end block”]

*hide

Predef: hide

[x^{hide} $\xrightarrow{\text{tex}}$ “#1.
{ } ^ {hide}”]

[*^{hide} $\xrightarrow{\text{pyk}}$ “" hide”]

MacroIndent(*)

[MacroIndent(x) $\xrightarrow{\text{name}}$ “
MacroIndent(#1.
)”]

[MacroIndent(x) $\xrightarrow{\text{macro}}$ λt.λs.λc. \tilde{M}_4 (t, s, c, [[MacroIndent(x) ≐ x]])]

$[\text{MacroIndent}(x) \xrightarrow{\text{tex}} “$
 $\$%$
 $\backslash\text{leftskip}=1em%$
 $\$#1.”]$

$[\text{MacroIndent}(*) \xrightarrow{\text{pyk}} “\text{macro indent } ”]$

$*'$

$[x' \xrightarrow{\text{tex}} “\#1.$
 $\{\}'”]$

$[*' \xrightarrow{\text{pyk}} “\text{ suc}”]$

$* = *$

$[x = y \xrightarrow{\text{tex}} “\#1.$
 $= \#2.”]$

$[* = * \xrightarrow{\text{pyk}} “\text{ equal } ”]$

$* \neq *$

$[x \neq y \xrightarrow{\text{tex}} “\#1.$
 $\backslash\text{neq } \#2.”]$

$[* \neq * \xrightarrow{\text{pyk}} “\text{ unequal } ”]$

$*\text{var}$

$[x^{\text{var}} \xrightarrow{\text{val}} x \stackrel{r}{=} [\bar{x}]]$

$[x^{\text{var}} \xrightarrow{\text{tex}} “\#1.$
 $\{\}^{\{\text{var}\}}”]$

$[*^{\text{var}} \xrightarrow{\text{pyk}} “\text{ is object var}”]$

$*\neq^0*$

$[x\neq^0y \xrightarrow{\text{val}} \lambda c.x^{\text{var}} \wedge y^c \wedge x\neq^1y]$

[x#⁰y $\xrightarrow{\text{tex}}$ “#1.
\#. ^0#2.”]

[*#⁰* $\xrightarrow{\text{pyk}}$ “" avoid zero ""]

#¹

[x#¹y $\xrightarrow{\text{val}}$ If(y^{var}, $\neg x \stackrel{t}{=} y$,
If($\neg y \stackrel{r}{=} [\forall_{\text{obj}} x: y]$, x#*y^t,
If(x $\stackrel{t}{=} y^1$, T, x#¹y²)))]

[x#¹y $\xrightarrow{\text{tex}}$ “#1.
\#. ^1#2.”]

[*#¹* $\xrightarrow{\text{pyk}}$ “" avoid one ""]

#^{*}*

[x#*y $\xrightarrow{\text{val}}$ x!If(y, T, If(x#¹y^h, x#*y^t, F))]

[x#*y $\xrightarrow{\text{tex}}$ “#1.
\#. ^*#2.”]

[*#*^{*}* $\xrightarrow{\text{pyk}}$ “" avoid star ""]

∃*: *

[∃x: y $\xrightarrow{\text{tex}}$ “
\exists exists #1.
\colon #2.”]

[∃*: * $\xrightarrow{\text{pyk}}$ “exist " indeed ""]

∀*: *

[∀x: y $\xrightarrow{\text{macro}}$ lt.ls.lc. $\tilde{\mathcal{M}}$ (If($\neg t^1 \stackrel{r}{=} [x, y]$, $\tilde{\mathcal{Q}}(t, [\forall_{\text{obj}} x: y], [x] :: t^1 :: [y] :: t^2 :: T)$,
 $\tilde{\mathcal{Q}}(t, [\forall_{\text{obj}} x: \forall y: z], [x] :: t^{11} :: [y] :: t^{12} :: [z] :: t^2 :: T)$), s, c)]

[∀x: y $\xrightarrow{\text{tex}}$ “
\forall forall #1.
\colon #2.”]

[∀*: * $\xrightarrow{\text{pyk}}$ “for all " indeed ""]

$\forall_{\text{obj}} * : *$

$[\forall_{\text{obj}} x : y \xrightarrow{\text{tex}} \text{"\#1.} \\ \backslash \text{forall_}\{\text{obj}\} \text{\#1.} \\ \backslash \text{colon \#2.}"]$

$[\forall_{\text{obj}} * : * \xrightarrow{\text{pyk}} \text{"for all objects " indeed "}]$

$* \Rightarrow *$

$[x \Rightarrow y \xrightarrow{\text{tex}} \text{"\#1.} \\ \backslash \text{Rightarrow \#2.}"]$

$[* \Rightarrow * \xrightarrow{\text{pyk}} \text{" imply "}]$

$* \Leftrightarrow *$

$[x \Leftrightarrow y \xrightarrow{\text{tex}} \text{"\#1.} \\ \backslash \text{Leftrightarrow \#2.}"]$

$[* \Leftrightarrow * \xrightarrow{\text{pyk}} \text{" if and only if "}]$

$* \# *$

$[x \# y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \# y \doteq [x] \#^0 [y]])]]$

$[x \# y \xrightarrow{\text{tex}} \text{"\#1.} \\ \backslash \text{\#.\#2.}"]$

$[* \# * \xrightarrow{\text{pyk}} \text{" avoid "}]$

$* \triangleright *$

$[x \triangleright y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \triangleright y \doteq \text{MP} \triangleright x \triangleright y]])]]$

$[x \triangleright y \xrightarrow{\text{tex}} \text{"\#1.} \\ \backslash \text{unrhd \#2.}"]$

$[* \triangleright * \xrightarrow{\text{pyk}} \text{" object modus ponens "}]$

$\Pi * : *$

$[\Pi x: y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{M}(\text{If}(\neg t^1 \stackrel{r}{=} [x, y], \tilde{Q}(t, [\forall x: y], [x] :: t^1 :: [y] :: t^2 :: T), \tilde{Q}(t, [\forall x: \Pi y: z], [x] :: t^{11} :: [y] :: t^{12} :: [z] :: t^2 :: T)), s, c)]$

$[\Pi x: y \xrightarrow{\text{tex}} “$
 $\backslash \text{Pi} \#1.$
 $\backslash \text{colon} \#2.”]$

$[\Pi * : * \xrightarrow{\text{pyk}} “\text{for all terms " indeed " ”]$

Begin * ; * : End ; *

$[\text{Begin } b; l : \text{End}; p \xrightarrow{\text{name}} “$
 $\text{Begin } \backslash, \#1.$
 $; \#2.$
 $: \text{End}; \#3.”]$

$[\text{Begin } b; l : \text{End}; p \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \text{Block}_1(t, s, c)]$

$[\text{Begin } b; l : \text{End}; p \xrightarrow{\text{tex}} “$
 $\backslash \text{newline} \backslash \text{makebox} [0.1 \backslash \text{textwidth}] \{ \} \%$
 $\backslash \text{parbox} [b] \{ 0.4 \backslash \text{textwidth} \} \{ \backslash \text{raggedright}$
 $\backslash \text{setlength} \{ \backslash \text{parindent} \} \{ -0.1 \backslash \text{textwidth} \} \%$
 $\backslash \text{makebox} [0.1 \backslash \text{textwidth}] [l] \{ \$$
 $\backslash \text{if} \backslash \text{relax} \backslash \text{cname} \text{lgwproofline} \backslash \text{endcsname} L_? \backslash \text{else}$
 $\backslash \text{global} \backslash \text{advance} \backslash \text{lgwproofline} \text{ by } 1$
 $L \text{ifnum} \backslash \text{lgwproofline} < 10 0 \backslash \text{fi} \backslash \text{number} \backslash \text{lgwproofline}$
 $\backslash \text{fi}$
 $\$: \} \$ \text{Block} \{ \} \backslash \text{gg} \{ \} \$ \} \backslash \text{quad}$
 $\backslash \text{parbox} [t] \{ 0.4 \backslash \text{textwidth} \} \{ \$ \text{Begin}$
 $\$ \backslash \text{hfill} \backslash \text{makebox} [0 \text{mm}] [l] \{ \backslash \text{quad} ; \} \} \#1.$
 $\backslash \text{newline} \backslash \text{makebox} [0.1 \backslash \text{textwidth}] \{ \} \%$
 $\backslash \text{parbox} [b] \{ 0.4 \backslash \text{textwidth} \} \{ \backslash \text{raggedright}$
 $\backslash \text{setlength} \{ \backslash \text{parindent} \} \{ -0.1 \backslash \text{textwidth} \} \%$
 $\backslash \text{makebox} [0.1 \backslash \text{textwidth}] [l] \{ \$ \#2.$
 $\$: \} \$ \text{Block} \{ \} \backslash \text{gg} \{ \} \$ \} \backslash \text{quad}$
 $\backslash \text{parbox} [t] \{ 0.4 \backslash \text{textwidth} \} \{ \$ \text{End}$
 $\$ \backslash \text{hfill} \backslash \text{makebox} [0 \text{mm}] [l] \{ \backslash \text{quad} ; \} \} \#3.”]$
 $[\text{Begin } * ; * : \text{End}; * \xrightarrow{\text{pyk}} “\text{block " line " end block " ”]$

Last block line * \gg * ;

```
[Last block line a  $\gg$  i ;  $\xrightarrow{\text{name}}$  “  
Last\ block\ line \, #1.  
\gg #2.  
\,;”]
```

```
[Last block line a  $\gg$  i ;  $\xrightarrow{\text{macro}}$  \t.\s.\c.\tilde{\mathcal{M}}_4(t,s,c, [[Last block line a  $\gg$  i ;  $\doteq$  (a  $\gg$   
i)])]
```

```
[Last block line a  $\gg$  i ;  $\xrightarrow{\text{tex}}$  “  
\newline \makebox [0.1\textwidth]{}%  
\parbox [b]{0.4\textwidth }{\raggedright  
\setlength {\parindent }{-0.1\textwidth }%  
\makebox [0.1\textwidth ][l]{$  
\if \relax \csname lgwprooflinep\endcsname L_? \else  
\global \advance \lgwproofline by 1  
L\ifnum \lgwproofline <10 0\fi \number \lgwproofline  
\fi  
$:}$#1.  
{ }\gg { }$}\quad  
\parbox [t]{0.4\textwidth }{ $#2.  
$\hfill \makebox [0mm][l]{\quad ; }”]
```

```
[Last block line *  $\gg$  * ;  $\xrightarrow{\text{pyk}}$  “because " indeed " end line”]
```

Arbitrary \gg * ; *

```
[Arbitrary  $\gg$  i ; p  $\xrightarrow{\text{name}}$  “  
Arbitrary \gg #1.  
; #2.”]
```

```
[Arbitrary  $\gg$  i ; p  $\xrightarrow{\text{macro}}$  \t.\s.\c.\tilde{\mathcal{M}}_4(t,s,c, [[Arbitrary  $\gg$  i ; p  $\doteq$   $\Pi$ i: p]])]
```

```
[Arbitrary  $\gg$  i ; p  $\xrightarrow{\text{tex}}$  “  
\newline \makebox [0.1\textwidth ][l]{$  
\if \relax \csname lgwprooflinep\endcsname L_? \else  
\global \advance \lgwproofline by 1  
L\ifnum \lgwproofline <10 0\fi \number \lgwproofline  
\fi  
$:}\makebox [0.4\textwidth ][l]{$Arbitrary}\gg{ }$}\quad  
\parbox [t]{0.4\textwidth }{ $#1.  
$\hfill \makebox [0mm][l]{\quad ; }#2.”]
```

```
[Arbitrary  $\gg$  * ; *  $\xrightarrow{\text{pyk}}$  “any term " end line ”]
```


* | *

[x | y $\xrightarrow{\text{tex}}$ "#1.
\mathrel{} #2.]

[* | * $\xrightarrow{\text{pyk}}$ "# alternative "]

→

[→ $\xrightarrow{\text{tex}}$ "
\rightarrow "]

[→ $\xrightarrow{\text{pyk}}$ "evaluates to"]

* \ \ *

[x \ \ y $\xrightarrow{\text{name}}$ "#1.
\backslash \backslash #2.]

[x \ \ y $\xrightarrow{\text{tex}}$ "#1.
\{\} #2.]

[* \ \ * $\xrightarrow{\text{pyk}}$ "# safe row "]

*The pyk compiler, version 0.grue.20060417+ by Klaus Grue
GRD-2006-05-04.UTC:15:42:31.918490 = MJD-53859.TAI:15:43:04.918490 =
LGT-4653474184918490e-6*