Logiweb codex of sup

Up Help

Nat(∗), $\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$, $\langle * \equiv^{1} * \mid * :== * \rangle_{\mathrm{Me}}$, $\langle * \equiv^{*} * \mid * :== * \rangle_{\mathrm{Me}}$, lemma eqLeq(R), ∗(exp)∗, sup, ToNegatedAnd(1), UniqueNegative, DoubleMinus, MinusNegated, eqReflexivity, eqSymmetry, eqTransitivity, eqTransitivity4, eqTransitivity5, eqTransitivity6, AddEquations, SubtractEquations, SubtractEquationsLeft, MultiplyEquations, EqNegated, PositiveToRight(Eq), PositiveToLeft(Eq)(1term), NegativeToLeft(Eq), NonreciprocalToRight(Eq)(1term), PlusAssociativity(4terms), LessNeq, NeqSymmetry, NeqNegated, SubNeqRight, SubNeqLeft, NegativeToRight(Neq)(1term), NeqAddition, NeqMultiplication, NonzeroProduct(2), UStelescope(+1), TelescopeBound(Base), TelescopeBound(Indu), TelescopeBound, IntervalSize(Base), IntervalSize(Indu), IntervalSize, XS < US, lemma USdecreasing(+1), CloseUS, CloseUS(n + 1), AllNegated(Imply), ExistNegated(Imply), IntroExist(Helper), IntroExist, ExistMP, ExistMP2, TwiceExistMP, TwiceExistMP2, EAE − MP, AddAll, AddExist(Helper1), AddExist(Helper2), AddExist, AddExist(SimpleAnt), AddExist(Simple), AddEAE, AEA − negated, EEA − negated, Induction, leqAntisymmetry, leqTransitivity, leqAddition, leqMultiplication, Reciprocal, Equality, eqLeq, eqAddition, eqMultiplication, LeqMultiplicationLeft, LeqLessEq, LessLeq, FromLeqGeq, subLeqRight, subLeqLeft, Leq + 1, PositiveToRight(Leq), PositiveToRight(Leq)(1term), lemma negativeToRight(Leq), PositiveToLeft(Leq), negativeToLeft(Leq), negativeToLeft(Leq)(1term), LeqAdditionLeft, leqSubtraction, leqSubtractionLeft, thirdGeq, LeqNegated, AddEquations(Leq), MultiplyEquations(Leq), ThirdGeqSeries, LeqNeqLess, FromLess, ToLess, fromNotLess, toNotLess, NegativeLessPositive, leqLessTransitivity, LessLeqTransitivity, LessTransitivity, LessTotality, SubLessRight, SubLessLeft, SwitchTerms(x < y − z), SwitchTerms(x − y < z), LessAddition, LessAdditionLeft, LessMultiplication, LessMultiplicationLeft, LessDivision, PositiveToRight(Less), PositiveToLeft(Less), NegativeToLeft(Less), NegativeToRight(Less), AddEquations(Less), AddEquations(LeqLess), reciprocalToLeft(Less), LessNegated, PositiveNonzero, PositiveNegated, NonpositiveNegated, NegativeNegated, NonnegativeNegated, PositiveHalved, PositiveInverted, NonnegativeNumerical, NegativeNumerical, PositiveNumerical, lemma nonpositiveNumerical, |0| = 0, 0 <= |x|, x <= |x|, FromPositiveNumerical, SameNumerical, SignNumerical(+), SignNumerical, ToNumericalLess, FromNumericalGreater, NumericalDifference, NumericalDifferenceLess(Helper), NumericalDifferenceLess, SplitNumericalSumHelper, splitNumericalSum(++), splitNumericalSum(−−), splitNumericalSum(+ − small), splitNumericalSum(+ − big), splitNumericalSum(+−), splitNumericalSum(−+), splitNumericalSum, SplitNumericalProduct(++), SplitNumericalProduct(+−), SplitNumericalProduct, insertMiddleTerm(Numerical), insertTwoMiddleTerms(Numerical), Three2twoTerms, Three2threeTerms, Three2twoFactors, Three2threeFactors, Times(−1), Times(−1)Left, MaxLeq(1), MaxLeq(2), LessThanMax, x + y = zBackwards, x * y = zBackwards, x = x + (y − y), x = x + y − y, x = x * y * (1/y), insertMiddleTerm(Sum), insertTwoMiddleTerms(Sum), insertMiddleTerm(Difference), x * 0 + x = x, x * 0 = 0, NonnegativeFactors, NonzeroFactors, PositiveFactors, PlusTimesMinus, MinusTimesMinus, (−1) * (−1) + (−1) * 1 = 0, (−1) * (−1) = 1, 0 < 1Helper, 0 < 1, 0 < 2, 0 < 3, 0 < 1/2, 0 < 1/3, TwoWholes, ThreeWholes, TwoHalves, ThreeThirds, −x − y = −(x + y), −x * y = −(x * y), −0 = 0, SFsymmetry, SFtransitivity, f2R(Plus), f2R(Times), << TransitivityHelper(Q), << Transitivity, <<== Reflexivity, <<== AntisymmetryHelper(Q), FromNot < f(Weak)(Helper), FromNot < f(Weak), FromNot < f(Strong)(Helper2), FromNot < f(Strong)(Helper), FromNot < f(Strong), fromNotSameF(Strongest)(Helper2), fromNotSameF(Strongest)(Helper), fromNotSameF(Strongest), ToLess(F)(Helper), ToLess(F), FromNot <<, ToLess(R), FromNotSameF(Weak)(Helper), FromNotSameF(Weak), FromNotLess(F), == Addition, == AdditionLeft, Fpart − Bounded(Base), Fpart − Bounded(InduHelper), Fpart − Bounded(Indu), Fpart − Bounded, F − Bounded(Helper), F − Bounded, SameFmultiplication(Helper), SameFmultiplication, EqMultiplication(R), EqMultiplicationLeft(R), x * 0 = 0(F), x * 0 = 0(R), LessMultiplication(F)(Helper2), LessMultiplication(F)(Helper), LessMultiplication(F), LessMultiplication(R), LeqMultiplication(R), PlusAssociativity(F), Plus0(F), PlusCommutativity(F), TimesAssociativity(F), Times1f, Cauchy(2)(Helper), Cauchy(2), ReciprocalFnonzero, (Eventually = f)2sameF(Helper), (Eventually = f)2sameF, FromNotSameF(Strong)(Helper2), FromNotSameF(Strong)(Helper), FromNotSameF(Strong), SameFreciprocal(Helper), SameFreciprocal, From!! ==, Reciprocal(R), TimesCommutativity(F), Distribution(F), FromMax(1), FromMax(2), ToNegatedAnd, DistributionOut, DistributionOutLeft, DistributionLeft, FromNotLess(R), CartProdIsRelation, FromSubset, SubsetIsRelation, ToSeries, FromSeries, SeriesSubsetCP, ValueType, RemoveOr, FromSingleton, InPair(1), InPair(2), SameMember(2), ToBinaryUnion(1), ToBinaryUnion(2), FromOrderedPair(TwoLevels), ToCartProd(Helper), ToCartProd, NonreciprocalToRight(Eq), NonreciprocalToLeft(Eq)(1term), SameReciprocal, CPseparationIsRelation, OrderedPairEquality, ReciprocalIsFunction, ReciprocalIsTotal, ReciprocalIsRationalSeries, CrsIsRelation, CrsIsFunction, CrsIsTotal, CrsIsSeries, CrsLookup, 0f, 1f, ToSingleton, FromSameSingleton, SingletonmembersEqual, UnequalsNotInSingleton, NonsingletonmembersUnequal, FromOrderedPair, FromOrderedPair(1), FromOrderedPair(2), FromCartProd, FromCartProd(1), FromCartProd(2), sameOrderedPair, InSeriesHelper, InSeries, To = f(Subset)(Helper), To = f(Subset), To = f, productIsFunction, productIsTotal, ProductIsRationalSeries, TimesF, $-x + (1/2)x = -(1/2)x$, PositiveTripled, PositiveDividedBy3, $|x - x| = 0$, $1 < 2$, $1/3 < 2/3$, $(1/3)x + (1/3)x = (2/3)x$, $(2/3)x + (1/3)x = x$, $-x + (2/3)x = -(1/3)x$, $-(1/3)x - (1/3)x = -(2/3)x$, $-x + (1/3)x = -(2/3)x$, PreserveLessGreater, ClosetolessIsLess, SubLessLeft(F), SubLessLeft(R), ClosetogreaterIsGreater, SubLessRight(F), SubLessRight(R), plus0Left, times1Left, EqAdditionLeft, EqMultiplicationLeft, PlusF(Sym), TimesF(Sym), SameSeries(Gen), EqualsSameF, LeqReflexivity(R), Tester1, Tester2, Tester3, Tester4, Tester5, Tester6,

Nat(∗)

$[\mathrm{Nat}(\mathsf{x}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{Nat}(\mathsf{x}) \ddot{=} \lambda \mathsf{c}.\lceil \mathsf{x}\rceil \in_{\mathrm{t}} (\lceil \mathrm{V}_{2n}\rceil :: \lceil \mathcal{M}\rceil :: \lceil \mathcal{N}\rceil :: \mathsf{T})]\rceil)]$

$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$

$[\langle \mathsf{a} \equiv \mathsf{b} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\langle \mathsf{a} \equiv \mathsf{b} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}} \ddot{=} \langle \lceil \mathsf{a}\rceil \equiv^1 \lceil \mathsf{b}\rceil | \lceil \mathsf{x}\rceil :== \lceil \mathsf{t}\rceil\rangle_{\mathrm{Me}}]\rceil)]$

$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}}$

$[\langle \mathsf{a} \equiv^1 \mathsf{b} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}} \stackrel{\mathrm{val}}{\to} \mathsf{a}!\mathsf{x}!\mathsf{t}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \stackrel{\mathrm{r}}{=} \lceil \forall_{\mathrm{obj}} \mathsf{u} \colon \mathsf{v}\rceil, \mathsf{b}^1 \stackrel{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b} \stackrel{\mathrm{t}}{=} \mathsf{x}, \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{t}, \mathrm{If}($
$\mathsf{a} \stackrel{\mathrm{r}}{=} \mathsf{b}, \langle \mathsf{a}^{\mathrm{t}} \equiv^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}}, \mathsf{F})))]$

$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}}$

$[\langle \mathsf{a} \equiv^* \mathsf{b} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}} \stackrel{\mathrm{val}}{\to} \mathsf{b}!\mathsf{x}!\mathsf{t}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\langle \mathsf{a}^{\mathrm{h}} \equiv^1 \mathsf{b}^{\mathrm{h}} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}}, \langle \mathsf{a}^{\mathrm{t}} \equiv^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} :== \mathsf{t}\rangle_{\mathrm{Me}}, \mathsf{F}))]$

lemma eqLeq(R)

$[\text{lemma eqLeq(R)} \stackrel{\mathrm{tex}}{\to} \text{“eqLeq(R)”}]$

$*$(exp)$*$

[x(exp)y $\stackrel{\text{tex}}{\to}$ “(#1.
(expARGH!) #2.
)”]

sup

[sup $\stackrel{\text{prio}}{\to}$
Preassociative
[sup], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [$ $*$ $], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\to} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if($*, *, *$)], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\bot$], [f($*$)], [$(*)^{\text{I}}$], [F], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\text{M}}$], [If($*, *, *$)], [array{$*$} $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\text{M}}(*)$], [**apply**($*, *$)], [$\textbf{apply}_1(*, *)$], [identifier($*$)], [$\text{identifier}_1(*, *)$], [array-plus($*, *$)], [array-remove($*, *, *$)], [array-put($*, *, *, *$)], [array-add($*, *, *, *, *$)], [bit($*, *$)], [$\text{bit}_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**($*, *, *$)], [**abstract**($*, *, *, *$)], [[$\lceil * \rceil$]], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [$\text{s}_0$], [**zip**($*, *$)], [$\textbf{assoc}_1(*, *, *)$], [$(*)^{\text{P}}$], [self], [[$* \ddot{=} *$]], [[$* \doteq *$]], [[$* \acute{=} *$]], [[$* \stackrel{\text{pyk}}{=} *$]], [[$* \stackrel{\text{tex}}{=} *$]], [[$* \stackrel{\text{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [($*$)], [($*$)], [display($*$)], [statement($*$)], [[$[*]^{\cdot}$]], [[$[*]^{-}$]], [**aspect**($*, *$)], [**aspect**($*, *, *$)], [$\langle * \rangle$], [$\textbf{tuple}_1(*)$], [$\textbf{tuple}_2(*)$], [$\text{let}_2(*, *)$], [$\text{let}_1(*, *)$], [[$* \stackrel{\text{claim}}{=} *$]], [checker], [**check**($*, *$)], [$\textbf{check}_2(*, *, *)$], [$\textbf{check}_3(*, *, *)$], [$\textbf{check}^*(*, *)$], [$\textbf{check}^*_2(*, *, *)$], [[$[*]^{\cdot}$]], [[$[*]^{-}$]], [[$[*]^{\circ}$]], [msg], [[$* \stackrel{\text{msg}}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\text{stmt}}{=} *$]], [HeadNil$'$], [HeadPair$'$], [Transitivity$'$], [$\perp\!\!\!\perp$], [Contra$'$], [$\text{T}'_{\text{E}}$], [$\text{L}_1$], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder], [$(*)^{\text{v}}$], [intro($*, *, *, *$)], [intro($*, *, *$)], [error($*, *$)], [$\text{error}_2(*, *)$], [proof($*, *, *$)], [$\text{proof}_2(*, *)$], [$\mathcal{S}(*, *)$], [$\mathcal{S}^{\text{I}}(*, *)$], [$\mathcal{S}^{\rhd}(*, *)$], [$\mathcal{S}^{\rhd}_1(*, *, *)$], [$\mathcal{S}^{\text{E}}(*, *)$], [$\mathcal{S}^{\text{E}}_1(*, *, *)$], [$\mathcal{S}^{+}(*, *)$], [$\mathcal{S}^{+}_1(*, *, *)$], [$\mathcal{S}^{-}(*, *)$], [$\mathcal{S}^{-}_1(*, *, *)$], [$\mathcal{S}^{*}(*, *)$], [$\mathcal{S}^{*}_1(*, *, *)$], [$\mathcal{S}^{*}_2(*, *, *, *)$], [$\mathcal{S}^{@}(*, *)$], [$\mathcal{S}^{@}_1(*, *, *)$], [$\mathcal{S}^{\vdash}(*, *)$], [$\mathcal{S}^{\vdash}_1(*, *, *, *)$], [$\mathcal{S}^{\Vdash}(*, *)$], [$\mathcal{S}^{\Vdash}_1(*, *, *, *)$], [$\mathcal{S}^{\text{i.e.}}(*, *)$], [$\mathcal{S}^{\text{i.e.}}_1(*, *, *, *)$], [$\mathcal{S}^{\text{i.e.}}_2(*, *, *, *, *)$], [$\mathcal{S}^{\forall}(*, *)$], [$\mathcal{S}^{\forall}_1(*, *, *, *)$], [$\mathcal{S}^{;}(*, *)$], [$\mathcal{S}^{;}_1(*, *, *)$], [$\mathcal{S}^{;}_2(*, *, *, *)$], [$\mathcal{T}(*)$], [claims($*, *, *$)], [$\text{claims}_2(*, *, *)$], [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]],

[[$*$ **lemma** $*\!:*$]], [[$*$ **antilemma** $*\!:*$]], [[$*$ **rule** $*\!:*$]], [[$*$ **antirule** $*\!:*$]], [verifier], [$\mathcal{V}_1(*)$], [$\mathcal{V}_2(*,*)$], [$\mathcal{V}_3(*,*,*,*)$], [$\mathcal{V}_4(*,*)$], [$\mathcal{V}_5(*,*,*,*)$], [$\mathcal{V}_6(*,*,*,*)$], [$\mathcal{V}_7(*,*,*,*)$], [Cut$(*,*)$], [Head$_\oplus(*)$], [Tail$_\oplus(*)$], [rule$_1(*,*)$], [rule$(*,*)$], [Rule tactic], [Plus$(*,*)$], [[**Theory** $*$]], [theory$_2(*,*)$], [theory$_3(*,*)$], [theory$_4(*,*,*)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [$\mathrm{T_E}$], [ragged right], [ragged right expansion], [parm$(*,*,*)$], [parm$^*(*,*,*)$], [inst$(*,*)$], [inst$^*(*,*)$], [occur$(*,*,*)$], [occur$^*(*,*,*)$], [unify$(*=*,*)$], [unify$^*(*=*,*)$], [unify$_2(*=*,*)$], [$\mathrm{L_a}$], [$\mathrm{L_b}$], [$\mathrm{L_c}$], [$\mathrm{L_d}$], [$\mathrm{L_e}$], [$\mathrm{L_f}$], [$\mathrm{L_g}$], [$\mathrm{L_h}$], [$\mathrm{L_i}$], [$\mathrm{L_j}$], [$\mathrm{L_k}$], [$\mathrm{L_l}$], [$\mathrm{L_m}$], [$\mathrm{L_n}$], [$\mathrm{L_o}$], [$\mathrm{L_p}$], [$\mathrm{L_q}$], [$\mathrm{L_r}$], [$\mathrm{L_s}$], [$\mathrm{L_t}$], [$\mathrm{L_u}$], [$\mathrm{L_v}$], [$\mathrm{L_w}$], [$\mathrm{L_x}$], [$\mathrm{L_y}$], [$\mathrm{L_z}$], [$\mathrm{L_A}$], [$\mathrm{L_B}$], [$\mathrm{L_C}$], [$\mathrm{L_D}$], [$\mathrm{L_E}$], [$\mathrm{L_F}$], [$\mathrm{L_G}$], [$\mathrm{L_H}$], [$\mathrm{L_I}$], [$\mathrm{L_J}$], [$\mathrm{L_K}$], [$\mathrm{L_L}$], [$\mathrm{L_M}$], [$\mathrm{L_N}$], [$\mathrm{L_O}$], [$\mathrm{L_P}$], [$\mathrm{L_Q}$], [$\mathrm{L_R}$], [$\mathrm{L_S}$], [$\mathrm{L_T}$], [$\mathrm{L_U}$], [$\mathrm{L_V}$], [$\mathrm{L_W}$], [$\mathrm{L_X}$], [$\mathrm{L_Y}$], [$\mathrm{L_Z}$], [$\mathrm{L_?}$], [Reflexivity], [Reflexivity$_1$], [Commutativity], [Commutativity$_1$], [<tactic>], [tactic], [[$* \overset{\text{tactic}}{=} *$]], [$\mathcal{P}(*,*,*)$], [$\mathcal{P}^*(*,*,*)$], [$\mathrm{p_0}$], [conclude$_1(*,*)$], [conclude$_2(*,*,*)$], [conclude$_3(*,*,*,*)$], [conclude$_4(*,*)$], [check], [[$* \overset{\circ}{=} *$]], [RootVisible$(*)$], [A], [R], [C], [T], [L], [$\{*\}$], [$\overline{*}$], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [x], [y], [z], [$\langle * \equiv * \mid * := * \rangle$], [$\langle * \equiv^0 * \mid * := * \rangle$], [$\langle * \equiv^1 * \mid * := * \rangle$], [$\langle * \equiv^* * \mid * := * \rangle$], [Ded$(*,*)$], [Ded$_0(*,*)$], [Ded$_1(*,*,*)$], [Ded$_2(*,*,*)$], [Ded$_3(*,*,*,*)$], [Ded$_4(*,*,*,*)$], [Ded$^*_4(*,*,*,*)$], [Ded$_5(*,*,*)$], [Ded$_6(*,*,*,*)$], [Ded$^*_6(*,*,*,*)$], [Ded$_7(*)$], [Ded$_8(*,*)$], [Ded$^*_8(*,*)$], [S], [Neg], [MP], [Gen], [Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1$'$], [A2$'$], [A4$'$], [A5$'$], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop 3.2e$_1$], [Prop 3.2e$_2$], [Prop 3.2e], [Prop 3.2f$_1$], [Prop 3.2f$_2$], [Prop 3.2f], [Prop 3.2g$_1$], [Prop 3.2g$_2$], [Prop 3.2g], [Prop 3.2h$_1$], [Prop 3.2h$_2$], [Prop 3.2h], [Block$_1(*,*,*)$], [Block$_2(*)$], [kvanti], [UniqueMember], [UniqueMember(Type)], [SameSeries], [A4], [SameMember], [Qclosed(Addition)], [Qclosed(Multiplication)], [FromCartProd(1)], [1rule fromCartProd(2)], [constantRationalSeries$(*)$], [cartProd$(*)$], [Power$(*)$], [binaryUnion$(*,*)$], [SetOfRationalSeries], [IsSubset$(*,*)$], [(p$*,*$)], [(s$*$)], [$(\cdots)$], [`Objekt-var`], [`Ex-var`], [`Ph-var`], [`Værdi`], [`Variabel`], [Op$(*)$], [Op$(*,*)$], [$* \ddot{=} *$], [ContainsEmpty$(*)$], [Nat$(*)$], [Dedu$(*,*)$], [Dedu$_0(*,*)$], [Dedu$_\mathrm{s}(*,*,*)$], [Dedu$_1(*,*,*)$], [Dedu$_2(*,*,*)$], [Dedu$_3(*,*,*,*)$], [Dedu$_4(*,*,*,*)$], [Dedu$^*_4(*,*,*,*)$], [Dedu$_5(*,*,*)$], [Dedu$_6(*,*,*,*)$], [Dedu$^*_6(*,*,*,*)$], [Dedu$_7(*)$], [Dedu$_8(*,*)$], [Dedu$^*_8(*,*)$], [Ex$_1$], [Ex$_2$], [Ex3], [Ex$_{10}$], [Ex$_{20}$], [$*_{\mathrm{Ex}}$], [$*^{\mathrm{Ex}}$], [$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ex}}$], [$\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ex}}$], [$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ex}}$], [$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ex}}$], [ph$_1$], [ph$_2$], [ph$_3$], [$*_{\mathrm{Ph}}$], [$*^{\mathrm{Ph}}$], [$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ph}}$], [$\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ph}}$], [$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ph}}$], [$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ph}}$], [$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$], [$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}}$], [$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}}$], [bs], [OBS], [$\mathcal{BS}$], [Ø], [SystemQ], [MP], [Gen], [Repetition], [Neg], [Ded], [ExistIntro], [Extensionality], [Ødef], [PairDef], [UnionDef], [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg], [AndCommutativity], [AutoImply], [Contrapositive], [FirstConjunct], [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity], [IffFirst], [IffSecond], [ImplyTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4], [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1], [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],

[Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power], [SubsetInPower], [HelperPowerIsSub], [PowerIsSub], [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality], [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality], [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry], [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive], [ERisSymmetric], [ERisTransitive], [∅isSubset], [HelperMemberNot∅], [MemberNot∅], [HelperUnique∅], [Unique∅], [==Reflexivity], [==Symmetry], [Helper==Transitivity], [==Transitivity], [HelperTransferNotEq], [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset], [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset], [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection], [AutoMember], [HelperEqSysNot∅], [EqSysNot∅], [HelperEqSubset], [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary], [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset], [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImply], [BSsubset], [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)], [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ϵ)], [$(\epsilon)_1$], [$(\epsilon 2)$], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)], [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)], [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X_1], [X_2], [Y_1], [Y_2], [V_1], [V_2], [V_3], [V_4], [V_{2n}], [M_1], [M_2], [N_1], [N_2], [N_3], [ϵ], [$\epsilon 1$], [$\epsilon 2$], [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)], [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)], [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)], [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)], [(S1ob)], [(S2ob)], [ph_4], [ph_5], [ph_6], [NAT], [$\mathrm{RATIONAL}_\mathrm{S}$ERIES], [SERIES], [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1], [(−1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(− − 01)], [02], [01//02], [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)], [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)], [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)], [Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom], [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom], [QisClosed(Reciprocal)(Imply)], [QisClosed(Reciprocal)], [QisClosed(Negative)(Imply)], [QisClosed(Negative)], [leqReflexivity], [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality], [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity], [plusCommutativity], [Negative], [plus0], [timesAssociativity], [timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1], [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)], [(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [UStelescope(Zero)], [UStelescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [NatType], [RationalType], [SeriesType], [Max], [Numerical], [NumericalF], [MemberOfSeries(Imply)], [JoinConjuncts(2conditions)], [prop lemma imply negation], [TND], [FromNegatedImply], [ToNegatedImply], [FromNegated(2 * Imply)], [FromNegatedAnd], [FromNegatedOr], [ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 * 2Disjuncts], [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2], [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)], [ToNegatedAnd(1)], [UniqueNegative], [DoubleMinus], [MinusNegated], [eqReflexivity], [eqSymmetry], [eqTransitivity], [eqTransitivity4], [eqTransitivity5], [eqTransitivity6], [AddEquations], [SubtractEquations], [SubtractEquationsLeft], [MultiplyEquations], [EqNegated], [PositiveToRight(Eq)], [PositiveToLeft(Eq)(1term)], [NegativeToLeft(Eq)], [NonreciprocalToRight(Eq)(1term)], [PlusAssociativity(4terms)], [LessNeq], [NeqSymmetry], [NeqNegated], [SubNeqRight], [SubNeqLeft], [NegativeToRight(Neq)(1term)], [NeqAddition], [NeqMultiplication], [NonzeroProduct(2)], [UStelescope(+1)], [TelescopeBound(Base)], [TelescopeBound(Indu)], [TelescopeBound], [IntervalSize(Base)], [IntervalSize(Indu)], [IntervalSize], [XS < US], [lemma USdecreasing(+1)], [CloseUS], [CloseUS(n + 1)], [AllNegated(Imply)], [ExistNegated(Imply)], [IntroExist(Helper)], [IntroExist], [ExistMP], [ExistMP2], [TwiceExistMP], [TwiceExistMP2], [EAE − MP], [AddAll], [AddExist(Helper1)], [AddExist(Helper2)], [AddExist], [AddExist(SimpleAnt)], [AddExist(Simple)], [AddEAE], [AEA − negated], [EEA − negated], [Induction], [leqAntisymmetry], [leqTransitivity], [leqAddition], [leqMultiplication], [Reciprocal], [Equality], [eqLeq], [eqAddition], [eqMultiplication], [LeqMultiplicationLeft], [LeqLessEq], [LessLeq], [FromLeqGeq], [subLeqRight], [subLeqLeft], [Leq + 1], [PositiveToRight(Leq)], [PositiveToRight(Leq)(1term)], [lemma negativeToRight(Leq)], [PositiveToLeft(Leq)], [negativeToLeft(Leq)], [negativeToLeft(Leq)(1term)], [LeqAdditionLeft], [leqSubtraction], [leqSubtractionLeft], [thirdGeq], [LeqNegated], [AddEquations(Leq)], [MultiplyEquations(Leq)], [ThirdGeqSeries], [LeqNeqLess], [FromLess], [ToLess], [fromNotLess], [toNotLess], [NegativeLessPositive], [leqLessTransitivity], [LessLeqTransitivity], [LessTransitivity], [LessTotality], [SubLessRight], [SubLessLeft], [SwitchTerms(x < y − z)], [SwitchTerms(x − y < z)], [LessAddition], [LessAdditionLeft], [LessMultiplication], [LessMultiplicationLeft], [LessDivision], [PositiveToRight(Less)], [PositiveToLeft(Less)], [NegativeToLeft(Less)], [NegativeToRight(Less)], [AddEquations(Less)], [AddEquations(LeqLess)], [reciprocalToLeft(Less)], [LessNegated], [PositiveNonzero], [PositiveNegated], [NonpositiveNegated], [NegativeNegated], [NonnegativeNegated], [PositiveHalved], [PositiveInverted], [NonnegativeNumerical], [NegativeNumerical], [PositiveNumerical], [lemma nonpositiveNumerical], [|0| = 0], [0 <= |x|], [x <= |x|], [FromPositiveNumerical], [SameNumerical], [SignNumerical(+)], [SignNumerical], [ToNumericalLess], [FromNumericalGreater], [NumericalDifference], [NumericalDifferenceLess(Helper)], [NumericalDifferenceLess], [SplitNumericalSumHelper], [splitNumericalSum(++)], [splitNumericalSum(−−)], [splitNumericalSum(+ − small)], [splitNumericalSum(+ − big)], [splitNumericalSum(+−)], [splitNumericalSum(−+)], [splitNumericalSum], [SplitNumericalProduct(++)], [SplitNumericalProduct(+−)], [SplitNumericalProduct], [insertMiddleTerm(Numerical)], [insertTwoMiddleTerms(Numerical)], [Three2twoTerms], [Three2threeTerms], [Three2twoFactors], [Three2threeFactors], [Times(−1)], [Times(−1)Left], [MaxLeq(1)], [MaxLeq(2)], [LessThanMax], [x + y = zBackwards], [x ∗ y = zBackwards], [x = x + (y − y)], [x = x + y − y], [x = x ∗ y ∗ (1/y)], [insertMiddleTerm(Sum)], [insertTwoMiddleTerms(Sum)], [insertMiddleTerm(Difference)], [x ∗ 0 + x = x], [x ∗ 0 = 0], [NonnegativeFactors], [NonzeroFactors], [PositiveFactors], [PlusTimesMinus], [MinusTimesMinus], [(−1) ∗ (−1) + (−1) ∗ 1 = 0], [(−1) ∗ (−1) = 1], [0 < 1Helper], [0 < 1], [0 < 2], [0 < 3], [0 < 1/2], [0 < 1/3], [TwoWholes], [ThreeWholes], [TwoHalves], [ThreeThirds], [−x − y = −(x + y)], [−x ∗ y = −(x ∗ y)], [−0 = 0], [SFsymmetry], [SFtransitivity], [f2R(Plus)], [f2R(Times)], [<< TransitivityHelper(Q)], [<< Transitivity], [<<== Reflexivity], [<<== AntisymmetryHelper(Q)], [FromNot < f(Weak)(Helper)], [FromNot < f(Weak)], [FromNot < f(Strong)(Helper2)], [FromNot < f(Strong)(Helper)], [FromNot < f(Strong)], [fromNotSameF(Strongest)(Helper2)], [fromNotSameF(Strongest)(Helper)], [fromNotSameF(Strongest)], [ToLess(F)(Helper)], [ToLess(F)], [FromNot <<], [ToLess(R)], [FromNotSameF(Weak)(Helper)], [FromNotSameF(Weak)], [FromNotLess(F)], [== Addition], [== AdditionLeft], [Fpart − Bounded(Base)], [Fpart − Bounded(InduHelper)], [Fpart − Bounded(Indu)], [Fpart − Bounded], [F − Bounded(Helper)], [F − Bounded], [SameFmultiplication(Helper)], [SameFmultiplication], [EqMultiplication(R)], [EqMultiplicationLeft(R)], [x ∗ 0 = 0(F)], [x ∗ 0 = 0(R)], [LessMultiplication(F)(Helper2)], [LessMultiplication(F)(Helper)], [LessMultiplication(F)], [LessMultiplication(R)], [LeqMultiplication(R)], [PlusAssociativity(F)], [Plus0(F)], [PlusCommutativity(F)], [TimesAssociativity(F)], [Times1f], [Cauchy(2)(Helper)], [Cauchy(2)], [ReciprocalFnonzero], [(Eventually = f)2sameF(Helper)], [(Eventually = f)2sameF], [FromNotSameF(Strong)(Helper2)], [FromNotSameF(Strong)(Helper)], [FromNotSameF(Strong)], [SameFreciprocal(Helper)], [SameFreciprocal], [From!! ==], [Reciprocal(R)], [TimesCommutativity(F)], [Distribution(F)], [FromMax(1)], [FromMax(2)], [ToNegatedAnd], [DistributionOut], [DistributionOutLeft], [DistributionLeft], [FromNotLess(R)], [CartProdIsRelation], [FromSubset], [SubsetIsRelation], [ToSeries], [FromSeries], [SeriesSubsetCP], [ValueType], [RemoveOr], [FromSingleton], [InPair(1)], [InPair(2)], [SameMember(2)], [ToBinaryUnion(1)], [ToBinaryUnion(2)], [FromOrderedPair(TwoLevels)], [ToCartProd(Helper)], [ToCartProd], [NonreciprocalToRight(Eq)], [NonreciprocalToLeft(Eq)(1term)], [SameReciprocal], [CPseparationIsRelation], [OrderedPairEquality], [ReciprocalIsFunction], [ReciprocalIsTotal], [ReciprocalIsRationalSeries], [CrsIsRelation], [CrsIsFunction], [CrsIsTotal], [CrsIsSeries], [CrsLookup], [0f], [1f], [ToSingleton], [FromSameSingleton], [SingletonmembersEqual], [UnequalsNotInSingleton], [NonsingletonmembersUnequal], [FromOrderedPair], [FromOrderedPair(1)], [FromOrderedPair(2)], [FromCartProd], [FromCartProd(1)], [FromCartProd(2)], [sameOrderedPair], [InSeriesHelper], [InSeries], [To = f(Subset)(Helper)], [To = f(Subset)], [To = f], [productIsFunction], [productIsTotal], [ProductIsRationalSeries], [TimesF], [$-$x + (1/2)x = $-$(1/2)x], [PositiveTripled], [PositiveDividedBy3], [|x $-$ x| = 0], [1 < 2], [1/3 < 2/3], [(1/3)x + (1/3)x = (2/3)x], [(2/3)x + (1/3)x = x], [$-$x + (2/3)x = $-$(1/3)x], [$-$(1/3)x $-$ (1/3)x = $-$(2/3)x], [$-$x + (1/3)x = $-$(2/3)x], [PreserveLessGreater], [ClosetolessIsLess], [SubLessLeft(F)], [SubLessLeft(R)], [ClosetogreaterIsGreater], [SubLessRight(F)], [SubLessRight(R)], [plus0Left], [times1Left], [EqAdditionLeft], [EqMultiplicationLeft], [PlusF(Sym)], [TimesF(Sym)], [SameSeries(Gen)], [EqualsSameF], [LeqReflexivity(R)];

Preassociative

[Tester1], [Tester2], [Tester3], [Tester4], [Tester5], [Tester6];

Preassociative

[$*_{\{*\}}$], [*/indexintro(*, *, *, *)], [*/intro(*, *, *)], [*/bothintro(*, *, *, *, *)], [*/nameintro(*, *, *, *)], [$*'$], [*[*]], [*[*→*]], [*[*⇒*]], [*0], [*1], [0b], [*-color(*)], [$*$-color*(*)], [$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^{0}$], [$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*^{\mathrm{hide}}$];

Preassociative

[“ * ”], [], [$(*)^{\mathbf{t}}$], [string(*) + *], [string(*) ++ *], [
*], [*], [!*], ["*], [#*], [$*], [%*], [&*], [’*], [(*], [)*], [**], [+*], [, *], [-*], [.*], [/*], [0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [:*], [; *], [<*], [=*], [>*], [?*], [@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*], [O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [*], []*], [^*], [_*], [‘*], [a*], [b*], [c*], [d*], [e*], [f*], [g*], [h*], [i*], [j*], [k*], [l*], [m*], [n*], [o*], [p*], [q*], [r*], [s*], [t*], [u*], [v*], [w*], [x*], [y*], [z*], [{*], [|*], [}*], [~*], [**Preassociative** *; *], [**Postassociative** *; *], [[*], *], [priority * end], [newline *], [macro newline *], [MacroIndent(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*(exp)*];

Preassociative

[$*'$], [R(*)], [$-$ $-$ R(*)], [rec*];

Preassociative

[*/*], [* ∩ *], [*[*]];

Preassociative

$[\cup *], [* \cup *], [\mathrm{P}(*)]$;
Preassociative
$[\{*\}], [\mathrm{StateExpand}(*,*,*)], [\mathrm{extractSeries}(*)], [\mathrm{SetOfSeries}(*)], [--\mathrm{Macro}(*)],$
$[\mathrm{ExpandList}(*,*,*)], [**\mathrm{Macro}(*)], [++\mathrm{Macro}(*)], [<<\mathrm{Macro}(*)],$
$[||\mathrm{Macro}(*)], [01//\mathrm{Macro}(*)], [\mathrm{UB}(*,*)], [\mathrm{LUB}(*,*)], [\mathrm{BS}(*,*)],$
$[\mathrm{UStelescope}(*,*)], [(*)], [|\mathrm{f} * |], [|\mathrm{r} * |], [\mathrm{Limit}(*,*)], [\mathrm{Union}(*)],$
$[\mathrm{IsOrderedPair}(*,*,*)], [\mathrm{IsRelation}(*,*,*)], [\mathrm{isFunction}(*,*,*)], [\mathrm{IsSeries}(*,*)],$
$[\mathrm{IsNatural}(*,*)], [\mathrm{OrderedPair}(*,*)], [\mathrm{TypeNat}(*)], [\mathrm{TypeNat0}(*)],$
$[\mathrm{TypeRational}(*)], [\mathrm{TypeRational0}(*)], [\mathrm{TypeSeries}(*,*)], [\mathrm{Typeseries0}(*,*)]$;
Preassociative
$[\{*,*\}], [\langle *,* \rangle], [(-\mathrm{u}*)], [-_{\mathrm{f}}*], [(--*)], [1\mathrm{f}/*], [01//\mathrm{temp}*]$;
Preassociative
$[*(*,*)], [\mathrm{ReflRel}(*,*)], [\mathrm{SymRel}(*,*)], [\mathrm{TransRel}(*,*)], [\mathrm{EqRel}(*,*)], [[* \in *]_*],$
$[\mathrm{Partition}(*,*)]$;
Preassociative
$[* \cdot *], [* \cdot_0 *], [(* * *)], [* *_{\mathrm{f}} *], [* * **]$;
Preassociative
$[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [(* + *)], [(* - *)], [* +_{\mathrm{f}} *],$
$[* -_{\mathrm{f}} *], [* + + *], [\mathrm{R}(*) - -\mathrm{R}(*)]$;
Preassociative
$[* \in *]$;
Preassociative
$[| * |], [\mathrm{if}(*,*,*)], [\mathrm{Max}(*,*)], [\mathrm{Max}(*,*)]$;
Preassociative
$[* = *], [* \neq *], [* <= *], [* < *], [* <_{\mathrm{f}} *], [* \leq_{\mathrm{f}} *], [\mathrm{SF}(*,*)], [* == *],$
$[*!! == *], [* << *], [* <<== *]$;
Preassociative
$[* \cup \{*\}], [* \cup *], [* \backslash \{*\}]$;
Postassociative
$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *]$;
Postassociative
$[*,*]$;
Preassociative
$[* \stackrel{\mathrm{B}}{\approx} *], [* \stackrel{\mathrm{D}}{\approx} *], [* \stackrel{\mathrm{C}}{\approx} *], [* \stackrel{\mathrm{P}}{\approx} *], [* \approx *], [* = *], [* \stackrel{+}{\rightarrow} *], [* \stackrel{\mathrm{t}}{=} *], [* \stackrel{\mathrm{t}^*}{=} *], [* \stackrel{\mathrm{r}}{=} *],$
$[* \in_{\mathrm{t}} *], [* \subseteq_{\mathrm{T}} *], [* \stackrel{\mathrm{T}}{=} *], [* \stackrel{\mathrm{s}}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *],$
$[* \text{ free for}^* * \text{ in } *], [* \in_{\mathrm{c}} *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\mathrm{var}}],$
$[* \#^0 *], [* \#^1 *], [* \#^* *], [* {=}{=} *], [* \subseteq *]$;
Preassociative
$[\neg *], [\dot{\neg}(*)\mathrm{n}], [* \notin *], [* \neq *]$;
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_{\mathrm{c}} *], [* \dot{\wedge} *]$;
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *]$;
Postassociative
$[* \dot{\vee} *]$;

Preassociative
$[\exists *\colon *], [\forall *\colon *], [\forall_{\mathrm{obj}} *\colon *], [\exists *\colon *];$
Postassociative
$[* \ddot{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \dot{\Leftrightarrow} *];$
Preassociative
$[\{ \mathrm{ph} \in * \mid * \}];$
Postassociative
$[* : *], [* \text{ spy } *], [* ! *];$
Preassociative
$[* \begin{cases} * \\ * \end{cases}];$
Preassociative
$[\lambda * . *], [\Lambda * . *], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *];$
Preassociative
$[* \# *];$
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}];$
Preassociative
$[* @ *], [* \triangleright *], [* \Vvdash *], [* \gg *], [* \trianglerighteq *];$
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
$[\forall *\colon *], [\Pi *\colon *];$
Postassociative
$[* \oplus *];$
Postassociative
$[*; *];$
Preassociative
$[* \text{ proves } *];$
Preassociative
$[* \textbf{ proof of } * : *], [\text{Line} * : * \gg *; *], [\text{Last line} * \gg * \square],$
$[\text{Line} * : \text{Premise} \gg *; *], [\text{Line} * : \text{Side-condition} \gg *; *], [\text{Arbitrary} \gg *; *],$
$[\text{Local} \gg * = *; *], [\text{Begin} *; * : \text{End}; *], [\text{Last block line} * \gg *;],$
$[\text{Arbitrary} \gg *; *];$
Postassociative
$[* \mid *];$
Postassociative
$[* , *], [* [*] *];$
Preassociative
$[* \& *];$
Preassociative
$[* \backslash\backslash *], [* \text{ linebreak}[4] *], [* \backslash\backslash *];]$

$[\text{sup} \overset{\text{tex}}{\rightarrow} \text{``sup''}]$

$[\text{sup} \overset{\text{pyk}}{\rightarrow} \text{``sup''}]$

ToNegatedAnd(1)

$[\mathrm{ToNegatedAnd(1)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \vdash \underline{\mathsf{a}} \vdash \mathrm{FromContradiction} \rhd \underline{\mathsf{a}} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \gg \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \mathrm{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \vdash \underline{\mathsf{a}} \vdash \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \gg \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n}; \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \vdash \mathrm{MP} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \gg \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n}; \mathrm{ToNegatedAnd} \rhd \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToNegatedAnd(1)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \vdash \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{ToNegatedAnd(1)} \stackrel{\mathrm{tex}}{\rightarrow}$ "ToNegatedAnd(1)"]

$[\mathrm{ToNegatedAnd(1)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma to negated and(1)"]

UniqueNegative

$[\mathrm{UniqueNegative} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = 0 \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = 0 \vdash \mathrm{plusCommutativity} \gg (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{y}}); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{y}}) \rhd (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = 0 \gg (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = 0; \mathrm{PositiveToRight(Eq)} \rhd (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = 0 \gg \underline{\mathsf{y}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})); \mathrm{plusCommutativity} \gg (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{z}}); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{z}}) \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = 0 \gg (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = 0; \mathrm{PositiveToRight(Eq)} \rhd (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = 0 \gg \underline{\mathsf{z}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})); \mathrm{eqSymmetry} \rhd \underline{\mathsf{z}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})) \gg (0 + (-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{z}}; \mathrm{eqTransitivity} \rhd \underline{\mathsf{y}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})) \rhd (0 + (-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{z}} \gg \underline{\mathsf{y}} = \underline{\mathsf{z}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{UniqueNegative} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = 0 \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = 0 \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}}]$

$[\mathrm{UniqueNegative} \stackrel{\mathrm{tex}}{\rightarrow}$ "UniqueNegative"]

$[\mathrm{UniqueNegative} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma uniqueNegative"]

DoubleMinus

$[\mathrm{DoubleMinus} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \mathrm{Negative} \gg ((-\mathrm{u}\underline{\mathsf{x}}) + (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}}))) = 0; \mathrm{x} + \mathrm{y} = \mathrm{zBackwards} \rhd ((-\mathrm{u}\underline{\mathsf{x}}) + (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}}))) = 0 \gg 0 = ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) + (-\mathrm{u}\underline{\mathsf{x}})); \mathrm{NegativeToLeft(Eq)} \rhd 0 = ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) + (-\mathrm{u}\underline{\mathsf{x}})) \gg (0 + \underline{\mathsf{x}}) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})); \mathrm{plus0Left} \gg (0 + \underline{\mathsf{x}}) = \underline{\mathsf{x}}; \mathrm{Equality} \rhd (0 + \underline{\mathsf{x}}) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) \rhd (0 + \underline{\mathsf{x}}) = \underline{\mathsf{x}} \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{DoubleMinus} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{x}}]$

$[\mathrm{DoubleMinus} \stackrel{\mathrm{tex}}{\rightarrow}$ "DoubleMinus"]

$[\mathrm{DoubleMinus} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma doubleMinus"]

MinusNegated

$[\mathrm{MinusNegated} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \mathrm{DoubleMinus} \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}}; \mathrm{eqAddition} \rhd (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}} \gg ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) + (-\mathrm{u}\underline{\mathsf{x}})) = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})); \mathrm{eqSymmetry} \rhd ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) + (-\mathrm{u}\underline{\mathsf{x}})) = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) \gg (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) = ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) + (-\mathrm{u}\underline{\mathsf{x}})); -\mathrm{x} - \mathrm{y} = -(\mathrm{x} + \mathrm{y}) \gg ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) + (-\mathrm{u}\underline{\mathsf{x}})) = (-\mathrm{u}((-\mathrm{u}\underline{\mathsf{y}}) + \underline{\mathsf{x}})); \mathrm{plusCommutativity} \gg ((-\mathrm{u}\underline{\mathsf{y}}) + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})); \mathrm{EqNegated} \rhd ((-\mathrm{u}\underline{\mathsf{y}}) + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) \gg (-\mathrm{u}((-\mathrm{u}\underline{\mathsf{y}}) + \underline{\mathsf{x}})) = (-\mathrm{u}(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))); \mathrm{eqTransitivity4} \rhd (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) = ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) + (-\mathrm{u}\underline{\mathsf{x}})) \rhd ((-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) + (-\mathrm{u}\underline{\mathsf{x}})) = (-\mathrm{u}((-\mathrm{u}\underline{\mathsf{y}}) + \underline{\mathsf{x}})) \rhd (-\mathrm{u}((-\mathrm{u}\underline{\mathsf{y}}) + \underline{\mathsf{x}})) = (-\mathrm{u}(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))) \gg (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) = (-\mathrm{u}(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))); \mathrm{eqSymmetry} \rhd (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) = (-\mathrm{u}(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))) \gg (-\mathrm{u}(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MinusNegated} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon (-\mathrm{u}(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))]$

$[\mathrm{MinusNegated} \stackrel{\mathrm{tex}}{\rightarrow}$ "MinusNegated"$]$

$[\mathrm{MinusNegated} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma minusNegated"$]$

eqReflexivity

$[\mathrm{eqReflexivity} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \mathrm{leqReflexivity} \gg \underline{\mathsf{x}} <= \underline{\mathsf{x}}; \mathrm{leqAntisymmetry} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{x}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{eqReflexivity} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \underline{\mathsf{x}} = \underline{\mathsf{x}}]$

$[\mathrm{eqReflexivity} \stackrel{\mathrm{tex}}{\rightarrow}$ "eqReflexivity"$]$

$[\mathrm{eqReflexivity} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma eqReflexivity"$]$

eqSymmetry

$[\mathrm{eqSymmetry} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \mathrm{eqReflexivity} \gg \underline{\mathsf{x}} = \underline{\mathsf{x}}; \mathrm{Equality} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{x}} = \underline{\mathsf{x}} \gg \underline{\mathsf{y}} = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{eqSymmetry} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{x}}]$

$[\mathrm{eqSymmetry} \stackrel{\mathrm{tex}}{\rightarrow}$ "eqSymmetry"$]$

$[\mathrm{eqSymmetry} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma eqSymmetry"$]$

eqTransitivity

[eqTransitivity $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash$ $\text{eqSymmetry} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{y}} = \underline{\mathsf{x}}; \text{Equality} \rhd \underline{\mathsf{y}} = \underline{\mathsf{x}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \gg \underline{\mathsf{x}} = \underline{\mathsf{z}}\rceil, \text{p}_0, \mathsf{c})$]

[eqTransitivity $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{z}}$]

[eqTransitivity $\stackrel{\text{tex}}{\rightarrow}$ "eqTransitivity"]

[eqTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma eqTransitivity"]

eqTransitivity4

[eqTransitivity4 $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \text{eqTransitivity} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \gg \underline{\mathsf{x}} = \underline{\mathsf{z}}; \text{eqTransitivity} \rhd \underline{\mathsf{x}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{z}} = \underline{\mathsf{u}} \gg \underline{\mathsf{x}} = \underline{\mathsf{u}}\rceil, \text{p}_0, \mathsf{c})$]

[eqTransitivity4 $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{u}}$]

[eqTransitivity4 $\stackrel{\text{tex}}{\rightarrow}$ "eqTransitivity4"]

[eqTransitivity4 $\stackrel{\text{pyk}}{\rightarrow}$ "lemma eqTransitivity4"]

eqTransitivity5

[eqTransitivity5 $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \text{eqTransitivity4} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{z}} = \underline{\mathsf{u}} \gg \underline{\mathsf{x}} = \underline{\mathsf{u}}; \text{eqTransitivity} \rhd \underline{\mathsf{x}} = \underline{\mathsf{u}} \rhd \underline{\mathsf{u}} = \underline{\mathsf{v}} \gg \underline{\mathsf{x}} = \underline{\mathsf{v}}\rceil, \text{p}_0, \mathsf{c})$]

[eqTransitivity5 $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{v}}$]

[eqTransitivity5 $\stackrel{\text{tex}}{\rightarrow}$ "eqTransitivity5"]

[eqTransitivity5 $\stackrel{\text{pyk}}{\rightarrow}$ "lemma eqTransitivity5"]

eqTransitivity6

[eqTransitivity6 $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \forall \underline{\mathsf{w}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \underline{\mathsf{v}} = \underline{\mathsf{w}} \vdash \text{eqTransitivity5} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{z}} = \underline{\mathsf{u}} \rhd \underline{\mathsf{u}} = \underline{\mathsf{v}} \gg \underline{\mathsf{x}} = \underline{\mathsf{v}}; \text{eqTransitivity} \rhd \underline{\mathsf{x}} = \underline{\mathsf{v}} \rhd \underline{\mathsf{v}} = \underline{\mathsf{w}} \gg \underline{\mathsf{x}} = \underline{\mathsf{w}}\rceil, \text{p}_0, \mathsf{c})$]

[eqTransitivity6 $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \forall \underline{\mathsf{w}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \underline{\mathsf{v}} = \underline{\mathsf{w}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{w}}$]

[eqTransitivity6 $\stackrel{\text{tex}}{\to}$ "eqTransitivity6"]

[eqTransitivity6 $\stackrel{\text{pyk}}{\to}$ "lemma eqTransitivity6"]

AddEquations

[AddEquations $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash \text{eqAddition} \rhd \underline{x} = \underline{y} \gg (\underline{x}+\underline{z}) = (\underline{y}+\underline{z}); \text{EqAdditionLeft} \rhd \underline{z} = \underline{u} \gg (\underline{y}+\underline{z}) = (\underline{y}+\underline{u}); \text{eqTransitivity} \rhd (\underline{x}+\underline{z}) = (\underline{y}+\underline{z}) \rhd (\underline{y}+\underline{z}) = (\underline{y}+\underline{u}) \gg (\underline{x}+\underline{z}) = (\underline{y}+\underline{u})\rceil, p_0, c)$]

[AddEquations $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash (\underline{x}+\underline{z}) = (\underline{y}+\underline{u})$]

[AddEquations $\stackrel{\text{tex}}{\to}$ "AddEquations"]

[AddEquations $\stackrel{\text{pyk}}{\to}$ "lemma addEquations"]

SubtractEquations

[SubtractEquations $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{x}+\underline{z}) = (\underline{y}+\underline{u}) \vdash \underline{z} = \underline{u} \vdash \text{eqAddition} \rhd (\underline{x}+\underline{z}) = (\underline{y}+\underline{u}) \gg ((\underline{x}+\underline{z}) + (-\text{u}\underline{z})) = ((\underline{y}+\underline{u}) + (-\text{u}\underline{z})); \text{plus0Left} \gg (0+\underline{z}) = \underline{z}; \text{eqTransitivity} \rhd (0+\underline{z}) = \underline{z} \rhd \underline{z} = \underline{u} \gg (0+\underline{z}) = \underline{u}; \text{PositiveToRight(Eq)} \rhd (0+\underline{z}) = \underline{u} \gg 0 = (\underline{u} + (-\text{u}\underline{z})); \text{eqSymmetry} \rhd 0 = (\underline{u} + (-\text{u}\underline{z})) \gg (\underline{u} + (-\text{u}\underline{z})) = 0; \text{EqAdditionLeft} \rhd (\underline{u} + (-\text{u}\underline{z})) = 0 \gg (\underline{y} + (\underline{u} + (-\text{u}\underline{z}))) = (\underline{y}+0); \text{plusAssociativity} \gg ((\underline{y}+\underline{u})+(-\text{u}\underline{z})) = (\underline{y}+(\underline{u}+(-\text{u}\underline{z}))); \text{plus0} \gg (\underline{y}+0) = \underline{y}; \text{eqTransitivity4} \rhd ((\underline{y}+\underline{u}) + (-\text{u}\underline{z})) = (\underline{y} + (\underline{u} + (-\text{u}\underline{z}))) \rhd (\underline{y} + (\underline{u} + (-\text{u}\underline{z}))) = (\underline{y}+0) \rhd (\underline{y}+0) = \underline{y} \gg ((\underline{y}+\underline{u}) + (-\text{u}\underline{z})) = \underline{y}; \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{x} = ((\underline{x}+\underline{z}) + (-\text{u}\underline{z})); \text{eqTransitivity4} \rhd \underline{x} = ((\underline{x}+\underline{z}) + (-\text{u}\underline{z})) \rhd ((\underline{x}+\underline{z}) + (-\text{u}\underline{z})) = ((\underline{y}+\underline{u}) + (-\text{u}\underline{z})) \rhd ((\underline{y}+\underline{u}) + (-\text{u}\underline{z})) = \underline{y} \gg \underline{x} = \underline{y}\rceil, p_0, c)$]

[SubtractEquations $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{x}+\underline{z}) = (\underline{y}+\underline{u}) \vdash \underline{z} = \underline{u} \vdash \underline{x} = \underline{y}$]

[SubtractEquations $\stackrel{\text{tex}}{\to}$ "SubtractEquations"]

[SubtractEquations $\stackrel{\text{pyk}}{\to}$ "lemma subtractEquations"]

SubtractEquationsLeft

[SubtractEquationsLeft $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{x}+\underline{z}) = (\underline{y}+\underline{u}) \vdash \underline{x} = \underline{y} \vdash \text{plusCommutativity} \gg (\underline{z}+\underline{x}) = (\underline{x}+\underline{z}); \text{plusCommutativity} \gg (\underline{y}+\underline{u}) = (\underline{u}+\underline{y}); \text{eqTransitivity4} \rhd (\underline{z}+\underline{x}) =$

$(\underline{x} + \underline{z}) \rhd (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \rhd (\underline{y} + \underline{u}) = (\underline{u} + \underline{y}) \gg (\underline{z} + \underline{x}) = (\underline{u} + \underline{y}); \mathrm{SubtractEquations} \rhd (\underline{z} + \underline{x}) = (\underline{u} + \underline{y}) \rhd \underline{x} = \underline{y} \gg \underline{z} = \underline{u}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SubtractEquationsLeft} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{x} = \underline{y} \vdash \underline{z} = \underline{u}]$

[SubtractEquationsLeft $\stackrel{\mathrm{tex}}{\rightarrow}$ "SubtractEquationsLeft"]

[SubtractEquationsLeft $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma subtractEquationsLeft"]

MultiplyEquations

$[\mathrm{MultiplyEquations} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash \mathrm{eqMultiplication} \rhd \underline{x} = \underline{y} \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}); \mathrm{EqMultiplicationLeft} \rhd \underline{z} = \underline{u} \gg (\underline{y} * \underline{z}) = (\underline{y} * \underline{u}); \mathrm{eqTransitivity} \rhd (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \rhd (\underline{y} * \underline{z}) = (\underline{y} * \underline{u}) \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{u})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MultiplyEquations} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash (\underline{x} * \underline{z}) = (\underline{y} * \underline{u})]$

[MultiplyEquations $\stackrel{\mathrm{tex}}{\rightarrow}$ "MultiplyEquations"]

[MultiplyEquations $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma multiplyEquations"]

EqNegated

$[\mathrm{EqNegated} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \underline{x} = \underline{y} \vdash \mathrm{Negative} \gg (\underline{x} + (-\mathrm{u}\underline{x})) = 0; \mathrm{Negative} \gg (\underline{y} + (-\mathrm{u}\underline{y})) = 0; \mathrm{eqSymmetry} \rhd (\underline{y} + (-\mathrm{u}\underline{y})) = 0 \gg 0 = (\underline{y} + (-\mathrm{u}\underline{y})); \mathrm{eqTransitivity} \rhd (\underline{x} + (-\mathrm{u}\underline{x})) = 0 \rhd 0 = (\underline{y} + (-\mathrm{u}\underline{y})) \gg (\underline{x} + (-\mathrm{u}\underline{x})) = (\underline{y} + (-\mathrm{u}\underline{y})); \mathrm{SubtractEquationsLeft} \rhd (\underline{x} + (-\mathrm{u}\underline{x})) = (\underline{y} + (-\mathrm{u}\underline{y})) \rhd \underline{x} = \underline{y} \gg (-\mathrm{u}\underline{x}) = (-\mathrm{u}\underline{y})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{EqNegated} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \underline{x} = \underline{y} \vdash (-\mathrm{u}\underline{x}) = (-\mathrm{u}\underline{y})]$

[EqNegated $\stackrel{\mathrm{tex}}{\rightarrow}$ "EqNegated"]

[EqNegated $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma eqNegated"]

PositiveToRight(Eq)

$[\mathrm{PositiveToRight(Eq)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon (\underline{x} + \underline{y}) = \underline{z} \vdash \mathrm{eqAddition} \rhd (\underline{x} + \underline{y}) = \underline{z} \gg ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) = (\underline{z} + (-\mathrm{u}\underline{y})); \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})); \mathrm{eqTransitivity} \rhd \underline{x} = ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) \rhd ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) = (\underline{z} + (-\mathrm{u}\underline{y})) \gg \underline{x} = (\underline{z} + (-\mathrm{u}\underline{y}))\rceil, \mathrm{p}_0, \mathsf{c})]$

[PositiveToRight(Eq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = \underline{z} \vdash \underline{x} = (\underline{z} + (-u\underline{y}))$]

[PositiveToRight(Eq) $\stackrel{\text{tex}}{\rightarrow}$ "PositiveToRight(Eq)"]

[PositiveToRight(Eq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma positiveToRight(Eq)"]

PositiveToLeft(Eq)(1term)

[PositiveToLeft(Eq)(1term) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash$ eqAddition $\rhd \underline{x} = \underline{y} \gg (\underline{x} + (-u\underline{y})) = (\underline{y} + (-u\underline{y}))$; Negative $\gg (\underline{y} + (-u\underline{y})) = 0$; eqTransitivity $\rhd (\underline{x} + (-u\underline{y})) = (\underline{y} + (-u\underline{y})) \rhd (\underline{y} + (-u\underline{y})) = 0 \gg (\underline{x} + (-u\underline{y})) = 0\rceil, p_0, c)$]

[PositiveToLeft(Eq)(1term) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash (\underline{x} + (-u\underline{y})) = 0$]

[PositiveToLeft(Eq)(1term) $\stackrel{\text{tex}}{\rightarrow}$ "PositiveToLeft(Eq)(1 term)"]

[PositiveToLeft(Eq)(1term) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma positiveToLeft(Eq)(1 term)"]

NegativeToLeft(Eq)

[NegativeToLeft(Eq) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = (\underline{y} + (-u\underline{z})) \vdash$ eqAddition $\rhd \underline{x} = (\underline{y} + (-u\underline{z})) \gg (\underline{x} + \underline{z}) = ((\underline{y} + (-u\underline{z})) + \underline{z})$; Three2threeTerms $\gg ((\underline{y} + (-u\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-u\underline{z}))$; x = x + y − y $\gg \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z}))$; eqSymmetry $\rhd \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y}$; eqTransitivity4 $\rhd (\underline{x} + \underline{z}) = ((\underline{y} + (-u\underline{z})) + \underline{z}) \rhd ((\underline{y} + (-u\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-u\underline{z})) \rhd ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y} \gg (\underline{x} + \underline{z}) = \underline{y}\rceil, p_0, c)$]

[NegativeToLeft(Eq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = (\underline{y} + (-u\underline{z})) \vdash (\underline{x} + \underline{z}) = \underline{y}$]

[NegativeToLeft(Eq) $\stackrel{\text{tex}}{\rightarrow}$ "NegativeToLeft(Eq)"]

[NegativeToLeft(Eq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeToLeft(Eq)"]

NonreciprocalToRight(Eq)(1term)

[NonreciprocalToRight(Eq)(1term) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon (\underline{x} * \underline{y}) = 1 \vdash$ eqMultiplication $\rhd (\underline{x} * \underline{y}) = 1 \gg ((\underline{x} * \underline{y}) * \text{rec}\underline{y}) = (1 * \text{rec}\underline{y})$; $0 < 1 \gg \dot{\neg}\,(0 <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = 1)n)n)n$; PositiveNonzero $\rhd \dot{\neg}\,(0 <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = 1)n)n)n \gg \dot{\neg}\,(1 = 0)n$; eqSymmetry $\rhd (\underline{x} * \underline{y}) = 1 \gg 1 = (\underline{x} * \underline{y})$; SubNeqLeft $\rhd 1 = (\underline{x} * \underline{y}) \rhd \dot{\neg}\,(1 = 0)n \gg \dot{\neg}\,((\underline{x} * \underline{y}) = 0)n$; NonzeroProduct(2) $\rhd \dot{\neg}\,((\underline{x} * \underline{y}) = 0)n \gg \dot{\neg}\,(\underline{y} = 0)n$; x = x ∗ y ∗ (1/y) $\rhd \dot{\neg}\,(\underline{y} = 0)n \gg \underline{x} = ((\underline{x} * \underline{y}) * \text{rec}\underline{y})$; times1Left $\gg$

$(1 * \mathrm{rec}\underline{y}) = \mathrm{rec}\underline{y}; \mathrm{eqTransitivity4} \rhd \underline{x} = ((\underline{x} * \underline{y}) * \mathrm{rec}\underline{y}) \rhd ((\underline{x} * \underline{y}) * \mathrm{rec}\underline{y}) = (1 * \mathrm{rec}\underline{y}) \rhd (1 * \mathrm{rec}\underline{y}) = \mathrm{rec}\underline{y} \gg \underline{x} = \mathrm{rec}\underline{y}\rceil, p_0, \mathsf{c})]$

$[\mathrm{NonreciprocalToRight(Eq)(1term)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon (\underline{x} * \underline{y}) = 1 \vdash \underline{x} = \mathrm{rec}\underline{y}]$

$[\mathrm{NonreciprocalToRight(Eq)(1term)} \stackrel{\mathrm{tex}}{\rightarrow}$ "NonreciprocalToRight(Eq)(1 term)"]

$[\mathrm{NonreciprocalToRight(Eq)(1term)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma nonreciprocalToRight(Eq)(1 term)"]

PlusAssociativity(4terms)

$[\mathrm{PlusAssociativity(4terms)} \stackrel{\mathrm{tex}}{\rightarrow}$ "PlusAssociativity(4 terms)"]

$[\mathrm{PlusAssociativity(4terms)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma plusAssociativity(4 terms)"]

LessNeq

$[\mathrm{LessNeq} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \mathrm{Repetition} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} = \underline{y})n\rceil, p_0, \mathsf{c})]$

$[\mathrm{LessNeq} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(\underline{x} = \underline{y})n]$

$[\mathrm{LessNeq} \stackrel{\mathrm{tex}}{\rightarrow}$ "LessNeq"]

$[\mathrm{LessNeq} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma lessNeq"]

NeqSymmetry

$[\mathrm{NeqSymmetry} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \underline{y} = \underline{x} \vdash \mathrm{eqSymmetry} \rhd \underline{y} = \underline{x} \gg \underline{x} = \underline{y}; \forall \underline{x} \colon \forall \underline{y} \colon \mathrm{Ded} \rhd \forall \underline{x} \colon \forall \underline{y} \colon \underline{y} = \underline{x} \vdash \underline{x} = \underline{y} \gg \underline{y} = \underline{x} \Rightarrow \underline{x} = \underline{y}; \dot{\neg}(\underline{x} = \underline{y})n \vdash \mathrm{MT} \rhd \underline{y} = \underline{x} \Rightarrow \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}(\underline{y} = \underline{x})n\rceil, p_0, \mathsf{c})]$

$[\mathrm{NeqSymmetry} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \dot{\neg}(\underline{x} = \underline{y})n \vdash \dot{\neg}(\underline{y} = \underline{x})n]$

$[\mathrm{NeqSymmetry} \stackrel{\mathrm{tex}}{\rightarrow}$ "NeqSymmetry"]

$[\mathrm{NeqSymmetry} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma neqSymmetry"]

NeqNegated

[NeqNegated $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} = \underline{y})n \vdash (-u\underline{x}) = (-u\underline{y}) \vdash \text{EqNegated} \rhd (-u\underline{x}) = (-u\underline{y}) \gg (-u(-u\underline{x})) = (-u(-u\underline{y})); \text{DoubleMinus} \gg (-u(-u\underline{x})) = \underline{x}; \text{eqSymmetry} \rhd (-u(-u\underline{x})) = \underline{x} \gg \underline{x} = (-u(-u\underline{x})); \text{DoubleMinus} \gg (-u(-u\underline{y})) = \underline{y}; \text{eqTransitivity4} \rhd \underline{x} = (-u(-u\underline{x})) \rhd (-u(-u\underline{x})) = (-u(-u\underline{y})) \rhd (-u(-u\underline{y})) = \underline{y} \gg \underline{x} = \underline{y}; \text{FromContradiction} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} = \underline{y})n \vdash (-u\underline{x}) = (-u\underline{y}) \vdash \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n \gg \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow (-u\underline{x}) = (-u\underline{y}) \Rightarrow \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n; \dot{\neg}(\underline{x} = \underline{y})n \vdash \text{MP} \rhd \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow (-u\underline{x}) = (-u\underline{y}) \Rightarrow \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg (-u\underline{x}) = (-u\underline{y}) \Rightarrow \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n; \text{prop lemma imply negation} \rhd (-u\underline{x}) = (-u\underline{y}) \Rightarrow \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n \gg \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n\rceil, p_0, c)$]

[NeqNegated $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} = \underline{y})n \vdash \dot{\neg}((-u\underline{x}) = (-u\underline{y}))n$]

[NeqNegated $\stackrel{\text{tex}}{\rightarrow}$ "NeqNegated"]

[NeqNegated $\stackrel{\text{pyk}}{\rightarrow}$ "lemma neqNegated"]

SubNeqRight

[SubNeqRight $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{z} = \underline{x})n \vdash \text{NeqSymmetry} \rhd \dot{\neg}(\underline{z} = \underline{x})n \gg \dot{\neg}(\underline{x} = \underline{z})n; \text{SubNeqLeft} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{y} = \underline{z})n; \text{NeqSymmetry} \rhd \dot{\neg}(\underline{y} = \underline{z})n \gg \dot{\neg}(\underline{z} = \underline{y})n\rceil, p_0, c)$]

[SubNeqRight $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{z} = \underline{x})n \vdash \dot{\neg}(\underline{z} = \underline{y})n$]

[SubNeqRight $\stackrel{\text{tex}}{\rightarrow}$ "SubNeqRight"]

[SubNeqRight $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subNeqRight"]

SubNeqLeft

[SubNeqLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{x} = \underline{z})n \vdash \text{EqualityAxiom} \gg \underline{y} = \underline{x} \Rightarrow \underline{y} = \underline{z} \Rightarrow \underline{x} = \underline{z}; \text{eqSymmetry} \rhd \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \text{MP} \rhd \underline{y} = \underline{x} \Rightarrow \underline{y} = \underline{z} \Rightarrow \underline{x} = \underline{z} \rhd \underline{y} = \underline{x} \gg \underline{y} = \underline{z} \Rightarrow \underline{x} = \underline{z}; \text{Contrapositive} \rhd \underline{y} = \underline{z} \Rightarrow \underline{x} = \underline{z} \gg \dot{\neg}(\underline{x} = \underline{z})n \Rightarrow \dot{\neg}(\underline{y} = \underline{z})n; \text{MP} \rhd \dot{\neg}(\underline{x} = \underline{z})n \Rightarrow \dot{\neg}(\underline{y} = \underline{z})n \rhd \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{y} = \underline{z})n\rceil, p_0, c)$]

[SubNeqLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{x} = \underline{z})n \vdash \dot{\neg}(\underline{y} = \underline{z})n$]

[SubNeqLeft $\stackrel{\text{tex}}{\rightarrow}$ "SubNeqLeft"]

[SubNeqLeft $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subNeqLeft"]

NegativeToRight(Neq)(1term)

[NegativeToRight(Neq)(1term) $\overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathrm{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash \mathrm{PositiveToLeft(Eq)(1term)} \rhd \underline{x} = \underline{y} \gg (\underline{x} + (-\mathrm{u}\underline{y})) = 0; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash (\underline{x} + (-\mathrm{u}\underline{y})) = 0 \gg \underline{x} = \underline{y} \Rightarrow (\underline{x} + (-\mathrm{u}\underline{y})) = 0; \dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) = 0)\mathrm{n} \vdash \mathrm{MT} \rhd \underline{x} = \underline{y} \Rightarrow (\underline{x} + (-\mathrm{u}\underline{y})) = 0 \rhd \dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) = 0)\mathrm{n} \gg \dot{\neg}(\underline{x} = \underline{y})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[NegativeToRight(Neq)(1term) $\overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) = 0)\mathrm{n} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n}$]

[NegativeToRight(Neq)(1term) $\overset{\text{tex}}{\rightarrow}$ “NegativeToRight(Neq)(1 term)”]

[NegativeToRight(Neq)(1term) $\overset{\text{pyk}}{\rightarrow}$ “lemma negativeToRight(Neq)(1 term)”]

NeqAddition

[NeqAddition $\overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathrm{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \vdash \mathrm{eqReflexivity} \gg \underline{z} = \underline{z}; \mathrm{SubtractEquations} \rhd (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \rhd \underline{z} = \underline{z} \gg \underline{x} = \underline{y}; \mathrm{FromContradiction} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \gg \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n}; \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \vdash \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n} \gg \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n}; \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash \mathrm{MP} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \gg (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n}; \mathrm{prop\ lemma\ imply\ negation} \rhd (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n} \gg \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[NeqAddition $\overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n}$]

[NeqAddition $\overset{\text{tex}}{\rightarrow}$ “NeqAddition”]

[NeqAddition $\overset{\text{pyk}}{\rightarrow}$ “lemma neqAddition”]

NeqMultiplication

[NeqMultiplication $\overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathrm{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{z} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \vdash \mathrm{x} = \mathrm{x} * \mathrm{y} * (1/\mathrm{y}) \rhd \dot{\neg}(\underline{z} = 0)\mathrm{n} \gg \underline{x} = ((\underline{x} * \underline{z}) * \mathrm{rec}\underline{z}); \mathrm{eqMultiplication} \rhd (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \gg ((\underline{x} * \underline{z}) * \mathrm{rec}\underline{z}) = ((\underline{y} * \underline{z}) * \mathrm{rec}\underline{z}); \mathrm{x} = \mathrm{x} * \mathrm{y} * (1/\mathrm{y}) \rhd \dot{\neg}(\underline{z} = 0)\mathrm{n} \gg \underline{y} = ((\underline{y} * \underline{z}) * \mathrm{rec}\underline{z}); \mathrm{eqSymmetry} \rhd \underline{y} = ((\underline{y} * \underline{z}) * \mathrm{rec}\underline{z}) \gg ((\underline{y} * \underline{z}) * \mathrm{rec}\underline{z}) = \underline{y}; \mathrm{eqTransitivity4} \rhd \underline{x} = ((\underline{x} * \underline{z}) * \mathrm{rec}\underline{z}) \rhd ((\underline{x} * \underline{z}) * \mathrm{rec}\underline{z}) = ((\underline{y} * \underline{z}) * \mathrm{rec}\underline{z}) \rhd ((\underline{y} * \underline{z}) * \mathrm{rec}\underline{z}) = \underline{y} \gg \underline{x} = \underline{y}; \mathrm{FromContradiction} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \gg \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n}; \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{z} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \vdash \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n} \gg \dot{\neg}(\underline{z} = 0)\mathrm{n} \Rightarrow \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n}; \dot{\neg}(\underline{z} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash \mathrm{MP2} \rhd \dot{\neg}(\underline{z} = 0)\mathrm{n} \Rightarrow \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n} \rhd \dot{\neg}(\underline{z} = 0)\mathrm{n} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n}; \mathrm{prop\ lemma\ imply\ negation} \rhd (\underline{x} * \underline{z}) =$

$(\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}\, ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n} \gg \dot{\neg}\, ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NeqMultiplication} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \forall \underline{\mathsf{z}} \colon \dot{\neg}\, (\underline{\mathsf{z}} = 0)\mathrm{n} \vdash \dot{\neg}\, (\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash \dot{\neg}\, ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n}]$

$[\mathrm{NeqMultiplication} \overset{\mathrm{tex}}{\rightarrow}$ "NeqMultiplication"]

$[\mathrm{NeqMultiplication} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma neqMultiplication"]

NonzeroProduct(2)

$[\mathrm{NonzeroProduct(2)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{y}} = 0 \vdash \mathrm{EqMultiplicationLeft} \rhd \underline{\mathsf{y}} = 0 \gg (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (\underline{\mathsf{x}} * 0); \mathrm{x} * 0 = 0 \gg (\underline{\mathsf{x}} * 0) = 0; \mathrm{eqTransitivity} \rhd (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (\underline{\mathsf{x}} * 0) \rhd (\underline{\mathsf{x}} * 0) = 0 \gg (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0; \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \mathrm{Ded} \rhd \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{y}} = 0 \vdash (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0 \gg \underline{\mathsf{y}} = 0 \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0; \dot{\neg}\, ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n} \vdash \mathrm{MT} \rhd \underline{\mathsf{y}} = 0 \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0 \rhd \dot{\neg}\, ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n} \gg \dot{\neg}\, (\underline{\mathsf{y}} = 0)\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NonzeroProduct(2)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \dot{\neg}\, ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n} \vdash \dot{\neg}\, (\underline{\mathsf{y}} = 0)\mathrm{n}]$

$[\mathrm{NonzeroProduct(2)} \overset{\mathrm{tex}}{\rightarrow}$ "NonzeroProduct(2)"]

$[\mathrm{NonzeroProduct(2)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma nonzeroProduct(2)"]

UStelescope(+1)

$[\mathrm{UStelescope(+1)} \overset{\mathrm{tex}}{\rightarrow}$ "UStelescope(+1)"]

$[\mathrm{UStelescope(+1)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma UStelescope(+1)"]

TelescopeBound(Base)

$[\mathrm{TelescopeBound(Base)} \overset{\mathrm{tex}}{\rightarrow}$ "TelescopeBound(Base)"]

$[\mathrm{TelescopeBound(Base)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma telescopeBound base"]

TelescopeBound(Indu)

$[\mathrm{TelescopeBound(Indu)} \overset{\mathrm{tex}}{\rightarrow}$ "TelescopeBound(Indu)"]

$[\mathrm{TelescopeBound(Indu)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma telescopeBound indu"]

TelescopeBound

[TelescopeBound $\overset{\text{tex}}{\to}$ "TelescopeBound"]

[TelescopeBound $\overset{\text{pyk}}{\to}$ "lemma telescopeBound"]

IntervalSize(Base)

[IntervalSize(Base) $\overset{\text{tex}}{\to}$ "IntervalSize(Base)"]

[IntervalSize(Base) $\overset{\text{pyk}}{\to}$ "lemma intervalSize base"]

IntervalSize(Indu)

[IntervalSize(Indu) $\overset{\text{tex}}{\to}$ "IntervalSize(Indu)"]

[IntervalSize(Indu) $\overset{\text{pyk}}{\to}$ "lemma intervalSize indu"]

IntervalSize

[IntervalSize $\overset{\text{tex}}{\to}$ "IntervalSize"]

[IntervalSize $\overset{\text{pyk}}{\to}$ "lemma intervalSize"]

XS < US

[XS < US $\overset{\text{tex}}{\to}$ "XS<US"]

[XS < US $\overset{\text{pyk}}{\to}$ "lemma XSlessUS"]

lemma USdecreasing(+1)

[lemma USdecreasing(+1) $\overset{\text{pyk}}{\to}$ "lemma USdecreasing(+1)"]

CloseUS

[CloseUS $\overset{\text{tex}}{\to}$ "CloseUS"]

[CloseUS $\overset{\text{pyk}}{\to}$ "lemma closeUS"]

CloseUS(n + 1)

[CloseUS(n + 1) $\stackrel{\text{tex}}{\to}$ "CloseUS(n+1)"]

[CloseUS(n + 1) $\stackrel{\text{pyk}}{\to}$ "lemma closeUS(n+1)"]

AllNegated(Imply)

[AllNegated(Imply) $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash$ $\mathrm{A4} \mathbin{@} \underline{\mathsf{x}} \rhd \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n}; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg$ $\underline{\mathsf{a}}; \mathrm{Gen} \rhd \underline{\mathsf{a}} \gg \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}}; \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \mathrm{Ded} \rhd \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash$ $\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}} \gg \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow$ $\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}}; \mathrm{Contrapositive} \rhd \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}} \gg$ $\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow$ $\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow$ $\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[AllNegated(Imply) $\stackrel{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow$ $\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}$]

[AllNegated(Imply) $\stackrel{\text{tex}}{\to}$ "AllNegated(Imply)"]

[AllNegated(Imply) $\stackrel{\text{pyk}}{\to}$ "pred lemma allNegated(Imply)"]

ExistNegated(Imply)

[ExistNegated(Imply) $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$ $\forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n}; \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \mathrm{Ded} \rhd \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ $\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[ExistNegated(Imply) $\stackrel{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{(\mathrm{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$ $\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n}$]

[ExistNegated(Imply) $\stackrel{\text{tex}}{\to}$ "ExistNegated(Imply)"]

[ExistNegated(Imply) $\stackrel{\text{pyk}}{\to}$ "pred lemma existNegated(Imply)"]

IntroExist(Helper)

[IntroExist(Helper) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \text{A4} @ \underline{\mathsf{x}} \rhd\!\!\rhd \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \rhd \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \gg \dot{\neg}(\underline{\mathsf{a}})\text{n}; \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \dot{\neg}(\underline{\mathsf{a}})\text{n} \gg \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[IntroExist(Helper) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}$]

[IntroExist(Helper) $\stackrel{\text{tex}}{\rightarrow}$ "IntroExist(Helper)"]

[IntroExist(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma intro exist helper"]

IntroExist

[IntroExist $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \text{IntroExist(Helper)} @ \underline{\mathsf{x}} \rhd\!\!\rhd \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \gg \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}; \underline{\mathsf{a}} \vdash \text{AddDoubleNeg} \rhd \underline{\mathsf{a}} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n}; \text{MT} \rhd \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}; \text{Repetition} \rhd \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[IntroExist $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} :== \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \vdash \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}$]

[IntroExist $\stackrel{\text{tex}}{\rightarrow}$ "IntroExist"]

[IntroExist $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma intro exist"]

ExistMP

[ExistMP $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \vdash \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \text{MT} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \gg \dot{\neg}(\underline{\mathsf{a}})\text{n}; \text{Gen} \rhd \dot{\neg}(\underline{\mathsf{a}})\text{n} \gg \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n}; \text{Repetition} \rhd \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n}; \text{FromContradiction} \rhd \forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n} \rhd \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}; \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \vdash \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}; \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \vdash \text{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}(\forall_{\text{obj}}\underline{(\text{v1})}\colon \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}; \text{prop lemma imply negation} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}; \text{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \underline{\mathsf{b}}\rceil, \text{p}_0, \mathsf{c})$]

[ExistMP $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(v1)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\underline{a})n)n \vdash \underline{b}$]

[ExistMP $\stackrel{\text{tex}}{\rightarrow}$ "ExistMP"]

[ExistMP $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma exist mp"]

ExistMP2

[ExistMP2 $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{(v1)} \colon \forall \underline{(v2)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\underline{a})n)n \vdash \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{b})n)n \vdash$ ExistMP $\rhd \underline{a} \Rightarrow \underline{b} \Rightarrow$ $\underline{c} \rhd \dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\underline{a})n)n \gg \underline{b} \Rightarrow \underline{c}$; ExistMP $\rhd \underline{b} \Rightarrow \underline{c} \rhd \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{b})n)n \gg$ $\underline{c}\rceil, p_0, c)$]

[ExistMP2 $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(v1)} \colon \forall \underline{(v2)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\underline{a})n)n \vdash \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{b})n)n \vdash \underline{c}$]

[ExistMP2 $\stackrel{\text{tex}}{\rightarrow}$ "ExistMP2"]

[ExistMP2 $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma exist mp2"]

TwiceExistMP

[TwiceExistMP $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{(v2)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \underline{a} \Rightarrow \underline{b} \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \vdash$ ExistMP $\rhd \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \gg$ $\underline{b}; \forall \underline{(v1)} \colon \forall \underline{(v2)} \colon \forall \underline{a} \colon \forall \underline{b} \colon$ Ded $\rhd \forall \underline{(v2)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \vdash \underline{b} \gg$ $\underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \Rightarrow \underline{b}; \underline{a} \Rightarrow \underline{b} \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n)n)n \vdash$ MP $\rhd \underline{a} \Rightarrow \underline{b} \Rightarrow$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \Rightarrow \underline{b} \rhd \underline{a} \Rightarrow \underline{b} \gg \dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \Rightarrow \underline{b}$; ExistMP $\rhd$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n \Rightarrow \underline{b} \rhd \dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n)n)n \gg \underline{b}\rceil, p_0, c)$]

[TwiceExistMP $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(v1)} \colon \forall \underline{(v2)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \underline{a} \Rightarrow \underline{b} \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n)n)n \vdash \underline{b}$]

[TwiceExistMP $\stackrel{\text{tex}}{\rightarrow}$ "TwiceExistMP"]

[TwiceExistMP $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma 2exist mp"]

TwiceExistMP2

[TwiceExistMP2 $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall \underline{(v1)} \colon \forall \underline{(v2)} \colon \forall \underline{(v3)} \colon \forall \underline{(v4)} \colon \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v1)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(v2)} \colon \dot{\neg}\,(\underline{a})n)n)n)n \vdash$ $\dot{\neg}\,(\forall_{\text{obj}} \underline{(v3)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(v4)} \colon \dot{\neg}\,(\underline{b})n)n)n)n \vdash$ TwiceExistMP $\rhd \underline{a} \Rightarrow \underline{b} \Rightarrow$

$\underline{c} \rhd \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\underline{a})n)n)n)n \gg \underline{b} \Rightarrow \underline{c}; \mathrm{TwiceExistMP} \rhd \underline{b} \Rightarrow \underline{c} \rhd \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v4)}: \dot{\neg}(\underline{b})n)n)n)n \gg \underline{c}\rceil, p_0, \mathsf{c})]$

[TwiceExistMP2 $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{(v4)}: \forall\underline{a}: \forall\underline{b}: \forall\underline{c}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\underline{a})n)n)n)n \vdash \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v4)}: \dot{\neg}(\underline{b})n)n)n)n \vdash \underline{c}$]

[TwiceExistMP2 $\overset{\text{tex}}{\rightarrow}$ "TwiceExistMP2"]

[TwiceExistMP2 $\overset{\text{pyk}}{\rightarrow}$ "pred lemma 2exist mp2"]

EAE – MP

[EAE – MP $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{a}: \forall\underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \vdash \mathrm{A4} @ \underline{(v2)} \rhd \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \gg \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n; \mathrm{ExistMP} \rhd \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \gg \underline{b}; \forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{a}: \forall\underline{b}: \mathrm{Ded} \rhd \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{a}: \forall\underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \vdash \underline{b} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b}; \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n)n)n \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \underline{b} \Rightarrow \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b} \rhd \underline{a} \Rightarrow \underline{b} \gg \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b}; \mathrm{ExistMP} \rhd \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b} \rhd \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n)n)n \gg \underline{b}\rceil, p_0, \mathsf{c})$]

[EAE – MP $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{a}: \forall\underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\underline{a})n)n)n)n \vdash \underline{b}$]

[EAE – MP $\overset{\text{tex}}{\rightarrow}$ "EAE-MP"]

[EAE – MP $\overset{\text{pyk}}{\rightarrow}$ "pred lemma EAE mp"]

AddAll

[AddAll $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(v1)}: \forall\underline{a}: \forall\underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \forall_{obj}\underline{(v1)}: \underline{a} \vdash \mathrm{A4} \rhd \forall_{obj}\underline{(v1)}: \underline{a} \gg \underline{a}; \mathrm{MP} \rhd \underline{a} \Rightarrow \underline{b} \rhd \underline{a} \gg \underline{b}; \mathrm{Gen} \rhd \underline{b} \gg \forall_{obj}\underline{(v1)}: \underline{b}; \forall\underline{(v1)}: \forall\underline{a}: \forall\underline{b}: \mathrm{Ded} \rhd \forall\underline{(v1)}: \forall\underline{a}: \forall\underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \forall_{obj}\underline{(v1)}: \underline{a} \vdash \forall_{obj}\underline{(v1)}: \underline{b} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \forall_{obj}\underline{(v1)}: \underline{a} \Rightarrow \forall_{obj}\underline{(v1)}: \underline{b}; \underline{a} \Rightarrow \underline{b} \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \underline{b} \Rightarrow \forall_{obj}\underline{(v1)}: \underline{a} \Rightarrow \forall_{obj}\underline{(v1)}: \underline{b} \rhd \underline{a} \Rightarrow \underline{b} \gg \forall_{obj}\underline{(v1)}: \underline{a} \Rightarrow \forall_{obj}\underline{(v1)}: \underline{b}\rceil, p_0, \mathsf{c})$]

[AddAll $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall\underline{(v1)}: \forall\underline{a}: \forall\underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \forall_{obj}\underline{(v1)}: \underline{a} \Rightarrow \forall_{obj}\underline{(v1)}: \underline{b}$]

[AddAll $\overset{\text{tex}}{\rightarrow}$ "AddAll "]

[AddAll $\overset{\text{pyk}}{\rightarrow}$ "pred lemma addAll"]

AddExist(Helper1)

[AddExist(Helper1) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \vdash \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \vdash \text{A4} \mathbin{@} \underline{\mathsf{y}} \rhd \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \gg \dot{\neg}\, (\underline{\mathsf{b}})\text{n}; \text{MT} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \gg \dot{\neg}\, (\underline{\mathsf{a}})\text{n}; \text{MT} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \rhd \dot{\neg}\, (\underline{\mathsf{a}})\text{n} \gg \dot{\neg}\, (\underline{\mathsf{c}})\text{n}; \text{Gen} \rhd \dot{\neg}\, (\underline{\mathsf{c}})\text{n} \gg \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n}; \text{Repetition} \rhd \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \gg \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n}; \text{FromContradiction} \rhd \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n} \rhd \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \gg \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}; \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \text{Ded} \rhd \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \vdash \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \vdash \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \gg \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \rceil, \text{p}_0, \mathsf{c})$]

[AddExist(Helper1) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}$]

[AddExist(Helper1) $\stackrel{\text{tex}}{\rightarrow}$ "AddExist(Helper1)"]

[AddExist(Helper1) $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma addExist helper1"]

AddExist(Helper2)

[AddExist(Helper2) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \vdash \text{AddExist(Helper1)} \unrhd \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}; \text{MP3} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \rhd \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \gg \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}; \text{prop lemma imply negation} \rhd \forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \gg \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}; \text{Repetition} \rhd \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \gg \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}; \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \text{Ded} \rhd \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \vdash \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \gg \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n} \rceil, \text{p}_0, \mathsf{c})$]

[AddExist(Helper2) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{y}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{(\text{v2})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\text{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\text{n} | \underline{(\text{v2})} \text{:==} \underline{\mathsf{y}} \rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\, (\underline{\mathsf{c}})\text{n})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} \underline{(\text{v2})}\colon \dot{\neg}\, (\underline{\mathsf{d}})\text{n})\text{n}$]

[AddExist(Helper2) $\stackrel{\text{tex}}{\rightarrow}$ "AddExist(Helper2)"]

[AddExist(Helper2) $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma addExist helper2"]

AddExist

[AddExist $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{\mathsf{y}} \colon \forall \underline{(\mathrm{v1})} \colon \forall \underline{(\mathrm{v2})} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{d}} \colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} \mathrel{:==} \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash$
$\mathrm{AddExist(Helper2)} \rhd\!\!\rhd \langle \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} \mathrel{:==} \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \mathrm{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \gg$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \rceil, \mathrm{p}_0, \mathsf{c})$]

[AddExist $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{\mathsf{y}} \colon \forall \underline{(\mathrm{v1})} \colon \forall \underline{(\mathrm{v2})} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{c}} \colon \forall \underline{\mathsf{d}} \colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} \mathrel{:==} \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n})\mathrm{n}$]

[AddExist $\stackrel{\text{tex}}{\rightarrow}$ "AddExist"]

[AddExist $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma addExist"]

AddExist(SimpleAnt)

[AddExist(SimpleAnt) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{\mathsf{y}} \colon \forall \underline{(\mathrm{v1})} \colon \forall \underline{(\mathrm{v2})} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{d}} \colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} \mathrel{:==} \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\mathrm{AutoImply} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{AddExist} @ \underline{\mathsf{y}} \rhd\!\!\rhd \langle \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} \mathrel{:==} \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \rhd \underline{\mathsf{a}} \Rightarrow$
$\underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \rceil, \mathrm{p}_0, \mathsf{c})$]

[AddExist(SimpleAnt) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{\mathsf{y}} \colon \forall \underline{(\mathrm{v1})} \colon \forall \underline{(\mathrm{v2})} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{d}} \colon \langle \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} \mathrel{:==} \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{d}})\mathrm{n})\mathrm{n}$]

[AddExist(SimpleAnt) $\stackrel{\text{tex}}{\rightarrow}$ "AddExist(SimpleAnt)"]

[AddExist(SimpleAnt) $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma addExist(SimpleAnt)"]

AddExist(Simple)

[AddExist(Simple) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{v1})} \colon \forall \underline{(\mathrm{v2})} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\mathrm{AutoImply} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{AddExist} @ \underline{(\mathrm{v2})} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \rceil, \mathrm{p}_0, \mathsf{c})$]

[AddExist(Simple) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\mathrm{v1})} \colon \forall \underline{(\mathrm{v2})} \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v1})} \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{v2})} \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n}$]

$[\mathrm{AddExist(Simple)} \stackrel{\mathrm{tex}}{\rightarrow} \text{“AddExist(Simple)”}]$

$[\mathrm{AddExist(Simple)} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“pred lemma addExist(Simple)”}]$

AddEAE

$[\mathrm{AddEAE} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall (\mathrm{v1}) \colon \forall (\mathrm{v2}) \colon \forall (\mathrm{v3}) \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\mathrm{AddExist(Simple)} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n}; \mathrm{AddAll} \rhd \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n}; \mathrm{AddExist(Simple)} \rhd$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{AddEAE} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall (\mathrm{v1}) \colon \forall (\mathrm{v2}) \colon \forall (\mathrm{v3}) \colon \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{AddEAE} \stackrel{\mathrm{tex}}{\rightarrow} \text{“AddEAE”}]$

$[\mathrm{AddEAE} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“pred lemma addEAE”}]$

AEA – negated

$[\mathrm{AEA} - \mathrm{negated} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall (\mathrm{v1}) \colon \forall (\mathrm{v2}) \colon \forall (\mathrm{v3}) \colon \forall \underline{\mathsf{a}} \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{AllNegated(Imply)} \gg \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddAll} \rhd$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistNegated(Imply)} \gg$
$\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n}; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n} \rhd \forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddExist(Simple)} \rhd$
$\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AllNegated(Imply)} \gg$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v1}) \colon \dot{\neg}\, (\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v2}) \colon \dot{\neg}\, (\forall_{\mathrm{obj}} (\mathrm{v3}) \colon \underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{AEA-negated} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1}){:}\,\forall(\mathrm{v2}){:}\,\forall(\mathrm{v3}){:}\,\forall\underline{\mathsf{a}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{AEA-negated} \stackrel{\mathrm{tex}}{\rightarrow}$ "AEA-negated"$]$

$[\mathrm{AEA-negated} \stackrel{\mathrm{pyk}}{\rightarrow}$ "pred lemma AEAnegated"$]$

EEA – negated

$[\mathrm{EEA-negated} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1}){:}\,\forall(\mathrm{v2}){:}\,\forall(\mathrm{v3}){:}\,\forall\underline{\mathsf{a}}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{AllNegated}(\mathrm{Imply}) \gg \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddAll} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistNegated}(\mathrm{Imply}) \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n}; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n} \rhd \forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddAll} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistNegated}(\mathrm{Imply}) \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v3}){:}\,\dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[EEA − negated $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall\underline{(v1)}\colon\forall\underline{(v2)}\colon\forall\underline{(v3)}\colon\forall\underline{\mathsf{a}}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\underline{(v1)}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\underline{(v2)}\colon\dot\neg\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon\underline{\mathsf{a}})n)n)n)n)n \vdash \forall_{\mathrm{obj}}\underline{(v1)}\colon\forall_{\mathrm{obj}}\underline{(v2)}\colon\dot\neg\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon\dot\neg\,(\dot\neg\,(\underline{\mathsf{a}})n)n)n$]

[EEA − negated $\overset{\mathrm{tex}}{\rightarrow}$ "EEA-negated"]

[EEA − negated $\overset{\mathrm{pyk}}{\rightarrow}$ "pred lemma EEAnegated"]

Induction

[Induction $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(v1)}\colon\forall\underline{\mathsf{a}}\colon\forall\underline{\mathsf{b}}\colon\forall\underline{\mathsf{c}}\colon\langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}0\rangle_{\mathrm{Me}} \Vdash \langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}(\underline{(v1)}+1)\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \mathrm{Gen} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \gg \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}; \mathrm{InductionAxiom} \unrhd \langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}0\rangle_{\mathrm{Me}} \unrhd \langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}(\underline{(v1)}+1)\rangle_{\mathrm{Me}} \gg \underline{\mathsf{b}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}}; \mathrm{MP2} \rhd \underline{\mathsf{b}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \rhd \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \gg \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}}; \mathrm{A4}\,@\,\underline{(v1)} \rhd \forall_{\mathrm{obj}}\underline{(v1)}\colon\underline{\mathsf{a}} \gg \underline{\mathsf{a}}\rceil, p_0, \mathsf{c})$]

[Induction $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{(v1)}\colon\forall\underline{\mathsf{a}}\colon\forall\underline{\mathsf{b}}\colon\forall\underline{\mathsf{c}}\colon\langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}0\rangle_{\mathrm{Me}} \Vdash \langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}(\underline{(v1)}+1)\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}}$]

[Induction $\overset{\mathrm{tex}}{\rightarrow}$ "Induction"]

[Induction $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma induction"]

leqAntisymmetry

[leqAntisymmetry $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon\underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{x}} \vdash \mathrm{leqAntisymmetryAxiom} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}}; \mathrm{MP2} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}\rceil, p_0, \mathsf{c})$]

[leqAntisymmetry $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon\underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{x}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{y}}$]

[leqAntisymmetry $\overset{\mathrm{tex}}{\rightarrow}$ "leqAntisymmetry"]

[leqAntisymmetry $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqAntisymmetry"]

leqTransitivity

[leqTransitivity $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon\forall\underline{\mathsf{z}}\colon\underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{z}} \vdash \mathrm{leqTransitivityAxiom} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{z}}; \mathrm{MP2} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{z}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{z}} \gg \underline{\mathsf{x}} <= \underline{\mathsf{z}}\rceil, p_0, \mathsf{c})$]

[leqTransitivity $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon\forall\underline{\mathsf{z}}\colon\underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{z}}$]

[leqTransitivity $\overset{\mathrm{tex}}{\rightarrow}$ "leqTransitivity"]

[leqTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqTransitivity"]

leqAddition

[leqAddition $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash$ leqAdditionAxiom $\gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}); \text{MP} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}})\rceil, \text{p}_0, \mathsf{c})$]

[leqAddition $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}})$]

[leqAddition $\stackrel{\text{tex}}{\rightarrow}$ "leqAddition"]

[leqAddition $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqAddition"]

leqMultiplication

[leqMultiplication $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon 0 <= \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash$ leqMultiplicationAxiom $\gg 0 <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}); \text{MP2} \rhd 0 <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \rhd 0 <= \underline{\mathsf{z}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}})\rceil, \text{p}_0, \mathsf{c})$]

[leqMultiplication $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon 0 <= \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash (\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}})$]

[leqMultiplication $\stackrel{\text{tex}}{\rightarrow}$ "leqMultiplication"]

[leqMultiplication $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqMultiplication"]

Reciprocal

[Reciprocal $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}\, (\underline{\mathsf{x}} = 0)\text{n} \vdash$ ReciprocalAxiom $\gg \dot{\neg}\, (\underline{\mathsf{x}} = 0)\text{n} \Rightarrow (\underline{\mathsf{x}} * \text{rec}\underline{\mathsf{x}}) = 1; \text{MP} \rhd \dot{\neg}\, (\underline{\mathsf{x}} = 0)\text{n} \Rightarrow (\underline{\mathsf{x}} * \text{rec}\underline{\mathsf{x}}) = 1 \rhd \dot{\neg}\, (\underline{\mathsf{x}} = 0)\text{n} \gg (\underline{\mathsf{x}} * \text{rec}\underline{\mathsf{x}}) = 1\rceil, \text{p}_0, \mathsf{c})$]

[Reciprocal $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}\, (\underline{\mathsf{x}} = 0)\text{n} \vdash (\underline{\mathsf{x}} * \text{rec}\underline{\mathsf{x}}) = 1$]

[Reciprocal $\stackrel{\text{tex}}{\rightarrow}$ "Reciprocal"]

[Reciprocal $\stackrel{\text{pyk}}{\rightarrow}$ "lemma reciprocal"]

Equality

[Equality $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{z}} \vdash$ EqualityAxiom $\gg \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{y}} = \underline{\mathsf{z}}; \text{MP2} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{y}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{x}} =$

$\underline{y} \rhd \underline{x} = \underline{z} \gg \underline{y} = \underline{z}\rceil, p_0, c)]$

[Equality $\stackrel{stmt}{\rightarrow}$ SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \underline{x} = \underline{y} \vdash \underline{x} = \underline{z} \vdash \underline{y} = \underline{z}$]

[Equality $\stackrel{tex}{\rightarrow}$ "Equality"]

[Equality $\stackrel{pyk}{\rightarrow}$ "lemma equality"]

eqLeq

[eqLeq $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} = \underline{y} \vdash$ EqLeqAxiom $\gg \underline{x} = \underline{y} \Rightarrow \underline{x} <= \underline{y};$ MP $\rhd \underline{x} = \underline{y} \Rightarrow \underline{x} <= \underline{y} \rhd \underline{x} = \underline{y} \gg \underline{x} <= \underline{y}\rceil, p_0, c)$]

[eqLeq $\stackrel{stmt}{\rightarrow}$ SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} = \underline{y} \vdash \underline{x} <= \underline{y}$]

[eqLeq $\stackrel{tex}{\rightarrow}$ "eqLeq"]

[eqLeq $\stackrel{pyk}{\rightarrow}$ "lemma eqLeq"]

eqAddition

[eqAddition $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \underline{x} = \underline{y} \vdash$ EqAdditionAxiom $\gg \underline{x} = \underline{y} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z});$ MP $\rhd \underline{x} = \underline{y} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \rhd \underline{x} = \underline{y} \gg (\underline{x} + \underline{z}) = (\underline{y} + \underline{z})\rceil, p_0, c)$]

[eqAddition $\stackrel{stmt}{\rightarrow}$ SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \underline{x} = \underline{y} \vdash (\underline{x} + \underline{z}) = (\underline{y} + \underline{z})$]

[eqAddition $\stackrel{tex}{\rightarrow}$ "eqAddition"]

[eqAddition $\stackrel{pyk}{\rightarrow}$ "lemma eqAddition"]

eqMultiplication

[eqMultiplication $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \underline{x} = \underline{y} \vdash$ EqMultiplicationAxiom $\gg \underline{x} = \underline{y} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z});$ MP $\rhd \underline{x} = \underline{y} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \rhd \underline{x} = \underline{y} \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})\rceil, p_0, c)$]

[eqMultiplication $\stackrel{stmt}{\rightarrow}$ SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \underline{x} = \underline{y} \vdash (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})$]

[eqMultiplication $\stackrel{tex}{\rightarrow}$ "eqMultiplication"]

[eqMultiplication $\stackrel{pyk}{\rightarrow}$ "lemma eqMultiplication"]

LeqMultiplicationLeft

$[\mathrm{LeqMultiplicationLeft} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \forall \underline{\mathsf{z}} \colon 0 <= \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \mathrm{leqMultiplication} \rhd 0 <= \underline{\mathsf{z}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}); \mathrm{timesCommutativity} \gg (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{x}}); \mathrm{subLeqLeft} \rhd (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{x}}) \rhd (\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \gg (\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}); \mathrm{timesCommutativity} \gg (\underline{\mathsf{y}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{y}}); \mathrm{subLeqRight} \rhd (\underline{\mathsf{y}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{y}}) \rhd (\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \gg (\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{z}} * \underline{\mathsf{y}}) \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LeqMultiplicationLeft} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \forall \underline{\mathsf{z}} \colon 0 <= \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash (\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{z}} * \underline{\mathsf{y}})]$

[LeqMultiplicationLeft $\stackrel{\mathrm{tex}}{\rightarrow}$ “LeqMultiplicationLeft ”]

[LeqMultiplicationLeft $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma leqMultiplicationLeft”]

LeqLessEq

$[\mathrm{LeqLessEq} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{fromNotLess} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{y}} <= \underline{\mathsf{x}}; \mathrm{leqAntisymmetry} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}; \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \mathrm{Ded} \rhd \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}}; \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \mathrm{MP} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}}; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LeqLessEq} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}}]$

[LeqLessEq $\stackrel{\mathrm{tex}}{\rightarrow}$ “LeqLessEq”]

[LeqLessEq $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma leqLessEq”]

LessLeq

$[\mathrm{LessLeq} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessLeq} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{y}}]$

[LessLeq $\stackrel{\mathrm{tex}}{\rightarrow}$ “LessLeq”]

[LessLeq $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma lessLeq”]

FromLeqGeq

[FromLeqGeq $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{a}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{a}} \vdash \text{leqTotality} \gg \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}})\text{n} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}}; \text{FromDisjuncts} \rhd \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}})\text{n} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{a}} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \rceil, \text{p}_0, \mathsf{c})$]

[FromLeqGeq $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{a}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{a}} \vdash \underline{\mathsf{a}}$]

[FromLeqGeq $\stackrel{\text{tex}}{\rightarrow}$ “FromLeqGeq”]

[FromLeqGeq $\stackrel{\text{pyk}}{\rightarrow}$ “lemma from leqGeq”]

subLeqRight

[subLeqRight $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{z}} <= \underline{\mathsf{x}} \vdash \text{eqLeq} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}}; \text{leqTransitivity} \rhd \underline{\mathsf{z}} <= \underline{\mathsf{x}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg \underline{\mathsf{z}} <= \underline{\mathsf{y}} \rceil, \text{p}_0, \mathsf{c})$]

[subLeqRight $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{z}} <= \underline{\mathsf{x}} \vdash \underline{\mathsf{z}} <= \underline{\mathsf{y}}$]

[subLeqRight $\stackrel{\text{tex}}{\rightarrow}$ “subLeqRight”]

[subLeqRight $\stackrel{\text{pyk}}{\rightarrow}$ “lemma subLeqRight”]

subLeqLeft

[subLeqLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{z}} \vdash \text{eqSymmetry} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{y}} = \underline{\mathsf{x}}; \text{eqLeq} \rhd \underline{\mathsf{y}} = \underline{\mathsf{x}} \gg \underline{\mathsf{y}} <= \underline{\mathsf{x}}; \text{leqTransitivity} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{x}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{z}} \gg \underline{\mathsf{y}} <= \underline{\mathsf{z}} \rceil, \text{p}_0, \mathsf{c})$]

[subLeqLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{z}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{z}}$]

[subLeqLeft $\stackrel{\text{tex}}{\rightarrow}$ “subLeqLeft”]

[subLeqLeft $\stackrel{\text{pyk}}{\rightarrow}$ “lemma subLeqLeft”]

Leq + 1

[Leq + 1 $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash 0 < 1 \gg \dot{\neg}\,(0 <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = 1)\text{n})\text{n})\text{n}; \text{LessAdditionLeft} \rhd \dot{\neg}\,(0 <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = 1)\text{n})\text{n})\text{n} \gg \dot{\neg}\,((\underline{\mathsf{y}} + 0) <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{y}} + 0) = (\underline{\mathsf{y}} + 1))\text{n})\text{n})\text{n}; \text{plus0} \gg (\underline{\mathsf{y}} + 0) = \underline{\mathsf{y}}; \text{SubLessLeft} \rhd (\underline{\mathsf{y}} + 0) = \underline{\mathsf{y}} \rhd \dot{\neg}\,((\underline{\mathsf{y}} + 0) <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{y}} + 0) = (\underline{\mathsf{y}} + 1))\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{y}} <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{y}} = (\underline{\mathsf{y}} + 1))\text{n})\text{n})\text{n}; \text{leqLessTransitivity} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \dot{\neg}\,(\underline{\mathsf{y}} <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{y}} =$

$(\underline{y} + 1))n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y} + 1) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + 1))n)n)n\rceil, p_0, c)]$

$[\mathrm{Leq} + 1 \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{x} <= (\underline{y} + 1) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + 1))n)n)n]$

$[\mathrm{Leq} + 1 \stackrel{\mathrm{tex}}{\to}$ "Leq+1"$]$

$[\mathrm{Leq} + 1 \stackrel{\mathrm{pyk}}{\to}$ "lemma leqPlus1"$]$

PositiveToRight(Leq)

$[\mathrm{PositiveToRight(Leq)} \stackrel{\mathrm{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) <= \underline{z} \vdash \mathrm{leqAddition} \rhd (\underline{x} + \underline{y}) <= \underline{z} \gg ((\underline{x} + \underline{y}) + (-u\underline{y})) <= (\underline{z} + (-u\underline{y})); x = x + y - y \gg \underline{x} = ((\underline{x} + \underline{y}) + (-u\underline{y})); \mathrm{eqSymmetry} \rhd \underline{x} = ((\underline{x} + \underline{y}) + (-u\underline{y})) \gg ((\underline{x} + \underline{y}) + (-u\underline{y})) = \underline{x}; \mathrm{subLeqLeft} \rhd ((\underline{x} + \underline{y}) + (-u\underline{y})) = \underline{x} \rhd ((\underline{x} + \underline{y}) + (-u\underline{y})) <= (\underline{z} + (-u\underline{y})) \gg \underline{x} <= (\underline{z} + (-u\underline{y}))\rceil, p_0, c)]$

$[\mathrm{PositiveToRight(Leq)} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) <= \underline{z} \vdash \underline{x} <= (\underline{z} + (-u\underline{y}))]$

$[\mathrm{PositiveToRight(Leq)} \stackrel{\mathrm{tex}}{\to}$ "PositiveToRight(Leq)"$]$

$[\mathrm{PositiveToRight(Leq)} \stackrel{\mathrm{pyk}}{\to}$ "lemma positiveToRight(Leq)"$]$

PositiveToRight(Leq)(1term)

$[\mathrm{PositiveToRight(Leq)(1term)} \stackrel{\mathrm{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{y}\colon \forall \underline{z}\colon \underline{y} <= \underline{z} \vdash \mathrm{plus0Left} \gg (0 + \underline{y}) = \underline{y}; \mathrm{eqSymmetry} \rhd (0 + \underline{y}) = \underline{y} \gg \underline{y} = (0 + \underline{y}); \mathrm{subLeqLeft} \rhd \underline{y} = (0 + \underline{y}) \rhd \underline{y} <= \underline{z} \gg (0 + \underline{y}) <= \underline{z}; \mathrm{PositiveToRight(Leq)} \rhd (0 + \underline{y}) <= \underline{z} \gg 0 <= (\underline{z} + (-u\underline{y}))\rceil, p_0, c)]$

$[\mathrm{PositiveToRight(Leq)(1term)} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{y}\colon \forall \underline{z}\colon \underline{y} <= \underline{z} \vdash 0 <= (\underline{z} + (-u\underline{y}))]$

$[\mathrm{PositiveToRight(Leq)(1term)} \stackrel{\mathrm{tex}}{\to}$ "PositiveToRight(Leq)(1 term)"$]$

$[\mathrm{PositiveToRight(Leq)(1term)} \stackrel{\mathrm{pyk}}{\to}$ "lemma positiveToRight(Leq)(1 term)"$]$

lemma negativeToRight(Leq)

$[\mathrm{lemma\ negativeToRight(Leq)} \stackrel{\mathrm{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + (-u\underline{y})) <= \underline{z} \vdash \mathrm{leqAddition} \rhd (\underline{x} + (-u\underline{y})) <= \underline{z} \gg ((\underline{x} + (-u\underline{y})) + \underline{y}) <= (\underline{z} + \underline{y}); x = x + y - y \gg \underline{x} = ((\underline{x} + \underline{y}) + (-u\underline{y})); \mathrm{Three2threeTerms} \gg ((\underline{x} + \underline{y}) + (-u\underline{y})) =$

$((\underline{x} + (-u\underline{y})) + \underline{y})$; eqTransitivity $\rhd \underline{x} = ((\underline{x} + \underline{y}) + (-u\underline{y})) \rhd ((\underline{x} + \underline{y}) + (-u\underline{y})) = ((\underline{x} + (-u\underline{y})) + \underline{y}) \gg \underline{x} = ((\underline{x} + (-u\underline{y})) + \underline{y})$; eqSymmetry $\rhd \underline{x} = ((\underline{x} + (-u\underline{y})) + \underline{y}) \gg ((\underline{x} + (-u\underline{y})) + \underline{y}) = \underline{x}$; subLeqLeft $\rhd ((\underline{x} + (-u\underline{y})) + \underline{y}) = \underline{x} \rhd ((\underline{x} + (-u\underline{y})) + \underline{y}) <= (\underline{z} + \underline{y}) \gg \underline{x} <= (\underline{z} + \underline{y})\rceil, p_0, c)]$

[lemma negativeToRight(Leq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon (\underline{x} + (-u\underline{y})) <= \underline{z} \vdash \underline{x} <= (\underline{z} + \underline{y})$]

[lemma negativeToRight(Leq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeToRight(Leq)"]

PositiveToLeft(Leq)

[PositiveToLeft(Leq) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= (\underline{y} + \underline{z}) \vdash$ leqAddition $\rhd \underline{x} <= (\underline{y} + \underline{z}) \gg (\underline{x} + (-u\underline{z})) <= ((\underline{y} + \underline{z}) + (-u\underline{z}))$; $x = x + y - y \gg \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z}))$; eqSymmetry $\rhd \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y}$; subLeqRight $\rhd ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y} \rhd (\underline{x} + (-u\underline{z})) <= ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg (\underline{x} + (-u\underline{z})) <= \underline{y}\rceil, p_0, c)]$

[PositiveToLeft(Leq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= (\underline{y}+\underline{z}) \vdash (\underline{x}+(-u\underline{z})) <= \underline{y}$]

[PositiveToLeft(Leq) $\stackrel{\text{tex}}{\rightarrow}$ "PositiveToLeft(Leq)"]

[PositiveToLeft(Leq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma positiveToLeft(Leq)"]

negativeToLeft(Leq)

[negativeToLeft(Leq) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= (\underline{y} + (-u\underline{z})) \vdash$ leqAddition $\rhd \underline{x} <= (\underline{y} + (-u\underline{z})) \gg (\underline{x} + \underline{z}) <= ((\underline{y} + (-u\underline{z})) + \underline{z})$; $x = x + y - y \gg \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z}))$; Three2threeTerms $\gg ((\underline{y} + \underline{z}) + (-u\underline{z})) = ((\underline{y} + (-u\underline{z})) + \underline{z})$; eqTransitivity $\rhd \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})) \rhd ((\underline{y} + \underline{z}) + (-u\underline{z})) = ((\underline{y} + (-u\underline{z})) + \underline{z}) \gg \underline{y} = ((\underline{y} + (-u\underline{z})) + \underline{z})$; eqSymmetry $\rhd \underline{y} = ((\underline{y} + (-u\underline{z})) + \underline{z}) \gg ((\underline{y} + (-u\underline{z})) + \underline{z}) = \underline{y}$; subLeqRight $\rhd ((\underline{y} + (-u\underline{z})) + \underline{z}) = \underline{y} \rhd (\underline{x} + \underline{z}) <= ((\underline{y} + (-u\underline{z})) + \underline{z}) \gg (\underline{x} + \underline{z}) <= \underline{y}\rceil, p_0, c)]$

[negativeToLeft(Leq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= (\underline{y}+(-u\underline{z})) \vdash (\underline{x}+\underline{z}) <= \underline{y}$]

[negativeToLeft(Leq) $\stackrel{\text{tex}}{\rightarrow}$ "negativeToLeft(Leq)"]

[negativeToLeft(Leq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeToLeft(Leq)"]

negativeToLeft(Leq)(1term)

[negativeToLeft(Leq)(1term) $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{y} + (-u\underline{z})) \vdash \text{negativeToLeft(Leq)} \rhd 0 <= (\underline{y} + (-u\underline{z})) \gg (0 + \underline{z}) <= \underline{y}; \text{plus0Left} \gg (0 + \underline{z}) = \underline{z}; \text{subLeqLeft} \rhd (0 + \underline{z}) = \underline{z} \gg \underline{z} <= \underline{y}\rceil, p_0, c)$]

[negativeToLeft(Leq)(1term) $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{y} + (-u\underline{z})) \vdash \underline{z} <= \underline{y}$]

[negativeToLeft(Leq)(1term) $\stackrel{\text{tex}}{\to}$ “negativeToLeft(Leq)(1 term)”]

[negativeToLeft(Leq)(1term) $\stackrel{\text{pyk}}{\to}$ “lemma negativeToLeft(Leq)(1 term)”]

LeqAdditionLeft

[LeqAdditionLeft $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \text{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}); \text{plusCommutativity} \gg (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}); \text{plusCommutativity} \gg (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}); \text{subLeqLeft} \rhd (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}) \rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \gg (\underline{z} + \underline{x}) <= (\underline{y} + \underline{z}); \text{subLeqRight} \rhd (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}) \rhd (\underline{z} + \underline{x}) <= (\underline{y} + \underline{z}) \gg (\underline{z} + \underline{x}) <= (\underline{z} + \underline{y})\rceil, p_0, c)$]

[LeqAdditionLeft $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash (\underline{z} + \underline{x}) <= (\underline{z} + \underline{y})$]

[LeqAdditionLeft $\stackrel{\text{tex}}{\to}$ “LeqAdditionLeft”]

[LeqAdditionLeft $\stackrel{\text{pyk}}{\to}$ “lemma leqAdditionLeft”]

leqSubtraction

[leqSubtraction $\stackrel{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \vdash \text{leqAddition} \rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \gg ((\underline{x} + \underline{z}) + (-u\underline{z})) <= ((\underline{y} + \underline{z}) + (-u\underline{z})); \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{x} = ((\underline{x} + \underline{z}) + (-u\underline{z})); \text{eqSymmetry} \rhd \underline{x} = ((\underline{x} + \underline{z}) + (-u\underline{z})) \gg ((\underline{x} + \underline{z}) + (-u\underline{z})) = \underline{x}; \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})); \text{eqSymmetry} \rhd \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y}; \text{subLeqLeft} \rhd ((\underline{x} + \underline{z}) + (-u\underline{z})) = \underline{x} \rhd ((\underline{x} + \underline{z}) + (-u\underline{z})) <= ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg \underline{x} <= ((\underline{y} + \underline{z}) + (-u\underline{z})); \text{subLeqRight} \rhd ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y} \rhd \underline{x} <= ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg \underline{x} <= \underline{y}\rceil, p_0, c)$]

[leqSubtraction $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \vdash \underline{x} <= \underline{y}$]

[leqSubtraction $\stackrel{\text{tex}}{\to}$ “leqSubtraction”]

[leqSubtraction $\stackrel{\text{pyk}}{\to}$ “lemma leqSubtraction”]

leqSubtractionLeft

$[\text{leqSubtractionLeft} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{z}+\underline{x}) <= (\underline{z}+\underline{y}) \vdash \text{plusCommutativity} \gg (\underline{z}+\underline{x}) = (\underline{x}+\underline{z}); \text{plusCommutativity} \gg (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}); \text{subLeqLeft} \rhd (\underline{z}+\underline{x}) = (\underline{x}+\underline{z}) \rhd (\underline{z}+\underline{x}) <= (\underline{z}+\underline{y}) \gg (\underline{x}+\underline{z}) <= (\underline{z}+\underline{y}); \text{subLeqRight} \rhd (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}) \rhd (\underline{x}+\underline{z}) <= (\underline{z}+\underline{y}) \gg (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}); \text{leqSubtraction} \rhd (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \gg \underline{x} <= \underline{y}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{leqSubtractionLeft} \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{z}+\underline{x}) <= (\underline{z}+\underline{y}) \vdash \underline{x} <= \underline{y}]$

$[\text{leqSubtractionLeft} \overset{\text{tex}}{\rightarrow}$ “leqSubtractionLeft”$]$

$[\text{leqSubtractionLeft} \overset{\text{pyk}}{\rightarrow}$ “lemma leqSubtractionLeft”$]$

thirdGeq

$[\text{thirdGeq} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \text{leqReflexivity} \gg \underline{y} <= \underline{y}; \text{JoinConjuncts} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{y} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{y})\mathrm{n})\mathrm{n}; \text{ExistIntro} @ \mathsf{c}_{\mathrm{Ex}} @ \underline{y} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{y})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n}; \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \text{leqReflexivity} \gg \underline{x} <= \underline{x}; \text{JoinConjuncts} \rhd \underline{x} <= \underline{x} \rhd \underline{y} <= \underline{x} \gg \dot{\neg}(\underline{x} <= \underline{x} \Rightarrow \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n})\mathrm{n}; \text{ExistIntro} @ \mathsf{c}_{\mathrm{Ex}} @ \underline{x} \rhd \dot{\neg}(\underline{x} <= \underline{x} \Rightarrow \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n}; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n} \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n}; \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n} \gg \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n}; \text{leqTotality} \gg \dot{\neg}(\underline{x} <= \underline{y})\mathrm{n} \Rightarrow \underline{y} <= \underline{x}; \text{FromDisjuncts} \rhd \dot{\neg}(\underline{x} <= \underline{y})\mathrm{n} \Rightarrow \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n} \rhd \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{thirdGeq} \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= \mathsf{c}_{\mathrm{Ex}} \Rightarrow \dot{\neg}(\underline{y} <= \mathsf{c}_{\mathrm{Ex}})\mathrm{n})\mathrm{n}]$

$[\text{thirdGeq} \overset{\text{tex}}{\rightarrow}$ “thirdGeq”$]$

$[\text{thirdGeq} \overset{\text{pyk}}{\rightarrow}$ “lemma thirdGeq”$]$

LeqNegated

$[\text{LeqNegated} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \text{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x} + (-\mathrm{u}\underline{x})) <= (\underline{y} + (-\mathrm{u}\underline{x})); \text{Negative} \gg (\underline{x} + (-\mathrm{u}\underline{x})) = 0; \text{subLeqLeft} \rhd (\underline{x} + (-\mathrm{u}\underline{x})) = 0 \rhd (\underline{x} + (-\mathrm{u}\underline{x})) <= (\underline{y} + (-\mathrm{u}\underline{x})) \gg 0 <= (\underline{y} + (-\mathrm{u}\underline{x})); \text{plusCommutativity} \gg (\underline{y} + (-\mathrm{u}\underline{x})) = ((-\mathrm{u}\underline{x}) + \underline{y}); \text{subLeqRight} \rhd (\underline{y} + (-\mathrm{u}\underline{x})) = ((-\mathrm{u}\underline{x}) + \underline{y}) \rhd 0 <= (\underline{y} + (-\mathrm{u}\underline{x})) \gg 0 <= ((-\mathrm{u}\underline{x}) + \underline{y}); \text{leqAddition} \rhd 0 <= ((-\mathrm{u}\underline{x}) + \underline{y}) \gg (0 + (-\mathrm{u}\underline{y})) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})); \text{plus0Left} \gg (0 + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{y}); \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg$

$(-\mathrm{u}\underline{x}) = (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y}))$; eqSymmetry $\rhd (-\mathrm{u}\underline{x}) = (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) \gg (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{x})$; subLeqLeft $\rhd (0 + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{y}) \rhd (0 + (-\mathrm{u}\underline{y})) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) \gg (-\mathrm{u}\underline{y}) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y}))$; subLeqRight $\rhd (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{x}) \rhd (-\mathrm{u}\underline{y}) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) \gg (-\mathrm{u}\underline{y}) <= (-\mathrm{u}\underline{x})\rceil, \mathrm{p}_0, \mathsf{c})]$

[LeqNegated $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash (-\mathrm{u}\underline{y}) <= (-\mathrm{u}\underline{x})]$

[LeqNegated $\stackrel{\text{tex}}{\rightarrow}$ “LeqNegated”]

[LeqNegated $\stackrel{\text{pyk}}{\rightarrow}$ “lemma leqNegated”]

AddEquations(Leq)

[AddEquations(Leq) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash$ leqAddition $\rhd \underline{x} <= \underline{y} \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z})$; LeqAdditionLeft $\rhd \underline{z} <= \underline{u} \gg (\underline{y} + \underline{z}) <= (\underline{y} + \underline{u})$; leqTransitivity $\rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \rhd (\underline{y} + \underline{z}) <= (\underline{y} + \underline{u}) \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{u})\rceil, \mathrm{p}_0, \mathsf{c})]$

[AddEquations(Leq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash (\underline{x} + \underline{z}) <= (\underline{y} + \underline{u})]$

[AddEquations(Leq) $\stackrel{\text{tex}}{\rightarrow}$ “AddEquations(Leq)”]

[AddEquations(Leq) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma addEquations(Leq)”]

MultiplyEquations(Leq)

[MultiplyEquations(Leq) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon 0 <= \underline{x} \vdash 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash$ leqMultiplication $\rhd 0 <= \underline{z} \rhd \underline{x} <= \underline{y} \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z})$; leqTransitivity $\rhd 0 <= \underline{x} \rhd \underline{x} <= \underline{y} \gg 0 <= \underline{y}$; LeqMultiplicationLeft $\rhd 0 <= \underline{y} \rhd \underline{z} <= \underline{u} \gg (\underline{y} * \underline{z}) <= (\underline{y} * \underline{u})$; leqTransitivity $\rhd (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \rhd (\underline{y} * \underline{z}) <= (\underline{y} * \underline{u}) \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{u})\rceil, \mathrm{p}_0, \mathsf{c})]$

[MultiplyEquations(Leq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon 0 <= \underline{x} \vdash 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash (\underline{x} * \underline{z}) <= (\underline{y} * \underline{u})]$

[MultiplyEquations(Leq) $\stackrel{\text{tex}}{\rightarrow}$ “MultiplyEquations(Leq)”]

[MultiplyEquations(Leq) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma multiplyEquations(Leq)”]

ThirdGeqSeries

[ThirdGeqSeries $\stackrel{\text{tex}}{\rightarrow}$ “ThirdGeqSeries”]

[ThirdGeqSeries $\stackrel{\text{pyk}}{\rightarrow}$ "lemma thirdGeqSeries"]

LeqNeqLess

[LeqNeqLess $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}\,(\underline{x} = \underline{y})\text{n} \vdash \text{JoinConjuncts} \rhd \underline{x} <= \underline{y} \rhd \dot{\neg}\,(\underline{x} = \underline{y})\text{n} \gg \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n}; \text{Repetition} \rhd \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[LeqNeqLess $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}\,(\underline{x} = \underline{y})\text{n} \vdash \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n}$]

[LeqNeqLess $\stackrel{\text{tex}}{\rightarrow}$ "LeqNeqLess"]

[LeqNeqLess $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqNeqLess"]

FromLess

[FromLess $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \text{toNotLess} \rhd \underline{y} <= \underline{x} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n}; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n} \gg \underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n}; \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \text{AddDoubleNeg} \rhd \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n})\text{n}; \text{MT} \rhd \underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n} \rhd \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\underline{y} <= \underline{x})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[FromLess $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \dot{\neg}\,(\underline{y} <= \underline{x})\text{n}$]

[FromLess $\stackrel{\text{tex}}{\rightarrow}$ "FromLess"]

[FromLess $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromLess"]

ToLess

[ToLess $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n})\text{n} \vdash \text{fromNotLess} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n})\text{n} \gg \underline{x} <= \underline{y}; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n})\text{n} \vdash \underline{x} <= \underline{y} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n})\text{n} \Rightarrow \underline{x} <= \underline{y}; \dot{\neg}\,(\underline{x} <= \underline{y})\text{n} \vdash \text{NegativeMT} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n})\text{n} \Rightarrow \underline{x} <= \underline{y} \rhd \dot{\neg}\,(\underline{x} <= \underline{y})\text{n} \gg \dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[ToLess $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\,(\underline{x} <= \underline{y})\text{n} \vdash \dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} = \underline{x})\text{n})\text{n})\text{n}$]

$[\mathrm{ToLess} \stackrel{\mathrm{tex}}{\rightarrow} \text{“ToLess”}]$

$[\mathrm{ToLess} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“lemma toLess”}]$

fromNotLess

$[\mathrm{fromNotLess} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \underline{x} <= \underline{y} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n; \mathrm{MP} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n \rhd \underline{x} <= \underline{y} \gg \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n \gg \underline{x} = \underline{y}; \mathrm{eqSymmetry} \rhd \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \mathrm{eqLeq} \rhd \underline{y} = \underline{x} \gg \underline{y} <= \underline{x}; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{x} \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x}; \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x}; \mathrm{AutoImply} \gg \underline{y} <= \underline{x} \Rightarrow \underline{y} <= \underline{x}; \mathrm{leqTotality} \gg \dot{\neg}(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x}; \mathrm{FromDisjuncts} \rhd \dot{\neg}(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \rhd \underline{y} <= \underline{x} \Rightarrow \underline{y} <= \underline{x} \gg \underline{y} <= \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{fromNotLess} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \underline{y} <= \underline{x}]$

$[\mathrm{fromNotLess} \stackrel{\mathrm{tex}}{\rightarrow} \text{“fromNotLess”}]$

$[\mathrm{fromNotLess} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“lemma fromNotLess”}]$

toNotLess

$[\mathrm{toNotLess} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{x} \vdash \mathrm{leqAntisymmetry} \rhd \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \gg \underline{y} = \underline{x}; \mathrm{AddDoubleNeg} \rhd \underline{y} = \underline{x} \gg \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{x} \vdash \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n \gg \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n; \underline{x} <= \underline{y} \vdash \mathrm{MP} \rhd \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n \rhd \underline{x} <= \underline{y} \gg \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n; \mathrm{AddDoubleNeg} \rhd \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{toNotLess} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n]$

$[\mathrm{toNotLess} \stackrel{\mathrm{tex}}{\rightarrow} \text{“toNotLess”}]$

$[\mathrm{toNotLess} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“lemma toNotLess”}]$

NegativeLessPositive

[NegativeLessPositive $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \text{FirstConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg 0 <= \underline{x}; \text{leqAddition} \rhd 0 <= \underline{x} \gg (0 + (-u\underline{x})) <= (\underline{x} + (-u\underline{x})); \text{plus0Left} \gg (0 + (-u\underline{x})) = (-u\underline{x}); \text{Negative} \gg (\underline{x} + (-u\underline{x})) = 0; \text{subLeqLeft} \rhd (0 + (-u\underline{x})) = (-u\underline{x}) \rhd (0 + (-u\underline{x})) <= (\underline{x} + (-u\underline{x})) \gg (-u\underline{x}) <= (\underline{x} + (-u\underline{x})); \text{subLeqRight} \rhd (\underline{x} + (-u\underline{x})) = 0 \rhd (-u\underline{x}) <= (\underline{x} + (-u\underline{x})) \gg (-u\underline{x}) <= 0; \text{leqLessTransitivity} \rhd (-u\underline{x}) <= 0 \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg \dot{\neg}((-u\underline{x}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = \underline{x})n)n)n \rceil, p_0, c)$]

[NegativeLessPositive $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \dot{\neg}((-u\underline{x}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = \underline{x})n)n)n$]

[NegativeLessPositive $\stackrel{\text{tex}}{\rightarrow}$ "NegativeLessPositive"]

[NegativeLessPositive $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeLessPositive"]

leqLessTransitivity

[leqLessTransitivity $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \underline{x} = \underline{z} \vdash \text{FirstConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \underline{y} <= \underline{z}; \text{SecondConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \dot{\neg}(\underline{y} = \underline{z})n; \text{subLeqLeft} \rhd \underline{x} = \underline{z} \rhd \underline{x} <= \underline{y} \gg \underline{z} <= \underline{y}; \text{leqAntisymmetry} \rhd \underline{y} <= \underline{z} \rhd \underline{z} <= \underline{y} \gg \underline{y} = \underline{z}; \text{FromContradiction} \rhd \underline{y} = \underline{z} \rhd \dot{\neg}(\underline{y} = \underline{z})n \gg \dot{\neg}(\underline{x} = \underline{z})n; \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \underline{x} = \underline{z} \vdash \dot{\neg}(\underline{x} = \underline{z})n \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \Rightarrow \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n; \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \text{MP2} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \Rightarrow \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n \rhd \underline{x} <= \underline{y} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n; \text{prop lemma imply negation} \rhd \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{x} = \underline{z})n; \text{FirstConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \underline{y} <= \underline{z}; \text{leqTransitivity} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{z} \gg \underline{x} <= \underline{z}; \text{JoinConjuncts} \rhd \underline{x} <= \underline{z} \rhd \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \rceil, p_0, c)$]

[leqLessTransitivity $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n$]

[leqLessTransitivity $\stackrel{\text{tex}}{\rightarrow}$ "leqLessTransitivity"]

[leqLessTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqLessTransitivity"]

LessLeqTransitivity

$[\text{LessLeqTransitivity} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \underline{y} <= \underline{z} \vdash \underline{z} = \underline{x} \vdash \text{FirstConjunct} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \underline{x} <= \underline{y}; \text{SecondConjunct} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \dot\neg(\underline{x} = \underline{y})\text{n}; \text{subLeqRight} \rhd \underline{z} = \underline{x} \rhd \underline{y} <= \underline{z} \gg \underline{y} <= \underline{x}; \text{leqAntisymmetry} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{x} \gg \underline{x} = \underline{y}; \text{FromContradiction} \rhd \underline{x} = \underline{y} \rhd \dot\neg(\underline{x} = \underline{y})\text{n} \gg \dot\neg(\underline{z} = \underline{x})\text{n}; \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \underline{y} <= \underline{z} \vdash \underline{z} = \underline{x} \vdash \dot\neg(\underline{z} = \underline{x})\text{n} \gg \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{z} = \underline{x} \Rightarrow \dot\neg(\underline{z} = \underline{x})\text{n}; \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \underline{y} <= \underline{z} \vdash \text{MP2} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{z} = \underline{x} \Rightarrow \dot\neg(\underline{z} = \underline{x})\text{n} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \rhd \underline{y} <= \underline{z} \gg \underline{z} = \underline{x} \Rightarrow \dot\neg(\underline{z} = \underline{x})\text{n}; \text{prop lemma imply negation} \rhd \underline{z} = \underline{x} \Rightarrow \dot\neg(\underline{z} = \underline{x})\text{n} \gg \dot\neg(\underline{z} = \underline{x})\text{n}; \text{NeqSymmetry} \rhd \dot\neg(\underline{z} = \underline{x})\text{n} \gg \dot\neg(\underline{x} = \underline{z})\text{n}; \text{FirstConjunct} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \underline{x} <= \underline{y}; \text{leqTransitivity} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{z} \gg \underline{x} <= \underline{z}; \text{JoinConjuncts} \rhd \underline{x} <= \underline{z} \rhd \dot\neg(\underline{x} = \underline{z})\text{n} \gg \dot\neg(\underline{x} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{z})\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})]$

$[\text{LessLeqTransitivity} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \underline{y} <= \underline{z} \vdash \dot\neg(\underline{x} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{z})\text{n})\text{n})\text{n}]$

$[\text{LessLeqTransitivity} \stackrel{\text{tex}}{\rightarrow}$ "LessLeqTransitivity"$]$

$[\text{LessLeqTransitivity} \stackrel{\text{pyk}}{\rightarrow}$ "lemma lessLeqTransitivity"$]$

LessTransitivity

$[\text{LessTransitivity} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \dot\neg(\underline{y} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{y} = \underline{z})\text{n})\text{n})\text{n} \vdash \text{FirstConjunct} \rhd \dot\neg(\underline{y} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{y} = \underline{z})\text{n})\text{n})\text{n} \gg \underline{y} <= \underline{z}; \text{LessLeqTransitivity} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \rhd \underline{y} <= \underline{z} \gg \dot\neg(\underline{x} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{z})\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})]$

$[\text{LessTransitivity} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \dot\neg(\underline{y} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{y} = \underline{z})\text{n})\text{n})\text{n} \vdash \dot\neg(\underline{x} <= \underline{z} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{z})\text{n})\text{n})\text{n}]$

$[\text{LessTransitivity} \stackrel{\text{tex}}{\rightarrow}$ "LessTransitivity"$]$

$[\text{LessTransitivity} \stackrel{\text{pyk}}{\rightarrow}$ "lemma lessTransitivity"$]$

LessTotality

$[\text{LessTotality} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot\neg(\dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n} \vdash \dot\neg(\underline{x} = \underline{y})\text{n} \vdash \text{fromNotLess} \rhd \dot\neg(\dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\text{n})\text{n})\text{n})\text{n} \gg \underline{y} <= \underline{x}; \text{NeqSymmetry} \rhd \dot\neg(\underline{x} = \underline{y})\text{n} \gg \dot\neg(\underline{y} = \underline{x})\text{n}; \text{LeqNeqLess} \rhd \underline{y} <= \underline{x} \rhd \dot\neg(\underline{y} = \underline{x})\text{n} \gg \dot\neg(\underline{y} <= \underline{x} \Rightarrow \dot\neg(\dot\neg(\underline{y} = \underline{x})\text{n})\text{n})\text{n}; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot\neg(\dot\neg(\underline{x} <= \underline{y} \Rightarrow$

$\dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \dot{\neg}(\underline{x} = \underline{y})n \vdash \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n$; Repetition $\rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n\rceil, p_0, c)]$

[LessTotality $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n$]

[LessTotality $\overset{\text{tex}}{\rightarrow}$ "LessTotality"]

[LessTotality $\overset{\text{pyk}}{\rightarrow}$ "lemma lessTotality"]

SubLessRight

[SubLessRight $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \vdash$ Repetition $\rhd \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \gg \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n$; FirstConjunct $\rhd \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \gg \underline{z} <= \underline{x}$; subLeqRight $\rhd \underline{x} = \underline{y} \rhd \underline{z} <= \underline{x} \gg \underline{z} <= \underline{y}$; SecondConjunct $\rhd \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \gg \dot{\neg}(\underline{z} = \underline{x})n$; SubNeqRight $\rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{z} = \underline{x})n \gg \dot{\neg}(\underline{z} = \underline{y})n$; JoinConjuncts $\rhd \underline{z} <= \underline{y} \rhd \dot{\neg}(\underline{z} = \underline{y})n \gg \dot{\neg}(\underline{z} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{y})n)n)n\rceil, p_0, c)]$

[SubLessRight $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \vdash \dot{\neg}(\underline{z} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{y})n)n)n$]

[SubLessRight $\overset{\text{tex}}{\rightarrow}$ "SubLessRight"]

[SubLessRight $\overset{\text{pyk}}{\rightarrow}$ "lemma subLessRight"]

SubLessLeft

[SubLessLeft $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash$ Repetition $\rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n$; FirstConjunct $\rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \underline{x} <= \underline{z}$; subLeqLeft $\rhd \underline{x} = \underline{y} \rhd \underline{x} <= \underline{z} \gg \underline{y} <= \underline{z}$; SecondConjunct $\rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \dot{\neg}(\underline{x} = \underline{z})n$; SubNeqLeft $\rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{y} = \underline{z})n$; JoinConjuncts $\rhd \underline{y} <= \underline{z} \rhd \dot{\neg}(\underline{y} = \underline{z})n \gg \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n\rceil, p_0, c)]$

[SubLessLeft $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n$]

[SubLessLeft $\overset{\text{tex}}{\rightarrow}$ "SubLessLeft"]

[SubLessLeft $\overset{\text{pyk}}{\rightarrow}$ "lemma subLessLeft"]

SwitchTerms(x < y − z)

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + (-u\underline{z})))n)n)n \vdash \mathrm{NegativeToLeft}(\mathrm{Less}) \rhd \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + (-u\underline{z})))n)n)n \gg \dot{\neg}((\underline{x} + \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = \underline{y})n)n)n; \mathrm{plusCommutativity} \gg (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}); \mathrm{SubLessLeft} \rhd (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}) \rhd \dot{\neg}((\underline{x} + \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = \underline{y})n)n)n \gg \dot{\neg}((\underline{z} + \underline{x}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z}+\underline{x}) = \underline{y})n)n)n; \mathrm{PositiveToRight}(\mathrm{Less}) \rhd \dot{\neg}((\underline{z}+\underline{x}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z}+\underline{x}) = \underline{y})n)n)n \gg \dot{\neg}(\underline{z} <= (\underline{y} + (-u\underline{x})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = (\underline{y} + (-u\underline{x})))n)n)n\rceil, p_0, c)]$

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + (-u\underline{z})))n)n)n \vdash \dot{\neg}(\underline{z} <= (\underline{y} + (-u\underline{x})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = (\underline{y} + (-u\underline{x})))n)n)n]$

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{tex}}{\rightarrow}$ "SwitchTerms(x<y-z)"]

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma switchTerms(x<y-z)"]

SwitchTerms(x − y < z)

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x} + (-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{y})) = \underline{z})n)n)n \vdash \mathrm{NegativeToRight}(\mathrm{Less}) \rhd \dot{\neg}((\underline{x} + (-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{y})) = \underline{z})n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z} + \underline{y}))n)n)n; \mathrm{plusCommutativity} \gg (\underline{z} + \underline{y}) = (\underline{y} + \underline{z}); \mathrm{SubLessRight} \rhd (\underline{z} + \underline{y}) = (\underline{y} + \underline{z}) \rhd \dot{\neg}(\underline{x} <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z} + \underline{y}))n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n; \mathrm{PositiveToLeft}(\mathrm{Less}) \rhd \dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n \gg \dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) = \underline{y})n)n)n\rceil, p_0, c)]$

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x} + (-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{y})) = \underline{z})n)n)n \vdash \dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) = \underline{y})n)n)n]$

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{tex}}{\rightarrow}$ "SwitchTerms(x-y<z)"]

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma switchTerms(x-y<z)"]

LessAddition

$[\mathrm{LessAddition} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \mathrm{LessLeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \underline{x} <= \underline{y}; \mathrm{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}); \mathrm{LessNeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} = \underline{y})n; \mathrm{NeqAddition} \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n; \mathrm{JoinConjuncts} \rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \rhd \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n \gg \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n)n)n\rceil, p_0, c)]$

$[\mathrm{LessAddition} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{LessAddition} \stackrel{\mathrm{tex}}{\rightarrow}$ “LessAddition”$]$

$[\mathrm{LessAddition} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma lessAddition”$]$

LessAdditionLeft

$[\mathrm{LessAdditionLeft} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessAddition} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plusCommutativity} \gg (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}); \mathrm{SubLessLeft} \rhd (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}) \rhd \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plusCommutativity} \gg (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}); \mathrm{SubLessRight} \rhd (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}) \rhd \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{z} + \underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessAdditionLeft} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{z} + \underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{LessAdditionLeft} \stackrel{\mathrm{tex}}{\rightarrow}$ “LessAdditionLeft”$]$

$[\mathrm{LessAdditionLeft} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma lessAdditionLeft”$]$

LessMultiplication

$[\mathrm{LessMultiplication} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessLeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{x} <= \underline{y}; \mathrm{LessLeq} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{z}; \mathrm{leqMultiplication} \rhd 0 <= \underline{z} \rhd \underline{x} <= \underline{y} \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}); \mathrm{LessNeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{x} = \underline{y})\mathrm{n}; \mathrm{LessNeq} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = \underline{z})\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{z})\mathrm{n} \gg \dot{\neg}(\underline{z} = 0)\mathrm{n}; \mathrm{NeqMultiplication} \rhd \dot{\neg}(\underline{z} = 0)\mathrm{n} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \gg \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n}; \mathrm{LeqNeqLess} \rhd (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \rhd \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n} \gg \dot{\neg}((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessMultiplication} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{LessMultiplication} \stackrel{\mathrm{tex}}{\rightarrow}$ “LessMultiplication”$]$

$[\mathrm{LessMultiplication} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma lessMultiplication”$]$

LessMultiplicationLeft

$[\mathrm{LessMultiplicationLeft} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessMultiplication} \rhd \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{timesCommutativity} \gg (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{x}}); \mathrm{timesCommutativity} \gg (\underline{\mathsf{y}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{y}}); \mathrm{SubLessLeft} \rhd (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{x}}) \rhd \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubLessRight} \rhd (\underline{\mathsf{y}} * \underline{\mathsf{z}}) = (\underline{\mathsf{z}} * \underline{\mathsf{y}}) \rhd \dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{z}} * \underline{\mathsf{y}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) = (\underline{\mathsf{z}} * \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessMultiplicationLeft} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) <= (\underline{\mathsf{z}} * \underline{\mathsf{y}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{z}} * \underline{\mathsf{x}}) = (\underline{\mathsf{z}} * \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{LessMultiplicationLeft} \overset{\mathrm{tex}}{\rightarrow}$ “LessMultiplicationLeft”]

$[\mathrm{LessMultiplicationLeft} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma lessMultiplicationLeft”]

LessDivision

$[\mathrm{LessDivision} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon 0 <= \underline{\mathsf{z}} \vdash \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{FromLess} \rhd \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}))\mathrm{n}; \mathrm{leqMultiplicationAxiom} \gg 0 <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow (\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}); \mathrm{MP} \rhd 0 <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow (\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}) \rhd 0 <= \underline{\mathsf{z}} \gg \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow (\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}); \mathrm{Contrapositive} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow (\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}) \gg \dot{\neg}((\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}))\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{x}})\mathrm{n}; \mathrm{MP} \rhd \dot{\neg}((\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}))\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{x}})\mathrm{n} \rhd \dot{\neg}((\underline{\mathsf{y}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{z}}))\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{x}})\mathrm{n}; \mathrm{ToLess} \rhd \dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{x}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessDivision} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon 0 <= \underline{\mathsf{z}} \vdash \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{LessDivision} \overset{\mathrm{tex}}{\rightarrow}$ “LessDivision”]

$[\mathrm{LessDivision} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma lessDivision”]

PositiveToRight(Less)

$[\mathrm{PositiveToRight(Less)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{y}}) <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{y}}) = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessAddition} \rhd \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{y}}) <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{y}}) = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\mathrm{u}\underline{\mathsf{y}})) <= (\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{y}})) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{y}})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\mathrm{u}\underline{\mathsf{y}})); \mathrm{eqSymmetry} \rhd \underline{\mathsf{x}} =$

$((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) \gg ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) = \underline{x}; \mathrm{SubLessLeft} \rhd ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) = \underline{x} \rhd \dot{\neg}\,(((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) <= (\underline{z} + (-\mathrm{u}\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) = (\underline{z} + (-\mathrm{u}\underline{y})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\underline{x} <= (\underline{z} + (-\mathrm{u}\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{z} + (-\mathrm{u}\underline{y})))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveToRight(Less)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,((\underline{x} + \underline{y}) <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + \underline{y}) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\underline{x} <= (\underline{z} + (-\mathrm{u}\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{z} + (-\mathrm{u}\underline{y})))\mathrm{n})\mathrm{n})\mathrm{n}]$

[PositiveToRight(Less) $\stackrel{\mathrm{tex}}{\rightarrow}$ "PositiveToRight(Less)"]

[PositiveToRight(Less) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma positiveToRight(Less)"]

PositiveToLeft(Less)

$[\mathrm{PositiveToLeft(Less)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessAddition} \rhd \dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) <= ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})); \mathrm{eqSymmetry} \rhd \underline{y} = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) \gg ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) = \underline{y}; \mathrm{SubLessRight} \rhd ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) = \underline{y} \rhd \dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) <= ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveToLeft(Less)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + (-\mathrm{u}\underline{z})) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}]$

[PositiveToLeft(Less) $\stackrel{\mathrm{tex}}{\rightarrow}$ "PositiveToLeft(Less)"]

[PositiveToLeft(Less) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma positiveToLeft(Less)"]

NegativeToLeft(Less)

$[\mathrm{NegativeToLeft(Less)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(\underline{x} <= (\underline{y} + (-\mathrm{u}\underline{z})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + (-\mathrm{u}\underline{z})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessAddition} \rhd \dot{\neg}\,(\underline{x} <= (\underline{y} + (-\mathrm{u}\underline{z})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + (-\mathrm{u}\underline{z})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{x} + \underline{z}) <= ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + \underline{z}) = ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Three2threeTerms} \gg ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})); \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})); \mathrm{eqSymmetry} \rhd \underline{y} = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) \gg ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) = \underline{y}; \mathrm{eqTransitivity} \rhd ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) \rhd ((\underline{y} + \underline{z}) + (-\mathrm{u}\underline{z})) = \underline{y} \gg ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) = \underline{y}; \mathrm{SubLessRight} \rhd ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) = \underline{y} \rhd \dot{\neg}\,((\underline{x} + \underline{z}) <= ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + \underline{z}) = ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{x} + \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + \underline{z}) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NegativeToLeft(Less)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(\underline{x} <= (\underline{y} + (-\mathrm{u}\underline{z})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + (-\mathrm{u}\underline{z})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,((\underline{x} + \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} + \underline{z}) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}]$

[NegativeToLeft(Less) $\stackrel{\mathrm{tex}}{\rightarrow}$ "NegativeToLeft(Less)"]

[NegativeToLeft(Less) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma negativeToLeft(Less)”]

NegativeToRight(Less)

[NegativeToRight(Less) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x}+(-\text{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\text{u}\underline{y})) = \underline{z})\text{n})\text{n})\text{n} \vdash \text{LessAddition} \rhd \dot{\neg}((\underline{x}+(-\text{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\text{u}\underline{y})) = \underline{z})\text{n})\text{n})\text{n} \gg \dot{\neg}(((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) = (\underline{z}+\underline{y}))\text{n})\text{n})\text{n}; \text{Three2threeTerms} \gg ((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) = ((\underline{x}+\underline{y})+(-\text{u}\underline{y})); \text{x} = \text{x}+\text{y}-\text{y} \gg \underline{x} = ((\underline{x}+\underline{y})+(-\text{u}\underline{y})); \text{eqSymmetry} \rhd \underline{x} = ((\underline{x}+\underline{y})+(-\text{u}\underline{y})) \gg ((\underline{x}+\underline{y})+(-\text{u}\underline{y})) = \underline{x}; \text{eqTransitivity} \rhd ((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) = ((\underline{x}+\underline{y})+(-\text{u}\underline{y})) \rhd ((\underline{x}+\underline{y})+(-\text{u}\underline{y})) = \underline{x} \gg ((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) = \underline{x}; \text{SubLessLeft} \rhd ((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) = \underline{x} \rhd \dot{\neg}(((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+(-\text{u}\underline{y}))+\underline{y}) = (\underline{z}+\underline{y}))\text{n})\text{n})\text{n} \gg \dot{\neg}(\underline{x} <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z}+\underline{y}))\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[NegativeToRight(Less) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x}+(-\text{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\text{u}\underline{y})) = \underline{z})\text{n})\text{n})\text{n} \vdash \dot{\neg}(\underline{x} <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z}+\underline{y}))\text{n})\text{n})\text{n}$]

[NegativeToRight(Less) $\stackrel{\text{tex}}{\rightarrow}$ “NegativeToRight(Less)”]

[NegativeToRight(Less) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma negativeToRight(Less)”]

AddEquations(Less)

[AddEquations(Less) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})\text{n})\text{n})\text{n} \vdash \text{LessAddition} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \dot{\neg}((\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = (\underline{y}+\underline{z}))\text{n})\text{n})\text{n}; \text{LessAdditionLeft} \rhd \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})\text{n})\text{n})\text{n} \gg \dot{\neg}((\underline{y}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+\underline{z}) = (\underline{y}+\underline{u}))\text{n})\text{n})\text{n}; \text{LessTransitivity} \rhd \dot{\neg}((\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = (\underline{y}+\underline{z}))\text{n})\text{n})\text{n} \rhd \dot{\neg}((\underline{y}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+\underline{z}) = (\underline{y}+\underline{u}))\text{n})\text{n})\text{n} \gg \dot{\neg}((\underline{x}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = (\underline{y}+\underline{u}))\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[AddEquations(Less) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})\text{n})\text{n})\text{n} \vdash \dot{\neg}((\underline{x}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = (\underline{y}+\underline{u}))\text{n})\text{n})\text{n}$]

[AddEquations(Less) $\stackrel{\text{tex}}{\rightarrow}$ “AddEquations(Less)”]

[AddEquations(Less) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma addEquations(Less)”]

AddEquations(LeqLess)

[AddEquations(LeqLess) $\overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})n)n)n \vdash \text{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}); \text{LessAdditionLeft} \rhd \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z}=\underline{u})n)n)n \gg \dot{\neg}((\underline{y}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+\underline{z}) = (\underline{y}+\underline{u}))n)n)n; \text{leqLessTransitivity} \rhd (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \rhd \dot{\neg}((\underline{y}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+\underline{z}) = (\underline{y}+\underline{u}))n)n)n \gg \dot{\neg}((\underline{x}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = (\underline{y}+\underline{u}))n)n)n\rceil, p_0, c)$]

[AddEquations(LeqLess) $\overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})n)n)n \vdash \dot{\neg}((\underline{x}+\underline{z}) <= (\underline{y}+\underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = (\underline{y}+\underline{u}))n)n)n$]

[AddEquations(LeqLess) $\overset{\text{tex}}{\rightarrow}$ "AddEquations(LeqLess)"]

[AddEquations(LeqLess) $\overset{\text{pyk}}{\rightarrow}$ "lemma addEquations(LeqLess)"]

reciprocalToLeft(Less)

[reciprocalToLeft(Less) $\overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= (\underline{y} * \text{rec}\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} * \text{rec}\underline{z}))n)n)n \vdash \text{LessMultiplication} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \rhd \dot{\neg}(\underline{x} <= (\underline{y} * \text{rec}\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} * \text{rec}\underline{z}))n)n)n \gg \dot{\neg}((\underline{x} * \underline{z}) <= ((\underline{y} * \text{rec}\underline{z}) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = ((\underline{y} * \text{rec}\underline{z}) * \underline{z}))n)n)n; \text{Three2threeFactors} \gg ((\underline{y} * \text{rec}\underline{z}) * \underline{z}) = ((\underline{y} * \underline{z}) * \text{rec}\underline{z}); \text{PositiveNonzero} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \gg \dot{\neg}(\underline{z} = 0)n; x = x * y * (1/y) \rhd \dot{\neg}(\underline{z} = 0)n \gg \underline{y} = ((\underline{y} * \underline{z}) * \text{rec}\underline{z}); \text{eqSymmetry} \rhd \underline{y} = ((\underline{y} * \underline{z}) * \text{rec}\underline{z}) \gg ((\underline{y} * \underline{z}) * \text{rec}\underline{z}) = \underline{y}; \text{eqTransitivity} \rhd ((\underline{y} * \text{rec}\underline{z}) * \underline{z}) = ((\underline{y} * \underline{z}) * \text{rec}\underline{z}) \rhd ((\underline{y} * \underline{z}) * \text{rec}\underline{z}) = \underline{y} \gg ((\underline{y} * \text{rec}\underline{z}) * \underline{z}) = \underline{y}; \text{SubLessRight} \rhd ((\underline{y} * \text{rec}\underline{z}) * \underline{z}) = \underline{y} \rhd \dot{\neg}((\underline{x} * \underline{z}) <= ((\underline{y} * \text{rec}\underline{z}) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = ((\underline{y} * \text{rec}\underline{z}) * \underline{z}))n)n)n \gg \dot{\neg}((\underline{x} * \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = \underline{y})n)n)n\rceil, p_0, c)$]

[reciprocalToLeft(Less) $\overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= (\underline{y} * \text{rec}\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} * \text{rec}\underline{z}))n)n)n \vdash \dot{\neg}((\underline{x} * \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = \underline{y})n)n)n$]

[reciprocalToLeft(Less) $\overset{\text{tex}}{\rightarrow}$ "reciprocalToLeft(Less)"]

[reciprocalToLeft(Less) $\overset{\text{pyk}}{\rightarrow}$ "lemma reciprocalToLeft(Less)"]

LessNegated

[LessNegated $\overset{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \text{LessLeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \underline{x} <= \underline{y}; \text{LeqNegated} \rhd \underline{x} <= \underline{y} \gg (-u\underline{y}) <= (-u\underline{x}); \text{LessNeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} = \underline{y})n; \text{NeqNegated} \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}((-u\underline{x}) =$

$(-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}$; NeqSymmetry $\rhd \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n}$; LeqNeqLess $\rhd (-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}) \rhd \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[LessNegated $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}$]

[LessNegated $\overset{\text{tex}}{\rightarrow}$ "LessNegated"]

[LessNegated $\overset{\text{pyk}}{\rightarrow}$ "lemma lessNegated"]

PositiveNonzero

[PositiveNonzero $\overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ Repetition $\rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n}$; SecondConjunct $\rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n}$; NeqSymmetry $\rhd \dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[PositiveNonzero $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n}$]

[PositiveNonzero $\overset{\text{tex}}{\rightarrow}$ "PositiveNonzero"]

[PositiveNonzero $\overset{\text{pyk}}{\rightarrow}$ "lemma positiveNonzero"]

PositiveNegated

[PositiveNegated $\overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ LessNegated $\rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= (-\mathrm{u}0) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}0))\mathrm{n})\mathrm{n})\mathrm{n}; -0 = 0 \gg (-\mathrm{u}0) = 0$; SubLessRight $\rhd (-\mathrm{u}0) = 0 \rhd \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= (-\mathrm{u}0) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}0))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = 0)\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[PositiveNegated $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = 0)\mathrm{n})\mathrm{n})\mathrm{n}$]

[PositiveNegated $\overset{\text{tex}}{\rightarrow}$ "PositiveNegated"]

[PositiveNegated $\overset{\text{pyk}}{\rightarrow}$ "lemma positiveNegated"]

NonpositiveNegated

[NonpositiveNegated $\overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \underline{\mathsf{x}} <= 0 \vdash$ LeqNegated $\rhd \underline{\mathsf{x}} <= 0 \gg (-\mathrm{u}0) <= (-\mathrm{u}\underline{\mathsf{x}}); -0 = 0 \gg (-\mathrm{u}0) = 0$; subLeqLeft $\rhd (-\mathrm{u}0) = 0 \rhd (-\mathrm{u}0) <= (-\mathrm{u}\underline{\mathsf{x}}) \gg 0 <= (-\mathrm{u}\underline{\mathsf{x}})\rceil, \mathrm{p}_0, \mathsf{c})]$

[NonpositiveNegated $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \underline{\mathsf{x}} <= 0 \vdash 0 <= (-\mathrm{u}\underline{\mathsf{x}})$]

[NonpositiveNegated $\overset{\text{tex}}{\rightarrow}$ "NonpositiveNegated"]

[NonpositiveNegated $\overset{\text{pyk}}{\rightarrow}$ "lemma nonpositiveNegated"]

NegativeNegated

[NegativeNegated $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ LessNegated $\rhd \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}0) <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}0) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}; -0 = 0 \gg (-\mathrm{u}0) = 0;$ SubLessLeft $\rhd (-\mathrm{u}0) = 0 \rhd \dot{\neg}((-\mathrm{u}0) <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}0) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[NegativeNegated $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}$]

[NegativeNegated $\overset{\text{tex}}{\rightarrow}$ "NegativeNegated"]

[NegativeNegated $\overset{\text{pyk}}{\rightarrow}$ "lemma negativeNegated"]

NonnegativeNegated

[NonnegativeNegated $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{\mathsf{x}}\colon 0 <= \underline{\mathsf{x}} \vdash$ LeqNegated $\rhd 0 <= \underline{\mathsf{x}} \gg (-\mathrm{u}\underline{\mathsf{x}}) <= (-\mathrm{u}0); -0 = 0 \gg (-\mathrm{u}0) = 0;$ subLeqRight $\rhd (-\mathrm{u}0) = 0 \rhd (-\mathrm{u}\underline{\mathsf{x}}) <= (-\mathrm{u}0) \gg (-\mathrm{u}\underline{\mathsf{x}}) <= 0\rceil, \mathrm{p}_0, \mathsf{c})$]

[NonnegativeNegated $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon 0 <= \underline{\mathsf{x}} \vdash (-\mathrm{u}\underline{\mathsf{x}}) <= 0$]

[NonnegativeNegated $\overset{\text{tex}}{\rightarrow}$ "NonnegativeNegated"]

[NonnegativeNegated $\overset{\text{pyk}}{\rightarrow}$ "lemma nonnegativeNegated"]

PositiveHalved

[PositiveHalved $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash 0 < 1/2 \gg \dot{\neg}(0 <= \mathrm{rec}(1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n};$ LessMultiplicationLeft $\rhd \dot{\neg}(0 <= \mathrm{rec}(1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\mathrm{rec}(1+1) * 0) <= (\mathrm{rec}(1+1) * \underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}(1+1) * 0) = (\mathrm{rec}(1+1) * \underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} * 0 = 0 \gg (\mathrm{rec}(1+1) * 0) = 0;$ SubLessLeft $\rhd (\mathrm{rec}(1+1) * 0) = 0 \rhd \dot{\neg}((\mathrm{rec}(1+1) * 0) <= (\mathrm{rec}(1+1) * \underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}(1+1) * 0) = (\mathrm{rec}(1+1) * \underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}(1+1) * \underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1) * \underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

$[\mathrm{PositiveHalved} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (\mathrm{rec}(1+1) * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1) * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{PositiveHalved} \overset{\mathrm{tex}}{\rightarrow}$ “PositiveHalved”$]$

$[\mathrm{PositiveHalved} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma positiveHalved”$]$

PositiveInverted

$[\mathrm{PositiveInverted} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{x}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = \underline{x})\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{x})\mathrm{n} \gg \dot{\neg}(\underline{x} = 0)\mathrm{n}; 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} * 0 = 0 \gg (\underline{x} * 0) = 0; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd (\underline{x} * 0) = 0 \gg 0 = (0 * \underline{x}); \mathrm{SubLessLeft} \rhd 0 = (0 * \underline{x}) \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * \underline{x}) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Reciprocal} \rhd \dot{\neg}(\underline{x} = 0)\mathrm{n} \gg (\underline{x} * \mathrm{rec}\underline{x}) = 1; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd (\underline{x} * \mathrm{rec}\underline{x}) = 1 \gg 1 = (\mathrm{rec}\underline{x} * \underline{x}); \mathrm{SubLessRight} \rhd 1 = (\mathrm{rec}\underline{x} * \underline{x}) \rhd \dot{\neg}((0 * \underline{x}) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * \underline{x}) <= (\mathrm{rec}\underline{x} * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = (\mathrm{rec}\underline{x} * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessDivision} \rhd 0 <= \underline{x} \rhd \dot{\neg}((0 * \underline{x}) <= (\mathrm{rec}\underline{x} * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = (\mathrm{rec}\underline{x} * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \mathrm{rec}\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}\underline{x})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveInverted} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= \mathrm{rec}\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}\underline{x})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{PositiveInverted} \overset{\mathrm{tex}}{\rightarrow}$ “PositiveInverted”$]$

$[\mathrm{PositiveInverted} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma positiveInverted”$]$

NonnegativeNumerical

$[\mathrm{NonnegativeNumerical} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \mathrm{Numerical} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n}; \mathrm{AddDoubleNeg} \rhd 0 <= \underline{x} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n})\mathrm{n}; \mathrm{ToNegatedAnd}(1) \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegateDisjunct2} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n} \gg |\underline{x}| = \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NonnegativeNumerical} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash |\underline{x}| = \underline{x}]$

$[\mathrm{NonnegativeNumerical} \overset{\mathrm{tex}}{\rightarrow}$ “NonnegativeNumerical”$]$

$[\mathrm{NonnegativeNumerical} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma nonnegativeNumerical”$]$

NegativeNumerical

$[\mathrm{NegativeNumerical} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{Numerical} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n}; \mathrm{FromLess} \rhd \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{\mathsf{x}})\mathrm{n}; \mathrm{ToNegatedAnd}(1) \rhd \dot{\neg}(0 <= \underline{\mathsf{x}})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegateDisjunct1} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{\mathsf{x}})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n} \gg |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NegativeNumerical} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \vdash |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}})]$

$[\mathrm{NegativeNumerical} \overset{\mathrm{tex}}{\rightarrow}$ "NegativeNumerical"$]$

$[\mathrm{NegativeNumerical} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma negativeNumerical"$]$

PositiveNumerical

$[\mathrm{PositiveNumerical} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessLeq} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{\mathsf{x}}; \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{\mathsf{x}} \gg |\underline{\mathsf{x}}| = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveNumerical} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash |\underline{\mathsf{x}}| = \underline{\mathsf{x}}]$

$[\mathrm{PositiveNumerical} \overset{\mathrm{tex}}{\rightarrow}$ "PositiveNumerical"$]$

$[\mathrm{PositiveNumerical} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma positiveNumerical"$]$

lemma nonpositiveNumerical

$[\mathrm{lemma\ nonpositiveNumerical} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{NegativeNumerical} \rhd \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}); \forall \underline{\mathsf{x}}\colon \underline{\mathsf{x}} = 0 \vdash \mathrm{eqSymmetry} \rhd \underline{\mathsf{x}} = 0 \gg 0 = \underline{\mathsf{x}}; \mathrm{eqLeq} \rhd 0 = \underline{\mathsf{x}} \gg 0 <= \underline{\mathsf{x}}; \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{\mathsf{x}} \gg |\underline{\mathsf{x}}| = \underline{\mathsf{x}}; -0 = 0 \gg (-\mathrm{u}0) = 0; \mathrm{eqSymmetry} \rhd (-\mathrm{u}0) = 0 \gg 0 = (-\mathrm{u}0); \mathrm{EqNegated} \rhd 0 = \underline{\mathsf{x}} \gg (-\mathrm{u}0) = (-\mathrm{u}\underline{\mathsf{x}}); \mathrm{eqTransitivity5} \rhd |\underline{\mathsf{x}}| = \underline{\mathsf{x}} \rhd \underline{\mathsf{x}} = 0 \rhd 0 = (-\mathrm{u}0) \rhd (-\mathrm{u}0) = (-\mathrm{u}\underline{\mathsf{x}}) \gg |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}); \forall \underline{\mathsf{x}}\colon \mathrm{Ded} \rhd \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \vdash |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}) \gg \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}); \mathrm{Ded} \rhd \forall \underline{\mathsf{x}}\colon \underline{\mathsf{x}} = 0 \vdash |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}) \gg \underline{\mathsf{x}} = 0 \Rightarrow |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}); \underline{\mathsf{x}} <= 0 \vdash \mathrm{LeqLessEq} \rhd \underline{\mathsf{x}} <= 0 \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = 0; \mathrm{FromDisjuncts} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} = 0 \rhd \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}) \rhd \underline{\mathsf{x}} = 0 \Rightarrow |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}}) \gg |\underline{\mathsf{x}}| = (-\mathrm{u}\underline{\mathsf{x}})\rceil, \mathrm{p}_0, \mathsf{c})]$

[lemma nonpositiveNumerical $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \underline{x} <= 0 \vdash |\underline{x}| = (-\mathrm{u}\underline{x})$]

[lemma nonpositiveNumerical $\overset{\text{pyk}}{\rightarrow}$ "lemma nonpositiveNumerical"]

$|0| = 0$

$[|0| = 0 \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{leqReflexivity} \gg 0 <= 0; \mathrm{NonnegativeNumerical} \rhd 0 <= 0 \gg |0| = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[|0| = 0 \overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash |0| = 0]$

[$|0| = 0 \overset{\text{tex}}{\rightarrow}$ "|0|=0"]

[$|0| = 0 \overset{\text{pyk}}{\rightarrow}$ "lemma |0|=0"]

$0 <= |x|$

$[0 <= |x| \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{eqSymmetry} \rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \mathrm{subLeqRight} \rhd \underline{x} = |\underline{x}| \rhd 0 <= \underline{x} \gg 0 <= |\underline{x}|; \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x})\mathrm{n} \vdash \mathrm{ToLess} \rhd \dot{\neg}(0 <= \underline{x})\mathrm{n} \gg \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegativeNumerical} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg |\underline{x}| = (-\mathrm{u}\underline{x}); \mathrm{eqSymmetry} \rhd |\underline{x}| = (-\mathrm{u}\underline{x}) \gg (-\mathrm{u}\underline{x}) = |\underline{x}|; \mathrm{NegativeNegated} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (-\mathrm{u}\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessLeq} \rhd \dot{\neg}(0 <= (-\mathrm{u}\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= (-\mathrm{u}\underline{x}); \mathrm{subLeqRight} \rhd (-\mathrm{u}\underline{x}) = |\underline{x}| \rhd 0 <= (-\mathrm{u}\underline{x}) \gg 0 <= |\underline{x}|; \forall \underline{x}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon 0 <= \underline{x} \vdash 0 <= |\underline{x}| \gg 0 <= \underline{x} \Rightarrow 0 <= |\underline{x}|; \mathrm{Ded} \rhd \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x})\mathrm{n} \vdash 0 <= |\underline{x}| \gg \dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow 0 <= |\underline{x}|; \mathrm{FromNegations} \rhd 0 <= \underline{x} \Rightarrow 0 <= |\underline{x}| \rhd \dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow 0 <= |\underline{x}| \gg 0 <= |\underline{x}|\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 <= |x| \overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= |\underline{x}|]$

[$0 <= |x| \overset{\text{tex}}{\rightarrow}$ "0<=|x|"]

[$0 <= |x| \overset{\text{pyk}}{\rightarrow}$ "lemma 0<=|x|"]

$x <= |x|$

$[x <= |x| \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \mathrm{NonnegativeNumerical} \gg |\underline{x}| = \underline{x}; \mathrm{eqSymmetry} \rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \mathrm{eqLeq} \rhd \underline{x} = |\underline{x}| \gg \underline{x} <= |\underline{x}|; \forall \underline{x}\colon \underline{x} <= 0 \vdash 0 <= |x| \gg 0 <= |\underline{x}|; \mathrm{leqTransitivity} \rhd \underline{x} <= 0 \rhd 0 <= |\underline{x}| \gg \underline{x} <= |\underline{x}|; \forall \underline{x}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon 0 <= \underline{x} \vdash \underline{x} <= |\underline{x}| \gg 0 <= \underline{x} \Rightarrow \underline{x} <= |\underline{x}|; \mathrm{Ded} \rhd \forall \underline{x}\colon \underline{x} <= 0 \vdash \underline{x} <= |\underline{x}| \gg \underline{x} <= 0 \Rightarrow \underline{x} <= |\underline{x}|; \mathrm{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \underline{x} <= |\underline{x}| \rhd \underline{x} <= 0 \Rightarrow \underline{x} <= |\underline{x}| \gg \underline{x} <= |\underline{x}|\rceil, \mathrm{p}_0, \mathsf{c})]$

$[x <= |x| \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \underline{x} <= |\underline{x}|]$

$[x <= |x| \stackrel{\mathrm{tex}}{\to}$ “x<=|x|”$]$

$[x <= |x| \stackrel{\mathrm{pyk}}{\to}$ “lemma x<=|x|”$]$

FromPositiveNumerical

$[\mathrm{FromPositiveNumerical} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon 0 <= \underline{x} \vdash \dot\neg (0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0 = |\underline{x}|)n)n)n \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{SubLessRight} \rhd |\underline{x}| = \underline{x} \rhd \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0 = |\underline{x}|)n)n)n \gg \dot\neg(0 <= \underline{x} \Rightarrow \dot\neg(\dot\neg(0 = \underline{x})n)n)n; \mathrm{LessNeq} \rhd \dot\neg(0 <= \underline{x} \Rightarrow \dot\neg(\dot\neg(0=\underline{x})n)n)n \gg \dot\neg(0 = \underline{x})n; \mathrm{NeqSymmetry} \rhd \dot\neg(0=\underline{x})n \gg \dot\neg(\underline{x} = 0)n; \forall\underline{x}\colon \underline{x} <= 0 \vdash \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0 = |\underline{x}|)n)n)n \vdash \mathrm{lemma\ nonpositiveNumerical} \rhd \underline{x} <= 0 \gg |\underline{x}| = (-u\underline{x}); \mathrm{SubLessRight} \rhd |\underline{x}| = (-u\underline{x}) \rhd \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \gg \dot\neg(0 <= (-u\underline{x}) \Rightarrow \dot\neg(\dot\neg(0 = (-u\underline{x}))n)n)n; \mathrm{PositiveNegated} \rhd \dot\neg(0 <= (-u\underline{x}) \Rightarrow \dot\neg(\dot\neg(0=(-u\underline{x}))n)n)n \gg \dot\neg((-u(-u\underline{x})) <= 0 \Rightarrow \dot\neg(\dot\neg((-u(-u\underline{x})) = 0)n)n)n; \mathrm{DoubleMinus} \gg (-u(-u\underline{x})) = \underline{x}; \mathrm{SubLessLeft} \rhd (-u(-u\underline{x})) = \underline{x} \rhd \dot\neg((-u(-u\underline{x})) <= 0 \Rightarrow \dot\neg(\dot\neg((-u(-u\underline{x})) = 0)n)n)n \gg \dot\neg(\underline{x} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{x} = 0)n)n)n; \mathrm{LessNeq} \rhd \dot\neg(\underline{x} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{x}=0)n)n)n \gg \dot\neg(\underline{x} = 0)n; \forall\underline{x}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon 0 <= \underline{x} \vdash \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \vdash \dot\neg(\underline{x}=0)n \gg 0 <= \underline{x} \Rightarrow \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \Rightarrow \dot\neg(\underline{x}=0)n; \mathrm{Ded} \rhd \forall\underline{x}\colon \underline{x} <= 0 \vdash \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \vdash \dot\neg(\underline{x}=0)n \gg \underline{x} <= 0 \Rightarrow \dot\neg(0<=|\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \Rightarrow \dot\neg(\underline{x}=0)n; \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \vdash \mathrm{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \dot\neg(0<=|\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \Rightarrow \dot\neg(\underline{x}=0)n \rhd \underline{x} <= 0 \Rightarrow \dot\neg(0<=|\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \Rightarrow \dot\neg(\underline{x}=0)n \gg \dot\neg(0<=|\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \Rightarrow \dot\neg(\underline{x}=0)n; \mathrm{MP} \rhd \dot\neg(0<=|\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \Rightarrow \dot\neg(\underline{x}=0)n \rhd \dot\neg(0<=|\underline{x}| \Rightarrow \dot\neg(\dot\neg(0=|\underline{x}|)n)n)n \gg \dot\neg(\underline{x}=0)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{FromPositiveNumerical} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \dot\neg(0 <= |\underline{x}| \Rightarrow \dot\neg(\dot\neg(0 = |\underline{x}|)n)n)n \vdash \dot\neg(\underline{x} = 0)n]$

$[\mathrm{FromPositiveNumerical} \stackrel{\mathrm{tex}}{\to}$ “FromPositiveNumerical”$]$

$[\mathrm{FromPositiveNumerical} \stackrel{\mathrm{pyk}}{\to}$ “lemma fromPositiveNumerical”$]$

SameNumerical

$[\mathrm{SameNumerical} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon 0 <= \underline{x} \vdash \underline{x} = \underline{y} \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{subLeqRight} \rhd \underline{x} = \underline{y} \rhd 0 <= \underline{x} \gg 0 <= \underline{y}; \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \mathrm{eqSymmetry} \rhd |\underline{y}| = \underline{y} \gg \underline{y} = |\underline{y}|; \mathrm{eqTransitivity4} \rhd |\underline{x}| = \underline{x} \rhd \underline{x} = \underline{y} \rhd \underline{y} = |\underline{y}| \gg |\underline{x}| = |\underline{y}|; \forall \underline{x} \colon \forall \underline{y} \colon \dot\neg(0 <= \underline{x})n \vdash \underline{x} = \underline{y} \vdash \mathrm{ToLess} \rhd \dot\neg(0 <= \underline{x})n \gg \dot\neg(\underline{x} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{x} = 0)n)n)n; \mathrm{NegativeNumerical} \rhd \dot\neg(\underline{x} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{x} = 0)n)n)n \gg |\underline{x}| = (-u\underline{x}); \mathrm{SubLessLeft} \rhd \underline{x} = \underline{y} \rhd \dot\neg(\underline{x} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{x} = 0)n)n)n \gg \dot\neg(\underline{y} <= 0 \Rightarrow$

$\dot{\neg}(\dot{\neg}(\underline{y}=0)n)n)n; \mathrm{NegativeNumerical} \rhd \dot{\neg}(\underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y}=0)n)n)n \gg |\underline{y}| = (-u\underline{y}); \mathrm{eqSymmetry} \rhd |\underline{y}| = (-u\underline{y}) \gg (-u\underline{y}) = |\underline{y}|; \mathrm{EqNegated} \rhd \underline{x} = \underline{y} \gg (-u\underline{x}) = (-u\underline{y}); \mathrm{eqTransitivity4} \rhd |\underline{x}| = (-u\underline{x}) \rhd (-u\underline{x}) = (-u\underline{y}) \rhd (-u\underline{y}) = |\underline{y}| \gg |\underline{x}| = |\underline{y}|; \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} = \underline{y} \vdash \mathrm{Ded} \rhd \forall\underline{x}\colon\forall\underline{y}\colon 0 <= \underline{x} \vdash \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}| \gg 0 <= \underline{x} \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \mathrm{Ded} \rhd \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(0 <= \underline{x})n \vdash \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}| \gg \dot{\neg}(0 <= \underline{x})n \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \mathrm{FromNegations} \rhd 0 <= \underline{x} \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \rhd \dot{\neg}(0 <= \underline{x})n \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \gg \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \mathrm{MP} \rhd \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \rhd \underline{x} = \underline{y} \gg |\underline{x}| = |\underline{y}|\rceil, p_0, \mathsf{c})]$

$[\mathrm{SameNumerical} \xrightarrow{\mathrm{stmt}} \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}|]$

[SameNumerical $\xrightarrow{\mathrm{tex}}$ "SameNumerical"]

[SameNumerical $\xrightarrow{\mathrm{pyk}}$ "lemma sameNumerical"]

SignNumerical(+)

$[\mathrm{SignNumerical}(+) \xrightarrow{\mathrm{proof}} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \mathrm{PositiveNumerical} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg |\underline{x}| = \underline{x}; \mathrm{PositiveNegated} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg \dot{\neg}((-u\underline{x}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = 0)n)n)n; \mathrm{NegativeNumerical} \rhd \dot{\neg}((-u\underline{x}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = 0)n)n)n \gg |(-u\underline{x})| = (-u(-u\underline{x})); \mathrm{DoubleMinus} \gg (-u(-u\underline{x})) = \underline{x}; \mathrm{eqTransitivity} \rhd |(-u\underline{x})| = (-u(-u\underline{x})) \rhd (-u(-u\underline{x})) = \underline{x} \gg |(-u\underline{x})| = \underline{x}; \mathrm{eqSymmetry} \rhd |(-u\underline{x})| = \underline{x} \gg \underline{x} = |(-u\underline{x})|; \mathrm{eqTransitivity} \rhd |\underline{x}| = \underline{x} \rhd \underline{x} = |(-u\underline{x})| \gg |\underline{x}| = |(-u\underline{x})|\rceil, p_0, \mathsf{c})]$

$[\mathrm{SignNumerical}(+) \xrightarrow{\mathrm{stmt}} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash |\underline{x}| = |(-u\underline{x})|]$

[SignNumerical(+) $\xrightarrow{\mathrm{tex}}$ "SignNumerical(+)"]

[SignNumerical(+) $\xrightarrow{\mathrm{pyk}}$ "lemma signNumerical(+)"]

SignNumerical

$[\mathrm{SignNumerical} \xrightarrow{\mathrm{proof}} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n \vdash \mathrm{NegativeNegated} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n \gg \dot{\neg}(0 <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-u\underline{x}))n)n)n; \mathrm{SignNumerical}(+) \rhd \dot{\neg}(0 <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-u\underline{x}))n)n)n \gg |(-u\underline{x})| = |(-u(-u\underline{x}))|; \mathrm{DoubleMinus} \gg (-u(-u\underline{x})) = \underline{x}; \mathrm{SameNumerical} \rhd (-u(-u\underline{x})) = \underline{x} \gg |(-u(-u\underline{x}))| = |\underline{x}|; \mathrm{eqTransitivity} \rhd |(-u\underline{x})| = |(-u(-u\underline{x}))| \rhd |(-u(-u\underline{x}))| = |\underline{x}| \gg |(-u\underline{x})| = |\underline{x}|; \mathrm{eqSymmetry} \rhd |(-u\underline{x})| = |\underline{x}| \gg |\underline{x}| = |(-u\underline{x})|; \forall\underline{x}\colon \underline{x} = 0 \vdash \mathrm{EqNegated} \rhd \underline{x} = 0 \gg (-u\underline{x}) = (-u0); -0 = 0 \gg (-u0) = 0; \mathrm{eqSymmetry} \rhd \underline{x} = 0 \gg 0 = \underline{x}; \mathrm{eqTransitivity4} \rhd (-u\underline{x}) = (-u0) \rhd (-u0) = 0 \rhd 0 = \underline{x} \gg (-u\underline{x}) = \underline{x}; \mathrm{eqSymmetry} \rhd (-u\underline{x}) = \underline{x} \gg \underline{x} = (-u\underline{x}); \mathrm{SameNumerical} \rhd \underline{x} = (-u\underline{x}) \gg |\underline{x}| = |(-u\underline{x})|; \forall\underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \mathrm{SignNumerical}(+) \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow$

$\dot{\neg}\,(\dot{\neg}\,(0 = \underline{x})n)n)n \gg |\underline{x}| = |(-u\underline{x})|; \forall\underline{x}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon \dot{\neg}\,(\underline{x} <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = 0)n)n)n \vdash |\underline{x}| = |(-u\underline{x})| \gg \dot{\neg}\,(\underline{x} <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = 0)n)n)n \Rightarrow |\underline{x}| = |(-u\underline{x})|; \mathrm{Ded} \rhd \forall\underline{x}\colon \underline{x} = 0 \vdash |\underline{x}| = |(-u\underline{x})| \gg \underline{x} = 0 \Rightarrow |\underline{x}| = |(-u\underline{x})|; \mathrm{Ded} \rhd \forall\underline{x}\colon \dot{\neg}\,(0 <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{x})n)n)n \vdash |\underline{x}| = |(-u\underline{x})| \gg \dot{\neg}\,(0 <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{x})n)n)n \Rightarrow |\underline{x}| = |(-u\underline{x})|; \mathrm{LessTotality} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = 0)n)n)n)n \Rightarrow \dot{\neg}\,(\underline{x} = 0)n \Rightarrow \dot{\neg}\,(0 <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{x})n)n)n; \mathrm{From3Disjuncts} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = 0)n)n)n)n \Rightarrow \dot{\neg}\,(\underline{x} = 0)n \Rightarrow \dot{\neg}\,(0 <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{x})n)n)n \rhd \dot{\neg}\,(\underline{x} <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = 0)n)n)n \Rightarrow |\underline{x}| = |(-u\underline{x})| \rhd \underline{x} = 0 \Rightarrow |\underline{x}| = |(-u\underline{x})| \rhd \dot{\neg}\,(0 <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{x})n)n)n \Rightarrow |\underline{x}| = |(-u\underline{x})| \gg |\underline{x}| = |(-u\underline{x})|\rceil, p_0, \mathsf{c})]$

$[\mathrm{SignNumerical} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{x}\colon |\underline{x}| = |(-u\underline{x})|]$

$[\mathrm{SignNumerical} \stackrel{\mathrm{tex}}{\to}$ “SignNumerical”$]$

$[\mathrm{SignNumerical} \stackrel{\mathrm{pyk}}{\to}$ “lemma signNumerical”$]$

ToNumericalLess

$[\mathrm{ToNumericalLess} \stackrel{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \vdash 0 <= \underline{x} \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{eqSymmetry} \rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \mathrm{SubLessLeft} \rhd \underline{x} = |\underline{x}| \rhd \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n; \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \vdash \underline{x} <= 0 \vdash \mathrm{LessNegated} \rhd \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \gg \dot{\neg}\,((-u\underline{x}) <= (-u(-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{x}) = (-u(-u\underline{y})))n)n)n; \mathrm{lemma\ nonpositiveNumerical} \rhd \underline{x} <= 0 \gg |\underline{x}| = (-u\underline{x}); \mathrm{eqSymmetry} \rhd |\underline{x}| = (-u\underline{x}) \gg (-u\underline{x}) = |\underline{x}|; \mathrm{SubLessLeft} \rhd (-u\underline{x}) = |\underline{x}| \rhd \dot{\neg}\,((-u\underline{x}) <= (-u(-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{x}) = (-u(-u\underline{y})))n)n)n \gg \dot{\neg}\,(|\underline{x}| <= (-u(-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = (-u(-u\underline{y})))n)n)n; \mathrm{DoubleMinus} \gg (-u(-u\underline{y})) = \underline{y}; \mathrm{SubLessRight} \rhd (-u(-u\underline{y})) = \underline{y} \rhd \dot{\neg}\,(|\underline{x}| <= (-u(-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = (-u(-u\underline{y})))n)n)n \gg \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n; \forall\underline{x}\colon \forall\underline{y}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \vdash 0 <= \underline{x} \vdash \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n \gg \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \Rightarrow 0 <= \underline{x} \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n; \mathrm{Ded} \rhd \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \vdash \underline{x} <= 0 \vdash \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n \gg \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \Rightarrow \underline{x} <= 0 \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n; \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \vdash \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \Rightarrow 0 <= \underline{x} \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n \rhd \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})n)n)n \gg 0 <= \underline{x} \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n; \mathrm{MP} \rhd \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \Rightarrow \underline{x} <= 0 \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n \rhd \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) = \underline{x})n)n)n \gg \underline{x} <= 0 \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n; \mathrm{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n \rhd \underline{x} <= 0 \Rightarrow \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n \gg \dot{\neg}\,(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = \underline{y})n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{ToNumericalLess} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}\,((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((-u\underline{y}) =$

$\underline{x})n)n \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n]$

[ToNumericalLess $\stackrel{\text{tex}}{\to}$ “ToNumericalLess”]

[ToNumericalLess $\stackrel{\text{pyk}}{\to}$ “lemma toNumericalLess”]

FromNumericalGreater

[FromNumericalGreater $\stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash 0 <= \underline{y} \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \mathrm{SubLessRight} \rhd |\underline{y}| = \underline{y} \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{WeakenOr1} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \underline{y} <= 0 \vdash \mathrm{lemma\ nonpositiveNumerical} \rhd \underline{y} <= 0 \gg |\underline{y}| = (-u\underline{y}); \mathrm{SubLessRight} \rhd |\underline{y}| = (-u\underline{y}) \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \gg \dot{\neg}(\underline{x} <= (-u\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (-u\underline{y}))n)n)n; \mathrm{LessNegated} \rhd \dot{\neg}(\underline{x} <= (-u\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (-u\underline{y}))n)n)n \gg \dot{\neg}((-u(-u\underline{y})) <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((-u(-u\underline{y})) = (-u\underline{x}))n)n)n; \mathrm{DoubleMinus} \gg (-u(-u\underline{y})) = \underline{y}; \mathrm{SubLessLeft} \rhd (-u(-u\underline{y})) = \underline{y} \rhd \dot{\neg}((-u(-u\underline{y})) <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((-u(-u\underline{y})) = (-u\underline{x}))n)n)n \gg \dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n; \mathrm{WeakenOr2} \rhd \dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \forall\underline{x}\colon\forall\underline{y}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash 0 <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow 0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{Ded} \rhd \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \underline{y} <= 0 \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow \underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow 0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \gg 0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{MP} \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow \underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \gg \underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{FromLeqGeq} \rhd 0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \rhd \underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n\rceil, p_0, c)]$

[FromNumericalGreater $\stackrel{\text{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow$

$\dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n]$

[FromNumericalGreater $\stackrel{\text{tex}}{\rightarrow}$ “FromNumericalGreater”]

[FromNumericalGreater $\stackrel{\text{pyk}}{\rightarrow}$ “lemma fromNumericalGreater”]

NumericalDifference

[NumericalDifference $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \text{SignNumerical} \gg |(\underline{x}+(-u\underline{y}))| = |(-u(\underline{x}+(-u\underline{y})))|; \text{MinusNegated} \gg (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x})); \text{SameNumerical} \rhd (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x})) \gg |(-u(\underline{x}+(-u\underline{y})))| = |(\underline{y}+(-u\underline{x}))|; \text{eqTransitivity} \rhd |(\underline{x}+(-u\underline{y}))| = |(-u(\underline{x}+(-u\underline{y})))| \rhd |(-u(\underline{x}+(-u\underline{y})))| = |(\underline{y}+(-u\underline{x}))| \gg |(\underline{x}+(-u\underline{y}))| = |(\underline{y}+(-u\underline{x}))|\rceil, p_0, c)$]

[NumericalDifference $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |(\underline{x}+(-u\underline{y}))| = |(\underline{y}+(-u\underline{x}))|$]

[NumericalDifference $\stackrel{\text{tex}}{\rightarrow}$ “NumericalDifference”]

[NumericalDifference $\stackrel{\text{pyk}}{\rightarrow}$ “lemma numericalDifference”]

NumericalDifferenceLess(Helper)

[NumericalDifferenceLess(Helper) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{x}+(-u\underline{y})) \vdash \dot{\neg}((\underline{x}+(-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = \underline{z})n)n)n \vdash \text{leqLessTransitivity} \rhd 0 <= (\underline{x}+(-u\underline{y})) \rhd \dot{\neg}((\underline{x}+(-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = \underline{z})n)n)n \gg \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n; \text{PositiveNegated} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \gg \dot{\neg}((-u\underline{z}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{z}) = 0)n)n)n; \text{LessAdditionLeft} \rhd \dot{\neg}((-u\underline{z}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{z}) = 0)n)n)n \gg \dot{\neg}((\underline{y}+(-u\underline{z})) <= (\underline{y}+0) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = (\underline{y}+0))n)n)n; \text{plus0} \gg (\underline{y}+0) = \underline{y}; \text{SubLessRight} \rhd (\underline{y}+0) = \underline{y} \rhd \dot{\neg}((\underline{y}+(-u\underline{z})) <= (\underline{y}+0) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = (\underline{y}+0))n)n)n \gg \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{y})n)n)n; \text{negativeToLeft(Leq)(1term)} \rhd 0 <= (\underline{x}+(-u\underline{y})) \gg \underline{y} <= \underline{x}; \text{LessLeqTransitivity} \rhd \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{y})n)n)n \rhd \underline{y} <= \underline{x} \gg \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n; \text{NegativeToRight(Less)} \rhd \dot{\neg}((\underline{x}+(-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = \underline{z})n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z}+\underline{y}))n)n)n; \text{plusCommutativity} \gg (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}); \text{SubLessRight} \rhd (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}) \rhd \dot{\neg}(\underline{x} <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z}+\underline{y}))n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n; \text{JoinConjuncts} \rhd \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \rhd \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n\rceil, p_0, c)$]

[NumericalDifferenceLess(Helper) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{x}+(-u\underline{y})) \vdash \dot{\neg}((\underline{x}+(-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = \underline{z})n)n)n \vdash \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow$

$\dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n$]

[NumericalDifferenceLess(Helper) $\stackrel{\text{tex}}{\to}$ "NumericalDifferenceLess(Helper)"]

[NumericalDifferenceLess(Helper) $\stackrel{\text{pyk}}{\to}$ "lemma numericalDifferenceLess helper"]

NumericalDifferenceLess

[NumericalDifferenceLess $\stackrel{\text{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon\dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \vdash 0 <= (\underline{x}+(-u\underline{y})) \vdash$ NonnegativeNumerical $\rhd 0 <= (\underline{x}+(-u\underline{y})) \gg |(\underline{x}+(-u\underline{y}))| = (\underline{x}+(-u\underline{y}))$; SubLessLeft $\rhd |(\underline{x}+(-u\underline{y}))| = (\underline{x}+(-u\underline{y})) \rhd \dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \gg \dot{\neg}((\underline{x}+(-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = \underline{z})n)n)n$; NumericalDifferenceLess(Helper) $\rhd 0 <= (\underline{x}+(-u\underline{y})) \rhd \dot{\neg}((\underline{x}+(-u\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = \underline{z})n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n$; $\forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon\dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \vdash$ ToLess $\rhd \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \gg \dot{\neg}((\underline{x}+(-u\underline{y})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = 0)n)n)n$; NegativeNumerical $\rhd \dot{\neg}((\underline{x}+(-u\underline{y})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = 0)n)n)n \gg |(\underline{x}+(-u\underline{y}))| = (-u(\underline{x}+(-u\underline{y})))$; MinusNegated $\gg (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x}))$; eqTransitivity $\rhd |(\underline{x}+(-u\underline{y}))| = (-u(\underline{x}+(-u\underline{y}))) \rhd (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x})) \gg |(\underline{x}+(-u\underline{y}))| = (\underline{y}+(-u\underline{x}))$; SubLessLeft $\rhd |(\underline{x}+(-u\underline{y}))| = (\underline{y}+(-u\underline{x})) \rhd \dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \gg \dot{\neg}((\underline{y}+(-u\underline{x})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{x})) = \underline{z})n)n)n$; NegativeNegated $\rhd \dot{\neg}((\underline{x}+(-u\underline{y})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = 0)n)n)n \gg \dot{\neg}(0 <= (-u(\underline{x}+(-u\underline{y}))) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-u(\underline{x}+(-u\underline{y}))))n)n)n$; SubLessRight $\rhd (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x})) \rhd \dot{\neg}(0 <= (-u(\underline{x}+(-u\underline{y}))) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-u(\underline{x}+(-u\underline{y}))))n)n)n \gg \dot{\neg}(0 <= (\underline{y}+(-u\underline{x})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{y}+(-u\underline{x})))n)n)n$; LessLeq $\rhd \dot{\neg}(0 <= (\underline{y}+(-u\underline{x})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{y}+(-u\underline{x})))n)n)n \gg 0 <= (\underline{y}+(-u\underline{x}))$; NumericalDifferenceLess(Helper) $\rhd 0 <= (\underline{y}+(-u\underline{x})) \rhd \dot{\neg}((\underline{y}+(-u\underline{x})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{x})) = \underline{z})n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n)n)n$; FirstConjunct $\rhd \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n)n)n \gg \dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n$; NegativeToRight(Less) $\rhd \dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n$; SecondConjunct $\rhd \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n)n)n \gg \dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n$; PositiveToLeft(Less) $\rhd \dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n \gg \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n$; JoinConjuncts $\rhd \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \rhd \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$

$(\underline{y} + \underline{z}))n)n)n)n)n; \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash 0 <= (\underline{x} + (-u\underline{y})) \vdash \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \Rightarrow$
$0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n; \mathrm{Ded} \rhd \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}\,(0 <= (\underline{x} + (-u\underline{y})))n \vdash \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \Rightarrow$
$\dot{\neg}\,(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n; \dot{\neg}\,(|(\underline{x}+(-u\underline{y}))| <=$
$\underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \Rightarrow 0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \rhd \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \gg$
$0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n; \mathrm{MP} \rhd \dot{\neg}\,(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \Rightarrow \dot{\neg}\,(0 <= (\underline{x}+(-u\underline{y})))n \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \rhd \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \gg$
$\dot{\neg}\,(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n; \mathrm{FromNegations} \rhd$
$0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n \rhd \dot{\neg}\,(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{NumericalDifferenceLess} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n]$

$[\mathrm{NumericalDifferenceLess} \overset{\mathrm{tex}}{\rightarrow}$ "NumericalDifferenceLess"$]$

$[\mathrm{NumericalDifferenceLess} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma numericalDifferenceLess"$]$

SplitNumericalSumHelper

$[\mathrm{SplitNumericalSumHelper} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall\underline{x}\colon\forall\underline{y}\colon |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \vdash \mathrm{SignNumerical} \gg |\underline{x}| =$
$|(-u\underline{x})|; \mathrm{SignNumerical} \gg |\underline{y}| = |(-u\underline{y})|; \mathrm{AddEquations} \rhd |\underline{x}| = |(-u\underline{x})| \rhd |\underline{y}| =$
$|(-u\underline{y})| \gg (|\underline{x}| + |\underline{y}|) = (|(-u\underline{x})| + |(-u\underline{y})|); \mathrm{eqSymmetry} \rhd (|\underline{x}| + |\underline{y}|) =$
$(|(-u\underline{x})| + |(-u\underline{y})|) \gg (|(-u\underline{x})| + |(-u\underline{y})|) = (|\underline{x}| + |\underline{y}|); -x - y = -(x + y) \gg$
$((-u\underline{x}) + (-u\underline{y})) = (-u(\underline{x} + \underline{y})); \mathrm{SameNumerical} \rhd ((-u\underline{x}) + (-u\underline{y})) =$

$(-u(\underline{x}+\underline{y})) \gg |((-u\underline{x})+(-u\underline{y}))| = |(-u(\underline{x}+\underline{y}))|; \mathrm{SignNumerical} \gg |(\underline{x}+\underline{y})| = |(-u(\underline{x}+\underline{y}))|; \mathrm{eqSymmetry} \rhd |(\underline{x}+\underline{y})| = |(-u(\underline{x}+\underline{y}))| \gg |(-u(\underline{x}+\underline{y}))| = |(\underline{x}+\underline{y})|; \mathrm{eqTransitivity} \rhd |((-u\underline{x})+(-u\underline{y}))| = |(-u(\underline{x}+\underline{y}))| \rhd |(-u(\underline{x}+\underline{y}))| = |(\underline{x}+\underline{y})| \gg |((-u\underline{x})+(-u\underline{y}))| = |(\underline{x}+\underline{y})|; \mathrm{subLeqRight} \rhd (|(-u\underline{x})|+|(-u\underline{y})|) = (|\underline{x}|+|\underline{y}|) \rhd |((-u\underline{x})+(-u\underline{y}))| <= (|(-u\underline{x})|+|(-u\underline{y})|) \gg |((-u\underline{x})+(-u\underline{y}))| <= (|\underline{x}|+|\underline{y}|); \mathrm{subLeqLeft} \rhd |((-u\underline{x})+(-u\underline{y}))| = |(\underline{x}+\underline{y})| \rhd |((-u\underline{x})+(-u\underline{y}))| <= (|\underline{x}|+|\underline{y}|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, p_0, c)]$

$[\mathrm{SplitNumericalSumHelper} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |((-u\underline{x})+(-u\underline{y}))| <= (|(-u\underline{x})|+|(-u\underline{y})|) \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

$[\mathrm{SplitNumericalSumHelper} \overset{\mathrm{tex}}{\to}$ "SplitNumericalSumHelper"]

$[\mathrm{SplitNumericalSumHelper} \overset{\mathrm{pyk}}{\to}$ "lemma splitNumericalSumHelper"]

splitNumericalSum(++)

$[\mathrm{splitNumericalSum}(++) \overset{\mathrm{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash \mathrm{AddEquations(Leq)} \rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg (0+0) <= (\underline{x}+\underline{y}); \mathrm{plus0} \gg (0+0) = 0; \mathrm{subLeqLeft} \rhd (0+0) = 0 \rhd (0+0) <= (\underline{x}+\underline{y}) \gg 0 <= (\underline{x}+\underline{y}); \mathrm{NonnegativeNumerical} \rhd 0 <= (\underline{x}+\underline{y}) \gg |(\underline{x}+\underline{y})| = (\underline{x}+\underline{y}); \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \mathrm{AddEquations} \rhd |\underline{x}| = \underline{x} \rhd |\underline{y}| = \underline{y} \gg (|\underline{x}|+|\underline{y}|) = (\underline{x}+\underline{y}); \mathrm{eqSymmetry} \rhd (|\underline{x}|+|\underline{y}|) = (\underline{x}+\underline{y}) \gg (\underline{x}+\underline{y}) = (|\underline{x}|+|\underline{y}|); \mathrm{eqTransitivity} \rhd |(\underline{x}+\underline{y})| = (\underline{x}+\underline{y}) \rhd (\underline{x}+\underline{y}) = (|\underline{x}|+|\underline{y}|) \gg |(\underline{x}+\underline{y})| = (|\underline{x}|+|\underline{y}|); \mathrm{eqLeq} \rhd |(\underline{x}+\underline{y})| = (|\underline{x}|+|\underline{y}|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, p_0, c)]$

$[\mathrm{splitNumericalSum}(++) \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

$[\mathrm{splitNumericalSum}(++) \overset{\mathrm{tex}}{\to}$ "splitNumericalSum(++)"]

$[\mathrm{splitNumericalSum}(++) \overset{\mathrm{pyk}}{\to}$ "lemma splitNumericalSum(++)"]

splitNumericalSum(−−)

$[\mathrm{splitNumericalSum}(--) \overset{\mathrm{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash \mathrm{NonpositiveNegated} \rhd \underline{x} <= 0 \gg 0 <= (-u\underline{x}); \mathrm{NonpositiveNegated} \rhd \underline{y} <= 0 \gg 0 <= (-u\underline{y}); \mathrm{splitNumericalSum}(++) \rhd 0 <= (-u\underline{x}) \rhd 0 <= (-u\underline{y}) \gg |((-u\underline{x})+(-u\underline{y}))| <= (|(-u\underline{x})|+|(-u\underline{y})|); \mathrm{SplitNumericalSumHelper} \rhd |((-u\underline{x})+(-u\underline{y}))| <= (|(-u\underline{x})|+|(-u\underline{y})|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, p_0, c)]$

$[\mathrm{splitNumericalSum}(--) \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

[splitNumericalSum(−−) $\stackrel{\text{tex}}{\rightarrow}$ “splitNumericalSum(--)”]

[splitNumericalSum(−−) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSum(--)”]

splitNumericalSum(+ − small)

[splitNumericalSum(+ − small) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash \underline{\mathsf{y}} <= 0 \vdash |\underline{\mathsf{y}}| <= |\underline{\mathsf{x}}| \vdash \text{LeqAdditionLeft} \rhd \underline{\mathsf{y}} <= 0 \gg (\underline{\mathsf{x}}+\underline{\mathsf{y}}) <= (\underline{\mathsf{x}}+0); \text{plus0} \gg (\underline{\mathsf{x}}+0) = \underline{\mathsf{x}}; \text{subLeqRight} \rhd (\underline{\mathsf{x}}+0) = \underline{\mathsf{x}} \rhd (\underline{\mathsf{x}}+\underline{\mathsf{y}}) <= (\underline{\mathsf{x}}+0) \gg (\underline{\mathsf{x}}+\underline{\mathsf{y}}) <= \underline{\mathsf{x}}; \text{PositiveToRight(Leq)(1term)} \rhd |\underline{\mathsf{y}}| <= |\underline{\mathsf{x}}| \gg 0 <= (|\underline{\mathsf{x}}| + (-\mathrm{u}|\underline{\mathsf{y}}|)); \text{lemma nonpositiveNumerical} \rhd \underline{\mathsf{y}} <= 0 \gg |\underline{\mathsf{y}}| = (-\mathrm{u}\underline{\mathsf{y}}); \text{EqNegated} \rhd |\underline{\mathsf{y}}| = (-\mathrm{u}\underline{\mathsf{y}}) \gg (-\mathrm{u}|\underline{\mathsf{y}}|) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})); \text{DoubleMinus} \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}}; \text{eqTransitivity} \rhd (-\mathrm{u}|\underline{\mathsf{y}}|) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) \rhd (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}} \gg (-\mathrm{u}|\underline{\mathsf{y}}|) = \underline{\mathsf{y}}; \text{NonnegativeNumerical} \rhd 0 <= \underline{\mathsf{x}} \gg |\underline{\mathsf{x}}| = \underline{\mathsf{x}}; \text{AddEquations} \rhd |\underline{\mathsf{x}}| = \underline{\mathsf{x}} \rhd (-\mathrm{u}|\underline{\mathsf{y}}|) = \underline{\mathsf{y}} \gg (|\underline{\mathsf{x}}| + (-\mathrm{u}|\underline{\mathsf{y}}|)) = (\underline{\mathsf{x}}+\underline{\mathsf{y}}); \text{subLeqRight} \rhd (|\underline{\mathsf{x}}| + (-\mathrm{u}|\underline{\mathsf{y}}|)) = (\underline{\mathsf{x}}+\underline{\mathsf{y}}) \rhd 0 <= (|\underline{\mathsf{x}}| + (-\mathrm{u}|\underline{\mathsf{y}}|)) \gg 0 <= (\underline{\mathsf{x}}+\underline{\mathsf{y}}); \text{NonnegativeNumerical} \rhd 0 <= (\underline{\mathsf{x}}+\underline{\mathsf{y}}) \gg |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| = (\underline{\mathsf{x}}+\underline{\mathsf{y}}); \text{eqSymmetry} \rhd |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| = (\underline{\mathsf{x}}+\underline{\mathsf{y}}) \gg (\underline{\mathsf{x}}+\underline{\mathsf{y}}) = |(\underline{\mathsf{x}}+\underline{\mathsf{y}})|; \text{eqSymmetry} \rhd |\underline{\mathsf{x}}| = \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = |\underline{\mathsf{x}}|; \text{subLeqLeft} \rhd (\underline{\mathsf{x}}+\underline{\mathsf{y}}) = |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| \rhd (\underline{\mathsf{x}}+\underline{\mathsf{y}}) <= \underline{\mathsf{x}} \gg |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| <= \underline{\mathsf{x}}; \text{subLeqRight} \rhd \underline{\mathsf{x}} = |\underline{\mathsf{x}}| \rhd |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| <= \underline{\mathsf{x}} \gg |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| <= |\underline{\mathsf{x}}|\rceil, \mathrm{p}_0, \mathsf{c})$]

[splitNumericalSum(+ − small) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash \underline{\mathsf{y}} <= 0 \vdash |\underline{\mathsf{y}}| <= |\underline{\mathsf{x}}| \vdash |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| <= |\underline{\mathsf{x}}|$]

[splitNumericalSum(+ − small) $\stackrel{\text{tex}}{\rightarrow}$ “splitNumericalSum(+-small)”]

[splitNumericalSum(+ − small) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSum(+-, smallNegative)”]

splitNumericalSum(+ − big)

[splitNumericalSum(+ − big) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash \underline{\mathsf{y}} <= 0 \vdash \dot{\neg}(|\underline{\mathsf{x}}| <= |\underline{\mathsf{y}}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{\mathsf{x}}| = |\underline{\mathsf{y}}|)\mathrm{n})\mathrm{n})\mathrm{n} \vdash \text{NonnegativeNegated} \rhd 0 <= \underline{\mathsf{x}} \gg (-\mathrm{u}\underline{\mathsf{x}}) <= 0; \text{NonpositiveNegated} \rhd \underline{\mathsf{y}} <= 0 \gg 0 <= (-\mathrm{u}\underline{\mathsf{y}}); \text{SignNumerical} \gg |\underline{\mathsf{x}}| = |(-\mathrm{u}\underline{\mathsf{x}})|; \text{SubLessLeft} \rhd |\underline{\mathsf{x}}| = |(-\mathrm{u}\underline{\mathsf{x}})| \rhd \dot{\neg}(|\underline{\mathsf{x}}| <= |\underline{\mathsf{y}}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{\mathsf{x}}| = |\underline{\mathsf{y}}|)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| <= |\underline{\mathsf{y}}| \Rightarrow \dot{\neg}(\dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| = |\underline{\mathsf{y}}|)\mathrm{n})\mathrm{n})\mathrm{n}; \text{SignNumerical} \gg |\underline{\mathsf{y}}| = |(-\mathrm{u}\underline{\mathsf{y}})|; \text{SubLessRight} \rhd |\underline{\mathsf{y}}| = |(-\mathrm{u}\underline{\mathsf{y}})| \rhd \dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| <= |\underline{\mathsf{y}}| \Rightarrow \dot{\neg}(\dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| = |\underline{\mathsf{y}}|)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| <= |(-\mathrm{u}\underline{\mathsf{y}})| \Rightarrow \dot{\neg}(\dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| = |(-\mathrm{u}\underline{\mathsf{y}})|)\mathrm{n})\mathrm{n})\mathrm{n}; \text{LessLeq} \rhd \dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| <= |(-\mathrm{u}\underline{\mathsf{y}})| \Rightarrow \dot{\neg}(\dot{\neg}(|(-\mathrm{u}\underline{\mathsf{x}})| = |(-\mathrm{u}\underline{\mathsf{y}})|)\mathrm{n})\mathrm{n})\mathrm{n} \gg |(-\mathrm{u}\underline{\mathsf{x}})| <= |(-\mathrm{u}\underline{\mathsf{y}})|; \text{splitNumericalSum}(+-\text{small}) \rhd 0 <= (-\mathrm{u}\underline{\mathsf{y}}) \rhd (-\mathrm{u}\underline{\mathsf{x}}) <= 0 \rhd |(-\mathrm{u}\underline{\mathsf{x}})| <= |(-\mathrm{u}\underline{\mathsf{y}})| \gg |((-\mathrm{u}\underline{\mathsf{y}}) + (-\mathrm{u}\underline{\mathsf{x}}))| <= |(-\mathrm{u}\underline{\mathsf{y}})|; \text{SignNumerical} \gg |(\underline{\mathsf{x}}+\underline{\mathsf{y}})| = |(-\mathrm{u}(\underline{\mathsf{x}}+\underline{\mathsf{y}}))|; -\mathrm{x} - \mathrm{y} = -(\mathrm{x}+\mathrm{y}) \gg ((-\mathrm{u}\underline{\mathsf{x}}) + (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}}+\underline{\mathsf{y}})); \text{plusCommutativity} \gg ((-\mathrm{u}\underline{\mathsf{x}}) + (-\mathrm{u}\underline{\mathsf{y}})) = ((-\mathrm{u}\underline{\mathsf{y}}) + (-\mathrm{u}\underline{\mathsf{x}})); \text{Equality} \rhd ((-\mathrm{u}\underline{\mathsf{x}}) + (-\mathrm{u}\underline{\mathsf{y}})) =$

$(-\mathrm{u}(\underline{x}+\underline{y})) \rhd ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})) = ((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x})) \gg (-\mathrm{u}(\underline{x}+\underline{y})) = ((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x})); \mathrm{SameNumerical} \rhd (-\mathrm{u}(\underline{x}+\underline{y})) = ((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x})) \gg |(-\mathrm{u}(\underline{x}+\underline{y}))| = |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))|; \mathrm{eqTransitivity} \rhd |(\underline{x}+\underline{y})| = |(-\mathrm{u}(\underline{x}+\underline{y}))| \rhd |(-\mathrm{u}(\underline{x}+\underline{y}))| = |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))| \gg |(\underline{x}+\underline{y})| = |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))|; \mathrm{eqSymmetry} \rhd |(\underline{x}+\underline{y})| = |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))| \gg |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))| = |(\underline{x}+\underline{y})|; \mathrm{eqSymmetry} \rhd |\underline{y}| = |(-\mathrm{u}\underline{y})| \gg |(-\mathrm{u}\underline{y})| = |\underline{y}|; \mathrm{subLeqLeft} \rhd |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))| = |(\underline{x}+\underline{y})| \rhd |((-\mathrm{u}\underline{y})+(-\mathrm{u}\underline{x}))| <= |(-\mathrm{u}\underline{y})| \gg |(\underline{x}+\underline{y})| <= |(-\mathrm{u}\underline{y})|; \mathrm{subLeqRight} \rhd |(-\mathrm{u}\underline{y})| = |\underline{y}| \rhd |(\underline{x}+\underline{y})| <= |(-\mathrm{u}\underline{y})| \gg |(\underline{x}+\underline{y})| <= |\underline{y}|\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{splitNumericalSum}(+-\mathrm{big}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n} \vdash |(\underline{x}+\underline{y})| <= |\underline{y}|]$

$[\mathrm{splitNumericalSum}(+-\mathrm{big}) \overset{\mathrm{tex}}{\rightarrow}$ "splitNumericalSum(+-big)"]

$[\mathrm{splitNumericalSum}(+-\mathrm{big}) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma splitNumericalSum(+-, bigNegative)"]

splitNumericalSum(+−)

$[\mathrm{splitNumericalSum}(+-) \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon |\underline{y}| <= |\underline{x}| \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \mathrm{splitNumericalSum}(+-\mathrm{small}) \rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \rhd |\underline{y}| <= |\underline{x}| \gg |(\underline{x}+\underline{y})| <= |\underline{x}|; 0 <= |\mathrm{x}| \gg 0 <= |\underline{y}|; \mathrm{LeqAdditionLeft} \rhd 0 <= |\underline{y}| \gg (|\underline{x}|+0) <= (|\underline{x}|+|\underline{y}|); \mathrm{plus0} \gg (|\underline{x}|+0) = |\underline{x}|; \mathrm{subLeqLeft} \rhd (|\underline{x}|+0) = |\underline{x}| \rhd (|\underline{x}|+0) <= (|\underline{x}|+|\underline{y}|) \gg |\underline{x}| <= (|\underline{x}|+|\underline{y}|); \mathrm{leqTransitivity} \rhd |(\underline{x}+\underline{y})| <= |\underline{x}| \rhd |\underline{x}| <= (|\underline{x}|+|\underline{y}|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}(|\underline{y}| <= |\underline{x}|)\mathrm{n} \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \mathrm{ToLess} \rhd \dot{\neg}(|\underline{y}| <= |\underline{x}|)\mathrm{n} \gg \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{splitNumericalSum}(+-\mathrm{big}) \rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \rhd \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n} \gg |(\underline{x}+\underline{y})| <= |\underline{y}|; 0 <= |\mathrm{x}| \gg 0 <= |\underline{x}|; \mathrm{leqAddition} \rhd 0 <= |\underline{x}| \gg (0+|\underline{y}|) <= (|\underline{x}|+|\underline{y}|); \mathrm{plus0Left} \gg (0+|\underline{y}|) = |\underline{y}|; \mathrm{subLeqLeft} \rhd (0+|\underline{y}|) = |\underline{y}| \rhd (0+|\underline{y}|) <= (|\underline{x}|+|\underline{y}|) \gg |\underline{y}| <= (|\underline{x}|+|\underline{y}|); \mathrm{leqTransitivity} \rhd |(\underline{x}+\underline{y})| <= |\underline{y}| \rhd |\underline{y}| <= (|\underline{x}|+|\underline{y}|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \forall\underline{x}\colon \forall\underline{y}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon \forall\underline{y}\colon |\underline{y}| <= |\underline{x}| \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg |\underline{y}| <= |\underline{x}| \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{Ded} \rhd \forall\underline{x}\colon \forall\underline{y}\colon \dot{\neg}(|\underline{y}| <= |\underline{x}|)\mathrm{n} \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg \dot{\neg}(|\underline{y}| <= |\underline{x}|)\mathrm{n} \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \mathrm{FromNegations} \rhd |\underline{y}| <= |\underline{x}| \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd \dot{\neg}(|\underline{y}| <= |\underline{x}|)\mathrm{n} \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{MP2} \rhd 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{splitNumericalSum}(+-) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

$[\mathrm{splitNumericalSum}(+-) \overset{\mathrm{tex}}{\rightarrow}$ "splitNumericalSum(+-)"]

$[\mathrm{splitNumericalSum}(+-) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma splitNumericalSum(+-)"]

splitNumericalSum(−+)

$[\text{splitNumericalSum}(-+) \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash \text{NonpositiveNegated} \rhd \underline{x} <= 0 \gg 0 <= (-u\underline{x}); \text{NonnegativeNegated} \rhd 0 <= \underline{y} \gg (-u\underline{y}) <= 0; \text{splitNumericalSum}(+-) \rhd 0 <= (-u\underline{x}) \rhd (-u\underline{y}) <= 0 \gg |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|); \text{SplitNumericalSumHelper} \rhd |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)\rceil, p_0, c)]$

$[\text{splitNumericalSum}(-+) \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)]$

$[\text{splitNumericalSum}(-+) \stackrel{\text{tex}}{\rightarrow}$ “splitNumericalSum(-+)”$]$

$[\text{splitNumericalSum}(-+) \stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSum(-+)”$]$

splitNumericalSum

$[\text{splitNumericalSum} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash \text{splitNumericalSum}(++) \rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \text{splitNumericalSum}(+-) \rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash \text{splitNumericalSum}(-+) \rhd \underline{x} <= 0 \rhd 0 <= \underline{y} \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash \text{splitNumericalSum}(--) \rhd \underline{x} <= 0 \rhd \underline{y} <= 0 \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \text{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg 0 <= \underline{y} \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \text{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|); \text{FromLeqGeq} \rhd 0 <= \underline{y} \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \rhd \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)\rceil, p_0, c)]$

$[\text{splitNumericalSum} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)]$

$[\text{splitNumericalSum} \stackrel{\text{tex}}{\rightarrow}$ “splitNumericalSum”$]$

$[\text{splitNumericalSum} \stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSum”$]$

SplitNumericalProduct(++)

[SplitNumericalProduct(++) $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash 0 <= \underline{\mathsf{y}} \vdash \text{NonnegativeFactors} \rhd 0 <= \underline{\mathsf{x}} \rhd 0 <= \underline{\mathsf{y}} \gg 0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \text{NonnegativeNumerical} \rhd 0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \gg |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \text{NonnegativeNumerical} \rhd 0 <= \underline{\mathsf{x}} \gg |\underline{\mathsf{x}}| = \underline{\mathsf{x}}; \text{NonnegativeNumerical} \rhd 0 <= \underline{\mathsf{y}} \gg |\underline{\mathsf{y}}| = \underline{\mathsf{y}}; \text{MultiplyEquations} \rhd |\underline{\mathsf{x}}| = \underline{\mathsf{x}} \rhd |\underline{\mathsf{y}}| = \underline{\mathsf{y}} \gg (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \text{eqSymmetry} \rhd (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \gg (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|); \text{eqTransitivity} \rhd |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \rhd (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|) \gg |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|)\rceil, \mathrm{p}_0, \mathsf{c})$]

[SplitNumericalProduct(++) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash 0 <= \underline{\mathsf{y}} \vdash |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|)$]

[SplitNumericalProduct(++) $\stackrel{\text{tex}}{\rightarrow}$ "SplitNumericalProduct(++)"]

[SplitNumericalProduct(++) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma splitNumericalProduct(++)"]

SplitNumericalProduct(+−)

[SplitNumericalProduct(+−) $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash \underline{\mathsf{y}} <= 0 \vdash \text{SignNumerical} \gg |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = |(-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))|; \text{eqSymmetry} \rhd |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = |(-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))| \gg |(-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))| = |(\underline{\mathsf{x}} * \underline{\mathsf{y}})|; \text{PlusTimesMinus} \gg (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})); \text{SameNumerical} \rhd (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) \gg |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = |(-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))|; \text{eqTransitivity} \rhd |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = |(-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))| \rhd |(-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))| = |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| \gg |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = |(\underline{\mathsf{x}} * \underline{\mathsf{y}})|; \text{SignNumerical} \gg |\underline{\mathsf{y}}| = |(-\mathrm{u}\underline{\mathsf{y}})|; \text{eqSymmetry} \rhd |\underline{\mathsf{y}}| = |(-\mathrm{u}\underline{\mathsf{y}})| \gg |(-\mathrm{u}\underline{\mathsf{y}})| = |\underline{\mathsf{y}}|; \text{EqMultiplicationLeft} \rhd |(-\mathrm{u}\underline{\mathsf{y}})| = |\underline{\mathsf{y}}| \gg (|\underline{\mathsf{x}}| * |(-\mathrm{u}\underline{\mathsf{y}})|) = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|); \text{NonpositiveNegated} \rhd \underline{\mathsf{y}} <= 0 \gg 0 <= (-\mathrm{u}\underline{\mathsf{y}}); \text{SplitNumericalProduct}(++) \rhd 0 <= \underline{\mathsf{x}} \rhd 0 <= (-\mathrm{u}\underline{\mathsf{y}}) \gg |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = (|\underline{\mathsf{x}}| * |(-\mathrm{u}\underline{\mathsf{y}})|); \text{eqTransitivity} \rhd |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = (|\underline{\mathsf{x}}| * |(-\mathrm{u}\underline{\mathsf{y}})|) \rhd (|\underline{\mathsf{x}}| * |(-\mathrm{u}\underline{\mathsf{y}})|) = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|) \gg |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|); \text{Equality} \rhd |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| \rhd |(\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}}))| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|) \gg |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|)\rceil, \mathrm{p}_0, \mathsf{c})$]

[SplitNumericalProduct(+−) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash \underline{\mathsf{y}} <= 0 \vdash |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|)$]

[SplitNumericalProduct(+−) $\stackrel{\text{tex}}{\rightarrow}$ "SplitNumericalProduct(+-)"]

[SplitNumericalProduct(+−) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma splitNumericalProduct(+-)"]

SplitNumericalProduct

[SplitNumericalProduct $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash 0 <= \underline{\mathsf{y}} \vdash \text{SplitNumericalProduct}(++) \rhd 0 <= \underline{\mathsf{x}} \rhd 0 <= \underline{\mathsf{y}} \gg |(\underline{\mathsf{x}} * \underline{\mathsf{y}})| = (|\underline{\mathsf{x}}| * |\underline{\mathsf{y}}|); \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash \underline{\mathsf{y}} <= 0 \vdash \text{SplitNumericalProduct}(+-) \rhd 0 <=$

$\underline{x} \rhd \underline{y} <= 0 \gg |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash$
$\text{SplitNumericalProduct}(+-) \rhd 0 <= \underline{y} \rhd \underline{x} <= 0 \gg |(\underline{y} \ast \underline{x})| =$
$(|\underline{y}| \ast |\underline{x}|); \text{timesCommutativity} \gg (\underline{x} \ast \underline{y}) = (\underline{y} \ast \underline{x}); \text{SameNumerical} \rhd (\underline{x} \ast \underline{y}) =$
$(\underline{y} \ast \underline{x}) \gg |(\underline{x} \ast \underline{y})| = |(\underline{y} \ast \underline{x})|; \text{timesCommutativity} \gg (|\underline{y}| \ast |\underline{x}|) =$
$(|\underline{x}| \ast |\underline{y}|); \text{eqTransitivity4} \rhd |(\underline{x} \ast \underline{y})| = |(\underline{y} \ast \underline{x})| \rhd |(\underline{y} \ast \underline{x})| =$
$(|\underline{y}| \ast |\underline{x}|) \rhd (|\underline{y}| \ast |\underline{x}|) = (|\underline{x}| \ast |\underline{y}|) \gg |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <=$
$0 \vdash \text{NonpositiveNegated} \rhd \underline{x} <= 0 \gg 0 <= (-\text{u}\underline{x}); \text{NonpositiveNegated} \rhd \underline{y} <=$
$0 \gg 0 <= (-\text{u}\underline{y}); \text{SplitNumericalProduct}(++) \rhd 0 <= (-\text{u}\underline{x}) \rhd 0 <= (-\text{u}\underline{y}) \gg$
$|((-\text{u}\underline{x}) \ast (-\text{u}\underline{y}))| = (|(-\text{u}\underline{x})| \ast |(-\text{u}\underline{y})|); \text{MinusTimesMinus} \gg ((-\text{u}\underline{x}) \ast (-\text{u}\underline{y})) =$
$(\underline{x} \ast \underline{y}); \text{SameNumerical} \rhd ((-\text{u}\underline{x}) \ast (-\text{u}\underline{y})) = (\underline{x} \ast \underline{y}) \gg |((-\text{u}\underline{x}) \ast (-\text{u}\underline{y}))| =$
$|(\underline{x} \ast \underline{y})|; \text{eqSymmetry} \rhd |((-\text{u}\underline{x}) \ast (-\text{u}\underline{y}))| = |(\underline{x} \ast \underline{y})| \gg |(\underline{x} \ast \underline{y})| =$
$|((-\text{u}\underline{x}) \ast (-\text{u}\underline{y}))|; \text{SignNumerical} \gg |\underline{x}| = |(-\text{u}\underline{x})|; \text{SignNumerical} \gg |\underline{y}| =$
$|(-\text{u}\underline{y})|; \text{MultiplyEquations} \rhd |\underline{x}| = |(-\text{u}\underline{x})| \rhd |\underline{y}| = |(-\text{u}\underline{y})| \gg (|\underline{x}| \ast |\underline{y}|) =$
$(|(-\text{u}\underline{x})| \ast |(-\text{u}\underline{y})|); \text{eqSymmetry} \rhd (|\underline{x}| \ast |\underline{y}|) = (|(-\text{u}\underline{x})| \ast |(-\text{u}\underline{y})|) \gg$
$(|(-\text{u}\underline{x})| \ast |(-\text{u}\underline{y})|) = (|\underline{x}| \ast |\underline{y}|); \text{eqTransitivity4} \rhd |(\underline{x} \ast \underline{y})| =$
$|((-\text{u}\underline{x}) \ast (-\text{u}\underline{y}))| \rhd |((-\text{u}\underline{x}) \ast (-\text{u}\underline{y}))| = (|(-\text{u}\underline{x})| \ast |(-\text{u}\underline{y})|) \rhd (|(-\text{u}\underline{x})| \ast |(-\text{u}\underline{y})|) =$
$(|\underline{x}| \ast |\underline{y}|) \gg |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x} \ast \underline{y})| =$
$(|\underline{x}| \ast |\underline{y}|) \gg 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash$
$\underline{y} <= 0 \vdash |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|) \gg 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} \ast \underline{y})| =$
$(|\underline{x}| \ast |\underline{y}|); \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|) \gg \underline{x} <= 0 \Rightarrow$
$0 <= \underline{y} \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash |(\underline{x} \ast \underline{y})| =$
$(|\underline{x}| \ast |\underline{y}|) \gg \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \text{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow$
$0 <= \underline{y} \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|) \gg$
$0 <= \underline{y} \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|); \text{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} \ast \underline{y})| =$
$(|\underline{x}| \ast |\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|) \gg \underline{y} <= 0 \Rightarrow |(\underline{x} \ast \underline{y})| =$
$(|\underline{x}| \ast |\underline{y}|); \text{FromLeqGeq} \rhd 0 <= \underline{y} \Rightarrow |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|) \rhd \underline{y} <= 0 \Rightarrow |(\underline{x} \ast \underline{y})| =$
$(|\underline{x}| \ast |\underline{y}|) \gg |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|)\rceil, \text{p}_0, \mathsf{c})]$

$[\text{SplitNumericalProduct} \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |(\underline{x} \ast \underline{y})| = (|\underline{x}| \ast |\underline{y}|)]$

$[\text{SplitNumericalProduct} \overset{\text{tex}}{\rightarrow}$ "SplitNumericalProduct"$]$

$[\text{SplitNumericalProduct} \overset{\text{pyk}}{\rightarrow}$ "lemma splitNumericalProduct"$]$

insertMiddleTerm(Numerical)

$[\text{insertMiddleTerm(Numerical)} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash$
$\forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{splitNumericalSum} \gg |((\underline{x} + (-\text{u}\underline{z})) + (\underline{z} + \underline{y}))| <=$
$(|(\underline{x} + (-\text{u}\underline{z}))| + |(\underline{z} + \underline{y})|); \text{insertMiddleTerm(Sum)} \gg (\underline{x} + \underline{y}) =$
$((\underline{x} + (-\text{u}\underline{z})) + (\underline{z} + \underline{y})); \text{SameNumerical} \rhd (\underline{x} + \underline{y}) = ((\underline{x} + (-\text{u}\underline{z})) + (\underline{z} + \underline{y})) \gg$
$|(\underline{x} + \underline{y})| = |((\underline{x} + (-\text{u}\underline{z})) + (\underline{z} + \underline{y}))|; \text{eqSymmetry} \rhd |(\underline{x} + \underline{y})| =$
$|((\underline{x} + (-\text{u}\underline{z})) + (\underline{z} + \underline{y}))| \gg |((\underline{x} + (-\text{u}\underline{z})) + (\underline{z} + \underline{y}))| =$
$|(\underline{x}+\underline{y})|; \text{subLeqLeft} \rhd |((\underline{x}+(-\text{u}\underline{z}))+(\underline{z}+\underline{y}))| = |(\underline{x}+\underline{y})| \rhd |((\underline{x}+(-\text{u}\underline{z}))+(\underline{z}+\underline{y}))| <=$
$(|(\underline{x} + (-\text{u}\underline{z}))| + |(\underline{z} + \underline{y})|) \gg |(\underline{x} + \underline{y})| <= (|(\underline{x} + (-\text{u}\underline{z}))| + |(\underline{z} + \underline{y})|)\rceil, \text{p}_0, \mathsf{c})]$

[insertMiddleTerm(Numerical) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon |(\underline{x}+\underline{y})| <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+\underline{y})|)$]

[insertMiddleTerm(Numerical) $\stackrel{\text{tex}}{\rightarrow}$ "insertMiddleTerm(Numerical)"]

[insertMiddleTerm(Numerical) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma insertMiddleTerm(Numerical)"]

insertTwoMiddleTerms(Numerical)

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon$ insertMiddleTerm(Numerical) $\gg |(\underline{x}+\underline{y})| <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+\underline{y})|)$; insertMiddleTerm(Numerical) $\gg |(\underline{z}+\underline{y})| <= (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|)$; LeqAdditionLeft $\rhd |(\underline{z}+\underline{y})| <= (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|) \gg (|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+\underline{y})|) <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|))$; leqTransitivity $\rhd |(\underline{x}+\underline{y})| <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+\underline{y})|) \rhd (|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+\underline{y})|) <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|)) \gg |(\underline{x}+\underline{y})| <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|))$; plusAssociativity $\gg ((|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+(-\mathrm{u}\underline{u}))|) + |(\underline{u}+\underline{y})|) = (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|))$; eqSymmetry $\rhd ((|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+(-\mathrm{u}\underline{u}))|) + |(\underline{u}+\underline{y})|) = (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|)) \gg (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|)) = ((|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+(-\mathrm{u}\underline{u}))|) + |(\underline{u}+\underline{y})|)$; subLeqRight $\rhd (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|)) = ((|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+(-\mathrm{u}\underline{u}))|) + |(\underline{u}+\underline{y})|) \rhd |(\underline{x}+\underline{y})| <= (|(\underline{x}+(-\mathrm{u}\underline{z}))| + (|(\underline{z}+(-\mathrm{u}\underline{u}))| + |(\underline{u}+\underline{y})|)) \gg |(\underline{x}+\underline{y})| <= ((|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+(-\mathrm{u}\underline{u}))|) + |(\underline{u}+\underline{y})|)\rceil, p_0, c)$]

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon |(\underline{x}+\underline{y})| <= ((|(\underline{x}+(-\mathrm{u}\underline{z}))| + |(\underline{z}+(-\mathrm{u}\underline{u}))|) + |(\underline{u}+\underline{y})|)$]

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{tex}}{\rightarrow}$ "insertTwoMiddleTerms(Numerical)"]

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma insertTwoMiddleTerms(Numerical)"]

Three2twoTerms

[Three2twoTerms $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{y}+\underline{z}) = \underline{u} \vdash$ EqAdditionLeft $\rhd (\underline{y}+\underline{z}) = \underline{u} \gg (\underline{x}+(\underline{y}+\underline{z})) = (\underline{x}+\underline{u})$; plusAssociativity $\gg ((\underline{x}+\underline{y})+\underline{z}) = (\underline{x}+(\underline{y}+\underline{z}))$; eqTransitivity $\rhd ((\underline{x}+\underline{y})+\underline{z}) = (\underline{x}+(\underline{y}+\underline{z})) \rhd (\underline{x}+(\underline{y}+\underline{z})) = (\underline{x}+\underline{u}) \gg ((\underline{x}+\underline{y})+\underline{z}) = (\underline{x}+\underline{u})\rceil, p_0, c)$]

[Three2twoTerms $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{y}+\underline{z}) = \underline{u} \vdash ((\underline{x}+\underline{y})+\underline{z}) = (\underline{x}+\underline{u})$]

[Three2twoTerms $\stackrel{\text{tex}}{\rightarrow}$ "Three2twoTerms"]

[Three2twoTerms $\stackrel{\text{pyk}}{\rightarrow}$ "lemma three2twoTerms"]

Three2threeTerms

[Three2threeTerms $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{plusCommutativity} \gg (\underline{y}+\underline{z}) = (\underline{z}+\underline{y}); \text{Three2twoTerms} \rhd (\underline{y}+\underline{z}) = (\underline{z}+\underline{y}) \gg ((\underline{x}+\underline{y})+\underline{z}) = (\underline{x}+(\underline{z}+\underline{y})); \text{plusAssociativity} \gg ((\underline{x}+\underline{z})+\underline{y}) = (\underline{x}+(\underline{z}+\underline{y})); \text{eqSymmetry} \rhd ((\underline{x}+\underline{z})+\underline{y}) = (\underline{x}+(\underline{z}+\underline{y})) \gg (\underline{x}+(\underline{z}+\underline{y})) = ((\underline{x}+\underline{z})+\underline{y}); \text{eqTransitivity} \rhd ((\underline{x}+\underline{y})+\underline{z}) = (\underline{x}+(\underline{z}+\underline{y})) \rhd (\underline{x}+(\underline{z}+\underline{y})) = ((\underline{x}+\underline{z})+\underline{y}) \gg ((\underline{x}+\underline{y})+\underline{z}) = ((\underline{x}+\underline{z})+\underline{y})\rceil, p_0, c)$]

[Three2threeTerms $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon ((\underline{x}+\underline{y})+\underline{z}) = ((\underline{x}+\underline{z})+\underline{y})$]

[Three2threeTerms $\stackrel{\text{tex}}{\rightarrow}$ "Three2threeTerms"]

[Three2threeTerms $\stackrel{\text{pyk}}{\rightarrow}$ "lemma three2threeTerms"]

Three2twoFactors

[Three2twoFactors $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{y}*\underline{z}) = \underline{u} \vdash \text{EqMultiplicationLeft} \rhd (\underline{y}*\underline{z}) = \underline{u} \gg (\underline{x}*(\underline{y}*\underline{z})) = (\underline{x}*\underline{u}); \text{timesAssociativity} \gg ((\underline{x}*\underline{y})*\underline{z}) = (\underline{x}*(\underline{y}*\underline{z})); \text{eqTransitivity} \rhd ((\underline{x}*\underline{y})*\underline{z}) = (\underline{x}*(\underline{y}*\underline{z})) \rhd (\underline{x}*(\underline{y}*\underline{z})) = (\underline{x}*\underline{u}) \gg ((\underline{x}*\underline{y})*\underline{z}) = (\underline{x}*\underline{u})\rceil, p_0, c)$]

[Three2twoFactors $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{y}*\underline{z}) = \underline{u} \vdash ((\underline{x}*\underline{y})*\underline{z}) = (\underline{x}*\underline{u})$]

[Three2twoFactors $\stackrel{\text{tex}}{\rightarrow}$ "Three2twoFactors"]

[Three2twoFactors $\stackrel{\text{pyk}}{\rightarrow}$ "lemma three2twoFactors"]

Three2threeFactors

[Three2threeFactors $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{timesCommutativity} \gg (\underline{y}*\underline{z}) = (\underline{z}*\underline{y}); \text{Three2twoFactors} \rhd (\underline{y}*\underline{z}) = (\underline{z}*\underline{y}) \gg ((\underline{x}*\underline{y})*\underline{z}) = (\underline{x}*(\underline{z}*\underline{y})); \text{timesAssociativity} \gg ((\underline{x}*\underline{z})*\underline{y}) = (\underline{x}*(\underline{z}*\underline{y})); \text{eqSymmetry} \rhd ((\underline{x}*\underline{z})*\underline{y}) = (\underline{x}*(\underline{z}*\underline{y})) \gg (\underline{x}*(\underline{z}*\underline{y})) = ((\underline{x}*\underline{z})*\underline{y}); \text{eqTransitivity} \rhd ((\underline{x}*\underline{y})*\underline{z}) = (\underline{x}*(\underline{z}*\underline{y})) \rhd (\underline{x}*(\underline{z}*\underline{y})) = ((\underline{x}*\underline{z})*\underline{y}) \gg ((\underline{x}*\underline{y})*\underline{z}) = ((\underline{x}*\underline{z})*\underline{y})\rceil, p_0, c)$]

[Three2threeFactors $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon ((\underline{x}*\underline{y})*\underline{z}) = ((\underline{x}*\underline{z})*\underline{y})$]

[Three2threeFactors $\stackrel{\text{tex}}{\rightarrow}$ "Three2threeFactors"]

[Three2threeFactors $\stackrel{\text{pyk}}{\rightarrow}$ "lemma three2threeFactors"]

Times(−1)

$[\mathrm{Times}(-1) \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \mathrm{Negative} \gg (1 + (-\mathrm{u}1)) = 0; \mathrm{plusCommutativity} \gg ((-\mathrm{u}1) + 1) = (1 + (-\mathrm{u}1)); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}1) + 1) = (1 + (-\mathrm{u}1)) \rhd (1 + (-\mathrm{u}1)) = 0 \gg ((-\mathrm{u}1) + 1) = 0; \mathrm{EqMultiplicationLeft} \rhd ((-\mathrm{u}1) + 1) = 0 \gg (\underline{x} * ((-\mathrm{u}1) + 1)) = (\underline{x} * 0); \mathrm{x} * 0 = 0 \gg (\underline{x} * 0) = 0; \mathrm{eqTransitivity} \rhd (\underline{x} * ((-\mathrm{u}1) + 1)) = (\underline{x} * 0) \rhd (\underline{x} * 0) = 0 \gg (\underline{x} * ((-\mathrm{u}1) + 1)) = 0; \mathrm{Distribution} \gg (\underline{x} * ((-\mathrm{u}1) + 1)) = ((\underline{x} * (-\mathrm{u}1)) + (\underline{x} * 1)); \mathrm{eqSymmetry} \rhd (\underline{x} * ((-\mathrm{u}1) + 1)) = ((\underline{x} * (-\mathrm{u}1)) + (\underline{x} * 1)) \gg ((\underline{x} * (-\mathrm{u}1)) + (\underline{x} * 1)) = (\underline{x} * ((-\mathrm{u}1) + 1)); \mathrm{eqTransitivity} \rhd ((\underline{x} * (-\mathrm{u}1)) + (\underline{x} * 1)) = (\underline{x} * ((-\mathrm{u}1) + 1)) \rhd (\underline{x} * ((-\mathrm{u}1) + 1)) = 0 \gg ((\underline{x} * (-\mathrm{u}1)) + (\underline{x} * 1)) = 0; \mathrm{PositiveToRight(Eq)} \rhd ((\underline{x} * (-\mathrm{u}1)) + (\underline{x} * 1)) = 0 \gg (\underline{x} * (-\mathrm{u}1)) = (0 + (-\mathrm{u}(\underline{x} * 1))); \mathrm{plus0Left} \gg (0 + (-\mathrm{u}(\underline{x} * 1))) = (-\mathrm{u}(\underline{x} * 1)); \mathrm{eqTransitivity} \rhd (\underline{x} * (-\mathrm{u}1)) = (0 + (-\mathrm{u}(\underline{x} * 1))) \rhd (0 + (-\mathrm{u}(\underline{x} * 1))) = (-\mathrm{u}(\underline{x} * 1)) \gg (\underline{x} * (-\mathrm{u}1)) = (-\mathrm{u}(\underline{x} * 1)); \mathrm{times1} \gg (\underline{x} * 1) = \underline{x}; \mathrm{EqNegated} \rhd (\underline{x} * 1) = \underline{x} \gg (-\mathrm{u}(\underline{x} * 1)) = (-\mathrm{u}\underline{x}); \mathrm{eqTransitivity} \rhd (\underline{x} * (-\mathrm{u}1)) = (-\mathrm{u}(\underline{x} * 1)) \rhd (-\mathrm{u}(\underline{x} * 1)) = (-\mathrm{u}\underline{x}) \gg (\underline{x} * (-\mathrm{u}1)) = (-\mathrm{u}\underline{x})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Times}(-1) \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon (\underline{x} * (-\mathrm{u}1)) = (-\mathrm{u}\underline{x})]$

[Times(−1) $\stackrel{\mathrm{tex}}{\to}$ “Times(-1)”]

[Times(−1) $\stackrel{\mathrm{pyk}}{\to}$ “lemma times(-1)”]

Times(−1)Left

$[\mathrm{Times}(-1)\mathrm{Left} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \mathrm{Times}(-1) \gg (\underline{x} * (-\mathrm{u}1)) = (-\mathrm{u}\underline{x}); \mathrm{timesCommutativity} \gg ((-\mathrm{u}1) * \underline{x}) = (\underline{x} * (-\mathrm{u}1)); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}1) * \underline{x}) = (\underline{x} * (-\mathrm{u}1)) \rhd (\underline{x} * (-\mathrm{u}1)) = (-\mathrm{u}\underline{x}) \gg ((-\mathrm{u}1) * \underline{x}) = (-\mathrm{u}\underline{x})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Times}(-1)\mathrm{Left} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon ((-\mathrm{u}1) * \underline{x}) = (-\mathrm{u}\underline{x})]$

[Times(−1)Left $\stackrel{\mathrm{tex}}{\to}$ “Times(-1)Left”]

[Times(−1)Left $\stackrel{\mathrm{pyk}}{\to}$ “lemma times(-1)Left”]

MaxLeq(1)

$[\mathrm{MaxLeq}(1) \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \mathrm{FromMax}(1) \rhd \underline{y} <= \underline{x} \gg \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x}; \mathrm{eqSymmetry} \rhd \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x} \gg \underline{x} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{eqLeq} \rhd \underline{x} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \vdash \mathrm{FromMax}(2) \rhd \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \gg \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y}; \mathrm{eqSymmetry} \rhd \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y} \gg \underline{y} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{ToLess} \rhd \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessLeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{x} <=$

$\underline{y};\mathrm{subLeqRight} \rhd \underline{y} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \underline{x} <= \underline{y} \gg \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <= \underline{x} \Rightarrow \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \vdash \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \Rightarrow \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{FromNegations} \rhd \underline{y} <= \underline{x} \Rightarrow \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \Rightarrow \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y})\rceil, \mathrm{p}_0, \mathsf{c})]$

[MaxLeq(1) $\overset{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$]

[MaxLeq(1) $\overset{\mathrm{tex}}{\rightarrow}$ “MaxLeq(1)”]

[MaxLeq(1) $\overset{\mathrm{pyk}}{\rightarrow}$ “lemma leqMax1”]

MaxLeq(2)

[MaxLeq(2) $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \mathrm{FromMax}(1) \rhd \underline{y} <= \underline{x} \gg \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x}; \mathrm{eqSymmetry} \rhd \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x} \gg \underline{x} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{subLeqRight} \rhd \underline{x} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \underline{y} <= \underline{x} \gg \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \vdash \mathrm{FromMax}(2) \rhd \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \gg \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y}; \mathrm{eqSymmetry} \rhd \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y} \gg \underline{y} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{eqLeq} \rhd \underline{y} = \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <= \underline{x} \Rightarrow \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \vdash \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \Rightarrow \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}); \mathrm{FromNegations} \rhd \underline{y} <= \underline{x} \Rightarrow \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n} \Rightarrow \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y})\rceil, \mathrm{p}_0, \mathsf{c})$]

[MaxLeq(2) $\overset{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$]

[MaxLeq(2) $\overset{\mathrm{tex}}{\rightarrow}$ “MaxLeq(2)”]

[MaxLeq(2) $\overset{\mathrm{pyk}}{\rightarrow}$ “lemma leqMax2”]

LessThanMax

[LessThanMax $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{a} \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{a} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MaxLeq}(1) \gg \underline{y} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}); \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{y} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \gg \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{a})\mathrm{n} \vdash \mathrm{MP} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\underline{a})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MaxLeq}(2) \gg \underline{z} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}); \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{z} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \gg \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{a} \vdash \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{a} \Rightarrow \dot{\neg}(\underline{x} <=$

$\mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{a})n \vdash \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \Rightarrow \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \mathrm{MP} \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \mathrm{FromNegations} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \gg \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n\rceil, p_0, c)]$

$[\mathrm{LessThanMax} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n]$

[LessThanMax $\stackrel{\mathrm{tex}}{\rightarrow}$ "LessThanMax"]

[LessThanMax $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma lessThanMax"]

x + y = zBackwards

$[x + y = \mathrm{zBackwards} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = \underline{z} \vdash \mathrm{plusCommutativity} \gg (\underline{x} + \underline{y}) = (\underline{y} + \underline{x}); \mathrm{Equality} \rhd (\underline{x} + \underline{y}) = \underline{z} \gg \underline{z} = (\underline{y} + \underline{x})\rceil, p_0, c)]$

$[x + y = \mathrm{zBackwards} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = \underline{z} \vdash \underline{z} = (\underline{y} + \underline{x})]$

[x + y = zBackwards $\stackrel{\mathrm{tex}}{\rightarrow}$ "x+y=zBackwards"]

[x + y = zBackwards $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma x+y=zBackwards"]

x ∗ y = zBackwards

$[x * y = \mathrm{zBackwards} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} * \underline{y}) = \underline{z} \vdash \mathrm{timesCommutativity} \gg (\underline{x} * \underline{y}) = (\underline{y} * \underline{x}); \mathrm{Equality} \rhd (\underline{x} * \underline{y}) = \underline{z} \gg \underline{z} = (\underline{y} * \underline{x})\rceil, p_0, c)]$

$[x * y = \mathrm{zBackwards} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} * \underline{y}) = \underline{z} \vdash \underline{z} = (\underline{y} * \underline{x})]$

[x ∗ y = zBackwards $\stackrel{\mathrm{tex}}{\rightarrow}$ "x∗y=zBackwards"]

[x ∗ y = zBackwards $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma x∗y=zBackwards"]

$x = x + (y - y)$

$[x = x + (y - y) \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \text{plus0} \gg (\underline{\mathsf{x}} + 0) = \underline{\mathsf{x}}; \text{Negative} \gg (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) = 0; \text{eqSymmetry} \rhd (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) = 0 \gg 0 = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})); \text{EqAdditionLeft} \rhd 0 = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) \gg (\underline{\mathsf{x}} + 0) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))); \text{Equality} \rhd (\underline{\mathsf{x}} + 0) = \underline{\mathsf{x}} \rhd (\underline{\mathsf{x}} + 0) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))) \gg \underline{\mathsf{x}} = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})))\rceil, \text{p}_0, \mathsf{c})]$

$[x = x + (y - y) \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{x}} = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})))]$

$[x = x + (y - y) \overset{\text{tex}}{\to}$ "x=x+(y-y)"]

$[x = x + (y - y) \overset{\text{pyk}}{\to}$ "lemma x=x+(y-y)"]

$x = x + y - y$

$[x = x + y - y \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon x = x + (y - y) \gg \underline{\mathsf{x}} = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))); \text{plusAssociativity} \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))); \text{eqSymmetry} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))) \gg (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))) = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})); \text{eqTransitivity} \rhd \underline{\mathsf{x}} = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))) \rhd (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}}))) = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})) \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}}))\rceil, \text{p}_0, \mathsf{c})]$

$[x = x + y - y \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}}))]$

$[x = x + y - y \overset{\text{tex}}{\to}$ "x=x+y-y"]

$[x = x + y - y \overset{\text{pyk}}{\to}$ "lemma x=x+y-y"]

$x = x * y * (1/y)$

$[x = x * y * (1/y) \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \dot{\neg}\, (\underline{\mathsf{y}} = 0)\text{n} \vdash \text{times1} \gg (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}}; \text{Reciprocal} \rhd \dot{\neg}\, (\underline{\mathsf{y}} = 0)\text{n} \gg (\underline{\mathsf{y}} * \text{rec}\underline{\mathsf{y}}) = 1; \text{Three2twoFactors} \rhd (\underline{\mathsf{y}} * \text{rec}\underline{\mathsf{y}}) = 1 \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \text{rec}\underline{\mathsf{y}}) = (\underline{\mathsf{x}} * 1); \text{eqTransitivity} \rhd ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \text{rec}\underline{\mathsf{y}}) = (\underline{\mathsf{x}} * 1) \rhd (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}} \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \text{rec}\underline{\mathsf{y}}) = \underline{\mathsf{x}}; \text{eqSymmetry} \rhd ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \text{rec}\underline{\mathsf{y}}) = \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \text{rec}\underline{\mathsf{y}})\rceil, \text{p}_0, \mathsf{c})]$

$[x = x * y * (1/y) \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \dot{\neg}\, (\underline{\mathsf{y}} = 0)\text{n} \vdash \underline{\mathsf{x}} = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \text{rec}\underline{\mathsf{y}})]$

$[x = x * y * (1/y) \overset{\text{tex}}{\to}$ "x=x*y*(1/y)"]

$[x = x * y * (1/y) \overset{\text{pyk}}{\to}$ "lemma x=x*y*(1/y)"]

insertMiddleTerm(Sum)

[insertMiddleTerm(Sum) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{x} = ((\underline{x} + \underline{z}) + (-\mathrm{u}\underline{z})); \text{Three2threeTerms} \gg ((\underline{x} + \underline{z}) + (-\mathrm{u}\underline{z})) = ((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}); \text{eqTransitivity} \rhd \underline{x} = ((\underline{x} + \underline{z}) + (-\mathrm{u}\underline{z})) \rhd ((\underline{x} + \underline{z}) + (-\mathrm{u}\underline{z})) = ((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}) \gg \underline{x} = ((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}); \text{eqAddition} \rhd \underline{x} = ((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}) \gg (\underline{x} + \underline{y}) = (((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}) + \underline{y}); \text{plusAssociativity} \gg (((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}) + \underline{y}) = ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y})); \text{eqTransitivity} \rhd (\underline{x} + \underline{y}) = (((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}) + \underline{y}) \rhd (((\underline{x} + (-\mathrm{u}\underline{z})) + \underline{z}) + \underline{y}) = ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y})) \gg (\underline{x} + \underline{y}) = ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y}))\rceil, \mathrm{p}_0, c)$]

[insertMiddleTerm(Sum) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y}))$]

[insertMiddleTerm(Sum) $\stackrel{\text{tex}}{\rightarrow}$ "insertMiddleTerm(Sum)"]

[insertMiddleTerm(Sum) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma insertMiddleTerm(Sum)"]

insertTwoMiddleTerms(Sum)

[insertTwoMiddleTerms(Sum) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \text{insertMiddleTerm(Sum)} \gg (\underline{x} + \underline{y}) = ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y})); \text{insertMiddleTerm(Sum)} \gg (\underline{z} + \underline{y}) = ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y})); \text{EqAdditionLeft} \rhd (\underline{z} + \underline{y}) = ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y})) \gg ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y})) = ((\underline{x} + (-\mathrm{u}\underline{z})) + ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y}))); \text{plusAssociativity} \gg (((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + (-\mathrm{u}\underline{u}))) + (\underline{u} + \underline{y})) = ((\underline{x} + (-\mathrm{u}\underline{z})) + ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y}))); \text{eqSymmetry} \rhd (((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + (-\mathrm{u}\underline{u}))) + (\underline{u} + \underline{y})) = ((\underline{x} + (-\mathrm{u}\underline{z})) + ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y}))) \gg ((\underline{x} + (-\mathrm{u}\underline{z})) + ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y}))) = (((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + (-\mathrm{u}\underline{u}))) + (\underline{u} + \underline{y})); \text{eqTransitivity4} \rhd (\underline{x} + \underline{y}) = ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y})) \rhd ((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + \underline{y})) = ((\underline{x} + (-\mathrm{u}\underline{z})) + ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y}))) \rhd ((\underline{x} + (-\mathrm{u}\underline{z})) + ((\underline{z} + (-\mathrm{u}\underline{u})) + (\underline{u} + \underline{y}))) = (((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + (-\mathrm{u}\underline{u}))) + (\underline{u} + \underline{y})) \gg (\underline{x} + \underline{y}) = (((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + (-\mathrm{u}\underline{u}))) + (\underline{u} + \underline{y}))\rceil, \mathrm{p}_0, c)$]

[insertTwoMiddleTerms(Sum) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon (\underline{x} + \underline{y}) = (((\underline{x} + (-\mathrm{u}\underline{z})) + (\underline{z} + (-\mathrm{u}\underline{u}))) + (\underline{u} + \underline{y}))$]

[insertTwoMiddleTerms(Sum) $\stackrel{\text{tex}}{\rightarrow}$ "insertTwoMiddleTerms(Sum)"]

[insertTwoMiddleTerms(Sum) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma insertTwoMiddleTerms(Sum)"]

insertMiddleTerm(Difference)

[insertMiddleTerm(Difference) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{insertMiddleTerm(Sum)} \gg (\underline{x} + (-\mathrm{u}\underline{y})) =$

$((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))); \mathrm{DoubleMinus} \gg (-u(-u\underline{z})) = \underline{z}; \mathrm{EqAdditionLeft} \rhd (-u(-u\underline{z})) = \underline{z} \gg (\underline{x} + (-u(-u\underline{z}))) = (\underline{x} + \underline{z}); \mathrm{plusCommutativity} \gg ((-u\underline{z}) + (-u\underline{y})) = ((-u\underline{y}) + (-u\underline{z})); -x - y = -(x + y) \gg ((-u\underline{y}) + (-u\underline{z})) = (-u(\underline{y} + \underline{z})); \mathrm{eqTransitivity} \rhd ((-u\underline{z}) + (-u\underline{y})) = ((-u\underline{y}) + (-u\underline{z})) \rhd ((-u\underline{y}) + (-u\underline{z})) = (-u(\underline{y} + \underline{z})) \gg ((-u\underline{z}) + (-u\underline{y})) = (-u(\underline{y} + \underline{z})); \mathrm{AddEquations} \rhd (\underline{x} + (-u(-u\underline{z}))) = (\underline{x} + \underline{z}) \rhd ((-u\underline{z}) + (-u\underline{y})) = (-u(\underline{y} + \underline{z})) \gg ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z}))); \mathrm{eqTransitivity} \rhd (\underline{x} + (-u\underline{y})) = ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))) \rhd ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z}))) \gg (\underline{x} + (-u\underline{y})) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z})))\rceil, p_0, \mathsf{c})]$

$[\mathrm{insertMiddleTerm(Difference)} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon (\underline{x} + (-u\underline{y})) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z})))]$

$[\mathrm{insertMiddleTerm(Difference)} \stackrel{\mathrm{tex}}{\to}$ "insertMiddleTerm(Difference)"$]$

$[\mathrm{insertMiddleTerm(Difference)} \stackrel{\mathrm{pyk}}{\to}$ "lemma insertMiddleTerm(Difference)"$]$

$x * 0 + x = x$

$[x * 0 + x = x \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \mathrm{times1} \gg (\underline{x} * 1) = \underline{x}; \mathrm{eqSymmetry} \rhd (\underline{x} * 1) = \underline{x} \gg \underline{x} = (\underline{x} * 1); \mathrm{EqAdditionLeft} \rhd \underline{x} = (\underline{x} * 1) \gg ((\underline{x} * 0) + \underline{x}) = ((\underline{x} * 0) + (\underline{x} * 1)); \mathrm{Distribution} \gg (\underline{x} * (0 + 1)) = ((\underline{x}*0)+(\underline{x}*1)); \mathrm{eqSymmetry} \rhd (\underline{x}*(0+1)) = ((\underline{x}*0)+(\underline{x}*1)) \gg ((\underline{x}*0)+(\underline{x}*1)) = (\underline{x} * (0 + 1)); \mathrm{plus0Left} \gg (0 + 1) = 1; \mathrm{EqMultiplicationLeft} \rhd (0 + 1) = 1 \gg (\underline{x}*(0+1)) = (\underline{x}*1); \mathrm{eqTransitivity5} \rhd ((\underline{x}*0)+\underline{x}) = ((\underline{x}*0)+(\underline{x}*1)) \rhd ((\underline{x}*0)+(\underline{x}*1)) = (\underline{x}*(0+1)) \rhd (\underline{x}*(0+1)) = (\underline{x}*1) \rhd (\underline{x}*1) = \underline{x} \gg ((\underline{x}*0)+\underline{x}) = \underline{x}\rceil, p_0, \mathsf{c})]$

$[x * 0 + x = x \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x} \colon ((\underline{x} * 0) + \underline{x}) = \underline{x}]$

$[x * 0 + x = x \stackrel{\mathrm{tex}}{\to}$ "x∗0+x=x"$]$

$[x * 0 + x = x \stackrel{\mathrm{pyk}}{\to}$ "lemma x∗0+x=x"$]$

$x * 0 = 0$

$[x * 0 = 0 \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon x = x + (y - y) \gg (\underline{x} * 0) = ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))); \mathrm{plusAssociativity} \gg (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))); \mathrm{eqSymmetry} \rhd (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = ((\underline{x}*0)+(\underline{x}+(-u\underline{x}))) \gg ((\underline{x}*0)+(\underline{x}+(-u\underline{x}))) = (((\underline{x}*0)+\underline{x})+(-u\underline{x})); x*0+x = x \gg ((\underline{x} * 0) + \underline{x}) = \underline{x}; \mathrm{eqAddition} \rhd ((\underline{x} * 0) + \underline{x}) = \underline{x} \gg (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = (\underline{x} + (-u\underline{x})); \mathrm{Negative} \gg (\underline{x} + (-u\underline{x})) = 0; \mathrm{eqTransitivity5} \rhd (\underline{x} * 0) = ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))) \rhd ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))) = (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) \rhd (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = (\underline{x} + (-u\underline{x})) \rhd (\underline{x} + (-u\underline{x})) = 0 \gg (\underline{x} * 0) = 0\rceil, p_0, \mathsf{c})]$

$[\mathrm{x} * 0 = 0 \stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon (\underline{\mathsf{x}} * 0) = 0]$

$[\mathrm{x} * 0 = 0 \stackrel{\text{tex}}{\rightarrow} \text{“x*0=0”}]$

$[\mathrm{x} * 0 = 0 \stackrel{\text{pyk}}{\rightarrow} \text{“lemma x*0=0”}]$

NonnegativeFactors

$[\mathrm{NonnegativeFactors} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash 0 <= \underline{\mathsf{y}} \vdash \mathrm{leqMultiplication} \rhd 0 <= \underline{\mathsf{y}} \rhd 0 <= \underline{\mathsf{x}} \gg (0 * \underline{\mathsf{y}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \mathrm{timesCommutativity} \gg (0 * \underline{\mathsf{y}}) = (\underline{\mathsf{y}} * 0); \mathrm{x} * 0 = 0 \gg (\underline{\mathsf{y}} * 0) = 0; \mathrm{eqTransitivity} \rhd (0 * \underline{\mathsf{y}}) = (\underline{\mathsf{y}} * 0) \rhd (\underline{\mathsf{y}} * 0) = 0 \gg (0 * \underline{\mathsf{y}}) = 0; \mathrm{subLeqLeft} \rhd (0 * \underline{\mathsf{y}}) = 0 \rhd (0 * \underline{\mathsf{y}}) <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \gg 0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NonnegativeFactors} \stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon 0 <= \underline{\mathsf{x}} \vdash 0 <= \underline{\mathsf{y}} \vdash 0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}})]$

$[\mathrm{NonnegativeFactors} \stackrel{\text{tex}}{\rightarrow} \text{“NonnegativeFactors”}]$

$[\mathrm{NonnegativeFactors} \stackrel{\text{pyk}}{\rightarrow} \text{“lemma nonnegativeFactors”}]$

NonzeroFactors

$[\mathrm{NonzeroFactors} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{y}} = 0)\mathrm{n} \vdash \mathrm{NeqMultiplication} \rhd \dot{\neg}(\underline{\mathsf{y}} = 0)\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (0 * \underline{\mathsf{y}}))\mathrm{n}; \mathrm{timesCommutativity} \gg (0 * \underline{\mathsf{y}}) = (\underline{\mathsf{y}} * 0); \mathrm{x} * 0 = 0 \gg (\underline{\mathsf{y}} * 0) = 0; \mathrm{eqTransitivity} \rhd (0 * \underline{\mathsf{y}}) = (\underline{\mathsf{y}} * 0) \rhd (\underline{\mathsf{y}} * 0) = 0 \gg (0 * \underline{\mathsf{y}}) = 0; \mathrm{SubNeqRight} \rhd (0 * \underline{\mathsf{y}}) = 0 \rhd \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (0 * \underline{\mathsf{y}}))\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NonzeroFactors} \stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{y}} = 0)\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n}]$

$[\mathrm{NonzeroFactors} \stackrel{\text{tex}}{\rightarrow} \text{“NonzeroFactors”}]$

$[\mathrm{NonzeroFactors} \stackrel{\text{pyk}}{\rightarrow} \text{“lemma nonzeroFactors”}]$

PositiveFactors

$[\mathrm{PositiveFactors} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{\mathsf{x}}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(0 <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{\mathsf{y}}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 =$

$\underline{y})n; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{y})n \gg \dot{\neg}(\underline{y} = 0)n; \mathrm{NonnegativeFactors} \rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg 0 <= (\underline{x} * \underline{y}); \mathrm{NonzeroFactors} \rhd \dot{\neg}(\underline{x} = 0)n \rhd \dot{\neg}(\underline{y} = 0)n \gg \dot{\neg}((\underline{x} * \underline{y}) = 0)n; \mathrm{NeqSymmetry} \rhd \dot{\neg}((\underline{x} * \underline{y}) = 0)n \gg \dot{\neg}(0 = (\underline{x} * \underline{y}))n; \mathrm{JoinConjuncts} \rhd 0 <= (\underline{x} * \underline{y}) \rhd \dot{\neg}(0 = (\underline{x} * \underline{y}))n \gg \dot{\neg}(0 <= (\underline{x} * \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{x} * \underline{y}))n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(0 <= (\underline{x} * \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{x} * \underline{y}))n)n)n \gg \dot{\neg}(0 <= (\underline{x} * \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{x} * \underline{y}))n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{PositiveFactors} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \dot{\neg}(0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{y})n)n)n \vdash \dot{\neg}(0 <= (\underline{x} * \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{x} * \underline{y}))n)n)n]$

$[\mathrm{PositiveFactors} \overset{\mathrm{tex}}{\rightarrow}$ "PositiveFactors"$]$

$[\mathrm{PositiveFactors} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma positiveFactors"$]$

PlusTimesMinus

$[\mathrm{PlusTimesMinus} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Times}(-1)\mathrm{Left} \gg ((-u1) * \underline{y}) = (-u\underline{y}); \mathrm{EqMultiplicationLeft} \rhd ((-u1) * \underline{y}) = (-u\underline{y}) \gg (\underline{x} * ((-u1) * \underline{y})) = (\underline{x} * (-u\underline{y})); \mathrm{timesAssociativity} \gg ((\underline{x} * (-u1)) * \underline{y}) = (\underline{x} * ((-u1) * \underline{y})); \mathrm{timesCommutativity} \gg (\underline{x} * (-u1)) = ((-u1) * \underline{x}); \mathrm{eqMultiplication} \rhd (\underline{x} * (-u1)) = ((-u1) * \underline{x}) \gg ((\underline{x} * (-u1)) * \underline{y}) = (((-u1) * \underline{x}) * \underline{y}); \mathrm{timesAssociativity} \gg (((-u1) * \underline{x}) * \underline{y}) = ((-u1) * (\underline{x} * \underline{y})); \mathrm{Times}(-1)\mathrm{Left} \gg ((-u1) * (\underline{x} * \underline{y})) = (-u(\underline{x} * \underline{y})); \mathrm{eqTransitivity4} \rhd ((\underline{x} * (-u1)) * \underline{y}) = (((-u1) * \underline{x}) * \underline{y}) \rhd (((-u1) * \underline{x}) * \underline{y}) = ((-u1) * (\underline{x} * \underline{y})) \rhd ((-u1) * (\underline{x} * \underline{y})) = (-u(\underline{x} * \underline{y})) \gg ((\underline{x} * (-u1)) * \underline{y}) = (-u(\underline{x} * \underline{y})); \mathrm{Equality} \rhd ((\underline{x} * (-u1)) * \underline{y}) = (-u(\underline{x} * \underline{y})) \rhd ((\underline{x} * (-u1)) * \underline{y}) = (\underline{x} * ((-u1) * \underline{y})) \gg (-u(\underline{x} * \underline{y})) = (\underline{x} * ((-u1) * \underline{y})); \mathrm{eqTransitivity} \rhd (-u(\underline{x} * \underline{y})) = (\underline{x} * ((-u1) * \underline{y})) \rhd (\underline{x} * ((-u1) * \underline{y})) = (\underline{x} * (-u\underline{y})) \gg (-u(\underline{x} * \underline{y})) = (\underline{x} * (-u\underline{y})); \mathrm{eqSymmetry} \rhd (-u(\underline{x} * \underline{y})) = (\underline{x} * (-u\underline{y})) \gg (\underline{x} * (-u\underline{y})) = (-u(\underline{x} * \underline{y}))\rceil, p_0, \mathsf{c})]$

$[\mathrm{PlusTimesMinus} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon (\underline{x} * (-u\underline{y})) = (-u(\underline{x} * \underline{y}))]$

$[\mathrm{PlusTimesMinus} \overset{\mathrm{tex}}{\rightarrow}$ "PlusTimesMinus"$]$

$[\mathrm{PlusTimesMinus} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma plusTimesMinus"$]$

MinusTimesMinus

$[\mathrm{MinusTimesMinus} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{DoubleMinus} \gg (-u(-u\underline{y})) = \underline{y}; \mathrm{Times}(-1)\mathrm{Left} \gg ((-u1) * (-u\underline{y})) = (-u(-u\underline{y})); \mathrm{eqTransitivity} \rhd ((-u1) * (-u\underline{y})) = (-u(-u\underline{y})) \rhd (-u(-u\underline{y})) = \underline{y} \gg ((-u1) * (-u\underline{y})) = \underline{y}; \mathrm{EqMultiplicationLeft} \rhd ((-u1) * (-u\underline{y})) = \underline{y} \gg (\underline{x} * ((-u1) * (-u\underline{y}))) = (\underline{x} * \underline{y}); \mathrm{Times}(-1) \gg (\underline{x} * (-u1)) = (-u\underline{x}); \mathrm{eqMultiplication} \rhd (\underline{x} * (-u1)) = (-u\underline{x}) \gg ((\underline{x} * (-u1)) * (-u\underline{y})) =$

$((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})); \mathrm{timesAssociativity} \gg ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))); \mathrm{Equality} \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) \gg ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) \rhd (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \gg ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * \underline{\mathsf{y}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * \underline{\mathsf{y}})]$

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{tex}}{\rightarrow}$ "MinusTimesMinus"$]$

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma minusTimesMinus"$]$

$(-1) * (-1) + (-1) * 1 = 0$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{DistributionOut} \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)); \mathrm{Negative} \gg (1 + (-\mathrm{u}1)) = 0; \mathrm{plusCommutativity} \gg ((-\mathrm{u}1) + 1) = (1 + (-\mathrm{u}1)); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}1) + 1) = (1 + (-\mathrm{u}1)) \rhd (1 + (-\mathrm{u}1)) = 0 \gg ((-\mathrm{u}1) + 1) = 0; \mathrm{EqMultiplicationLeft} \rhd ((-\mathrm{u}1) + 1) = 0 \gg ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)) = ((-\mathrm{u}1) * 0); \mathrm{x} * 0 = 0 \gg ((-\mathrm{u}1) * 0) = 0; \mathrm{eqTransitivity4} \rhd (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)) \rhd ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)) = ((-\mathrm{u}1) * 0) \rhd ((-\mathrm{u}1) * 0) = 0 \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = 0]$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{tex}}{\rightarrow}$ "(-1)*(-1)+(-1)*1=0"$]$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma (-1)*(-1)+(-1)*1=0"$]$

$(-1) * (-1) = 1$

$[(-1) * (-1) = 1 \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{x} = \mathrm{x} + (\mathrm{y} - \mathrm{y}) \gg ((-\mathrm{u}1) * (-\mathrm{u}1)) = (((-\mathrm{u}1) * (-\mathrm{u}1)) + (1 + (-\mathrm{u}1))); \mathrm{times1} \gg ((-\mathrm{u}1) * 1) = (-\mathrm{u}1); \mathrm{eqSymmetry} \rhd ((-\mathrm{u}1) * 1) = (-\mathrm{u}1) \gg (-\mathrm{u}1) = ((-\mathrm{u}1) * 1); \mathrm{EqAdditionLeft} \rhd (-\mathrm{u}1) = ((-\mathrm{u}1) * 1) \gg (1 + (-\mathrm{u}1)) = (1 + ((-\mathrm{u}1) * 1)); \mathrm{EqAdditionLeft} \rhd (1 + (-\mathrm{u}1)) = (1 + ((-\mathrm{u}1) * 1)) \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + (1 + (-\mathrm{u}1))) = (((-\mathrm{u}1) * (-\mathrm{u}1)) + (1 + ((-\mathrm{u}1) * 1))); \mathrm{plusCommutativity} \gg (1 + ((-\mathrm{u}1) * 1)) = (((-\mathrm{u}1) * 1) + 1); \mathrm{EqAdditionLeft} \rhd (1 + ((-\mathrm{u}1) * 1)) = (((-\mathrm{u}1) * 1) + 1) \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + (1 + ((-\mathrm{u}1) * 1))) = (((-\mathrm{u}1) * (-\mathrm{u}1)) + (((-\mathrm{u}1) * 1) + 1)); \mathrm{plusAssociativity} \gg ((((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) + 1) = (((-\mathrm{u}1) * (-\mathrm{u}1)) + (((-\mathrm{u}1) * 1) + 1)); \mathrm{eqSymmetry} \rhd ((((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) + 1) = (((-\mathrm{u}1) * (-\mathrm{u}1)) + (((-\mathrm{u}1) * 1) + 1)) \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + (((-\mathrm{u}1) * 1) + 1)) = ((((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) + 1); (-1) * (-1) + (-1) * 1 = 0 \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = 0; \mathrm{eqAddition} \rhd (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) =$

$0 \gg ((((-\mathrm{u1}) * (-\mathrm{u1})) + ((-\mathrm{u1}) * 1)) + 1) = (0 + 1); \mathrm{plus0Left} \gg (0 + 1) = 1; \mathrm{eqTransitivity5} \rhd ((-\mathrm{u1}) * (-\mathrm{u1})) = (((-\mathrm{u1}) * (-\mathrm{u1})) + (1 + (-\mathrm{u1}))) \rhd (((-\mathrm{u1}) * (-\mathrm{u1})) + (1 + (-\mathrm{u1}))) = (((-\mathrm{u1}) * (-\mathrm{u1})) + (1 + ((-\mathrm{u1}) * 1))) \rhd (((-\mathrm{u1}) * (-\mathrm{u1})) + (1 + ((-\mathrm{u1}) * 1))) = (((-\mathrm{u1}) * (-\mathrm{u1})) + (((-\mathrm{u1}) * 1) + 1)) \rhd (((-\mathrm{u1}) * (-\mathrm{u1})) + (((-\mathrm{u1}) * 1) + 1)) = ((((-\mathrm{u1}) * (-\mathrm{u1})) + ((-\mathrm{u1}) * 1)) + 1) \gg ((-\mathrm{u1}) * (-\mathrm{u1})) = ((((-\mathrm{u1}) * (-\mathrm{u1})) + ((-\mathrm{u1}) * 1)) + 1); \mathrm{eqTransitivity4} \rhd ((-\mathrm{u1}) * (-\mathrm{u1})) = ((((-\mathrm{u1}) * (-\mathrm{u1})) + ((-\mathrm{u1}) * 1)) + 1) \rhd ((((-\mathrm{u1}) * (-\mathrm{u1})) + ((-\mathrm{u1}) * 1)) + 1) = (0 + 1) \rhd (0 + 1) = 1 \gg ((-\mathrm{u1}) * (-\mathrm{u1})) = 1\rceil, \mathrm{p}_0, \mathsf{c})]$

$[(-1) * (-1) = 1 \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash ((-\mathrm{u1}) * (-\mathrm{u1})) = 1]$

$[(-1) * (-1) = 1 \stackrel{\mathrm{tex}}{\to}$ "(-1)*(-1)=1"$]$

$[(-1) * (-1) = 1 \stackrel{\mathrm{pyk}}{\to}$ "lemma (-1)*(-1)=1"$]$

0 < 1Helper

$[0 < 1\mathrm{Helper} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 1 <= 0 \vdash \mathrm{leqAddition} \rhd 1 <= 0 \gg (1 + (-\mathrm{u1})) <= (0 + (-\mathrm{u1})); \mathrm{Negative} \gg (1 + (-\mathrm{u1})) = 0; \mathrm{subLeqLeft} \rhd (1 + (-\mathrm{u1})) = 0 \rhd (1 + (-\mathrm{u1})) <= (0 + (-\mathrm{u1})) \gg 0 <= (0 + (-\mathrm{u1})); \mathrm{plus0Left} \gg (0 + (-\mathrm{u1})) = (-\mathrm{u1}); \mathrm{subLeqRight} \rhd (0 + (-\mathrm{u1})) = (-\mathrm{u1}) \rhd 0 <= (0 + (-\mathrm{u1})) \gg 0 <= (-\mathrm{u1}); \mathrm{leqMultiplication} \rhd 0 <= (-\mathrm{u1}) \rhd 0 <= (-\mathrm{u1}) \gg (0 * (-\mathrm{u1})) <= ((-\mathrm{u1}) * (-\mathrm{u1})); \mathrm{x} * 0 = 0 \gg ((-\mathrm{u1}) * 0) = 0; \mathrm{timesCommutativity} \gg (0 * (-\mathrm{u1})) = ((-\mathrm{u1}) * 0); \mathrm{eqTransitivity} \rhd (0 * (-\mathrm{u1})) = ((-\mathrm{u1}) * 0) \rhd ((-\mathrm{u1}) * 0) = 0 \gg (0 * (-\mathrm{u1})) = 0; \mathrm{subLeqLeft} \rhd (0 * (-\mathrm{u1})) = 0 \rhd (0 * (-\mathrm{u1})) <= ((-\mathrm{u1}) * (-\mathrm{u1})) \gg 0 <= ((-\mathrm{u1}) * (-\mathrm{u1})); (-1) * (-1) = 1 \gg ((-\mathrm{u1}) * (-\mathrm{u1})) = 1; \mathrm{subLeqRight} \rhd ((-\mathrm{u1}) * (-\mathrm{u1})) = 1 \rhd 0 <= ((-\mathrm{u1}) * (-\mathrm{u1})) \gg 0 <= 1; \mathrm{Ded} \rhd 1 <= 0 \vdash 0 <= 1 \gg 1 <= 0 \Rightarrow 0 <= 1\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 < 1\mathrm{Helper} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash 1 <= 0 \Rightarrow 0 <= 1]$

$[0 < 1\mathrm{Helper} \stackrel{\mathrm{tex}}{\to}$ "0<1Helper"$]$

$[0 < 1\mathrm{Helper} \stackrel{\mathrm{pyk}}{\to}$ "lemma 0<1Helper"$]$

0 < 1

$[0 < 1 \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{leqTotality} \gg \dot\neg (0 <= 1)\mathrm{n} \Rightarrow 1 <= 0; \mathrm{AutoImply} \gg 0 <= 1 \Rightarrow 0 <= 1; 0 < 1\mathrm{Helper} \gg 1 <= 0 \Rightarrow 0 <= 1; \mathrm{FromDisjuncts} \rhd \dot\neg (0 <= 1)\mathrm{n} \Rightarrow 1 <= 0 \rhd 0 <= 1 \Rightarrow 0 <= 1 \rhd 1 <= 0 \Rightarrow 0 <= 1 \gg 0 <= 1; 0\mathrm{not1} \gg \dot\neg (0 = 1)\mathrm{n}; \mathrm{JoinConjuncts} \rhd 0 <= 1 \rhd \dot\neg (0 = 1)\mathrm{n} \gg \dot\neg (0 <= 1 \Rightarrow \dot\neg (\dot\neg (0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 < 1 \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \dot\neg (0 <= 1 \Rightarrow \dot\neg (\dot\neg (0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0 < 1 \stackrel{\text{tex}}{\rightarrow} \text{“0<1”}]$

$[0 < 1 \stackrel{\text{pyk}}{\rightarrow} \text{“lemma 0<1”}]$

$0 < 2$

$[0 < 2 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessAddition} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0+1) <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}((0+1) = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plus0Left} \gg (0+1) = 1; \mathrm{SubLessLeft} \rhd (0+1) = 1 \rhd \dot{\neg}((0+1) <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}((0+1) = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(1 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessTransitivity} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(1 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 < 2 \stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0 < 2 \stackrel{\text{tex}}{\rightarrow} \text{“0<2”}]$

$[0 < 2 \stackrel{\text{pyk}}{\rightarrow} \text{“lemma 0<2”}]$

$0 < 3$

$[0 < 3 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0 < 2 \gg \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessLeq} \rhd \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= (1+1); \mathrm{Leq}+1 \rhd 0 <= (1+1) \gg \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 < 3 \stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0 < 3 \stackrel{\text{tex}}{\rightarrow} \text{“0<3”}]$

$[0 < 3 \stackrel{\text{pyk}}{\rightarrow} \text{“lemma 0<3”}]$

$0 < 1/2$

$[0 < 1/2 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0 < 2 \gg \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= (1+1); \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = (1+1))\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = (1+1))\mathrm{n} \gg \dot{\neg}((1+1) = 0)\mathrm{n}; 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} * 0 = 0 \gg ((1+1) * 0) = 0; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd ((1+1) * 0) = 0 \gg 0 = (0 * (1+1)); \mathrm{SubLessLeft} \rhd 0 = (0 * (1+1)) \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * (1+1)) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * (1+1)) = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Reciprocal} \rhd \dot{\neg}((1+1) = 0)\mathrm{n} \gg$

$((1+1)*\mathrm{rec}(1+1))=1; \mathrm{x}*\mathrm{y}=\mathrm{zBackwards} \rhd ((1+1)*\mathrm{rec}(1+1))=1 \gg 1=(\mathrm{rec}(1+1)*(1+1)); \mathrm{SubLessRight} \rhd 1=(\mathrm{rec}(1+1)*(1+1)) \rhd \dot{\neg}\,((0*(1+1)) <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,((0*(1+1))=1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((0*(1+1)) <= (\mathrm{rec}(1+1)*(1+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((0*(1+1))=(\mathrm{rec}(1+1)*(1+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessDivision} \rhd 0 <= (1+1) \rhd \dot{\neg}\,((0*(1+1)) <= (\mathrm{rec}(1+1)*(1+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((0*(1+1))=(\mathrm{rec}(1+1)*(1+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= \mathrm{rec}(1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0=\mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0<1/2 \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \dot{\neg}\,(0 <= \mathrm{rec}(1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0=\mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0<1/2 \overset{\mathrm{tex}}{\to}$ "0<1/2"]

$[0<1/2 \overset{\mathrm{pyk}}{\to}$ "lemma 0<1/2"]

$0<1/3$

$[0<1/3 \overset{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0<3 \gg \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0=((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveInverted} \rhd \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0=((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0=\mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0<1/3 \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \dot{\neg}\,(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0=\mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0<1/3 \overset{\mathrm{tex}}{\to}$ "0<1/3"]

$[0<1/3 \overset{\mathrm{pyk}}{\to}$ "lemma 0<1/3"]

TwoWholes

$[\mathrm{TwoWholes} \overset{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \mathrm{times1} \gg (\underline{\mathsf{x}}*1)=\underline{\mathsf{x}}; \mathrm{eqSymmetry} \gg \underline{\mathsf{x}}=(\underline{\mathsf{x}}*1); \mathrm{EqAdditionLeft} \rhd \underline{\mathsf{x}}=(\underline{\mathsf{x}}*1) \gg (\underline{\mathsf{x}}+\underline{\mathsf{x}})=(\underline{\mathsf{x}}+(\underline{\mathsf{x}}*1)); \mathrm{eqAddition} \rhd \underline{\mathsf{x}}=(\underline{\mathsf{x}}*1) \gg (\underline{\mathsf{x}}+(\underline{\mathsf{x}}*1))=((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1)); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{x}}+\underline{\mathsf{x}})=(\underline{\mathsf{x}}+(\underline{\mathsf{x}}*1)) \rhd (\underline{\mathsf{x}}+(\underline{\mathsf{x}}*1))=((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1)) \gg (\underline{\mathsf{x}}+\underline{\mathsf{x}})=((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1)); \mathrm{DistributionOut} \gg ((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1))=(\underline{\mathsf{x}}*(1+1)); \mathrm{Repetition} \rhd ((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1))=(\underline{\mathsf{x}}*(1+1)) \gg ((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1))=(\underline{\mathsf{x}}*(1+1)); \mathrm{timesCommutativity} \gg (\underline{\mathsf{x}}*(1+1))=((1+1)*\underline{\mathsf{x}}); \mathrm{eqTransitivity4} \rhd (\underline{\mathsf{x}}+\underline{\mathsf{x}})=((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1)) \rhd ((\underline{\mathsf{x}}*1)+(\underline{\mathsf{x}}*1))=(\underline{\mathsf{x}}*(1+1)) \rhd (\underline{\mathsf{x}}*(1+1))=((1+1)*\underline{\mathsf{x}}) \gg (\underline{\mathsf{x}}+\underline{\mathsf{x}})=((1+1)*\underline{\mathsf{x}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{TwoWholes} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon (\underline{\mathsf{x}}+\underline{\mathsf{x}})=((1+1)*\underline{\mathsf{x}})]$

$[\mathrm{TwoWholes} \overset{\mathrm{tex}}{\to}$ "TwoWholes"]

$[\mathrm{TwoWholes} \overset{\mathrm{pyk}}{\to}$ "lemma x+x=2*x"]

ThreeWholes

$[\mathrm{ThreeWholes} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \mathrm{TwoWholes} \gg (\underline{\mathsf{x}} + \underline{\mathsf{x}}) = ((1+1) * \underline{\mathsf{x}}); \mathrm{times1Left} \gg (1 * \underline{\mathsf{x}}) = \underline{\mathsf{x}}; \mathrm{eqSymmetry} \rhd (1 * \underline{\mathsf{x}}) = \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = (1 * \underline{\mathsf{x}}); \mathrm{AddEquations} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{x}}) = ((1+1) * \underline{\mathsf{x}}) \rhd \underline{\mathsf{x}} = (1 * \underline{\mathsf{x}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{x}}) + \underline{\mathsf{x}}) = (((1+1) * \underline{\mathsf{x}}) + (1 * \underline{\mathsf{x}})); \mathrm{DistributionOutLeft} \gg (((1+1) * \underline{\mathsf{x}}) + (1 * \underline{\mathsf{x}})) = (\underline{\mathsf{x}} * ((1+1)+1)); \mathrm{timesCommutativity} \gg (\underline{\mathsf{x}} * ((1+1)+1)) = (((1+1)+1) * \underline{\mathsf{x}}); \mathrm{eqTransitivity4} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{x}}) + \underline{\mathsf{x}}) = (((1+1) * \underline{\mathsf{x}}) + (1 * \underline{\mathsf{x}})) \rhd (((1+1) * \underline{\mathsf{x}}) + (1 * \underline{\mathsf{x}})) = (\underline{\mathsf{x}} * ((1+1)+1)) \rhd (\underline{\mathsf{x}} * ((1+1)+1)) = (((1+1)+1) * \underline{\mathsf{x}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{x}}) + \underline{\mathsf{x}}) = (((1+1)+1) * \underline{\mathsf{x}}); \mathrm{Repetition} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{x}}) + \underline{\mathsf{x}}) = (((1+1)+1) * \underline{\mathsf{x}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{x}}) + \underline{\mathsf{x}}) = (((1+1)+1) * \underline{\mathsf{x}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ThreeWholes} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon ((\underline{\mathsf{x}} + \underline{\mathsf{x}}) + \underline{\mathsf{x}}) = (((1+1)+1) * \underline{\mathsf{x}})]$

$[\mathrm{ThreeWholes} \stackrel{\mathrm{tex}}{\to}$ "ThreeWholes"$]$

$[\mathrm{ThreeWholes} \stackrel{\mathrm{pyk}}{\to}$ "lemma x+x+x=3*x"$]$

TwoHalves

$[\mathrm{TwoHalves} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon 0 < 2 \gg \dot\neg (0 <= (1+1) \Rightarrow \dot\neg (\dot\neg (0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessNeq} \rhd \dot\neg (0 <= (1+1) \Rightarrow \dot\neg (\dot\neg (0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg (0 = (1+1))\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot\neg (0 = (1+1))\mathrm{n} \gg \dot\neg ((1+1) = 0)\mathrm{n}; \mathrm{TwoWholes} \gg ((\mathrm{rec}(1+1) * \underline{\mathsf{x}}) + (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) = ((1+1) * (\mathrm{rec}(1+1) * \underline{\mathsf{x}})); \mathrm{timesAssociativity} \gg (((1+1) * \mathrm{rec}(1+1)) * \underline{\mathsf{x}}) = ((1+1) * (\mathrm{rec}(1+1) * \underline{\mathsf{x}})); \mathrm{eqSymmetry} \rhd (((1+1) * \mathrm{rec}(1+1)) * \underline{\mathsf{x}}) = ((1+1) * (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) \gg ((1+1) * (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) = (((1+1) * \mathrm{rec}(1+1)) * \underline{\mathsf{x}}); \mathrm{Reciprocal} \rhd \dot\neg ((1+1) = 0)\mathrm{n} \gg ((1+1) * \mathrm{rec}(1+1)) = 1; \mathrm{eqMultiplication} \rhd ((1+1) * \mathrm{rec}(1+1)) = 1 \gg (((1+1) * \mathrm{rec}(1+1)) * \underline{\mathsf{x}}) = (1 * \underline{\mathsf{x}}); \mathrm{times1Left} \gg (1 * \underline{\mathsf{x}}) = \underline{\mathsf{x}}; \mathrm{eqTransitivity5} \rhd ((\mathrm{rec}(1+1) * \underline{\mathsf{x}}) + (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) = ((1+1) * (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) \rhd ((1+1) * (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) = (((1+1) * \mathrm{rec}(1+1)) * \underline{\mathsf{x}}) \rhd (((1+1) * \mathrm{rec}(1+1)) * \underline{\mathsf{x}}) = (1 * \underline{\mathsf{x}}) \rhd (1 * \underline{\mathsf{x}}) = \underline{\mathsf{x}} \gg ((\mathrm{rec}(1+1) * \underline{\mathsf{x}}) + (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{TwoHalves} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon ((\mathrm{rec}(1+1) * \underline{\mathsf{x}}) + (\mathrm{rec}(1+1) * \underline{\mathsf{x}})) = \underline{\mathsf{x}}]$

$[\mathrm{TwoHalves} \stackrel{\mathrm{tex}}{\to}$ "TwoHalves"$]$

$[\mathrm{TwoHalves} \stackrel{\mathrm{pyk}}{\to}$ "lemma (1/2)x+(1/2)x=x"$]$

ThreeThirds

$[\mathrm{ThreeThirds} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon 0 < 3 \gg \dot\neg (0 <= ((1+1)+1) \Rightarrow \dot\neg (\dot\neg (0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveNonzero} \rhd \dot\neg (0 <= ((1+1)+1) \Rightarrow \dot\neg (\dot\neg (0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg (((1+1)+1) = 0)\mathrm{n}; \mathrm{ThreeWholes} \gg (((\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}})) + (\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}})) = (((1+1)+1) *$

$(\mathrm{rec}((1+1)+1)*\underline{x})); \mathrm{timesAssociativity} \gg ((((1+1)+1)*\mathrm{rec}((1+1)+1))*\underline{x}) = (((1+1)+1)*(\mathrm{rec}((1+1)+1)*\underline{x})); \mathrm{eqSymmetry} \rhd ((((1+1)+1)*\mathrm{rec}((1+1)+1))*\underline{x}) = (((1+1)+1)*(\mathrm{rec}((1+1)+1)*\underline{x})) \gg (((1+1)+1)*(\mathrm{rec}((1+1)+1)*\underline{x})) = ((((1+1)+1)*\mathrm{rec}((1+1)+1))*\underline{x}); \mathrm{Reciprocal} \rhd \dot{\neg}(((1+1)+1) = 0)\mathrm{n} \gg (((1+1)+1)*\mathrm{rec}((1+1)+1)) = 1; \mathrm{eqMultiplication} \rhd (((1+1)+1)*\mathrm{rec}((1+1)+1)) = 1 \gg ((((1+1)+1)*\mathrm{rec}((1+1)+1))*\underline{x}) = (1*\underline{x}); \mathrm{times1Left} \gg (1*\underline{x}) = \underline{x}; \mathrm{eqTransitivity5} \rhd (((\mathrm{rec}((1+1)+1)*\underline{x})+(\mathrm{rec}((1+1)+1)*\underline{x}))+(\mathrm{rec}((1+1)+1)*\underline{x})) = (((1+1)+1)*(\mathrm{rec}((1+1)+1)*\underline{x})) \rhd (((1+1)+1)*(\mathrm{rec}((1+1)+1)*\underline{x})) = ((((1+1)+1)*\mathrm{rec}((1+1)+1))*\underline{x}) \rhd ((((1+1)+1)*\mathrm{rec}((1+1)+1))*\underline{x}) = (1*\underline{x}) \rhd (1*\underline{x}) = \underline{x} \gg$
$(((\mathrm{rec}((1+1)+1)*\underline{x})+(\mathrm{rec}((1+1)+1)*\underline{x}))+(\mathrm{rec}((1+1)+1)*\underline{x})) = \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ThreeThirds} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{x}\colon (((\mathrm{rec}((1+1)+1)*\underline{x})+(\mathrm{rec}((1+1)+1)*\underline{x}))+(\mathrm{rec}((1+1)+1)*\underline{x})) = \underline{x}]$

$[\mathrm{ThreeThirds} \overset{\mathrm{tex}}{\rightarrow}$ "ThreeThirds"$]$

$[\mathrm{ThreeThirds} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma (1/3)x+(1/3)x+(1/3)x=x"$]$

$-\mathrm{x} - \mathrm{y} = -(\mathrm{x}+\mathrm{y})$

$[-\mathrm{x} - \mathrm{y} = -(\mathrm{x}+\mathrm{y}) \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon \mathrm{Times(-1)Left} \gg ((-\mathrm{u}1)*\underline{x}) = (-\mathrm{u}\underline{x}); \mathrm{Times(-1)Left} \gg ((-\mathrm{u}1)*\underline{y}) = (-\mathrm{u}\underline{y}); \mathrm{AddEquations} \rhd ((-\mathrm{u}1)*\underline{x}) = (-\mathrm{u}\underline{x}) \rhd ((-\mathrm{u}1)*\underline{y}) = (-\mathrm{u}\underline{y}) \gg (((-\mathrm{u}1)*\underline{x})+((-\mathrm{u}1)*\underline{y})) = ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})); \mathrm{eqSymmetry} \rhd (((-\mathrm{u}1)*\underline{x})+((-\mathrm{u}1)*\underline{y})) = ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})) \gg ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})) = (((-\mathrm{u}1)*\underline{x})+((-\mathrm{u}1)*\underline{y})); \mathrm{DistributionOut} \gg (((-\mathrm{u}1)*\underline{x})+((-\mathrm{u}1)*\underline{y})) = ((-\mathrm{u}1)*(\underline{x}+\underline{y})); \mathrm{Times(-1)Left} \gg ((-\mathrm{u}1)*(\underline{x}+\underline{y})) = (-\mathrm{u}(\underline{x}+\underline{y})); \mathrm{eqTransitivity4} \rhd ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})) = (((-\mathrm{u}1)*\underline{x})+((-\mathrm{u}1)*\underline{y})) \rhd (((-\mathrm{u}1)*\underline{x})+((-\mathrm{u}1)*\underline{y})) = ((-\mathrm{u}1)*(\underline{x}+\underline{y})) \rhd ((-\mathrm{u}1)*(\underline{x}+\underline{y})) = (-\mathrm{u}(\underline{x}+\underline{y})) \gg ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})) = (-\mathrm{u}(\underline{x}+\underline{y}))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[-\mathrm{x} - \mathrm{y} = -(\mathrm{x}+\mathrm{y}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon ((-\mathrm{u}\underline{x})+(-\mathrm{u}\underline{y})) = (-\mathrm{u}(\underline{x}+\underline{y}))]$

$[-\mathrm{x} - \mathrm{y} = -(\mathrm{x}+\mathrm{y}) \overset{\mathrm{tex}}{\rightarrow}$ "-x-y=-(x+y)"$]$

$[-\mathrm{x} - \mathrm{y} = -(\mathrm{x}+\mathrm{y}) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma -x-y=-(x+y)"$]$

$-\mathrm{x} * \mathrm{y} = -(\mathrm{x} * \mathrm{y})$

$[-\mathrm{x} * \mathrm{y} = -(\mathrm{x} * \mathrm{y}) \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon \mathrm{Times(-1)Left} \gg ((-\mathrm{u}1)*\underline{x}) = (-\mathrm{u}\underline{x}); \mathrm{eqMultiplication} \rhd ((-\mathrm{u}1)*\underline{x}) = (-\mathrm{u}\underline{x}) \gg (((-\mathrm{u}1)*\underline{x})*\underline{y}) = ((-\mathrm{u}\underline{x})*\underline{y}); \mathrm{eqSymmetry} \rhd (((-\mathrm{u}1)*\underline{x})*\underline{y}) = ((-\mathrm{u}\underline{x})*\underline{y}) \gg ((-\mathrm{u}\underline{x})*\underline{y}) = (((-\mathrm{u}1)*\underline{x})*\underline{y}); \mathrm{timesAssociativity} \gg (((-\mathrm{u}1)*\underline{x})*\underline{y}) = ((-\mathrm{u}1)*(\underline{x}*\underline{y})); \mathrm{Times(-1)Left} \gg ((-\mathrm{u}1)*(\underline{x}*\underline{y})) =$

$(-u(\underline{x} * \underline{y})); \mathrm{eqTransitivity4} \rhd ((-u\underline{x}) * \underline{y}) = (((-u1) * \underline{x}) * \underline{y}) \rhd (((-u1) * \underline{x}) * \underline{y}) = ((-u1) * (\underline{x} * \underline{y})) \rhd ((-u1) * (\underline{x} * \underline{y})) = (-u(\underline{x} * \underline{y})) \gg ((-u\underline{x}) * \underline{y}) = (-u(\underline{x} * \underline{y}))\rceil, p_0, \mathsf{c})]$

$[-x * y = -(x * y) \overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon ((-u\underline{x}) * \underline{y}) = (-u(\underline{x} * \underline{y}))]$

$[-x * y = -(x * y) \overset{\text{tex}}{\rightarrow}$ "-x*y=-(x*y)"]

$[-x * y = -(x * y) \overset{\text{pyk}}{\rightarrow}$ "lemma -x*y=-(x*y)"]

$-0 = 0$

$[-0 = 0 \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{Negative} \gg (0 + (-u0)) = 0; \mathrm{plus0} \gg (0+0) = 0; \mathrm{UniqueNegative} \rhd (0+(-u0)) = 0 \rhd (0+0) = 0 \gg (-u0) = 0\rceil, p_0, \mathsf{c})]$

$[-0 = 0 \overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash (-u0) = 0]$

$[-0 = 0 \overset{\text{tex}}{\rightarrow}$ "-0=0"]

$[-0 = 0 \overset{\text{pyk}}{\rightarrow}$ "lemma -0=0"]

SFsymmetry

[SFsymmetry $\overset{\text{tex}}{\rightarrow}$ "SFsymmetry"]

[SFsymmetry $\overset{\text{pyk}}{\rightarrow}$ "lemma sameFsymmetry"]

SFtransitivity

[SFtransitivity $\overset{\text{tex}}{\rightarrow}$ "SFtransitivity"]

[SFtransitivity $\overset{\text{pyk}}{\rightarrow}$ "lemma sameFtransitivity"]

f2R(Plus)

[f2R(Plus) $\overset{\text{tex}}{\rightarrow}$ "f2R(Plus)"]

[f2R(Plus) $\overset{\text{pyk}}{\rightarrow}$ "lemma f2R(Plus)"]

f2R(Times)

[f2R(Times) $\overset{\text{tex}}{\rightarrow}$ "f2R(Times)"] [f2R(Times) $\stackrel{\text{pyk}}{\to}$ "lemma f2R(Times)"]

<< TransitivityHelper(Q)

[<< TransitivityHelper(Q) $\stackrel{\text{tex}}{\to}$ "<<TransitivityHelper(Q)"]

[<< TransitivityHelper(Q) $\stackrel{\text{pyk}}{\to}$ "lemma <<TransitivityHelper(Q)"]

<< Transitivity

[<< Transitivity $\stackrel{\text{tex}}{\to}$ "<<Transitivity"]

[<< Transitivity $\stackrel{\text{pyk}}{\to}$ "lemma <<Transitivity"]

<<== Reflexivity

[<<== Reflexivity $\stackrel{\text{tex}}{\to}$ "<<==Reflexivity"]

[<<== Reflexivity $\stackrel{\text{pyk}}{\to}$ "lemma <<==Reflexivity"]

<<== AntisymmetryHelper(Q)

[<<== AntisymmetryHelper(Q) $\stackrel{\text{tex}}{\to}$ "<<==AntisymmetryHelper(Q)"]

[<<== AntisymmetryHelper(Q) $\stackrel{\text{pyk}}{\to}$ "lemma <<==AntisymmetryHelper(Q)"]

FromNot < f(Weak)(Helper)

[FromNot < f(Weak)(Helper) $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{\mathsf{m}} \colon \forall \underline{\mathsf{n}} \colon \forall \underline{(\epsilon)} \colon \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{FromNegatedAnd} \triangleright \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n} \triangleright \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n}; \mathrm{FromNegatedImply} \triangleright \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \triangleright \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{n}} <= \underline{\mathsf{m}}; \mathrm{SecondConjunct} \triangleright \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$

∸(∸((fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n ≫ ∸((fx)[m] <= ((fy)[m] + (−u(ϵ))))n; ToLess ⊳ ∸((fx)[m] <= ((fy)[m] + (−u(ϵ))))n ≫ ∸(((fy)[m] + (−u(ϵ))) <= (fx)[m] ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) = (fx)[m])n)n)n; JoinConjuncts ⊳ n <= m ⊳ ∸(((fy)[m] + (−u(ϵ))) <= (fx)[m] ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) = (fx)[m])n)n)n ≫ ∸(n <= m ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) <= (fx)[m] ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) = (fx)[m])n)n)n)n)n; ∀m: ∀n: ∀(ϵ): ∀(fx): ∀(fy): Ded ⊳ ∀m: ∀n: ∀(ϵ): ∀(fx): ∀(fy): ∸(∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⊢ ∸(n <= m ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) <= (fx)[m] ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) = (fx)[m])n)n)n)n)n ≫ ∸(∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) <= (fx)[m] ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) = (fx)[m])n)n)n)n)n⌉, p_0, c)]

[FromNot < f(Weak)(Helper) $\overset{\text{stmt}}{\to}$ SystemQ ⊢ ∀m: ∀n: ∀(ϵ): ∀(fx): ∀(fy): ∸(∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) <= (fx)[m] ⇒ ∸(∸(((fy)[m] + (−u(ϵ))) = (fx)[m])n)n)n)n)n]

[FromNot < f(Weak)(Helper) $\overset{\text{tex}}{\to}$ "FromNot<f(Weak)(Helper)"]

[FromNot < f(Weak)(Helper) $\overset{\text{pyk}}{\to}$ "lemma fromNot<f(Weak) helper"]

FromNot < f(Weak)

[FromNot < f(Weak) $\overset{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢ ∀m: ∀n: ∀(ϵ): ∀(fx): ∀(fy): ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n: ∸($\forall_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n)n)n)n ⊢ Ded ⊳ ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n: ∸($\forall_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n)n)n)n ≫ ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n̄: ∸($\forall_{obj}$m̄: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n̄ <= m̄ ⇒ (fx)[m̄] <= ((fy)[m̄] + (−u(ϵ))))n)n)n)n)n)n; Repetition ⊳ ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n̄: ∸($\forall_{obj}$m̄: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n̄ <= m̄ ⇒ (fx)[m̄] <= ((fy)[m̄] + (−u(ϵ))))n)n)n)n)n)n ≫ ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n̄: ∸($\forall_{obj}$m̄: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n̄ <= m̄ ⇒ (fx)[m̄] <= ((fy)[m̄] + (−u(ϵ))))n)n)n)n)n)n; ∀m: ∀n: ∀(ϵ): ∀(fx): ∀(fy): Ded ⊳ ∀m: ∀n: ∀(ϵ): ∀(fx): ∀(fy): ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n: ∸($\forall_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ))))n)n)n)n)n)n ⊢ ∸($\forall_{obj}$(ϵ): ∸(∸($\forall_{obj}$n̄: ∸($\forall_{obj}$m̄: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n̄ <= m̄ ⇒ (fx)[m̄] <=

$((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{MT} \rhd \dot\neg(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{EEA} - \mathrm{negated} \rhd$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ \underline{(\epsilon)} \rhd$
$\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ \underline{\mathsf{n}} \rhd$
$\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNot} < \mathrm{f(Weak)(Helper)} \gg \dot\neg(\dot\neg(\dot\neg(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) =$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistMP} \rhd \dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow$
$\dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) =$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @ \underline{\mathsf{m}} \rhd \dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow$
$\dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow$
$\dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <=$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot\neg(\dot\neg(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$

$\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNot} < \mathrm{f(Weak)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNot} < \mathrm{f(Weak)} \stackrel{\mathrm{tex}}{\rightarrow}$ "FromNot<f(Weak)"]

$[\mathrm{FromNot} < \mathrm{f(Weak)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromNot<f(Weak)"]

FromNot < f(Strong)(Helper2)

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon \forall\underline{\mathsf{u}}\colon \forall\underline{\mathsf{v}}\colon \dot{\neg}\,(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,((\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}})) = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{x} <= |\mathrm{x}| \gg (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))|; \mathrm{leqLessTransitivity} \rhd (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| \rhd \dot{\neg}\,(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NumericalDifference} \gg |(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| = |(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))|; \mathrm{SubLessLeft} \rhd |(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| = |(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))| \rhd \dot{\neg}\,(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(|(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} <= |\mathrm{x}| \gg (\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}})) <= |(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))|; \mathrm{leqLessTransitivity} \rhd (\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}})) <= |(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))| \rhd \dot{\neg}\,(|(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{u}} + (-\mathrm{u}\underline{\mathsf{z}})) = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddEquations(Less)} \rhd \dot{\neg}\,((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,((\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}})) = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) + (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}}))) <= ((\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) + (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) + (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}}))) = ((\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) + (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddEquations(Less)} \rhd \dot{\neg}\,(((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) + (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}}))) <=$

$((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u}))) = ((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}((\underline{u}+(-\mathrm{u}\underline{z})) <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{u}+(-\mathrm{u}\underline{z})) = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) <= (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) = (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{insertTwoMiddleTerms}(\mathrm{Sum}) \gg (\underline{x}+(-\mathrm{u}\underline{z})) = (((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))); \mathrm{eqSymmetry} \rhd (\underline{x}+(-\mathrm{u}\underline{z})) = (((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) \gg (((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) = (\underline{x}+(-\mathrm{u}\underline{z})); \mathrm{SubLessLeft} \rhd (((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) = (\underline{x}+(-\mathrm{u}\underline{z})) \rhd \dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) <= (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) = (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ThreeThirds} \gg (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) = \underline{v}; \mathrm{SubLessRight} \rhd (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) = \underline{v} \rhd \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \forall\underline{u}\colon \forall\underline{v}\colon \dot{\neg}(|(\underline{x}+(-\mathrm{u}\underline{y}))| <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-\mathrm{u}\underline{y}))| = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(|(\underline{z}+(-\mathrm{u}\underline{u}))| <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{z}+(-\mathrm{u}\underline{u}))| = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{y}+(-\mathrm{u}\underline{u})) <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-\mathrm{u}\underline{u})) = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{tex}}{\to}$ “FromNot<f(Strong)(Helper2)”]

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{pyk}}{\to}$ “lemma fromNot<f(Strong) helper2”]

FromNot < f(Strong)(Helper)

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper)} \stackrel{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{m}\colon \forall\underline{(\mathrm{n1})}\colon \forall\underline{(\mathrm{n2})}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{n1})} <= \underline{(\mathrm{n2})} \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]+(-\mathrm{u}(\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}))) <= \underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]+(-\mathrm{u}(\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}))) = \underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{v1})} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{v2})} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}]+(-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| <= (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}]+(-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| = (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}]+(-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| <= (\mathrm{rec}((1+1)+1)*\underline{(\epsilon)}) \Rightarrow$

$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n)n)n \vdash \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \vdash (n2) <= \mathsf{m} \vdash 0 < 3 \gg \dot{\neg}(0 <= ((1 + 1) + 1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1 + 1) + 1))n)n)n; \mathrm{PositiveInverted} \rhd \dot{\neg}(0 <= ((1 + 1) + 1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1 + 1) + 1))n)n)n \gg \dot{\neg}(0 <= rec((1 + 1) + 1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = rec((1 + 1) + 1))n)n)n; \mathrm{PositiveFactors} \rhd \dot{\neg}(0 <= rec((1 + 1) + 1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = rec((1+1)+1))n)n)n \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \gg \dot{\neg}(0 <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1 + 1) + 1) * (\epsilon)))n)n)n; \mathrm{MP} \rhd \dot{\neg}(0 <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1 + 1) + 1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}((n1) <= (n2) \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n \rhd \dot{\neg}(0 <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1 + 1) + 1) * (\epsilon)))n)n)n \gg \dot{\neg}((n1) <= (n2) \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n; \mathrm{FirstConjunct} \rhd \dot{\neg}((n1) <= (n2) \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1+1)+1)*(\epsilon)))) = (fx)[(n2)])n)n)n)n)n \gg (n1) <= (n2); \mathrm{SecondConjunct} \rhd \dot{\neg}((n1) <= (n2) \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n \gg \dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) = (fx)[(n2)])n)n)n; \mathrm{SwitchTerms}(x - y < z) \rhd \dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)] + (-u(rec((1 + 1) + 1) * (\epsilon)))) = (fx)[(n2)])n)n)n \gg \dot{\neg}(((fy)[(n2)] + (-u(fx)[(n2)])) <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(((fy)[(n2)]+(-u(fx)[(n2)])) = (rec((1+1)+1)*(\epsilon)))n)n)n; \mathrm{leqTransitivity} \rhd (n1) <= (n2) \rhd (n2) <= \mathsf{m} \gg (n1) <= \mathsf{m}; \mathrm{A4} @ \mathsf{m} \rhd \forall_{\mathrm{obj}}(v1)\colon \forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)]+(-u(fx)[(v2)]))| <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n)n)n \gg \forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= \mathsf{m} \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{m}] + (-u(fx)[(v2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{m}] + (-u(fx)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[\mathsf{m}] + (-u(fy)[(v2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[\mathsf{m}] + (-u(fy)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n)n)n; \mathrm{A4} @ (n2) \rhd \forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= \mathsf{m} \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{m}] + (-u(fx)[(v2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{m}] + (-u(fx)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[\mathsf{m}] + (-u(fy)[(v2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[\mathsf{m}] + (-u(fy)[(v2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n)n)n \gg \dot{\neg}(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= \mathsf{m} \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{m}] + (-u(fx)[(n2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{m}] + (-u(fx)[(n2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[\mathsf{m}] + (-u(fy)[(n2)]))| <= (rec((1 + 1) + 1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[\mathsf{m}] + (-u(fy)[(n2)]))| = (rec((1 + 1) + 1) * (\epsilon)))n)n)n)n)n; \mathrm{MP3} \rhd \dot{\neg}(0 <=$

$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= \mathsf{m} \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\mathrm{n1}) <=$
$\mathsf{m} \rhd (\mathrm{n1}) <= (\mathrm{n2}) \gg \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNot} <$
$\mathrm{f}(\mathrm{Strong})(\mathrm{Helper2}) \rhd \dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(|((\mathrm{fx})[\mathsf{m}] +$
$(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})])) <= (\mathrm{rec}((1+1)+$
$1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})])) = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[\mathsf{m}])) <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[\mathsf{m}])) =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \forall(\mathrm{v1})\colon \forall(\mathrm{v2})\colon \forall\mathsf{m}\colon \forall(\mathrm{n1})\colon \forall(\mathrm{n2})\colon \forall(\epsilon)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \mathrm{Ded} \rhd$
$\forall(\mathrm{v1})\colon \forall(\mathrm{v2})\colon \forall\mathsf{m}\colon \forall(\mathrm{n1})\colon \forall(\mathrm{n2})\colon \forall(\epsilon)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \dot\neg(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow$
$\dot\neg(\dot\neg(((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*(\epsilon)))) <= (\mathrm{fx})[(\mathrm{n2})] \Rightarrow \dot\neg(\dot\neg(((\mathrm{fy})[(\mathrm{n2})] +$
$(-\mathrm{u}(\mathrm{rec}((1+1)+1)*(\epsilon)))) = (\mathrm{fx})[(\mathrm{n2})])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}(\mathrm{v1})\colon \forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot\neg(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot\neg(0 <=$
$(\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \vdash (\mathrm{n2}) <= \mathsf{m} \vdash \dot\neg(((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[\mathsf{m}])) <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg(((\mathrm{fy})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fx})[\mathsf{m}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow$
$\dot\neg(\dot\neg(((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*(\epsilon)))) <= (\mathrm{fx})[(\mathrm{n2})] \Rightarrow \dot\neg(\dot\neg(((\mathrm{fy})[(\mathrm{n2})] +$
$(-\mathrm{u}(\mathrm{rec}((1+1)+1)*(\epsilon)))) = (\mathrm{fx})[(\mathrm{n2})])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1})\colon \forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot\neg(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\dot\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((fy)[(v1)]+(-u(fy)[(v2)]))| = (rec((1+1)+1)*(\epsilon)))n)n)n)n)n \Rightarrow \dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow (n2) <= \mathsf{m} \Rightarrow \dot\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) = (\epsilon))n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall(v1){:}\forall(v2){:}\forall\mathsf{m}{:}\forall(n1){:}\forall(n2){:}\forall(\epsilon){:}\forall(fx){:}\forall(fy){:}\dot\neg(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow \dot\neg((n1) <= (n2) \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)]+(-u(rec((1+1)+1)*(\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)]+(-u(rec((1+1)+1)*(\epsilon)))) = (fx)[(n2)])n)n)n)n)n \Rightarrow \forall_{obj}(v1){:}\forall_{obj}(v2){:}\dot\neg(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot\neg(\dot\neg(|((fx)[(v1)]+(-u(fx)[(v2)]))| <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow \dot\neg(\dot\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg(\dot\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1)*(\epsilon)))n)n)n)n)n \Rightarrow \dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow (n2) <= \mathsf{m} \Rightarrow \dot\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) = (\epsilon))n)n)n]$

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper)} \overset{\mathrm{tex}}{\rightarrow}$ "FromNot<f(Strong)(Helper)"]

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma fromNot<f(Strong) helper"]

FromNot < f(Strong)

$[\mathrm{FromNot} < \mathrm{f(Strong)} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall(v1){:}\forall(v2){:}\forall\mathsf{m}{:}\forall(n2){:}\forall(\epsilon){:}\forall(fx){:}\forall(fy){:}\dot\neg(\dot\neg(\forall_{obj}(\epsilon){:}\dot\neg(\dot\neg(\forall_{obj}\mathsf{n}{:}\dot\neg(\forall_{obj}\mathsf{m}{:}\dot\neg(\dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow \dot\neg(\mathsf{n} <= \mathsf{m} \Rightarrow (fx)[\mathsf{m}] <= ((fy)[\mathsf{m}] + (-u(\epsilon))))n)n)n)n)n)n)n \vdash \mathrm{FromNot} < \mathrm{f(Weak)} \rhd \dot\neg(\dot\neg(\forall_{obj}(\epsilon){:}\dot\neg(\dot\neg(\forall_{obj}\mathsf{n}{:}\dot\neg(\forall_{obj}\mathsf{m}{:}\dot\neg(\dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow \dot\neg(\mathsf{n} <= \mathsf{m} \Rightarrow (fx)[\mathsf{m}] <= ((fy)[\mathsf{m}] + (-u(\epsilon))))n)n)n)n)n)n)n \gg \forall_{obj}(\epsilon){:}\forall_{obj}\mathsf{n}{:}\dot\neg(\forall_{obj}(n2){:}\dot\neg(\dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow \dot\neg(\mathsf{n} <= (n2) \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(\epsilon))) <= (fx)[(n2)] \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(\epsilon))) = (fx)[(n2)])n)n)n)n)n)n)n; \mathrm{A4} @ (rec((1+1)+1) * (\epsilon)) \rhd \forall_{obj}(\epsilon){:}\forall_{obj}\mathsf{n}{:}\dot\neg(\forall_{obj}(n2){:}\dot\neg(\dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow \dot\neg(\mathsf{n} <= (n2) \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(\epsilon))) <= (fx)[(n2)] \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(\epsilon))) = (fx)[(n2)])n)n)n)n)n)n)n \gg \forall_{obj}\mathsf{n}{:}\dot\neg(\forall_{obj}(n2){:}\dot\neg(\dot\neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \dot\neg(\mathsf{n} <= (n2) \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n)n)n; \mathrm{A4} @ (n1) \rhd \forall_{obj}\mathsf{n}{:}\dot\neg(\forall_{obj}(n2){:}\dot\neg(\dot\neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \dot\neg(\mathsf{n} <= (n2) \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \dot\neg(\dot\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n)n)n \gg \dot\neg(\forall_{obj}(n2){:}\dot\neg(\dot\neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot\neg(\dot\neg(0 =$

(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸((n1) <= (n2) ⇒
∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒ ∸(∸(((fy)[(n2)] +
(−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n)n)n; Cauchy(2) ≫
$\forall_{obj}$(ϵ): ∸($\forall_{obj}$(n1): ∸($\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
(n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ∸(∸(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ϵ) ⇒ ∸(∸(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ϵ))n)n)n)n)n)n)n; A4 @(rec((1 + 1) + 1) ∗ (ϵ)) ▷
$\forall_{obj}$(ϵ): ∸($\forall_{obj}$(n1): ∸($\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
(n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ∸(∸(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ϵ) ⇒ ∸(∸(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ϵ))n)n)n)n)n)n)n ≫ ∸($\forall_{obj}$(n1): ∸($\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <=
(v1) ⇒ (n1) <= (v2) ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸(∸(|((fy)[(v1)] + (−u(fy)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n)n)n; FromNot < f(Strong)(Helper) ≫
∸(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸((n1) <= (n2) ⇒ ∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <=
(fx)[(n2)] ⇒ ∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) =
(fx)[(n2)])n)n)n)n)n ⇒ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸(∸(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸(∸(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸(∸(|((fy)[(v1)]+(−u(fy)[(v2)]))| = (rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⇒ ∸(0 <=
(ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <=
(ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n; ExistMP2 ▷ ∸(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸((n1) <=
(n2) ⇒ ∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒
∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n ⇒
$\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸(∸(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸(∸(|((fy)[(v1)]+(−u(fy)[(v2)]))| = (rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⇒ ∸(0 <=
(ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <=
(ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ▷ ∸($\forall_{obj}$(n2): ∸(∸(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸(∸(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸((n1) <=
(n2) ⇒ ∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒
∸(∸(((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) =
(fx)[(n2)])n)n)n)n)n)n)n ▷ ∸($\forall_{obj}$(n1): ∸($\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸(0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ∸(∸(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒

$\underline{(n1)} <= \underline{(v2)} \Rightarrow \dot\neg(\dot\neg(|((\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| <= (rec((1+1)+1) * \underline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(|((\overline{(fx)}[\overline{(v1)}] + (-u\overline{(fx)}[\overline{(v2)}]))| = (rec((1+1)+1) * \overline{(\epsilon)}))n)n)n \Rightarrow \dot\neg(\dot\neg(|((\overline{(fy)}[\overline{(v1)}] + (-u\overline{(fy)}[\overline{(v2)}]))| <= (rec((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(|((\overline{(fy)}[\overline{(v1)}] + (-u\overline{(fy)}[\overline{(v2)}]))| = (rec((1+1)+1) * \overline{(\epsilon)}))n)n)n)n)n)n)n \gg \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \underline{(n2)} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(((\overline{(fy)}[\underline{\mathsf{m}}] + (-u\underline{(fx)}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(((\underline{(fy)}[\underline{\mathsf{m}}] + (-u\underline{(fx)}[\underline{\mathsf{m}}])) = \underline{(\epsilon)})n)n)n\rceil, p_0, \mathsf{c})]$

[FromNot < f(Strong) $\overset{stmt}{\rightarrow}$ SystemQ $\vdash$ $\forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{\mathsf{m}}: \forall\underline{(n2)}: \forall\underline{(\epsilon)}: \forall\underline{(fx)}: \forall\underline{(fy)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}: \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \vdash \dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{(n2)} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(((\underline{(fy)}[\underline{\mathsf{m}}] + (-u\underline{(fx)}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(((\underline{(fy)}[\underline{\mathsf{m}}] + (-u\underline{(fx)}[\underline{\mathsf{m}}])) = \underline{(\epsilon)})n)n)n]$

[FromNot < f(Strong) $\overset{tex}{\rightarrow}$ "FromNot<f(Strong)"]

[FromNot < f(Strong) $\overset{pyk}{\rightarrow}$ "lemma fromNot<f(Strong)"]

fromNotSameF(Strongest)(Helper2)

[fromNotSameF(Strongest)(Helper2) $\overset{proof}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall\underline{\mathsf{x}}: \forall\underline{\mathsf{y}}: \forall\underline{\mathsf{z}}: \forall\underline{\mathsf{u}}: \forall\underline{\mathsf{v}}: \dot\neg(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| <= (rec((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot\neg(\dot\neg(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| = (rec((1+1)+1) * \underline{\mathsf{v}}))n)n)n \vdash \dot\neg(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| <= (rec((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot\neg(\dot\neg(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| = (rec((1+1)+1) * \underline{\mathsf{v}}))n)n)n \vdash \underline{\mathsf{v}} <= |(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| \vdash \text{NumericalDifference} \gg |(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| = |(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|; \text{SubLessLeft} \rhd |(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| = |(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))| \rhd \dot\neg(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| <= (rec((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot\neg(\dot\neg(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| = (rec((1+1)+1) * \underline{\mathsf{v}}))n)n)n \gg \dot\neg(|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))| <= (rec((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot\neg(\dot\neg(|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))| = (rec((1+1)+1) * \underline{\mathsf{v}}))n)n)n; \text{LessNegated} \rhd \dot\neg(|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))| <= (rec((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot\neg(\dot\neg(|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))| = (rec((1+1)+1) * \underline{\mathsf{v}}))n)n)n \gg \dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|) \Rightarrow \dot\neg(\dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|))n)n)n; \text{LessNegated} \rhd \dot\neg(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| <= (rec((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot\neg(\dot\neg(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| = (rec((1+1)+1) * \underline{\mathsf{v}}))n)n)n \gg \dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))|) \Rightarrow \dot\neg(\dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))|))n)n)n; \text{AddEquations(LeqLess)} \rhd \underline{\mathsf{v}} <= |(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| \rhd \dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|) \Rightarrow \dot\neg(\dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|))n)n)n \gg \dot\neg((\underline{\mathsf{v}} + (-u(rec((1+1)+1) * \underline{\mathsf{v}}))) <= (|(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| + (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|)) \Rightarrow \dot\neg(\dot\neg((\underline{\mathsf{v}} + (-u(rec((1+1)+1) * \underline{\mathsf{v}}))) = (|(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| + (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|)))n)n)n; \text{AddEquations(Less)} \rhd \dot\neg((\underline{\mathsf{v}} + (-u(rec((1+1)+1) * \underline{\mathsf{v}}))) <= (|(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| + (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|)) \Rightarrow \dot\neg(\dot\neg((\underline{\mathsf{v}} + (-u(rec((1+1)+1) * \underline{\mathsf{v}}))) = (|(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| + (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|)))n)n)n \rhd \dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))|) \Rightarrow \dot\neg(\dot\neg((-u(rec((1+1)+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))|))n)n)n \gg \dot\neg(((\underline{\mathsf{v}} + (-u(rec((1+1)+1) * \underline{\mathsf{v}}))) + (-u(rec((1+1)+1) * \underline{\mathsf{v}}))) <= ((|(\underline{\mathsf{y}} + (-u\underline{\mathsf{u}}))| + (-u|(\underline{\mathsf{y}} + (-u\underline{\mathsf{x}}))|)) + (-u|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))|)) \Rightarrow$

$\dot\neg(\dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) =$
$(((|(\underline{y}+(-u\underline{u}))|+(-u|(\underline{y}+(-u\underline{x}))|))+(-u|(\underline{z}+$
$(-u\underline{u}))|)))n)n)n;$ insertTwoMiddleTerms(Numerical) $\gg |(\underline{y}+(-u\underline{u}))| <=$
$(((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|);$ plusAssociativity $\gg$
$(((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|) =$
$(|(\underline{y}+(-u\underline{x}))|+(|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|));$ plusCommutativity $\gg$
$(|(\underline{y}+(-u\underline{x}))|+(|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)) = (((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+$
$|(\underline{y}+(-u\underline{x}))|);$ eqTransitivity $\rhd (((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|) =$
$(|(\underline{y}+(-u\underline{x}))|+(|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)) \rhd (|(\underline{y}+(-u\underline{x}))|+(|(\underline{x}+$
$(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)) = (((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{y}+(-u\underline{x}))|) \gg$
$(((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|) = (((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+$
$|(\underline{y}+(-u\underline{x}))|);$ subLeqRight $\rhd (((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|) =$
$(((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{y}+(-u\underline{x}))|) \rhd |(\underline{y}+(-u\underline{u}))| <=$
$(((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|) \gg |(\underline{y}+(-u\underline{u}))| <=$
$(((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{y}+(-u\underline{x}))|);$ PositiveToLeft(Leq) $\rhd |(\underline{y}+$
$(-u\underline{u}))| <= (((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{y}+(-u\underline{x}))|) \gg (|(\underline{y}+(-u\underline{u}))|+$
$(-u|(\underline{y}+(-u\underline{x}))|)) <= (|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|);$ PositiveToLeft(Leq) $\rhd$
$(|(\underline{y}+(-u\underline{u}))|+(-u|(\underline{y}+(-u\underline{x}))|)) <= (|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|) \gg$
$(((|(\underline{y}+(-u\underline{u}))|+(-u|(\underline{y}+(-u\underline{x}))|))+(-u|(\underline{z}+(-u\underline{u}))|)) <= |(\underline{x}+$
$(-u\underline{z}))|;$ LessLeqTransitivity $\rhd \dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+$
$1)+1)*\underline{v}))) <= (((|(\underline{y}+(-u\underline{u}))|+(-u|(\underline{y}+(-u\underline{x}))|))+(-u|(\underline{z}+(-u\underline{u}))|)) \Rightarrow$
$\dot\neg(\dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) =$
$(((|(\underline{y}+(-u\underline{u}))|+(-u|(\underline{y}+(-u\underline{x}))|))+(-u|(\underline{z}+(-u\underline{u}))|)))n)n)n \rhd (((|(\underline{y}+$
$(-u\underline{u}))|+(-u|(\underline{y}+(-u\underline{x}))|))+(-u|(\underline{z}+(-u\underline{u}))|)) <= |(\underline{x}+(-u\underline{z}))| \gg$
$\dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) <= |(\underline{x}+(-u\underline{z}))| \Rightarrow$
$\dot\neg(\dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) =$
$|(\underline{x}+(-u\underline{z}))|)n)n)n;$ ThreeThirds $\gg$
$(((rec((1+1)+1)*\underline{v})+(rec((1+1)+1)*\underline{v}))+(rec((1+1)+1)*\underline{v})) =$
$\underline{v};$ PositiveToRight(Eq) $\rhd (((rec((1+1)+1)*\underline{v})+(rec((1+1)+1)*\underline{v}))+(rec((1+$
$1)+1)*\underline{v})) = \underline{v} \gg ((rec((1+1)+1)*\underline{v})+(rec((1+1)+1)*\underline{v})) = (\underline{v}+(-u(rec((1+$
$1)+1)*\underline{v})));$ PositiveToRight(Eq) $\rhd ((rec((1+1)+1)*\underline{v})+(rec((1+1)+1)*\underline{v})) =$
$(\underline{v}+(-u(rec((1+1)+1)*\underline{v}))) \gg (rec((1+1)+1)*\underline{v}) = ((\underline{v}+(-u(rec((1+$
$1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v})));$ eqSymmetry $\rhd (rec((1+1)+1)*\underline{v}) =$
$((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) \gg$
$((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) = (rec((1+1)+1)*$
$\underline{v});$ SubLessLeft $\rhd ((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+1)*\underline{v}))) =$
$(rec((1+1)+1)*\underline{v}) \rhd \dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+(-u(rec((1+1)+$
$1)*\underline{v}))) <= |(\underline{x}+(-u\underline{z}))| \Rightarrow \dot\neg(\dot\neg(((\underline{v}+(-u(rec((1+1)+1)*\underline{v})))+$
$(-u(rec((1+1)+1)*\underline{v}))) = |(\underline{x}+(-u\underline{z}))|)n)n)n \gg \dot\neg((rec((1+1)+1)*\underline{v}) <=$
$|(\underline{x}+(-u\underline{z}))| \Rightarrow \dot\neg(\dot\neg((rec((1+1)+1)*\underline{v}) = |(\underline{x}+(-u\underline{z}))|)n)n)n\rceil, p_0, c)]$

[fromNotSameF(Strongest)(Helper2) $\overset{stmt}{\to}$ SystemQ $\vdash$
$\forall\underline{x}: \forall\underline{y}: \forall\underline{z}: \forall\underline{u}: \forall\underline{v}: \dot\neg(|(\underline{x}+(-u\underline{y}))| <= (rec((1+1)+1)*\underline{v}) \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{x}+(-u\underline{y}))| = (rec((1+1)+1)*\underline{v}))n)n)n \vdash \dot\neg(|(\underline{z}+(-u\underline{u}))| <=$
$(rec((1+1)+1)*\underline{v}) \Rightarrow \dot\neg(\dot\neg(|(\underline{z}+(-u\underline{u}))| = (rec((1+1)+1)*\underline{v}))n)n)n \vdash$
$\underline{v} <= |(\underline{y}+(-u\underline{u}))| \vdash \dot\neg((rec((1+1)+1)*\underline{v}) <= |(\underline{x}+(-u\underline{z}))| \Rightarrow$

$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) = |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}}))|)\mathrm{n})\mathrm{n})\mathrm{n}]$

[fromNotSameF(Strongest)(Helper2) $\overset{\mathrm{tex}}{\to}$ "fromNotSameF(Strongest)(Helper2)"]

[fromNotSameF(Strongest)(Helper2) $\overset{\mathrm{pyk}}{\to}$ "lemma fromNotSameF(Strongest) helper2"]

fromNotSameF(Strongest)(Helper)

[fromNotSameF(Strongest)(Helper) $\overset{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall(\mathrm{v1})\colon\forall(\mathrm{v2})\colon\forall\underline{\mathsf{m}}\colon\forall(\mathrm{n1})\colon\forall(\mathrm{n2})\colon\forall(\epsilon)\colon\forall(\mathrm{fx})\colon\forall(\mathrm{fy})\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}(\mathrm{v1})\colon\forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(0 <= (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})]{+}(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash (\mathrm{n2}) <= \underline{\mathsf{m}} \vdash \mathrm{FromNegated}(2*\mathrm{Imply}) \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}))\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}))\mathrm{n})\mathrm{n} \gg (\mathrm{n1}) <= (\mathrm{n2}); \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{fromNotLess} \rhd \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg (\epsilon) <= |((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))|; \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}(\mathrm{v1})\colon\forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(0 <= (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})]{+}(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(0 <= (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1{+}1){+}1){*}(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <=$

$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};\ \mathrm{A4} @ (\mathrm{n2}) \rhd \forall_{\mathrm{obj}}(\mathrm{v2}){:}\ \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+$
$(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};\ 0 < 1/3 \gg \dot{\neg}(0 <=$
$\mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n};\ \mathrm{PositiveFactors} \rhd \dot{\neg}(0 <=$
$\mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n};\ \mathrm{leqTransitivity} \rhd (\mathrm{n1}) <= (\mathrm{n2}) \rhd (\mathrm{n2}) <= \underline{\mathsf{m}} \gg$
$(\mathrm{n1}) <= \underline{\mathsf{m}};\ \mathrm{MP3} \rhd \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\mathrm{n1}) <=$
$\underline{\mathsf{m}} \rhd (\mathrm{n1}) <= (\mathrm{n2}) \gg \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};\ \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n};\ \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n};\ \mathrm{fromNotSameF}(\mathrm{Strongest})(\mathrm{Helper2}) \rhd \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+$
$(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{n2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\epsilon) <= |((\mathrm{fx})[(\mathrm{n2})]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| \gg \dot{\neg}((\mathrm{rec}((1+1)+1)*(\epsilon)) <=$

|((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n; ∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⊢ FromNegatedImply ▷ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ≫ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇(¬̇((n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n; FirstConjunct ▷ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇(¬̇((n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n ≫ ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n; 0 < 1/3 ≫ ¬̇(0 <= rec((1 + 1) + 1) ⇒ ¬̇(¬̇(0 = rec((1 + 1) + 1))n)n)n; PositiveFactors ▷ ¬̇(0 <= rec((1 + 1) + 1) ⇒ ¬̇(¬̇(0 = rec((1 + 1) + 1))n)n)n ▷ ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ≫ ¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n; ∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): Ded ▷ ∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⊢ (n2) <= m ⊢ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ≫ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⇒ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⇒ (n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n; Ded ▷ ∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=

$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\, \forall_{\mathrm{obj}}(\mathrm{v2}){:}\, \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*$
$(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts}(2\mathrm{conditions}) \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\, \forall_{\mathrm{obj}}(\mathrm{v2}){:}\, \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <=$
$(\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\, \forall_{\mathrm{obj}}(\mathrm{v2}){:}\, \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <=$
$\mathsf{m} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1)*(\epsilon)) <= |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+$
$1)+1)*(\epsilon)) = |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))|)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\, \forall_{\mathrm{obj}}(\mathrm{v2}){:}\, \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n2}) <= \mathsf{m} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1)*(\epsilon)) <= |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1)*(\epsilon)) = |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{fromNotSameF}(\mathrm{Strongest})(\mathrm{Helper}) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1}){:}\, \forall(\mathrm{v2}){:}\, \forall\mathsf{m}{:}\, \forall(\mathrm{n1}){:}\, \forall(\mathrm{n2}){:}\, \forall(\epsilon){:}\, \forall(\mathrm{fx}){:}\, \forall(\mathrm{fy}){:}\, \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$

$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n \Rightarrow \forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1)*(\epsilon)))n)n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow \dot{\neg}((n2) <= \mathsf{m} \Rightarrow \dot{\neg}((rec((1+1)+1)*(\epsilon)) <= |((fx)[\mathsf{m}] + (-u(fy)[\mathsf{m}]))| \Rightarrow \dot{\neg}(\dot{\neg}((rec((1+1)+1)*(\epsilon)) = |((fx)[\mathsf{m}] + (-u(fy)[\mathsf{m}]))|)n)n)n)n)n]$

[fromNotSameF(Strongest)(Helper) $\stackrel{tex}{\rightarrow}$ "fromNotSameF(Strongest)(Helper)"]

[fromNotSameF(Strongest)(Helper) $\stackrel{pyk}{\rightarrow}$ "lemma fromNotSameF(Strongest) helper"]

fromNotSameF(Strongest)

[fromNotSameF(Strongest) $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall(v1){:}\,\forall(v2){:}\,\forall\mathsf{m}{:}\,\forall\mathsf{n}{:}\,\forall(n1){:}\,\forall(n2){:}\,\forall(\epsilon){:}\,\forall(fx){:}\,\forall(fy){:}\,\dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\,\dot{\neg}($ $\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$ Repetition $\rhd$ $\dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg$ $\dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n;$ Ded $\rhd$ $\dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(n2){:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n)n)n;$ AEA $-$ negated $\rhd$ $\dot{\neg}(\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(n2){:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(n2){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n)n)n)n)n;$ Cauchy(2) $\gg$ $\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n)n)n)n;$ A4 @(rec((1 + 1) + 1) $*$ (ϵ)) $\rhd$

$\forall_{\mathrm{obj}}(\epsilon){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] +$
$(-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <=$
$(\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{fromNotSameF}(\mathrm{Strongest})(\mathrm{Helper}) \gg$
$\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow$
$\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n2}) <= \mathsf{m} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <= |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) =$
$|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{EAE}-\mathrm{MP} \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n2}) <= \mathsf{m} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <= |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) = |((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\epsilon){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] +$
$(-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <=$
$(\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n2}) <= \mathsf{m} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <=$
$|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) =$
$|((\mathrm{fx})[\mathsf{m}] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}}(\mathrm{v1}){:}\,\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <=$

$(\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| =$
$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <=$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) =$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1}) \colon \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1}) \colon \forall_{\mathrm{obj}}(\mathrm{v2}) \colon \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd$
$\dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <=$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) =$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @ (\mathrm{n2}) \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n2}) <=$
$\underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <=$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) =$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @(\mathrm{rec}((1+1)+1) * (\epsilon)) \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * (\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) <=$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * (\epsilon)) =$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(\epsilon) \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1}) \colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\epsilon) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\epsilon) =$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{fromNotSameF}(\mathrm{Strongest}) \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1}) \colon \forall(\mathrm{v2}) \colon \forall\underline{\mathsf{m}} \colon \forall\underline{\mathsf{n}} \colon \forall(\mathrm{n1}) \colon \forall(\mathrm{n2}) \colon \forall(\epsilon) \colon \forall(\mathrm{fx}) \colon \forall(\mathrm{fy}) \colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)} \colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}} \colon \dot{\neg}($
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}(\epsilon) \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1}) \colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\epsilon) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\epsilon) =$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[fromNotSameF(Strongest) $\stackrel{\text{tex}}{\to}$ "fromNotSameF(Strongest)"]

[fromNotSameF(Strongest) $\stackrel{\text{pyk}}{\to}$ "lemma fromNotSameF(Strongest)"]

ToLess(F)(Helper)

[ToLess(F)(Helper) $\stackrel{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢
∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 =
(ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
(n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒
∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ if((n2) <= (n1), (n1), (n2)) <= m ⊢
MaxLeq(1) ≫ (n1) <= if((n2) <= (n1), (n1), (n2)); leqTransitivity ▷ (n1) <=
if((n2) <= (n1), (n1), (n2)) ▷ if((n2) <= (n1), (n1), (n2)) <= m ≫ (n1) <=
m; MaxLeq(2) ≫ (n2) <= if((n2) <= (n1), (n1), (n2)); leqTransitivity ▷
(n2) <= if((n2) <= (n1), (n1), (n2)) ▷ if((n2) <= (n1), (n1), (n2)) <= m ≫
(n2) <= m; A4 @ m ▷ $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n; FirstConjunct ▷ ∸(∸(0 <= (ϵ) ⇒
∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒
∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ ∸(0 <= (ϵ) ⇒ ∸(∸(0 =
(ϵ))n)n)n; SecondConjunct ▷ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] +
(−u(fy)[m]))|)n)n)n)n)n ≫ (n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒
∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n; MP ▷ (n1) <= m ⇒ ∸((ϵ) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ▷ (n1) <=
m ≫ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n; FromNumericalGreater ▷ ∸((ϵ) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ≫
∸(∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) =
(−u(ϵ)))n)n)n)n ⇒ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] +
(−u(fy)[m])))n)n)n; MP2 ▷ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒
∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) =
(ϵ))n)n)n ▷ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ▷ (n2) <= m ≫
∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) =
(ϵ))n)n)n; LessNegated ▷ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] +
(−u(fx)[m])) = (ϵ))n)n)n ≫ ∸((−u(ϵ)) <= (−u((fy)[m] + (−u(fx)[m]))) ⇒
∸(∸((−u(ϵ)) = (−u((fy)[m] + (−u(fx)[m]))))n)n)n; MinusNegated ≫
(−u((fy)[m] + (−u(fx)[m]))) = ((fx)[m] + (−u(fy)[m])); SubLessRight ▷
(−u((fy)[m] + (−u(fx)[m]))) = ((fx)[m] + (−u(fy)[m])) ▷ ∸((−u(ϵ)) <=
(−u((fy)[m] + (−u(fx)[m]))) ⇒ ∸(∸((−u(ϵ)) =
(−u((fy)[m] + (−u(fx)[m]))))n)n)n ≫ ∸((−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])) ⇒

∸(∸((−u(ϵ)) = ((fx)[m] + (−u(fy)[m])))n)n)n; LessLeq ⊳ ∸((−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((−u(ϵ)) = ((fx)[m] + (−u(fy)[m])))n)n)n ≫ (−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])); toNotLess ⊳ (−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])) ≫ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n)n; NegateDisjunct1 ⊳ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n)n ⇒ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] + (−u(fy)[m])))n)n)n ⊳ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n)n ≫ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] + (−u(fy)[m])))n)n)n; SwitchTerms(x < y − z) ⊳ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] + (−u(fy)[m])))n)n)n ≫ ∸((fy)[m] <= ((fx)[m] + (−u(ϵ))) ⇒ ∸(∸((fy)[m] = ((fx)[m] + (−u(ϵ))))n)n)n; LessLeq ⊳ ∸((fy)[m] <= ((fx)[m] + (−u(ϵ))) ⇒ ∸(∸((fy)[m] = ((fx)[m] + (−u(ϵ))))n)n)n ≫ (fy)[m] <= ((fx)[m] + (−u(ϵ))); ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ A4 @ m ⊳ ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ (ϵ) <= |((fx)[m] + (−u(fy)[m]))|)n)n; FirstConjunct ⊳ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ (ϵ) <= |((fx)[m] + (−u(fy)[m]))|)n)n ≫ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n; ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): Ded ⊳ ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ if((n2) <= (n1), (n1), (n2)) <= m ⊢ (fy)[m] <= ((fx)[m] + (−u(ϵ))) ≫ ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ϵ))); Ded ⊳ ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ≫ ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n; JoinConjuncts(2conditions) ⊳ ∀$_{obj}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 =

$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{n1})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\underline{(\epsilon)} <= |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\epsilon)} = |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{n1})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\underline{(\epsilon)} <= |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\epsilon)} = |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}]+(-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}]+(-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\underline{(\mathrm{n2})} <= \underline{(\mathrm{n1})}, \underline{(\mathrm{n1})}, \underline{(\mathrm{n2})}) <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{n1})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\underline{(\epsilon)} <= |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\epsilon)} = |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}(\underline{(\mathrm{n2})} <= \underline{(\mathrm{n1})}, \underline{(\mathrm{n1})}, \underline{(\mathrm{n2})}) <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToLess(F)(Helper)} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{n1})}\colon \forall\underline{(\mathrm{n2})}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{n1})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\underline{(\epsilon)} <= |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\epsilon)} = |((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}(\underline{(\mathrm{n2})} <= \underline{(\mathrm{n1})}, \underline{(\mathrm{n1})}, \underline{(\mathrm{n2})}) <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n}]$

$[\mathrm{ToLess(F)(Helper)} \overset{\mathrm{tex}}{\to}$ "ToLess(F)(Helper)"]

$[\mathrm{ToLess(F)(Helper)} \overset{\mathrm{pyk}}{\to}$ "lemma toLess(F) helper"]

ToLess(F)

$[\mathrm{ToLess(F)} \overset{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\mathrm{n1})}\colon \forall\underline{(\mathrm{n2})}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ Repetition $\rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$

$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNegatedOr} \rhd$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNot} <$
$\mathrm{f(Strong)} \rhd \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <= \underline{\mathsf{m}} \Rightarrow$
$\dot{-}(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])) =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{fromNotSameF(Strongest)} \rhd \dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{-}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}((\underline{(\mathrm{n1})} <= \underline{\mathsf{m}} \Rightarrow \dot{-}((\underline{(\epsilon)} <= |((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \Rightarrow \dot{-}(\dot{-}((\underline{(\epsilon)} =$
$|((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ToLess(F)(Helper)} \gg$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}((\underline{(\mathrm{n1})} <= \underline{\mathsf{m}} \Rightarrow \dot{-}((\underline{(\epsilon)} <=$
$|((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \Rightarrow \dot{-}(\dot{-}((\underline{(\epsilon)} = |((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <= \underline{\mathsf{m}} \Rightarrow \dot{-}(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] +$

$(-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow$
$(\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n}; \mathrm{TwiceExistMP} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\epsilon) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\epsilon) = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <=$
$((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\epsilon) <= |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((\epsilon) = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <=$
$((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <=$
$\underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <=$
$(\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <=$
$((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <=$
$(\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] +$
$(-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@} \mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}((\mathrm{n2}) <= (\mathrm{n1}), (\mathrm{n1}), (\mathrm{n2})) <= \underline{\mathsf{m}} \Rightarrow$
$(\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] +$
$(-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@} (\epsilon) \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\underline{\mathsf{m}}] <= ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\overline{\mathsf{m}}] <= ((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\overline{\mathsf{m}}] <= ((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fy})[\overline{\mathsf{m}}] <= ((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}], \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToLess}(\mathrm{F}) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\mathrm{n1})\colon \forall(\mathrm{n2})\colon \forall(\epsilon)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0$

$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[ToLess(F) $\stackrel{\mathrm{tex}}{\rightarrow}$ “ToLess(F)”]

[ToLess(F) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma toLess(F)”]

FromNot <<

[FromNot $<<\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon \mathrm{AutoImply} \gg$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{MT} \rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[FromNot $<<\stackrel{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash$
$\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[FromNot $<<\stackrel{\mathrm{tex}}{\rightarrow}$ “FromNot<<”]

[FromNot $<<\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma fromNot<<”]

ToLess(R)

$[\mathrm{ToLess(R)} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \overline{(\mathrm{fx})}\colon \forall \overline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \vdash$
$\mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$

$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \gg$
$\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n}; \mathrm{FromNegatedOr} \rhd$
$\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \gg$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \dot{-}(\dot{-}(\{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n)n)n; \mathrm{FirstConjunct} \rhd$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \dot{-}(\dot{-}(\{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\bar{\mathsf{n}} <= \bar{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\bar{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\bar{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\bar{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\bar{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\bar{\mathsf{n}} <= \bar{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\bar{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\bar{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\bar{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\bar{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\bar{\mathsf{n}} <= \bar{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\bar{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\bar{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)}))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \triangleright$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\bar{\mathsf{n}} <= \bar{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\bar{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\bar{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)}))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\bar{\mathsf{n}} <= \bar{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\bar{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\bar{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\bar{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\bar{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\bar{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n)n)n \gg \dot\neg(\{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n;$ FromNot <<
$\rhd\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \gg$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n;$ From!! ==
$\rhd\dot\neg(\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\mathsf{n}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \gg$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ToNegatedOr} \rhd$
$\dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ToLess(F)} \rhd \dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}], \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToLess(R)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{ToLess(R)} \stackrel{\mathrm{tex}}{\rightarrow}$ "ToLess(R)"]

$[\mathrm{ToLess(R)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma toLess(R)"]

FromNotSameF(Weak)(Helper)

[FromNotSameF(Weak)(Helper) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{z} <= |(\underline{x}+(-u\underline{y}))| \vdash 0 <= (\underline{x}+(-u\underline{y})) \vdash \text{NonnegativeNumerical} \rhd 0 <= (\underline{x}+(-u\underline{y})) \gg |(\underline{x}+(-u\underline{y}))| = (\underline{x}+(-u\underline{y}));$ $\text{subLeqRight} \rhd |(\underline{x}+(-u\underline{y}))| = (\underline{x}+(-u\underline{y})) \rhd \underline{z} <= |(\underline{x}+(-u\underline{y}))| \gg \underline{z} <= (\underline{x}+(-u\underline{y}));$ $\text{negativeToLeft(Leq)} \rhd \underline{z} <= (\underline{x}+(-u\underline{y})) \gg (\underline{z}+\underline{y}) <= \underline{x};$ $\text{plusCommutativity} \gg (\underline{z}+\underline{y}) = (\underline{y}+\underline{z});$ $\text{subLeqLeft} \rhd (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}) \rhd (\underline{z}+\underline{y}) <= \underline{x} \gg (\underline{y}+\underline{z}) <= \underline{x};$ $\text{PositiveToRight(Leq)} \rhd (\underline{y}+\underline{z}) <= \underline{x} \gg \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\text{WeakenOr1} \rhd \underline{y} <= (\underline{x}+(-u\underline{z})) \gg \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{z} <= |(\underline{x}+(-u\underline{y}))| \vdash \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \vdash \text{ToLess} \rhd \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \gg \dot{\neg}((\underline{x}+(-u\underline{y})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{y})) = 0)n)n)n;$ $\text{NegativeNumerical} \gg |(\underline{x}+(-u\underline{y}))| = (-u(\underline{x}+(-u\underline{y})));$ $\text{MinusNegated} \gg (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x}));$ $\text{eqTransitivity} \rhd |(\underline{x}+(-u\underline{y}))| = (-u(\underline{x}+(-u\underline{y}))) \rhd (-u(\underline{x}+(-u\underline{y}))) = (\underline{y}+(-u\underline{x})) \gg |(\underline{x}+(-u\underline{y}))| = (\underline{y}+(-u\underline{x}));$ $\text{subLeqRight} \rhd |(\underline{x}+(-u\underline{y}))| = (\underline{y}+(-u\underline{x})) \rhd \underline{z} <= |(\underline{x}+(-u\underline{y}))| \gg \underline{z} <= (\underline{y}+(-u\underline{x}));$ $\text{negativeToLeft(Leq)} \rhd \underline{z} <= (\underline{y}+(-u\underline{x})) \gg (\underline{z}+\underline{x}) <= \underline{y};$ $\text{plusCommutativity} \gg (\underline{z}+\underline{x}) = (\underline{x}+\underline{z});$ $\text{subLeqLeft} \rhd (\underline{z}+\underline{x}) = (\underline{x}+\underline{z}) \rhd (\underline{z}+\underline{x}) <= \underline{y} \gg (\underline{x}+\underline{z}) <= \underline{y};$ $\text{PositiveToRight(Leq)} \rhd (\underline{x}+\underline{z}) <= \underline{y} \gg \underline{x} <= (\underline{y}+(-u\underline{z}));$ $\text{WeakenOr2} \rhd \underline{x} <= (\underline{y}+(-u\underline{z})) \gg \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{z} <= |(\underline{x}+(-u\underline{y}))| \vdash 0 <= (\underline{x}+(-u\underline{y})) \vdash \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z})) \gg \underline{z} <= |(\underline{x}+(-u\underline{y}))| \Rightarrow 0 <= (\underline{x}+(-u\underline{y})) \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{z} <= |(\underline{x}+(-u\underline{y}))| \vdash \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \vdash \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z})) \gg \underline{z} <= |(\underline{x}+(-u\underline{y}))| \Rightarrow \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n)n \vdash \text{fromNotLess} \rhd \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n)n \gg \underline{z} <= |(\underline{x}+(-u\underline{y}))|;$ $\text{MP} \rhd \underline{z} <= |(\underline{x}+(-u\underline{y}))| \Rightarrow 0 <= (\underline{x}+(-u\underline{y})) \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z})) \rhd \underline{z} <= |(\underline{x}+(-u\underline{y}))| \gg 0 <= (\underline{x}+(-u\underline{y})) \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\text{MP} \rhd \underline{z} <= |(\underline{x}+(-u\underline{y}))| \Rightarrow \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z})) \rhd \underline{z} <= |(\underline{x}+(-u\underline{y}))| \gg \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}));$ $\text{FromNegations} \rhd 0 <= (\underline{x}+(-u\underline{y})) \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z})) \rhd \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z})) \gg \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}))\urcorner, p_0, c)$]

[FromNotSameF(Weak)(Helper) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n)n \vdash \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x}+(-u\underline{z}))$]

[FromNotSameF(Weak)(Helper) $\stackrel{\text{tex}}{\rightarrow}$ "FromNotSameF(Weak)(Helper)"]

[FromNotSameF(Weak)(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromNotSameF(Weak)(Helper)"]

FromNotSameF(Weak)

[FromNotSameF(Weak) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$ $\forall \underline{\mathsf{m}}: \forall \underline{\mathsf{n}}: \forall \underline{(\epsilon)}: \forall \underline{(\mathrm{fx})}: \forall \underline{(\mathrm{fy})}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$

Repetition $\rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; Ded $\rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}(\forall_{\mathrm{obj}} \underline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}: \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; AEA − negated $\rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}: \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}(\forall_{\mathrm{obj}} \underline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; $\forall \underline{\mathsf{m}}: \forall \underline{\mathsf{n}}: \forall \underline{(\epsilon)}: \forall \underline{(\mathrm{fx})}: \forall \underline{(\mathrm{fy})}: \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$

FromNegated($2 * $ Imply) $\rhd \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; FirstConjunct $\rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})\mathrm{n})\mathrm{n}$; SecondConjunct $\rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; FromNotSameF(Weak)(Helper) $\rhd \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n} \Rightarrow \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)}))$; JoinConjuncts $\rhd \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n} \Rightarrow \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n} \Rightarrow \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n}$; Ded $\rhd \forall \underline{\mathsf{m}}: \forall \underline{\mathsf{n}}: \forall \underline{(\epsilon)}: \forall \underline{(\mathrm{fx})}: \forall \underline{(\mathrm{fy})}: \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| = (\underline{\epsilon}))n)n)n)n \vdash \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <= ((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <= ((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n \gg \dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| = (\underline{\epsilon}))n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <= ((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <= ((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n; \mathrm{AddEAE} \rhd \dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| = (\underline{\epsilon}))n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <= ((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <= ((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| = (\underline{\epsilon}))n)n)n)n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <=$
$((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <= ((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n)n)n)n)n; \mathrm{MP} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| = (\underline{\epsilon}))n)n)n)n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <=$
$((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <= ((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n)n)n)n)n \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{fy})[\underline{\mathsf{m}}]))| = (\underline{\epsilon}))n)n)n)n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <=$
$((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <= ((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[FromNotSameF(Weak) $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\forall(\underline{\epsilon})\colon\forall(\underline{fx})\colon\forall(\underline{fy})\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}(\underline{\epsilon})\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\underline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{\epsilon}))n)n)n \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}})n)n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{fx})[\underline{\mathsf{m}}] <= ((\underline{fy})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n \Rightarrow (\underline{fy})[\underline{\mathsf{m}}] <=$
$((\underline{fx})[\underline{\mathsf{m}}] + (-u(\underline{\epsilon}))))n)n)n)n)n)n]$

[FromNotSameF(Weak) $\stackrel{\mathrm{tex}}{\rightarrow}$ "FromNotSameF(Weak)"]

[FromNotSameF(Weak) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromNotSameF(Weak)"]

FromNotLess(F)

[FromNotLess(F) $\stackrel{\mathrm{tex}}{\rightarrow}$ "FromNotLess(F)"]

[FromNotLess(F) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromNotLess(F)"]

== Addition

[== Addition $\stackrel{\text{tex}}{\rightarrow}$ “==Addition”]

[== Addition $\stackrel{\text{pyk}}{\rightarrow}$ “lemma ==Addition”]

== AdditionLeft

[== AdditionLeft $\stackrel{\text{tex}}{\rightarrow}$ “==AdditionLeft”]

[== AdditionLeft $\stackrel{\text{pyk}}{\rightarrow}$ “lemma ==AdditionLeft”]

Fpart − Bounded(Base)

[Fpart − Bounded(Base) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall(\text{v1}){:}\,\forall(\text{v2n}){:}\,\forall(\text{fx}){:}\,(\text{v2n}) <= 0 \vdash \text{LeqLessEq} \rhd (\text{v2n}) <= 0 \gg \dot{\neg}(\dot{\neg}((\text{v2n}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\text{v2n}) = 0)\text{n})\text{n})\text{n})\text{n} \Rightarrow (\text{v2n}) = 0; \text{Nonnegative(N)} \gg 0 <= (\text{v2n}); \text{toNotLess} \rhd 0 <= (\text{v2n}) \gg \dot{\neg}(\dot{\neg}((\text{v2n}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\text{v2n}) = 0)\text{n})\text{n})\text{n})\text{n}; \text{NegateDisjunct1} \rhd \dot{\neg}(\dot{\neg}((\text{v2n}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\text{v2n}) = 0)\text{n})\text{n})\text{n})\text{n} \Rightarrow (\text{v2n}) = 0 \rhd \dot{\neg}(\dot{\neg}((\text{v2n}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\text{v2n}) = 0)\text{n})\text{n})\text{n})\text{n} \gg (\text{v2n}) = 0; \text{SameSeries}\rhd(\text{v2n}) = 0 \gg (\text{fx})[(\text{v2n})] = (\text{fx})[0]; \text{SameNumerical}\rhd(\text{fx})[(\text{v2n})] = (\text{fx})[0] \gg |(\text{fx})[(\text{v2n})]| = |(\text{fx})[0]|; \text{eqAddition} \rhd |(\text{fx})[(\text{v2n})]| = |(\text{fx})[0]| \gg (|(\text{fx})[(\text{v2n})]| + 1) = (|(\text{fx})[0]| + 1); \text{leqReflexivity} \gg |(\text{fx})[(\text{v2n})]| <= |(\text{fx})[(\text{v2n})]|; \text{Leq} + 1 \rhd |(\text{fx})[(\text{v2n})]| <= |(\text{fx})[(\text{v2n})]| \gg \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[(\text{v2n})]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[(\text{v2n})]| + 1))\text{n})\text{n})\text{n}; \text{SubLessRight} \rhd (|(\text{fx})[(\text{v2n})]| + 1) = (|(\text{fx})[0]| + 1) \rhd \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[(\text{v2n})]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[(\text{v2n})]| + 1))\text{n})\text{n})\text{n} \gg \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[0]| + 1))\text{n})\text{n})\text{n}; \forall(\text{v1}){:}\,\forall(\text{v2n}){:}\,\forall(\text{fx}){:}\,\text{Ded} \rhd \forall(\text{v1}){:}\,\forall(\text{v2n}){:}\,\forall(\text{fx}){:}\,(\text{v2n}) <= 0 \vdash \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[0]| + 1))\text{n})\text{n})\text{n} \gg (\text{v2n}) <= 0 \Rightarrow \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[0]| + 1))\text{n})\text{n})\text{n}; \text{Gen} \rhd (\text{v2n}) <= 0 \Rightarrow \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[0]| + 1))\text{n})\text{n})\text{n} \gg \forall_{\text{obj}}(\text{v2n}){:}\,(\text{v2n}) <= 0 \Rightarrow \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[0]| + 1))\text{n})\text{n})\text{n}; \text{IntroExist}\,@(|(\text{fx})[0]| + 1) \rhd \forall_{\text{obj}}(\text{v2n}){:}\,(\text{v2n}) <= 0 \Rightarrow \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (|(\text{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (|(\text{fx})[0]| + 1))\text{n})\text{n})\text{n} \gg \dot{\neg}(\forall_{\text{obj}}(\text{v1}){:}\,\dot{\neg}(\forall_{\text{obj}}(\text{v2n}){:}\,(\text{v2n}) <= 0 \Rightarrow \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (\text{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (\text{v1}))\text{n})\text{n})\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[Fpart − Bounded(Base) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall(\text{v1}){:}\,\forall(\text{v2n}){:}\,\forall(\text{fx}){:}\,\dot{\neg}(\forall_{\text{obj}}(\text{v1}){:}\,\dot{\neg}(\forall_{\text{obj}}(\text{v2n}){:}\,(\text{v2n}) <= 0 \Rightarrow \dot{\neg}(|(\text{fx})[(\text{v2n})]| <= (\text{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\text{fx})[(\text{v2n})]| = (\text{v1}))\text{n})\text{n})\text{n})\text{n})\text{n}$]

[Fpart − Bounded(Base) $\stackrel{\text{tex}}{\rightarrow}$ “Fpart-Bounded(Base)”]

[Fpart – Bounded(Base) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fpart-Bounded base"]

Fpart – Bounded(InduHelper)

[Fpart – Bounded(InduHelper) $\stackrel{\text{proof}}{\rightarrow}$ λc.λx.𝒫(⌈SystemQ ⊢
∀(v1): ∀(v2n): ∀n: ∀(fx): (v2n) <= n ⇒ ∸(|(fx)[(v2n)]| <= (v1) ⇒
∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⊢ (v2n) <= (n + 1) ⊢ LeqLessEq ⊳ (v2n) <=
(n + 1) ≫ ∸(∸((v2n) <= (n + 1) ⇒ ∸(∸((v2n) = (n + 1))n)n)n)n ⇒ (v2n) =
(n + 1); ∀(v1): ∀(v2n): ∀n: ∀(fx): (v2n) <= n ⇒ ∸(|(fx)[(v2n)]| <= (v1) ⇒
∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⊢ ∸((v2n) <= (n + 1) ⇒ ∸(∸((v2n) =
(n + 1))n)n)n ⊢ LessMinus1(N) ⊳ ∸((v2n) <= (n + 1) ⇒ ∸(∸((v2n) =
(n + 1))n)n)n ≫ (v2n) <= n; MP ⊳ (v2n) <= n ⇒ ∸(|(fx)[(v2n)]| <= (v1) ⇒
∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⊳ (v2n) <= n ≫ ∸(|(fx)[(v2n)]| <= (v1) ⇒
∸(∸(|(fx)[(v2n)]| = (v1))n)n)n; MaxLeq(1) ≫ (v1) <= if((|(fx)[(n+1)]|+1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)); LessLeqTransitivity ⊳ ∸(|(fx)[(v2n)]| <=
(v1) ⇒ ∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⊳ (v1) <= if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)) ≫ ∸(|(fx)[(v2n)]| <= if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)) ⇒ ∸(∸(|(fx)[(v2n)]| = if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)))n)n)n; ∀(v1): ∀(v2n): ∀n: ∀(fx): (v2n) = (n + 1) ⊢
SameSeries ⊳ (v2n) = (n + 1) ≫ (fx)[(v2n)] =
(fx)[(n + 1)]; SameNumerical ⊳ (fx)[(v2n)] = (fx)[(n + 1)] ≫ |(fx)[(v2n)]| =
|(fx)[(n + 1)]|; eqLeq ⊳ |(fx)[(v2n)]| = |(fx)[(n + 1)]| ≫ |(fx)[(v2n)]| <=
|(fx)[(n + 1)]|; Leq + 1 ⊳ |(fx)[(v2n)]| <= |(fx)[(n + 1)]| ≫ ∸(|(fx)[(v2n)]| <=
(|(fx)[(n+1)]|+1) ⇒ ∸(∸(|(fx)[(v2n)]| = (|(fx)[(n+1)]|+1))n)n)n; MaxLeq(2) ≫
(|(fx)[(n + 1)]| + 1) <= if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)); LessLeqTransitivity ⊳ ∸(|(fx)[(v2n)]| <=
(|(fx)[(n + 1)]| + 1) ⇒ ∸(∸(|(fx)[(v2n)]| =
(|(fx)[(n + 1)]| + 1))n)n)n ⊳ (|(fx)[(n + 1)]| + 1) <= if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)) ≫ ∸(|(fx)[(v2n)]| <= if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)) ⇒ ∸(∸(|(fx)[(v2n)]| = if((|(fx)[(n + 1)]| + 1) <=
(v1), (v1), (|(fx)[(n + 1)]| + 1)))n)n)n; Ded ⊳ ∀(v1): ∀(v2n): ∀n: ∀(fx): (v2n) <=
n ⇒ ∸(|(fx)[(v2n)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⊢ ∸((v2n) <=
(n + 1) ⇒ ∸(∸((v2n) = (n + 1))n)n)n ⊢ ∸(|(fx)[(v2n)]| <=
if((|(fx)[(n + 1)]| + 1) <= (v1), (v1), (|(fx)[(n + 1)]| + 1)) ⇒ ∸(∸(|(fx)[(v2n)]| =
if((|(fx)[(n + 1)]| + 1) <= (v1), (v1), (|(fx)[(n + 1)]| + 1)))n)n)n ≫ (v2n) <=
n ⇒ ∸(|(fx)[(v2n)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⇒ ∸((v2n) <=
(n + 1) ⇒ ∸(∸((v2n) = (n + 1))n)n)n ⇒ ∸(|(fx)[(v2n)]| <=
if((|(fx)[(n + 1)]| + 1) <= (v1), (v1), (|(fx)[(n + 1)]| + 1)) ⇒ ∸(∸(|(fx)[(v2n)]| =
if((|(fx)[(n + 1)]| + 1) <= (v1), (v1), (|(fx)[(n + 1)]| + 1)))n)n)n; MP ⊳ (v2n) <=
n ⇒ ∸(|(fx)[(v2n)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ⇒ ∸((v2n) <=
(n + 1) ⇒ ∸(∸((v2n) = (n + 1))n)n)n ⇒ ∸(|(fx)[(v2n)]| <=
if((|(fx)[(n + 1)]| + 1) <= (v1), (v1), (|(fx)[(n + 1)]| + 1)) ⇒ ∸(∸(|(fx)[(v2n)]| =
if((|(fx)[(n + 1)]| + 1) <= (v1), (v1), (|(fx)[(n + 1)]| + 1)))n)n)n ⊳ (v2n) <= n ⇒
∸(|(fx)[(v2n)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2n)]| = (v1))n)n)n ≫ ∸((v2n) <=

$(\underline{n} + 1) \Rightarrow \dot{\neg}(\dot{\neg}((v2n) = (\underline{n} + 1))n)n \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n; \mathrm{Ded} \rhd$
$\forall(v1){:}\,\forall(v2n){:}\,\forall\underline{n}{:}\,\forall(fx){:}\,(v2n) = (\underline{n} + 1) \vdash \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n \gg (v2n) =$
$(\underline{n} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n; \mathrm{FromDisjuncts} \rhd \dot{\neg}(\dot{\neg}((v2n) <=$
$(\underline{n} + 1) \Rightarrow \dot{\neg}(\dot{\neg}((v2n) = (\underline{n} + 1))n)n)n)n \Rightarrow (v2n) = (\underline{n} + 1) \rhd \dot{\neg}((v2n) <=$
$(\underline{n} + 1) \Rightarrow \dot{\neg}(\dot{\neg}((v2n) = (\underline{n} + 1))n)n)n \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n \rhd (v2n) =$
$(\underline{n} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n \gg \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n\rceil, p_0, c)]$

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{InduHelper}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(v1){:}\,\forall(v2n){:}\,\forall\underline{n}{:}\,\forall(fx){:}\,(v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash (v2n) <= (\underline{n} + 1) \vdash \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)))n)n)n]$

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{InduHelper}) \overset{\mathrm{tex}}{\rightarrow}$ "Fpart-Bounded(InduHelper)"]

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{InduHelper}) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma fpart-Bounded indu helper"]

Fpart − Bounded(Indu)

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{Indu}) \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall(v1){:}\,\forall(v2n){:}\,\forall\underline{n}{:}\,\forall(fx){:}\,(v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash (v2n) <= (\underline{n} + 1) \vdash$
$\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{InduHelper}) \rhd (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \rhd (v2n) <= (\underline{n} + 1) \gg \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| +$
$1)))n)n)n; \forall(v1){:}\,\forall(v2n){:}\,\forall\underline{n}{:}\,\forall(fx){:}\,\mathrm{Ded} \rhd \forall(v1){:}\,\forall(v2n){:}\,\forall\underline{n}{:}\,\forall(fx){:}\,(v2n) <= \underline{n} \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash (v2n) <= (\underline{n}+1) \vdash$
$\dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{n} + 1)]| +$
$1)))n)n)n \gg (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$(v1))n)n)n \Rightarrow (v2n) <= (\underline{n} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{n} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n} + 1)]| + 1) <=$

$(v1), (v1), (|(fx)[(\underline{n}+1)]|+1)))n)n)n; \mathrm{AddAll} \rhd (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \Rightarrow (v2n) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)))n)n)n \gg \forall_{\mathrm{obj}}(v2n) \colon (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \Rightarrow \forall_{\mathrm{obj}}(v2n) \colon (v2n) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)))n)n)n; \mathrm{AddExist}(\mathrm{SimpleAnt}) @ \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)) \rhd \forall_{\mathrm{obj}}(v2n) \colon (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \Rightarrow \forall_{\mathrm{obj}}(v2n) \colon (v2n) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{n}+1)]|+1) <= (v1), (v1), (|(fx)[(\underline{n}+1)]|+1)))n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n) \colon (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n) \colon (v2n) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n\rceil, p_0, c)]$

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{Indu}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall(v1) \colon \forall(v2n) \colon \forall \underline{n} \colon \forall(fx) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n) \colon (v2n) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n) \colon (v2n) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n]$

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{Indu}) \overset{\mathrm{tex}}{\rightarrow}$ "Fpart-Bounded(Indu)"]

$[\mathrm{Fpart} - \mathrm{Bounded}(\mathrm{Indu}) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma fpart-Bounded indu"]

Fpart − Bounded

$[\mathrm{Fpart} - \mathrm{Bounded} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall(v1) \colon \forall(v2) \colon \forall \underline{n} \colon \forall(fx) \colon \mathrm{Fpart} - \mathrm{Bounded}(\mathrm{Base}) \gg \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= 0 \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n; \mathrm{Fpart} - \mathrm{Bounded}(\mathrm{Indu}) \gg \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n; \mathrm{Induction} \rhd \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= 0 \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n \rhd \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= (\underline{n}+1) \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n\rceil, p_0, c)]$

$[\mathrm{Fpart} - \mathrm{Bounded} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$

$\forall\underline{(v1)}\colon\forall\underline{(v2)}\colon\forall\underline{n}\colon\forall\underline{(fx)}\colon\dot{\neg}(\forall_{obj}\underline{(v1)}\colon\dot{\neg}(\forall_{obj}\underline{(v2)}\colon\underline{(v2)} <= \underline{n} \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n]$

$[\mathrm{Fpart}-\mathrm{Bounded} \stackrel{tex}{\to}$ "Fpart-Bounded"]

$[\mathrm{Fpart}-\mathrm{Bounded} \stackrel{pyk}{\to}$ "lemma fpart-Bounded"]

F − Bounded(Helper)

$[\mathrm{F}-\mathrm{Bounded(Helper)} \stackrel{proof}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon\dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \vdash$
NumericalDifferenceLess $\rhd \dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n;$ FirstConjunct $\rhd \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n \gg \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n;$ NegativeToRight(Less) $\rhd \dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \gg \dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n; x <= |x| \gg (\underline{x}+\underline{z}) <= |(\underline{x}+\underline{z})|;$ LessLeqTransitivity $\rhd \dot{\neg}(\underline{y} <= (\underline{x}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x}+\underline{z}))n)n)n \rhd (\underline{x}+\underline{z}) <= |(\underline{x}+\underline{z})| \gg \dot{\neg}(\underline{y} <= |(\underline{x}+\underline{z})| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = |(\underline{x}+\underline{z})|)n)n)n;$ MaxLeq(1) $\gg |(\underline{x}+\underline{z})| <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|);$ LessLeqTransitivity $\rhd \dot{\neg}(\underline{y} <= |(\underline{x}+\underline{z})| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = |(\underline{x}+\underline{z})|)n)n)n \rhd |(\underline{x}+\underline{z})| <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|) \gg \dot{\neg}(\underline{y} <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|))n)n)n;$ SecondConjunct $\rhd \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n;$ PositiveToLeft(Less) $\rhd \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n \gg \dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n; x <= |x| \gg (\underline{z}+(-u\underline{x})) <= |(\underline{z}+(-u\underline{x}))|;$ NumericalDifference $\gg |(\underline{z}+(-u\underline{x}))| = |(\underline{x}+(-u\underline{z}))|;$ subLeqRight $\rhd |(\underline{z}+(-u\underline{x}))| = |(\underline{x}+(-u\underline{z}))| \rhd (\underline{z}+(-u\underline{x})) <= |(\underline{z}+(-u\underline{x}))| \gg (\underline{z}+(-u\underline{x})) <= |(\underline{x}+(-u\underline{z}))|;$ LeqNegated $\rhd (\underline{z}+(-u\underline{x})) <= |(\underline{x}+(-u\underline{z}))| \gg (-u|(\underline{x}+(-u\underline{z}))|) <= (-u(\underline{z}+(-u\underline{x})));$ MinusNegated $\gg (-u(\underline{z}+(-u\underline{x}))) = (\underline{x}+(-u\underline{z}));$ subLeqRight $\rhd (-u(\underline{z}+(-u\underline{x}))) = (\underline{x}+(-u\underline{z})) \rhd (-u|(\underline{x}+(-u\underline{z}))|) <= (-u(\underline{z}+(-u\underline{x}))) \gg (-u|(\underline{x}+(-u\underline{z}))|) <= (\underline{x}+(-u\underline{z}));$ leqLessTransitivity $\rhd (-u|(\underline{x}+(-u\underline{z}))|) <= (\underline{x}+(-u\underline{z})) \rhd \dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n \gg \dot{\neg}((-u|(\underline{x}+(-u\underline{z}))|) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-u|(\underline{x}+(-u\underline{z}))|) = \underline{y})n)n)n;$ MaxLeq(2) $\gg |(\underline{x}+(-u\underline{z}))| <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|);$ LeqNegated $\rhd |(\underline{x}+(-u\underline{z}))| <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|) \gg (-u\mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) <= (-u|(\underline{x}+(-u\underline{z}))|);$ leqLessTransitivity $\rhd (-u\mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) <= (-u|(\underline{x}+(-u\underline{z}))|) \rhd \dot{\neg}((-u|(\underline{x}+(-u\underline{z}))|) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-u|(\underline{x}+(-u\underline{z}))|) = \underline{y})n)n)n \gg \dot{\neg}((-u\mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) <= \underline{y} \Rightarrow$

$\dot{\neg}(\dot{\neg}((-\mathrm{uif}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) = \underline{y})n)n)n; \mathrm{ToNumericalLess} \rhd \dot{\neg}((-\mathrm{uif}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{uif}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) = \underline{y})n)n)n \rhd \dot{\neg}(\underline{y} <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|))n)n)n \gg \dot{\neg}(|\underline{y}| <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{y}| = \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|))n)n)n\rceil, p_0, c)]$

$[\mathrm{F}-\mathrm{Bounded(Helper)} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon \dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}(|\underline{y}| <= \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{y}| = \mathrm{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|))n)n)n]$

$[\mathrm{F}-\mathrm{Bounded(Helper)} \overset{\mathrm{tex}}{\to}$ "F-Bounded(Helper)"$]$

$[\mathrm{F}-\mathrm{Bounded(Helper)} \overset{\mathrm{pyk}}{\to}$ "lemma f-Bounded helper"$]$

F − Bounded

$[\mathrm{F}-\mathrm{Bounded} \overset{\mathrm{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall(v1)\colon\forall(v2)\colon\forall\underline{n}\colon\forall(fx)\colon\forall_{\mathrm{obj}}(v1)\colon\forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= (v1) \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = 1)n)n)n \vdash \underline{n} <= (v2) \vdash \mathrm{A4} @ \underline{n} \rhd \forall_{\mathrm{obj}}(v1)\colon\forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= (v1) \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = 1)n)n)n \gg \forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{n} \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| = 1)n)n)n; \mathrm{A4} @ (v2) \rhd \forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{n} \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| = 1)n)n)n \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{n} \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| = 1)n)n)n; 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n; \mathrm{leqReflexivity} \gg \underline{n} <= \underline{n}; \mathrm{MP3} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{n} \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| = 1)n)n)n \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \rhd \underline{n} <= \underline{n} \rhd \underline{n} <= (v2) \gg \dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| = 1)n)n)n; \mathrm{F}-\mathrm{Bounded(Helper)} \rhd \dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{n}] + (-u(fx)[(v2)]))| = 1)n)n)n \gg \dot{\neg}(|(fx)[(v2)]| <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|))n)n)n; \forall(v1)\colon\forall(v2)\colon\forall\underline{n}\colon\forall(fx)\colon\forall_{\mathrm{obj}}(v2)\colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \vdash \dot{\neg}(\underline{n} <= (v2))n \vdash \mathrm{A4} @ (v2) \rhd \forall_{\mathrm{obj}}(v2)\colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \gg (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$

∸(∸(|(fx)[(v2)]| = (v1))n)n)n; ToLess ⊳ ∸(n <= (v2))n ≫ ∸((v2) <= n ⇒
∸(∸((v2) = n)n)n)n; LessLeq ⊳ ∸((v2) <= n ⇒ ∸(∸((v2) = n)n)n)n ≫
(v2) <= n; MP ⊳ (v2) <= n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| =
(v1))n)n)n ⊳ (v2) <= n ≫ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| =
(v1))n)n)n; ∀(v1): ∀(v2): ∀n: ∀(ϵ): ∀(fx): Ded ⊳
∀(v1): ∀(v2): ∀n: ∀(fx): ∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= 1 ⇒ ∸(∸(0 = 1)n)n)n ⇒
n <= (v1) ⇒ n <= (v2) ⇒ ∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= 1 ⇒
∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| = 1)n)n)n ⊢ n <= (v2) ⊢ ∸(|(fx)[(v2)]| <=
if(|((fx)[n] + (−u1))| <= |((fx)[n] + 1)|, |((fx)[n] + 1)|, |((fx)[n] + (−u1))|) ⇒
∸(∸(|(fx)[(v2)]| = if(|((fx)[n] + (−u1))| <= |((fx)[n] + 1)|, |((fx)[n] +
1)|, |((fx)[n] + (−u1))|))n)n)n ≫ ∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= 1 ⇒ ∸(∸(0 =
1)n)n)n ⇒ n <= (v1) ⇒ n <= (v2) ⇒ ∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= 1 ⇒
∸(∸(|((fx)[(v1)]+(−u(fx)[(v2)]))| = 1)n)n)n ⇒ n <= (v2) ⇒ ∸(|(fx)[(v2)]| <=
if(|((fx)[n] + (−u1))| <= |((fx)[n] + 1)|, |((fx)[n] + 1)|, |((fx)[n] + (−u1))|) ⇒
∸(∸(|(fx)[(v2)]| = if(|((fx)[n] + (−u1))| <= |((fx)[n] + 1)|, |((fx)[n] +
1)|, |((fx)[n] + (−u1))|))n)n)n; Ded ⊳ ∀(v1): ∀(v2): ∀n: ∀(fx): ∀$_{obj}$(v2): (v2) <=
n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| = (v1))n)n)n ⊢ ∸(n <=
(v2))n ⊢ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| = (v1))n)n)n ≫
∀$_{obj}$(v2): (v2) <= n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| =
(v1))n)n)n ⇒ ∸(n <= (v2))n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| =
(v1))n)n)n; Cauchy ≫ ∀$_{obj}$(ϵ): ∸(∀$_{obj}$n: ∸(∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= (ϵ) ⇒
∸(∸(0 = (ϵ))n)n)n ⇒ n <= (v1) ⇒ n <= (v2) ⇒
∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n)n)n; A4 @ 1 ⊳ ∀$_{obj}$(ϵ): ∸(∀$_{obj}$n: ∸(∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= (ϵ) ⇒
∸(∸(0 = (ϵ))n)n)n ⇒ n <= (v1) ⇒ n <= (v2) ⇒
∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n)n)n ≫ ∸(∀$_{obj}$n: ∸(∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= 1 ⇒ ∸(∸(0 =
1)n)n)n ⇒ n <= (v1) ⇒ n <= (v2) ⇒ ∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
1 ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| = 1)n)n)n)n)n; Fpart − Bounded ≫
∸(∀$_{obj}$(v1): ∸(∀$_{obj}$(v2): (v2) <= n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒
∸(∸(|(fx)[(v2)]| = (v1))n)n)n)n)n; ExistMP ⊳ ∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= 1 ⇒
∸(∸(0 = 1)n)n)n ⇒ n <= (v1) ⇒ n <= (v2) ⇒
∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= 1 ⇒ ∸(∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
1)n)n)n ⇒ n <= (v2) ⇒ ∸(|(fx)[(v2)]| <= if(|((fx)[n] + (−u1))| <=
|((fx)[n] + 1)|, |((fx)[n] + 1)|, |((fx)[n] + (−u1))|) ⇒ ∸(∸(|(fx)[(v2)]| =
if(|((fx)[n]+(−u1))| <= |((fx)[n]+1)|, |((fx)[n]+1)|, |((fx)[n]+(−u1))|))n)n)n ⊳
∸(∀$_{obj}$n: ∸(∀$_{obj}$(v1): ∀$_{obj}$(v2): ∸(0 <= 1 ⇒ ∸(∸(0 = 1)n)n)n ⇒ n <= (v1) ⇒
n <= (v2) ⇒ ∸(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= 1 ⇒ ∸(∸(|((fx)[(v1)] +
(−u(fx)[(v2)]))| = 1)n)n)n)n)n ≫ n <= (v2) ⇒ ∸(|(fx)[(v2)]| <=
if(|((fx)[n] + (−u1))| <= |((fx)[n] + 1)|, |((fx)[n] + 1)|, |((fx)[n] + (−u1))|) ⇒
∸(∸(|(fx)[(v2)]| = if(|((fx)[n]+(−u1))| <= |((fx)[n]+1)|, |((fx)[n]+1)|, |((fx)[n]+
(−u1))|))n)n)n; ExistMP ⊳ ∀$_{obj}$(v2): (v2) <= n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒
∸(∸(|(fx)[(v2)]| = (v1))n)n)n ⇒ ∸(n <= (v2))n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒
∸(∸(|(fx)[(v2)]| = (v1))n)n)n ⊳ ∸(∀$_{obj}$(v1): ∸(∀$_{obj}$(v2): (v2) <= n ⇒
∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| = (v1))n)n)n)n)n ≫ ∸(n <=
(v2))n ⇒ ∸(|(fx)[(v2)]| <= (v1) ⇒ ∸(∸(|(fx)[(v2)]| =

$\underline{(v1)})n)n)n; \mathrm{LessThanMax} \rhd \underline{\mathsf{n}} <= \underline{(v2)} \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <=$
$\overline{\mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|))n)n)n \rhd \dot{\neg}(\underline{\mathsf{n}} <= \underline{(v2)})n \Rightarrow$
$\dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \gg \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)))n)n)n; \mathrm{Gen} \rhd$
$\dot{\neg}(|(fx)[(v2)]| <= \mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)))n)n)n \gg$
$\forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(|(fx)[(v2)]| <= \mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] +$
$(-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] +$
$(-u1))|), (v1)))n)n)n; \mathrm{IntroExist} @ \mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}]+1)|, |((fx)[\mathsf{n}]+1)|, |((fx)[\mathsf{n}]+(-u1))|), (v1)) \rhd \forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <=$
$|((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|), (v1)))n)n)n \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$(v1))n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{F-Bounded} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(v1)\colon \forall(v2)\colon \forall \mathsf{n}\colon \forall(\epsilon)\colon \forall(fx)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2)\colon \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n]$

$[\mathrm{F-Bounded} \stackrel{\mathrm{tex}}{\rightarrow}$ "F-Bounded"$]$

$[\mathrm{F-Bounded} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma f-Bounded"$]$

SameFmultiplication(Helper)

$[\mathrm{SameFmultiplication(Helper)} \overset{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall(v1){:}\, \forall(v2){:}\, \forall\underline{\mathsf{m}}{:}\, \forall\underline{\mathsf{n}}{:}\, \forall(\epsilon){:}\, \forall(fx){:}\, \forall(fy){:}\, \forall(fz){:}\, \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\, \dot{\neg}(0 <= ((\epsilon) * \mathrm{rec}(v1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| <=$
$((\epsilon) * \mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \vdash$
$\forall_{\mathrm{obj}}(v2){:}\, \dot{\neg}(|(fz)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fz)[(v2)]| = (v1))n)n)n \vdash \dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash \mathrm{A4} @ (v2) \rhd \forall_{\mathrm{obj}}(v2){:}\, \dot{\neg}(|(fz)[(v2)]| <=$
$(v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fz)[(v2)]| = (v1))n)n)n \gg \dot{\neg}(|(fz)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fz)[(v2)]| = (v1))n)n)n; 0 <= |x| \gg 0 <=$
$|(fz)[(v2)]|; \mathrm{leqLessTransitivity} \rhd 0 <= |(fz)[(v2)]| \rhd \dot{\neg}(|(fz)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fz)[(v2)]| = (v1))n)n)n \gg \dot{\neg}(0 <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(v1))n)n)n; \mathrm{PositiveInverted} \rhd \dot{\neg}(0 <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (v1))n)n)n \gg$
$\dot{\neg}(0 <= \mathrm{rec}(v1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}(v1))n)n)n; \mathrm{PositiveFactors} \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \rhd \dot{\neg}(0 <= \mathrm{rec}(v1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}(v1))n)n)n \gg \dot{\neg}(0 <=$
$((\epsilon) * \mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(v1)))n)n)n; \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\, \dot{\neg}(0 <=$
$((\epsilon) * \mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| <= ((\epsilon) * \mathrm{rec}(v1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \gg \dot{\neg}(0 <=$
$((\epsilon) * \mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((fx)[\underline{\mathsf{m}}]+(-u(fy)[\underline{\mathsf{m}}]))| <= ((\epsilon)*\mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{\mathsf{m}}]+(-u(fy)[\underline{\mathsf{m}}]))| =$
$((\epsilon) * \mathrm{rec}(v1)))n)n)n; \mathrm{MP2} \rhd \dot{\neg}(0 <= ((\epsilon) * \mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon) * \mathrm{rec}(v1)))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| <=$
$((\epsilon)*\mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{\mathsf{m}}]+(-u(fy)[\underline{\mathsf{m}}]))| = ((\epsilon)*\mathrm{rec}(v1)))n)n)n \rhd \dot{\neg}(0 <=$
$((\epsilon) * \mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \gg$
$\dot{\neg}(|((fx)[\underline{\mathsf{m}}]+(-u(fy)[\underline{\mathsf{m}}]))| <= ((\epsilon)*\mathrm{rec}(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{\mathsf{m}}]+(-u(fy)[\underline{\mathsf{m}}]))| =$
$((\epsilon) * \mathrm{rec}(v1)))n)n)n; \mathrm{reciprocalToLeft(Less)} \rhd \dot{\neg}(0 <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(v1))n)n)n \rhd \dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| <= ((\epsilon) * \mathrm{rec}(v1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(v1)))n)n)n \gg$
$\dot{\neg}((|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| * (v1)) <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}((|((fx)[\underline{\mathsf{m}}] + (-u(fy)[\underline{\mathsf{m}}]))| * (v1)) = (\epsilon))n)n)n; \mathrm{TimesF} \gg \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1){:}\, \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2){:}\, \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\, \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] =$
$((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}]); \mathrm{TimesF} \gg \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(op1){:}\, \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2){:}\, \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\, \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] =$
$((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}]); \mathrm{EqNegated} \rhd \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(op1){:}\, \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2){:}\, \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\, \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}]) \gg (-u\{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1){:}\, \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2){:}\, \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$

$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) = (-\mathrm{u}(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])); -\mathrm{x} * \mathrm{y} =$
$-(\mathrm{x} * \mathrm{y}) \gg ((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) = (-\mathrm{u}(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])); \mathrm{eqSymmetry} \rhd$
$((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) = (-\mathrm{u}(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) \gg (-\mathrm{u}(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) =$
$((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]); \mathrm{eqTransitivity} \rhd (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) = (-\mathrm{u}(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] *$
$\underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) \rhd (-\mathrm{u}(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) = ((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) \gg (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) =$
$((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]); \mathrm{AddEquations} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) \rhd (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) =$
$((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) \gg (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])) =$
$(((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + ((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])); \mathrm{DistributionOutLeft} \gg$
$(((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + ((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) =$
$((\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))); \mathrm{eqTransitivity} \rhd (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])) =$
$(((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + ((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) \rhd (((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) +$
$((-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])) = ((\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))) \gg (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])) =$
$(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))); \mathrm{SameNumerical} \rhd (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])) =$
$(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))) \gg |(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$|(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}])))|; \mathrm{SplitNumericalProduct} \gg$
$|(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}])))| = (|\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]| * |(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|); \mathrm{timesCommutativity} \gg (|\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]| * |(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|) =$
$(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]|); \mathrm{eqTransitivity4} \rhd |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$|(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}])))| \rhd |(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}])))| =$
$(|\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]| * |(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|) \rhd (|\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]| * |(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|) =$
$(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) \gg |(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\underline{(\mathrm{fz})}[\underline{\mathsf{m}}]|); \mathrm{eqSymmetry} \rhd |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| = (|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] +$

$(-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|*|(\mathrm{fz})[\underline{\mathsf{m}}]|) \gg (|((\mathrm{fx})[\underline{\mathsf{m}}]+(-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|*|(\mathrm{fz})[\underline{\mathsf{m}}]|) = |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))|; \mathrm{A4}\,@\,(\mathrm{fz})[\underline{\mathsf{m}}] \rhd$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot\neg(|(\mathrm{fz})[(\mathrm{v2})]| <= (\mathrm{v1}) \Rightarrow \dot\neg(\dot\neg(|(\mathrm{fz})[(\mathrm{v2})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$|(\mathrm{fz})[\underline{\mathsf{m}}]| <= (\mathrm{v1}); 0 <= |\mathrm{x}| \gg 0 <=$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|; \mathrm{LeqMultiplicationLeft} \rhd 0 <=$
$|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \rhd |(\mathrm{fz})[\underline{\mathsf{m}}]| <= (\mathrm{v1}) \gg$
$(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) <= (|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| *$
$(\mathrm{v1})); \mathrm{leqLessTransitivity} \rhd (|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) <=$
$(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * (\mathrm{v1})) \rhd \dot\neg((|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * (\mathrm{v1})) <=$
$(\epsilon) \Rightarrow \dot\neg(\dot\neg((|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * (\mathrm{v1})) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg((|((\mathrm{fx})[\underline{\mathsf{m}}] +$
$(-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) <= (\epsilon) \Rightarrow \dot\neg(\dot\neg((|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubLessLeft} \rhd (|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) = |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| \rhd$
$\dot\neg((|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg((|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| * |(\mathrm{fz})[\underline{\mathsf{m}}]|) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \forall(\mathrm{v1}){:}\,\forall(\mathrm{v2}){:}\,\forall\underline{\mathsf{m}}{:}\,\forall\underline{\mathsf{n}}{:}\,\forall(\epsilon){:}\,\forall(\mathrm{fx}){:}\,\forall(\mathrm{fy}){:}\,\forall(\mathrm{fz}){:}\,\mathrm{Ded} \rhd$
$\forall(\mathrm{v1}){:}\,\forall(\mathrm{v2}){:}\,\forall\underline{\mathsf{m}}{:}\,\forall\underline{\mathsf{n}}{:}\,\forall(\epsilon){:}\,\forall(\mathrm{fx}){:}\,\forall(\mathrm{fy}){:}\,\forall(\mathrm{fz}){:}\,\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(0 <= ((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow$
$\dot\neg(\dot\neg(0 = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <=$
$((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow \dot\neg(\dot\neg(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot\neg(|(\mathrm{fz})[(\mathrm{v2})]| <= (\mathrm{v1}) \Rightarrow \dot\neg(\dot\neg(|(\mathrm{fz})[(\mathrm{v2})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot\neg(0 <=$

$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})n)n)n \vdash \underline{n} <= \underline{m} \vdash \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| = \underline{(\epsilon)})n)n)n \gg$
$\forall_{obj}\underline{m}: \dot\neg(0 <= (\underline{(\epsilon)} * rec(v1)) \Rightarrow \dot\neg(\dot\neg(0 = (\underline{(\epsilon)} * rec(v1)))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow$
$\dot\neg(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= (\underline{(\epsilon)} * rec(v1)) \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = (\underline{(\epsilon)} * rec(v1)))n)n)n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot\neg(|\underline{(fz)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot\neg(\dot\neg(|\underline{(fz)}[\underline{(v2)}]| = \underline{(v1)})n)n)n \Rightarrow \dot\neg(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$\underline{(\epsilon)})n)n)n\rceil, p_0, c)]$

$[SameFmultiplication(Helper) \overset{stmt}{\rightarrow} SystemQ \vdash$
$\forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{m}: \forall\underline{n}: \forall\underline{(\epsilon)}: \forall\underline{(fx)}: \forall\underline{(fy)}: \forall\underline{(fz)}: \forall_{obj}\underline{m}: \dot\neg(0 <= (\underline{(\epsilon)} * rec(v1)) \Rightarrow$
$\dot\neg(\dot\neg(0 = (\underline{(\epsilon)} * rec(v1)))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <=$
$(\underline{(\epsilon)} * rec(v1)) \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = (\underline{(\epsilon)} * rec(v1)))n)n)n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot\neg(|\underline{(fz)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot\neg(\dot\neg(|\underline{(fz)}[\underline{(v2)}]| = \underline{(v1)})n)n)n \Rightarrow \dot\neg(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{SameFmultiplication(Helper)} \overset{\mathrm{tex}}{\to}$ "SameFmultiplication(Helper)"$]$

$[\mathrm{SameFmultiplication(Helper)} \overset{\mathrm{pyk}}{\to}$ "lemma sameFmultiplication helper"$]$

SameFmultiplication

$[\mathrm{SameFmultiplication} \overset{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ Repetition $\rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \rhd \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}))\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{F} - \mathrm{Bounded} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}(|\underline{(\mathrm{fz})}[\underline{(\mathrm{v2})}]| <= \underline{(\mathrm{v1})} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(\mathrm{fz})}[\underline{(\mathrm{v2})}]| = \underline{(\mathrm{v1})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SameFmultiplication(Helper)} \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}))\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}))\mathrm{n})\mathrm{n} \Rightarrow$

$\forall_{obj}(v2)\colon \dot\neg(|(fz)[(v2)]| <= (v1) \Rightarrow \dot\neg(\dot\neg(|(fz)[(v2)]| = (v1))n)n)n \Rightarrow \dot\neg(0 <=$
$(\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$(\epsilon))n)n)n; ExistMP2 \rhd \forall_{obj}\underline{m}\colon \dot\neg(0 <= ((\epsilon) * rec(v1)) \Rightarrow \dot\neg(\dot\neg(0 =$
$((\epsilon) * rec(v1)))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| <=$
$((\epsilon) * rec(v1)) \Rightarrow \dot\neg(\dot\neg(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| = ((\epsilon) * rec(v1)))n)n)n \Rightarrow$
$\forall_{obj}(v2)\colon \dot\neg(|(fz)[(v2)]| <= (v1) \Rightarrow \dot\neg(\dot\neg(|(fz)[(v2)]| = (v1))n)n)n \Rightarrow \dot\neg(0 <=$
$(\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$(\epsilon))n)n)n \rhd \dot\neg(\forall_{obj}\underline{n}\colon \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(0 <= ((\epsilon) * rec(v1)) \Rightarrow \dot\neg(\dot\neg(0 =$
$((\epsilon) * rec(v1)))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot\neg(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| <=$
$((\epsilon) * rec(v1)) \Rightarrow \dot\neg(\dot\neg(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| =$
$((\epsilon) * rec(v1)))n)n)n)n)n \rhd \dot\neg(\forall_{obj}(v1)\colon \dot\neg(\forall_{obj}(v2)\colon \dot\neg(|(fz)[(v2)]| <= (v1) \Rightarrow$
$\dot\neg(\dot\neg(|(fz)[(v2)]| = (v1))n)n)n)n)n \gg \dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow$
$\underline{n} <= \underline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$

$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow$
$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$\underline{(\epsilon)})n)n)n; \mathrm{Gen} \triangleright \dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in$
$N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} =$
$\{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow$
$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow$
$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| = \underline{(\epsilon)})n)n)n \gg$
$\forall_{obj}\underline{m}: \dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{-}(|(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow$
$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow$
$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow$
$\dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$\underline{(\epsilon)})n)n)n; \mathrm{IntroExist} @ \underline{n} \triangleright \forall_{obj}\underline{m}: \dot{-}(0 <= \underline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <=$
$\underline{m} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1): \dot{-}(\dot{-}(\forall_{obj}(op2): \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{m}: \dot{-}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \underline{(\epsilon)} \colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})n)n)n)n)n; \mathrm{Ded} \rhd \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})n)n)n)n)n \gg \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{\overline{(\epsilon)}})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$

$\overline{(\epsilon)})n)n)n)n)n; \mathrm{Repetition} \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n \gg \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{SameFmultiplication} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(fz)}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \vdash$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n]$

[SameFmultiplication $\stackrel{tex}{\to}$ "SameFmultiplication"]

[SameFmultiplication $\stackrel{pyk}{\to}$ "lemma sameFmultiplication"]

EqMultiplication(R)

[EqMultiplication(R) $\stackrel{proof}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(fz)}\colon \{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(fy)}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash From == \rhd\{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SameFmultiplication} \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{To} == \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-\mathsf{ud}_{Ph}[\overline{m}]))| =$
$(\epsilon))n)n)n)n)n\}; \mathrm{eqReflexivity} \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}(r1)\colon (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot\neg(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot\neg(\forall_{obj}(s2)\colon \dot\neg(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot\neg(\forall_{obj}\overline{n}\colon \dot\neg(\forall_{obj}\overline{m}\colon \dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-\mathsf{ud}_{Ph}[\overline{m}]))| <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-\mathsf{ud}_{Ph}[\overline{m}]))| = (\epsilon))n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}(r1)\colon (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot\neg(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot\neg(\forall_{obj}(s2)\colon \dot\neg(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot\neg(\forall_{obj}\overline{n}\colon \dot\neg(\forall_{obj}\overline{m}\colon \dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-\mathsf{ud}_{Ph}[\overline{m}]))| <= (\epsilon) \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-\mathsf{ud}_{Ph}[\overline{m}]))| =$
$(\epsilon))n)n)n)n)n\}; \mathrm{eqReflexivity} \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}(r1)\colon (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot\neg(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot\neg(\forall_{obj}(s2)\colon \dot\neg(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqTransitivity4} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$

$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1): (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1): \forall_{obj}(f2): \forall_{obj}(f3): \forall_{obj}(f4): \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2): \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon): \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1): (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1): \forall_{obj}(f2): \forall_{obj}(f3): \forall_{obj}(f4): \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2): \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon): \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{EqMultiplication(R)} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{ph \in \{ph \in P(\overline{P(Union(\{N, Q\})))} \,|$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

$[\mathrm{EqMultiplication(R)} \overset{\mathrm{tex}}{\rightarrow}$ "EqMultiplication(R)"]

$[\mathrm{EqMultiplication(R)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma eqMultiplication(R)"]

EqMultiplicationLeft(R)

$[\mathrm{EqMultiplicationLeft(R)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \forall \underline{(fz)}\colon \{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \,|$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \mathrm{EqMultiplication(R)} \rhd \{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \,|$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

∸(∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀obj m: ∸(ePh =
{{m, m}, {m, ((fy)[m] ∗ (fz)[m])}})n)n}[m] + (−udPh[m]))| =
(ϵ))n)n)n)n)n}; TimesCommutativity(R) ≫ {ph ∈ P({ph ∈ P({ph ∈
P(P(Union({N, Q}))) | ∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) |
∸(∸(∀obj(r1): (r1) ∈ fPh ⇒ ∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸(∀obj(f1): ∀obj(f2): ∀obj(f3): ∀obj(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ fPh ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ fPh ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸(∀obj(s1): (s1) ∈ N ⇒ ∸(∀obj(s2): ∸({{(s1), (s1)}, {(s1), (s2)}} ∈
fPh)n)n)n)n}) | ∀obj(ϵ): ∸(∀obj n: ∸(∀obj m: ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
n <= m ⇒ ∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀obj m: ∸(ePh =
{{m, m}, {m, ((fz)[m] ∗ (fx)[m])}})n)n}[m] + (−udPh[m]))| <= (ϵ) ⇒
∸(∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀obj m: ∸(ePh =
{{m, m}, {m, ((fz)[m] ∗ (fx)[m])}})n)n}[m] + (−udPh[m]))| = (ϵ))n)n)n)n)n} =
{ph ∈ P({ph ∈ P({ph ∈ P(P(Union({N, Q}))) |
∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) | ∸(∸(∀obj(r1): (r1) ∈
fPh ⇒ ∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸(∀obj(f1): ∀obj(f2): ∀obj(f3): ∀obj(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ fPh ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ fPh ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸(∀obj(s1): (s1) ∈ N ⇒ ∸(∀obj(s2): ∸({{(s1), (s1)}, {(s1), (s2)}} ∈
fPh)n)n)n)n}) | ∀obj(ϵ): ∸(∀obj n: ∸(∀obj m: ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
n <= m ⇒ ∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀obj m: ∸(ePh =
{{m, m}, {m, ((fx)[m] ∗ (fz)[m])}})n)n}[m] + (−udPh[m]))| <= (ϵ) ⇒
∸(∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀obj m: ∸(ePh =
{{m, m}, {m, ((fx)[m] ∗ (fz)[m])}})n)n}[m] + (−udPh[m]))| =
(ϵ))n)n)n)n)n}; TimesCommutativity(R) ≫ {ph ∈ P({ph ∈ P({ph ∈
P(P(Union({N, Q}))) | ∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(aPh = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) |
∸(∸(∀obj(r1): (r1) ∈ fPh ⇒ ∸(∀obj(op1): ∸(∸(∀obj(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒

$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\,\forall_{\mathrm{obj}}(\mathrm{f2}){:}\,\forall_{\mathrm{obj}}(\mathrm{f3}){:}\,\forall_{\mathrm{obj}}(\mathrm{f4}){:}\,\{\{(\mathrm{f1}),(\mathrm{f1})\},\{(\mathrm{f1}),(\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}),(\mathrm{f3})\},\{(\mathrm{f3}),(\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\,(\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\,\dot{\neg}(\{\{(\mathrm{s1}),(\mathrm{s1})\},\{(\mathrm{s1}),(\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon){:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\,(\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\,\forall_{\mathrm{obj}}(\mathrm{f2}){:}\,\forall_{\mathrm{obj}}(\mathrm{f3}){:}\,\forall_{\mathrm{obj}}(\mathrm{f4}){:}\,\{\{(\mathrm{f1}),(\mathrm{f1})\},\{(\mathrm{f1}),(\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}),(\mathrm{f3})\},\{(\mathrm{f3}),(\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\,(\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\,\dot{\neg}(\{\{(\mathrm{s1}),(\mathrm{s1})\},\{(\mathrm{s1}),(\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon){:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\};$ $==\mathrm{Transitivity} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\,(\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\,\forall_{\mathrm{obj}}(\mathrm{f2}){:}\,\forall_{\mathrm{obj}}(\mathrm{f3}){:}\,\forall_{\mathrm{obj}}(\mathrm{f4}){:}\,\{\{(\mathrm{f1}),(\mathrm{f1})\},\{(\mathrm{f1}),(\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}),(\mathrm{f3})\},\{(\mathrm{f3}),(\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\,(\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\,\dot{\neg}(\{\{(\mathrm{s1}),(\mathrm{s1})\},\{(\mathrm{s1}),(\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon){:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; == \mathrm{Transitivity} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fz})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fz})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$

$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fz}})[\underline{\mathsf{m}}]*(\underline{\mathrm{fx}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=$
$\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fz}})[\underline{\mathsf{m}}]*(\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fz}})[\underline{\mathsf{m}}]*(\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil,\mathrm{p}_0,\mathsf{c})]$

$[\mathrm{EqMultiplicationLeft(R)}\overset{\mathrm{stmt}}{\to}\mathrm{SystemQ}\vdash\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\forall\underline{(\mathrm{fz})}\colon\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in$
$\mathrm{P}(\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in$
$\mathrm{N}\Rightarrow\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|((\underline{\mathrm{fx}})[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{\mathrm{fx}})[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})}\in \mathrm{N}\Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})}\in \mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in \mathrm{N}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|((\underline{\mathrm{fy}})[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{\mathrm{fy}})[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\vdash\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in$

$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

[EqMultiplicationLeft(R) $\overset{\text{tex}}{\to}$ “EqMultiplicationLeft(R)”]

[EqMultiplicationLeft(R) $\overset{\text{pyk}}{\to}$ “lemma eqMultiplicationLeft(R)”]

$x * 0 = 0(F)$

[$x * 0 = 0(F) \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}: \forall \underline{(fx)}: \mathrm{TimesF} \gg \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]); \mathrm{NatType} \gg \underline{\mathsf{m}} \in$

$\mathrm{N}; 0\mathrm{f} \rhd \underline{\mathsf{m}} \in \mathrm{N} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0; \mathrm{EqMultiplicationLeft} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \gg$

$(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * 0); \mathrm{x} * 0 = 0 \gg$

$(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * 0) = 0; \mathrm{eqSymmetry} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \gg 0 = \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{eqTransitivity5} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) \rhd (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in$

$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$

$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})} \colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * 0) \rhd (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * 0) = 0 \rhd 0 =$

$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{Gen} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{To} =$
$\mathrm{f} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}\rceil, p_0, \mathsf{c})]$

$[x * 0 = 0(F) \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(fx)}\colon \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}]$

$[x * 0 = 0(F) \stackrel{tex}{\rightarrow}$ "x*0=0(F)"$]$

$[x * 0 = 0(F) \stackrel{pyk}{\rightarrow}$ "lemma x*0=0(F)"$]$

$x * 0 = 0(R)$

$[x * 0 = 0(R) \stackrel{proof}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall\underline{(fx)}\colon x * 0 = 0(F) \gg \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}; EqualsSameF \rhd \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\} = \{ph \in \{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{To} == \rhd\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$

$\overline{(\epsilon)})n)n)n)n)n\}; \mathrm{eqReflexivity} \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\overline{(\text{crs1})}\colon\dot\neg(\mathsf{c}_{\text{Ph}}=$
$\{\{\overline{(\text{crs1})},\overline{(\text{crs1})}\},\{\overline{(\text{crs1})},0\}\})\text{n})\text{n}\}[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))|=$
$(\epsilon))\text{n})\text{n})\text{n})\text{n})\text{n}\};==\text{Transitivity}\rhd\{\text{ph}\in \text{P}(\{\text{ph}\in \text{P}(\{\text{ph}\in$
$\text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow$
$\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\})\mid$
$\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{r1})}\colon\overline{(\text{r1})}\in\mathsf{f}_{\text{Ph}}\Rightarrow\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow$
$\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow\dot\neg(\overline{(\text{r1})}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\Rightarrow$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{f1})}\colon\forall_{\text{obj}}\overline{(\text{f2})}\colon\forall_{\text{obj}}\overline{(\text{f3})}\colon\forall_{\text{obj}}\overline{(\text{f4})}\colon\{\{\overline{(\text{f1})},\overline{(\text{f1})}\},\{\overline{(\text{f1})},\overline{(\text{f2})}\}\}\in\mathsf{f}_{\text{Ph}}\Rightarrow$
$\{\{\overline{(\text{f3})},\overline{(\text{f3})}\},\{\overline{(\text{f3})},\overline{(\text{f4})}\}\}\in\mathsf{f}_{\text{Ph}}\Rightarrow\overline{(\text{f1})}=\overline{(\text{f3})}\Rightarrow\overline{(\text{f2})}=\overline{(\text{f4})})\text{n})\text{n}\Rightarrow$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{s1})}\colon\overline{(\text{s1})}\in \text{N}\Rightarrow\dot\neg(\forall_{\text{obj}}\overline{(\text{s2})}\colon\dot\neg(\{\{\overline{(\text{s1})},\overline{(\text{s1})}\},\{\overline{(\text{s1})},\overline{(\text{s2})}\}\}\in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\})\mid\forall_{\text{obj}}\overline{(\epsilon)}\colon\dot\neg(\forall_{\text{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\text{obj}}\overline{\mathsf{m}}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))\text{n})\text{n})\text{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg(|(\{\text{ph}\in\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{e}_{\text{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\text{fx})[\underline{\mathsf{m}}]*\{\text{ph}\in\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\overline{(\text{crs1})}\colon\dot\neg(\mathsf{c}_{\text{Ph}}=$
$\{\{\overline{(\text{crs1})},\overline{(\text{crs1})}\},\{\overline{(\text{crs1})},0\}\})\text{n})\text{n}\}[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg(\dot\neg(|(\{\text{ph}\in\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{e}_{\text{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\text{fx})[\underline{\mathsf{m}}]*\{\text{ph}\in\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\overline{(\text{crs1})}\colon\dot\neg(\mathsf{c}_{\text{Ph}}=$
$\{\{\overline{(\text{crs1})},\overline{(\text{crs1})}\},\{\overline{(\text{crs1})},0\}\})\text{n})\text{n}\}[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))|=$
$(\epsilon))\text{n})\text{n})\text{n})\text{n})\text{n}\}=\{\text{ph}\in \text{P}(\{\text{ph}\in \text{P}(\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\})\mid\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{r1})}\colon\overline{(\text{r1})}\in$
$\mathsf{f}_{\text{Ph}}\Rightarrow\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\overline{(\text{r1})}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\Rightarrow$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{f1})}\colon\forall_{\text{obj}}\overline{(\text{f2})}\colon\forall_{\text{obj}}\overline{(\text{f3})}\colon\forall_{\text{obj}}\overline{(\text{f4})}\colon\{\{\overline{(\text{f1})},\overline{(\text{f1})}\},\{\overline{(\text{f1})},\overline{(\text{f2})}\}\}\in\mathsf{f}_{\text{Ph}}\Rightarrow$
$\{\{\overline{(\text{f3})},\overline{(\text{f3})}\},\{\overline{(\text{f3})},\overline{(\text{f4})}\}\}\in\mathsf{f}_{\text{Ph}}\Rightarrow\overline{(\text{f1})}=\overline{(\text{f3})}\Rightarrow\overline{(\text{f2})}=\overline{(\text{f4})})\text{n})\text{n}\Rightarrow$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{s1})}\colon\overline{(\text{s1})}\in \text{N}\Rightarrow\dot\neg(\forall_{\text{obj}}\overline{(\text{s2})}\colon\dot\neg(\{\{\overline{(\text{s1})},\overline{(\text{s1})}\},\{\overline{(\text{s1})},\overline{(\text{s2})}\}\}\in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\})\mid\forall_{\text{obj}}\overline{(\epsilon)}\colon\dot\neg(\forall_{\text{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\text{obj}}\overline{\mathsf{m}}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))\text{n})\text{n})\text{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg(|(\{\text{ph}\in\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{e}_{\text{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\text{fx})[\underline{\mathsf{m}}]*\{\text{ph}\in\{\text{ph}\in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\})))\mid$
$\dot\neg(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot\neg(\dot\neg(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})}\in \text{N}\Rightarrow\dot\neg(\overline{(\text{op2})}\in \text{Q})\text{n})\text{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\text{Ph}}=\{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}\mid\dot\neg(\forall_{\text{obj}}\overline{(\text{crs1})}\colon\dot\neg(\mathsf{c}_{\text{Ph}}=$
$\{\{\overline{(\text{crs1})},\overline{(\text{crs1})}\},\{\overline{(\text{crs1})},0\}\})\text{n})\text{n}\}[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$

$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(|(\{ph\in$
$\{ph\in P(P(Union(\{N,Q\})))\mid\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in$
$N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\mathsf{a}_{Ph}=$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon\dot{\neg}(\mathsf{c}_{Ph}=$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}\gg$
$\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}]*\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon\dot{\neg}(\mathsf{c}_{Ph}=$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}]*\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon\dot{\neg}(\mathsf{c}_{Ph}=$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=$
$(\epsilon))n)n)n)n)n\}=\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon\dot{\neg}(\mathsf{c}_{Ph}=$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(|(\{ph\in$
$\{ph\in P(P(Union(\{N,Q\})))\mid\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in$
$N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\mathsf{a}_{Ph}=$

$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; \text{Repetition} \triangleright \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}], p_0, \mathsf{c})]$

$[x * 0 = 0(R) \overset{stmt}{\to} SystemQ \vdash \forall\underline{(fx)}: \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$

$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

$[x * 0 = 0(R) \stackrel{tex}{\rightarrow}$ "x*0=0(R)"$]$

$[x * 0 = 0(R) \stackrel{pyk}{\rightarrow}$ "lemma x*0=0(R)"$]$

LessMultiplication(F)(Helper2)

$[\mathrm{LessMultiplication(F)(Helper2)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \forall \underline{v}\colon \dot{\neg}(0 <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{u})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{x} <= (\underline{y} + (-\mathrm{u}\underline{u})) \vdash 0 <= (\underline{z} + (-\mathrm{u}\underline{v})) \vdash \mathrm{negativeToLeft(Leq)} \rhd \underline{x} <= (\underline{y} + (-\mathrm{u}\underline{u})) \gg (\underline{x} + \underline{u}) <= \underline{y}; \mathrm{plusCommutativity} \gg (\underline{x} + \underline{u}) = (\underline{u} + \underline{x}); \mathrm{subLeqLeft} \rhd (\underline{x} + \underline{u}) = (\underline{u} + \underline{x}) \rhd (\underline{x} + \underline{u}) <= \underline{y} \gg (\underline{u} + \underline{x}) <= \underline{y}; \mathrm{PositiveToRight(Leq)} \rhd (\underline{u} + \underline{x}) <= \underline{y} \gg \underline{u} <= (\underline{y} + (-\mathrm{u}\underline{x})); \mathrm{negativeToLeft(Leq)(1term)} \rhd 0 <= (\underline{z} + (-\mathrm{u}\underline{v})) \gg \underline{v} <= \underline{z}; \mathrm{LessLeq} \rhd \dot{\neg}(0 <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{u})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{u}; \mathrm{LessLeq} \rhd \dot{\neg}(0 <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{v}; \mathrm{MultiplyEquations(Leq)} \rhd 0 <= \underline{u} \rhd 0 <= \underline{v} \rhd \underline{u} <= (\underline{y} + (-\mathrm{u}\underline{x})) \rhd \underline{v} <= \underline{z} \gg (\underline{u} * \underline{v}) <= ((\underline{y} + (-\mathrm{u}\underline{x})) * \underline{z}); \mathrm{timesCommutativity} \gg ((\underline{y} + (-\mathrm{u}\underline{x})) * \underline{z}) = (\underline{z} * (\underline{y} + (-\mathrm{u}\underline{x}))); \mathrm{DistributionLeft} \gg (\underline{z} * (\underline{y} + (-\mathrm{u}\underline{x}))) = ((\underline{y} * \underline{z}) + ((-\mathrm{u}\underline{x}) * \underline{z})); -\mathrm{x} * \mathrm{y} = -(\mathrm{x} * \mathrm{y}) \gg ((-\mathrm{u}\underline{x}) * \underline{z}) = (-\mathrm{u}(\underline{x} * \underline{z})); \mathrm{EqAdditionLeft} \rhd ((-\mathrm{u}\underline{x}) * \underline{z}) = (-\mathrm{u}(\underline{x} * \underline{z})) \gg ((\underline{y} * \underline{z}) + ((-\mathrm{u}\underline{x}) * \underline{z})) = ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{x} * \underline{z}))); \mathrm{eqTransitivity4} \rhd ((\underline{y} + (-\mathrm{u}\underline{x})) * \underline{z}) = (\underline{z} * (\underline{y} + (-\mathrm{u}\underline{x}))) \rhd (\underline{z} * (\underline{y} + (-\mathrm{u}\underline{x}))) = ((\underline{y} * \underline{z}) + ((-\mathrm{u}\underline{x}) * \underline{z})) \rhd ((\underline{y} * \underline{z}) + ((-\mathrm{u}\underline{x}) * \underline{z})) = ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{x} * \underline{z}))) \gg ((\underline{y} + (-\mathrm{u}\underline{x})) * \underline{z}) = ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{x} * \underline{z}))); \mathrm{subLeqRight} \rhd ((\underline{y} + (-\mathrm{u}\underline{x})) * \underline{z}) = ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{x} * \underline{z}))) \rhd (\underline{u} * \underline{v}) <= ((\underline{y} + (-\mathrm{u}\underline{x})) * \underline{z}) \gg (\underline{u} * \underline{v}) <= ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{x} * \underline{z}))); \mathrm{negativeToLeft(Leq)} \rhd (\underline{u} * \underline{v}) <= ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{x} * \underline{z}))) \gg ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) <= (\underline{y} * \underline{z}); \mathrm{plusCommutativity} \gg ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) = ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})); \mathrm{subLeqLeft} \rhd ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) = ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})) \rhd ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) <= (\underline{y} * \underline{z}) \gg ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})) <= (\underline{y} * \underline{z}); \mathrm{PositiveToRight(Leq)} \rhd ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})) <= (\underline{y} * \underline{z}) \gg (\underline{x} * \underline{z}) <= ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{u} * \underline{v})))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessMultiplication(F)(Helper2)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \forall \underline{v}\colon \dot{\neg}(0 <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{u})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{x} <= (\underline{y} + (-\mathrm{u}\underline{u})) \vdash 0 <= (\underline{z} + (-\mathrm{u}\underline{v})) \vdash (\underline{x} * \underline{z}) <= ((\underline{y} * \underline{z}) + (-\mathrm{u}(\underline{u} * \underline{v})))]$

$[\mathrm{LessMultiplication(F)(Helper2)} \overset{\mathrm{tex}}{\rightarrow}$ "LessMultiplication(F)(Helper2)"$]$

$[\mathrm{LessMultiplication(F)(Helper2)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma lessMultiplication(F) helper2"$]$

LessMultiplication(F)(Helper)

$[\mathrm{LessMultiplication(F)(Helper)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{m}\colon \forall \underline{n}\colon \forall \overline{(\epsilon)_1}\colon \forall \overline{(\epsilon 2)}\colon \forall \overline{(\mathrm{fx})}\colon \forall \overline{(\mathrm{fy})}\colon \forall \overline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}} \underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \overline{(\mathrm{fx})}[\underline{m}] <= (\overline{(\mathrm{fy})}[\underline{m}] + (-\mathrm{u}\overline{(\epsilon)_1})))\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}} \underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon 2)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon 2)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{m}] <= (\overline{(\mathrm{fz})}[\underline{m}] + (-\mathrm{u}\overline{(\epsilon 2)})))\mathrm{n})\mathrm{n} \vdash \mathrm{A4} @ \underline{m} \rhd \forall_{\mathrm{obj}} \underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \overline{(\mathrm{fx})}[\underline{m}] <= (\overline{(\mathrm{fy})}[\underline{m}] + (-\mathrm{u}\overline{(\epsilon)_1})))\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <=$

$(\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <=$
$((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n}; \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot\neg(\dot\neg(0 <= (\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n} \gg \dot\neg(0 <=$
$(\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n} \gg \dot\neg(0 <=$
$(\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveFactors} \rhd \dot\neg(0 <= (\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow$
$\dot\neg(\dot\neg(0 = ((\epsilon)_1 *$
$(\epsilon 2)))\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\epsilon)_1\colon \forall(\epsilon 2)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \forall(\mathrm{fz})\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\epsilon)_1 \Rightarrow$
$\dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n} \vdash$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\epsilon)_1 \Rightarrow$
$\dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(0 <= (\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <=$
$((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n}; \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot\neg(\dot\neg(0 <= (\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n} \gg \dot\neg(0 <=$
$(\epsilon)_1 \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot\neg(\dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n} \gg \dot\neg\,(0 <=$
$(\epsilon 2) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot\neg\,(\dot\neg\,(0 <= (\epsilon)_1 \Rightarrow \dot\neg\,(\dot\neg\,(0 =$
$(\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))\mathrm{n})\mathrm{n} \gg \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$(\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)); \mathrm{MP} \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <=$
$((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)) \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \gg (\mathrm{fx})[\underline{\mathsf{m}}] <=$
$((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)); \mathrm{SecondConjunct} \rhd \dot\neg\,(\dot\neg\,(0 <= (\epsilon 2) \Rightarrow \dot\neg\,(\dot\neg\,(0 =$
$(\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))))\mathrm{n})\mathrm{n} \gg \underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))); \mathrm{MP} \rhd \underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))) \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \gg$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))); \mathrm{NatType} \gg \underline{\mathsf{m}} \in$
$\mathrm{N}; 0\mathrm{f} \rhd \underline{\mathsf{m}} \in \mathrm{N} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0; \mathrm{subLeqLeft} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))) \gg 0 <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))); \mathrm{LessMultiplication(F)(Helper2)} \rhd \dot\neg\,(0 <= (\epsilon)_1 \Rightarrow$
$\dot\neg\,(\dot\neg\,(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg\,(0 <= (\epsilon 2) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\mathrm{fx})[\underline{\mathsf{m}}] <=$
$((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)) \rhd 0 <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))) \gg ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) <=$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))); \mathrm{TimesF(Sym)} \gg ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{subLeqLeft} \rhd ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \rhd ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) <=$
$(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)})))) \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)})))); \mathrm{TimesF}(\mathrm{Sym}) \gg (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{eqAddition} \rhd (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg$
$(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)})))) = (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))); \mathrm{subLeqRight} \rhd$
$(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)})))) = (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))) \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$(((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}]) + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)})))) \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)}))); \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\epsilon)_1\colon \forall(\epsilon 2)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \forall(\mathrm{fz})\colon \mathrm{Ded} \rhd$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\epsilon)_1\colon \forall(\epsilon 2)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \forall(\mathrm{fz})\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)_1} \Rightarrow \dot\neg(\dot\neg(0 =$
$\underline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)_1})))\mathrm{n})\mathrm{n} \vdash$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon 2)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon 2)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon 2)})))\mathrm{n})\mathrm{n} \vdash \dot\neg(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)_1} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)_1})))\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon 2)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <= ((fz)[\underline{\mathsf{m}}] + (-u(\epsilon 2))))n)n \Rightarrow \dot{\neg}(0 <=$
$((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon)_1 * (\epsilon 2)))n)n)n; Ded \rhd$
$\forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)_1\colon \forall (\epsilon 2)\colon \forall (fx)\colon \forall (fy)\colon \forall (fz)\colon \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (fx)[\underline{\mathsf{m}}] <= ((fy)[\underline{\mathsf{m}}] + (-u(\epsilon)_1)))n)n \vdash$
$\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <=$
$((fz)[\underline{\mathsf{m}}] + (-u(\epsilon 2))))n)n \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u((\epsilon)_1 *$
$(\epsilon 2)))) \gg \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$(fx)[\underline{\mathsf{m}}] <= ((fy)[\underline{\mathsf{m}}] + (-u(\epsilon)_1)))n)n \Rightarrow \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <= ((fz)[\underline{\mathsf{m}}] + (-u(\epsilon 2))))n)n \Rightarrow \underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u((\epsilon)_1 *$
$(\epsilon 2)))); JoinConjuncts(2conditions) \rhd \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (fx)[\underline{\mathsf{m}}] <= ((fy)[\underline{\mathsf{m}}] + (-u(\epsilon)_1)))n)n \Rightarrow$
$\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <=$
$((fz)[\underline{\mathsf{m}}] + (-u(\epsilon 2))))n)n \Rightarrow \dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon)_1 * (\epsilon 2)))n)n)n \rhd \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow (fx)[\underline{\mathsf{m}}] <= ((fy)[\underline{\mathsf{m}}] + (-u(\epsilon)_1)))n)n \Rightarrow \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon 2)})))\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)}))) \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)_1})))\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon 2)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon 2)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon 2)})))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))))\mathrm{n})\mathrm{n}], \mathrm{p}_0, \mathsf{c})]$

[LessMultiplication(F)(Helper) $\stackrel{\mathrm{stmt}}{\to}$ SystemQ $\vdash$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)_1}\colon \forall\underline{(\epsilon 2)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)_1})))\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon 2)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon 2)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon 2)})))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))))\mathrm{n})\mathrm{n}]$

[LessMultiplication(F)(Helper) $\stackrel{\mathrm{tex}}{\to}$ “LessMultiplication(F)(Helper)”]

[LessMultiplication(F)(Helper) $\overset{\text{pyk}}{\rightarrow}$ “lemma lessMultiplication(F) helper”]

LessMultiplication(F)

[LessMultiplication(F) $\overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\forall\underline{(\epsilon)_1}\colon\forall\underline{(\epsilon 2)}\colon\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\forall\underline{(\mathrm{fz})}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \vdash$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \vdash$
Repetition $\rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)}))n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n;$ Ded $\rhd$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot\neg(\forall_{\mathrm{obj}}\underline{(\epsilon)_1}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \underline{(\epsilon)_1} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)_1}))n)n)n \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)_1})))n)n)n)n)n)n;$ Repetition $\rhd$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n;$ Ded $\rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot\neg(\forall_{\mathrm{obj}}\underline{(\epsilon 2)}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\dot\neg(0 <= \underline{(\epsilon 2)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon 2)}))n)n)n \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\underline{\mathsf{m}}] <=$

$((fz)[\underline{m}] + (-u(\epsilon 2)))n)n)n)n)n)n; \mathrm{LessMultiplication}(F)(\mathrm{Helper}) \gg$
$\forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow (fx)[\underline{m}] <=$
$((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n \Rightarrow \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow$
$\dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{m}] <= ((fz)[\underline{m}] + (-u(\epsilon 2))))n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon)_1 * (\epsilon 2)))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$
$N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u((\epsilon)_1 *$
$(\epsilon 2)))))n)n; \mathrm{TwiceExistMP2} \rhd \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow (fx)[\underline{m}] <= ((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n \Rightarrow$
$\forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{m}] <=$
$((fz)[\underline{m}] + (-u(\epsilon 2))))n)n \Rightarrow \dot{\neg}(\dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon)_1 * (\epsilon 2)))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u((\epsilon)_1 *$
$(\epsilon 2)))))n)n \rhd \dot{\neg}(\forall_{obj}(\epsilon)_1\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{n}\colon \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow (fx)[\underline{m}] <=$
$((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n)n)n)n)n \rhd \dot{\neg}(\forall_{obj}(\epsilon 2)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{n}\colon \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <=$
$(\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{m}] <=$
$((fz)[\underline{m}] + (-u(\epsilon 2))))n)n)n)n)n)n \gg \dot{\neg}(\dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon)_1 * (\epsilon 2)))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$

$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)})))))n)n; \mathrm{Gen} \rhd \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow$
$\dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)})))))n)n \gg \forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow$
$\dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)})))))n)n; \mathrm{IntroExist} \,@\, \underline{n} \rhd \forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)})))))n)n \gg \dot{\neg}(\forall_{obj}\underline{n}: \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)})))))n)n)n)n; \mathrm{IntroExist} \,@(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \rhd \dot{\neg}(\forall_{obj}\underline{n}: \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(0 <=$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u(\underline{(\epsilon)_1} *$

$\underline{(\epsilon 2)}))))n)n)n)n \gg \dot\neg(\forall_{obj}\underline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\underline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\underline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] +$
$(-u(\epsilon))))n)n)n)n)n)n; Ded \rhd \dot\neg(\forall_{obj}\underline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\underline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \underline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] +$
$(-u\underline{(\epsilon)}))))n)n)n)n)n)n \gg \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)}))))n)n)n)n)n)n; Repetition \rhd \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)}))))n)n)n)n)n)n \gg \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)}))))n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[LessMultiplication(F) $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall \underline{(\epsilon)_1}\colon \forall \underline{(\epsilon 2)}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \forall \underline{(\mathrm{fz})}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[LessMultiplication(F) $\overset{\mathrm{tex}}{\rightarrow}$ “LessMultiplication(F)”]

[LessMultiplication(F) $\overset{\mathrm{pyk}}{\rightarrow}$ “lemma lessMultiplication(F)”]

LessMultiplication(R)

[LessMultiplication(R) $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \forall \underline{(\mathrm{fz})}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
Repetition $\rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{LessMultiplication(F)} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{eqReflexivity} \gg \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{SubLessLeft}(\mathrm{R}) \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)}))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)}))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{eqReflexivity} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon\forall_{\mathrm{obj}}\overline{(f2)}\colon\forall_{\mathrm{obj}}\overline{(f3)}\colon\forall_{\mathrm{obj}}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon\overline{(s1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}]*\overline{(fz)}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}]*\overline{(fz)}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=$
$\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon\forall_{\mathrm{obj}}\overline{(f2)}\colon\forall_{\mathrm{obj}}\overline{(f3)}\colon\forall_{\mathrm{obj}}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon\overline{(s1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}]*\overline{(fz)}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}]*\overline{(fz)}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\};\mathrm{SubLessRight}(\mathrm{R})\rhd\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon\forall_{\mathrm{obj}}\overline{(f2)}\colon\forall_{\mathrm{obj}}\overline{(f3)}\colon\forall_{\mathrm{obj}}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon\overline{(s1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in\mathrm{N}\Rightarrow\dot{\neg}(\overline{(op2)}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}]*\overline{(fz)}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessMultiplication}(\mathrm{R}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\overline{(\mathrm{fx})}\colon \forall\overline{(\mathrm{fy})}\colon \forall\overline{(\mathrm{fz})}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[LessMultiplication(R) $\stackrel{\mathrm{tex}}{\to}$ "LessMultiplication(R)"]

[LessMultiplication(R) $\stackrel{\mathrm{pyk}}{\to}$ "lemma lessMultiplication(R)"]

LeqMultiplication(R)

[LeqMultiplication(R) $\stackrel{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash\!== \mathrm{Symmetry} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$

$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{\mathrm{ph} \in$

$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in$

$\mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$

$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{EqMultiplicationLeft}(\mathrm{R}) \triangleright \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$

$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$

$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{\mathrm{ph} \in$

$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in$

$N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$

$(\epsilon))n)n)n)n)n\}; x * 0 = 0(R) \gg \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}(r1){:}\ (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}(f1){:}\ \forall_{obj}(f2){:}\ \forall_{obj}(f3){:}\ \forall_{obj}(f4){:}\ \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{-}(\forall_{obj}(s1){:}\ (s1) \in N \Rightarrow \dot{-}(\forall_{obj}(s2){:}\ \dot{-}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon){:}\ \dot{-}(\forall_{obj}\overline{\mathsf{n}}{:}\ \dot{-}(\forall_{obj}\overline{\mathsf{m}}{:}\ \dot{-}(0 <= (\epsilon) \Rightarrow \dot{-}(\dot{-}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}(crs1){:}\ \dot{-}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}(crs1){:}\ \dot{-}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}(r1){:}\ (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}(f1){:}\ \forall_{obj}(f2){:}\ \forall_{obj}(f3){:}\ \forall_{obj}(f4){:}\ \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{-}(\forall_{obj}(s1){:}\ (s1) \in N \Rightarrow \dot{-}(\forall_{obj}(s2){:}\ \dot{-}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon){:}\ \dot{-}(\forall_{obj}\overline{\mathsf{n}}{:}\ \dot{-}(\forall_{obj}\overline{\mathsf{m}}{:}\ \dot{-}(0 <= (\epsilon) \Rightarrow \dot{-}(\dot{-}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}(crs1){:}\ \dot{-}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{-}(\dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in$
$N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}(crs1){:}\ \dot{-}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\}; x * 0 =$
$0(R) \gg \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}(op1){:}\ \dot{-}(\dot{-}(\forall_{obj}(op2){:}\ \dot{-}(\dot{-}(\dot{-}((op1) \in N \Rightarrow \dot{-}((op2) \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}(r1){:}\ (r1) \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; ==$
$\mathrm{Symmetry} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(|(\{ph \in$
$\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; EqMultiplicationLeft(R) \rhd \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; ==$ Transitivity $\rhd \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\overline{fx})[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\overline{fx})[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\overline{fx})[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in$
$N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in$
$N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}; ==$
Transitivity $\triangleright \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \rhd \{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(crs1)}: \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(crs1)}: \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}: \forall_{\mathrm{obj}}\overline{(f2)}: \forall_{\mathrm{obj}}\overline{(f3)}: \forall_{\mathrm{obj}}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(crs1)}: \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(crs1)}: \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} \gg \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}: \forall_{\mathrm{obj}}\overline{(f2)}: \forall_{\mathrm{obj}}\overline{(f3)}: \forall_{\mathrm{obj}}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) | \dot{\neg}(\dot{\neg}(\forall_{obj}(r1) \colon (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1) \colon \forall_{obj}(f2) \colon \forall_{obj}(f3) \colon \forall_{obj}(f4) \colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1) \colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2) \colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}(\epsilon) \colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}} \colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}} \colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}(crs1) \colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}(crs1) \colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}; ==$ Transitivity $\triangleright \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) | \dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) |$
$\dot{\neg}(\dot{\neg}(\forall_{obj}(r1) \colon (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1) \colon \forall_{obj}(f2) \colon \forall_{obj}(f3) \colon \forall_{obj}(f4) \colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1) \colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2) \colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}(\epsilon) \colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}} \colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}} \colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{Ph} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot\neg\, (\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot\neg\, (\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot\neg\, (\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot\neg\, (0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\, (\dot\neg\, (0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\, (|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\, (\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg\, (\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\, (\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\, (\dot\neg\, (|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\, (\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg\, (\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\, (\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot\neg\, (\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot\neg\, (\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot\neg\, (\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot\neg\, (0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\, (\dot\neg\, (0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\, (|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\, (\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg\, (\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\, (\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg\, (\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\, (\dot\neg\, (|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg\, (\dot\neg\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg\, (\dot\neg\, (\dot\neg\, (\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\, (\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\};$ lemma eqLeq(R) $\rhd \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{n} <= \overline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] +$
$(-u\overline{(\epsilon)}))))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}; \forall(fx)\colon \forall(fy)\colon \forall(fz)\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\} \vdash \text{EqMultiplication}(R) \rhd \{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}{:}\ \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}{:}\ \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\ \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}{:}\ \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}{:}\ \forall_{\mathrm{obj}}\overline{(f2)}{:}\ \forall_{\mathrm{obj}}\overline{(f3)}{:}\ \forall_{\mathrm{obj}}\overline{(f4)}{:}\ \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}{:}\ \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}{:}\ \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\ \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}{:}\ \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}{:}\ \forall_{\mathrm{obj}}\overline{(f2)}{:}\ \forall_{\mathrm{obj}}\overline{(f3)}{:}\ \forall_{\mathrm{obj}}\overline{(f4)}{:}\ \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}{:}\ \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}{:}\ \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\ \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\};$ lemma eqLeq(R) $\rhd \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}{:}\ \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg\,(\forall_{obj}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg\,(\forall_{obj}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(fz)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{obj}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n)n \vdash$
$\text{FirstConjunct} \rhd \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{obj}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$

$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <=$
$(\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{SecondConjunct} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{LessMultiplication}(R) \rhd$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <=$
$(\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{LessLeq}(R) \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$

$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$

$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$

$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$

$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$

$\overline{(\epsilon)})n)n)n)n)n\}; \forall\overline{(fx)}: \forall\overline{(fy)}: \forall\overline{(fz)}: \mathrm{Ded} \rhd \forall\overline{(fx)}: \forall\overline{(fy)}: \forall\overline{(fz)}: \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}: \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u(\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fx)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fx)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}] * \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$

$\mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}: \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}: \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)}))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}: \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(\dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \Rightarrow$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Ded} \rhd$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((fz)[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$

Repetition $\rhd \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)}))n)n)n \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\overline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\overline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}(s1): \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}: \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}(s1): \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}$; Repetition $\triangleright \dot{-}(\dot{-}(\forall_{obj}\overline{(\epsilon)}: \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(\dot{-}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <=$
$(\overline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}(s1): \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}; \mathrm{ExpandDisjuncts} \triangleright$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \dot{-}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$

$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \Rightarrow \dot{\neg}(\{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \Rightarrow$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] <= (\overline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fy)}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n)n)n;$ From3Disjuncts $\triangleright \dot{-}(\{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\overline{(fz)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \Rightarrow \dot{-}(\{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$

$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \triangleright \{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$

$\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((fz)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((fz)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \Rightarrow$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}: \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow$
$\dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(f1)}\colon \forall_{\mathrm{obj}} \overline{(f2)}\colon \forall_{\mathrm{obj}} \overline{(f3)}\colon \forall_{\mathrm{obj}} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(f1)}\colon \forall_{\mathrm{obj}} \overline{(f2)}\colon \forall_{\mathrm{obj}} \overline{(f3)}\colon \forall_{\mathrm{obj}} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \Rightarrow$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}\rceil, p_0, \mathsf{c})]$

$[LeqMultiplication(R) \overset{stmt}{\rightarrow} SystemQ \vdash$
$\forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(fz)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1})\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1})\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1})\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)}))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[LeqMultiplication(R) $\stackrel{\text{tex}}{\to}$ "LeqMultiplication(R)"]

[LeqMultiplication(R) $\stackrel{\text{pyk}}{\to}$ "lemma leqMultiplication(R)"]

PlusAssociativity(F)

[PlusAssociativity(F) $\stackrel{\text{tex}}{\to}$ "PlusAssociativity(F)"]

[PlusAssociativity(F) $\stackrel{\text{pyk}}{\to}$ "lemma plusAssociativity(F)"]

Plus0(F)

[Plus0(F) $\stackrel{\text{tex}}{\to}$ "Plus0(F)"]

[Plus0(F) $\stackrel{\text{pyk}}{\to}$ "lemma plus0(F)"]

PlusCommutativity(F)

[PlusCommutativity(F) $\stackrel{\text{tex}}{\to}$ "PlusCommutativity(F)"]

[PlusCommutativity(F) $\stackrel{\text{pyk}}{\to}$ "lemma plusCommutativity(F)"]

TimesAssociativity(F)

[TimesAssociativity(F) $\stackrel{\text{tex}}{\to}$ "TimesAssociativity(F)"]

[TimesAssociativity(F) $\stackrel{\text{pyk}}{\to}$ "lemma timesAssociativity(F)"]

Times1f

[Times1f $\stackrel{\text{tex}}{\rightarrow}$ “Times1f”]

[Times1f $\stackrel{\text{pyk}}{\rightarrow}$ “lemma times1f”]

Cauchy(2)(Helper)

[Cauchy(2)(Helper) $\stackrel{\text{proof}}{\rightarrow}$ λc.λx.𝒫(⌈SystemQ ⊢
∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
$\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n ⊢ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒ (n2) <=
(v1) ⇒ (n2) <= (v2) ⇒ $\dot\neg$(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n ⊢ $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 =
(ϵ))n)n)n ⊢ if((n2) <= (n1), (n1), (n2)) <= (v1) ⊢ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⊢ MaxLeq(1) ≫ (n1) <= if((n2) <=
(n1), (n1), (n2)); leqTransitivity ▷ (n1) <= if((n2) <=
(n1), (n1), (n2)) ▷ if((n2) <= (n1), (n1), (n2)) <= (v1) ≫ (n1) <=
(v1); leqTransitivity ▷ (n1) <= if((n2) <= (n1), (n1), (n2)) ▷ if((n2) <=
(n1), (n1), (n2)) <= (v2) ≫ (n1) <= (v2); MaxLeq(2) ≫ (n2) <= if((n2) <=
(n1), (n1), (n2)); leqTransitivity ▷ (n2) <= if((n2) <= (n1), (n1), (n2)) ▷
if((n2) <= (n1), (n1), (n2)) <= (v1) ≫ (n2) <= (v1); leqTransitivity ▷ (n2) <=
if((n2) <= (n1), (n1), (n2)) ▷ if((n2) <= (n1), (n1), (n2)) <= (v2) ≫ (n2) <=
(v2); A4 @ (v1) ▷ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒
(n1) <= (v1) ⇒ (n1) <= (v2) ⇒ $\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ≫ $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
$\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n; A4 @ (v2) ▷ $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒ (n1) <=
(v1) ⇒ (n1) <= (v2) ⇒ $\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ≫ $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 =
(ϵ))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
$\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n; MP3 ▷ $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ $\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ▷ $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 =
(ϵ))n)n)n ▷ (n1) <= (v1) ▷ (n1) <= (v2) ≫
$\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n; A4 @ (v1) ▷ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒
(n2) <= (v1) ⇒ (n2) <= (v2) ⇒ $\dot\neg$(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n ≫ $\forall_{\mathrm{obj}}$(v2): $\dot\neg$(0 <= (ϵ) ⇒
$\dot\neg$($\dot\neg$(0 = (ϵ))n)n)n ⇒ (n2) <= (v1) ⇒ (n2) <= (v2) ⇒
$\dot\neg$(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒ $\dot\neg$($\dot\neg$(|((fy)[(v1)] + (−u(fy)[(v2)]))| =

(ε))n)n)n; A4 @ (v2) ▷ $\forall_{obj}$(v2): ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒ (n2) <=
(v1) ⇒ (n2) <= (v2) ⇒ ¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n ≫ ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 =
(ε))n)n)n ⇒ (n2) <= (v1) ⇒ (n2) <= (v2) ⇒
¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ε))n)n)n; MP3 ▷ ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒ (n2) <= (v1) ⇒
(n2) <= (v2) ⇒ ¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n ▷ ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 =
(ε))n)n)n ▷ (n2) <= (v1) ▷ (n2) <= (v2) ≫ ¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <=
(ε) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n; JoinConjuncts ▷
¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ε))n)n)n ▷ ¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| = (ε))n)n)n ≫ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ε))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ε))n)n)n)n)n; ∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ε): ∀(fx): ∀(fy): Ded ▷
∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ε): ∀(fx): ∀(fy): $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <= (ε) ⇒
¬̇(¬̇(0 = (ε))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ε))n)n)n ⊢ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒ (n2) <=
(v1) ⇒ (n2) <= (v2) ⇒ ¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n ⊢ ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 =
(ε))n)n)n ⊢ if((n2) <= (n1), (n1), (n2)) <= (v1) ⊢ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⊢ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ε))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ε))n)n)n)n)n ≫ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒
(n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ε))n)n)n ⇒ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <=
(ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒ (n2) <= (v1) ⇒ (n2) <= (v2) ⇒ ¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n ⇒
¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒
if((n2) <= (n1), (n1), (n2)) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(ε) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ε))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n)n)n⌉, p_0, c)]

[Cauchy(2)(Helper) $\stackrel{stmt}{\rightarrow}$ SystemQ ⊢
∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ε): ∀(fx): ∀(fy): $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <= (ε) ⇒
¬̇(¬̇(0 = (ε))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ε))n)n)n ⇒ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 = (ε))n)n)n ⇒ (n2) <=
(v1) ⇒ (n2) <= (v2) ⇒ ¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ε) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ε))n)n)n ⇒ ¬̇(0 <= (ε) ⇒ ¬̇(¬̇(0 =
(ε))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ε) ⇒

$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] +$
$(-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n)n)n]$

[Cauchy(2)(Helper) $\overset{\text{tex}}{\to}$ "Cauchy(2)(Helper)"]

[Cauchy(2)(Helper) $\overset{\text{pyk}}{\to}$ "lemma 2cauchy helper"]

Cauchy(2)

[Cauchy(2) $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash$
$\forall(v1)\colon\forall(v2)\colon\forall \underline{n}\colon\forall(n1)\colon\forall(n2)\colon\forall(\epsilon)\colon\forall(fx)\colon\forall(fy)\colon \text{Cauchy} \gg$
$\forall_{obj}(\epsilon)\colon\dot{\neg}(\forall_{obj}(n1)\colon\dot{\neg}(\forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$(n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n)n)n; \text{A4} @ (\epsilon) \rhd \forall_{obj}(\epsilon)\colon\dot{\neg}(\forall_{obj}(n1)\colon\dot{\neg}(\forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n)n)n \gg \dot{\neg}(\forall_{obj}(n1)\colon\dot{\neg}(\forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n)n)n; \text{Cauchy} \gg \forall_{obj}(\epsilon)\colon\dot{\neg}(\forall_{obj}(n2)\colon\dot{\neg}(\forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n2) <= (v1) \Rightarrow (n2) <= (v2) \Rightarrow$
$\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(\epsilon))n)n)n)n)n; \text{A4} @ (\epsilon) \rhd \forall_{obj}(\epsilon)\colon\dot{\neg}(\forall_{obj}(n2)\colon\dot{\neg}(\forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n2) <= (v1) \Rightarrow (n2) <= (v2) \Rightarrow$
$\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(\epsilon))n)n)n)n)n \gg \dot{\neg}(\forall_{obj}(n2)\colon\dot{\neg}(\forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))n)n)n \Rightarrow (n2) <= (v1) \Rightarrow (n2) <= (v2) \Rightarrow$
$\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(\epsilon))n)n)n)n)n; \text{Cauchy(2)(Helper)} \gg \forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n \Rightarrow \forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n2) <=$
$(v1) \Rightarrow (n2) <= (v2) \Rightarrow \dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))n)n)n \Rightarrow \text{if}((n2) <= (n1), (n1), (n2)) <= (v1) \Rightarrow \text{if}((n2) <=$
$(n1), (n1), (n2)) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] +$
$(-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(\epsilon))n)n)n)n)n; \text{ExistMP2} \rhd \forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n \Rightarrow \forall_{obj}(v1)\colon\forall_{obj}(v2)\colon\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n2) <=$
$(v1) \Rightarrow (n2) <= (v2) \Rightarrow \dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$

(ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ϵ))n)n)n)n ⊳ ¬̇(∀$_{\mathrm{obj}}$(n1): ¬̇(∀$_{\mathrm{obj}}$(v1): ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(ϵ))n)n)n)n ⊳ ¬̇(∀$_{\mathrm{obj}}$(n2): ¬̇(∀$_{\mathrm{obj}}$(v1): ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ (n2) <= (v1) ⇒ (n2) <= (v2) ⇒
¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ϵ))n)n)n)n ≫ ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ if((n2) <=
(n1), (n1), (n2)) <= (v1) ⇒ if((n2) <= (n1), (n1), (n2)) <= (v2) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(v1)] +
(−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n; Gen ⊳ ¬̇(0 <= (ϵ) ⇒
¬̇(¬̇(0 = (ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n ≫ ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒
¬̇(¬̇(0 = (ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n; Gen ⊳ ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <=
(ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <=
(n1), (n1), (n2)) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ϵ))n)n)n)n)n ≫ ∀$_{\mathrm{obj}}$(v1): ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒
if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <= (n1), (n1), (n2)) <= (v2) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(v1)] +
(−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n; IntroExist @ if((n2) <=
(n1), (n1), (n2)) ⊳ ∀$_{\mathrm{obj}}$(v1): ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒
if((n2) <= (n1), (n1), (n2)) <= (v1) ⇒ if((n2) <= (n1), (n1), (n2)) <= (v2) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(v1)] +
(−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n ≫
¬̇(∀$_{\mathrm{obj}}$n: ¬̇(∀$_{\mathrm{obj}}$(v1): ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ n <=
(v1) ⇒ n <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] +
(−u(fy)[(v2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(ϵ))n)n)n)n)n)n)n; Gen ⊳ ¬̇(∀$_{\mathrm{obj}}$n: ¬̇(∀$_{\mathrm{obj}}$(v1): ∀$_{\mathrm{obj}}$(v2): ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ n <= (v1) ⇒ n <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(ϵ) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒

$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n)n)n)n)n \gg$
$\forall_{obj}(\epsilon) \colon \dot{\neg}(\forall_{obj}\underline{n} \colon \dot{\neg}(\forall_{obj}(v1) \colon \forall_{obj}(v2) \colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\underline{n} <= (v1) \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[Cauchy(2) $\stackrel{stmt}{\rightarrow}$ SystemQ $\vdash$
$\forall(v1) \colon \forall(v2) \colon \forall\underline{n} \colon \forall(n1) \colon \forall(n2) \colon \forall(\epsilon) \colon \forall(fx) \colon \forall(fy) \colon \forall_{obj}(\epsilon) \colon \dot{\neg}(\forall_{obj}\underline{n} \colon \dot{\neg}(\forall_{obj}(v1) \colon \forall_{obj}(v2$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \underline{n} <= (v1) \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] +$
$(-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (\epsilon))n)n)n)n)n)n)n]$

[Cauchy(2) $\stackrel{tex}{\rightarrow}$ "Cauchy(2)"]

[Cauchy(2) $\stackrel{pyk}{\rightarrow}$ "lemma 2cauchy"]

ReciprocalFnonzero

[ReciprocalFnonzero $\stackrel{proof}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{m} \colon \forall(m1) \colon \forall(fx) \colon \dot{\neg}((fx)[\underline{m}] =$
$0)n \vdash$ NatType $\gg \underline{m} \in N$; ReciprocalIsRationalSeries $\gg \dot{\neg}(\dot{\neg}(\forall_{obj}(r1) \colon (r1) \in$
$\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n)n\} \Rightarrow \dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$
$N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) =$
$\{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1) \colon \forall_{obj}(f2) \colon \forall_{obj}(f3) \colon \forall_{obj}(f4) \colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n)n\} \Rightarrow \{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1) \colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2) \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n)n\} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1) \colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2) \colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in \{ph \in \{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MemberOfSeries} \rhd \underline{\mathsf{m}} \in$
$\mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Repetition} \rhd \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Separation2formula}(2) \rhd \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{m1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[(\mathrm{m1})] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),\mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow$
$\dot{\neg}\,(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{\mathsf{m}}{:}\,\forall(\mathrm{m1}){:}\,\forall(\mathrm{fx}){:}\,\dot{\neg}\,((\mathrm{fx})[(\mathrm{m1})] = 0)\mathrm{n} \vdash$
$\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{fx})[(\mathrm{m1})] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow$
$\dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg(((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg(((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} = \{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), 0\}\})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{ToNegatedAnd}(1) \rhd \dot\neg(((\mathrm{fx})[(\mathrm{m1})] = 0)\mathrm{n} \gg \dot\neg(\dot\neg(((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow$
$\dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg(((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg(((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), 0\}\})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegateDisjunct2} \rhd \dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[(\mathrm{m1})] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg(((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg(((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow$
$\dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg(((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg(((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), 0\}\})\mathrm{n})\mathrm{n} \rhd \dot\neg(\dot\neg(((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow \dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg(((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot\neg(((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} = \{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), 0\}\})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\dot\neg(((\mathrm{fx})[(\mathrm{m1})] = 0)\mathrm{n} \Rightarrow \dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg(((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg(((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot\neg(\dot\neg(((\mathrm{fx})[(\mathrm{m1})] =$

$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})\mathrm{n})\mathrm{n} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\}; \mathrm{FromOrderedPair}(2) \rhd$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} = \{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\} \gg$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] =$
$\mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]; \mathrm{FromOrderedPair}(1) \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} = \{\{(\mathrm{m1}), (\mathrm{m1})\}, \{(\mathrm{m1}), \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\} \gg$
$\underline{\mathsf{m}} = (\mathrm{m1}); \mathrm{SameSeries} \rhd \underline{\mathsf{m}} = (\mathrm{m1}) \gg (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fx})[(\mathrm{m1})]; \mathrm{eqSymmetry} \rhd$
$(\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fx})[(\mathrm{m1})] \gg (\mathrm{fx})[(\mathrm{m1})] = (\mathrm{fx})[\underline{\mathsf{m}}]; \mathrm{SameReciprocal} \rhd \dot\neg((\mathrm{fx})[(\mathrm{m1})] =$
$0)\mathrm{n} \rhd (\mathrm{fx})[(\mathrm{m1})] = (\mathrm{fx})[\underline{\mathsf{m}}] \gg \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})] = \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]; \mathrm{eqTransitivity} \rhd \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})] \rhd \mathrm{rec}(\mathrm{fx})[(\mathrm{m1})] = \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}] \gg$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg(\dot\neg(\dot\neg((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}] =$
$\mathrm{rec}(\mathrm{fx})[\underline{m}]; \mathrm{Ded} \rhd \forall \underline{m}\colon \forall(\mathrm{m1})\colon \forall(\mathrm{fx})\colon \dot{\neg}((\mathrm{fx})[(\mathrm{m1})] = 0)n \vdash \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[(\mathrm{m1})] =$
$0)n \Rightarrow \dot{\neg}(\{\{\underline{m},\underline{m}\},\{\underline{m},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}]\}\} =$
$\{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),\mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow$
$\dot{\neg}(\{\{\underline{m},\underline{m}\},\{\underline{m},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}]\}\} = \{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),0\}\})n)n \vdash \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] =$
$0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}] = \mathrm{rec}(\mathrm{fx})[\underline{m}] \gg \dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[(\mathrm{m1})] = 0)n \Rightarrow \dot{\neg}(\{\{\underline{m},\underline{m}\},\{\underline{m},\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}]\}\} =$
$\{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),\mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow$
$\dot{\neg}(\{\{\underline{m},\underline{m}\},\{\underline{m},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}]\}\} = \{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),0\}\})n)n \Rightarrow \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] =$
$0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}] = \mathrm{rec}(\mathrm{fx})[\underline{m}]; \mathrm{MP} \rhd \dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[(\mathrm{m1})] = 0)n \Rightarrow \dot{\neg}(\{\{\underline{m},\underline{m}\},\{\underline{m},\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}(\mathrm{fx})[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}]\}\} =$

$\{\{(m1),(m1)\},\{(m1),rec(fx)[(m1)]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[(m1)] = 0 \Rightarrow$
$\dot{\neg}(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}]\}\} = \{\{(m1),(m1)\},\{(m1),0\}\})n)n \Rightarrow \{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$
$N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] =$
$0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}] = rec(fx)[\underline{\mathsf{m}}] \rhd \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0)n \gg$
$\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[(m1)] = 0)n \Rightarrow \dot{\neg}(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(m1),(m1)\},\{(m1),rec(fx)[(m1)]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[(m1)] = 0 \Rightarrow$
$\dot{\neg}(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}]\}\} = \{\{(m1),(m1)\},\{(m1),0\}\})n)n \Rightarrow \{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$
$N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] =$
$0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}] = rec(fx)[\underline{\mathsf{m}}]; ExistMP \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[(m1)] =$
$0)n \Rightarrow \dot{\neg}(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}]\}\} =$
$\{\{(m1),(m1)\},\{(m1),rec(fx)[(m1)]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[(m1)] = 0 \Rightarrow$
$\dot{\neg}(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}]\}\} = \{\{(m1),(m1)\},\{(m1),0\}\})n)n \Rightarrow \{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$

$\mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}] \rhd \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{m1})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[(\mathrm{m1})] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} = \{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),\mathrm{rec}(\mathrm{fx})[(\mathrm{m1})]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\mathrm{fx})[(\mathrm{m1})] = 0 \Rightarrow \dot\neg\,(\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} = \{\{(\mathrm{m1}),(\mathrm{m1})\},\{(\mathrm{m1}),0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ReciprocalFnonzero} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall(\mathrm{m1})\colon \forall(\mathrm{fx})\colon \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \vdash \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]]$

$[\mathrm{ReciprocalFnonzero} \stackrel{\mathrm{tex}}{\rightarrow}$ "ReciprocalFnonzero"]

$[\mathrm{ReciprocalFnonzero} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma reciprocalF nonzero"]

(Eventually = f)2sameF(Helper)

$[(\mathrm{Eventually} = \mathrm{f})\mathrm{2sameF}(\mathrm{Helper}) \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\epsilon)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \vdash \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \gg \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}]; \dot\neg\,(0 <= (\epsilon) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash \mathrm{MP} \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \gg (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}]; \mathrm{PositiveToLeft}(\mathrm{Eq})(\mathrm{1term}) \rhd (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \gg ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}])) = 0; \mathrm{SameNumerical} \rhd ((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}])) = 0 \gg |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = |0|; |0| = 0 \gg |0| = 0; \mathrm{eqTransitivity} \rhd |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = |0| \rhd |0| =$

$0 \gg |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = 0; \mathrm{eqSymmetry} \rhd |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = 0 \gg 0 = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))|; \mathrm{SubLessLeft} \rhd 0 = |((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\mathrm{fx})\colon \forall (\mathrm{fy})\colon \mathrm{Ded} \rhd \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\mathrm{fx})\colon \forall (\mathrm{fy})\colon \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \vdash \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[(\mathrm{Eventually} = \mathrm{f})\mathrm{2sameF}(\mathrm{Helper}) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\mathrm{fx})\colon \forall (\mathrm{fy})\colon \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[(\mathrm{Eventually} = \mathrm{f})\mathrm{2sameF}(\mathrm{Helper}) \stackrel{\mathrm{tex}}{\rightarrow}$ “(Eventually=f)2sameF(Helper)”]

$[(\mathrm{Eventually} = \mathrm{f})\mathrm{2sameF}(\mathrm{Helper}) \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma eventually=f to sameF helper”]

(Eventually = f)2sameF

$[(\mathrm{Eventually} = \mathrm{f})\mathrm{2sameF} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\mathrm{fx})\colon \forall (\mathrm{fy})\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n} \vdash (\mathrm{Eventually} = \mathrm{f})\mathrm{2sameF}(\mathrm{Helper}) \gg \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] = (\mathrm{fy})[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @ \underline{\mathsf{n}} \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} (\epsilon)\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| =$

$\underline{(\epsilon)})n)n)n)n)n$; Ded $\rhd$ $\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{\mathsf{m}}] + (-u\underline{(fy)}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{\mathsf{m}}] + (-u\underline{(fy)}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})n)n)n)n)n \gg$ $\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n$; Repetition $\rhd$ $\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \gg$ $\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[(Eventually = f)2sameF $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\forall\underline{(fx)}\colon\forall\underline{(fy)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fy)}[\underline{\mathsf{m}}])n)n \vdash$ $\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n]$

[(Eventually = f)2sameF $\stackrel{\mathrm{tex}}{\rightarrow}$ "(Eventually=f)2sameF"]

[(Eventually = f)2sameF $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma eventually=f to sameF"]

FromNotSameF(Strong)(Helper2)

[FromNotSameF(Strong)(Helper2) $\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon\forall\underline{\mathsf{z}}\colon\forall\underline{\mathsf{u}}\colon\forall\underline{\mathsf{v}}\colon \underline{\mathsf{v}} <= |(\underline{\mathsf{x}} + (-u\underline{\mathsf{z}}))| \vdash \dot{\neg}(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| <= (rec(1+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| = (rec(1+1) * \underline{\mathsf{v}}))n)n)n \vdash \dot{\neg}(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| <= (rec(1+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| = (rec(1+1) * \underline{\mathsf{v}}))n)n)n \vdash$ LessNegated $\rhd$ $\dot{\neg}(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| <= (rec(1+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))| = (rec(1+1) * \underline{\mathsf{v}}))n)n)n \gg \dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))|))n)n)n$; NumericalDifference $\gg$ $|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| = |(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))|$; SubLessLeft $\rhd$ $|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| = |(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))| \rhd \dot{\neg}(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| <= (rec(1+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{z}} + (-u\underline{\mathsf{u}}))| = (rec(1+1) * \underline{\mathsf{v}}))n)n)n \gg \dot{\neg}(|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))| <= (rec(1+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))| = (rec(1+1) * \underline{\mathsf{v}}))n)n)n$; LessNegated $\rhd$ $\dot{\neg}(|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))| <= (rec(1+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))| = (rec(1+1) * \underline{\mathsf{v}}))n)n)n \gg \dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))|))n)n)n$; AddEquations(Less) $\rhd$ $\dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))|))n)n)n \rhd \dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) <= (-u|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-u(rec(1+1) * \underline{\mathsf{v}})) = (-u|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))|))n)n)n \gg \dot{\neg}(((-u(rec(1+1) * \underline{\mathsf{v}})) + (-u(rec(1+1) * \underline{\mathsf{v}}))) <= ((-u|(\underline{\mathsf{x}}+(-u\underline{\mathsf{y}}))|)+(-u|(\underline{\mathsf{u}}+(-u\underline{\mathsf{z}}))|)) \Rightarrow \dot{\neg}(\dot{\neg}(((-u(rec(1+1)*\underline{\mathsf{v}}))+(-u(rec(1+1) * \underline{\mathsf{v}}))) = ((-u|(\underline{\mathsf{x}} + (-u\underline{\mathsf{y}}))|) + (-u|(\underline{\mathsf{u}} + (-u\underline{\mathsf{z}}))|)))n)n)n$; TwoHalves $\gg$ $((rec(1+1)*\underline{\mathsf{v}})+(rec(1+1)*\underline{\mathsf{v}})) = \underline{\mathsf{v}}$; EqNegated$\rhd((rec(1+1)*\underline{\mathsf{v}})+(rec(1+1)*\underline{\mathsf{v}})) =$

v̲ ≫ (−u((rec(1 + 1) ∗ v̲) + (rec(1 + 1) ∗ v̲))) = (−uv̲); −x − y = −(x + y) ≫
((−u(rec(1 + 1) ∗ v̲)) + (−u(rec(1 + 1) ∗ v̲))) = (−u((rec(1 + 1) ∗ v̲) + (rec(1 +
1) ∗ v̲))); eqTransitivity ▷ ((−u(rec(1 + 1) ∗ v̲)) + (−u(rec(1 + 1) ∗ v̲))) =
(−u((rec(1+1)∗v̲) + (rec(1+1)∗v̲))) ▷ (−u((rec(1+1)∗v̲) + (rec(1+1)∗v̲))) =
(−uv̲) ≫ ((−u(rec(1 + 1) ∗ v̲)) + (−u(rec(1 + 1) ∗ v̲))) =
(−uv̲); SubLessLeft ▷ ((−u(rec(1 + 1) ∗ v̲)) + (−u(rec(1 + 1) ∗ v̲))) =
(−uv̲) ▷ ¬̇ (((−u(rec(1 + 1) ∗ v̲)) + (−u(rec(1 + 1) ∗ v̲))) <=
((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)) ⇒ ¬̇ (¬̇ (((−u(rec(1 + 1) ∗ v̲)) +
(−u(rec(1 + 1) ∗ v̲))) = ((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)))n)n)n ≫
¬̇ ((−uv̲) <= ((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)) ⇒ ¬̇ (¬̇ ((−uv̲) =
((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)))n)n)n; plusCommutativity ≫
((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)) = ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ +
(−uy̲))|)); SubLessRight ▷ ((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)) =
((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|)) ▷ ¬̇ ((−uv̲) <=
((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)) ⇒ ¬̇ (¬̇ ((−uv̲) =
((−u|(x̲ + (−uy̲))|) + (−u|(u̲ + (−uz̲))|)))n)n)n ≫ ¬̇ ((−uv̲) <=
((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|)) ⇒ ¬̇ (¬̇ ((−uv̲) =
((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|)))n)n)n; AddEquations(LeqLess) ▷ v̲ <=
|(x̲ + (−uz̲))| ▷ ¬̇ ((−uv̲) <= ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|)) ⇒
¬̇ (¬̇ ((−uv̲) = ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|)))n)n)n ≫
¬̇ ((v̲ + (−uv̲)) <= (|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) ⇒
¬̇ (¬̇ ((v̲ + (−uv̲)) =
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))))n)n)n; Negative ≫
(v̲ + (−uv̲)) = 0; SubLessLeft ▷ (v̲ + (−uv̲)) = 0 ▷ ¬̇ ((v̲ + (−uv̲)) <=
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) ⇒ ¬̇ (¬̇ ((v̲ + (−uv̲)) =
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))))n)n)n ≫ ¬̇ (0 <=
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) ⇒ ¬̇ (¬̇ (0 = (|(x̲ +
(−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))))n)n)n; plusAssociativity ≫
(((|(x̲ + (−uz̲))| + (−u|(u̲ + (−uz̲))|)) + (−u|(x̲ + (−uy̲))|)) =
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))); eqSymmetry ▷
(((|(x̲ + (−uz̲))| + (−u|(u̲ + (−uz̲))|)) + (−u|(x̲ + (−uy̲))|)) =
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) ≫
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) =
(((|(x̲ + (−uz̲))| + (−u|(u̲ + (−uz̲))|)) + (−u|(x̲ + (−uy̲))|)); SubLessRight ▷
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) =
(((|(x̲ + (−uz̲))| + (−u|(u̲ + (−uz̲))|)) + (−u|(x̲ + (−uy̲))|)) ▷ ¬̇ (0 <=
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))) ⇒ ¬̇ (¬̇ (0 =
(|(x̲ + (−uz̲))| + ((−u|(u̲ + (−uz̲))|) + (−u|(x̲ + (−uy̲))|))))n)n)n ≫ ¬̇ (0 <=
(((|(x̲ + (−uz̲))| + (−u|(u̲ + (−uz̲))|)) + (−u|(x̲ + (−uy̲))|)) ⇒ ¬̇ (¬̇ (0 =
(((|(x̲ + (−uz̲))| + (−u|(u̲ + (−uz̲))|)) + (−u|(x̲ +
(−uy̲))|)))n)n)n; insertTwoMiddleTerms(Numerical) ≫ |(x̲ + (−uz̲))| <=
(((|(x̲ + (−uy̲))| + |(y̲ + (−uu̲))|) + |(u̲ + (−uz̲))|); PositiveToLeft(Leq) ▷ |(x̲ +
(−uz̲))| <= (((|(x̲ + (−uy̲))| + |(y̲ + (−uu̲))|) + |(u̲ + (−uz̲))|) ≫ (|(x̲ + (−uz̲))| +
(−u|(u̲ + (−uz̲))|)) <= (|(x̲ + (−uy̲))| + |(y̲ + (−uu̲))|); plusCommutativity ≫
(|(x̲ + (−uy̲))| + |(y̲ + (−uu̲))|) =
(|(y̲ + (−uu̲))| + |(x̲ + (−uy̲))|); subLeqRight ▷ (|(x̲ + (−uy̲))| + |(y̲ + (−uu̲))|) =

(|(y + (−uu))| + |(x + (−uy))|) ⊳ (|(x + (−uz))| + (−u|(u + (−uz))|)) <= (|(x + (−uy))| + |(y + (−uu))|) ≫ (|(x + (−uz))| + (−u|(u + (−uz))|)) <= (|(y + (−uu))| + |(x + (−uy))|); PositiveToLeft(Leq) ⊳ (|(x + (−uz))| + (−u|(u + (−uz))|)) <= (|(y + (−uu))| + |(x + (−uy))|) ≫ ((|(x + (−uz))| + (−u|(u + (−uz))|)) + (−u|(x + (−uy))|)) <= |(y + (−uu))|; LessLeqTransitivity ⊳ ∸ (0 <= ((|(x + (−uz))| + (−u|(u + (−uz))|)) + (−u|(x + (−uy))|)) ⇒ ∸ (∸ (0 = ((|(x + (−uz))| + (−u|(u + (−uz))|)) + (−u|(x + (−uy))|)))n)n)n ⊳ ((|(x + (−uz))| + (−u|(u + (−uz))|)) + (−u|(x + (−uy))|)) <= |(y + (−uu))| ≫ ∸ (0 <= |(y + (−uu))| ⇒ ∸ (∸ (0 = |(y + (−uu))|)n)n)n; FromPositiveNumerical ⊳ ∸ (0 <= |(y + (−uu))| ⇒ ∸ (∸ (0 = |(y + (−uu))|)n)n)n ≫ ∸ ((y + (−uu)) = 0)n; NegativeToRight(Neq)(1term) ⊳ ∸ ((y + (−uu)) = 0)n ≫ ∸ (y = u)n⌉, p_0, c)]

[FromNotSameF(Strong)(Helper2) $\stackrel{\text{stmt}}{\to}$ SystemQ ⊢ ∀x: ∀y: ∀z: ∀u: ∀v: v <= |(x + (−uz))| ⊢ ∸ (|(x + (−uy))| <= (rec(1 + 1) ∗ v) ⇒ ∸ (∸ (|(x + (−uy))| = (rec(1 + 1) ∗ v))n)n)n ⊢ ∸ (|(z + (−uu))| <= (rec(1 + 1) ∗ v) ⇒ ∸ (∸ (|(z + (−uu))| = (rec(1 + 1) ∗ v))n)n)n ⊢ ∸ (y = u)n]

[FromNotSameF(Strong)(Helper2) $\stackrel{\text{tex}}{\to}$ "FromNotSameF(Strong)(Helper2)"]

[FromNotSameF(Strong)(Helper2) $\stackrel{\text{pyk}}{\to}$ "lemma fromNotSameF(Strong) helper2"]

FromNotSameF(Strong)(Helper)

[FromNotSameF(Strong)(Helper) $\stackrel{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢ ∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\text{obj}}$(v1): $\forall_{\text{obj}}$(v2): ∸ (0 <= (rec(1 + 1) ∗ (ϵ)) ⇒ ∸ (∸ (0 = (rec(1 + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ∸ (∸ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec(1 + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec(1 + 1) ∗ (ϵ)))n)n)n ⇒ ∸ (∸ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec(1 + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec(1 + 1) ∗ (ϵ)))n)n)n)n)n ⊢ (n2) <= m ⊢ FromNegated(2 ∗ Imply) ⊳ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ≫ ∸ (∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ ∸ ((n1) <= (n2))n)n ⇒ ∸ (∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n; FirstConjunct ⊳ ∸ (∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ ∸ ((n1) <= (n2))n)n ⇒ ∸ (∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n ≫ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ ∸ ((n1) <= (n2))n)n; FirstConjunct ⊳ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ ∸ ((n1) <= (n2))n)n ≫ ∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n; SecondConjunct ⊳ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ ∸ ((n1) <= (n2))n)n ≫ (n1) <= (n2); SecondConjunct ⊳ ∸ (∸ (∸ (0 <= (ϵ) ⇒

$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \dot{\neg}((n1) <= (n2))n)n \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|((fx)[(n2)] +$
$(-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| =$
$(\epsilon))n)n)n)n)n)n \gg \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| =$
$(\epsilon))n)n)n)n$; fromNotLess $\rhd \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n \gg (\epsilon) <=$
$|((fx)[(n2)] + (-u(fy)[(n2)]))|$; A4 @ $(n2) \rhd \forall_{obj}(v1){:}\ \forall_{obj}(v2){:}\ \dot{\neg}(0 <=$
$(rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow$
$(n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n \gg$
$\forall_{obj}(v2){:}\ \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow$
$(n1) <= (n2) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[(v2)]))| <=$
$(rec(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)]+(-u(fx)[(v2)]))| = (rec(1+1)*(\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(n2)] +$
$(-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n$; A4 @ $\mathsf{m} \rhd \forall_{obj}(v2){:}\ \dot{\neg}(0 <=$
$(rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow$
$(n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n \gg \dot{\neg}(0 <=$
$(rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow$
$(n1) <= \mathsf{m} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(n2)] +$
$(-u(fy)[\mathsf{m}]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n$; PositiveHalved $\rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \gg \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(rec(1+1) * (\epsilon)))n)n)n$; leqTransitivity $\rhd (n1) <= (n2) \rhd (n2) <= \mathsf{m} \gg (n1) <=$
m; MP3 $\rhd \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow$
$(n1) <= (n2) \Rightarrow (n1) <= \mathsf{m} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| <=$
$(rec(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)]+(-u(fx)[\mathsf{m}]))| = (rec(1+1)*(\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[\mathsf{m}]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n \rhd \dot{\neg}(0 <=$
$(rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \rhd (n1) <= (n2) \rhd (n1) <=$
$\mathsf{m} \gg \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| = (rec(1+1)*(\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(n2)] +$
$(-u(fy)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[\mathsf{m}]))| =$
$(rec(1+1)*(\epsilon)))n)n)n)n)n$; FirstConjunct $\rhd \dot{\neg}(\dot{\neg}(|((fx)[(n2)]+(-u(fx)[\mathsf{m}]))| <=$
$(rec(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)]+(-u(fx)[\mathsf{m}]))| = (rec(1+1)*(\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(n2)] +$
$(-u(fy)[\mathsf{m}]))| = (rec(1+1)*(\epsilon)))n)n)n)n)n \gg \dot{\neg}(|((fx)[(n2)]+(-u(fx)[\mathsf{m}]))| <=$
$(rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| =$
$(rec(1+1) * (\epsilon)))n)n)n$; SecondConjunct $\rhd \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fx)[\mathsf{m}]))| <=$
$(rec(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)]+(-u(fx)[\mathsf{m}]))| = (rec(1+1)*(\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(n2)] + (-u(fy)[\mathsf{m}]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(n2)] +$

$(-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|((\mathrm{fy})[(\mathrm{n2})]+(-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| <=$
$(\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| =$
$(\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNotSameF(Strong)(Helper2)} \rhd (\epsilon) <=$
$|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| \rhd \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fx})[\mathsf{m}]))| <=$
$(\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fx})[\mathsf{m}]))| =$
$(\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(|((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[\mathsf{m}]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\mathrm{fx})[\mathsf{m}] =$
$(\mathrm{fy})[\mathsf{m}])\mathrm{n}; \forall(\mathrm{v1}){:}\forall(\mathrm{v2}){:}\forall\mathsf{m}{:}\forall(\mathrm{n1}){:}\forall(\mathrm{n2}){:}\forall(\epsilon){:}\forall(\mathrm{fx}){:}\forall(\mathrm{fy}){:}\,\mathrm{Ded} \rhd$
$\forall(\mathrm{v1}){:}\forall(\mathrm{v2}){:}\forall\mathsf{m}{:}\forall(\mathrm{n1}){:}\forall(\mathrm{n2}){:}\forall(\epsilon){:}\forall(\mathrm{fx}){:}\forall(\mathrm{fy}){:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}(\mathrm{v1}){:}\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash (\mathrm{n2}) <=$
$\mathsf{m} \vdash \dot{\neg}((\mathrm{fx})[\mathsf{m}] = (\mathrm{fy})[\mathsf{m}])\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <=$
$(\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <=$
$\mathsf{m} \Rightarrow \dot{\neg}((\mathrm{fx})[\mathsf{m}] = (\mathrm{fy})[\mathsf{m}])\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNotSameF(Strong)(Helper)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1}){:}\forall(\mathrm{v2}){:}\forall\mathsf{m}{:}\forall(\mathrm{n1}){:}\forall(\mathrm{n2}){:}\forall(\epsilon){:}\forall(\mathrm{fx}){:}\forall(\mathrm{fy}){:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{n2})]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}(\mathrm{v1}){:}\forall_{\mathrm{obj}}(\mathrm{v2}){:}\,\dot{\neg}(0 <=$
$(\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow$
$(\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}(1+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}(1+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <=$
$\mathsf{m} \Rightarrow \dot{\neg}((\mathrm{fx})[\mathsf{m}] = (\mathrm{fy})[\mathsf{m}])\mathrm{n}]$

[FromNotSameF(Strong)(Helper) $\stackrel{\mathrm{tex}}{\rightarrow}$ "FromNotSameF(Strong)(Helper)"]

[FromNotSameF(Strong)(Helper) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromNotSameF(Strong) helper"]

FromNotSameF(Strong)

$[\mathrm{FromNotSameF(Strong)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1}){:}\forall(\mathrm{v2}){:}\forall\mathsf{m}{:}\forall\mathsf{n}{:}\forall(\mathrm{n1}){:}\forall(\mathrm{n2}){:}\forall(\epsilon){:}\forall(\mathrm{fx}){:}\forall(\mathrm{fy}){:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}($

$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n2})}\colon \dot{\neg}(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{n2})} \Rightarrow$
$\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AEA-negated} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n2})}\colon \dot{\neg}(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{n2})} \Rightarrow$
$\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{n2})} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Cauchy}(2) \gg$
$\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{(\mathrm{n1})} <= \underline{(\mathrm{v1})} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{v2})} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ (\mathrm{rec}(1+1) * \underline{(\epsilon)}) \rhd$
$\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{(\mathrm{n1})} <= \underline{(\mathrm{v1})} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{v2})} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{n1})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}(0 <=$
$(\mathrm{rec}(1+1) * \underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1) * \underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{v1})} \Rightarrow$
$\underline{(\mathrm{n1})} <= \underline{(\mathrm{v2})} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| <= (\mathrm{rec}(1+1) * \underline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| = (\mathrm{rec}(1+1) * \underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| <= (\mathrm{rec}(1+1) * \underline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| =$
$(\mathrm{rec}(1+1) * \underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNotSameF(Strong)(Helper)} \gg$
$\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{n2})} \Rightarrow$
$\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}(0 <=$
$(\mathrm{rec}(1+1) * \underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1) * \underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{v1})} \Rightarrow$
$\underline{(\mathrm{n1})} <= \underline{(\mathrm{v2})} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| <= (\mathrm{rec}(1+1) * \underline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{(\mathrm{v2})}]))| = (\mathrm{rec}(1+1) * \underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| <= (\mathrm{rec}(1+1) * \underline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\underline{(\mathrm{v1})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{v2})}]))| = (\mathrm{rec}(1+1) * \underline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n2})} <=$
$\underline{\mathsf{m}} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\mathrm{n}; \mathrm{EAE-MP} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{n1})} <= \underline{(\mathrm{n2})} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{(\mathrm{n2})}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{(\mathrm{n2})}]))| <= \underline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n \Rightarrow \forall_{obj}(v1){:}\, \forall_{obj}(v2){:}\, \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n \Rightarrow (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n \rhd \dot{\neg}(\forall_{obj}(\epsilon){:}\, \dot{\neg}(\forall_{obj}(n1){:}\, \dot{\neg}(\forall_{obj}(n2){:}\, \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n)n)n)n)n \gg \forall_{obj}(v1){:}\, \forall_{obj}(v2){:}\, \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n \Rightarrow (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n; \mathrm{ExistMP} \rhd \forall_{obj}(v1){:}\, \forall_{obj}(v2){:}\, \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n \Rightarrow (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n \rhd \dot{\neg}(\forall_{obj}(n1){:}\, \dot{\neg}(\forall_{obj}(v1){:}\, \forall_{obj}(v2){:}\, \dot{\neg}(0 <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec(1+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec(1+1) * (\epsilon)))n)n)n)n)n)n)n \gg (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n; \mathrm{Gen} \rhd (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n \gg \forall_{obj}\underline{\mathsf{m}}{:}\, (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n; \mathrm{IntroExist} @ (n2) \rhd \forall_{obj}\underline{\mathsf{m}}{:}\, (n2) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n \gg \dot{\neg}(\forall_{obj}\underline{\mathsf{n}}{:}\, \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\, \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n)n)n\rceil, p_0, \mathsf{c})]$

[FromNotSameF(Strong) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall(v1){:}\, \forall(v2){:}\, \forall\underline{\mathsf{m}}{:}\, \forall\underline{\mathsf{n}}{:}\, \forall(n1){:}\, \forall(n2){:}\, \forall(\epsilon){:}\, \forall(fx){:}\, \forall(fy){:}\, \dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\, \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\, \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\, \dot{\neg}($ $\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash \dot{\neg}(\forall_{obj}\underline{\mathsf{n}}{:}\, \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\, \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((fx)[\underline{\mathsf{m}}] = (fy)[\underline{\mathsf{m}}])n)n)n]$

[FromNotSameF(Strong) $\stackrel{\text{tex}}{\rightarrow}$ "FromNotSameF(Strong)"]

[FromNotSameF(Strong) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromNotSameF(Strong)"]

SameFreciprocal(Helper)

[SameFreciprocal(Helper) $\xrightarrow{\mathrm{proof}} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\overline{\underline{(\mathrm{fx})}}\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n} \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash \mathrm{A4} \mathbin{@} \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n} \gg \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n}; \mathrm{MP} \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n} \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \gg \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n}; \mathrm{NatType} \gg \underline{\mathsf{m}} \in \mathrm{N}; 0\mathrm{f} \rhd \underline{\mathsf{m}} \in \mathrm{N} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0; \mathrm{SubNeqRight} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \rhd \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n} \gg \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n}; \mathrm{ReciprocalFnonzero} \rhd \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]; \mathrm{EqMultiplicationLeft} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$

0)n ⇒ ∸(f_{Ph} = {{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ∸((fx)[m] = 0 ⇒ ∸(f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n}[m] = rec(fx)[m] ≫ ((fx)[m] ∗ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$m: ∸(∸(∸(∸((fx)[m] = 0)n ⇒ ∸(f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ∸((fx)[m] = 0 ⇒ ∸(f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n}[m]) = ((fx)[m] ∗ rec(fx)[m]); Reciprocal ▷ ∸((fx)[m] =
0)n ≫ ((fx)[m] ∗ rec(fx)[m]) = 1; 1f ▷ m ∈ N ≫ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$(crs1): ∸(c_{Ph} = {{(crs1), (crs1)}, {(crs1), 1}})n)n}[m] =
1; eqSymmetry ▷ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$(crs1): ∸(c_{Ph} =
{{(crs1), (crs1)}, {(crs1), 1}})n)n}[m] = 1 ≫ 1 = {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$(crs1): ∸(c_{Ph} = {{(crs1), (crs1)}, {(crs1), 1}})n)n}[m]; eqTransitivity ▷
((fx)[m] ∗ rec(fx)[m]) = 1 ▷ 1 = {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$(crs1): ∸(c_{Ph} =
{{(crs1), (crs1)}, {(crs1), 1}})n)n}[m] ≫ ((fx)[m] ∗ rec(fx)[m]) = {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$(crs1): ∸(c_{Ph} = {{(crs1), (crs1)}, {(crs1), 1}})n)n}[m]; TimesF ≫ {ph ∈
{ph ∈ P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈
N ⇒ ∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} =
{{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$m: ∸(e_{Ph} =
{{m, m}, {m, ((fx)[m] ∗ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$m: ∸(∸(∸(∸((fx)[m] = 0)n ⇒ ∸(f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ∸((fx)[m] = 0 ⇒ ∸(f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n}[m])}})n)n}[m] = ((fx)[m] ∗ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$m: ∸(∸(∸(∸((fx)[m] = 0)n ⇒ ∸(f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ∸((fx)[m] = 0 ⇒ ∸(f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n}[m]); eqTransitivity4 ▷ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸($\forall_{obj}$m: ∸(e_{Ph} = {{m, m}, {m, ((fx)[m] ∗ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = ((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) \rhd ((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) = ((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]) \rhd ((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\underline{\mathrm{fx}})\colon \mathrm{Ded} \rhd$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall(\underline{\mathrm{fx}})\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n} \vdash \underline{\mathsf{n}} <= \underline{\mathsf{m}} \vdash$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\underline{\mathrm{fx}})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = \{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},1\}\})n)n\}[\underline{m}] \gg \forall_{obj}\underline{m}: \underline{n} <=$
$\underline{m} \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{m}])n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)n \Rightarrow \dot{\neg}(f_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},rec\underline{(fx)}[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow \dot{\neg}(f_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = \{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},1\}\})n)n\}[\underline{m}]\rceil, p_0, c)]$

[SameFreciprocal(Helper) $\overset{\text{stmt}}{\to}$ SystemQ $\vdash \forall\underline{m}: \forall\underline{n}: \forall\underline{(fx)}: \forall_{obj}\underline{m}: \underline{n} <= \underline{m} \Rightarrow$
$\dot{\neg}(\underline{(fx)}[\underline{m}] = \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{m}])n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)n \Rightarrow \dot{\neg}(f_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},rec\underline{(fx)}[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow \dot{\neg}(f_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = \{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},1\}\})n)n\}[\underline{m}]]$

[SameFreciprocal(Helper) $\overset{\text{tex}}{\to}$ "SameFreciprocal(Helper)"]

[SameFreciprocal(Helper) $\overset{\text{pyk}}{\to}$ "lemma sameFreciprocal helper"]

SameFreciprocal

[SameFreciprocal $\overset{\text{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall\underline{(fx)}\colon \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$
$\mathrm{FromNotSameF(Strong)} \rhd \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg$
$\dot\neg(\forall_{obj}\underline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(fx)}[\underline{\mathsf{m}}] = \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])n)n)n; \mathrm{SameFreciprocal(Helper)} \gg$
$\forall_{obj}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(fx)}[\underline{\mathsf{m}}] = \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |\ \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\ |$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}]; \mathrm{ExistMP} \rhd$
$\forall_{obj}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\underline{(fx)}[\underline{\mathsf{m}}] = \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |$

$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(\dot{-}(\dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\overline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] \rhd$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])n)n \gg \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(\dot{-}(\dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\overline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}]; Gen \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(\dot{-}(\dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\overline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{-}(\overline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{IntroExist} @ \underline{\mathsf{n}} \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n}; (\mathrm{Eventually} =$
$\mathrm{f})2\mathrm{sameF} \rhd \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SameFreciprocal} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{m},\underline{m}\},\{\underline{m},((fx)[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1): \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\overline{m}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},((fx)[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((fx)[\underline{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}((fx)[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1): \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\overline{m}]))| =$
$(\epsilon))n)n)n)n)n]$

[SameFreciprocal $\overset{tex}{\rightarrow}$ "SameFreciprocal"]

[SameFreciprocal $\overset{pyk}{\rightarrow}$ "lemma sameFreciprocal"]

From!! ==

[From!! == $\overset{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash$
$\forall\underline{m}: \forall\underline{n}: \forall(\epsilon): \forall(fx): \forall(fy): \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|((fx)[\overline{m}] + (-u(fy)[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\overline{m}] +$
$(-u(fy)[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n \vdash To == \rhd\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|((fx)[\overline{m}] + (-u(fy)[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{m}] + (-u(fy)[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}(r1): (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1): \forall_{obj}(f2): \forall_{obj}(f3): \forall_{obj}(f4): \{\{(f1),(f1)\},\{(f1),(f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3),(f3)\},\{(f3),(f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2): \dot{\neg}(\{\{(s1),(s1)\},\{(s1),(s2)\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{Ded} \rhd$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$

$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \dot{\neg}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \vdash \mathrm{MT} \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot{\neg}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$$[\mathrm{From!!} == \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \dot\neg(\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \vdash \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n]$$

$$[\mathrm{From!!} == \overset{\mathrm{tex}}{\rightarrow} \text{“From!!==”}]$$

$$[\mathrm{From!!} == \overset{\mathrm{pyk}}{\rightarrow} \text{“lemma from!!==”}]$$

Reciprocal(R)

$$[\mathrm{Reciprocal(R)} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(fx)}\colon \dot\neg(\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \vdash$
$\mathrm{From!!} == \rhd \dot{\neg}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \gg$

$\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n; SameFreciprocal \rhd \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg$
$\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$

$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n; \text{To} == \triangleright\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n \gg \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec(fx)[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(((fx)[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; eqReflexivity \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) |$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} |$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(((fx)[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg\,((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\};$ == Transitivity $\rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(f1)}{:}\,\forall_{obj}\overline{(f2)}{:}\,\forall_{obj}\overline{(f3)}{:}\,\forall_{obj}\overline{(f4)}{:}\,\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}{:}\,\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}{:}\,\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}{:}\,\dot\neg(\forall_{obj}\overline{\mathsf{n}}{:}\,\dot\neg(\forall_{obj}\overline{\mathsf{m}}{:}\,\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot\neg(\forall_{obj}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}]*\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}{:}\,\dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}]=0)n\Rightarrow\dot\neg(\mathsf{f}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n\Rightarrow\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}]=0\Rightarrow\dot\neg(\mathsf{f}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg(\dot\neg(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot\neg(\forall_{obj}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}]*\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}{:}\,\dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}]=0)n\Rightarrow\dot\neg(\mathsf{f}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n\Rightarrow\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}]=0\Rightarrow\dot\neg(\mathsf{f}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}=$
$\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}{:}\,\overline{(r1)}\in$
$\mathsf{f}_{Ph}\Rightarrow\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}{:}\,\forall_{obj}\overline{(f2)}{:}\,\forall_{obj}\overline{(f3)}{:}\,\forall_{obj}\overline{(f4)}{:}\,\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}{:}\,\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}{:}\,\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}{:}\,\dot\neg(\forall_{obj}\overline{\mathsf{n}}{:}\,\dot\neg(\forall_{obj}\overline{\mathsf{m}}{:}\,\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot\neg(\forall_{obj}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}]*\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}{:}\,\dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}]=0)n\Rightarrow\dot\neg(\mathsf{f}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n\Rightarrow\dot\neg((\underline{(fx)}[\underline{\mathsf{m}}]=0\Rightarrow\dot\neg(\mathsf{f}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg(\dot\neg(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot\neg(\forall_{obj}\overline{(op1)}{:}\,\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Repetition} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\overline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\overline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\overline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\overline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}], p_0, \mathsf{c})]$

$[\mathrm{Reciprocal}(R) \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall(fx)\colon \dot{\neg}(\{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \vdash$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}(\mathrm{fx})[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{fx})[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[Reciprocal(R) $\stackrel{\mathrm{tex}}{\to}$ “Reciprocal(R)”]

[Reciprocal(R) $\stackrel{\mathrm{pyk}}{\to}$ “lemma reciprocal(R)”]

TimesCommutativity(F)

[TimesCommutativity(F) $\stackrel{\text{tex}}{\rightarrow}$ "TimesCommutativity(F)"]

[TimesCommutativity(F) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma timesCommutativity(F)"]

Distribution(F)

[Distribution(F) $\stackrel{\text{tex}}{\rightarrow}$ "Distribution(F)"]

[Distribution(F) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma distribution(F)"]

FromMax(1)

[FromMax(1) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \text{Max} \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n; \text{AddDoubleNeg} \rhd \underline{y} <= \underline{x} \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n)n; \text{ToNegatedAnd}(1) \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n)n \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n)n; \text{NegateDisjunct2} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n)n \gg \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n; \text{SecondConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n \gg \text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x}\rceil, p_0, c)$]

[FromMax(1) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x}$]

[FromMax(1) $\stackrel{\text{tex}}{\rightarrow}$ "FromMax(1)"]

[FromMax(1) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromMax(1)"]

FromMax(2)

[FromMax(2) $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})n \vdash \text{Max} \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n; \text{ToNegatedAnd}(1) \rhd \dot{\neg}(\underline{y} <= \underline{x})n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n)n; \text{NegateDisjunct1} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{x})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n; \text{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x})n \Rightarrow \dot{\neg}(\text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y})n)n \gg \text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y}\rceil, p_0, c)$]

[FromMax(2) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})n \vdash \text{if}(\underline{y} <= \underline{x},\underline{x},\underline{y}) = \underline{y}$]

$[\mathrm{FromMax}(2) \stackrel{\mathrm{tex}}{\to}$ "FromMax(2)"$]$

$[\mathrm{FromMax}(2) \stackrel{\mathrm{pyk}}{\to}$ "lemma fromMax(2)"$]$

ToNegatedAnd

$[\mathrm{ToNegatedAnd} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n} \vdash \mathrm{AddDoubleNeg} \rhd \underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n} \gg \dot\neg(\dot\neg(\underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot\neg(\dot\neg(\underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\dot\neg(\underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToNegatedAnd} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n} \vdash \dot\neg(\dot\neg(\underline{\mathsf{a}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{ToNegatedAnd} \stackrel{\mathrm{tex}}{\to}$ "ToNegatedAnd"$]$

$[\mathrm{ToNegatedAnd} \stackrel{\mathrm{pyk}}{\to}$ "prop lemma to negated and"$]$

DistributionOut

$[\mathrm{DistributionOut} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \mathrm{Distribution} \gg (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}})) = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})); \mathrm{eqSymmetry} \rhd (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}})) = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{DistributionOut} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}}))]$

$[\mathrm{DistributionOut} \stackrel{\mathrm{tex}}{\to}$ "DistributionOut"$]$

$[\mathrm{DistributionOut} \stackrel{\mathrm{pyk}}{\to}$ "lemma distributionOut"$]$

DistributionOutLeft

$[\mathrm{DistributionOutLeft} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \mathrm{timesCommutativity} \gg (\underline{\mathsf{y}} * \underline{\mathsf{x}}) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \mathrm{timesCommutativity} \gg (\underline{\mathsf{z}} * \underline{\mathsf{x}}) = (\underline{\mathsf{x}} * \underline{\mathsf{z}}); \mathrm{AddEquations} \rhd (\underline{\mathsf{y}} * \underline{\mathsf{x}}) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \rhd (\underline{\mathsf{z}} * \underline{\mathsf{x}}) = (\underline{\mathsf{x}} * \underline{\mathsf{z}}) \gg ((\underline{\mathsf{y}} * \underline{\mathsf{x}}) + (\underline{\mathsf{z}} * \underline{\mathsf{x}})) = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})); \mathrm{DistributionOut} \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}})); \mathrm{eqTransitivity} \rhd ((\underline{\mathsf{y}} * \underline{\mathsf{x}}) + (\underline{\mathsf{z}} * \underline{\mathsf{x}})) = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) \rhd ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}})) \gg ((\underline{\mathsf{y}} * \underline{\mathsf{x}}) + (\underline{\mathsf{z}} * \underline{\mathsf{x}})) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{DistributionOutLeft} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon ((\underline{\mathsf{y}} * \underline{\mathsf{x}}) + (\underline{\mathsf{z}} * \underline{\mathsf{x}})) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}}))]$

$[\mathrm{DistributionOutLeft} \stackrel{\mathrm{tex}}{\to}$ "DistributionOutLeft"$]$

$[\mathrm{DistributionOutLeft} \stackrel{\mathrm{pyk}}{\to}$ "lemma distributionOutLeft"$]$

DistributionLeft

[DistributionLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{DistributionOutLeft} \gg$ $((\underline{y} * \underline{x}) + (\underline{z} * \underline{x})) = (\underline{x} * (\underline{y} + \underline{z})); \text{eqSymmetry} \rhd ((\underline{y} * \underline{x}) + (\underline{z} * \underline{x})) =$ $(\underline{x} * (\underline{y} + \underline{z})) \gg (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{y} * \underline{x}) + (\underline{z} * \underline{x}))\rceil, p_0, c)$]

[DistributionLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{y} * \underline{x}) + (\underline{z} * \underline{x}))$]

[DistributionLeft $\stackrel{\text{tex}}{\rightarrow}$ "DistributionLeft"]

[DistributionLeft $\stackrel{\text{pyk}}{\rightarrow}$ "lemma distributionLeft"]

FromNotLess(R)

[FromNotLess(R) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash$
$\forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{n} <= \overline{m} \Rightarrow \underline{(fy)}[\overline{m}] <= (\underline{(fx)}[\overline{m}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\text{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fy)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fy)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\text{Union}(\{N, Q\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \vdash$
$\text{ToLess(R)} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{n} <= \overline{m} \Rightarrow \underline{(fy)}[\overline{m}] <= (\underline{(fx)}[\overline{m}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\text{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \gg$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{Ded} \rhd$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \vdash$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n; \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \vdash$
$\mathrm{NegativeMT} \rhd \dot\neg(\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

[FromNotLess(R) $\overset{\mathrm{stmt}}{\to}$ SystemQ $\vdash$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[FromNotLess(R) $\overset{\mathrm{tex}}{\to}$ “FromNotLess(R)”]

[FromNotLess(R) $\overset{\mathrm{pyk}}{\to}$ “lemma fromNotLess(R)”]

CartProdIsRelation

[CartProdIsRelation $\overset{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \mathrm{Sep2Formula} \rhd \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$

$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}\,(\overline{(\mathrm{r1})} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}\,(\overline{(\mathrm{r1})} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \mathrm{Ded} \rhd$
$\forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{CartProdIsRelation} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[CartProdIsRelation $\stackrel{\mathrm{tex}}{\to}$ "CartProdIsRelation"]

[CartProdIsRelation $\stackrel{\mathrm{pyk}}{\to}$ "lemma cartProdIsRelation"]

FromSubset

$[\mathrm{FromSubset} \stackrel{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sz})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \vdash \underline{(\mathrm{sz})} \in \underline{(\mathrm{sx})} \vdash \mathrm{Repetition} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})}; \mathrm{A4} @ \underline{(\mathrm{sz})} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sz})} \in \underline{(\mathrm{sx})} \Rightarrow \underline{(\mathrm{sz})} \in \underline{(\mathrm{sy})}; \mathrm{MP} \rhd \underline{(\mathrm{sz})} \in \underline{(\mathrm{sx})} \Rightarrow \underline{(\mathrm{sz})} \in \underline{(\mathrm{sy})} \rhd \underline{(\mathrm{sz})} \in \underline{(\mathrm{sx})} \gg \underline{(\mathrm{sz})} \in \underline{(\mathrm{sy})}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromSubset} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sz})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \vdash \underline{(\mathrm{sz})} \in \underline{(\mathrm{sx})} \vdash \underline{(\mathrm{sz})} \in \underline{(\mathrm{sy})}]$

[FromSubset $\stackrel{\mathrm{tex}}{\to}$ "FromSubset"]

[FromSubset $\stackrel{\mathrm{pyk}}{\to}$ "lemma fromSubset"]

SubsetIsRelation

$[\mathrm{SubsetIsRelation} \stackrel{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sz})}\colon \forall\underline{(\mathrm{su})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \vdash \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \vdash \mathrm{Repetition} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ \overline{(\mathrm{r1})} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromSubset} \rhd$
$\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \rhd \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \gg \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})}; \mathrm{MP} \rhd \overline{(\mathrm{r1})} \in$
$\underline{(\mathrm{sx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \overline{(\mathrm{r1})} \in$
$\underline{(\mathrm{sx})} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sz})}\colon \forall\underline{(\mathrm{su})}\colon \mathrm{Ded} \rhd$
$\forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sz})}\colon \forall\underline{(\mathrm{su})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \Rightarrow$
$\overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \vdash \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in$
$\underline{(\mathrm{sy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \Rightarrow$
$\overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \vdash \mathrm{MP2} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in$
$\underline{(\mathrm{sy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \Rightarrow$
$\overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \gg \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\underline{(\mathrm{sy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$

$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SubsetIsRelation} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \forall \underline{(\mathrm{su})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{sy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{sx})} \vdash \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{sy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sz})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{su})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{SubsetIsRelation} \overset{\mathrm{tex}}{\rightarrow}$ "SubsetIsRelation"]

$[\mathrm{SubsetIsRelation} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma subsetIsRelation"]

ToSeries

$[\mathrm{ToSeries} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})} \vdash \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n} \vdash \mathrm{Repetition} \rhd \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})} \gg \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToSeries} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{sy})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$

$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \vdash$
$\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} \vdash \forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in$
$N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})n)n \vdash \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in$
$\underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})n)n)n)n]$

[ToSeries $\overset{tex}{\rightarrow}$ "ToSeries"]

[ToSeries $\overset{pyk}{\rightarrow}$ "lemma toSeries"]

FromSeries

[FromSeries $\overset{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall\underline{(fx)}\colon \forall\underline{(sy)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash Repetition \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; Repetition \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{FromSeries} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(fx)}: \forall\underline{(sy)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\underline{(fx)})n)n)n)n\vdash\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\underline{(fx)}\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in\underline{(sy)})n)n\Rightarrow$
$\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\underline{(fx)}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\underline{(fx)}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in\underline{(fx)})n)n)n)n]$

[FromSeries $\stackrel{tex}{\rightarrow}$ "FromSeries"]

[FromSeries $\stackrel{pyk}{\rightarrow}$ "lemma fromSeries"]

SeriesSubsetCP

[SeriesSubsetCP $\stackrel{proof}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil SystemQ\vdash\forall\underline{(fx)}\colon\forall\underline{(sy)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in$
$\underline{(fx)}\Rightarrow\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in$
$\underline{(sy)})n)n\Rightarrow\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\underline{(fx)}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\underline{(fx)}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\underline{(fx)})n)n)n)n\vdash\overline{(s1)}\in\underline{(fx)}\vdash$ FromSeries $\rhd\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\underline{(fx)}\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in\underline{(sy)})n)n\Rightarrow$
$\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\underline{(fx)}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\underline{(fx)}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\underline{(fx)})n)n)n)n\gg\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\underline{(fx)}\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in\underline{(sy)})n)n\Rightarrow$
$\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\underline{(fx)}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\underline{(fx)}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\underline{(fx)})n)n)n)n;$ FirstConjunct $\rhd\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\underline{(fx)}\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in\underline{(sy)})n)n\Rightarrow$
$\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\underline{(fx)}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\underline{(fx)}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\underline{(fx)})n)n)n)n\gg\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\underline{(fx)}\Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} \mathbin{@} \overline{(\mathrm{s1})} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{s1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{s1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{s1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{sy})}\colon \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{s1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{s1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{sy})})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{op1})} \in$
$\mathrm{N}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{sy})}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{s1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{s1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\}; \mathrm{eqSymmetry} \rhd \overline{(\mathrm{s1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} \gg \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} =$
$\overline{(\mathrm{s1})}; \mathrm{ToCartProd} \rhd \overline{(\mathrm{op1})} \in \mathrm{N} \rhd \overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})} \gg$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{SameMember} \rhd$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} = \overline{(\mathrm{s1})} \rhd \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}; \mathrm{Ded} \rhd \forall\underline{(sy)}\colon \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(s1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \vdash \overline{(s1)} \in$
$\{ph \in P(P(Union(\{\mathrm{N}, \underline{(sy)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \gg \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in$
$\underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(s1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow \overline{(s1)} \in \{ph \in$
$P(P(Union(\{\mathrm{N}, \underline{(sy)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}; \mathrm{TwiceExistMP} \rhd \dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(s1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow$
$\overline{(s1)} \in \{ph \in P(P(Union(\{\mathrm{N}, \underline{(sy)}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(s1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \gg \overline{(s1)} \in \{ph \in$
$P(P(Union(\{\mathrm{N}, \underline{(sy)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}; \forall\underline{(fx)}\colon \forall\underline{(sy)}\colon \mathrm{Ded} \rhd$
$\forall\underline{(fx)}\colon \forall\underline{(sy)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash \overline{(s1)} \in \underline{(fx)} \vdash \overline{(s1)} \in \{ph \in P(P(Union(\{\mathrm{N}, \underline{(sy)}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\underline{(fx)} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in$
$\underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \{ph \in P(P(Union(\{\mathrm{N}, \underline{(sy)}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}; \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\underline{(fx)} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in$
$\underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash MP \rhd \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \forall_{obj}\overline{(s1)}: \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \{ph \in P(P(Union(\{N, \underline{(sy)}\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \rhd \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in$
$\underline{(sy)})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \forall_{obj}\overline{(s1)}: \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \{ph \in P(P(Union(\{N, \underline{(sy)}\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}; Repetition \rhd$
$\forall_{obj}\overline{(s1)}: \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \{ph \in P(P(Union(\{N, \underline{(sy)}\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \gg \forall_{obj}\overline{(s1)}: \overline{(s1)} \in$
$\underline{(fx)} \Rightarrow \overline{(s1)} \in \{ph \in P(P(Union(\{N, \underline{(sy)}\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}\rceil, p_0, \mathsf{c})]$

$[SeriesSubsetCP \overset{stmt}{\rightarrow} SystemQ \vdash \forall\underline{(fx)}: \forall\underline{(sy)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash \forall_{obj}\overline{(s1)}: \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \{ph \in P(P(Union(\{N, \underline{(sy)}\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}]$

$[SeriesSubsetCP \overset{tex}{\rightarrow}$ "SeriesSubsetCP"]

$[SeriesSubsetCP \overset{pyk}{\rightarrow}$ "lemma seriesSubsetCP"]

ValueType

$[\mathrm{ValueType} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}: \forall \underline{(\mathrm{fx})}: \forall \underline{(\mathrm{sy})}: \underline{\mathsf{m}} \in \mathrm{N} \vdash$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}: \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{MemberOfSeries} \rhd \underline{\mathsf{m}} \in \mathrm{N} \rhd \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}: \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \in$
$\underline{(\mathrm{fx})}; \mathrm{SeriesSubsetCP} \rhd \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}: \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}: \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{FromSubset} \rhd$
$\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \in$
$\underline{(\mathrm{fx})} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{FromCartProd}(2) \rhd$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \in$
$\underline{(\mathrm{sy})}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ValueType} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}: \forall \underline{(\mathrm{fx})}: \forall \underline{(\mathrm{sy})}: \underline{\mathsf{m}} \in \mathrm{N} \vdash \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in$
$\underline{(\mathrm{fx})} \Rightarrow \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}: \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}: \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in$

$\underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \in \underline{(\mathrm{sy})}]$

[ValueType $\overset{\mathrm{tex}}{\to}$ “ValueType”]

[ValueType $\overset{\mathrm{pyk}}{\to}$ “lemma valueType”]

RemoveOr

[RemoveOr $\overset{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}} \vdash \mathrm{Repetition} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}} \gg \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}}; \mathrm{AutoImply} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{FromNegations} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}} \gg \underline{\mathsf{a}}\rceil, \mathrm{p}_0, \mathsf{c})$]

[RemoveOr $\overset{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{a}} \vdash \underline{\mathsf{a}}$]

[RemoveOr $\overset{\mathrm{tex}}{\to}$ “RemoveOr”]

[RemoveOr $\overset{\mathrm{pyk}}{\to}$ “prop lemma remove or”]

FromSingleton

[FromSingleton $\overset{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \vdash \mathrm{Repetition} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\}; \mathrm{Pair2Formula} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \gg \dot{\neg}\,(\underline{(\mathrm{sx})} = \underline{(\mathrm{sy})})\mathrm{n} \Rightarrow \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}; \mathrm{RemoveOr} \rhd \dot{\neg}\,(\underline{(\mathrm{sx})} = \underline{(\mathrm{sy})})\mathrm{n} \Rightarrow \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}\rceil, \mathrm{p}_0, \mathsf{c})$]

[FromSingleton $\overset{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \vdash \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}$]

[FromSingleton $\overset{\mathrm{tex}}{\to}$ “FromSingleton”]

[FromSingleton $\overset{\mathrm{pyk}}{\to}$ “lemma fromSingleton”]

InPair(1)

[InPair(1) $\overset{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \mathrm{eqReflexivity} \gg \underline{(\mathrm{sx})} = \underline{(\mathrm{sx})}; \mathrm{WeakenOr2} \rhd \underline{(\mathrm{sx})} = \underline{(\mathrm{sx})} \gg \dot{\neg}\,(\underline{(\mathrm{sx})} = \underline{(\mathrm{sx})})\mathrm{n} \Rightarrow \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}; \mathrm{Formula2Pair} \rhd \dot{\neg}\,(\underline{(\mathrm{sx})} = \underline{(\mathrm{sx})})\mathrm{n} \Rightarrow \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\rceil, \mathrm{p}_0, \mathsf{c})$]

[InPair(1) $\overset{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}$]

$[\mathrm{InPair}(1) \stackrel{\mathrm{tex}}{\to} \text{“InPair(1)”}]$

$[\mathrm{InPair}(1) \stackrel{\mathrm{pyk}}{\to} \text{“lemma inPair(1)”}]$

InPair(2)

$[\mathrm{InPair}(2) \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \mathrm{eqReflexivity} \gg \underline{(\mathrm{sy})} = \underline{(\mathrm{sy})}; \mathrm{WeakenOr1} \rhd \underline{(\mathrm{sy})} = \underline{(\mathrm{sy})} \gg \dot{\neg}\,(\underline{(\mathrm{sy})} = \underline{(\mathrm{sx})})\mathrm{n} \Rightarrow \underline{(\mathrm{sy})} = \underline{(\mathrm{sy})}; \mathrm{Formula2Pair} \rhd \dot{\neg}\,(\underline{(\mathrm{sy})} = \underline{(\mathrm{sx})})\mathrm{n} \Rightarrow \underline{(\mathrm{sy})} = \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{InPair}(2) \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}]$

$[\mathrm{InPair}(2) \stackrel{\mathrm{tex}}{\to} \text{“InPair(2)”}]$

$[\mathrm{InPair}(2) \stackrel{\mathrm{pyk}}{\to} \text{“lemma inPair(2)”}]$

SameMember(2)

$[\mathrm{SameMember}(2) \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \vdash \underline{(\mathrm{sy})} \in \underline{(\mathrm{sz})} \vdash \mathrm{eqSymmetry} \rhd \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sy})} = \underline{(\mathrm{sx})}; \mathrm{SameMember} \rhd \underline{(\mathrm{sy})} = \underline{(\mathrm{sx})} \rhd \underline{(\mathrm{sy})} \in \underline{(\mathrm{sz})} \gg \underline{(\mathrm{sx})} \in \underline{(\mathrm{sz})}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SameMember}(2) \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \vdash \underline{(\mathrm{sy})} \in \underline{(\mathrm{sz})} \vdash \underline{(\mathrm{sx})} \in \underline{(\mathrm{sz})}]$

$[\mathrm{SameMember}(2) \stackrel{\mathrm{tex}}{\to} \text{“SameMember(2)”}]$

$[\mathrm{SameMember}(2) \stackrel{\mathrm{pyk}}{\to} \text{“lemma sameMember(2)”}]$

ToBinaryUnion(1)

$[\mathrm{ToBinaryUnion}(1) \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \forall \underline{(\mathrm{su})}\colon \underline{(\mathrm{sx})} \in \underline{(\mathrm{sy})} \vdash \mathrm{InPair}(1) \gg \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\}; \mathrm{JoinConjuncts} \rhd \underline{(\mathrm{sx})} \in \underline{(\mathrm{sy})} \rhd \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\} \gg \dot{\neg}\,(\underline{(\mathrm{sx})} \in \underline{(\mathrm{sy})} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@} \underline{(\mathrm{sy})} \rhd \dot{\neg}\,(\underline{(\mathrm{sx})} \in \underline{(\mathrm{sy})} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{su})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{sx})} \in \underline{(\mathrm{su})} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{su})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Formula2Union} \rhd \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{(\mathrm{su})}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{sx})} \in \underline{(\mathrm{su})} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{su})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{sx})} \in \mathrm{Union}(\{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\}); \mathrm{Repetition} \rhd \underline{(\mathrm{sx})} \in \mathrm{Union}(\{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\}) \gg \underline{(\mathrm{sx})} \in \mathrm{Union}(\{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToBinaryUnion}(1) \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \forall \underline{(\mathrm{su})}\colon \underline{(\mathrm{sx})} \in \underline{(\mathrm{sy})} \vdash \underline{(\mathrm{sx})} \in \mathrm{Union}(\{\underline{(\mathrm{sy})}, \underline{(\mathrm{sz})}\})]$

[ToBinaryUnion(1) $\overset{\text{tex}}{\to}$ "ToBinaryUnion(1)"]

[ToBinaryUnion(1) $\overset{\text{pyk}}{\to}$ "lemma toBinaryUnion(1)"]

ToBinaryUnion(2)

[ToBinaryUnion(2) $\overset{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢ ∀(sx): ∀(sy): ∀(sz): ∀(su): (sx) ∈ (sz) ⊢ InPair(2) ≫ (sz) ∈ {(sy), (sz)}; JoinConjuncts ▷ (sx) ∈ (sz) ▷ (sz) ∈ {(sy), (sz)} ≫ ∸((sx) ∈ (sz) ⇒ ∸((sz) ∈ {(sy), (sz)})n)n; IntroExist @ (sz) ▷ ∸((sx) ∈ (sz) ⇒ ∸((sz) ∈ {(sy), (sz)})n)n ≫ ∸(∀obj(su): ∸(∸((sx) ∈ (su) ⇒ ∸((su) ∈ {(sy), (sz)})n)n)n)n; Formula2Union ▷ ∸(∀obj(su): ∸(∸((sx) ∈ (su) ⇒ ∸((su) ∈ {(sy), (sz)})n)n)n)n ≫ (sx) ∈ Union({(sy), (sz)}); Repetition ▷ (sx) ∈ Union({(sy), (sz)}) ≫ (sx) ∈ Union({(sy), (sz)})⌉, p_0, c)]

[ToBinaryUnion(2) $\overset{\text{stmt}}{\to}$ SystemQ ⊢ ∀(sx): ∀(sy): ∀(sz): ∀(su): (sx) ∈ (sz) ⊢ (sx) ∈ Union({(sy), (sz)})]

[ToBinaryUnion(2) $\overset{\text{tex}}{\to}$ "ToBinaryUnion(2)"]

[ToBinaryUnion(2) $\overset{\text{pyk}}{\to}$ "lemma toBinaryUnion(2)"]

FromOrderedPair(TwoLevels)

[FromOrderedPair(TwoLevels) $\overset{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢ ∀(sx): ∀(sy): ∀(sz): ∀(su): (sx) ∈ (sy) ⊢ (sy) ∈ {{(sz), (sz)}, {(sz), (su)}} ⊢ Repetition ▷ (sy) ∈ {{(sz), (sz)}, {(sz), (su)}} ≫ (sy) ∈ {{(sz), (sz)}, {(sz), (su)}}; Pair2Formula ▷ (sy) ∈ {{(sz), (sz)}, {(sz), (su)}} ≫ ∸((sy) = {(sz), (sz)})n ⇒ (sy) = {(sz), (su)}; ∀(sx): ∀(sy): ∀(sz): ∀(su): (sy) = {(sz), (sz)} ⊢ (sx) ∈ (sy) ⊢ SENC1 ▷ (sy) = {(sz), (sz)} ▷ (sx) ∈ (sy) ≫ (sx) ∈ {(sz), (sz)}; FromSingleton ▷ (sx) ∈ {(sz), (sz)} ≫ (sx) = (sz); WeakenOr2 ▷ (sx) = (sz) ≫ ∸((sx) = (sz))n ⇒ (sx) = (su); ∀(sx): ∀(sy): ∀(sz): ∀(su): (sy) = {(sz), (su)} ⊢ (sx) ∈ (sy) ⊢ SENC1 ▷ (sy) = {(sz), (su)} ▷ (sx) ∈ (sy) ≫ (sx) ∈ {(sz), (su)}; Pair2Formula ▷ (sx) ∈ {(sz), (su)} ≫ ∸((sx) = (sz))n ⇒ (sx) = (su); Ded ▷ ∀(sx): ∀(sy): ∀(sz): ∀(su): (sy) = {(sz), (sz)} ⊢ (sx) ∈ (sy) ⊢ ∸((sx) = (sz))n ⇒ (sx) = (su) ≫ (sy) = {(sz), (sz)} ⇒ (sx) ∈ (sy) ⇒ ∸((sx) = (sz))n ⇒ (sx) = (su); Ded ▷ ∀(sx): ∀(sy): ∀(sz): ∀(su): (sy) = {(sz), (su)} ⊢ (sx) ∈ (sy) ⊢ ∸((sx) = (sz))n ⇒ (sx) = (su) ≫ (sy) = {(sz), (su)} ⇒ (sx) ∈ (sy) ⇒ ∸((sx) = (sz))n ⇒ (sx) = (su); FromDisjuncts ▷ ∸((sy) = {(sz), (sz)})n ⇒ (sy) = {(sz), (su)} ▷ (sy) = {(sz), (sz)} ⇒ (sx) ∈ (sy) ⇒ ∸((sx) = (sz))n ⇒ (sx) = (su) ▷ (sy) = {(sz), (su)} ⇒ (sx) ∈ (sy) ⇒ ∸((sx) = (sz))n ⇒ (sx) = (su) ≫ (sx) ∈ (sy) ⇒ ∸((sx) = (sz))n ⇒ (sx) = (su); MP ▷ (sx) ∈ (sy) ⇒ ∸((sx) = (sz))n ⇒ (sx) = (su) ▷ (sx) ∈ (sy) ≫ ∸((sx) = (sz))n ⇒ (sx) = (su)⌉, p_0, c)]

[FromOrderedPair(TwoLevels) $\overset{\text{stmt}}{\to}$ SystemQ ⊢ ∀(sx): ∀(sy): ∀(sz): ∀(su): (sx) ∈

$\underline{(sy)} \vdash \underline{(sy)} \in \{\{\underline{(sz)}, \underline{(sz)}\}, \{\underline{(sz)}, \underline{(su)}\}\} \vdash \dot{\neg}(\underline{(sx)} = \underline{(sz)})\mathrm{n} \Rightarrow \underline{(sx)} = \underline{(su)}]$

[FromOrderedPair(TwoLevels) $\stackrel{\mathrm{tex}}{\rightarrow}$ "FromOrderedPair(TwoLevels)"]

[FromOrderedPair(TwoLevels) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromOrderedPair(twoLevels)"]

ToCartProd(Helper)

[ToCartProd(Helper) $\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \forall \underline{(sz)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \underline{(sz)} \in$
$\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \vdash \overline{(s1)} \in \underline{(sz)} \vdash \mathrm{FromOrderedPair(TwoLevels)} \rhd \overline{(s1)} \in$
$\underline{(sz)} \rhd \underline{(sz)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \gg \dot{\neg}(\overline{(s1)} = \underline{(sx)})\mathrm{n} \Rightarrow \overline{(s1)} =$
$\underline{(sy)}; \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy1)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \overline{(s1)} = \underline{(sx)} \vdash \mathrm{SameMember}(2) \rhd \overline{(s1)} =$
$\underline{(sx)} \rhd \underline{(sx)} \in \underline{(sx1)} \gg \overline{(s1)} \in \underline{(sx1)}; \mathrm{ToBinaryUnion}(1) \rhd \overline{(s1)} \in \underline{(sx1)} \gg \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \underline{(sy)} \in \underline{(sy1)} \vdash \overline{(s1)} = \underline{(sy)} \vdash$
$\mathrm{SameMember}(2) \rhd \overline{(s1)} = \underline{(sy)} \rhd \underline{(sy)} \in \underline{(sy1)} \gg \overline{(s1)} \in$
$\underline{(sy1)}; \mathrm{ToBinaryUnion}(2) \rhd \overline{(s1)} \in \underline{(sy1)} \gg \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{Ded} \rhd \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy1)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \overline{(s1)} = \underline{(sx)} \vdash$
$\overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg \underline{(sx)} \in \underline{(sx1)} \Rightarrow \overline{(s1)} = \underline{(sx)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{MP} \rhd \underline{(sx)} \in \underline{(sx1)} \Rightarrow \overline{(s1)} = \underline{(sx)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \rhd \underline{(sx)} \in \underline{(sx1)} \gg \overline{(s1)} = \underline{(sx)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{Ded} \rhd \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \underline{(sy)} \in \underline{(sy1)} \vdash \overline{(s1)} = \underline{(sy)} \vdash$
$\overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg \underline{(sy)} \in \underline{(sy1)} \Rightarrow \overline{(s1)} = \underline{(sy)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{MP} \rhd \underline{(sy)} \in \underline{(sy1)} \Rightarrow \overline{(s1)} = \underline{(sy)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \rhd \underline{(sy)} \in \underline{(sy1)} \gg \overline{(s1)} = \underline{(sy)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{FromDisjuncts} \rhd \dot{\neg}(\overline{(s1)} = \underline{(sx)})\mathrm{n} \Rightarrow \overline{(s1)} = \underline{(sy)} \rhd \overline{(s1)} =$
$\underline{(sx)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \rhd \overline{(s1)} = \underline{(sy)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \forall \underline{(sz)}\colon \mathrm{Ded} \rhd$
$\forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \forall \underline{(sz)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \underline{(sz)} \in$
$\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \vdash \overline{(s1)} \in \underline{(sz)} \vdash \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg \underline{(sx)} \in$
$\underline{(sx1)} \Rightarrow \underline{(sy)} \in \underline{(sy1)} \Rightarrow \underline{(sz)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \underline{(sz)} \Rightarrow$
$\overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \underline{(sz)} \in$
$\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \vdash \mathrm{MP3} \rhd \underline{(sx)} \in \underline{(sx1)} \Rightarrow \underline{(sy)} \in \underline{(sy1)} \Rightarrow \underline{(sz)} \in$
$\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \underline{(sz)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \rhd \underline{(sx)} \in$
$\underline{(sx1)} \rhd \underline{(sy)} \in \underline{(sy1)} \rhd \underline{(sz)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \gg \overline{(s1)} \in \underline{(sz)} \Rightarrow \overline{(s1)} \in$
$\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{Gen} \rhd \overline{(s1)} \in \underline{(sz)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg$
$\forall_{\mathrm{obj}} \overline{(s1)}\colon \overline{(s1)} \in \underline{(sz)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{Repetition} \rhd \forall_{\mathrm{obj}} \overline{(s1)}\colon \overline{(s1)} \in$

$\underline{(sz)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \underline{(sz)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToCartProd(Helper)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \forall\underline{(sz)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \underline{(sz)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \vdash \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \underline{(sz)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})]$

[ToCartProd(Helper) $\stackrel{\mathrm{tex}}{\rightarrow}$ "ToCartProd(Helper)"]

[ToCartProd(Helper) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma toCartProd helper"]

ToCartProd

$[\mathrm{ToCartProd} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \vdash$ $\mathrm{ToCartProd(Helper)} \rhd \underline{(sx)} \in \underline{(sx1)} \rhd \underline{(sy)} \in \underline{(sy1)} \rhd \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \gg \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \overline{(s1)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}); \mathrm{Formula2Power} \rhd \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \overline{(s1)} \Rightarrow \overline{(s1)} \in \mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}) \gg \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})); \forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \mathrm{Ded} \rhd$ $\forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \vdash \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})) \gg \underline{(sx)} \in \underline{(sx1)} \Rightarrow \underline{(sy)} \in \underline{(sy1)} \Rightarrow \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})); \underline{(sx)} \in \underline{(sx1)} \vdash \underline{(sy)} \in \underline{(sy1)} \vdash \mathrm{MP2} \rhd \underline{(sx)} \in \underline{(sx1)} \Rightarrow \underline{(sy)} \in \underline{(sy1)} \Rightarrow \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})) \rhd \underline{(sx)} \in \underline{(sx1)} \rhd \underline{(sy)} \in \underline{(sy1)} \gg \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})); \mathrm{Gen} \rhd \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})) \gg \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})); \mathrm{Repetition} \rhd$ $\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})) \gg \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})); \mathrm{Formula2Power} \rhd \forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \Rightarrow \overline{(s1)} \in \mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\})) \gg \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}))); \mathrm{eqReflexivity} \gg \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\}; \mathrm{JoinConjuncts} \rhd \underline{(sx)} \in \underline{(sx1)} \rhd \underline{(sy)} \in \underline{(sy1)} \gg \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})\mathrm{n})\mathrm{n} \rhd \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \gg \dot{\neg}(\dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @ \underline{(sy)} \rhd \dot{\neg}(\dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow$

$\dot{\neg}\,(\underline{(\mathrm{sy})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} =$
$\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{sx})} \in \underline{(\mathrm{sx1})} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} =$
$\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@} \underline{(\mathrm{sx})} \rhd$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{sx})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} = \{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Formula2Sep} \rhd$
$\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})},\underline{(\mathrm{sy1})}\}))) \rhd$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})},\underline{(\mathrm{sy1})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Repetition} \rhd$
$\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})},\underline{(\mathrm{sy1})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})},\underline{(\mathrm{sy1})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToCartProd} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sx1})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sy1})}\colon \underline{(\mathrm{sx})} \in \underline{(\mathrm{sx1})} \vdash \underline{(\mathrm{sy})} \in$
$\underline{(\mathrm{sy1})} \vdash \{\{\underline{(\mathrm{sx})},\underline{(\mathrm{sx})}\},\{\underline{(\mathrm{sx})},\underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})},\underline{(\mathrm{sy1})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

$[\mathrm{ToCartProd} \overset{\mathrm{tex}}{\to}$ “ToCartProd”$]$

$[\mathrm{ToCartProd} \overset{\mathrm{pyk}}{\to}$ “lemma toCartProd”$]$

NonreciprocalToRight(Eq)

$[\mathrm{NonreciprocalToRight(Eq)} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon \dot{\neg}\,(\underline{\mathsf{y}} = 0)\mathrm{n} \vdash$
$(\underline{\mathsf{x}} * \underline{\mathsf{y}}) = \underline{\mathsf{z}} \vdash \mathrm{eqMultiplication} \rhd (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = \underline{\mathsf{z}} \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}}) = (\underline{\mathsf{z}} * \mathrm{rec}\underline{\mathsf{y}}); \mathrm{x} =$

$x * y * (1/y) \rhd \dot{\neg}\, (\underline{y} = 0)n \gg \underline{x} = ((\underline{x} * \underline{y}) * rec\underline{y}); eqTransitivity \rhd \underline{x} = ((\underline{x} * \underline{y}) * rec\underline{y}) \rhd ((\underline{x} * \underline{y}) * rec\underline{y}) = (\underline{z} * rec\underline{y}) \gg \underline{x} = (\underline{z} * rec\underline{y})\rceil, p_0, c)]$

$[NonreciprocalToRight(Eq) \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\, (\underline{y} = 0)n \vdash (\underline{x} * \underline{y}) = \underline{z} \vdash \underline{x} = (\underline{z} * rec\underline{y})]$

$[NonreciprocalToRight(Eq) \stackrel{tex}{\rightarrow}$ "NonreciprocalToRight(Eq)"]

$[NonreciprocalToRight(Eq) \stackrel{pyk}{\rightarrow}$ "lemma nonreciprocalToRight(Eq)"]

NonreciprocalToLeft(Eq)(1term)

$[NonreciprocalToLeft(Eq)(1term) \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon 1 = (\underline{x} * \underline{y}) \vdash eqSymmetry \rhd 1 = (\underline{x} * \underline{y}) \gg (\underline{x} * \underline{y}) = 1; NonreciprocalToRight(Eq)(1term) \rhd (\underline{x} * \underline{y}) = 1 \gg \underline{x} = rec\underline{y}; eqSymmetry \rhd \underline{x} = rec\underline{y} \gg rec\underline{y} = \underline{x}\rceil, p_0, c)]$

$[NonreciprocalToLeft(Eq)(1term) \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon 1 = (\underline{x} * \underline{y}) \vdash rec\underline{y} = \underline{x}]$

$[NonreciprocalToLeft(Eq)(1term) \stackrel{tex}{\rightarrow}$ "NonreciprocalToLeft(Eq)(1 term)"]

$[NonreciprocalToLeft(Eq)(1term) \stackrel{pyk}{\rightarrow}$ "lemma nonreciprocalToLeft(Eq)(1 term)"]

SameReciprocal

$[SameReciprocal \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\, (\underline{x} = 0)n \vdash \underline{x} = \underline{y} \vdash times1Left \gg (1 * \underline{x}) = \underline{x}; eqTransitivity \rhd (1 * \underline{x}) = \underline{x} \rhd \underline{x} = \underline{y} \gg (1 * \underline{x}) = \underline{y}; NonreciprocalToRight(Eq) \rhd \dot{\neg}\, (\underline{x} = 0)n \rhd (1 * \underline{x}) = \underline{y} \gg 1 = (\underline{y} * rec\underline{x}); timesCommutativity \gg (\underline{y} * rec\underline{x}) = (rec\underline{x} * \underline{y}); eqTransitivity \rhd 1 = (\underline{y} * rec\underline{x}) \rhd (\underline{y} * rec\underline{x}) = (rec\underline{x} * \underline{y}) \gg 1 = (rec\underline{x} * \underline{y}); NonreciprocalToLeft(Eq)(1term) \rhd 1 = (rec\underline{x} * \underline{y}) \gg rec\underline{y} = rec\underline{x}; eqSymmetry \rhd rec\underline{y} = rec\underline{x} \gg rec\underline{x} = rec\underline{y}\rceil, p_0, c)]$

$[SameReciprocal \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\, (\underline{x} = 0)n \vdash \underline{x} = \underline{y} \vdash rec\underline{x} = rec\underline{y}]$

$[SameReciprocal \stackrel{tex}{\rightarrow}$ "SameReciprocal"]

$[SameReciprocal \stackrel{pyk}{\rightarrow}$ "lemma sameReciprocal"]

CPseparationIsRelation

$[CPseparationIsRelation \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{a}\colon \forall \underline{(sx)}\colon \forall \underline{(sy)}\colon \overline{(s1)} \in \{ph \in \{ph \in P(P(Union(\{\underline{(sx)}, \underline{(sy)}\}))) \mid$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \underline{\mathsf{a}}\} \vdash$
$\mathrm{Separation2formula}(1) \rhd \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \underline{\mathsf{a}}\} \gg \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \forall\underline{\mathsf{a}}\colon \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \mathrm{Ded} \rhd$
$\forall\underline{\mathsf{a}}\colon \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sy})}\colon \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \underline{\mathsf{a}}\} \vdash \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \underline{\mathsf{a}}\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Repetition} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \underline{\mathsf{a}}\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \underline{\mathsf{a}}\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{CartProdIsRelation} \gg$
$\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubsetIsRelation} \rhd$
$\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in \underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \rhd \forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \{ph \in$
$\{ph \in P(P(Union(\{\underline{(sx)}, \underline{(sy)}\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$\underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n\} \mid \underline{\mathsf{a}}\} \Rightarrow \overline{(s1)} \in \{ph \in$
$P(P(Union(\{\underline{(sx)}, \underline{(sy)}\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$\underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n\} \gg \forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \{ph \in \{ph \in$
$P(P(Union(\{\underline{(sx)}, \underline{(sy)}\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$\underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n\} \mid \underline{\mathsf{a}}\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in \underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[CPseparationIsRelation $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{a}}\colon \forall\underline{(sx)}\colon \forall\underline{(sy)}\colon \forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \{ph \in$
$\{ph \in P(P(Union(\{\underline{(sx)}, \underline{(sy)}\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$\underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n\} \mid \underline{\mathsf{a}}\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in \underline{(sx)} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n]$

[CPseparationIsRelation $\overset{\mathrm{tex}}{\rightarrow}$ "CPseparationIsRelation"]

[CPseparationIsRelation $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma CPseparationIsRelation"]

OrderedPairEquality

[OrderedPairEquality $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \forall\underline{(sz)}\colon \forall\underline{(sz1)}\colon \forall\underline{(su)}\colon \forall\underline{(su1)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} =$
$\{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \vdash \{\{\underline{(sz)}, \underline{(sz)}\}, \{\underline{(sz)}, \underline{(sz1)}\}\} =$
$\{\{\underline{(su)}, \underline{(su)}\}, \{\underline{(su)}, \underline{(su1)}\}\} \vdash \underline{(sx)} = \underline{(sz)} \vdash$
FromOrderedPair(1) $\rhd \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} =$
$\{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \gg \underline{(sx)} = \underline{(sy)}$; eqSymmetry $\rhd \underline{(sx)} = \underline{(sy)} \gg \underline{(sy)} =$
$\underline{(sx)}$; FromOrderedPair(1) $\rhd \{\{\underline{(sz)}, \underline{(sz)}\}, \{\underline{(sz)}, \underline{(sz1)}\}\} =$
$\{\{\underline{(su)}, \underline{(su)}\}, \{\underline{(su)}, \underline{(su1)}\}\} \gg \underline{(sz)} = \underline{(su)}$; eqTransitivity4 $\rhd \underline{(sy)} =$
$\underline{(sx)} \rhd \underline{(sx)} = \underline{(sz)} \rhd \underline{(sz)} = \underline{(su)} \gg \underline{(sy)} = \underline{(su)}\rceil, p_0, \mathsf{c})]$

[OrderedPairEquality $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \forall\underline{(sz)}\colon \forall\underline{(sz1)}\colon \forall\underline{(su)}\colon \forall\underline{(su1)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} =$
$\{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \vdash \{\{\underline{(sz)}, \underline{(sz)}\}, \{\underline{(sz)}, \underline{(sz1)}\}\} =$
$\{\{\underline{(su)}, \underline{(su)}\}, \{\underline{(su)}, \underline{(su1)}\}\} \vdash \underline{(sx)} = \underline{(sz)} \vdash \underline{(sy)} = \underline{(su)}]$

[OrderedPairEquality $\overset{\text{tex}}{\to}$ “OrderedPairEquality”]

[OrderedPairEquality $\overset{\text{pyk}}{\to}$ “lemma orderedPairEquality”]

ReciprocalIsFunction

[ReciprocalIsFunction $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash$
$\mathrm{Repetition} \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Separation2formula}(2) \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \gg \{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N,Q\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}; \mathrm{Separation2formula}(2) \rhd \{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in$
$\{ph \in \{ph \in P(P(\mathrm{Union}(\{N,Q\})))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \gg \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n; \forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{m}}\colon\forall\underline{(fx)}\colon \overline{(f1)} = \overline{(f3)} \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] =$
$0)n \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \vdash$
$\mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \gg \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] =$
$0)n; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \gg \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \gg$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\}; \mathrm{FromOrderedPair}(1) \rhd \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\} \gg \overline{(f1)} = \underline{\mathsf{m}}; \mathrm{eqSymmetry} \rhd \overline{(f1)} = \underline{\mathsf{m}} \gg \underline{\mathsf{m}} =$
$\overline{(f1)}; \mathrm{FromOrderedPair}(1) \rhd \{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\} \gg \overline{(f3)} = \underline{\mathsf{m}}; \mathrm{eqTransitivity4} \rhd \underline{\mathsf{m}} = \overline{(f1)} \rhd \overline{(f1)} =$
$\overline{(f3)} \rhd \overline{(f3)} = \underline{\mathsf{m}} \gg \underline{\mathsf{m}} = \underline{\mathsf{m}}; \mathrm{SameSeries} \rhd \underline{\mathsf{m}} = \underline{\mathsf{m}} \gg \underline{(fx)}[\underline{\mathsf{m}}] =$
$\underline{(fx)}[\underline{\mathsf{m}}]; \mathrm{SameReciprocal} \rhd \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \rhd \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fx)}[\underline{\mathsf{m}}] \gg rec\underline{(fx)}[\underline{\mathsf{m}}] =$
$rec\underline{(fx)}[\underline{\mathsf{m}}]; \mathrm{FromOrderedPair}(2) \rhd \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\} \gg \overline{(f2)} = rec\underline{(fx)}[\underline{\mathsf{m}}]; \mathrm{FromOrderedPair}(2) \rhd$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\} \gg \overline{(f4)} =$
$rec\underline{(fx)}[\underline{\mathsf{m}}]; \mathrm{eqSymmetry} \rhd \overline{(f4)} = rec\underline{(fx)}[\underline{\mathsf{m}}] \gg rec\underline{(fx)}[\underline{\mathsf{m}}] =$
$\overline{(f4)}; \mathrm{eqTransitivity4} \rhd \overline{(f2)} = rec\underline{(fx)}[\underline{\mathsf{m}}] \rhd rec\underline{(fx)}[\underline{\mathsf{m}}] = rec\underline{(fx)}[\underline{\mathsf{m}}] \rhd rec\underline{(fx)}[\underline{\mathsf{m}}] =$
$\overline{(f4)} \gg \overline{(f2)} = \overline{(f4)}; \forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{m}}\colon\forall\underline{(fx)}\colon \overline{(f1)} = \overline{(f3)} \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \vdash \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n \vdash$
$\mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},rec\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \gg \dot{\neg}(\underline{(fx)}[\underline{\mathsf{m}}] =$

$0)\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \gg \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \gg \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\}; \mathrm{OrderedPairEquality} \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\} \rhd \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \gg \underline{\mathsf{m}} = \underline{\mathsf{m}}; \mathrm{SameSeries} \rhd \underline{\mathsf{m}} = \underline{\mathsf{m}} \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]; \mathrm{eqTransitivity} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0; \mathrm{FromContradiction} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \gg \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \gg \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \gg \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\}; \mathrm{OrderedPairEquality} \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\} \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \rhd \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \gg \underline{\mathsf{m}} = \underline{\mathsf{m}}; \mathrm{SameSeries} \rhd \underline{\mathsf{m}} = \underline{\mathsf{m}} \gg$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]; \mathrm{eqSymmetry} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]; \mathrm{eqTransitivity} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0; \mathrm{FromContradiction} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \gg \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \vdash \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \gg$
$\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \gg$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\}; \mathrm{FromOrderedPair}(2) \rhd$
$\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\} \gg \overline{(\mathrm{f2})} =$
$0; \mathrm{FromOrderedPair}(2) \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\} \gg \overline{(\mathrm{f4})} =$
$0; \mathrm{eqSymmetry} \rhd \overline{(\mathrm{f4})} = 0 \gg 0 = \overline{(\mathrm{f4})}; \mathrm{eqTransitivity} \rhd \overline{(\mathrm{f2})} = 0 \rhd 0 = \overline{(\mathrm{f4})} \gg$
$\overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$

$0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{Ded} \rhd \forall\underline{m}\colon \forall\underline{m}\colon \forall\underline{(fx)}\colon \overline{(f1)} = \overline{(f3)} \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] =$
$0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \vdash \overline{(f2)} =$
$\overline{(f4)} \gg \overline{(f1)} = \overline{(f3)} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} =$
$\overline{(f4)}; \mathrm{MP} \rhd \overline{(f1)} = \overline{(f3)} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} =$
$\overline{(f4)} \rhd \overline{(f1)} = \overline{(f3)} \gg \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} =$
$\overline{(f4)}; \mathrm{Ded} \rhd \forall\underline{m}\colon \forall\underline{m}\colon \forall\underline{(fx)}\colon \overline{(f1)} = \overline{(f3)} \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{(fx)}[\underline{m}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \vdash \overline{(f2)} = \overline{(f4)} \gg \overline{(f1)} =$
$\overline{(f3)} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} = \overline{(f4)}; \mathrm{MP} \rhd \overline{(f1)} =$
$\overline{(f3)} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} = \overline{(f4)} \rhd \overline{(f1)} =$
$\overline{(f3)} \gg \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} =$
$\overline{(f4)}; \mathrm{Ded} \rhd \forall\underline{m}\colon \forall\underline{m}\colon \forall\underline{(fx)}\colon \overline{(f1)} = \overline{(f3)} \vdash \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \vdash \overline{(f2)} = \overline{(f4)} \gg$
$\overline{(f1)} = \overline{(f3)} \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} = \overline{(f4)}; \mathrm{MP} \rhd \overline{(f1)} = \overline{(f3)} \Rightarrow \dot{\neg}(\underline{(fx)}[\underline{m}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] =$
$0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} =$
$\overline{(f4)} \rhd \overline{(f1)} = \overline{(f3)} \gg \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(fx)}[\underline{m}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},\mathrm{rec}\underline{(fx)}[\underline{m}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(f2)} = \overline{(f4)}; \mathrm{Ded} \rhd \forall\underline{m}\colon \forall\underline{m}\colon \forall\underline{(fx)}\colon \dot{\neg}(\underline{(fx)}[\underline{m}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{(fx)}[\underline{m}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})\mathrm{n})\mathrm{n} \vdash \overline{(f2)} = \overline{(f4)} \gg$

$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})}; \mathrm{From2} * \mathrm{2Disjuncts} \rhd \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \rhd \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \rhd \dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \rhd \dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})} \rhd \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \rhd \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \gg \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}; \mathrm{Ded} \rhd \forall \underline{\mathsf{m}} \colon \forall \underline{\mathsf{m}} \colon \forall \underline{(\mathrm{fx})} \colon \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \vdash \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \vdash \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \gg \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})}; \mathrm{MP} \rhd \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \rhd \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \gg \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{f2})} =$

$\overline{(f4)}$; ExistMP2 $\rhd \dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] =$
$0)n \Rightarrow \dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow$
$\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n \Rightarrow \overline{(f2)} =$
$\overline{(f4)} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow$
$\dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] =$
$0 \Rightarrow \dot{\neg}(\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n \gg \overline{(f2)} =$
$\overline{(f4)}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{(fx)}\colon \mathrm{Ded} \rhd \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{(fx)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] =$
$0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\} \vdash \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\} \vdash \overline{(f1)} = \overline{(f3)} \vdash \overline{(f2)} = \overline{(f4)} \gg$
$\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)}], p_0, \mathsf{c})]$

$[\mathrm{ReciprocalIsFunction} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{(fx)}\colon \forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in$
$\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}((\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}]$

[ReciprocalIsFunction $\overset{\mathrm{tex}}{\rightarrow}$ "ReciprocalIsFunction"]

[ReciprocalIsFunction $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma reciprocalIsFunction"]

ReciprocalIsTotal

[ReciprocalIsTotal $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \vdash \overline{(\mathrm{s1})} \in \mathrm{N} \vdash$
$\mathrm{RationalType} \gg 0 \in \mathrm{Q}; \mathrm{ToCartProd} \rhd \overline{(\mathrm{s1})} \in \mathrm{N} \rhd 0 \in \mathrm{Q} \gg$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqReflexivity} \gg$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\}; \mathrm{JoinConjuncts} \rhd \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] =$
$0 \rhd \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} \gg \dot\neg((\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} =$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\})\mathrm{n})\mathrm{n}; \mathrm{WeakenOr1} \rhd \dot\neg((\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\})\mathrm{n})\mathrm{n} \gg \dot\neg(\dot\neg(\dot\neg((\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg((\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} =$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @ \overline{(\mathrm{s1})} \rhd \dot\neg(\dot\neg(\dot\neg((\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg((\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Formula2Sep} \rhd \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}$; IntroExist @ $0 \rhd \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, 0\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{fx})}\colon \dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \vdash \overline{(\mathrm{s1})} \in \mathrm{N} \vdash$ SeriesType $\gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; ValueType $\rhd \overline{(\mathrm{s1})} \in \mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] \in \mathrm{Q}$; QisClosed(Reciprocal) $\rhd \dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \rhd \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] \in \mathrm{Q} \gg \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] \in \mathrm{Q}$; ToCartProd $\rhd \overline{(\mathrm{s1})} \in \mathrm{N} \rhd \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] \in \mathrm{Q} \gg \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}$; eqReflexivity $\gg \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\}$; JoinConjuncts $\rhd \dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \rhd \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} \gg \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\})\mathrm{n})\mathrm{n}$; WeakenOr2 $\rhd \dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, 0\}\})\mathrm{n})\mathrm{n}$; IntroExist @ $\overline{(\mathrm{s1})} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow$

$\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} =$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},0\}\})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Formula2Sep} \rhd \{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow$
$\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{IntroExist} @ \mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] \rhd$
$\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\mathrm{rec}\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{fx})}\colon \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}\colon \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \vdash \overline{(\mathrm{s1})} \in \mathrm{N} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}\colon \dot{\neg}\,(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \vdash \overline{(\mathrm{s1})} \in \mathrm{N} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \mathrm{FromNegations} \rhd \underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0 \Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] = 0)\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})\rceil$

$[\mathrm{ReciprocalIsTotal} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}]$

$[\mathrm{ReciprocalIsTotal} \overset{\mathrm{tex}}{\to}$ "ReciprocalIsTotal"$]$

[ReciprocalIsTotal $\stackrel{\text{pyk}}{\to}$ “lemma reciprocalIsTotal”]

ReciprocalIsRationalSeries

[ReciprocalIsRationalSeries $\stackrel{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢
∀(fx): CPseparationIsRelation ≫ $\forall_{\mathrm{obj}}$(r1): (r1) ∈ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\mathrm{obj}}$m: ¬̇ (¬̇ (¬̇ (¬̇ ((fx)[m] = 0)n ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ¬̇ ((fx)[m] = 0 ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n} ⇒ ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈
N ⇒ ¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ ((r1) =
{{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n; ReciprocalIsFunction ≫
$\forall_{\mathrm{obj}}$(f1): $\forall_{\mathrm{obj}}$(f2): $\forall_{\mathrm{obj}}$(f3): $\forall_{\mathrm{obj}}$(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\mathrm{obj}}$m: ¬̇ (¬̇ (¬̇ (¬̇ ((fx)[m] = 0)n ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ¬̇ ((fx)[m] = 0 ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n} ⇒ {{(f3), (f3)}, {(f3), (f4)}} ∈ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\mathrm{obj}}$m: ¬̇ (¬̇ (¬̇ (¬̇ ((fx)[m] = 0)n ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ¬̇ ((fx)[m] = 0 ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n} ⇒ (f1) = (f3) ⇒ (f2) = (f4); ReciprocalIsTotal ≫
$\forall_{\mathrm{obj}}$(s1): (s1) ∈ N ⇒ ¬̇ ($\forall_{\mathrm{obj}}$(s2): ¬̇ ({{(s1), (s1)}, {(s1), (s2)}} ∈ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\mathrm{obj}}$m: ¬̇ (¬̇ (¬̇ (¬̇ ((fx)[m] = 0)n ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ¬̇ ((fx)[m] = 0 ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n})n)n; ToSeries ⊳ $\forall_{\mathrm{obj}}$(r1): (r1) ∈ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\mathrm{obj}}$m: ¬̇ (¬̇ (¬̇ (¬̇ ((fx)[m] = 0)n ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, rec(fx)[m]}})n)n)n ⇒ ¬̇ ((fx)[m] = 0 ⇒ ¬̇ (f_{Ph} =
{{m, m}, {m, 0}})n)n)n)n} ⇒ ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈
N ⇒ ¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ ((r1) =
{{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⊳
$\forall_{\mathrm{obj}}$(f1): $\forall_{\mathrm{obj}}$(f2): $\forall_{\mathrm{obj}}$(f3): $\forall_{\mathrm{obj}}$(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\mathrm{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\mathrm{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\mathrm{obj}}$m: ¬̇ (¬̇ (¬̇ (¬̇ ((fx)[m] = 0)n ⇒ ¬̇ (f_{Ph} =

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\})n)n \gg \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\})n)n)n)n\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ReciprocalIsRationalSeries} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] =$
$0)n \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\mathrm{rec}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$

$\{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, 0\}\})n)n)n)n\} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}): \forall_{\mathrm{obj}}(\mathrm{f2}): \forall_{\mathrm{obj}}(\mathrm{f3}): \forall_{\mathrm{obj}}(\mathrm{f4}): \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\mathsf{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{fx})[\mathsf{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, \mathrm{rec}(\mathrm{fx})[\mathsf{m}]\}\})n)n)n \Rightarrow \dot{\neg}(((\mathrm{fx})[\mathsf{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, 0\}\})n)n)n)n\} \Rightarrow \{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\mathsf{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{fx})[\mathsf{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, \mathrm{rec}(\mathrm{fx})[\mathsf{m}]\}\})n)n)n \Rightarrow \dot{\neg}(((\mathrm{fx})[\mathsf{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, 0\}\})n)n)n)n\} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}): (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}): \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\mathsf{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{fx})[\mathsf{m}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, \mathrm{rec}(\mathrm{fx})[\mathsf{m}]\}\})n)n)n \Rightarrow \dot{\neg}(((\mathrm{fx})[\mathsf{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\mathsf{m}, \mathsf{m}\}, \{\mathsf{m}, 0\}\})n)n)n)n\})n)n)n)n]$

[ReciprocalIsRationalSeries $\stackrel{\mathrm{tex}}{\to}$ "ReciprocalIsRationalSeries"]

[ReciprocalIsRationalSeries $\stackrel{\mathrm{pyk}}{\to}$ "lemma reciprocalIsRationalSeries"]

CrsIsRelation

[CrsIsRelation $\stackrel{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}: (\mathrm{s1}) \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}): \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})n)n\} \vdash \mathrm{Repetition} \rhd (\mathrm{s1}) \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}): \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})n)n\} \gg (\mathrm{s1}) \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}): \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})n)n\}; \mathrm{Sep2Formula} \rhd (\mathrm{s1}) \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}): \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})n)n\} \gg \dot{\neg}((\mathrm{s1}) \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\overline{(\mathrm{s1})} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n)n)n; \mathrm{FirstConjunct} \triangleright \dot{\neg}(\overline{(\mathrm{s1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\overline{(\mathrm{s1})} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n)n)n \gg \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}; \forall\underline{x}\colon \mathrm{Ded} \triangleright \forall\underline{x}\colon \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n\} \vdash \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \gg$
$\overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}; \mathrm{Gen} \triangleright \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}; \mathrm{Repetition} \triangleright$
$\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{x}\}\})n)n\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{CartProdIsRelation} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubsetIsRelation} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{s1})} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{CrsIsRelation} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{CrsIsRelation} \stackrel{\mathrm{tex}}{\to}$ "CrsIsRelation"$]$

$[\mathrm{CrsIsRelation} \stackrel{\mathrm{pyk}}{\to}$ "lemma crsIsRelation"$]$

CrsIsFunction

$[\mathrm{CrsIsFunction} \stackrel{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\} \vdash \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\} \vdash \mathrm{FromOrderedPair} \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\} \gg \dot{\neg}\,(\overline{(\mathrm{f1})} = \overline{(\mathrm{crs1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{f2})} = \underline{\mathsf{x}})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}\,(\overline{(\mathrm{f1})} = \overline{(\mathrm{crs1})} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{f2})} = \underline{\mathsf{x}})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{f2})} = \underline{\mathsf{x}}; \mathrm{FromOrderedPair} \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \gg \dot{\neg}(\overline{(\mathrm{f3})} = \overline{(\mathrm{crs1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{f4})} =$
$\underline{\mathrm{x}})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\overline{(\mathrm{f3})} = \overline{(\mathrm{crs1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{f4})} = \underline{\mathrm{x}})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{f4})} =$
$\underline{\mathrm{x}}; \mathrm{eqSymmetry} \rhd \overline{(\mathrm{f4})} = \underline{\mathrm{x}} \gg \underline{\mathrm{x}} = \overline{(\mathrm{f4})}; \mathrm{eqTransitivity} \rhd \overline{(\mathrm{f2})} = \underline{\mathrm{x}} \rhd \underline{\mathrm{x}} = \overline{(\mathrm{f4})} \gg$
$\overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}; \forall \underline{\mathrm{x}}\colon \mathrm{Ded} \rhd \forall \underline{\mathrm{x}}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \vdash \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \vdash \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})} \gg \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}; \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \vdash \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \vdash \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash$
$\mathrm{Sep2Formula} \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd$
$\dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}; \mathrm{Sep2Formula} \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd$
$\dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n; \mathrm{ExistMP2} \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n \rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n \gg \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}; \forall \underline{\mathrm{x}}\colon \mathrm{Ded} \rhd$
$\forall \underline{\mathrm{x}}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \vdash \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \vdash \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})} \gg \forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow$
$\overline{(\mathrm{f2})} = \overline{(\mathrm{f4})}; \mathrm{CrsIsRelation} \gg \forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n; \mathrm{JoinConjuncts} \rhd$
$\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \rhd$
$\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})n)n\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{CrsIsFunction} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}]$

$[\mathrm{CrsIsFunction} \stackrel{\mathrm{tex}}{\rightarrow}$ "CrsIsFunction "]

$[\mathrm{CrsIsFunction} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma crsIsFunction"]

CrsIsTotal

$[\mathrm{CrsIsTotal} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{x}}\colon \lambda\mathsf{c}.\mathrm{TypeRational0}(\lceil\underline{\mathsf{x}}\rceil) \Vdash \underline{\mathsf{m}} \in$
$\mathrm{N} \vdash \mathrm{RationalType} \rhd \lambda\mathsf{c}.\mathrm{TypeRational0}(\lceil\underline{\mathsf{x}}\rceil) \gg \underline{\mathsf{x}} \in \mathrm{Q}; \mathrm{ToCartProd} \rhd \underline{\mathsf{m}} \in$
$\mathrm{N} \rhd \underline{\mathsf{x}} \in \mathrm{Q} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqReflexivity} \gg$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\}; \mathrm{IntroExist} @ \underline{\mathsf{m}} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} \gg \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}; \mathrm{Formula2Sep} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} = \{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n} \gg$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

[CrsIsTotal $\overset{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{x}}\colon \lambda \mathsf{c}.\mathrm{TypeRational0}(\lceil \underline{\mathsf{x}} \rceil) \Vdash \underline{\mathsf{m}} \in \mathrm{N} \vdash$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\}$]

[CrsIsTotal $\overset{\mathrm{tex}}{\to}$ “CrsIsTotal”]

[CrsIsTotal $\overset{\mathrm{pyk}}{\to}$ “lemma crsIsTotal”]

CrsIsSeries

[CrsIsSeries $\overset{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon (\mathrm{s1}) \in \mathrm{N} \vdash \mathrm{CrsIsTotal} \rhd (\mathrm{s1}) \in \mathrm{N} \gg$
$\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), \underline{\mathsf{x}}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\}; \mathrm{IntroExist} @ \underline{\mathsf{x}} \rhd (\mathrm{s1}) \in \mathrm{N} \gg$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \forall \underline{\mathsf{x}}\colon \mathrm{Ded} \rhd \forall \underline{\mathsf{x}}\colon (\mathrm{s1}) \in \mathrm{N} \vdash$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg (\mathrm{s1}) \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd (\mathrm{s1}) \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), \underline{\mathsf{x}}\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \mathrm{CrsIsFunction} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} =$
$\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \rhd \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathrm{x}}\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\})n)n)n)n\rceil, p_0, \mathsf{c})]$

$[CrsIsSeries \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall\underline{x}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\})n)n)n)n]$

$[CrsIsSeries \stackrel{tex}{\rightarrow}$ "CrsIsSeries"$]$

$[CrsIsSeries \stackrel{pyk}{\rightarrow}$ "lemma crsIsSeries"$]$

CrsLookup

$[CrsLookup \stackrel{proof}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall\underline{m}\colon \forall\underline{x}\colon \underline{m} \in N \vdash CrsIsSeries \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, \underline{x}\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\})n)n)n)n; \mathrm{MemberOfSeries} \rhd \underline{\mathsf{m}} \in$
$N \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\})n)n)n)n \gg$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\}[\underline{\mathsf{m}}]\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\}; \mathrm{CrsIsTotal} \rhd \underline{\mathsf{m}} \in N \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{x}\}\} \in$
$\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), \underline{x}\}\})n)n\}; \mathrm{eqReflexivity} \gg \underline{\mathsf{m}} =$
$\underline{\mathsf{m}}; \mathrm{UniqueMember} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\})n)n)n)n \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{\mathsf{x}}\}\} \in$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\} \rhd \underline{\mathsf{m}} = \underline{\mathsf{m}} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\}[\underline{\mathsf{m}}] = \underline{\mathsf{x}}], \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{CrsLookup} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{x}}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, \underline{\mathsf{x}}\}\})n)n\}[\underline{\mathsf{m}}] = \underline{\mathsf{x}}]$

$[\mathrm{CrsLookup} \stackrel{\mathrm{tex}}{\rightarrow}$ “CrsLookup”$]$

$[\mathrm{CrsLookup} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma crsLookup”$]$

0f

$[\mathrm{0f} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \mathrm{CrsLookup} \rhd \underline{\mathsf{m}} \in \mathrm{N} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] =$
$0; \mathrm{Repetition} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0\mathrm{f} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0]$

$[0\mathrm{f} \stackrel{\mathrm{tex}}{\rightarrow}$ “0f”]

$[0\mathrm{f} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma 0f”]

1f

$[1\mathrm{f} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \mathrm{CrsLookup} \rhd \underline{\mathsf{m}} \in \mathrm{N} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] =$
$1; \mathrm{Repetition} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 1 \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 1\rceil, \mathrm{p}_0, \mathsf{c})]$

$[1\mathrm{f} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 1]$

$[1\mathrm{f} \stackrel{\mathrm{tex}}{\rightarrow}$ “1f”]

$[1\mathrm{f} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma 1f”]

ToSingleton

$[\mathrm{ToSingleton} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \vdash \mathrm{WeakenOr1} \rhd \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \gg \dot{\neg}\,(\underline{(\mathrm{sx})} = \underline{(\mathrm{sy})})\mathrm{n} \Rightarrow \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}; \mathrm{Formula2Pair} \rhd \dot{\neg}\,(\underline{(\mathrm{sx})} = \underline{(\mathrm{sy})})\mathrm{n} \Rightarrow \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\}; \mathrm{Repetition} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToSingleton} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})} \vdash \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\}]$

[ToSingleton $\stackrel{\mathrm{tex}}{\rightarrow}$ “ToSingleton”]

[ToSingleton $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma toSingleton”]

FromSameSingleton

$[\mathrm{FromSameSingleton} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\} = \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \vdash \mathrm{eqReflexivity} \gg \underline{(\mathrm{sx})} = \underline{(\mathrm{sx})}; \mathrm{ToSingleton} \rhd \underline{(\mathrm{sx})} = \underline{(\mathrm{sx})} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}; \mathrm{SENC1} \rhd \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\} = \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\}; \mathrm{FromSingleton} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \gg \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromSameSingleton} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\} = \{\underline{(\mathrm{sy})}, \underline{(\mathrm{sy})}\} \vdash \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}]$

[FromSameSingleton $\stackrel{\mathrm{tex}}{\rightarrow}$ “FromSameSingleton”]

[FromSameSingleton $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma fromSameSingleton”]

SingletonmembersEqual

$[\mathrm{SingletonmembersEqual} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\} = \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\} \vdash \mathrm{InPair}(1) \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}; \mathrm{SENC1} \rhd \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\} = \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\} \gg \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\}; \mathrm{FromSingleton} \rhd \underline{(\mathrm{sx})} \in \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\} \gg \underline{(\mathrm{sx})} = \underline{(\mathrm{sz})}; \mathrm{InPair}(2) \gg \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}; \mathrm{SENC1} \rhd \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\} = \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\} \rhd \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\} \gg \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\}; \mathrm{FromSingleton} \rhd \underline{(\mathrm{sy})} \in \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\} \gg \underline{(\mathrm{sy})} = \underline{(\mathrm{sz})}; \mathrm{eqSymmetry} \rhd \underline{(\mathrm{sy})} = \underline{(\mathrm{sz})} \gg \underline{(\mathrm{sz})} = \underline{(\mathrm{sy})}; \mathrm{eqTransitivity} \rhd \underline{(\mathrm{sx})} = \underline{(\mathrm{sz})} \rhd \underline{(\mathrm{sz})} = \underline{(\mathrm{sy})} \gg \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SingletonmembersEqual} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sz})}\colon \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\} = \{\underline{(\mathrm{sz})}, \underline{(\mathrm{sz})}\} \vdash \underline{(\mathrm{sx})} = \underline{(\mathrm{sy})}]$

[SingletonmembersEqual $\stackrel{\mathrm{tex}}{\rightarrow}$ “SingletonmembersEqual”]

[SingletonmembersEqual $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma singletonmembersEqual”]

UnequalsNotInSingleton

[UnequalsNotInSingleton $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \forall \underline{\mathrm{(sz)}}\colon \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\} \vdash \mathrm{SingletonmembersEqual} \rhd \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\} \gg \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}}; \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \forall \underline{\mathrm{(sz)}}\colon \mathrm{Ded} \rhd \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \forall \underline{\mathrm{(sz)}}\colon \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\} \vdash \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \gg \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\} \Rightarrow \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}}; \dot{\neg}\,(\underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}})\mathrm{n} \vdash \mathrm{MT} \rhd \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\} \Rightarrow \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \rhd \dot{\neg}\,(\underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}})\mathrm{n} \gg \dot{\neg}\,(\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[UnequalsNotInSingleton $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \forall \underline{\mathrm{(sz)}}\colon \dot{\neg}\,(\underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}})\mathrm{n} \vdash \dot{\neg}\,(\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sz)}}, \underline{\mathrm{(sz)}}\})\mathrm{n}$]

[UnequalsNotInSingleton $\stackrel{\text{tex}}{\rightarrow}$ "UnequalsNotInSingleton"]

[UnequalsNotInSingleton $\stackrel{\text{pyk}}{\rightarrow}$ "lemma unequalsNotInSingleton"]

NonsingletonmembersUnequal

[NonsingletonmembersUnequal $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \vdash \mathrm{eqReflexivity} \gg \underline{\mathrm{(sx)}} = \underline{\mathrm{(sx)}}; \mathrm{SamePair} \rhd \underline{\mathrm{(sx)}} = \underline{\mathrm{(sx)}} \rhd \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \gg \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\}; \mathrm{Repetition} \rhd \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} \gg \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\}; \mathrm{eqSymmetry} \rhd \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} \gg \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\}; \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \mathrm{Ded} \rhd \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \vdash \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\} \gg \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \Rightarrow \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\}; \dot{\neg}\,(\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\})\mathrm{n} \vdash \mathrm{MT} \rhd \underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}} \Rightarrow \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\} \rhd \dot{\neg}\,(\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\})\mathrm{n} \gg \dot{\neg}\,(\underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[NonsingletonmembersUnequal $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sy)}}\colon \dot{\neg}\,(\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\} = \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\})\mathrm{n} \vdash \dot{\neg}\,(\underline{\mathrm{(sx)}} = \underline{\mathrm{(sy)}})\mathrm{n}$]

[NonsingletonmembersUnequal $\stackrel{\text{tex}}{\rightarrow}$ "NonsingletonmembersUnequal"]

[NonsingletonmembersUnequal $\stackrel{\text{pyk}}{\rightarrow}$ "lemma nonsingletonmembersUnequal"]

FromOrderedPair

[FromOrderedPair $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathrm{(sx)}}\colon \forall \underline{\mathrm{(sx1)}}\colon \forall \underline{\mathrm{(sy)}}\colon \forall \underline{\mathrm{(sy1)}}\colon \underline{\mathrm{(sx1)}} = \underline{\mathrm{(sy1)}} \vdash \{\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\}, \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\}\} = \{\{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sx1)}}\}, \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sy1)}}\}\} \vdash \mathrm{Repetition} \rhd \{\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\}, \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\}\} = \{\{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sx1)}}\}, \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sy1)}}\}\} \gg \{\{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sx)}}\}, \{\underline{\mathrm{(sx)}}, \underline{\mathrm{(sy)}}\}\} = \{\{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sx1)}}\}, \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sy1)}}\}\}; \mathrm{eqReflexivity} \gg \underline{\mathrm{(sx1)}} = \underline{\mathrm{(sx1)}}; \mathrm{SamePair} \rhd \underline{\mathrm{(sx1)}} = \underline{\mathrm{(sx1)}} \rhd \underline{\mathrm{(sx1)}} = \underline{\mathrm{(sy1)}} \gg \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sx1)}}\} = \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sy1)}}\}; \mathrm{Repetition} \rhd \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sx1)}}\} = \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sy1)}}\} \gg \{\underline{\mathrm{(sx1)}}, \underline{\mathrm{(sx1)}}\} =$

{(sx1), (sy1)}; eqReflexivity ≫ {(sx1), (sx1)} =
{(sx1), (sx1)}; SamePair ▷ {(sx1), (sx1)} = {(sx1), (sx1)} ▷ {(sx1), (sx1)} =
{(sx1), (sy1)} ≫ {{(sx1), (sx1)}, {(sx1), (sx1)}} =
{{(sx1), (sx1)}, {(sx1), (sy1)}}; Repetition ▷ {{(sx1), (sx1)}, {(sx1), (sx1)}} =
{{(sx1), (sx1)}, {(sx1), (sy1)}} ≫ {{(sx1), (sx1)}, {(sx1), (sx1)}} =
{{(sx1), (sx1)}, {(sx1), (sy1)}}; eqSymmetry ▷ {{(sx1), (sx1)}, {(sx1), (sx1)}} =
{{(sx1), (sx1)}, {(sx1), (sy1)}} ≫ {{(sx1), (sx1)}, {(sx1), (sy1)}} =
{{(sx1), (sx1)}, {(sx1), (sx1)}}; eqTransitivity ▷ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx1), (sx1)}, {(sx1), (sy1)}} ▷ {{(sx1), (sx1)}, {(sx1), (sy1)}} =
{{(sx1), (sx1)}, {(sx1), (sx1)}} ≫ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx1), (sx1)}, {(sx1), (sx1)}}; InPair(1) ≫ {(sx), (sx)} ∈
{{(sx), (sx)}, {(sx), (sy)}}; SENC1 ▷ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx1), (sx1)}, {(sx1), (sx1)}} ▷ {(sx), (sx)} ∈ {{(sx), (sx)}, {(sx), (sy)}} ≫
{(sx), (sx)} ∈ {{(sx1), (sx1)}, {(sx1), (sx1)}}; FromSingleton ▷ {(sx), (sx)} ∈
{{(sx1), (sx1)}, {(sx1), (sx1)}} ≫ {(sx), (sx)} =
{(sx1), (sx1)}; FromSameSingleton ▷ {(sx), (sx)} = {(sx1), (sx1)} ≫ (sx) =
(sx1); eqSymmetry ▷ {(sx), (sx)} = {(sx1), (sx1)} ≫ {(sx1), (sx1)} =
{(sx), (sx)}; SameSingleton ▷ {(sx1), (sx1)} = {(sx), (sx)} ≫
{{(sx1), (sx1)}, {(sx1), (sx1)}} =
{{(sx), (sx)}, {(sx), (sx)}}; eqTransitivity ▷ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx1), (sx1)}, {(sx1), (sx1)}} ▷ {{(sx1), (sx1)}, {(sx1), (sx1)}} =
{{(sx), (sx)}, {(sx), (sx)}} ≫ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx), (sx)}, {(sx), (sx)}}; InPair(2) ≫ {(sx), (sy)} ∈
{{(sx), (sx)}, {(sx), (sy)}}; SENC1 ▷ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx), (sx)}, {(sx), (sx)}} ▷ {(sx), (sy)} ∈ {{(sx), (sx)}, {(sx), (sy)}} ≫
{(sx), (sy)} ∈ {{(sx), (sx)}, {(sx), (sx)}}; FromSingleton ▷ {(sx), (sy)} ∈
{{(sx), (sx)}, {(sx), (sx)}} ≫ {(sx), (sy)} =
{(sx), (sx)}; SingletonmembersEqual ▷ {(sx), (sy)} = {(sx), (sx)} ≫ (sx) =
(sy); eqSymmetry ▷ (sx) = (sy) ≫ (sy) = (sx); eqTransitivity4 ▷ (sy) =
(sx) ▷ (sx) = (sx1) ▷ (sx1) = (sy1) ≫ (sy) = (sy1); JoinConjuncts ▷ (sx) =
(sx1) ▷ (sy) = (sy1) ≫ ¬̇((sx) = (sx1) ⇒ ¬̇((sy) =
(sy1))n)n; ∀(sx): ∀(sx1): ∀(sy): ∀(sy1): ¬̇((sx1) = (sy1))n ⊢
{{(sx), (sx)}, {(sx), (sy)}} = {{(sx1), (sx1)}, {(sx1), (sy1)}} ⊢
Repetition ▷ {{(sx), (sx)}, {(sx), (sy)}} = {{(sx1), (sx1)}, {(sx1), (sy1)}} ≫
{{(sx), (sx)}, {(sx), (sy)}} = {{(sx1), (sx1)}, {(sx1), (sy1)}}; InPair(1) ≫
{(sx), (sx)} ∈ {{(sx), (sx)}, {(sx), (sy)}}; SENC1 ▷ {{(sx), (sx)}, {(sx), (sy)}} =
{{(sx1), (sx1)}, {(sx1), (sy1)}} ▷ {(sx), (sx)} ∈ {{(sx), (sx)}, {(sx), (sy)}} ≫
{(sx), (sx)} ∈ {{(sx1), (sx1)}, {(sx1), (sy1)}}; Pair2Formula ▷ {(sx), (sx)} ∈
{{(sx1), (sx1)}, {(sx1), (sy1)}} ≫ ¬̇({(sx), (sx)} = {(sx1), (sx1)})n ⇒
{(sx), (sx)} = {(sx1), (sy1)}; UnequalsNotInSingleton ▷ ¬̇((sx1) = (sy1))n ≫
¬̇({(sx1), (sy1)} = {(sx), (sx)})n; NeqSymmetry ▷ ¬̇({(sx1), (sy1)} =
{(sx), (sx)})n ≫ ¬̇({(sx), (sx)} =
{(sx1), (sy1)})n; NegateDisjunct2 ▷ ¬̇({(sx), (sx)} = {(sx1), (sx1)})n ⇒
{(sx), (sx)} = {(sx1), (sy1)} ▷ ¬̇({(sx), (sx)} = {(sx1), (sy1)})n ≫ {(sx), (sx)} =
{(sx1), (sx1)}; FromSameSingleton ▷ {(sx), (sx)} = {(sx1), (sx1)} ≫ (sx) =
(sx1); InPair(2) ≫ {(sx1), (sy1)} ∈ {{(sx1), (sx1)}, {(sx1), (sy1)}}; SENC2 ▷

$$\{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \rhd \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} \in \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \gg \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} \in \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\}; \mathrm{Pair2Formula} \rhd \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} \in \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} \gg \dot{\neg}(\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n} \Rightarrow \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}; \mathrm{NegateDisjunct1} \rhd \dot{\neg}(\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n} \Rightarrow \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\} \rhd \dot{\neg}(\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n} \gg \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}; \mathrm{InPair(2)} \gg \overline{(\mathrm{sy})} \in \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}; \mathrm{SENC2} \rhd \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\} \rhd \overline{(\mathrm{sy})} \in \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\} \gg \overline{(\mathrm{sy})} \in \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}; \mathrm{Pair2Formula} \rhd \overline{(\mathrm{sy})} \in \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} \gg \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sx1})})\mathrm{n} \Rightarrow \overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})}; \mathrm{UnequalsNotInSingleton} \rhd \dot{\neg}(\overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})})\mathrm{n} \gg \dot{\neg}(\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n}; \mathrm{SubNeqLeft} \rhd \{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\} \rhd \dot{\neg}(\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n} \gg \dot{\neg}(\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n}; \mathrm{NonsingletonmembersUnequal} \rhd \dot{\neg}(\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\} = \{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\})\mathrm{n} \gg \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sy})})\mathrm{n}; \mathrm{SubNeqLeft} \rhd \overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \rhd \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sy})})\mathrm{n} \gg \dot{\neg}(\overline{(\mathrm{sx1})} = \overline{(\mathrm{sy})})\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(\overline{(\mathrm{sx1})} = \overline{(\mathrm{sy})})\mathrm{n} \gg \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sx1})})\mathrm{n}; \mathrm{NegateDisjunct1} \rhd \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sx1})})\mathrm{n} \Rightarrow \overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})} \rhd \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sx1})})\mathrm{n} \gg \overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})}; \mathrm{JoinConjuncts} \rhd \overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \rhd \overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})} \gg \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}; \forall\overline{(\mathrm{sx})}\colon\forall\overline{(\mathrm{sx1})}\colon\forall\overline{(\mathrm{sy})}\colon\forall\overline{(\mathrm{sy1})}\colon \mathrm{Ded} \rhd \forall\overline{(\mathrm{sx})}\colon\forall\overline{(\mathrm{sx1})}\colon\forall\overline{(\mathrm{sy})}\colon\forall\overline{(\mathrm{sy1})}\colon \overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})} \vdash \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \vdash \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})} \Rightarrow \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \Rightarrow \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \forall\overline{(\mathrm{sx})}\colon\forall\overline{(\mathrm{sx1})}\colon\forall\overline{(\mathrm{sy})}\colon\forall\overline{(\mathrm{sy1})}\colon \dot{\neg}(\overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})})\mathrm{n} \vdash \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \vdash \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})})\mathrm{n} \Rightarrow \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \Rightarrow \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}; \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \vdash \mathrm{FromNegations} \rhd \overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})} \Rightarrow \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \Rightarrow \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\overline{(\mathrm{sx1})} = \overline{(\mathrm{sy1})})\mathrm{n} \Rightarrow \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \Rightarrow \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \gg \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \Rightarrow \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \Rightarrow \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \rhd \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \gg \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}], \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{FromOrderedPair} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\overline{(\mathrm{sx})}\colon\forall\overline{(\mathrm{sx1})}\colon\forall\overline{(\mathrm{sy})}\colon\forall\overline{(\mathrm{sy1})}\colon \{\{\overline{(\mathrm{sx})},\overline{(\mathrm{sx})}\},\{\overline{(\mathrm{sx})},\overline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sx1})}\},\{\overline{(\mathrm{sx1})},\overline{(\mathrm{sy1})}\}\} \vdash \dot{\neg}(\overline{(\mathrm{sx})} = \overline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{sy})} = \overline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}]$$

$$[\mathrm{FromOrderedPair} \stackrel{\mathrm{tex}}{\rightarrow} \text{“FromOrderedPair”}]$$

$$[\mathrm{FromOrderedPair} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“lemma fromOrderedPair”}]$$

FromOrderedPair(1)

[FromOrderedPair(1) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sx1})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})}, \underline{(\text{sx1})}\}, \{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}\} \vdash \text{FromOrderedPair} \rhd \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})}, \underline{(\text{sx1})}\}, \{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}\} \gg \dot{\neg}\,(\underline{(\text{sx})} = \underline{(\text{sx1})} \Rightarrow \dot{\neg}\,(\underline{(\text{sy})} = \underline{(\text{sy1})})\text{n})\text{n}; \text{FirstConjunct} \rhd \dot{\neg}\,(\underline{(\text{sx})} = \underline{(\text{sx1})} \Rightarrow \dot{\neg}\,(\underline{(\text{sy})} = \underline{(\text{sy1})})\text{n})\text{n} \gg \underline{(\text{sx})} = \underline{(\text{sx1})}\rceil, \text{p}_0, \mathsf{c})$]

[FromOrderedPair(1) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sx1})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})}, \underline{(\text{sx1})}\}, \{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}\} \vdash \underline{(\text{sx})} = \underline{(\text{sx1})}$]

[FromOrderedPair(1) $\stackrel{\text{tex}}{\rightarrow}$ “FromOrderedPair(1)”]

[FromOrderedPair(1) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma fromOrderedPair(1)”]

FromOrderedPair(2)

[FromOrderedPair(2) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sx1})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})}, \underline{(\text{sx1})}\}, \{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}\} \vdash \text{FromOrderedPair} \rhd \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})}, \underline{(\text{sx1})}\}, \{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}\} \gg \dot{\neg}\,(\underline{(\text{sx})} = \underline{(\text{sx1})} \Rightarrow \dot{\neg}\,(\underline{(\text{sy})} = \underline{(\text{sy1})})\text{n})\text{n}; \text{SecondConjunct} \rhd \dot{\neg}\,(\underline{(\text{sx})} = \underline{(\text{sx1})} \Rightarrow \dot{\neg}\,(\underline{(\text{sy})} = \underline{(\text{sy1})})\text{n})\text{n} \gg \underline{(\text{sy})} = \underline{(\text{sy1})}\rceil, \text{p}_0, \mathsf{c})$]

[FromOrderedPair(2) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sx1})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})}, \underline{(\text{sx1})}\}, \{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}\} \vdash \underline{(\text{sy})} = \underline{(\text{sy1})}$]

[FromOrderedPair(2) $\stackrel{\text{tex}}{\rightarrow}$ “FromOrderedPair(2)”]

[FromOrderedPair(2) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma fromOrderedPair(2)”]

FromCartProd

[FromCartProd $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sx1})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}))) \mid \dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\text{op1})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\text{op2})} \in \underline{(\text{sy1})})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \vdash \text{Repetition} \rhd \{\{\underline{(\text{sx})}, \underline{(\text{sx})}\}, \{\underline{(\text{sx})}, \underline{(\text{sy})}\}\} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\underline{(\text{sx1})}, \underline{(\text{sy1})}\}))) \mid \dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\text{op1})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}\,(\overline{(\text{op2})} \in \underline{(\text{sy1})})\text{n})\text{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})}, \underline{(\mathrm{sy1})}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{Sep2Formula} \rhd$
$\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})}, \underline{(\mathrm{sy1})}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})}, \underline{(\mathrm{sy1})}\}))) \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd$
$\dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})}, \underline{(\mathrm{sy1})}\}))) \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{sx})}\colon \forall\underline{(\mathrm{sx1})}\colon \forall\underline{(\mathrm{sy})}\colon \forall\underline{(\mathrm{sy1})}\colon \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx1})}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{op2})} \in$
$\underline{(\mathrm{sy1})}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \gg$
$\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\}; \mathrm{FromOrderedPair}(1) \rhd$
$\{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} \gg \underline{(\mathrm{sx})} =$
$\overline{(\mathrm{op1})}; \mathrm{SameMember}(2) \rhd \underline{(\mathrm{sx})} = \overline{(\mathrm{op1})} \rhd \overline{(\mathrm{op1})} \in \underline{(\mathrm{sx1})} \gg \underline{(\mathrm{sx})} \in$
$\underline{(\mathrm{sx1})}; \mathrm{FromOrderedPair}(2) \rhd \{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} \gg \underline{(\mathrm{sy})} = \overline{(\mathrm{op2})}; \mathrm{SameMember}(2) \rhd \underline{(\mathrm{sy})} =$
$\overline{(\mathrm{op2})} \rhd \overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})} \gg \underline{(\mathrm{sy})} \in \underline{(\mathrm{sy1})}; \mathrm{JoinConjuncts} \rhd \underline{(\mathrm{sx})} \in \underline{(\mathrm{sx1})} \rhd \underline{(\mathrm{sy})} \in$
$\underline{(\mathrm{sy1})} \gg \dot{\neg}(\underline{(\mathrm{sx})} \in \underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}(\underline{(\mathrm{sy})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd$

$\forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \vdash \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n \gg \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n; \mathrm{TwiceExistMP} \rhd \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n \rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \gg \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n\rceil, p_0, \mathsf{c})]$

[FromCartProd $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \vdash \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n$]

[FromCartProd $\overset{\mathrm{tex}}{\rightarrow}$ "FromCartProd"]

[FromCartProd $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma fromCartProd"]

FromCartProd(1)

[FromCartProd(1) $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \vdash \mathrm{FromCartProd} \rhd \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \gg \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n; \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{(sx)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\underline{(sy)} \in \underline{(sy1)})n)n \gg \underline{(sx)} \in \underline{(sx1)}\rceil, p_0, \mathsf{c})]$

[FromCartProd(1) $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(sx1)}, \underline{(sy1)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \vdash \underline{(sx)} \in \underline{(sx1)}$]

[FromCartProd(1) $\overset{\text{tex}}{\rightarrow}$ “FromCartProd(1)”]

[FromCartProd(1) $\overset{\text{pyk}}{\rightarrow}$ “lemma fromCartProd(1)”]

FromCartProd(2)

[FromCartProd(2) $\overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{(\text{sx})}\colon\forall\underline{(\text{sx1})}\colon\forall\underline{(\text{sy})}\colon\forall\underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}))) \mid \dot{\neg}(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\text{op1})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}((\overline{(\text{op2})} \in \underline{(\text{sy1})})\text{n})\text{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \vdash \text{FromCartProd} \rhd \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}))) \mid \dot{\neg}(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\text{op1})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}((\overline{(\text{op2})} \in \underline{(\text{sy1})})\text{n})\text{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \gg \dot{\neg}((\underline{(\text{sx})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}((\underline{(\text{sy})} \in \underline{(\text{sy1})})\text{n})\text{n}; \text{SecondConjunct} \rhd \dot{\neg}((\underline{(\text{sx})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}((\underline{(\text{sy})} \in \underline{(\text{sy1})})\text{n})\text{n} \gg \underline{(\text{sy})} \in \underline{(\text{sy1})}\rceil, \text{p}_0, \mathsf{c})$]

[FromCartProd(2) $\overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{(\text{sx})}\colon\forall\underline{(\text{sx1})}\colon\forall\underline{(\text{sy})}\colon\forall\underline{(\text{sy1})}\colon \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}))) \mid \dot{\neg}(\forall_{\text{obj}}\overline{(\text{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\text{obj}}\overline{(\text{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\text{op1})} \in \underline{(\text{sx1})} \Rightarrow \dot{\neg}((\overline{(\text{op2})} \in \underline{(\text{sy1})})\text{n})\text{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})},\overline{(\text{op1})}\},\{\overline{(\text{op1})},\overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \vdash \underline{(\text{sy})} \in \underline{(\text{sy1})}$]

[FromCartProd(2) $\overset{\text{tex}}{\rightarrow}$ “FromCartProd(2)”]

[FromCartProd(2) $\overset{\text{pyk}}{\rightarrow}$ “lemma fromCartProd(2)”]

sameOrderedPair

[sameOrderedPair $\overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{(\text{sx})}\colon\forall\underline{(\text{sx1})}\colon\forall\underline{(\text{sy})}\colon\forall\underline{(\text{sy1})}\colon \underline{(\text{sx})} = \underline{(\text{sx1})} \vdash \underline{(\text{sy})} = \underline{(\text{sy1})} \vdash \text{SameSingleton} \rhd \underline{(\text{sx})} = \underline{(\text{sx1})} \gg \{\underline{(\text{sx})},\underline{(\text{sx})}\} = \{\underline{(\text{sx1})},\underline{(\text{sx1})}\}; \text{SamePair} \rhd \underline{(\text{sx})} = \underline{(\text{sx1})} \rhd \underline{(\text{sy})} = \underline{(\text{sy1})} \gg \{\underline{(\text{sx})},\underline{(\text{sy})}\} = \{\underline{(\text{sx1})},\underline{(\text{sy1})}\}; \text{SamePair} \rhd \{\underline{(\text{sx})},\underline{(\text{sx})}\} = \{\underline{(\text{sx1})},\underline{(\text{sx1})}\} \rhd \{\underline{(\text{sx})},\underline{(\text{sy})}\} = \{\underline{(\text{sx1})},\underline{(\text{sy1})}\} \gg \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})},\underline{(\text{sx1})}\},\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}\}; \text{Repetition} \rhd \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})},\underline{(\text{sx1})}\},\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}\} \gg \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})},\underline{(\text{sx1})}\},\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}\}\rceil, \text{p}_0, \mathsf{c})$]

[sameOrderedPair $\overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{(\text{sx})}\colon\forall\underline{(\text{sx1})}\colon\forall\underline{(\text{sy})}\colon\forall\underline{(\text{sy1})}\colon \underline{(\text{sx})} = \underline{(\text{sx1})} \vdash \underline{(\text{sy})} = \underline{(\text{sy1})} \vdash \{\{\underline{(\text{sx})},\underline{(\text{sx})}\},\{\underline{(\text{sx})},\underline{(\text{sy})}\}\} = \{\{\underline{(\text{sx1})},\underline{(\text{sx1})}\},\{\underline{(\text{sx1})},\underline{(\text{sy1})}\}\}$]

[sameOrderedPair $\overset{\text{tex}}{\rightarrow}$ “sameOrderedPair”]

[sameOrderedPair $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameOrderedPair”]

InSeriesHelper

[InSeriesHelper $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{(fx)}: \forall \underline{(sx)}: \forall \underline{(sy)}: \underline{(sy)} \in \underline{(fx)} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash \dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\underline{(sy)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \vdash \text{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \gg$
$\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n; \text{FirstConjunct} \rhd \dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \gg \overline{(op1)} \in N; \text{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \gg \underline{(sy)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}; \text{SameMember} \rhd \underline{(sy)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\} \rhd \underline{(sy)} \in \underline{(fx)} \gg$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\} \in \underline{(fx)}; \text{MemberOfSeries} \rhd \overline{(op1)} \in$
$N \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \underline{(fx)}[\overline{(op1)}]\}\} \in \underline{(fx)}; \text{eqReflexivity} \gg$
$\overline{(op1)} = \overline{(op1)}; \text{UniqueMember} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \rhd \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\} \in$
$\underline{(fx)} \rhd \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \underline{(fx)}[\overline{(op1)}]\}\} \in \underline{(fx)} \rhd \overline{(op1)} = \overline{(op1)} \gg \overline{(op2)} =$
$\underline{(fx)}[\overline{(op1)}]; \text{sameOrderedPair} \rhd \overline{(op1)} = \overline{(op1)} \rhd \overline{(op2)} = \underline{(fx)}[\overline{(op1)}] \gg$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\} =$

$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \underline{(\mathrm{fx})}[\overline{(\mathrm{op1})}]\}\}; \mathrm{eqTransitivity} \rhd \underline{(\mathrm{sy})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} \rhd \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \underline{(\mathrm{fx})}[\overline{(\mathrm{op1})}]\}\} \gg \underline{(\mathrm{sy})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \underline{(\mathrm{fx})}[\overline{(\mathrm{op1})}]\}\}; \mathrm{IntroExist} @ \overline{(\mathrm{op1})} \rhd \underline{(\mathrm{sy})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \underline{(\mathrm{fx})}[\overline{(\mathrm{op1})}]\}\} \gg \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n}; \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \mathrm{Ded} \rhd$
$\forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \underline{(\mathrm{sy})} \in \underline{(\mathrm{fx})} \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sx})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sx})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{sy})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sx})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sx})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{InSeriesHelper} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sy})}\colon \underline{(\mathrm{sy})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sx})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sx})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\underline{(\mathrm{sy})} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n}]$

$[\mathrm{InSeriesHelper} \overset{\mathrm{tex}}{\rightarrow}$ “InSeriesHelper”]

$[\mathrm{InSeriesHelper} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma inSeries helper”]

InSeries

$[\mathrm{InSeries} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{m}\colon \forall \underline{(fx)}\colon \forall \underline{(sx)}\colon \forall \underline{(sy)}\colon \lambda c.\mathrm{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sx)} \rceil) \Vdash \underline{(sy)} \in \underline{(fx)} \vdash$
$\mathrm{SeriesType} \rhd \lambda c.\mathrm{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sx)} \rceil) \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} =$
$\overline{(f4)})n)n; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \gg$
$\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot\neg(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} =$
$\overline{(f4)})n)n; \mathrm{FirstConjunct} \rhd \dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \gg$
$\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot\neg(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n; \mathrm{Repetition} \rhd \forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in$
$\underline{(sx)})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \gg$
$\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot\neg(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n; \mathrm{A4} \,@\, \underline{(sy)} \rhd \forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \gg \underline{(sy)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n; \mathrm{MP} \rhd \underline{(sy)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \rhd \underline{(sy)} \in \underline{(fx)} \gg$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n; \mathrm{Repetition} \rhd$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \gg$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n; \mathrm{InSeriesHelper} \gg$
$\underline{(sy)} \in \underline{(fx)} \Rightarrow \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot\neg(\underline{(sy)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\underline{(sy)} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n; \mathrm{MP2} \rhd \underline{(sy)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\underline{(sy)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\underline{(sy)} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \rhd \underline{(sy)} \in \underline{(fx)} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow \dot{\neg}(\underline{(sy)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\underline{(sy)} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n; \mathrm{TwiceExistMP} \rhd \dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in$
$\underline{(sx)})n)n \Rightarrow \dot{\neg}(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\underline{(sy)} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n \rhd$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sx)})n)n \Rightarrow$
$\dot{\neg}(\underline{(sy)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\underline{(sy)} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{InSeries} \stackrel{stmt}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}: \forall\underline{(fx)}: \forall\underline{(sx)}: \forall\underline{(sy)}: \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(fx)}\rceil, \lceil\underline{(sx)}\rceil) \Vdash$
$\underline{(sy)} \in \underline{(fx)} \vdash \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\underline{(sy)} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\})n)n]$

[InSeries $\stackrel{tex}{\rightarrow}$ "InSeries"]

[InSeries $\stackrel{pyk}{\rightarrow}$ "lemma inSeries"]

To = f(Subset)(Helper)

$[\mathrm{To} = \mathrm{f(Subset)(Helper)} \stackrel{proof}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}: \forall\underline{(fx)}: \forall\underline{(fy)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \vdash \forall_{obj}\underline{\mathsf{m}}: \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fy)}[\underline{\mathsf{m}}] \vdash \overline{(s1)} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \vdash$

$\mathrm{eqReflexivity} \gg \underline{\mathsf{m}} = \underline{\mathsf{m}}; \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]; \mathrm{sameOrderedPair} \rhd \underline{\mathsf{m}} = \underline{\mathsf{m}} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\}; \mathrm{eqTransitivity} \rhd \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\} \gg \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\}; \mathrm{eqSymmetry} \rhd \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\} = \overline{(\mathrm{s1})}; \mathrm{NatType} \gg \underline{\mathsf{m}} \in \mathrm{N}; \mathrm{MemberOfSeries} \rhd \underline{\mathsf{m}} \in \mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\} \in \underline{(\mathrm{fy})}; \mathrm{SameMember} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\} = \overline{(\mathrm{s1})} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]\}\} \in \underline{(\mathrm{fy})} \gg \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})}; \forall \underline{\mathsf{m}} \colon \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \mathrm{Ded} \rhd \forall \underline{\mathsf{m}} \colon \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \vdash \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})} \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \Rightarrow \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})}], \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{To} = \mathrm{f}(\mathrm{Subset})(\mathrm{Helper}) \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}} \colon \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \Rightarrow \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})}]$

$[\mathrm{To} = \mathrm{f}(\mathrm{Subset})(\mathrm{Helper}) \stackrel{\mathrm{tex}}{\to}$ “To=f(Subset)(Helper)”]

[To = f(Subset)(Helper) $\overset{\text{pyk}}{\rightarrow}$ “lemma to=f subset helper”]

To = f(Subset)

[To = f(Subset) $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}\colon \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \vdash \forall_{obj}\underline{m}\colon \underline{(fx)}[\underline{m}] = \underline{(fy)}[\underline{m}] \vdash \overline{(s1)} \in \underline{(fx)} \vdash \text{InSeries} \gg$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(\overline{(s1)} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(fx)}[\underline{m}]\}\})n)n; \text{To = f(Subset)(Helper)} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \Rightarrow \forall_{obj}\underline{m}\colon \underline{(fx)}[\underline{m}] = \underline{(fy)}[\underline{m}] \Rightarrow \overline{(s1)} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(fx)}[\underline{m}]\}\} \Rightarrow$
$\overline{(s1)} \in \underline{(fy)}; \text{MP2} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \Rightarrow \forall_{obj}\underline{m}\colon \underline{(fx)}[\underline{m}] = \underline{(fy)}[\underline{m}] \Rightarrow \overline{(s1)} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(fx)}[\underline{m}]\}\} \Rightarrow$
$\overline{(s1)} \in \underline{(fy)} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \rhd \forall_{obj}\underline{m}\colon \underline{(fx)}[\underline{m}] = \underline{(fy)}[\underline{m}] \gg \overline{(s1)} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(fx)}[\underline{m}]\}\} \Rightarrow$

$\overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})}; \mathrm{ExistMP} \rhd \overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \Rightarrow \overline{(\mathrm{s1})} \in$
$\underline{(\mathrm{fy})} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\overline{(\mathrm{s1})} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{s1})} \in$
$\underline{(\mathrm{fy})}; \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{Ded} \rhd \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fy})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \vdash \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fy})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{s1})} \in$
$\underline{(\mathrm{fy})}; \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fx})}\rceil, \lceil\mathrm{Q}\rceil) \Vdash \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fy})}\rceil, \lceil\mathrm{Q}\rceil) \Vdash$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \mathrm{SeriesType} \mathrel{|\!\rhd} \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fx})}\rceil, \lceil\mathrm{Q}\rceil) \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SeriesType} \mathrel{|\!\rhd} \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fy})}\rceil, \lceil\mathrm{Q}\rceil) \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n; MP3 \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \Rightarrow \forall_{obj}\underline{\mathsf{m}}\colon \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fy)}[\underline{\mathsf{m}}] \Rightarrow \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \rhd \forall_{obj}\underline{\mathsf{m}}\colon \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fy)}[\underline{\mathsf{m}}] \gg \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)}; Gen \rhd \overline{(s1)} \in$
$\underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)} \gg \forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)}; Repetition \rhd$
$\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)} \gg \forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)}\rceil, p_0, \mathsf{c})]$

$[To = f(Subset) \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \lambda \mathsf{c}.Typeseries0(\lceil \underline{(fx)} \rceil, \lceil Q \rceil) \Vdash \lambda \mathsf{c}.Typeseries0(\lceil \underline{(fy)} \rceil, \lceil Q \rceil) \Vdash \forall_{obj}\underline{\mathsf{m}}\colon \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fy)}[\underline{\mathsf{m}}] \vdash \forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)}]$

$[To = f(Subset) \stackrel{tex}{\rightarrow}$ "To=f(Subset)"$]$

$[To = f(Subset) \stackrel{pyk}{\rightarrow}$ "lemma to=f subset"$]$

To = f

$[\mathrm{To}=\mathrm{f} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fx})} \rceil, \lceil \mathrm{Q} \rceil) \Vdash \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fy})} \rceil, \lceil \mathrm{Q} \rceil) \Vdash \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \mathrm{A4} @ \underline{\mathsf{m}} \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \gg \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]; \mathrm{eqSymmetry} \rhd \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \gg \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]; \mathrm{Gen} \rhd \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \gg \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}]; \mathrm{To}=\mathrm{f}(\mathrm{Subset}) \unrhd \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fx})} \rceil, \lceil \mathrm{Q} \rceil) \unrhd \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fy})} \rceil, \lceil \mathrm{Q} \rceil) \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \gg \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})}; \mathrm{To}=\mathrm{f}(\mathrm{Subset}) \unrhd \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fy})} \rceil, \lceil \mathrm{Q} \rceil) \unrhd \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fx})} \rceil, \lceil \mathrm{Q} \rceil) \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \gg \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})}; \mathrm{ToSetEquality} \rhd \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})} \rhd \forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \underline{(\mathrm{fy})} \Rightarrow \overline{(\mathrm{s1})} \in \underline{(\mathrm{fx})} \gg \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{To}=\mathrm{f} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fx})} \rceil, \lceil \mathrm{Q} \rceil) \Vdash \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fy})} \rceil, \lceil \mathrm{Q} \rceil) \Vdash \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})}]$

$[\mathrm{To}=\mathrm{f} \stackrel{\mathrm{tex}}{\rightarrow}$ "To=f"$]$

$[\mathrm{To}=\mathrm{f} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma to=f"$]$

productIsFunction

$[\mathrm{productIsFunction} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{m1})}\colon \forall \underline{(\mathrm{m2})}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \vdash \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{(\mathrm{m1})}, \underline{(\mathrm{m1})}\}, \{\underline{(\mathrm{m1})}, (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}])\}\} \vdash \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{(\mathrm{m2})}, \underline{(\mathrm{m2})}\}, \{\underline{(\mathrm{m2})}, (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}])\}\} \vdash \mathrm{FromOrderedPair}(1) \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{(\mathrm{m1})}, \underline{(\mathrm{m1})}\}, \{\underline{(\mathrm{m1})}, (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}])\}\} \gg \overline{(\mathrm{f1})} = \underline{(\mathrm{m1})}; \mathrm{eqSymmetry} \rhd \overline{(\mathrm{f1})} = \underline{(\mathrm{m1})} \gg \underline{(\mathrm{m1})} = \overline{(\mathrm{f1})}; \mathrm{FromOrderedPair}(1) \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{(\mathrm{m2})}, \underline{(\mathrm{m2})}\}, \{\underline{(\mathrm{m2})}, (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}])\}\} \gg \overline{(\mathrm{f3})} = \underline{(\mathrm{m2})}; \mathrm{eqTransitivity4} \rhd \underline{(\mathrm{m1})} = \overline{(\mathrm{f1})} \rhd \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \rhd \overline{(\mathrm{f3})} = \underline{(\mathrm{m2})} \gg \underline{(\mathrm{m1})} = \underline{(\mathrm{m2})}; \mathrm{SameSeries} \rhd \underline{(\mathrm{m1})} = \underline{(\mathrm{m2})} \gg \underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] = \underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}]; \mathrm{SameSeries} \rhd \underline{(\mathrm{m1})} = \underline{(\mathrm{m2})} \gg \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}] = \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]; \mathrm{MultiplyEquations} \rhd \underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] = \underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] \rhd \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}] = \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}] \gg (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}]) = (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]); \mathrm{FromOrderedPair}(2) \rhd \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} = \{\{\underline{(\mathrm{m1})}, \underline{(\mathrm{m1})}\}, \{\underline{(\mathrm{m1})}, (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}])\}\} \gg \overline{(\mathrm{f2})} = (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}]); \mathrm{FromOrderedPair}(2) \rhd \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} = \{\{\underline{(\mathrm{m2})}, \underline{(\mathrm{m2})}\}, \{\underline{(\mathrm{m2})}, (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}])\}\} \gg \overline{(\mathrm{f4})} = (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]); \mathrm{eqSymmetry} \rhd \overline{(\mathrm{f4})} = (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]) \gg (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]) = \overline{(\mathrm{f4})}; \mathrm{eqTransitivity4} \rhd \overline{(\mathrm{f2})} = (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}]) \rhd (\underline{(\mathrm{fx})}[\underline{(\mathrm{m1})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m1})}]) = (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]) \rhd (\underline{(\mathrm{fx})}[\underline{(\mathrm{m2})}] * \underline{(\mathrm{fy})}[\underline{(\mathrm{m2})}]) = \overline{(\mathrm{f4})} \gg \overline{(\mathrm{f2})} =$

$\overline{(f4)}; \forall \underline{(m1)}: \forall \underline{(m2)}: \forall \underline{(fx)}: \forall \underline{(fy)}: \mathrm{Ded} \rhd \forall \underline{(m1)}: \forall \underline{(m2)}: \forall \underline{(fx)}: \forall \underline{(fy)}: \overline{(f1)} = \overline{(f3)} \vdash$
$\{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} = \{\{\underline{(m1)}, \underline{(m1)}\}, \{\underline{(m1)}, (\underline{(fx)}[\underline{(m1)}] * \underline{(fy)}[\underline{(m1)}])\}\} \vdash$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} = \{\{\underline{(m2)}, \underline{(m2)}\}, \{\underline{(m2)}, (\underline{(fx)}[\underline{(m2)}] * \underline{(fy)}[\underline{(m2)}])\}\} \vdash$
$\overline{(f2)} = \overline{(f4)} \gg \forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in$
$\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)}\rceil, p_0, \mathsf{c})]$

$[productIsFunction \overset{stmt}{\rightarrow} SystemQ \vdash$
$\forall \underline{(m1)}: \forall \underline{(m2)}: \forall \underline{(fx)}: \forall \underline{(fy)}: \forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2}$
$\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)}]$

$[productIsFunction \overset{tex}{\rightarrow}$ "productIsFunction"$]$

$[productIsFunction \overset{pyk}{\rightarrow}$ "lemma productIsFunction"$]$

productIsTotal

$[productIsTotal \overset{proof}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{\mathsf{m}}: \forall \underline{(fx)}: \forall \underline{(fy)}: \overline{(s1)} \in N \vdash$
$SeriesType \gg \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; SeriesType \gg \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n; \mathrm{ValueType} \rhd \overline{(s1)} \in N \rhd \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \underline{(fx)}[\overline{(s1)}] \in Q; \mathrm{ValueType} \rhd \overline{(s1)} \in N \rhd \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\underline{(fy)} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n \gg \underline{(fy)}[\overline{(s1)}] \in Q; \mathrm{Qclosed(Multiplication)} \rhd \underline{(fx)}[\overline{(s1)}] \in$
$Q \rhd \underline{(fy)}[\overline{(s1)}] \in Q \gg (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}]) \in Q; \mathrm{ToCartProd} \rhd \overline{(s1)} \in$
$N \rhd (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}]) \in Q \gg \{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} \in$
$\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}; \mathrm{eqReflexivity} \gg$
$\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} = \{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] *$
$\underline{(fy)}[\overline{(s1)}])\}\}; \mathrm{IntroExist} @ \overline{(s1)} \rhd \{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} =$
$\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} \gg$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n; \mathrm{Formula2Sep} \rhd$
$\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \rhd$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n \gg$
$\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} \in \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\}; \mathrm{Repetition} \rhd$
$\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} \in \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \gg$
$\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, (\underline{(fx)}[\overline{(s1)}] * \underline{(fy)}[\overline{(s1)}])\}\} \in \{ph \in \{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}; \mathrm{IntroExist} @(\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] * \underline{(\mathrm{fy})}[\overline{(\mathrm{s1})}]) \rhd \{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, (\underline{(\mathrm{fx})}[\overline{(\mathrm{s1})}] * \underline{(\mathrm{fy})}[\overline{(\mathrm{s1})}])\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{Ded} \rhd \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{productIsTotal} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}]$

$[\mathrm{productIsTotal} \stackrel{\mathrm{tex}}{\rightarrow}$ “productIsTotal”$]$

$[\mathrm{productIsTotal} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma productIsTotal”$]$

ProductIsRationalSeries

$[\mathrm{ProductIsRationalSeries} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{m1})}\colon \forall\underline{(\mathrm{m2})}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{CPseparationIsRelation} \gg \forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{productIsFunction} \gg$
$\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) =$
$(\mathrm{f4}); \mathrm{productIsTotal} \gg \forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n}; \mathrm{ToSeries} \rhd \forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) =$
$(\mathrm{f4}) \rhd \forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ProductIsRationalSeries} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}\colon\forall\underline{(\mathrm{m1})}\colon\forall\underline{(\mathrm{m2})}\colon\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\underline{\mathrm{fx}})[\underline{\mathsf{m}}] * (\underline{\mathrm{fy}})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{ProductIsRationalSeries} \stackrel{\mathrm{tex}}{\to}$ "ProductIsRationalSeries"]

$[\mathrm{ProductIsRationalSeries} \stackrel{\mathrm{pyk}}{\to}$ "lemma productIsRationalSeries"]

TimesF

$[\mathrm{TimesF} \stackrel{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\lambda\mathsf{c}.\mathrm{TypeNat0}(\lceil\underline{\mathsf{m}}\rceil) \Vdash$
$\lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fx})}\rceil,\lceil\mathrm{Q}\rceil) \Vdash \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fy})}\rceil,\lceil\mathrm{Q}\rceil) \Vdash \mathrm{NatType} \gg \underline{\mathsf{m}} \in$
$\mathrm{N};\mathrm{SeriesType} \gg \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; \mathrm{SeriesType} \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n)n; \mathrm{ProductIsRationalSeries} \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\})n)n)n)n; \mathrm{MemberOfSeries} \rhd \underline{\mathsf{m}} \in$
$N \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}; \mathrm{ValueType} \rhd \underline{\mathsf{m}} \in \mathrm{N} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in (\mathrm{fx}) \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in (\mathrm{fx}) \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in (\mathrm{fx}) \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$(\mathrm{fx}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg (\mathrm{fx})[\underline{\mathsf{m}}] \in \mathrm{Q}; \mathrm{ValueType} \rhd \underline{\mathsf{m}} \in \mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in (\mathrm{fy}) \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in (\mathrm{fy}) \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in (\mathrm{fy}) \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$(\mathrm{fy}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg (\mathrm{fy})[\underline{\mathsf{m}}] \in \mathrm{Q}; \mathrm{Qclosed}(\mathrm{Multiplication}) \rhd (\mathrm{fx})[\underline{\mathsf{m}}] \in \mathrm{Q} \rhd (\mathrm{fy})[\underline{\mathsf{m}}] \in$
$\mathrm{Q} \gg ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}]) \in \mathrm{Q}; \mathrm{ToCartProd} \rhd \underline{\mathsf{m}} \in \mathrm{N} \rhd ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}]) \in \mathrm{Q} \gg$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqReflexivity} \gg$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] *$
$(\mathrm{fy})[\underline{\mathsf{m}}])\}\}; \mathrm{IntroExist} @ \underline{\mathsf{m}} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] *$
$(\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}; \mathrm{Formula2Sep} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] *$
$(\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}; \mathrm{eqReflexivity} \gg \underline{\mathsf{m}} =$
$\underline{\mathsf{m}}; \mathrm{UniqueMember} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \rhd$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\} \in \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\} \rhd \underline{\mathsf{m}} = \underline{\mathsf{m}} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] =$
$(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{TimesF} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \lambda\mathsf{c}.\mathrm{TypeNat0}(\lceil\underline{\mathsf{m}}\rceil) \Vdash$
$\lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fx})}\rceil, \lceil\mathrm{Q}\rceil) \Vdash \lambda\mathsf{c}.\mathrm{Typeseries0}(\lceil\underline{(\mathrm{fy})}\rceil, \lceil\mathrm{Q}\rceil) \Vdash \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] =$
$(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])]$

$[\mathrm{TimesF} \stackrel{\mathrm{tex}}{\rightarrow}$ “TimesF”$]$

$[\mathrm{TimesF} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma timesF”$]$

$-\mathrm{x}+(1/2)\mathrm{x}=-(1/2)\mathrm{x}$

[$-\mathrm{x}+(1/2)\mathrm{x}=-(1/2)\mathrm{x} \stackrel{\mathrm{tex}}{\rightarrow}$ "-x+(1/2)x=-(1/2)x"]

[$-\mathrm{x}+(1/2)\mathrm{x}=-(1/2)\mathrm{x} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma -x+(1/2)x=-(1/2)x"]

PositiveTripled

[$\mathrm{PositiveTripled} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathrm{x}}\colon \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash 0 < 3 \gg \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveFactors} \rhd \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{timesAssociativity} \gg ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{\mathrm{x}}) = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})); \mathrm{eqSymmetry} \rhd ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{\mathrm{x}}) = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) \gg (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) = ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{\mathrm{x}}); \mathrm{PositiveNonzero} \rhd \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(((1+1)+1) = 0)\mathrm{n}; \mathrm{Reciprocal} \rhd \dot{\neg}\,(((1+1)+1) = 0)\mathrm{n} \gg (((1+1)+1) * \mathrm{rec}((1+1)+1)) = 1; \mathrm{eqMultiplication} \rhd (((1+1)+1) * \mathrm{rec}((1+1)+1)) = 1 \gg ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{\mathrm{x}}) = (1 * \underline{\mathrm{x}}); \mathrm{times1Left} \gg (1 * \underline{\mathrm{x}}) = \underline{\mathrm{x}}; \mathrm{eqTransitivity4} \rhd (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) = ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{\mathrm{x}}) \rhd ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{\mathrm{x}}) = (1 * \underline{\mathrm{x}}) \rhd (1 * \underline{\mathrm{x}}) = \underline{\mathrm{x}} \gg (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) = \underline{\mathrm{x}}; \mathrm{SubLessRight} \rhd (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) = \underline{\mathrm{x}} \rhd \dot{\neg}\,(0 <= (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= \underline{\mathrm{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathrm{x}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[$\mathrm{PositiveTripled} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathrm{x}}\colon \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\mathrm{rec}((1+1)+1) * \underline{\mathrm{x}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(0 <= \underline{\mathrm{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathrm{x}})\mathrm{n})\mathrm{n})\mathrm{n}$]

[$\mathrm{PositiveTripled} \stackrel{\mathrm{tex}}{\rightarrow}$ "PositiveTripled"]

[$\mathrm{PositiveTripled} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma positiveTripled"]

PositiveDividedBy3

[$\mathrm{PositiveDividedBy3} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathrm{x}}\colon \dot{\neg}\,(0 <= \underline{\mathrm{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathrm{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash 0 < 3 \gg \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveInverted} \rhd \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveFactors} \rhd \dot{\neg}\,(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$

$\mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveDividedBy3} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}\,(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{PositiveDividedBy3} \stackrel{\mathrm{tex}}{\rightarrow}$ "PositiveDividedBy3"$]$

$[\mathrm{PositiveDividedBy3} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma positiveDividedBy3"$]$

$|\mathrm{x} - \mathrm{x}| = 0$

$[|\mathrm{x} - \mathrm{x}| = 0 \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \mathrm{eqReflexivity} \gg \underline{\mathsf{x}} = \underline{\mathsf{x}}; \mathrm{PositiveToLeft(Eq)(1term)} \rhd \underline{\mathsf{x}} = \underline{\mathsf{x}} \gg (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{x}})) = 0; \mathrm{SameNumerical} \rhd (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{x}})) = 0 \gg |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{x}}))| = |0|; |0| = 0 \gg |0| = 0; \mathrm{eqTransitivity} \rhd |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{x}}))| = |0| \rhd |0| = 0 \gg |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{x}}))| = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[|\mathrm{x} - \mathrm{x}| = 0 \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{x}}))| = 0]$

$[|\mathrm{x} - \mathrm{x}| = 0 \stackrel{\mathrm{tex}}{\rightarrow}$ "|x-x|=0"$]$

$[|\mathrm{x} - \mathrm{x}| = 0 \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma |x-x|=0"$]$

$1 < 2$

$[1 < 2 \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0 < 1 \gg \dot{\neg}\,(0 <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessAddition} \rhd \dot{\neg}\,(0 <= 1 \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((0+1) <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((0+1) = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plus0Left} \gg (0+1) = 1; \mathrm{SubLessLeft} \rhd (0+1) = 1 \rhd \dot{\neg}\,((0+1) <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((0+1) = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(1 <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}\,(1 <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(1 <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[1 < 2 \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \dot{\neg}\,(1 <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[1 < 2 \stackrel{\mathrm{tex}}{\rightarrow}$ "1<2"$]$

$[1 < 2 \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma 1<2"$]$

$1/3 < 2/3$

$[1/3 < 2/3 \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 1 < 2 \gg \dot{\neg}\,(1 <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; 0 < 1/3 \gg \dot{\neg}\,(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessMultiplication} \rhd \dot{\neg}\,(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,(1 <= (1+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(1 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((1 * \mathrm{rec}((1+1)+1)) <= ((1+1) * \mathrm{rec}((1+1)+1)) \Rightarrow$

$\dot{\neg}\,(\dot{\neg}\,((1*\mathrm{rec}((1+1)+1)) = ((1+1)*\mathrm{rec}((1+1)+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{times1Left} \gg (1*\mathrm{rec}((1+1)+1)) = \mathrm{rec}((1+1)+1); \mathrm{SubLessLeft} \rhd (1*\mathrm{rec}((1+1)+1)) = \mathrm{rec}((1+1)+1) \rhd \dot{\neg}\,((1*\mathrm{rec}((1+1)+1)) <= ((1+1)*\mathrm{rec}((1+1)+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((1*\mathrm{rec}((1+1)+1)) = ((1+1)*\mathrm{rec}((1+1)+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\mathrm{rec}((1+1)+1) <= ((1+1)*\mathrm{rec}((1+1)+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\mathrm{rec}((1+1)+1) = ((1+1)*\mathrm{rec}((1+1)+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}\,(\mathrm{rec}((1+1)+1) <= ((1+1)*\mathrm{rec}((1+1)+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\mathrm{rec}((1+1)+1) = ((1+1)*\mathrm{rec}((1+1)+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\mathrm{rec}((1+1)+1) <= ((1+1)*\mathrm{rec}((1+1)+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\mathrm{rec}((1+1)+1) = ((1+1)*\mathrm{rec}((1+1)+1)))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[1/3 < 2/3 \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \dot{\neg}\,(\mathrm{rec}((1+1)+1) <= ((1+1)*\mathrm{rec}((1+1)+1)) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\mathrm{rec}((1+1)+1) = ((1+1)*\mathrm{rec}((1+1)+1)))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[1/3 < 2/3 \stackrel{\mathrm{tex}}{\rightarrow}$ "1/3<2/3"]

$[1/3 < 2/3 \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma 1/3<2/3"]

(1/3)x + (1/3)x = (2/3)x

$[(1/3)\mathrm{x} + (1/3)\mathrm{x} = (2/3)\mathrm{x} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \mathrm{TwoWholes} \gg ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})); \mathrm{timesAssociativity} \gg (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})); \mathrm{eqSymmetry} \rhd (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) \gg ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}); \mathrm{eqTransitivity} \rhd ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) \rhd ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}) \gg ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}); \mathrm{Repetition} \rhd ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}) \gg ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[(1/3)\mathrm{x} + (1/3)\mathrm{x} = (2/3)\mathrm{x} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}})]$

$[(1/3)\mathrm{x} + (1/3)\mathrm{x} = (2/3)\mathrm{x} \stackrel{\mathrm{tex}}{\rightarrow}$ "(1/3)x+(1/3)x=(2/3)x"]

$[(1/3)\mathrm{x} + (1/3)\mathrm{x} = (2/3)\mathrm{x} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma (1/3)x+(1/3)x=(2/3)x"]

(2/3)x + (1/3)x = x

$[(2/3)\mathrm{x} + (1/3)\mathrm{x} = \mathrm{x} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \mathrm{TwoWholes} \gg ((\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}}) + (\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})); \mathrm{timesAssociativity} \gg (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})); \mathrm{eqSymmetry} \rhd (((1+1)*\mathrm{rec}((1+1)+1))*\underline{\mathsf{x}}) = ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) \gg ((1+1)*(\mathrm{rec}((1+1)+1)*\underline{\mathsf{x}})) =$

$(((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}); \mathrm{eqTransitivity} \triangleright ((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = ((1+1)\ast(\mathrm{rec}((1+1)+1)\ast \underline{x})) \triangleright ((1+1)\ast(\mathrm{rec}((1+1)+1)\ast \underline{x})) = (((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}) \gg ((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = (((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}); \mathrm{eqAddition} \triangleright ((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = (((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}) \gg$
$(((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = ((((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})); \mathrm{ThreeThirds} \gg$
$(((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = \underline{x}; \mathrm{Equality} \triangleright (((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = ((((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) \triangleright (((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = \underline{x} \gg$
$((((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[(2/3)\mathrm{x}+(1/3)\mathrm{x} = \mathrm{x} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash$
$\forall \underline{x} \colon ((((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = \underline{x}]$

$[(2/3)\mathrm{x}+(1/3)\mathrm{x} = \mathrm{x} \overset{\mathrm{tex}}{\to}$ "(2/3)x+(1/3)x=x"]

$[(2/3)\mathrm{x}+(1/3)\mathrm{x} = \mathrm{x} \overset{\mathrm{pyk}}{\to}$ "lemma (2/3)x+(1/3)x=x"]

$-\mathrm{x}+(2/3)\mathrm{x} = -(1/3)\mathrm{x}$

$[-\mathrm{x}+(2/3)\mathrm{x} = -(1/3)\mathrm{x} \overset{\mathrm{tex}}{\to}$ "-x+(2/3)x=-(1/3)x"]

$[-\mathrm{x}+(2/3)\mathrm{x} = -(1/3)\mathrm{x} \overset{\mathrm{pyk}}{\to}$ "lemma -x+(2/3)x=-(1/3)x"]

$-(1/3)\mathrm{x}-(1/3)\mathrm{x} = -(2/3)\mathrm{x}$

$[-(1/3)\mathrm{x}-(1/3)\mathrm{x} = -(2/3)\mathrm{x} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon (1/3)\mathrm{x}+(1/3)\mathrm{x} = (2/3)\mathrm{x} \gg ((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = (((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}); \mathrm{EqNegated} \triangleright ((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x})) = (((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}) \gg (-\mathrm{u}((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))) = (-\mathrm{u}(((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x})); -\mathrm{x}-\mathrm{y} = -(\mathrm{x}+\mathrm{y}) \gg ((-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))) = (-\mathrm{u}((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))); \mathrm{eqTransitivity} \triangleright ((-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))) = (-\mathrm{u}((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))) \triangleright (-\mathrm{u}((\mathrm{rec}((1+1)+1)\ast \underline{x})+(\mathrm{rec}((1+1)+1)\ast \underline{x}))) = (-\mathrm{u}(((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x})) \gg ((-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))) = (-\mathrm{u}(((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[-(1/3)\mathrm{x}-(1/3)\mathrm{x} = -(2/3)\mathrm{x} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x} \colon ((-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))+(-\mathrm{u}(\mathrm{rec}((1+1)+1)\ast \underline{x}))) = (-\mathrm{u}(((1+1)\ast \mathrm{rec}((1+1)+1))\ast \underline{x}))]$

$[-(1/3)\mathrm{x}-(1/3)\mathrm{x} = -(2/3)\mathrm{x} \overset{\mathrm{tex}}{\to}$ "-(1/3)x-(1/3)x=-(2/3)x"]

$[-(1/3)\mathrm{x}-(1/3)\mathrm{x} = -(2/3)\mathrm{x} \overset{\mathrm{pyk}}{\to}$ "lemma -(1/3)x-(1/3)x=-(2/3)x"]

$-x + (1/3)x = -(2/3)x$

$[-x + (1/3)x = -(2/3)x \overset{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon (2/3)x + (1/3)x = x \gg ((((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) = \underline{x}; \text{PositiveToRight}(\text{Eq}) \rhd ((((1+1)*\text{rec}((1+1)+1))*\underline{x})+(\text{rec}((1+1)+1)*\underline{x})) = \underline{x} \gg (((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}) = (\underline{x} + (-u(\text{rec}((1 + 1) + 1) * \underline{x}))); \text{EqNegated} \rhd (((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}) = (\underline{x} + (-u(\text{rec}((1 + 1) + 1) * \underline{x}))) \gg (-u(((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x})) = (-u(\underline{x} + (-u(\text{rec}((1 + 1) + 1) * \underline{x})))); \text{MinusNegated} \gg (-u(\underline{x} + (-u(\text{rec}((1 + 1) + 1) * \underline{x})))) = ((\text{rec}((1 + 1) + 1) * \underline{x}) + (-u\underline{x})); \text{plusCommutativity} \gg ((\text{rec}((1 + 1) + 1) * \underline{x}) + (-u\underline{x})) = ((-u\underline{x})+(\text{rec}((1+1)+1)*\underline{x})); \text{eqTransitivity4} \rhd (-u(((1+1)*\text{rec}((1+1)+1))*\underline{x})) = (-u(\underline{x} + (-u(\text{rec}((1 + 1) + 1) * \underline{x})))) \rhd (-u(\underline{x} + (-u(\text{rec}((1 + 1) + 1) * \underline{x})))) = ((\text{rec}((1 + 1) + 1) * \underline{x}) + (-u\underline{x})) \rhd ((\text{rec}((1 + 1) + 1) * \underline{x}) + (-u\underline{x})) = ((-u\underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) \gg (-u(((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x})) = ((-u\underline{x})+(\text{rec}((1+1)+1)*\underline{x})); \text{eqSymmetry} \rhd (-u(((1+1)*\text{rec}((1+1)+1))*\underline{x})) = ((-u\underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) \gg ((-u\underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) = (-u(((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}))\rceil, p_0, c)]$

$[-x + (1/3)x = -(2/3)x \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon ((-u\underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) = (-u(((1 + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}))]$

$[-x + (1/3)x = -(2/3)x \overset{\text{tex}}{\to}$ "-x+(1/3)x=-(2/3)x"$]$

$[-x + (1/3)x = -(2/3)x \overset{\text{pyk}}{\to}$ "lemma -x+(1/3)x=-(2/3)x"$]$

PreserveLessGreater

$[\text{PreserveLessGreater} \overset{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(x1)}\colon \forall \underline{(x2)}\colon \forall \underline{(y1)}\colon \forall \underline{(y2)}\colon \forall \underline{z}\colon \underline{(x1)} <= (\underline{(y1)} + (-u\underline{z})) \vdash \dot{\neg}(|(\underline{(x1)} + (-u\underline{(x2)}))| <= (\text{rec}((1 + 1) + 1) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(x1)} + (-u\underline{(x2)}))| = (\text{rec}((1 + 1) + 1) * \underline{z}))n)n)n \vdash \dot{\neg}(|(\underline{(y1)} + (-u\underline{(y2)}))| <= (\text{rec}((1 + 1) + 1) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(y1)} + (-u\underline{(y2)}))| = (\text{rec}((1 + 1) + 1) * \underline{z}))n)n)n \vdash 0 <= |x| \gg 0 <= |(\underline{(x1)} + (-u\underline{(x2)}))|; \text{leqLessTransitivity} \rhd 0 <= |(\underline{(x1)} + (-u\underline{(x2)}))| \rhd \dot{\neg}(|(\underline{(x1)} + (-u\underline{(x2)}))| <= (\text{rec}((1 + 1) + 1) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(x1)} + (-u\underline{(x2)}))| = (\text{rec}((1 + 1) + 1) * \underline{z}))n)n)n \gg \dot{\neg}(0 <= (\text{rec}((1 + 1) + 1) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\text{rec}((1 + 1) + 1) * \underline{z}))n)n)n; \text{PositiveTripled} \rhd \dot{\neg}(0 <= (\text{rec}((1 + 1) + 1) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\text{rec}((1 + 1) + 1) * \underline{z}))n)n)n \gg \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n; \text{NumericalDifferenceLess} \rhd \dot{\neg}(|(\underline{(x1)} + (-u\underline{(x2)}))| <= (\text{rec}((1 + 1) + 1) * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(x1)} + (-u\underline{(x2)}))| = (\text{rec}((1 + 1) + 1) * \underline{z}))n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{(x2)} + (-u(\text{rec}((1 + 1) + 1) * \underline{z}))) <= \underline{(x1)} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{(x2)} + (-u(\text{rec}((1 + 1) + 1) * \underline{z}))) = \underline{(x1)})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(x1)} <= (\underline{(x2)} + (\text{rec}((1 + 1) + 1) * \underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(x1)} = (\underline{(x2)}+(\text{rec}((1+1)+1)*\underline{z})))n)n)n)n)n; \text{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}((\underline{(x2)}+(-u(\text{rec}((1 + 1) + 1) * \underline{z}))) <= \underline{(x1)} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{(x2)} + (-u(\text{rec}((1 + 1) + 1) * \underline{z}))) = \underline{(x1)})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(x1)} <= (\underline{(x2)} + (\text{rec}((1 + 1) + 1) * \underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{(x1)} =$

((x2)+(rec((1+1)+1)∗z)))n)n)n)n)n ≫ ∸(((x2)+(−u(rec((1+1)+1)∗z))) <=
(x1) ⇒ ∸(∸(((x2) + (−u(rec((1 + 1) + 1) ∗ z))) =
(x1))n)n)n; NegativeToRight(Less) ⊳ ∸(((x2) + (−u(rec((1 + 1) + 1) ∗ z))) <=
(x1) ⇒ ∸(∸(((x2) + (−u(rec((1 + 1) + 1) ∗ z))) = (x1))n)n)n ≫ ∸((x2) <=
((x1) + (rec((1 + 1) + 1) ∗ z)) ⇒ ∸(∸((x2) =
((x1) + (rec((1 + 1) + 1) ∗ z)))n)n)n; leqAddition ⊳ (x1) <= ((y1) + (−uz)) ≫
((x1) + (rec((1 + 1) + 1) ∗ z)) <= (((y1) + (−uz)) + (rec((1 + 1) + 1) ∗ z)); −x +
(1/3)x = −(2/3)x ≫ ((−uz) + (rec((1 + 1) + 1) ∗ z)) = (−u(((1 + 1) ∗ rec((1 +
1) + 1)) ∗ z)); Three2twoTerms ⊳ ((−uz) + (rec((1 + 1) + 1) ∗ z)) = (−u(((1 + 1) ∗
rec((1+1)+1))∗z)) ≫ (((y1)+(−uz))+(rec((1+1)+1)∗z)) = ((y1)+(−u(((1+
1)∗rec((1+1)+1))∗z))); subLeqRight ⊳ (((y1)+(−uz))+(rec((1+1)+1)∗z)) =
((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) ⊳ ((x1) + (rec((1 + 1) + 1) ∗ z)) <=
(((y1) + (−uz)) + (rec((1 + 1) + 1) ∗ z)) ≫ ((x1) + (rec((1 + 1) + 1) ∗ z)) <=
((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))); NumericalDifferenceLess ⊳
∸(|((y1) + (−u(y2)))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|((y1) + (−u(y2)))| =
(rec((1 + 1) + 1) ∗ z))n)n)n ≫ ∸(∸(((y2) + (−u(rec((1 + 1) + 1) ∗ z))) <=
(y1) ⇒ ∸(∸(((y2) + (−u(rec((1 + 1) + 1) ∗ z))) = (y1))n)n)n ⇒ ∸(∸((y1) <=
((y2) + (rec((1 + 1) + 1) ∗ z)) ⇒ ∸(∸((y1) = ((y2) + (rec((1 + 1) + 1) ∗
z)))n)n)n)n)n; SecondConjunct ⊳ ∸(∸(((y2) + (−u(rec((1 + 1) + 1) ∗ z))) <=
(y1) ⇒ ∸(∸(((y2) + (−u(rec((1 + 1) + 1) ∗ z))) = (y1))n)n)n ⇒ ∸(∸((y1) <=
((y2) + (rec((1 + 1) + 1) ∗ z)) ⇒ ∸(∸((y1) = ((y2) + (rec((1 + 1) + 1) ∗
z)))n)n)n)n)n ≫ ∸((y1) <= ((y2) + (rec((1 + 1) + 1) ∗ z)) ⇒ ∸(∸((y1) =
((y2) + (rec((1 + 1) + 1) ∗ z)))n)n)n; PositiveToLeft(Less) ⊳ ∸((y1) <=
((y2)+(rec((1+1)+1)∗z)) ⇒ ∸(∸((y1) = ((y2)+(rec((1+1)+1)∗z)))n)n)n ≫
∸(((y1)+(−u(rec((1+1)+1)∗z))) <= (y2) ⇒ ∸(∸(((y1)+(−u(rec((1+1)+1)∗
z))) = (y2))n)n)n; LessAddition ⊳ ∸(((y1)+(−u(rec((1+1)+1)∗z))) <= (y2) ⇒
∸(∸(((y1)+(−u(rec((1+1)+1)∗z))) = (y2))n)n)n ≫ ∸((((y1)+(−u(rec((1+
1)+1)∗z)))+(−u(rec((1+1)+1)∗z))) <= ((y2)+(−u(rec((1+1)+1)∗z))) ⇒
∸(∸((((y1) + (−u(rec((1 + 1) + 1) ∗ z))) + (−u(rec((1 + 1) + 1) ∗ z))) =
((y2) + (−u(rec((1 + 1) + 1) ∗ z))))n)n)n; plusAssociativity ≫
(((y1) + (−u(rec((1 + 1) + 1) ∗ z))) + (−u(rec((1 + 1) + 1) ∗ z))) =
((y1)+((−u(rec((1+1)+1)∗z))+(−u(rec((1+1)+1)∗z)))); −(1/3)x−(1/3)x =
−(2/3)x ≫ ((−u(rec((1 + 1) + 1) ∗ z)) + (−u(rec((1 + 1) + 1) ∗ z))) =
(−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z)); EqAdditionLeft ⊳ ((−u(rec((1 + 1) + 1) ∗
z)) + (−u(rec((1 + 1) + 1) ∗ z))) = (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z)) ≫
((y1) + ((−u(rec((1 + 1) + 1) ∗ z)) + (−u(rec((1 + 1) + 1) ∗ z)))) =
((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))); eqTransitivity ⊳ (((y1) +
(−u(rec((1 + 1) + 1) ∗ z))) + (−u(rec((1 + 1) + 1) ∗ z))) = ((y1) + ((−u(rec((1 +
1) + 1) ∗ z)) + (−u(rec((1 + 1) + 1) ∗ z)))) ⊳ ((y1) + ((−u(rec((1 + 1) + 1) ∗ z)) +
(−u(rec((1 + 1) + 1) ∗ z)))) = ((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) ≫
(((y1) + (−u(rec((1 + 1) + 1) ∗ z))) + (−u(rec((1 + 1) + 1) ∗ z))) =
((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))); SubLessLeft ⊳ (((y1) + (−u(rec((1 +
1) + 1) ∗ z))) + (−u(rec((1 + 1) + 1) ∗ z))) = ((y1) + (−u(((1 + 1) ∗ rec((1 + 1) +
1)) ∗ z))) ⊳ ∸((((y1) + (−u(rec((1 + 1) + 1) ∗ z))) + (−u(rec((1 + 1) + 1) ∗ z))) <=
((y2) + (−u(rec((1 + 1) + 1) ∗ z))) ⇒ ∸(∸((((y1) + (−u(rec((1 + 1) + 1) ∗ z))) +
(−u(rec((1 + 1) + 1) ∗ z))) = ((y2) + (−u(rec((1 + 1) + 1) ∗ z))))n)n)n ≫

∸(((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) <= ((y2) + (−u(rec((1 + 1) + 1) ∗ z))) ⇒ ∸(∸(((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) = ((y2) + (−u(rec((1 + 1) + 1) ∗ z))))n)n)n; LessLeqTransitivity ⊳ ∸((x2) <= ((x1) + (rec((1+1)+1) ∗ z)) ⇒ ∸(∸((x2) = ((x1) + (rec((1+1)+1) ∗ z)))n)n)n ⊳ ((x1) + (rec((1 + 1) + 1) ∗ z)) <= ((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) ≫ ∸((x2) <= ((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) ⇒ ∸(∸((x2) = ((y1) + (−u(((1+1) ∗ rec((1+1)+1)) ∗ z))))n)n)n; LessTransitivity ⊳ ∸((x2) <= ((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) ⇒ ∸(∸((x2) = ((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))))n)n)n ⊳ ∸(((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) <= ((y2) + (−u(rec((1 + 1) + 1) ∗ z))) ⇒ ∸(∸(((y1) + (−u(((1 + 1) ∗ rec((1 + 1) + 1)) ∗ z))) = ((y2) + (−u(rec((1 + 1) + 1) ∗ z))))n)n)n ≫ ∸((x2) <= ((y2) + (−u(rec((1 + 1) + 1) ∗ z))) ⇒ ∸(∸((x2) = ((y2) + (−u(rec((1 + 1) + 1) ∗ z))))n)n)n; LessLeq ⊳ ∸((x2) <= ((y2) + (−u(rec((1 + 1) + 1) ∗ z))) ⇒ ∸(∸((x2) = ((y2) + (−u(rec((1 + 1) + 1) ∗ z))))n)n)n ≫ (x2) <= ((y2) + (−u(rec((1 + 1) + 1) ∗ z)))⌉, p_0, c)]

[PreserveLessGreater $\overset{\text{stmt}}{\to}$ SystemQ ⊢ ∀(x1): ∀(x2): ∀(y1): ∀(y2): ∀z: (x1) <= ((y1) + (−uz)) ⊢ ∸(|((x1) + (−u(x2)))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|((x1) + (−u(x2)))| = (rec((1 + 1) + 1) ∗ z))n)n)n ⊢ ∸(|((y1) + (−u(y2)))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|((y1) + (−u(y2)))| = (rec((1 + 1) + 1) ∗ z))n)n)n ⊢ (x2) <= ((y2) + (−u(rec((1 + 1) + 1) ∗ z)))]

[PreserveLessGreater $\overset{\text{tex}}{\to}$ “PreserveLessGreater”]

[PreserveLessGreater $\overset{\text{pyk}}{\to}$ “lemma preserveLessGreater”]

ClosetolessIsLess

[ClosetolessIsLess $\overset{\text{proof}}{\to}$ λc.λx.𝒫(⌈SystemQ ⊢ ∀(x1): ∀(x2): ∀y: ∀z: (x1) <= (y + (−uz)) ⊢ ∸(|((x1) + (−u(x2)))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|((x1) + (−u(x2)))| = (rec((1 + 1) + 1) ∗ z))n)n)n ⊢ 0 <= |x| ≫ 0 <= |((x1) + (−u(x2)))|; leqLessTransitivity ⊳ 0 <= |((x1) + (−u(x2)))| ⊳ ∸(|((x1) + (−u(x2)))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|((x1) + (−u(x2)))| = (rec((1 + 1) + 1) ∗ z))n)n)n ≫ ∸(0 <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(0 = (rec((1 + 1) + 1) ∗ z))n)n)n; |x − x| = 0 ≫ |(y + (−uy))| = 0; eqSymmetry ⊳ |(y + (−uy))| = 0 ≫ 0 = |(y + (−uy))|; SubLessLeft ⊳ 0 = |(y + (−uy))| ⊳ ∸(0 <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(0 = (rec((1+1)+1) ∗ z))n)n)n ≫ ∸(|(y + (−uy))| <= (rec((1+1)+1) ∗ z) ⇒ ∸(∸(|(y + (−uy))| = (rec((1 + 1) + 1) ∗ z))n)n)n; PreserveLessGreater ⊳ (x1) <= (y + (−uz)) ⊳ ∸(|((x1) + (−u(x2)))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|((x1) + (−u(x2)))| = (rec((1 + 1) + 1) ∗ z))n)n)n ⊳ ∸(|(y + (−uy))| <= (rec((1 + 1) + 1) ∗ z) ⇒ ∸(∸(|(y + (−uy))| = (rec((1 + 1) + 1) ∗ z))n)n)n ≫ (x2) <= (y + (−u(rec((1 + 1) + 1) ∗ z)))⌉, p_0, c)]

[ClosetolessIsLess $\overset{\text{stmt}}{\to}$ SystemQ ⊢ ∀(x1): ∀(x2): ∀y: ∀z: (x1) <= (y + (−uz)) ⊢

$\dot{\neg}(|((\underline{x1}) + (-u(\underline{x2})))| <= (rec((1+1)+1) * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{x1}) + (-u(\underline{x2})))| = (rec((1+1)+1) * \underline{\mathsf{z}}))n)n)n \vdash \underline{(x2)} <= (\underline{\mathsf{y}} + (-u(rec((1+1)+1) * \underline{\mathsf{z}})))]$

[ClosetolessIsLess $\stackrel{tex}{\rightarrow}$ "ClosetolessIsLess"]

[ClosetolessIsLess $\stackrel{pyk}{\rightarrow}$ "lemma closetolessIsLess"]

SubLessLeft(F)

[SubLessLeft(F) $\stackrel{proof}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \forall \underline{(fz)}\colon \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \vdash$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{fz})[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \vdash$
Repetition $\rhd \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \gg$
$\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n$; Ded $\rhd \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \gg$
$\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{(n1)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{(n1)} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n$; A4 @$(rec((1+1)+1) * \overline{(\epsilon)}) \rhd \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\overline{(n1)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (rec((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1) * \overline{(\epsilon)}))n)n)n \Rightarrow \overline{(n1)} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| <= (rec((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\overline{\mathsf{m}}] + (-u(\underline{fy})[\overline{\mathsf{m}}]))| = (rec((1+1)+1) * \overline{(\epsilon)}))n)n)n)n)n$; Repetition $\rhd$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{fz})[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{fz})[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n$; Ded $\rhd$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{fz})[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(n2)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{(n2)} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$

$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash$
$\mathrm{A4} \mathbin{@} \overline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveDividedBy3} \rhd \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \vdash \mathrm{MaxLeq}(1) \gg$
$\overline{(\mathrm{n1})} <= \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}); \mathrm{leqTransitivity} \rhd \overline{(\mathrm{n1})} <= \mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) \rhd \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \gg \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}}; \mathrm{MaxLeq}(2) \gg \overline{(\mathrm{n2})} <= \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}); \mathrm{leqTransitivity} \rhd$
$\overline{(\mathrm{n2})} <= \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) \rhd \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \gg$
$\overline{(\mathrm{n2})} <= \overline{\mathsf{m}}; \mathrm{A4} \mathbin{@} \overline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{n2})} <=$
$\overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})); \mathrm{MP} \rhd \overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})) \rhd \overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \gg \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})); \mathrm{A4} \mathbin{@} \overline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveDividedBy3} \rhd \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP2} \rhd \dot{\neg}\,(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$

$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \gg \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ClosetolessIsLess} \rhd \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)}))) \rhd \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))); \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}: \forall\underline{(\mathrm{fx})}: \forall\underline{(\mathrm{fz})}: \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{TwiceExistMP} \rhd$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{n2})}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}: \forall\underline{(\mathrm{fy})}: \forall\underline{(\mathrm{fz})}: \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash \mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \vdash \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))) \gg$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))); \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))) \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{n1})}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$

$\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))); \mathrm{TwiceExistMP} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow$
$\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+$
$1) * \overline{(\epsilon)}))) \rhd \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{n2})}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+$
$1) * \overline{(\epsilon)}))); \mathrm{JoinConjuncts} \rhd \dot\neg(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))) \gg \dot\neg(\dot\neg(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow$
$\dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <=$
$\overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot\neg(\dot\neg(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@} \mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+$
$1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n} \gg \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@}(\mathrm{rec}((1+$
$1)+1) * \overline{(\epsilon)}) \rhd \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow$
$\dot\neg(\dot\neg(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SubLessLeft}(\mathrm{F}) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$

$\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n]$

[SubLessLeft(F) $\stackrel{\text{tex}}{\to}$ “SubLessLeft(F)”]

[SubLessLeft(F) $\stackrel{\text{pyk}}{\to}$ “lemma subLessLeft(F)”]

SubLessLeft(R)

[SubLessLeft(R) $\stackrel{\text{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(fx)}\colon\forall\underline{(fy)}\colon\forall\underline{(fz)}\colon \{ph \in P(\{ph \in$
$P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \vdash \mathrm{From} == \rhd\{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubLessLeft(F)} \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SubLessLeft(R)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\dot\neg(\forall_{obj}\overline{(\epsilon)}: \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \vdash$
$\dot\neg(\forall_{obj}\overline{(\epsilon)}: \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= ((\underline{(fz)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n]$

[SubLessLeft(R) $\overset{tex}{\to}$ "SubLessLeft(R)"]

[SubLessLeft(R) $\overset{pyk}{\to}$ "lemma subLessLeft(R)"]

ClosetogreaterIsGreater

[ClosetogreaterIsGreater $\overset{proof}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall\underline{x}: \forall\underline{(y1)}: \forall\underline{(y2)}: \forall\underline{z}: \underline{x} <=$
$((\underline{(y1)} + (-u\underline{z})) \vdash \dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| <= (rec((1+1)+1) * \underline{z}) \Rightarrow$
$\dot\neg(\dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| = (rec((1+1)+1) * \underline{z}))n)n)n \vdash 0 <= |x| \gg 0 <=$
$|((\underline{(y1)} + (-u\underline{(y2)}))|; leqLessTransitivity \rhd 0 <=$
$|((\underline{(y1)} + (-u\underline{(y2)}))| \rhd \dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| <= (rec((1+1)+1) * \underline{z}) \Rightarrow$
$\dot\neg(\dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| = (rec((1+1)+1) * \underline{z}))n)n)n \gg \dot\neg(0 <=$
$(rec((1+1)+1) * \underline{z}) \Rightarrow \dot\neg(\dot\neg(0 = (rec((1+1)+1) * \underline{z}))n)n)n; |x - x| = 0 \gg$
$|(\underline{x} + (-u\underline{x}))| = 0; eqSymmetry \rhd |(\underline{x} + (-u\underline{x}))| = 0 \gg 0 =$
$|(\underline{x} + (-u\underline{x}))|; SubLessLeft \rhd 0 = |(\underline{x} + (-u\underline{x}))| \rhd \dot\neg(0 <= (rec((1+1)+1) * \underline{z}) \Rightarrow$
$\dot\neg(\dot\neg(0 = (rec((1+1)+1) * \underline{z}))n)n)n \gg \dot\neg(|(\underline{x} + (-u\underline{x}))| <= (rec((1+1)+1) * \underline{z}) \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{x} + (-u\underline{x}))| = (rec((1+1)+1) * \underline{z}))n)n)n; PreserveLessGreater \rhd \underline{x} <=$
$((\underline{(y1)} + (-u\underline{z})) \rhd \dot\neg(|(\underline{x} + (-u\underline{x}))| <= (rec((1+1)+1) * \underline{z}) \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{x} + (-u\underline{x}))| = (rec((1+1)+1) * \underline{z}))n)n)n \rhd \dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| <=$
$(rec((1+1)+1) * \underline{z}) \Rightarrow \dot\neg(\dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| =$
$(rec((1+1)+1) * \underline{z}))n)n)n \gg \underline{x} <= ((\underline{(y2)} + (-u(rec((1+1)+1) * \underline{z})))\rceil, p_0, \mathsf{c})]$

[ClosetogreaterIsGreater $\overset{stmt}{\to}$ $SystemQ \vdash \forall\underline{x}: \forall\underline{(y1)}: \forall\underline{(y2)}: \forall\underline{z}: \underline{x} <=$
$((\underline{(y1)} + (-u\underline{z})) \vdash \dot\neg(|((\underline{(y1)} + (-u\underline{(y2)}))| <= (rec((1+1)+1) * \underline{z}) \Rightarrow \dot\neg(\dot\neg(|((\underline{(y1)} +$
$(-u\underline{(y2)}))| = (rec((1+1)+1) * \underline{z}))n)n)n \vdash \underline{x} <= ((\underline{(y2)} + (-u(rec((1+1)+1) * \underline{z})))$]

[ClosetogreaterIsGreater $\overset{tex}{\to}$ "ClosetogreaterIsGreater"]

[ClosetogreaterIsGreater $\stackrel{\text{pyk}}{\rightarrow}$ "lemma closetogreaterIsGreater"]

SubLessRight(F)

[SubLessRight(F) $\stackrel{\text{proof}}{\rightarrow}$ $\underline{\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash}$
$\forall \underline{(\text{fx})}\colon \forall \underline{(\text{fy})}\colon \forall \underline{(\text{fz})}\colon \forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n} \vdash$
$\dot{\neg}\,(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <= (\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \vdash$
Repetition $\rhd$ $\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n} \gg$
$\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n}$; Ded $\rhd$ $\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n} \gg$
$\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{n1})}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \overline{(\text{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n}$; A4 @ $(\text{rec}((1+1)+1) * \overline{(\epsilon)})$ $\rhd$ $\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{n1})}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\text{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\text{rec}((1+1)+1) * \overline{(\epsilon)}))\text{n})\text{n})\text{n} \Rightarrow \overline{(\text{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| <= (\text{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\underline{(\text{fy})}[\overline{\mathsf{m}}]))| = (\text{rec}((1+1)+1) * \overline{(\epsilon)}))\text{n})\text{n})\text{n})\text{n})\text{n}$; Repetition $\rhd$
$\dot{\neg}\,(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <= (\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \gg$
$\dot{\neg}\,(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <= (\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}$; Ded $\rhd$
$\dot{\neg}\,(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <= (\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \gg$
$\dot{\neg}\,(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(\text{n2})}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{(\text{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}$; $\forall \underline{(\text{fx})}\colon \forall \underline{(\text{fx})}\colon \forall \underline{(\text{fz})}\colon \forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{(\text{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <= (\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n} \vdash$
A4 @ $\overline{\mathsf{m}}$ $\rhd$ $\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{(\text{n2})} <= \overline{\mathsf{m}} \Rightarrow$

$\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveDividedBy3} \rhd \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \vdash \mathrm{MaxLeq}(1) \gg$
$\overline{(\mathrm{n1})} <= \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}); \mathrm{leqTransitivity} \rhd \overline{(\mathrm{n1})} <= \mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) \rhd \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \gg \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}}; \mathrm{MaxLeq}(2) \gg \overline{(\mathrm{n2})} <= \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}); \mathrm{leqTransitivity} \rhd$
$\overline{(\mathrm{n2})} <= \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) \rhd \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \gg$
$\overline{(\mathrm{n2})} <= \overline{\mathsf{m}}; \mathrm{A4} @ \overline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \gg \overline{(\mathrm{n2})} <=$
$\overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})); \mathrm{MP} \rhd \overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})) \rhd \overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \gg \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})); \mathrm{A4} @ \overline{\mathsf{m}} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveDividedBy3} \rhd \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP2} \rhd \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \gg \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$

$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ClosetogreaterIsGreater} \rhd \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})) \rhd \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)})))); \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}: \forall\underline{(\mathrm{fx})}: \forall\underline{(\mathrm{fz})}: \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{TwiceExistMP} \rhd$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{n2})}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \forall\underline{(\mathrm{fx})}: \forall\underline{(\mathrm{fy})}: \forall\underline{(\mathrm{fz})}: \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \vdash \mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \vdash \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)})))) \gg$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{(\mathrm{n1})} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)})))); \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)})))) \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{n1})}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{(\mathrm{n1})} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = (\mathrm{rec}((1+1)+1)*\overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*\overline{(\epsilon)})))); \mathrm{TwiceExistMP} \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n} \Rightarrow$
$\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+$
$1) * \overline{(\epsilon)})))) \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{n2})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{n2})} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+$
$1) * \overline{(\epsilon)}))))$; JoinConjuncts $\rhd \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))) \gg \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <=$
$\overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n}$; Gen $\rhd \dot{\neg}(\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n}$; IntroExist @ $\mathrm{if}(\overline{(\mathrm{n2})} <=$
$\overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) \rhd \forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+$
$1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathrm{if}(\overline{(\mathrm{n2})} <= \overline{(\mathrm{n1})}, \overline{(\mathrm{n1})}, \overline{(\mathrm{n2})}) <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; IntroExist @$(\mathrm{rec}((1+$
$1)+1) * \overline{(\epsilon)}) \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{rec}((1+1)+1) * \overline{(\epsilon)})))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$; Repetition $\rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[SubLessRight(F) $\overset{\mathrm{stmt}}{\to}$ SystemQ $\vdash$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[SubLessRight(F) $\overset{\mathrm{tex}}{\to}$ "SubLessRight(F)"]

[SubLessRight(F) $\stackrel{\text{pyk}}{\to}$ "lemma subLessRight(F)"]

SubLessRight(R)

[SubLessRight(R) $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\text{fx})}\colon \forall \underline{(\text{fy})}\colon \forall \underline{(\text{fz})}\colon \{\text{ph} \in \text{P}(\{\text{ph} \in$
$\text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in$
$\text{N} \Rightarrow \dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} =$
$\{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{r1})}\colon \overline{(\text{r1})} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in \text{N} \Rightarrow \dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot\neg(\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{f1})}\colon \forall_{\text{obj}} \overline{(\text{f2})}\colon \forall_{\text{obj}} \overline{(\text{f3})}\colon \forall_{\text{obj}} \overline{(\text{f4})}\colon \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})})\text{n})\text{n} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{s1})}\colon \overline{(\text{s1})} \in \text{N} \Rightarrow \dot\neg(\forall_{\text{obj}} \overline{(\text{s2})}\colon \dot\neg(\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}} \overline{(\epsilon)}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n}\} = \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in$
$\text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in \text{N} \Rightarrow$
$\dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{r1})}\colon \overline{(\text{r1})} \in \mathsf{f}_{\text{Ph}} \Rightarrow \dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in \text{N} \Rightarrow$
$\dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{f1})}\colon \forall_{\text{obj}} \overline{(\text{f2})}\colon \forall_{\text{obj}} \overline{(\text{f3})}\colon \forall_{\text{obj}} \overline{(\text{f4})}\colon \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})})\text{n})\text{n} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{s1})}\colon \overline{(\text{s1})} \in \text{N} \Rightarrow \dot\neg(\forall_{\text{obj}} \overline{(\text{s2})}\colon \dot\neg(\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}} \overline{(\epsilon)}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\text{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(\text{fy})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n}\} \vdash \dot\neg(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\text{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \vdash \text{From} == \rhd \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in$
$\text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in \text{N} \Rightarrow$
$\dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{r1})}\colon \overline{(\text{r1})} \in \mathsf{f}_{\text{Ph}} \Rightarrow \dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in \text{N} \Rightarrow$
$\dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{f1})}\colon \forall_{\text{obj}} \overline{(\text{f2})}\colon \forall_{\text{obj}} \overline{(\text{f3})}\colon \forall_{\text{obj}} \overline{(\text{f4})}\colon \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})})\text{n})\text{n} \Rightarrow$
$\dot\neg(\forall_{\text{obj}} \overline{(\text{s1})}\colon \overline{(\text{s1})} \in \text{N} \Rightarrow \dot\neg(\forall_{\text{obj}} \overline{(\text{s2})}\colon \dot\neg(\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}} \overline{(\epsilon)}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot\neg(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\text{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\text{n})\text{n})\text{n})\text{n})\text{n}\} = \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in$
$\text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}} \overline{(\text{op1})}\colon \dot\neg(\dot\neg(\forall_{\text{obj}} \overline{(\text{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\text{op1})} \in \text{N} \Rightarrow$
$\dot\neg(\overline{(\text{op2})} \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid$

$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubLessRight(F)} \rhd \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SubLessRight(R)} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fz})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[SubLessRight(R) $\stackrel{\text{tex}}{\to}$ "SubLessRight(R)"]

[SubLessRight(R) $\stackrel{\text{pyk}}{\to}$ "lemma subLessRight(R)"]

plus0Left

[plus0Left $\stackrel{\text{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \mathrm{plus0} \gg (\underline{\mathsf{x}} + 0) = \underline{\mathsf{x}}; \mathrm{plusCommutativity} \gg (0 + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + 0); \mathrm{eqTransitivity} \rhd (0 + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + 0) \rhd (\underline{\mathsf{x}} + 0) = \underline{\mathsf{x}} \gg (0 + \underline{\mathsf{x}}) = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})$]

[plus0Left $\stackrel{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon (0 + \underline{\mathsf{x}}) = \underline{\mathsf{x}}$]

[plus0Left $\stackrel{\text{tex}}{\to}$ "plus0Left"]

[plus0Left $\stackrel{\text{pyk}}{\to}$ "lemma plus0Left"]

times1Left

[times1Left $\stackrel{\text{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \mathrm{times1} \gg (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}}; \mathrm{timesCommutativity} \gg (1 * \underline{\mathsf{x}}) = (\underline{\mathsf{x}} * 1); \mathrm{eqTransitivity} \rhd (1 * \underline{\mathsf{x}}) = (\underline{\mathsf{x}} * 1) \rhd (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}} \gg (1 * \underline{\mathsf{x}}) = \underline{\mathsf{x}}\rceil, \mathrm{p}_0, \mathsf{c})$]

[times1Left $\stackrel{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon (1 * \underline{\mathsf{x}}) = \underline{\mathsf{x}}$]

[times1Left $\stackrel{\text{tex}}{\to}$ "times1Left"]

[times1Left $\stackrel{\text{pyk}}{\to}$ "lemma times1Left"]

EqAdditionLeft

[EqAdditionLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \text{eqAddition} \rhd \underline{x} = \underline{y} \gg (\underline{x}+\underline{z}) = (\underline{y}+\underline{z}); \text{plusCommutativity} \gg (\underline{z}+\underline{x}) = (\underline{x}+\underline{z}); \text{plusCommutativity} \gg (\underline{y}+\underline{z}) = (\underline{z}+\underline{y}); \text{eqTransitivity4} \rhd (\underline{z}+\underline{x}) = (\underline{x}+\underline{z}) \rhd (\underline{x}+\underline{z}) = (\underline{y}+\underline{z}) \rhd (\underline{y}+\underline{z}) = (\underline{z}+\underline{y}) \gg (\underline{z}+\underline{x}) = (\underline{z}+\underline{y})\rceil, p_0, c)$]

[EqAdditionLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash (\underline{z}+\underline{x}) = (\underline{z}+\underline{y})$]

[EqAdditionLeft $\stackrel{\text{tex}}{\rightarrow}$ “EqAdditionLeft”]

[EqAdditionLeft $\stackrel{\text{pyk}}{\rightarrow}$ “lemma eqAdditionLeft”]

EqMultiplicationLeft

[EqMultiplicationLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \text{eqMultiplication} \rhd \underline{x} = \underline{y} \gg (\underline{x}*\underline{z}) = (\underline{y}*\underline{z}); \text{timesCommutativity} \gg (\underline{z}*\underline{x}) = (\underline{x}*\underline{z}); \text{timesCommutativity} \gg (\underline{y}*\underline{z}) = (\underline{z}*\underline{y}); \text{eqTransitivity4} \rhd (\underline{z}*\underline{x}) = (\underline{x}*\underline{z}) \rhd (\underline{x}*\underline{z}) = (\underline{y}*\underline{z}) \rhd (\underline{y}*\underline{z}) = (\underline{z}*\underline{y}) \gg (\underline{z}*\underline{x}) = (\underline{z}*\underline{y})\rceil, p_0, c)$]

[EqMultiplicationLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash (\underline{z}*\underline{x}) = (\underline{z}*\underline{y})$]

[EqMultiplicationLeft $\stackrel{\text{tex}}{\rightarrow}$ “EqMultiplicationLeft”]

[EqMultiplicationLeft $\stackrel{\text{pyk}}{\rightarrow}$ “lemma eqMultiplicationLeft”]

PlusF(Sym)

[PlusF(Sym) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{m}\colon \forall \overline{(fx)}\colon \forall \overline{(fy)}\colon \text{PlusF} \gg \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{m}\colon \dot{\neg}(d_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}])\}\})n)n\}[\underline{m}] = (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}]); \text{eqSymmetry} \rhd \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{m}\colon \dot{\neg}(d_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}])\}\})n)n\}[\underline{m}] = (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}]) \gg (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}]) = \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj} \underline{m}\colon \dot{\neg}(d_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}])\}\})n)n\}[\underline{m}]\rceil, p_0, c)$]

[PlusF(Sym) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{m}\colon \forall \overline{(fx)}\colon \forall \overline{(fy)}\colon (\overline{(fx)}[\underline{m}] + \overline{(fy)}[\underline{m}]) = \{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]]$

[PlusF(Sym) $\stackrel{\mathrm{tex}}{\to}$ "PlusF(Sym)"]

[PlusF(Sym) $\stackrel{\mathrm{pyk}}{\to}$ "lemma plusF(Sym)"]

TimesF(Sym)

[TimesF(Sym) $\stackrel{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{TimesF} \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]); \mathrm{eqSymmetry} \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) \gg (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\rceil, \mathrm{p}_0, \mathsf{c})$]

[TimesF(Sym) $\stackrel{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}]) = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]$]

[TimesF(Sym) $\stackrel{\mathrm{tex}}{\to}$ "TimesF(Sym)"]

[TimesF(Sym) $\stackrel{\mathrm{pyk}}{\to}$ "lemma timesF(Sym)"]

SameSeries(Gen)

[SameSeries(Gen) $\stackrel{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{sz})}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \underline{\mathsf{n}} \in \mathrm{N} \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sz})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\underline{(fx)})n)n)n \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n \vdash \underline{\mathsf{m}} = \underline{\mathsf{n}} \vdash \underline{(fx)} = \underline{(fy)} \vdash \mathrm{MemberOfSeries} \rhd \underline{\mathsf{m}} \in$
$\mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)}; \mathrm{SENC1} \rhd \underline{(fx)} =$
$\underline{(fy)} \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)} \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in$
$\underline{(fy)}; \mathrm{MemberOfSeries} \rhd \underline{\mathsf{n}} \in \mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n \gg \{\{\underline{\mathsf{n}}, \underline{\mathsf{n}}\}, \{\underline{\mathsf{n}}, \underline{(fy)}[\underline{\mathsf{n}}]\}\} \in$
$\underline{(fy)}; \mathrm{UniqueMember} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fy)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fy)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fy)})n)n)n \rhd \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fy)} \rhd \{\{\underline{\mathsf{n}}, \underline{\mathsf{n}}\}, \{\underline{\mathsf{n}}, \underline{(fy)}[\underline{\mathsf{n}}]\}\} \in$
$\underline{(fy)} \rhd \underline{\mathsf{m}} = \underline{\mathsf{n}} \gg \underline{(fx)}[\underline{\mathsf{m}}] = \underline{(fy)}[\underline{\mathsf{n}}]\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SameSeries(Gen)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(sz)}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \underline{\mathsf{n}} \in \mathrm{N} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fy)} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\underline{(\mathrm{sz})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\underline{(\mathrm{fy})}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\underline{(\mathrm{fy})}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\vdash\underline{\mathsf{m}}=\underline{\mathsf{n}}\vdash\underline{(\mathrm{fx})}=\underline{(\mathrm{fy})}\vdash\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]=\underline{(\mathrm{fy})}[\underline{\mathsf{n}}]]$

[SameSeries(Gen) $\overset{\mathrm{tex}}{\to}$ "SameSeries(Gen)"]

[SameSeries(Gen) $\overset{\mathrm{pyk}}{\to}$ "lemma sameSeries(Gen)"]

EqualsSameF

[EqualsSameF $\overset{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ}\vdash\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\underline{(\mathrm{fx})}=\underline{(\mathrm{fy})}\vdash\dot\neg(0<=$
$\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\vdash 0<=\overline{\mathsf{m}}\vdash\mathrm{NatType}\gg\overline{\mathsf{m}}\in\mathrm{N};\mathrm{SeriesType}\gg$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\underline{(\mathrm{fx})}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\underline{(\mathrm{fx})}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\underline{(\mathrm{fx})}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};\mathrm{SeriesType}\gg\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\underline{(\mathrm{fy})}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\underline{(\mathrm{fy})}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\underline{(\mathrm{fy})}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};\mathrm{eqReflexivity}\gg\overline{\mathsf{m}}=\overline{\mathsf{m}};\mathrm{SameSeries(Gen)}\rhd\overline{\mathsf{m}}\in\mathrm{N}\rhd\overline{\mathsf{m}}\in\mathrm{N}\rhd$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\underline{(\mathrm{fx})}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\underline{(\mathrm{fx})}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\underline{(\mathrm{fx})}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rhd\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\underline{(\mathrm{fy})}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\underline{(\mathrm{fy})}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\underline{(\mathrm{fy})}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\underline{(\mathrm{fy})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rhd\overline{\mathsf{m}}=\overline{\mathsf{m}}\rhd\underline{(\mathrm{fx})}=\underline{(\mathrm{fy})}\gg\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]=$
$\underline{(\mathrm{fy})}[\overline{\mathsf{m}}];\mathrm{PositiveToLeft(Eq)(1term)}\rhd\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]=\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]\gg$

$((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}])) = 0; \mathrm{SameNumerical} \rhd ((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}])) = 0 \gg$
$|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = |0|; |0| = 0 \gg |0| =$
$0; \mathrm{eqTransitivity} \rhd |((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = |0| \rhd |0| = 0 \gg$
$|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = 0; \mathrm{eqSymmetry} \rhd |((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = 0 \gg 0 =$
$|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))|; \mathrm{SubLessLeft} \rhd 0 = |((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| \rhd \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \mathrm{Ded} \rhd \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \underline{(\mathrm{fx})} =$
$\underline{(\mathrm{fy})} \vdash \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \vdash 0 <= \overline{\mathsf{m}} \vdash$
$\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow 0 <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \vdash \mathrm{MP} \rhd \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \Rightarrow \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow 0 <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \gg \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow 0 <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow 0 <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow 0 <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} @ 0 \rhd \forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow 0 <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{EqualsSameF} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \vdash$
$\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[EqualsSameF $\overset{\text{tex}}{\rightarrow}$ "EqualsSameF"]

[EqualsSameF $\overset{\text{pyk}}{\rightarrow}$ "lemma equalsSameF"]

LeqReflexivity(R)

[LeqReflexivity(R) $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \overline{(\mathrm{fx})} \colon \mathrm{eqReflexivity} \gg \{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{lemma\ eqLeq(R)} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}], p_0, \mathsf{c})]$

$[\mathrm{LeqReflexivity}(\underline{R}) \overset{stmt}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\overline{(fx)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[LeqReflexivity(R) $\overset{\mathrm{tex}}{\to}$ “LeqReflexivity(R)”]

[LeqReflexivity(R) $\overset{\mathrm{pyk}}{\to}$ “lemma leqReflexivity(R)”]

Tester1

[Tester1 $\overset{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \mathrm{To} = \mathrm{f} \rhd \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \gg \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})}\rceil, \mathrm{p}_0, \mathsf{c})$]

[Tester1 $\overset{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] = \underline{(\mathrm{fy})}[\underline{\mathsf{m}}] \vdash \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})}$]

[Tester1 $\overset{\mathrm{tex}}{\to}$ “Tester1”]

[Tester1 $\overset{\mathrm{pyk}}{\to}$ “tester1”]

Tester2

[Tester2 $\overset{\mathrm{tex}}{\to}$ “Tester2”]

[Tester2 $\overset{\mathrm{pyk}}{\to}$ “tester2”]

Tester3

[Tester3 $\overset{\mathrm{tex}}{\to}$ “Tester3”]

[Tester3 $\overset{\mathrm{pyk}}{\to}$ “tester3”]

Tester4

[Tester4 $\stackrel{\text{tex}}{\to}$ “Tester4”]

[Tester4 $\stackrel{\text{pyk}}{\to}$ “tester4”]

Tester5

[Tester5 $\stackrel{\text{tex}}{\to}$ “Tester5”]

[Tester5 $\stackrel{\text{pyk}}{\to}$ “tester5”]

Tester6

[Tester6 $\stackrel{\text{tex}}{\to}$ “Tester6”]

[Tester6 $\stackrel{\text{pyk}}{\to}$ “tester6”]

The pyk compiler, version 0.grue.20060417+ by Klaus Grue
GRD-2006-12-29.UTC:09:42:35.018035 = MJD-54098.TAI:09:43:08.018035 = LGT-467410218801 8035e-6