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[SystemQ **lemma** ToNegatedDoubleImply: $\Pi\mathcal{A},\mathcal{B},\mathcal{C}\colon \mathcal{A} \vdash \mathcal{B} \vdash \dot\neg(\mathcal{C})\mathrm{n} \vdash \dot\neg((\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}))\mathrm{n}$]

SystemQ **proof of** ToNegatedDoubleImply:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A},\mathcal{B},\mathcal{C}$	;
L03:	Premise $\gg$	$\mathcal{A}$	;
L04:	Premise $\gg$	$\mathcal{B}$	;
L05:	Premise $\gg$	$\dot\neg(\mathcal{C})\mathrm{n}$	;
L06:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}$	;
L07:	MP2 $\rhd$ L06 $\rhd$ L03 $\rhd$ L04 $\gg$	$\mathcal{C}$	;
L08:	FromContradiction $\rhd$ L07 $\rhd$ L05 $\gg$	$\dot\neg((\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}))\mathrm{n}$	;
L09:	Block $\gg$	End	;
L10:	Arbitrary $\gg$	$\mathcal{A},\mathcal{B},\mathcal{C}$	;
L03:	Ded $\rhd$ L09 $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \dot\neg(\mathcal{C})\mathrm{n} \Rightarrow (\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}) \Rightarrow \dot\neg((\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}))\mathrm{n}$	;
L04:	Premise $\gg$	$\mathcal{A}$	;
L05:	Premise $\gg$	$\mathcal{B}$	;
L06:	Premise $\gg$	$\dot\neg(\mathcal{C})\mathrm{n}$	;
L07:	MP3 $\rhd$ L03 $\rhd$ L04 $\rhd$ L05 $\rhd$ L06 $\gg$	$(\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}) \Rightarrow \dot\neg((\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}))\mathrm{n}$	;
L11:	prop lemma imply negation $\rhd$ L07 $\gg$	$\dot\neg((\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}))\mathrm{n}$	$\Box$

[SystemQ **lemma** FromNegatedAnd(Imply): $\Pi\mathcal{A},\mathcal{B}\colon \dot\neg((\mathcal{A} \dot\wedge \mathcal{B}))\mathrm{n} \Rightarrow \mathcal{A} \Rightarrow \dot\neg(\mathcal{B})\mathrm{n}$]

SystemQ **proof of** FromNegatedAnd(Imply):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A},\mathcal{B}$	;
L03:	Premise $\gg$	$\dot\neg((\mathcal{A} \dot\wedge \mathcal{B}))\mathrm{n}$	;
L04:	Premise $\gg$	$\mathcal{A}$	;
L05:	FromNegatedAnd $\rhd$ L03 $\rhd$ L04 $\gg$	$\dot\neg(\mathcal{B})\mathrm{n}$	;
L06:	Block $\gg$	End	;
L07:	Arbitrary $\gg$	$\mathcal{A},\mathcal{B}$	;
L08:	Ded $\rhd$ L06 $\gg$	$\dot\neg((\mathcal{A} \dot\wedge \mathcal{B}))\mathrm{n} \Rightarrow \mathcal{A} \Rightarrow \dot\neg(\mathcal{B})\mathrm{n}$	$\Box$

[SystemQ **lemma** RemoveDoubleNeg(Consequent): $\Pi\mathcal{A},\mathcal{B}\colon \mathcal{A} \Rightarrow \dot\neg(\dot\neg(\mathcal{B})\mathrm{n})\mathrm{n} \vdash \mathcal{A} \Rightarrow \mathcal{B}$]

SystemQ **proof of** RemoveDoubleNeg(Consequent):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A},\mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{A} \Rightarrow \dot\neg(\dot\neg(\mathcal{B})\mathrm{n})\mathrm{n}$	;
L04:	Premise $\gg$	$\mathcal{A}$	;
L05:	MP $\rhd$ L03 $\rhd$ L04 $\gg$	$\dot\neg(\dot\neg(\mathcal{B})\mathrm{n})\mathrm{n}$	;
L06:	RemoveDoubleNeg $\rhd$ L05 $\gg$	$\mathcal{B}$	;
L07:	Block $\gg$	End	;

L08: Arbitrary $\gg$ $\mathcal{A}, \mathcal{B}$;

L03: Ded $\rhd$ L07 $\gg$ $(\mathcal{A} \Rightarrow \dot{\neg}(\dot{\neg}(\mathcal{B})n)n) \Rightarrow \mathcal{A} \Rightarrow \mathcal{B}$;

L04: Premise $\gg$ $\mathcal{A} \Rightarrow \dot{\neg}(\dot{\neg}(\mathcal{B})n)n$;

L09: MP $\rhd$ L03 $\rhd$ L04 $\gg$ $\mathcal{A} \Rightarrow \mathcal{B}$ $\square$

[SystemQ **lemma** (A)to(E)(Imply): $\Pi V_1, \mathcal{A}\colon \forall V_1\colon \mathcal{A} \Rightarrow \dot{\neg}(\exists V_1\colon \dot{\neg}(\mathcal{A})n)n$]

SystemQ **proof of** (A)to(E)(Imply):

L01: Block $\gg$ Begin ;

L02: Arbitrary $\gg$ $V_1, \mathcal{A}$;

L03: Premise $\gg$ $\forall V_1\colon \mathcal{A}$;

L04: A4 @ V_1 $\rhd$ L03 $\gg$ $\mathcal{A}$;

L05: AddDoubleNeg $\rhd$ L04 $\gg$ $\dot{\neg}(\dot{\neg}(\mathcal{A})n)n$;

L06: Gen $\rhd$ L05 $\gg$ $\forall V_1\colon \dot{\neg}(\dot{\neg}(\mathcal{A})n)n$;

L07: AddDoubleNeg $\rhd$ L06 $\gg$ $\dot{\neg}(\dot{\neg}(\forall V_1\colon \dot{\neg}(\dot{\neg}(\mathcal{A})n)n)n)n$;

L08: Repetition $\rhd$ L07 $\gg$ $\dot{\neg}(\exists V_1\colon \dot{\neg}(\mathcal{A})n)n$;

L09: Block $\gg$ End ;

L10: Arbitrary $\gg$ $V_1, \mathcal{A}$;

L11: Ded $\rhd$ L09 $\gg$ $\forall V_1\colon \mathcal{A} \Rightarrow \dot{\neg}(\exists V_1\colon \dot{\neg}(\mathcal{A})n)n$ $\square$

[SystemQ **lemma** (E)to(A)(Imply): $\Pi V_1, \mathcal{A}\colon \exists V_1\colon \mathcal{A} \Rightarrow \dot{\neg}(\forall V_1\colon \dot{\neg}(\mathcal{A})n)n$]

SystemQ **proof of** (E)to(A)(Imply):

L01: Arbitrary $\gg$ $V_1, \mathcal{A}$;

L02: AutoImply $\gg$ $\dot{\neg}(\forall V_1\colon \dot{\neg}(\mathcal{A})n)n \Rightarrow$
$\dot{\neg}(\forall V_1\colon \dot{\neg}(\mathcal{A})n)n$;

L03: Repetition $\rhd$ L02 $\gg$ $\exists V_1\colon \mathcal{A} \Rightarrow \dot{\neg}(\forall V_1\colon \dot{\neg}(\mathcal{A})n)n$ $\square$

[SystemQ **lemma** (E)to(A)(Imply): $\Pi V_1, \mathcal{A}\colon \exists V_1\colon \dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\forall V_1\colon \mathcal{A})n$]

SystemQ **proof of** (E)to(A)(Imply):

L01: Block $\gg$ Begin ;

L02: Arbitrary $\gg$ $V_1, \mathcal{A}$;

L03: Premise $\gg$ $\exists V_1\colon \dot{\neg}(\mathcal{A})n$;

L04: AddDoubleNeg $\rhd$ L03 $\gg$ $\dot{\neg}(\dot{\neg}(\exists V_1\colon \dot{\neg}(\mathcal{A})n)n)n$;

L05: (A)to(E)(Imply) $\gg$ $\forall V_1\colon \mathcal{A} \Rightarrow \dot{\neg}(\exists V_1\colon \dot{\neg}(\mathcal{A})n)n$;

L06: MT $\rhd$ L05 $\rhd$ L04 $\gg$ $\dot{\neg}(\forall V_1\colon \mathcal{A})n$;

L07: Block $\gg$ End ;

L08: Arbitrary $\gg$ $V_1, \mathcal{A}$;

L09: Ded $\rhd$ L07 $\gg$ $\exists V_1\colon \dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\forall V_1\colon \mathcal{A})n$ $\square$

[SystemQ **lemma** AddNegatedAll: $\Pi V_1, \mathcal{A}, \mathcal{B}\colon \mathcal{B} \Rightarrow \mathcal{A} \vdash \dot{\neg}(\forall V_1\colon \mathcal{A})n \Rightarrow \dot{\neg}(\forall V_1\colon$

SystemQ **proof of** AddNegatedAll:

L01: Arbitrary $\gg$ $V_1, \mathcal{A}, \mathcal{B}$;

L02: Premise $\gg$ $\mathcal{B} \Rightarrow \mathcal{A}$;

L03: AddAll $\rhd$ L02 $\gg$ $\forall V_1\colon \mathcal{B} \Rightarrow \forall V_1\colon \mathcal{A}$;

L04: Contrapositive $\rhd$ L03 $\gg$ $\dot{\neg}(\forall V_1\colon \mathcal{A})n \Rightarrow \dot{\neg}(\forall V_1\colon \mathcal{B})n$ $\square$

[SystemQ **lemma** ToNegatedAEA: $\Pi V_1, V_2, V_3, \mathcal{A}\colon \exists V_1\colon \forall V_2\colon \exists V_3\colon \dot{\neg}(\mathcal{A})n \vdash \dot{\neg}(\forall V_1\colon \exists V_2\colon \forall V_3\colon \mathcal{A})n$]

SystemQ **proof of** ToNegatedAEA:

L01: Arbitrary $\gg$ $V_1, V_2, V_3, \mathcal{A}$;

L02: Premise $\gg$ $\exists V_1\colon \forall V_2\colon \exists V_3\colon \dot{\neg}(\mathcal{A})n$;

L03: (E)to(A)(Imply) $\gg$ $\exists V_3\colon \dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\forall V_3\colon \mathcal{A})n$;

L04: AddNegatedAll $\rhd$ L03 $\gg$ $\dot{\neg}(\forall V_2 : \dot{\neg}(\forall V_3 : \mathcal{A})n)n \Rightarrow \dot{\neg}(\forall V_2 : \exists V_3 : \dot{\neg}(\mathcal{A})n)n$;

L05: (E)to(A)(Imply) $\gg$ $\exists V_2 : \forall V_3 : \mathcal{A} \Rightarrow \dot{\neg}(\forall V_2 : \dot{\neg}(\forall V_3 : \mathcal{A})n)n$;

L06: ImplyTransitivity$\rhd$L05$\rhd$L04 $\gg$ $\exists V_2 : \forall V_3 : \mathcal{A} \Rightarrow \dot{\neg}(\forall V_2 : \exists V_3 : \dot{\neg}(\mathcal{A})n)n$;

L07: AddNegatedAll $\rhd$ L06 $\gg$ $\dot{\neg}(\forall V_1 : \dot{\neg}(\forall V_2 : \exists V_3 : \dot{\neg}(\mathcal{A})n)n)n \Rightarrow \dot{\neg}(\forall V_1 : \exists V_2 : \forall V_3 : \mathcal{A})n$;

L08: (E)to(A)(Imply) $\gg$ $\exists V_1 : \forall V_2 : \exists V_3 : \dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\forall V_1 : \dot{\neg}(\forall V_2 : \exists V_3 : \dot{\neg}(\mathcal{A})n)n)n$;

L09: ImplyTransitivity$\rhd$L08$\rhd$L07 $\gg$ $\exists V_1 : \forall V_2 : \exists V_3 : \dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\forall V_1 : \exists V_2 : \forall V_3 : \mathcal{A})n$;

L10: MP $\rhd$ L09 $\rhd$ L02 $\gg$ $\dot{\neg}(\forall V_1 : \exists V_2 : \forall V_3 : \mathcal{A})n$ □

[SystemQ **lemma** (A)to(E): $\Pi V_1, \mathcal{A} : \forall V_1 : \dot{\neg}(\mathcal{A})n \vdash \dot{\neg}(\exists V_1 : \mathcal{A})n$]

SystemQ **proof of** (A)to(E):

L01: Arbitrary $\gg$ $V_1, \mathcal{A}$;

L02: Premise $\gg$ $\forall V_1 : \dot{\neg}(\mathcal{A})n$;

L03: AddDoubleNeg $\rhd$ L02 $\gg$ $\dot{\neg}(\dot{\neg}(\forall V_1 : \dot{\neg}(\mathcal{A})n)n)n$;

L04: Repetition $\rhd$ L03 $\gg$ $\dot{\neg}(\exists V_1 : \mathcal{A})n$ □

[SystemQ **lemma** ExistMP3: $\Pi V_1, V_2, V_3, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D} : \mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C} \Rightarrow \mathcal{D} \vdash \exists V_1 : \mathcal{A} \vdash \exists V_2 : \mathcal{B} \vdash \exists V_3 : \mathcal{C} \vdash \mathcal{D}$]

SystemQ **proof of** ExistMP3:

L01: Arbitrary $\gg$ $V_1, V_2, V_3, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$;

L02: Premise $\gg$ $\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C} \Rightarrow \mathcal{D}$;

L03: Premise $\gg$ $\exists V_1 : \mathcal{A}$;

L04: Premise $\gg$ $\exists V_2 : \mathcal{B}$;

L05: Premise $\gg$ $\exists V_3 : \mathcal{C}$;

L06: ExistMP2 $\rhd$ L02 $\rhd$ L03 $\rhd$ L04 $\gg$ $\mathcal{C} \Rightarrow \mathcal{D}$;

L07: ExistMP $\rhd$ L06 $\rhd$ L05 $\gg$ $\mathcal{D}$ □

[SystemQ **lemma** PositiveToLeft(Eq): $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z} : \mathcal{X} = (\mathcal{Y} + \mathcal{Z}) \vdash (\mathcal{X} - \mathcal{Z}) = \mathcal{Y}$]

SystemQ **proof of** PositiveToLeft(Eq):

L01: Arbitrary $\gg$ $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$;

L02: Premise $\gg$ $\mathcal{X} = (\mathcal{Y} + \mathcal{Z})$;

L03: eqAddition $\rhd$ L02 $\gg$ $(\mathcal{X} - \mathcal{Z}) = ((\mathcal{Y} + \mathcal{Z}) - \mathcal{Z})$;

L04: x = x + y − y $\gg$ $\mathcal{Y} = ((\mathcal{Y} + \mathcal{Z}) - \mathcal{Z})$;

L05: eqSymmetry $\rhd$ L04 $\gg$ $((\mathcal{Y} + \mathcal{Z}) - \mathcal{Z}) = \mathcal{Y}$;

L06: eqTransitivity $\rhd$ L03 $\rhd$ L05 $\gg$ $(\mathcal{X} - \mathcal{Z}) = \mathcal{Y}$ □

[SystemQ **lemma** ExpZero(Exact): $\Pi \mathcal{X} : \mathcal{X}(\exp)0 = 1$]

SystemQ **proof of** ExpZero(Exact):

L01: Arbitrary $\gg$ $\mathcal{X}$;

L02: eqReflexivity $\gg$ $0 = 0$;

L03:	ExpZero $\rhd$ L02 $\gg$	$\mathcal{X}(\text{exp})0 = 1$	□

[SystemQ **lemma** (+1)IsPositive(N): $\Pi\mathcal{M}\colon \text{Nat}(\mathcal{M}) \Vdash 0 < (\mathcal{M}+1)$]

SystemQ **proof of** (+1)IsPositive(N):

L01:	Arbitrary $\gg$	$\mathcal{M}$	;
L02:	Side-condition $\gg$	$\text{Nat}(\mathcal{M})$	;
L03:	Nonnegative(N) $\unrhd$ L02 $\gg$	$0 <= \mathcal{M}$	;
L04:	Leq + 1 $\rhd$ L03 $\gg$	$0 < (\mathcal{M}+1)$	□

[SystemQ **lemma** SameExp(Base): $\Pi\mathcal{N},\mathcal{X}\colon \forall\mathcal{N}\colon (0 = \mathcal{N} \Rightarrow \mathcal{X}(\text{exp})0 = \mathcal{X}(\text{exp})\mathcal{N}$

SystemQ **proof of** SameExp(Base):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{N},\mathcal{X}$	;
L03:	Premise $\gg$	$0 = \mathcal{N}$	;
L04:	ExpZero(Exact) $\gg$	$\mathcal{X}(\text{exp})0 = 1$	;
L05:	eqSymmetry $\rhd$ L03 $\gg$	$\mathcal{N} = 0$	;
L06:	ExpZero $\rhd$ L05 $\gg$	$\mathcal{X}(\text{exp})\mathcal{N} = 1$	;
L07:	eqSymmetry $\rhd$ L06 $\gg$	$1 = \mathcal{X}(\text{exp})\mathcal{N}$	;
L08:	eqTransitivity $\rhd$ L04 $\rhd$ L07 $\gg$	$\mathcal{X}(\text{exp})0 = \mathcal{X}(\text{exp})\mathcal{N}$	;
L09:	Block $\gg$	End	;
L10:	Arbitrary $\gg$	$\mathcal{N},\mathcal{X}$	;
L03:	Ded $\rhd$ L09 $\gg$	$0 = \mathcal{N} \Rightarrow \mathcal{X}(\text{exp})0 = \mathcal{X}(\text{exp})\mathcal{N}$	;
L11:	Gen $\rhd$ L03 $\gg$	$\forall\mathcal{N}\colon (0 = \mathcal{N} \Rightarrow \mathcal{X}(\text{exp})0 = \mathcal{X}(\text{exp})\mathcal{N})$	□

[SystemQ **lemma** SameExp(Indu): $\Pi\mathcal{M},\mathcal{N},\mathcal{X}\colon \forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\text{exp})\mathcal{M} = \mathcal{X}(\text{exp})\mathcal{N}) \Rightarrow \forall\mathcal{N}\colon ((\mathcal{M}+1) = \mathcal{N} \Rightarrow \mathcal{X}(\text{exp})((\mathcal{M}+1)) = \mathcal{X}(\text{exp})\mathcal{N})$]

SystemQ **proof of** SameExp(Indu):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{M},\mathcal{N},\mathcal{X}$	;
L03:	Block $\gg$	Begin	;
L04:	Arbitrary $\gg$	$\mathcal{M},\mathcal{N},\mathcal{X}$	;
L05:	Premise $\gg$	$\forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\text{exp})\mathcal{M} = \mathcal{X}(\text{exp})\mathcal{N})$	;
L06:	Premise $\gg$	$(\mathcal{M}+1) = \mathcal{N}$	;
L07:	(+1)IsPositive(N) $\gg$	$0 < (\mathcal{M}+1)$	;
L08:	ExpPositive $\rhd$ L07 $\gg$	$\mathcal{X}(\text{exp})((\mathcal{M}+1)) = (\mathcal{X} * \mathcal{X}(\text{exp})(((\mathcal{M}+1)-1)))$	;
L09:	x = x + y − y $\gg$	$\mathcal{M} = ((\mathcal{M}+1)-1)$	;
L10:	A4 @$((\mathcal{M}+1)-1)$ $\rhd$ L05 $\gg$	$\mathcal{M} = ((\mathcal{M}+1)-1) \Rightarrow \mathcal{X}(\text{exp})\mathcal{M} = \mathcal{X}(\text{exp})(((\mathcal{M}+1)-1))$	;
L11:	MP $\rhd$ L10 $\rhd$ L09 $\gg$	$\mathcal{X}(\text{exp})\mathcal{M} = \mathcal{X}(\text{exp})(((\mathcal{M}+1)-1))$	;
L12:	eqSymmetry $\rhd$ L11 $\gg$	$\mathcal{X}(\text{exp})(((\mathcal{M}+1)-1)) = \mathcal{X}(\text{exp})\mathcal{M}$	;
L13:	EqMultiplicationLeft $\rhd$ L12 $\gg$	$(\mathcal{X} * \mathcal{X}(\text{exp})(((\mathcal{M}+1)-1))) = (\mathcal{X} * \mathcal{X}(\text{exp})\mathcal{M})$	;

L14:	A4 @$(\mathcal{N}-1)$ ▷ L05 ≫	$\mathcal{M} = (\mathcal{N}-1) \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)((\mathcal{N}-1))$	;
L15:	PositiveToRight(Eq) ▷ L06 ≫	$\mathcal{M} = (\mathcal{N}-1)$	;
L16:	MP ▷ L14 ▷ L15 ≫	$\mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)((\mathcal{N}-1))$	;
L17:	EqMultiplicationLeft ▷ L16 ≫	$(\mathcal{X} * \mathcal{X}(\exp)\mathcal{M}) = (\mathcal{X} * \mathcal{X}(\exp)((\mathcal{N}-1)))$	;
L18:	SubLessRight ▷ L06 ▷ L07 ≫	$0 < \mathcal{N}$	;
L19:	ExpPositive ▷ L18 ≫	$\mathcal{X}(\exp)\mathcal{N} = (\mathcal{X} * \mathcal{X}(\exp)((\mathcal{N}-1)))$	;
L20:	eqSymmetry ▷ L19 ≫	$(\mathcal{X} * \mathcal{X}(\exp)((\mathcal{N}-1))) = \mathcal{X}(\exp)\mathcal{N}$	;
L21:	eqTransitivity5 ▷ L08 ▷ L13 ▷ L17 ▷ L20 ≫	$\mathcal{X}(\exp)((\mathcal{M}+1)) = \mathcal{X}(\exp)\mathcal{N}$	;
L22:	Block ≫	End	;
L05:	Ded ▷ L22 ≫	$\forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N}) \Rightarrow (\mathcal{M}+1) = \mathcal{N} \Rightarrow \mathcal{X}(\exp)((\mathcal{M}+1)) = \mathcal{X}(\exp)\mathcal{N}$	;
L06:	Premise ≫	$\forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N})$	;
L07:	MP ▷ L05 ▷ L06 ≫	$(\mathcal{M}+1) = \mathcal{N} \Rightarrow \mathcal{X}(\exp)((\mathcal{M}+1)) = \mathcal{X}(\exp)\mathcal{N}$	;
L23:	Gen ▷ L07 ≫	$\forall\mathcal{N}\colon ((\mathcal{M}+1) = \mathcal{N} \Rightarrow \mathcal{X}(\exp)((\mathcal{M}+1)) = \mathcal{X}(\exp)\mathcal{N})$	;
L24:	Block ≫	End	;
L25:	Arbitrary ≫	$\mathcal{M}, \mathcal{N}, \mathcal{X}$	;
L26:	Ded ▷ L24 ≫	$\forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N}) \Rightarrow \forall\mathcal{N}\colon ((\mathcal{M}+1) = \mathcal{N} \Rightarrow \mathcal{X}(\exp)((\mathcal{M}+1)) = \mathcal{X}(\exp)\mathcal{N})$	□

[SystemQ **lemma** SameExp: $\Pi\mathcal{M}, \mathcal{N}, \mathcal{X}\colon \mathcal{M} = \mathcal{N} \vdash \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N}$]

SystemQ **proof of** SameExp:

L01:	Arbitrary ≫	$\mathcal{M}, \mathcal{N}, \mathcal{X}$	;
L02:	Premise ≫	$\mathcal{M} = \mathcal{N}$	;
L03:	SameExp(Base) ≫	$\forall\mathcal{N}\colon (0 = \mathcal{N} \Rightarrow \mathcal{X}(\exp)0 = \mathcal{X}(\exp)\mathcal{N})$	;
L04:	SameExp(Indu) ≫	$\forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N}) \Rightarrow \forall\mathcal{N}\colon ((\mathcal{M}+1) = \mathcal{N} \Rightarrow \mathcal{X}(\exp)((\mathcal{M}+1)) = \mathcal{X}(\exp)\mathcal{N})$	;
L05:	Induction ▷ L03 ▷ L04 ≫	$\forall\mathcal{N}\colon (\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N})$	;
L06:	A4 @$\mathcal{N}$ ▷ L05 ≫	$\mathcal{M} = \mathcal{N} \Rightarrow \mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N}$	;
L07:	MP ▷ L06 ▷ L02 ≫	$\mathcal{X}(\exp)\mathcal{M} = \mathcal{X}(\exp)\mathcal{N}$	□

[SystemQ **lemma** Exp(+1): $\Pi\mathcal{M}, \mathcal{X}\colon \mathcal{X}(\exp)((\mathcal{M}+1)) = (\mathcal{X} * \mathcal{X}(\exp)\mathcal{M})$]

SystemQ **proof of** Exp(+1):

L01:	Arbitrary $\gg$	$\mathcal{M},\mathcal{X}$	;
L02:	(+1)IsPositive(N) $\gg$	$0<(\mathcal{M}+1)$	;
L03:	ExpPositive $\rhd$ L02 $\gg$	$\mathcal{X}(\exp)((\mathcal{M}+1))=(\mathcal{X}*\mathcal{X}(\exp)(((\mathcal{M}+1)-1)))$	;
L04:	$\mathrm{x}=\mathrm{x}+\mathrm{y}-\mathrm{y}$ $\gg$	$\mathcal{M}=((\mathcal{M}+1)-1)$	;
L05:	SameExp $\rhd$ L04 $\gg$	$\mathcal{X}(\exp)\mathcal{M}=\mathcal{X}(\exp)(((\mathcal{M}+1)-1))$	;
L06:	EqMultiplicationLeft $\rhd$ L05 $\gg$	$(\mathcal{X}*\mathcal{X}(\exp)\mathcal{M})=(\mathcal{X}*\mathcal{X}(\exp)(((\mathcal{M}+1)-1)))$	;
L07:	eqSymmetry $\rhd$ L06 $\gg$	$(\mathcal{X}*\mathcal{X}(\exp)(((\mathcal{M}+1)-1)))=(\mathcal{X}*\mathcal{X}(\exp)\mathcal{M})$	;
L08:	eqTransitivity $\rhd$ L03 $\rhd$ L07 $\gg$	$\mathcal{X}(\exp)((\mathcal{M}+1))=(\mathcal{X}*\mathcal{X}(\exp)\mathcal{M})$	$\square$

[SystemQ **lemma** DistributionOut(Minus): $\Pi\mathcal{X},\mathcal{Y},\mathcal{Z}$: $((\mathcal{X}*\mathcal{Y})-(\mathcal{X}*\mathcal{Z}))=(\mathcal{X}*((\mathcal{Y}-\mathcal{Z})))$]

SystemQ **proof of** DistributionOut(Minus):

L01:	Arbitrary $\gg$	$\mathcal{X},\mathcal{Y},\mathcal{Z}$	;
L02:	Times(−1)Left $\gg$	$((-1)*((\mathcal{X}*\mathcal{Z})))=(-\mathrm{u}((\mathcal{X}*\mathcal{Z})))$	;
L03:	eqSymmetry $\rhd$ L02 $\gg$	$(-\mathrm{u}((\mathcal{X}*\mathcal{Z})))=((-1)*((\mathcal{X}*\mathcal{Z})))$	;
L04:	timesCommutativity $\gg$	$((-1)*((\mathcal{X}*\mathcal{Z})))=((\mathcal{X}*\mathcal{Z})*(-1))$	;
L05:	timesAssociativity $\gg$	$((\mathcal{X}*\mathcal{Z})*(-1))=(\mathcal{X}*((\mathcal{Z}*(-1))))$	;
L06:	Times(−1) $\gg$	$(\mathcal{Z}*(-1))=(-\mathrm{u}\mathcal{Z})$	;
L07:	EqMultiplicationLeft $\rhd$ L06 $\gg$	$(\mathcal{X}*((\mathcal{Z}*(-1))))=(\mathcal{X}*((-\mathrm{u}\mathcal{Z})))$	;
L08:	eqTransitivity5 $\rhd$ L03 $\rhd$ L04 $\rhd$ L05 $\rhd$ L07 $\gg$	$(-\mathrm{u}((\mathcal{X}*\mathcal{Z})))=(\mathcal{X}*((-\mathrm{u}\mathcal{Z})))$	;
L09:	EqAdditionLeft $\rhd$ L08 $\gg$	$((\mathcal{X}*\mathcal{Y})-(\mathcal{X}*\mathcal{Z}))=((\mathcal{X}*\mathcal{Y})+(\mathcal{X}*((-\mathrm{u}\mathcal{Z}))))$	;
L10:	DistributionOut $\gg$	$((\mathcal{X}*\mathcal{Y})+(\mathcal{X}*((-\mathrm{u}\mathcal{Z}))))=(\mathcal{X}*((\mathcal{Y}-\mathcal{Z})))$	;
L11:	eqTransitivity $\rhd$ L09 $\rhd$ L10 $\gg$	$((\mathcal{X}*\mathcal{Y})-(\mathcal{X}*\mathcal{Z}))=(\mathcal{X}*((\mathcal{Y}-\mathcal{Z})))$	$\square$

[SystemQ **lemma** (1/2)(x+y)−x = (1/2)(y−x): $\Pi\mathcal{X},\mathcal{Y}$: $((1/2*((\mathcal{X}+\mathcal{Y})))-\mathcal{X})=(1/2*((\mathcal{Y}-\mathcal{X})))$]

SystemQ **proof of** (1/2)(x + y) − x = (1/2)(y − x):

L01:	Arbitrary $\gg$	$\mathcal{X},\mathcal{Y}$	;
L02:	Distribution $\gg$	$(1/2*((\mathcal{X}+\mathcal{Y})))=((1/2*\mathcal{X})+(1/2*\mathcal{Y}))$	;
L03:	eqAddition $\rhd$ L02 $\gg$	$((1/2*((\mathcal{X}+\mathcal{Y})))-\mathcal{X})=(((1/2*\mathcal{X})+(1/2*\mathcal{Y}))-\mathcal{X})$	;
L04:	plusCommutativity $\gg$	$((1/2*\mathcal{X})+(1/2*\mathcal{Y}))=((1/2*\mathcal{Y})+(1/2*\mathcal{X}))$	;

L05: eqAddition $\rhd$ L04 $\gg$ $(((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y})) - \mathcal{X}) = (((1/2 * \mathcal{Y}) + (1/2 * \mathcal{X})) - \mathcal{X})$;

L06: plusAssociativity $\gg$ $(((1/2 * \mathcal{Y}) + (1/2 * \mathcal{X})) - \mathcal{X}) = ((1/2 * \mathcal{Y}) + (((1/2 * \mathcal{X}) - \mathcal{X})))$;

L07: TwoHalves $\gg$ $((1/2 * \mathcal{X}) + (1/2 * \mathcal{X})) = \mathcal{X}$;

L08: PositiveToRight(Eq) $\rhd$ L07 $\gg$ $(1/2 * \mathcal{X}) = (\mathcal{X} - (1/2 * \mathcal{X}))$;

L09: EqNegated $\rhd$ L08 $\gg$ $(-\mathrm{u}((1/2 * \mathcal{X}))) = (-\mathrm{u}((\mathcal{X} - (1/2 * \mathcal{X}))))$;

L10: MinusNegated $\gg$ $(-\mathrm{u}((\mathcal{X} - (1/2 * \mathcal{X})))) = ((1/2 * \mathcal{X}) - \mathcal{X})$;

L11: eqTransitivity $\rhd$ L09 $\rhd$ L10 $\gg$ $(-\mathrm{u}((1/2 * \mathcal{X}))) = ((1/2 * \mathcal{X}) - \mathcal{X})$;

L12: eqSymmetry $\rhd$ L11 $\gg$ $((1/2 * \mathcal{X}) - \mathcal{X}) = (-\mathrm{u}((1/2 * \mathcal{X})))$;

L13: EqAdditionLeft $\rhd$ L12 $\gg$ $((1/2 * \mathcal{Y}) + (((1/2 * \mathcal{X}) - \mathcal{X}))) = ((1/2 * \mathcal{Y}) - (1/2 * \mathcal{X}))$;

L14: DistributionOut(Minus) $\gg$ $((1/2 * \mathcal{Y}) - (1/2 * \mathcal{X})) = (1/2 * ((\mathcal{Y} - \mathcal{X})))$;

L15: eqTransitivity6 $\rhd$ L03 $\rhd$ L05 $\rhd$ L06 $\rhd$ L13 $\rhd$ L14 $\gg$ $((1/2 * ((\mathcal{X} + \mathcal{Y}))) - \mathcal{X}) = (1/2 * ((\mathcal{Y} - \mathcal{X})))$ □

[SystemQ **lemma** $\mathrm{y} - (1/2)(\mathrm{x} + \mathrm{y}) = (1/2)(\mathrm{y} - \mathrm{x})$: $\Pi \mathcal{X}, \mathcal{Y}$: $(\mathcal{Y} - (1/2 * ((\mathcal{X} + \mathcal{Y})))) = (1/2 * ((\mathcal{Y} - \mathcal{X})))$]

SystemQ **proof of** $\mathrm{y} - (1/2)(\mathrm{x} + \mathrm{y}) = (1/2)(\mathrm{y} - \mathrm{x})$:

L01: Arbitrary $\gg$ $\mathcal{X}, \mathcal{Y}$;

L02: Distribution $\gg$ $(1/2 * ((\mathcal{X} + \mathcal{Y}))) = ((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y}))$;

L03: EqNegated $\rhd$ L02 $\gg$ $(-\mathrm{u}((1/2 * ((\mathcal{X} + \mathcal{Y}))))) = (-\mathrm{u}(((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y}))))$;

L04: EqAdditionLeft $\rhd$ L03 $\gg$ $(\mathcal{Y} - (1/2 * ((\mathcal{X} + \mathcal{Y})))) = (\mathcal{Y} - (((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y}))))$;

L05: plusCommutativity $\gg$ $((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y})) = ((1/2 * \mathcal{Y}) + (1/2 * \mathcal{X}))$;

L06: EqNegated $\rhd$ L05 $\gg$ $(-\mathrm{u}(((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y})))) = (-\mathrm{u}(((1/2 * \mathcal{Y}) + (1/2 * \mathcal{X}))))$;

L07: $-\mathrm{x} - \mathrm{y} = -(\mathrm{x} + \mathrm{y})$ $\gg$ $((-\mathrm{u}((1/2 * \mathcal{Y}))) - (1/2 * \mathcal{X})) = (-\mathrm{u}(((1/2 * \mathcal{Y}) + (1/2 * \mathcal{X}))))$;

L08: eqSymmetry $\rhd$ L07 $\gg$ $(-\mathrm{u}(((1/2 * \mathcal{Y}) + (1/2 * \mathcal{X})))) = ((-\mathrm{u}((1/2 * \mathcal{Y}))) - (1/2 * \mathcal{X}))$;

L09: eqTransitivity $\rhd$ L06 $\rhd$ L08 $\gg$ $(-\mathrm{u}(((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y})))) = ((-\mathrm{u}((1/2 * \mathcal{Y}))) - (1/2 * \mathcal{X}))$;

L10: EqAdditionLeft $\rhd$ L09 $\gg$ $(\mathcal{Y} - (((1/2 * \mathcal{X}) + (1/2 * \mathcal{Y})))) = (\mathcal{Y} + (((-\mathrm{u}((1/2 * \mathcal{Y}))) - (1/2 * \mathcal{X}))))$;

L11: plusAssociativity $\gg$ $((\mathcal{Y} - (1/2 * \mathcal{Y})) - (1/2 * \mathcal{X})) = (\mathcal{Y} + (((-\mathrm{u}((1/2 * \mathcal{Y}))) - (1/2 * \mathcal{X}))))$;

L12:	eqSymmetry ▷ L11 ≫	$(\mathcal{Y}+(((-\mathrm{u}((1/2*\mathcal{Y})))-(1/2*\mathcal{X}))))=((\mathcal{Y}-(1/2*\mathcal{Y}))-(1/2*\mathcal{X}))$	;
L13:	TwoHalves ≫	$((1/2*\mathcal{Y})+(1/2*\mathcal{Y}))=\mathcal{Y}$	;
L14:	PositiveToRight(Eq) ▷ L13 ≫	$(1/2*\mathcal{Y})=(\mathcal{Y}-(1/2*\mathcal{Y}))$	;
L15:	eqSymmetry ▷ L14 ≫	$(\mathcal{Y}-(1/2*\mathcal{Y}))=(1/2*\mathcal{Y})$	;
L16:	eqAddition ▷ L15 ≫	$((\mathcal{Y}-(1/2*\mathcal{Y}))-(1/2*\mathcal{X}))=((1/2*\mathcal{Y})-(1/2*\mathcal{X}))$	;
L17:	DistributionOut(Minus) ≫	$((1/2*\mathcal{Y})-(1/2*\mathcal{X}))=(1/2*((\mathcal{Y}-\mathcal{X})))$	;
L18:	eqTransitivity6 ▷ L04 ▷ L10 ▷ L12 ▷ L16 ▷ L17 ≫	$(\mathcal{Y}-(1/2*((\mathcal{X}+\mathcal{Y}))))=(1/2*((\mathcal{Y}-\mathcal{X})))$	□

[SystemQ **lemma** PositiveBase(Base): $\Pi\mathcal{X}: 0<\mathcal{X}(\exp)0$]

SystemQ **proof of** PositiveBase(Base):

L01:	Arbitrary ≫	$\mathcal{X}$	;
L02:	ExpZero(Exact) ≫	$\mathcal{X}(\exp)0=1$	;
L03:	eqSymmetry ▷ L02 ≫	$1=\mathcal{X}(\exp)0$	;
L04:	0 < 1 ≫	$0<1$	;
L05:	SubLessRight ▷ L03 ▷ L04 ≫	$0<\mathcal{X}(\exp)0$	□

[SystemQ **lemma** PositiveBase(Indu): $\Pi\mathcal{M},\mathcal{X}: 0<\mathcal{X}\vdash 0<\mathcal{X}(\exp)\mathcal{M}\Rightarrow 0<\mathcal{X}(\exp)((\mathcal{M}+1))$]

SystemQ **proof of** PositiveBase(Indu):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	$\mathcal{M},\mathcal{X}$	;
L03:	Premise ≫	$0<\mathcal{X}$	;
L04:	Premise ≫	$0<\mathcal{X}(\exp)\mathcal{M}$	;
L05:	Exp(+1) ≫	$\mathcal{X}(\exp)((\mathcal{M}+1))=(\mathcal{X}*\mathcal{X}(\exp)\mathcal{M})$	;
L06:	eqSymmetry ▷ L05 ≫	$(\mathcal{X}*\mathcal{X}(\exp)\mathcal{M})=\mathcal{X}(\exp)((\mathcal{M}+1))$	;
L07:	PositiveFactors ▷ L03 ▷ L04 ≫	$0<(\mathcal{X}*\mathcal{X}(\exp)\mathcal{M})$	;
L08:	SubLessRight ▷ L06 ▷ L07 ≫	$0<\mathcal{X}(\exp)((\mathcal{M}+1))$	;
L09:	Block ≫	End	;
L10:	Arbitrary ≫	$\mathcal{M},\mathcal{X}$	;
L03:	Ded ▷ L09 ≫	$0<\mathcal{X}\Rightarrow 0<\mathcal{X}(\exp)\mathcal{M}\Rightarrow 0<\mathcal{X}(\exp)((\mathcal{M}+1))$	;
L04:	Premise ≫	$0<\mathcal{X}$	;
L11:	MP ▷ L03 ▷ L04 ≫	$0<\mathcal{X}(\exp)\mathcal{M}\Rightarrow 0<\mathcal{X}(\exp)((\mathcal{M}+1))$	□

[SystemQ **lemma** PositiveBase: $\Pi\mathcal{M},\mathcal{X}: 0<\mathcal{X}\vdash 0<\mathcal{X}(\exp)\mathcal{M}$]

SystemQ **proof of** PositiveBase:

L01:	Arbitrary ≫	$\mathcal{M},\mathcal{X}$	;
L02:	Premise ≫	$0<\mathcal{X}$	;
L03:	PositiveBase(Base) ≫	$0<\mathcal{X}(\exp)0$	;

L04: PositiveBase(Indu) $\rhd$ L02 $\gg$ $0 < \mathcal{X}(\text{exp})\mathcal{M} \Rightarrow 0 < \mathcal{X}(\text{exp})((\mathcal{M}+1))$;

L05: Induction $\rhd$ L03 $\rhd$ L04 $\gg$ $0 < \mathcal{X}(\text{exp})\mathcal{M}$ $\Box$

[SystemQ **lemma** PositiveToRight(Eq)(1term): $\Pi\mathcal{X},\mathcal{Y}\colon \mathcal{X} = \mathcal{Y} \vdash 0 = (\mathcal{Y} - \mathcal{X})$]

SystemQ **proof of** PositiveToRight(Eq)(1term):

L01: Arbitrary $\gg$ $\mathcal{X},\mathcal{Y}$;

L02: Premise $\gg$ $\mathcal{X} = \mathcal{Y}$;

L03: eqSymmetry $\rhd$ L02 $\gg$ $\mathcal{Y} = \mathcal{X}$;

L04: PositiveToLeft(Eq)(1term) $\rhd$ L03 $\gg$ $(\mathcal{Y} - \mathcal{X}) = 0$;

L05: eqSymmetry $\rhd$ L04 $\gg$ $0 = (\mathcal{Y} - \mathcal{X})$ $\Box$

[SystemQ **lemma** BSzero(Exact): $\Pi\mathcal{M}\colon \text{BS}(\mathcal{M},0) = 1/2(\text{exp})\mathcal{M}$]

SystemQ **proof of** BSzero(Exact):

L01: Arbitrary $\gg$ $\mathcal{M}$;

L02: eqReflexivity $\gg$ $0 = 0$;

L03: BSzero $\rhd$ L02 $\gg$ $\text{BS}(\mathcal{M},0) = 1/2(\text{exp})\mathcal{M}$ $\Box$

[SystemQ **lemma** SameBS(2)(Base): $\Pi\mathcal{M},\text{N}_2\colon \forall\text{N}_2\colon (0 = \text{N}_2 \Rightarrow \text{BS}(\mathcal{M},0) = \text{BS}(\mathcal{M},\text{N}_2))$]

SystemQ **proof of** SameBS(2)(Base):

L01: Block $\gg$ Begin ;

L02: Arbitrary $\gg$ $\mathcal{M},\text{N}_2$;

L03: Premise $\gg$ $0 = \text{N}_2$;

L04: eqSymmetry $\rhd$ L03 $\gg$ $\text{N}_2 = 0$;

L05: BSzero $\rhd$ L04 $\gg$ $\text{BS}(\mathcal{M},\text{N}_2) = 1/2(\text{exp})(\mathcal{M})$;

L06: eqSymmetry $\rhd$ L05 $\gg$ $1/2(\text{exp})(\mathcal{M}) = \text{BS}(\mathcal{M},\text{N}_2)$;

L07: BSzero(Exact) $\gg$ $\text{BS}(\mathcal{M},0) = 1/2(\text{exp})(\mathcal{M})$;

L08: eqTransitivity $\rhd$ L07 $\rhd$ L06 $\gg$ $\text{BS}(\mathcal{M},0) = \text{BS}(\mathcal{M},\text{N}_2)$;

L09: Block $\gg$ End ;

L10: Arbitrary $\gg$ $\mathcal{M},\text{N}_2$;

L03: Ded $\rhd$ L09 $\gg$ $0 = \text{N}_2 \Rightarrow \text{BS}(\mathcal{M},0) = \text{BS}(\mathcal{M},\text{N}_2)$;

L11: Gen $\rhd$ L03 $\gg$ $\forall\text{N}_2\colon (0 = \text{N}_2 \Rightarrow \text{BS}(\mathcal{M},0) = \text{BS}(\mathcal{M},\text{N}_2))$ $\Box$

[SystemQ **lemma** SameBS(2)(Indu): $\Pi\mathcal{M},\text{N}_1,\text{N}_2\colon \forall\text{N}_2\colon (\text{N}_1 = \text{N}_2 \Rightarrow \text{BS}(\mathcal{M},\text{N}_1$
$\text{BS}(\mathcal{M},\text{N}_2)) \Rightarrow \forall\text{N}_2\colon ((\text{N}_1+1) = \text{N}_2 \Rightarrow \text{BS}(\mathcal{M},(\text{N}_1+1)) = \text{BS}(\mathcal{M},\text{N}_2))$]

SystemQ **proof of** SameBS(2)(Indu):

L01: Block $\gg$ Begin ;

L02: Arbitrary $\gg$ $\mathcal{M},\text{N}_1,\text{N}_2$;

L03: Block $\gg$ Begin ;

L04: Arbitrary $\gg$ $\mathcal{M},\text{N}_1,\text{N}_2$;

L05: Premise $\gg$ $\forall\text{N}_2\colon (\text{N}_1 = \text{N}_2 \Rightarrow \text{BS}(\mathcal{M},\text{N}_1) = \text{BS}(\mathcal{M},\text{N}_2))$;

L06: Premise $\gg$ $(\text{N}_1+1) = \text{N}_2$;

L07:	(+1)IsPositive(N) ≫	$0 < (N_1 + 1)$	;
L08:	BSpositive ▷ L07 ≫	$BS(\mathcal{M}, (N_1 + 1)) = (1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) + BS(\mathcal{M}, ((N_1 + 1) - 1)))$	;
L09:	x = x + y − y ≫	$N_1 = ((N_1 + 1) - 1)$	;
L10:	A4 @$((N_1 + 1) - 1)$ ▷ L05 ≫	$N_1 = ((N_1 + 1) - 1) \Rightarrow BS(\mathcal{M}, N_1) = BS(\mathcal{M}, ((N_1 + 1) - 1))$	;
L11:	MP ▷ L10 ▷ L09 ≫	$BS(\mathcal{M}, N_1) = BS(\mathcal{M}, ((N_1 + 1) - 1))$	;
L12:	eqSymmetry ▷ L11 ≫	$BS(\mathcal{M}, ((N_1 + 1) - 1)) = BS(\mathcal{M}, N_1)$	;
L13:	EqAdditionLeft ▷ L12 ≫	$(1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) + BS(\mathcal{M}, ((N_1 + 1) - 1))) = (1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) + BS(\mathcal{M}, N_1))$	;
L14:	PositiveToRight(Eq) ▷ L06 ≫	$N_1 = (N_2 - 1)$	;
L15:	A4 @$(N_2 - 1)$ ▷ L05 ≫	$N_1 = (N_2 - 1) \Rightarrow BS(\mathcal{M}, N_1) = BS(\mathcal{M}, (N_2 - 1))$	;
L16:	MP ▷ L15 ▷ L14 ≫	$BS(\mathcal{M}, N_1) = BS(\mathcal{M}, (N_2 - 1))$	;
L17:	EqAdditionLeft ▷ L16 ≫	$(1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) + BS(\mathcal{M}, N_1)) = (1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) + BS(\mathcal{M}, (N_2 - 1)))$	;
L18:	EqAdditionLeft ▷ L06 ≫	$(\mathcal{M} + ((N_1 + 1))) = (\mathcal{M} + N_2)$	;
L19:	SameExp ▷ L18 ≫	$1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) = 1/2(\exp)((\mathcal{M} + N_2))$	;
L20:	eqAddition ▷ L19 ≫	$(1/2(\exp)((\mathcal{M} + ((N_1 + 1)))) + BS(\mathcal{M}, (N_2 - 1))) = (1/2(\exp)((\mathcal{M} + N_2)) + BS(\mathcal{M}, (N_2 - 1)))$	;
L21:	SubLessRight ▷ L06 ▷ L07 ≫	$0 < N_2$	;
L22:	BSpositive ▷ L21 ≫	$BS(\mathcal{M}, N_2) = (1/2(\exp)((\mathcal{M} + N_2)) + BS(\mathcal{M}, (N_2 - 1)))$	;
L23:	eqSymmetry ▷ L22 ≫	$(1/2(\exp)((\mathcal{M} + N_2)) + BS(\mathcal{M}, (N_2 - 1))) = BS(\mathcal{M}, N_2)$	;
L24:	eqTransitivity6 ▷ L08 ▷ L13 ▷ L17 ▷ L20 ▷ L23 ≫	$BS(\mathcal{M}, (N_1 + 1)) = BS(\mathcal{M}, N_2)$	;
L25:	Block ≫	End	;
L05:	Ded ▷ L25 ≫	$\forall N_2 : (N_1 = N_2 \Rightarrow BS(\mathcal{M}, N_1) = BS(\mathcal{M}, N_2)) \Rightarrow (N_1 + 1) = N_2 \Rightarrow BS(\mathcal{M}, (N_1 + 1)) = BS(\mathcal{M}, N_2)$	;
L06:	Premise ≫	$\forall N_2 : (N_1 = N_2 \Rightarrow BS(\mathcal{M}, N_1) = BS(\mathcal{M}, N_2))$	;
L07:	MP ▷ L05 ▷ L06 ≫	$(N_1 + 1) = N_2 \Rightarrow BS(\mathcal{M}, (N_1 + 1)) = BS(\mathcal{M}, N_2)$	;

L26: Gen $\rhd$ L07 $\gg$ $\forall \mathrm{N}_2\colon ((\mathrm{N}_1 + 1) = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, (\mathrm{N}_1 + 1)) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2))$;
L27: Block $\gg$ End ;
L28: Arbitrary $\gg$ $\mathcal{M}, \mathrm{N}_1, \mathrm{N}_2$;
L29: Ded $\rhd$ L27 $\gg$ $\forall \mathrm{N}_2\colon (\mathrm{N}_1 = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, \mathrm{N}_1) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2)) \Rightarrow \forall \mathrm{N}_2\colon ((\mathrm{N}_1 + 1) = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, (\mathrm{N}_1 + 1)) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2))$ □

[SystemQ **lemma** SameBS(2): $\Pi \mathcal{M}, \mathrm{N}_1, \mathrm{N}_2\colon \mathrm{N}_1 = \mathrm{N}_2 \vdash \mathrm{BS}(\mathcal{M}, \mathrm{N}_1) = \mathrm{BS}(\mathcal{M}, \mathrm{N}$

SystemQ **proof of** SameBS(2):

L01: Arbitrary $\gg$ $\mathcal{M}, \mathrm{N}_1, \mathrm{N}_2$;
L02: Premise $\gg$ $\mathrm{N}_1 = \mathrm{N}_2$;
L03: SameBS(2)(Base) $\gg$ $\forall \mathrm{N}_2\colon (0 = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, 0) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2))$;
L04: SameBS(2)(Indu) $\gg$ $\forall \mathrm{N}_2\colon (\mathrm{N}_1 = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, \mathrm{N}_1) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2)) \Rightarrow \forall \mathrm{N}_2\colon ((\mathrm{N}_1 + 1) = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, (\mathrm{N}_1 + 1)) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2))$;
L05: Induction $\rhd$ L03 $\rhd$ L04 $\gg$ $\forall \mathrm{N}_2\colon (\mathrm{N}_1 = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, \mathrm{N}_1) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2))$;
L06: A4 @ N_2 $\rhd$ L05 $\gg$ $\mathrm{N}_1 = \mathrm{N}_2 \Rightarrow \mathrm{BS}(\mathcal{M}, \mathrm{N}_1) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2)$;
L07: MP $\rhd$ L06 $\rhd$ L02 $\gg$ $\mathrm{BS}(\mathcal{M}, \mathrm{N}_1) = \mathrm{BS}(\mathcal{M}, \mathrm{N}_2)$ □

[SystemQ **lemma** NegativeToLeft(Less)(1term): $\Pi \mathcal{X}, \mathcal{Y}\colon 0 < (\mathcal{X} - \mathcal{Y}) \vdash \mathcal{Y} < \mathcal{X}$]

SystemQ **proof of** NegativeToLeft(Less)(1term):

L01: Arbitrary $\gg$ $\mathcal{X}, \mathcal{Y}$;
L02: Premise $\gg$ $0 < (\mathcal{X} - \mathcal{Y})$;
L03: LessAddition $\rhd$ L02 $\gg$ $(0 + \mathcal{Y}) < ((\mathcal{X} - \mathcal{Y}) + \mathcal{Y})$;
L04: plus0Left $\gg$ $(0 + \mathcal{Y}) = \mathcal{Y}$;
L05: SubLessLeft $\rhd$ L04 $\rhd$ L03 $\gg$ $\mathcal{Y} < ((\mathcal{X} - \mathcal{Y}) + \mathcal{Y})$;
L06: Three2threeTerms $\gg$ $((\mathcal{X} - \mathcal{Y}) + \mathcal{Y}) = ((\mathcal{X} + \mathcal{Y}) - \mathcal{Y})$;
L07: $\mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y}$ $\gg$ $\mathcal{X} = ((\mathcal{X} + \mathcal{Y}) - \mathcal{Y})$;
L08: eqSymmetry $\rhd$ L07 $\gg$ $((\mathcal{X} + \mathcal{Y}) - \mathcal{Y}) = \mathcal{X}$;
L09: eqTransitivity $\rhd$ L06 $\rhd$ L08 $\gg$ $((\mathcal{X} - \mathcal{Y}) + \mathcal{Y}) = \mathcal{X}$;
L10: SubLessRight $\rhd$ L09 $\rhd$ L05 $\gg$ $\mathcal{Y} < \mathcal{X}$ □

[SystemQ **lemma** BS(+1): $\Pi \mathcal{M}, \mathcal{N}\colon \mathrm{BS}(\mathcal{M}, (\mathcal{N} + 1)) = (1/2(\exp)(((\mathcal{M} + \mathcal{N}) + 1)) + \mathrm{BS}(\mathcal{M}, \mathcal{N}))$]

SystemQ **proof of** BS(+1):

L01: Arbitrary $\gg$ $\mathcal{M}, \mathcal{N}$;
L02: (+1)IsPositive(N) $\gg$ $0 < (\mathcal{N} + 1)$;
L03: BSpositive $\rhd$ L02 $\gg$ $\mathrm{BS}(\mathcal{M}, (\mathcal{N} + 1)) = (1/2(\exp)((\mathcal{M} + ((\mathcal{N} + 1)))) + \mathrm{BS}(\mathcal{M}, ((\mathcal{N} + 1) - 1)))$;

L04: plusAssociativity $\gg$ $((\mathcal{M}+\mathcal{N})+1) = (\mathcal{M}+((\mathcal{N}+1)))$;
L05: SameExp $\rhd$ L04 $\gg$ $1/2(\exp)(((\mathcal{M}+\mathcal{N})+1)) = 1/2(\exp)((\mathcal{M}+((\mathcal{N}+1))))$;
L06: eqSymmetry $\rhd$ L05 $\gg$ $1/2(\exp)((\mathcal{M}+((\mathcal{N}+1)))) = 1/2(\exp)(((\mathcal{M}+\mathcal{N})+1))$;
L07: $\mathrm{x} = \mathrm{x}+\mathrm{y}-\mathrm{y}$ $\gg$ $\mathcal{N} = ((\mathcal{N}+1)-1)$;
L08: SameBS(2) $\rhd$ L07 $\gg$ $\mathrm{BS}(\mathcal{M},\mathcal{N}) = \mathrm{BS}(\mathcal{M},((\mathcal{N}+1)-1))$;
L09: eqSymmetry $\rhd$ L08 $\gg$ $\mathrm{BS}(\mathcal{M},((\mathcal{N}+1)-1)) = \mathrm{BS}(\mathcal{M},\mathcal{N})$;
L10: AddEquations $\rhd$ L06 $\rhd$ L09 $\gg$ $(1/2(\exp)((\mathcal{M}+((\mathcal{N}+1)))) + \mathrm{BS}(\mathcal{M},((\mathcal{N}+1)-1))) = (1/2(\exp)(((\mathcal{M}+\mathcal{N})+1)) + \mathrm{BS}(\mathcal{M},\mathcal{N}))$;
L11: eqTransitivity $\rhd$ L03 $\rhd$ L10 $\gg$ $\mathrm{BS}(\mathcal{M},(\mathcal{N}+1)) = (1/2(\exp)(((\mathcal{M}+\mathcal{N})+1)) + \mathrm{BS}(\mathcal{M},\mathcal{N}))$ □

[SystemQ **lemma** BSbound(Exact)(Base): $\Pi\mathcal{M}$: $(\mathrm{BS}((\mathcal{M}+1),0)+1/2(\exp)(((\mathcal{M}$ $1)+0))) = 1/2(\exp)\mathcal{M}$]

SystemQ **proof of** BSbound(Exact)(Base):

L01: Arbitrary $\gg$ $\mathcal{M}$;
L02: BSzero(Exact) $\gg$ $\mathrm{BS}((\mathcal{M}+1),0) = 1/2(\exp)((\mathcal{M}+1))$;
L03: Exp(+1) $\gg$ $1/2(\exp)((\mathcal{M}+1)) = (1/2 * 1/2(\exp)\mathcal{M})$;
L04: eqTransitivity $\rhd$ L02 $\rhd$ L03 $\gg$ $\mathrm{BS}((\mathcal{M}+1),0) = (1/2 * 1/2(\exp)\mathcal{M})$;
L05: plus0 $\gg$ $((\mathcal{M}+1)+0) = (\mathcal{M}+1)$;
L06: SameExp $\rhd$ L05 $\gg$ $1/2(\exp)(((\mathcal{M}+1)+0)) = 1/2(\exp)((\mathcal{M}+1))$;
L07: eqTransitivity $\rhd$ L06 $\rhd$ L03 $\gg$ $1/2(\exp)(((\mathcal{M}+1)+0)) = (1/2 * 1/2(\exp)\mathcal{M})$;
L08: AddEquations $\rhd$ L04 $\rhd$ L07 $\gg$ $(\mathrm{BS}((\mathcal{M}+1),0) + 1/2(\exp)(((\mathcal{M}+1)+0))) = ((1/2 * 1/2(\exp)\mathcal{M}) + (1/2 * 1/2(\exp)\mathcal{M}))$;
L09: TwoHalves $\gg$ $((1/2 * 1/2(\exp)\mathcal{M}) + (1/2 * 1/2(\exp)\mathcal{M})) = 1/2(\exp)\mathcal{M}$;
L10: eqTransitivity $\rhd$ L08 $\rhd$ L09 $\gg$ $(\mathrm{BS}((\mathcal{M}+1),0) + 1/2(\exp)(((\mathcal{M}+1)+0))) = 1/2(\exp)\mathcal{M}$ □

[SystemQ **lemma** BSbound(Exact)(Indu): $\Pi\mathcal{M},\mathcal{N}$: $(\mathrm{BS}((\mathcal{M}+1),\mathcal{N})+1/2(\exp)$ $1)+\mathcal{N}))) = 1/2(\exp)\mathcal{M} \Rightarrow (\mathrm{BS}((\mathcal{M}+1),(\mathcal{N}+1))+1/2(\exp)((((\mathcal{M}+1)+\mathcal{N})+1))) = 1/2(\exp)\mathcal{M}$]

SystemQ **proof of** BSbound(Exact)(Indu):

L01: Block $\gg$ Begin ;
L02: Arbitrary $\gg$ $\mathcal{M},\mathcal{N}$;

L03:	Premise $\gg$	$(\mathrm{BS}((\mathcal{M}+1),\mathcal{N})+1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N})))=1/2(\mathrm{exp})\mathcal{M}$	;
L04:	BS(+1) $\gg$	$\mathrm{BS}((\mathcal{M}+1),(\mathcal{N}+1))=(1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))+\mathrm{BS}((\mathcal{M}+1),\mathcal{N}))$	;
L05:	eqAddition $\triangleright$ L04 $\gg$	$(\mathrm{BS}((\mathcal{M}+1),(\mathcal{N}+1))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))=((1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))+\mathrm{BS}((\mathcal{M}+1),\mathcal{N}))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))$	;
L06:	plusCommutativity $\gg$	$(1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))+\mathrm{BS}((\mathcal{M}+1),\mathcal{N}))=(\mathrm{BS}((\mathcal{M}+1),\mathcal{N})+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))$	;
L07:	eqAddition $\triangleright$ L06 $\gg$	$((1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))+\mathrm{BS}((\mathcal{M}+1),\mathcal{N}))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))=((\mathrm{BS}((\mathcal{M}+1),\mathcal{N})+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))$	;
L08:	Exp(+1) $\gg$	$1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))=(1/2*1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N})))$	;
L09:	AddEquations $\triangleright$ L08 $\triangleright$ L08 $\gg$	$(1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))=((1/2*1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N})))+(1/2*1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N}))))$	;
L10:	TwoHalves $\gg$	$((1/2*1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N})))+(1/2*1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N}))))=1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N}))$	;
L11:	eqTransitivity $\triangleright$ L09 $\triangleright$ L10 $\gg$	$(1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))=1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N}))$	;
L12:	Three2twoTerms $\triangleright$ L11 $\gg$	$((\mathrm{BS}((\mathcal{M}+1),\mathcal{N})+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))=(\mathrm{BS}((\mathcal{M}+1),\mathcal{N})+1/2(\mathrm{exp})(((\mathcal{M}+1)+\mathcal{N})))$	;
L13:	eqTransitivity5 $\triangleright$ L05 $\triangleright$ L07 $\triangleright$ L12 $\triangleright$ L03 $\gg$	$(\mathrm{BS}((\mathcal{M}+1),(\mathcal{N}+1))+1/2(\mathrm{exp})((((\mathcal{M}+1)+\mathcal{N})+1)))=1/2(\mathrm{exp})\mathcal{M}$	;
L14:	Block $\gg$	End	;
L15:	Arbitrary $\gg$	$\mathcal{M},\mathcal{N}$	;

L16: Ded ▷ L14 ≫ $(BS((\mathcal{M}+1),\mathcal{N})+1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))) = 1/2(exp)\mathcal{M} \Rightarrow (BS((\mathcal{M}+1),(\mathcal{N}+1))+1/2(exp)((((\mathcal{M}+1)+\mathcal{N})+1))) = 1/2(exp)\mathcal{M}$ □

[SystemQ **lemma** BSbound(Exact): $\Pi\mathcal{M},\mathcal{N}$: $(BS((\mathcal{M}+1),\mathcal{N})+1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))) = 1/2(exp)\mathcal{M}$]

SystemQ **proof of** BSbound(Exact):

L01: Arbitrary ≫ $\mathcal{M},\mathcal{N}$;
L02: BSbound(Exact)(Base) ≫ $(BS((\mathcal{M}+1),0)+1/2(exp)(((\mathcal{M}+1)+0))) = 1/2(exp)\mathcal{M}$;
L03: BSbound(Exact)(Indu) ≫ $(BS((\mathcal{M}+1),\mathcal{N})+1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))) = 1/2(exp)\mathcal{M} \Rightarrow (BS((\mathcal{M}+1),(\mathcal{N}+1))+1/2(exp)((((\mathcal{M}+1)+\mathcal{N})+1))) = 1/2(exp)\mathcal{M}$;
L04: Induction ▷ L02 ▷ L03 ≫ $(BS((\mathcal{M}+1),\mathcal{N})+1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))) = 1/2(exp)\mathcal{M}$ □

[SystemQ **lemma** BSbound: $\Pi\mathcal{M},\mathcal{N}$: $BS((\mathcal{M}+1),\mathcal{N}) < 1/2(exp)\mathcal{M}$]

SystemQ **proof of** BSbound:

L01: Arbitrary ≫ $\mathcal{M},\mathcal{N}$;
L02: BSbound(Exact) ≫ $(BS((\mathcal{M}+1),\mathcal{N})+1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))) = 1/2(exp)\mathcal{M}$;
L03: plusCommutativity ≫ $(1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))+BS((\mathcal{M}+1),\mathcal{N})) = (BS((\mathcal{M}+1),\mathcal{N})+1/2(exp)(((\mathcal{M}+1)+\mathcal{N})))$;
L04: eqTransitivity ▷ L03 ▷ L02 ≫ $(1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))+BS((\mathcal{M}+1),\mathcal{N})) = 1/2(exp)\mathcal{M}$;
L05: PositiveToRight(Eq) ▷ L04 ≫ $1/2(exp)(((\mathcal{M}+1)+\mathcal{N})) = (1/2(exp)\mathcal{M} - BS((\mathcal{M}+1),\mathcal{N}))$;
L06: $0 < 1/2$ ≫ $0 < 1/2$;
L07: PositiveBase ▷ L06 ≫ $0 < 1/2(exp)(((\mathcal{M}+1)+\mathcal{N}))$;
L08: SubLessRight ▷ L05 ▷ L07 ≫ $0 < (1/2(exp)\mathcal{M} - BS((\mathcal{M}+1),\mathcal{N}))$;
L09: NegativeToLeft(Less)(1term) ▷ L08 ≫ $BS((\mathcal{M}+1),\mathcal{N}) < 1/2(exp)\mathcal{M}$ □

[SystemQ **lemma** SameSeries(NumDiff): $\Pi FX, FY, \mathcal{O}, \mathcal{P}, N_1, N_2$: $\mathcal{O} = \mathcal{P} \vdash N_1 = N_2 \vdash |(FX[\mathcal{O}] - FY[N_1])| = |(FX[\mathcal{P}] - FY[N_2])|$]

SystemQ **proof of** SameSeries(NumDiff):

L01: Arbitrary ≫ $FX, FY, \mathcal{O}, \mathcal{P}, N_1, N_2$;
L02: Premise ≫ $\mathcal{O} = \mathcal{P}$;

L03: Premise ≫ $N_1 = N_2$;
L04: SameSeries ⊳ L02 ≫ $FX[\mathcal{O}] = FX[\mathcal{P}]$;
L05: SameSeries ⊳ L03 ≫ $FY[N_1] = FY[N_2]$;
L06: EqNegated ⊳ L05 ≫ $(-uFY[N_1]) = (-uFY[N_2])$;
L07: AddEquations ⊳ L04 ⊳ L06 ≫ $(FX[\mathcal{O}] - FY[N_1]) = (FX[\mathcal{P}] - FY[N_2])$;
L08: SameNumerical ⊳ L07 ≫ $|(FX[\mathcal{O}] - FY[N_1])| = |(FX[\mathcal{P}] - FY[N_2])|$ □

[SystemQ **lemma** UStelescope(Zero)(Exact): $\Pi\mathcal{M}$: UStelescope$(\mathcal{M}, 0) = |(us[\mathcal{M}] - us[(\mathcal{M} + 1)])|$]

SystemQ **proof of** UStelescope(Zero)(Exact):

L01: Arbitrary ≫ $\mathcal{M}$;
L02: eqReflexivity ≫ $0 = 0$;
L03: UStelescope(Zero) ⊳ L02 ≫ UStelescope$(\mathcal{M}, 0) = |(us[\mathcal{M}] - us[(\mathcal{M} + 1)])|$ □

[SystemQ **lemma** SameTelescope(2)(Base): $\Pi\mathcal{M}, N_2$: $\forall N_2$: $(0 = N_2 \Rightarrow$ UStelescope$(\mathcal{M}, 0) =$ UStelescope$(\mathcal{M}, N_2))$]

SystemQ **proof of** SameTelescope(2)(Base):

L01: Block ≫ Begin ;
L02: Arbitrary ≫ $\mathcal{M}, N_2$;
L03: Premise ≫ $0 = N_2$;
L04: eqSymmetry ⊳ L03 ≫ $N_2 = 0$;
L05: UStelescope(Zero) ⊳ L04 ≫ UStelescope$(\mathcal{M}, N_2) = |(us[\mathcal{M}] - us[(\mathcal{M} + 1)])|$;
L06: eqSymmetry ⊳ L05 ≫ $|(us[\mathcal{M}] - us[(\mathcal{M} + 1)])| =$ UStelescope$(\mathcal{M}, N_2)$;
L07: UStelescope(Zero)(Exact) ≫ UStelescope$(\mathcal{M}, 0) = |(us[\mathcal{M}] - us[(\mathcal{M} + 1)])|$;
L08: eqTransitivity ⊳ L07 ⊳ L06 ≫ UStelescope$(\mathcal{M}, 0) =$ UStelescope$(\mathcal{M}, N_2)$;
L09: Block ≫ End ;
L10: Arbitrary ≫ $\mathcal{M}, N_2$;
L03: Ded ⊳ L09 ≫ $0 = N_2 \Rightarrow$ UStelescope$(\mathcal{M}, 0) =$ UStelescope$(\mathcal{M}, N_2)$;
L11: Gen ⊳ L03 ≫ $\forall N_2$: $(0 = N_2 \Rightarrow$ UStelescope$(\mathcal{M}, 0) =$ UStelescope$(\mathcal{M}, N_2))$ □

[SystemQ **lemma** SameTelescope(2)(Indu): $\Pi\mathcal{M}, N_1, N_2$: $\forall N_2$: $(N_1 = N_2 \Rightarrow$ UStelescope$(\mathcal{M}, N_1) =$ UStelescope$(\mathcal{M}, N_2)) \Rightarrow \forall N_2$: $((N_1 + 1) = N_2 \Rightarrow$ UStelescope$(\mathcal{M}, (N_1 + 1)) =$ UStelescope$(\mathcal{M}, N_2))$]

SystemQ **proof of** SameTelescope(2)(Indu):

L01: Block ≫ Begin ;
L02: Arbitrary ≫ $\mathcal{M}, N_1, N_2$;
L03: Block ≫ Begin ;
L04: Arbitrary ≫ $\mathcal{M}, N_1, N_2$;

L05:	Premise $\gg$	$\forall N_2\colon (N_1 = N_2 \Rightarrow \mathrm{UStelescope}(\mathcal{M}, N_1) = \mathrm{UStelescope}(\mathcal{M}, N_2))$	;
L06:	Premise $\gg$	$(N_1 + 1) = N_2$	;
L07:	$(+1)\mathrm{IsPositive}(N) \gg$	$0 < (N_1 + 1)$	;
L08:	$\mathrm{UStelescope}(\mathrm{Positive}) \rhd \mathrm{L07} \gg$	$\mathrm{UStelescope}(\mathcal{M}, (N_1 + 1)) = (\lvert(\mathrm{us}[(\mathcal{M} + ((N_1 + 1)))] - \mathrm{us}[(\mathcal{M} + (((N_1 + 1) + 1)))])\rvert + \mathrm{UStelescope}(\mathcal{M}, ((N_1 + 1) - 1)))$	;
L09:	$x = x + y - y \gg$	$N_1 = ((N_1 + 1) - 1)$	;
L10:	$\mathrm{A4}\,@((N_1 + 1) - 1) \rhd \mathrm{L05} \gg$	$N_1 = ((N_1 + 1) - 1) \Rightarrow \mathrm{UStelescope}(\mathcal{M}, N_1) = \mathrm{UStelescope}(\mathcal{M}, ((N_1 + 1) - 1))$	;
L11:	$\mathrm{MP} \rhd \mathrm{L10} \rhd \mathrm{L09} \gg$	$\mathrm{UStelescope}(\mathcal{M}, N_1) = \mathrm{UStelescope}(\mathcal{M}, ((N_1 + 1) - 1))$	;
L12:	$\mathrm{eqSymmetry} \rhd \mathrm{L11} \gg$	$\mathrm{UStelescope}(\mathcal{M}, ((N_1 + 1) - 1)) = \mathrm{UStelescope}(\mathcal{M}, N_1)$	;
L13:	$\mathrm{PositiveToRight}(\mathrm{Eq}) \rhd \mathrm{L06} \gg$	$N_1 = (N_2 - 1)$	;
L14:	$\mathrm{A4}\,@(N_2 - 1) \rhd \mathrm{L05} \gg$	$N_1 = (N_2 - 1) \Rightarrow \mathrm{UStelescope}(\mathcal{M}, N_1) = \mathrm{UStelescope}(\mathcal{M}, (N_2 - 1))$;	
L15:	$\mathrm{MP} \rhd \mathrm{L14} \rhd \mathrm{L13} \gg$	$\mathrm{UStelescope}(\mathcal{M}, N_1) = \mathrm{UStelescope}(\mathcal{M}, (N_2 - 1))$;	
L16:	$\mathrm{eqTransitivity} \rhd \mathrm{L12} \rhd \mathrm{L15} \gg$	$\mathrm{UStelescope}(\mathcal{M}, ((N_1 + 1) - 1)) = \mathrm{UStelescope}(\mathcal{M}, (N_2 - 1))$	;
L17:	$\mathrm{EqAdditionLeft} \rhd \mathrm{L06} \gg$	$(\mathcal{M} + ((N_1 + 1))) = (\mathcal{M} + N_2)$	;
L18:	$\mathrm{eqAddition} \rhd \mathrm{L06} \gg$	$((N_1 + 1) + 1) = (N_2 + 1)$	;
L19:	$\mathrm{EqAdditionLeft} \rhd \mathrm{L18} \gg$	$(\mathcal{M} + (((N_1 + 1) + 1))) = (\mathcal{M} + ((N_2 + 1)))$	;
L20:	$\mathrm{SameSeries}(\mathrm{NumDiff}) \rhd \mathrm{L17} \rhd \mathrm{L19} \gg$	$\lvert(\mathrm{us}[(\mathcal{M} + ((N_1 + 1)))] - \mathrm{us}[(\mathcal{M} + (((N_1 + 1) + 1)))])\rvert = \lvert(\mathrm{us}[(\mathcal{M} + N_2)] - \mathrm{us}[(\mathcal{M} + ((N_2 + 1)))])\rvert$	;
L21:	$\mathrm{AddEquations} \rhd \mathrm{L20} \rhd \mathrm{L16} \gg$	$(\lvert(\mathrm{us}[(\mathcal{M} + ((N_1 + 1)))] - \mathrm{us}[(\mathcal{M} + (((N_1 + 1) + 1)))])\rvert + \mathrm{UStelescope}(\mathcal{M}, ((N_1 + 1) - 1))) = (\lvert(\mathrm{us}[(\mathcal{M} + N_2)] - \mathrm{us}[(\mathcal{M} + ((N_2 + 1)))])\rvert + \mathrm{UStelescope}(\mathcal{M}, (N_2 - 1)))$	;
L22:	$\mathrm{SubLessRight} \rhd \mathrm{L06} \rhd \mathrm{L07} \gg$	$0 < N_2$	;
L23:	$\mathrm{UStelescope}(\mathrm{Positive}) \rhd \mathrm{L22} \gg$	$\mathrm{UStelescope}(\mathcal{M}, N_2) = (\lvert(\mathrm{us}[(\mathcal{M} + N_2)] - \mathrm{us}[(\mathcal{M} + ((N_2 + 1)))])\rvert + \mathrm{UStelescope}(\mathcal{M}, (N_2 - 1)))$	;

L24: eqSymmetry ▷ L23 ≫ $(|(us[(\mathcal{M}+N_2)]-us[(\mathcal{M}+((N_2+1)))])| + UStelescope(\mathcal{M},(N_2-1))) = UStelescope(\mathcal{M},N_2)$;

L25: eqTransitivity4 ▷ L08 ▷ L21 ▷ L24 ≫ $UStelescope(\mathcal{M},(N_1+1)) = UStelescope(\mathcal{M},N_2)$;

L26: Block ≫ End ;

L05: Ded ▷ L26 ≫ $\forall N_2\colon (N_1 = N_2 \Rightarrow UStelescope(\mathcal{M},N_1) = UStelescope(\mathcal{M},N_2)) \Rightarrow (N_1+1) = N_2 \Rightarrow UStelescope(\mathcal{M},(N_1+1)) = UStelescope(\mathcal{M},N_2)$;

L06: Premise ≫ $\forall N_2\colon (N_1 = N_2 \Rightarrow UStelescope(\mathcal{M},N_1) = UStelescope(\mathcal{M},N_2))$;

L07: MP ▷ L05 ▷ L06 ≫ $(N_1+1) = N_2 \Rightarrow UStelescope(\mathcal{M},(N_1+1)) = UStelescope(\mathcal{M},N_2)$;

L27: Gen ▷ L07 ≫ $\forall N_2\colon ((N_1+1) = N_2 \Rightarrow UStelescope(\mathcal{M},(N_1+1)) = UStelescope(\mathcal{M},N_2))$;

L28: Block ≫ End ;

L29: Arbitrary ≫ $\mathcal{M},N_1,N_2$;

L30: Ded ▷ L28 ≫ $\forall N_2\colon (N_1 = N_2 \Rightarrow UStelescope(\mathcal{M},N_1) = UStelescope(\mathcal{M},N_2)) \Rightarrow \forall N_2\colon ((N_1+1) = N_2 \Rightarrow UStelescope(\mathcal{M},(N_1+1)) = UStelescope(\mathcal{M},N_2))$ □

[SystemQ **lemma** SameTelescope(2): $\Pi \mathcal{M},N_1,N_2\colon N_1 = N_2 \vdash UStelescope(\mathcal{M},$ $UStelescope(\mathcal{M},N_2)$]

SystemQ **proof of** SameTelescope(2):

L01: Arbitrary ≫ $\mathcal{M},N_1,N_2$;

L02: Premise ≫ $N_1 = N_2$;

L03: SameTelescope(2)(Base) ≫ $\forall N_2\colon (0 = N_2 \Rightarrow UStelescope(\mathcal{M},0) = UStelescope(\mathcal{M},N_2))$;

L04: SameTelescope(2)(Indu) ≫ $\forall N_2\colon (N_1 = N_2 \Rightarrow UStelescope(\mathcal{M},N_1) = UStelescope(\mathcal{M},N_2)) \Rightarrow \forall N_2\colon ((N_1+1) = N_2 \Rightarrow UStelescope(\mathcal{M},(N_1+1)) = UStelescope(\mathcal{M},N_2))$;

L05: Induction ▷ L03 ▷ L04 ≫ $\forall N_2\colon (N_1 = N_2 \Rightarrow UStelescope(\mathcal{M},N_1) = UStelescope(\mathcal{M},N_2))$;

L06: A4 @ $N_2 \triangleright$ L05 $\gg$ $N_1 = N_2 \Rightarrow \text{UStelescope}(\mathcal{M}, N_1) = \text{UStelescope}(\mathcal{M}, N_2)$;

L07: MP $\triangleright$ L06 $\triangleright$ L02 $\gg$ $\text{UStelescope}(\mathcal{M}, N_1) = \text{UStelescope}(\mathcal{M}, N_2)$ $\Box$

[SystemQ **lemma** TelescopeNumerical(Base): $\Pi\mathcal{M}$: $|(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((0 + 1)))])| <= \text{UStelescope}(\mathcal{M}, 0)$]

SystemQ **proof of** TelescopeNumerical(Base):

L01: Arbitrary $\gg$ $\mathcal{M}$;

L02: eqReflexivity $\gg$ $0 = 0$;

L03: UStelescope(Zero) $\triangleright$ L02 $\gg$ $\text{UStelescope}(\mathcal{M}, 0) = |(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + 1)])|$;

L04: eqReflexivity $\gg$ $\mathcal{M} = \mathcal{M}$;

L05: plus0Left $\gg$ $(0 + 1) = 1$;

L06: EqAdditionLeft $\triangleright$ L05 $\gg$ $(\mathcal{M} + ((0 + 1))) = (\mathcal{M} + 1)$;

L07: eqSymmetry $\triangleright$ L06 $\gg$ $(\mathcal{M} + 1) = (\mathcal{M} + ((0 + 1)))$;

L08: SameSeries(NumDiff) $\triangleright$ L04 $\triangleright$ L07 $\gg$ $|(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + 1)])| = |(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((0 + 1)))])|$;

L09: eqTransitivity $\triangleright$ L03 $\triangleright$ L08 $\gg$ $\text{UStelescope}(\mathcal{M}, 0) = |(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((0 + 1)))])|$;

L10: eqSymmetry $\triangleright$ L09 $\gg$ $|(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((0+1)))])| = \text{UStelescope}(\mathcal{M}, 0)$;

L11: eqLeq $\triangleright$ L10 $\gg$ $|(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((0 + 1)))])| <= \text{UStelescope}(\mathcal{M}, 0)$ $\Box$

[SystemQ **lemma** TelescopeNumerical(Indu): $\Pi\mathcal{M}, \mathcal{N}$: $|(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| <= \text{UStelescope}(\mathcal{M}, \mathcal{N}) \Rightarrow |(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| <= \text{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$]

SystemQ **proof of** TelescopeNumerical(Indu):

L01: Block $\gg$ Begin ;

L02: Arbitrary $\gg$ $\mathcal{M}, \mathcal{N}$;

L03: Premise $\gg$ $|(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| <= \text{UStelescope}(\mathcal{M}, \mathcal{N})$;

L04: (+1)IsPositive(N) $\gg$ $0 < (\mathcal{N} + 1)$;

L05: UStelescope(Positive) $\triangleright$ L04 $\gg$ $\text{UStelescope}(\mathcal{M}, (\mathcal{N} + 1)) = (|(\text{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \text{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| + \text{UStelescope}(\mathcal{M}, ((\mathcal{N} + 1) - 1)))$;

L06: $x = x + y - y$ $\gg$ $\mathcal{N} = ((\mathcal{N} + 1) - 1)$;

L07: eqSymmetry $\triangleright$ L06 $\gg$ $((\mathcal{N} + 1) - 1) = \mathcal{N}$;

L08: SameTelescope(2) $\triangleright$ L07 $\gg$ $\text{UStelescope}(\mathcal{M}, ((\mathcal{N}+1)-1)) = \text{UStelescope}(\mathcal{M}, \mathcal{N})$;

L09: EqAdditionLeft $\triangleright$ L08 $\gg$ $(|(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| + \mathrm{UStelescope}(\mathcal{M}, ((\mathcal{N} + 1) - 1))) = (|(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| + \mathrm{UStelescope}(\mathcal{M}, \mathcal{N}))$;

L10: eqTransitivity $\triangleright$ L05 $\triangleright$ L09 $\gg$ $\mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1)) = (|(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| + \mathrm{UStelescope}(\mathcal{M}, \mathcal{N}))$;

L11: eqSymmetry $\triangleright$ L10 $\gg$ $(|(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| + \mathrm{UStelescope}(\mathcal{M}, \mathcal{N})) = \mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$;

L12: LeqAdditionLeft $\triangleright$ L03 $\gg$ $(|(\mathrm{us}[(\mathcal{M}+((\mathcal{N}+1)))]-\mathrm{us}[(\mathcal{M}+(((\mathcal{N} + 1) + 1)))])| + |(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])|) <= (|(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| + \mathrm{UStelescope}(\mathcal{M}, \mathcal{N}))$;

L13: subLeqRight $\triangleright$ L11 $\triangleright$ L12 $\gg$ $(|(\mathrm{us}[(\mathcal{M}+((\mathcal{N}+1)))]-\mathrm{us}[(\mathcal{M}+(((\mathcal{N} + 1) + 1)))])| + |(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])|) <= \mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$;

L14: plusCommutativity $\gg$ $(|(\mathrm{us}[(\mathcal{M}+((\mathcal{N}+1)))]-\mathrm{us}[(\mathcal{M}+(((\mathcal{N} + 1) + 1)))])| + |(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])|) = (|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| + |(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])|)$;

L15: subLeqLeft $\triangleright$ L14 $\triangleright$ L13 $\gg$ $(|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| + |(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])|) <= \mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$;

L16: insertMiddleTerm(Numerical) $\gg$ $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| <= (|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| + |(\mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])|)$;

L17: leqTransitivity $\triangleright$ L16 $\triangleright$ L15 $\gg$ $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$;

L18: Block $\gg$ End ;

L19: Arbitrary $\gg$ $\mathcal{M}, \mathcal{N}$;

L20: Ded $\rhd$ L18 $\gg$ $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, \mathcal{N}) \Rightarrow |(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$ □

[SystemQ **lemma** TelescopeNumerical: $\Pi\mathcal{M}, \mathcal{N}$: $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, \mathcal{N})$]

SystemQ **proof of** TelescopeNumerical:

L01: Arbitrary $\gg$ $\mathcal{M}, \mathcal{N}$;

L02: TelescopeNumerical(Base) $\gg$ $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((0 + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, 0)$;

L03: TelescopeNumerical(Indu) $\gg$ $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, \mathcal{N}) \Rightarrow |(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + (((\mathcal{N} + 1) + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, (\mathcal{N} + 1))$;

L04: Induction $\rhd$ L02 $\rhd$ L03 $\gg$ $|(\mathrm{us}[\mathcal{M}] - \mathrm{us}[(\mathcal{M} + ((\mathcal{N} + 1)))])| <= \mathrm{UStelescope}(\mathcal{M}, \mathcal{N})$ □

————————(21.10.06)

[SystemQ **lemma** EqAdditionLeft(R): ΠFX, FY, FZ: R(FX) == R(FY) $\vdash$ R(FZ) + +R(FX) == R(FZ) + +R(FY)]

SystemQ **proof of** EqAdditionLeft(R):

L01: Arbitrary $\gg$ FX, FY, FZ ;

L02: Premise $\gg$ R(FX) == R(FY) ;

L03: EqAddition(R) $\rhd$ L02 $\gg$ R(FX) + +R(FZ) == R(FY) + +R(FZ) ;

L04: PlusCommutativity(R) $\gg$ R(FZ) + +R(FX) == R(FX) + +R(FZ) ;

L05: PlusCommutativity(R) $\gg$ R(FY) + +R(FZ) == R(FZ) + +R(FY) ;

L06: eqTransitivity4 $\rhd$ L04 $\rhd$ L03 $\rhd$ L05 $\gg$ R(FZ) + +R(FX) == R(FZ) + +R(FY) □

[SystemQ **lemma** x = x+(y−y)(R): ΠFX, FY: R(FX) = R(FX)++(R(FY)++(− − R(FY)))]

SystemQ **proof of** x = x + (y − y)(R):

L01: Arbitrary $\gg$ FX, FY ;

L02: Plus0(R) $\gg$ R(FX) + +00 == R(FX) ;

L03: ==Symmetry $\rhd$ L02 $\gg$ R(FX) == R(FX) + +00 ;

L04: Negative(R) $\gg$ R(FY) + +(− − R(FY)) == 00 ;

L05: ==Symmetry $\rhd$ L04 $\gg$ 00 == R(FY) + +(− − R(FY)) ;

L06:	EqAdditionLeft(R) ▷ L05 ≫	R(FX) + +00 == R(FX) + +(R(FY) + +(− − R(FY)))	;
L07:	eqTransitivity ▷ L03 ▷ L06 ≫	R(FX) == R(FX)++(R(FY)+ +(− − R(FY)))	□

[SystemQ **lemma** x = x + y − y(R): ΠFX, FY: R(FX) = R(FX) + +R(FY) + +(− − R(FY))]

SystemQ **proof of** x = x + y − y(R):

L01:	Arbitrary ≫	FX, FY	;
L02:	x = x + (y − y)(R) ≫	R(FX) == R(FX)++(R(FY)+ +(− − R(FY)))	;
L03:	PlusAssociativity(R) ≫	R(FX) + +R(FY) + +(− − R(FY)) == R(FX) + +(R(FY) + +(− − R(FY)))	;
L04:	==Symmetry ▷ L03 ≫	R(FX) + +(R(FY) + +(− − R(FY))) == R(FX) + +R(FY) + +(− − R(FY))	;
L05:	eqTransitivity ▷ L02 ▷ L04 ≫	R(FX) == R(FX) + +R(FY) + +(− − R(FY))	□

———-(22.10.06)

[SystemQ **lemma** Three2twoTerms(R): ΠFX, FY, FZ, FU: R(FY)++R(FZ) == R(FU) ⊢ R(FX) + +R(FY) + +R(FZ) == R(FX) + +R(FU)]

SystemQ **proof of** Three2twoTerms(R):

L01:	Arbitrary ≫	FX, FY, FZ, FU	;
L02:	Premise ≫	R(FY) + +R(FZ) == R(FU)	;
L03:	EqAdditionLeft(R) ▷ L02 ≫	R(FX) + +(R(FY) + +R(FZ)) == R(FX) + +R(FU)	;
L04:	PlusAssociativity(R) ≫	R(FX)++R(FY)++R(FZ) == R(FX) + +(R(FY) + +R(FZ))	;
L05:	eqTransitivity ▷ L04 ▷ L03 ≫	R(FX)++R(FY)++R(FZ) == R(FX) + +R(FU)	□

[SystemQ **lemma** PositiveToRight(Less)(R): ΠFX, FY, FZ: R(FX)++R(FY) < R(FZ) ⊢ R(FX) << R(FZ) + +(− − R(FY))]

SystemQ **proof of** PositiveToRight(Less)(R):

L01:	Arbitrary ≫	FX, FY, FZ	;
L02:	Premise ≫	R(FX) + +R(FY) << R(FZ)	;
L03:	lessAddition(R) ▷ L02 ≫	R(FX) + +R(FY) + +(− − R(FY)) << R(FZ) + +(− − R(FY))	;
L04:	x = x + y − y(R) ≫	R(FX) == R(FX) + +R(FY) + +(− − R(FY))	;
L05:	==Symmetry ▷ L04 ≫	R(FX) + +R(FY) + +(− − R(FY)) == R(FX)	;
L06:	SubLessLeft(R) ▷ L05 ▷ L03 ≫	R(FX) << R(FZ) + +(− − R(FY))	□

[SystemQ **lemma** Three2threeTerms(R): ΠFX, FY, FZ: R(FX) + +R(FY) + +R(FZ) == R(FX) + +R(FZ) + +R(FY)]

SystemQ **proof of** Three2threeTerms(R):

L01: Arbitrary $\gg$ FX, FY, FZ ;

L02: PlusCommutativity(R) $\gg$ R(FY) + +R(FZ) == R(FZ) + +R(FY) ;

L03: Three2twoTerms(R) $\rhd$ L02 $\gg$ R(FX)++R(FY)++R(FZ) == R(FX) + +(R(FZ) + +R(FY)) ;

L04: PlusAssociativity(R) $\gg$ R(FX)++R(FZ)++R(FY) == R(FX) + +(R(FZ) + +R(FY)) ;

L05: ==Symmetry $\rhd$ L04 $\gg$ R(FX) + +(R(FZ) + +R(FY)) == R(FX) + +R(FZ) + +R(FY) ;

L06: eqTransitivity $\rhd$ L03 $\rhd$ L05 $\gg$ R(FX)++R(FY)++R(FZ) == R(FX) + +R(FZ) + +R(FY) □

———(22.10.06)

[SystemQ **lemma** Plus0Left(R): ΠFX: 00 + +R(FX) == R(FX)]

SystemQ **proof of** Plus0Left(R):

L01: Arbitrary $\gg$ FX ;

L02: Plus0(R) $\gg$ R(FX) + +00 == R(FX) ;

L03: PlusCommutativity(R) $\gg$ 00 + +R(FX) == R(FX) + +00 ;

L04: eqTransitivity $\rhd$ L03 $\rhd$ L02 $\gg$ 00 + +R(FX) == R(FX) □

[SystemQ **lemma** PositiveToRight(Eq)(R): ΠFX, FY, FZ: R(FX)++R(FY) == R(FZ) ⊢ R(FX) == R(FZ) + +(− − R(FY))]

SystemQ **proof of** PositiveToRight(Eq)(R):

L01: Arbitrary $\gg$ FX, FY, FZ ;

L02: Premise $\gg$ R(FX) + +R(FY) == R(FZ) ;

L03: EqAddition(R) $\rhd$ L02 $\gg$ R(FX) + +R(FY) + +(− − R(FY)) == R(FZ) + +(− − R(FY)) ;

L04: x = x + y − y(R) $\gg$ R(FX) == R(FX) + +R(FY) + +(− − R(FY)) ;

L05: eqTransitivity $\rhd$ L04 $\rhd$ L03 $\gg$ R(FX) == R(FZ) + +(− − R(FY)) □

[SystemQ **lemma** SubtractEquations(R): ΠFX, FY, FZ, FU: R(FX)++R(FZ) = R(FY) + +R(FU) ⊢ R(FZ) == R(FU) ⊢ R(FX) == R(FY)]

SystemQ **proof of** SubtractEquations(R):

L01: Arbitrary $\gg$ FX, FY, FZ, FU ;

L02: Premise $\gg$ R(FX) + +R(FZ) == R(FY) + +R(FU) ;

L03: Premise $\gg$ R(FZ) == R(FU) ;

L04: EqAddition(R) $\rhd$ L02 $\gg$ R(FX) + +R(FZ) + +(− − R(FZ)) == R(FY) + +R(FU) + +(− − R(FZ)) ;

L05: Plus0Left(R) $\gg$ 00 + +R(FZ) == R(FZ) ;

L06: eqTransitivity $\rhd$ L05 $\rhd$ L03 $\gg$ 00 + +R(FZ) == R(FU) ;

L07: PositiveToRight(Eq)(R) $\rhd$ L06 $\gg$ 00 == R(FU) + +(− − R(FZ)) ;

L08:	==Symmetry ⊳ L07 ≫	R(FU) + +(− − R(FZ)) == 00	;
L09:	EqAdditionLeft(R) ⊳ L08 ≫	R(FY) + +(R(FU) + +(− − R(FZ))) == R(FY) + +00	;
L10:	PlusAssociativity(R) ≫	R(FY) + +R(FU) + +(− − R(FZ)) == R(FY) + +(R(FU) + +(− − R(FZ)))	;
L11:	Plus0(R) ≫	R(FY) + +00 == R(FY)	;
L12:	eqTransitivity4 ⊳ L10 ⊳ L09 ⊳ L11 ≫	R(FY) + +R(FU) + +(− − R(FZ)) == R(FY)	;
L13:	x = x + y − y(R) ≫	R(FX) == R(FX) + +R(FZ) + +(− − R(FZ))	;
L14:	eqTransitivity4 ⊳ L13 ⊳ L04 ⊳ L12 ≫	R(FX) == R(FY)	□

[SystemQ **lemma** NeqAddition(R): ΠFX, FY, FZ: R(FX) ≠ R(FY) ⊢ R(FX)+ +R(FZ) ≠ R(FY) + +R(FZ)]

SystemQ **proof of** NeqAddition(R):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	FX, FY, FZ	;
L03:	Premise ≫	R(FX) ≠ R(FY)	;
L04:	Premise ≫	R(FX) + +R(FZ) == R(FY) + +R(FZ)	;
L05:	==Reflexivity ≫	R(FZ) == R(FZ)	;
L06:	SubtractEquations(R) ⊳ L04 ⊳ L05 ≫	R(FX) == R(FY)	;
L07:	FromContradiction ⊳ L06 ⊳ L03 ≫	R(FX) + +R(FZ) ≠ R(FY) + +R(FZ)	;
L08:	Block ≫	End	;
L09:	Arbitrary ≫	FX, FY, FZ	;
L10:	Ded ⊳ L08 ≫	R(FX) ≠ R(FY) ⇒ R(FX) + +R(FZ) == R(FY) + +R(FZ) ⇒ R(FX) + +R(FZ) ≠ R(FY) + +R(FZ)	;
L11:	Premise ≫	R(FX) ≠ R(FY)	;
L12:	MP ⊳ L10 ⊳ L11 ≫	R(FX) + +R(FZ) == R(FY) + +R(FZ) ⇒ R(FX) + +R(FZ) ≠ R(FY) + +R(FZ)	;
L13:	prop lemma imply negation ⊳ L12 ≫	R(FX) + +R(FZ) ≠ R(FY) + +R(FZ)	□

————(22.10.06)

[SystemQ **lemma** PositiveToRight(Less)(1term)(R): ΠFX, FY: R(FX) << R(F 00 << R(FY) + +(− − R(FX))]

SystemQ **proof of** PositiveToRight(Less)(1term)(R):

L01:	Arbitrary ≫	FX, FY	;

L02:	Premise $\gg$	R(FX) << R(FY)	;
L03:	Plus0Left(R) $\gg$	00 + +R(FX) == R(FX)	;
L04:	==Symmetry $\rhd$ L03 $\gg$	R(FX) == 00 + +R(FX)	;
L05:	SubLessLeft(R) $\rhd$ L04 $\rhd$ L02 $\gg$	00 + +R(FX) << R(FY)	;
L06:	PositiveToRight(Less)(R) $\rhd$ L05 $\gg$	00 << R(FY) + +(− − R(FX))	$\square$

————(22.10.06)

[SystemQ **lemma** To!! ==: $\Pi\mathcal{M},\mathcal{N},\epsilon$, FX, FY: $\dot{\neg}$ (SF(FX, FY))n $\vdash$ R(FX)!! == R(FY)]

SystemQ **proof of** To!! ==:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{M},\mathcal{N},\epsilon$, FX, FY	;
L03:	Premise $\gg$	R(FX) == R(FY)	;
L04:	From == $\rhd$L03 $\gg$	SF(FX, FY)	;
L05:	Block $\gg$	End	;
L06:	Arbitrary $\gg$	$\mathcal{M},\mathcal{N},\epsilon$, FX, FY	;
L03:	Ded $\rhd$ L05 $\gg$	R(FX) == R(FY) $\Rightarrow$ SF(FX, FY)	;
L07:	Premise $\gg$	$\dot{\neg}$ (SF(FX, FY))n	;
L08:	MT $\rhd$ L03 $\rhd$ L07 $\gg$	R(FX)!! == R(FY)	$\square$

[SystemQ **lemma** SwitchTerms(x <= y − z): $\Pi\mathcal{X},\mathcal{Y},\mathcal{Z}$: $\mathcal{X}$ <= ($\mathcal{Y}$ − $\mathcal{Z}$) $\vdash$ $\mathcal{Z}$ <= ($\mathcal{Y}$ − $\mathcal{X}$)]

SystemQ **proof of** SwitchTerms(x <= y − z):

L01:	Arbitrary $\gg$	$\mathcal{X},\mathcal{Y},\mathcal{Z}$	;
L02:	Premise $\gg$	$\mathcal{X}$ <= ($\mathcal{Y}$ − $\mathcal{Z}$)	;
L03:	negativeToLeft(Leq) $\rhd$ L02 $\gg$	($\mathcal{X}$ + $\mathcal{Z}$) <= $\mathcal{Y}$	;
L04:	plusCommutativity $\gg$	($\mathcal{X}$ + $\mathcal{Z}$) = ($\mathcal{Z}$ + $\mathcal{X}$)	;
L05:	subLeqLeft $\rhd$ L04 $\rhd$ L03 $\gg$	($\mathcal{Z}$ + $\mathcal{X}$) <= $\mathcal{Y}$	;
L06:	PositiveToRight(Leq) $\rhd$ L05 $\gg$	$\mathcal{Z}$ <= ($\mathcal{Y}$ − $\mathcal{X}$)	$\square$

[SystemQ **lemma** LessNeq(F)(Helper): $\Pi\mathcal{M},\mathcal{N},\epsilon$, FX, FY: $\forall\mathcal{M}$: 0 < ϵ $\dot{\wedge}$ ($\mathcal{N}$ <= $\mathcal{M}$ $\Rightarrow$ FX[$\mathcal{M}$] <= (FY[$\mathcal{M}$] − ϵ)) $\Rightarrow$ $\dot{\neg}$((0 < ϵ $\Rightarrow$ $\mathcal{N}$ <= $\mathcal{N}$ $\Rightarrow$ \|(FX[$\mathcal{N}$] − FY[$\mathcal{N}$])\| < ϵ))n]

SystemQ **proof of** LessNeq(F)(Helper):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{M},\mathcal{N},\epsilon$, FX, FY	;
L03:	Premise $\gg$	$\forall\mathcal{M}$: 0 < ϵ $\dot{\wedge}$ ($\mathcal{N}$ <= $\mathcal{M}$ $\Rightarrow$ FX[$\mathcal{M}$] <= (FY[$\mathcal{M}$] − ϵ))	;
L04:	A4 @ $\mathcal{N}$ $\rhd$ L03 $\gg$	0 < ϵ $\dot{\wedge}$ ($\mathcal{N}$ <= $\mathcal{N}$ $\Rightarrow$ FX[$\mathcal{N}$] <= (FY[$\mathcal{N}$] − ϵ))	;
L05:	FirstConjunct $\rhd$ L04 $\gg$	0 < ϵ	;
L06:	SecondConjunct $\rhd$ L04 $\gg$	$\mathcal{N}$ <= $\mathcal{N}$ $\Rightarrow$ FX[$\mathcal{N}$] <= (FY[$\mathcal{N}$] − ϵ)	;
L07:	leqReflexivity $\gg$	$\mathcal{N}$ <= $\mathcal{N}$	;
L08:	MP $\rhd$ L06 $\rhd$ L07 $\gg$	FX[$\mathcal{N}$] <= (FY[$\mathcal{N}$] − ϵ)	;
L09:	SwitchTerms(x <= y − z) $\rhd$ L08 $\gg$	ϵ <= (FY[$\mathcal{N}$] − FX[$\mathcal{N}$])	;

L10: $x <= |x| \gg$ $(FY[\mathcal{N}] - FX[\mathcal{N}]) <= |(FY[\mathcal{N}] - FX[\mathcal{N}])|$;

L11: leqTransitivity $\triangleright$ L09 $\triangleright$ L10 $\gg$ $\epsilon <= |(FY[\mathcal{N}] - FX[\mathcal{N}])|$;

L12: NumericalDifference $\gg$ $|(FY[\mathcal{N}] - FX[\mathcal{N}])| = |(FX[\mathcal{N}] - FY[\mathcal{N}])|$;

L13: subLeqRight $\triangleright$ L12 $\triangleright$ L11 $\gg$ $\epsilon <= |(FX[\mathcal{N}] - FY[\mathcal{N}])|$;

L14: toNotLess $\triangleright$ L13 $\gg$ $\dot{\neg}\,(|(FX[\mathcal{N}] - FY[\mathcal{N}])| < \epsilon)n$;

L15: ToNegatedDoubleImply $\triangleright$ L05 $\triangleright$ L07 $\triangleright$ L14 $\gg$ $\dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{N} \Rightarrow |(FX[\mathcal{N}] - FY[\mathcal{N}])| < \epsilon))n$;

L16: Block $\gg$ End ;

L17: Arbitrary $\gg$ $\mathcal{M}, \mathcal{N}, \epsilon, FX, FY$;

L18: Ded $\triangleright$ L16 $\gg$ $\forall \mathcal{M}\colon 0 < \epsilon \mathbin{\dot{\wedge}} (\mathcal{N} <= \mathcal{M} \Rightarrow FX[\mathcal{M}] <= (FY[\mathcal{M}] - \epsilon)) \Rightarrow \dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{N} \Rightarrow |(FX[\mathcal{N}] - FY[\mathcal{N}])| < \epsilon))n$ □

[SystemQ **lemma** LessNeq(F): $\Pi \mathcal{M}, \mathcal{N}, \epsilon, FX, FY\colon FX <_{\mathrm{f}} FY \vdash \dot{\neg}\,(SF(FX, FY)$

SystemQ **proof of** LessNeq(F):

L01: Arbitrary $\gg$ $\mathcal{M}, \mathcal{N}, \epsilon, FX, FY$;

L02: Premise $\gg$ $FX <_{\mathrm{f}} FY$;

L03: Repetition $\triangleright$ L02 $\gg$ $\exists (EPob)\colon \exists n\colon \forall m\colon 0 < (EPob) \mathbin{\dot{\wedge}} (n <= m \Rightarrow FX[m] <= (FY[m] - (EPob)))$;

L04: Ded $\triangleright$ L03 $\gg$ $\exists \epsilon\colon \exists \mathcal{N}\colon \forall \mathcal{M}\colon 0 < \epsilon \mathbin{\dot{\wedge}} (\mathcal{N} <= \mathcal{M} \Rightarrow FX[\mathcal{M}] <= (FY[\mathcal{M}] - \epsilon))$;

L05: LessNeq(F)(Helper) $\gg$ $\forall \mathcal{M}\colon 0 < \epsilon \mathbin{\dot{\wedge}} (\mathcal{N} <= \mathcal{M} \Rightarrow FX[\mathcal{M}] <= (FY[\mathcal{M}] - \epsilon)) \Rightarrow \dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{N} \Rightarrow |(FX[\mathcal{N}] - FY[\mathcal{N}])| < \epsilon))n$;

L06: TwiceExistMP $\triangleright$ L05 $\triangleright$ L04 $\gg$ $\dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{N} \Rightarrow |(FX[\mathcal{N}] - FY[\mathcal{N}])| < \epsilon))n$;

L07: IntroExist @ $\mathcal{N}$ $\triangleright$ L06 $\gg$ $\exists \mathcal{M}\colon \dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{M} \Rightarrow |(FX[\mathcal{M}] - FY[\mathcal{M}])| < \epsilon))n$;

L08: Gen $\triangleright$ L07 $\gg$ $\forall \mathcal{N}\colon \exists \mathcal{M}\colon \dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{M} \Rightarrow |(FX[\mathcal{M}] - FY[\mathcal{M}])| < \epsilon))n$;

L09: IntroExist @ ϵ $\triangleright$ L08 $\gg$ $\exists \epsilon\colon \forall \mathcal{N}\colon \exists \mathcal{M}\colon \dot{\neg}\,((0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{M} \Rightarrow |(FX[\mathcal{M}] - FY[\mathcal{M}])| < \epsilon))n$;

L10: ToNegatedAEA $\triangleright$ L09 $\gg$ $\dot{\neg}\,(\forall \epsilon\colon \exists \mathcal{N}\colon \forall \mathcal{M}\colon (0 < \epsilon \Rightarrow \mathcal{N} <= \mathcal{M} \Rightarrow |(FX[\mathcal{M}] - FY[\mathcal{M}])| < \epsilon))n$;

L11: Ded $\triangleright$ L10 $\gg$ $\dot{\neg}\,(\forall (EPob)\colon \exists n\colon \forall m\colon (0 < (EPob) \Rightarrow n <= m \Rightarrow |(FX[m] - FY[m])| < (EPob)))n$;

L12: Repetition $\triangleright$ L11 $\gg$ $\dot{\neg}\,(SF(FX, FY))n$ □

[SystemQ **lemma** LessNeq(R): $\Pi FX, FY\colon R(FX) << R(FY) \vdash R(FX)!! ==$

R(FY)]

SystemQ **proof of** LessNeq(R):

L01:	Arbitrary $\gg$	FX, FY	;
L02:	Premise $\gg$	$\mathrm{R(FX)} << \mathrm{R(FY)}$	;
L03:	Repetition $\rhd$ L02 $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY}$	;
L04:	LessNeq(F) $\rhd$ L03 $\gg$	$\dot{\neg}\,(\mathrm{SF}(\mathrm{FX}, \mathrm{FY}))\mathrm{n}$	;
L05:	To!! == $\rhd$L04 $\gg$	$\mathrm{R(FX)!!} == \mathrm{R(FY)}$	$\square$

[SystemQ **lemma** PositiveToRight(Less)(1term): $\Pi\mathcal{X}, \mathcal{Y}\colon \mathcal{X} < \mathcal{Y} \vdash 0 < (\mathcal{Y} - \mathcal{X})$]

SystemQ **proof of** PositiveToRight(Less)(1term):

L01:	Arbitrary $\gg$	$\mathcal{X}, \mathcal{Y}$	;
L02:	Premise $\gg$	$\mathcal{X} < \mathcal{Y}$	;
L03:	plus0Left $\gg$	$(0 + \mathcal{X}) = \mathcal{X}$	;
L04:	eqSymmetry $\rhd$ L03 $\gg$	$\mathcal{X} = (0 + \mathcal{X})$	;
L05:	SubLessLeft $\rhd$ L04 $\rhd$ L02 $\gg$	$(0 + \mathcal{X}) < \mathcal{Y}$	;
L06:	PositiveToRight(Less) $\rhd$ L05 $\gg$	$0 < (\mathcal{Y} - \mathcal{X})$	$\square$

[SystemQ **lemma** ToLeq(Advanced)(R): $\Pi\mathrm{FEP}, \mathrm{FX}, \mathrm{FY}\colon 00 << \mathrm{R(FEP)} \Rightarrow \mathrm{R(FX)} + +\mathrm{R(FEP)} \neq \mathrm{R(FY)} \vdash \mathrm{R(FY)} <<== \mathrm{R(FX)}$]

SystemQ **proof of** ToLeq(Advanced)(R):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathrm{FEP}, \mathrm{FX}, \mathrm{FY}$	;
L03:	Premise $\gg$	$\mathrm{R(FX)} << \mathrm{R(FY)}$	;
L04:	PositiveToRight(Less)(1term)(R) $\rhd$ L03 $\gg$	$00 << \mathrm{R(FY)} + +(- - \mathrm{R(FX)})$	;
L05:	PlusCommutativity(R) $\gg$	$\mathrm{R(FX)} + +(\mathrm{R(FY)} + +(- - \mathrm{R(FX)})) = \mathrm{R(FY)} + +(- - \mathrm{R(FX)}) + +\mathrm{R(FX)}$	;
L06:	Three2threeTerms(R) $\gg$	$\mathrm{R(FY)} + +(- - \mathrm{R(FX)}) + +\mathrm{R(FX)} = \mathrm{R(FY)} + +\mathrm{R(FX)} + +(- - \mathrm{R(FX)})$	;
L07:	$\mathrm{x = x + y - y(R)}$ $\gg$	$\mathrm{R(FY)} = \mathrm{R(FY)} + +\mathrm{R(FX)} + +(- - \mathrm{R(FX)})$	;
L08:	==Symmetry $\rhd$ L07 $\gg$	$\mathrm{R(FY)} + +\mathrm{R(FX)} + +(- - \mathrm{R(FX)}) == \mathrm{R(FY)}$	;
L09:	eqTransitivity4 $\rhd$ L05 $\rhd$ L06 $\rhd$ L08 $\gg$	$\mathrm{R(FX)} + +(\mathrm{R(FY)} + +(- - \mathrm{R(FX)})) == \mathrm{R(FY)}$	;
L10:	JoinConjuncts $\rhd$ L04 $\rhd$ L09 $\gg$	$00 << \mathrm{R(FY)}++(--\mathrm{R(FX)}) \mathrel{\dot{\wedge}} \mathrm{R(FX)} + +(\mathrm{R(FY)} + +(- - \mathrm{R(FX)})) = \mathrm{R(FY)}$	;
L11:	IntroExist @ $\mathrm{R(FY)} + +(- - \mathrm{R(FX)})$ $\rhd$ L10 $\gg$	$\exists \mathrm{FEP}\colon 00 << \mathrm{R(FEP)} \mathrel{\dot{\wedge}} \mathrm{R(FX)} + +\mathrm{R(FEP)} == \mathrm{R(FY)}$	;
L12:	Block $\gg$	End	;
L13:	Arbitrary $\gg$	$\mathrm{FEP}, \mathrm{FX}, \mathrm{FY}$	;

L03: Ded ▷ L12 ≫ R(FX) << R(FY) ⇒ ∃FEP: 00 << R(FEP) ∧̇ R(FX) + +R(FEP) == R(FY) ;
L04: Premise ≫ 00 << R(FEP) ⇒ R(FX) + +R(FEP)!! == R(FY) ;
L05: ToNegatedAnd ▷ L04 ≫ ¬̇ ((00 << R(FEP) ∧̇ R(FX) + +R(FEP) == R(FY)))n ;
L06: Gen ▷ L05 ≫ ∀FEP: ¬̇ ((00 << R(FEP) ∧̇ R(FX) + +R(FEP) == R(FY)))n ;
L07: (A)to(E) ▷ L06 ≫ ¬̇ (∃FEP: 00 << R(FEP) ∧̇ R(FX) + +R(FEP) == R(FY))n ;
L08: MT ▷ L03 ▷ L07 ≫ ¬̇ (R(FX) << R(FY))n ;
L14: FromNotLess(R) ▷ L08 ≫ R(FY) <<== R(FX) □

———(23.10.06)

[SystemQ **lemma** LeqNeqLess(R): ΠFX, FY: R(FX) <<== R(FY) ⊢ R(FX)!! R(FY) ⊢ R(FX) << R(FY)]

SystemQ **proof of** LeqNeqLess(R):

L01: Block ≫ Begin ;
L02: Arbitrary ≫ FX, FY ;
L03: Premise ≫ R(FX) <<== R(FY) ;
L04: Premise ≫ R(FX)!! == R(FY) ;
L05: Premise ≫ R(FY) <<== R(FX) ;
L06: LeqAntisymmetry(R) ▷ L03 ▷ L05 ≫ R(FX) == R(FY) ;
L07: FromContradiction ▷ L06 ▷ L04 ≫ ¬̇ (R(FY) <<== R(FX))n ;
L08: Block ≫ End ;
L09: Arbitrary ≫ FX, FY ;
L03: Ded ▷ L08 ≫ R(FX) <<== R(FY) ⇒ R(FX)!! == R(FY) ⇒ R(FY) <<== R(FX) ⇒ ¬̇ (R(FY) <<== R(FX))n ;
L04: Premise ≫ R(FX) <<== R(FY) ;
L05: Premise ≫ R(FX)!! == R(FY) ;
L06: MP2 ▷ L03 ▷ L04 ▷ L05 ≫ R(FY) <<== R(FX) ⇒ ¬̇ (R(FY) <<== R(FX))n ;
L10: prop lemma imply negation ▷ L06 ≫ ¬̇ (R(FY) <<== R(FX))n ;
L11: ToLess(R) ▷ L10 ≫ R(FX) << R(FY) □

[SystemQ **lemma** SubLeqLeft(R): ΠFX, FY, FZ: R(FX) == R(FY) ⊢ R(FX) R(FZ) ⊢ R(FY) <<== R(FZ)]

SystemQ **proof of** SubLeqLeft(R):

L01: Arbitrary ≫ FX, FY, FZ ;
L02: Premise ≫ R(FX) == R(FY) ;

L03:	Premise ≫	R(FX) <<== R(FZ)	;
L04:	==Symmetry ▷ L02 ≫	R(FY) == R(FX)	;
L05:	lemma eqLeq(R) ▷ L04 ≫	R(FY) <<== R(FX)	;
L06:	LeqTransitivity(R) ▷ L05 ▷ L03 ≫	R(FY) <<== R(FZ)	□

[SystemQ **lemma** LeqLessTransitivity(R): ΠFX, FY, FZ: R(FX) <<== R(FY)
R(FY) << R(FZ) ⊢ R(FX) << R(FZ)]

SystemQ **proof of** LeqLessTransitivity(R):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	FX, FY, FZ	;
L03:	Premise ≫	R(FX) <<== R(FY)	;
L04:	Premise ≫	R(FY) << R(FZ)	;
L05:	Premise ≫	R(FX) == R(FZ)	;
L06:	LessLeq(R) ▷ L04 ≫	R(FY) <<== R(FZ)	;
L07:	LessNeq(R) ▷ L04 ≫	R(FY)!! == R(FZ)	;
L08:	SubLeqLeft(R) ▷ L05 ▷ L03 ≫	R(FZ) <<== R(FY)	;
L09:	LeqAntisymmetry(R) ▷ L06 ▷ L08 ≫	R(FY) == R(FZ)	;
L10:	FromContradiction ▷ L09 ▷ L07 ≫	R(FX)!! == R(FZ)	;
L11:	Block ≫	End	;
L12:	Arbitrary ≫	FX, FY, FZ	;
L13:	Ded ▷ L11 ≫	R(FX) <<== R(FY) ⇒ R(FY) << R(FZ) ⇒ R(FX) == R(FZ) ⇒ R(FX)!! == R(FZ)	;
L14:	Premise ≫	R(FX) <<== R(FY)	;
L15:	Premise ≫	R(FY) << R(FZ)	;
L16:	MP2 ▷ L13 ▷ L14 ▷ L15 ≫	R(FX) == R(FZ) ⇒ R(FX)!! == R(FZ)	;
L17:	prop lemma imply negation ▷ L16 ≫	R(FX)!! == R(FZ)	;
L18:	LessLeq(R) ▷ L15 ≫	R(FY) <<== R(FZ)	;
L19:	LeqTransitivity(R) ▷ L14 ▷ L18 ≫	R(FX) <<== R(FZ)	;
L20:	LeqNeqLess(R) ▷ L19 ▷ L17 ≫	R(FX) << R(FZ)	□

————(23.10.06)

[SystemQ **lemma** NegativeToLeft(Eq)(R): ΠFX, FY, FZ: R(FX) == R(FY)+
+(− − R(FZ)) ⊢ R(FX) + +R(FZ) == R(FY)]

SystemQ **proof of** NegativeToLeft(Eq)(R):

L01:	Arbitrary ≫	FX, FY, FZ	;
L02:	Premise ≫	R(FX) == R(FY) + +(− − R(FZ))	;
L03:	EqAddition(R) ▷ L02 ≫	R(FX) + +R(FZ) == R(FY) + +(− − R(FZ)) + +R(FZ)	;

L04: Three2threeTerms(R) ≫ $R(FY) + +(- - R(FZ)) + +R(FZ) == R(FY)++R(FZ)+ +(- - R(FZ))$;
L05: $x = x + y - y(R)$ ≫ $R(FY) == R(FY) + +R(FZ) + +(- - R(FZ))$;
L06: ==Symmetry ▷ L05 ≫ $R(FY) + +R(FZ) + +(- - R(FZ)) == R(FY)$;
L07: eqTransitivity4 ▷ L03 ▷ L04 ▷ L06 ≫ $R(FX) + +R(FZ) == R(FY)$ □

[SystemQ **lemma** NegativeToRight(Less)(R): ΠFX, FY, FZ: $R(FX)++(- - R(FY)) << R(FZ) \vdash R(FX) << R(FZ) + +R(FY)$]

SystemQ **proof of** NegativeToRight(Less)(R):

L01: Arbitrary ≫ FX, FY, FZ ;
L02: Premise ≫ $R(FX) + +(- - R(FY)) << R(FZ)$;
L03: lessAddition(R) ▷ L02 ≫ $R(FX) + +(- - R(FY)) + +R(FY) << R(FZ) + +R(FY)$;
L04: Three2threeTerms(R) ≫ $R(FX) + +(- - R(FY)) + +R(FY) == R(FX) + +R(FY) + +(- - R(FY))$;
L05: $x = x + y - y(R)$ ≫ $R(FX) == R(FX) + +R(FY) + +(- - R(FY))$;
L06: ==Symmetry ▷ L05 ≫ $R(FX) + +R(FY) + +(- - R(FY)) == R(FX)$;
L07: eqTransitivity ▷ L04 ▷ L06 ≫ $R(FX) + +(- - R(FY)) + +R(FY) == R(FX)$;
L08: SubLessLeft(R) ▷ L07 ▷ L03 ≫ $R(FX) << R(FZ) + +R(FY)$ □

[SystemQ **lemma** !! == Symmetry: ΠFX, FY: $R(FX)!! == R(FY) \vdash R(FY)!! = R(FX)$]

SystemQ **proof of** !! == Symmetry:

L01: Block ≫ Begin ;
L02: Arbitrary ≫ FX, FY ;
L03: Premise ≫ $R(FY) == R(FX)$;
L04: ==Symmetry ▷ L03 ≫ $R(FX) == R(FY)$;
L05: Block ≫ End ;
L06: Arbitrary ≫ FX, FY ;
L07: Ded ▷ L05 ≫ $R(FY) == R(FX) \Rightarrow R(FX) == R(FY)$;
L08: Premise ≫ $R(FX)!! == R(FY)$;
L09: MT ▷ L07 ▷ L08 ≫ $R(FY)!! == R(FX)$ □

————-(23.10.06)

[SystemQ **lemma** NegativeToRight(Eq)(R): ΠFX, FY, FZ: $R(FX) + +(- - R(FY)) == R(FZ) \vdash R(FX) == R(FZ) + +R(FY)$]

SystemQ **proof of** NegativeToRight(Eq)(R):

L01: Arbitrary ≫ FX, FY, FZ ;

L02: Premise $\gg$ $R(FX) + +(- - R(FY)) == R(FZ)$;

L03: EqAddition(R) $\rhd$ L02 $\gg$ $R(FX) + +(- - R(FY)) + +R(FY) == R(FZ) + +R(FY)$;

L04: $x = x + y - y(R) \gg$ $R(FX) == R(FX) + +R(FY) + +(- - R(FY))$;

L05: Three2threeTerms(R) $\gg$ $R(FX) + +R(FY) + +(- - R(FY)) == R(FX) + +(- - R(FY)) + +R(FY)$;

L06: eqTransitivity4 $\rhd$ L04 $\rhd$ L05 $\rhd$ L03 $\gg$ $R(FX) == R(FZ) + +R(FY)$ □

[SystemQ **lemma** NegativeToRight(Eq)(1term)(R): $\Pi FX, FY\colon R(FX) + +(- - R(FY)) == 00 \vdash R(FX) == R(FY)$]

SystemQ **proof of** NegativeToRight(Eq)(1term)(R):

L01: Arbitrary $\gg$ FX, FY ;

L02: Premise $\gg$ $R(FX) + +(- - R(FY)) == 00$;

L03: NegativeToRight(Eq)(R) $\rhd$ L02 $\gg$ $R(FX) == 00 + +R(FY)$;

L04: Plus0Left(R) $\gg$ $00 + +R(FY) == R(FY)$;

L05: eqTransitivity $\rhd$ L03 $\rhd$ L04 $\gg$ $R(FX) == R(FY)$ □

[SystemQ **lemma** DoubleMinus(R): $\Pi FX\colon (- - (- - R(FX))) == R(FX)$]

SystemQ **proof of** DoubleMinus(R):

L01: Arbitrary $\gg$ FX ;

L02: Negative(R) $\gg$ $(- - R(FX)) + +(- - (- - R(FX))) == 00$;

L03: PlusCommutativity(R) $\gg$ $(- - R(FX)) + +(- - (- - R(FX))) == (- - (- - R(FX))) + +(- - R(FX))$;

L04: ==Symmetry $\rhd$ L03 $\gg$ $(- - (- - R(FX))) + +(- - R(FX)) == (- - R(FX)) + +(- - (- - R(FX)))$;

L05: eqTransitivity $\rhd$ L04 $\rhd$ L02 $\gg$ $(- - (- - R(FX))) + +(- - R(FX)) == 00$;

L06: NegativeToRight(Eq)(1term)(R) $\rhd$ L05 $\gg$ $(- - (- - R(FX))) == R(FX)$ □

[SystemQ **lemma** UniqueNegative(R): $\Pi FX, FY, FZ\colon R(FX) + +R(FY) == 00 \vdash R(FX) + +R(FZ) == 00 \vdash R(FY) == R(FZ)$]

SystemQ **proof of** UniqueNegative(R):

L01: Arbitrary $\gg$ FX, FY, FZ ;

L02: Premise $\gg$ $R(FX) + +R(FY) == 00$;

L03: Premise $\gg$ $R(FX) + +R(FZ) == 00$;

L04: PlusCommutativity(R) $\gg$ $R(FY) + +R(FX) == R(FX) + +R(FY)$;

L05: eqTransitivity $\rhd$ L04 $\rhd$ L02 $\gg$ $R(FY) + +R(FX) == 00$;

L06: PositiveToRight(Eq)(R) $\rhd$ L05 $\gg$ $R(FY) == 00 + +(- - R(FX))$;

L07:	PlusCommutativity(R) ≫	R(FZ) + +R(FX) == R(FX) + +R(FZ)	;
L08:	eqTransitivity ▷ L07 ▷ L03 ≫	R(FZ) + +R(FX) == 00	;
L09:	PositiveToRight(Eq)(R) ▷ L08 ≫	R(FZ) == 00 + +(− − R(FX))	;
L10:	==Symmetry ▷ L09 ≫	00 + +(− − R(FX)) == R(FZ)	;
L11:	eqTransitivity ▷ L06 ▷ L10 ≫	R(FY) == R(FZ)	□

[SystemQ **lemma** SubtractEquationsLeft(R): ΠFX, FY, FZ, FU: R(FX)++R(F
R(FY) + +R(FU) ⊢ R(FX) == R(FY) ⊢ R(FZ) == R(FU)]

SystemQ **proof of** SubtractEquationsLeft(R):

L01:	Arbitrary ≫	FX, FY, FZ, FU	;
L02:	Premise ≫	R(FX) + +R(FZ) == R(FY) + +R(FU)	;
L03:	Premise ≫	R(FX) == R(FY)	;
L04:	PlusCommutativity(R) ≫	R(FZ) + +R(FX) == R(FX) + +R(FZ)	;
L05:	PlusCommutativity(R) ≫	R(FY) + +R(FU) == R(FU) + +R(FY)	;
L06:	eqTransitivity4 ▷ L04 ▷ L02 ▷ L05 ≫	R(FZ) + +R(FX) == R(FU) + +R(FY)	;
L07:	SubtractEquations(R) ▷ L06 ▷ L03 ≫	R(FZ) == R(FU)	□

[SystemQ **lemma** EqNegated(R): ΠFX, FY: R(FX) == R(FY) ⊢ (−−R(FX)) =
(− − R(FY))]

SystemQ **proof of** EqNegated(R):

L01:	Arbitrary ≫	FX, FY	;
L02:	Premise ≫	R(FX) == R(FY)	;
L03:	Negative(R) ≫	R(FX) + +(− − R(FX)) == 00	;
L04:	Negative(R) ≫	R(FY) + +(− − R(FY)) == 00	;
L05:	==Symmetry ▷ L04 ≫	00 == R(FY) + +(− − R(FY))	;
L06:	eqTransitivity ▷ L03 ▷ L05 ≫	R(FX) + +(− − R(FX)) == R(FY) + +(− − R(FY))	;
L07:	SubtractEquationsLeft(R) ▷ L06 ▷ L02 ≫	(− − R(FX)) == (− − R(FY))	□

[SystemQ **lemma** NeqNegated(R): ΠFX, FY: R(FX)!! == R(FY) ⊢ (− − R(FX))!! == (− − R(FY))]

SystemQ **proof of** NeqNegated(R):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	FX, FY	;
L03:	Premise ≫	R(FX)!! == R(FY)	;
L04:	Premise ≫	(− − R(FX)) == (− − R(FY))	;
L05:	EqNegated(R) ▷ L04 ≫	(−−(−−R(FX))) == (−−(−−R(FY)))	;
L06:	DoubleMinus(R) ≫	(− − (− − R(FX))) == R(FX)	;
L07:	==Symmetry ▷ L06 ≫	R(FX) == (− − (− − R(FX)))	;

L08:	DoubleMinus(R) ≫	$(--(--R(FY))) == R(FY)$	;
L09:	eqTransitivity4 ▷ L07 ▷ L05 ▷ L08 ≫	$R(FX) == R(FY)$	;
L10:	FromContradiction ▷ L09 ▷ L03 ≫	$(--R(FX))!! == (--R(FY))$	;
L11:	Block ≫	End	;
L12:	Arbitrary ≫	FX, FY	;
L13:	Ded ▷ L11 ≫	$R(FX)!! == R(FY) \Rightarrow (--R(FX)) == (--R(FY)) \Rightarrow \dot{\neg}((--R(FX)) == (--R(FY)))n$	;
L14:	Premise ≫	$R(FX)!! == R(FY)$	;
L15:	MP ▷ L13 ▷ L14 ≫	$(--R(FX)) == (--R(FY)) \Rightarrow \dot{\neg}((--R(FX)) == (--R(FY)))n$	;
L16:	prop lemma imply negation ▷ L15 ≫	$\dot{\neg}((--R(FX)) == (--R(FY)))n$	□

————(23.10.06)

[SystemQ **lemma** SubLeqRight(R): ΠFX, FY, FZ: R(FX) == R(FY) ⊢ R(FZ) R(FX) ⊢ R(FZ) <<== R(FY)]

SystemQ **proof of** SubLeqRight(R):

L01:	Arbitrary ≫	FX, FY, FZ	;
L02:	Premise ≫	$R(FX) == R(FY)$	;
L03:	Premise ≫	$R(FZ) <<== R(FX)$	;
L04:	lemma eqLeq(R) ▷ L02 ≫	$R(FX) <<== R(FY)$	;
L05:	LeqTransitivity(R) ▷ L03 ▷ L04 ≫	$R(FZ) <<== R(FY)$	□

[SystemQ **lemma** LeqNegated(R): ΠFX, FY: R(FX) <<== R(FY) ⊢ (−− R(FY)) <<== (−− R(FX))]

SystemQ **proof of** LeqNegated(R):

L01:	Arbitrary ≫	FX, FY	;
L02:	Premise ≫	$R(FX) <<== R(FY)$	;
L03:	leqAddition(R) ▷ L02 ≫	$R(FX)++(--R(FX)) <<== R(FY)++(--R(FX))$	;
L04:	Negative(R) ≫	$R(FX)++(--R(FX)) == 00$	;
L05:	SubLeqLeft(R) ▷ L04 ▷ L03 ≫	$00 <<== R(FY)++(--R(FX))$	;
L06:	PlusCommutativity(R) ≫	$R(FY)++(--R(FX)) == (--R(FX))++R(FY)$	;
L07:	SubLeqRight(R) ▷ L06 ▷ L05 ≫	$00 <<== (--R(FX))++R(FY)$	;
L08:	leqAddition(R) ▷ L07 ≫	$00++(--R(FY)) <<== (--R(FX))++R(FY)++(--R(FY))$	;
L09:	Plus0Left(R) ≫	$00++(--R(FY)) == (--R(FY))$	;

L10: x = x + y − y(R) ≫ (−−R(FX)) == (−−R(FX))+ +R(FY) + +(− − R(FY)) ;
L11: ==Symmetry ▷ L10 ≫ (−−R(FX))++R(FY)++(−− R(FY)) == (− − R(FX)) ;
L12: SubLeqLeft(R) ▷ L09 ▷ L08 ≫ (− − R(FY)) <<== (− − R(FX)) + +R(FY) + +(− − R(FY)) ;
L13: SubLeqRight(R) ▷ L11 ▷ L12 ≫ (− − R(FY)) <<== (− − R(FX)) □

[SystemQ **lemma** LessNegated(R): ΠFX, FY: R(FX) << R(FY) ⊢ (−−R(FY)) (− − R(FX))]

SystemQ **proof of** LessNegated(R):

L01: Arbitrary ≫ FX, FY ;
L02: Premise ≫ R(FX) << R(FY) ;
L03: LessLeq(R) ▷ L02 ≫ R(FX) <<== R(FY) ;
L04: LeqNegated(R) ▷ L03 ≫ (− − R(FY)) <<== (− − R(FX)) ;
L05: LessNeq(R) ▷ L02 ≫ R(FX)!! == R(FY) ;
L06: NeqNegated(R) ▷ L05 ≫ (− − R(FX))!! == (− − R(FY)) ;
L07: !! == Symmetry ▷ L06 ≫ (− − R(FY))!! == (− − R(FX)) ;
L08: LeqNeqLess(R) ▷ L04 ▷ L07 ≫ (− − R(FY)) << (− − R(FX)) □

[SystemQ **lemma** − 0 = 0(R): (− − 00) == 00]

SystemQ **proof of** − 0 = 0(R):

L01: Negative(R) ≫ 00 + +(− − 00) == 00 ;
L02: Plus0(R) ≫ 00 + +00 == 00 ;
L03: UniqueNegative(R) ▷ L01 ▷ L02 ≫ (− − 00) == 00 □

[SystemQ **lemma** NegativeNegated(R): ΠFX: R(FX) << 00 ⊢ 00 << (− − R(FX))]

SystemQ **proof of** NegativeNegated(R):

L01: Arbitrary ≫ FX ;
L02: Premise ≫ R(FX) << 00 ;
L03: LessNegated(R) ▷ L02 ≫ (− − 00) << (− − R(FX)) ;
L04: −0 = 0(R) ≫ (− − 00) == 00 ;
L05: SubLessLeft(R) ▷ L04 ▷ L03 ≫ 00 << (− − R(FX)) □

[SystemQ **lemma** LeqTotality(R): ΠFX, FY: R(FX) <<== R(FY) $\dot{\vee}$ R(FY) < R(FX)]

SystemQ **proof of** LeqTotality(R):

L01: Block ≫ Begin ;
L02: Arbitrary ≫ FX, FY ;
L03: Premise ≫ $\dot{\neg}$ (R(FX) <<== R(FY))n ;
L04: ToLess(R) ▷ L03 ≫ R(FY) << R(FX) ;
L05: LessLeq(R) ▷ L04 ≫ R(FY) <<== R(FX) ;
L06: Block ≫ End ;
L07: Arbitrary ≫ FX, FY ;

L03: Ded $\rhd$ L06 $\gg$ $\dot{\neg}$ (R(FX) <<== R(FY))n $\Rightarrow$ R(FY) <<== R(FX) ;
L08: Repetition $\rhd$ L03 $\gg$ R(FX) <<== R(FY) $\dot{\vee}$ R(FY) <<== R(FX) □

[SystemQ **lemma** FromLeqGeq(R): $\Pi\mathcal{A}$, FX, FY: R(FX) <<== R(FY) $\Rightarrow \mathcal{A} \vdash$ R(FY) <<== R(FX) $\Rightarrow \mathcal{A} \vdash \mathcal{A}$]

SystemQ **proof of** FromLeqGeq(R):

L01: Arbitrary $\gg$ $\mathcal{A}$, FX, FY ;
L02: Premise $\gg$ R(FX) <<== R(FY) $\Rightarrow \mathcal{A}$;
L03: Premise $\gg$ R(FY) <<== R(FX) $\Rightarrow \mathcal{A}$;
L04: LeqTotality(R) $\gg$ R(FX) <<== R(FY) $\dot{\vee}$ R(FY) <<== R(FX) ;
L05: FromDisjuncts $\rhd$ L04 $\rhd$ L02 $\rhd$ L03 $\gg$ $\mathcal{A}$ □

———(24.10.06)

[SystemQ **lemma** FromLess(R): ΠFX, FY: R(FX) << R(FY) $\vdash \dot{\neg}$ (R(FY) <<= R(FX))n]

SystemQ **proof of** FromLess(R):

L01: Block $\gg$ Begin ;
L02: Arbitrary $\gg$ FX, FY ;
L03: Premise $\gg$ R(FX) << R(FY) ;
L04: Premise $\gg$ R(FY) <<== R(FX) ;
L05: LessLeq(R) $\rhd$ L03 $\gg$ R(FX) <<== R(FY) ;
L06: LeqAntisymmetry(R) $\rhd$ L05 $\rhd$ L04 $\gg$ R(FX) == R(FY) ;
L07: LessNeq(R) $\rhd$ L03 $\gg$ R(FX)!! == R(FY) ;
L08: FromContradiction $\rhd$ L06 $\rhd$ L07 $\gg$ $\dot{\neg}$ (R(FY) <<== R(FX))n ;
L09: Block $\gg$ End ;
L10: Arbitrary $\gg$ FX, FY ;
L03: Ded $\rhd$ L09 $\gg$ R(FX) << R(FY) $\Rightarrow$ R(FY) <<== R(FX) $\Rightarrow$ $\dot{\neg}$ (R(FY) <<== R(FX))n ;
L04: Premise $\gg$ R(FX) << R(FY) ;
L05: MP $\rhd$ L03 $\rhd$ L04 $\gg$ R(FY) <<== R(FX) $\Rightarrow$ $\dot{\neg}$ (R(FY) <<== R(FX))n ;
L11: prop lemma imply negation $\rhd$ L05 $\gg$ $\dot{\neg}$ (R(FY) <<== R(FX))n □

[SystemQ **lemma** from <<==: ΠFX, FY: R(FX) <<== R(FY) $\vdash$ FX $\leq_{\mathrm{f}}$ FY]

SystemQ **proof of** from <<==:

L01: Arbitrary $\gg$ FX, FY ;
L02: Premise $\gg$ R(FX) <<== R(FY) ;
L03: Repetition $\rhd$ L02 $\gg$ R(FX) << R(FY) $\dot{\vee}$ R(FX) = R(FY) ;
L04: Repetition $\rhd$ L03 $\gg$ FX $<_{\mathrm{f}}$ FY $\dot{\vee}$ R(FX) = R(FY) ;

L05:	Block $\gg$	Begin	;
L06:	Arbitrary $\gg$	FX, FY	;
L02:	Premise $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY}$	;
L07:	WeakenOr2 $\rhd$ L02 $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \dot{\vee} \mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L08:	Block $\gg$	End	;
L09:	Block $\gg$	Begin	;
L10:	Arbitrary $\gg$	FX, FY	;
L02:	Premise $\gg$	$\mathrm{R}(\mathrm{FX}) = \mathrm{R}(\mathrm{FY})$	;
L03:	From == $\gg$	$\mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L11:	WeakenOr1 $\rhd$ L03 $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \dot{\vee} \mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L12:	Block $\gg$	End	;
L13:	Ded $\rhd$ L08 $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \Rightarrow \mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \dot{\vee} \mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L14:	Ded $\rhd$ L12 $\gg$	$\mathrm{R}(\mathrm{FX}) = \mathrm{R}(\mathrm{FY}) \Rightarrow \mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \dot{\vee} \mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L15:	FromDisjuncts $\rhd$ L04 $\rhd$ L13 $\rhd$ L14 $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \dot{\vee} \mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L16:	Repetition $\rhd$ L15 $\gg$	$\mathrm{FX} \leq_{\mathrm{f}} \mathrm{FY}$	$\square$

[SystemQ **lemma** NonnegativeNumerical(F): $\Pi \mathrm{FX}\colon 0\mathrm{f} \leq_{\mathrm{f}} \mathrm{FX} \vdash |\mathrm{fFX}| = \mathrm{FX}$]

SystemQ **proof of** NonnegativeNumerical(F):

L01:	Arbitrary $\gg$	FX	;
L02:	Premise $\gg$	$0\mathrm{f} \leq_{\mathrm{f}} \mathrm{FX}$	;
L03:	NumericalF $\gg$	$(0\mathrm{f} \leq_{\mathrm{f}} \mathrm{FX} \Rightarrow |\mathrm{fFX}| = \mathrm{FX}) \dot{\wedge} (\dot{\neg}(0\mathrm{f} \leq_{\mathrm{f}} \mathrm{FX})\mathrm{n} \Rightarrow |\mathrm{fFX}| = -_{\mathrm{f}}\mathrm{FX})$	;
L04:	FirstConjunct $\rhd$ L03 $\gg$	$0\mathrm{f} \leq_{\mathrm{f}} \mathrm{FX} \Rightarrow |\mathrm{fFX}| = \mathrm{FX}$	;
L05:	MP $\rhd$ L04 $\rhd$ L02 $\gg$	$|\mathrm{fFX}| = \mathrm{FX}$	$\square$

[SystemQ **lemma** NonnegativeNumerical(R): $\Pi \mathrm{FX}\colon 00 <<== \mathrm{R}(\mathrm{FX}) \vdash |\mathrm{rR}(\mathrm{FX}$
$\mathrm{R}(\mathrm{FX})$]

SystemQ **proof of** NonnegativeNumerical(R):

L01:	Arbitrary $\gg$	FX	;
L02:	Premise $\gg$	$00 <<== \mathrm{R}(\mathrm{FX})$	;
L03:	from <<== $\rhd$L02 $\gg$	$0\mathrm{f} \leq_{\mathrm{f}} \mathrm{FX}$	;
L04:	NonnegativeNumerical(F) $\rhd$ L03 $\gg$	$|\mathrm{fFX}| = \mathrm{FX}$	;
L05:	(Adgic)SameR $\rhd$ L04 $\gg$	$\mathrm{R}(|\mathrm{fFX}|) = \mathrm{R}(\mathrm{FX})$	;
L06:	Repetition $\rhd$ L05 $\gg$	$|\mathrm{rR}(\mathrm{FX})| = \mathrm{R}(\mathrm{FX})$	$\square$

[SystemQ **lemma** to <<==: $\Pi \mathrm{FX}, \mathrm{FY}\colon \mathrm{FX} \leq_{\mathrm{f}} \mathrm{FY} \vdash \mathrm{R}(\mathrm{FX}) <<== \mathrm{R}(\mathrm{FY})$]

SystemQ **proof of** to <<==:

L01:	Arbitrary $\gg$	FX, FY	;
L02:	Premise $\gg$	$\mathrm{FX} \leq_{\mathrm{f}} \mathrm{FY}$	;
L03:	Repetition $\rhd$ L02 $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY} \dot{\vee} \mathrm{SF}(\mathrm{FX}, \mathrm{FY})$	;
L04:	Block $\gg$	Begin	;
L05:	Arbitrary $\gg$	FX, FY	;
L06:	Premise $\gg$	$\mathrm{FX} <_{\mathrm{f}} \mathrm{FY}$	;
L07:	Repetition $\rhd$ L06 $\gg$	$\mathrm{R}(\mathrm{FX}) << \mathrm{R}(\mathrm{FY})$	;

L08:	WeakenOr2 $\rhd$ L07 $\gg$	$R(FX) << R(FY) \dot{\vee} R(FX) = R(FY)$	;
L09:	Block $\gg$	End	;
L10:	Block $\gg$	Begin	;
L11:	Arbitrary $\gg$	FX, FY	;
L02:	Premise $\gg$	$SF(FX, FY)$	;
L03:	To $==$ $\rhd$L02 $\gg$	$R(FX) = R(FY)$	;
L12:	WeakenOr1 $\rhd$ L03 $\gg$	$R(FX) << R(FY) \dot{\vee} R(FX) = R(FY)$	;
L13:	Block $\gg$	End	;
L06:	Ded $\rhd$ L09 $\gg$	$FX <_f FY \Rightarrow R(FX) << R(FY) \dot{\vee} R(FX) = R(FY)$	;
L07:	Ded $\rhd$ L13 $\gg$	$SF(FX, FY) \Rightarrow R(FX) << R(FY) \dot{\vee} R(FX) = R(FY)$	;
L14:	FromDisjuncts $\rhd$ L03 $\rhd$ L06 $\rhd$ L07 $\gg$	$R(FX) << R(FY) \dot{\vee} R(FX) = R(FY)$	;
L15:	Repetition $\rhd$ L14 $\gg$	$R(FX) <<== R(FY)$	$\square$

[SystemQ **lemma** NegativeNumerical(F): $\Pi FX\colon \dot{\neg}\,(0f \leq_f FX)n \vdash |fFX| = -_f FX$]

SystemQ **proof of** NegativeNumerical(F):

L01:	Arbitrary $\gg$	FX	;
L02:	Premise $\gg$	$\dot{\neg}\,(0f \leq_f FX)n$	;
L03:	NumericalF $\gg$	$(0f \leq_f FX \Rightarrow |fFX| = FX) \dot{\wedge} (\dot{\neg}\,(0f \leq_f FX)n \Rightarrow |fFX| = -_f FX)$	;
L04:	SecondConjunct $\rhd$ L03 $\gg$	$\dot{\neg}\,(0f \leq_f FX)n \Rightarrow |fFX| = -_f FX$	;
L05:	MP $\rhd$ L04 $\rhd$ L02 $\gg$	$|fFX| = -_f FX$	$\square$

[SystemQ **lemma** NegativeNumerical(R): $\Pi FX\colon R(FX) << 00 \vdash |rR(FX)| == (- - R(FX))$]

SystemQ **proof of** NegativeNumerical(R):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	FX	;
L03:	Premise $\gg$	$R(FX) << 00$	;
L04:	Premise $\gg$	$0f \leq_f FX$	;
L05:	FromLess(R) $\rhd$ L03 $\gg$	$\dot{\neg}\,(00 <<== R(FX))n$	;
L06:	to $<<==$ $\rhd$L04 $\gg$	$00 <<== R(FX)$	;
L07:	FromContradiction $\rhd$ L06 $\rhd$ L05 $\gg$	$\dot{\neg}\,(0f \leq_f FX)n$	;
L08:	Block $\gg$	End	;
L09:	Arbitrary $\gg$	FX	;
L03:	Ded $\rhd$ L08 $\gg$	$R(FX) << 00 \Rightarrow 0f \leq_f FX \Rightarrow \dot{\neg}\,(0f \leq_f FX)n$	;
L04:	Premise $\gg$	$R(FX) << 00$	;
L05:	MP $\rhd$ L03 $\rhd$ L04 $\gg$	$0f \leq_f FX \Rightarrow \dot{\neg}\,(0f \leq_f FX)n$	;

L06: prop lemma imply negation $\triangleright$

L05 $\gg$ $\dot\neg$ (0f $\leq_{\mathrm{f}}$ FX)n ;

L10: NegativeNumerical(F) $\gg$ |fFX| = $-_{\mathrm{f}}$FX ;

L11: (Adgic)SameR $\triangleright$ L10 $\gg$ R(|fFX|) = R($-_{\mathrm{f}}$FX) ;

L12: Repetition $\triangleright$ L11 $\gg$ |rR(FX)| = (− − R(FX)) $\Box$

[SystemQ **lemma** 0 <= |x|(R): ΠFX: 00 <<== |rR(FX)|]

SystemQ **proof of** 0 <= |x|(R):

L01: Block $\gg$ Begin ;

L02: Arbitrary $\gg$ FX ;

L03: Premise $\gg$ 00 <<== R(FX) ;

L04: NonnegativeNumerical(R) $\triangleright$

L03 $\gg$ |rR(FX)| == R(FX) ;

L05: ==Symmetry $\triangleright$ L04 $\gg$ R(FX) == |rR(FX)| ;

L06: SubLeqRight(R) $\triangleright$ L05 $\triangleright$ L03 $\gg$ 00 <<== |rR(FX)| ;

L07: Block $\gg$ End ;

L08: Block $\gg$ Begin ;

L09: Arbitrary $\gg$ FX ;

L10: Premise $\gg$ $\dot\neg$ (00 <<== R(FX))n ;

L11: ToLess(R) $\triangleright$ L10 $\gg$ R(FX) << 00 ;

L12: NegativeNumerical(R) $\triangleright$ L11 $\gg$ |rR(FX)| == (− − R(FX)) ;

L13: ==Symmetry $\triangleright$ L12 $\gg$ (− − R(FX)) == |rR(FX)| ;

L14: NegativeNegated(R) $\triangleright$ L11 $\gg$ 00 << (− − R(FX)) ;

L15: LessLeq(R) $\triangleright$ L14 $\gg$ 00 <<== (− − R(FX)) ;

L16: SubLeqRight(R) $\triangleright$ L13 $\triangleright$ L15 $\gg$ 00 <<== |rR(FX)| ;

L17: Block $\gg$ End ;

L18: Arbitrary $\gg$ FX ;

L19: Ded $\triangleright$ L07 $\gg$ 00 <<== R(FX) $\Rightarrow$ 00 <<== |rR(FX)| ;

L20: Ded $\triangleright$ L17 $\gg$ $\dot\neg$ (00 <<== R(FX))n $\Rightarrow$ 00 <<== |rR(FX)| ;

L21: FromNegations $\triangleright$ L19 $\triangleright$ L20 $\gg$ 00 <<== |rR(FX)| $\Box$

————(24.10.06)

[SystemQ **lemma** PositiveNegated(R): ΠFX: 00 << R(FX) $\vdash$ (−−R(FX)) << 00]

SystemQ **proof of** PositiveNegated(R):

L01: Arbitrary $\gg$ FX ;

L02: Premise $\gg$ 00 << R(FX) ;

L03: LessNegated(R) $\triangleright$ L02 $\gg$ (− − R(FX)) << (− − 00) ;

L04: −0 = 0(R) $\gg$ (− − 00) == 00 ;

L05: SubLessRight(R) $\triangleright$ L04 $\triangleright$ L03 $\gg$

(− − R(FX)) << 00 $\Box$

[SystemQ **lemma** AddEquations(R): ΠFX, FY, FZ, FU: R(FX) == R(FY) $\vdash$ R(FZ) == R(FU) $\vdash$ R(FX) + +R(FZ) == R(FY) + +R(FU)]

SystemQ **proof of** AddEquations(R):

L01: Arbitrary $\gg$ FX, FY, FZ, FU ;

L02: Premise $\gg$ R(FX) == R(FY) ;

L03:	Premise ≫	R(FZ) == R(FU)	;
L04:	EqAddition(R) ▷ L02 ≫	R(FX) + +R(FZ) == R(FY) + +R(FZ)	;
L05:	EqAdditionLeft(R) ▷ L03 ≫	R(FY) + +R(FZ) == R(FY) + +R(FU)	;
L06:	eqTransitivity ▷ L04 ▷ L05 ≫	R(FX) + +R(FZ) == R(FY) + +R(FU)	□

[SystemQ **lemma** DistributionOut(R): ΠFX, FY, FZ: R(FX)∗∗R(FY)++R(FX ∗R(FZ) == R(FX) ∗ ∗(R(FY) + +R(FZ))]

SystemQ **proof of** DistributionOut(R):

L01:	Arbitrary ≫	FX, FY, FZ	;
L02:	Distribution(R) ≫	R(FX)∗∗(R(FY)++R(FZ)) == R(FX) ∗ ∗R(FY) + +R(FX) ∗ ∗R(FZ)	;
L03:	==Symmetry ▷ L02 ≫	R(FX) ∗ ∗R(FY) + +R(FX) ∗ ∗R(FZ) == R(FX) ∗ ∗(R(FY) + +R(FZ))	□

[SystemQ **lemma** x ∗ 0 + x = x(R): ΠFX: R(FX) ∗ ∗00 + +R(FX) == R(FX)]

SystemQ **proof of** x ∗ 0 + x = x(R):

L01:	Arbitrary ≫	FX	;
L02:	Times1(R) ≫	R(FX) ∗ ∗01 == R(FX)	;
L03:	==Symmetry ▷ L02 ≫	R(FX) == R(FX) ∗ ∗01	;
L04:	EqAdditionLeft(R) ▷ L03 ≫	R(FX) ∗ ∗00 + +R(FX) == R(FX) ∗ ∗00 + +R(FX) ∗ ∗01	;
L05:	Distribution(R) ≫	R(FX)∗∗(00++01) == R(FX)∗ ∗00 + +R(FX) ∗ ∗01	;
L06:	==Symmetry ▷ L05 ≫	R(FX) ∗ ∗00 + +R(FX) ∗ ∗01 == R(FX) ∗ ∗(00 + +01)	;
L07:	Plus0Left(R) ≫	00 + +01 == 01	;
L08:	EqMultiplicationLeft(R) ▷ L07 ≫	R(FX)∗∗(00++01) == R(FX)∗ ∗01	;
L09:	eqTransitivity5 ▷ L04 ▷ L06 ▷ L08 ▷ L02 ≫	R(FX) ∗ ∗00 + +R(FX) == R(FX)	□

———(24.10.06)

[SystemQ **lemma** x ∗ 0 = 0(R)(fff): ΠFX: R(FX) ∗ ∗00 == 00]

SystemQ **proof of** x ∗ 0 = 0(R)(fff):

L01:	Arbitrary ≫	FX	;
L02:	x = x + (y − y)(R) ≫	R(FX) ∗ ∗00 == R(FX) ∗ ∗00 + +(R(FX) + +(− − R(FX)))	;
L03:	PlusAssociativity(R) ≫	R(FX) ∗ ∗00 + +R(FX) + +(− − R(FX)) == R(FX) ∗ ∗00 + +(R(FX) + +(− − R(FX)))	;
L04:	==Symmetry ▷ L03 ≫	R(FX) ∗ ∗00 + +(R(FX) + +(− − R(FX))) == R(FX) ∗ ∗00 + +R(FX) + +(− − R(FX))	;

L05:	x ∗ 0 + x = x(R) ≫	R(FX) ∗ ∗00 + +R(FX) == R(FX)	;
L06:	EqAddition(R) ▷ L05 ≫	R(FX) ∗ ∗00 + +R(FX) + +(− − R(FX)) == R(FX) + +(− − R(FX))	;
L07:	Negative(R) ≫	R(FX) + +(− − R(FX)) == 00	;
L08:	eqTransitivity5 ▷ L02 ▷ L04 ▷ L06 ▷ L07 ≫	R(FX) ∗ ∗00 == 00	□

[SystemQ **lemma** Times(−1)(R): ΠFX: R(FX) ∗ ∗(− − 01) == (− − R(FX))]

SystemQ **proof of** Times(−1)(R):

L01:	Arbitrary ≫	FX	;
L02:	Negative(R) ≫	01 + +(− − 01) == 00	;
L03:	PlusCommutativity(R) ≫	(− − 01) + +01 == 01 + +(− − 01)	;
L04:	eqTransitivity ▷ L03 ▷ L02 ≫	(− − 01) + +01 == 00	;
L05:	EqMultiplicationLeft(R) ▷ L04 ≫	R(FX) ∗ ∗((− − 01) + +01) == R(FX) ∗ ∗00	;
L06:	x ∗ 0 = 0(R)(fff) ≫	R(FX) ∗ ∗00 == 00	;
L07:	eqTransitivity ▷ L05 ▷ L06 ≫	R(FX)∗∗((− −01)+ +01) == 00	;
L08:	Distribution(R) ≫	R(FX) ∗ ∗((− − 01) + +01) == R(FX)∗∗(− −01)+ +R(FX)∗∗01	;
L09:	==Symmetry ▷ L08 ≫	R(FX) ∗ ∗(− − 01) + +R(FX) ∗ ∗01 == R(FX) ∗ ∗((− − 01) + +01)	;
L10:	eqTransitivity ▷ L09 ▷ L07 ≫	R(FX) ∗ ∗(− − 01) + +R(FX) ∗ ∗01 == 00	;
L11:	PositiveToRight(Eq)(R) ▷ L10 ≫	R(FX) ∗ ∗(− − 01) == 00 + +(− − (R(FX) ∗ ∗01))	;
L12:	Plus0Left(R) ≫	00 + +(− − (R(FX) ∗ ∗01)) == (− − (R(FX) ∗ ∗01))	;
L13:	eqTransitivity ▷ L11 ▷ L12 ≫	R(FX) ∗ ∗(− − 01) == (− − (R(FX) ∗ ∗01))	;
L14:	Times1(R) ≫	R(FX) ∗ ∗01 == R(FX)	;
L15:	EqNegated(R) ▷ L14 ≫	(− − (R(FX) ∗ ∗01)) == (− − R(FX))	;
L16:	eqTransitivity ▷ L13 ▷ L15 ≫	R(FX) ∗ ∗(− − 01) == (− − R(FX))	□

[SystemQ **lemma** Times(−1)Left(R): ΠFX: (− − 01) ∗ ∗R(FX) == (− − R(FX))]

SystemQ **proof of** Times(−1)Left(R):

L01:	Arbitrary ≫	FX	;
L02:	Times(−1)(R) ≫	R(FX) ∗ ∗(− − 01) == (− − R(FX))	;
L03:	TimesCommutativity(R) ≫	(− − 01) ∗ ∗R(FX) == R(FX) ∗ ∗(− − 01)	;

L04: eqTransitivity ▷ L03 ▷ L02 ≫ (− − 01) ∗ ∗R(FX) == (− − R(FX)) □

(************)

[SystemQ **lemma** − x − y = −(x + y)(R): ΠFX, FY: (− − R(FX)) + +(− − R(FY)) == (− − (R(FX) + +R(FY)))]

SystemQ **proof of** − x − y = −(x + y)(R):

L01:	Arbitrary ≫	FX, FY	;
L02:	Times(−1)Left(R) ≫	(− − 01) ∗ ∗R(FX) == (− − R(FX))	;
L03:	Times(−1)Left(R) ≫	(− − 01) ∗ ∗R(FY) == (− − R(FY))	;
L04:	AddEquations(R) ▷ L02 ▷ L03 ≫	(− − 01) ∗ ∗R(FX) + +(− − 01) ∗ ∗R(FY) == (− − R(FX)) + +(− − R(FY))	;
L05:	==Symmetry ▷ L04 ≫	(− − R(FX)) + +(− − R(FY)) == (− − 01) ∗ ∗R(FX) + +(− − 01) ∗ ∗R(FY)	;
L06:	DistributionOut(R) ≫	(− − 01) ∗ ∗R(FX) + +(− − 01) ∗ ∗R(FY) == (− − 01) ∗ ∗(R(FX) + +R(FY))	;
L07:	Times(−1)Left(R) ≫	(− − 01) ∗ ∗(R(FX) + +R(FY)) == (− − (R(FX) + +R(FY)))	;
L08:	eqTransitivity4 ▷ L05 ▷ L06 ▷ L07 ≫	(− − R(FX)) + +(− − R(FY)) == (− − (R(FX) + +R(FY)))	□

[SystemQ **lemma** LessTotality(R): ΠFX, FY: R(FX) << R(FY) $\dot{\vee}$ R(FX) == R(FY) $\dot{\vee}$ R(FY) << R(FX)]

SystemQ **proof of** LessTotality(R):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	FX, FY	;
L03:	Premise ≫	$\dot{\neg}$ (R(FX) << R(FY))n	;
L04:	Premise ≫	R(FX)!! == R(FY)	;
L05:	FromNotLess(R) ▷ L03 ≫	R(FY) <<== R(FX)	;
L06:	!! == Symmetry ▷ L04 ≫	R(FY)!! == R(FX)	;
L07:	LeqNeqLess(R) ▷ L05 ▷ L06 ≫	R(FY) << R(FX)	;
L08:	Block ≫	End	;
L09:	Arbitrary ≫	FX, FY	;
L10:	Ded ▷ L08 ≫	$\dot{\neg}$ (R(FX) << R(FY))n ⇒ R(FX)!! == R(FY) ⇒ R(FY) << R(FX)	;
L11:	Repetition ▷ L10 ≫	R(FX) << R(FY) $\dot{\vee}$ R(FX) == R(FY) $\dot{\vee}$ R(FY) << R(FX)	□

[SystemQ **lemma** SameNumerical(R): ΠFX, FY: R(FX) == R(FY) ⊢ |rR(FX)| |rR(FY)|]

SystemQ **proof of** SameNumerical(R):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	FX, FY	;
L03:	Premise ≫	00 <<== R(FX)	;
L04:	Premise ≫	R(FX) == R(FY)	;
L05:	NonnegativeNumerical(R) ▷ L03 ≫	|rR(FX)| == R(FX)	;
L06:	SubLeqRight(R) ▷ L04 ▷ L03 ≫	00 <<== R(FY)	;
L07:	NonnegativeNumerical(R) ▷ L06 ≫	|rR(FY)| == R(FY)	;
L08:	==Symmetry ▷ L07 ≫	R(FY) == |rR(FY)|	;
L09:	eqTransitivity4 ▷ L05 ▷ L04 ▷ L08 ≫	|rR(FX)| == |rR(FY)|	;
L10:	Block ≫	End	;
L11:	Block ≫	Begin	;
L12:	Arbitrary ≫	FX, FY	;
L13:	Premise ≫	¬̇ (00 <<== R(FX))n	;
L14:	Premise ≫	R(FX) == R(FY)	;
L15:	ToLess(R) ▷ L13 ≫	R(FX) << 00	;
L16:	NegativeNumerical(R) ▷ L15 ≫	|rR(FX)| == (− − R(FX))	;
L17:	SubLessLeft(R) ▷ L14 ▷ L15 ≫	R(FY) << 00	;
L18:	NegativeNumerical(R) ▷ L17 ≫	|rR(FY)| == (− − R(FY))	;
L19:	==Symmetry ▷ L18 ≫	(− − R(FY)) == |rR(FY)|	;
L20:	EqNegated(R) ▷ L14 ≫	(− − R(FX)) == (− − R(FY))	;
L21:	eqTransitivity4 ▷ L16 ▷ L20 ▷ L19 ≫	|rR(FX)| == |rR(FY)|	;
L22:	Block ≫	End	;
L23:	Arbitrary ≫	FX, FY	;
L24:	Premise ≫	R(FX) == R(FY)	;
L25:	Ded ▷ L10 ≫	00 <<== R(FX) ⇒ R(FX) == R(FY) ⇒ |rR(FX)| == |rR(FY)|	;
L26:	Ded ▷ L22 ≫	¬̇ (00 <<== R(FX))n ⇒ R(FX) == R(FY) ⇒ |rR(FX)| == |rR(FY)|	;
L27:	FromNegations ▷ L25 ▷ L26 ≫	R(FX) == R(FY) ⇒ |rR(FX)| == |rR(FY)|	;
L28:	MP ▷ L27 ▷ L24 ≫	|rR(FX)| == |rR(FY)|	□

[SystemQ **lemma** MinusNegated(R): ΠFX, FY: (−−(R(FX)++(−−R(FY)))) R(FY) + +(− − R(FX))]

SystemQ **proof of** MinusNegated(R):

L01:	Arbitrary ≫	FX, FY	;
L02:	DoubleMinus(R) ≫	(− − (− − R(FY))) == R(FY)	;

L03:	EqAddition(R) ▷ L02 ≫	(− − (− − R(FY))) + +(− − R(FX)) == R(FY) + +(− − R(FX))	;
L04:	==Symmetry ▷ L03 ≫	R(FY)++(−−R(FX)) == (−− (− − R(FY))) + +(− − R(FX))	;
L05:	−x − y = −(x + y)(R) ≫	(− − (− − R(FY))) + +(− − R(FX)) == (−−((−−R(FY))+ +R(FX)))	;
L06:	PlusCommutativity(R) ≫	(− − R(FY)) + +R(FX) == R(FX) + +(− − R(FY))	;
L07:	EqNegated(R) ▷ L06 ≫	(− − ((− − R(FY)) + +R(FX))) == (− − (R(FX) + +(− − R(FY))))	;
L08:	eqTransitivity4 ▷ L04 ▷ L05 ▷ L07 ≫	R(FY)++(−−R(FX)) == (−− (R(FX) + +(− − R(FY))))	;
L09:	==Symmetry ▷ L08 ≫	(− − (R(FX) + +(− − R(FY)))) == R(FY) + +(− − R(FX))	□

————(24.10.06)

[SystemQ **lemma** PositiveNumerical(R): ΠFX: 00 << R(FX) ⊢ |rR(FX)| == R(FX)]

SystemQ **proof of** PositiveNumerical(R):

L01:	Arbitrary ≫	FX	;
L02:	Premise ≫	00 << R(FX)	;
L03:	LessLeq(R) ▷ L02 ≫	00 <<== R(FX)	;
L04:	NonnegativeNumerical(R) ▷ L03 ≫	|rR(FX)| == R(FX)	□

[SystemQ **lemma** SignNumerical(+)(R): ΠFX: 00 << R(FX) ⊢ |rR(FX)| == |r(− − R(FX))|]

SystemQ **proof of** SignNumerical(+)(R):

L01:	Arbitrary ≫	FX	;
L02:	Premise ≫	00 << R(FX)	;
L03:	PositiveNumerical(R) ▷ L02 ≫	|rR(FX)| == R(FX)	;
L04:	PositiveNegated(R) ▷ L02 ≫	(− − R(FX)) << 00	;
L05:	NegativeNumerical(R)▷L04 ≫	|r(− − R(FX))| == (− − (− − R(FX)))	;
L06:	DoubleMinus(R) ≫	(− − (− − R(FX))) == R(FX)	;
L07:	eqTransitivity ▷ L05 ▷ L06 ≫	|r(− − R(FX))| == R(FX)	;
L08:	==Symmetry ▷ L07 ≫	R(FX) == |r(− − R(FX))|	;
L09:	eqTransitivity ▷ L03 ▷ L08 ≫	|rR(FX)| == |r(− − R(FX))|	□

[SystemQ **lemma** SignNumerical(R): ΠFX: |rR(FX)| == |r(− − R(FX))|]

SystemQ **proof of** SignNumerical(R):

L01:	Block ≫	Begin	;
L02:	Arbitrary ≫	FX	;
L03:	Premise ≫	R(FX) << 00	;

L04:	NegativeNegated(R) ▷ L03 ≫	00 << (− − R(FX))	;
L05:	SignNumerical(+)(R) ▷ L04 ≫	|r(− − R(FX))| == |r(− − (− − R(FX)))|	;
L06:	DoubleMinus(R) ≫	(− − (− − R(FX))) == R(FX)	;
L07:	SameNumerical(R) ▷ L06 ≫	|r(− − (− − R(FX)))| == |rR(FX)|	;
L08:	eqTransitivity ▷ L05 ▷ L07 ≫	|r(− − R(FX))| == |rR(FX)|	;
L09:	==Symmetry ▷ L08 ≫	|rR(FX)| == |r(− − R(FX))|	;
L10:	Block ≫	End	;
L11:	Block ≫	Begin	;
L12:	Arbitrary ≫	FX	;
L03:	Premise ≫	R(FX) == 00	;
L04:	EqNegated(R) ▷ L03 ≫	(− − R(FX)) == (− − 00)	;
L05:	−0 = 0(R) ≫	(− − 00) == 00	;
L06:	==Symmetry ▷ L03 ≫	00 == R(FX)	;
L07:	eqTransitivity4 ▷ L04 ▷ L05 ▷ L06 ≫	(− − R(FX)) == R(FX)	;
L08:	==Symmetry ▷ L07 ≫	R(FX) == (− − R(FX))	;
L13:	SameNumerical(R) ▷ L08 ≫	|rR(FX)| == |r(− − R(FX))|	;
L14:	Block ≫	End	;
L15:	Block ≫	Begin	;
L16:	Arbitrary ≫	FX	;
L03:	Premise ≫	00 << R(FX)	;
L17:	SignNumerical(+)(R) ▷ L03 ≫	|rR(FX)| == |r(− − R(FX))|	;
L18:	Block ≫	End	;
L19:	Arbitrary ≫	FX	;
L20:	Ded ▷ L10 ≫	R(FX) << 00 ⇒ |rR(FX)| == |r(− − R(FX))|	;
L21:	Ded ▷ L14 ≫	R(FX) == 00 ⇒ |rR(FX)| == |r(− − R(FX))|	;
L22:	Ded ▷ L18 ≫	00 << R(FX) ⇒ |rR(FX)| == |r(− − R(FX))|	;
L23:	LessTotality(R) ≫	R(FX) << 00 ∨̇ R(FX) == 00 ∨̇ 00 << R(FX)	;
L24:	From3Disjuncts ▷ L23 ▷ L20 ▷ L21 ▷ L22 ≫	|rR(FX)| == |r(− − R(FX))|	□

[SystemQ **lemma** NumericalDifference(R): ΠFX, FY: |rR(FX)++(−−R(FY))| |rR(FY) + +(− − R(FX))|]

SystemQ **proof of** NumericalDifference(R):

L01:	Arbitrary ≫	FX, FY	;
L02:	SignNumerical(R) ≫	|rR(FX) + +(− − R(FY))| == |r(−−(R(FX)++(−−R(FY))))|	;
L03:	MinusNegated(R) ≫	(− − (R(FX) + +(− − R(FY)))) == R(FY) + +(− − R(FX))	;

L04:	SameNumerical(R) ▷ L03 ≫	|r(− − (R(FX) + +(− − R(FY))))| == |rR(FY) + +(− − R(FX))| ;
L05:	eqTransitivity ▷ L02 ▷ L04 ≫	|rR(FX) + +(− − R(FY))| == |rR(FY) + +(− − R(FX))| □

———(25.10.06)

[SystemQ **lemma** x <= |x|(R): ΠFX: R(FX) <<== |rR(FX)|]

SystemQ **proof of** x <= |x|(R):

L01:	Block ≫	Begin ;
L02:	Arbitrary ≫	FX ;
L03:	Premise ≫	00 <<== R(FX) ;
L04:	NonnegativeNumerical(R) ≫	|rR(FX)| == R(FX) ;
L05:	==Symmetry ▷ L04 ≫	R(FX) == |rR(FX)| ;
L06:	lemma eqLeq(R) ▷ L05 ≫	R(FX) <<== |rR(FX)| ;
L07:	Block ≫	End ;
L08:	Block ≫	Begin ;
L09:	Arbitrary ≫	FX ;
L03:	Premise ≫	R(FX) <<== 00 ;
L04:	0 <= |x|(R) ≫	00 <<== |rR(FX)| ;
L10:	LeqTransitivity(R) ▷ L03 ▷ L04 ≫	R(FX) <<== |rR(FX)| ;
L11:	Block ≫	End ;
L12:	Arbitrary ≫	FX ;
L03:	Ded ▷ L07 ≫	00 <<== R(FX) ⇒ R(FX) <<== |rR(FX)| ;
L04:	Ded ▷ L11 ≫	R(FX) <<== 00 ⇒ R(FX) <<== |rR(FX)| ;
L13:	FromLeqGeq(R) ▷ L03 ▷ L04 ≫	R(FX) <<== |rR(FX)| □

a

venter—-

Priority table

Preassociative

[sup2], [base], [bracket ∗ end bracket], [big bracket ∗ end bracket], [\$ ∗ \$], [**flush left** [∗]], [x], [y], [z], [[∗ ⋈ ∗]], [[∗ $\xrightarrow{*}$ ∗]], [pyk], [tex], [name], [prio], [∗], [T], [if(∗, ∗, ∗)], [[∗ $\stackrel{*}{\Rightarrow}$ ∗]], [val], [claim], [⊥], [f(∗)], [(∗)$^{\mathrm{I}}$], [F], [0̲], [1̲], [2̲], [3̲], [4̲], [5̲], [6̲], [7̲], [8̲], [9̲], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(∗)$^{\mathrm{M}}$], [If(∗, ∗, ∗)], [array{∗} ∗ end array], [l], [c], [r], [empty], [⟨∗ | ∗ := ∗⟩], [$\mathcal{M}$(∗)], [$\tilde{\mathcal{U}}$(∗)], [$\mathcal{U}$(∗)], [$\mathcal{U}^{\mathrm{M}}$(∗)], [**apply**(∗, ∗)], [$\mathbf{apply}_1$(∗, ∗)], [identifier(∗)], [$\mathrm{identifier}_1$(∗, ∗)], [array-plus(∗, ∗)], [array-remove(∗, ∗, ∗)], [array-put(∗, ∗, ∗, ∗)], [array-add(∗, ∗, ∗, ∗, ∗)], [bit(∗, ∗)], [bit_1(∗, ∗)], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],

["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*,*,*)$], [$\mathcal{E}_2(*,*,*,*,*)$], [$\mathcal{E}_3(*,*,*,*)$], [$\mathcal{E}_4(*,*,*,*)$], [$\mathbf{lookup}(*,*,*)$], [$\mathbf{abstract}(*,*,*,*)$], [$\lceil * \rceil$], [$\mathcal{M}(*,*,*)$], [$\mathcal{M}_2(*,*,*,*)$], [$\mathcal{M}^*(*,*,*)$], [macro], [$\mathrm{s}_0$], [$\mathbf{zip}(*,*)$], [$\mathbf{assoc}_1(*,*,*)$], [$(*)^{\mathbf{P}}$], [self], [[$* \ddot{=} *$]], [[$* \dot{=} *$]], [[$* \acute{=} *$]], [[$* \overset{\text{pyk}}{=} *$]], [[$* \overset{\text{tex}}{=} *$]], [[$* \overset{\text{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*,*,*,*)$], [$\tilde{\mathcal{M}}(*,*,*)$], [$\tilde{\mathcal{Q}}(*,*,*)$], [$\tilde{\mathcal{Q}}_2(*,*,*)$], [$\tilde{\mathcal{Q}}_3(*,*,*,*)$], [$\tilde{\mathcal{Q}}^*(*,*,*)$], [$(*)$], [$(*)$], [display($*$)], [statement($*$)], [$[*]^{\cdot}$], [$[*]^{-}$], [$\mathbf{aspect}(*,*)$], [$\mathbf{aspect}(*,*,*)$], [$\langle * \rangle$], [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], [$\mathrm{let}_2(*,*)$], [$\mathrm{let}_1(*,*)$], [[$* \overset{\text{claim}}{=} *$]], [checker], [$\mathbf{check}(*,*)$], [$\mathbf{check}_2(*,*,*)$], [$\mathbf{check}_3(*,*,*)$], [$\mathbf{check}^*(*,*)$], [$\mathbf{check}_2^*(*,*,*)$], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \overset{\text{msg}}{=} *$]], [<stmt>], [stmt], [[$* \overset{\text{stmt}}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\perp\!\!\!\perp$], [Contra′], [T'_{E}], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\mathbf{V}}$], [intro($*,*,*,*$)], [intro($*,*,*$)], [error($*,*$)], [$\mathrm{error}_2(*,*)$], [proof($*,*,*$)], [$\mathrm{proof}_2(*,*)$], [$\mathcal{S}(*,*)$], [$\mathcal{S}^{\mathrm{I}}(*,*)$], [$\mathcal{S}^{\rhd}(*,*)$], [$\mathcal{S}_1^{\rhd}(*,*,*)$], [$\mathcal{S}^{\mathrm{E}}(*,*)$], [$\mathcal{S}_1^{\mathrm{E}}(*,*,*)$], [$\mathcal{S}^{+}(*,*)$], [$\mathcal{S}_1^{+}(*,*,*)$], [$\mathcal{S}^{-}(*,*)$], [$\mathcal{S}_1^{-}(*,*,*)$], [$\mathcal{S}^{*}(*,*)$], [$\mathcal{S}_1^{*}(*,*,*)$], [$\mathcal{S}_2^{*}(*,*,*,*)$], [$\mathcal{S}^{@}(*,*)$], [$\mathcal{S}_1^{@}(*,*,*)$], [$\mathcal{S}^{\vdash}(*,*)$], [$\mathcal{S}_1^{\vdash}(*,*,*,*)$], [$\mathcal{S}^{\Vdash}(*,*)$], [$\mathcal{S}_1^{\Vdash}(*,*,*,*)$], [$\mathcal{S}^{\mathrm{i.e.}}(*,*)$], [$\mathcal{S}_1^{\mathrm{i.e.}}(*,*,*,*)$], [$\mathcal{S}_2^{\mathrm{i.e.}}(*,*,*,*,*)$], [$\mathcal{S}^{\forall}(*,*)$], [$\mathcal{S}_1^{\forall}(*,*,*,*)$], [$\mathcal{S}^{;}(*,*)$], [$\mathcal{S}_1^{;}(*,*,*)$], [$\mathcal{S}_2^{;}(*,*,*,*)$], [$\mathcal{T}(*)$], [claims($*,*,*$)], [$\mathrm{claims}_2(*,*,*)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]], [[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut($*,*$)], [$\mathrm{Head}_{\oplus}(*)$], [$\mathrm{Tail}_{\oplus}(*)$], [$\mathrm{rule}_1(*,*)$], [rule($*,*$)], [Rule tactic], [Plus($*,*$)], [[**Theory** $*$]], [$\mathrm{theory}_2(*,*)$], [$\mathrm{theory}_3(*,*)$], [$\mathrm{theory}_4(*,*,*)$], [HeadNil″], [HeadPair″], [Transitivity″], [Contra″], [HeadNil], [HeadPair], [Transitivity], [Contra], [T_{E}], [ragged right], [ragged right expansion], [parm($*,*,*$)], [$\mathrm{parm}^*(*,*,*)$], [inst($*,*$)], [$\mathrm{inst}^*(*,*)$], [occur($*,*,*$)], [$\mathrm{occur}^*(*,*,*)$], [unify($* = *,*$)], [$\mathrm{unify}^*(* = *,*)$], [$\mathrm{unify}_2(* = *,*)$], [$\mathrm{L}_{\mathrm{a}}$], [$\mathrm{L}_{\mathrm{b}}$], [$\mathrm{L}_{\mathrm{c}}$], [$\mathrm{L}_{\mathrm{d}}$], [$\mathrm{L}_{\mathrm{e}}$], [$\mathrm{L}_{\mathrm{f}}$], [$\mathrm{L}_{\mathrm{g}}$], [$\mathrm{L}_{\mathrm{h}}$], [$\mathrm{L}_{\mathrm{i}}$], [$\mathrm{L}_{\mathrm{j}}$], [$\mathrm{L}_{\mathrm{k}}$], [$\mathrm{L}_{\mathrm{l}}$], [$\mathrm{L}_{\mathrm{m}}$], [$\mathrm{L}_{\mathrm{n}}$], [$\mathrm{L}_{\mathrm{o}}$], [$\mathrm{L}_{\mathrm{p}}$], [$\mathrm{L}_{\mathrm{q}}$], [$\mathrm{L}_{\mathrm{r}}$], [$\mathrm{L}_{\mathrm{s}}$], [$\mathrm{L}_{\mathrm{t}}$], [$\mathrm{L}_{\mathrm{u}}$], [$\mathrm{L}_{\mathrm{v}}$], [$\mathrm{L}_{\mathrm{w}}$], [$\mathrm{L}_{\mathrm{x}}$], [$\mathrm{L}_{\mathrm{y}}$], [$\mathrm{L}_{\mathrm{z}}$], [$\mathrm{L}_{\mathrm{A}}$], [$\mathrm{L}_{\mathrm{B}}$], [$\mathrm{L}_{\mathrm{C}}$], [$\mathrm{L}_{\mathrm{D}}$], [$\mathrm{L}_{\mathrm{E}}$], [$\mathrm{L}_{\mathrm{F}}$], [$\mathrm{L}_{\mathrm{G}}$], [$\mathrm{L}_{\mathrm{H}}$], [$\mathrm{L}_{\mathrm{I}}$], [$\mathrm{L}_{\mathrm{J}}$], [$\mathrm{L}_{\mathrm{K}}$], [$\mathrm{L}_{\mathrm{L}}$], [$\mathrm{L}_{\mathrm{M}}$], [$\mathrm{L}_{\mathrm{N}}$], [$\mathrm{L}_{\mathrm{O}}$], [$\mathrm{L}_{\mathrm{P}}$], [$\mathrm{L}_{\mathrm{Q}}$], [$\mathrm{L}_{\mathrm{R}}$], [$\mathrm{L}_{\mathrm{S}}$], [$\mathrm{L}_{\mathrm{T}}$], [$\mathrm{L}_{\mathrm{U}}$], [$\mathrm{L}_{\mathrm{V}}$], [$\mathrm{L}_{\mathrm{W}}$], [$\mathrm{L}_{\mathrm{X}}$], [$\mathrm{L}_{\mathrm{Y}}$], [$\mathrm{L}_{\mathrm{Z}}$], [$\mathrm{L}_{?}$], [Reflexivity], [$\mathrm{Reflexivity}_1$], [Commutativity], [$\mathrm{Commutativity}_1$], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$\mathrm{p}_0$], [$\mathrm{conclude}_1(*,*)$], [$\mathrm{conclude}_2(*,*,*)$], [$\mathrm{conclude}_3(*,*,*,*)$], [$\mathrm{conclude}_4(*,*)$], [check], [[$* \overset{\circ}{=} *$]], [RootVisible($*$)], [A], [R], [C], [T], [L], [$\{*\}$], [$\overline{*}$], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [x], [y], [z], [$\langle * \equiv * \mid * := * \rangle$], [$\langle * \equiv^0 * \mid * := * \rangle$], [$\langle * \equiv^1 * \mid * := * \rangle$], [$\langle * \equiv^* * \mid * := * \rangle$], [Ded($*,*$)], [$\mathrm{Ded}_0(*,*)$], [$\mathrm{Ded}_1(*,*,*)$], [$\mathrm{Ded}_2(*,*,*)$], [$\mathrm{Ded}_3(*,*,*,*)$], [$\mathrm{Ded}_4(*,*,*,*)$], [$\mathrm{Ded}_4^*(*,*,*,*)$], [$\mathrm{Ded}_5(*,*,*)$], [$\mathrm{Ded}_6(*,*,*,*)$], [$\mathrm{Ded}_6^*(*,*,*,*)$], [$\mathrm{Ded}_7(*)$], [$\mathrm{Ded}_8(*,*)$], [$\mathrm{Ded}_8^*(*,*)$], [S], [Neg], [MP], [Gen], [Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1′], [A2′], [A4′], [A5′], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop $3.2\mathrm{e}_1$], [Prop $3.2\mathrm{e}_2$], [Prop 3.2e], [Prop $3.2\mathrm{f}_1$], [Prop $3.2\mathrm{f}_2$], [Prop 3.2f], [Prop $3.2\mathrm{g}_1$], [Prop $3.2\mathrm{g}_2$], [Prop 3.2g], [Prop 3.2h$_1$], [Prop 3.2h$_2$], [Prop 3.2h], [Block$_1(*, *, *)$], [Block$_2(*)$], [kvanti], [UniqueMember], [UniqueMember(Type)], [SameSeries], [A4], [SameMember], [Qclosed(Addition)], [Qclosed(Multiplication)], [FromCartProd(1)], [1rule fromCartProd(2)], [constantRationalSeries($*$)], [cartProd($*$)], [Power($*$)], [binaryUnion($*, *$)], [SetOfRationalSeries], [IsSubset($*, *$)], [(p$*, *$)], [(s$*$)], [($\cdots$)], [`Objekt-var`], [`Ex-var`], [`Ph-var`], [`Værdi`], [`Variabel`], [Op($*$)], [Op($*, *$)], [$* \ddot{=\!=} *$], [ContainsEmpty($*$)], [Nat($*$)], [Dedu($*, *$)], [Dedu$_0(*, *)$], [Dedu$_\mathrm{s}(*, *, *)$], [Dedu$_1(*, *, *)$], [Dedu$_2(*, *, *)$], [Dedu$_3(*, *, *, *)$], [Dedu$_4(*, *, *, *)$], [Dedu$^*_4(*, *, *, *)$], [Dedu$_5(*, *, *)$], [Dedu$_6(*, *, *, *)$], [Dedu$^*_6(*, *, *, *)$], [Dedu$_7(*)$], [Dedu$_8(*, *)$], [Dedu$^*_8(*, *)$], [Ex$_1$], [Ex$_2$], [Ex3], [Ex$_{10}$], [Ex$_{20}$], [$*_\mathrm{Ex}$], [$*^\mathrm{Ex}$], [$\langle * \equiv * \mid * :== * \rangle_\mathrm{Ex}$], [$\langle * \equiv^0 * \mid * :== * \rangle_\mathrm{Ex}$], [$\langle * \equiv^1 * \mid * :== * \rangle_\mathrm{Ex}$], [$\langle * \equiv^* * \mid * :== * \rangle_\mathrm{Ex}$], [ph$_1$], [ph$_2$], [ph$_3$], [$*_\mathrm{Ph}$], [$*^\mathrm{Ph}$], [$\langle * \equiv * \mid * :== * \rangle_\mathrm{Ph}$], [$\langle * \equiv^0 * \mid * :== * \rangle_\mathrm{Ph}$], [$\langle * \equiv^1 * \mid * :== * \rangle_\mathrm{Ph}$], [$\langle * \equiv^* * \mid * :== * \rangle_\mathrm{Ph}$], [$\langle * \equiv * \mid * :== * \rangle_\mathrm{Me}$], [$\langle * \equiv^1 * \mid * :== * \rangle_\mathrm{Me}$], [$\langle * \equiv^* * \mid * :== * \rangle_\mathrm{Me}$], [bs], [OBS], [$\mathcal{BS}$], [Ø], [SystemQ], [MP], [Gen], [Repetition], [Neg], [Ded], [ExistIntro], [Extensionality], [Ødef], [PairDef], [UnionDef], [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg], [AndCommutativity], [AutoImply], [Contrapositive], [FirstConjunct], [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity], [IffFirst], [IffSecond], [ImplyTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4], [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1], [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union], [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power], [SubsetInPower], [HelperPowerIsSub], [PowerIsSub], [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality], [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality], [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry], [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive], [ERisSymmetric], [ERisTransitive], [ØisSubset], [HelperMemberNotØ], [MemberNotØ], [HelperUniqueØ], [UniqueØ], [== Reflexivity], [== Symmetry], [Helper == Transitivity], [== Transitivity], [HelperTransferNotEq], [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset], [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset], [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection], [AutoMember], [HelperEqSysNotØ], [EqSysNotØ], [HelperEqSubset], [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary], [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset], [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImply], [BSsubset], [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)], [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ϵ)], [(ϵ)$_1$], [(ϵ2)], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)], [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)], [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X$_1$], [X$_2$], [Y$_1$], [Y$_2$], [V$_1$], [V$_2$], [V$_3$], [V$_4$], [V$_{2\mathrm{n}}$], [M$_1$], [M$_2$], [N$_1$], [N$_2$], [N$_3$], [ϵ], [ϵ1], [ϵ2], [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)], [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)], [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)], [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)], [(S1ob)], [(S2ob)], [ph_4], [ph_5], [ph_6], [NAT], [$\mathrm{RATIONAL}_\mathrm{S}$ERIES], [SERIES], [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1], [(-1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [$(--01)$], [02], [01//02], [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)], [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)], [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)], [Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom], [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom], [QisClosed(Reciprocal)(Imply)], [QisClosed(Reciprocal)], [QisClosed(Negative)(Imply)], [QisClosed(Negative)], [leqReflexivity], [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality], [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity], [plusCommutativity], [Negative], [plus0], [timesAssociativity], [timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1], [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)], [(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [UStelescope(Zero)], [UStelescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [NatType], [RationalType], [SeriesType], [Max], [Numerical], [NumericalF], [MemberOfSeries(Imply)], [JoinConjuncts(2conditions)], [prop lemma imply negation], [TND], [FromNegatedImply], [ToNegatedImply], [FromNegated($2 * $Imply)], [FromNegatedAnd], [FromNegatedOr], [ToNegatedOr], [FromNegations], [From3Disjuncts], [From$2 * $2Disjuncts], [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2], [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)], [sup], [ToNegatedAnd(1)], [UniqueNegative], [DoubleMinus], [MinusNegated], [eqReflexivity], [eqSymmetry], [eqTransitivity], [eqTransitivity4], [eqTransitivity5], [eqTransitivity6], [AddEquations], [SubtractEquations], [SubtractEquationsLeft], [MultiplyEquations], [EqNegated], [PositiveToRight(Eq)], [PositiveToLeft(Eq)(1term)], [NegativeToLeft(Eq)], [NonreciprocalToRight(Eq)(1term)], [PlusAssociativity(4terms)], [LessNeq], [NeqSymmetry], [NeqNegated], [SubNeqRight], [SubNeqLeft], [NegativeToRight(Neq)(1term)], [NeqAddition], [NeqMultiplication], [NonzeroProduct(2)], [UStelescope($+1$)], [TelescopeBound(Base)], [TelescopeBound(Indu)], [TelescopeBound], [IntervalSize(Base)], [IntervalSize(Indu)], [IntervalSize], [$\mathrm{XS} < \mathrm{US}$], [lemma USdecreasing($+1$)], [CloseUS], [CloseUS($\mathrm{n}+1$)], [AllNegated(Imply)], [ExistNegated(Imply)], [IntroExist(Helper)], [IntroExist], [ExistMP], [ExistMP2], [TwiceExistMP], [TwiceExistMP2], [EAE − MP], [AddAll], [AddExist(Helper1)], [AddExist(Helper2)], [AddExist], [AddExist(SimpleAnt)], [AddExist(Simple)], [AddEAE], [AEA − negated], [EEA − negated], [Induction], [leqAntisymmetry], [leqTransitivity], [leqAddition], [leqMultiplication], [Reciprocal], [Equality], [eqLeq], [eqAddition], [eqMultiplication], [LeqMultiplicationLeft], [LeqLessEq], [LessLeq], [FromLeqGeq], [subLeqRight], [subLeqLeft], [Leq + 1], [PositiveToRight(Leq)], [PositiveToRight(Leq)(1term)], [lemma negativeToRight(Leq)], [PositiveToLeft(Leq)], [negativeToLeft(Leq)], [negativeToLeft(Leq)(1term)], [LeqAdditionLeft], [leqSubtraction], [leqSubtractionLeft], [thirdGeq], [LeqNegated], [AddEquations(Leq)], [MultiplyEquations(Leq)], [ThirdGeqSeries], [LeqNeqLess], [FromLess], [ToLess], [fromNotLess], [toNotLess], [NegativeLessPositive], [leqLessTransitivity], [LessLeqTransitivity], [LessTransitivity], [LessTotality], [SubLessRight], [SubLessLeft], [SwitchTerms(x < y − z)], [SwitchTerms(x − y < z)], [LessAddition], [LessAdditionLeft], [LessMultiplication], [LessMultiplicationLeft], [LessDivision], [PositiveToRight(Less)], [PositiveToLeft(Less)], [NegativeToLeft(Less)], [NegativeToRight(Less)], [AddEquations(Less)], [AddEquations(LeqLess)], [reciprocalToLeft(Less)], [LessNegated], [PositiveNonzero], [PositiveNegated], [NonpositiveNegated], [NegativeNegated], [NonnegativeNegated], [PositiveHalved], [PositiveInverted], [NonnegativeNumerical], [NegativeNumerical], [PositiveNumerical], [lemma nonpositiveNumerical], [|0| = 0], [0 <= |x|], [x <= |x|], [FromPositiveNumerical], [SameNumerical], [SignNumerical(+)], [SignNumerical], [ToNumericalLess], [FromNumericalGreater], [NumericalDifference], [NumericalDifferenceLess(Helper)], [NumericalDifferenceLess], [SplitNumericalSumHelper], [splitNumericalSum(++)], [splitNumericalSum(−−)], [splitNumericalSum(+ − small)], [splitNumericalSum(+ − big)], [splitNumericalSum(+−)], [splitNumericalSum(−+)], [splitNumericalSum], [SplitNumericalProduct(++)], [SplitNumericalProduct(+−)], [SplitNumericalProduct], [insertMiddleTerm(Numerical)], [insertTwoMiddleTerms(Numerical)], [Three2twoTerms], [Three2threeTerms], [Three2twoFactors], [Three2threeFactors], [Times(−1)], [Times(−1)Left], [MaxLeq(1)], [MaxLeq(2)], [LessThanMax], [x + y = zBackwards], [x * y = zBackwards], [x = x + (y − y)], [x = x + y − y], [x = x * y * (1/y)], [insertMiddleTerm(Sum)], [insertTwoMiddleTerms(Sum)], [insertMiddleTerm(Difference)], [x * 0 + x = x], [x * 0 = 0], [NonnegativeFactors], [NonzeroFactors], [PositiveFactors], [PlusTimesMinus], [MinusTimesMinus], [(−1) * (−1) + (−1) * 1 = 0], [(−1) * (−1) = 1], [0 < 1Helper], [0 < 1], [0 < 2], [0 < 3], [0 < 1/2], [0 < 1/3], [TwoWholes], [ThreeWholes], [TwoHalves], [ThreeThirds], [−x − y = −(x + y)], [−x * y = −(x * y)], [−0 = 0], [SFsymmetry], [SFtransitivity], [f2R(Plus)], [f2R(Times)], [<< TransitivityHelper(Q)], [<< Transitivity], [<<== Reflexivity], [<<== AntisymmetryHelper(Q)], [FromNot < f(Weak)(Helper)], [FromNot < f(Weak)], [FromNot < f(Strong)(Helper2)], [FromNot < f(Strong)(Helper)], [FromNot < f(Strong)], [fromNotSameF(Strongest)(Helper2)], [fromNotSameF(Strongest)(Helper)], [fromNotSameF(Strongest)], [ToLess(F)(Helper)], [ToLess(F)], [FromNot <<], [ToLess(R)], [FromNotSameF(Weak)(Helper)], [FromNotSameF(Weak)], [FromNotLess(F)], [== Addition], [== AdditionLeft], [Fpart − Bounded(Base)], [Fpart − Bounded(InduHelper)], [Fpart − Bounded(Indu)], [Fpart − Bounded], [F − Bounded(Helper)], [F − Bounded], [SameFmultiplication(Helper)], [SameFmultiplication], [EqMultiplication(R)], [EqMultiplicationLeft(R)], [x ∗ 0 = 0(F)], [x ∗ 0 = 0(R)], [LessMultiplication(F)(Helper2)], [LessMultiplication(F)(Helper)], [LessMultiplication(F)], [LessMultiplication(R)], [LeqMultiplication(R)], [PlusAssociativity(F)], [Plus0(F)], [PlusCommutativity(F)], [TimesAssociativity(F)], [Times1f], [Cauchy(2)(Helper)], [Cauchy(2)], [ReciprocalFnonzero], [(Eventually = f)2sameF(Helper)], [(Eventually = f)2sameF], [FromNotSameF(Strong)(Helper2)], [FromNotSameF(Strong)(Helper)], [FromNotSameF(Strong)], [SameFreciprocal(Helper)], [SameFreciprocal], [From!! ==], [Reciprocal(R)], [TimesCommutativity(F)], [Distribution(F)], [FromMax(1)], [FromMax(2)], [ToNegatedAnd], [DistributionOut], [DistributionOutLeft], [DistributionLeft], [FromNotLess(R)], [CartProdIsRelation], [FromSubset], [SubsetIsRelation], [ToSeries], [FromSeries], [SeriesSubsetCP], [ValueType], [RemoveOr], [FromSingleton], [InPair(1)], [InPair(2)], [SameMember(2)], [ToBinaryUnion(1)], [ToBinaryUnion(2)], [FromOrderedPair(TwoLevels)], [ToCartProd(Helper)], [ToCartProd], [NonreciprocalToRight(Eq)], [NonreciprocalToLeft(Eq)(1term)], [SameReciprocal], [CPseparationIsRelation], [OrderedPairEquality], [ReciprocalIsFunction], [ReciprocalIsTotal], [ReciprocalIsRationalSeries], [CrsIsRelation], [CrsIsFunction], [CrsIsTotal], [CrsIsSeries], [CrsLookup], [0f], [1f], [ToSingleton], [FromSameSingleton], [SingletonmembersEqual], [UnequalsNotInSingleton], [NonsingletonmembersUnequal], [FromOrderedPair], [FromOrderedPair(1)], [FromOrderedPair(2)], [FromCartProd], [FromCartProd(1)], [FromCartProd(2)], [sameOrderedPair], [InSeriesHelper], [InSeries], [To = f(Subset)(Helper)], [To = f(Subset)], [To = f], [productIsFunction], [productIsTotal], [ProductIsRationalSeries], [TimesF], [−x + (1/2)x = −(1/2)x], [PositiveTripled], [PositiveDividedBy3], [|x − x| = 0], [1 < 2], [1/3 < 2/3], [(1/3)x + (1/3)x = (2/3)x], [(2/3)x + (1/3)x = x], [−x + (2/3)x = −(1/3)x], [−(1/3)x − (1/3)x = −(2/3)x], [−x + (1/3)x = −(2/3)x], [PreserveLessGreater], [ClosetolessIsLess], [SubLessLeft(F)], [SubLessLeft(R)], [ClosetogreaterIsGreater], [SubLessRight(F)], [SubLessRight(R)], [plus0Left], [times1Left], [EqAdditionLeft], [EqMultiplicationLeft], [PlusF(Sym)], [TimesF(Sym)], [SameSeries(Gen)], [EqualsSameF], [LeqReflexivity(R)], [LeqTotality(R)], [PositiveToLeft(Eq)], [ExpZero(Exact)], [SameExp(Base)], [SameExp(Indu)], [SameExp], [(1/2)(x + y) − x = (1/2)(y − x)], [y − (1/2)(x + y) = (1/2)(y − x)], [BSzero(Exact)], [SameBS(2)(Base)], [SameBS(2)(Indu)], [SameBS(2)], [NegativeToLeft(Less)(1term)], [BS(+1)], [BSbound(Exact)(Base)], [BSbound(Exact)(Indu)], [BSbound(Exact)], [BSbound], [UStelescope(Zero)(Exact)], [SameTelescope(2)(Base)], [SameTelescope(2)(Indu)], [SameTelescope(2)], [Exp(+1)], [PositiveBase(Base)], [PositiveBase(Indu)], [PositiveBase], [TelescopeNumerical(Base)], [TelescopeNumerical(Indu)], [TelescopeNumerical], [(+1)IsPositive(N)], [DistributionOut(Minus)], [PositiveToRight(Eq)(1term)], [SameSeries(NumDiff)], [ToNegatedDoubleImply], [AddNegatedAll], [(A)to(E)(Imply)], [(E)to(A)(Imply)], [(E)to(A)(Imply)], [ToNegatedAEA], [Three2threeTerms(R)], [LessNeq(F)(Helper)], [LessNeq(F)], [LessNeq(R)], [x = x + (y − y)(R)], [x = x + y − y(R)], [SubtractEquations(R)], [NeqAddition(R)], [PositiveToRight(Less)(R)], [PositiveToRight(Less)(1term)(R)], [LeqNeqLess(R)], [SubLeqLeft(R)], [ToLeq(Advanced)(R)], [LeqLessTransitivity(R)], [NegativeToLeft(Eq)(R)], [NegativeToRight(Less)(R)], [!! == Symmetry], [SwitchTerms(x <= y − z)], [Plus0Left(R)], [PositiveToRight(Eq)(R)], [EqAdditionLeft(R)], [Three2twoTerms(R)], [To!! ==], [PositiveToRight(Less)(1term)], [(A)to(E)], [NegativeToRight(Eq)(R)], [NegativeToRight(Eq)(1term)(R)], [DoubleMinus(R)], [UniqueNegative(R)], [SubtractEquationsLeft(R)], [EqNegated(R)], [NeqNegated(R)], [−0 = 0(R)], [NegativeNegated(R)], [FromLeqGeq(R)], [0 <= |x|(R)], [PositiveNegated(R)], [AddEquations(R)], [Times(−1)(R)], [Times(−1)Left(R)], [−x − y = −(x + y)(R)], [LessTotality(R)], [SameNumerical(R)], [MinusNegated(R)], [PositiveNumerical(R)], [SignNumerical(+)(R)], [NonnegativeNumerical(R)], [NegativeNumerical(R)], [LeqNegated(R)], [LessNegated(R)], [SubLeqRight(R)], [FromLess(R)], [DistributionOut(R)], [x ∗ 0 + x = x(R)], [x ∗ 0 = 0(R)(fff)], [SignNumerical(R)], [NumericalDifference(R)], [x <= |x|(R)], [USlimitIsUpperBound(Helper)], [USlimitIsUpperBound], [(−1) ∗ (−1) + (−1) ∗ 1 = 0(R)], [(−1) ∗ (−1) = 1(R)], [0 < 1Helper(R)], [0 < 1(R)], [ExpZero(Exact)(R)], [PositiveBase(R)(Base)], [Three2twoFactors(R)], [x = x ∗ y ∗ (1/y)(R)], [NeqMultiplication(R)], [LessTransitivity(R)], [0 < 2(R)], [SameExp(R)(Base)], [SameExp(R)(Indu)], [SameExp(R)], [SubNeqLeft(R)], [SubNeqRight(R)], [NonzeroFactors(R)], [NonnegativeFactors(R)], [PositiveFactors(R)], [LessDivision(R)], [0 < 1/2(R)], [PositiveToRight(Eq)(1term)(R)], [Exp(+1)(R)], [PositiveBase(R)(Indu)], [PositiveBase(R)], [−x ∗ y = −(x ∗ y)(R)], [PositiveToLeft(Eq)(R)], [Times1Left(R)], [x + x = 2 ∗ x(R)], [(1/2)x + (1/2)x = x(R)], [DistributionOut(Minus)(R)], [(1/2)(x + y) − x = (1/2)(y − x)(R)], [IntervalSize(R)(Base)], [LessMultiplicationLeft(R)], [NegativeToLeft(Less)(R)], [NegativeToLeft(Less)(1term)(R)], [y − (1/2)(x + y) = (1/2)(y − x)(R)], [IntervalSize(R)(Indu)], [IntervalSize(R)], [XSlessUS(R)], [USdecreasing(+1)(R)], [ExpUnbounded(Base)], [ExpUnbounded(Indu)], [ExpUnbounded], [1 <= x + 1(N)], [ExpNonzero(Base)], [ExpNonzero(Indu)], [ExpNonzero], [ExpNonzero(2)], [HalfBase(Base)], [HalfBase(Indu)], [MultiplyEquations(R)], [NonreciprocalToRight(Eq)(1term)(R)], [PositiveNonzero(R)], [NonzeroProduct(2)(R)], [HalfBase], [Three2threeFactors(R)], [x ∗ y = zBackwards(R)], [PositiveInverted(R)], [ReciprocalToRight(Less)(R)], [ReciprocalToRight(Less)(1term)(R)], [NonreciprocalToLeft(Less)(R)], [1 < x ∗ y(R)], [SwitchFactors(1/x < y)(R)],

[SmallHalving], [IntervalSize(anyPositive)], [USdecreasing(+n)(Base)], [USdecreasing(+n)(Indu)], [USdecreasing(+n)], [USdecreasing], [LeqAdditionLeft(R)], [ToNotLess(R)], [LimitOfUSIsLeq], [SubtractEquations(Less)(R)], [SubtractEquationsLeft(Less)(R)], [LessNegated(Negative)(R)], [FromNegatedAnd(Imply)], [RemoveDoubleNeg(Consequent)], [FromNotUpperBound], [LeqNUB], [USlimitIsLeastUpperBound(Helper)], [USlimitIsLeastUpperBound], [ExistMP3], [GreaterPositive(N)], [ysFClose(Helper)], [ysFClose], [ysFCauchy(Helper)], [ysFCauchy], [from <<==], [to <<==], [NonnegativeNumerical(F)], [NegativeNumerical(F)];
Preassociative
[tester1], [tester2], [tester3], [tester4], [tester5], [tester6];
Preassociative
[∗_{∗}], [∗/indexintro(∗, ∗, ∗, ∗)], [∗/intro(∗, ∗, ∗)], [∗/bothintro(∗, ∗, ∗, ∗, ∗)], [∗/nameintro(∗, ∗, ∗, ∗)], [∗′], [∗[∗]], [∗[∗→∗]], [∗[∗⇒∗]], [∗0], [∗1], [0b], [∗-color(∗)], [∗-color*(∗)], $[*^{\mathrm{H}}]$, $[*^{\mathrm{T}}]$, $[*^{\mathrm{U}}]$, $[*^{\mathrm{h}}]$, $[*^{\mathrm{t}}]$, $[*^{\mathrm{s}}]$, $[*^{\mathrm{c}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{a}}]$, $[*^{\mathrm{C}}]$, $[*^{\mathrm{M}}]$, $[*^{\mathrm{B}}]$, $[*^{\mathrm{r}}]$, $[*^{\mathrm{i}}]$, $[*^{\mathrm{d}}]$, $[*^{\mathrm{R}}]$, $[*^{0}]$, $[*^{1}]$, $[*^{2}]$, $[*^{3}]$, $[*^{4}]$, $[*^{5}]$, $[*^{6}]$, $[*^{7}]$, $[*^{8}]$, $[*^{9}]$, $[*^{\mathrm{E}}]$, $[*^{\mathcal{V}}]$, $[*^{\mathcal{C}}]$, $[*^{\mathcal{C}^*}]$, $[*^{\mathrm{hide}}]$;
Preassociative
[“ ∗ ”], [], [(∗)$^{\mathbf{t}}$], [string(∗) + ∗], [string(∗) ++ ∗], [
∗], [∗], [!∗], ["∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗], [0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗], [@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗], [O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [^∗], [_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗], [p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [~∗], [**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end], [newline ∗], [macro newline ∗], [MacroIndent(∗)];
Preassociative
[∗ ’ ∗], [∗ ‘ ∗];
Preassociative
[∗(exp)∗];
Preassociative
[∗′], [R(∗)], [− − R(∗)], [rec∗];
Preassociative
[∗/∗], [∗ ∩ ∗], [∗[∗]];
Preassociative
[∪∗], [∗ ∪ ∗], [P(∗)];
Preassociative
[{∗}], [StateExpand(∗, ∗, ∗)], [extractSeries(∗)], [SetOfSeries(∗)], [− − Macro(∗)], [ExpandList(∗, ∗, ∗)], [∗ ∗ Macro(∗)], [+ + Macro(∗)], [<< Macro(∗)], [||Macro(∗)], [01//Macro(∗)], [UB(∗, ∗)], [LUB(∗, ∗)], [BS(∗, ∗)], [UStelescope(∗, ∗)], [(∗)], [|f ∗ |], [|r ∗ |], [Limit(∗, ∗)], [Union(∗)], [IsOrderedPair(∗, ∗, ∗)], [IsRelation(∗, ∗, ∗)], [isFunction(∗, ∗, ∗)], [IsSeries(∗, ∗)], [IsNatural(∗, ∗)], [OrderedPair(∗, ∗)], [TypeNat(∗)], [TypeNat0(∗)], [TypeRational(∗)], [TypeRational0(∗)], [TypeSeries(∗, ∗)], [Typeseries0(∗, ∗)];

Preassociative
$[\{*,*\}], [\langle *,*\rangle], [(-\mathrm{u}*)], [-_{\mathrm{f}}*], [(--*)], [1\mathrm{f}/*], [01//\mathrm{temp}*]$;
Preassociative
$[*(*,*)], [\mathrm{ReflRel}(*,*)], [\mathrm{SymRel}(*,*)], [\mathrm{TransRel}(*,*)], [\mathrm{EqRel}(*,*)], [[*\in *]_{*}]$, $[\mathrm{Partition}(*,*)]$;
Preassociative
$[*\cdot *], [*\cdot_{0} *], [(***)], [**_{\mathrm{f}} *], [****]$;
Preassociative
$[*+*], [*+_{0}*], [*+_{1}*], [*-*], [*-_{0}*], [*-_{1}*], [(*+*)], [(*-*)], [*+_{\mathrm{f}}*]$, $[*-_{\mathrm{f}}*], [*++*], [\mathrm{R}(*)--\mathrm{R}(*)]$;
Preassociative
$[*\in *]$;
Preassociative
$[|*|], [\mathrm{if}(*,*,*)], [\mathrm{Max}(*,*)], [\mathrm{Max}(*,*)]$;
Preassociative
$[*=*], [*\neq *], [*<=*], [*<*], [*<_{\mathrm{f}}*], [*\leq_{\mathrm{f}}*], [\mathrm{SF}(*,*)], [*==*]$, $[*!!==*], [*<<*], [*<<==*]$;
Preassociative
$[*\cup\{*\}], [*\cup *], [*\backslash\{*\}]$;
Postassociative
$[*\therefore *], [*\underline{\therefore} *], [*\underline{::} *], [*\underline{+2*} *], [*::*], [*+2**]$;
Postassociative
$[*,*]$;
Preassociative
$[*\stackrel{\mathrm{B}}{\approx}*], [*\stackrel{\mathrm{D}}{\approx}*], [*\stackrel{\mathrm{C}}{\approx}*], [*\stackrel{\mathrm{P}}{\approx}*], [*\approx *], [*=*], [*\stackrel{+}{\to}*], [*\stackrel{\mathrm{t}}{=}*], [*\stackrel{\mathrm{t}^*}{=}*], [*\stackrel{\mathrm{r}}{=}*]$, $[*\in_{\mathrm{t}}*], [*\subseteq_{\mathrm{T}}*], [*\stackrel{\mathrm{T}}{=}*], [*\stackrel{\mathrm{s}}{=}*], [*\text{ free in }*], [*\text{ free in}^{*}\ *], [*\text{ free for }*\text{ in }*]$, $[*\text{ free for}^{*}\ *\text{ in }*], [*\in_{\mathrm{c}}*], [*<*], [*<'*], [*\leq' *], [*=*], [*\neq *], [*^{\mathrm{var}}]$, $[*\#^{0}*], [*\#^{1}*], [*\#^{*}*], [*==*], [*\subseteq *]$;
Preassociative
$[\neg *], [\dot{\neg}(*)\mathrm{n}], [*\notin *], [*\neq *]$;
Preassociative
$[*\wedge *], [*\ddot{\wedge}*], [*\tilde{\wedge}*], [*\wedge_{\mathrm{c}}*], [*\dot{\wedge}*]$;
Preassociative
$[*\vee *], [*\parallel *], [*\ddot{\vee}*]$;
Postassociative
$[*\dot{\vee}*]$;
Preassociative
$[\exists *\colon *], [\forall *\colon *], [\forall_{\mathrm{obj}}*\colon *], [\exists *\colon *]$;
Postassociative
$[*\stackrel{..}{\Rightarrow}*], [*\Rightarrow *], [*\Leftrightarrow *], [*\dot{\Leftrightarrow}*]$;
Preassociative
$[\{\mathrm{ph}\in *\mid *\}]$;
Postassociative
$[*:*], [*\text{ spy }*], [*!*]$;
Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$
Preassociative
$[\lambda * .*], [\Lambda * .*], [\Lambda *]$, [**if** $*$ **then** $*$ **else** $*$], [**let** $* = *$ **in** $*$], [**let** $* \stackrel{..}{=} *$ in $*$];
Preassociative
$[*\#*];$
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}];$
Preassociative
$[* @ *], [* \rhd *], [* \gg\!\!\rhd *], [* \gg *], [* \unrhd *];$
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
$[\forall *\colon *], [\Pi *\colon *];$
Postassociative
$[* \oplus *];$
Postassociative
$[*; *];$
Preassociative
[$*$ proves $*$];
Preassociative
[$*$ **proof of** $* : *$], [Line $* : * \gg *; *$], [Last line $* \gg * \square$],
[Line $*$: Premise $\gg *; *$], [Line $*$: Side-condition $\gg *; *$], [Arbitrary $\gg *; *$],
[Local $\gg * = *; *$], [Begin $*; *$: End; $*$], [Last block line $* \gg *$;],
[Arbitrary $\gg *; *$];
Postassociative
$[* \mid *];$
Postassociative
$[* \, , *], [*[\, * \,]*];$
Preassociative
$[*\&*];$
Preassociative
$[*\backslash\backslash*]$, [$*$ linebreak[4] $*$], $[*\backslash\backslash*]$; **End table**

A Pyk definitioner

([LeqTotality(R) $\stackrel{\text{pyk}}{\to}$ “lemma leqTotality(R)”]

[PositiveToLeft(Eq) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToLeft(Eq)”]

[ExpZero(Exact) $\stackrel{\text{pyk}}{\to}$ “lemma expZero exact”]

[SameExp(Base) $\stackrel{\text{pyk}}{\to}$ “lemma sameExp base”]

[SameExp(Indu) $\stackrel{\text{pyk}}{\to}$ “lemma sameExp indu”]

[SameExp $\stackrel{\text{pyk}}{\to}$ “lemma sameExp”]

[$(1/2)(\mathrm{x}+\mathrm{y}) - \mathrm{x} = (1/2)(\mathrm{y} - \mathrm{x})$ $\stackrel{\text{pyk}}{\to}$ “lemma (1/2)(x+y)-x=(1/2)(y-x)”]

[$y - (1/2)(x + y) = (1/2)(y - x)$ $\stackrel{\text{pyk}}{\rightarrow}$ “lemma y-(1/2)(x+y)=(1/2)(y-x)”]
[BSzero(Exact) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum zero exact”]
[SameBS(2)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameBase(1/2)Sum second base”]
[SameBS(2)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameBase(1/2)Sum second indu”]
[SameBS(2) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameBase(1/2)Sum second”]
[NegativeToLeft(Less)(1term) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma negativeToLeft(Less)(1 term)”]
[BS(+1) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum(+1)”]
[BSbound(Exact)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum exact bound base”]
[BSbound(Exact)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum exact bound indu”]
[BSbound(Exact) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum exact bound”]
[BSbound $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum bound”]
[UStelescope(Zero)(Exact) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma UStelescope zero exact”]
[SameTelescope(2)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameTelescope second base”]
[SameTelescope(2)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameTelescope second indu”]
[SameTelescope(2) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameTelescope second”]
[Exp(+1) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma exp(+1)”]
[PositiveBase(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveBase base”]
[PositiveBase(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveBase indu”]
[PositiveBase $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveBase”]
[TelescopeNumerical(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma telescopeNumerical base”]
[TelescopeNumerical(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma telescopeNumerical indu”]
[TelescopeNumerical $\stackrel{\text{pyk}}{\rightarrow}$ “lemma telescopeNumerical”]
[(+1)IsPositive(N) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma +1IsPositive(N)”]
[DistributionOut(Minus) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma distributionOut(Minus)”]
[PositiveToRight(Eq)(1term) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveToRight(Eq)(1 term)”]
[SameSeries(NumDiff) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameSeries(NumDiff)”]
[ToNegatedDoubleImply $\stackrel{\text{pyk}}{\rightarrow}$ “prop lemma to negated double imply”]
[AddNegatedAll $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma addNegatedAll”]
[(A)to(E)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma (A)to(˜E˜)(Imply)”]
[(E)to(A)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma (E)to(˜A˜)(Imply)”]
[(E)to(A)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma (E˜)to(˜A)(Imply)”]
[ToNegatedAEA $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma toNegatedAEA”]
[Three2threeTerms(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma three2threeTerms(R)”]
[LessNeq(F)(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma lessNeq(F) helper”]
[LessNeq(F) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma lessNeq(F)”]

[LessNeq(R) $\stackrel{\text{pyk}}{\to}$ “lemma lessNeq(R)”]

[x = x + (y − y)(R) $\stackrel{\text{pyk}}{\to}$ “lemma x=x+(y-y)(R)”]

[x = x + y − y(R) $\stackrel{\text{pyk}}{\to}$ “lemma x=x+y-y(R)”]

[SubtractEquations(R) $\stackrel{\text{pyk}}{\to}$ “lemma subtractEquations(R)”]

[NeqAddition(R) $\stackrel{\text{pyk}}{\to}$ “lemma neqAddition(R)”]

[PositiveToRight(Less)(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToRight(Less)(R)”]

[PositiveToRight(Less)(1term)(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToRight(Less)(1 term)(R)”]

[LeqNeqLess(R) $\stackrel{\text{pyk}}{\to}$ “lemma leqNeqLess(R)”]

[SubLeqLeft(R) $\stackrel{\text{pyk}}{\to}$ “lemma subLeqLeft(R)”]

[ToLeq(Advanced)(R) $\stackrel{\text{pyk}}{\to}$ “lemma toLeq(Advanced)(R)”]

[LeqLessTransitivity(R) $\stackrel{\text{pyk}}{\to}$ “lemma leqLessTransitivity(R)”]

[NegativeToLeft(Eq)(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToLeft(Eq)(R)”]

[NegativeToRight(Less)(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToRight(Less)(R)”]

[!! == Symmetry $\stackrel{\text{pyk}}{\to}$ “lemma !!==Symmetry”]

[SwitchTerms(x <= y − z) $\stackrel{\text{pyk}}{\to}$ “lemma switchTerms(x<=y-z)”]

[Plus0Left(R) $\stackrel{\text{pyk}}{\to}$ “lemma plus0Left(R)”]

[PositiveToRight(Eq)(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToRight(Eq)(R)”]

[EqAdditionLeft(R) $\stackrel{\text{pyk}}{\to}$ “lemma eqAdditionLeft(R)”]

[Three2twoTerms(R) $\stackrel{\text{pyk}}{\to}$ “lemma three2twoTerms(R)”]

[To!! ==$\stackrel{\text{pyk}}{\to}$ “lemma to!!==”]

[PositiveToRight(Less)(1term) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToRight(Less)(1 term)”]

[(A)to(E) $\stackrel{\text{pyk}}{\to}$ “pred lemma (A˜)to(˜E)”]

[NegativeToRight(Eq)(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToRight(Eq)(R)”]

[NegativeToRight(Eq)(1term)(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToRight(Eq)(1 term)(R)”]

[DoubleMinus(R) $\stackrel{\text{pyk}}{\to}$ “lemma doubleMinus(R)”]

[UniqueNegative(R) $\stackrel{\text{pyk}}{\to}$ “lemma uniqueNegative(R)”]

[SubtractEquationsLeft(R) $\stackrel{\text{pyk}}{\to}$ “lemma subtractEquationsLeft(R)”]

[EqNegated(R) $\stackrel{\text{pyk}}{\to}$ “lemma eqNegated(R)”]

[NeqNegated(R) $\stackrel{\text{pyk}}{\to}$ “lemma neqNegated(R)”]

[−0 = 0(R) $\stackrel{\text{pyk}}{\to}$ “lemma -0=0(R)”]

[NegativeNegated(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeNegated(R)”]

[FromLeqGeq(R) $\stackrel{\text{pyk}}{\to}$ “lemma from leqGeq(R)”]

[0 <= |x|(R) $\stackrel{\text{pyk}}{\to}$ “lemma 0<=|x|(R)”]

[PositiveNegated(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveNegated(R)”]

[AddEquations(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma addEquations(R)”]

[Times(-1)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma times(-1)(R)”]

[Times(-1)Left(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma times(-1)Left(R)”]

[$-x - y = -(x + y)$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma -x-y=-(x+y)(R)”]

[LessTotality(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma lessTotality(R)”]

[SameNumerical(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameNumerical(R)”]

[MinusNegated(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma minusNegated(R)”]

[PositiveNumerical(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveNumerical(R)”]

[SignNumerical(+)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma signNumerical(+)(R)”]

[NonnegativeNumerical(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonnegativeNumerical(R)”]

[NegativeNumerical(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma negativeNumerical(R)”]

[LeqNegated(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma leqNegated(R)”]

[LessNegated(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma lessNegated(R)”]

[SubLeqRight(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma subLeqRight(R)”]

[FromLess(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma fromLess(R)”]

[DistributionOut(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma distributionOut(R)”]

[$x * 0 + x = x$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma x*0+x=x(R)”]

[$x * 0 = 0$(R)(fff) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma x*0=0(R)fff”]

[SignNumerical(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma signNumerical(R)”]

[NumericalDifference(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma numericalDifference(R)”]

[$x <= |x|$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma x<=|x|(R)”]

[USlimitIsUpperBound(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USlimitIsUpperBound helper”]

[USlimitIsUpperBound $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USlimitIsUpperBound”]

[$(-1) * (-1) + (-1) * 1 = 0$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma (-1)*(-1)+(-1)*1=0(R)”]

[$(-1) * (-1) = 1$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma (-1)*(-1)=1(R)”]

[$0 < 1$Helper(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma 0<1Helper(R)”]

[$0 < 1$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma 0<1(R)”]

[ExpZero(Exact)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma expZero exact(R)”]

[PositiveBase(R)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveBase(R) base”]

[Three2twoFactors(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma three2twoFactors(R)”]

[$x = x * y * (1/y)$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma x=x*y*(1/y)(R)”]

[NeqMultiplication(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma neqMultiplication(R)”]

[LessTransitivity(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma lessTransitivity(R)”]

[$0 < 2$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma 0<2(R)”]

[SameExp(R)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameExp(R) base”]

[SameExp(R)(Indu) $\stackrel{\text{pyk}}{\to}$ “lemma sameExp(R) indu”]

[SameExp(R) $\stackrel{\text{pyk}}{\to}$ “lemma sameExp(R)”]

[SubNeqLeft(R) $\stackrel{\text{pyk}}{\to}$ “lemma subNeqLeft(R)”]

[SubNeqRight(R) $\stackrel{\text{pyk}}{\to}$ “lemma subNeqRight(R)”]

[NonzeroFactors(R) $\stackrel{\text{pyk}}{\to}$ “lemma nonzeroFactors(R)”]

[NonnegativeFactors(R) $\stackrel{\text{pyk}}{\to}$ “lemma nonnegativeFactors(R)”]

[PositiveFactors(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveFactors(R)”]

[LessDivision(R) $\stackrel{\text{pyk}}{\to}$ “lemma lessDivision(R)”]

[$0 < 1/2$(R) $\stackrel{\text{pyk}}{\to}$ “lemma 0<1/2(R)”]

[PositiveToRight(Eq)(1term)(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToRight(Eq)(1 term)(R)”]

[Exp(+1)(R) $\stackrel{\text{pyk}}{\to}$ “lemma exp(+1)(R)”]

[PositiveBase(R)(Indu) $\stackrel{\text{pyk}}{\to}$ “lemma positiveBase(R) indu”]

[PositiveBase(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveBase(R)”]

[$-\mathrm{x} * \mathrm{y} = -(\mathrm{x} * \mathrm{y})$(R) $\stackrel{\text{pyk}}{\to}$ “lemma -x∗y=-(x∗y)(R)”]

[PositiveToLeft(Eq)(R) $\stackrel{\text{pyk}}{\to}$ “lemma positiveToLeft(Eq)(R)”]

[Times1Left(R) $\stackrel{\text{pyk}}{\to}$ “lemma times1Left(R)”]

[$\mathrm{x} + \mathrm{x} = 2 * \mathrm{x}$(R) $\stackrel{\text{pyk}}{\to}$ “lemma x+x=2∗x(R)”]

[$(1/2)\mathrm{x} + (1/2)\mathrm{x} = \mathrm{x}$(R) $\stackrel{\text{pyk}}{\to}$ “lemma (1/2)x+(1/2)x=x(R)”]

[DistributionOut(Minus)(R) $\stackrel{\text{pyk}}{\to}$ “lemma distributionOut(Minus)(R)”]

[$(1/2)(\mathrm{x} + \mathrm{y}) - \mathrm{x} = (1/2)(\mathrm{y} - \mathrm{x})$(R) $\stackrel{\text{pyk}}{\to}$ “lemma (1/2)(x+y)-x=(1/2)(y-x)(R)”]

[IntervalSize(R)(Base) $\stackrel{\text{pyk}}{\to}$ “lemma intervalSize(R) base”]

[LessMultiplicationLeft(R) $\stackrel{\text{pyk}}{\to}$ “lemma lessMultiplicationLeft(R)”]

[NegativeToLeft(Less)(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToLeft(Less)(R)”]

[NegativeToLeft(Less)(1term)(R) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToLeft(Less)(1 term)(R)”]

[$\mathrm{y} - (1/2)(\mathrm{x} + \mathrm{y}) = (1/2)(\mathrm{y} - \mathrm{x})$(R) $\stackrel{\text{pyk}}{\to}$ “lemma y-(1/2)(x+y)=(1/2)(y-x)(R)”]

[IntervalSize(R)(Indu) $\stackrel{\text{pyk}}{\to}$ “lemma intervalSize(R) indu”]

[IntervalSize(R) $\stackrel{\text{pyk}}{\to}$ “lemma intervalSize(R)”]

[XSlessUS(R) $\stackrel{\text{pyk}}{\to}$ “lemma XSlessUS(R)”]

[USdecreasing(+1)(R) $\stackrel{\text{pyk}}{\to}$ “lemma USdecreasing(+1)(R)”]

[ExpUnbounded(Base) $\stackrel{\text{pyk}}{\to}$ “lemma expUnbounded base”]

[ExpUnbounded(Indu) $\stackrel{\text{pyk}}{\to}$ “lemma expUnbounded indu”]

[ExpUnbounded $\stackrel{\text{pyk}}{\to}$ “lemma expUnbounded”]

[$1 <= \mathrm{x} + 1$(N) $\stackrel{\text{pyk}}{\to}$ “lemma 1<=x+1(N)”]

[ExpNonzero(Base) $\stackrel{\text{pyk}}{\to}$ “lemma expNonzero base”]

[ExpNonzero(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma expNonzero indu”]

[ExpNonzero $\stackrel{\text{pyk}}{\rightarrow}$ “lemma expNonzero”]

[ExpNonzero(2) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma expNonzero(2)”]

[HalfBase(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma halfBase base”]

[HalfBase(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma halfBase indu”]

[MultiplyEquations(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma multiplyEquations(R)”]

[NonreciprocalToRight(Eq)(1term)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonreciprocalToRight(Eq)(1 term)(R)”]

[PositiveNonzero(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveNonzero(R)”]

[NonzeroProduct(2)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonzeroProduct(2)(R)”]

[HalfBase $\stackrel{\text{pyk}}{\rightarrow}$ “lemma halfBase”]

[Three2threeFactors(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma three2threeFactors(R)”]

[$\mathrm{x} * \mathrm{y} = $ zBackwards(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma x∗y=zBackwards(R)”]

[PositiveInverted(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma positiveInverted(R)”]

[ReciprocalToRight(Less)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma reciprocalToRight(Less)(R)”]

[ReciprocalToRight(Less)(1term)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma reciprocalToRight(Less)(1 term)(R)”]

[NonreciprocalToLeft(Less)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonreciprocalToLeft(Less)(R)”]

[$1 < \mathrm{x} * \mathrm{y}$(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma 1<x∗y(R)”]

[SwitchFactors($1/\mathrm{x} < \mathrm{y}$)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma switchFactors(1/x<y)(R)”]

[SmallHalving $\stackrel{\text{pyk}}{\rightarrow}$ “lemma smallHalving”]

[IntervalSize(anyPositive) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma intervalSize(anyPositive)”]

[USdecreasing(+n)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USdecreasing(+n) base”]

[USdecreasing(+n)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USdecreasing(+n) indu”]

[USdecreasing(+n) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USdecreasing(+n)”]

[USdecreasing $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USdecreasing”]

[LeqAdditionLeft(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma leqAdditionLeft(R)”]

[ToNotLess(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma toNotLess(R)”]

[LimitOfUSIsLeq $\stackrel{\text{pyk}}{\rightarrow}$ “lemma limitOfUSIsLeq”]

[SubtractEquations(Less)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma subtractEquations(Less)(R)”]

[SubtractEquationsLeft(Less)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma subtractEquationsLeft(Less)(R)”]

[LessNegated(Negative)(R) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma lessNegated(Negative)(R)”]

[FromNegatedAnd(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “prop lemma from negated and (imply)”]

[RemoveDoubleNeg(Consequent) $\stackrel{\text{pyk}}{\rightarrow}$ “prop lemma remove double neg (consequent)”]

[FromNotUpperBound $\stackrel{\text{pyk}}{\rightarrow}$ “lemma fromNotUpperBound”]

[LeqNUB $\stackrel{\text{pyk}}{\rightarrow}$ “lemma leqNUB”]

[USlimitIsLeastUpperBound(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USlimitIsLeastUpperBound helper”]

[USlimitIsLeastUpperBound $\stackrel{\text{pyk}}{\rightarrow}$ “lemma USlimitIsLeastUpperBound”]

[ExistMP3 $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma exist mp3”]

[GreaterPositive(N) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma greaterPositive(N)”]

[ysFClose(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma ysFClose helper”]

[ysFClose $\stackrel{\text{pyk}}{\rightarrow}$ “lemma ysFClose”]

[ysFCauchy(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma ysFCauchy helper”]

[ysFCauchy $\stackrel{\text{pyk}}{\rightarrow}$ “lemma ysFCauchy”]

[from <<==$\stackrel{\text{pyk}}{\rightarrow}$ “lemma from<<==”]

[to <<==$\stackrel{\text{pyk}}{\rightarrow}$ “lemma to<<==”]

[NonnegativeNumerical(F) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonnegativeNumerical(F)”]

[NegativeNumerical(F) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma negativeNumerical(F)”]

[tester1 $\stackrel{\text{pyk}}{\rightarrow}$ “tester1”]

[tester2 $\stackrel{\text{pyk}}{\rightarrow}$ “tester2”]

[tester3 $\stackrel{\text{pyk}}{\rightarrow}$ “tester3”]

[tester4 $\stackrel{\text{pyk}}{\rightarrow}$ “tester4”]

[tester5 $\stackrel{\text{pyk}}{\rightarrow}$ “tester5”]

[tester6 $\stackrel{\text{pyk}}{\rightarrow}$ “tester6”]

[sup2 $\stackrel{\text{pyk}}{\rightarrow}$ “sup2”]

$)^{\mathbf{P}}$

[sup2 $\stackrel{\text{tex}}{=}$ "sup2"]

[LeqTotality(R) $\stackrel{\text{tex}}{=}$ "LeqTotality(R)"]

[PositiveToLeft(Eq) $\stackrel{\text{tex}}{=}$ "PositiveToLeft(Eq)"]

[ExpZero(Exact) $\stackrel{\text{tex}}{=}$ "ExpZero(Exact) "]

[(+1)IsPositive(N) $\stackrel{\text{tex}}{=}$ "(+1)IsPositive(N)"]

[SameExp(Base) $\stackrel{\text{tex}}{=}$ "SameExp(Base)"]

[SameExp(Indu) $\stackrel{\text{tex}}{=}$ "SameExp(Indu)"]

[SameExp $\stackrel{\text{tex}}{=}$ "SameExp"]

[Exp(+1) $\stackrel{\text{tex}}{=}$ "Exp(+1)"]

[DistributionOut(Minus) $\stackrel{\text{tex}}{=}$ "DistributionOut(Minus)"]

[$(1/2)(x + y) - x = (1/2)(y - x)$ $\stackrel{\text{tex}}{=}$ "(1/2)(x+y)-x=(1/2)(y-x)"]

[$y - (1/2)(x + y) = (1/2)(y - x)$ $\stackrel{\text{tex}}{=}$ "y-(1/2)(x+y)=(1/2)(y-x)"]

[PositiveBase(Base) $\stackrel{\text{tex}}{=}$ "PositiveBase(Base)"]

[PositiveBase(Indu) $\stackrel{\text{tex}}{=}$ "PositiveBase(Indu)"]

[PositiveBase $\stackrel{\text{tex}}{=}$ "PositiveBase"]

[PositiveToRight(Eq)(1term) $\stackrel{\text{tex}}{=}$ "PositiveToRight(Eq)(1 term)"]

[BSzero(Exact) $\stackrel{\text{tex}}{=}$ "BSzero(Exact)"]

[SameBS(2)(Base) $\stackrel{\text{tex}}{=}$ "SameBS(2)(Base)"]

[SameBS(2)(Indu) $\stackrel{\text{tex}}{=}$ "SameBS(2)(Indu)"]

[SameBS(2) $\stackrel{\text{tex}}{=}$ "SameBS(2)"]

[NegativeToLeft(Less)(1term) $\stackrel{\text{tex}}{=}$ "NegativeToLeft(Less)(1 term)"]

[BS(+1) $\stackrel{\text{tex}}{=}$ "BS(+1)"]

[BSbound(Exact)(Base) $\stackrel{\text{tex}}{=}$ "BSbound(Exact)(Base)"]

[BSbound(Exact)(Indu) $\stackrel{\text{tex}}{=}$ "BSbound(Exact)(Indu)"]

[BSbound(Exact) $\stackrel{\text{tex}}{=}$ "BSbound(Exact)"]

[BSbound $\stackrel{\text{tex}}{=}$ “BSbound”]

[SameSeries(NumDiff) $\stackrel{\text{tex}}{=}$ “SameSeries(NumDiff)”]

[UStelescope(Zero)(Exact) $\stackrel{\text{tex}}{=}$ “UStelescope(Zero)(Exact)”]

[SameTelescope(2)(Base) $\stackrel{\text{tex}}{=}$ “SameTelescope(2)(Base)”]

[SameTelescope(2)(Indu) $\stackrel{\text{tex}}{=}$ “SameTelescope(2)(Indu)”]

[SameTelescope(2) $\stackrel{\text{tex}}{=}$ “SameTelescope(2)”]

[TelescopeNumerical(Base) $\stackrel{\text{tex}}{=}$ “TelescopeNumerical(Base)”]

[TelescopeNumerical(Indu) $\stackrel{\text{tex}}{=}$ “TelescopeNumerical(Indu)”]

[TelescopeNumerical $\stackrel{\text{tex}}{=}$ “TelescopeNumerical”]

[ToNegatedDoubleImply $\stackrel{\text{tex}}{=}$ “ToNegatedDoubleImply”]

[EqAdditionLeft(R) $\stackrel{\text{tex}}{=}$ “EqAdditionLeft(R)”]

[$\mathrm{x = x + (y - y)(R)}$ $\stackrel{\text{tex}}{=}$ “x=x+(y-y)(R)”]

[$\mathrm{x = x + y - y(R)}$ $\stackrel{\text{tex}}{=}$ “x=x+y-y(R)”]

[Three2twoTerms(R) $\stackrel{\text{tex}}{=}$ “Three2twoTerms(R)”]

[PositiveToRight(Less)(R) $\stackrel{\text{tex}}{=}$ “PositiveToRight(Less)(R)”]

[Three2threeTerms(R) $\stackrel{\text{tex}}{=}$ “Three2threeTerms(R)”]

[Plus0Left(R) $\stackrel{\text{tex}}{=}$ “Plus0Left(R)”]

[PositiveToRight(Eq)(R) $\stackrel{\text{tex}}{=}$ “PositiveToRight(Eq)(R)”]

[SubtractEquations(R) $\stackrel{\text{tex}}{=}$ “SubtractEquations(R)”]

[NeqAddition(R) $\stackrel{\text{tex}}{=}$ “NeqAddition(R)”]

[PositiveToRight(Less)(R) $\stackrel{\text{tex}}{=}$ “PositiveToRight(Less)(R)”]

[PositiveToRight(Less)(1term)(R) $\stackrel{\text{tex}}{=}$ “PositiveToRight(Less)(1 term)(R)”]

[To!! $==\stackrel{\text{tex}}{=}$ “To!!==”]

[SwitchTerms($\mathrm{x <= y - z}$) $\stackrel{\text{tex}}{=}$ “SwitchTerms(x<=y-z)”]

[(A)to(E)(Imply) $\stackrel{\text{tex}}{=}$ “(A)to(˜E˜)(Imply)”]

[(E)to(A)(Imply) $\stackrel{\text{tex}}{=}$ "(E)to(˜A˜)(Imply)"]

[(E)to(A)(Imply) $\stackrel{\text{tex}}{=}$ "(E˜)to(˜A)(Imply)"]

[AddNegatedAll $\stackrel{\text{tex}}{=}$ "AddNegatedAll"]

[ToNegatedAEA $\stackrel{\text{tex}}{=}$ "ToNegatedAEA "]

[LessNeq(F)(Helper) $\stackrel{\text{tex}}{=}$ "LessNeq(F)(Helper)"]

[LessNeq(F) $\stackrel{\text{tex}}{=}$ "LessNeq(F)"]

[LessNeq(R) $\stackrel{\text{tex}}{=}$ "LessNeq(R)"]

[PositiveToRight(Less)(1term) $\stackrel{\text{tex}}{=}$ "PositiveToRight(Less)(1 term)"]

[(A)to(E) $\stackrel{\text{tex}}{=}$ "(A˜)to(˜E)"]

[ToLeq(Advanced)(R) $\stackrel{\text{tex}}{=}$ "ToLeq(Advanced)(R)"]

[LeqNeqLess(R) $\stackrel{\text{tex}}{=}$ "LeqNeqLess(R)"]

[SubLeqLeft(R) $\stackrel{\text{tex}}{=}$ "SubLeqLeft(R)"]

[LeqLessTransitivity(R) $\stackrel{\text{tex}}{=}$ "LeqLessTransitivity(R)"]

[NegativeToLeft(Eq)(R) $\stackrel{\text{tex}}{=}$ "NegativeToLeft(Eq)(R)"]

[NegativeToRight(Less)(R) $\stackrel{\text{tex}}{=}$ "NegativeToRight(Less)(R)"]

[!! == Symmetry $\stackrel{\text{tex}}{=}$ "!!==Symmetry"]

[NegativeToRight(Eq)(R) $\stackrel{\text{tex}}{=}$ "NegativeToRight(Eq)(R)"]

[NegativeToRight(Eq)(1term)(R) $\stackrel{\text{tex}}{=}$ "NegativeToRight(Eq)(1 term)(R)"]

[DoubleMinus(R) $\stackrel{\text{tex}}{=}$ "DoubleMinus(R)"]

[UniqueNegative(R) $\stackrel{\text{tex}}{=}$ "UniqueNegative(R)"]

[SubtractEquationsLeft(R) $\stackrel{\text{tex}}{=}$ "SubtractEquationsLeft(R)"]

[EqNegated(R) $\stackrel{\text{tex}}{=}$ "EqNegated(R)"]

[NeqNegated(R) $\stackrel{\text{tex}}{=}$ "NeqNegated(R)"]

[SubLeqRight(R) $\stackrel{\text{tex}}{=}$ "SubLeqRight(R)"]

[LeqNegated(R) $\stackrel{\text{tex}}{=}$ "LeqNegated(R)"]

[LessNegated(R) $\stackrel{\text{tex}}{=}$ “LessNegated(R)”]

[$-0 = 0$(R) $\stackrel{\text{tex}}{=}$ “-0=0(R)”]

[NegativeNegated(R) $\stackrel{\text{tex}}{=}$ “NegativeNegated(R)”]

[FromLeqGeq(R) $\stackrel{\text{tex}}{=}$ “FromLeqGeq(R)”]

[FromLess(R) $\stackrel{\text{tex}}{=}$ “FromLess(R)”]

[NonnegativeNumerical(R) $\stackrel{\text{tex}}{=}$ “NonnegativeNumerical(R)”]

[NegativeNumerical(R) $\stackrel{\text{tex}}{=}$ “NegativeNumerical(R)”]

[$0 <= |x|$(R) $\stackrel{\text{tex}}{=}$ “0<=|x|(R)”]

[PositiveNegated(R) $\stackrel{\text{tex}}{=}$ “PositiveNegated(R)”]

[AddEquations(R) $\stackrel{\text{tex}}{=}$ “AddEquations(R)”]

[DistributionOut(R) $\stackrel{\text{tex}}{=}$ “DistributionOut(R)”]

[$x * 0 + x = x$(R) $\stackrel{\text{tex}}{=}$ “x*0+x=x(R)”]

[$x * 0 = 0$(R)(fff) $\stackrel{\text{tex}}{=}$ “x*0=0(R)(fff)”]

[Times(-1)(R) $\stackrel{\text{tex}}{=}$ “Times(-1)(R)”]

[Times(-1)Left(R) $\stackrel{\text{tex}}{=}$ “Times(-1)Left(R)”]

[$-x - y = -(x + y)$(R) $\stackrel{\text{tex}}{=}$ “-x-y=-(x+y)(R)”]

[LessTotality(R) $\stackrel{\text{tex}}{=}$ “LessTotality(R)”]

[SameNumerical(R) $\stackrel{\text{tex}}{=}$ “SameNumerical(R)”]

[MinusNegated(R) $\stackrel{\text{tex}}{=}$ “MinusNegated(R)”]

[PositiveNumerical(R) $\stackrel{\text{tex}}{=}$ “PositiveNumerical(R)”]

[SignNumerical($+$)(R) $\stackrel{\text{tex}}{=}$ “SignNumerical(+)(R)”]

[SignNumerical(R) $\stackrel{\text{tex}}{=}$ “SignNumerical(R)”]

[NumericalDifference(R) $\stackrel{\text{tex}}{=}$ “NumericalDifference(R)”]

[$x <= |x|$(R) $\stackrel{\text{tex}}{=}$ “x<=|x|(R)”]

[USlimitIsUpperBound(Helper) $\stackrel{\text{tex}}{=}$ “USlimitIsUpperBound(Helper)”]

[USlimitIsUpperBound $\stackrel{\text{tex}}{=}$ “USlimitIsUpperBound”]

[$(-1)*(-1)+(-1)*1=0$(R) $\stackrel{\text{tex}}{=}$ “(-1)*(-1)+(-1)*1=0(R)”]

[$(-1)*(-1)=1$(R) $\stackrel{\text{tex}}{=}$ “(-1)*(-1)=1(R)”]

[$0<1$Helper(R) $\stackrel{\text{tex}}{=}$ “0<1Helper(R)”]

[$0<1$(R) $\stackrel{\text{tex}}{=}$ “0<1(R)”]

[ExpZero(Exact)(R) $\stackrel{\text{tex}}{=}$ “ExpZero(Exact)(R)”]

[PositiveBase(R)(Base) $\stackrel{\text{tex}}{=}$ “PositiveBase(R)(Base)”]

[Three2twoFactors(R) $\stackrel{\text{tex}}{=}$ “Three2twoFactors(R)”]

[$\mathrm{x}=\mathrm{x}*\mathrm{y}*(1/\mathrm{y})$(R) $\stackrel{\text{tex}}{=}$ “x=x*y*(1/y)(R)”]

[NeqMultiplication(R) $\stackrel{\text{tex}}{=}$ “NeqMultiplication(R)”]

[LessTransitivity(R) $\stackrel{\text{tex}}{=}$ “LessTransitivity(R)”]

[$0<2$(R) $\stackrel{\text{tex}}{=}$ “0<2(R)”]

[SameExp(R)(Base) $\stackrel{\text{tex}}{=}$ “SameExp(R)(Base)”]

[SameExp(R)(Indu) $\stackrel{\text{tex}}{=}$ “SameExp(R)(Indu)”]

[SameExp(R) $\stackrel{\text{tex}}{=}$ “SameExp(R)”]

[SubNeqLeft(R) $\stackrel{\text{tex}}{=}$ “SubNeqLeft(R)”]

[SubNeqRight(R) $\stackrel{\text{tex}}{=}$ “SubNeqRight(R)”]

[NonzeroFactors(R) $\stackrel{\text{tex}}{=}$ “NonzeroFactors(R)”]

[NonnegativeFactors(R) $\stackrel{\text{tex}}{=}$ “NonnegativeFactors(R)”]

[PositiveFactors(R) $\stackrel{\text{tex}}{=}$ “PositiveFactors(R)”]

[LessDivision(R) $\stackrel{\text{tex}}{=}$ “LessDivision(R)”]

[$0<1/2$(R) $\stackrel{\text{tex}}{=}$ “0<1/2(R)”]

[PositiveToRight(Eq)(1term)(R) $\stackrel{\text{tex}}{=}$ “PositiveToRight(Eq)(1 term)(R)”]

[Exp(+1)(R) $\stackrel{\text{tex}}{=}$ “Exp(+1)(R)”]

[PositiveBase(R)(Indu) $\stackrel{\text{tex}}{=}$ “PositiveBase(R)(Indu)”]

[PositiveBase(R) $\overset{\text{tex}}{=}$ “PositiveBase(R)”]

[$-x * y = -(x * y)$(R) $\overset{\text{tex}}{=}$ “-x*y=-(x*y)(R)”]

[PositiveToLeft(Eq)(R) $\overset{\text{tex}}{=}$ “PositiveToLeft(Eq)(R)”]

[Times1Left(R) $\overset{\text{tex}}{=}$ “Times1Left(R)”]

[$x + x = 2 * x$(R) $\overset{\text{tex}}{=}$ “x+x=2*x(R)”]

[$(1/2)x + (1/2)x = x$(R) $\overset{\text{tex}}{=}$ “(1/2)x+(1/2)x=x(R)”]

[DistributionOut(Minus)(R) $\overset{\text{tex}}{=}$ “DistributionOut(Minus)(R)”]

[$(1/2)(x + y) - x = (1/2)(y - x)$(R) $\overset{\text{tex}}{=}$ “(1/2)(x+y)-x=(1/2)(y-x)(R)”]

[IntervalSize(R)(Base) $\overset{\text{tex}}{=}$ “IntervalSize(R)(Base)”]

[LessMultiplicationLeft(R) $\overset{\text{tex}}{=}$ “LessMultiplicationLeft(R)”]

[NegativeToLeft(Less)(R) $\overset{\text{tex}}{=}$ “NegativeToLeft(Less)(R)”]

[NegativeToLeft(Less)(1term)(R) $\overset{\text{tex}}{=}$ “NegativeToLeft(Less)(1 term)(R)”]

[$y - (1/2)(x + y) = (1/2)(y - x)$(R) $\overset{\text{tex}}{=}$ “y-(1/2)(x+y)=(1/2)(y-x)(R)”]

[IntervalSize(R)(Indu) $\overset{\text{tex}}{=}$ “IntervalSize(R)(Indu)”]

[IntervalSize(R) $\overset{\text{tex}}{=}$ “IntervalSize(R)”]

[XSlessUS(R) $\overset{\text{tex}}{=}$ “XSlessUS(R)”]

[USdecreasing(+1)(R) $\overset{\text{tex}}{=}$ “USdecreasing(+1)(R)”]

[$1 <= x + 1$(N) $\overset{\text{tex}}{=}$ “1<=x+1(N)”]

[ExpUnbounded(Base) $\overset{\text{tex}}{=}$ “ExpUnbounded(Base)”]

[ExpUnbounded(Indu) $\overset{\text{tex}}{=}$ “ExpUnbounded(Indu)”]

[ExpUnbounded $\overset{\text{tex}}{=}$ “ExpUnbounded”]

[NonzeroProduct(2)(R) $\overset{\text{tex}}{=}$ “NonzeroProduct(2)(R)”]

[PositiveNonzero(R) $\overset{\text{tex}}{=}$ “PositiveNonzero(R)”]

[NonreciprocalToRight(Eq)(1term)(R) $\overset{\text{tex}}{=}$ “NonreciprocalToRight(Eq)(1 term)(R)”

[ExpNonzero(Base) $\overset{\text{tex}}{=}$ “ExpNonzero(Base)”]

[ExpNonzero(Indu) $\stackrel{\text{tex}}{=}$ “ExpNonzero(Indu)”]

[ExpNonzero $\stackrel{\text{tex}}{=}$ “ExpNonzero”]

[MultiplyEquations(R) $\stackrel{\text{tex}}{=}$ “MultiplyEquations(R)”]

[ExpNonzero(2) $\stackrel{\text{tex}}{=}$ “ExpNonzero(2)”]

[HalfBase(Base) $\stackrel{\text{tex}}{=}$ “HalfBase(Base)”]

[HalfBase(Indu) $\stackrel{\text{tex}}{=}$ “HalfBase(Indu)”]

[HalfBase $\stackrel{\text{tex}}{=}$ “HalfBase”]

[Three2threeFactors(R) $\stackrel{\text{tex}}{=}$ “Three2threeFactors(R)”]

[x ∗ y = zBackwards(R) $\stackrel{\text{tex}}{=}$ “x∗y=zBackwards(R)”]

[PositiveInverted(R) $\stackrel{\text{tex}}{=}$ “PositiveInverted(R)”]

[ReciprocalToRight(Less)(R) $\stackrel{\text{tex}}{=}$ “ReciprocalToRight(Less)(R)”]

[ReciprocalToRight(Less)(1term)(R) $\stackrel{\text{tex}}{=}$ “ReciprocalToRight(Less)(1 term)(R)”]

[NonreciprocalToLeft(Less)(R) $\stackrel{\text{tex}}{=}$ “NonreciprocalToLeft(Less)(R)”]

[1 < x ∗ y(R) $\stackrel{\text{tex}}{=}$ “1<x∗y(R)”]

[SwitchFactors(1/x < y)(R) $\stackrel{\text{tex}}{=}$ “SwitchFactors(1/x<y)(R)”]

[SmallHalving $\stackrel{\text{tex}}{=}$ “SmallHalving”]

[IntervalSize(anyPositive) $\stackrel{\text{tex}}{=}$ “IntervalSize(anyPositive)”]

[USdecreasing(+n)(Base) $\stackrel{\text{tex}}{=}$ “USdecreasing(+n)(Base)”]

[USdecreasing(+n)(Indu) $\stackrel{\text{tex}}{=}$ “USdecreasing(+n)(Indu)”]

[USdecreasing(+n) $\stackrel{\text{tex}}{=}$ “USdecreasing(+n)”]

[USdecreasing $\stackrel{\text{tex}}{=}$ “USdecreasing”]

[LeqAdditionLeft(R) $\stackrel{\text{tex}}{=}$ “LeqAdditionLeft(R)”]

[ToNotLess(R) $\stackrel{\text{tex}}{=}$ “ToNotLess(R)”]

[LimitOfUSIsLeq $\stackrel{\text{tex}}{=}$ “LimitOfUSIsLeq”]

[SubtractEquations(Less)(R) $\stackrel{\text{tex}}{=}$ “SubtractEquations(Less)(R)”]

[SubtractEquationsLeft(Less)(R) $\stackrel{\text{tex}}{=}$ "SubtractEquationsLeft(Less)(R)"]

[LessNegated(Negative)(R) $\stackrel{\text{tex}}{=}$ "LessNegated(Negative)(R)"]

[FromNegatedAnd(Imply) $\stackrel{\text{tex}}{=}$ "FromNegatedAnd(Imply)"]

[RemoveDoubleNeg(Consequent) $\stackrel{\text{tex}}{=}$ "RemoveDoubleNeg(Consequent)"]

[FromNotUpperBound $\stackrel{\text{tex}}{=}$ "FromNotUpperBound"]

[LeqNUB $\stackrel{\text{tex}}{=}$ "LeqNUB"]

[USlimitIsLeastUpperBound(Helper) $\stackrel{\text{tex}}{=}$ "USlimitIsLeastUpperBound(Helper)"]

[USlimitIsLeastUpperBound $\stackrel{\text{tex}}{=}$ "USlimitIsLeastUpperBound"]

[ExistMP3 $\stackrel{\text{tex}}{=}$ "ExistMP3"]

[GreaterPositive(N) $\stackrel{\text{tex}}{=}$ "GreaterPositive(N)"]

[ysFClose(Helper) $\stackrel{\text{tex}}{=}$ "ysFClose(Helper)"]

[ysFClose $\stackrel{\text{tex}}{=}$ "ysFClose"]

[ysFCauchy(Helper) $\stackrel{\text{tex}}{=}$ "ysFCauchy(Helper)"]

[ysFCauchy $\stackrel{\text{tex}}{=}$ "ysFCauchy"]

[from <<== $\stackrel{\text{tex}}{=}$ "from<<=="]

[NonnegativeNumerical(F) $\stackrel{\text{tex}}{=}$ "NonnegativeNumerical(F)"]

[to <<== $\stackrel{\text{tex}}{=}$ "to<<=="]

[NegativeNumerical(F) $\stackrel{\text{tex}}{=}$ "NegativeNumerical(F)"]