



## Up Help

sup2, LeqTotality(R), PositiveToLeft(Eq), ExpZero(Exact), SameExp(Base), SameExp(Indu), SameExp,  $(1/2)(x + y) - x = (1/2)(y - x)$ ,  $y - (1/2)(x + y) = (1/2)(y - x)$ , BSzero(Exact), SameBS(2)(Base), SameBS(2)(Indu), SameBS(2), NegativeToLeft(Less)(1term), BS(+1), BSbound(Exact)(Base), BSbound(Exact)(Indu), BSbound(Exact), BSbound, USteleScope(Zero)(Exact), SameTelescope(2)(Base), SameTelescope(2)(Indu), SameTelescope(2), Exp(+1), PositiveBase(Base), PositiveBase(Indu), PositiveBase, TelescopeNumerical(Base), TelescopeNumerical(Indu), TelescopeNumerical, (+1)IsPositive(N), DistributionOut(Minus), PositiveToRight(Eq)(1term), SameSeries(NumDiff), ToNegatedDoubleImPLY, AddNegatedAll, (A)to( E )(ImPLY), (E)to( A )(ImPLY), (E )to( A )(ImPLY), ToNegatedAEA, Three2threeTerms(R), LessNeq(F)(Helper), LessNeq(F), LessNeq(R),  $x = x + (y - y)$ (R),  $x = x + y - y$ (R), SubtractEquations(R), NeqAddition(R), PositiveToRight(Less)(R), PositiveToRight(Less)(1term)(R), LeqNeqLess(R), SubLeqLeft(R), ToLeq(Advanced)(R), LeqLessTransitivity(R), NegativeToLeft(Eq)(R), NegativeToRight(Less)(R), !! == Symmetry, SwitchTerms( $x \leq y - z$ ), Plus0Left(R), PositiveToRight(Eq)(R), EqAdditionLeft(R), Three2twoTerms(R), To!! ==, PositiveToRight(Less)(1term), (A )to( E ), NegativeToRight(Eq)(R), NegativeToRight(Eq)(1term)(R), DoubleMinus(R), UniqueNegative(R), SubtractEquationsLeft(R), EqNegated(R), NeqNegated(R),  $-0 = 0$ (R), NegativeNegated(R), FromLeqGeq(R),  $0 \leq |x|$ (R), PositiveNegated(R), AddEquations(R), Times(-1)(R), Times(-1)Left(R),  $-x - y = -(x + y)$ (R), LessTotality(R), SameNumerical(R), MinusNegated(R), PositiveNumerical(R), SignNumerical(+)(R), NonnegativeNumerical(R), NegativeNumerical(R), LeqNegated(R), LessNegated(R), SubLeqRight(R), FromLess(R), DistributionOut(R),  $x * 0 + x = x$ (R),  $x * 0 = 0$ (R)(fff), SignNumerical(R), NumericalDifference(R),  $x \leq |x|$ (R), USlimitIsUpperBound(Helper), USlimitIsUpperBound,  $(-1) * (-1) + (-1) * 1 = 0$ (R),  $(-1) * (-1) = 1$ (R),  $0 < 1$ Helper(R),  $0 < 1$ (R), ExpZero(Exact)(R), PositiveBase(R)(Base), Three2twoFactors(R),  $x = x * y * (1/y)$ (R), NeqMultiplication(R), LessTransitivity(R),  $0 < 2$ (R), SameExp(R)(Base), SameExp(R)(Indu), SameExp(R), SubNeqLeft(R), SubNeqRight(R), NonzeroFactors(R), NonnegativeFactors(R), PositiveFactors(R), LessDivision(R),  $0 < 1/2$ (R), PositiveToRight(Eq)(1term)(R), Exp(+1)(R), PositiveBase(R)(Indu), PositiveBase(R),  $-x * y = -(x * y)$ (R), PositiveToLeft(Eq)(R), Times1Left(R),  $x + x = 2 * x$ (R),  $(1/2)x + (1/2)x = x$ (R), DistributionOut(Minus)(R),  $(1/2)(x + y) - x = (1/2)(y - x)$ (R), IntervalSize(R)(Base), LessMultiplicationLeft(R), NegativeToLeft(Less)(R),

NegativeToLeft(Less)(1term)(R),  $y - (1/2)(x + y) = (1/2)(y - x)(R)$ ,  
 IntervalSize(R)(Indu), IntervalSize(R), XSlessUS(R), USdecreasing(+1)(R),  
 ExpUnbounded(Base), ExpUnbounded(Indu), ExpUnbounded,  $1 \leq x + 1(N)$ ,  
 ExpNonzero(Base), ExpNonzero(Indu), ExpNonzero, ExpNonzero(2),  
 HalfBase(Base), HalfBase(Indu), MultiplyEquations(R),  
 NonreciprocalToRight(Eq)(1term)(R), PositiveNonzero(R),  
 NonzeroProduct(2)(R), HalfBase, Three2threeFactors(R),  
 $x * y = z$ Backwards(R), PositiveInverted(R), ReciprocalToRight(Less)(R),  
 ReciprocalToRight(Less)(1term)(R), NonreciprocalToLeft(Less)(R),  
 $1 < x * y(R)$ , SwitchFactors( $1/x < y$ )(R), SmallHalving,  
 IntervalSize(anyPositive), USdecreasing(+n)(Base), USdecreasing(+n)(Indu),  
 USdecreasing(+n), USdecreasing, LeqAdditionLeft(R), ToNotLess(R),  
 LimitOfUSIsLeq, SubtractEquations(Less)(R),  
 SubtractEquationsLeft(Less)(R), LessNegated(Negative)(R),  
 FromNegatedAnd(ImPLY), RemoveDoubleNeg(Consequent),  
 FromNotUpperBound, LeqNUB, USlimitIsLeastUpperBound(Helper),  
 USlimitIsLeastUpperBound, ExistMP3, GreaterPositive(N), ysFClose(Helper),  
 ysFClose, ysFCauchy(Helper), ysFCauchy, from  $\ll =$ , to  $\ll =$ ,  
 NonnegativeNumerical(F), NegativeNumerical(F), tester1, tester2, tester3,  
 tester4, tester5, tester6,

sup2

$[\text{sup2} \xrightarrow{\text{prio}}$

**Preassociative**

[sup2], [base], [bracket \* end bracket], [big bracket \* end bracket], [ \$ \* \$ ],  
**[flush left** [\*], [x], [y], [z], [ $* \bowtie *$ ], [ $* \xrightarrow{*} *$ ], [pyk], [tex], [name], [prio], [\*], [T],  
 [if(\*, \*, \*)], [ $* \xrightarrow{*} *$ ], [val], [claim], [ $\perp$ ], [f(\*)], [ $(*)^I$ ], [F], [0], [1], [2], [3], [4], [5], [6],  
 [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],  
 [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [ $(*)^M$ ], [If(\*, \*, \*)],  
 [array{\*} \* end array], [l], [c], [r], [empty], [ $* | * := *$ ], [ $\mathcal{M}(*)$ ], [ $\tilde{\mathcal{U}}(*)$ ], [ $\mathcal{U}(*)$ ],  
 [ $\mathcal{U}^M(*)$ ], [**apply**(\*, \*)], [**apply**<sub>1</sub>(\*, \*)], [identifier(\*)], [identifier<sub>1</sub>(\*, \*)], [array-  
 plus(\*, \*)], [array-remove(\*, \*, \*)], [array-put(\*, \*, \*, \*)], [array-add(\*, \*, \*, \*, \*)],  
 [bit(\*, \*)], [bit<sub>1</sub>(\*, \*)], [rack], ["vector"], ["bibliography"], ["dictionary"],  
 ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],  
 ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],  
 ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],  
 ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],  
 [ $\mathcal{E}(*, *, *)$ ], [ $\mathcal{E}_2(*, *, *, *, *)$ ], [ $\mathcal{E}_3(*, *, *, *, *)$ ], [ $\mathcal{E}_4(*, *, *, *, *)$ ], [**lookup**(\*, \*, \*)],  
 [**abstract**(\*, \*, \*, \*)], [ $[*]$ ], [ $\mathcal{M}(*, *, *)$ ], [ $\mathcal{M}_2(*, *, *, *)$ ], [ $\mathcal{M}^(*, *, *)$ ], [macro],  
 [s<sub>0</sub>], [**zip**(\*, \*)], [**assoc**<sub>1</sub>(\*, \*, \*)], [ $(*)^P$ ], [self], [ $* \doteq *$ ], [ $* \doteq *$ ], [ $* \doteq *$ ],  
 [ $* \xrightarrow{\text{pyk}} *$ ], [ $* \xrightarrow{\text{tex}} *$ ], [ $* \xrightarrow{\text{name}} *$ ], [**Priority table**[\*]], [ $\tilde{\mathcal{M}}_1$ ], [ $\tilde{\mathcal{M}}_2(*)$ ], [ $\tilde{\mathcal{M}}_3(*)$ ],  
 [ $\tilde{\mathcal{M}}_4(*, *, *, *)$ ], [ $\mathcal{M}(*, *, *)$ ], [ $\mathcal{Q}(*, *, *)$ ], [ $\tilde{\mathcal{Q}}_2(*, *, *)$ ], [ $\tilde{\mathcal{Q}}_3(*, *, *, *)$ ], [ $\tilde{\mathcal{Q}}^(*, *, *, *)$ ],

$[(*)], [(*), [\text{display}(*)], [\text{statement}(*)], [[*]'], [[*]^-], [\text{aspect}(*, *)],$   
 $[\text{aspect}(*, *, *)], [(\langle * \rangle)], [\text{tuple}_1(*)], [\text{tuple}_2(*)], [\text{let}_2(*, *)], [\text{let}_1(*, *, *)],$   
 $[[* \stackrel{\text{claim}}{=} *]], [\text{checker}], [\text{check}(*, *)], [\text{check}_2(*, *, *)], [\text{check}_3(*, *, *, *)],$   
 $[\text{check}^*(*, *)], [\text{check}_2^*(*, *, *)], [[*]'], [[*]^-], [[*]^\circ], [\text{msg}], [[* \stackrel{\text{msg}}{=} *]], [\langle \text{stmt} \rangle],$   
 $[\text{stmt}], [[* \stackrel{\text{stmt}}{=} *]], [\text{HeadNil}'], [\text{HeadPair}'], [\text{Transitivity}'], [\perp], [\text{Contra}'], [\text{T}_E],$   
 $[\text{L}_1], [*, [\mathcal{A}], [\mathcal{B}], [\mathcal{C}], [\mathcal{D}], [\mathcal{E}], [\mathcal{F}], [\mathcal{G}], [\mathcal{H}], [\mathcal{I}], [\mathcal{J}], [\mathcal{K}], [\mathcal{L}], [\mathcal{M}], [\mathcal{N}], [\mathcal{O}], [\mathcal{P}], [\mathcal{Q}],$   
 $[\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [(* * := *)], [(\langle * * := * \rangle)], [\emptyset], [\text{Remainder}],$   
 $[(*^\vee)], [\text{intro}(*, *, *, *)], [\text{intro}(*, *, *)], [\text{error}(*, *)], [\text{error}_2(*, *)], [\text{proof}(*, *, *)],$   
 $[\text{proof}_2(*, *)], [\mathcal{S}(*, *)], [\mathcal{S}^I(*, *)], [\mathcal{S}^\triangleright(*, *)], [\mathcal{S}^\triangleright^I(*, *, *)], [\mathcal{S}^E(*, *)], [\mathcal{S}_I^E(*, *, *)],$   
 $[\mathcal{S}^+(*, *)], [\mathcal{S}_I^+(*, *, *)], [\mathcal{S}^-(*, *)], [\mathcal{S}_I^-(*, *, *)], [\mathcal{S}^*(*, *)], [\mathcal{S}_I^*(*, *, *)],$   
 $[\mathcal{S}_2^*(*, *, *, *)], [\mathcal{S}^\oplus(*, *)], [\mathcal{S}_I^\oplus(*, *, *)], [\mathcal{S}^+(*, *)], [\mathcal{S}_I^+(*, *, *, *)], [\mathcal{S}^{\#}(*, *)],$   
 $[\mathcal{S}_I^{\#}(*, *, *, *)], [\mathcal{S}^{\text{i.e.}}(*, *)], [\mathcal{S}_I^{\text{i.e.}}(*, *, *, *)], [\mathcal{S}_2^{\text{i.e.}}(*, *, *, *, *)], [\mathcal{S}^\vee(*, *)],$   
 $[\mathcal{S}_I^\vee(*, *, *, *)], [\mathcal{S}^i(*, *)], [\mathcal{S}_I^i(*, *, *)], [\mathcal{S}_2^i(*, *, *, *)], [\mathcal{T}(*)], [\text{claims}(*, *, *)],$   
 $[\text{claims}_2(*, *, *)], [\langle \text{proof} \rangle], [\text{proof}], [[\text{Lemma} * : *]], [[\text{Proof of} * : *]],$   
 $[[* \text{ lemma} * : *]], [[* \text{ antilemma} * : *]], [[* \text{ rule} * : *]], [[* \text{ antirule} * : *]],$   
 $[\text{verifier}], [\mathcal{V}_1(*)], [\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)],$   
 $[\mathcal{V}_7(*, *, *, *)], [\text{Cut}(*, *)], [\text{Head}_\oplus(*)], [\text{Tail}_\oplus(*)], [\text{rule}_1(*, *)], [\text{rule}(*, *)],$   
 $[\text{Rule tactic}], [\text{Plus}(*, *)], [[\text{Theory} *]], [\text{theory}_2(*, *)], [\text{theory}_3(*, *)],$   
 $[\text{theory}_4(*, *, *)], [\text{HeadNil}''], [\text{HeadPair}''], [\text{Transitivity}''], [\text{Contra}''], [\text{HeadNil}],$   
 $[\text{HeadPair}], [\text{Transitivity}], [\text{Contra}], [\text{T}_E], [\text{ragged right}],$   
 $[\text{ragged right expansion}], [\text{parm}(*, *, *)], [\text{parm}^*(*, *, *)], [\text{inst}(*, *)],$   
 $[\text{inst}^*(*, *)], [\text{occur}(*, *, *)], [\text{occur}^*(*, *, *)], [\text{unify}(* = *, *)], [\text{unify}^*( * = *, *)],$   
 $[\text{unify}_2(* = *, *)], [\text{L}_a], [\text{L}_b], [\text{L}_c], [\text{L}_d], [\text{L}_e], [\text{L}_f], [\text{L}_g], [\text{L}_h], [\text{L}_i], [\text{L}_j], [\text{L}_k], [\text{L}_l], [\text{L}_m],$   
 $[\text{L}_n], [\text{L}_o], [\text{L}_p], [\text{L}_q], [\text{L}_r], [\text{L}_s], [\text{L}_t], [\text{L}_u], [\text{L}_v], [\text{L}_w], [\text{L}_x], [\text{L}_y], [\text{L}_z], [\text{L}_A], [\text{L}_B], [\text{L}_C],$   
 $[\text{L}_D], [\text{L}_E], [\text{L}_F], [\text{L}_G], [\text{L}_H], [\text{L}_I], [\text{L}_J], [\text{L}_K], [\text{L}_L], [\text{L}_M], [\text{L}_N], [\text{L}_O], [\text{L}_P], [\text{L}_Q], [\text{L}_R],$   
 $[\text{L}_S], [\text{L}_T], [\text{L}_U], [\text{L}_V], [\text{L}_W], [\text{L}_X], [\text{L}_Y], [\text{L}_Z], [\text{L}_?], [\text{Reflexivity}], [\text{Reflexivity}_1],$   
 $[\text{Commutativity}], [\text{Commutativity}_1], [\langle \text{tactic} \rangle], [\text{tactic}], [[* \stackrel{\text{tactic}}{=} *]], [\mathcal{P}(*, *, *)],$   
 $[\mathcal{P}^*(*, *, *)], [\text{p}_0], [\text{conclude}_1(*, *)], [\text{conclude}_2(*, *, *)], [\text{conclude}_3(*, *, *, *)],$   
 $[\text{conclude}_4(*, *)], [\text{check}], [[* \stackrel{\circ}{=} *]], [\text{RootVisible}(*)], [\text{A}], [\text{R}], [\text{C}], [\text{T}], [\text{L}], [\{*\}], [*\],$   
 $[a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v],$   
 $[w], [x], [y], [z], [( * \equiv * | * := *)], [( * \equiv^0 * | * := *)], [( * \equiv^1 * | * := *)], [( * \equiv^* * | * := *)],$   
 $[\text{Ded}(*, *)], [\text{Ded}_0(*, *)], [\text{Ded}_1(*, *, *)], [\text{Ded}_2(*, *, *)], [\text{Ded}_3(*, *, *, *)],$   
 $[\text{Ded}_4(*, *, *, *)], [\text{Ded}_4^*(*, *, *, *)], [\text{Ded}_5(*, *, *)], [\text{Ded}_6(*, *, *, *)],$   
 $[\text{Ded}_6^*(*, *, *, *)], [\text{Ded}_7(*, *)], [\text{Ded}_8(*, *)], [\text{Ded}_8^*(*, *)], [\text{S}], [\text{Neg}], [\text{MP}], [\text{Gen}],$   
 $[\text{Ded}], [\text{S}_1], [\text{S}_2], [\text{S}_3], [\text{S}_4], [\text{S}_5], [\text{S}_6], [\text{S}_7], [\text{S}_8], [\text{S}_9], [\text{Repetition}], [\text{A1}'], [\text{A2}'], [\text{A4}'],$   
 $[\text{A5}'], [\text{Prop 3.2a}], [\text{Prop 3.2b}], [\text{Prop 3.2c}], [\text{Prop 3.2d}], [\text{Prop 3.2e}_1], [\text{Prop 3.2e}_2],$   
 $[\text{Prop 3.2e}], [\text{Prop 3.2f}_1], [\text{Prop 3.2f}_2], [\text{Prop 3.2f}], [\text{Prop 3.2g}_1], [\text{Prop 3.2g}_2],$   
 $[\text{Prop 3.2g}], [\text{Prop 3.2h}_1], [\text{Prop 3.2h}_2], [\text{Prop 3.2h}], [\text{Block}_1(*, *, *)], [\text{Block}_2(*, *)],$   
 $[\text{kvanti}], [\text{UniqueMember}], [\text{UniqueMember}(\text{Type})], [\text{SameSeries}], [\text{A4}],$   
 $[\text{SameMember}], [\text{Qclosed}(\text{Addition})], [\text{Qclosed}(\text{Multiplication})],$   
 $[\text{FromCartProd}(1)], [\text{1rule fromCartProd}(2)], [\text{constantRationalSeries}(*)],$   
 $[\text{cartProd}(*)], [\text{Power}(*)], [\text{binaryUnion}(*, *)], [\text{SetOfRationalSeries}],$   
 $[\text{IsSubset}(*, *)], [(\text{p}(*, *)), [(\text{s}(*, *)), [(\cdot \cdot \cdot)], [\text{Objekt-var}], [\text{Ex-var}], [\text{Ph-var}], [\text{Vardi}],$   
 $[\text{Variabel}], [\text{Op}(*)], [\text{Op}(*, *)], [* \stackrel{=}{=} *], [\text{ContainsEmpty}(*)], [\text{Nat}(*)],$   
 $[\text{Dedu}(*, *)], [\text{Dedu}_0(*, *)], [\text{Dedu}_s(*, *, *)], [\text{Dedu}_1(*, *, *)], [\text{Dedu}_2(*, *, *)],$

[Dedu<sub>3</sub>(\* , \* , \* , \*)], [Dedu<sub>4</sub>(\* , \* , \* , \*)], [Dedu<sub>4</sub><sup>\*</sup>(\* , \* , \* , \*)], [Dedu<sub>5</sub>(\* , \* , \*)],  
 [Dedu<sub>6</sub>(\* , \* , \* , \*)], [Dedu<sub>6</sub><sup>\*</sup>(\* , \* , \* , \*)], [Dedu<sub>7</sub>(\* )], [Dedu<sub>8</sub>(\* , \*)], [Dedu<sub>8</sub><sup>\*</sup>(\* , \*)],  
 [EX<sub>1</sub>], [EX<sub>2</sub>], [EX<sub>3</sub>], [EX<sub>10</sub>], [EX<sub>20</sub>], [\*EX], [\*EX], [ $\langle * \equiv * \mid * := * \rangle_{EX}$ ],  
 [ $\langle * \equiv^0 * \mid * := * \rangle_{EX}$ ], [ $\langle * \equiv^1 * \mid * := * \rangle_{EX}$ ], [ $\langle * \equiv^* * \mid * := * \rangle_{EX}$ ], [ph<sub>1</sub>], [ph<sub>2</sub>], [ph<sub>3</sub>],  
 [\*Ph], [\*Ph], [ $\langle * \equiv * \mid * := * \rangle_{Ph}$ ], [ $\langle * \equiv^0 * \mid * := * \rangle_{Ph}$ ], [ $\langle * \equiv^1 * \mid * := * \rangle_{Ph}$ ], [ $\langle * \equiv^* * \mid * := * \rangle_{Ph}$ ],  
 [ $\langle * \equiv^* * \mid * := * \rangle_{Ph}$ ], [ $\langle * \equiv * \mid * := * \rangle_{Me}$ ], [ $\langle * \equiv^1 * \mid * := * \rangle_{Me}$ ],  
 [ $\langle * \equiv^* * \mid * := * \rangle_{Me}$ ], [bs], [OBS], [BS], [∅], [SystemQ], [MP], [Gen], [Repetition],  
 [Neg], [Ded], [ExistIntro], [Extensionality], [∅def], [PairDef], [UnionDef],  
 [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg],  
 [AndCommutativity], [AutoImPLY], [Contrapositive], [FirstConjunct],  
 [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity],  
 [IffFirst], [IffSecond], [ImPLYTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4],  
 [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1],  
 [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],  
 [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power],  
 [SubsetInPower], [HelperPowerIsSub], [PowerIsSub],  
 [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality],  
 [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality],  
 [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry],  
 [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive],  
 [ERisSymmetric], [ERisTransitive], [∅isSubset], [HelperMemberNot∅],  
 [MemberNot∅], [HelperUnique∅], [Unique∅], [= Reflexivity], [= Symmetry],  
 [Helper == Transitivity], [= Transitivity], [HelperTransferNotEq],  
 [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset],  
 [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset],  
 [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection],  
 [AutoMember], [HelperEqSysNot∅], [EqSysNot∅], [HelperEqSubset],  
 [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary],  
 [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset],  
 [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImPLY], [BSsubset],  
 [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)],  
 [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ε)],  
 [(ε<sub>1</sub>)], [(ε<sub>2</sub>)], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)],  
 [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)],  
 [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X<sub>1</sub>], [X<sub>2</sub>],  
 [Y<sub>1</sub>], [Y<sub>2</sub>], [V<sub>1</sub>], [V<sub>2</sub>], [V<sub>3</sub>], [V<sub>4</sub>], [V<sub>2n</sub>], [M<sub>1</sub>], [M<sub>2</sub>], [N<sub>1</sub>], [N<sub>2</sub>], [N<sub>3</sub>], [ε], [ε<sub>1</sub>], [ε<sub>2</sub>],  
 [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)],  
 [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)],  
 [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)],  
 [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)],  
 [(S1ob)], [(S2ob)], [ph<sub>4</sub>], [ph<sub>5</sub>], [ph<sub>6</sub>], [NAT], [RATIONALSERIES], [SERIES],  
 [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1],  
 [(-1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(- - 01)], [02], [01/02],  
 [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)],  
 [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)],  
 [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)],

[Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom],  
 [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom],  
 [QisClosed(Reciprocal)(ImPLY)], [QisClosed(Reciprocal)],  
 [QisClosed(Negative)(ImPLY)], [QisClosed(Negative)], [leqReflexivity],  
 [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality],  
 [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity],  
 [plusCommutativity], [Negative], [plus0], [timesAssociativity],  
 [timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1],  
 [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)],  
 [(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy],  
 [PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)],  
 [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0],  
 [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)],  
 [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)],  
 [ExpPositive(R)], [BSzero], [BSpositive], [UStescope(Zero)],  
 [UStescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound],  
 [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)],  
 [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound],  
 [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [NatType],  
 [RationalType], [SeriesType], [Max], [Numerical], [NumericalF],  
 [MemberOfSeries(ImPLY)], [JoinConjuncts(2conditions)],  
 [prop lemma imply negation], [TND], [FromNegatedImPLY], [ToNegatedImPLY],  
 [FromNegated(2 \* ImPLY)], [FromNegatedAnd], [FromNegatedOr],  
 [ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 \* 2Disjuncts],  
 [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2],  
 [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)], [sup],  
 [ToNegatedAnd(1)], [UniqueNegative], [DoubleMinus], [MinusNegated],  
 [eqReflexivity], [eqSymmetry], [eqTransitivity], [eqTransitivity4],  
 [eqTransitivity5], [eqTransitivity6], [AddEquations], [SubtractEquations],  
 [SubtractEquationsLeft], [MultiplyEquations], [EqNegated],  
 [PositiveToRight(Eq)], [PositiveToLeft(Eq)(1term)], [NegativeToLeft(Eq)],  
 [NonreciprocalToRight(Eq)(1term)], [PlusAssociativity(4terms)], [LessNeq],  
 [NeqSymmetry], [NeqNegated], [SubNeqRight], [SubNeqLeft],  
 [NegativeToRight(Neq)(1term)], [NeqAddition], [NeqMultiplication],  
 [NonzeroProduct(2)], [UStescope(+1)], [TelescopeBound(Base)],  
 [TelescopeBound(Indu)], [TelescopeBound], [IntervalSize(Base)],  
 [IntervalSize(Indu)], [IntervalSize], [XS < US], [lemma USdecreasing(+1)],  
 [CloseUS], [CloseUS(n + 1)], [AllNegated(ImPLY)], [ExistNegated(ImPLY)],  
 [IntroExist(Helper)], [IntroExist], [ExistMP], [ExistMP2], [TwiceExistMP],  
 [TwiceExistMP2], [EAE - MP], [AddAll], [AddExist(Helper1)],  
 [AddExist(Helper2)], [AddExist], [AddExist(SimpleAnt)], [AddExist(Simple)],  
 [AddEAE], [AEA - negated], [EEA - negated], [Induction], [leqAntisymmetry],  
 [leqTransitivity], [leqAddition], [leqMultiplication], [Reciprocal], [Equality],  
 [eqLeq], [eqAddition], [eqMultiplication], [LeqMultiplicationLeft], [LeqLessEq],  
 [LessLeq], [FromLeqGeq], [subLeqRight], [subLeqLeft], [Leq + 1],  
 [PositiveToRight(Leq)], [PositiveToRight(Leq)(1term)],

[lemma negativeToRight(Leq)], [PositiveToLeft(Leq)], [negativeToLeft(Leq)],  
 [negativeToLeft(Leq)(1term)], [LeqAdditionLeft], [leqSubtraction],  
 [leqSubtractionLeft], [thirdGeq], [LeqNegated], [AddEquations(Leq)],  
 [MultiplyEquations(Leq)], [ThirdGeqSeries], [LeqNeqLess], [FromLess], [ToLess],  
 [fromNotLess], [toNotLess], [NegativeLessPositive], [leqLessTransitivity],  
 [LessLeqTransitivity], [LessTransitivity], [LessTotality], [SubLessRight],  
 [SubLessLeft], [SwitchTerms( $x < y - z$ )], [SwitchTerms( $x - y < z$ )],  
 [LessAddition], [LessAdditionLeft], [LessMultiplication],  
 [LessMultiplicationLeft], [LessDivision], [PositiveToRight(Less)],  
 [PositiveToLeft(Less)], [NegativeToLeft(Less)], [NegativeToRight(Less)],  
 [AddEquations(Less)], [AddEquations(LeqLess)], [reciprocalToLeft(Less)],  
 [LessNegated], [PositiveNonzero], [PositiveNegated], [NonpositiveNegated],  
 [NegativeNegated], [NonnegativeNegated], [PositiveHalved], [PositiveInverted],  
 [NonnegativeNumerical], [NegativeNumerical], [PositiveNumerical],  
 [lemma nonpositiveNumerical], [ $|0| = 0$ ], [ $0 \leq |x|$ ], [ $x \leq |x|$ ],  
 [FromPositiveNumerical], [SameNumerical], [SignNumerical(+)],  
 [SignNumerical], [ToNumericalLess], [FromNumericalGreater],  
 [NumericalDifference], [NumericalDifferenceLess(Helper)],  
 [NumericalDifferenceLess], [SplitNumericalSumHelper],  
 [splitNumericalSum(++)], [splitNumericalSum(--)],  
 [splitNumericalSum(+ - small)], [splitNumericalSum(+ - big)],  
 [splitNumericalSum(+ -)], [splitNumericalSum(- +)], [splitNumericalSum],  
 [SplitNumericalProduct(++)], [SplitNumericalProduct(+ -)],  
 [SplitNumericalProduct], [insertMiddleTerm(Numerical)],  
 [insertTwoMiddleTerms(Numerical)], [Three2twoTerms], [Three2threeTerms],  
 [Three2twoFactors], [Three2threeFactors], [Times(-1)], [Times(-1)Left],  
 [MaxLeq(1)], [MaxLeq(2)], [LessThanMax], [ $x + y = z$ Backwards],  
 [ $x * y = z$ Backwards], [ $x = x + (y - y)$ ], [ $x = x + y - y$ ], [ $x = x * y * (1/y)$ ],  
 [insertMiddleTerm(Sum)], [insertTwoMiddleTerms(Sum)],  
 [insertMiddleTerm(Difference)], [ $x * 0 + x = x$ ], [ $x * 0 = 0$ ], [NonnegativeFactors],  
 [NonzeroFactors], [PositiveFactors], [PlusTimesMinus], [MinusTimesMinus],  
 [ $(-1) * (-1) + (-1) * 1 = 0$ ], [ $(-1) * (-1) = 1$ ], [ $0 < 1$ Helper], [ $0 < 1$ ], [ $0 < 2$ ],  
 [ $0 < 3$ ], [ $0 < 1/2$ ], [ $0 < 1/3$ ], [TwoWholes], [ThreeWholes], [TwoHalves],  
 [ThreeThirds], [ $-x - y = -(x + y)$ ], [ $-x * y = -(x * y)$ ], [ $-0 = 0$ ],  
 [SFSymmetry], [SFtransitivity], [f2R(Plus)], [f2R(Times)],  
 [<< TransitivityHelper(Q)], [<< Transitivity], [<<== Reflexivity],  
 [<<== AntisymmetryHelper(Q)], [FromNot < f(Weak)(Helper)],  
 [FromNot < f(Weak)], [FromNot < f(Strong)(Helper2)],  
 [FromNot < f(Strong)(Helper)], [FromNot < f(Strong)],  
 [fromNotSameF(Strongest)(Helper2)], [fromNotSameF(Strongest)(Helper)],  
 [fromNotSameF(Strongest)], [ToLess(F)(Helper)], [ToLess(F)], [FromNot <<],  
 [ToLess(R)], [FromNotSameF(Weak)(Helper)], [FromNotSameF(Weak)],  
 [FromNotLess(F)], [== Addition], [== AdditionLeft],  
 [Fpart - Bounded(Base)], [Fpart - Bounded(InduHelper)],  
 [Fpart - Bounded(Indu)], [Fpart - Bounded], [F - Bounded(Helper)],  
 [F - Bounded], [SameFmultiplication(Helper)], [SameFmultiplication],

[EqMultiplication(R)], [EqMultiplicationLeft(R)], [ $x * 0 = 0(F)$ ], [ $x * 0 = 0(R)$ ],  
 [LessMultiplication(F)(Helper2)], [LessMultiplication(F)(Helper)],  
 [LessMultiplication(F)], [LessMultiplication(R)], [LeqMultiplication(R)],  
 [PlusAssociativity(F)], [Plus0(F)], [PlusCommutativity(F)],  
 [TimesAssociativity(F)], [Times1f], [Cauchy(2)(Helper)], [Cauchy(2)],  
 [ReciprocalFnonzero], [(Eventually = f)2sameF(Helper)],  
 [(Eventually = f)2sameF], [FromNotSameF(Strong)(Helper2)],  
 [FromNotSameF(Strong)(Helper)], [FromNotSameF(Strong)],  
 [SameFreciprocal(Helper)], [SameFreciprocal], [From!! ==], [Reciprocal(R)],  
 [TimesCommutativity(F)], [Distribution(F)], [FromMax(1)], [FromMax(2)],  
 [ToNegatedAnd], [DistributionOut], [DistributionOutLeft], [DistributionLeft],  
 [FromNotLess(R)], [CartProdIsRelation], [FromSubset], [SubsetIsRelation],  
 [ToSeries], [FromSeries], [SeriesSubsetCP], [ValueType], [RemoveOr],  
 [FromSingleton], [InPair(1)], [InPair(2)], [SameMember(2)], [ToBinaryUnion(1)],  
 [ToBinaryUnion(2)], [FromOrderedPair(TwoLevels)], [ToCartProd(Helper)],  
 [ToCartProd], [NonreciprocalToRight(Eq)], [NonreciprocalToLeft(Eq)(1term)],  
 [SameReciprocal], [CPseparationIsRelation], [OrderedPairEquality],  
 [ReciprocalIsFunction], [ReciprocalIsTotal], [ReciprocalIsRationalSeries],  
 [CrsIsRelation], [CrsIsFunction], [CrsIsTotal], [CrsIsSeries], [CrsLookup], [0f],  
 [1f], [ToSingleton], [FromSameSingleton], [SingletonmembersEqual],  
 [UnequalsNotInSingleton], [NonsingletonmembersUnequal], [FromOrderedPair],  
 [FromOrderedPair(1)], [FromOrderedPair(2)], [FromCartProd],  
 [FromCartProd(1)], [FromCartProd(2)], [sameOrderedPair], [InSeriesHelper],  
 [InSeries], [To = f(Subset)(Helper)], [To = f(Subset)], [To = f],  
 [productIsFunction], [productIsTotal], [ProductIsRationalSeries], [TimesF],  
 [ $-x + (1/2)x = -(1/2)x$ ], [PositiveTripled], [PositiveDividedBy3], [ $|x - x| = 0$ ],  
 [ $1 < 2$ ], [ $1/3 < 2/3$ ], [ $(1/3)x + (1/3)x = (2/3)x$ ], [ $(2/3)x + (1/3)x = x$ ],  
 [ $-x + (2/3)x = -(1/3)x$ ], [ $-(1/3)x - (1/3)x = -(2/3)x$ ],  
 [ $-x + (1/3)x = -(2/3)x$ ], [PreserveLessGreater], [ClosestolesIsLess],  
 [SubLessLeft(F)], [SubLessLeft(R)], [ClosestogreaterIsGreater],  
 [SubLessRight(F)], [SubLessRight(R)], [plus0Left], [times1Left],  
 [EqAdditionLeft], [EqMultiplicationLeft], [PlusF(Sym)], [TimesF(Sym)],  
 [SameSeries(Gen)], [EqualsSameF], [LeqReflexivity(R)], [LeqTotality(R)],  
 [PositiveToLeft(Eq)], [ExpZero(Exact)], [SameExp(Base)], [SameExp(Indu)],  
 [SameExp], [ $(1/2)(x + y) - x = (1/2)(y - x)$ ], [ $y - (1/2)(x + y) = (1/2)(y - x)$ ],  
 [BSzero(Exact)], [SameBS(2)(Base)], [SameBS(2)(Indu)], [SameBS(2)],  
 [NegativeToLeft(Less)(1term)], [BS(+1)], [BSbound(Exact)(Base)],  
 [BSbound(Exact)(Indu)], [BSbound(Exact)], [BSbound],  
 [UStescope(Zero)(Exact)], [SameTelescope(2)(Base)],  
 [SameTelescope(2)(Indu)], [SameTelescope(2)], [Exp(+1)], [PositiveBase(Base)],  
 [PositiveBase(Indu)], [PositiveBase], [TelescopeNumerical(Base)],  
 [TelescopeNumerical(Indu)], [TelescopeNumerical], [(+1)IsPositive(N)],  
 [DistributionOut(Minus)], [PositiveToRight(Eq)(1term)],  
 [SameSeries(NumDiff)], [ToNegatedDoubleImply], [AddNegatedAll],  
 [(A)to( E )(Imply)], [(E)to( A )(Imply)], [(E)to( A )(Imply)], [ToNegatedAEA],  
 [Three2threeTerms(R)], [LessNeq(F)(Helper)], [LessNeq(F)], [LessNeq(R)],

$[x = x + (y - y)(R)]$ ,  $[x = x + y - y(R)]$ ,  $[SubtractEquations(R)]$ ,  
 $[NeqAddition(R)]$ ,  $[PositiveToRight(Less)(R)]$ ,  
 $[PositiveToRight(Less)(1term)(R)]$ ,  $[LeqNeqLess(R)]$ ,  $[SubLeqLeft(R)]$ ,  
 $[ToLeq(Advanced)(R)]$ ,  $[LeqLessTransitivity(R)]$ ,  $[NegativeToLeft(Eq)(R)]$ ,  
 $[NegativeToRight(Less)(R)]$ ,  $[! = Symmetry]$ ,  $[SwitchTerms(x <= y - z)]$ ,  
 $[Plus0Left(R)]$ ,  $[PositiveToRight(Eq)(R)]$ ,  $[EqAdditionLeft(R)]$ ,  
 $[Three2twoTerms(R)]$ ,  $[To! =]$ ,  $[PositiveToRight(Less)(1term)]$ ,  $[(A \text{ to } E)]$ ,  
 $[NegativeToRight(Eq)(R)]$ ,  $[NegativeToRight(Eq)(1term)(R)]$ ,  
 $[DoubleMinus(R)]$ ,  $[UniqueNegative(R)]$ ,  $[SubtractEquationsLeft(R)]$ ,  
 $[EqNegated(R)]$ ,  $[NeqNegated(R)]$ ,  $[-0 = 0(R)]$ ,  $[NegativeNegated(R)]$ ,  
 $[FromLeqGeq(R)]$ ,  $[0 <= |x|(R)]$ ,  $[PositiveNegated(R)]$ ,  $[AddEquations(R)]$ ,  
 $[Times(-1)(R)]$ ,  $[Times(-1)Left(R)]$ ,  $[-x - y = -(x + y)(R)]$ ,  $[LessTotality(R)]$ ,  
 $[SameNumerical(R)]$ ,  $[MinusNegated(R)]$ ,  $[PositiveNumerical(R)]$ ,  
 $[SignNumerical(+)(R)]$ ,  $[NonnegativeNumerical(R)]$ ,  $[NegativeNumerical(R)]$ ,  
 $[LeqNegated(R)]$ ,  $[LessNegated(R)]$ ,  $[SubLeqRight(R)]$ ,  $[FromLess(R)]$ ,  
 $[DistributionOut(R)]$ ,  $[x * 0 + x = x(R)]$ ,  $[x * 0 = 0(R)(fff)]$ ,  $[SignNumerical(R)]$ ,  
 $[NumericalDifference(R)]$ ,  $[x <= |x|(R)]$ ,  $[USlimitIsUpperBound(Helper)]$ ,  
 $[USlimitIsUpperBound]$ ,  $[(-1) * (-1) + (-1) * 1 = 0(R)]$ ,  $[(-1) * (-1) = 1(R)]$ ,  
 $[0 < 1Helper(R)]$ ,  $[0 < 1(R)]$ ,  $[ExpZero(Exact)(R)]$ ,  $[PositiveBase(R)(Base)]$ ,  
 $[Three2twoFactors(R)]$ ,  $[x = x * y * (1/y)(R)]$ ,  $[NeqMultiplication(R)]$ ,  
 $[LessTransitivity(R)]$ ,  $[0 < 2(R)]$ ,  $[SameExp(R)(Base)]$ ,  $[SameExp(R)(Indu)]$ ,  
 $[SameExp(R)]$ ,  $[SubNeqLeft(R)]$ ,  $[SubNeqRight(R)]$ ,  $[NonzeroFactors(R)]$ ,  
 $[NonnegativeFactors(R)]$ ,  $[PositiveFactors(R)]$ ,  $[LessDivision(R)]$ ,  $[0 < 1/2(R)]$ ,  
 $[PositiveToRight(Eq)(1term)(R)]$ ,  $[Exp(+1)(R)]$ ,  $[PositiveBase(R)(Indu)]$ ,  
 $[PositiveBase(R)]$ ,  $[-x * y = -(x * y)(R)]$ ,  $[PositiveToLeft(Eq)(R)]$ ,  
 $[Times1Left(R)]$ ,  $[x + x = 2 * x(R)]$ ,  $[(1/2)x + (1/2)x = x(R)]$ ,  
 $[DistributionOut(Minus)(R)]$ ,  $[(1/2)(x + y) - x = (1/2)(y - x)(R)]$ ,  
 $[IntervalSize(R)(Base)]$ ,  $[LessMultiplicationLeft(R)]$ ,  $[NegativeToLeft(Less)(R)]$ ,  
 $[NegativeToLeft(Less)(1term)(R)]$ ,  $[y - (1/2)(x + y) = (1/2)(y - x)(R)]$ ,  
 $[IntervalSize(R)(Indu)]$ ,  $[IntervalSize(R)]$ ,  $[XSlessUS(R)]$ ,  
 $[USdecreasing(+1)(R)]$ ,  $[ExpUnbounded(Base)]$ ,  $[ExpUnbounded(Indu)]$ ,  
 $[ExpUnbounded]$ ,  $[1 <= x + 1(N)]$ ,  $[ExpNonzero(Base)]$ ,  $[ExpNonzero(Indu)]$ ,  
 $[ExpNonzero]$ ,  $[ExpNonzero(2)]$ ,  $[HalfBase(Base)]$ ,  $[HalfBase(Indu)]$ ,  
 $[MultiplyEquations(R)]$ ,  $[NonreciprocalToRight(Eq)(1term)(R)]$ ,  
 $[PositiveNonzero(R)]$ ,  $[NonzeroProduct(2)(R)]$ ,  $[HalfBase]$ ,  
 $[Three2threeFactors(R)]$ ,  $[x * y = zBackwards(R)]$ ,  $[PositiveInverted(R)]$ ,  
 $[ReciprocalToRight(Less)(R)]$ ,  $[ReciprocalToRight(Less)(1term)(R)]$ ,  
 $[NonreciprocalToLeft(Less)(R)]$ ,  $[1 < x * y(R)]$ ,  $[SwitchFactors(1/x < y)(R)]$ ,  
 $[SmallHalving]$ ,  $[IntervalSize(anyPositive)]$ ,  $[USdecreasing(+n)(Base)]$ ,  
 $[USdecreasing(+n)(Indu)]$ ,  $[USdecreasing(+n)]$ ,  $[USdecreasing]$ ,  
 $[LeqAdditionLeft(R)]$ ,  $[ToNotLess(R)]$ ,  $[LimitOfUSIsLeq]$ ,  
 $[SubtractEquations(Less)(R)]$ ,  $[SubtractEquationsLeft(Less)(R)]$ ,  
 $[LessNegated(Negative)(R)]$ ,  $[FromNegatedAnd(Implied)]$ ,  
 $[RemoveDoubleNeg(Consequent)]$ ,  $[FromNotUpperBound]$ ,  $[LeqNUB]$ ,  
 $[USlimitIsLeastUpperBound(Helper)]$ ,  $[USlimitIsLeastUpperBound]$ ,  
 $[ExistMP3]$ ,  $[GreaterPositive(N)]$ ,  $[ysFClose(Helper)]$ ,  $[ysFClose]$ ,





$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [( * + *)], [( * - *)], [* +_f *],$   
 $[* -_f *], [* + + *], [R(* ) - R(* )];$

**Preassociative**

$[* \in *];$

**Preassociative**

$[| * |], [if(*, *, *)], [Max(*, *)], [Max(*, *)];$

**Preassociative**

$[* = *], [* \neq *], [* \leq *], [* < *], [* <_f *], [* \leq_f *], [SF(*, *)], [* == *],$   
 $[*!! == *], [* << *], [* <<== *];$

**Preassociative**

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$

**Postassociative**

$[* \dot{:} *], [* \dot{:} *], [* \dot{:} : *], [* \underline{+2*} *], [* :: *], [* +2* *];$

**Postassociative**

$[*, *];$

**Preassociative**

$[* \overset{B}{\sim} *], [* \overset{D}{\sim} *], [* \overset{C}{\sim} *], [* \overset{P}{\sim} *], [* \sim *], [* = *], [* \dagger *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$   
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{ free in } *], [* \text{ free in }^* *], [* \text{ free for } * \text{ in } *],$   
 $[* \text{ free for }^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\text{var}}],$   
 $[* \#^0 *], [* \#^1 *], [* \#^* *], [* == *], [* \subseteq *];$

**Preassociative**

$[ \neg *], [ \dot{\neg} (* )n], [* \notin *], [* \neq *];$

**Preassociative**

$[* \wedge *], [* \overset{\cdot}{\wedge} *], [* \overset{\cdot}{\wedge} *], [* \wedge_c *], [* \overset{\cdot}{\wedge} *];$

**Preassociative**

$[* \vee *], [* \parallel *], [* \overset{\cdot}{\vee} *];$

**Postassociative**

$[* \overset{\cdot}{\vee} *];$

**Preassociative**

$[ \exists *: *], [ \forall *: *], [ \forall_{\text{obj}} *: *], [ \exists *: *];$

**Postassociative**

$[* \overset{\cdot}{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \overset{\cdot}{\Leftrightarrow} *];$

**Preassociative**

$[ \{ \text{ph} \in * \mid * \}];$

**Postassociative**

$[* : *], [* \text{ spy } *], [* ! *];$

**Preassociative**

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$

**Preassociative**

$[ \lambda * . *], [ \Lambda * . *], [ \Lambda * ], [ \text{ if } * \text{ then } * \text{ else } *], [ \text{ let } * = * \text{ in } *], [ \text{ let } * \ddot{=} * \text{ in } *];$

**Preassociative**

$[* \# *];$

**Preassociative**

$[*^1], [*^\triangleright], [*^\vee], [*^+], [*^-], [*^*];$

**Preassociative**

[\* @ \*], [\* ▷ \*], [\* ▷ \*], [\* ▷ \*], [\* ▷ \*], [\* ▷ \*];

**Postassociative**

[\* ⊢ \*], [\* ⊢ \*], [\* i.e. \*];

**Preassociative**

[∀\*: \*], [Π\*: \*];

**Postassociative**

[\* ⊕ \*];

**Postassociative**

[\*; \*];

**Preassociative**

[\* proves \*];

**Preassociative**

[\* **proof of** \* : \*], [Line \* : \* ▷ \*; \*], [Last line \* ▷ \* □],  
 [Line \* : Premise ▷ \*; \*], [Line \* : Side-condition ▷ \*; \*], [Arbitrary ▷ \*; \*],  
 [Local ▷ \* = \*; \*], [Begin \*; \* : End; \*], [Last block line \* ▷ \*; \*],  
 [Arbitrary ▷ \*; \*];

**Postassociative**

[\* | \*];

**Postassociative**

[\* , \*], [\* | \* \*];

**Preassociative**

[\*&\*];

**Preassociative**

[\* \\ \*], [\* linebreak[4] \*], [\* \\ \*];

[sup2  $\xrightarrow{\text{tex}}$  "sup2"]

[sup2  $\xrightarrow{\text{pyk}}$  "sup2"]

**LeqTotality(R)**

[LeqTotality(R)  $\xrightarrow{\text{proof}}$  λc.λx.℘([SystemQ ⊢  
 ∀(fx):∀(fy):⋄(⋄(⋄(∀<sub>obj</sub>ε):⋄(⋄(∀<sub>obj</sub>ñ:⋄(∀<sub>obj</sub>m̄:⋄(⋄(0 <= ε̄) ⇒ ⋄(⋄(0 =  
 ε̄)n)n)n) ⇒ ⋄(ñ <= m̄ ⇒ (fx)[m̄] <= ((fy)[m̄] + (-u(ε̄))))n)n)n)n)n) ⇒  
 {ph ∈ P({ph ∈ P({ph ∈ P(P(Union({N, Q}))) |  
 ⋄(∀<sub>obj</sub>(op1):⋄(⋄(∀<sub>obj</sub>(op2):⋄(⋄(⋄((op1) ∈ N ⇒ ⋄((op2) ∈ Q)n)n) ⇒  
 ⋄(a<sub>Ph</sub> = {{(op1), (op1)}, {(op1), (op2)}}n)n)n)n)n) | ⋄(⋄(∀<sub>obj</sub>(r1): (r1) ∈  
 f<sub>Ph</sub> ⇒ ⋄(∀<sub>obj</sub>(op1):⋄(⋄(∀<sub>obj</sub>(op2):⋄(⋄(⋄((op1) ∈ N ⇒ ⋄((op2) ∈ Q)n)n) ⇒  
 ⋄((r1) = {{(op1), (op1)}, {(op1), (op2)}}n)n)n)n)n) ⇒  
 ⋄(∀<sub>obj</sub>(f1):∀<sub>obj</sub>(f2):∀<sub>obj</sub>(f3):∀<sub>obj</sub>(f4):{{(f1), (f1)}, {(f1), (f2)}} ∈ f<sub>Ph</sub> ⇒  
 {{(f3), (f3)}, {(f3), (f4)}} ∈ f<sub>Ph</sub> ⇒ (f1) = (f3) ⇒ (f2) = (f4)n)n) ⇒  
 ⋄(∀<sub>obj</sub>(s1): (s1) ∈ N ⇒ ⋄(∀<sub>obj</sub>(s2):⋄(⋄(⋄(s1), (s1)}, {(s1), (s2)})) ∈  
 f<sub>Ph</sub>n)n)n)n) | ∀<sub>obj</sub>ε̄:⋄(⋄(∀<sub>obj</sub>ñ:⋄(∀<sub>obj</sub>m̄:⋄(0 <= ε̄) ⇒ ⋄(⋄(0 = ε̄)n)n)n) ⇒





















$$\underline{x}(\text{exp})\underline{n} \Rightarrow \forall_{\text{obj}}\underline{n}: (\underline{m} + 1) = \underline{n} \Rightarrow \underline{x}(\text{exp})(\underline{m} + 1) = \underline{x}(\text{exp})\underline{n}]$$

$$[\text{SameExp}(\text{Indu}) \xrightarrow{\text{tex}} \text{“SameExp}(\text{Indu})\text{”}]$$

$$[\text{SameExp}(\text{Indu}) \xrightarrow{\text{pyk}} \text{“lemma sameExp indu”}]$$

## SameExp

$$\begin{aligned} & [\text{SameExp} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \forall \underline{x}: \underline{m} = \underline{n} \vdash \text{SameExp}(\text{Base}) \gg \\ & \forall_{\text{obj}}\underline{n}: 0 = \underline{n} \Rightarrow \underline{x}(\text{exp})0 = \underline{x}(\text{exp})\underline{n}; \text{SameExp}(\text{Indu}) \gg \forall_{\text{obj}}\underline{n}: \underline{m} = \underline{n} \Rightarrow \\ & \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})\underline{n} \Rightarrow \forall_{\text{obj}}\underline{n}: (\underline{m} + 1) = \underline{n} \Rightarrow \underline{x}(\text{exp})(\underline{m} + 1) = \\ & \underline{x}(\text{exp})\underline{n}; \text{Induction} \triangleright \forall_{\text{obj}}\underline{n}: 0 = \underline{n} \Rightarrow \underline{x}(\text{exp})0 = \underline{x}(\text{exp})\underline{n} \triangleright \forall_{\text{obj}}\underline{n}: \underline{m} = \underline{n} \Rightarrow \\ & \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})\underline{n} \Rightarrow \forall_{\text{obj}}\underline{n}: (\underline{m} + 1) = \underline{n} \Rightarrow \underline{x}(\text{exp})(\underline{m} + 1) = \underline{x}(\text{exp})\underline{n} \gg \\ & \forall_{\text{obj}}\underline{n}: \underline{m} = \underline{n} \Rightarrow \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})\underline{n}; \text{A4} @ \underline{n} \triangleright \forall_{\text{obj}}\underline{n}: \underline{m} = \underline{n} \Rightarrow \underline{x}(\text{exp})\underline{m} = \\ & \underline{x}(\text{exp})\underline{n} \gg \underline{m} = \underline{n} \Rightarrow \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})\underline{n}; \text{MP} \triangleright \underline{m} = \underline{n} \Rightarrow \underline{x}(\text{exp})\underline{m} = \\ & \underline{x}(\text{exp})\underline{n} \triangleright \underline{m} = \underline{n} \gg \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})\underline{n} \rceil, p_0, c)] \end{aligned}$$

$$[\text{SameExp} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \forall \underline{x}: \underline{m} = \underline{n} \vdash \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})\underline{n}]$$

$$[\text{SameExp} \xrightarrow{\text{tex}} \text{“SameExp”}]$$

$$[\text{SameExp} \xrightarrow{\text{pyk}} \text{“lemma sameExp”}]$$

$$(1/2)(x + y) - x = (1/2)(y - x)$$

$$\begin{aligned} & [(1/2)(x+y) - x = (1/2)(y-x) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \text{Distribution} \gg \\ & (\text{rec}(1+1) * (\underline{x} + \underline{y})) = ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})); \text{eqAddition} \triangleright (\text{rec}(1+1) * \\ & (\underline{x} + \underline{y})) = ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) \gg ((\text{rec}(1+1) * (\underline{x} + \underline{y})) + (-\underline{ux})) = \\ & (((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) + (-\underline{ux})); \text{plusCommutativity} \gg \\ & ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) = \\ & ((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})); \text{eqAddition} \triangleright ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) = \\ & ((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})) \gg (((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) + (-\underline{ux})) = \\ & (((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})) + (-\underline{ux})); \text{plusAssociativity} \gg \\ & (((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})) + (-\underline{ux})) = ((\text{rec}(1+1) * \underline{y}) + ((\text{rec}(1+1) * \\ & \underline{x}) + (-\underline{ux}))); \text{TwoHalves} \gg ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{x})) = \\ & \underline{x}; \text{PositiveToRight}(\text{Eq}) \triangleright ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{x})) = \underline{x} \gg \\ & (\text{rec}(1+1) * \underline{x}) = (\underline{x} + (-\underline{u}(\text{rec}(1+1) * \underline{x}))); \text{EqNegated} \triangleright (\text{rec}(1+1) * \underline{x}) = \\ & (\underline{x} + (-\underline{u}(\text{rec}(1+1) * \underline{x}))) \gg (-\underline{u}(\text{rec}(1+1) * \underline{x})) = \\ & (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}(1+1) * \underline{x}))))); \text{MinusNegated} \gg (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}(1+1) * \underline{x})))) = \\ & ((\text{rec}(1+1) * \underline{x}) + (-\underline{ux})); \text{eqTransitivity} \triangleright (-\underline{u}(\text{rec}(1+1) * \underline{x})) = (-\underline{u}(\underline{x} + \\ & (-\underline{u}(\text{rec}(1+1) * \underline{x})))) \triangleright (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}(1+1) * \underline{x})))) = ((\text{rec}(1+1) * \underline{x}) + (-\underline{ux})) \gg \\ & (-\underline{u}(\text{rec}(1+1) * \underline{x})) = ((\text{rec}(1+1) * \underline{x}) + (-\underline{ux})); \text{eqSymmetry} \triangleright (-\underline{u}(\text{rec}(1+1) * \underline{x})) = \\ & ((\text{rec}(1+1) * \underline{x}) + (-\underline{ux})) \gg ((\text{rec}(1+1) * \underline{x}) + (-\underline{ux})) = \\ & (-\underline{u}(\text{rec}(1+1) * \underline{x})); \text{EqAdditionLeft} \triangleright ((\text{rec}(1+1) * \underline{x}) + (-\underline{ux})) = \end{aligned}$$

$$\begin{aligned}
& (-u(\text{rec}(1+1) * \underline{x})) \gg ((\text{rec}(1+1) * \underline{y}) + ((\text{rec}(1+1) * \underline{x}) + (-u\underline{x}))) = \\
& ((\text{rec}(1+1) * \underline{y}) + (-u(\text{rec}(1+1) * \underline{x}))); \text{DistributionOut(Minus)} \gg \\
& ((\text{rec}(1+1) * \underline{y}) + (-u(\text{rec}(1+1) * \underline{x}))) = \\
& (\text{rec}(1+1) * (\underline{y} + (-u\underline{x}))); \text{eqTransitivity6} \triangleright ((\text{rec}(1+1) * (\underline{x} + \underline{y})) + (-u\underline{x})) = \\
& (((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) + (-u\underline{x})) \triangleright (((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \\
& \underline{y})) + (-u\underline{x})) = (((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})) + (-u\underline{x})) \triangleright (((\text{rec}(1+1) * \\
& \underline{y}) + (\text{rec}(1+1) * \underline{x})) + (-u\underline{x})) = ((\text{rec}(1+1) * \underline{y}) + ((\text{rec}(1+1) * \underline{x}) + (-u\underline{x}))) \triangleright \\
& ((\text{rec}(1+1) * \underline{y}) + ((\text{rec}(1+1) * \underline{x}) + (-u\underline{x}))) = ((\text{rec}(1+1) * \underline{y}) + (-u(\text{rec}(1+1) * \\
& \underline{x}))) \triangleright ((\text{rec}(1+1) * \underline{y}) + (-u(\text{rec}(1+1) * \underline{x}))) = (\text{rec}(1+1) * (\underline{y} + (-u\underline{x}))) \gg \\
& ((\text{rec}(1+1) * (\underline{x} + \underline{y})) + (-u\underline{x})) = (\text{rec}(1+1) * (\underline{y} + (-u\underline{x})))], p_0, c]
\end{aligned}$$

$$\begin{aligned}
& [(1/2)(x+y) - x = (1/2)(y-x) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
& \forall \underline{x}. \forall \underline{y}. ((\text{rec}(1+1) * (\underline{x} + \underline{y})) + (-u\underline{x})) = (\text{rec}(1+1) * (\underline{y} + (-u\underline{x})))]
\end{aligned}$$

$$[(1/2)(x+y) - x = (1/2)(y-x) \xrightarrow{\text{tex}} "(1/2)(x+y)-x=(1/2)(y-x)"]$$

$$[(1/2)(x+y) - x = (1/2)(y-x) \xrightarrow{\text{pyk}} "\text{lemma } (1/2)(x+y)-x=(1/2)(y-x)"]$$

$$y - (1/2)(x+y) = (1/2)(y-x)$$

$$\begin{aligned}
& [y - (1/2)(x+y) = (1/2)(y-x) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. \text{Distribution} \gg \\
& (\text{rec}(1+1) * (\underline{x} + \underline{y})) = ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))); \text{EqNegated} \triangleright (\text{rec}(1+1) * \\
& (\underline{x} + \underline{y})) = ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) \gg (-u(\text{rec}(1+1) * (\underline{x} + \underline{y}))) = \\
& (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))); \text{EqAdditionLeft} \triangleright (-u(\text{rec}(1+1) * (\underline{x} + \underline{y}))) = \\
& (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))) \gg (\underline{y} + (-u(\text{rec}(1+1) * (\underline{x} + \underline{y})))) = \\
& (\underline{y} + (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))))]; \text{plusCommutativity} \gg \\
& ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) = \\
& ((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})); \text{EqNegated} \triangleright ((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})) = \\
& ((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x})) \gg (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))) = \\
& (-u((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x}))); -x - y = -(x+y) \gg \\
& ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))) = (-u((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \\
& \underline{x}))); \text{eqSymmetry} \triangleright ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))) = (-u((\text{rec}(1+1) * \\
& \underline{y}) + (\text{rec}(1+1) * \underline{x}))) \gg (-u((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x}))) = ((-u(\text{rec}(1+1) * \\
& \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))); \text{eqTransitivity} \triangleright (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))) = \\
& (-u((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x}))) \triangleright (-u((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{x}))) = \\
& ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))) \gg (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y}))) = \\
& ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))); \text{EqAdditionLeft} \triangleright (-u((\text{rec}(1+1) * \\
& \underline{x}) + (\text{rec}(1+1) * \underline{y}))) = ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))) \gg \\
& (\underline{y} + (-u((\text{rec}(1+1) * \underline{x}) + (\text{rec}(1+1) * \underline{y})))) = (\underline{y} + ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \\
& \underline{x}))))]; \text{plusAssociativity} \gg ((\underline{y} + (-u(\text{rec}(1+1) * \underline{y}))) + (-u(\text{rec}(1+1) * \underline{x}))) = \\
& (\underline{y} + ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x}))))]; \text{eqSymmetry} \triangleright ((\underline{y} + (-u(\text{rec}(1+1) * \\
& \underline{y}))) + (-u(\text{rec}(1+1) * \underline{x}))) = (\underline{y} + ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x})))) \gg \\
& (\underline{y} + ((-u(\text{rec}(1+1) * \underline{y})) + (-u(\text{rec}(1+1) * \underline{x})))) = ((\underline{y} + (-u(\text{rec}(1+1) * \\
& \underline{y}))) + (-u(\text{rec}(1+1) * \underline{x}))); \text{TwoHalves} \gg ((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{y})) = \\
& \underline{y}; \text{PositiveToRight(Eq)} \triangleright ((\text{rec}(1+1) * \underline{y}) + (\text{rec}(1+1) * \underline{y})) = \underline{y} \gg (\text{rec}(1+1) * \underline{y}) =
\end{aligned}$$

$$\begin{aligned}
& (\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))); \text{eqSymmetry} \triangleright (\text{rec}(1+1)*\underline{y}) = (\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))) \gg \\
& (\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))) = (\text{rec}(1+1)*\underline{y}); \text{eqAddition} \triangleright (\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))) = \\
& (\text{rec}(1+1)*\underline{y}) \gg ((\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))) + (-u(\text{rec}(1+1)*\underline{x}))) = \\
& ((\text{rec}(1+1)*\underline{y}) + (-u(\text{rec}(1+1)*\underline{x}))); \text{DistributionOut(Minus)} \gg \\
& ((\text{rec}(1+1)*\underline{y}) + (-u(\text{rec}(1+1)*\underline{x}))) = \\
& (\text{rec}(1+1)*(\underline{y} + (-u\underline{x}))); \text{eqTransitivity6} \triangleright (\underline{y} + (-u(\text{rec}(1+1)*(\underline{x} + \underline{y})))) = \\
& (\underline{y} + (-u((\text{rec}(1+1)*\underline{x}) + (\text{rec}(1+1)*\underline{y})))) \triangleright (\underline{y} + (-u((\text{rec}(1+1)*\underline{x}) + (\text{rec}(1+1)* \\
& \underline{y})))) = (\underline{y} + ((-u(\text{rec}(1+1)*\underline{y})) + (-u(\text{rec}(1+1)*\underline{x})))) \triangleright (\underline{y} + ((-u(\text{rec}(1+1)* \\
& \underline{y})) + (-u(\text{rec}(1+1)*\underline{x})))) = ((\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))) + (-u(\text{rec}(1+1)*\underline{x}))) \triangleright \\
& ((\underline{y} + (-u(\text{rec}(1+1)*\underline{y}))) + (-u(\text{rec}(1+1)*\underline{x}))) = ((\text{rec}(1+1)*\underline{y}) + (-u(\text{rec}(1+1) \\
& * \underline{x}))) \triangleright ((\text{rec}(1+1)*\underline{y}) + (-u(\text{rec}(1+1)*\underline{x}))) = (\text{rec}(1+1)*(\underline{y} + (-u\underline{x}))) \gg \\
& (\underline{y} + (-u(\text{rec}(1+1)*(\underline{x} + \underline{y})))) = (\text{rec}(1+1)*(\underline{y} + (-u\underline{x}))), p_0, c]
\end{aligned}$$

$$\begin{aligned}
& [\underline{y} - (1/2)(\underline{x} + \underline{y}) = (1/2)(\underline{y} - \underline{x}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
& \forall \underline{x}: \forall \underline{y}: (\underline{y} + (-u(\text{rec}(1+1)*(\underline{x} + \underline{y})))) = (\text{rec}(1+1)*(\underline{y} + (-u\underline{x})))] \\
& [\underline{y} - (1/2)(\underline{x} + \underline{y}) = (1/2)(\underline{y} - \underline{x}) \xrightarrow{\text{tex}} \text{"y-(1/2)(x+y)=(1/2)(y-x)"}] \\
& [\underline{y} - (1/2)(\underline{x} + \underline{y}) = (1/2)(\underline{y} - \underline{x}) \xrightarrow{\text{pyk}} \text{"lemma y-(1/2)(x+y)=(1/2)(y-x)"}]
\end{aligned}$$

## BSzero(Exact)

$$\begin{aligned}
& [\text{BSzero(Exact)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{m}: \text{eqReflexivity} \gg 0 = \\
& 0; \text{BSzero} \triangleright 0 = 0 \gg \text{BS}(\underline{m}, 0) = \text{rec}(1+1)(\text{exp}\underline{m})], p_0, c)] \\
& [\text{BSzero(Exact)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \text{BS}(\underline{m}, 0) = \text{rec}(1+1)(\text{exp}\underline{m})] \\
& [\text{BSzero(Exact)} \xrightarrow{\text{tex}} \text{"BSzero(Exact)"}] \\
& [\text{BSzero(Exact)} \xrightarrow{\text{pyk}} \text{"lemma base(1/2)Sum zero exact"}]
\end{aligned}$$

## SameBS(2)(Base)

$$\begin{aligned}
& [\text{SameBS(2)(Base)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{m}: \forall (\underline{n2}): 0 = (\underline{n2}) \vdash \\
& \text{eqSymmetry} \triangleright 0 = (\underline{n2}) \gg (\underline{n2}) = 0; \text{BSzero} \triangleright (\underline{n2}) = 0 \gg \text{BS}(\underline{m}, (\underline{n2})) = \\
& \text{rec}(1+1)(\text{exp}\underline{m}); \text{eqSymmetry} \triangleright \text{BS}(\underline{m}, (\underline{n2})) = \text{rec}(1+1)(\text{exp}\underline{m}) \gg \\
& \text{rec}(1+1)(\text{exp}\underline{m}) = \text{BS}(\underline{m}, (\underline{n2})); \text{BSzero(Exact)} \gg \text{BS}(\underline{m}, 0) = \\
& \text{rec}(1+1)(\text{exp}\underline{m}); \text{eqTransitivity} \triangleright \text{BS}(\underline{m}, 0) = \\
& \text{rec}(1+1)(\text{exp}\underline{m}) \triangleright \text{rec}(1+1)(\text{exp}\underline{m}) = \text{BS}(\underline{m}, (\underline{n2})) \gg \text{BS}(\underline{m}, 0) = \\
& \text{BS}(\underline{m}, (\underline{n2})); \forall \underline{m}: \forall (\underline{n2}): \text{Ded} \triangleright \forall \underline{m}: \forall (\underline{n2}): 0 = (\underline{n2}) \vdash \text{BS}(\underline{m}, 0) = \text{BS}(\underline{m}, (\underline{n2})) \gg \\
& 0 = (\underline{n2}) \Rightarrow \text{BS}(\underline{m}, 0) = \text{BS}(\underline{m}, (\underline{n2})); \text{Gen} \triangleright 0 = (\underline{n2}) \Rightarrow \text{BS}(\underline{m}, 0) = \\
& \text{BS}(\underline{m}, (\underline{n2})) \gg \forall_{\text{obj}} \underline{n2}: 0 = (\underline{n2}) \Rightarrow \text{BS}(\underline{m}, 0) = \text{BS}(\underline{m}, (\underline{n2}))], p_0, c)] \\
& [\text{SameBS(2)(Base)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall (\underline{n2}): \forall_{\text{obj}} \underline{n2}: 0 = (\underline{n2}) \Rightarrow \text{BS}(\underline{m}, 0) = \\
& \text{BS}(\underline{m}, (\underline{n2}))]
\end{aligned}$$

[SameBS(2)(Base)  $\xrightarrow{\text{tex}}$  “SameBS(2)(Base)”]

[SameBS(2)(Base)  $\xrightarrow{\text{pyk}}$  “lemma sameBase(1/2)Sum second base”]

## SameBS(2)(Indu)

[SameBS(2)(Indu)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash$   
 $\forall \mathbf{m}: \forall (\mathbf{n1}): \forall (\mathbf{n2}): \forall \mathbf{m}: \forall (\mathbf{n1}): \forall (\mathbf{n2}): \forall_{\text{obj}} (\mathbf{n2}): (\mathbf{n1} = \mathbf{n2}) \Rightarrow \text{BS}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, (\mathbf{n2})) \urcorner \vdash ((\mathbf{n1}) + \bar{1}) = (\mathbf{n2}) \urcorner \vdash (+1)\text{IsPositive}(\bar{N}) \gg \dot{\vdash} (0 <= ((\mathbf{n1}) + 1) \Rightarrow$   
 $\dot{\vdash} (\dot{\vdash} (0 = ((\mathbf{n1}) + 1))\mathbf{n})\mathbf{n}; \bar{\text{BS}}\text{positive} \triangleright \dot{\vdash} (0 <= ((\mathbf{n1}) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$   
 $((\mathbf{n1}) + 1))\mathbf{n})\mathbf{n} \gg \text{BS}(\mathbf{m}, ((\mathbf{n1}) + 1)) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))))); \mathbf{x} = \mathbf{x} + \mathbf{y} - \mathbf{y} \gg$   
 $(\mathbf{n1}) = (((\mathbf{n1}) + 1) + (-\mathbf{u1})); \text{A4} @(((\mathbf{n1}) + 1) + (-\mathbf{u1})) \triangleright \forall_{\text{obj}} (\mathbf{n2}): (\mathbf{n1} = \mathbf{n2}) \Rightarrow$   
 $\bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) = \text{BS}(\mathbf{m}, (\mathbf{n2})) \gg (\mathbf{n1}) = (((\mathbf{n1}) + 1) + (-\mathbf{u1})) \Rightarrow \bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))); \text{MP} \triangleright (\mathbf{n1}) = (((\mathbf{n1}) + 1) + (-\mathbf{u1})) \Rightarrow$   
 $\text{BS}(\mathbf{m}, (\mathbf{n1})) = \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))) \triangleright (\mathbf{n1}) = (((\mathbf{n1}) + 1) + (-\mathbf{u1})) \gg$   
 $\text{BS}(\mathbf{m}, (\mathbf{n1})) = \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))); \text{eqSymmetry} \triangleright \text{BS}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))) \gg \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))) =$   
 $\text{BS}(\mathbf{m}, (\mathbf{n1})); \text{EqAdditionLeft} \triangleright \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1}))) = \text{BS}(\mathbf{m}, (\mathbf{n1})) \gg$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1})))) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (\mathbf{n1}))); \text{PositiveToRight}(\text{Eq}) \triangleright ((\mathbf{n1}) + 1) =$   
 $(\mathbf{n2}) \gg (\mathbf{n1}) = ((\mathbf{n2}) + (-\mathbf{u1})); \text{A4} @(((\mathbf{n2}) + (-\mathbf{u1})) \triangleright \forall_{\text{obj}} (\mathbf{n2}): (\mathbf{n1}) = (\mathbf{n2}) \Rightarrow$   
 $\bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) = \text{BS}(\mathbf{m}, (\mathbf{n2})) \gg (\mathbf{n1}) = ((\mathbf{n2}) + (-\mathbf{u1})) \Rightarrow \bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))); \text{MP} \triangleright (\mathbf{n1}) = ((\mathbf{n2}) + (-\mathbf{u1})) \Rightarrow \bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))) \triangleright (\mathbf{n1}) = ((\mathbf{n2}) + (-\mathbf{u1})) \gg \bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))); \text{EqAdditionLeft} \triangleright \bar{\text{BS}}(\mathbf{m}, (\mathbf{n1})) =$   
 $\text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))) \gg (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (\mathbf{n1}))) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))))); \text{EqAdditionLeft} \triangleright$   
 $((\mathbf{n1}) + 1) = (\mathbf{n2}) \gg (\mathbf{m} + ((\mathbf{n1}) + 1)) = (\mathbf{m} + (\mathbf{n2})); \text{SameExp} \triangleright (\mathbf{m} + ((\mathbf{n1}) + 1)) =$   
 $(\mathbf{m} + (\mathbf{n2})) \gg \text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1))) = \text{rec}(1 + 1)(\text{exp}(\mathbf{m} +$   
 $(\mathbf{n2}))); \text{eqAddition} \triangleright \text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1))) = \text{rec}(1 + 1)(\text{exp}(\mathbf{m} + (\mathbf{n2})) \gg$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1})))) = (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} +$   
 $(\mathbf{n2})) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))))); \text{SubLessRight} \triangleright ((\mathbf{n1}) + 1) = (\mathbf{n2}) \triangleright \dot{\vdash} (0 <=$   
 $((\mathbf{n1}) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = ((\mathbf{n1}) + 1))\mathbf{n})\mathbf{n} \gg \dot{\vdash} (0 <= (\mathbf{n2}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$   
 $(\mathbf{n2})\mathbf{n})\mathbf{n}; \bar{\text{BS}}\text{positive} \triangleright \dot{\vdash} (0 <= (\mathbf{n2}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\mathbf{n2}))\mathbf{n})\mathbf{n} \gg \text{BS}(\mathbf{m}, (\mathbf{n2})) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + (\mathbf{n2})) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1}))))); \text{eqSymmetry} \triangleright \text{BS}(\mathbf{m}, (\mathbf{n2})) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + (\mathbf{n2})) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1})))) \gg (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + (\mathbf{n2})) +$   
 $\text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1})))) = \text{BS}(\mathbf{m}, (\mathbf{n2})); \text{eqTransitivity6} \triangleright \text{BS}(\mathbf{m}, ((\mathbf{n1}) + 1)) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1})))) \triangleright (\text{rec}(1 +$   
 $1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (((\mathbf{n1}) + 1) + (-\mathbf{u1})))) = (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} +$   
 $((\mathbf{n1}) + 1)) + \bar{\text{BS}}(\mathbf{m}, (\mathbf{n1}))) \triangleright (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, (\mathbf{n1}))) =$   
 $(\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + ((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1})))) \triangleright (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} +$   
 $((\mathbf{n1}) + 1)) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1})))) = (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + (\mathbf{n2})) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) +$   
 $(-\mathbf{u1})))) \triangleright (\text{rec}(1 + 1)(\text{exp}(\mathbf{m} + (\mathbf{n2})) + \text{BS}(\mathbf{m}, ((\mathbf{n2}) + (-\mathbf{u1})))) = \text{BS}(\mathbf{m}, (\mathbf{n2})) \gg$





## NegativeToLeft(Less)(1term)

$$\begin{aligned}
 & [\text{NegativeToLeft(Less)(1term)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: \dot{\vdash} (0 \leq = \\
 & (\underline{x} + (-\underline{uy})) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{x} + (-\underline{uy})))n)n)n \vdash \text{LessAddition} \triangleright \dot{\vdash} (0 \leq = \\
 & (\underline{x} + (-\underline{uy})) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{x} + (-\underline{uy})))n)n)n \gg \dot{\vdash} ((0 + \underline{y}) < = \\
 & ((\underline{x} + (-\underline{uy})) + \underline{y}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 + \underline{y}) = ((\underline{x} + (-\underline{uy})) + \underline{y}))n)n)n; \text{plus0Left} \gg \\
 & (0 + \underline{y}) = \underline{y}; \text{SubLessLeft} \triangleright (0 + \underline{y}) = \underline{y} \triangleright \dot{\vdash} ((0 + \underline{y}) \leq = ((\underline{x} + (-\underline{uy})) + \underline{y}) \Rightarrow \\
 & \dot{\vdash} (\dot{\vdash} ((0 + \underline{y}) = ((\underline{x} + (-\underline{uy})) + \underline{y}))n)n)n \gg \dot{\vdash} (\underline{y} \leq = ((\underline{x} + (-\underline{uy})) + \underline{y}) \Rightarrow \\
 & \dot{\vdash} (\dot{\vdash} (\underline{y} = ((\underline{x} + (-\underline{uy})) + \underline{y}))n)n)n; \text{Three2threeTerms} \gg ((\underline{x} + (-\underline{uy})) + \underline{y}) = \\
 & ((\underline{x} + \underline{y}) + (-\underline{uy})); x = x + y - y \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{uy})); \text{eqSymmetry} \triangleright \underline{x} = \\
 & ((\underline{x} + \underline{y}) + (-\underline{uy})) \gg ((\underline{x} + \underline{y}) + (-\underline{uy})) = \underline{x}; \text{eqTransitivity} \triangleright ((\underline{x} + (-\underline{uy})) + \underline{y}) = \\
 & ((\underline{x} + \underline{y}) + (-\underline{uy})) \triangleright ((\underline{x} + \underline{y}) + (-\underline{uy})) = \underline{x} \gg ((\underline{x} + (-\underline{uy})) + \underline{y}) = \\
 & \underline{x}; \text{SubLessRight} \triangleright ((\underline{x} + (-\underline{uy})) + \underline{y}) = \underline{x} \triangleright \dot{\vdash} (\underline{y} \leq = ((\underline{x} + (-\underline{uy})) + \underline{y}) \Rightarrow \\
 & \dot{\vdash} (\dot{\vdash} (\underline{y} = ((\underline{x} + (-\underline{uy})) + \underline{y}))n)n)n \gg \dot{\vdash} (\underline{y} \leq = \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{y} = \underline{x})n)n)n \rceil, p_0, c)]
 \end{aligned}$$

$$\begin{aligned}
 & [\text{NegativeToLeft(Less)(1term)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: \forall y: \dot{\vdash} (0 \leq = (\underline{x} + (-\underline{uy})) \Rightarrow \\
 & \dot{\vdash} (\dot{\vdash} (0 = (\underline{x} + (-\underline{uy})))n)n)n \vdash \dot{\vdash} (\underline{y} \leq = \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{y} = \underline{x})n)n)n]
 \end{aligned}$$

$$[\text{NegativeToLeft(Less)(1term)} \xrightarrow{\text{tex}} \text{“NegativeToLeft(Less)(1 term)”}]$$

$$[\text{NegativeToLeft(Less)(1term)} \xrightarrow{\text{pyk}} \text{“lemma negativeToLeft(Less)(1 term)”}]$$

## BS(+1)

$$\begin{aligned}
 & [\text{BS(+1)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: (+1)\text{IsPositive(N)} \gg \dot{\vdash} (0 \leq = \\
 & (\underline{n} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{n} + 1))n)n)n; \text{BSpositive} \triangleright \dot{\vdash} (0 \leq = (\underline{n} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\
 & (\underline{n} + 1))n)n)n \gg \text{BS}(\underline{m}, (\underline{n} + 1)) = \\
 & (\text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + 1)) + \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1}))))); \text{plusAssociativity} \gg \\
 & ((\underline{m} + \underline{n}) + 1) = (\underline{m} + (\underline{n} + 1)); \text{SameExp} \triangleright ((\underline{m} + \underline{n}) + 1) = (\underline{m} + (\underline{n} + 1)) \gg \\
 & \text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) = \text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + 1)); \text{eqSymmetry} \triangleright \\
 & \text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) = \text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + 1)) \gg \\
 & \text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + 1)) = \text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1); x = x + y - y \gg \\
 & \underline{n} = ((\underline{n} + 1) + (-\underline{u1})); \text{SameBS}(2) \triangleright \underline{n} = ((\underline{n} + 1) + (-\underline{u1})) \gg \text{BS}(\underline{m}, \underline{n}) = \\
 & \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1}))); \text{eqSymmetry} \triangleright \text{BS}(\underline{m}, \underline{n}) = \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1}))) \gg \\
 & \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1}))) = \text{BS}(\underline{m}, \underline{n}); \text{AddEquations} \triangleright \text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + \\
 & 1)) = \text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) \triangleright \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1}))) = \text{BS}(\underline{m}, \underline{n}) \gg \\
 & (\text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + 1)) + \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1})))) = \\
 & (\text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) + \text{BS}(\underline{m}, \underline{n})); \text{eqTransitivity} \triangleright \text{BS}(\underline{m}, (\underline{n} + 1)) = \\
 & (\text{rec}(1 + 1)(\text{exp})(\underline{m} + (\underline{n} + 1)) + \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1})))) \triangleright (\text{rec}(1 + 1)(\text{exp})(\underline{m} + \\
 & (\underline{n} + 1)) + \text{BS}(\underline{m}, ((\underline{n} + 1) + (-\underline{u1})))) = (\text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) + \text{BS}(\underline{m}, \underline{n})) \gg \\
 & \text{BS}(\underline{m}, (\underline{n} + 1)) = (\text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) + \text{BS}(\underline{m}, \underline{n})) \rceil, p_0, c)]
 \end{aligned}$$

$$\begin{aligned}
 & [\text{BS(+1)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \text{BS}(\underline{m}, (\underline{n} + 1)) = \\
 & (\text{rec}(1 + 1)(\text{exp})((\underline{m} + \underline{n}) + 1) + \text{BS}(\underline{m}, \underline{n}))]
 \end{aligned}$$

[BS(+1)  $\xrightarrow{\text{tex}}$  “BS(+1)”]

[BS(+1)  $\xrightarrow{\text{pyk}}$  “lemma base(1/2)Sum(+1)”]

## BSbound(Exact)(Base)

[BSbound(Exact)(Base)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{m}: \text{BSzero(Exact)} \gg \text{BS}(\underline{m} + 1, 0) = \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1); \text{Exp}(+1) \gg \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}); \text{eqTransitivity} \triangleright \text{BS}(\underline{m} + 1, 0) = \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1) \triangleright \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) \gg \text{BS}(\underline{m} + 1, 0) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}); \text{plus0} \gg ((\underline{m} + 1) + 0) = (\underline{m} + 1); \text{SameExp} \triangleright ((\underline{m} + 1) + 0) = (\underline{m} + 1) \gg \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0) = \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1); \text{eqTransitivity} \triangleright \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0) = \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1) \triangleright \text{rec}(1 + 1)(\text{exp})(\underline{m} + 1) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) \gg \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}); \text{AddEquations} \triangleright \text{BS}(\underline{m} + 1, 0) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) \triangleright \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0) = (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) \gg (\text{BS}(\underline{m} + 1, 0) + \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0)) = ((\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) + (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m})); \text{TwoHalves} \gg ((\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) + (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m})) = \text{rec}(1 + 1)(\text{exp})\underline{m}; \text{eqTransitivity} \triangleright (\text{BS}(\underline{m} + 1, 0) + \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0)) = ((\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) + (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m})) \triangleright ((\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m}) + (\text{rec}(1 + 1) * \text{rec}(1 + 1)(\text{exp})\underline{m})) = \text{rec}(1 + 1)(\text{exp})\underline{m} \gg (\text{BS}(\underline{m} + 1, 0) + \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0)) = \text{rec}(1 + 1)(\text{exp})\underline{m} \rfloor, p_0, c]$ ]

[BSbound(Exact)(Base)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$   
 $\forall \underline{m}: (\text{BS}(\underline{m} + 1, 0) + \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + 0)) = \text{rec}(1 + 1)(\text{exp})\underline{m}$ ]

[BSbound(Exact)(Base)  $\xrightarrow{\text{tex}}$  “BSbound(Exact)(Base)”]

[BSbound(Exact)(Base)  $\xrightarrow{\text{pyk}}$  “lemma base(1/2)Sum exact bound base”]

## BSbound(Exact)(Indu)

[BSbound(Exact)(Indu)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash$   
 $\forall \underline{m}: \forall \underline{n}: (\text{BS}(\underline{m} + 1, \underline{n}) + \text{rec}(1 + 1)(\text{exp})((\underline{m} + 1) + \underline{n})) = \text{rec}(1 + 1)(\text{exp})\underline{m} \vdash$   
 $\text{BS}(+1) \gg \text{BS}(\underline{m} + 1, \underline{n} + 1) = (\text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1) + \text{BS}(\underline{m} + 1, \underline{n})); \text{eqAddition} \triangleright \text{BS}(\underline{m} + 1, \underline{n} + 1) = (\text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1) + \text{BS}(\underline{m} + 1, \underline{n})) \gg (\text{BS}(\underline{m} + 1, \underline{n} + 1) + \text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1)) = ((\text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1) + \text{BS}(\underline{m} + 1, \underline{n})) + \text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1)); \text{plusCommutativity} \gg (\text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1) + \text{BS}(\underline{m} + 1, \underline{n})) = (\text{BS}(\underline{m} + 1, \underline{n}) + \text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1)); \text{eqAddition} \triangleright (\text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1) + \text{BS}(\underline{m} + 1, \underline{n})) = (\text{BS}(\underline{m} + 1, \underline{n}) + \text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1)) \gg ((\text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1) + \text{BS}(\underline{m} + 1, \underline{n})) + \text{rec}(1 + 1)(\text{exp})(((\underline{m} + 1) + \underline{n}) + 1)) =$



$$\begin{aligned}
& \text{rec}(1+1)(\text{exp})\underline{m}; \text{Induction} \triangleright (\text{BS}((\underline{m}+1), 0) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + 0)) = \\
& \text{rec}(1+1)(\text{exp})\underline{m} \triangleright (\text{BS}((\underline{m}+1), \underline{n}) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) = \\
& \text{rec}(1+1)(\text{exp})\underline{m} \Rightarrow (\text{BS}((\underline{m}+1), (\underline{n}+1)) + \text{rec}(1+1)(\text{exp})(((\underline{m}+1) + \underline{n}) + 1)) = \\
& \text{rec}(1+1)(\text{exp})\underline{m} \gg (\text{BS}((\underline{m}+1), \underline{n}) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) = \\
& \text{rec}(1+1)(\text{exp})\underline{m}], p_0, c]
\end{aligned}$$

$$[\text{BSbound}(\text{Exact}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash$$

$$\forall \underline{m}: \forall \underline{n}: (\text{BS}((\underline{m}+1), \underline{n}) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) = \text{rec}(1+1)(\text{exp})\underline{m}]$$

$$[\text{BSbound}(\text{Exact}) \xrightarrow{\text{tex}} \text{“BSbound}(\text{Exact})\text{”}]$$

$$[\text{BSbound}(\text{Exact}) \xrightarrow{\text{pyk}} \text{“lemma base}(1/2)\text{Sum exact bound”}]$$

## BSbound

$$\begin{aligned}
& [\text{BSbound} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \text{BSbound}(\text{Exact}) \gg (\text{BS}((\underline{m}+1), \underline{n}) \\
& + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) = \text{rec}(1+1)(\text{exp})\underline{m}; \text{plusCommutativity} \gg \\
& (\text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}) + \text{BS}((\underline{m}+1), \underline{n})) = \\
& (\text{BS}((\underline{m}+1), \underline{n}) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})); \text{eqTransitivity} \triangleright (\text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}) + \text{BS}((\underline{m}+1), \underline{n})) = \\
& (\text{BS}((\underline{m}+1), \underline{n}) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) \triangleright (\text{BS}((\underline{m}+1), \underline{n}) + \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) = \\
& \text{rec}(1+1)(\text{exp})\underline{m} \gg (\text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}) + \text{BS}((\underline{m}+1), \underline{n})) = \\
& \text{rec}(1+1)(\text{exp})\underline{m}; \text{PositiveToRight}(\text{Eq}) \triangleright (\text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}) + \\
& \text{BS}((\underline{m}+1), \underline{n})) = \text{rec}(1+1)(\text{exp})\underline{m} \gg \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}) = \\
& (\text{rec}(1+1)(\text{exp})\underline{m} + (-\text{uBS}((\underline{m}+1), \underline{n}))); 0 < 1/2 \gg \dot{\vdash} (0 \leq \text{rec}(1+1) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (0 = \text{rec}(1+1))\underline{n})\underline{n}); \text{PositiveBase} \triangleright \dot{\vdash} (0 \leq \text{rec}(1+1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\
& \text{rec}(1+1))\underline{n})\underline{n}) \gg \dot{\vdash} (0 \leq \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\
& \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}))\underline{n})\underline{n}); \text{SubLessRight} \triangleright \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}) = \\
& (\text{rec}(1+1)(\text{exp})\underline{m} + (-\text{uBS}((\underline{m}+1), \underline{n}))) \triangleright \dot{\vdash} (0 \leq \\
& \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n})) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \text{rec}(1+1)(\text{exp})((\underline{m}+1) + \underline{n}))\underline{n})\underline{n}) \gg \\
& \dot{\vdash} (0 \leq (\text{rec}(1+1)(\text{exp})\underline{m} + (-\text{uBS}((\underline{m}+1), \underline{n})))) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\
& (\text{rec}(1+1)(\text{exp})\underline{m} + (-\text{uBS}((\underline{m}+1), \underline{n}))))\underline{n})\underline{n}); \text{NegativeToLeft}(\text{Less})(1\text{term}) \triangleright \\
& \dot{\vdash} (0 \leq (\text{rec}(1+1)(\text{exp})\underline{m} + (-\text{uBS}((\underline{m}+1), \underline{n})))) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\
& (\text{rec}(1+1)(\text{exp})\underline{m} + (-\text{uBS}((\underline{m}+1), \underline{n}))))\underline{n})\underline{n}) \gg \dot{\vdash} (\text{BS}((\underline{m}+1), \underline{n}) \leq \\
& \text{rec}(1+1)(\text{exp})\underline{m}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\text{BS}((\underline{m}+1), \underline{n}) = \text{rec}(1+1)(\text{exp})\underline{m})\underline{n})\underline{n}], p_0, c)]
\end{aligned}$$

$$[\text{BSbound} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (\text{BS}((\underline{m}+1), \underline{n}) \leq \text{rec}(1+1)(\text{exp})\underline{m}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\text{BS}((\underline{m}+1), \underline{n}) = \text{rec}(1+1)(\text{exp})\underline{m})\underline{n})\underline{n})]$$

$$[\text{BSbound} \xrightarrow{\text{tex}} \text{“BSbound”}]$$

$$[\text{BSbound} \xrightarrow{\text{pyk}} \text{“lemma base}(1/2)\text{Sum bound”}]$$

## UStelescope(Zero)(Exact)

$$\begin{aligned} & [\text{UStelescope(Zero)(Exact)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}. \text{eqReflexivity} \gg 0 = \\ & 0; \text{UStelescope(Zero)} \triangleright 0 = 0 \gg \text{UStelescope}(\underline{m}, 0) = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))|, p_0, c)] \end{aligned}$$

$$[\text{UStelescope(Zero)(Exact)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}. \text{UStelescope}(\underline{m}, 0) = |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))|]$$

$$[\text{UStelescope(Zero)(Exact)} \xrightarrow{\text{tex}} \text{“UStelescope(Zero)(Exact)”}]$$

$$[\text{UStelescope(Zero)(Exact)} \xrightarrow{\text{pyk}} \text{“lemma UStelescope zero exact”}]$$

## SameTelescope(2)(Base)

$$\begin{aligned} & [\text{SameTelescope(2)(Base)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}. \forall (\underline{n2}): 0 = (\underline{n2}) \vdash \\ & \text{eqSymmetry} \triangleright 0 = (\underline{n2}) \gg (\underline{n2}) = 0; \text{UStelescope(Zero)} \triangleright (\underline{n2}) = 0 \gg \\ & \text{UStelescope}(\underline{m}, (\underline{n2})) = |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))|; \text{eqSymmetry} \triangleright \text{UStelescope}(\underline{m}, (\underline{n2})) = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))| \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))| = \\ & \text{UStelescope}(\underline{m}, (\underline{n2})); \text{UStelescope(Zero)(Exact)} \gg \text{UStelescope}(\underline{m}, 0) = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))|; \text{eqTransitivity} \triangleright \text{UStelescope}(\underline{m}, 0) = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))| \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + 1)]))| = \text{UStelescope}(\underline{m}, (\underline{n2})) \gg \\ & \text{UStelescope}(\underline{m}, 0) = \text{UStelescope}(\underline{m}, (\underline{n2})); \forall \underline{m}. \forall (\underline{n2}): \text{Ded} \triangleright \forall \underline{m}. \forall (\underline{n2}): 0 = \\ & (\underline{n2}) \vdash \text{UStelescope}(\underline{m}, 0) = \text{UStelescope}(\underline{m}, (\underline{n2})) \gg 0 = (\underline{n2}) \Rightarrow \\ & \text{UStelescope}(\underline{m}, 0) = \text{UStelescope}(\underline{m}, (\underline{n2})); \text{Gen} \triangleright 0 = (\underline{n2}) \Rightarrow \\ & \text{UStelescope}(\underline{m}, 0) = \text{UStelescope}(\underline{m}, (\underline{n2})) \gg \forall_{\text{obj}} (\underline{n2}): 0 = (\underline{n2}) \Rightarrow \\ & \text{UStelescope}(\underline{m}, 0) = \text{UStelescope}(\underline{m}, (\underline{n2})) \rceil, p_0, c)] \end{aligned}$$

$$[\text{SameTelescope(2)(Base)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}. \forall (\underline{n2}): \forall_{\text{obj}} (\underline{n2}): 0 = (\underline{n2}) \Rightarrow \text{UStelescope}(\underline{m}, 0) = \text{UStelescope}(\underline{m}, (\underline{n2}))]$$

$$[\text{SameTelescope(2)(Base)} \xrightarrow{\text{tex}} \text{“SameTelescope(2)(Base)”}]$$

$$[\text{SameTelescope(2)(Base)} \xrightarrow{\text{pyk}} \text{“lemma sameTelescope second base”}]$$

## SameTelescope(2)(Indu)

$$\begin{aligned} & [\text{SameTelescope(2)(Indu)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \\ & \forall \underline{m}. \forall (\underline{n1}): \forall (\underline{n2}): \forall \underline{m}. \forall (\underline{n1}): \forall (\underline{n2}): \forall_{\text{obj}} (\underline{n2}): (\underline{n1}) = (\underline{n2}) \Rightarrow \\ & \text{UStelescope}(\underline{m}, (\underline{n1})) = \text{UStelescope}(\underline{m}, (\underline{n2})) \vdash ((\underline{n1}) + 1) = (\underline{n2}) \vdash \\ & (+1)\text{IsPositive}(\underline{N}) \gg \dot{\vdash} (0 \leq ((\underline{n1}) + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & ((\underline{n1}) + 1))\underline{n})\underline{n}; \text{UStelescope(Positive)} \triangleright \dot{\vdash} (0 \leq ((\underline{n1}) + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & ((\underline{n1}) + 1))\underline{n})\underline{n} \gg \text{UStelescope}(\underline{m}, ((\underline{n1}) + 1)) = (|(\text{us}[\underline{m}] + ((\underline{n1}) + 1))| + \end{aligned}$$







## Exp(+1)

[Exp(+1)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: (+1)\text{IsPositive}(N) \gg \dot{\vdash} (0 <= (\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{m} + 1))n)n)n; \text{ExpPositive} \triangleright \dot{\vdash} (0 <= (\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{m} + 1))n)n)n \gg \underline{x}(\text{exp})(\underline{m} + 1) = (\underline{x} * \underline{x}(\text{exp}))((\underline{m} + 1) + (-u1)); x = x + y - y \gg \underline{m} = ((\underline{m} + 1) + (-u1)); \text{SameExp} \triangleright \underline{m} = ((\underline{m} + 1) + (-u1)) \gg \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})((\underline{m} + 1) + (-u1)); \text{EqMultiplicationLeft} \triangleright \underline{x}(\text{exp})\underline{m} = \underline{x}(\text{exp})((\underline{m} + 1) + (-u1)) \gg (\underline{x} * \underline{x}(\text{exp})\underline{m}) = (\underline{x} * \underline{x}(\text{exp}))((\underline{m} + 1) + (-u1)); \text{eqSymmetry} \triangleright (\underline{x} * \underline{x}(\text{exp})\underline{m}) = (\underline{x} * \underline{x}(\text{exp}))((\underline{m} + 1) + (-u1)) \gg (\underline{x} * \underline{x}(\text{exp}))((\underline{m} + 1) + (-u1)) = (\underline{x} * \underline{x}(\text{exp})\underline{m}); \text{eqTransitivity} \triangleright \underline{x}(\text{exp})(\underline{m} + 1) = (\underline{x} * \underline{x}(\text{exp}))((\underline{m} + 1) + (-u1)) \triangleright (\underline{x} * \underline{x}(\text{exp}))((\underline{m} + 1) + (-u1)) = (\underline{x} * \underline{x}(\text{exp})\underline{m}) \gg \underline{x}(\text{exp})(\underline{m} + 1) = (\underline{x} * \underline{x}(\text{exp})\underline{m}) \rceil, p_0, c]$

[Exp(+1)  $\xrightarrow{\text{stnt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{x}: \underline{x}(\text{exp})(\underline{m} + 1) = (\underline{x} * \underline{x}(\text{exp})\underline{m})]$

[Exp(+1)  $\xrightarrow{\text{tex}}$  “Exp(+1)”]

[Exp(+1)  $\xrightarrow{\text{pyk}}$  “lemma exp(+1)”]

## PositiveBase(Base)

[PositiveBase(Base)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \text{ExpZero}(\text{Exact}) \gg \underline{x}(\text{exp})0 = 1; \text{eqSymmetry} \triangleright \underline{x}(\text{exp})0 = 1 \gg 1 = \underline{x}(\text{exp})0; 0 < 1 \gg \dot{\vdash} (0 <= 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n; \text{SubLessRight} \triangleright 1 = \underline{x}(\text{exp})0 \triangleright \dot{\vdash} (0 <= 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n \gg \dot{\vdash} (0 <= \underline{x}(\text{exp})0 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})0)n)n)n \rceil, p_0, c]$

[PositiveBase(Base)  $\xrightarrow{\text{stnt}}$  SystemQ  $\vdash \forall \underline{x}: \dot{\vdash} (0 <= \underline{x}(\text{exp})0 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})0)n)n)n]$

[PositiveBase(Base)  $\xrightarrow{\text{tex}}$  “PositiveBase(Base)”]

[PositiveBase(Base)  $\xrightarrow{\text{pyk}}$  “lemma positiveBase base”]

## PositiveBase(Indu)

[PositiveBase(Indu)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 <= \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash \dot{\vdash} (0 <= \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})n)n)n \vdash \text{Exp}(+1) \gg \underline{x}(\text{exp})(\underline{m} + 1) = (\underline{x} * \underline{x}(\text{exp})\underline{m}); \text{eqSymmetry} \triangleright \underline{x}(\text{exp})(\underline{m} + 1) = (\underline{x} * \underline{x}(\text{exp})\underline{m}) \gg (\underline{x} * \underline{x}(\text{exp})\underline{m}) = \underline{x}(\text{exp})(\underline{m} + 1); \text{PositiveFactors} \triangleright \dot{\vdash} (0 <= \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \triangleright \dot{\vdash} (0 <= \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})n)n)n \gg \dot{\vdash} (0 <= (\underline{x} * \underline{x}(\text{exp})\underline{m}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{x} * \underline{x}(\text{exp})\underline{m}))n)n)n; \text{SubLessRight} \triangleright (\underline{x} * \underline{x}(\text{exp})\underline{m}) = \underline{x}(\text{exp})(\underline{m} + 1) \triangleright \dot{\vdash} (0 <= (\underline{x} * \underline{x}(\text{exp})\underline{m}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{x} * \underline{x}(\text{exp})\underline{m}))n)n)n \gg \dot{\vdash} (0 <= \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))n)n)n; \forall \underline{m}: \forall \underline{x}: \text{Ded} \triangleright \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 <= \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash$

$$\begin{aligned} & \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \vdash \dot{\vdash} (0 \leq \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n}) \gg \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \\ & \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n}); \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})\underline{n})\underline{n}) \vdash \text{MP} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \\ & \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n}) \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x})\underline{n})\underline{n}) \gg \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \\ & \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n}], p_0, c) \end{aligned}$$

$$\begin{aligned} & [\text{PositiveBase}(\text{Indu}) \xrightarrow{\text{stnt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})\underline{n})\underline{n}) \vdash \\ & \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n})] \end{aligned}$$

$$[\text{PositiveBase}(\text{Indu}) \xrightarrow{\text{tex}} \text{“PositiveBase}(\text{Indu})\text{”}]$$

$$[\text{PositiveBase}(\text{Indu}) \xrightarrow{\text{pyk}} \text{“lemma positiveBase indu”}]$$

## PositiveBase

$$\begin{aligned} & [\text{PositiveBase} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x})\underline{n})\underline{n}) \vdash \text{PositiveBase}(\text{Base}) \gg \dot{\vdash} (0 \leq \underline{x}(\text{exp})0 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x}(\text{exp})0)\underline{n})\underline{n}); \text{PositiveBase}(\text{Indu}) \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})\underline{n})\underline{n}) \gg \\ & \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n}); \text{Induction} \triangleright \dot{\vdash} (0 \leq \underline{x}(\text{exp})0 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x}(\text{exp})0)\underline{n})\underline{n}) \triangleright \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \Rightarrow \dot{\vdash} (0 \leq \\ & \underline{x}(\text{exp})(\underline{m} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})(\underline{m} + 1))\underline{n})\underline{n}) \gg \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n}) \rceil, p_0, c) \end{aligned}$$

$$[\text{PositiveBase} \xrightarrow{\text{stnt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})\underline{n})\underline{n}) \vdash \dot{\vdash} (0 \leq \underline{x}(\text{exp})\underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x}(\text{exp})\underline{m})\underline{n})\underline{n})]$$

$$[\text{PositiveBase} \xrightarrow{\text{tex}} \text{“PositiveBase”}]$$

$$[\text{PositiveBase} \xrightarrow{\text{pyk}} \text{“lemma positiveBase”}]$$

## TelescopeNumerical(Base)

$$\begin{aligned} & [\text{TelescopeNumerical}(\text{Base}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \text{eqReflexivity} \gg 0 = \\ & 0; \text{UStelescope}(\text{Zero}) \triangleright 0 = 0 \gg \text{UStelescope}(\underline{m}, 0) = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + 1]))|; \text{eqReflexivity} \gg \underline{m} = \underline{m}; \text{plus0Left} \gg (0 + 1) = \\ & 1; \text{EqAdditionLeft} \triangleright (0 + 1) = 1 \gg (\underline{m} + (0 + 1)) = (\underline{m} + 1); \text{eqSymmetry} \triangleright (\underline{m} + \\ & (0 + 1)) = (\underline{m} + 1) \gg (\underline{m} + 1) = (\underline{m} + (0 + 1)); \text{SameSeries}(\text{NumDiff}) \triangleright \underline{m} = \\ & \underline{m} \triangleright (\underline{m} + 1) = (\underline{m} + (0 + 1)) \gg |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + 1]))| = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + (0 + 1)]))|; \text{eqTransitivity} \triangleright \text{UStelescope}(\underline{m}, 0) = \\ & |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + 1]))| \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + 1]))| = \end{aligned}$$

$$\begin{aligned}
& |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| \gg \text{UStelescope}(\underline{m}, 0) = \\
& |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))|; \text{eqSymmetry} \triangleright \text{UStelescope}(\underline{m}, 0) = \\
& |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| = \\
& \text{UStelescope}(\underline{m}, 0); \text{eqLeq} \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| = \\
& \text{UStelescope}(\underline{m}, 0) \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| \leq = \\
& \text{UStelescope}(\underline{m}, 0], p_0, c]
\end{aligned}$$

$$\begin{aligned}
& [\text{TelescopeNumerical}(\text{Base}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
& \forall \underline{m}: |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| \leq = \text{UStelescope}(\underline{m}, 0)] \\
& [\text{TelescopeNumerical}(\text{Base}) \xrightarrow{\text{tex}} \text{“TelescopeNumerical}(\text{Base})\text{”}] \\
& [\text{TelescopeNumerical}(\text{Base}) \xrightarrow{\text{pyk}} \text{“lemma telescopeNumerical base”}]
\end{aligned}$$

## TelescopeNumerical(Indu)

$$\begin{aligned}
& [\text{TelescopeNumerical}(\text{Indu}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\uparrow \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: |(\text{us}[\underline{m}] + \\
& (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq = \text{UStelescope}(\underline{m}, \underline{n}) \vdash (+1)\text{IsPositive}(\underline{N}) \gg \dot{\vdash} (0 \leq = \\
& (\underline{n} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{n} + 1))\underline{n})\underline{n}); \text{UStelescope}(\text{Positive}) \triangleright \dot{\vdash} (0 \leq = (\underline{n} + 1) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (0 = (\underline{n} + 1))\underline{n})\underline{n}) \gg \text{UStelescope}(\underline{m}, (\underline{n} + 1)) = (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + \\
& (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, ((\underline{n} + 1) + (-\underline{u}1))))); x = \\
& x + y - y \gg \underline{n} = ((\underline{n} + 1) + (-\underline{u}1)); \text{eqSymmetry} \triangleright \underline{n} = ((\underline{n} + 1) + (-\underline{u}1)) \gg \\
& ((\underline{n} + 1) + (-\underline{u}1)) = \underline{n}; \text{SameTelescope}(2) \triangleright ((\underline{n} + 1) + (-\underline{u}1)) = \underline{n} \gg \\
& \text{UStelescope}(\underline{m}, ((\underline{n} + 1) + (-\underline{u}1))) = \text{UStelescope}(\underline{m}, \underline{n}); \text{EqAdditionLeft} \triangleright \\
& \text{UStelescope}(\underline{m}, ((\underline{n} + 1) + (-\underline{u}1))) = \text{UStelescope}(\underline{m}, \underline{n}) \gg (|(\text{us}[\underline{m} + (\underline{n} + \\
& 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, ((\underline{n} + 1) + (-\underline{u}1)))) = \\
& (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \\
& \text{UStelescope}(\underline{m}, \underline{n}); \text{eqTransitivity} \triangleright \text{UStelescope}(\underline{m}, (\underline{n} + 1)) = (|(\text{us}[\underline{m} + (\underline{n} + \\
& 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, ((\underline{n} + 1) + (-\underline{u}1)))) \triangleright (|(\text{us}[\underline{m} + \\
& (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, ((\underline{n} + 1) + (-\underline{u}1)))) = \\
& (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, \underline{n}) \gg \\
& \text{UStelescope}(\underline{m}, (\underline{n} + 1)) = (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \\
& \text{UStelescope}(\underline{m}, \underline{n}); \text{eqSymmetry} \triangleright \text{UStelescope}(\underline{m}, (\underline{n} + 1)) = \\
& (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, \underline{n}) \gg \\
& (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, \underline{n}) = \\
& \text{UStelescope}(\underline{m}, (\underline{n} + 1)); \text{LeqAdditionLeft} \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq = \\
& \text{UStelescope}(\underline{m}, \underline{n}) \gg (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \\
& |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))|) \leq = (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + \\
& 1))]))| + \text{UStelescope}(\underline{m}, \underline{n}); \text{subLeqRight} \triangleright (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + \\
& ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, \underline{n}) = \text{UStelescope}(\underline{m}, (\underline{n} + 1)) \triangleright (|(\text{us}[\underline{m} + \\
& (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))|) \leq = \\
& (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + \text{UStelescope}(\underline{m}, \underline{n}) \gg \\
& (|(\text{us}[\underline{m} + (\underline{n} + 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + \\
& 1))]))|) \leq = \text{UStelescope}(\underline{m}, (\underline{n} + 1)); \text{plusCommutativity} \gg (|(\text{us}[\underline{m} + (\underline{n} + \\
& 1))]| + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))|) =
\end{aligned}$$

$(|(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + |(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))|); \text{subLeqLeft} \triangleright (|(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))|) = (|(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + |(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))|) \triangleright (|(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| + |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))|) \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)) \gg (|(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + |(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))|) \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)); \text{insertMiddleTerm(Numerical)} \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq (|(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + |(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))|); \text{leqTransitivity} \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq (|(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + |(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))|) \triangleright (|(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + |(\text{us}[(\underline{m} + (\underline{n} + 1))] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))|) \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)) \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)); \forall \underline{m}: \forall \underline{n}: \text{Ded} \triangleright \forall \underline{m}: \forall \underline{n}: |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n}) \vdash |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)) \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n}) \Rightarrow |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)), p_0, c]$

$[\text{TelescopeNumerical}(\text{Indu}) \xrightarrow{\text{stnt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n}) \Rightarrow |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1))]$

$[\text{TelescopeNumerical}(\text{Indu}) \xrightarrow{\text{tex}} \text{“TelescopeNumerical}(\text{Indu})\text{”}]$

$[\text{TelescopeNumerical}(\text{Indu}) \xrightarrow{\text{pyk}} \text{“lemma telescopeNumerical indu”}]$

## TelescopeNumerical

$[\text{TelescopeNumerical} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\vdash \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \text{TelescopeNumerical}(\text{Base}) \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| \leq \text{UStelescope}(\underline{m}, 0); \text{TelescopeNumerical}(\text{Indu}) \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n}) \Rightarrow |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)); \text{Induction} \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (0 + 1))]))| \leq \text{UStelescope}(\underline{m}, 0) \triangleright |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n}) \Rightarrow |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + ((\underline{n} + 1) + 1))]))| \leq \text{UStelescope}(\underline{m}, (\underline{n} + 1)) \gg |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n}), p_0, c)]$

$[\text{TelescopeNumerical} \xrightarrow{\text{stnt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: |(\text{us}[\underline{m}] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| \leq \text{UStelescope}(\underline{m}, \underline{n})]$

$[\text{TelescopeNumerical} \xrightarrow{\text{tex}} \text{“TelescopeNumerical”}]$

$[\text{TelescopeNumerical} \xrightarrow{\text{pyk}} \text{“lemma telescopeNumerical”}]$

## (+1)IsPositive(N)

$[(+1)IsPositive(N) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \text{Nat}(\underline{m}) \# \text{Nonnegative}(N) \triangleright \text{Nat}(\underline{m}) \gg 0 \leq \underline{m}; \text{Leq} + 1 \triangleright 0 \leq \underline{m} \gg \dot{\vdash} (0 \leq (\underline{m} + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{m} + 1))n)n \rceil, p_0, c)]$

$[(+1)IsPositive(N) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \text{Nat}(\underline{m}) \# \dot{\vdash} (0 \leq (\underline{m} + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{m} + 1))n)n]$

$[(+1)IsPositive(N) \xrightarrow{\text{tex}} \text{"(+1)IsPositive(N)"}]$

$[(+1)IsPositive(N) \xrightarrow{\text{pyk}} \text{"lemma +1IsPositive(N)"}]$

## DistributionOut(Minus)

$[\text{DistributionOut(Minus)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \text{Times}(-1)\text{Left} \gg ((-u1) * (\underline{x} * \underline{z})) = (-u(\underline{x} * \underline{z})); \text{eqSymmetry} \triangleright ((-u1) * (\underline{x} * \underline{z})) = (-u(\underline{x} * \underline{z})) \gg (-u(\underline{x} * \underline{z})) = ((-u1) * (\underline{x} * \underline{z})); \text{timesCommutativity} \gg ((-u1) * (\underline{x} * \underline{z})) = ((\underline{x} * \underline{z}) * (-u1)); \text{timesAssociativity} \gg ((\underline{x} * \underline{z}) * (-u1)) = (\underline{x} * (\underline{z} * (-u1))); \text{Times}(-1) \gg (\underline{z} * (-u1)) = (-u\underline{z}); \text{EqMultiplicationLeft} \triangleright (\underline{z} * (-u1)) = (-u\underline{z}) \gg (\underline{x} * (\underline{z} * (-u1))) = (\underline{x} * (-u\underline{z})); \text{eqTransitivity5} \triangleright (-u(\underline{x} * \underline{z})) = ((-u1) * (\underline{x} * \underline{z})) \triangleright ((-u1) * (\underline{x} * \underline{z})) = ((\underline{x} * \underline{z}) * (-u1)) \triangleright ((\underline{x} * \underline{z}) * (-u1)) = (\underline{x} * (\underline{z} * (-u1))) \triangleright (\underline{x} * (\underline{z} * (-u1))) = (\underline{x} * (-u\underline{z})) \gg (-u(\underline{x} * \underline{z})) = (\underline{x} * (-u\underline{z})); \text{EqAdditionLeft} \triangleright (-u(\underline{x} * \underline{z})) = (\underline{x} * (-u\underline{z})) \gg ((\underline{x} * \underline{y}) + (-u(\underline{x} * \underline{z}))) = ((\underline{x} * \underline{y}) + (\underline{x} * (-u\underline{z}))); \text{DistributionOut} \gg ((\underline{x} * \underline{y}) + (\underline{x} * (-u\underline{z}))) = (\underline{x} * (\underline{y} + (-u\underline{z}))); \text{eqTransitivity} \triangleright ((\underline{x} * \underline{y}) + (-u(\underline{x} * \underline{z}))) = ((\underline{x} * \underline{y}) + (\underline{x} * (-u\underline{z}))) \triangleright ((\underline{x} * \underline{y}) + (\underline{x} * (-u\underline{z}))) = (\underline{x} * (\underline{y} + (-u\underline{z}))) \gg ((\underline{x} * \underline{y}) + (-u(\underline{x} * \underline{z}))) = (\underline{x} * (\underline{y} + (-u\underline{z}))) \rceil, p_0, c)]$

$[\text{DistributionOut(Minus)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} * \underline{y}) + (-u(\underline{x} * \underline{z}))) = (\underline{x} * (\underline{y} + (-u\underline{z})))]$

$[\text{DistributionOut(Minus)} \xrightarrow{\text{tex}} \text{"DistributionOut(Minus)"}]$

$[\text{DistributionOut(Minus)} \xrightarrow{\text{pyk}} \text{"lemma distributionOut(Minus)"}]$

## PositiveToRight(Eq)(1term)

$[\text{PositiveToRight(Eq)(1term)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash \text{eqSymmetry} \triangleright \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \text{PositiveToLeft(Eq)(1term)} \triangleright \underline{y} = \underline{x} \gg (\underline{y} + (-u\underline{x})) = 0; \text{eqSymmetry} \triangleright (\underline{y} + (-u\underline{x})) = 0 \gg 0 = (\underline{y} + (-u\underline{x})) \rceil, p_0, c)]$

$[\text{PositiveToRight(Eq)(1term)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash 0 = (\underline{y} + (-u\underline{x}))]$



# AddNegatedAll

$$[\text{AddNegatedAll} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \forall \underline{b}: \underline{b} \Rightarrow \underline{a} \vdash \text{AddAll} \triangleright \underline{b} \Rightarrow \underline{a} \gg \forall_{\text{obj}}(\underline{v1}): \underline{b} \Rightarrow \forall_{\text{obj}}(\underline{v1}): \underline{a}; \text{Contrapositive} \triangleright \forall_{\text{obj}}(\underline{v1}): \underline{b} \Rightarrow \forall_{\text{obj}}(\underline{v1}): \underline{a} \gg \neg(\forall_{\text{obj}}(\underline{v1}): \underline{a})n \Rightarrow \neg(\forall_{\text{obj}}(\underline{v1}): \underline{b})n \rceil, p_0, c)]$$

$$[\text{AddNegatedAll} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \forall \underline{b}: \underline{b} \Rightarrow \underline{a} \vdash \neg(\forall_{\text{obj}}(\underline{v1}): \underline{a})n \Rightarrow \neg(\forall_{\text{obj}}(\underline{v1}): \underline{b})n]$$

$$[\text{AddNegatedAll} \xrightarrow{\text{tex}} \text{“AddNegatedAll”}]$$

$$[\text{AddNegatedAll} \xrightarrow{\text{pyk}} \text{“pred lemma addNegatedAll”}]$$

## (A)to( E )(ImPLY)

$$[(\text{A})\text{to}(\text{E})(\text{ImPLY}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \forall_{\text{obj}}(\underline{v1}): \underline{a} \vdash \text{A4} @ (\underline{v1}) \triangleright \forall_{\text{obj}}(\underline{v1}): \underline{a} \gg \underline{a}; \text{AddDoubleNeg} \triangleright \underline{a} \gg \neg(\neg(\underline{a})n)n; \text{Gen} \triangleright \neg(\neg(\underline{a})n)n \gg \forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n; \text{AddDoubleNeg} \triangleright \forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n \gg \neg(\neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n)n); \text{Repetition} \triangleright \neg(\neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n)n) \gg \neg(\neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n)n); \forall \underline{(v1)}: \forall \underline{a}: \text{Ded} \triangleright \forall \underline{(v1)}: \forall \underline{a}: \forall_{\text{obj}}(\underline{v1}): \underline{a} \vdash \neg(\neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n)n) \gg \forall_{\text{obj}}(\underline{v1}): \underline{a} \Rightarrow \neg(\neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n)n) \rceil, p_0, c)]$$

$$[(\text{A})\text{to}(\text{E})(\text{ImPLY}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \forall_{\text{obj}}(\underline{v1}): \underline{a} \Rightarrow \neg(\neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\underline{a})n)n)n)]$$

$$[(\text{A})\text{to}(\text{E})(\text{ImPLY}) \xrightarrow{\text{tex}} \text{“}(\text{A})\text{to}(\sim \text{E}^{\sim})(\text{ImPLY})\text{”}]$$

$$[(\text{A})\text{to}(\text{E})(\text{ImPLY}) \xrightarrow{\text{pyk}} \text{“pred lemma } (\text{A})\text{to}(\sim \text{E}^{\sim})(\text{ImPLY})\text{”}]$$

## (E)to( A )(ImPLY)

$$[(\text{E})\text{to}(\text{A})(\text{ImPLY}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \text{AutoImPLY} \gg \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n \Rightarrow \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n; \text{Repetition} \triangleright \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n \Rightarrow \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n \gg \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n \rceil, p_0, c)]$$

$$[(\text{E})\text{to}(\text{A})(\text{ImPLY}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n \Rightarrow \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\underline{a})n)n]$$

$$[(\text{E})\text{to}(\text{A})(\text{ImPLY}) \xrightarrow{\text{tex}} \text{“}(\text{E})\text{to}(\sim \text{A}^{\sim})(\text{ImPLY})\text{”}]$$

$$[(\text{E})\text{to}(\text{A})(\text{ImPLY}) \xrightarrow{\text{pyk}} \text{“pred lemma } (\text{E})\text{to}(\sim \text{A}^{\sim})(\text{ImPLY})\text{”}]$$







































































































































































































































































































































































































$$\begin{aligned}
& f_{\text{Ph}}(n)n)n)n) \mid \forall_{\text{obj}}(\overline{\epsilon}): \dot{\dashv}(\forall_{\text{obj}}\overline{n}: \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\dashv}(\dot{\dashv}(0 = \overline{\epsilon})n)n)n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\dashv}(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \mid \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\mathbf{d}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\text{fx})[\underline{m}] + (\text{fz})[\underline{m}])\})n)n)\overline{m} + (-\text{ud}_{\text{Ph}}[\overline{m}]|) \leq (\overline{\epsilon}) \Rightarrow \\
& \dot{\dashv}(\dot{\dashv}(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \mid \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\mathbf{d}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\text{fx})[\underline{m}] + (\text{fz})[\underline{m}])\})n)n)\overline{m} + (-\text{ud}_{\text{Ph}}[\overline{m}]|) = (\overline{\epsilon})n)n)n)n) = \\
& \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \mid \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{r1}): (\overline{r1}) \in \\
& \mathbf{f}_{\text{Ph}} \Rightarrow \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\overline{r1}) = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \Rightarrow \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{f1}): \forall_{\text{obj}}(\overline{f2}): \forall_{\text{obj}}(\overline{f3}): \forall_{\text{obj}}(\overline{f4}): \{\{\overline{f1}, \overline{f1}\}, \{\overline{f1}, \overline{f2}\}\} \in \mathbf{f}_{\text{Ph}} \Rightarrow \\
& \{\{\overline{f3}, \overline{f3}\}, \{\overline{f3}, \overline{f4}\}\} \in \mathbf{f}_{\text{Ph}} \Rightarrow \overline{f1} = \overline{f3} \Rightarrow \overline{f2} = \overline{f4})n)n) \Rightarrow \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{s1}): (\overline{s1}) \in N \Rightarrow \dot{\dashv}(\forall_{\text{obj}}(\overline{s2}): \dot{\dashv}(\{\{\overline{s1}, \overline{s1}\}, \{\overline{s1}, \overline{s2}\}\} \in \\
& \mathbf{f}_{\text{Ph}})n)n)n)n) \mid \forall_{\text{obj}}(\overline{\epsilon}): \dot{\dashv}(\forall_{\text{obj}}\overline{n}: \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\dashv}(\dot{\dashv}(0 = \overline{\epsilon})n)n)n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\dashv}(|((\underline{fy})[\overline{m}] + (-\text{ud}_{\text{Ph}}[\overline{m}]|) \leq (\overline{\epsilon}) \Rightarrow \\
& \dot{\dashv}(\dot{\dashv}(|((\underline{fy})[\overline{m}] + (-\text{ud}_{\text{Ph}}[\overline{m}]|) = (\overline{\epsilon})n)n)n)n) \mid
\end{aligned}$$

$$[\text{NegativeToLeft}(\text{Eq})(\text{R}) \xrightarrow{\text{tex}} \text{“NegativeToLeft}(\text{Eq})(\text{R})\text{”}]$$

$$[\text{NegativeToLeft}(\text{Eq})(\text{R}) \xrightarrow{\text{pyk}} \text{“lemma negativeToLeft}(\text{Eq})(\text{R})\text{”}]$$

## NegativeToRight(Less)(R)

$$\begin{aligned}
& [\text{NegativeToRight}(\text{Less})(\text{R}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \\
& \forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \dot{\dashv}(\forall_{\text{obj}}(\overline{\epsilon}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}\overline{n}: \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\dot{\dashv}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\dashv}(\dot{\dashv}(0 = \\
& \overline{\epsilon})n)n)n) \Rightarrow \dot{\dashv}(\overline{n} \leq \overline{m} \Rightarrow \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \mid \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\mathbf{d}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\text{fx})[\underline{m}] + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \mid \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\mathbf{f}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (-\text{u}(\underline{fy})[\underline{m}])\})n)n)\overline{m}\})n)n)\overline{m}] \leq = \\
& ((\underline{fz})[\overline{m}] + (-\text{u}(\overline{\epsilon})))n)n)n)n) \vdash \\
& \text{lessAddition}(\text{R}) \triangleright \dot{\dashv}(\forall_{\text{obj}}(\overline{\epsilon}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}\overline{n}: \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\dot{\dashv}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\dashv}(\dot{\dashv}(0 = \\
& \overline{\epsilon})n)n)n) \Rightarrow \dot{\dashv}(\overline{n} \leq \overline{m} \Rightarrow \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid \\
& \dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\dashv}(\dot{\dashv}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\dashv}(\dot{\dashv}(\dot{\dashv}(\overline{\text{op1}}) \in N \Rightarrow \dot{\dashv}(\overline{\text{op2}}) \in Q)n)n) \Rightarrow \\
& \dot{\dashv}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n) \mid \dot{\dashv}(\forall_{\text{obj}}\overline{m}: \dot{\dashv}(\mathbf{d}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\text{fx})[\underline{m}] + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\}) \mid
\end{aligned}$$





















$$\begin{aligned}
& (\{ph \in \{ph \in P(P(\text{Union}(\{N, Q\})))\} \mid \\
& \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(\mathbf{d}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\underline{fz})[\underline{m}] + (\underline{fy})[\underline{m}])\})n)n)\}[\overline{m}] + (-u(\overline{\epsilon})))n)n)n)n) \gg \\
& \dot{\neg}(\forall_{obj} \overline{\epsilon}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(\dot{\neg}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{\epsilon})n)n) \Rightarrow \\
& \dot{\neg}(\overline{n} \leq \overline{m} \Rightarrow (\underline{fx})[\overline{m}] \leq (\{ph \in \{ph \in P(P(\text{Union}(\{N, Q\})))\} \mid \\
& \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(\mathbf{d}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\underline{fz})[\underline{m}] + (\underline{fy})[\underline{m}])\})n)n)\}[\overline{m}] + (-u(\overline{\epsilon})))n)n)n)n) \mid p_0, c) \\
& [\text{NegativeToRight(Less)}(R) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
& \forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \dot{\neg}(\forall_{obj} \overline{\epsilon}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(\dot{\neg}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \\
& \overline{\epsilon})n)n) \Rightarrow \dot{\neg}(\overline{n} \leq \overline{m} \Rightarrow \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\})))\} \mid \\
& \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(\mathbf{d}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\underline{fx})[\underline{m}] + \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\})))\} \mid \\
& \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(\mathbf{f}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (-u(\underline{fy})[\underline{m}])\})n)n)\}[\underline{m}])\})n)n)\}[\overline{m}] \leq = \\
& ((\underline{fz})[\overline{m}] + (-u(\overline{\epsilon})))n)n)n)n) \vdash \dot{\neg}(\forall_{obj} \overline{\epsilon}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(\dot{\neg}(0 \leq = \\
& \overline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{\epsilon})n)n) \Rightarrow \dot{\neg}(\overline{n} \leq \overline{m} \Rightarrow (\underline{fx})[\overline{m}] \leq (\{ph \in \{ph \in \\
& P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \\
& \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \\
& \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(\mathbf{d}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\underline{fz})[\underline{m}] + (\underline{fy})[\underline{m}])\})n)n)\}[\overline{m}] + (-u(\overline{\epsilon})))n)n)n)n) \\
& [\text{NegativeToRight(Less)}(R) \xrightarrow{\text{tex}} \text{“NegativeToRight(Less)}(R)\text{”}] \\
& [\text{NegativeToRight(Less)}(R) \xrightarrow{\text{pyk}} \text{“lemma negativeToRight(Less)}(R)\text{”}]
\end{aligned}$$

## !! == Symmetry

$$\begin{aligned}
& [!! == \text{Symmetry} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(\underline{fx}): \forall(\underline{fy}): \{ph \in P(\{ph \in \\
& P(\{ph \in P(P(\text{Union}(\{N, Q\})))\} \mid \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \\
& N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \dot{\neg}(\mathbf{a}_{Ph} = \\
& \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}): \overline{(r1)} \in \mathbf{f}_{Ph} \Rightarrow \\
& \dot{\neg}(\forall_{obj} \overline{(op1)}): \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n) \Rightarrow \\
& \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \Rightarrow \\
& \dot{\neg}(\forall_{obj} \overline{(f1)}): \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathbf{f}_{Ph} \Rightarrow \\
& \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathbf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n) \Rightarrow \\
& \dot{\neg}(\forall_{obj} \overline{(s1)}): \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj} \overline{(s2)}): \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in
\end{aligned}$$































































































$$\begin{aligned} & \dot{\neg} (\forall_{\text{obj}} \overline{(\mathbf{f1})}: \forall_{\text{obj}} \overline{(\mathbf{f2})}: \forall_{\text{obj}} \overline{(\mathbf{f3})}: \forall_{\text{obj}} \overline{(\mathbf{f4})}: \{ \{ \overline{(\mathbf{f1})}, \overline{(\mathbf{f1})} \}, \{ \overline{(\mathbf{f1})}, \overline{(\mathbf{f2})} \} \} \in \mathbf{f}_{\text{Ph}} \Rightarrow \\ & \{ \{ \overline{(\mathbf{f3})}, \overline{(\mathbf{f3})} \}, \{ \overline{(\mathbf{f3})}, \overline{(\mathbf{f4})} \} \} \in \mathbf{f}_{\text{Ph}} \Rightarrow \overline{(\mathbf{f1})} = \overline{(\mathbf{f3})} \Rightarrow \overline{(\mathbf{f2})} = \overline{(\mathbf{f4})}) \mathbf{n}) \mathbf{n} \Rightarrow \\ & \dot{\neg} (\forall_{\text{obj}} \overline{(\mathbf{s1})}: (\mathbf{s1}) \in \mathbf{N} \Rightarrow \dot{\neg} (\forall_{\text{obj}} \overline{(\mathbf{s2})}: \dot{\neg} (\{ \{ \overline{(\mathbf{s1})}, \overline{(\mathbf{s1})} \}, \{ \overline{(\mathbf{s1})}, \overline{(\mathbf{s2})} \} \} \in \\ & \mathbf{f}_{\text{Ph}}) \mathbf{n}) \mathbf{n}) \mathbf{n}) \mathbf{n}) \mid \forall_{\text{obj}} \overline{(\epsilon)}: \dot{\neg} (\forall_{\text{obj}} \overline{\mathbf{n}}: \dot{\neg} (\forall_{\text{obj}} \overline{\mathbf{m}}: \dot{\neg} (\mathbf{0} \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (\mathbf{0} = \overline{(\epsilon)}) \mathbf{n}) \mathbf{n}) \mathbf{n} \Rightarrow \\ & \overline{\mathbf{n}} \leq \overline{\mathbf{m}} \Rightarrow \dot{\neg} (|(\underline{(\mathbf{fy})}[\overline{\mathbf{m}}] + (-\text{ud}_{\text{Ph}}[\overline{\mathbf{m}}])| \leq \overline{(\epsilon)}) \Rightarrow \dot{\neg} (\dot{\neg} (|(\underline{(\mathbf{fy})}[\overline{\mathbf{m}}] + (-\text{ud}_{\text{Ph}}[\overline{\mathbf{m}}])| = \overline{(\epsilon)}) \mathbf{n}) \mathbf{n}) \mathbf{n}) \mathbf{n}) \mathbf{n}) \mathbf{n}) \end{aligned}$$

$$[\text{To!!} == \xrightarrow{\text{tex}} \text{"To!!=="}]$$

$$[\text{To!!} == \xrightarrow{\text{pyk}} \text{"lemma to!!=="}]$$

## PositiveToRight(Less)(1term)

$$\begin{aligned} & [\text{PositiveToRight(Less)(1term)} \xrightarrow{\text{proof}} \lambda \mathbf{c}. \lambda \mathbf{x}. \lambda \mathbf{y}. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathbf{x}}: \forall \underline{\mathbf{y}}: \dot{\neg} (\underline{\mathbf{x}} \leq \underline{\mathbf{y}} \Rightarrow \\ & \dot{\neg} (\dot{\neg} (\underline{\mathbf{x}} = \underline{\mathbf{y}}) \mathbf{n}) \mathbf{n}) \vdash \text{plus0Left} \gg (\mathbf{0} + \underline{\mathbf{x}}) = \underline{\mathbf{x}}; \text{eqSymmetry} \triangleright (\mathbf{0} + \underline{\mathbf{x}}) = \underline{\mathbf{x}} \gg \underline{\mathbf{x}} = \\ & (\mathbf{0} + \underline{\mathbf{x}}); \text{SubLessLeft} \triangleright \underline{\mathbf{x}} = (\mathbf{0} + \underline{\mathbf{x}}) \triangleright \dot{\neg} (\underline{\mathbf{x}} \leq \underline{\mathbf{y}} \Rightarrow \dot{\neg} (\dot{\neg} (\underline{\mathbf{x}} = \underline{\mathbf{y}}) \mathbf{n}) \mathbf{n}) \mathbf{n} \gg \\ & \dot{\neg} ((\mathbf{0} + \underline{\mathbf{x}}) \leq \underline{\mathbf{y}} \Rightarrow \dot{\neg} (\dot{\neg} ((\mathbf{0} + \underline{\mathbf{x}}) = \\ & \underline{\mathbf{y}}) \mathbf{n}) \mathbf{n}); \text{PositiveToRight(Less)} \triangleright \dot{\neg} ((\mathbf{0} + \underline{\mathbf{x}}) \leq \underline{\mathbf{y}} \Rightarrow \dot{\neg} (\dot{\neg} ((\mathbf{0} + \underline{\mathbf{x}}) = \underline{\mathbf{y}}) \mathbf{n}) \mathbf{n}) \mathbf{n} \gg \\ & \dot{\neg} (\mathbf{0} \leq (\underline{\mathbf{y}} + (-\underline{\mathbf{ux}})) \Rightarrow \dot{\neg} (\dot{\neg} (\mathbf{0} = (\underline{\mathbf{y}} + (-\underline{\mathbf{ux}}))) \mathbf{n}) \mathbf{n}) \mathbf{n} \rceil, \mathbf{p}_0, \mathbf{c})] \end{aligned}$$

$$[\text{PositiveToRight(Less)(1term)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{\mathbf{x}}: \forall \underline{\mathbf{y}}: \dot{\neg} (\underline{\mathbf{x}} \leq \underline{\mathbf{y}} \Rightarrow \dot{\neg} (\dot{\neg} (\underline{\mathbf{x}} = \underline{\mathbf{y}}) \mathbf{n}) \mathbf{n}) \mathbf{n} \vdash \dot{\neg} (\mathbf{0} \leq (\underline{\mathbf{y}} + (-\underline{\mathbf{ux}})) \Rightarrow \dot{\neg} (\dot{\neg} (\mathbf{0} = (\underline{\mathbf{y}} + (-\underline{\mathbf{ux}}))) \mathbf{n}) \mathbf{n}) \mathbf{n}]$$

$$[\text{PositiveToRight(Less)(1term)} \xrightarrow{\text{tex}} \text{"PositiveToRight(Less)(1 term)"}]$$

$$[\text{PositiveToRight(Less)(1term)} \xrightarrow{\text{pyk}} \text{"lemma positiveToRight(Less)(1 term)"}]$$

## (A )to( E)

$$\begin{aligned} & [(A )\text{to}( E) \xrightarrow{\text{proof}} \lambda \mathbf{c}. \lambda \mathbf{x}. \lambda \mathbf{y}. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\mathbf{v1})}: \forall \underline{\mathbf{a}}: \forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n} \vdash \\ & \text{AddDoubleNeg} \triangleright \forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n} \gg \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n}) \mathbf{n}); \text{Repetition} \triangleright \\ & \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n}) \mathbf{n} \gg \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n}) \mathbf{n}) \mathbf{n} \rceil, \mathbf{p}_0, \mathbf{c})] \end{aligned}$$

$$[(A )\text{to}( E) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{(\mathbf{v1})}: \forall \underline{\mathbf{a}}: \forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n} \vdash \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} \underline{(\mathbf{v1})}: \dot{\neg} (\underline{\mathbf{a}}) \mathbf{n}) \mathbf{n}) \mathbf{n}]$$

$$[(A )\text{to}( E) \xrightarrow{\text{tex}} \text{"(A~)to(~E)"}]$$

$$[(A )\text{to}( E) \xrightarrow{\text{pyk}} \text{"pred lemma (A~)to(~E)"}]$$

























































































































































































$$\begin{aligned}
& f_{Ph} \Rightarrow \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (\overline{(r1)} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{ \{ \{ (f1), \overline{(f1)} \}, \{ (f1), \overline{(f2)} \} \} \in f_{Ph} \Rightarrow \\
& \{ \{ \{ (f3), \overline{(f3)} \}, \{ (f3), \overline{(f4)} \} \} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg} (\forall_{obj} \overline{(s2)}: \dot{\neg} (\{ \{ (s1), \overline{(s1)} \}, \{ (s1), \overline{(s2)} \} \} \in \\
& f_{Ph} n) n) n) n) | \forall_{obj} \overline{(\epsilon)}: \dot{\neg} (\forall_{obj} \overline{n}: \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{(\epsilon)}) n) n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\neg} (| \{ \{ ph \in \{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (f_{Ph} = \\
& \{ \{ \underline{m}, \underline{m} \}, \{ \underline{m}, (-u(fy)[\underline{m}]) \} \} n) n) n) [\overline{m}] + (-ud_{Ph}[\overline{m}]) | \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (| \{ \{ ph \in \\
& \{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} | \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\overline{(op1)} \in \\
& N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \dot{\neg} (a_{Ph} = \\
& \{ \{ \overline{(op1)}, \overline{(op1)} \}, \{ \overline{(op1)}, \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (f_{Ph} = \\
& \{ \{ \underline{m}, \underline{m} \}, \{ \underline{m}, (-u(fy)[\underline{m}]) \} \} n) n) n) [\overline{m}] + (-ud_{Ph}[\overline{m}]) | = \overline{(\epsilon)} n) n) n) n) \vdash \\
& \dot{\neg} (\{ \{ ph \in P(\{ \{ ph \in P(\{ \{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \\
& f_{Ph} \Rightarrow \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (\overline{(r1)} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{ \{ \{ (f1), \overline{(f1)} \}, \{ (f1), \overline{(f2)} \} \} \in f_{Ph} \Rightarrow \\
& \{ \{ \{ (f3), \overline{(f3)} \}, \{ (f3), \overline{(f4)} \} \} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg} (\forall_{obj} \overline{(s2)}: \dot{\neg} (\{ \{ (s1), \overline{(s1)} \}, \{ (s1), \overline{(s2)} \} \} \in \\
& f_{Ph} n) n) n) n) | \forall_{obj} \overline{(\epsilon)}: \dot{\neg} (\forall_{obj} \overline{n}: \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{(\epsilon)}) n) n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\neg} (| \{ \{ ph \in \{ \{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (f_{Ph} =
\end{aligned}$$































































































































































































































































































































































































































































































































































































































































































































































































































































































$0 < 1(\mathbf{R})$

$[0 < 1(\mathbf{R}) \xrightarrow{\text{tex}} \text{"0<1(\mathbf{R})"}]$

$[0 < 1(\mathbf{R}) \xrightarrow{\text{pyk}} \text{"lemma 0<1(\mathbf{R})"}]$

$\text{ExpZero}(\text{Exact})(\mathbf{R})$

$[\text{ExpZero}(\text{Exact})(\mathbf{R}) \xrightarrow{\text{tex}} \text{"ExpZero(Exact)(\mathbf{R})"}]$

$[\text{ExpZero}(\text{Exact})(\mathbf{R}) \xrightarrow{\text{pyk}} \text{"lemma expZero exact(\mathbf{R})"}]$

$\text{PositiveBase}(\mathbf{R})(\text{Base})$

$[\text{PositiveBase}(\mathbf{R})(\text{Base}) \xrightarrow{\text{tex}} \text{"PositiveBase(\mathbf{R})(\text{Base})"}]$

$[\text{PositiveBase}(\mathbf{R})(\text{Base}) \xrightarrow{\text{pyk}} \text{"lemma positiveBase(\mathbf{R}) base"}]$

$\text{Three2twoFactors}(\mathbf{R})$

$[\text{Three2twoFactors}(\mathbf{R}) \xrightarrow{\text{tex}} \text{"Three2twoFactors(\mathbf{R})"}]$

$[\text{Three2twoFactors}(\mathbf{R}) \xrightarrow{\text{pyk}} \text{"lemma three2twoFactors(\mathbf{R})"}]$

$x = x * y * (1/y)(\mathbf{R})$

$[x = x * y * (1/y)(\mathbf{R}) \xrightarrow{\text{tex}} \text{"x=x*y*(1/y)(\mathbf{R})"}]$

$[x = x * y * (1/y)(\mathbf{R}) \xrightarrow{\text{pyk}} \text{"lemma x=x*y*(1/y)(\mathbf{R})"}]$

$\text{NeqMultiplication}(\mathbf{R})$

$[\text{NeqMultiplication}(\mathbf{R}) \xrightarrow{\text{tex}} \text{"NeqMultiplication(\mathbf{R})"}]$

$[\text{NeqMultiplication}(\mathbf{R}) \xrightarrow{\text{pyk}} \text{"lemma neqMultiplication(\mathbf{R})"}]$

## LessTransitivity( $\mathbf{R}$ )

[LessTransitivity( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “LessTransitivity( $\mathbf{R}$ )”]

[LessTransitivity( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma lessTransitivity( $\mathbf{R}$ )”]

## $0 < 2(\mathbf{R})$

[ $0 < 2(\mathbf{R})$   $\xrightarrow{\text{tex}}$  “ $0 < 2(\mathbf{R})$ ”]

[ $0 < 2(\mathbf{R})$   $\xrightarrow{\text{pyk}}$  “lemma  $0 < 2(\mathbf{R})$ ”]

## SameExp( $\mathbf{R}$ )(Base)

[SameExp( $\mathbf{R}$ )(Base)  $\xrightarrow{\text{tex}}$  “SameExp( $\mathbf{R}$ )(Base)”]

[SameExp( $\mathbf{R}$ )(Base)  $\xrightarrow{\text{pyk}}$  “lemma sameExp( $\mathbf{R}$ ) base”]

## SameExp( $\mathbf{R}$ )(Indu)

[SameExp( $\mathbf{R}$ )(Indu)  $\xrightarrow{\text{tex}}$  “SameExp( $\mathbf{R}$ )(Indu)”]

[SameExp( $\mathbf{R}$ )(Indu)  $\xrightarrow{\text{pyk}}$  “lemma sameExp( $\mathbf{R}$ ) indu”]

## SameExp( $\mathbf{R}$ )

[SameExp( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “SameExp( $\mathbf{R}$ )”]

[SameExp( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma sameExp( $\mathbf{R}$ )”]

## SubNeqLeft( $\mathbf{R}$ )

[SubNeqLeft( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “SubNeqLeft( $\mathbf{R}$ )”]

[SubNeqLeft( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma subNeqLeft( $\mathbf{R}$ )”]

## SubNeqRight( $\mathbf{R}$ )

[SubNeqRight( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “SubNeqRight( $\mathbf{R}$ )”]

[SubNeqRight( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma subNeqRight( $\mathbf{R}$ )”]

## NonzeroFactors( $\mathbf{R}$ )

[NonzeroFactors( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “NonzeroFactors( $\mathbf{R}$ )”]

[NonzeroFactors( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma nonzeroFactors( $\mathbf{R}$ )”]

## NonnegativeFactors( $\mathbf{R}$ )

[NonnegativeFactors( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “NonnegativeFactors( $\mathbf{R}$ )”]

[NonnegativeFactors( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma nonnegativeFactors( $\mathbf{R}$ )”]

## PositiveFactors( $\mathbf{R}$ )

[PositiveFactors( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “PositiveFactors( $\mathbf{R}$ )”]

[PositiveFactors( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma positiveFactors( $\mathbf{R}$ )”]

## LessDivision( $\mathbf{R}$ )

[LessDivision( $\mathbf{R}$ )  $\xrightarrow{\text{tex}}$  “LessDivision( $\mathbf{R}$ )”]

[LessDivision( $\mathbf{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma lessDivision( $\mathbf{R}$ )”]

## $0 < 1/2(\mathbf{R})$

[ $0 < 1/2(\mathbf{R})$   $\xrightarrow{\text{tex}}$  “ $0 < 1/2(\mathbf{R})$ ”]

[ $0 < 1/2(\mathbf{R})$   $\xrightarrow{\text{pyk}}$  “lemma  $0 < 1/2(\mathbf{R})$ ”]

## PositiveToRight(Eq)(1term)(R)

[PositiveToRight(Eq)(1term)(R)  $\xrightarrow{\text{tex}}$  "PositiveToRight(Eq)(1 term)(R)"]

[PositiveToRight(Eq)(1term)(R)  $\xrightarrow{\text{pyk}}$  "lemma positiveToRight(Eq)(1 term)(R)"]

## Exp(+1)(R)

[Exp(+1)(R)  $\xrightarrow{\text{tex}}$  "Exp(+1)(R)"]

[Exp(+1)(R)  $\xrightarrow{\text{pyk}}$  "lemma exp(+1)(R)"]

## PositiveBase(R)(Indu)

[PositiveBase(R)(Indu)  $\xrightarrow{\text{tex}}$  "PositiveBase(R)(Indu)"]

[PositiveBase(R)(Indu)  $\xrightarrow{\text{pyk}}$  "lemma positiveBase(R) indu"]

## PositiveBase(R)

[PositiveBase(R)  $\xrightarrow{\text{tex}}$  "PositiveBase(R)"]

[PositiveBase(R)  $\xrightarrow{\text{pyk}}$  "lemma positiveBase(R)"]

## $-x * y = -(x * y)(R)$

$[-x * y = -(x * y)(R) \xrightarrow{\text{tex}}$  " $-x*y=-(x*y)(R)$ "]

$[-x * y = -(x * y)(R) \xrightarrow{\text{pyk}}$  "lemma  $-x*y=-(x*y)(R)$ "]

## PositiveToLeft(Eq)(R)

[PositiveToLeft(Eq)(R)  $\xrightarrow{\text{tex}}$  "PositiveToLeft(Eq)(R)"]

[PositiveToLeft(Eq)(R)  $\xrightarrow{\text{pyk}}$  "lemma positiveToLeft(Eq)(R)"]

## Times1Left( $\mathbb{R}$ )

[Times1Left( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “Times1Left( $\mathbb{R}$ )”]

[Times1Left( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma times1Left( $\mathbb{R}$ )”]

$$x + x = 2 * x(\mathbb{R})$$

[ $x + x = 2 * x(\mathbb{R})$   $\xrightarrow{\text{tex}}$  “ $x+x=2*x(\mathbb{R})$ ”]

[ $x + x = 2 * x(\mathbb{R})$   $\xrightarrow{\text{pyk}}$  “lemma  $x+x=2*x(\mathbb{R})$ ”]

$$(1/2)x + (1/2)x = x(\mathbb{R})$$

[(1/2)x + (1/2)x = x( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “(1/2)x+(1/2)x=x( $\mathbb{R}$ )”]

[(1/2)x + (1/2)x = x( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma (1/2)x+(1/2)x=x( $\mathbb{R}$ )”]

## DistributionOut(Minus)( $\mathbb{R}$ )

[DistributionOut(Minus)( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “DistributionOut(Minus)( $\mathbb{R}$ )”]

[DistributionOut(Minus)( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma distributionOut(Minus)( $\mathbb{R}$ )”]

$$(1/2)(x + y) - x = (1/2)(y - x)(\mathbb{R})$$

[(1/2)(x + y) - x = (1/2)(y - x)( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “(1/2)(x+y)-x=(1/2)(y-x)( $\mathbb{R}$ )”]

[(1/2)(x + y) - x = (1/2)(y - x)( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma (1/2)(x+y)-x=(1/2)(y-x)( $\mathbb{R}$ )”]

## IntervalSize( $\mathbb{R}$ )(Base)

[IntervalSize( $\mathbb{R}$ )(Base)  $\xrightarrow{\text{tex}}$  “IntervalSize( $\mathbb{R}$ )(Base)”]

[IntervalSize( $\mathbb{R}$ )(Base)  $\xrightarrow{\text{pyk}}$  “lemma intervalSize( $\mathbb{R}$ ) base”]



## LessMultiplicationLeft(R)

[LessMultiplicationLeft(R)  $\xrightarrow{\text{tex}}$  "LessMultiplicationLeft(R)"]

[LessMultiplicationLeft(R)  $\xrightarrow{\text{pyk}}$  "lemma lessMultiplicationLeft(R)"]

## NegativeToLeft(Less)(R)

[NegativeToLeft(Less)(R)  $\xrightarrow{\text{tex}}$  "NegativeToLeft(Less)(R)"]

[NegativeToLeft(Less)(R)  $\xrightarrow{\text{pyk}}$  "lemma negativeToLeft(Less)(R)"]

## NegativeToLeft(Less)(1term)(R)

[NegativeToLeft(Less)(1term)(R)  $\xrightarrow{\text{tex}}$  "NegativeToLeft(Less)(1 term)(R)"]

[NegativeToLeft(Less)(1term)(R)  $\xrightarrow{\text{pyk}}$  "lemma negativeToLeft(Less)(1 term)(R)"]

## $y - (1/2)(x + y) = (1/2)(y - x)(R)$

[ $y - (1/2)(x + y) = (1/2)(y - x)(R)$   $\xrightarrow{\text{tex}}$  "y-(1/2)(x+y)=(1/2)(y-x)(R)"]

[ $y - (1/2)(x + y) = (1/2)(y - x)(R)$   $\xrightarrow{\text{pyk}}$  "lemma y-(1/2)(x+y)=(1/2)(y-x)(R)"]

## IntervalSize(R)(Indu)

[IntervalSize(R)(Indu)  $\xrightarrow{\text{tex}}$  "IntervalSize(R)(Indu)"]

[IntervalSize(R)(Indu)  $\xrightarrow{\text{pyk}}$  "lemma intervalSize(R) indu"]

## IntervalSize(R)

[IntervalSize(R)  $\xrightarrow{\text{tex}}$  "IntervalSize(R)"]

[IntervalSize(R)  $\xrightarrow{\text{pyk}}$  "lemma intervalSize(R)"]

XSlessUS(R)

[XSlessUS(R)  $\xrightarrow{\text{tex}}$  "XSlessUS(R)"]

[XSlessUS(R)  $\xrightarrow{\text{pyk}}$  "lemma XSlessUS(R)"]

USdecreasing(+1)(R)

[USdecreasing(+1)(R)  $\xrightarrow{\text{tex}}$  "USdecreasing(+1)(R)"]

[USdecreasing(+1)(R)  $\xrightarrow{\text{pyk}}$  "lemma USdecreasing(+1)(R)"]

ExpUnbounded(Base)

[ExpUnbounded(Base)  $\xrightarrow{\text{tex}}$  "ExpUnbounded(Base)"]

[ExpUnbounded(Base)  $\xrightarrow{\text{pyk}}$  "lemma expUnbounded base"]

ExpUnbounded(Indu)

[ExpUnbounded(Indu)  $\xrightarrow{\text{tex}}$  "ExpUnbounded(Indu)"]

[ExpUnbounded(Indu)  $\xrightarrow{\text{pyk}}$  "lemma expUnbounded indu"]

ExpUnbounded

[ExpUnbounded  $\xrightarrow{\text{tex}}$  "ExpUnbounded"]

[ExpUnbounded  $\xrightarrow{\text{pyk}}$  "lemma expUnbounded"]

1 <= x + 1(N)

[1 <= x + 1(N)  $\xrightarrow{\text{tex}}$  "1<=x+1(N)"]

[1 <= x + 1(N)  $\xrightarrow{\text{pyk}}$  "lemma 1<=x+1(N)"]

## ExpNonzero(Base)

[ExpNonzero(Base)  $\xrightarrow{\text{tex}}$  “ExpNonzero(Base)”]

[ExpNonzero(Base)  $\xrightarrow{\text{pyk}}$  “lemma expNonzero base”]

## ExpNonzero(Indu)

[ExpNonzero(Indu)  $\xrightarrow{\text{tex}}$  “ExpNonzero(Indu)”]

[ExpNonzero(Indu)  $\xrightarrow{\text{pyk}}$  “lemma expNonzero indu”]

## ExpNonzero

[ExpNonzero  $\xrightarrow{\text{tex}}$  “ExpNonzero”]

[ExpNonzero  $\xrightarrow{\text{pyk}}$  “lemma expNonzero”]

## ExpNonzero(2)

[ExpNonzero(2)  $\xrightarrow{\text{tex}}$  “ExpNonzero(2)”]

[ExpNonzero(2)  $\xrightarrow{\text{pyk}}$  “lemma expNonzero(2)”]

## HalfBase(Base)

[HalfBase(Base)  $\xrightarrow{\text{tex}}$  “HalfBase(Base)”]

[HalfBase(Base)  $\xrightarrow{\text{pyk}}$  “lemma halfBase base”]

## HalfBase(Indu)

[HalfBase(Indu)  $\xrightarrow{\text{tex}}$  “HalfBase(Indu)”]

[HalfBase(Indu)  $\xrightarrow{\text{pyk}}$  “lemma halfBase indu”]

## MultiplyEquations(R)

[MultiplyEquations(R)  $\xrightarrow{\text{tex}}$  "MultiplyEquations(R)"]

[MultiplyEquations(R)  $\xrightarrow{\text{pyk}}$  "lemma multiplyEquations(R)"]

## NonreciprocalToRight(Eq)(1term)(R)

[NonreciprocalToRight(Eq)(1term)(R)  $\xrightarrow{\text{tex}}$  "NonreciprocalToRight(Eq)(1term)(R)"]

[NonreciprocalToRight(Eq)(1term)(R)  $\xrightarrow{\text{pyk}}$  "lemma nonreciprocalToRight(Eq)(1 term)(R)"]

## PositiveNonzero(R)

[PositiveNonzero(R)  $\xrightarrow{\text{tex}}$  "PositiveNonzero(R)"]

[PositiveNonzero(R)  $\xrightarrow{\text{pyk}}$  "lemma positiveNonzero(R)"]

## NonzeroProduct(2)(R)

[NonzeroProduct(2)(R)  $\xrightarrow{\text{tex}}$  "NonzeroProduct(2)(R)"]

[NonzeroProduct(2)(R)  $\xrightarrow{\text{pyk}}$  "lemma nonzeroProduct(2)(R)"]

## HalfBase

[HalfBase  $\xrightarrow{\text{tex}}$  "HalfBase"]

[HalfBase  $\xrightarrow{\text{pyk}}$  "lemma halfBase"]

## Three2threeFactors(R)

[Three2threeFactors(R)  $\xrightarrow{\text{tex}}$  "Three2threeFactors(R)"]

[Three2threeFactors(R)  $\xrightarrow{\text{pyk}}$  "lemma three2threeFactors(R)"]

$x * y = z$ Backwards( $\mathbb{R}$ )

[ $x * y = z$ Backwards( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “ $x*y=z$ Backwards( $\mathbb{R}$ )”]

[ $x * y = z$ Backwards( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma  $x*y=z$ Backwards( $\mathbb{R}$ )”]

PositiveInverted( $\mathbb{R}$ )

[PositiveInverted( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “PositiveInverted( $\mathbb{R}$ )”]

[PositiveInverted( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma positiveInverted( $\mathbb{R}$ )”]

ReciprocalToRight(Less)( $\mathbb{R}$ )

[ReciprocalToRight(Less)( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “ReciprocalToRight(Less)( $\mathbb{R}$ )”]

[ReciprocalToRight(Less)( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma reciprocalToRight(Less)( $\mathbb{R}$ )”]

ReciprocalToRight(Less)(1term)( $\mathbb{R}$ )

[ReciprocalToRight(Less)(1term)( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “ReciprocalToRight(Less)(1term)( $\mathbb{R}$ )”]

[ReciprocalToRight(Less)(1term)( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma reciprocalToRight(Less)(1term)( $\mathbb{R}$ )”]

NonreciprocalToLeft(Less)( $\mathbb{R}$ )

[NonreciprocalToLeft(Less)( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “NonreciprocalToLeft(Less)( $\mathbb{R}$ )”]

[NonreciprocalToLeft(Less)( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma nonreciprocalToLeft(Less)( $\mathbb{R}$ )”]

$1 < x * y$ ( $\mathbb{R}$ )

[ $1 < x * y$ ( $\mathbb{R}$ )  $\xrightarrow{\text{tex}}$  “ $1<x*y$ ( $\mathbb{R}$ )”]

[ $1 < x * y$ ( $\mathbb{R}$ )  $\xrightarrow{\text{pyk}}$  “lemma  $1<x*y$ ( $\mathbb{R}$ )”]

## SwitchFactors(1/x < y)(R)

[SwitchFactors(1/x < y)(R)  $\xrightarrow{\text{tex}}$  “SwitchFactors(1/x<y)(R)”]

[SwitchFactors(1/x < y)(R)  $\xrightarrow{\text{pyk}}$  “lemma switchFactors(1/x<y)(R)”]

## SmallHalving

[SmallHalving  $\xrightarrow{\text{tex}}$  “SmallHalving”]

[SmallHalving  $\xrightarrow{\text{pyk}}$  “lemma smallHalving”]

## IntervalSize(anyPositive)

[IntervalSize(anyPositive)  $\xrightarrow{\text{tex}}$  “IntervalSize(anyPositive)”]

[IntervalSize(anyPositive)  $\xrightarrow{\text{pyk}}$  “lemma intervalSize(anyPositive)”]

## USdecreasing(+n)(Base)

[USdecreasing(+n)(Base)  $\xrightarrow{\text{tex}}$  “USdecreasing(+n)(Base)”]

[USdecreasing(+n)(Base)  $\xrightarrow{\text{pyk}}$  “lemma USdecreasing(+n) base”]

## USdecreasing(+n)(Indu)

[USdecreasing(+n)(Indu)  $\xrightarrow{\text{tex}}$  “USdecreasing(+n)(Indu)”]

[USdecreasing(+n)(Indu)  $\xrightarrow{\text{pyk}}$  “lemma USdecreasing(+n) indu”]

## USdecreasing(+n)

[USdecreasing(+n)  $\xrightarrow{\text{tex}}$  “USdecreasing(+n)”]

[USdecreasing(+n)  $\xrightarrow{\text{pyk}}$  “lemma USdecreasing(+n)”]

## USdecreasing

[USdecreasing  $\xrightarrow{\text{tex}}$  “USdecreasing”]

[USdecreasing  $\xrightarrow{\text{pyk}}$  “lemma USdecreasing”]

## LeqAdditionLeft(R)

[LeqAdditionLeft(R)  $\xrightarrow{\text{tex}}$  “LeqAdditionLeft(R)”]

[LeqAdditionLeft(R)  $\xrightarrow{\text{pyk}}$  “lemma leqAdditionLeft(R)”]

## ToNotLess(R)

[ToNotLess(R)  $\xrightarrow{\text{tex}}$  “ToNotLess(R)”]

[ToNotLess(R)  $\xrightarrow{\text{pyk}}$  “lemma toNotLess(R)”]

## LimitOfUSIsLeq

[LimitOfUSIsLeq  $\xrightarrow{\text{tex}}$  “LimitOfUSIsLeq”]

[LimitOfUSIsLeq  $\xrightarrow{\text{pyk}}$  “lemma limitOfUSIsLeq”]

## SubtractEquations(Less)(R)

[SubtractEquations(Less)(R)  $\xrightarrow{\text{tex}}$  “SubtractEquations(Less)(R)”]

[SubtractEquations(Less)(R)  $\xrightarrow{\text{pyk}}$  “lemma subtractEquations(Less)(R)”]

## SubtractEquationsLeft(Less)(R)

[SubtractEquationsLeft(Less)(R)  $\xrightarrow{\text{tex}}$  “SubtractEquationsLeft(Less)(R)”]

[SubtractEquationsLeft(Less)(R)  $\xrightarrow{\text{pyk}}$  “lemma  
subtractEquationsLeft(Less)(R)”]

## LessNegated(Negative)(R)

[LessNegated(Negative)(R)  $\xrightarrow{\text{tex}}$  “LessNegated(Negative)(R)”]

[LessNegated(Negative)(R)  $\xrightarrow{\text{pyk}}$  “lemma lessNegated(Negative)(R)”]

## FromNegatedAnd(ImPLY)

[FromNegatedAnd(ImPLY)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \vdash \underline{a} \vdash \text{FromNegatedAnd} \triangleright \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \triangleright \underline{a} \gg \dot{\neg}(\underline{b})n; \forall \underline{a}: \forall \underline{b}: \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \vdash \underline{a} \vdash \dot{\neg}(\underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \Rightarrow \underline{a} \Rightarrow \dot{\neg}(\underline{b})n \rrbracket, p_0, c)$ ]

[FromNegatedAnd(ImPLY)  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \Rightarrow \underline{a} \Rightarrow \dot{\neg}(\underline{b})n$ ]

[FromNegatedAnd(ImPLY)  $\xrightarrow{\text{tex}}$  “FromNegatedAnd(ImPLY)”]

[FromNegatedAnd(ImPLY)  $\xrightarrow{\text{pyk}}$  “prop lemma from negated and (imply)”]

## RemoveDoubleNeg(Consequent)

[RemoveDoubleNeg(Consequent)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \vdash \underline{a} \vdash \text{MP} \triangleright \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \triangleright \underline{a} \gg \dot{\neg}(\dot{\neg}(\underline{b})n)n; \text{RemoveDoubleNeg} \triangleright \dot{\neg}(\dot{\neg}(\underline{b})n)n \gg \underline{b}; \forall \underline{a}: \forall \underline{b}: \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \vdash \underline{a} \vdash \underline{b} \gg \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \Rightarrow \underline{a} \Rightarrow \underline{b}; \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \vdash \text{MP} \triangleright \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \Rightarrow \underline{a} \Rightarrow \underline{b} \triangleright \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \gg \underline{a} \Rightarrow \underline{b} \rrbracket, p_0, c)$ ]

[RemoveDoubleNeg(Consequent)  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \vdash \underline{a} \Rightarrow \underline{b}$ ]

[RemoveDoubleNeg(Consequent)  $\xrightarrow{\text{tex}}$  “RemoveDoubleNeg(Consequent)”]

[RemoveDoubleNeg(Consequent)  $\xrightarrow{\text{pyk}}$  “prop lemma remove double neg (consequent)”]

## FromNotUpperBound

[FromNotUpperBound  $\xrightarrow{\text{tex}}$  “FromNotUpperBound”]

[FromNotUpperBound  $\xrightarrow{\text{pyk}}$  “lemma fromNotUpperBound”]



## LeqNUB

[LeqNUB  $\xrightarrow{\text{tex}}$  “LeqNUB”]

[LeqNUB  $\xrightarrow{\text{pyk}}$  “lemma leqNUB”]

## USlimitIsLeastUpperBound(Helper)

[USlimitIsLeastUpperBound(Helper)  $\xrightarrow{\text{tex}}$   
“USlimitIsLeastUpperBound(Helper)”]

[USlimitIsLeastUpperBound(Helper)  $\xrightarrow{\text{pyk}}$  “lemma USlimitIsLeastUpperBound  
helper”]

## USlimitIsLeastUpperBound

[USlimitIsLeastUpperBound  $\xrightarrow{\text{tex}}$  “USlimitIsLeastUpperBound”]

[USlimitIsLeastUpperBound  $\xrightarrow{\text{pyk}}$  “lemma USlimitIsLeastUpperBound”]

## ExistMP3

[ExistMP3  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall (v1): \forall (v2): \forall (v3): \forall a: \forall b: \forall c: \forall d: a \Rightarrow b \Rightarrow c \Rightarrow d \vdash \dot{\vdash} (\forall_{\text{obj}}(v1): \dot{\vdash} (a)n) \dot{\vdash} (\forall_{\text{obj}}(v2): \dot{\vdash} (b)n) \dot{\vdash} \dot{\vdash} (\forall_{\text{obj}}(v3): \dot{\vdash} (c)n) \vdash \text{ExistMP2} \triangleright a \Rightarrow b \Rightarrow c \Rightarrow d \triangleright \dot{\vdash} (\forall_{\text{obj}}(v1): \dot{\vdash} (a)n) \triangleright \dot{\vdash} (\forall_{\text{obj}}(v2): \dot{\vdash} (b)n) \triangleright c \Rightarrow d; \text{ExistMP} \triangleright c \Rightarrow d \triangleright \dot{\vdash} (\forall_{\text{obj}}(v3): \dot{\vdash} (c)n) \triangleright d \rceil, p0, c)$ ]

[ExistMP3  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall (v1): \forall (v2): \forall (v3): \forall a: \forall b: \forall c: \forall d: a \Rightarrow b \Rightarrow c \Rightarrow d \vdash \dot{\vdash} (\forall_{\text{obj}}(v1): \dot{\vdash} (a)n) \dot{\vdash} (\forall_{\text{obj}}(v2): \dot{\vdash} (b)n) \dot{\vdash} \dot{\vdash} (\forall_{\text{obj}}(v3): \dot{\vdash} (c)n) \vdash d$ ]

[ExistMP3  $\xrightarrow{\text{tex}}$  “ExistMP3”]

[ExistMP3  $\xrightarrow{\text{pyk}}$  “pred lemma exist mp3”]

## GreaterPositive(N)

[GreaterPositive(N)  $\xrightarrow{\text{tex}}$  “GreaterPositive(N)”]

[GreaterPositive(N)  $\xrightarrow{\text{pyk}}$  “lemma greaterPositive(N)”]

## ysFClose(Helper)

$[\text{ysFClose}(\text{Helper}) \xrightarrow{\text{tex}} \text{“ysFClose}(\text{Helper})\text{”}]$

$[\text{ysFClose}(\text{Helper}) \xrightarrow{\text{pyk}} \text{“lemma ysFClose helper”}]$

## ysFClose

$[\text{ysFClose} \xrightarrow{\text{tex}} \text{“ysFClose”}]$

$[\text{ysFClose} \xrightarrow{\text{pyk}} \text{“lemma ysFClose”}]$

## ysFCAuchy(Helper)

$[\text{ysFCAuchy}(\text{Helper}) \xrightarrow{\text{tex}} \text{“ysFCAuchy}(\text{Helper})\text{”}]$

$[\text{ysFCAuchy}(\text{Helper}) \xrightarrow{\text{pyk}} \text{“lemma ysFCAuchy helper”}]$

## ysFCAuchy

$[\text{ysFCAuchy} \xrightarrow{\text{tex}} \text{“ysFCAuchy”}]$

$[\text{ysFCAuchy} \xrightarrow{\text{pyk}} \text{“lemma ysFCAuchy”}]$

## from <<==

$[\text{from } <<== \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash$   
 $\forall(\underline{\text{fx}}): \forall(\underline{\text{fy}}): \dot{\rightarrow}(\dot{\rightarrow}(\forall_{\text{obj}} \overline{\epsilon}): \dot{\rightarrow}(\dot{\rightarrow}(\forall_{\text{obj}} \overline{n}: \dot{\rightarrow}(\forall_{\text{obj}} \overline{m}: \dot{\rightarrow}(\dot{\rightarrow}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\rightarrow}(\dot{\rightarrow}(0 =$   
 $\overline{\epsilon}))n)n)n \Rightarrow \dot{\rightarrow}(\overline{n} \leq \overline{m} \Rightarrow (\underline{\text{fx}})[\overline{m}] \leq ((\underline{\text{fy}})[\overline{m}] + (-u(\overline{\epsilon})))n)n)n)n)n \Rightarrow$   
 $\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\overline{\text{P}}(\overline{\text{Union}}(\{N, Q\})))\}) \mid$   
 $\dot{\rightarrow}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\rightarrow}(\dot{\rightarrow}(\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\rightarrow}(\dot{\rightarrow}(\dot{\rightarrow}((\overline{\text{op1}}) \in N \Rightarrow \dot{\rightarrow}(\overline{\text{op2}}) \in Q)n)n \Rightarrow$   
 $\dot{\rightarrow}(\underline{\text{aPh}} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n)n \mid \dot{\rightarrow}(\dot{\rightarrow}(\forall_{\text{obj}} \overline{\text{r1}}): \overline{\text{r1}}) \in$   
 $\underline{\text{fPh}} \Rightarrow \dot{\rightarrow}(\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\rightarrow}(\dot{\rightarrow}(\forall_{\text{obj}} \overline{\text{op2}}): \dot{\rightarrow}(\dot{\rightarrow}(\dot{\rightarrow}((\overline{\text{op1}}) \in N \Rightarrow \dot{\rightarrow}(\overline{\text{op2}}) \in Q)n)n \Rightarrow$   
 $\dot{\rightarrow}(\overline{\text{r1}}) = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\})n)n)n)n)n \Rightarrow$   
 $\dot{\rightarrow}(\forall_{\text{obj}}(\overline{\text{f1}}): \forall_{\text{obj}}(\overline{\text{f2}}): \forall_{\text{obj}}(\overline{\text{f3}}): \forall_{\text{obj}}(\overline{\text{f4}}): \{\{\overline{\text{f1}}, \overline{\text{f1}}\}, \{\overline{\text{f1}}, \overline{\text{f2}}\}\} \in \underline{\text{fPh}} \Rightarrow$   
 $\{\{\overline{\text{f3}}, \overline{\text{f3}}\}, \{\overline{\text{f3}}, \overline{\text{f4}}\}\} \in \underline{\text{fPh}} \Rightarrow \overline{\text{f1}} = \overline{\text{f3}} \Rightarrow \overline{\text{f2}} = \overline{\text{f4}})n)n \Rightarrow$   
 $\dot{\rightarrow}(\forall_{\text{obj}}(\overline{\text{s1}}): \overline{\text{s1}}) \in N \Rightarrow \dot{\rightarrow}(\forall_{\text{obj}}(\overline{\text{s2}}): \dot{\rightarrow}(\{\{\overline{\text{s1}}, \overline{\text{s1}}\}, \{\overline{\text{s1}}, \overline{\text{s2}}\}\}) \in$   
 $\underline{\text{fPh}})n)n)n) \mid \forall_{\text{obj}}(\overline{\epsilon}): \dot{\rightarrow}(\forall_{\text{obj}} \overline{n}: \dot{\rightarrow}(\forall_{\text{obj}} \overline{m}: \dot{\rightarrow}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\rightarrow}(\dot{\rightarrow}(0 = \overline{\epsilon}))n)n) \Rightarrow$   
 $\overline{n} \leq \overline{m} \Rightarrow \dot{\rightarrow}(\lvert(\underline{\text{fx}})[\overline{m}] + (-u_{\text{Ph}}[\overline{m}])\rvert) \leq \overline{\epsilon}) \Rightarrow$























































$$|f(\underline{fx})| = \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid$$

$$\dot{\neg} (\forall_{\text{obj}} \overline{(\text{op1})}) : \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} \overline{(\text{op2})}) : \dot{\neg} (\dot{\neg} (\overline{(\text{op1})}) \in N \Rightarrow \dot{\neg} (\overline{(\text{op2})}) \in Q) \text{n}) \text{n} \Rightarrow$$

$$\dot{\neg} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \text{n} \} \mid \dot{\neg} (\forall_{\text{obj}} \underline{m} : \dot{\neg} (\text{f}_{\text{Ph}} =$$

$$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (-\text{u}(\underline{fx})[\underline{m}])\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \text{n} \}$$

$$[\text{NegativeNumerical}(F) \xrightarrow{\text{tex}} \text{“NegativeNumerical}(F)\text{”}]$$

$$[\text{NegativeNumerical}(F) \xrightarrow{\text{pyk}} \text{“lemma negativeNumerical}(F)\text{”}]$$

## tester1

$$[\text{tester1} \xrightarrow{\text{pyk}} \text{“tester1”}]$$

## tester2

$$[\text{tester2} \xrightarrow{\text{pyk}} \text{“tester2”}]$$

## tester3

$$[\text{tester3} \xrightarrow{\text{pyk}} \text{“tester3”}]$$

## tester4

$$[\text{tester4} \xrightarrow{\text{pyk}} \text{“tester4”}]$$

## tester5

$$[\text{tester5} \xrightarrow{\text{pyk}} \text{“tester5”}]$$

## tester6

$$[\text{tester6} \xrightarrow{\text{pyk}} \text{“tester6”}]$$

*The pyk compiler, version 0.grue.20060417+ by Klaus Grue*  
*GRD-2006-12-29.UTC:10:12:14.905583 = MJD-54098.TAI:10:12:47.905583 =*  
*LGT-4674103967905583e-6*