

[ExistMP2 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(v1): \forall(v2): \forall a: \forall b: \forall c: a \Rightarrow b \Rightarrow c \vdash \neg(\forall_{\text{obj}}(v1): \neg(a)n \vdash \neg(\forall_{\text{obj}}(v2): \neg(b)n) \vdash \text{ExistMP} \triangleright a \Rightarrow b \Rightarrow c \triangleright \neg(\forall_{\text{obj}}(\underline{v1}): \neg(a)n \gg \underline{b} \Rightarrow \underline{c}; \text{ExistMP} \triangleright \underline{b} \Rightarrow \underline{c} \triangleright \neg(\forall_{\text{obj}}(\underline{v2}): \neg(b)n) \gg \underline{c}]$, p_0, c)]

[TwiceExistMP $\xrightarrow{\text{stmt}}$ $\text{SystemQ} \vdash \forall(v1): \forall(v2): \forall a: \forall b: a \Rightarrow b \vdash \neg(\forall_{\text{obj}}(v1): \neg(\neg(\forall_{\text{obj}}(v2): \neg(a)n)n) \vdash \underline{b}]$

[TwiceExistMP $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(v2): \forall a: \forall b: a \Rightarrow b \vdash \neg(\forall_{\text{obj}}(v2): \neg(a)n \vdash \text{ExistMP} \triangleright a \Rightarrow b \triangleright \neg(\forall_{\text{obj}}(v2): \neg(a)n \gg \underline{b}; \forall(v1): \forall(v2): \forall a: \forall b: \text{Ded} \triangleright \forall(v2): \forall a: \forall b: a \Rightarrow b \vdash \neg(\forall_{\text{obj}}(\underline{v2}): \neg(a)n \vdash \underline{b} \gg a \Rightarrow \underline{b} \Rightarrow \neg(\forall_{\text{obj}}(\underline{v2}): \neg(a)n \Rightarrow \underline{b}; a \Rightarrow \underline{b} \vdash \neg(\forall_{\text{obj}}(v1): \neg(\neg(\forall_{\text{obj}}(v2): \neg(a)n)n) \vdash \text{MP} \triangleright a \Rightarrow b \Rightarrow \neg(\forall_{\text{obj}}(\underline{v2}): \neg(a)n \Rightarrow \underline{b} \triangleright a \Rightarrow \underline{b} \gg \neg(\forall_{\text{obj}}(v2): \neg(a)n \Rightarrow \underline{b}; \text{ExistMP} \triangleright \neg(\forall_{\text{obj}}(\underline{v2}): \neg(a)n \Rightarrow \underline{b} \triangleright \neg(\forall_{\text{obj}}(v1): \neg(\neg(\forall_{\text{obj}}(\underline{v2}): \neg(a)n)n) \gg \underline{b}]$, p_0, c)]

[TwiceExistMP2 $\xrightarrow{\text{stmt}}$ $\text{SystemQ} \vdash \forall(v1): \forall(v2): \forall(v3): \forall(v4): \forall a: \forall b: \forall c: a \Rightarrow b \Rightarrow c \vdash \neg(\forall_{\text{obj}}(v1): \neg(\neg(\forall_{\text{obj}}(v2): \neg(a)n)n) \vdash \neg(\forall_{\text{obj}}(v3): \neg(\neg(\forall_{\text{obj}}(v4): \neg(b)n)n) \vdash \underline{c}]$

[TwiceExistMP2 $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(v1): \forall(v2): \forall(v3): \forall(v4): \forall a: \forall b: \forall c: a \Rightarrow b \Rightarrow c \vdash \neg(\forall_{\text{obj}}(\underline{v1}): \neg(\neg(\forall_{\text{obj}}(\underline{v2}): \neg(a)n)n) \vdash \neg(\forall_{\text{obj}}(\underline{v3}): \neg(\neg(\forall_{\text{obj}}(\underline{v4}): \neg(b)n)n) \vdash \text{TwiceExistMP} \triangleright a \Rightarrow b \Rightarrow c \triangleright \neg(\forall_{\text{obj}}(v1): \neg(\neg(\forall_{\text{obj}}(v2): \neg(a)n)n) \gg \underline{b} \Rightarrow \underline{c}; \text{TwiceExistMP} \triangleright \underline{b} \Rightarrow \underline{c} \triangleright \neg(\forall_{\text{obj}}(\underline{v3}): \neg(\neg(\forall_{\text{obj}}(\underline{v4}): \neg(b)n)n) \gg \underline{c}]$, p_0, c)]

[NeqSymmetry $\xrightarrow{\text{stmt}}$ $\text{SystemQ} \vdash \forall x: \forall y: \neg(x = y) \vdash \neg(y = x)n]$

[NeqSymmetry $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: y = x \vdash \text{eqSymmetry} \triangleright y = x \gg x = y; \forall x: \forall y: \text{Ded} \triangleright \forall x: \forall y: y = x \vdash x = y \gg y = x \Rightarrow x = y; \neg(x = y) \vdash \text{MT} \triangleright y = x \Rightarrow x = y \triangleright \neg(x = y)n \gg \neg(y = x)n \rceil$, p_0, c)]

[SubNeqLeft $\xrightarrow{\text{stmt}}$ $\text{SystemQ} \vdash \forall x: \forall y: \forall z: x = y \vdash \neg(x = z) \vdash \neg(y = z)n]$

[SubNeqLeft $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: \forall z: x = y \vdash \neg(x = z) \vdash \text{EqualityAxiom} \gg y = x \Rightarrow y = z \Rightarrow x = z; \text{eqSymmetry} \triangleright x = y \gg y = x; \text{MP} \triangleright y = x \Rightarrow y = z \Rightarrow x = z \triangleright y = x \gg y = z \Rightarrow x = z; \text{Contrapositive} \triangleright y = z \Rightarrow x = z \gg \neg(x = z) \Rightarrow \neg(y = z)n; \text{MP} \triangleright \neg(x = z) \Rightarrow \neg(y = z) \triangleright \neg(x = z)n \gg \neg(y = z)n \rceil$, p_0, c)]

[InPair(1) $\xrightarrow{\text{stmt}}$ $\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): (\underline{sx}) \in \{(\underline{sx}), (\underline{sy})\}$

[InPair(1) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \text{eqReflexivity} \gg (\underline{sx}) = (\underline{sx}); \text{WeakenOr2} \triangleright (\underline{sx}) = (\underline{sx}) \gg \neg((\underline{sx}) = (\underline{sx})n \Rightarrow (\underline{sx}) = (\underline{sy}); \text{Formula2Pair} \triangleright \neg((\underline{sx}) = (\underline{sx})n \Rightarrow (\underline{sx}) = (\underline{sy}) \gg (\underline{sx}) \in \{(\underline{sx}), (\underline{sy})\}$, p_0, c)]

[InPair(2) $\xrightarrow{\text{stmt}}$ $\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): (\underline{sy}) \in \{(\underline{sx}), (\underline{sy})\}$

[InPair(2) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \text{eqReflexivity} \gg (\underline{sy}) = (\underline{sy}); \text{WeakenOr1} \triangleright (\underline{sy}) = (\underline{sy}) \gg \neg((\underline{sy}) = (\underline{sx})n \Rightarrow (\underline{sy}) = (\underline{sy}); \text{Formula2Pair} \triangleright \neg((\underline{sy}) = (\underline{sx})n \Rightarrow (\underline{sy}) = (\underline{sy}) \gg (\underline{sy}) \in \{(\underline{sx}), (\underline{sy})\}$, p_0, c)]

$\{(\underline{sx}), (\underline{sy})\}, p_0, c]$

$[\text{FromSingleton} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\} \vdash \underline{(sx)} = \underline{(sy)}]$

$[\text{FromSingleton} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\} \vdash \text{Repetition} \triangleright \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\} \gg \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\}; \text{Pair2Formula} \triangleright \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\} \gg \neg(\underline{(sx)} = \underline{(sy)})n \Rightarrow \underline{(sx)} = \underline{(sy)}; \text{RemoveOr} \triangleright \neg(\underline{(sx)} = \underline{(sy)})n \Rightarrow \underline{(sx)} = \underline{(sy)} \gg \underline{(sx)} = \underline{(sy)}], p_0, c)]$

$[\text{ToSingleton} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \underline{(sx)} = \underline{(sy)} \vdash \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\}]$

$[\text{ToSingleton} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \underline{(sx)} = \underline{(sy)} \vdash \text{WeakenOr1} \triangleright \underline{(sx)} = \underline{(sy)} \gg \neg(\underline{(sx)} = \underline{(sy)})n \Rightarrow \underline{(sx)} = \underline{(sy)}; \text{Formula2Pair} \triangleright \neg(\underline{(sx)} = \underline{(sy)})n \Rightarrow \underline{(sx)} = \underline{(sy)} \gg \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\}; \text{Repetition} \triangleright \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\} \gg \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\}], p_0, c)]$

$[\text{FromSameSingleton} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \{(\underline{sx}), (\underline{sx})\} = \{(\underline{sy}), (\underline{sy})\} \vdash \underline{(sx)} = \underline{(sy)}]$

$[\text{FromSameSingleton} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \{(\underline{sx}), (\underline{sx})\} = \{(\underline{sy}), (\underline{sy})\} \vdash \text{eqReflexivity} \gg \underline{(sx)} = \underline{(sx)}; \text{ToSingleton} \triangleright \underline{(sx)} = \underline{(sx)} \gg \underline{(sx)} \in \{(\underline{sx}), (\underline{sx})\}; \text{SENC1} \triangleright \{(\underline{sx}), (\underline{sx})\} = \{(\underline{sy}), (\underline{sy})\} \triangleright \underline{(sx)} \in \{(\underline{sx}), (\underline{sx})\} \gg \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\}; \text{FromSingleton} \triangleright \underline{(sx)} \in \{(\underline{sy}), (\underline{sy})\} \gg \underline{(sx)} = \underline{(sy)}], p_0, c)]$

$[\text{SingletonmembersEqual} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \forall(\underline{sz}): \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \vdash \underline{(sx)} = \underline{(sy)}]$

$[\text{SingletonmembersEqual} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \forall(\underline{sz}): \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \vdash \text{InPair}(1) \gg \underline{(sx)} \in \{(\underline{sx}), (\underline{sy})\}; \text{SENC1} \triangleright \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \triangleright \underline{(sx)} \in \{(\underline{sx}), (\underline{sy})\} \gg \underline{(sx)} \in \{(\underline{sz}), (\underline{sz})\}; \text{FromSingleton} \triangleright \underline{(sx)} \in \{(\underline{sz}), (\underline{sz})\} \gg \underline{(sx)} = \underline{(sz)}; \text{InPair}(2) \gg \underline{(sy)} \in \{(\underline{sx}), (\underline{sy})\}; \text{SENC1} \triangleright \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \triangleright \underline{(sy)} \in \{(\underline{sx}), (\underline{sy})\} \gg \underline{(sy)} \in \{(\underline{sz}), (\underline{sz})\}; \text{FromSingleton} \triangleright \underline{(sy)} \in \{(\underline{sz}), (\underline{sz})\} \gg \underline{(sy)} = \underline{(sz)}; \text{eqSymmetry} \triangleright \underline{(sy)} = \underline{(sz)} \gg \underline{(sz)} = \underline{(sy)}; \text{eqTransitivity} \triangleright \underline{(sx)} = \underline{(sz)} \triangleright \underline{(sz)} = \underline{(sy)} \gg \underline{(sx)} = \underline{(sy)}], p_0, c)]$

$[\text{UnequalsNotInSingleton} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \forall(\underline{sz}): \neg(\underline{(sx)} = \underline{(sy)})n \vdash \neg(\{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\})n]$

$[\text{UnequalsNotInSingleton} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \forall(\underline{sz}): \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \vdash \text{SingletonmembersEqual} \triangleright \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \gg \underline{(sx)} = \underline{(sy)}; \forall(\underline{sx}): \forall(\underline{sy}): \forall(\underline{sz}): \text{Ded} \triangleright \forall(\underline{sx}): \forall(\underline{sy}): \forall(\underline{sz}): \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \vdash \underline{(sx)} = \underline{(sy)} \gg \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \Rightarrow \underline{(sx)} = \underline{(sy)}; \neg(\underline{(sx)} = \underline{(sy)})n \vdash \text{MT} \triangleright \{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\} \Rightarrow \underline{(sx)} = \underline{(sy)} \triangleright \neg(\underline{(sx)} = \underline{(sy)})n \gg \neg(\{(\underline{sx}), (\underline{sy})\} = \{(\underline{sz}), (\underline{sz})\})n], p_0, c)]$

$[\text{NonsingletonmembersUnequal} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \neg(\{(\underline{sx}), (\underline{sy})\} = \{(\underline{sx}), (\underline{sx})\})n \vdash \neg(\underline{(sx)} = \underline{(sy)})n]$

$[\text{NonsingletonmembersUnequal} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall(\underline{sx}): \forall(\underline{sy}): \underline{(sx)} = \underline{(sy)} \vdash \text{eqReflexivity} \gg \underline{(sx)} = \underline{(sx)}; \text{SamePair} \triangleright \underline{(sx)} = \underline{(sx)} \triangleright \underline{(sx)} = \underline{(sy)} \gg \{(\underline{sx}), (\underline{sx})\} = \{(\underline{sx}), (\underline{sy})\}; \text{Repetition} \triangleright \{(\underline{sx}), (\underline{sx})\} = \{(\underline{sx}), (\underline{sy})\} \gg$

$$\begin{aligned} & \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \Rightarrow \dot{\neg} (\underline{(sx)} = \\ & \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \underline{(sy1)})n)n \gg \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \\ & \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \Rightarrow \dot{\neg} (\underline{(sx)} = \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \\ & \underline{(sy1)})n)n; \text{MP} \triangleright \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \Rightarrow \\ & \dot{\neg} (\underline{(sx)} = \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \underline{(sy1)})n)n \triangleright \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \\ & \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \gg \dot{\neg} (\underline{(sx)} = \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \underline{(sy1)})n)n], p_0, c) \end{aligned}$$

[FromOrderedPair(1) $\xrightarrow{\text{stmt}}$ SystemQ \vdash

$$\forall (sx): \forall (sx1): \forall (sy): \forall (sy1): \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \vdash \underline{(sx)} = \underline{(sx1)}$$

[FromOrderedPair(1) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash$

$$\begin{aligned} & \forall (sx): \forall (sx1): \forall (sy): \forall (sy1): \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \\ & \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \vdash \text{FromOrderedPair} \triangleright \\ & \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \gg \dot{\neg} (\underline{(sx)} = \\ & \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \underline{(sy1)})n)n; \text{FirstConjunct} \triangleright \dot{\neg} (\underline{(sx)} = \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \\ & \underline{(sy1)})n)n \gg \underline{(sx)} = \underline{(sx1)}], p_0, c) \end{aligned}$$

[FromOrderedPair(2) $\xrightarrow{\text{stmt}}$ SystemQ \vdash

$$\forall (sx): \forall (sx1): \forall (sy): \forall (sy1): \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \vdash \underline{(sy)} = \underline{(sy1)}$$

[FromOrderedPair(2) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash$

$$\begin{aligned} & \forall (sx): \forall (sx1): \forall (sy): \forall (sy1): \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \\ & \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \vdash \text{FromOrderedPair} \triangleright \\ & \{ \{ \underline{(sx)}, \underline{(sx)} \}, \{ \underline{(sx)}, \underline{(sy)} \} \} = \{ \{ \underline{(sx1)}, \underline{(sx1)} \}, \{ \underline{(sx1)}, \underline{(sy1)} \} \} \gg \dot{\neg} (\underline{(sx)} = \\ & \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \underline{(sy1)})n)n; \text{SecondConjunct} \triangleright \dot{\neg} (\underline{(sx)} = \underline{(sx1)} \Rightarrow \dot{\neg} (\underline{(sy)} = \\ & \underline{(sy1)})n)n \gg \underline{(sy)} = \underline{(sy1)}], p_0, c) \end{aligned}$$

[SameMember(2) $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall (sx): \forall (sy): \forall (sz): \underline{(sx)} = \underline{(sy)} \vdash \underline{(sy)} \in \underline{(sz)} \vdash \underline{(sx)} \in \underline{(sz)}$]

[SameMember(2) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall (sx): \forall (sy): \forall (sz): \underline{(sx)} = \underline{(sy)} \vdash \underline{(sy)} \in \underline{(sz)} \vdash \text{eqSymmetry} \triangleright \underline{(sx)} = \underline{(sy)} \gg \underline{(sy)} = \underline{(sx)}; \text{SameMember} \triangleright \underline{(sy)} = \underline{(sx)} \triangleright \underline{(sy)} \in \underline{(sz)} \gg \underline{(sx)} \in \underline{(sz)} \urcorner, p_0, c)$]

[ToBinaryUnion(1) $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall (sx): \forall (sy): \forall (sz): \forall (su): \underline{(sx)} \in \underline{(sy)} \vdash \underline{(sx)} \in \text{Union}(\{ \underline{(sy)}, \underline{(sz)} \})$]

[ToBinaryUnion(1) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall (sx): \forall (sy): \forall (sz): \forall (su): \underline{(sx)} \in \underline{(sy)} \vdash \text{InPair}(1) \gg \underline{(sy)} \in \{ \underline{(sy)}, \underline{(sz)} \}; \text{JoinConjuncts} \triangleright \underline{(sx)} \in \underline{(sy)} \triangleright \underline{(sy)} \in \{ \underline{(sy)}, \underline{(sz)} \} \gg \dot{\neg} (\underline{(sx)} \in \underline{(sy)} \Rightarrow \dot{\neg} (\underline{(sy)} \in \{ \underline{(sy)}, \underline{(sz)} \})n)n; \text{IntroExist} @ (\underline{(sy)} \triangleright \dot{\neg} (\underline{(sx)} \in \underline{(sy)} \Rightarrow \dot{\neg} (\underline{(sy)} \in \{ \underline{(sy)}, \underline{(sz)} \})n)n \gg \dot{\neg} (\forall_{\text{obj}} (su): \dot{\neg} (\dot{\neg} (\underline{(sx)} \in \underline{(su)}) \Rightarrow \dot{\neg} (\underline{(su)} \in \{ \underline{(sy)}, \underline{(sz)} \})n)n)n); \text{Formula2Union} \triangleright \dot{\neg} (\forall_{\text{obj}} (su): \dot{\neg} (\dot{\neg} (\underline{(sx)} \in \underline{(su)}) \Rightarrow \dot{\neg} (\underline{(su)} \in \{ \underline{(sy)}, \underline{(sz)} \})n)n)n) \gg \underline{(sx)} \in \text{Union}(\{ \underline{(sy)}, \underline{(sz)} \}); \text{Repetition} \triangleright \underline{(sx)} \in \text{Union}(\{ \underline{(sy)}, \underline{(sz)} \}) \gg \underline{(sx)} \in \text{Union}(\{ \underline{(sy)}, \underline{(sz)} \}) \urcorner, p_0, c)$]

[ToBinaryUnion(2) $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall (sx): \forall (sy): \forall (sz): \forall (su): \underline{(sx)} \in \underline{(sz)} \vdash \underline{(sx)} \in \text{Union}(\{ \underline{(sy)}, \underline{(sz)} \})$]

[ToBinaryUnion(2) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall (sx): \forall (sy): \forall (sz): \forall (su): \underline{(sx)} \in$

$$((\underline{x} * \underline{y}) + (\underline{x} * \underline{z})) \gg ((\underline{x} * \underline{y}) + (\underline{x} * \underline{z})) = (\underline{x} * (\underline{y} + \underline{z})), p_0, c]$$

$$[\text{Three2twoTerms} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{y} + \underline{z}) = \underline{u} \vdash ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + \underline{u})]$$

$$[\text{Three2twoTerms} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{y} + \underline{z}) = \underline{u} \vdash \text{lemma eqAdditionLeft} \triangleright (\underline{y} + \underline{z}) = \underline{u} \gg (\underline{x} + (\underline{y} + \underline{z})) = (\underline{x} + \underline{u}); \text{plusAssociativity} \gg ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z})); \text{eqTransitivity} \triangleright ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z})) \triangleright (\underline{x} + (\underline{y} + \underline{z})) = (\underline{x} + \underline{u}) \gg ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + \underline{u})], p_0, c)]$$

$$[\text{Three2threeTerms} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} + \underline{y}) + \underline{z}) = ((\underline{x} + \underline{z}) + \underline{y})]$$

$$[\text{Three2threeTerms} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \text{plusCommutativity} \gg (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}); \text{Three2twoTerms} \triangleright (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}) \gg ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{z} + \underline{y})); \text{plusAssociativity} \gg ((\underline{x} + \underline{z}) + \underline{y}) = (\underline{x} + (\underline{z} + \underline{y})); \text{eqSymmetry} \triangleright ((\underline{x} + \underline{z}) + \underline{y}) = (\underline{x} + (\underline{z} + \underline{y})) \gg (\underline{x} + (\underline{z} + \underline{y})) = ((\underline{x} + \underline{z}) + \underline{y}); \text{eqTransitivity} \triangleright ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{z} + \underline{y})) \triangleright (\underline{x} + (\underline{z} + \underline{y})) = ((\underline{x} + \underline{z}) + \underline{y}) \gg ((\underline{x} + \underline{y}) + \underline{z}) = ((\underline{x} + \underline{z}) + \underline{y})], p_0, c)]$$

$$[\text{Three2twoFactors} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{y} * \underline{z}) = \underline{u} \vdash ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * \underline{u})]$$

$$[\text{Three2twoFactors} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{y} * \underline{z}) = \underline{u} \vdash \text{lemma eqMultiplicationLeft} \triangleright (\underline{y} * \underline{z}) = \underline{u} \gg (\underline{x} * (\underline{y} * \underline{z})) = (\underline{x} * \underline{u}); \text{timesAssociativity} \gg ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{y} * \underline{z})); \text{eqTransitivity} \triangleright ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{y} * \underline{z})) \triangleright (\underline{x} * (\underline{y} * \underline{z})) = (\underline{x} * \underline{u}) \gg ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * \underline{u})], p_0, c)]$$

$$[\underline{x} = \underline{x} + (\underline{y} - \underline{y}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y})))]$$

$$[\underline{x} = \underline{x} + (\underline{y} - \underline{y}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \text{plus0} \gg (\underline{x} + 0) = \underline{x}; \text{Negative} \gg (\underline{y} + (-\underline{u}\underline{y})) = 0; \text{eqSymmetry} \triangleright (\underline{y} + (-\underline{u}\underline{y})) = 0 \gg 0 = (\underline{y} + (-\underline{u}\underline{y})); \text{lemma eqAdditionLeft} \triangleright 0 = (\underline{y} + (-\underline{u}\underline{y})) \gg (\underline{x} + 0) = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))); \text{Equality} \triangleright (\underline{x} + 0) = \underline{x} \triangleright (\underline{x} + 0) = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))) \gg \underline{x} = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))), p_0, c)]$$

$$[\underline{x} = \underline{x} + \underline{y} - \underline{y} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y}))]$$

$$[\underline{x} = \underline{x} + \underline{y} - \underline{y} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{x} + (\underline{y} - \underline{y}) \gg \underline{x} = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))); \text{plusAssociativity} \gg ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y})) = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))); \text{eqSymmetry} \triangleright ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y})) = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))) \gg ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y})) = ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y})); \text{eqTransitivity} \triangleright \underline{x} = (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))) \triangleright (\underline{x} + (\underline{y} + (-\underline{u}\underline{y}))) = ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y})) \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{u}\underline{y}))), p_0, c)]$$

$$[\underline{x} = \underline{x} * \underline{y} * (1/\underline{y}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{y} = 0) \text{n} \vdash \underline{x} = ((\underline{x} * \underline{y}) * \text{recy})]$$

$$[\underline{x} = \underline{x} * \underline{y} * (1/\underline{y}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{y} = 0) \text{n} \vdash \text{times1} \gg (\underline{x} * 1) = \underline{x}; \text{Reciprocal} \triangleright \dot{\vdash} (\underline{y} = 0) \text{n} \gg (\underline{y} * \text{recy}) = 1; \text{Three2twoFactors} \triangleright (\underline{y} * \text{recy}) = 1 \gg ((\underline{x} * \underline{y}) * \text{recy}) = (\underline{x} * 1); \text{eqTransitivity} \triangleright ((\underline{x} * \underline{y}) * \text{recy}) = (\underline{x} * 1) \triangleright (\underline{x} * 1) = \underline{x} \gg ((\underline{x} * \underline{y}) * \text{recy}) = \underline{x}; \text{eqSymmetry} \triangleright ((\underline{x} * \underline{y}) * \text{recy}) = \underline{x} \gg \underline{x} = ((\underline{x} * \underline{y}) * \text{recy})], p_0, c)]$$

$$[\underline{x} * 0 + \underline{x} = \underline{x} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: ((\underline{x} * 0) + \underline{x}) = \underline{x}]$$

$$[\underline{x} * 0 + \underline{x} = \underline{x} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \text{times1} \gg (\underline{x} * 1) = \underline{x}; \text{eqSymmetry} \triangleright (\underline{x} * 1) = \underline{x} \gg \underline{x} = (\underline{x} * 1); \text{lemma eqAdditionLeft} \triangleright \underline{x} = (\underline{x} * 1) \gg ((\underline{x} * 0) + \underline{x}) = ((\underline{x} * 0) + (\underline{x} * 1)); \text{Distribution} \gg (\underline{x} * (0 + 1)) = ((\underline{x} * 0) + (\underline{x} * 1)); \text{eqSymmetry} \triangleright (\underline{x} * (0 + 1)) = ((\underline{x} * 0) + (\underline{x} * 1)) \gg ((\underline{x} * 0) + (\underline{x} * 1)) = (\underline{x} * (0 + 1)); \text{lemma plus0Left} \gg (0 + 1) =$$

1 ; lemma eqMultiplicationLeft $\triangleright (0 + 1) = 1 \gg (\underline{x} * (0 + 1)) =$
 $(\underline{x} * 1)$; eqTransitivity5 $\triangleright ((\underline{x} * 0) + \underline{x}) = ((\underline{x} * 0) + (\underline{x} * 1)) \triangleright ((\underline{x} * 0) + (\underline{x} * 1)) =$
 $(\underline{x} * (0 + 1)) \triangleright (\underline{x} * (0 + 1)) = (\underline{x} * 1) \triangleright (\underline{x} * 1) = \underline{x} \gg ((\underline{x} * 0) + \underline{x}) = \underline{x}]$, p_0 , $c]$
 $[\underline{x} * 0 = 0 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: (\underline{x} * 0) = 0]$
 $[\underline{x} * 0 = 0 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: x = x + (y - y) \gg (\underline{x} * 0) =$
 $((\underline{x} * 0) + (\underline{x} + (-\underline{x})))$; plusAssociativity $\gg (((\underline{x} * 0) + \underline{x}) + (-\underline{x})) =$
 $((\underline{x} * 0) + (\underline{x} + (-\underline{x})))$; eqSymmetry $\triangleright (((\underline{x} * 0) + \underline{x}) + (-\underline{x})) =$
 $((\underline{x} * 0) + (\underline{x} + (-\underline{x}))) \gg ((\underline{x} * 0) + (\underline{x} + (-\underline{x}))) = (((\underline{x} * 0) + \underline{x}) + (-\underline{x}))$; $x * 0 + x =$
 $x \gg ((\underline{x} * 0) + \underline{x}) = \underline{x}$; eqAddition $\triangleright ((\underline{x} * 0) + \underline{x}) = \underline{x} \gg (((\underline{x} * 0) + \underline{x}) + (-\underline{x})) =$
 $(\underline{x} + (-\underline{x}))$; Negative $\gg (\underline{x} + (-\underline{x})) = 0$; eqTransitivity5 $\triangleright (\underline{x} * 0) =$
 $((\underline{x} * 0) + (\underline{x} + (-\underline{x}))) \triangleright ((\underline{x} * 0) + (\underline{x} + (-\underline{x}))) = (((\underline{x} * 0) + \underline{x}) + (-\underline{x})) \triangleright (((\underline{x} * 0) + \underline{x}) + (-\underline{x})) = (\underline{x} + (-\underline{x})) \triangleright (\underline{x} + (-\underline{x})) = 0 \gg (\underline{x} * 0) = 0]$, p_0 , $c]$
 $[(-1) * (-1) + (-1) * 1 = 0 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash (((-u1) * (-u1)) + ((-u1) * 1)) = 0]$
 $[(-1) * (-1) + (-1) * 1 = 0 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \text{DistributionOut} \gg$
 $(((-u1) * (-u1)) + ((-u1) * 1)) = ((-u1) * ((-u1) + 1))$; Negative \gg
 $(1 + (-u1)) = 0$; plusCommutativity $\gg ((-u1) + 1) =$
 $(1 + (-u1))$; eqTransitivity $\triangleright ((-u1) + 1) = (1 + (-u1)) \triangleright (1 + (-u1)) = 0 \gg$
 $((-u1) + 1) = 0$; lemma eqMultiplicationLeft $\triangleright ((-u1) + 1) = 0 \gg$
 $((-u1) * ((-u1) + 1)) = ((-u1) * 0)$; $x * 0 = 0 \gg ((-u1) * 0) = 0$; eqTransitivity4 \triangleright
 $(((-u1) * (-u1)) + ((-u1) * 1)) = ((-u1) * ((-u1) + 1)) \triangleright ((-u1) * ((-u1) + 1)) =$
 $((-u1) * 0) \triangleright ((-u1) * 0) = 0 \gg (((-u1) * (-u1)) + ((-u1) * 1)) = 0]$, p_0 , $c]$
 $[(-1) * (-1) = 1 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash ((-u1) * (-u1)) = 1]$
 $[(-1) * (-1) = 1 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash x = x + (y - y) \gg$
 $((-u1) * (-u1)) = (((-u1) * (-u1)) + (1 + (-u1)))$; times1 $\gg ((-u1) * 1) =$
 $(-u1)$; eqSymmetry $\triangleright ((-u1) * 1) = (-u1) \gg (-u1) =$
 $((-u1) * 1)$; lemma eqAdditionLeft $\triangleright (-u1) = ((-u1) * 1) \gg (1 + (-u1)) =$
 $(1 + ((-u1) * 1))$; lemma eqAdditionLeft $\triangleright (1 + (-u1)) = (1 + ((-u1) * 1)) \gg$
 $(((-u1) * (-u1)) + (1 + (-u1))) =$
 $(((-u1) * (-u1)) + (1 + ((-u1) * 1)))$; plusCommutativity $\gg (1 + ((-u1) * 1)) =$
 $(((-u1) * 1) + 1)$; lemma eqAdditionLeft $\triangleright (1 + ((-u1) * 1)) = (((-u1) * 1) + 1) \gg$
 $(((-u1) * (-u1)) + (1 + ((-u1) * 1))) = (((-u1) * (-u1)) + (((-u1) * 1) +$
 $1))$; plusAssociativity $\gg ((((-u1) * (-u1)) + ((-u1) * 1)) + 1) = (((-u1) * (-u1)) +$
 $((-u1) * 1) + 1)$; eqSymmetry $\triangleright ((((-u1) * (-u1)) + ((-u1) * 1)) + 1) =$
 $(((-u1) * (-u1)) + (((-u1) * 1) + 1)) \gg ((((-u1) * (-u1)) + (((-u1) * 1) + 1)) =$
 $((((-u1) * (-u1)) + ((-u1) * 1)) + 1)$; $(-1) * (-1) + (-1) * 1 = 0 \gg$
 $(((-u1) * (-u1)) + ((-u1) * 1)) = 0$; eqAddition $\triangleright ((((-u1) * (-u1)) + ((-u1) * 1)) =$
 $0 \gg ((((-u1) * (-u1)) + ((-u1) * 1)) + 1) = (0 + 1)$; lemma plus0Left \gg
 $(0 + 1) = 1$; eqTransitivity5 $\triangleright ((-u1) * (-u1)) =$
 $(((-u1) * (-u1)) + (1 + (-u1))) \triangleright (((-u1) * (-u1)) + (1 + (-u1))) =$
 $(((-u1) * (-u1)) + (1 + ((-u1) * 1))) \triangleright (((-u1) * (-u1)) + (1 + ((-u1) * 1))) =$
 $(((-u1) * (-u1)) + (((-u1) * 1) + 1)) \triangleright (((-u1) * (-u1)) + (((-u1) * 1) + 1)) =$
 $((((-u1) * (-u1)) + ((-u1) * 1)) + 1) \gg ((-u1) * (-u1)) =$
 $((((-u1) * (-u1)) + ((-u1) * 1)) + 1)$; eqTransitivity4 $\triangleright ((-u1) * (-u1)) =$
 $((((-u1) * (-u1)) + ((-u1) * 1)) + 1) \triangleright ((((-u1) * (-u1)) + ((-u1) * 1)) + 1) =$

$$(0 + 1) \triangleright (0 + 1) = 1 \gg ((-u1) * (-u1)) = 1], p_0, c)]$$

$$[\text{subLeqRight} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \vdash \underline{z} <= \underline{x} \vdash \underline{z} <= \underline{y}]$$

$$[\text{subLeqRight} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \vdash \underline{z} <= \underline{x} \vdash \text{eqLeq} \triangleright \underline{x} = \underline{y} \gg \underline{x} <= \underline{y}; \text{leqTransitivity} \triangleright \underline{z} <= \underline{x} \triangleright \underline{x} <= \underline{y} \gg \underline{z} <= \underline{y}], p_0, c)]$$

$$[\text{subLeqLeft} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \vdash \underline{x} <= \underline{z} \vdash \underline{y} <= \underline{z}]$$

$$[\text{subLeqLeft} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \vdash \underline{x} <= \underline{z} \vdash \text{eqSymmetry} \triangleright \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \text{eqLeq} \triangleright \underline{y} = \underline{x} \gg \underline{y} <= \underline{x}; \text{leqTransitivity} \triangleright \underline{y} <= \underline{x} \triangleright \underline{x} <= \underline{z} \gg \underline{y} <= \underline{z}], p_0, c)]$$

$$[0 < 1 \text{Helper} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash 1 <= 0 \Rightarrow 0 <= 1]$$

$$[0 < 1 \text{Helper} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash 1 <= 0 \vdash \text{leqAddition} \triangleright 1 <= 0 \gg (1 + (-u1)) <= (0 + (-u1)); \text{Negative} \gg (1 + (-u1)) = 0; \text{subLeqLeft} \triangleright (1 + (-u1)) = 0 \triangleright (1 + (-u1)) <= (0 + (-u1)) \gg 0 <= (0 + (-u1)); \text{lemma plus0Left} \gg (0 + (-u1)) = (-u1); \text{subLeqRight} \triangleright (0 + (-u1)) = (-u1) \triangleright 0 <= (0 + (-u1)) \gg 0 <= (-u1); \text{leqMultiplication} \triangleright 0 <= (-u1) \triangleright 0 <= (-u1) \gg (0 * (-u1)) <= ((-u1) * (-u1)); x * 0 = 0 \gg ((-u1) * 0) = 0; \text{timesCommutativity} \gg (0 * (-u1)) = ((-u1) * 0); \text{eqTransitivity} \triangleright (0 * (-u1)) = ((-u1) * 0) \triangleright ((-u1) * 0) = 0 \gg (0 * (-u1)) = 0; \text{subLeqLeft} \triangleright (0 * (-u1)) = 0 \triangleright (0 * (-u1)) <= ((-u1) * (-u1)) \gg 0 <= ((-u1) * (-u1)); (-1) * (-1) = 1 \gg ((-u1) * (-u1)) = 1; \text{subLeqRight} \triangleright ((-u1) * (-u1)) = 1 \triangleright 0 <= ((-u1) * (-u1)) \gg 0 <= 1; \text{Ded} \triangleright 1 <= 0 \vdash 0 <= 1 \gg 1 <= 0 \Rightarrow 0 <= 1], p_0, c)]$$

$$[0 < 1 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash} (0 <= 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1) n) n) n]$$

$$[0 < 1 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \text{leqTotality} \gg \dot{\vdash} (0 <= 1) n \Rightarrow 1 <= 0; \text{AutoImPLY} \gg 0 <= 1 \Rightarrow 0 <= 1; 0 < 1 \text{Helper} \gg 1 <= 0 \Rightarrow 0 <= 1; \text{FromDisjuncts} \triangleright \dot{\vdash} (0 <= 1) n \Rightarrow 1 <= 0 \triangleright 0 <= 1 \Rightarrow 0 <= 1 \triangleright 1 <= 0 \Rightarrow 0 <= 1 \gg 0 <= 1; 0 \text{not} 1 \gg \dot{\vdash} (0 = 1) n; \text{JoinConjuncts} \triangleright 0 <= 1 \triangleright \dot{\vdash} (0 = 1) n \gg \dot{\vdash} (0 <= 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1) n) n) n], p_0, c)]$$

$$[\text{AddEquations} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash (\underline{x} + \underline{z}) = (\underline{y} + \underline{u})]$$

$$[\text{AddEquations} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash \text{eqAddition} \triangleright \underline{x} = \underline{y} \gg (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}); \text{lemma eqAdditionLeft} \triangleright \underline{z} = \underline{u} \gg (\underline{y} + \underline{z}) = (\underline{y} + \underline{u}); \text{eqTransitivity} \triangleright (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \triangleright (\underline{y} + \underline{z}) = (\underline{y} + \underline{u}) \gg (\underline{x} + \underline{z}) = (\underline{y} + \underline{u})], p_0, c)]$$

$$[\text{SubtractEquations} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{z} = \underline{u} \vdash \underline{x} = \underline{y}]$$

$$[\text{SubtractEquations} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{z} = \underline{u} \vdash \text{eqAddition} \triangleright (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \gg ((\underline{x} + \underline{z}) + (-\underline{uz})) = ((\underline{y} + \underline{u}) + (-\underline{uz})); \text{lemma plus0Left} \gg (0 + \underline{z}) = \underline{z}; \text{eqTransitivity} \triangleright (0 + \underline{z}) = \underline{z} \triangleright \underline{z} \triangleright \underline{z} = \underline{u} \gg (0 + \underline{z}) = \underline{u}; \text{PositiveToRight}(\text{Eq}) \triangleright (0 + \underline{z}) = \underline{u} \gg 0 = (\underline{u} + (-\underline{uz})); \text{eqSymmetry} \triangleright 0 = (\underline{u} + (-\underline{uz})) \gg (\underline{u} + (-\underline{uz})) = 0; \text{lemma eqAdditionLeft} \triangleright (\underline{u} + (-\underline{uz})) = 0 \gg (\underline{y} + (\underline{u} + (-\underline{uz}))) = (\underline{y} + 0); \text{plusAssociativity} \gg ((\underline{y} + \underline{u}) + (-\underline{uz})) = (\underline{y} + (\underline{u} + (-\underline{uz}))); \text{plus0} \gg (\underline{y} + 0) = \underline{y}; \text{eqTransitivity4} \triangleright ((\underline{y} + \underline{u}) + (-\underline{uz})) = (\underline{y} + (\underline{u} + (-\underline{uz}))) \triangleright (\underline{y} + (\underline{u} + (-\underline{uz}))) = (\underline{y} + 0) \triangleright (\underline{y} + 0) = \underline{y} \gg ((\underline{y} + \underline{u}) + (-\underline{uz})) = \underline{y}; x = x + y - y \gg \underline{x} =$$

$(\underline{x} + \underline{z}) + (-\underline{uz}); \text{eqTransitivity4} \triangleright \underline{x} = ((\underline{x} + \underline{z}) + (-\underline{uz})) \triangleright ((\underline{x} + \underline{z}) + (-\underline{uz})) = ((\underline{y} + \underline{u}) + (-\underline{uz})) \triangleright ((\underline{y} + \underline{u}) + (-\underline{uz})) = \underline{y} \gg \underline{x} = \underline{y}], p_0, c]$

$[\text{SubtractEquationsLeft} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{x} = \underline{y} \vdash \underline{z} = \underline{u}]$

$[\text{SubtractEquationsLeft} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{x} = \underline{y} \vdash \text{plusCommutativity} \gg (\underline{z} + \underline{x}) = (\underline{x} + \underline{z}); \text{plusCommutativity} \gg (\underline{y} + \underline{u}) = (\underline{u} + \underline{y}); \text{eqTransitivity4} \triangleright (\underline{z} + \underline{x}) = (\underline{x} + \underline{z}) \triangleright (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \triangleright (\underline{y} + \underline{u}) = (\underline{u} + \underline{y}) \gg (\underline{z} + \underline{x}) = (\underline{u} + \underline{y}); \text{SubtractEquations} \triangleright (\underline{z} + \underline{x}) = (\underline{u} + \underline{y}) \triangleright \underline{x} = \underline{y} \gg \underline{z} = \underline{u}], p_0, c)]$

$[\text{EqNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash (-\underline{ux}) = (-\underline{uy})]$

$[\text{EqNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash \text{Negative} \gg (\underline{x} + (-\underline{ux})) = 0; \text{Negative} \gg (\underline{y} + (-\underline{uy})) = 0; \text{eqSymmetry} \triangleright (\underline{y} + (-\underline{uy})) = 0 \gg 0 = (\underline{y} + (-\underline{uy})); \text{eqTransitivity} \triangleright (\underline{x} + (-\underline{ux})) = 0 \triangleright 0 = (\underline{y} + (-\underline{uy})) \gg (\underline{x} + (-\underline{ux})) = (\underline{y} + (-\underline{uy})); \text{SubtractEquationsLeft} \triangleright (\underline{x} + (-\underline{ux})) = (\underline{y} + (-\underline{uy})) \triangleright \underline{x} = \underline{y} \gg (-\underline{ux}) = (-\underline{uy})], p_0, c)]$

$[\text{PositiveToRight}(\text{Eq}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) = \underline{z} \vdash \underline{x} = (\underline{z} + (-\underline{uy}))]$

$[\text{PositiveToRight}(\text{Eq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) = \underline{z} \vdash \text{eqAddition} \triangleright (\underline{x} + \underline{y}) = \underline{z} \gg ((\underline{x} + \underline{y}) + (-\underline{uy})) = (\underline{z} + (-\underline{uy})); \underline{x} = \underline{x} + \underline{y} - \underline{y} \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{uy})); \text{eqTransitivity} \triangleright \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{uy})) \triangleright ((\underline{x} + \underline{y}) + (-\underline{uy})) = (\underline{z} + (-\underline{uy})) \gg \underline{x} = (\underline{z} + (-\underline{uy}))], p_0, c)]$

$[\text{PositiveToLeft}(\text{Eq})(1\text{term}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash (\underline{x} + (-\underline{uy})) = 0]$

$[\text{PositiveToLeft}(\text{Eq})(1\text{term}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash \text{eqAddition} \triangleright \underline{x} = \underline{y} \gg (\underline{x} + (-\underline{uy})) = (\underline{y} + (-\underline{uy})); \text{Negative} \gg (\underline{y} + (-\underline{uy})) = 0; \text{eqTransitivity} \triangleright (\underline{x} + (-\underline{uy})) = (\underline{y} + (-\underline{uy})) \triangleright (\underline{y} + (-\underline{uy})) = 0 \gg (\underline{x} + (-\underline{uy})) = 0], p_0, c)]$

$[\text{PositiveToRight}(\text{Leq}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) \leq \underline{z} \vdash \underline{x} \leq (\underline{z} + (-\underline{uy}))]$

$[\text{PositiveToRight}(\text{Leq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) \leq \underline{z} \vdash \text{leqAddition} \triangleright (\underline{x} + \underline{y}) \leq \underline{z} \gg ((\underline{x} + \underline{y}) + (-\underline{uy})) \leq (\underline{z} + (-\underline{uy})); \underline{x} = \underline{x} + \underline{y} - \underline{y} \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{uy})); \text{eqSymmetry} \triangleright \underline{x} = ((\underline{x} + \underline{y}) + (-\underline{uy})) \gg ((\underline{x} + \underline{y}) + (-\underline{uy})) = \underline{x}; \text{subLeqLeft} \triangleright ((\underline{x} + \underline{y}) + (-\underline{uy})) = \underline{x} \triangleright ((\underline{x} + \underline{y}) + (-\underline{uy})) \leq (\underline{z} + (-\underline{uy})) \gg \underline{x} \leq (\underline{z} + (-\underline{uy}))], p_0, c)]$

$[\text{PositiveToRight}(\text{Leq})(1\text{term}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{y}: \forall \underline{z}: \underline{y} \leq \underline{z} \vdash 0 \leq (\underline{z} + (-\underline{uy}))]$

$[\text{PositiveToRight}(\text{Leq})(1\text{term}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{y}: \forall \underline{z}: \underline{y} \leq \underline{z} \vdash \text{lemma plus0Left} \gg (0 + \underline{y}) = \underline{y}; \text{eqSymmetry} \triangleright (0 + \underline{y}) = \underline{y} \gg \underline{y} = (0 + \underline{y}); \text{subLeqLeft} \triangleright \underline{y} = (0 + \underline{y}) \triangleright \underline{y} \leq \underline{z} \gg (0 + \underline{y}) \leq \underline{z}; \text{PositiveToRight}(\text{Leq}) \triangleright (0 + \underline{y}) \leq \underline{z} \gg 0 \leq (\underline{z} + (-\underline{uy}))], p_0, c)]$

$[\text{NegativeToLeft}(\text{Eq}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = (\underline{y} + (-\underline{uz})) \vdash (\underline{x} + \underline{z}) = \underline{y}]$

$[\text{NegativeToLeft}(\text{Eq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = (\underline{y} + (-\underline{uz})) \vdash$

eqAddition $\triangleright \underline{x} = (\underline{y} + (-\underline{u}\underline{z})) \gg (\underline{x} + \underline{z}) = ((\underline{y} + (-\underline{u}\underline{z})) + \underline{z})$; Three2threeTerms \gg
 $((\underline{y} + (-\underline{u}\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-\underline{u}\underline{z}))$; $\underline{x} = \underline{x} + \underline{y} - \underline{y} \gg \underline{y} =$
 $((\underline{y} + \underline{z}) + (-\underline{u}\underline{z}))$; eqSymmetry $\triangleright \underline{y} = ((\underline{y} + \underline{z}) + (-\underline{u}\underline{z})) \gg ((\underline{y} + \underline{z}) + (-\underline{u}\underline{z})) =$
 \underline{y} ; eqTransitivity4 $\triangleright (\underline{x} + \underline{z}) = ((\underline{y} + (-\underline{u}\underline{z})) + \underline{z}) \triangleright ((\underline{y} + (-\underline{u}\underline{z})) + \underline{z}) =$
 $((\underline{y} + \underline{z}) + (-\underline{u}\underline{z})) \triangleright ((\underline{y} + \underline{z}) + (-\underline{u}\underline{z})) = \underline{y} \gg (\underline{x} + \underline{z}) = \underline{y}$, p_0, c]
 (** NO EQUALITY **)

[LessNeq $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} <= \underline{y}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n \vdash \dot{\vdash} (\underline{x} = \underline{y})n$]

[LessNeq $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} <= \underline{y}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n \vdash$
 Repetition $\triangleright \dot{\vdash} (\underline{x} <= \underline{y}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n \gg \dot{\vdash} (\underline{x} <= \underline{y}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} =$
 $\underline{y})n)n$; SecondConjunct $\triangleright \dot{\vdash} (\underline{x} <= \underline{y}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n \gg \dot{\vdash} (\underline{x} = \underline{y})n$, p_0, c)]

[$\underline{x} + \underline{y} = \underline{z}$ Backwards $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) = \underline{z} \vdash \underline{z} = (\underline{y} + \underline{x})$]

[$\underline{x} + \underline{y} = \underline{z}$ Backwards $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) = \underline{z} \vdash$
 plusCommutativity $\gg (\underline{x} + \underline{y}) = (\underline{y} + \underline{x})$; Equality $\triangleright (\underline{x} + \underline{y}) = \underline{z} \gg \underline{z} =$
 $(\underline{y} + \underline{x})$, p_0, c]

[$\underline{x} * \underline{y} = \underline{z}$ Backwards $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * \underline{y}) = \underline{z} \vdash \underline{z} = (\underline{y} * \underline{x})$]

[$\underline{x} * \underline{y} = \underline{z}$ Backwards $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * \underline{y}) = \underline{z} \vdash$
 timesCommutativity $\gg (\underline{x} * \underline{y}) = (\underline{y} * \underline{x})$; Equality $\triangleright (\underline{x} * \underline{y}) = \underline{z} \gg \underline{z} = (\underline{y} * \underline{x})$, p_0, c]

[DoubleMinus $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: (-u(-\underline{u}\underline{x})) = \underline{x}$]

[DoubleMinus $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \text{Negative} \gg$
 $((-\underline{u}\underline{x}) + (-u(-\underline{u}\underline{x}))) = 0$; $\underline{x} + \underline{y} = \underline{z}$ Backwards $\triangleright ((-\underline{u}\underline{x}) + (-u(-\underline{u}\underline{x}))) = 0 \gg$
 $0 = ((-\underline{u}\underline{x}) + (-u(-\underline{u}\underline{x})))$; NegativeToLeft(Eq) $\triangleright 0 = ((-\underline{u}\underline{x}) + (-u(-\underline{u}\underline{x}))) \gg$
 $(0 + \underline{x}) = (-u(-\underline{u}\underline{x}))$; lemma plus0Left $\gg (0 + \underline{x}) = \underline{x}$; Equality $\triangleright (0 + \underline{x}) =$
 $(-u(-\underline{u}\underline{x})) \triangleright (0 + \underline{x}) = \underline{x} \gg (-u(-\underline{u}\underline{x})) = \underline{x}$, p_0, c]

[NeqNegated $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} = \underline{y})n \vdash \dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}\underline{y}))n$]

[NeqNegated $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} = \underline{y})n \vdash (-\underline{u}\underline{x}) = (-\underline{u}\underline{y}) \vdash$
 EqNegated $\triangleright (-\underline{u}\underline{x}) = (-\underline{u}\underline{y}) \gg (-u(-\underline{u}\underline{x})) = (-u(-\underline{u}\underline{y}))$; DoubleMinus \gg
 $(-u(-\underline{u}\underline{x})) = \underline{x}$; eqSymmetry $\triangleright (-u(-\underline{u}\underline{x})) = \underline{x} \gg \underline{x} =$
 $(-u(-\underline{u}\underline{x}))$; DoubleMinus $\gg (-u(-\underline{u}\underline{y})) = \underline{y}$; eqTransitivity4 $\triangleright \underline{x} =$
 $(-u(-\underline{u}\underline{x})) \triangleright (-u(-\underline{u}\underline{x})) = (-u(-\underline{u}\underline{y})) \triangleright (-u(-\underline{u}\underline{y})) = \underline{y} \gg \underline{x} =$
 \underline{y} ; FromContradiction $\triangleright \underline{x} = \underline{y} \triangleright \dot{\vdash} (\underline{x} = \underline{y})n \gg \dot{\vdash} ((-\underline{u}\underline{x}) =$
 $(-\underline{u}\underline{y}))n$; $\forall \underline{x}: \forall \underline{y}: \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} = \underline{y})n \vdash (-\underline{u}\underline{x}) = (-\underline{u}\underline{y}) \vdash \dot{\vdash} ((-\underline{u}\underline{x}) =$
 $(-\underline{u}\underline{y}))n \gg \dot{\vdash} (\underline{x} = \underline{y})n \Rightarrow (-\underline{u}\underline{x}) = (-\underline{u}\underline{y}) \Rightarrow \dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}\underline{y}))n$; $\dot{\vdash} (\underline{x} = \underline{y})n \vdash$
 MP $\triangleright \dot{\vdash} (\underline{x} = \underline{y})n \Rightarrow (-\underline{u}\underline{x}) = (-\underline{u}\underline{y}) \Rightarrow \dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}\underline{y}))n \triangleright \dot{\vdash} (\underline{x} = \underline{y})n \gg$
 $(-\underline{u}\underline{x}) = (-\underline{u}\underline{y}) \Rightarrow \dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}\underline{y}))n$; prop lemma imply negation $\triangleright (-\underline{u}\underline{x}) =$
 $(-\underline{u}\underline{y}) \Rightarrow \dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}\underline{y}))n \gg \dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}\underline{y}))n$, p_0, c]

[SubNeqRight $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \vdash \dot{\vdash} (\underline{z} = \underline{x})n \vdash \dot{\vdash} (\underline{z} = \underline{y})n$]

[SubNeqRight $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \vdash \dot{\vdash} (\underline{z} = \underline{x})n \vdash$
 NeqSymmetry $\triangleright \dot{\vdash} (\underline{z} = \underline{x})n \gg \dot{\vdash} (\underline{x} = \underline{z})n$; SubNeqLeft $\triangleright \underline{x} = \underline{y} \vdash \dot{\vdash} (\underline{x} = \underline{z})n \gg$
 $\dot{\vdash} (\underline{y} = \underline{z})n$; NeqSymmetry $\triangleright \dot{\vdash} (\underline{y} = \underline{z})n \gg \dot{\vdash} (\underline{z} = \underline{y})n$, p_0, c]

[NeqAddition $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\vdash} (\underline{x} = \underline{y})n \vdash \dot{\vdash} ((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n$]

[NeqAddition $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\vdash} (\underline{x} = \underline{y})n \vdash (\underline{x} + \underline{z}) =$

$(\underline{y} + \underline{z}); \text{leqSubtraction} \triangleright (\underline{x} + \underline{z}) \leq (\underline{y} + \underline{z}) \gg \underline{x} \leq \underline{y}], p_0, c]$
 $[\text{negativeToLeft}(\text{Leq}) \xrightarrow{\text{stmtt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} \leq (\underline{y} + (-\underline{uz})) \vdash (\underline{x} + \underline{z}) \leq \underline{y}]$
 $[\text{negativeToLeft}(\text{Leq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} \leq (\underline{y} + (-\underline{uz})) \vdash$
 $\text{leqAddition} \triangleright \underline{x} \leq (\underline{y} + (-\underline{uz})) \gg (\underline{x} + \underline{z}) \leq ((\underline{y} + (-\underline{uz})) + \underline{z}); \underline{x} =$
 $\underline{x} + \underline{y} - \underline{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-\underline{uz})); \text{Three2threeTerms} \gg ((\underline{y} + \underline{z}) + (-\underline{uz})) =$
 $((\underline{y} + (-\underline{uz})) + \underline{z}); \text{eqTransitivity} \triangleright \underline{y} = ((\underline{y} + \underline{z}) + (-\underline{uz})) \triangleright ((\underline{y} + \underline{z}) + (-\underline{uz})) =$
 $((\underline{y} + (-\underline{uz})) + \underline{z}) \gg \underline{y} = ((\underline{y} + (-\underline{uz})) + \underline{z}); \text{eqSymmetry} \triangleright \underline{y} =$
 $((\underline{y} + (-\underline{uz})) + \underline{z}) \gg ((\underline{y} + (-\underline{uz})) + \underline{z}) = \underline{y}; \text{subLeqRight} \triangleright ((\underline{y} + (-\underline{uz})) + \underline{z}) =$
 $\underline{y} \triangleright (\underline{x} + \underline{z}) \leq ((\underline{y} + (-\underline{uz})) + \underline{z}) \gg (\underline{x} + \underline{z}) \leq \underline{y}], p_0, c)]$
 $[\text{thirdGeq} \xrightarrow{\text{stmtt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)]$
 $[\text{thirdGeq} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \leq \underline{y} \vdash \text{leqReflexivity} \gg \underline{y} \leq$
 $\underline{y}; \text{JoinConjuncts} \triangleright \underline{x} \leq \underline{y} \triangleright \underline{y} \leq \underline{y} \gg \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\underline{y} \leq$
 $\underline{y})n)n; \text{ExistIntro} @ \underline{c_{Ex}} @ \underline{x} \triangleright \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{y})n)n \gg \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow$
 $\dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n; \forall \underline{x}: \forall \underline{y}: \underline{y} \leq \underline{x} \vdash \text{leqReflexivity} \gg \underline{x} \leq$
 $\underline{x}; \text{JoinConjuncts} \triangleright \underline{x} \leq \underline{x} \triangleright \underline{y} \leq \underline{x} \gg \dot{\vdash} (\underline{x} \leq \underline{x} \Rightarrow \dot{\vdash} (\underline{y} \leq$
 $\underline{x})n)n; \text{ExistIntro} @ \underline{c_{Ex}} @ \underline{x} \triangleright \dot{\vdash} (\underline{x} \leq \underline{x} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{x})n)n \gg \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow$
 $\dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n; \forall \underline{x}: \forall \underline{y}: \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \underline{x} \leq \underline{y} \vdash \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq$
 $\underline{c_{Ex}})n)n \gg \underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n; \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \underline{y} \leq \underline{x} \vdash$
 $\dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n \gg \underline{y} \leq \underline{x} \Rightarrow \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq$
 $\underline{c_{Ex}})n)n; \text{leqTotality} \gg \dot{\vdash} (\underline{x} \leq \underline{y})n \Rightarrow \underline{y} \leq \underline{x}; \text{FromDisjuncts} \triangleright \dot{\vdash} (\underline{x} \leq \underline{y})n \Rightarrow$
 $\underline{y} \leq \underline{x} \triangleright \underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n \triangleright \underline{y} \leq \underline{x} \Rightarrow \dot{\vdash} (\underline{x} \leq$
 $\underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n \gg \dot{\vdash} (\underline{x} \leq \underline{c_{Ex}} \Rightarrow \dot{\vdash} (\underline{y} \leq \underline{c_{Ex}})n)n], p_0, c)]$
 $[\text{LeqNegated} \xrightarrow{\text{stmtt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \leq \underline{y} \vdash (-\underline{uy}) \leq (-\underline{ux})]$
 $[\text{LeqNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \leq \underline{y} \vdash \text{leqAddition} \triangleright \underline{x} \leq$
 $\underline{y} \gg (\underline{x} + (-\underline{ux})) \leq (\underline{y} + (-\underline{ux})); \text{Negative} \gg (\underline{x} + (-\underline{ux})) =$
 $0; \text{subLeqLeft} \triangleright (\underline{x} + (-\underline{ux})) = 0 \triangleright (\underline{x} + (-\underline{ux})) \leq (\underline{y} + (-\underline{ux})) \gg 0 \leq$
 $(\underline{y} + (-\underline{ux})); \text{plusCommutativity} \gg (\underline{y} + (-\underline{ux})) =$
 $((-\underline{ux}) + \underline{y}); \text{subLeqRight} \triangleright (\underline{y} + (-\underline{ux})) = ((-\underline{ux}) + \underline{y}) \triangleright 0 \leq (\underline{y} + (-\underline{ux})) \gg$
 $0 \leq ((-\underline{ux}) + \underline{y}); \text{leqAddition} \triangleright 0 \leq ((-\underline{ux}) + \underline{y}) \gg (0 + (-\underline{uy})) \leq$
 $((-\underline{ux}) + \underline{y}) + (-\underline{uy}); \text{lemma plus0Left} \gg (0 + (-\underline{uy})) = (-\underline{uy}); \underline{x} = \underline{x} + \underline{y} - \underline{y} \gg$
 $(-\underline{ux}) = (((-\underline{ux}) + \underline{y}) + (-\underline{uy})); \text{eqSymmetry} \triangleright (-\underline{ux}) = (((-\underline{ux}) + \underline{y}) + (-\underline{uy})) \gg$
 $(((-\underline{ux}) + \underline{y}) + (-\underline{uy})) = (-\underline{ux}); \text{subLeqLeft} \triangleright (0 + (-\underline{uy})) =$
 $(-\underline{uy}) \triangleright (0 + (-\underline{uy})) \leq (((-\underline{ux}) + \underline{y}) + (-\underline{uy})) \gg (-\underline{uy}) \leq$
 $(((-\underline{ux}) + \underline{y}) + (-\underline{uy})); \text{subLeqRight} \triangleright (((-\underline{ux}) + \underline{y}) + (-\underline{uy})) =$
 $(-\underline{ux}) \triangleright (-\underline{uy}) \leq (((-\underline{ux}) + \underline{y}) + (-\underline{uy})) \gg (-\underline{uy}) \leq (-\underline{ux})], p_0, c)]$
 $[\text{AddEquations}(\text{Leq}) \xrightarrow{\text{stmtt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \underline{x} \leq \underline{y} \vdash \underline{z} \leq \underline{u} \vdash$
 $(\underline{x} + \underline{z}) \leq (\underline{y} + \underline{u})]$
 $[\text{AddEquations}(\text{Leq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \underline{x} \leq \underline{y} \vdash \underline{z} \leq$
 $\underline{u} \vdash \text{leqAddition} \triangleright \underline{x} \leq \underline{y} \gg (\underline{x} + \underline{z}) \leq (\underline{y} + \underline{z}); \text{LeqAdditionLeft} \triangleright \underline{z} \leq \underline{u} \gg$
 $(\underline{y} + \underline{z}) \leq (\underline{y} + \underline{u}); \text{leqTransitivity} \triangleright (\underline{x} + \underline{z}) \leq (\underline{y} + \underline{z}) \triangleright (\underline{y} + \underline{z}) \leq (\underline{y} + \underline{u}) \gg$
 $(\underline{x} + \underline{z}) \leq (\underline{y} + \underline{u})], p_0, c)]$

(***) LESS (***)

$\underline{x})n)n) \vdash \text{FirstConjunct} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg 0 \leq \underline{x}; \text{leqAddition} \triangleright 0 \leq \underline{x} \gg (0 + (-\underline{ux})) \leq (\underline{x} + (-\underline{ux})); \text{lemma plus0Left} \gg (0 + (-\underline{ux})) = (-\underline{ux}); \text{Negative} \gg (\underline{x} + (-\underline{ux})) = 0; \text{subLeqLeft} \triangleright (0 + (-\underline{ux})) = (-\underline{ux}) \triangleright (0 + (-\underline{ux})) \leq (\underline{x} + (-\underline{ux})) \gg (-\underline{ux}) \leq (\underline{x} + (-\underline{ux})); \text{subLeqRight} \triangleright (\underline{x} + (-\underline{ux})) = 0 \triangleright (-\underline{ux}) \leq (\underline{x} + (-\underline{ux})) \gg (-\underline{ux}) \leq 0; \text{leqLessTransitivity} \triangleright (-\underline{ux}) \leq 0 \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg \dot{\vdash} ((-\underline{ux}) \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{ux}) = \underline{x})n)n), p_0, c)$

$[\text{LessNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n) \vdash \dot{\vdash} ((-\underline{uy}) \leq (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{uy}) = (-\underline{ux}))n)n)]$

$[\text{LessNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n) \vdash \text{LessLeq} \triangleright \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n) \gg \underline{x} \leq \underline{y}; \text{LeqNegated} \triangleright \underline{x} \leq \underline{y} \gg (-\underline{uy}) \leq (-\underline{ux}); \text{LessNeq} \triangleright \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n) \gg \dot{\vdash} (\underline{x} = \underline{y})n; \text{NeqNegated} \triangleright \dot{\vdash} (\underline{x} = \underline{y})n \gg \dot{\vdash} ((-\underline{ux}) = (-\underline{uy}))n; \text{NeqSymmetry} \triangleright \dot{\vdash} ((-\underline{ux}) = (-\underline{uy}))n \gg \dot{\vdash} ((-\underline{uy}) = (-\underline{ux}))n; \text{LeqNeqLess} \triangleright (-\underline{uy}) \leq (-\underline{ux}) \triangleright \dot{\vdash} ((-\underline{uy}) = (-\underline{ux}))n \gg \dot{\vdash} ((-\underline{uy}) \leq (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{uy}) = (-\underline{ux}))n)n), p_0, c)$

$[-0 = 0 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash (-u0) = 0]$

$[-0 = 0 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \text{Negative} \gg (0 + (-u0)) = 0; \text{plus0} \gg (0 + 0) = 0; \text{UniqueNegative} \triangleright (0 + (-u0)) = 0 \triangleright (0 + 0) = 0 \gg (-u0) = 0 \rceil, p_0, c)]$

$[\text{PositiveNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \vdash \dot{\vdash} ((-\underline{ux}) \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{ux}) = 0)n)n)]$

$[\text{PositiveNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \vdash \text{LessNegated} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg \dot{\vdash} ((-\underline{ux}) \leq (-u0) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{ux}) = (-u0))n)n); -0 = 0 \gg (-u0) = 0; \text{SubLessRight} \triangleright (-u0) = 0 \triangleright \dot{\vdash} ((-\underline{ux}) \leq (-u0) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{ux}) = (-u0))n)n) \gg \dot{\vdash} ((-\underline{ux}) \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{ux}) = 0)n)n), p_0, c)$

$[\text{NonpositiveNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \underline{x} \leq 0 \vdash 0 \leq (-\underline{ux})]$

$[\text{NonpositiveNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \underline{x} \leq 0 \vdash \text{LeqNegated} \triangleright \underline{x} \leq 0 \gg (-u0) \leq (-\underline{ux}); -0 = 0 \gg (-u0) = 0; \text{subLeqLeft} \triangleright (-u0) = 0 \triangleright (-u0) \leq (-\underline{ux}) \gg 0 \leq (-\underline{ux}) \rceil, p_0, c)$

$[\text{NegativeNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \vdash \dot{\vdash} (0 \leq (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (-\underline{ux}))n)n)]$

$[\text{NegativeNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \vdash \text{LessNegated} \triangleright \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \gg \dot{\vdash} ((-u0) \leq (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u0) = (-\underline{ux}))n)n); -0 = 0 \gg (-u0) = 0; \text{SubLessLeft} \triangleright (-u0) = 0 \triangleright \dot{\vdash} ((-u0) \leq (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u0) = (-\underline{ux}))n)n) \gg \dot{\vdash} (0 \leq (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (-\underline{ux}))n)n), p_0, c)$

$[\text{NonnegativeNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. 0 \leq \underline{x} \vdash (-\underline{ux}) \leq 0]$

$[\text{NonnegativeNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. 0 \leq \underline{x} \vdash \text{LeqNegated} \triangleright 0 \leq \underline{x} \gg (-\underline{ux}) \leq (-u0); -0 = 0 \gg (-u0) = 0; \text{subLeqRight} \triangleright (-u0) = 0 \triangleright (-\underline{ux}) \leq (-u0) \gg (-\underline{ux}) \leq 0 \rceil, p_0, c)$

$[0 < 2 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash} (0 \leq (1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (1 + 1))n)n)]$

$[0 < 2 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash 0 < 1 \gg \dot{\vdash}(0 <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = 1)n)n)n; \text{LessAddition} \triangleright \dot{\vdash}(0 <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = 1)n)n)n \gg \dot{\vdash}((0 + 1) <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}((0 + 1) = (1 + 1))n)n)n; \text{lemma plus0Left} \gg (0 + 1) = 1; \text{SubLessLeft} \triangleright (0 + 1) = 1 \triangleright \dot{\vdash}((0 + 1) <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}((0 + 1) = (1 + 1))n)n)n \gg \dot{\vdash}(1 <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(1 = (1 + 1))n)n)n; \text{LessTransitivity} \triangleright \dot{\vdash}(0 <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = 1)n)n)n \triangleright \dot{\vdash}(1 <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(1 = (1 + 1))n)n)n \gg \dot{\vdash}(0 <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (1 + 1))n)n)n \rceil, p_0, c)]$

$[0 < 1/2 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash}(0 <= \text{rec}(1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \text{rec}(1 + 1))n)n)n]$

$[0 < 1/2 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash 0 < 2 \gg \dot{\vdash}(0 <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (1 + 1))n)n)n; \text{FirstConjunct} \triangleright \dot{\vdash}(0 <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (1 + 1))n)n)n \gg 0 <= (1 + 1); \text{SecondConjunct} \triangleright \dot{\vdash}(0 <= (1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (1 + 1))n)n)n \gg \dot{\vdash}(0 = (1 + 1))n; \text{NeqSymmetry} \triangleright \dot{\vdash}(0 = (1 + 1))n \gg \dot{\vdash}((1 + 1) = 0)n; 0 < 1 \gg \dot{\vdash}(0 <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = 1)n)n)n; x * 0 = 0 \gg ((1 + 1) * 0) = 0; x * y = z \text{Backwards} \triangleright ((1 + 1) * 0) = 0 \gg 0 = (0 * (1 + 1)); \text{SubLessLeft} \triangleright 0 = (0 * (1 + 1)) \triangleright \dot{\vdash}(0 <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = 1)n)n)n \gg \dot{\vdash}((0 * (1 + 1)) <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}((0 * (1 + 1)) = 1)n)n)n; \text{Reciprocal} \triangleright \dot{\vdash}((1 + 1) = 0)n \gg ((1 + 1) * \text{rec}(1 + 1)) = 1; x * y = z \text{Backwards} \triangleright ((1 + 1) * \text{rec}(1 + 1)) = 1 \gg 1 = (\text{rec}(1 + 1) * (1 + 1)); \text{SubLessRight} \triangleright 1 = (\text{rec}(1 + 1) * (1 + 1)) \triangleright \dot{\vdash}((0 * (1 + 1)) <= 1 \Rightarrow \dot{\vdash}(\dot{\vdash}((0 * (1 + 1)) = 1)n)n)n \gg \dot{\vdash}((0 * (1 + 1)) <= (\text{rec}(1 + 1) * (1 + 1)) \Rightarrow \dot{\vdash}(\dot{\vdash}((0 * (1 + 1)) = (\text{rec}(1 + 1) * (1 + 1)))n)n)n; \text{LessDivision} \triangleright 0 <= (1 + 1) \triangleright \dot{\vdash}((0 * (1 + 1)) <= (\text{rec}(1 + 1) * (1 + 1)) \Rightarrow \dot{\vdash}(\dot{\vdash}((0 * (1 + 1)) = (\text{rec}(1 + 1) * (1 + 1)))n)n)n \gg \dot{\vdash}(0 <= \text{rec}(1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \text{rec}(1 + 1))n)n)n \rceil, p_0, c)]$

$[\text{PositiveHalved} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \dot{\vdash}(0 <= \underline{x} \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \underline{x})n)n)n \vdash \dot{\vdash}(0 <= (\text{rec}(1 + 1) * \underline{x}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (\text{rec}(1 + 1) * \underline{x}))n)n)n]$

$[\text{PositiveHalved} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \dot{\vdash}(0 <= \underline{x} \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \underline{x})n)n)n \vdash 0 < 1/2 \gg \dot{\vdash}(0 <= \text{rec}(1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \text{rec}(1 + 1))n)n)n; \text{LessMultiplicationLeft} \triangleright \dot{\vdash}(0 <= \text{rec}(1 + 1) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \text{rec}(1 + 1))n)n)n \triangleright \dot{\vdash}(0 <= \underline{x} \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \underline{x})n)n)n \gg \dot{\vdash}((\text{rec}(1 + 1) * 0) <= (\text{rec}(1 + 1) * \underline{x}) \Rightarrow \dot{\vdash}(\dot{\vdash}((\text{rec}(1 + 1) * 0) = (\text{rec}(1 + 1) * \underline{x}))n)n)n; x * 0 = 0 \gg (\text{rec}(1 + 1) * 0) = 0; \text{SubLessLeft} \triangleright (\text{rec}(1 + 1) * 0) = 0 \triangleright \dot{\vdash}((\text{rec}(1 + 1) * 0) <= (\text{rec}(1 + 1) * \underline{x}) \Rightarrow \dot{\vdash}(\dot{\vdash}((\text{rec}(1 + 1) * 0) = (\text{rec}(1 + 1) * \underline{x}))n)n)n \gg \dot{\vdash}(0 <= (\text{rec}(1 + 1) * \underline{x}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (\text{rec}(1 + 1) * \underline{x}))n)n)n \rceil, p_0, c)]$

$[\text{FromNot} << \xrightarrow{\text{stmt}} \text{SystemQ} \vdash$

$\forall(\underline{fx}): \forall(\underline{fy}): \dot{\vdash}(\dot{\vdash}(\forall_{\text{obj}} \overline{\epsilon}): \dot{\vdash}(\dot{\vdash}(\forall_{\text{obj}} \overline{n}: \dot{\vdash}(\forall_{\text{obj}} \overline{m}: \dot{\vdash}(\dot{\vdash}(0 <= \overline{\epsilon}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \overline{\epsilon})n)n)n \Rightarrow \dot{\vdash}(\overline{n} <= \overline{m} \Rightarrow \underline{fx})[\overline{m}] <= ((\underline{fy})[\overline{m}] + (-u(\overline{\epsilon}))))n)n)n)n)n) \vdash \dot{\vdash}(\dot{\vdash}(\forall_{\text{obj}} \overline{\epsilon}): \dot{\vdash}(\dot{\vdash}(\forall_{\text{obj}} \overline{n}: \dot{\vdash}(\forall_{\text{obj}} \overline{m}: \dot{\vdash}(\dot{\vdash}(0 <= \overline{\epsilon}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \overline{\epsilon})n)n)n \Rightarrow \dot{\vdash}(\overline{n} <= \overline{m} \Rightarrow \underline{fx})[\overline{m}] <= ((\underline{fy})[\overline{m}] + (-u(\overline{\epsilon}))))n)n)n)n)n)]$

$[\text{FromNot} << \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(\underline{fx}): \forall(\underline{fy}): \text{AutoImply} \gg$

$\dot{\vdash}(\forall_{\text{obj}} \overline{\epsilon}): \dot{\vdash}(\dot{\vdash}(\forall_{\text{obj}} \overline{n}: \dot{\vdash}(\forall_{\text{obj}} \overline{m}: \dot{\vdash}(\dot{\vdash}(0 <= \overline{\epsilon}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \overline{\epsilon})n)n)n \Rightarrow \dot{\vdash}(\overline{n} <= \overline{m} \Rightarrow \underline{fx})[\overline{m}] <= ((\underline{fy})[\overline{m}] + (-u(\overline{\epsilon}))))n)n)n)n)n) \Rightarrow$

$\dot{\vdash}(\forall_{\text{obj}} \overline{\epsilon}): \dot{\vdash}(\dot{\vdash}(\forall_{\text{obj}} \overline{n}: \dot{\vdash}(\forall_{\text{obj}} \overline{m}: \dot{\vdash}(\dot{\vdash}(0 <= \overline{\epsilon}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \overline{\epsilon})n)n)n \Rightarrow \dot{\vdash}(\overline{n} <= \overline{m} \Rightarrow \underline{fx})[\overline{m}] <=$

$\underline{x})n)n$; NegateDisjunct1 $\triangleright \dot{\vdash}(\dot{\vdash}(0 \leq \underline{x} \Rightarrow \dot{\vdash}(|\underline{x}| = \underline{x})n)n \Rightarrow \dot{\vdash}(\dot{\vdash}(0 \leq \underline{x})n \Rightarrow \dot{\vdash}(|\underline{x}| = (-\underline{ux}))n)n \triangleright \dot{\vdash}(\dot{\vdash}(0 \leq \underline{x} \Rightarrow \dot{\vdash}(|\underline{x}| = \underline{x})n)n) \gg \dot{\vdash}(\dot{\vdash}(0 \leq \underline{x})n \Rightarrow \dot{\vdash}(|\underline{x}| = (-\underline{ux}))n)n$; SecondConjunct $\triangleright \dot{\vdash}(\dot{\vdash}(0 \leq \underline{x})n \Rightarrow \dot{\vdash}(|\underline{x}| = (-\underline{ux}))n)n \gg |\underline{x}| = (-\underline{ux})]$, p_0, c]

[lemma nonpositiveNumerical $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \underline{x} \leq 0 \vdash |\underline{x}| = (-\underline{ux})]$

[lemma nonpositiveNumerical $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \vdash \text{NegativeNumerical} \triangleright \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \gg |\underline{x}| = (-\underline{ux}); \forall \underline{x}: \underline{x} = 0 \vdash \text{eqSymmetry} \triangleright \underline{x} = 0 \gg 0 = \underline{x}; \text{eqLeq} \triangleright 0 = \underline{x} \gg 0 \leq \underline{x}; \text{NonnegativeNumerical} \triangleright 0 \leq \underline{x} \gg |\underline{x}| = \underline{x}; -0 = 0 \gg (-u0) = 0; \text{eqSymmetry} \triangleright (-u0) = 0 \gg 0 = (-u0); \text{EqNegated} \triangleright 0 = \underline{x} \gg (-u0) = (-\underline{ux}); \text{eqTransitivity5} \triangleright |\underline{x}| = \underline{x} \triangleright \underline{x} = 0 \triangleright 0 = (-u0) \triangleright (-u0) = (-\underline{ux}) \gg |\underline{x}| = (-\underline{ux}); \forall \underline{x}: \text{Ded} \triangleright \forall \underline{x}: \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \vdash |\underline{x}| = (-\underline{ux}) \gg \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \Rightarrow |\underline{x}| = (-\underline{ux}); \text{Ded} \triangleright \forall \underline{x}: \underline{x} = 0 \vdash |\underline{x}| = (-\underline{ux}) \gg \underline{x} = 0 \Rightarrow |\underline{x}| = (-\underline{ux}); \underline{x} \leq 0 \vdash \text{LeqLessEq} \triangleright \underline{x} \leq 0 \gg \dot{\vdash}(\dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \Rightarrow \underline{x} = 0; \text{FromDisjuncts} \triangleright \dot{\vdash}(\dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \Rightarrow \underline{x} = 0 \triangleright \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \Rightarrow |\underline{x}| = (-\underline{ux}) \triangleright \underline{x} = 0 \Rightarrow |\underline{x}| = (-\underline{ux}) \gg |\underline{x}| = (-\underline{ux})]$, p_0, c]

[|0| = 0 $\xrightarrow{\text{stmt}}$ SystemQ $\vdash |0| = 0]$

[|0| = 0 $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \text{leqReflexivity} \gg 0 \leq 0; \text{NonnegativeNumerical} \triangleright 0 \leq 0 \gg |0| = 0]$, p_0, c]

[0 <= |x| $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: 0 \leq |\underline{x}|]$

[0 <= |x| $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: 0 \leq \underline{x} \vdash \text{NonnegativeNumerical} \triangleright 0 \leq \underline{x} \gg |\underline{x}| = \underline{x}; \text{eqSymmetry} \triangleright |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \text{subLeqRight} \triangleright \underline{x} = |\underline{x}| \triangleright 0 \leq \underline{x} \gg 0 \leq |\underline{x}|; \forall \underline{x}: \dot{\vdash}(0 \leq \underline{x})n \vdash \text{ToLess} \triangleright \dot{\vdash}(0 \leq \underline{x})n \gg \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n); \text{NegativeNumerical} \triangleright \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \gg |\underline{x}| = (-\underline{ux}); \text{eqSymmetry} \triangleright |\underline{x}| = (-\underline{ux}) \gg (-\underline{ux}) = |\underline{x}|; \text{NegativeNegated} \triangleright \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \gg \dot{\vdash}(0 \leq (-\underline{ux}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (-\underline{ux})n)n); \text{LessLeq} \triangleright \dot{\vdash}(0 \leq (-\underline{ux}) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = (-\underline{ux})n)n) \gg 0 \leq (-\underline{ux}); \text{subLeqRight} \triangleright (-\underline{ux}) = |\underline{x}| \triangleright 0 \leq (-\underline{ux}) \gg 0 \leq |\underline{x}|; \forall \underline{x}: \text{Ded} \triangleright \forall \underline{x}: 0 \leq \underline{x} \vdash 0 \leq |\underline{x}| \gg 0 \leq \underline{x} \Rightarrow 0 \leq |\underline{x}|; \text{Ded} \triangleright \forall \underline{x}: \dot{\vdash}(0 \leq \underline{x})n \vdash 0 \leq |\underline{x}| \gg \dot{\vdash}(0 \leq \underline{x})n \Rightarrow 0 \leq |\underline{x}|; \text{FromNegations} \triangleright 0 \leq \underline{x} \Rightarrow 0 \leq |\underline{x}| \triangleright \dot{\vdash}(0 \leq \underline{x})n \Rightarrow 0 \leq |\underline{x}| \gg 0 \leq |\underline{x}|]$, p_0, c]

[SameNumerical $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}|]$

[SameNumerical $\xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda y. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{x} = \underline{y} \vdash \text{NonnegativeNumerical} \triangleright 0 \leq \underline{x} \gg |\underline{x}| = \underline{x}; \text{subLeqRight} \triangleright \underline{x} = \underline{y} \triangleright 0 \leq \underline{x} \gg 0 \leq \underline{y}; \text{NonnegativeNumerical} \triangleright 0 \leq \underline{y} \gg |\underline{y}| = \underline{y}; \text{eqSymmetry} \triangleright |\underline{y}| = \underline{y} \gg \underline{y} = |\underline{y}|; \text{eqTransitivity4} \triangleright |\underline{x}| = \underline{x} \triangleright \underline{x} = \underline{y} \triangleright \underline{y} = |\underline{y}| \gg |\underline{x}| = |\underline{y}|; \forall \underline{x}: \forall \underline{y}: \dot{\vdash}(0 \leq \underline{x})n \vdash \underline{x} = \underline{y} \vdash \text{ToLess} \triangleright \dot{\vdash}(0 \leq \underline{x})n \gg \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n); \text{NegativeNumerical} \triangleright \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \gg |\underline{x}| = (-\underline{ux}); \text{SubLessLeft} \triangleright \underline{x} = \underline{y} \triangleright \dot{\vdash}(\underline{x} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{x} = 0)n)n) \gg \dot{\vdash}(\underline{y} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{y} = 0)n)n); \text{NegativeNumerical} \triangleright \dot{\vdash}(\underline{y} \leq 0 \Rightarrow \dot{\vdash}(\dot{\vdash}(\underline{y} = 0)n)n) \gg |\underline{y}| = (-\underline{uy}); \text{eqSymmetry} \triangleright |\underline{y}| = (-\underline{uy}) \gg (-\underline{uy}) = |\underline{y}|; \text{EqNegated} \triangleright \underline{x} = \underline{y} \gg (-\underline{ux}) = (-\underline{uy}); \text{eqTransitivity4} \triangleright |\underline{x}| = (-\underline{ux}) \triangleright (-\underline{ux}) = (-\underline{uy}) \triangleright (-\underline{uy}) = |\underline{y}| \gg |\underline{x}| =$

$$\begin{aligned} & \underline{y}; \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}| \gg 0 \leq \underline{x} \Rightarrow \underline{x} = \underline{y} \Rightarrow \\ & |\underline{x}| = |\underline{y}|; \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (0 \leq \underline{x})n \vdash \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}| \gg \dot{\vdash} (0 \leq \underline{x})n \Rightarrow \underline{x} = \underline{y} \Rightarrow \\ & |\underline{x}| = |\underline{y}|; \text{FromNegations} \triangleright 0 \leq \underline{x} \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \triangleright \dot{\vdash} (0 \leq \underline{x})n \Rightarrow \underline{x} = \underline{y} \Rightarrow \\ & |\underline{x}| = |\underline{y}| \gg \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \text{MP} \triangleright \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \triangleright \underline{x} = \underline{y} \gg |\underline{x}| = |\underline{y}|, p_0, c) \\ & [\text{SignNumerical}(+) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \vdash |\underline{x}| = \\ & |(-\underline{u}\underline{x})|] \end{aligned}$$

$$\begin{aligned} & [\text{SignNumerical}(+) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x})n)n) \vdash \text{PositiveNumerical} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg |\underline{x}| = \\ & \underline{x}; \text{PositiveNegated} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg \dot{\vdash} ((-\underline{u}\underline{x}) \leq 0 \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} ((-\underline{u}\underline{x}) = 0)n)n); \text{NegativeNumerical} \triangleright \dot{\vdash} ((-\underline{u}\underline{x}) \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{u}\underline{x}) = \\ & 0)n)n) \gg |(-\underline{u}\underline{x})| = (-\underline{u}(-\underline{u}\underline{x})); \text{DoubleMinus} \gg (-\underline{u}(-\underline{u}\underline{x})) = \\ & \underline{x}; \text{eqTransitivity} \triangleright |(-\underline{u}\underline{x})| = (-\underline{u}(-\underline{u}\underline{x})) \triangleright (-\underline{u}(-\underline{u}\underline{x})) = \underline{x} \gg |(-\underline{u}\underline{x})| = \\ & \underline{x}; \text{eqSymmetry} \triangleright |(-\underline{u}\underline{x})| = \underline{x} \gg \underline{x} = |(-\underline{u}\underline{x})|; \text{eqTransitivity} \triangleright |\underline{x}| = \underline{x} \triangleright \underline{x} = \\ & |(-\underline{u}\underline{x})| \gg |\underline{x}| = |(-\underline{u}\underline{x})|, p_0, c)] \end{aligned}$$

$$[\text{SignNumerical} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: |\underline{x}| = |(-\underline{u}\underline{x})|]$$

$$\begin{aligned} & [\text{SignNumerical} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \vdash \text{NegativeNegated} \triangleright \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \gg \dot{\vdash} (0 \leq (-\underline{u}\underline{x}) \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = (-\underline{u}\underline{x})n)n); \text{SignNumerical}(+) \triangleright \dot{\vdash} (0 \leq (-\underline{u}\underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & (-\underline{u}\underline{x})n)n) \gg |(-\underline{u}\underline{x})| = |(-\underline{u}(-\underline{u}\underline{x}))|; \text{DoubleMinus} \gg (-\underline{u}(-\underline{u}\underline{x})) = \\ & \underline{x}; \text{SameNumerical} \triangleright (-\underline{u}(-\underline{u}\underline{x})) = \underline{x} \gg |(-\underline{u}(-\underline{u}\underline{x}))| = \\ & |\underline{x}|; \text{eqTransitivity} \triangleright |(-\underline{u}\underline{x})| = |(-\underline{u}(-\underline{u}\underline{x}))| \triangleright |(-\underline{u}(-\underline{u}\underline{x}))| = |\underline{x}| \gg |(-\underline{u}\underline{x})| = \\ & |\underline{x}|; \text{eqSymmetry} \triangleright |(-\underline{u}\underline{x})| = |\underline{x}| \gg |\underline{x}| = |(-\underline{u}\underline{x})|; \forall \underline{x}: \underline{x} = 0 \vdash \text{EqNegated} \triangleright \underline{x} = \\ & 0 \gg (-\underline{u}\underline{x}) = (-\underline{u}0); -0 = 0 \gg (-\underline{u}0) = 0; \text{eqSymmetry} \triangleright \underline{x} = 0 \gg 0 = \\ & \underline{x}; \text{eqTransitivity4} \triangleright (-\underline{u}\underline{x}) = (-\underline{u}0) \triangleright (-\underline{u}0) = 0 \triangleright 0 = \underline{x} \gg (-\underline{u}\underline{x}) = \\ & \underline{x}; \text{eqSymmetry} \triangleright (-\underline{u}\underline{x}) = \underline{x} \gg \underline{x} = (-\underline{u}\underline{x}); \text{SameNumerical} \triangleright \underline{x} = (-\underline{u}\underline{x}) \gg |\underline{x}| = \\ & |(-\underline{u}\underline{x})|; \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \vdash \text{SignNumerical}(+) \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg |\underline{x}| = |(-\underline{u}\underline{x})|; \forall \underline{x}: \text{Ded} \triangleright \forall \underline{x}: \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \\ & 0)n)n) \vdash |\underline{x}| = |(-\underline{u}\underline{x})| \gg \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \Rightarrow |\underline{x}| = \\ & |(-\underline{u}\underline{x})|; \text{Ded} \triangleright \forall \underline{x}: \underline{x} = 0 \vdash |\underline{x}| = |(-\underline{u}\underline{x})| \gg \underline{x} = 0 \Rightarrow |\underline{x}| = \\ & |(-\underline{u}\underline{x})|; \text{Ded} \triangleright \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \vdash |\underline{x}| = |(-\underline{u}\underline{x})| \gg \dot{\vdash} (0 \leq \\ & \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \Rightarrow |\underline{x}| = |(-\underline{u}\underline{x})|; \text{LessTotality} \gg \dot{\vdash} (\dot{\vdash} (\underline{x} \leq 0 \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \Rightarrow \dot{\vdash} (\underline{x} = 0)n) \Rightarrow \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \underline{x})n)n); \text{From3Disjuncts} \triangleright \dot{\vdash} (\dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \Rightarrow \dot{\vdash} (\underline{x} = \\ & 0)n) \Rightarrow \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \triangleright \dot{\vdash} (\underline{x} \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \Rightarrow \\ & |\underline{x}| = |(-\underline{u}\underline{x})| \triangleright \underline{x} = 0 \Rightarrow |\underline{x}| = |(-\underline{u}\underline{x})| \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \Rightarrow |\underline{x}| = \\ & |(-\underline{u}\underline{x})| \gg |\underline{x}| = |(-\underline{u}\underline{x})|, p_0, c)] \end{aligned}$$

$$[-x - y = -(x + y) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = (-\underline{u}(\underline{x} + \underline{y}))]$$

$$\begin{aligned} & [-x - y = -(x + y) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \text{Times}(-1)\text{Left} \gg \\ & ((-\underline{u}1) * \underline{x}) = (-\underline{u}\underline{x}); \text{Times}(-1)\text{Left} \gg ((-\underline{u}1) * \underline{y}) = (-\underline{u}\underline{y}); \text{AddEquations} \triangleright \\ & ((-\underline{u}1) * \underline{x}) = (-\underline{u}\underline{x}) \triangleright ((-\underline{u}1) * \underline{y}) = (-\underline{u}\underline{y}) \gg (((-\underline{u}1) * \underline{x}) + ((-\underline{u}1) * \underline{y})) = \\ & ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})); \text{eqSymmetry} \triangleright (((-\underline{u}1) * \underline{x}) + ((-\underline{u}1) * \underline{y})) = ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) \gg \\ & ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = (((-\underline{u}1) * \underline{x}) + ((-\underline{u}1) * \underline{y})); \text{DistributionOut} \gg \\ & (((-\underline{u}1) * \underline{x}) + ((-\underline{u}1) * \underline{y})) = ((-\underline{u}1) * (\underline{x} + \underline{y})); \text{Times}(-1)\text{Left} \gg \\ & ((-\underline{u}1) * (\underline{x} + \underline{y})) = (-\underline{u}(\underline{x} + \underline{y})); \text{eqTransitivity4} \triangleright ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = \end{aligned}$$

$$\begin{aligned} &(((\neg u1) * \underline{x}) + ((\neg u1) * \underline{y})) \triangleright (((\neg u1) * \underline{x}) + ((\neg u1) * \underline{y})) = ((\neg u1) * (\underline{x} + \underline{y})) \triangleright \\ &((\neg u1) * (\underline{x} + \underline{y})) = (-u(\underline{x} + \underline{y})) \ggg ((-\underline{ux}) + (-\underline{uy})) = (-u(\underline{x} + \underline{y})), p_0, c] \end{aligned}$$

$$[\text{MinusNegated} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: (-u(\underline{x} + (-\underline{uy}))) = (\underline{y} + (-\underline{ux}))]$$

$$\begin{aligned} &[\text{MinusNegated} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \text{DoubleMinus} \ggg (-u(-\underline{uy})) = \\ &\underline{y}; \text{eqAddition} \triangleright (-u(-\underline{uy})) = \underline{y} \ggg ((-\underline{u}(-\underline{uy})) + (-\underline{ux})) = \\ &(\underline{y} + (-\underline{ux})); \text{eqSymmetry} \triangleright ((-\underline{u}(-\underline{uy})) + (-\underline{ux})) = (\underline{y} + (-\underline{ux})) \ggg \\ &(\underline{y} + (-\underline{ux})) = ((-\underline{u}(-\underline{uy})) + (-\underline{ux})); -x - y = -(x + y) \ggg \\ &((-\underline{u}(-\underline{uy})) + (-\underline{ux})) = (-u((-\underline{uy}) + \underline{x})); \text{plusCommutativity} \ggg ((-\underline{uy}) + \underline{x}) = \\ &(\underline{x} + (-\underline{uy})); \text{EqNegated} \triangleright ((-\underline{uy}) + \underline{x}) = (\underline{x} + (-\underline{uy})) \ggg (-u((-\underline{uy}) + \underline{x})) = \\ &(-u(\underline{x} + (-\underline{uy}))); \text{eqTransitivity4} \triangleright (\underline{y} + (-\underline{ux})) = ((-\underline{u}(-\underline{uy})) + (-\underline{ux})) \triangleright \\ &((-\underline{u}(-\underline{uy})) + (-\underline{ux})) = (-u((-\underline{uy}) + \underline{x})) \triangleright (-u((-\underline{uy}) + \underline{x})) = \\ &(-u(\underline{x} + (-\underline{uy}))) \ggg (\underline{y} + (-\underline{ux})) = (-u(\underline{x} + (-\underline{uy}))); \text{eqSymmetry} \triangleright (\underline{y} + (-\underline{ux})) = \\ &(-u(\underline{x} + (-\underline{uy}))) \ggg (-u(\underline{x} + (-\underline{uy}))) = (\underline{y} + (-\underline{ux})), p_0, c] \end{aligned}$$

$$[\text{NumericalDifference} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: |(\underline{x} + (-\underline{uy}))| = |(\underline{y} + (-\underline{ux}))|]$$

$$\begin{aligned} &[\text{NumericalDifference} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \text{SignNumerical} \ggg \\ &|(\underline{x} + (-\underline{uy}))| = |(-u(\underline{x} + (-\underline{uy})))|; \text{MinusNegated} \ggg (-u(\underline{x} + (-\underline{uy}))) = \\ &(\underline{y} + (-\underline{ux})); \text{SameNumerical} \triangleright (-u(\underline{x} + (-\underline{uy}))) = (\underline{y} + (-\underline{ux})) \ggg \\ &|(-u(\underline{x} + (-\underline{uy})))| = |(\underline{y} + (-\underline{ux}))|; \text{eqTransitivity} \triangleright |(\underline{x} + (-\underline{uy}))| = \\ &|(-u(\underline{x} + (-\underline{uy})))| \triangleright |(-u(\underline{x} + (-\underline{uy})))| = |(\underline{y} + (-\underline{ux}))| \ggg |(\underline{x} + (-\underline{uy}))| = \\ &|(\underline{y} + (-\underline{ux}))|], p_0, c] \end{aligned}$$

$$[\text{SplitNumericalSumHelper} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: |((-\underline{ux}) + (-\underline{uy}))| \leq = \\ |(-\underline{ux})| + |(-\underline{uy})| \vdash |(\underline{x} + \underline{y})| \leq = (|\underline{x}| + |\underline{y}|)]$$

$$\begin{aligned} &[\text{SplitNumericalSumHelper} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \\ &\forall \underline{x}: \forall \underline{y}: |((-\underline{ux}) + (-\underline{uy}))| \leq = (|(-\underline{ux})| + |(-\underline{uy})|) \vdash \text{SignNumerical} \ggg |\underline{x}| = \\ &|(-\underline{ux})|; \text{SignNumerical} \ggg |\underline{y}| = |(-\underline{uy})|; \text{AddEquations} \triangleright |\underline{x}| = |(-\underline{ux})| \triangleright |\underline{y}| = \\ &|(-\underline{uy})| \ggg (|\underline{x}| + |\underline{y}|) = (|(-\underline{ux})| + |(-\underline{uy})|); \text{eqSymmetry} \triangleright (|\underline{x}| + |\underline{y}|) = \\ &(|(-\underline{ux})| + |(-\underline{uy})|) \ggg (|(-\underline{ux})| + |(-\underline{uy})|) = (|\underline{x}| + |\underline{y}|); -x - y = -(x + y) \ggg \\ &((-\underline{ux}) + (-\underline{uy})) = (-u(\underline{x} + \underline{y})); \text{SameNumerical} \triangleright ((-\underline{ux}) + (-\underline{uy})) = \\ &(-u(\underline{x} + \underline{y})) \ggg |((-\underline{ux}) + (-\underline{uy}))| = |(-u(\underline{x} + \underline{y}))|; \text{SignNumerical} \ggg |(\underline{x} + \underline{y})| = \\ &|(-u(\underline{x} + \underline{y}))|; \text{eqSymmetry} \triangleright |(\underline{x} + \underline{y})| = |(-u(\underline{x} + \underline{y}))| \ggg |(-u(\underline{x} + \underline{y}))| = \\ &|(\underline{x} + \underline{y})|; \text{eqTransitivity} \triangleright |((-\underline{ux}) + (-\underline{uy}))| = |(-u(\underline{x} + \underline{y}))| \triangleright |(-u(\underline{x} + \underline{y}))| = \\ &|(\underline{x} + \underline{y})| \ggg |((-\underline{ux}) + (-\underline{uy}))| = |(\underline{x} + \underline{y})|; \text{subLeqRight} \triangleright (|(-\underline{ux})| + |(-\underline{uy})|) = \\ &(|\underline{x}| + |\underline{y}|) \triangleright |((-\underline{ux}) + (-\underline{uy}))| \leq = (|(-\underline{ux})| + |(-\underline{uy})|) \ggg |((-\underline{ux}) + (-\underline{uy}))| \leq = \\ &(|\underline{x}| + |\underline{y}|); \text{subLeqLeft} \triangleright |((-\underline{ux}) + (-\underline{uy}))| = |(\underline{x} + \underline{y})| \triangleright |((-\underline{ux}) + (-\underline{uy}))| \leq = \\ &(|\underline{x}| + |\underline{y}|) \ggg |(\underline{x} + \underline{y})| \leq = (|\underline{x}| + |\underline{y}|)], p_0, c] \end{aligned}$$

$$[\text{splitNumericalSum}(++) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq = \underline{x} \vdash 0 \leq = \underline{y} \vdash \\ |(\underline{x} + \underline{y})| \leq = (|\underline{x}| + |\underline{y}|)]$$

$$\begin{aligned} &[\text{splitNumericalSum}(++) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq = \underline{x} \vdash 0 \leq = \underline{y} \vdash \\ &\text{AddEquations}(\text{Leq}) \triangleright 0 \leq = \underline{x} \triangleright 0 \leq = \underline{y} \ggg (0 + 0) \leq = (\underline{x} + \underline{y}); \text{plus0} \ggg \\ &(0 + 0) = 0; \text{subLeqLeft} \triangleright (0 + 0) = 0 \triangleright (0 + 0) \leq = (\underline{x} + \underline{y}) \ggg 0 \leq = \\ &(\underline{x} + \underline{y}); \text{NonnegativeNumerical} \triangleright 0 \leq = (\underline{x} + \underline{y}) \ggg |(\underline{x} + \underline{y})| = \\ &(\underline{x} + \underline{y}); \text{NonnegativeNumerical} \triangleright 0 \leq = \underline{x} \ggg |\underline{x}| = \\ &\underline{x}; \text{NonnegativeNumerical} \triangleright 0 \leq = \underline{y} \ggg |\underline{y}| = \underline{y}; \text{AddEquations} \triangleright |\underline{x}| = \underline{x} \triangleright |\underline{y}| = \end{aligned}$$

$\underline{y} \gg (|\underline{x}| + |\underline{y}|) = (\underline{x} + \underline{y}); \text{eqSymmetry} \triangleright (|\underline{x}| + |\underline{y}|) = (\underline{x} + \underline{y}) \gg (\underline{x} + \underline{y}) = (|\underline{x}| + |\underline{y}|); \text{eqTransitivity} \triangleright |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y}) \triangleright (\underline{x} + \underline{y}) = (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| = (|\underline{x}| + |\underline{y}|); \text{eqLeq} \triangleright |(\underline{x} + \underline{y})| = (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| \leq (|\underline{x}| + |\underline{y}|), p_0, c)$

$[\text{splitNumericalSum}(\text{--}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \leq 0 \vdash \underline{y} \leq 0 \vdash (|\underline{x} + \underline{y}| \leq (|\underline{x}| + |\underline{y}|))]$

$[\text{splitNumericalSum}(\text{--}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \leq 0 \vdash \underline{y} \leq 0 \vdash \text{NonpositiveNegated} \triangleright \underline{x} \leq 0 \gg 0 \leq (-\underline{u}\underline{x}); \text{NonpositiveNegated} \triangleright \underline{y} \leq 0 \gg 0 \leq (-\underline{u}\underline{y}); \text{splitNumericalSum}(++) \triangleright 0 \leq (-\underline{u}\underline{x}) \triangleright 0 \leq (-\underline{u}\underline{y}) \gg |((-\underline{u}\underline{x}) + (-\underline{u}\underline{y}))| \leq (|(-\underline{u}\underline{x})| + |(-\underline{u}\underline{y})|); \text{SplitNumericalSumHelper} \triangleright |((-\underline{u}\underline{x}) + (-\underline{u}\underline{y}))| \leq (|(-\underline{u}\underline{x})| + |(-\underline{u}\underline{y})|) \gg |(\underline{x} + \underline{y})| \leq (|\underline{x}| + |\underline{y}|), p_0, c)]$

$[\text{splitNumericalSum}(+ - \text{small}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{y} \leq 0 \vdash |\underline{y}| \leq |\underline{x}| \vdash (|\underline{x} + \underline{y}| \leq |\underline{x}|)]$

$[\text{splitNumericalSum}(+ - \text{small}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{y} \leq 0 \vdash |\underline{y}| \leq |\underline{x}| \vdash \text{LeqAdditionLeft} \triangleright \underline{y} \leq 0 \gg (\underline{x} + \underline{y}) \leq (\underline{x} + 0); \text{plus0} \gg (\underline{x} + 0) = \underline{x}; \text{subLeqRight} \triangleright (\underline{x} + 0) = \underline{x} \triangleright (\underline{x} + \underline{y}) \leq (\underline{x} + 0) \gg (\underline{x} + \underline{y}) \leq \underline{x}; \text{PositiveToRight}(\text{Leq})(1\text{term}) \triangleright |\underline{y}| \leq |\underline{x}| \gg 0 \leq (|\underline{x}| + (-\underline{u}|\underline{y}|)); \text{lemma nonpositiveNumerical} \triangleright \underline{y} \leq 0 \gg |\underline{y}| = (-\underline{u}\underline{y}); \text{EqNegated} \triangleright |\underline{y}| = (-\underline{u}\underline{y}) \gg (-\underline{u}|\underline{y}|) = (-\underline{u}(-\underline{u}\underline{y})); \text{DoubleMinus} \gg (-\underline{u}(-\underline{u}\underline{y})) = \underline{y}; \text{eqTransitivity} \triangleright (-\underline{u}|\underline{y}|) = (-\underline{u}(-\underline{u}\underline{y})) \triangleright (-\underline{u}(-\underline{u}\underline{y})) = \underline{y} \gg (-\underline{u}|\underline{y}|) = \underline{y}; \text{NonnegativeNumerical} \triangleright 0 \leq \underline{x} \gg |\underline{x}| = \underline{x}; \text{AddEquations} \triangleright |\underline{x}| = \underline{x} \triangleright (-\underline{u}|\underline{y}|) = \underline{y} \gg (|\underline{x}| + (-\underline{u}|\underline{y}|)) = (\underline{x} + \underline{y}); \text{subLeqRight} \triangleright (|\underline{x}| + (-\underline{u}|\underline{y}|)) = (\underline{x} + \underline{y}) \triangleright 0 \leq (|\underline{x}| + (-\underline{u}|\underline{y}|)) \gg 0 \leq (\underline{x} + \underline{y}); \text{NonnegativeNumerical} \triangleright 0 \leq (\underline{x} + \underline{y}) \gg |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y}); \text{eqSymmetry} \triangleright |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y}) \gg (\underline{x} + \underline{y}) = |(\underline{x} + \underline{y})|; \text{eqSymmetry} \triangleright |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \text{subLeqLeft} \triangleright (\underline{x} + \underline{y}) = |(\underline{x} + \underline{y})| \triangleright (\underline{x} + \underline{y}) \leq \underline{x} \gg |(\underline{x} + \underline{y})| \leq \underline{x}; \text{subLeqRight} \triangleright \underline{x} = |\underline{x}| \triangleright |(\underline{x} + \underline{y})| \leq \underline{x} \gg |(\underline{x} + \underline{y})| \leq |\underline{x}|, p_0, c)]$

$[\text{splitNumericalSum}(+ - \text{big}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{y} \leq 0 \vdash \dot{\vdash} (|\underline{x}| \leq |\underline{y}| \Rightarrow \dot{\vdash} (\dot{\vdash} (|\underline{x}| = |\underline{y}|)n)n)n \vdash |(\underline{x} + \underline{y})| \leq |\underline{y}|]$

$[\text{splitNumericalSum}(+ - \text{big}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{y} \leq 0 \vdash \dot{\vdash} (|\underline{x}| \leq |\underline{y}| \Rightarrow \dot{\vdash} (\dot{\vdash} (|\underline{x}| = |\underline{y}|)n)n)n \vdash \text{NonnegativeNegated} \triangleright 0 \leq \underline{x} \gg (-\underline{u}\underline{x}) \leq 0; \text{NonpositiveNegated} \triangleright \underline{y} \leq 0 \gg 0 \leq (-\underline{u}\underline{y}); \text{SignNumerical} \gg |\underline{x}| = |(-\underline{u}\underline{x})|; \text{SubLessLeft} \triangleright |\underline{x}| = |(-\underline{u}\underline{x})| \triangleright \dot{\vdash} (|\underline{x}| \leq |\underline{y}| \Rightarrow \dot{\vdash} (\dot{\vdash} (|\underline{x}| = |\underline{y}|)n)n)n \gg \dot{\vdash} (|(-\underline{u}\underline{x})| \leq |\underline{y}| \Rightarrow \dot{\vdash} (\dot{\vdash} (|(-\underline{u}\underline{x})| = |\underline{y}|)n)n)n; \text{SignNumerical} \gg |\underline{y}| = |(-\underline{u}\underline{y})|; \text{SubLessRight} \triangleright |\underline{y}| = |(-\underline{u}\underline{y})| \triangleright \dot{\vdash} (|(-\underline{u}\underline{x})| \leq |\underline{y}| \Rightarrow \dot{\vdash} (\dot{\vdash} (|(-\underline{u}\underline{x})| = |\underline{y}|)n)n)n \gg \dot{\vdash} (|(-\underline{u}\underline{x})| \leq |(-\underline{u}\underline{y})| \Rightarrow \dot{\vdash} (\dot{\vdash} (|(-\underline{u}\underline{x})| = |(-\underline{u}\underline{y})|)n)n)n; \text{LessLeq} \triangleright \dot{\vdash} (|(-\underline{u}\underline{x})| \leq |(-\underline{u}\underline{y})| \Rightarrow \dot{\vdash} (\dot{\vdash} (|(-\underline{u}\underline{x})| = |(-\underline{u}\underline{y})|)n)n)n \gg |(-\underline{u}\underline{x})| \leq |(-\underline{u}\underline{y})|; \text{splitNumericalSum}(+ - \text{small}) \triangleright 0 \leq (-\underline{u}\underline{y}) \triangleright (-\underline{u}\underline{x}) \leq 0 \triangleright |(-\underline{u}\underline{x})| \leq |(-\underline{u}\underline{y})| \gg |((- \underline{u}\underline{y}) + (-\underline{u}\underline{x}))| \leq |(-\underline{u}\underline{y})|; \text{SignNumerical} \gg |(\underline{x} + \underline{y})| = |(-\underline{u}(\underline{x} + \underline{y}))|; -\underline{x} - \underline{y} = -(\underline{x} + \underline{y}) \gg ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = (-\underline{u}(\underline{x} + \underline{y})); \text{plusCommutativity} \gg ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = ((-\underline{u}\underline{y}) + (-\underline{u}\underline{x})); \text{Equality} \triangleright ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = (-\underline{u}(\underline{x} + \underline{y})) \triangleright ((-\underline{u}\underline{x}) + (-\underline{u}\underline{y})) = ((-\underline{u}\underline{y}) + (-\underline{u}\underline{x})) \gg (-\underline{u}(\underline{x} + \underline{y})) = ((-\underline{u}\underline{y}) + (-\underline{u}\underline{x})); \text{SameNumerical} \triangleright (-\underline{u}(\underline{x} + \underline{y})) = ((-\underline{u}\underline{y}) + (-\underline{u}\underline{x})) \gg |(-\underline{u}(\underline{x} + \underline{y}))| =$

$$\begin{aligned}
& |((-uy) + (-ux))|; \text{eqTransitivity} \triangleright |(x + y)| = |(-u(x + y))| \triangleright |(-u(x + y))| = \\
& |((-uy) + (-ux))| \gg |(x + y)| = |((-uy) + (-ux))|; \text{eqSymmetry} \triangleright |(x + y)| = \\
& |((-uy) + (-ux))| \gg |((-uy) + (-ux))| = |(x + y)|; \text{eqSymmetry} \triangleright |y| = |(-uy)| \gg \\
& |(-uy)| = |y|; \text{subLeqLeft} \triangleright |((-uy) + (-ux))| = |(x + y)| \triangleright |((-uy) + (-ux))| \leq = \\
& |(-uy)| \gg |(x + y)| \leq = |(-uy)|; \text{subLeqRight} \triangleright |(-uy)| = |y| \triangleright |(x + y)| \leq = \\
& |(-uy)| \gg |(x + y)| \leq = |y|, p_0, c)
\end{aligned}$$

$$\begin{aligned}
& [\text{splitNumericalSum}(+-) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: \forall y: 0 \leq x \vdash y \leq 0 \vdash \\
& |(x + y)| \leq = (|x| + |y|)]
\end{aligned}$$

$$\begin{aligned}
& [\text{splitNumericalSum}(+-) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: |y| \leq |x| \vdash 0 \leq = \\
& x \vdash y \leq = 0 \vdash \text{splitNumericalSum}(+ - \text{small}) \triangleright 0 \leq = x \triangleright y \leq = 0 \triangleright |y| \leq = |x| \gg \\
& |(x + y)| \leq = |x|; 0 \leq = |x| \gg 0 \leq = |y|; \text{LeqAdditionLeft} \triangleright 0 \leq = |y| \gg \\
& (|x| + 0) \leq = (|x| + |y|); \text{plus0} \gg (|x| + 0) = |x|; \text{subLeqLeft} \triangleright (|x| + 0) = \\
& |x| \triangleright (|x| + 0) \leq = (|x| + |y|) \gg |x| \leq = (|x| + |y|); \text{leqTransitivity} \triangleright |(x + y)| \leq = \\
& |x| \triangleright |x| \leq = (|x| + |y|) \gg |(x + y)| \leq = (|x| + |y|); \forall x: \forall y: \dot{\vdash} (|y| \leq = |x|) \text{n} \vdash 0 \leq = \\
& x \vdash y \leq = 0 \vdash \text{ToLess} \triangleright \dot{\vdash} (|y| \leq = |x|) \text{n} \gg \dot{\vdash} (|x| \leq = |y|) \Rightarrow \dot{\vdash} (\dot{\vdash} (|x| = \\
& |y|) \text{n}) \text{n}; \text{splitNumericalSum}(+ - \text{big}) \triangleright 0 \leq = x \triangleright y \leq = 0 \triangleright \dot{\vdash} (|x| \leq = |y|) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (|x| = |y|) \text{n}) \text{n} \gg |(x + y)| \leq = |y|; 0 \leq = |x| \gg 0 \leq = \\
& |x|; \text{leqAddition} \triangleright 0 \leq = |x| \gg (0 + |y|) \leq = (|x| + |y|); \text{lemma plus0Left} \gg \\
& (0 + |y|) = |y|; \text{subLeqLeft} \triangleright (0 + |y|) = |y| \triangleright (0 + |y|) \leq = (|x| + |y|) \gg |y| \leq = \\
& (|x| + |y|); \text{leqTransitivity} \triangleright |(x + y)| \leq = |y| \triangleright |y| \leq = (|x| + |y|) \gg |(x + y)| \leq = \\
& (|x| + |y|); \forall x: \forall y: \text{Ded} \triangleright \forall x: \forall y: |y| \leq = |x| \vdash 0 \leq = x \vdash y \leq = 0 \vdash |(x + y)| \leq = \\
& (|x| + |y|) \gg |y| \leq = |x| \Rightarrow 0 \leq = x \Rightarrow y \leq = 0 \Rightarrow |(x + y)| \leq = \\
& (|x| + |y|); \text{Ded} \triangleright \forall x: \forall y: \dot{\vdash} (|y| \leq = |x|) \text{n} \vdash 0 \leq = x \vdash y \leq = 0 \vdash |(x + y)| \leq = \\
& (|x| + |y|) \gg \dot{\vdash} (|y| \leq = |x|) \text{n} \Rightarrow 0 \leq = x \Rightarrow y \leq = 0 \Rightarrow |(x + y)| \leq = (|x| + |y|); 0 \leq = \\
& x \vdash y \leq = 0 \vdash \text{FromNegations} \triangleright |y| \leq = |x| \Rightarrow 0 \leq = x \Rightarrow y \leq = 0 \Rightarrow |(x + y)| \leq = \\
& (|x| + |y|) \triangleright \dot{\vdash} (|y| \leq = |x|) \text{n} \Rightarrow 0 \leq = x \Rightarrow y \leq = 0 \Rightarrow |(x + y)| \leq = (|x| + |y|) \gg \\
& 0 \leq = x \Rightarrow y \leq = 0 \Rightarrow |(x + y)| \leq = (|x| + |y|); \text{MP2} \triangleright 0 \leq = x \Rightarrow y \leq = 0 \Rightarrow \\
& |(x + y)| \leq = (|x| + |y|) \triangleright 0 \leq = x \triangleright y \leq = 0 \gg |(x + y)| \leq = (|x| + |y|), p_0, c)
\end{aligned}$$

$$\begin{aligned}
& [\text{splitNumericalSum}(+-) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: \forall y: x \leq = 0 \vdash 0 \leq = y \vdash \\
& |(x + y)| \leq = (|x| + |y|)]
\end{aligned}$$

$$\begin{aligned}
& [\text{splitNumericalSum}(+-) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: x \leq = 0 \vdash 0 \leq = y \vdash \\
& \text{NonpositiveNegated} \triangleright x \leq = 0 \gg 0 \leq = (-ux); \text{NonnegativeNegated} \triangleright 0 \leq = \\
& y \gg (-uy) \leq = 0; \text{splitNumericalSum}(+ -) \triangleright 0 \leq = (-ux) \triangleright (-uy) \leq = 0 \gg \\
& |((-ux) + (-uy))| \leq = (|(-ux)| + |(-uy)|); \text{SplitNumericalSumHelper} \triangleright \\
& |((-ux) + (-uy))| \leq = (|(-ux)| + |(-uy)|) \gg |(x + y)| \leq = (|x| + |y|), p_0, c)
\end{aligned}$$

$$\begin{aligned}
& [\text{splitNumericalSum} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: \forall y: |(x + y)| \leq = (|x| + |y|)]
\end{aligned}$$

$$\begin{aligned}
& [\text{splitNumericalSum} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: 0 \leq = x \vdash 0 \leq = y \vdash \\
& \text{splitNumericalSum}(++) \triangleright 0 \leq = x \triangleright 0 \leq = y \gg |(x + y)| \leq = \\
& (|x| + |y|); \forall x: \forall y: 0 \leq = x \vdash y \leq = 0 \vdash \text{splitNumericalSum}(+ -) \triangleright 0 \leq = x \triangleright y \leq = \\
& 0 \gg |(x + y)| \leq = (|x| + |y|); \forall x: \forall y: x \leq = 0 \vdash 0 \leq = y \vdash \\
& \text{splitNumericalSum}(+-) \triangleright x \leq = 0 \triangleright 0 \leq = y \gg |(x + y)| \leq = \\
& (|x| + |y|); \forall x: \forall y: x \leq = 0 \vdash y \leq = 0 \vdash \text{splitNumericalSum}(--) \triangleright x \leq = 0 \triangleright y \leq = \\
& 0 \gg |(x + y)| \leq = (|x| + |y|); \forall x: \forall y: \text{Ded} \triangleright \forall x: \forall y: 0 \leq = x \vdash 0 \leq = y \vdash |(x + y)| \leq =
\end{aligned}$$

$$\begin{aligned}
& (|\underline{x}| + |\underline{y}|) \gg 0 \leq \underline{x} \Rightarrow 0 \leq \underline{y} \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|); \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: 0 \leq \underline{x} \vdash \underline{y} \leq 0 \vdash (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \gg 0 \leq \underline{x} \Rightarrow \underline{y} \leq 0 \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|); \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \underline{x} \leq 0 \vdash 0 \leq \underline{y} \vdash (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \gg \underline{x} \leq 0 \Rightarrow 0 \leq \underline{y} \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|); \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \underline{x} \leq 0 \vdash \underline{y} \leq 0 \vdash (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \gg \underline{x} \leq 0 \Rightarrow \underline{y} \leq 0 \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|); \text{FromLeqGeq} \triangleright 0 \leq \underline{x} \Rightarrow 0 \leq \underline{y} \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \triangleright \underline{x} \leq 0 \Rightarrow 0 \leq \underline{y} \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \gg 0 \leq \underline{y} \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|); \text{FromLeqGeq} \triangleright 0 \leq \underline{x} \Rightarrow \underline{y} \leq 0 \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \triangleright \underline{x} \leq 0 \Rightarrow \underline{y} \leq 0 \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \gg \underline{y} \leq 0 \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|); \text{FromLeqGeq} \triangleright 0 \leq \underline{y} \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \triangleright \underline{y} \leq 0 \Rightarrow (|\underline{x} + \underline{y}|) \leq (|\underline{x}| + |\underline{y}|) \gg |\underline{x} + \underline{y}| \leq (|\underline{x}| + |\underline{y}|), \text{po}, \text{c}) \\
& [\text{insertMiddleTerm}(\text{Sum}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + \underline{y}) = (\underline{x} + (-\underline{uz})) + (\underline{z} + \underline{y})]
\end{aligned}$$

$$\begin{aligned}
& [\text{insertMiddleTerm}(\text{Sum}) \xrightarrow{\text{proof}} \lambda \text{c}. \lambda \underline{x}. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{x} + \underline{y} - \underline{y} \gg \underline{x} = (\underline{x} + \underline{z}) + (-\underline{uz}); \text{Three2threeTerms} \gg ((\underline{x} + \underline{z}) + (-\underline{uz})) = ((\underline{x} + (-\underline{uz})) + \underline{z}); \text{eqTransitivity} \triangleright \underline{x} = ((\underline{x} + \underline{z}) + (-\underline{uz})) \triangleright ((\underline{x} + \underline{z}) + (-\underline{uz})) = ((\underline{x} + (-\underline{uz})) + \underline{z}) \gg \underline{x} = ((\underline{x} + (-\underline{uz})) + \underline{z}); \text{eqAddition} \triangleright \underline{x} = ((\underline{x} + (-\underline{uz})) + \underline{z}) \gg (\underline{x} + \underline{y}) = (((\underline{x} + (-\underline{uz})) + \underline{z}) + \underline{y}); \text{plusAssociativity} \gg (((\underline{x} + (-\underline{uz})) + \underline{z}) + \underline{y}) = ((\underline{x} + (-\underline{uz})) + (\underline{z} + \underline{y})); \text{eqTransitivity} \triangleright (\underline{x} + \underline{y}) = (((\underline{x} + (-\underline{uz})) + \underline{z}) + \underline{y}) \triangleright (((\underline{x} + (-\underline{uz})) + \underline{z}) + \underline{y}) = ((\underline{x} + (-\underline{uz})) + (\underline{z} + \underline{y})) \gg (\underline{x} + \underline{y}) = ((\underline{x} + (-\underline{uz})) + (\underline{z} + \underline{y}))], \text{po}, \text{c})]
\end{aligned}$$

$$[\text{insertMiddleTerm}(\text{Numerical}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (|\underline{x} + \underline{y}|) \leq (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|)]$$

$$\begin{aligned}
& [\text{insertMiddleTerm}(\text{Numerical}) \xrightarrow{\text{proof}} \lambda \text{c}. \lambda \underline{x}. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \text{splitNumericalSum} \gg (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|) \leq (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|); \text{insertMiddleTerm}(\text{Sum}) \gg (\underline{x} + \underline{y}) = ((\underline{x} + (-\underline{uz})) + (\underline{z} + \underline{y})); \text{SameNumerical} \triangleright (\underline{x} + \underline{y}) = ((\underline{x} + (-\underline{uz})) + (\underline{z} + \underline{y})) \gg |\underline{x} + \underline{y}| = (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|); \text{eqSymmetry} \triangleright |\underline{x} + \underline{y}| = (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|) \gg (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|) = |\underline{x} + \underline{y}|]; \text{subLeqLeft} \triangleright (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|) = |\underline{x} + \underline{y}| \triangleright (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|) \leq (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|) \gg |\underline{x} + \underline{y}| \leq (|\underline{x} + (-\underline{uz})| + |\underline{z} + \underline{y}|), \text{po}, \text{c}) \\
& (***) \text{ REGNESTYKKER} (***)
\end{aligned}$$

$$[\text{insertMiddleTerm}(\text{Difference}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} + (-\underline{uy})) = (\underline{x} + \underline{z}) + (-\underline{u}(\underline{y} + \underline{z}))]$$

$$\begin{aligned}
& [\text{insertMiddleTerm}(\text{Difference}) \xrightarrow{\text{proof}} \lambda \text{c}. \lambda \underline{x}. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \text{insertMiddleTerm}(\text{Sum}) \gg (\underline{x} + (-\underline{uy})) = ((\underline{x} + (-\underline{u}(-\underline{uz}))) + ((-\underline{uz}) + (-\underline{uy}))); \text{DoubleMinus} \gg (-\underline{u}(-\underline{uz})) = \underline{z}; \text{lemma eqAdditionLeft} \triangleright (-\underline{u}(-\underline{uz})) = \underline{z} \gg (\underline{x} + (-\underline{u}(-\underline{uz}))) = (\underline{x} + \underline{z}); \text{plusCommutativity} \gg ((-\underline{uz}) + (-\underline{uy})) = ((-\underline{uy}) + (-\underline{uz})); -\underline{x} - \underline{y} = -(\underline{x} + \underline{y}) \gg ((-\underline{uy}) + (-\underline{uz})) = (-\underline{u}(\underline{y} + \underline{z})); \text{eqTransitivity} \triangleright ((-\underline{uz}) + (-\underline{uy})) = ((-\underline{uy}) + (-\underline{uz})) \triangleright ((-\underline{uy}) + (-\underline{uz})) = (-\underline{u}(\underline{y} + \underline{z})) \gg ((-\underline{uz}) + (-\underline{uy})) = (-\underline{u}(\underline{y} + \underline{z})); \text{AddEquations} \triangleright (\underline{x} + (-\underline{u}(-\underline{uz}))) = (\underline{x} + \underline{z}) \triangleright ((-\underline{uz}) + (-\underline{uy})) = (-\underline{u}(\underline{y} + \underline{z})) \gg ((\underline{x} + (-\underline{u}(-\underline{uz}))) + ((-\underline{uz}) + (-\underline{uy}))) = ((\underline{x} + \underline{z}) + (-\underline{u}(\underline{y} + \underline{z}))); \text{eqTransitivity} \triangleright (\underline{x} + (-\underline{uy})) = ((\underline{x} + (-\underline{u}(-\underline{uz}))) + ((-\underline{uz}) + (-\underline{uy}))) = ((\underline{x} + (-\underline{u}(-\underline{uz}))) + ((-\underline{uz}) + (-\underline{uy}))) \triangleright ((\underline{x} + (-\underline{u}(-\underline{uz}))) + ((-\underline{uz}) + (-\underline{uy}))) =
\end{aligned}$$

$(\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z})) \gg (\underline{x} + (-u\underline{y})) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z})))$, p_0, c]

[TwoWholes $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x})$]

[TwoWholes $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \text{times1} \gg (\underline{x} * 1) =$

$\underline{x}; \text{eqSymmetry} \gg \underline{x} = (\underline{x} * 1); \text{lemma eqAdditionLeft} \triangleright \underline{x} = (\underline{x} * 1) \gg (\underline{x} + \underline{x}) =$

$(\underline{x} + (\underline{x} * 1)); \text{eqAddition} \triangleright \underline{x} = (\underline{x} * 1) \gg (\underline{x} + (\underline{x} * 1)) =$

$((\underline{x} * 1) + (\underline{x} * 1)); \text{eqTransitivity} \triangleright (\underline{x} + \underline{x}) = (\underline{x} + (\underline{x} * 1)) \triangleright (\underline{x} + (\underline{x} * 1)) =$

$((\underline{x} * 1) + (\underline{x} * 1)) \gg (\underline{x} + \underline{x}) = ((\underline{x} * 1) + (\underline{x} * 1)); \text{DistributionOut} \gg$

$((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)); \text{Repetition} \triangleright ((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)) \gg$

$((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)); \text{timesCommutativity} \gg (\underline{x} * (1 + 1)) =$

$((1 + 1) * \underline{x}); \text{eqTransitivity4} \triangleright (\underline{x} + \underline{x}) = ((\underline{x} * 1) + (\underline{x} * 1)) \triangleright ((\underline{x} * 1) + (\underline{x} * 1)) =$

$(\underline{x} * (1 + 1)) \triangleright (\underline{x} * (1 + 1)) = ((1 + 1) * \underline{x}) \gg (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x})$, p_0, c]

[TwoHalves $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: ((\text{rec}(1 + 1) * \underline{x}) + (\text{rec}(1 + 1) * \underline{x})) = \underline{x}]$

[TwoHalves $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: 0 < 2 \gg \dot{\vdash} (0 <= (1 + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$

$(1 + 1))n)n)n; \text{LessNeq} \triangleright \dot{\vdash} (0 <= (1 + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (1 + 1))n)n)n \gg \dot{\vdash} (0 =$

$(1 + 1))n; \text{NeqSymmetry} \triangleright \dot{\vdash} (0 = (1 + 1))n \gg \dot{\vdash} ((1 + 1) = 0)n; \text{TwoWholes} \gg$

$((\text{rec}(1 + 1) * \underline{x}) + (\text{rec}(1 + 1) * \underline{x})) = ((1 + 1) * (\text{rec}(1 + 1) * \underline{x})); \text{timesAssociativity} \gg$

$((1 + 1) * \text{rec}(1 + 1)) * \underline{x} = ((1 + 1) * (\text{rec}(1 + 1) * \underline{x})); \text{eqSymmetry} \triangleright (((1 + 1) * \text{rec}(1 + 1)) * \underline{x}) =$

$((1 + 1) * (\text{rec}(1 + 1) * \underline{x})) \gg ((1 + 1) * (\text{rec}(1 + 1) * \underline{x})) =$

$((1 + 1) * \text{rec}(1 + 1)) * \underline{x}; \text{Reciprocal} \triangleright \dot{\vdash} ((1 + 1) = 0)n \gg ((1 + 1) * \text{rec}(1 + 1)) =$

$1; \text{eqMultiplication} \triangleright ((1 + 1) * \text{rec}(1 + 1)) = 1 \gg (((1 + 1) * \text{rec}(1 + 1)) * \underline{x}) =$

$(1 * \underline{x}); \text{lemma times1Left} \gg (1 * \underline{x}) = \underline{x}; \text{eqTransitivity5} \triangleright ((\text{rec}(1 + 1) * \underline{x}) +$

$(\text{rec}(1 + 1) * \underline{x})) = ((1 + 1) * (\text{rec}(1 + 1) * \underline{x})) \triangleright ((1 + 1) * (\text{rec}(1 + 1) * \underline{x})) =$

$((1 + 1) * \text{rec}(1 + 1)) * \underline{x} \triangleright (((1 + 1) * \text{rec}(1 + 1)) * \underline{x}) = (1 * \underline{x}) \triangleright (1 * \underline{x}) = \underline{x} \gg$

$((\text{rec}(1 + 1) * \underline{x}) + (\text{rec}(1 + 1) * \underline{x})) = \underline{x}$], p_0, c]

[ThreeWholes $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: ((\underline{x} + \underline{x}) + \underline{x}) = (((1 + 1) + 1) * \underline{x})$]

[ThreeWholes $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \text{TwoWholes} \gg (\underline{x} + \underline{x}) =$

$((1 + 1) * \underline{x}); \text{lemma times1Left} \gg (1 * \underline{x}) = \underline{x}; \text{eqSymmetry} \triangleright (1 * \underline{x}) = \underline{x} \gg \underline{x} =$

$(1 * \underline{x}); \text{AddEquations} \triangleright (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x}) \triangleright \underline{x} = (1 * \underline{x}) \gg ((\underline{x} + \underline{x}) + \underline{x}) =$

$((1 + 1) * \underline{x}) + (1 * \underline{x}); \text{DistributionOutLeft} \gg (((1 + 1) * \underline{x}) + (1 * \underline{x})) =$

$(\underline{x} * ((1 + 1) + 1)); \text{timesCommutativity} \gg (\underline{x} * ((1 + 1) + 1)) =$

$((1 + 1) + 1) * \underline{x}; \text{eqTransitivity4} \triangleright ((\underline{x} + \underline{x}) + \underline{x}) =$

$((1 + 1) * \underline{x}) + (1 * \underline{x}) \triangleright (((1 + 1) * \underline{x}) + (1 * \underline{x})) = (\underline{x} * ((1 + 1) + 1)) \triangleright (\underline{x} * ((1 + 1) + 1)) =$

$((1 + 1) + 1) * \underline{x} \gg ((\underline{x} + \underline{x}) + \underline{x}) = (((1 + 1) + 1) * \underline{x}); \text{Repetition} \triangleright ((\underline{x} + \underline{x}) + \underline{x}) =$

$((1 + 1) + 1) * \underline{x} \gg ((\underline{x} + \underline{x}) + \underline{x}) = (((1 + 1) + 1) * \underline{x})$], p_0, c]

$[0 < 3 \xrightarrow{\text{stmt}}$ SystemQ $\vdash \dot{\vdash} (0 <= ((1 + 1) + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n)n]$

$[0 < 3 \xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash 0 < 2 \gg \dot{\vdash} (0 <= (1 + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$

$(1 + 1))n)n)n; \text{LessLeq} \triangleright \dot{\vdash} (0 <= (1 + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (1 + 1))n)n)n \gg 0 <=$

$(1 + 1); \text{Leq} + 1 \triangleright 0 <= (1 + 1) \gg \dot{\vdash} (0 <= ((1 + 1) + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$

$((1 + 1) + 1))n)n)n; \text{Repetition} \triangleright \dot{\vdash} (0 <= ((1 + 1) + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$

$\dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n$; PositiveNonzero $\triangleright \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n \gg \dot{\vdash} (((1 + 1) + 1) = 0)n$; ThreeWholes \gg
 $((\text{rec}((1 + 1) + 1) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) = (((1 + 1) + 1) *$
 $(\text{rec}((1 + 1) + 1) * \underline{x})); \text{timesAssociativity} \gg (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x} =$
 $((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x}); \text{eqSymmetry} \triangleright (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}$
 $= (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) \gg (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) =$
 $((1 + 1) + 1) * \text{rec}((1 + 1) + 1) * \underline{x}; \text{Reciprocal} \triangleright \dot{\vdash} (((1 + 1) + 1) = 0)n \gg ((1 + 1) + 1) * \text{rec}((1 + 1) + 1) = 1; \text{eqMultiplication} \triangleright ((1 + 1) + 1) * \text{rec}((1 + 1) + 1) = 1 \gg (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x} = (1 * \underline{x}); \text{lemma times1Left} \gg (1 * \underline{x}) = \underline{x}; \text{eqTransitivity5} \triangleright ((\text{rec}((1 + 1) + 1) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) = (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) \triangleright (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) = (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x} \triangleright (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x} = (1 * \underline{x}) \triangleright (1 * \underline{x}) = \underline{x} \gg$
 $((\text{rec}((1 + 1) + 1) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x}) + (\text{rec}((1 + 1) + 1) * \underline{x})) = \underline{x}], p_0, c)]$
 $[0 < 1/3 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash} (0 \leq \text{rec}((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \text{rec}((1 + 1) + 1))n)n)]$
 $[0 < 1/3 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash 0 < 3 \gg \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n) \rceil; \text{PositiveInverted} \triangleright \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n) \gg \dot{\vdash} (0 \leq \text{rec}((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \text{rec}((1 + 1) + 1))n)n) \rceil, p_0, c)]$
 $[\text{Times}(-1) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. (\underline{x} * (-u1)) = (-u\underline{x})]$
 $[\text{Times}(-1) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \text{Negative} \gg (1 + (-u1)) = 0; \text{plusCommutativity} \gg ((-u1) + 1) = (1 + (-u1)); \text{eqTransitivity} \triangleright ((-u1) + 1) = (1 + (-u1)) \triangleright (1 + (-u1)) = 0 \gg ((-u1) + 1) = 0; \text{lemma eqMultiplicationLeft} \triangleright ((-u1) + 1) = 0 \gg (\underline{x} * ((-u1) + 1)) = (\underline{x} * 0); \underline{x} * 0 = 0 \gg (\underline{x} * 0) = 0; \text{eqTransitivity} \triangleright (\underline{x} * ((-u1) + 1)) = (\underline{x} * 0) \triangleright (\underline{x} * 0) = 0 \gg (\underline{x} * ((-u1) + 1)) = 0; \text{Distribution} \gg (\underline{x} * ((-u1) + 1)) = ((\underline{x} * (-u1)) + (\underline{x} * 1)); \text{eqSymmetry} \triangleright (\underline{x} * ((-u1) + 1)) = ((\underline{x} * (-u1)) + (\underline{x} * 1)) \gg ((\underline{x} * (-u1)) + (\underline{x} * 1)) = (\underline{x} * ((-u1) + 1)); \text{eqTransitivity} \triangleright ((\underline{x} * (-u1)) + (\underline{x} * 1)) = (\underline{x} * ((-u1) + 1)) \triangleright (\underline{x} * ((-u1) + 1)) = 0 \gg ((\underline{x} * (-u1)) + (\underline{x} * 1)) = 0; \text{PositiveToRight}(\text{Eq}) \triangleright ((\underline{x} * (-u1)) + (\underline{x} * 1)) = 0 \gg (\underline{x} * (-u1)) = (0 + (-u(\underline{x} * 1))); \text{lemma plus0Left} \gg (0 + (-u(\underline{x} * 1))) = (-u(\underline{x} * 1)); \text{eqTransitivity} \triangleright (\underline{x} * (-u1)) = (0 + (-u(\underline{x} * 1))) \triangleright (0 + (-u(\underline{x} * 1))) = (-u(\underline{x} * 1)) \gg (\underline{x} * (-u1)) = (-u(\underline{x} * 1)); \text{times1} \gg (\underline{x} * 1) = \underline{x}; \text{EqNegated} \triangleright (\underline{x} * 1) = \underline{x} \gg (-u(\underline{x} * 1)) = (-u\underline{x}); \text{eqTransitivity} \triangleright (\underline{x} * (-u1)) = (-u(\underline{x} * 1)) \triangleright (-u(\underline{x} * 1)) = (-u\underline{x}) \gg (\underline{x} * (-u1)) = (-u\underline{x}) \rceil, p_0, c)]$
 $[\text{Times}(-1)\text{Left} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. ((-u1) * \underline{x}) = (-u\underline{x})]$
 $[\text{Times}(-1)\text{Left} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \text{Times}(-1) \gg (\underline{x} * (-u1)) = (-u\underline{x}); \text{timesCommutativity} \gg ((-u1) * \underline{x}) = (\underline{x} * (-u1)); \text{eqTransitivity} \triangleright ((-u1) * \underline{x}) = (\underline{x} * (-u1)) \triangleright (\underline{x} * (-u1)) = (-u\underline{x}) \gg ((-u1) * \underline{x}) = (-u\underline{x}) \rceil, p_0, c)]$
 $(** KvantI **)$

$[\text{Nat}(x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil [\text{Nat}(x) \ddot{=} \lambda c. [x] \in_t (\lceil V_{2n} \rceil :: [\mathcal{M}] :: [\mathcal{N}] :: T) \rceil \rceil)]$

$[\langle a \equiv b \mid x : == t \rangle_{\text{Me}} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \lceil \langle a \equiv b \mid x : == t \rangle_{\text{Me}} \ddot{=} \rceil]$

$$\dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz}))]$$

$$\begin{aligned} & [\text{FromNotSameF(Weak)(Helper)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. \forall \underline{z}. \underline{z} \leq \\ & |(\underline{x} + (-\underline{uy}))| \vdash 0 \leq (\underline{x} + (-\underline{uy})) \vdash \text{NonnegativeNumerical} \triangleright 0 \leq (\underline{x} + (-\underline{uy})) \gg \\ & |(\underline{x} + (-\underline{uy}))| = (\underline{x} + (-\underline{uy})); \text{subLeqRight} \triangleright |(\underline{x} + (-\underline{uy}))| = (\underline{x} + (-\underline{uy})) \triangleright \underline{z} \leq \\ & |(\underline{x} + (-\underline{uy}))| \gg \underline{z} \leq (\underline{x} + (-\underline{uy})); \text{negativeToLeft(Leq)} \triangleright \underline{z} \leq (\underline{x} + (-\underline{uy})) \gg \\ & (\underline{z} + \underline{y}) \leq \underline{x}; \text{plusCommutativity} \gg (\underline{z} + \underline{y}) = (\underline{y} + \underline{z}); \text{subLeqLeft} \triangleright (\underline{z} + \underline{y}) = \\ & (\underline{y} + \underline{z}) \triangleright (\underline{z} + \underline{y}) \leq \underline{x} \gg (\underline{y} + \underline{z}) \leq \underline{x}; \text{PositiveToRight(Leq)} \triangleright (\underline{y} + \underline{z}) \leq \underline{x} \gg \\ & \underline{y} \leq (\underline{x} + (-\underline{uz})); \text{WeakenOr1} \triangleright \underline{y} \leq (\underline{x} + (-\underline{uz})) \gg \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \\ & \underline{y} \leq (\underline{x} + (-\underline{uz})); \forall \underline{x}. \forall \underline{y}. \forall \underline{z}. \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \vdash \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \vdash \\ & \text{ToLess} \triangleright \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \gg \dot{\vdash} ((\underline{x} + (-\underline{uy})) \leq 0 \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{x} + (-\underline{uy})) = \\ & 0)n)n)n; \text{NegativeNumerical} \gg |(\underline{x} + (-\underline{uy}))| = \\ & (-u(\underline{x} + (-\underline{uy}))); \text{MinusNegated} \gg (-u(\underline{x} + (-\underline{uy}))) = \\ & (\underline{y} + (-\underline{ux})); \text{eqTransitivity} \triangleright |(\underline{x} + (-\underline{uy}))| = (-u(\underline{x} + (-\underline{uy}))) \triangleright (-u(\underline{x} + (-\underline{uy}))) = \\ & (\underline{y} + (-\underline{ux})) \gg |(\underline{x} + (-\underline{uy}))| = (\underline{y} + (-\underline{ux})); \text{subLeqRight} \triangleright |(\underline{x} + (-\underline{uy}))| = \\ & (\underline{y} + (-\underline{ux})) \triangleright \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \gg \underline{z} \leq (\underline{y} + (-\underline{ux})); \text{negativeToLeft(Leq)} \triangleright \underline{z} \leq \\ & (\underline{y} + (-\underline{ux})) \gg (\underline{z} + \underline{x}) \leq \underline{y}; \text{plusCommutativity} \gg (\underline{z} + \underline{x}) = \\ & (\underline{x} + \underline{z}); \text{subLeqLeft} \triangleright (\underline{z} + \underline{x}) = (\underline{x} + \underline{z}) \triangleright (\underline{z} + \underline{x}) \leq \underline{y} \gg (\underline{x} + \underline{z}) \leq \\ & \underline{y}; \text{PositiveToRight(Leq)} \triangleright (\underline{x} + \underline{z}) \leq \underline{y} \gg \underline{x} \leq (\underline{y} + (-\underline{uz})); \text{WeakenOr2} \triangleright \underline{x} \leq \\ & (\underline{y} + (-\underline{uz})) \gg \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz})); \forall \underline{x}. \forall \underline{y}. \forall \underline{z}. \text{Ded} \triangleright \\ & \forall \underline{x}. \forall \underline{y}. \forall \underline{z}. \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \vdash 0 \leq (\underline{x} + (-\underline{uy})) \vdash \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \\ & \underline{y} \leq (\underline{x} + (-\underline{uz})) \gg \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \Rightarrow 0 \leq (\underline{x} + (-\underline{uy})) \Rightarrow \dot{\vdash} (\underline{x} \leq \\ & (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz})); \text{Ded} \triangleright \forall \underline{x}. \forall \underline{y}. \forall \underline{z}. \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \vdash \\ & \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \vdash \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz})) \gg \underline{z} \leq \\ & |(\underline{x} + (-\underline{uy}))| \Rightarrow \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \Rightarrow \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq \\ & (\underline{x} + (-\underline{uz})); \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| \leq \underline{z}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| = \underline{z})n)n)n \vdash \\ & \text{fromNotLess} \triangleright \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| \leq \underline{z}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| = \underline{z})n)n)n) \gg \\ & \underline{z} \leq |(\underline{x} + (-\underline{uy}))|; \text{MP} \triangleright \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \Rightarrow 0 \leq (\underline{x} + (-\underline{uy})) \Rightarrow \dot{\vdash} (\underline{x} \leq \\ & (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz})) \triangleright \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \gg 0 \leq (\underline{x} + (-\underline{uy})) \Rightarrow \\ & \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz})); \text{MP} \triangleright \underline{z} \leq |(\underline{x} + (-\underline{uy}))| \Rightarrow \\ & \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \Rightarrow \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz})) \triangleright \underline{z} \leq \\ & |(\underline{x} + (-\underline{uy}))| \gg \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \Rightarrow \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq \\ & (\underline{x} + (-\underline{uz})); \text{FromNegations} \triangleright 0 \leq (\underline{x} + (-\underline{uy})) \Rightarrow \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \\ & \underline{y} \leq (\underline{x} + (-\underline{uz})) \triangleright \dot{\vdash} (0 \leq (\underline{x} + (-\underline{uy})))n \Rightarrow \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq \\ & (\underline{x} + (-\underline{uz})) \gg \dot{\vdash} (\underline{x} \leq (\underline{y} + (-\underline{uz})))n \Rightarrow \underline{y} \leq (\underline{x} + (-\underline{uz}))], p_0, c)] \end{aligned}$$

$$\begin{aligned} & [\text{FromNotSameF(Weak)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\ & \forall \underline{m}. \forall \underline{n}. \forall (\underline{\epsilon}): \forall (\underline{fx}): \forall (\underline{fy}): \dot{\vdash} (\forall_{\text{obj}} \overline{(\underline{\epsilon})}): \dot{\vdash} (\forall_{\text{obj}} \overline{\underline{n}}): \dot{\vdash} (\forall_{\text{obj}} \overline{\underline{m}}): \dot{\vdash} (0 \leq \overline{(\underline{\epsilon})}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \overline{(\underline{\epsilon})})n)n) \Rightarrow \overline{\underline{n}} \leq \overline{\underline{m}} \Rightarrow \dot{\vdash} (|(\underline{fx}[\overline{\underline{m}}] + (-u(\underline{fy})[\overline{\underline{m}}]) | \leq \overline{(\underline{\epsilon})}) \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (|(\underline{fx}[\overline{\underline{m}}] + (-u(\underline{fy})[\overline{\underline{m}}]) | = \overline{(\underline{\epsilon})})n)n)n) \vdash \\ & \dot{\vdash} (\forall_{\text{obj}} \overline{(\underline{\epsilon})}): \dot{\vdash} (\forall_{\text{obj}} \underline{n}): \dot{\vdash} (\forall_{\text{obj}} \underline{m}): \dot{\vdash} (\dot{\vdash} (\dot{\vdash} (0 \leq \overline{(\underline{\epsilon})}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{(\underline{\epsilon})})n)n) \Rightarrow \\ & \dot{\vdash} (\underline{n} \leq \underline{m})n) \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{fx})[\underline{m}] \leq ((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))))n \Rightarrow \underline{fy}[\underline{m}] \leq \\ & ((\underline{fx})[\underline{m}] + (-u(\underline{\epsilon}))))n)n)n) \end{aligned}$$

$$\begin{aligned} & [\text{FromNotSameF(Weak)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \\ & \forall \underline{m}. \forall \underline{n}. \forall (\underline{\epsilon}): \forall (\underline{fx}): \forall (\underline{fy}): \dot{\vdash} (\forall_{\text{obj}} \overline{(\underline{\epsilon})}): \dot{\vdash} (\forall_{\text{obj}} \overline{\underline{n}}): \dot{\vdash} (\forall_{\text{obj}} \overline{\underline{m}}): \dot{\vdash} (0 \leq \overline{(\underline{\epsilon})}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\ & \overline{(\underline{\epsilon})})n)n) \Rightarrow \overline{\underline{n}} \leq \overline{\underline{m}} \Rightarrow \dot{\vdash} (|(\underline{fx}[\overline{\underline{m}}] + (-u(\underline{fy})[\overline{\underline{m}}]) | \leq \overline{(\underline{\epsilon})}) \Rightarrow \end{aligned}$$

$$\begin{aligned}
& (((-u1) * \underline{x}) * \underline{y}); \text{timesAssociativity} \gg (((-u1) * \underline{x}) * \underline{y}) = \\
& ((-u1) * (\underline{x} * \underline{y})); \text{Times}(-1)\text{Left} \gg ((-u1) * (\underline{x} * \underline{y})) = \\
& (-u(\underline{x} * \underline{y})); \text{eqTransitivity4} \triangleright ((\underline{x} * (-u1)) * \underline{y}) = \\
& (((-u1) * \underline{x}) * \underline{y}) \triangleright (((-u1) * \underline{x}) * \underline{y}) = ((-u1) * (\underline{x} * \underline{y})) \triangleright ((-u1) * (\underline{x} * \underline{y})) = \\
& (-u(\underline{x} * \underline{y})) \gg ((\underline{x} * (-u1)) * \underline{y}) = (-u(\underline{x} * \underline{y})); \text{Equality} \triangleright ((\underline{x} * (-u1)) * \underline{y}) = \\
& (-u(\underline{x} * \underline{y})) \triangleright ((\underline{x} * (-u1)) * \underline{y}) = (\underline{x} * ((-u1) * \underline{y})) \gg (-u(\underline{x} * \underline{y})) = \\
& (\underline{x} * ((-u1) * \underline{y})); \text{eqTransitivity} \triangleright (-u(\underline{x} * \underline{y})) = (\underline{x} * ((-u1) * \underline{y})) \triangleright (\underline{x} * ((-u1) * \underline{y})) = \\
& (\underline{x} * (-u\underline{y})) \gg (-u(\underline{x} * \underline{y})) = (\underline{x} * (-u\underline{y})); \text{eqSymmetry} \triangleright (-u(\underline{x} * \underline{y})) = \\
& (\underline{x} * (-u\underline{y})) \gg (\underline{x} * (-u\underline{y})) = (-u(\underline{x} * \underline{y})), \text{p0, c}]
\end{aligned}$$

$$[\text{SplitNumericalProduct}(++) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. 0 \leq \underline{x} \vdash 0 \leq \underline{y} \vdash |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)]$$

$$\begin{aligned}
& [\text{SplitNumericalProduct}(++) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. 0 \leq \underline{x} \vdash 0 \leq \underline{y} \vdash \text{NonnegativeFactors} \triangleright 0 \leq \underline{x} \triangleright 0 \leq \underline{y} \gg 0 \leq \\
& \underline{y} \vdash \text{NonnegativeNumerical} \triangleright 0 \leq (\underline{x} * \underline{y}) \gg |(\underline{x} * \underline{y})| = \\
& (\underline{x} * \underline{y}); \text{NonnegativeNumerical} \triangleright 0 \leq \underline{x} \gg |\underline{x}| = \underline{x}; \text{NonnegativeNumerical} \triangleright 0 \leq \\
& \underline{y} \gg |\underline{y}| = \underline{y}; \text{MultiplyEquations} \triangleright |\underline{x}| = \underline{x} \triangleright |\underline{y}| = \underline{y} \gg (|\underline{x}| * |\underline{y}|) = \\
& (\underline{x} * \underline{y}); \text{eqSymmetry} \triangleright (|\underline{x}| * |\underline{y}|) = (\underline{x} * \underline{y}) \gg (\underline{x} * \underline{y}) = (|\underline{x}| * |\underline{y}|); \text{eqTransitivity} \triangleright \\
& |(\underline{x} * \underline{y})| = (\underline{x} * \underline{y}) \triangleright (\underline{x} * \underline{y}) = (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|), \text{p0, c}]
\end{aligned}$$

$$[\text{SplitNumericalProduct}(+-) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. 0 \leq \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)]$$

$$\begin{aligned}
& [\text{SplitNumericalProduct}(+-) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. 0 \leq \underline{x} \vdash \underline{y} <= \\
& 0 \vdash \text{SignNumerical} \gg |(\underline{x} * \underline{y})| = |(-u(\underline{x} * \underline{y}))|; \text{eqSymmetry} \triangleright |(\underline{x} * \underline{y})| = \\
& |(-u(\underline{x} * \underline{y}))| \gg |(-u(\underline{x} * \underline{y}))| = |(\underline{x} * \underline{y})|; \text{PlusTimesMinus} \gg (\underline{x} * (-u\underline{y})) = \\
& (-u(\underline{x} * \underline{y})); \text{SameNumerical} \triangleright (\underline{x} * (-u\underline{y})) = (-u(\underline{x} * \underline{y})) \gg |(\underline{x} * (-u\underline{y}))| = \\
& |(-u(\underline{x} * \underline{y}))|; \text{eqTransitivity} \triangleright |(\underline{x} * (-u\underline{y}))| = |(-u(\underline{x} * \underline{y}))| \triangleright |(-u(\underline{x} * \underline{y}))| = |(\underline{x} * \underline{y})| \gg \\
& |(\underline{x} * (-u\underline{y}))| = |(\underline{x} * \underline{y})|; \text{SignNumerical} \gg |\underline{y}| = |(-u\underline{y})|; \text{eqSymmetry} \triangleright |\underline{y}| = \\
& |(-u\underline{y})| \gg |(-u\underline{y})| = |\underline{y}|; \text{lemma eqMultiplicationLeft} \triangleright |(-u\underline{y})| = |\underline{y}| \gg \\
& (|\underline{x}| * |(-u\underline{y})|) = (|\underline{x}| * |\underline{y}|); \text{NonpositiveNegated} \triangleright \underline{y} <= 0 \gg 0 <= \\
& (-u\underline{y}); \text{SplitNumericalProduct}(++) \triangleright 0 \leq \underline{x} \triangleright 0 \leq (-u\underline{y}) \gg |(\underline{x} * (-u\underline{y}))| = \\
& (|\underline{x}| * |(-u\underline{y})|); \text{eqTransitivity} \triangleright |(\underline{x} * (-u\underline{y}))| = (|\underline{x}| * |(-u\underline{y})|) \triangleright (|\underline{x}| * |(-u\underline{y})|) = \\
& (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * (-u\underline{y}))| = (|\underline{x}| * |\underline{y}|); \text{Equality} \triangleright |(\underline{x} * (-u\underline{y}))| = \\
& |(\underline{x} * \underline{y})| \triangleright |(\underline{x} * (-u\underline{y}))| = (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|), \text{p0, c}]
\end{aligned}$$

$$[\text{SplitNumericalProduct} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)]$$

$$\begin{aligned}
& [\text{SplitNumericalProduct} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \forall \underline{y}. 0 \leq \underline{x} \vdash 0 \leq \underline{y} \vdash \text{SplitNumericalProduct}(++) \triangleright 0 \leq \underline{x} \triangleright 0 \leq \underline{y} \gg |(\underline{x} * \underline{y})| = \\
& (|\underline{x}| * |\underline{y}|); \forall \underline{x}. \forall \underline{y}. 0 \leq \underline{x} \vdash \underline{y} <= 0 \vdash \text{SplitNumericalProduct}(+-) \triangleright 0 \leq \\
& \underline{x} \triangleright \underline{y} <= 0 \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|); \forall \underline{x}. \forall \underline{y}. \underline{x} <= 0 \vdash 0 \leq \underline{y} \vdash \\
& \text{SplitNumericalProduct}(+-) \triangleright 0 \leq \underline{y} \triangleright \underline{x} <= 0 \gg |(\underline{y} * \underline{x})| = \\
& (|\underline{y}| * |\underline{x}|); \text{timesCommutativity} \gg (\underline{x} * \underline{y}) = (\underline{y} * \underline{x}); \text{SameNumerical} \triangleright (\underline{x} * \underline{y}) = \\
& (\underline{y} * \underline{x}) \gg |(\underline{x} * \underline{y})| = |(\underline{y} * \underline{x})|; \text{timesCommutativity} \gg (|\underline{y}| * |\underline{x}|) = \\
& (|\underline{x}| * |\underline{y}|); \text{eqTransitivity4} \triangleright |(\underline{x} * \underline{y})| = |(\underline{y} * \underline{x})| \triangleright |(\underline{y} * \underline{x})| = \\
& (|\underline{y}| * |\underline{x}|) \triangleright (|\underline{y}| * |\underline{x}|) = (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|); \forall \underline{x}. \forall \underline{y}. \underline{x} <= 0 \vdash \underline{y} <= \\
& 0 \vdash \text{NonpositiveNegated} \triangleright \underline{x} <= 0 \gg 0 \leq (-u\underline{x}); \text{NonpositiveNegated} \triangleright \underline{y} <=
\end{aligned}$$

$0 \gg 0 \leq (-uy)$; SplitNumericalProduct $(++) \triangleright 0 \leq (-ux) \triangleright 0 \leq (-uy) \gg$
 $|((-ux) * (-uy))| = (|(-ux)| * |(-uy)|)$; MinusTimesMinus $\gg ((-ux) * (-uy)) =$
 $(x * y)$; SameNumerical $\triangleright ((-ux) * (-uy)) = (x * y) \gg |((-ux) * (-uy))| =$
 $|x * y|$; eqSymmetry $\triangleright |((-ux) * (-uy))| = |x * y| \gg |x * y| =$
 $|((-ux) * (-uy))|$; SignNumerical $\gg |x| = |(-ux)$; SignNumerical $\gg |y| =$
 $|(-uy)$; MultiplyEquations $\triangleright |x| = |(-ux) \triangleright |y| = |(-uy) \gg (|x| * |y|) =$
 $(|(-ux)| * |(-uy)|)$; eqSymmetry $\triangleright (|x| * |y|) = (|(-ux)| * |(-uy)|) \gg$
 $(|(-ux)| * |(-uy)|) = (|x| * |y|)$; eqTransitivity4 $\triangleright |x * y| =$
 $|((-ux) * (-uy)) \triangleright |((-ux) * (-uy))| = (|(-ux)| * |(-uy)|) \triangleright (|(-ux)| * |(-uy)|) =$
 $(|x| * |y|) \gg |x * y| = (|x| * |y|)$; $\forall x: \forall y: \text{Ded} \triangleright \forall x: \forall y: 0 \leq x \vdash 0 \leq y \vdash |x * y| =$
 $(|x| * |y|) \gg 0 \leq x \Rightarrow 0 \leq y \Rightarrow |x * y| = (|x| * |y|)$; Ded $\triangleright \forall x: \forall y: 0 \leq x \vdash$
 $y \leq 0 \vdash |x * y| = (|x| * |y|) \gg 0 \leq x \Rightarrow y \leq 0 \Rightarrow |x * y| =$
 $(|x| * |y|)$; Ded $\triangleright \forall x: \forall y: x \leq 0 \vdash 0 \leq y \vdash |x * y| = (|x| * |y|) \gg x \leq 0 \Rightarrow$
 $0 \leq y \Rightarrow |x * y| = (|x| * |y|)$; Ded $\triangleright \forall x: \forall y: x \leq 0 \vdash y \leq 0 \vdash |x * y| =$
 $(|x| * |y|) \gg x \leq 0 \Rightarrow y \leq 0 \Rightarrow |x * y| = (|x| * |y|)$; FromLeqGeq $\triangleright 0 \leq x \Rightarrow$
 $0 \leq y \Rightarrow |x * y| = (|x| * |y|) \triangleright x \leq 0 \Rightarrow 0 \leq y \Rightarrow |x * y| = (|x| * |y|) \gg$
 $0 \leq y \Rightarrow |x * y| = (|x| * |y|)$; FromLeqGeq $\triangleright 0 \leq x \Rightarrow y \leq 0 \Rightarrow |x * y| =$
 $(|x| * |y|) \triangleright x \leq 0 \Rightarrow y \leq 0 \Rightarrow |x * y| = (|x| * |y|) \gg y \leq 0 \Rightarrow |x * y| =$
 $(|x| * |y|)$; FromLeqGeq $\triangleright 0 \leq y \Rightarrow |x * y| = (|x| * |y|) \triangleright y \leq 0 \Rightarrow |x * y| =$
 $(|x| * |y|) \gg |x * y| = (|x| * |y|)$, $p_0, c]$

[Three2threeFactors $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall x: \forall y: \forall z: ((x * y) * z) = ((x * z) * y)$]

[Three2threeFactors $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}([\text{SystemQ} \vdash$
 $\forall x: \forall y: \forall z: \text{timesCommutativity} \gg (y * z) = (z * y)$; Three2twoFactors $\triangleright (y * z) =$
 $(z * y) \gg ((x * y) * z) = (x * (z * y))$; timesAssociativity $\gg ((x * z) * y) =$
 $(x * (z * y))$; eqSymmetry $\triangleright ((x * z) * y) = (x * (z * y)) \gg (x * (z * y)) =$
 $((x * z) * y)$; eqTransitivity $\triangleright ((x * y) * z) = (x * (z * y)) \triangleright (x * (z * y)) =$
 $((x * z) * y) \gg ((x * y) * z) = ((x * z) * y)$, $p_0, c]$

[DistributionOutLeft $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall x: \forall y: \forall z: ((y * x) + (z * x)) = (x * (y + z))]$

[DistributionOutLeft $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}([\text{SystemQ} \vdash$
 $\forall x: \forall y: \forall z: \text{timesCommutativity} \gg (y * x) = (x * y)$; timesCommutativity \gg
 $(z * x) = (x * z)$; AddEquations $\triangleright (y * x) = (x * y) \triangleright (z * x) = (x * z) \gg$
 $((y * x) + (z * x)) = ((x * y) + (x * z))$; DistributionOut $\gg ((x * y) + (x * z)) =$
 $(x * (y + z))$; eqTransitivity $\triangleright ((y * x) + (z * x)) = ((x * y) + (x * z)) \triangleright ((x * y) + (x * z)) =$
 $(x * (y + z)) \gg ((y * x) + (z * x)) = (x * (y + z))$, $p_0, c]$

[NonzeroFactors $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall x: \forall y: \dot{\vdash} (x = 0) \text{n} \vdash \dot{\vdash} (y = 0) \text{n} \vdash \dot{\vdash} ((x * y) = 0) \text{n}]$

[NonzeroFactors $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \lambda y. \mathcal{P}([\text{SystemQ} \vdash \forall x: \forall y: \dot{\vdash} (x = 0) \text{n} \vdash \dot{\vdash} (y = 0) \text{n} \vdash$
 $\text{NeqMultiplication} \triangleright \dot{\vdash} (y = 0) \text{n} \triangleright \dot{\vdash} (x = 0) \text{n} \gg \dot{\vdash} ((x * y) =$
 $(0 * y)) \text{n}$; timesCommutativity $\gg (0 * y) = (y * 0)$; $x * 0 = 0 \gg (y * 0) =$
 0 ; eqTransitivity $\triangleright (0 * y) = (y * 0) \triangleright (y * 0) = 0 \gg (0 * y) =$
 0 ; SubNeqRight $\triangleright (0 * y) = 0 \triangleright \dot{\vdash} ((x * y) = (0 * y)) \text{n} \gg \dot{\vdash} ((x * y) = 0) \text{n}]$, $p_0, c]$

[PositiveFactors $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall x: \forall y: \dot{\vdash} (0 \leq x \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = x) \text{n}) \text{n}) \text{n} \vdash$
 $\dot{\vdash} (0 \leq y \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = y) \text{n}) \text{n}) \text{n} \vdash \dot{\vdash} (0 \leq (x * y) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (x * y)) \text{n}) \text{n}) \text{n}]$

[PositiveNonzero $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash \dot{\vdash} (\underline{x} = 0)n]$

[PositiveNonzero $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash \text{Repetition} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \gg \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n; \text{SecondConjunct} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \gg \dot{\vdash} (0 = \underline{x})n; \text{NeqSymmetry} \triangleright \dot{\vdash} (0 = \underline{x})n \gg \dot{\vdash} (\underline{x} = 0)n \urcorner], p_0, c)]$

[reciprocalToLeft(Less) $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\vdash} (0 \leq \underline{z} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{z})n)n)n \vdash \dot{\vdash} (\underline{x} \leq (\underline{y} * \text{recz}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = (\underline{y} * \text{recz}))n)n)n \vdash \dot{\vdash} ((\underline{x} * \underline{z}) \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{x} * \underline{z}) = \underline{y})n)n)n]$

[reciprocalToLeft(Less) $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\vdash} (0 \leq \underline{z} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{z})n)n)n \vdash \dot{\vdash} (\underline{x} \leq (\underline{y} * \text{recz}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = (\underline{y} * \text{recz}))n)n)n \vdash \text{LessMultiplication} \triangleright \dot{\vdash} (0 \leq \underline{z} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{z})n)n)n \triangleright \dot{\vdash} (\underline{x} \leq (\underline{y} * \text{recz}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = (\underline{y} * \text{recz}))n)n)n \gg \dot{\vdash} ((\underline{x} * \underline{z}) \leq ((\underline{y} * \text{recz}) * \underline{z}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{x} * \underline{z}) = ((\underline{y} * \text{recz}) * \underline{z}))n)n)n; \text{Three2threeFactors} \gg ((\underline{y} * \text{recz}) * \underline{z}) = ((\underline{y} * \underline{z}) * \text{recz}); \text{PositiveNonzero} \triangleright \dot{\vdash} (0 \leq \underline{z} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{z})n)n)n \gg \dot{\vdash} (\underline{z} = 0)n; \underline{x} = \underline{x} * \underline{y} * (1/\underline{y}) \triangleright \dot{\vdash} (\underline{z} = 0)n \gg \underline{y} = ((\underline{y} * \underline{z}) * \text{recz}); \text{eqSymmetry} \triangleright \underline{y} = ((\underline{y} * \underline{z}) * \text{recz}) \gg ((\underline{y} * \underline{z}) * \text{recz}) = \underline{y}; \text{eqTransitivity} \triangleright ((\underline{y} * \text{recz}) * \underline{z}) = ((\underline{y} * \underline{z}) * \text{recz}) \triangleright ((\underline{y} * \underline{z}) * \text{recz}) = \underline{y} \gg ((\underline{y} * \text{recz}) * \underline{z}) = \underline{y}; \text{SubLessRight} \triangleright ((\underline{y} * \text{recz}) * \underline{z}) = \underline{y} \triangleright \dot{\vdash} ((\underline{x} * \underline{z}) \leq ((\underline{y} * \text{recz}) * \underline{z}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{x} * \underline{z}) = ((\underline{y} * \text{recz}) * \underline{z}))n)n)n \gg \dot{\vdash} ((\underline{x} * \underline{z}) \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{x} * \underline{z}) = \underline{y})n)n)n \urcorner], p_0, c)]$

XX 0;1/2 er et specialtilfaelde [PositiveInverted $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash \dot{\vdash} (0 \leq \text{recx} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \text{recx})n)n)n]$

[PositiveInverted $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash \text{FirstConjunct} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \gg 0 \leq \underline{x}; \text{SecondConjunct} \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \gg \dot{\vdash} (0 = \underline{x})n; \text{NeqSymmetry} \triangleright \dot{\vdash} (0 = \underline{x})n \gg \dot{\vdash} (\underline{x} = 0)n; 0 < 1 \gg \dot{\vdash} (0 \leq 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n; \underline{x} * 0 = 0 \gg (\underline{x} * 0) = 0; \underline{x} * \underline{y} = \underline{z} \text{Backwards} \triangleright (\underline{x} * 0) = 0 \gg 0 = (0 * \underline{x}); \text{SubLessLeft} \triangleright 0 = (0 * \underline{x}) \triangleright \dot{\vdash} (0 \leq 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n \gg \dot{\vdash} ((0 * \underline{x}) \leq 1 \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 * \underline{x}) = 1)n)n)n; \text{Reciprocal} \triangleright \dot{\vdash} (\underline{x} = 0)n \gg ((\underline{x} * \text{recx}) = 1; \underline{x} * \underline{y} = \underline{z} \text{Backwards} \triangleright (\underline{x} * \text{recx}) = 1 \gg 1 = (\text{recx} * \underline{x}); \text{SubLessRight} \triangleright 1 = (\text{recx} * \underline{x}) \triangleright \dot{\vdash} ((0 * \underline{x}) \leq 1 \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 * \underline{x}) = 1)n)n)n \gg \dot{\vdash} ((0 * \underline{x}) \leq (\text{recx} * \underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 * \underline{x}) = (\text{recx} * \underline{x}))n)n)n; \text{LessDivision} \triangleright 0 \leq \underline{x} \triangleright \dot{\vdash} ((0 * \underline{x}) \leq (\text{recx} * \underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 * \underline{x}) = (\text{recx} * \underline{x}))n)n)n \gg \dot{\vdash} (0 \leq \text{recx} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \text{recx})n)n)n \urcorner], p_0, c)]$

[ToNumericalLess $\xrightarrow{\text{stmt}}$ SystemQ $\vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} ((-\underline{u}\underline{y}) \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{u}\underline{y}) = \underline{x})n)n)n \vdash \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n)n \vdash \dot{\vdash} (|\underline{x}| \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (|\underline{x}| = \underline{y})n)n)n]$

[ToNumericalLess $\xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n)n \vdash 0 \leq \underline{x} \vdash \text{NonnegativeNumerical} \triangleright 0 \leq \underline{x} \gg |\underline{x}| = \underline{x}; \text{eqSymmetry} \triangleright |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \text{SubLessLeft} \triangleright \underline{x} = |\underline{x}| \triangleright \dot{\vdash} (\underline{x} \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = \underline{y})n)n)n \gg \dot{\vdash} (|\underline{x}| \leq \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (|\underline{x}| = \underline{y})n)n)n; \forall \underline{x}: \forall \underline{y}: \dot{\vdash} ((-\underline{u}\underline{y}) \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{u}\underline{y}) = \underline{x})n)n)n \vdash \underline{x} \leq 0 \vdash \text{LessNegated} \triangleright \dot{\vdash} ((-\underline{u}\underline{y}) \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{u}\underline{y}) = \underline{x})n)n)n \gg \dot{\vdash} ((-\underline{u}\underline{x}) \leq (-\underline{u}(-\underline{u}\underline{y}))) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{u}\underline{x}) = (-\underline{u}(-\underline{u}\underline{y})))n)n)n; \text{lemma nonpositiveNumerical} \triangleright \underline{x} \leq 0 \gg |\underline{x}| =$

$(-ux)$; eqSymmetry $\triangleright |x| = (-ux) \gg (-ux) = |x|$; SubLessLeft $\triangleright (-ux) = |x| \triangleright \dot{\neg}((-ux) \leq (-u(-uy)) \Rightarrow \dot{\neg}(\dot{\neg}((-ux) = (-u(-uy)))n)n)n \gg \dot{\neg}(|x| \leq (-u(-uy)) \Rightarrow \dot{\neg}(\dot{\neg}(|x| = (-u(-uy)))n)n)n$; DoubleMinus $\gg (-u(-uy)) = y$; SubLessRight $\triangleright (-u(-uy)) = \underline{y} \triangleright \dot{\neg}(|x| \leq (-u(-uy)) \Rightarrow \dot{\neg}(\dot{\neg}(|x| = (-u(-uy)))n)n)n \gg \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n$; $\forall x: \forall y: \text{Ded} \triangleright \forall x: \forall y: \dot{\neg}(x \leq y \Rightarrow \dot{\neg}(\dot{\neg}(x = y)n)n)n \vdash 0 \leq x \vdash \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n \gg \dot{\neg}(x \leq y \Rightarrow \dot{\neg}(\dot{\neg}(x = y)n)n)n \Rightarrow 0 \leq x \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n$; $\text{Ded} \triangleright \forall x: \forall y: \dot{\neg}((-uy) \leq x \Rightarrow \dot{\neg}(\dot{\neg}((-uy) = x)n)n)n \vdash x \leq 0 \vdash \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n \gg \dot{\neg}((-uy) \leq x \Rightarrow \dot{\neg}(\dot{\neg}((-uy) = x)n)n)n \Rightarrow x \leq 0 \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n$; $\dot{\neg}((-uy) \leq x \Rightarrow \dot{\neg}(\dot{\neg}((-uy) = x)n)n)n \vdash \dot{\neg}(x \leq y \Rightarrow \dot{\neg}(\dot{\neg}(x = y)n)n)n \vdash \text{MP} \triangleright \dot{\neg}(x \leq y \Rightarrow \dot{\neg}(\dot{\neg}(x = y)n)n)n \Rightarrow 0 \leq x \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n \triangleright \dot{\neg}(x \leq y \Rightarrow \dot{\neg}(\dot{\neg}(x = y)n)n)n \gg 0 \leq x \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n$; $\text{MP} \triangleright \dot{\neg}((-uy) \leq x \Rightarrow \dot{\neg}(\dot{\neg}((-uy) = x)n)n)n \Rightarrow x \leq 0 \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n \triangleright \dot{\neg}((-uy) \leq x \Rightarrow \dot{\neg}(\dot{\neg}((-uy) = x)n)n)n \gg x \leq 0 \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n$; $\text{FromLeqGeq} \triangleright 0 \leq x \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n \triangleright x \leq 0 \Rightarrow \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n \gg \dot{\neg}(|x| \leq y \Rightarrow \dot{\neg}(\dot{\neg}(|x| = y)n)n)n]$

$[x \leq |x| \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: x \leq |x|]$

$[x \leq |x| \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: 0 \leq x \vdash \text{NonnegativeNumerical} \gg |x| = x; \text{eqSymmetry} \triangleright |x| = x \gg x = |x|; \text{eqLeq} \triangleright x = |x| \gg x \leq |x|; \forall x: x \leq 0 \vdash 0 \leq |x| \gg 0 \leq |x|; \text{leqTransitivity} \triangleright x \leq 0 \triangleright 0 \leq |x| \gg x \leq |x|; \forall x: \text{Ded} \triangleright \forall x: 0 \leq x \vdash x \leq |x| \gg 0 \leq x \Rightarrow x \leq |x|; \text{Ded} \triangleright \forall x: x \leq 0 \vdash x \leq |x| \gg x \leq 0 \Rightarrow x \leq |x|; \text{FromLeqGeq} \triangleright 0 \leq x \Rightarrow x \leq |x| \triangleright x \leq 0 \Rightarrow x \leq |x| \gg x \leq |x|, p_0, c)]$

$[\text{PositiveToLeft}(\text{Less}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: \forall y: \forall z: \dot{\neg}(x \leq (y + z) \Rightarrow \dot{\neg}(\dot{\neg}(x = (y + z))n)n)n \vdash \dot{\neg}((x + (-uz)) \leq y \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uz)) = y)n)n)n]$

$[\text{PositiveToLeft}(\text{Less}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: \forall z: \dot{\neg}(x \leq (y + z) \Rightarrow \dot{\neg}(\dot{\neg}(x = (y + z))n)n)n \vdash \text{LessAddition} \triangleright \dot{\neg}(x \leq (y + z) \Rightarrow \dot{\neg}(\dot{\neg}(x = (y + z))n)n)n \gg \dot{\neg}((x + (-uz)) \leq ((y + z) + (-uz)) \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uz)) = ((y + z) + (-uz)))n)n)n; x = x + y - y \gg y = ((y + z) + (-uz)); \text{eqSymmetry} \triangleright y = ((y + z) + (-uz)) \gg ((y + z) + (-uz)) = y; \text{SubLessRight} \triangleright ((y + z) + (-uz)) = y \triangleright \dot{\neg}((x + (-uz)) \leq ((y + z) + (-uz)) \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uz)) = ((y + z) + (-uz)))n)n)n \gg \dot{\neg}((x + (-uz)) \leq y \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uz)) = y)n)n)n \rceil, p_0, c)]$

$[\text{NegativeToRight}(\text{Less}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall x: \forall y: \forall z: \dot{\neg}((x + (-uy)) \leq z \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uy)) = z)n)n)n \vdash \dot{\neg}(x \leq (z + y) \Rightarrow \dot{\neg}(\dot{\neg}(x = (z + y))n)n)n]$

$[\text{NegativeToRight}(\text{Less}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall x: \forall y: \forall z: \dot{\neg}((x + (-uy)) \leq z \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uy)) = z)n)n)n \vdash \text{LessAddition} \triangleright \dot{\neg}((x + (-uy)) \leq z \Rightarrow \dot{\neg}(\dot{\neg}((x + (-uy)) = z)n)n)n \gg \dot{\neg}(((x + (-uy)) + y) \leq (z + y) \Rightarrow \dot{\neg}(\dot{\neg}(((x + (-uy)) + y) = (z + y))n)n)n; \text{Three2threeTerms} \gg ((x + (-uy)) + y) = ((x + y) + (-uy)); x = x + y - y \gg x = ((x + y) + (-uy)); \text{eqSymmetry} \triangleright x = ((x + y) + (-uy)) \gg ((x + y) + (-uy)) = x; \text{eqTransitivity} \triangleright ((x + (-uy)) + y) = ((x + y) + (-uy)) \triangleright ((x + y) + (-uy)) = x \gg ((x + (-uy)) + y) = x; \text{SubLessLeft} \triangleright ((x + (-uy)) + y) = x \triangleright \dot{\neg}(((x + (-uy)) + y) \leq (z + y) \Rightarrow \dot{\neg}(\dot{\neg}(((x + (-uy)) + y) = (z + y))n)n)n \rceil, p_0, c)]$

$$\begin{aligned}
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), (\underline{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2)}]| = \\
& \text{if}(\underline{v1}) \leq \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), (\underline{v1}))\text{n)n)n} \gg \\
& \forall_{\text{obj}}(\underline{v2}): \dot{\neg}(|(\underline{fx})[\underline{(v2)}]| \leq \text{if}(\underline{v1}) \leq \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), (\underline{v1})) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2)}]| = \\
& \text{if}(\underline{v1}) \leq \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + \\
& (-\underline{u1}))|), \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + \\
& (-\underline{u1}))|), (\underline{v1}))\text{n)n)n}; \text{IntroExist @} \text{if}(\underline{v1}) \leq \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), (\underline{v1})) \triangleright \forall_{\text{obj}}(\underline{v2}): \dot{\neg}(|(\underline{fx})[\underline{(v2)}]| \leq = \\
& \text{if}(\underline{v1}) \leq \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq = \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq = \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), (\underline{v1})) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2)}]| = \\
& \text{if}(\underline{v1}) \leq \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq = \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), \text{if}(|(\underline{fx})[\underline{n}] + (-\underline{u1}))| \leq = \\
& |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + 1)|, |((\underline{fx})[\underline{n}] + (-\underline{u1}))|), (\underline{v1}))\text{n)n)n} \gg \\
& \dot{\neg}(\forall_{\text{obj}}(\underline{v1}): \dot{\neg}(\forall_{\text{obj}}(\underline{v2}): \dot{\neg}(|(\underline{fx})[\underline{(v2)}]| \leq = (\underline{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2)}]| = \\
& (\underline{v1})\text{n)n)n)n}), \text{p}_0, \text{c})]
\end{aligned}$$

[Fpart - Bounded(Base) $\xrightarrow{\text{stmt}}$ SystemQ] \vdash

$$\forall(\underline{v1}): \forall(\underline{v2n}): \forall(\underline{fx}): \dot{\neg}(\forall_{\text{obj}}(\underline{v1}): \dot{\neg}(\forall_{\text{obj}}(\underline{v2n}): (\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = \\
(\underline{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = (\underline{v1})\text{n)n)n)n})]$$

[Fpart - Bounded(Base) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \urcorner$

$$\begin{aligned}
& \forall(\underline{v1}): \forall(\underline{v2n}): \forall(\underline{fx}): (\underline{v2n}) \leq 0 \vdash \text{LeqLessEq} \triangleright (\underline{v2n}) \leq 0 \gg \dot{\neg}(\dot{\neg}((\underline{v2n}) \leq = \\
& 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{v2n}) = 0)\text{n)n)n}) \Rightarrow (\underline{v2n}) = 0; \text{Nonnegative}(\underline{N}) \gg 0 \leq = \\
& (\underline{v2n}); \text{toNotLess} \triangleright 0 \leq = (\underline{v2n}) \gg \dot{\neg}(\dot{\neg}((\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{v2n}) = \\
& 0)\text{n)n)n}); \text{NegateDisjunct1} \triangleright \dot{\neg}(\dot{\neg}((\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{v2n}) = 0)\text{n)n)n}) \Rightarrow \\
& (\underline{v2n}) = 0 \triangleright \dot{\neg}(\dot{\neg}((\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{v2n}) = 0)\text{n)n)n}) \gg (\underline{v2n}) = \\
& 0; \text{SameSeries} \triangleright (\underline{v2n}) = 0 \gg (\underline{fx})[\underline{(v2n)}] = (\underline{fx})[0]; \text{SameNumerical} \triangleright (\underline{fx})[\underline{(v2n)}] = \\
& (\underline{fx})[0] \gg |(\underline{fx})[\underline{(v2n)}]| = |(\underline{fx})[0]|; \text{eqAddition} \triangleright |(\underline{fx})[\underline{(v2n)}]| = |(\underline{fx})[0]| \gg \\
& |(\underline{fx})[\underline{(v2n)}]| + 1 = (|(\underline{fx})[0]| + 1); \text{leqReflexivity} \gg |(\underline{fx})[\underline{(v2n)}]| \leq = \\
& |(\underline{fx})[\underline{(v2n)}]|; \text{Leq} + 1 \triangleright |(\underline{fx})[\underline{(v2n)}]| \leq = |(\underline{fx})[\underline{(v2n)}]| \gg \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = \\
& |(\underline{fx})[\underline{(v2n)}]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = \\
& |(\underline{fx})[\underline{(v2n)}]| + 1)\text{n)n)n}; \text{SubLessRight} \triangleright (|(\underline{fx})[\underline{(v2n)}]| + 1) = \\
& (|(\underline{fx})[0]| + 1) \triangleright \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = (|(\underline{fx})[\underline{(v2n)}]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = \\
& (|(\underline{fx})[\underline{(v2n)}]| + 1)\text{n)n)n} \gg \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = (|(\underline{fx})[0]| + 1) \Rightarrow \\
& \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = (|(\underline{fx})[0]| + 1)\text{n)n)n}); \forall(\underline{v1}): \forall(\underline{v2n}): \forall(\underline{fx}): \text{Ded} \triangleright \\
& \forall(\underline{v1}): \forall(\underline{v2n}): \forall(\underline{fx}): (\underline{v2n}) \leq 0 \vdash \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = (|(\underline{fx})[0]| + 1) \Rightarrow \\
& \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = (|(\underline{fx})[0]| + 1)\text{n)n)n} \gg (\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = \\
& (|(\underline{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = (|(\underline{fx})[0]| + 1)\text{n)n)n}); \text{Gen} \triangleright (\underline{v2n}) \leq 0 \Rightarrow \\
& \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = (|(\underline{fx})[0]| + 1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = (|(\underline{fx})[0]| + 1)\text{n)n)n} \gg \\
& \forall_{\text{obj}}(\underline{v2n}): (\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = (|(\underline{fx})[0]| + 1) \Rightarrow \\
& \dot{\neg}(\dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| = (|(\underline{fx})[0]| + 1)\text{n)n)n}; \text{IntroExist @} (|(\underline{fx})[0]| + 1) \triangleright \\
& \forall_{\text{obj}}(\underline{v2n}): (\underline{v2n}) \leq 0 \Rightarrow \dot{\neg}(|(\underline{fx})[\underline{(v2n)}]| \leq = (|(\underline{fx})[0]| + 1) \Rightarrow
\end{aligned}$$

$(\text{rec}((1 + 1) + 1) * \underline{x})n)n \vdash \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n$
 $\llbracket \text{PositiveTripled} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq (\text{rec}((1 + 1) + 1) * \underline{x}) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = (\text{rec}((1 + 1) + 1) * \underline{x})n)n)n \vdash 0 < 3 \gg \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n)n; \text{PositiveFactors} \triangleright \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n)n \triangleright \dot{\vdash} (0 \leq (\text{rec}((1 + 1) + 1) * \underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $(\text{rec}((1 + 1) + 1) * \underline{x})n)n)n \gg \dot{\vdash} (0 \leq (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})))n)n)n; \text{timesAssociativity} \gg$
 $((((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}) = (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) =$
 $((((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) \gg (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) =$
 $((((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}); \text{PositiveNonzero} \triangleright \dot{\vdash} (0 \leq$
 $((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = ((1 + 1) + 1))n)n)n \gg \dot{\vdash} (((1 + 1) + 1) =$
 $0)n; \text{Reciprocal} \triangleright \dot{\vdash} (((1 + 1) + 1) = 0)n \gg (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) =$
 $1; \text{eqMultiplication} \triangleright (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) = 1 \gg$
 $((((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}) = (1 * \underline{x}); \text{lemma times1Left} \gg (1 * \underline{x}) =$
 $\underline{x}; \text{eqTransitivity4} \triangleright (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) =$
 $((((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x}) \triangleright (((1 + 1) + 1) * \text{rec}((1 + 1) + 1)) * \underline{x} =$
 $(1 * \underline{x}) \triangleright (1 * \underline{x}) = \underline{x} \gg (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) =$
 $\underline{x}; \text{SubLessRight} \triangleright (((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x})) = \underline{x} \triangleright \dot{\vdash} (0 \leq$
 $((1 + 1) + 1) * (\text{rec}((1 + 1) + 1) * \underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (((1 + 1) + 1) * (\text{rec}((1 + 1) +$
 $1) * \underline{x})))n)n)n \gg \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \rrbracket, p_0, c)$
 $\llbracket \text{PositiveDividedBy3} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \vdash$
 $\dot{\vdash} (0 \leq (\text{rec}((1 + 1) + 1) * \underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\text{rec}((1 + 1) + 1) * \underline{x})n)n)n) \rrbracket$
 $\llbracket \text{PositiveDividedBy3} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $\underline{x})n)n)n \vdash 0 < 3 \gg \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $((1 + 1) + 1))n)n)n; \text{PositiveInverted} \triangleright \dot{\vdash} (0 \leq ((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $((1 + 1) + 1))n)n)n \gg \dot{\vdash} (0 \leq \text{rec}((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $\text{rec}((1 + 1) + 1))n)n)n; \text{PositiveFactors} \triangleright \dot{\vdash} (0 \leq \text{rec}((1 + 1) + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $\text{rec}((1 + 1) + 1))n)n)n \triangleright \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n)n \gg \dot{\vdash} (0 \leq$
 $(\text{rec}((1 + 1) + 1) * \underline{x}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\text{rec}((1 + 1) + 1) * \underline{x})n)n)n \rrbracket, p_0, c)$
 $\llbracket |x - x| = 0 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: |(\underline{x} + (-\underline{ux}))| = 0 \rrbracket$
 $\llbracket |x - x| = 0 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{x}: \text{eqReflexivity} \gg \underline{x} =$
 $\underline{x}; \text{PositiveToLeft}(\text{Eq})(1\text{term}) \triangleright \underline{x} = \underline{x} \gg (\underline{x} + (-\underline{ux})) =$
 $0; \text{SameNumerical} \triangleright (\underline{x} + (-\underline{ux})) = 0 \gg |(\underline{x} + (-\underline{ux}))| = |0|; |0| = 0 \gg |0| =$
 $0; \text{eqTransitivity} \triangleright |(\underline{x} + (-\underline{ux}))| = |0| \triangleright |0| = 0 \gg |(\underline{x} + (-\underline{ux}))| = 0 \rrbracket, p_0, c)$
 $\llbracket 1 < 2 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash} (1 <= (1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (1 = (1 + 1))n)n)n \rrbracket$
 $\llbracket 1 < 2 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash 0 < 1 \gg \dot{\vdash} (0 <= 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $1)n)n)n; \text{LessAddition} \triangleright \dot{\vdash} (0 <= 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n \gg \dot{\vdash} ((0 + 1) <=$
 $(1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 + 1) = (1 + 1))n)n)n; \text{lemma plus0Left} \gg (0 + 1) =$
 $1; \text{SubLessLeft} \triangleright (0 + 1) = 1 \triangleright \dot{\vdash} ((0 + 1) <= (1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} ((0 + 1) =$
 $(1 + 1))n)n)n \gg \dot{\vdash} (1 <= (1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (1 =$
 $(1 + 1))n)n)n; \text{Repetition} \triangleright \dot{\vdash} (1 <= (1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (1 = (1 + 1))n)n)n \gg$
 $\dot{\vdash} (1 <= (1 + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (1 = (1 + 1))n)n)n \rrbracket, p_0, c)$
 $\llbracket 1/3 < 2/3 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash} (\text{rec}((1 + 1) + 1) <= ((1 + 1) * \text{rec}((1 + 1) + 1))) \Rightarrow$

$\dot{\vdash}(\dot{\vdash}(\text{rec}((1+1)+1) = ((1+1) * \text{rec}((1+1)+1)))n)n]n]$
 $[1/3 < 2/3 \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash 1 < 2 \gg \dot{\vdash}(1 \leq (1+1)) \Rightarrow \dot{\vdash}(\dot{\vdash}(1 = (1+1))n)n)n; 0 < 1/3 \gg \dot{\vdash}(0 \leq \text{rec}((1+1)+1)) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \text{rec}((1+1)+1))n)n)n; \text{LessMultiplication} \triangleright \dot{\vdash}(0 \leq \text{rec}((1+1)+1)) \Rightarrow \dot{\vdash}(\dot{\vdash}(0 = \text{rec}((1+1)+1))n)n)n \triangleright \dot{\vdash}(1 \leq (1+1)) \Rightarrow \dot{\vdash}(\dot{\vdash}(1 = (1+1))n)n)n \gg \dot{\vdash}((1 * \text{rec}((1+1)+1)) \leq ((1+1) * \text{rec}((1+1)+1))) \Rightarrow \dot{\vdash}(\dot{\vdash}((1 * \text{rec}((1+1)+1)) = ((1+1) * \text{rec}((1+1)+1)))n)n)n; \text{lemma times1Left} \gg (1 * \text{rec}((1+1)+1)) = \text{rec}((1+1)+1); \text{SubLessLeft} \triangleright (1 * \text{rec}((1+1)+1)) = \text{rec}((1+1)+1) \triangleright \dot{\vdash}((1 * \text{rec}((1+1)+1)) \leq ((1+1) * \text{rec}((1+1)+1))) \Rightarrow \dot{\vdash}(\dot{\vdash}((1 * \text{rec}((1+1)+1)) = ((1+1) * \text{rec}((1+1)+1)))n)n)n \gg \dot{\vdash}(\text{rec}((1+1)+1) \leq ((1+1) * \text{rec}((1+1)+1))) \Rightarrow \dot{\vdash}(\dot{\vdash}(\text{rec}((1+1)+1) = ((1+1) * \text{rec}((1+1)+1)))n)n)n; \text{Repetition} \triangleright \dot{\vdash}(\text{rec}((1+1)+1) \leq ((1+1) * \text{rec}((1+1)+1))) \Rightarrow \dot{\vdash}(\dot{\vdash}(\text{rec}((1+1)+1) = ((1+1) * \text{rec}((1+1)+1)))n)n)n \gg \dot{\vdash}(\text{rec}((1+1)+1) \leq ((1+1) * \text{rec}((1+1)+1))) \Rightarrow \dot{\vdash}(\dot{\vdash}(\text{rec}((1+1)+1) = ((1+1) * \text{rec}((1+1)+1)))n)n)n]$, $p_0, c]$
 $[(1/3)x + (1/3)x = (2/3)x \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x})]$
 $[(1/3)x + (1/3)x = (2/3)x \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \text{TwoWholes} \gg ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = ((1+1) * (\text{rec}((1+1)+1) * \underline{x})); \text{timesAssociativity} \gg (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) = (1+1) * (\text{rec}((1+1)+1) * \underline{x}); \text{eqSymmetry} \triangleright (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) = (1+1) * (\text{rec}((1+1)+1) * \underline{x}) \gg ((1+1) * (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x}); \text{eqTransitivity} \triangleright ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (1+1) * (\text{rec}((1+1)+1) * \underline{x}) \triangleright ((1+1) * (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) \gg ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x}); \text{Repetition} \triangleright ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) \gg ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x})]$, $p_0, c]$
 $[-(1/3)x - (1/3)x = -(2/3)x \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. ((-\text{u}(\text{rec}((1+1)+1) * \underline{x})) + (-\text{u}(\text{rec}((1+1)+1) * \underline{x}))) = (-\text{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x}))]$
 $[-(1/3)x - (1/3)x = -(2/3)x \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. (1/3)x + (1/3)x = (2/3)x \gg ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x}); \text{EqNegated} \triangleright ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) \gg (-\text{u}((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}))) = (-\text{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})); -x - y = -(x + y) \gg ((-\text{u}(\text{rec}((1+1)+1) * \underline{x})) + (-\text{u}(\text{rec}((1+1)+1) * \underline{x}))) = (-\text{u}((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}))) \triangleright ((-\text{u}(\text{rec}((1+1)+1) * \underline{x})) + (-\text{u}(\text{rec}((1+1)+1) * \underline{x}))) = (-\text{u}((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}))) \triangleright (-\text{u}((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}))) = (-\text{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})) \gg ((-\text{u}(\text{rec}((1+1)+1) * \underline{x})) + (-\text{u}(\text{rec}((1+1)+1) * \underline{x}))) = (-\text{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x}))]$, $p_0, c]$
 $[(2/3)x + (1/3)x = x \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}. (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}) = \underline{x}]$
 $[(2/3)x + (1/3)x = x \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}. \text{TwoWholes} \gg ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) =$

$$\begin{aligned}
& ((1+1) * (\text{rec}((1+1)+1) * \underline{x})); \text{timesAssociativity} \gg (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) = \\
& ((1+1) * (\text{rec}((1+1)+1) * \underline{x})); \text{eqSymmetry} \triangleright (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) = \\
& ((1+1) * (\text{rec}((1+1)+1) * \underline{x})) \gg ((1+1) * (\text{rec}((1+1)+1) * \underline{x})) = \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}); \text{eqTransitivity} \triangleright ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& ((1+1) * (\text{rec}((1+1)+1) * \underline{x})) \triangleright ((1+1) * (\text{rec}((1+1)+1) * \underline{x})) = \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) \gg ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}); \text{eqAddition} \triangleright ((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) \gg \\
& (((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}); \text{ThreeThirds} \gg \\
& (((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& \underline{x}; \text{Equality} \triangleright (((\text{rec}((1+1)+1) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}) \triangleright (((\text{rec}((1+1)+1) * \underline{x}) * \\
& \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) + (\text{rec}((1+1)+1) * \underline{x}) = \underline{x} \gg \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x}) = \underline{x}], p_0, c]
\end{aligned}$$

$$[-x + (1/3)x = -(2/3)x \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: ((-\underline{u}\underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = (-\underline{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x}))]$$

$$\begin{aligned}
& [-x + (1/3)x = -(2/3)x \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: (2/3)x + (1/3)x = x \gg \\
& (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& \underline{x}; \text{PositiveToRight}(\text{Eq}) \triangleright (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& \underline{x} \gg (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) = \\
& (\underline{x} + (-\underline{u}(\text{rec}((1+1)+1) * \underline{x}))); \text{EqNegated} \triangleright (((1+1) * \text{rec}((1+1)+1)) * \underline{x}) = \\
& (\underline{x} + (-\underline{u}(\text{rec}((1+1)+1) * \underline{x}))) \gg (-\underline{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})) = \\
& (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}((1+1)+1) * \underline{x}))))); \text{MinusNegated} \gg (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}((1+1)+1) * \underline{x})))) = \\
& ((\text{rec}((1+1)+1) * \underline{x}) + (-\underline{u}\underline{x})); \text{plusCommutativity} \gg \\
& ((\text{rec}((1+1)+1) * \underline{x}) + (-\underline{u}\underline{x})) = \\
& ((-\underline{u}\underline{x}) + (\text{rec}((1+1)+1) * \underline{x})); \text{eqTransitivity4} \triangleright (-\underline{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})) = \\
& (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}((1+1)+1) * \underline{x})))) \triangleright (-\underline{u}(\underline{x} + (-\underline{u}(\text{rec}((1+1)+1) * \underline{x})))) = \\
& ((\text{rec}((1+1)+1) * \underline{x}) + (-\underline{u}\underline{x})) \triangleright ((\text{rec}((1+1)+1) * \underline{x}) + (-\underline{u}\underline{x})) = \\
& ((-\underline{u}\underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) \gg (-\underline{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})) = \\
& ((-\underline{u}\underline{x}) + (\text{rec}((1+1)+1) * \underline{x})); \text{eqSymmetry} \triangleright (-\underline{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})) = \\
& ((-\underline{u}\underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) \gg ((-\underline{u}\underline{x}) + (\text{rec}((1+1)+1) * \underline{x})) = \\
& (-\underline{u}(((1+1) * \text{rec}((1+1)+1)) * \underline{x})), p_0, c]
\end{aligned}$$

line ell y because lemma (2/3)x+(1/3)x=x indeed 2/3 * meta x + 1/3 * meta x = meta x end line line ell a because lemma eqTransitivity modus ponens ell x modus ponens ell y indeed 1/3 * meta x + 2/3 * meta x = meta x end line

$$\begin{aligned}
& [\text{PreserveLessGreater} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{x1}): \forall (\underline{x2}): \forall (\underline{y1}): \forall (\underline{y2}): \forall \underline{z}: (\underline{x1}) <= \\
& (\underline{y1}) + (-\underline{u}\underline{z}) \vdash \dot{\neg} (|((\underline{x1}) + (-\underline{u}(\underline{x2})))| <= (\text{rec}((1+1)+1) * \underline{z}) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (|((\underline{x1}) + (-\underline{u}(\underline{x2})))| = (\text{rec}((1+1)+1) * \underline{z}))n)n \vdash \\
& \dot{\neg} (|((\underline{y1}) + (-\underline{u}(\underline{y2})))| <= (\text{rec}((1+1)+1) * \underline{z}) \Rightarrow \dot{\neg} (\dot{\neg} (|((\underline{y1}) + (-\underline{u}(\underline{y2})))| = \\
& (\text{rec}((1+1)+1) * \underline{z}))n)n \vdash \underline{x2} <= (\underline{y2}) + (-\underline{u}(\text{rec}((1+1)+1) * \underline{z})))]
\end{aligned}$$

$$\begin{aligned}
& [\text{PreserveLessGreater} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \\
& \forall (\underline{x1}): \forall (\underline{x2}): \forall (\underline{y1}): \forall (\underline{y2}): \forall \underline{z}: (\underline{x1}) <= ((\underline{y1}) + (-\underline{u}\underline{z})) \vdash \\
& \dot{\neg} (|((\underline{x1}) + (-\underline{u}(\underline{x2})))| <= (\text{rec}((1+1)+1) * \underline{z}) \Rightarrow \dot{\neg} (\dot{\neg} (|((\underline{x1}) + (-\underline{u}(\underline{x2})))| =
\end{aligned}$$

$$\begin{aligned}
& - (2/3)x \gg ((-u(\text{rec}((1+1)+1) * \underline{z})) + (-u(\text{rec}((1+1)+1) * \underline{z}))) = \\
& (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z})); \text{lemma eqAdditionLeft} \triangleright ((-u(\text{rec}((1+1)+1) * \underline{z})) + (-u(\text{rec}((1+1)+1) * \underline{z}))) = (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z})) \gg \\
& ((\underline{y1}) + ((-u(\text{rec}((1+1)+1) * \underline{z})) + (-u(\text{rec}((1+1)+1) * \underline{z})))) = \\
& ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))); \text{eqTransitivity} \triangleright (((\underline{y1}) + (-u(\text{rec}((1+1)+1) * \underline{z}))) + (-u(\text{rec}((1+1)+1) * \underline{z}))) = ((\underline{y1}) + ((-u(\text{rec}((1+1)+1) * \underline{z})) + (-u(\text{rec}((1+1)+1) * \underline{z})))) \triangleright ((\underline{y1}) + ((-u(\text{rec}((1+1)+1) * \underline{z})) + (-u(\text{rec}((1+1)+1) * \underline{z})))) = ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))) \gg \\
& (((\underline{y1}) + (-u(\text{rec}((1+1)+1) * \underline{z}))) + (-u(\text{rec}((1+1)+1) * \underline{z}))) = \\
& ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))); \text{SubLessLeft} \triangleright (((\underline{y1}) + (-u(\text{rec}((1+1)+1) * \underline{z}))) + (-u(\text{rec}((1+1)+1) * \underline{z}))) = ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))) \triangleright \dot{\dot{}}(((\underline{y1}) + (-u(\text{rec}((1+1)+1) * \underline{z}))) + (-u(\text{rec}((1+1)+1) * \underline{z}))) \leq = \\
& ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(((\underline{y1}) + (-u(\text{rec}((1+1)+1) * \underline{z}))) + (-u(\text{rec}((1+1)+1) * \underline{z})))) + (-u(\text{rec}((1+1)+1) * \underline{z}))) = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))))n)n)n \gg \\
& \dot{\dot{}}(((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))) \leq = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z})))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z})))) = \\
& ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))))n)n)n; \text{LessLeqTransitivity} \triangleright \dot{\dot{}}(\underline{(x2)} \leq = \\
& ((\underline{x1}) + (\text{rec}((1+1)+1) * \underline{z}))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(\underline{(x2)} = ((\underline{x1}) + (\text{rec}((1+1)+1) * \underline{z}))))n)n)n \triangleright \\
& ((\underline{x1}) + (\text{rec}((1+1)+1) * \underline{z})) \leq = ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))) \gg \\
& \dot{\dot{}}(\underline{(x2)} \leq = ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z})))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(\underline{(x2)} = \\
& ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))))n)n)n; \text{LessTransitivity} \triangleright \dot{\dot{}}(\underline{(x2)} \leq = \\
& ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z})))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(\underline{(x2)} = \\
& ((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z}))))n)n)n \triangleright \dot{\dot{}}(((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1) * \underline{z})))) \leq = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z})))) \Rightarrow \\
& \dot{\dot{}}(\dot{\dot{}}(((\underline{y1}) + (-u(((1+1) * \text{rec}((1+1)+1)) * \underline{z})))) = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))))n)n)n \gg \dot{\dot{}}(\underline{(x2)} \leq = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z})))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(\underline{(x2)} = \\
& ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))))n)n)n; \text{LessLeq} \triangleright \dot{\dot{}}(\underline{(x2)} \leq = \\
& ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z})))) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(\underline{(x2)} = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))))n)n)n \gg \underline{(x2)} \leq = ((\underline{y2}) + (-u(\text{rec}((1+1)+1) * \underline{z}))))], \text{po}, \text{c})]
\end{aligned}$$

$$\begin{aligned}
& [\text{ClosetolessIsLess} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{x1}): \forall(\underline{x2}): \forall \underline{y}: \forall \underline{z}: (\underline{x1} \leq = (\underline{y} + (-\underline{uz}))) \vdash \\
& \dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| \leq = (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| = \\
& (\text{rec}((1+1)+1) * \underline{z}))n)n)n \vdash \underline{(x2)} \leq = (\underline{y} + (-u(\text{rec}((1+1)+1) * \underline{z}))))]
\end{aligned}$$

$$\begin{aligned}
& [\text{ClosetolessIsLess} \xrightarrow{\text{proof}} \lambda \text{c}. \lambda \text{x}. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall(\underline{x1}): \forall(\underline{x2}): \forall \underline{y}: \forall \underline{z}: (\underline{x1} \leq = \\
& (\underline{y} + (-\underline{uz}))) \vdash \dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| \leq = (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \\
& \dot{\dot{}}(\dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| = (\text{rec}((1+1)+1) * \underline{z}))n)n)n \vdash 0 \leq = |\text{x}| \gg 0 \leq = \\
& |((\underline{x1}) + (-\underline{u}(\underline{x2})))| \rceil); \text{leqLessTransitivity} \triangleright 0 \leq = \\
& |((\underline{x1}) + (-\underline{u}(\underline{x2})))| \triangleright \dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| \leq = (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \\
& \dot{\dot{}}(\dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| = (\text{rec}((1+1)+1) * \underline{z}))n)n)n \gg \dot{\dot{}}(0 \leq = \\
& (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \dot{\dot{}}(\dot{\dot{}}(0 = (\text{rec}((1+1)+1) * \underline{z}))n)n)n; |\text{x} - \text{x}| = 0 \gg \\
& |(\underline{y} + (-\underline{uy}))| = 0; \text{eqSymmetry} \triangleright |(\underline{y} + (-\underline{uy}))| = 0 \gg 0 = \\
& |(\underline{y} + (-\underline{uy}))|; \text{SubLessLeft} \triangleright 0 = |(\underline{y} + (-\underline{uy}))| \triangleright \dot{\dot{}}(0 \leq = (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \\
& \dot{\dot{}}(\dot{\dot{}}(0 = (\text{rec}((1+1)+1) * \underline{z}))n)n)n \gg \dot{\dot{}}(|(\underline{y} + (-\underline{uy}))| \leq = (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \\
& \dot{\dot{}}(\dot{\dot{}}(|(\underline{y} + (-\underline{uy}))| = (\text{rec}((1+1)+1) * \underline{z}))n)n)n; \text{PreserveLessGreater} \triangleright \underline{(x1)} \leq = \\
& (\underline{y} + (-\underline{uz})) \triangleright \dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| \leq = (\text{rec}((1+1)+1) * \underline{z})) \Rightarrow \\
& \dot{\dot{}}(\dot{\dot{}}(|((\underline{x1}) + (-\underline{u}(\underline{x2})))| = (\text{rec}((1+1)+1) * \underline{z}))n)n)n \triangleright \dot{\dot{}}(|(\underline{y} + (-\underline{uy}))| \leq =
\end{aligned}$$

$$\begin{aligned}
& (\overline{\epsilon})n)n)n)n)n); \text{Repetition} \triangleright \dot{\dot{\dot{\dot{\dot{(\forall_{\text{obj}}\overline{\epsilon}): \dot{\dot{\dot{(\dot{(\dot{(\forall_{\text{obj}}\overline{n}): \dot{\dot{\dot{(\forall_{\text{obj}}\overline{m}): \dot{\dot{\dot{(\dot{(\dot{0} \leq \overline{\epsilon}) \Rightarrow \\
\dot{\dot{\dot{(\dot{0} = \overline{\epsilon})n)n)n} \Rightarrow \dot{\dot{\overline{n} \leq \overline{m} \Rightarrow \underline{\underline{(fx)}}[\overline{m}] \leq \underline{\underline{(fx)}}[\overline{m}] \leq \\
(\underline{\underline{(fx)}}[\overline{m}] + \underline{\underline{(-u(\overline{\epsilon}))}}))n)n)n)n)n)n) \gg \dot{\dot{\dot{(\forall_{\text{obj}}\overline{\epsilon}): \dot{\dot{\dot{(\dot{(\dot{(\forall_{\text{obj}}\overline{n}): \dot{\dot{\dot{(\forall_{\text{obj}}\overline{m}): \dot{\dot{\dot{(\dot{(\dot{0} \leq \\
\overline{\epsilon}) \Rightarrow \dot{\dot{\dot{(\dot{0} = \overline{\epsilon})n)n)n} \Rightarrow \dot{\dot{\overline{n} \leq \overline{m} \Rightarrow \dot{\dot{\dot{(\underline{\underline{((\underline{\underline{(fx)}}[\overline{m}] + \underline{\underline{(-u(\underline{\underline{(fy)}}[\overline{m}]))) \leq \overline{\epsilon}) \Rightarrow \\
\dot{\dot{\dot{(\underline{\underline{((\underline{\underline{(fx)}}[\overline{m}] + \underline{\underline{(-u(\underline{\underline{(fy)}}[\overline{m}]))) \leq \\
\overline{\epsilon})n)n)n)n)n) \triangleright \dot{\dot{\dot{(\forall_{\text{obj}}\overline{\epsilon}): \dot{\dot{\dot{(\dot{(\dot{(\forall_{\text{obj}}\overline{n}): \dot{\dot{\dot{(\forall_{\text{obj}}\overline{m}): \dot{\dot{\dot{(\dot{(\dot{0} \leq \overline{\epsilon}) \Rightarrow \dot{\dot{\dot{(\dot{(\dot{0} \leq \\
\overline{\epsilon})n)n)n} \Rightarrow \dot{\dot{\overline{n} \leq \overline{m} \Rightarrow \underline{\underline{(fx)}}[\overline{m}] \leq \underline{\underline{((\underline{\underline{(fx)}}[\overline{m}] + \underline{\underline{(-u(\overline{\epsilon}))}}))n)n)n)n)n) \gg \\
\dot{\dot{\dot{(\forall_{\text{obj}}\overline{\epsilon}): \dot{\dot{\dot{(\dot{(\dot{(\forall_{\text{obj}}\overline{n}): \dot{\dot{\dot{(\forall_{\text{obj}}\overline{m}): \dot{\dot{\dot{(\dot{(\dot{0} \leq \overline{\epsilon}) \Rightarrow \dot{\dot{\dot{(\dot{(\dot{0} = \overline{\epsilon})n)n)n} \Rightarrow \\
\dot{\dot{\overline{n} \leq \overline{m} \Rightarrow \underline{\underline{(fy)}}[\overline{m}] \leq \underline{\underline{((\underline{\underline{(fy)}}[\overline{m}] + \underline{\underline{(-u(\overline{\epsilon}))}}))n)n)n)n)n) \triangleright \\
\dot{\dot{\dot{(\forall_{\text{obj}}\overline{\epsilon}): \dot{\dot{\dot{(\dot{(\dot{(\forall_{\text{obj}}\overline{n}): \dot{\dot{\dot{(\forall_{\text{obj}}\overline{m}): \dot{\dot{\dot{(\dot{(\dot{0} \leq \overline{\epsilon}) \Rightarrow \dot{\dot{\dot{(\dot{(\dot{0} = \overline{\epsilon})n)n)n} \Rightarrow \\
\dot{\dot{\overline{n} \leq \overline{m} \Rightarrow \underline{\underline{(fx)}}[\overline{m}] \leq \underline{\underline{((\underline{\underline{(fy)}}[\overline{m}] + \underline{\underline{(-u(\overline{\epsilon}))}}))n)n)n)n)n}], p_0, c]} \\
[-x * y = -(x * y) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: ((-\underline{ux}) * \underline{y}) = (-\underline{u(x * y)})] \\
[-x * y = -(x * y) \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}. ([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \text{Times}(-1)\text{Left} \gg \\
(-\underline{u1}) * \underline{x}) = (-\underline{ux}); \text{eqMultiplication} \triangleright ((-\underline{u1}) * \underline{x}) = (-\underline{ux}) \gg \\
((-\underline{u1}) * \underline{x}) * \underline{y}) = ((-\underline{ux}) * \underline{y}); \text{eqSymmetry} \triangleright (((-\underline{u1}) * \underline{x}) * \underline{y}) = ((-\underline{ux}) * \underline{y}) \gg \\
((-\underline{ux}) * \underline{y}) = (((-\underline{u1}) * \underline{x}) * \underline{y}); \text{timesAssociativity} \gg (((-\underline{u1}) * \underline{x}) * \underline{y}) = \\
((-\underline{u1}) * (\underline{x * y})); \text{Times}(-1)\text{Left} \gg ((-\underline{u1}) * (\underline{x * y})) = \\
(-\underline{u(x * y)}); \text{eqTransitivity4} \triangleright ((-\underline{ux}) * \underline{y}) = (((-\underline{u1}) * \underline{x}) * \underline{y}) \triangleright (((-\underline{u1}) * \underline{x}) * \underline{y}) = \\
((-\underline{u1}) * (\underline{x * y})) \triangleright ((-\underline{u1}) * (\underline{x * y})) = (-\underline{u(x * y)}) \gg ((-\underline{ux}) * \underline{y}) = (-\underline{u(x * y)})], p_0, c)] \\
[\text{LeqMultiplicationLeft} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: 0 \leq \underline{z} \vdash \underline{x} \leq \underline{y} \vdash (\underline{z * x}) \leq \\
(\underline{z * y})] \\
[\text{LeqMultiplicationLeft} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}. ([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: 0 \leq \underline{z} \vdash \underline{x} \leq \underline{y} \vdash \\
\text{leqMultiplication} \triangleright 0 \leq \underline{z} \triangleright \underline{x} \leq \underline{y} \gg (\underline{x * z}) \leq (\underline{y * z}); \text{timesCommutativity} \gg \\
(\underline{x * z}) = (\underline{z * x}); \text{subLeqLeft} \triangleright (\underline{x * z}) = (\underline{z * x}) \triangleright (\underline{x * z}) \leq (\underline{y * z}) \gg (\underline{z * x}) \leq \\
(\underline{y * z}); \text{timesCommutativity} \gg (\underline{y * z}) = (\underline{z * y}); \text{subLeqRight} \triangleright (\underline{y * z}) = \\
(\underline{z * y}) \triangleright (\underline{z * x}) \leq (\underline{y * z}) \gg (\underline{z * x}) \leq (\underline{z * y})], p_0, c)] \\
[\text{SameFMultiplication}(\text{Helper}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
\forall \underline{v1}: \forall \underline{v2}: \forall \underline{m}: \forall \underline{n}: \forall \underline{\epsilon}: \forall \underline{(fx)}: \forall \underline{(fy)}: \forall \underline{(fz)}: \forall_{\text{obj}} \underline{m}: \dot{\dot{0} \leq ((\underline{\epsilon}) * \text{rec}(\underline{v1}))} \Rightarrow \\
\dot{\dot{(\dot{(\dot{0} = ((\underline{\epsilon}) * \text{rec}(\underline{v1}))n)n)n} \Rightarrow \underline{n} \leq \underline{m} \Rightarrow \dot{\dot{(\underline{\underline{((\underline{\underline{(fx)}}[\underline{m}] + \underline{\underline{(-u(\underline{\underline{(fy)}}[\underline{m}]))) \leq \\
((\underline{\epsilon}) * \text{rec}(\underline{v1})) \Rightarrow \dot{\dot{(\dot{(\underline{\underline{((\underline{\underline{(fx)}}[\underline{m}] + \underline{\underline{(-u(\underline{\underline{(fy)}}[\underline{m}]))) \leq \\
((\underline{\epsilon}) * \text{rec}(\underline{v1}))n)n)n} \Rightarrow \\
\forall_{\text{obj}} \underline{v2}: \dot{\dot{(\underline{\underline{(fz)}}[\underline{v2}])} \leq \underline{(v1)} \Rightarrow \dot{\dot{(\dot{(\underline{\underline{(fz)}}[\underline{v2}])} = \underline{(v1)})n)n)n} \Rightarrow \dot{\dot{0} \leq \\
\underline{\epsilon}) \Rightarrow \dot{\dot{(\dot{0} = \underline{\epsilon})n)n)n} \Rightarrow \underline{n} \leq \underline{m} \Rightarrow \dot{\dot{(\underline{\underline{\{ph} \in \{ph \in \\
\text{P}(\text{P}(\text{Union}(\{\underline{N}, \underline{Q}\})) \mid \dot{\dot{(\forall_{\text{obj}}(\underline{op1}): \dot{\dot{(\dot{(\forall_{\text{obj}}(\underline{op2}): \dot{\dot{(\dot{(\dot{(\dot{(\underline{op1}} \in \underline{N} \Rightarrow \\
\dot{\dot{(\underline{op2}} \in \underline{Q})n)n} \Rightarrow \dot{\dot{a_{\text{ph}}} = \{\{\{\underline{op1}, \underline{op1}\}\}, \{\{\underline{op1}, \underline{op2}\}\}\})n)n)n)n)n) \mid \\
\dot{\dot{(\forall_{\text{obj}} \underline{m}: \dot{\dot{e_{\text{ph}}} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(\underline{fx}}[\underline{m}] * \underline{(fz)}[\underline{m}]\})\}\})n)n)n}[\underline{m}] + \underline{-u}\{\text{ph} \in \{\text{ph} \in \\
\text{P}(\text{P}(\text{Union}(\{\underline{N}, \underline{Q}\})) \mid \dot{\dot{(\forall_{\text{obj}}(\underline{op1}): \dot{\dot{(\dot{(\forall_{\text{obj}}(\underline{op2}): \dot{\dot{(\dot{(\dot{(\dot{(\underline{op1}} \in \underline{N} \Rightarrow
\end{aligned}$$

$$\begin{aligned}
& f_{Ph} \Rightarrow \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (\overline{(r1)} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{ \{ \{ (f1), \overline{(f1)} \}, \{ (f1), \overline{(f2)} \} \} \in f_{Ph} \Rightarrow \\
& \{ \{ \{ (f3), \overline{(f3)} \}, \{ (f3), \overline{(f4)} \} \} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(s1)}: (s1) \in N \Rightarrow \dot{\neg} (\forall_{obj} \overline{(s2)}: \dot{\neg} (\{ \{ (s1), \overline{(s1)} \}, \{ (s1), \overline{(s2)} \} \} \in \\
& f_{Ph} n) n) n) n) | \forall_{obj} \overline{(\epsilon)}: \dot{\neg} (\forall_{obj} \overline{n}: \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{(\epsilon)}) n) n) n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\neg} (| \{ \{ ph \in \{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (e_{Ph} = \\
& \{ \{ \overline{m}, \overline{m} \}, \{ \overline{m}, ((fy) \overline{[m]} * (fz) \overline{[m]}) \} \} n) n) n) \overline{[m]} + (-ud_{Ph} \overline{[m]}) | \leq \overline{(\epsilon)} \Rightarrow \\
& \dot{\neg} (\dot{\neg} (| \{ \{ ph \in \{ ph \in P(\overline{P(\text{Union}(\{N, Q\})) \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (e_{Ph} = \\
& \{ \{ \overline{m}, \overline{m} \}, \{ \overline{m}, ((fy) \overline{[m]} * (fz) \overline{[m]}) \} \} n) n) n) \overline{[m]} + (-ud_{Ph} \overline{[m]}) | = \overline{(\epsilon)} n) n) n) n) = \\
& \{ ph \in P(\{ ph \in P(\{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \\
& f_{Ph} \Rightarrow \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (\overline{(r1)} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{ \{ \{ (f1), \overline{(f1)} \}, \{ (f1), \overline{(f2)} \} \} \in f_{Ph} \Rightarrow \\
& \{ \{ \{ (f3), \overline{(f3)} \}, \{ (f3), \overline{(f4)} \} \} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} n) n) \Rightarrow \\
& \dot{\neg} (\forall_{obj} \overline{(s1)}: (s1) \in N \Rightarrow \dot{\neg} (\forall_{obj} \overline{(s2)}: \dot{\neg} (\{ \{ (s1), \overline{(s1)} \}, \{ (s1), \overline{(s2)} \} \} \in \\
& f_{Ph} n) n) n) n) | \forall_{obj} \overline{(\epsilon)}: \dot{\neg} (\forall_{obj} \overline{n}: \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{(\epsilon)}) n) n) n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\neg} (| \{ \{ ph \in \{ ph \in P(P(\text{Union}(\{N, Q\})) \} \} \} | \\
& \dot{\neg} (\forall_{obj} \overline{(op1)}): \dot{\neg} (\dot{\neg} (\forall_{obj} \overline{(op2)}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{(op1)} \in N \Rightarrow \dot{\neg} (\overline{(op2)} \in Q) n) n) \Rightarrow \\
& \dot{\neg} (a_{Ph} = \{ \{ \{ (op1), \overline{(op1)} \}, \{ (op1), \overline{(op2)} \} \} n) n) n) n) n) | \dot{\neg} (\forall_{obj} \overline{m}: \dot{\neg} (e_{Ph} =
\end{aligned}$$

$f_{Ph} \Rightarrow \dot{\vdash} (\forall_{obj} \overline{(op1)}): \dot{\vdash} (\dot{\vdash} (\forall_{obj} \overline{(op2)}): \dot{\vdash} (\dot{\vdash} (\overline{(op1)} \in N \Rightarrow \dot{\vdash} (\overline{(op2)} \in Q)n)n) \Rightarrow$
 $\dot{\vdash} (\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \Rightarrow$
 $\dot{\vdash} (\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
 $\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n) \Rightarrow$
 $\dot{\vdash} (\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\vdash} (\forall_{obj} \overline{(s2)}: \dot{\vdash} (\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
 $f_{Ph})n)n)n) \mid \forall_{obj} \overline{(\epsilon)}: \dot{\vdash} (\forall_{obj} \overline{n}: \dot{\vdash} (\forall_{obj} \overline{m}: \dot{\vdash} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{(\epsilon)})n)n) \Rightarrow$
 $\overline{n} \leq \overline{m} \Rightarrow \dot{\vdash} (\{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))\})) \mid$
 $\dot{\vdash} (\forall_{obj} \overline{(op1)}: \dot{\vdash} (\dot{\vdash} (\forall_{obj} \overline{(op2)}: \dot{\vdash} (\dot{\vdash} (\overline{(op1)} \in N \Rightarrow \dot{\vdash} (\overline{(op2)} \in Q)n)n) \Rightarrow$
 $\dot{\vdash} (a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\vdash} (\forall_{obj} \overline{(crs1)}: \dot{\vdash} (c_{Ph} =$
 $\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n) \mid \overline{[m]} + (-ud_{Ph} \overline{[m]}) \leq \overline{(\epsilon)} \Rightarrow \dot{\vdash} (\dot{\vdash} (\{ph \in$
 $\{ph \in P(P(\text{Union}(\{N, Q\}))\})) \mid \dot{\vdash} (\forall_{obj} \overline{(op1)}: \dot{\vdash} (\dot{\vdash} (\forall_{obj} \overline{(op2)}: \dot{\vdash} (\dot{\vdash} (\overline{(op1)} \in$
 $N \Rightarrow \dot{\vdash} (\overline{(op2)} \in Q)n)n) \Rightarrow \dot{\vdash} (a_{Ph} =$
 $\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n) \mid \dot{\vdash} (\forall_{obj} \overline{(crs1)}: \dot{\vdash} (c_{Ph} =$
 $\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n) \mid \overline{[m]} + (-ud_{Ph} \overline{[m]}) = \overline{(\epsilon)})n)n)n)n) \mid p_0, c]$
 $[\text{DistributionLeft} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{y} * \underline{x}) + (\underline{z} * \underline{x}))]$
 $[\text{DistributionLeft} \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \text{DistributionOutLeft} \gg$
 $((\underline{y} * \underline{x}) + (\underline{z} * \underline{x})) = (\underline{x} * (\underline{y} + \underline{z})); \text{eqSymmetry} \triangleright ((\underline{y} * \underline{x}) + (\underline{z} * \underline{x})) =$
 $(\underline{x} * (\underline{y} + \underline{z})) \gg (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{y} * \underline{x}) + (\underline{z} * \underline{x}))], p_0, c)]$
XX 'nonnegativeFactors' er en konsekvens heraf
 $[\text{MultiplyEquations}(\text{Leq}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: 0 \leq \underline{x} \vdash 0 \leq \underline{z} \vdash$
 $\underline{x} \leq \underline{y} \vdash \underline{z} \leq \underline{u} \vdash (\underline{x} * \underline{z}) \leq (\underline{y} * \underline{u})]$
 $[\text{MultiplyEquations}(\text{Leq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: 0 \leq \underline{x} \vdash$
 $0 \leq \underline{z} \vdash \underline{x} \leq \underline{y} \vdash \underline{z} \leq \underline{u} \vdash \text{leqMultiplication} \triangleright 0 \leq \underline{z} \triangleright \underline{x} \leq \underline{y} \gg (\underline{x} * \underline{z}) \leq$
 $(\underline{y} * \underline{z}); \text{leqTransitivity} \triangleright 0 \leq \underline{x} \triangleright \underline{x} \leq \underline{y} \gg 0 \leq \underline{y}; \text{LeqMultiplicationLeft} \triangleright 0 \leq$
 $\underline{y} \triangleright \underline{z} \leq \underline{u} \gg (\underline{y} * \underline{z}) \leq (\underline{y} * \underline{u}); \text{leqTransitivity} \triangleright (\underline{x} * \underline{z}) \leq (\underline{y} * \underline{z}) \triangleright (\underline{y} * \underline{z}) \leq$
 $(\underline{y} * \underline{u}) \gg (\underline{x} * \underline{z}) \leq (\underline{y} * \underline{u})], p_0, c)]$
 $[\text{LessMultiplication}(\text{F})(\text{Helper2}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \forall \underline{v}: \dot{\vdash} (0 \leq \underline{u} \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = \underline{u})n)n) \vdash \dot{\vdash} (0 \leq \underline{v} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{v})n)n) \vdash \underline{x} \leq (\underline{y} + (-u\underline{u})) \vdash$
 $0 \leq (\underline{z} + (-u\underline{v})) \vdash (\underline{x} * \underline{z}) \leq ((\underline{y} * \underline{z}) + (-u(\underline{u} * \underline{v})))]$
 $[\text{LessMultiplication}(\text{F})(\text{Helper2}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda x. \mathcal{P}([\text{SystemQ} \vdash$
 $\forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \forall \underline{v}: \dot{\vdash} (0 \leq \underline{u} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{u})n)n) \vdash \dot{\vdash} (0 \leq \underline{v} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$
 $\underline{v})n)n) \vdash \underline{x} \leq (\underline{y} + (-u\underline{u})) \vdash 0 \leq (\underline{z} + (-u\underline{v})) \vdash \text{negativeToLeft}(\text{Leq}) \triangleright \underline{x} \leq$
 $(\underline{y} + (-u\underline{u})) \gg (\underline{x} + \underline{u}) \leq \underline{y}; \text{plusCommutativity} \gg (\underline{x} + \underline{u}) =$
 $(\underline{u} + \underline{x}); \text{subLeqLeft} \triangleright (\underline{x} + \underline{u}) = (\underline{u} + \underline{x}) \triangleright (\underline{x} + \underline{u}) \leq \underline{y} \gg (\underline{u} + \underline{x}) \leq$
 $\underline{y}; \text{PositiveToRight}(\text{Leq}) \triangleright (\underline{u} + \underline{x}) \leq \underline{y} \gg \underline{u} \leq$
 $(\underline{y} + (-u\underline{x})); \text{negativeToLeft}(\text{Leq})(\text{1term}) \triangleright 0 \leq (\underline{z} + (-u\underline{v})) \gg \underline{v} \leq$
 $\underline{z}; \text{LessLeq} \triangleright \dot{\vdash} (0 \leq \underline{u} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{u})n)n) \gg 0 \leq \underline{u}; \text{LessLeq} \triangleright \dot{\vdash} (0 \leq \underline{v} \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (0 = \underline{v})n)n) \gg 0 \leq \underline{v}; \text{MultiplyEquations}(\text{Leq}) \triangleright 0 \leq \underline{u} \triangleright 0 \leq \underline{v} \triangleright \underline{u} \leq$
 $(\underline{y} + (-u\underline{x})) \triangleright \underline{v} \leq \underline{z} \gg (\underline{u} * \underline{v}) \leq ((\underline{y} + (-u\underline{x})) * \underline{z}); \text{timesCommutativity} \gg$
 $((\underline{y} + (-u\underline{x})) * \underline{z}) = (\underline{z} * (\underline{y} + (-u\underline{x}))); \text{DistributionLeft} \gg (\underline{z} * (\underline{y} + (-u\underline{x}))) =$
 $((\underline{y} * \underline{z}) + ((-u\underline{x}) * \underline{z})); -x * y = -(x * y) \gg ((-u\underline{x}) * \underline{z}) =$
 $(-u(\underline{x} * \underline{z})); \text{lemma eqAdditionLeft} \triangleright ((-u\underline{x}) * \underline{z}) = (-u(\underline{x} * \underline{z})) \gg$
 $((\underline{y} * \underline{z}) + ((-u\underline{x}) * \underline{z})) = ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))); \text{eqTransitivity4} \triangleright ((\underline{y} + (-u\underline{x})) * \underline{z}) =$

$$\begin{aligned}
& \{\underline{\{m, m\}}, \underline{\{m, ((fy)[m] * (fz)[m])\}}\})n)n\}[\bar{m}] + (-ud_{Ph}[\bar{m}]) \mid = \\
& \overline{(\epsilon)}n)n)n)n\}; \text{lemma eqLeq(R)} \triangleright \{ph \in P(\{ph \in P(\{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n\}) \mid \\
& \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{r1}): (\overline{r1}) \in \mathbf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \dot{\neg}(\overline{r1}) = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n) \Rightarrow \\
& \dot{\neg}(\forall_{obj}(\overline{f1}): \forall_{obj}(\overline{f2}): \forall_{obj}(\overline{f3}): \forall_{obj}(\overline{f4}): \{\{\overline{f1}, \overline{f1}\}, \{\overline{f1}, \overline{f2}\}\}) \in \mathbf{f}_{Ph} \Rightarrow \\
& \{\{\overline{f3}, \overline{f3}\}, \{\overline{f3}, \overline{f4}\}\}) \in \mathbf{f}_{Ph} \Rightarrow \overline{f1} = \overline{f3} \Rightarrow \overline{f2} = \overline{f4})n)n \Rightarrow \\
& \dot{\neg}(\forall_{obj}(\overline{s1}): (\overline{s1}) \in N \Rightarrow \dot{\neg}(\forall_{obj}(\overline{s2}): \dot{\neg}(\{\{\overline{s1}, \overline{s1}\}, \{\overline{s1}, \overline{s2}\}\}) \in \\
& \mathbf{f}_{Ph})n)n)n\}) \mid \forall_{obj}(\overline{\epsilon}): \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{\epsilon})n)n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\neg}(\{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \\
& \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n\}) \mid \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(\mathbf{e}_{Ph} = \\
& \{\underline{\{m, m\}}, \underline{\{m, ((fx)[m] * (fz)[m])\}}\})n)n\}[\bar{m}] + (-ud_{Ph}[\bar{m}]) \mid \leq \overline{(\epsilon)} \Rightarrow \\
& \dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(\overline{P}(\text{Union}(\{N, Q\}))) \mid \\
& \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{r1}): (\overline{r1}) \in \\
& \mathbf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \\
& \dot{\neg}(\overline{r1}) = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n) \Rightarrow \\
& \dot{\neg}(\forall_{obj}(\overline{f1}): \forall_{obj}(\overline{f2}): \forall_{obj}(\overline{f3}): \forall_{obj}(\overline{f4}): \{\{\overline{f1}, \overline{f1}\}, \{\overline{f1}, \overline{f2}\}\}) \in \mathbf{f}_{Ph} \Rightarrow \\
& \{\{\overline{f3}, \overline{f3}\}, \{\overline{f3}, \overline{f4}\}\}) \in \mathbf{f}_{Ph} \Rightarrow \overline{f1} = \overline{f3} \Rightarrow \overline{f2} = \overline{f4})n)n \Rightarrow \\
& \dot{\neg}(\forall_{obj}(\overline{s1}): (\overline{s1}) \in N \Rightarrow \dot{\neg}(\forall_{obj}(\overline{s2}): \dot{\neg}(\{\{\overline{s1}, \overline{s1}\}, \{\overline{s1}, \overline{s2}\}\}) \in \\
& \mathbf{f}_{Ph})n)n)n\}) \mid \forall_{obj}(\overline{\epsilon}): \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 \leq \overline{\epsilon}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{\epsilon})n)n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\neg}(\{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid \\
& \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n\}) \mid \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(\mathbf{e}_{Ph} = \\
& \{\underline{\{m, m\}}, \underline{\{m, ((fy)[m] * (fz)[m])\}}\})n)n\}[\bar{m}] + (-ud_{Ph}[\bar{m}]) \mid \leq \overline{(\epsilon)} \Rightarrow \\
& \dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(\overline{P}(\text{Union}(\{N, Q\}))) \mid \\
& \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n\}) \mid \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(\mathbf{e}_{Ph} = \\
& \{\underline{\{m, m\}}, \underline{\{m, ((fx)[m] * (fz)[m])\}}\})n)n\}[\bar{m}] <= (\{ph \in \{ph \in \\
& P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(\overline{op1}): \dot{\neg}(\dot{\neg}(\forall_{obj}(\overline{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{op1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{op2}) \in Q)n)n \Rightarrow \dot{\neg}(\mathbf{a}_{Ph} = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n)n\}) \mid \\
& \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(\mathbf{e}_{Ph} = \{\underline{\{m, m\}}, \underline{\{m, ((fy)[m] * (fz)[m])\}}\})n)n\}[\bar{m}] +
\end{aligned}$$

$$(|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + (-\underline{uu}))|) + |(\underline{u} + \underline{y})|)$$

$$\begin{aligned} & [\text{insertTwoMiddleTerms}(\text{Numerical}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \\ & \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \text{insertMiddleTerm}(\text{Numerical}) \gg |(\underline{x} + \underline{y})| <= \\ & (|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + \underline{y})|); \text{insertMiddleTerm}(\text{Numerical}) \gg |(\underline{z} + \underline{y})| <= \\ & (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|); \text{LeqAdditionLeft} \triangleright |(\underline{z} + \underline{y})| <= \\ & (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|) \gg (|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + \underline{y})|) <= \\ & (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|)); \text{leqTransitivity} \triangleright |(\underline{x} + \underline{y})| <= \\ & (|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + \underline{y})|) \triangleright (|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + \underline{y})|) <= \\ & (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|)) \gg |(\underline{x} + \underline{y})| <= \\ & (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|)); \text{plusAssociativity} \gg \\ & ((|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + (-\underline{uu}))|) + |(\underline{u} + \underline{y})|) = (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \\ & \underline{y})|)); \text{eqSymmetry} \triangleright ((|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + (-\underline{uu}))|) + |(\underline{u} + \underline{y})|) = (|(\underline{x} + (-\underline{uz}))| + \\ & (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|)) \gg (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|)) = \\ & ((|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + (-\underline{uu}))|) + |(\underline{u} + \underline{y})|); \text{subLeqRight} \triangleright (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + \\ & (-\underline{uu}))| + |(\underline{u} + \underline{y})|)) = ((|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + (-\underline{uu}))|) + |(\underline{u} + \underline{y})|) \triangleright |(\underline{x} + \underline{y})| <= \\ & (|(\underline{x} + (-\underline{uz}))| + (|(\underline{z} + (-\underline{uu}))| + |(\underline{u} + \underline{y})|)) \gg |(\underline{x} + \underline{y})| <= \\ & ((|(\underline{x} + (-\underline{uz}))| + |(\underline{z} + (-\underline{uu}))|) + |(\underline{u} + \underline{y})|), p_0, c) \end{aligned}$$

$$[\text{FromPositiveNumerical} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \vdash \dot{\vdash} (\underline{x} = 0)n]$$

$$\begin{aligned} & [\text{FromPositiveNumerical} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: 0 <= \underline{x} \vdash \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \vdash \text{NonnegativeNumerical} \triangleright 0 <= \underline{x} \gg |\underline{x}| = \\ & \underline{x}; \text{SubLessRight} \triangleright |\underline{x}| = \underline{x} \triangleright \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \gg \dot{\vdash} (0 <= \underline{x} \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n); \text{LessNeq} \triangleright \dot{\vdash} (0 <= \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \gg \dot{\vdash} (0 = \\ & \underline{x})n; \text{NeqSymmetry} \triangleright \dot{\vdash} (0 = \underline{x})n \gg \dot{\vdash} (\underline{x} = 0)n; \forall \underline{x}: \underline{x} <= 0 \vdash \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \vdash \text{lemma nonpositiveNumerical} \triangleright \underline{x} <= 0 \gg |\underline{x}| = \\ & (-\underline{ux}); \text{SubLessRight} \triangleright |\underline{x}| = (-\underline{ux}) \triangleright \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \gg \\ & \dot{\vdash} (0 <= (-\underline{ux}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (-\underline{ux})n)n)); \text{PositiveNegated} \triangleright \dot{\vdash} (0 <= (-\underline{ux}) \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = (-\underline{ux})n)n) \gg \dot{\vdash} ((-u(-\underline{ux})) <= 0 \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u(-\underline{ux})) = \\ & 0)n)n); \text{DoubleMinus} \gg (-u(-\underline{ux})) = \underline{x}; \text{SubLessLeft} \triangleright (-u(-\underline{ux})) = \\ & \underline{x} \triangleright \dot{\vdash} ((-u(-\underline{ux})) <= 0 \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u(-\underline{ux})) = 0)n)n) \gg \dot{\vdash} (\underline{x} <= 0 \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n); \text{LessNeq} \triangleright \dot{\vdash} (\underline{x} <= 0 \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{x} = 0)n)n) \gg \dot{\vdash} (\underline{x} = \\ & 0)n; \forall \underline{x}: \text{Ded} \triangleright \forall \underline{x}: 0 <= \underline{x} \vdash \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \vdash \dot{\vdash} (\underline{x} = 0)n \gg \\ & 0 <= \underline{x} \Rightarrow \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \Rightarrow \dot{\vdash} (\underline{x} = 0)n; \text{Ded} \triangleright \forall \underline{x}: \underline{x} <= 0 \vdash \\ & \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \vdash \dot{\vdash} (\underline{x} = 0)n \gg \underline{x} <= 0 \Rightarrow \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \Rightarrow \dot{\vdash} (\underline{x} = 0)n; \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \vdash \\ & \text{FromLeqGeq} \triangleright 0 <= \underline{x} \Rightarrow \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \Rightarrow \dot{\vdash} (\underline{x} = 0)n \triangleright \\ & \underline{x} <= 0 \Rightarrow \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \Rightarrow \dot{\vdash} (\underline{x} = 0)n \gg \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \\ & \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \Rightarrow \dot{\vdash} (\underline{x} = 0)n; \text{MP} \triangleright \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \Rightarrow \\ & \dot{\vdash} (\underline{x} = 0)n \triangleright \dot{\vdash} (0 <= |\underline{x}| \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = |\underline{x}|)n)n) \gg \dot{\vdash} (\underline{x} = 0)n], p_0, c) \end{aligned}$$

$$[\text{NegativeToRight}(\text{Neq})(1\text{term}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} ((\underline{x} + (-\underline{uy})) = 0)n \vdash \dot{\vdash} (\underline{x} = \underline{y})n]$$

$$\begin{aligned} & [\text{NegativeToRight}(\text{Neq})(1\text{term}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \vdash \\ & \text{PositiveToLeft}(\text{Eq})(1\text{term}) \triangleright \underline{x} = \underline{y} \gg (\underline{x} + (-\underline{uy})) = 0; \forall \underline{x}: \forall \underline{y}: \text{Ded} \triangleright \forall \underline{x}: \forall \underline{y}: \underline{x} = \\ & \underline{y} \vdash (\underline{x} + (-\underline{uy})) = 0 \gg \underline{x} = \underline{y} \Rightarrow (\underline{x} + (-\underline{uy})) = 0; \dot{\vdash} ((\underline{x} + (-\underline{uy})) = 0)n \vdash \end{aligned}$$

$MT \triangleright \underline{x} = \underline{y} \Rightarrow (\underline{x} + (-\underline{uy})) = 0 \triangleright \dot{\vdash} ((\underline{x} + (-\underline{uy})) = 0)n \gg \dot{\vdash} (\underline{x} = \underline{y})n], p_0, c]$
 $[NonzeroProduct(2) \xrightarrow{stmt} SystemQ \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} ((\underline{x} * \underline{y}) = 0)n \vdash \dot{\vdash} (\underline{y} = 0)n]$
 $[NonzeroProduct(2) \xrightarrow{proof} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{y} = 0 \vdash$
 $lemma eqMultiplicationLeft \triangleright \underline{y} = 0 \gg (\underline{x} * \underline{y}) = (\underline{x} * 0); x * 0 = 0 \gg (\underline{x} * 0) =$
 $0; eqTransitivity \triangleright (\underline{x} * \underline{y}) = (\underline{x} * 0) \triangleright (\underline{x} * 0) = 0 \gg (\underline{x} * \underline{y}) =$
 $0; \forall \underline{x}: \forall \underline{y}: Ded \triangleright \forall \underline{x}: \forall \underline{y}: \underline{y} = 0 \vdash (\underline{x} * \underline{y}) = 0 \gg \underline{y} = 0 \Rightarrow (\underline{x} * \underline{y}) = 0; \dot{\vdash} ((\underline{x} * \underline{y}) =$
 $0)n \vdash MT \triangleright \underline{y} = 0 \Rightarrow (\underline{x} * \underline{y}) = 0 \triangleright \dot{\vdash} ((\underline{x} * \underline{y}) = 0)n \gg \dot{\vdash} (\underline{y} = 0)n], p_0, c)]$
 $[NonreciprocalToRight(Eq)(1term) \xrightarrow{stmt} SystemQ \vdash \forall \underline{x}: \forall \underline{y}: (\underline{x} * \underline{y}) = 1 \vdash \underline{x} = \text{recy}]$
 $[NonreciprocalToRight(Eq)(1term) \xrightarrow{proof} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: (\underline{x} * \underline{y}) =$
 $1 \vdash eqMultiplication \triangleright (\underline{x} * \underline{y}) = 1 \gg ((\underline{x} * \underline{y}) * \text{recy}) = (1 * \text{recy}); 0 < 1 \gg \dot{\vdash} (0 < =$
 $1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n; PositiveNonzero \triangleright \dot{\vdash} (0 < = 1 \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = 1)n)n)n \gg$
 $\dot{\vdash} (1 = 0)n; eqSymmetry \triangleright (\underline{x} * \underline{y}) = 1 \gg 1 = (\underline{x} * \underline{y}); SubNeqLeft \triangleright 1 =$
 $(\underline{x} * \underline{y}) \triangleright \dot{\vdash} (1 = 0)n \gg \dot{\vdash} ((\underline{x} * \underline{y}) = 0)n; NonzeroProduct(2) \triangleright \dot{\vdash} ((\underline{x} * \underline{y}) = 0)n \gg$
 $\dot{\vdash} (\underline{y} = 0)n; x = x * y * (1/y) \triangleright \dot{\vdash} (\underline{y} = 0)n \gg x =$
 $((\underline{x} * \underline{y}) * \text{recy}); lemma times1Left \gg (1 * \text{recy}) = \text{recy}; eqTransitivity4 \triangleright \underline{x} =$
 $((\underline{x} * \underline{y}) * \text{recy}) \triangleright ((\underline{x} * \underline{y}) * \text{recy}) = (1 * \text{recy}) \triangleright (1 * \text{recy}) = \text{recy} \gg \underline{x} = \text{recy}], p_0, c)]$
 $[NonreciprocalToRight(Eq) \xrightarrow{stmt} SystemQ \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\vdash} (\underline{y} = 0)n \vdash (\underline{x} * \underline{y}) = \underline{z} \vdash$
 $\underline{x} = (\underline{z} * \text{recy})]$

$[NonreciprocalToRight(Eq) \xrightarrow{proof} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\vdash} (\underline{y} = 0)n \vdash$
 $(\underline{x} * \underline{y}) = \underline{z} \vdash eqMultiplication \triangleright (\underline{x} * \underline{y}) = \underline{z} \gg ((\underline{x} * \underline{y}) * \text{recy}) = (\underline{z} * \text{recy}); x =$
 $x * y * (1/y) \triangleright \dot{\vdash} (\underline{y} = 0)n \gg \underline{x} = ((\underline{x} * \underline{y}) * \text{recy}); eqTransitivity \triangleright \underline{x} =$
 $((\underline{x} * \underline{y}) * \text{recy}) \triangleright ((\underline{x} * \underline{y}) * \text{recy}) = (\underline{z} * \text{recy}) \gg \underline{x} = (\underline{z} * \text{recy})], p_0, c)]$

$[NonreciprocalToLeft(Eq)(1term) \xrightarrow{stmt} SystemQ \vdash \forall \underline{x}: \forall \underline{y}: 1 = (\underline{x} * \underline{y}) \vdash \text{recy} = \underline{x}]$
 $[NonreciprocalToLeft(Eq)(1term) \xrightarrow{proof} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: 1 = (\underline{x} * \underline{y}) \vdash$
 $eqSymmetry \triangleright 1 = (\underline{x} * \underline{y}) \gg (\underline{x} * \underline{y}) = 1; NonreciprocalToRight(Eq)(1term) \triangleright$
 $(\underline{x} * \underline{y}) = 1 \gg \underline{x} = \text{recy}; eqSymmetry \triangleright \underline{x} = \text{recy} \gg \text{recy} = \underline{x}], p_0, c)]$

$[SameReciprocal \xrightarrow{stmt} SystemQ \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} = 0)n \vdash \underline{x} = \underline{y} \vdash \text{recx} = \text{recy}]$

$[SameReciprocal \xrightarrow{proof} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} = 0)n \vdash \underline{x} = \underline{y} \vdash$
 $lemma times1Left \gg (1 * \underline{x}) = \underline{x}; eqTransitivity \triangleright (1 * \underline{x}) = \underline{x} \triangleright \underline{x} = \underline{y} \gg$
 $(1 * \underline{x}) = \underline{y}; NonreciprocalToRight(Eq) \triangleright \dot{\vdash} (\underline{x} = 0)n \triangleright (1 * \underline{x}) = \underline{y} \gg 1 =$
 $(\underline{y} * \text{recx}); timesCommutativity \gg (\underline{y} * \text{recx}) = (\text{recx} * \underline{y}); eqTransitivity \triangleright 1 =$
 $(\underline{y} * \text{recx}) \triangleright (\underline{y} * \text{recx}) = (\text{recx} * \underline{y}) \gg 1 =$
 $(\text{recx} * \underline{y}); NonreciprocalToLeft(Eq)(1term) \triangleright 1 = (\text{recx} * \underline{y}) \gg \text{recy} =$
 $\text{recx}; eqSymmetry \triangleright \text{recy} = \text{recx} \gg \text{recx} = \text{recy}], p_0, c)]$

$[OrderedPairEquality \xrightarrow{stmt} SystemQ \vdash$

$\forall (\underline{sx}): \forall (\underline{sx1}): \forall (\underline{sy}): \forall (\underline{sy1}): \forall (\underline{sz}): \forall (\underline{sz1}): \forall (\underline{su}): \forall (\underline{su1}): \{ \{ (\underline{sx}), (\underline{sx}) \}, \{ (\underline{sx}), (\underline{sx1}) \} \} =$
 $\{ \{ (\underline{sy}), (\underline{sy}) \}, \{ (\underline{sy}), (\underline{sy1}) \} \} \vdash \{ \{ (\underline{sz}), (\underline{sz}) \}, \{ (\underline{sz}), (\underline{sz1}) \} \} =$
 $\{ \{ (\underline{su}), (\underline{su}) \}, \{ (\underline{su}), (\underline{su1}) \} \} \vdash \underline{sx} = \underline{sz} \vdash \underline{sy} = \underline{su}]$

$[OrderedPairEquality \xrightarrow{proof} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash$
 $\forall (\underline{sx}): \forall (\underline{sx1}): \forall (\underline{sy}): \forall (\underline{sy1}): \forall (\underline{sz}): \forall (\underline{sz1}): \forall (\underline{su}): \forall (\underline{su1}): \{ \{ (\underline{sx}), (\underline{sx}) \}, \{ (\underline{sx}), (\underline{sx1}) \} \} =$
 $\{ \{ (\underline{sy}), (\underline{sy}) \}, \{ (\underline{sy}), (\underline{sy1}) \} \} \vdash \{ \{ (\underline{sz}), (\underline{sz}) \}, \{ (\underline{sz}), (\underline{sz1}) \} \} =$

$$\begin{aligned}
& (\text{rec}((1+1)+1) * \underline{v})n)n \gg \dot{\neg} (|(\underline{u} + (-\underline{uz}))| \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (|(\underline{u} + (-\underline{uz}))| = (\text{rec}((1+1)+1) * \underline{v})n)n)n; x \leq |x| \gg (\underline{u} + (-\underline{uz})) \leq |(\underline{u} + \\
& (-\underline{uz}))|; \text{leqLessTransitivity} \triangleright (\underline{u} + (-\underline{uz})) \leq |(\underline{u} + (-\underline{uz}))| \triangleright \dot{\neg} (|(\underline{u} + (-\underline{uz}))| \leq \\
& (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\neg} (\dot{\neg} (|(\underline{u} + (-\underline{uz}))| = (\text{rec}((1+1)+1) * \underline{v})n)n)n \gg \\
& \dot{\neg} ((\underline{u} + (-\underline{uz})) \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\neg} (\dot{\neg} ((\underline{u} + (-\underline{uz})) = (\text{rec}((1+1)+1) + \\
& 1) * \underline{v})n)n)n; \text{AddEquations(Less)} \triangleright \dot{\neg} ((\underline{x} + (-\underline{uy})) \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \\
& \dot{\neg} (\dot{\neg} ((\underline{x} + (-\underline{uy})) = (\text{rec}((1+1)+1) * \underline{v})n)n)n \triangleright \dot{\neg} ((\underline{y} + (-\underline{uu})) \leq \\
& (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\neg} (\dot{\neg} ((\underline{y} + (-\underline{uu})) = (\text{rec}((1+1)+1) * \underline{v})n)n)n \gg \\
& \dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) \leq ((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) = ((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \\
& \underline{v}))n)n)n; \text{AddEquations(Less)} \triangleright \dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) \leq \\
& ((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) \Rightarrow \dot{\neg} (\dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) = \\
& ((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v}))n)n)n \triangleright \dot{\neg} ((\underline{u} + (-\underline{uz})) \leq \\
& (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\neg} (\dot{\neg} ((\underline{u} + (-\underline{uz})) = (\text{rec}((1+1)+1) * \underline{v})n)n)n \gg \\
& \dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))) \leq \\
& (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v})) \Rightarrow \dot{\neg} (\dot{\neg} (((\underline{x} + \\
& (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))) = (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \\
& \underline{v})) + (\text{rec}((1+1)+1) * \underline{v}))n)n)n; \text{insertTwoMiddleTerms(Sum)} \gg (\underline{x} + (-\underline{uz})) = \\
& (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))); \text{eqSymmetry} \triangleright (\underline{x} + (-\underline{uz})) = \\
& (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))) \gg (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + \\
& (-\underline{uz}))) = (\underline{x} + (-\underline{uz})); \text{SubLessLeft} \triangleright (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))) = \\
& (\underline{x} + (-\underline{uz})) \triangleright \dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))) \leq \\
& (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v})) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (((\underline{x} + (-\underline{uy})) + (\underline{y} + (-\underline{uu}))) + (\underline{u} + (-\underline{uz}))) = (((\text{rec}((1+1)+1) * \underline{v}) + \\
& (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v}))n)n)n \gg \dot{\neg} ((\underline{x} + (-\underline{uz})) \leq \\
& (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v})) \Rightarrow \\
& \dot{\neg} (\dot{\neg} ((\underline{x} + (-\underline{uz})) = (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1) + \\
& 1) * \underline{v}))n)n)n; \text{ThreeThirds} \gg (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \\
& \underline{v})) + (\text{rec}((1+1)+1) * \underline{v})) = \underline{v}; \text{SubLessRight} \triangleright (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1) + \\
& 1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v})) = \underline{v} \triangleright \dot{\neg} ((\underline{x} + (-\underline{uz})) \leq (((\text{rec}((1+1)+1) + \\
& 1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v})) \Rightarrow \dot{\neg} (\dot{\neg} ((\underline{x} + (-\underline{uz})) = \\
& (((\text{rec}((1+1)+1) * \underline{v}) + (\text{rec}((1+1)+1) * \underline{v})) + (\text{rec}((1+1)+1) * \underline{v}))n)n)n \gg \\
& \dot{\neg} ((\underline{x} + (-\underline{uz})) \leq \underline{v} \Rightarrow \dot{\neg} (\dot{\neg} ((\underline{x} + (-\underline{uz})) = \underline{v})n)n)n], p_0, c)]
\end{aligned}$$

$$\begin{aligned}
& [\text{FromNot} < f(\text{Strong})(\text{Helper}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
& \forall(\underline{v1}): \forall(\underline{v2}): \forall \underline{m}: \forall(\underline{n1}): \forall(\underline{n2}): \forall(\underline{\epsilon}): \forall(\underline{fx}): \forall(\underline{fy}): \dot{\neg} (0 \leq (\text{rec}((1+1)+1) * \underline{\epsilon})) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (0 = (\text{rec}((1+1)+1) * \underline{\epsilon}))n)n)n \Rightarrow \dot{\neg} (\underline{n1} \leq \underline{n2}) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (((\underline{fy})[\underline{n2}] + (-u \text{rec}((1+1)+1) * \underline{\epsilon}))) \leq \underline{fx}[\underline{n2}]) \Rightarrow \dot{\neg} (\dot{\neg} (((\underline{fy})[\underline{n2}]) + \\
& (-u \text{rec}((1+1)+1) * \underline{\epsilon}))) = \underline{fx}[\underline{n2}])n)n)n \Rightarrow \forall_{\text{obj}}(\underline{v1}): \forall_{\text{obj}}(\underline{v2}): \dot{\neg} (0 \leq \\
& \text{rec}((1+1)+1) * \underline{\epsilon})) \Rightarrow \dot{\neg} (\dot{\neg} (0 = (\text{rec}((1+1)+1) * \underline{\epsilon}))n)n)n \Rightarrow \underline{n1} \leq \underline{v1}) \Rightarrow \\
& \underline{n1} \leq \underline{v2}) \Rightarrow \dot{\neg} (\dot{\neg} (|((\underline{fx})[\underline{v1}] + (-u \underline{fx})[\underline{v2}]))| \leq (\text{rec}((1+1)+1) * \underline{\epsilon})) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (|((\underline{fx})[\underline{v1}] + (-u \underline{fx})[\underline{v2}]))| = (\text{rec}((1+1)+1) * \underline{\epsilon}))n)n)n \Rightarrow \\
& \dot{\neg} (\dot{\neg} (|((\underline{fy})[\underline{v1}] + (-u \underline{fy})[\underline{v2}]))| \leq (\text{rec}((1+1)+1) * \underline{\epsilon})) \Rightarrow \\
& \dot{\neg} (\dot{\neg} (|((\underline{fy})[\underline{v1}] + (-u \underline{fy})[\underline{v2}]))| = (\text{rec}((1+1)+1) * \underline{\epsilon}))n)n)n \Rightarrow \\
& \dot{\neg} (0 \leq \underline{\epsilon}) \Rightarrow \dot{\neg} (\dot{\neg} (0 = \underline{\epsilon})n)n)n \Rightarrow \underline{n2} \leq \underline{m} \Rightarrow \\
& \dot{\neg} (((\underline{fy})[\underline{m}] + (-u \underline{fx})[\underline{m}])) \leq \underline{\epsilon} \Rightarrow \dot{\neg} (\dot{\neg} (((\underline{fy})[\underline{m}] + (-u \underline{fx})[\underline{m}])) = \underline{\epsilon})n)n)n]
\end{aligned}$$

$\dot{\vdash} (0 \leq (\epsilon) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\epsilon))n)n) \Rightarrow (\underline{n2}) \leq \underline{m} \Rightarrow \dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{fx})[\underline{m}])) \leq (\epsilon) \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{fx})[\underline{m}])) = (\underline{\epsilon}))n)n), p_0, c)$
 [fromNotSameF(Strongest)(Helper2) $\xrightarrow{\text{stmt}}$ SystemQ \vdash
 $\forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \forall \underline{v}: \dot{\vdash} (|(\underline{x} + (-u\underline{y}))| \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-u\underline{y}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \vdash \dot{\vdash} (|(\underline{z} + (-u\underline{u}))| \leq$
 $(\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{z} + (-u\underline{u}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \vdash$
 $\underline{v} \leq |(\underline{y} + (-u\underline{u}))| \vdash \dot{\vdash} ((\text{rec}((1+1)+1) * \underline{v}) \leq |(\underline{x} + (-u\underline{z}))| \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} ((\text{rec}((1+1)+1) * \underline{v}) = |(\underline{x} + (-u\underline{z}))|)n)n)$
 [fromNotSameF(Strongest)(Helper2) $\xrightarrow{\text{proof}}$ $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash$
 $\forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \forall \underline{v}: \dot{\vdash} (|(\underline{x} + (-u\underline{y}))| \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-u\underline{y}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \vdash \dot{\vdash} (|(\underline{z} + (-u\underline{u}))| \leq$
 $(\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{z} + (-u\underline{u}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \vdash$
 $\underline{v} \leq |(\underline{y} + (-u\underline{u}))| \vdash \text{NumericalDifference} \gg |(\underline{x} + (-u\underline{y}))| =$
 $|(\underline{y} + (-u\underline{x}))|; \text{SubLessLeft} \triangleright |(\underline{x} + (-u\underline{y}))| = |(\underline{y} + (-u\underline{x}))| \triangleright \dot{\vdash} (|(\underline{x} + (-u\underline{y}))| \leq$
 $(\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-u\underline{y}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \gg$
 $\dot{\vdash} (|(\underline{y} + (-u\underline{x}))| \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{y} + (-u\underline{x}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n);$
 $\text{LessNegated} \triangleright \dot{\vdash} (|(\underline{y} + (-u\underline{x}))| \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (|(\underline{y} + (-u\underline{x}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \gg \dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) \leq$
 $(-u|(\underline{y} + (-u\underline{x}))|) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) =$
 $(-u|(\underline{y} + (-u\underline{x}))|)n)n); \text{LessNegated} \triangleright \dot{\vdash} (|(\underline{z} + (-u\underline{u}))| \leq (\text{rec}((1+1)+1) * \underline{v}) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} (|(\underline{z} + (-u\underline{u}))| = (\text{rec}((1+1)+1) * \underline{v}))n)n) \gg \dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) \leq$
 $(-u|(\underline{z} + (-u\underline{u}))|) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) =$
 $(-u|(\underline{z} + (-u\underline{u}))|)n)n); \text{AddEquations(LeqLess)} \triangleright \underline{v} \leq$
 $|(\underline{y} + (-u\underline{u}))| \triangleright \dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) \leq (-u|(\underline{y} + (-u\underline{x}))|) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) = (-u|(\underline{y} + (-u\underline{x}))|)n)n) \gg$
 $\dot{\vdash} ((\underline{v} + (-u(\text{rec}((1+1)+1) * \underline{v}))) \leq (|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|))) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} ((\underline{v} + (-u(\text{rec}((1+1)+1) * \underline{v}))) = (|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)))n)n);$
 $\text{AddEquations(Less)} \triangleright \dot{\vdash} ((\underline{v} + (-u(\text{rec}((1+1)+1) * \underline{v}))) \leq$
 $(|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|))) \Rightarrow \dot{\vdash} (\dot{\vdash} ((\underline{v} + (-u(\text{rec}((1+1)+1) * \underline{v}))) =$
 $(|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)))n)n) \triangleright \dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) \leq$
 $(-u|(\underline{z} + (-u\underline{u}))|) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-u(\text{rec}((1+1)+1) * \underline{v})) = (-u|(\underline{z} + (-u\underline{u}))|)n)n) \gg$
 $\dot{\vdash} ((\underline{v} + (-u(\text{rec}((1+1)+1) * \underline{v}))) + (-u(\text{rec}((1+1)+1) * \underline{v}))) \leq$
 $((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|)) \Rightarrow$
 $\dot{\vdash} (\dot{\vdash} ((\underline{v} + (-u(\text{rec}((1+1)+1) * \underline{v}))) + (-u(\text{rec}((1+1)+1) * \underline{v}))) =$
 $((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|))n)n); \text{insertTwoMiddleTerms(Numerical)} \gg |(\underline{y} + (-u\underline{u}))| \leq$
 $((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|); \text{plusAssociativity} \gg$
 $((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) =$
 $(|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)); \text{plusCommutativity} \gg$
 $(|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)) = ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) +$
 $|(\underline{y} + (-u\underline{x}))|); \text{eqTransitivity} \triangleright ((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) =$
 $(|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)) \triangleright (|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} +$
 $(-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)) = ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{y} + (-u\underline{x}))|) \gg$
 $((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) = ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) +$
 $|(\underline{y} + (-u\underline{x}))|); \text{subLeqRight} \triangleright ((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) =$

$\dot{\vdash} (\dot{\vdash} (|(\underline{fx})[\underline{m}] + (-\text{ud}_{\text{Ph}}[\underline{m}])| = \overline{(\epsilon)})n)n)n)n\dot{\vdash}], p_0, c)]$

$[\text{PositiveToLeft}(\text{Leq}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} <= (\underline{y} + \underline{z}) \vdash (\underline{x} + (-\underline{uz})) <= \underline{y}]$

$[\text{PositiveToLeft}(\text{Leq}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} <= (\underline{y} + \underline{z}) \vdash \text{leqAddition} \triangleright \underline{x} <= (\underline{y} + \underline{z}) \gg (\underline{x} + (-\underline{uz})) <= ((\underline{y} + \underline{z}) + (-\underline{uz})); x = x + y - y \gg \underline{y} = ((\underline{y} + \underline{z}) + (-\underline{uz})); \text{eqSymmetry} \triangleright \underline{y} = ((\underline{y} + \underline{z}) + (-\underline{uz})) \gg ((\underline{y} + \underline{z}) + (-\underline{uz})) = \underline{y}; \text{subLeqRight} \triangleright ((\underline{y} + \underline{z}) + (-\underline{uz})) = \underline{y} \triangleright (\underline{x} + (-\underline{uz})) <= ((\underline{y} + \underline{z}) + (-\underline{uz})) \gg (\underline{x} + (-\underline{uz})) <= \underline{y}], p_0, c)]$

$[\text{FromNotSameF}(\text{Strong})(\text{Helper2}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \forall \underline{v}: \underline{v} <= |(\underline{x} + (-\underline{uz}))| \vdash \dot{\vdash} (|(\underline{x} + (-\underline{uy}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \vdash \dot{\vdash} (|(\underline{z} + (-\underline{uu}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{z} + (-\underline{uu}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \vdash \dot{\vdash} (\underline{y} = \underline{u})n]$

$[\text{FromNotSameF}(\text{Strong})(\text{Helper2}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \forall \underline{u}: \forall \underline{v}: \underline{v} <= |(\underline{x} + (-\underline{uz}))| \vdash \dot{\vdash} (|(\underline{x} + (-\underline{uy}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \vdash \dot{\vdash} (|(\underline{z} + (-\underline{uu}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{z} + (-\underline{uu}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \vdash \text{LessNegated} \triangleright \dot{\vdash} (|(\underline{x} + (-\underline{uy}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{x} + (-\underline{uy}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \gg \dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) <= (-\text{u}|(\underline{x} + (-\underline{uy}))|)) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) = (-\text{u}|(\underline{x} + (-\underline{uy}))|))n)n)n; \text{NumericalDifference} \gg |(\underline{z} + (-\underline{uu}))| = |(\underline{u} + (-\underline{uz}))|; \text{SubLessLeft} \triangleright |(\underline{z} + (-\underline{uu}))| = |(\underline{u} + (-\underline{uz}))| \triangleright \dot{\vdash} (|(\underline{z} + (-\underline{uu}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{z} + (-\underline{uu}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \gg \dot{\vdash} (|(\underline{u} + (-\underline{uz}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{u} + (-\underline{uz}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n); \text{LessNegated} \triangleright \dot{\vdash} (|(\underline{u} + (-\underline{uz}))| <= (\text{rec}(1 + 1) * \underline{v}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\underline{u} + (-\underline{uz}))| = (\text{rec}(1 + 1) * \underline{v}))n)n)n) \gg \dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) <= (-\text{u}|(\underline{u} + (-\underline{uz}))|)) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) = (-\text{u}|(\underline{u} + (-\underline{uz}))|))n)n)n); \text{AddEquations(Less)} \triangleright \dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) <= (-\text{u}|(\underline{x} + (-\underline{uy}))|)) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) = (-\text{u}|(\underline{x} + (-\underline{uy}))|))n)n)n) \triangleright \dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) <= (-\text{u}|(\underline{u} + (-\underline{uz}))|)) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) = (-\text{u}|(\underline{u} + (-\underline{uz}))|))n)n)n) \gg \dot{\vdash} (((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) <= ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|))) \Rightarrow \dot{\vdash} (\dot{\vdash} (((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) = ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|)))n)n)n); \text{TwoHalves} \gg ((\text{rec}(1 + 1) * \underline{v}) + (\text{rec}(1 + 1) * \underline{v})) = \underline{v}; \text{EqNegated} \triangleright ((\text{rec}(1 + 1) * \underline{v}) + (\text{rec}(1 + 1) * \underline{v})) = \underline{v} \gg (-\text{u}((\text{rec}(1 + 1) * \underline{v}) + (\text{rec}(1 + 1) * \underline{v}))) = (-\underline{uv}); -x - y = -(x + y) \gg ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) = (-\text{u}((\text{rec}(1 + 1) * \underline{v}) + (\text{rec}(1 + 1) * \underline{v}))))); \text{eqTransitivity} \triangleright ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) = (-\text{u}((\text{rec}(1 + 1) * \underline{v}) + (\text{rec}(1 + 1) * \underline{v})))) \triangleright (-\text{u}((\text{rec}(1 + 1) * \underline{v}) + (\text{rec}(1 + 1) * \underline{v})))) = (-\underline{uv}) \gg ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) = (-\underline{uv}); \text{SubLessLeft} \triangleright ((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) = (-\underline{uv}) \triangleright \dot{\vdash} (((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) <= ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|))) \Rightarrow \dot{\vdash} (\dot{\vdash} (((-\text{u}(\text{rec}(1 + 1) * \underline{v})) + (-\text{u}(\text{rec}(1 + 1) * \underline{v}))) = ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|)))n)n)n) \gg \dot{\vdash} ((-\underline{uv}) <= ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|))) \Rightarrow \dot{\vdash} (\dot{\vdash} ((-\underline{uv}) = ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|)))n)n)n); \text{plusCommutativity} \gg ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|)) = ((-\text{u}|(\underline{u} + (-\underline{uz}))|) + (-\text{u}|(\underline{x} + (-\underline{uy}))|)); \text{SubLessRight} \triangleright ((-\text{u}|(\underline{x} + (-\underline{uy}))|) + (-\text{u}|(\underline{u} + (-\underline{uz}))|)) =$

$$\begin{aligned}
& N \Rightarrow \dot{\bar{((op2) \in Q)n}} \Rightarrow \dot{\bar{(a_{Ph} = \\
& \{ \{ \{ \overline{(op1)}, \overline{(op1)} \}, \{ \overline{(op1)}, \overline{(op2)} \} \})n)n)n)n} \mid \dot{\bar{(\forall_{obj} \overline{(crs1)} : \dot{\bar{(c_{Ph} = \\
& \{ \{ \{ \overline{(crs1)}, \overline{(crs1)} \}, \{ \overline{(crs1)}, \overline{1} \} \})n)n}[\bar{m}] + (-ud_{Ph}[\bar{m}]))} = \overline{(\epsilon)})n)n)n} \\
& [\text{Reciprocal}(R) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \overline{(fx)} : \dot{\bar{(\{ph \in P(\{ph \in P(\{ph \in \\
& P(P(\text{Union}(\{N, Q\}))) \mid \dot{\bar{(\forall_{obj} \overline{(op1)} : \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(op2)} : \dot{\bar{(\dot{\bar{(\overline{(op1)} \in N \Rightarrow \\
& \dot{\bar{((op2) \in Q)n}} \Rightarrow \dot{\bar{(a_{Ph} = \{ \{ \{ \overline{(op1)}, \overline{(op1)} \}, \{ \overline{(op1)}, \overline{(op2)} \} \})n)n)n)n} \} \mid \\
& \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(r1)} : \overline{(r1)} \in f_{Ph} \Rightarrow \dot{\bar{(\forall_{obj} \overline{(op1)} : \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(op2)} : \dot{\bar{(\dot{\bar{(\overline{(op1)} \in N \Rightarrow \\
& \dot{\bar{((op2) \in Q)n}} \Rightarrow \dot{\bar{(\overline{(r1)} = \{ \{ \{ \overline{(op1)}, \overline{(op1)} \}, \{ \overline{(op1)}, \overline{(op2)} \} \})n)n)n)n} \Rightarrow \\
& \dot{\bar{(\forall_{obj} \overline{(f1)} : \forall_{obj} \overline{(f2)} : \forall_{obj} \overline{(f3)} : \forall_{obj} \overline{(f4)} : \{ \{ \overline{(f1)}, \overline{(f1)} \}, \{ \overline{(f1)}, \overline{(f2)} \} \} \in f_{Ph} \Rightarrow \\
& \{ \{ \overline{(f3)}, \overline{(f3)} \}, \{ \overline{(f3)}, \overline{(f4)} \} \} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n} \Rightarrow \\
& \dot{\bar{(\forall_{obj} \overline{(s1)} : \overline{(s1)} \in N \Rightarrow \dot{\bar{(\forall_{obj} \overline{(s2)} : \dot{\bar{(\{ \{ \overline{(s1)}, \overline{(s1)} \}, \{ \overline{(s1)}, \overline{(s2)} \} \} \in \\
& f_{Ph})n)n)n} \mid \forall_{obj} \overline{(\epsilon)} : \dot{\bar{(\forall_{obj} \overline{n} : \dot{\bar{(\forall_{obj} \overline{m} : \dot{\bar{(0 \leq \overline{(\epsilon)} \Rightarrow \dot{\bar{(\dot{\bar{(0 = \overline{(\epsilon)})n)n} \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\bar{(\lceil (\overline{(fx)}[\bar{m}] + (-ud_{Ph}[\bar{m}]))} \leq \overline{(\epsilon)} \Rightarrow \\
& \dot{\bar{(\dot{\bar{(\lceil (\overline{(fx)}[\bar{m}] + (-ud_{Ph}[\bar{m}]))} = \overline{(\epsilon)})n)n)n} = \{ph \in P(\{ph \in P(\{ph \in \\
& P(P(\text{Union}(\{N, Q\}))) \mid \dot{\bar{(\forall_{obj} \overline{(op1)} : \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(op2)} : \dot{\bar{(\dot{\bar{(\overline{(op1)} \in N \Rightarrow \\
& \dot{\bar{((op2) \in Q)n}} \Rightarrow \dot{\bar{(a_{Ph} = \{ \{ \{ \overline{(op1)}, \overline{(op1)} \}, \{ \overline{(op1)}, \overline{(op2)} \} \})n)n)n)n} \} \mid \\
& \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(r1)} : \overline{(r1)} \in f_{Ph} \Rightarrow \dot{\bar{(\forall_{obj} \overline{(op1)} : \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(op2)} : \dot{\bar{(\dot{\bar{(\overline{(op1)} \in N \Rightarrow \\
& \dot{\bar{((op2) \in Q)n}} \Rightarrow \dot{\bar{(\overline{(r1)} = \{ \{ \{ \overline{(op1)}, \overline{(op1)} \}, \{ \overline{(op1)}, \overline{(op2)} \} \})n)n)n)n} \Rightarrow \\
& \dot{\bar{(\forall_{obj} \overline{(f1)} : \forall_{obj} \overline{(f2)} : \forall_{obj} \overline{(f3)} : \forall_{obj} \overline{(f4)} : \{ \{ \overline{(f1)}, \overline{(f1)} \}, \{ \overline{(f1)}, \overline{(f2)} \} \} \in f_{Ph} \Rightarrow \\
& \{ \{ \overline{(f3)}, \overline{(f3)} \}, \{ \overline{(f3)}, \overline{(f4)} \} \} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n} \Rightarrow \\
& \dot{\bar{(\forall_{obj} \overline{(s1)} : \overline{(s1)} \in N \Rightarrow \dot{\bar{(\forall_{obj} \overline{(s2)} : \dot{\bar{(\{ \{ \overline{(s1)}, \overline{(s1)} \}, \{ \overline{(s1)}, \overline{(s2)} \} \} \in \\
& f_{Ph})n)n)n} \mid \forall_{obj} \overline{(\epsilon)} : \dot{\bar{(\forall_{obj} \overline{n} : \dot{\bar{(\forall_{obj} \overline{m} : \dot{\bar{(0 \leq \overline{(\epsilon)} \Rightarrow \dot{\bar{(\dot{\bar{(0 = \overline{(\epsilon)})n)n} \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\bar{(\lceil (\overline{(fx)}[\bar{m}] + (-ud_{Ph}[\bar{m}]))} \leq \overline{(\epsilon)} \Rightarrow \\
& \dot{\bar{(\dot{\bar{(\lceil (\overline{(fx)}[\bar{m}] + (-ud_{Ph}[\bar{m}]))} = \overline{(\epsilon)})n)n)n} = \{ph \in P(\{ph \in P(\{ph \in \\
& P(P(\text{Union}(\{N, Q\}))) \mid \dot{\bar{(\forall_{obj} \overline{(op1)} : \dot{\bar{(\dot{\bar{(\forall_{obj} \overline{(op2)} : \dot{\bar{(\dot{\bar{(\overline{(op1)} \in N \Rightarrow \\
\end{aligned}$$

$$\begin{aligned}
& \dot{\vdash} (\underline{n} \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\epsilon))) <= (\underline{fx})[\underline{m}] \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) = (\underline{fx})[\underline{m}]) \underline{n}) \underline{n}) \underline{n}) \underline{n} \gg \dot{\vdash} (\forall_{\text{obj}} \underline{m}: \dot{\vdash} (\dot{\vdash} (0 \leq (\epsilon) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (0 = (\epsilon)) \underline{n}) \underline{n}) \underline{n} \Rightarrow \dot{\vdash} (\underline{n} \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) <= (\underline{fx})[\underline{m}]) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) = (\underline{fx})[\underline{m}]) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}; \text{Gen} \triangleright \dot{\vdash} (\forall_{\text{obj}} \underline{m}: \dot{\vdash} (\dot{\vdash} (0 <= \\
& (\epsilon) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\epsilon)) \underline{n}) \underline{n}) \underline{n} \Rightarrow \dot{\vdash} (\underline{n} \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) <= \\
& (\underline{fx})[\underline{m}]) \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) = (\underline{fx})[\underline{m}]) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n} \gg \\
& \forall_{\text{obj}} \underline{n}: \dot{\vdash} (\forall_{\text{obj}} \underline{m}: \dot{\vdash} (\dot{\vdash} (0 \leq (\epsilon) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\epsilon)) \underline{n}) \underline{n}) \underline{n} \Rightarrow \dot{\vdash} (\underline{n} \leq \underline{m} \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) <= (\underline{fx})[\underline{m}]) \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) = \\
& (\underline{fx})[\underline{m}]) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}; \text{Gen} \triangleright \forall_{\text{obj}} \underline{n}: \dot{\vdash} (\forall_{\text{obj}} \underline{m}: \dot{\vdash} (\dot{\vdash} (0 \leq (\epsilon) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \\
& (\epsilon)) \underline{n}) \underline{n}) \underline{n} \Rightarrow \dot{\vdash} (\underline{n} \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) <= (\underline{fx})[\underline{m}]) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) = (\underline{fx})[\underline{m}]) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n} \gg \\
& \forall_{\text{obj}} (\epsilon): \forall_{\text{obj}} \underline{n}: \dot{\vdash} (\forall_{\text{obj}} \underline{m}: \dot{\vdash} (\dot{\vdash} (0 \leq (\epsilon) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\epsilon)) \underline{n}) \underline{n}) \underline{n} \Rightarrow \dot{\vdash} (\underline{n} \leq \underline{m} \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) <= (\underline{fx})[\underline{m}]) \Rightarrow \dot{\vdash} (\dot{\vdash} (((\underline{fy})[\underline{m}] + (-u(\underline{\epsilon}))) = \\
& (\underline{fx})[\underline{m}]) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}) \underline{n}], p_0, c) \\
& (*****)
\end{aligned}$$

a
venter—

[sup $\xrightarrow{\text{prio}}$

Preassociative

[sup], [base], [bracket * end bracket], [big bracket * end bracket], [\$ * \$],
[flush left [*], [x], [y], [z], [[* \bowtie *]], [[* \rightarrow *]], [pyk], [tex], [name], [prio], [*], [T],
[if(*, *, *)], [[* $\xrightarrow{*}$ *]], [val], [claim], [\perp], [f(*)], [(*)¹], [F], [0], [1], [2], [3], [4], [5], [6],
[7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],
[k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(*)^M], [If(*, *, *)],
[array{*} * end array], [l], [c], [r], [empty], [(* | * := *)], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$],
 $\mathcal{U}^M(*)$, [**apply**(*, *)], [**apply**₁(*, *)], [identifier(*)], [identifier₁(*, *)], [array-
plus(*, *)], [array-remove(*, *, *)], [array-put(*, *, *, *)], [array-add(*, *, *, *, *)],
[bit(*, *)], [bit₁(*, *)], [rack], ["vector"], ["bibliography"], ["dictionary"],
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
 $\mathcal{E}(*, *, *)$, [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *, *)$], [$\mathcal{E}_4(*, *, *, *, *)$], [**lookup**(*, *, *)],
[**abstract**(*, *, *, *)], [[*]], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro],
[s₀], [**zip**(*, *)], [**assoc**₁(*, *, *)], [(*)^P], [self], [[* \doteq *]], [[* \doteq *]], [[* \doteq *]],
[[* $\stackrel{\text{pyk}}{=}$ *]], [[* $\stackrel{\text{tex}}{=}$ *]], [[* $\stackrel{\text{name}}{=}$ *]], [**Priority table**[*]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2$ (*)], [$\tilde{\mathcal{M}}_3$ (*)],
 $\tilde{\mathcal{M}}_4(*, *, *, *, *)$], [$\mathcal{M}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *, *)$],
[(*)], [(*)], [display(*)], [statement(*)], [[*]], [[*]], [**aspect**(*, *)],
[**aspect**(*, *, *)], [(*)], [**tuple**₁(*)], [**tuple**₂(*)], [let₂(*, *)], [let₁(*, *)],
[[* $\stackrel{\text{claim}}{=}$ *]], [checker], [**check**(*, *)], [**check**₂(*, *, *)], [**check**₃(*, *, *)],
[**check**^{*}(*, *)], [**check**₂^{*}(*, *, *)], [[*]], [[*]], [[*]], [msg], [[* $\stackrel{\text{msg}}{=}$ *]], [<stmt>],
[stmt], [[* $\stackrel{\text{stmt}}{=}$ *]], [HeadNil'], [HeadPair'], [Transitivity'], [\perp], [Contra'], [T'_E],
[L₁], [$\underline{*}$], [\underline{A}], [\underline{B}], [\underline{C}], [\underline{D}], [\underline{E}], [\underline{F}], [\underline{G}], [\underline{H}], [\underline{I}], [\underline{J}], [\underline{K}], [\underline{L}], [\underline{M}], [\underline{N}], [\underline{O}], [\underline{P}], [\underline{Q}],

$[\mathcal{R}], [\mathcal{S}], [\mathcal{T}], [\mathcal{U}], [\mathcal{V}], [\mathcal{W}], [\mathcal{X}], [\mathcal{Y}], [\mathcal{Z}], [(* * := *)], [(* | * := *)], [\emptyset], [\text{Remainder}],$
 $[(*)^\vee], [\text{intro}(*, *, *, *)], [\text{intro}(*, *, *)], [\text{error}(*, *)], [\text{error}_2(*, *)], [\text{proof}(*, *, *)],$
 $[\text{proof}_2(*, *)], [\mathcal{S}(*, *)], [\mathcal{S}^I(*, *)], [\mathcal{S}^{\triangleright}(*, *)], [\mathcal{S}_1^{\triangleright}(*, *, *)], [\mathcal{S}^E(*, *)], [\mathcal{S}_1^E(*, *, *)],$
 $[\mathcal{S}^+(*, *)], [\mathcal{S}_1^+(*, *, *)], [\mathcal{S}^-(*, *)], [\mathcal{S}_1^-(*, *, *)], [\mathcal{S}^*(*)], [\mathcal{S}_1^*(*)],$
 $[\mathcal{S}_2^*(*)], [\mathcal{S}_1^{\textcircled{a}}(*, *)], [\mathcal{S}_1^{\textcircled{b}}(*, *, *)], [\mathcal{S}^{\text{+}}(*, *)], [\mathcal{S}_1^{\text{+}}(*, *, *, *)], [\mathcal{S}^{\text{+}}(*, *)],$
 $[\mathcal{S}_1^{\text{+}}(*, *, *, *)], [\mathcal{S}^{\text{i.e.}}(*, *)], [\mathcal{S}_1^{\text{i.e.}}(*, *, *, *)], [\mathcal{S}_2^{\text{i.e.}}(*, *, *, *)], [\mathcal{S}^{\vee}(*, *)],$
 $[\mathcal{S}_1^{\vee}(*, *, *, *)], [\mathcal{S}^{\text{i}}(*, *)], [\mathcal{S}_1^{\text{i}}(*, *, *, *)], [\mathcal{S}_2^{\text{i}}(*, *, *, *)], [\mathcal{T}(*)], [\text{claims}(*, *, *)],$
 $[\text{claims}_2(*, *, *)], [<\text{proof}>], [\text{proof}], [[\text{Lemma} * : *]], [[\text{Proof of} * : *]],$
 $[[* \text{ lemma} * : *]], [[* \text{ antilemma} * : *]], [[* \text{ rule} * : *]], [[* \text{ antirule} * : *]],$
 $[\text{verifier}], [\mathcal{V}_1(*)], [\mathcal{V}_2(*, *)], [\mathcal{V}_3(*, *, *, *)], [\mathcal{V}_4(*, *)], [\mathcal{V}_5(*, *, *, *)], [\mathcal{V}_6(*, *, *, *)],$
 $[\mathcal{V}_7(*, *, *, *)], [\text{Cut}(*, *)], [\text{Head}_{\oplus}(*)], [\text{Tail}_{\oplus}(*)], [\text{rule}_1(*, *)], [\text{rule}(*, *)],$
 $[\text{Rule tactic}], [\text{Plus}(*, *)], [[\text{Theory} *]], [\text{theory}_2(*, *)], [\text{theory}_3(*, *)],$
 $[\text{theory}_4(*, *, *)], [\text{HeadNil}''], [\text{HeadPair}''], [\text{Transitivity}''], [\text{Contra}''], [\text{HeadNil}],$
 $[\text{HeadPair}], [\text{Transitivity}], [\text{Contra}], [\text{T}_E], [\text{ragged right}],$
 $[\text{ragged right expansion}], [\text{parm}(*, *, *)], [\text{parm}^*(*)], [\text{inst}(*, *)],$
 $[\text{inst}^*(*)], [\text{occur}(*, *, *)], [\text{occur}^*(*)], [\text{unify}(* = *, *)], [\text{unify}^*(* = *, *)],$
 $[\text{unify}_2(* = *, *)], [\text{L}_a], [\text{L}_b], [\text{L}_c], [\text{L}_d], [\text{L}_e], [\text{L}_f], [\text{L}_g], [\text{L}_h], [\text{L}_i], [\text{L}_j], [\text{L}_k], [\text{L}_l], [\text{L}_m],$
 $[\text{L}_n], [\text{L}_o], [\text{L}_p], [\text{L}_q], [\text{L}_r], [\text{L}_s], [\text{L}_t], [\text{L}_u], [\text{L}_v], [\text{L}_w], [\text{L}_x], [\text{L}_y], [\text{L}_z], [\text{L}_A], [\text{L}_B], [\text{L}_C],$
 $[\text{L}_D], [\text{L}_E], [\text{L}_F], [\text{L}_G], [\text{L}_H], [\text{L}_I], [\text{L}_J], [\text{L}_K], [\text{L}_L], [\text{L}_M], [\text{L}_N], [\text{L}_O], [\text{L}_P], [\text{L}_Q], [\text{L}_R],$
 $[\text{L}_S], [\text{L}_T], [\text{L}_U], [\text{L}_V], [\text{L}_W], [\text{L}_X], [\text{L}_Y], [\text{L}_Z], [\text{L}_?], [\text{Reflexivity}], [\text{Reflexivity}_1],$
 $[\text{Commutativity}], [\text{Commutativity}_1], [<\text{tactic}>], [\text{tactic}], [[* \stackrel{\text{tactic}}{=} *]], [\mathcal{P}(*, *, *)],$
 $[\mathcal{P}^*(*)], [\text{p}_0], [\text{conclude}_1(*, *)], [\text{conclude}_2(*, *, *)], [\text{conclude}_3(*, *, *, *)],$
 $[\text{conclude}_4(*, *)], [\text{check}], [[* \stackrel{\circ}{=} *]], [\text{RootVisible}(*)], [\text{A}], [\text{R}], [\text{C}], [\text{T}], [\text{L}], [\{ * \}], [\bar{*}],$
 $[a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v],$
 $[w], [x], [y], [z], [(* \equiv * | * := *)], [(* \equiv^0 * | * := *)], [(* \equiv^1 * | * := *)], [(* \equiv^* * | * := *)],$
 $[\text{Ded}(*, *)], [\text{Ded}_0(*, *)], [\text{Ded}_1(*, *, *)], [\text{Ded}_2(*, *, *)], [\text{Ded}_3(*, *, *, *)],$
 $[\text{Ded}_4(*, *, *, *)], [\text{Ded}_4^*(*)], [\text{Ded}_5(*, *, *)], [\text{Ded}_6(*, *, *, *)],$
 $[\text{Ded}_6^*(*)], [\text{Ded}_7(*)], [\text{Ded}_8(*, *)], [\text{Ded}_8^*(*)], [\text{S}], [\text{Neg}], [\text{MP}], [\text{Gen}],$
 $[\text{Ded}], [\text{S1}], [\text{S2}], [\text{S3}], [\text{S4}], [\text{S5}], [\text{S6}], [\text{S7}], [\text{S8}], [\text{S9}], [\text{Repetition}], [\text{A1}'], [\text{A2}'], [\text{A4}'],$
 $[\text{A5}'], [\text{Prop 3.2a}], [\text{Prop 3.2b}], [\text{Prop 3.2c}], [\text{Prop 3.2d}], [\text{Prop 3.2e}_1], [\text{Prop 3.2e}_2],$
 $[\text{Prop 3.2e}], [\text{Prop 3.2f}_1], [\text{Prop 3.2f}_2], [\text{Prop 3.2f}], [\text{Prop 3.2g}_1], [\text{Prop 3.2g}_2],$
 $[\text{Prop 3.2g}], [\text{Prop 3.2h}_1], [\text{Prop 3.2h}_2], [\text{Prop 3.2h}], [\text{Block}_1(*, *, *)], [\text{Block}_2(*)],$
 $[\text{kvanti}], [\text{UniqueMember}], [\text{UniqueMember}(\text{Type})], [\text{SameSeries}], [\text{A4}],$
 $[\text{SameMember}], [\text{Qclosed}(\text{Addition})], [\text{Qclosed}(\text{Multiplication})],$
 $[\text{FromCartProd}(1)], [\text{1rule fromCartProd}(2)], [\text{constantRationalSeries}(*)],$
 $[\text{cartProd}(*)], [\text{Power}(*)], [\text{binaryUnion}(*, *)], [\text{SetOfRationalSeries}],$
 $[\text{IsSubset}(*, *)], [(\text{p} * , *)], [(\text{s} *)], [(\cdot \cdot \cdot)], [\text{Objekt-var}], [\text{Ex-var}], [\text{Ph-var}], [\text{Værdi}],$
 $[\text{Variabel}], [\text{Op}(*)], [\text{Op}(*, *)], [* := *], [\text{ContainsEmpty}(*)], [\text{Nat}(*)],$
 $[\text{Dedu}(*, *)], [\text{Dedu}_0(*, *)], [\text{Dedu}_s(*, *, *)], [\text{Dedu}_1(*, *, *)], [\text{Dedu}_2(*, *, *)],$
 $[\text{Dedu}_3(*, *, *, *)], [\text{Dedu}_4(*, *, *, *)], [\text{Dedu}_4^*(*)], [\text{Dedu}_5(*, *, *)],$
 $[\text{Dedu}_6(*, *, *, *)], [\text{Dedu}_6^*(*)], [\text{Dedu}_7(*)], [\text{Dedu}_8(*, *)], [\text{Dedu}_8^*(*)],$
 $[\text{EX}_1], [\text{EX}_2], [\text{EX}_3], [\text{EX}_{10}], [\text{EX}_{20}], [*_{\text{EX}}], [*^{\text{EX}}], [(* \equiv * | * := *)_{\text{EX}}],$
 $[(* \equiv^0 * | * := *)_{\text{EX}}], [(* \equiv^1 * | * := *)_{\text{EX}}], [(* \equiv^* * | * := *)_{\text{EX}}], [\text{ph}_1], [\text{ph}_2], [\text{ph}_3],$
 $[*_{\text{Ph}}], [*^{\text{Ph}}], [(* \equiv * | * := *)_{\text{Ph}}], [(* \equiv^0 * | * := *)_{\text{Ph}}], [(* \equiv^1 * | * := *)_{\text{Ph}}],$
 $[(* \equiv^* * | * := *)_{\text{Ph}}], [(* \equiv * | * := *)_{\text{Me}}], [(* \equiv^1 * | * := *)_{\text{Me}}],$

$\langle \equiv * * | * := * \rangle_{Me}$, [bs], [OBS], [\mathcal{BS}], [\emptyset], [SystemQ], [MP], [Gen], [Repetition],
 [Neg], [Ded], [ExistIntro], [Extensionality], [\emptyset def], [PairDef], [UnionDef],
 [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg],
 [AndCommutativity], [AutoImPLY], [Contrapositive], [FirstConjunct],
 [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity],
 [IffFirst], [IffSecond], [ImPLYTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4],
 [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1],
 [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],
 [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power],
 [SubsetInPower], [HelperPowerIsSub], [PowerIsSub],
 [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality],
 [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality],
 [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry],
 [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive],
 [ERisSymmetric], [ERisTransitive], [\emptyset isSubset], [HelperMemberNot \emptyset],
 [MemberNot \emptyset], [HelperUnique \emptyset], [Unique \emptyset], [= Reflexivity], [= Symmetry],
 [Helper == Transitivity], [= Transitivity], [HelperTransferNotEq],
 [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset],
 [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset],
 [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection],
 [AutoMember], [HelperEqSysNot \emptyset], [EqSysNot \emptyset], [HelperEqSubset],
 [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary],
 [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset],
 [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImPLY], [BSsubset],
 [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)],
 [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ϵ)],
 [(ϵ)₁], [(ϵ)₂], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)],
 [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)],
 [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X₁], [X₂],
 [Y₁], [Y₂], [V₁], [V₂], [V₃], [V₄], [V_{2n}], [M₁], [M₂], [N₁], [N₂], [N₃], [ϵ], [ϵ]₁], [ϵ]₂],
 [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)],
 [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)],
 [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)],
 [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)],
 [(S1ob)], [(S2ob)], [ph₄], [ph₅], [ph₆], [NAT], [RATIONALSERIES], [SERIES],
 [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1],
 [(-1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(- - 01)], [02], [01/02],
 [lemma plus0Left], [lemma times1Left], [lemma eqAdditionLeft],
 [lemma eqMultiplicationLeft], [PlusAssociativity(R)],
 [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)], [Times1(R)],
 [lessAddition(R)], [PlusCommutativity(R)], [LeqAntisymmetry(R)],
 [LeqTransitivity(R)], [leqAddition(R)], [Distribution(R)], [A4(Axiom)],
 [InductionAxiom], [EqualityAxiom], [EqLeqAxiom], [EqAdditionAxiom],
 [EqMultiplicationAxiom], [QisClosed(Reciprocal)(ImPLY)],
 [QisClosed(Reciprocal)], [QisClosed(Negative)(ImPLY)], [QisClosed(Negative)],
 [leqReflexivity], [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality],

[leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity],
[plusCommutativity], [Negative], [plus0], [timesAssociativity],
[timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1],
[lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)],
[lemma =f to sameF], [lemma plusF(Sym)], [lemma timesF(Sym)],
[Separation2formula(1)], [Separation2formula(2)], [IfThenElse(T)],
[IfThenElse(F)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==],
[From <<], [to <<], [FromInR], [PlusR], [PlusR(Sym)], [TimesR],
[TimesR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)],
[US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)],
[NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive],
[ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [USTelescope(Zero)],
[USTelescope(Positive)], [EqAddition(R)], [Unminus(R)], [FromLimit],
[ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)],
[ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound],
[ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess],
[SmallInverse], [NatType], [RationalType], [SeriesType], [Max], [Numerical],
[MemberOfSeries(Implied)], [JoinConjuncts(2conditions)],
[prop lemma imply negation], [TND], [FromNegatedImplied], [ToNegatedImplied],
[FromNegated(2 * Implied)], [FromNegatedAnd], [FromNegatedOr],
[ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 * 2Disjuncts],
[NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2],
[LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)], [To!! ==],
[ToNegatedDoubleImplied], [ToNegatedAnd(1)], [AddNegatedAll],
[(A)to(E)(Implied)], [(E)to(A)(Implied)], [(E)to(A)(Implied)], [ToNegatedAEA],
[UniqueNegative], [DoubleMinus], [MinusNegated], [eqReflexivity],
[eqSymmetry], [eqTransitivity], [eqTransitivity4], [eqTransitivity5],
[eqTransitivity6], [AddEquations], [SubtractEquations],
[SubtractEquationsLeft], [MultiplyEquations], [EqNegated],
[PositiveToRight(Eq)], [PositiveToLeft(Eq)], [PositiveToLeft(Eq)(1term)],
[NegativeToLeft(Eq)], [NonreciprocalToRight(Eq)(1term)],
[DistributionOut(Minus)], [PositiveToRight(Eq)(1term)],
[SameSeries(NumDiff)], [PlusAssociativity(4terms)], [LessNeq], [NeqSymmetry],
[NeqNegated], [SubNeqRight], [SubNeqLeft], [NegativeToRight(Neq)(1term)],
[NeqAddition], [NeqMultiplication], [NonzeroProduct(2)],
[SwitchTerms(x <= y - z)], [NegativeToLeft(Less)(1term)], [(+1)IsPositive(N)],
[(1/2)(x + y) - x = (1/2)(y - x)], [y - (1/2)(x + y) = (1/2)(y - x)],
[ExpZero(Exact)], [SameExp(Base)], [SameExp(Indu)], [SameExp], [Exp(+1)],
[PositiveBase(Base)], [PositiveBase(Indu)], [PositiveBase], [BSzero(Exact)],
[SameBS(2)(Base)], [SameBS(2)(Indu)], [SameBS(2)], [BS(+1)],
[BSbound(Exact)(Base)], [BSbound(Exact)(Indu)], [BSbound(Exact)],
[BSbound], [USTelescope(Zero)(Exact)], [SameTelescope(2)(Base)],
[SameTelescope(2)(Indu)], [SameTelescope(2)], [USTelescope(+1)],
[TelescopeNumerical(Base)], [TelescopeNumerical(Indu)], [TelescopeNumerical],
[TelescopeBound(Base)], [TelescopeBound(Indu)], [TelescopeBound],
[LessNeq(F)(Helper)], [LessNeq(F)], [LessNeq(R)], [IntervalSize(Base)],

[IntervalSize(Indu)], [IntervalSize], [XS < US], [lemma USdecreasing(+1)],
 [CloseUS], [CloseUS(n + 1)], [AllNegated(ImPLY)], [ExistNegated(ImPLY)],
 [IntroExist(Helper)], [IntroExist], [ExistMP], [ExistMP2], [TwiceExistMP],
 [TwiceExistMP2], [EAE – MP], [AddAll], [AddExist(Helper1)],
 [AddExist(Helper2)], [AddExist], [AddExist(SimpleAnt)], [AddExist(Simple)],
 [AddEAE], [AEA – negated], [EEA – negated], [Induction], [leqAntisymmetry],
 [leqTransitivity], [leqAddition], [leqMultiplication], [Reciprocal], [Equality],
 [eqLeq], [eqAddition], [eqMultiplication], [LeqMultiplicationLeft], [LeqLessEq],
 [LessLeq], [FromLeqGeq], [subLeqRight], [subLeqLeft], [Leq + 1],
 [PositiveToRight(Leq)], [PositiveToRight(Leq)(1term)],
 [lemma negativeToRight(Leq)], [PositiveToLeft(Leq)], [negativeToLeft(Leq)],
 [negativeToLeft(Leq)(1term)], [LeqAdditionLeft], [leqSubtraction],
 [leqSubtractionLeft], [thirdGeq], [LeqNegated], [AddEquations(Leq)],
 [MultiplyEquations(Leq)], [ThirdGeqSeries], [LeqNeqLess], [FromLess], [ToLess],
 [fromNotLess], [toNotLess], [NegativeLessPositive], [leqLessTransitivity],
 [LessLeqTransitivity], [LessTransitivity], [LessTotality], [SubLessRight],
 [SubLessLeft], [SwitchTerms(x < y – z)], [SwitchTerms(x – y < z)],
 [LessAddition], [LessAdditionLeft], [LessMultiplication],
 [LessMultiplicationLeft], [LessDivision], [PositiveToRight(Less)],
 [PositiveToLeft(Less)], [NegativeToLeft(Less)], [NegativeToRight(Less)],
 [AddEquations(Less)], [AddEquations(LeqLess)], [reciprocalToLeft(Less)],
 [LessNegated], [PositiveNonzero], [PositiveNegated], [NonpositiveNegated],
 [NegativeNegated], [NonnegativeNegated], [PositiveHalved], [PositiveInverted],
 [NonnegativeNumerical], [NegativeNumerical], [PositiveNumerical],
 [lemma nonpositiveNumerical], [|0| = 0], [0 <= |x|], [x <= |x|],
 [FromPositiveNumerical], [SameNumerical], [SignNumerical(+)],
 [SignNumerical], [ToNumericalLess], [FromNumericalGreater],
 [NumericalDifference], [NumericalDifferenceLess(Helper)],
 [NumericalDifferenceLess], [SplitNumericalSumHelper],
 [splitNumericalSum(++)], [splitNumericalSum(--)],
 [splitNumericalSum(+ – small)], [splitNumericalSum(+ – big)],
 [splitNumericalSum(++–)], [splitNumericalSum(–+)], [splitNumericalSum],
 [SplitNumericalProduct(++)], [SplitNumericalProduct(+-)],
 [SplitNumericalProduct], [insertMiddleTerm(Numerical)],
 [insertTwoMiddleTerms(Numerical)], [Three2twoTerms], [Three2threeTerms],
 [Three2twoFactors], [Three2threeFactors], [Times(-1)], [Times(-1)Left],
 [MaxLeq(1)], [MaxLeq(2)], [LessThanMax], [x + y = zBackwards],
 [x * y = zBackwards], [x = x + (y – y)], [x = x + y – y], [x = x * y * (1/y)],
 [insertMiddleTerm(Sum)], [insertTwoMiddleTerms(Sum)],
 [insertMiddleTerm(Difference)], [x * 0 + x = x], [x * 0 = 0], [NonnegativeFactors],
 [NonzeroFactors], [PositiveFactors], [PlusTimesMinus], [MinusTimesMinus],
 [(-1) * (-1) + (-1) * 1 = 0], [(-1) * (-1) = 1], [0 < 1Helper], [0 < 1], [0 < 2],
 [0 < 3], [0 < 1/2], [0 < 1/3], [TwoWholes], [ThreeWholes], [TwoHalves],
 [ThreeThirds], [-x – y = -(x + y)], [-x * y = -(x * y)], [-0 = 0],
 [SFsymmetry], [SFtransitivity], [f2R(Plus)], [f2R(Times)],
 [<< TransitivityHelper(Q)], [<< Transitivity], [<<== Reflexivity],

[<=<= AntisymmetryHelper(Q)], [FromNot < f(Weak)(Helper)],
 [FromNot < f(Weak)], [FromNot < f(Strong)(Helper2)],
 [FromNot < f(Strong)(Helper)], [FromNot < f(Strong)],
 [fromNotSameF(Strongest)(Helper2)], [fromNotSameF(Strongest)(Helper)],
 [fromNotSameF(Strongest)], [ToLess(F)(Helper)], [ToLess(F)], [FromNot <<],
 [ToLess(R)], [LeqTotality(R)], [FromNotSameF(Weak)(Helper)],
 [FromNotSameF(Weak)], [FromNotLess(F)], [= Addition], [= AdditionLeft],
 [Fpart - Bounded(Base)], [Fpart - Bounded(InduHelper)],
 [Fpart - Bounded(Indu)], [Fpart - Bounded], [F - Bounded(Helper)],
 [F - Bounded], [SameFmultiplication(Helper)], [SameFmultiplication],
 [EqMultiplication(R)], [EqMultiplicationLeft(R)], [x * 0 = 0(F)], [x * 0 = 0(R)],
 [LessMultiplication(F)(Helper2)], [LessMultiplication(F)(Helper)],
 [LessMultiplication(F)], [LessMultiplication(R)], [LeqMultiplication(R)],
 [PlusAssociativity(F)], [Plus0(F)], [PlusCommutativity(F)],
 [TimesAssociativity(F)], [Times1f], [Cauchy(2)(Helper)], [Cauchy(2)],
 [ReciprocalFnonzero], [ReciprocalFnynonzero],
 [(Eventually = f)2sameF(Helper)], [(Eventually = f)2sameF],
 [FromNotSameF(Strong)(Helper2)], [FromNotSameF(Strong)(Helper)],
 [FromNotSameF(Strong)], [SameFreciprocal(Helper)], [SameFreciprocal],
 [From!! ==], [Reciprocal(R)], [TimesCommutativity(F)], [Distribution(F)],
 [FromMax(1)], [FromMax(2)], [ToNegatedAnd], [PositiveToRight(Less)(1term)],
 [(A) to (E)], [lemma ==Transitivity4], [Plus0Left(R)], [x = x + (y - y)(R)],
 [x = x + y - y(R)], [PositiveToRight(Eq)(R)], [SubtractEquations(R)],
 [NeqAddition(R)], [EqAdditionLeft(R)], [Three2twoTerms(R)],
 [PositiveToRight(Less)(R)], [Three2threeTerms(R)],
 [PositiveToRight(Less)(1term)(R)], [ToLeq(Advanced)(R)], [LeqNeqLess(R)],
 [SubLeqLeft(R)], [LeqLessTransitivity(R)], [NegativeToLeft(Eq)(R)],
 [NegativeToRight(Less)(R)], [!! == Symmetry], [NegativeToRight(Eq)(R)],
 [NegativeToRight(Eq)(1term)(R)], [DoubleMinus(R)], [UniqueNegative(R)],
 [SubtractEquationsLeft(R)], [EqNegated(R)], [NeqNegated(R)],
 [LeqNegated(R)], [LessNegated(R)], [-0 = 0(R)], [NegativeNegated(R)],
 [FromLeqGeq(R)], [SubLeqRight(R)], [FromLess(R)],
 [NonnegativeNumerical(R)], [NegativeNumerical(R)], [0 <= |x|(R)],
 [PositiveNegated(R)], [AddEquations(R)], [DistributionOut(R)],
 [= Transitivity5], [x * 0 + x = x(R)], [x * 0 = 0(R)(fff)], [Times(-1)(R)],
 [Times(-1)Left(R)], [-x - y = -(x + y)(R)], [LessTotality(R)],
 [PositiveNumerical(R)], [SignNumerical(+)(R)], [SameNumerical(R)],
 [MinusNegated(R)], [SignNumerical(R)], [NumericalDifference(R)],
 [x <= |x|(R)], [USlimitIsUpperBound(Helper)], [USlimitIsUpperBound],
 [(-1) * (-1) + (-1) * 1 = 0(R)], [(-1) * (-1) = 1(R)], [0 < 1Helper(R)],
 [0 < 1(R)], [ExpZero(Exact)(R)], [PositiveBase(R)(Base)],
 [Three2twoFactors(R)], [x = x * y * (1/y)(R)], [NeqMultiplication(R)],
 [LessTransitivity(R)], [0 < 2(R)], [SameExp(R)(Base)], [SameExp(R)(Indu)],
 [SameExp(R)], [SubNeqLeft(R)], [SubNeqRight(R)], [NonzeroFactors(R)],
 [NonnegativeFactors(R)], [PositiveFactors(R)], [LessDivision(R)], [0 < 1/2(R)],
 [PositiveToRight(Eq)(1term)(R)], [Exp(+1)(R)], [PositiveBase(R)(Indu)],

[PositiveBase(R)], [$-x * y = -(x * y)$ (R)], [PositiveToLeft(Eq)(R)],
 [Times1Left(R)], [= Transitivty6], [$x + x = 2 * x$ (R)],
 [(1/2)x + (1/2)x = x(R)], [DistributionOut(Minus)(R)],
 [(1/2)(x + y) - x = (1/2)(y - x)(R)], [IntervalSize(R)(Base)],
 [LessMultiplicationLeft(R)], [NegativeToLeft(Less)(R)],
 [NegativeToLeft(Less)(1term)(R)], [$y - (1/2)(x + y) = (1/2)(y - x)$ (R)],
 [IntervalSize(R)(Indu)], [IntervalSize(R)], [XSlessUS(R)],
 [USdecreasing(+1)(R)], [ExpUnbounded(Base)], [ExpUnbounded(Indu)],
 [ExpUnbounded], [$1 \leq x + 1$ (N)], [NonzeroProduct(2)(R)],
 [PositiveNonzero(R)], [NonreciprocalToRight(Eq)(1term)(R)],
 [ExpNonzero(Base)], [ExpNonzero(Indu)], [ExpNonzero], [ExpNonzero(2)],
 [MultiplyEquations(R)], [HalfBase(Base)], [HalfBase(Indu)], [HalfBase],
 [Three2threeFactors(R)], [$x * y = z$ Backwards(R)], [PositiveInverted(R)],
 [ReciprocalToRight(Less)(R)], [ReciprocalToRight(Less)(1term)(R)],
 [NonreciprocalToLeft(Less)(R)], [$1 < x * y$ (R)], [SwitchFactors(1/x < y)(R)],
 [SmallHalving], [IntervalSize(anyPositive)], [USdecreasing(+n)(Base)],
 [USdecreasing(+n)(Indu)], [USdecreasing(+n)], [USdecreasing],
 [LeqAdditionLeft(R)], [ToNotLess(R)], [LimitOfUSIsLeq],
 [SubtractEquations(Less)(R)], [SubtractEquationsLeft(Less)(R)],
 [LessNegated(Negative)(R)], [FromNegatedAnd(Implied)],
 [RemoveDoubleNeg(Consequent)], [FromNotUpperBound], [DistributionOut],
 [DistributionOutLeft], [DistributionLeft], [LeqNUB],
 [USlimitIsLeastUpperBound(Helper)], [USlimitIsLeastUpperBound],
 [FromNotLess(R)], [ExistMP3], [GreaterPositive(N)], [ysFClose(Helper)],
 [ysFClose], [ysFCAuchy(Helper)], [ysFCAuchy], [CartProdIsRelation],
 [FromSubset], [SubsetIsRelation], [ToSeries], [FromSeries], [SeriesSubsetCP],
 [ValueType], [RemoveOr], [FromSingleton], [InPair(1)], [InPair(2)],
 [SameMember(2)], [ToBinaryUnion(1)], [ToBinaryUnion(2)],
 [FromOrderedPair(TwoLevels)], [ToCartProd(Helper)], [ToCartProd],
 [NonreciprocalToRight(Eq)], [NonreciprocalToLeft(Eq)(1term)],
 [SameReciprocal], [CPseparationIsRelation], [OrderedPairEquality],
 [ReciprocalIsFunction], [ReciprocalIsTotal], [ReciprocalIsRationalSeries],
 [CrsIsRelation], [CrsIsFunction], [CrsIsTotal], [CrsIsSeries], [CrsLookup], [0f],
 [1f], [ToSingleton], [FromSameSingleton], [SingletonmembersEqual],
 [UnequalsNotInSingleton], [NonsingletonmembersUnequal], [FromOrderedPair],
 [FromOrderedPair(1)], [FromOrderedPair(2)], [FromCartProd],
 [FromCartProd(1)], [FromCartProd(2)], [sameOrderedPair], [InSeriesHelper],
 [InSeries], [To = f(Subset)(Helper)], [To = f(Subset)], [To = f],
 [productIsFunction], [productIsTotal], [ProductIsRationalSeries], [TimesF],
 [$-x + (1/2)x = -(1/2)x$], [PositiveTripled], [PositiveDividedBy3], [$|x - x| = 0$],
 [$1 < 2$], [$1/3 < 2/3$], [(1/3)x + (1/3)x = (2/3)x], [(2/3)x + (1/3)x = x],
 [$-x + (2/3)x = -(1/3)x$], [$-(1/3)x - (1/3)x = -(2/3)x$],
 [$-x + (1/3)x = -(2/3)x$], [PreserveLessGreater], [ClosestlessIsLess],
 [SubLessLeft(F)], [SubLessLeft(R)], [ClosestgreaterIsGreater],
 [SubLessRight(F)], [SubLessRight(R)];

Preassociative

[Tester1], [Tester2], [Tester3], [Tester4], [Tester5], [Tester6];

Preassociative

[*_{}*], [*/indexintro(*, *, *, *)], [*/intro(*, *, *)], [*/bothintro(*, *, *, *, *)],
[*/nameintro(*, *, *, *)], [*/], [*[*]], [*[* → *]], [*[* ⇒ *]], [*[0], [*[1], [0b], [*-color(*)],
[*-color* (*)], [*_H], [*_T], [*_U], [*_h], [*_t], [*_s], [*_c], [*_d], [*_a], [*_C], [*_M], [*_B], [*_r], [*_i],
[*_d], [*_R], [*_0], [*_1], [*_2], [*_3], [*_4], [*_5], [*_6], [*_7], [*_8], [*_9], [*_E], [*_V], [*_C], [*_C*],
[*_hide];

Preassociative

[“ * ”], [], [(*)^t], [string(*) + *], [string(*) ++ *], [
*, [*], [! *], [\" *], [# *], [\$ *], [% *], [& *], [’ *], [(*), (*)], [**], [+ *], [*], [- *], [. *], [/ *],
[0 *], [1 *], [2 *], [3 *], [4 *], [5 *], [6 *], [7 *], [8 *], [9 *], [: *], [; *], [< *], [= *], [> *], [? *],
[@ *], [A *], [B *], [C *], [D *], [E *], [F *], [G *], [H *], [I *], [J *], [K *], [L *], [M *], [N *],
[O *], [P *], [Q *], [R *], [S *], [T *], [U *], [V *], [W *], [X *], [Y *], [Z *], [[*], [\ *], [] *], [^ *],
[_ *], [‘ *], [a *], [b *], [c *], [d *], [e *], [f *], [g *], [h *], [i *], [j *], [k *], [l *], [m *], [n *], [o *],
[p *], [q *], [r *], [s *], [t *], [u *], [v *], [w *], [x *], [y *], [z *], [{ * }, [| *], [} *], [~ *],
[Preassociative * ; *], [Postassociative * ; *], [[*], *], [priority * end],
[newline *], [macro newline *], [MacroIndent(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*(exp)*];

Preassociative

[*], [R(*)], [— R(*)], [rec*];

Preassociative

[*/ *], [* ∩ *], [* [*]];

Preassociative

[∪ *], [* ∪ *], [P(*)];

Preassociative

[{ * }], [StateExpand(*, *, *)], [extractSeries(*)], [SetOfSeries(*)], [— Macro(*)],
[ExpandList(*, *, *)], [* Macro(*)], [+ + Macro(*)], [< < Macro(*)], [UB(*, *)],
[LUB(*, *)], [BS(*, *)], [USteelescope(*, *)], [(*)], [| r * |], [Limit(*, *)], [Union(*)],
[IsOrderedPair(*, *, *)], [IsRelation(*, *, *)], [isFunction(*, *, *)], [IsSeries(*, *)],
[IsNatural(*, *)], [OrderedPair(*, *)], [TypeNat(*)], [TypeNat0(*)],
[TypeRational(*)], [TypeRational0(*)], [TypeSeries(*, *)], [Typeseries0(*, *)];

Preassociative

[{* , * }], [(< * , *)], [(-u*)], [-_f *], [(- - *)], [1f / *], [1fny / *], [01 // temp *];

Preassociative

[*(* , *)], [RefRel(*, *)], [SymRel(*, *)], [TransRel(*, *)], [EqRel(*, *)], [[* ∈ *] *],
[Partition(*, *)];

Preassociative

[* · *], [* ·_0 *], [(* * *)], [* *_f *], [* * * *];

Preassociative

[* + *], [* +_0 *], [* +_1 *], [* - *], [* -_0 *], [* -_1 *], [(* + *)], [(* - *)], [* +_f *],
[* -_f *], [* + + *], [R(*) - -R(*)];

Preassociative

[* ∈ *];

Preassociative

$[[* \mid]], [\text{if}(*, *, *)], [\text{Max}(*, *)], [\text{Max}(*, *)];$

Preassociative

$[* = *], [* \neq *], [* \leq *], [* < *], [* <_r *], [* \leq_r *], [\text{SF}(*, *)], [* == *],$
 $[*!! == *], [* << *], [* << == *];$

Preassociative

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$

Postassociative

$[* \dot{:} *], [* \ddot{:} *], [* \vdash *], [* \underline{+2*} *], [* :: *], [* +2* *];$

Postassociative

$[*, *];$

Preassociative

$[* \overset{B}{\approx} *], [* \overset{D}{\approx} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \dashv *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{free in } *], [* \text{free in}^* *], [* \text{free for } * \text{ in } *],$
 $[* \text{free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\text{var}}],$
 $[* \#^0 *], [* \#^1 *], [* \#^* *], [* == *], [* \subseteq *];$

Preassociative

$[\neg *], [\dot{\neg} (*n)], [* \notin *], [* \neq *];$

Preassociative

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$

Preassociative

$[* \vee *], [* \parallel *], [* \ddot{\vee} *];$

Postassociative

$[* \dot{\vee} *];$

Preassociative

$[\exists *: *], [\forall *: *], [\forall_{\text{obj}} *: *], [\exists *: *];$

Postassociative

$[* \dot{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \dot{\Leftrightarrow} *];$

Preassociative

$[\{\text{ph} \in * \mid *\}];$

Postassociative

$[* : *], [* \text{spy } *], [*! *];$

Preassociative

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$

Preassociative

$[\lambda *. *], [\Lambda *. *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \dot{=} * \text{ in } *];$

Preassociative

$[* \# *];$

Preassociative

$[*^I], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$

Preassociative

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *], [* \triangleright *];$

Postassociative

$[* \vdash *], [* \# *], [* \text{i.e. } *];$

Preassociative

$[\forall *: *], [\Pi *: *];$

Postassociative

$[* \oplus *];$

Postassociative

$[*; *];$

Preassociative

$[* \text{ proves } *];$

Preassociative

$[* \text{ proof of } *: *], [\text{Line } *: * \gg *: *], [\text{Last line } * \gg * \square],$
 $[\text{Line } *: \text{Premise } \gg *: *], [\text{Line } *: \text{Side-condition } \gg *: *], [\text{Arbitrary } \gg *: *],$
 $[\text{Local } \gg * = *: *], [\text{Begin } *: * : \text{End}; *], [\text{Last block line } * \gg *: *],$
 $[\text{Arbitrary } \gg *: *];$

Postassociative

$[* | *];$

Postassociative

$[* , *], [* [*]*];$

Preassociative

$[*\&*];$

Preassociative

$[* \\ *], [* \text{ linebreak}[4] *], [* \\ *];$

A Pyk definitioner

$[\text{To!!} == \xrightarrow{\text{pyk}} \text{“lemma to!!==”}]$

$[\text{ToNegatedDoubleImply} \xrightarrow{\text{pyk}} \text{“prop lemma to negated double imply”}]$

$[\text{ToNegatedAnd}(1) \xrightarrow{\text{pyk}} \text{“prop lemma to negated and}(1)”]$

$[\text{AddNegatedAll} \xrightarrow{\text{pyk}} \text{“pred lemma addNegatedAll”}]$

$[(A)\text{to}(E)(\text{Imply}) \xrightarrow{\text{pyk}} \text{“pred lemma (A)to}(\sim E\sim)(\text{Imply})”]$

$[(E)\text{to}(A)(\text{Imply}) \xrightarrow{\text{pyk}} \text{“pred lemma (E)to}(\sim A\sim)(\text{Imply})”]$

$[(E)\text{to}(A)(\text{Imply}) \xrightarrow{\text{pyk}} \text{“pred lemma (E}\sim\text{)to}(\sim A)(\text{Imply})”]$

$[\text{ToNegatedAEA} \xrightarrow{\text{pyk}} \text{“pred lemma toNegatedAEA”}]$

$[\text{UniqueNegative} \xrightarrow{\text{pyk}} \text{“lemma uniqueNegative”}]$

$[\text{DoubleMinus} \xrightarrow{\text{pyk}} \text{“lemma doubleMinus”}]$

$[\text{MinusNegated} \xrightarrow{\text{pyk}} \text{“lemma minusNegated”}]$

$[\text{eqReflexivity} \xrightarrow{\text{pyk}} \text{“lemma eqReflexivity”}]$

$[\text{eqSymmetry} \xrightarrow{\text{pyk}} \text{“lemma eqSymmetry”}]$

$[\text{eqTransitivity} \xrightarrow{\text{pyk}} \text{“lemma eqTransitivity”}]$

$[\text{eqTransitivity4} \xrightarrow{\text{pyk}} \text{“lemma eqTransitivity4”}]$

$[\text{eqTransitivity5} \xrightarrow{\text{pyk}} \text{“lemma eqTransitivity5”}]$

[eqTransitivity6 $\xrightarrow{\text{pyk}}$ “lemma eqTransitivity6”]
 [AddEquations $\xrightarrow{\text{pyk}}$ “lemma addEquations”]
 [SubtractEquations $\xrightarrow{\text{pyk}}$ “lemma subtractEquations”]
 [SubtractEquationsLeft $\xrightarrow{\text{pyk}}$ “lemma subtractEquationsLeft”]
 [MultiplyEquations $\xrightarrow{\text{pyk}}$ “lemma multiplyEquations”]
 [EqNegated $\xrightarrow{\text{pyk}}$ “lemma eqNegated”]
 [PositiveToRight(Eq) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Eq)”]
 [PositiveToLeft(Eq) $\xrightarrow{\text{pyk}}$ “lemma positiveToLeft(Eq)”]
 [PositiveToLeft(Eq)(1term) $\xrightarrow{\text{pyk}}$ “lemma positiveToLeft(Eq)(1 term)”]
 [NegativeToLeft(Eq) $\xrightarrow{\text{pyk}}$ “lemma negativeToLeft(Eq)”]
 [NonreciprocalToRight(Eq)(1term) $\xrightarrow{\text{pyk}}$ “lemma nonreciprocalToRight(Eq)(1 term)”]
 [DistributionOut(Minus) $\xrightarrow{\text{pyk}}$ “lemma distributionOut(Minus)”]
 [PositiveToRight(Eq)(1term) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Eq)(1 term)”]
 [SameSeries(NumDiff) $\xrightarrow{\text{pyk}}$ “lemma sameSeries(NumDiff)”]
 [PlusAssociativity(4terms) $\xrightarrow{\text{pyk}}$ “lemma plusAssociativity(4 terms)”]
 [LessNeq $\xrightarrow{\text{pyk}}$ “lemma lessNeq”]
 [NeqSymmetry $\xrightarrow{\text{pyk}}$ “lemma neqSymmetry”]
 [NeqNegated $\xrightarrow{\text{pyk}}$ “lemma neqNegated”]
 [SubNeqRight $\xrightarrow{\text{pyk}}$ “lemma subNeqRight”]
 [SubNeqLeft $\xrightarrow{\text{pyk}}$ “lemma subNeqLeft”]
 [NegativeToRight(Neq)(1term) $\xrightarrow{\text{pyk}}$ “lemma negativeToRight(Neq)(1 term)”]
 [NeqAddition $\xrightarrow{\text{pyk}}$ “lemma neqAddition”]
 [NeqMultiplication $\xrightarrow{\text{pyk}}$ “lemma neqMultiplication”]
 [NonzeroProduct(2) $\xrightarrow{\text{pyk}}$ “lemma nonzeroProduct(2)”]
 [SwitchTerms(x <= y - z) $\xrightarrow{\text{pyk}}$ “lemma switchTerms(x<=y-z)”]
 [NegativeToLeft(Less)(1term) $\xrightarrow{\text{pyk}}$ “lemma negativeToLeft(Less)(1 term)”]
 [(+1)IsPositive(N) $\xrightarrow{\text{pyk}}$ “lemma +1IsPositive(N)”]
 [(1/2)(x + y) - x = (1/2)(y - x) $\xrightarrow{\text{pyk}}$ “lemma (1/2)(x+y)-x=(1/2)(y-x)”]
 [y - (1/2)(x + y) = (1/2)(y - x) $\xrightarrow{\text{pyk}}$ “lemma y-(1/2)(x+y)=(1/2)(y-x)”]
 [ExpZero(Exact) $\xrightarrow{\text{pyk}}$ “lemma expZero exact”]
 [SameExp(Base) $\xrightarrow{\text{pyk}}$ “lemma sameExp base”]
 [SameExp(Indu) $\xrightarrow{\text{pyk}}$ “lemma sameExp indu”]
 [SameExp $\xrightarrow{\text{pyk}}$ “lemma sameExp”]
 [Exp(+1) $\xrightarrow{\text{pyk}}$ “lemma exp(+1)”]

[PositiveBase(Base) $\xrightarrow{\text{pyk}}$ “lemma positiveBase base”]
 [PositiveBase(Indu) $\xrightarrow{\text{pyk}}$ “lemma positiveBase indu”]
 [PositiveBase $\xrightarrow{\text{pyk}}$ “lemma positiveBase”]
 [BSzero(Exact) $\xrightarrow{\text{pyk}}$ “lemma base(1/2)Sum zero exact”]
 [SameBS(2)(Base) $\xrightarrow{\text{pyk}}$ “lemma sameBase(1/2)Sum second base”]
 [SameBS(2)(Indu) $\xrightarrow{\text{pyk}}$ “lemma sameBase(1/2)Sum second indu”]
 [SameBS(2) $\xrightarrow{\text{pyk}}$ “lemma sameBase(1/2)Sum second”]
 [BS(+1) $\xrightarrow{\text{pyk}}$ “lemma base(1/2)Sum(+1)”]
 [BSbound(Exact)(Base) $\xrightarrow{\text{pyk}}$ “lemma base(1/2)Sum exact bound base”]
 [BSbound(Exact)(Indu) $\xrightarrow{\text{pyk}}$ “lemma base(1/2)Sum exact bound indu”]
 [BSbound(Exact) $\xrightarrow{\text{pyk}}$ “lemma base(1/2)Sum exact bound”]
 [BSbound $\xrightarrow{\text{pyk}}$ “lemma base(1/2)Sum bound”]
 [USTelescope(Zero)(Exact) $\xrightarrow{\text{pyk}}$ “lemma UTelescope zero exact”]
 [SameTelescope(2)(Base) $\xrightarrow{\text{pyk}}$ “lemma sameTelescope second base”]
 [SameTelescope(2)(Indu) $\xrightarrow{\text{pyk}}$ “lemma sameTelescope second indu”]
 [SameTelescope(2) $\xrightarrow{\text{pyk}}$ “lemma sameTelescope second”]
 [USTelescope(+1) $\xrightarrow{\text{pyk}}$ “lemma UTelescope(+1)”]
 [TelescopeNumerical(Base) $\xrightarrow{\text{pyk}}$ “lemma telescopeNumerical base”]
 [TelescopeNumerical(Indu) $\xrightarrow{\text{pyk}}$ “lemma telescopeNumerical indu”]
 [TelescopeNumerical $\xrightarrow{\text{pyk}}$ “lemma telescopeNumerical”]
 [TelescopeBound(Base) $\xrightarrow{\text{pyk}}$ “lemma telescopeBound base”]
 [TelescopeBound(Indu) $\xrightarrow{\text{pyk}}$ “lemma telescopeBound indu”]
 [TelescopeBound $\xrightarrow{\text{pyk}}$ “lemma telescopeBound”]
 [LessNeq(F)(Helper) $\xrightarrow{\text{pyk}}$ “lemma lessNeq(F) helper”]
 [LessNeq(F) $\xrightarrow{\text{pyk}}$ “lemma lessNeq(F)”]
 [LessNeq(R) $\xrightarrow{\text{pyk}}$ “lemma lessNeq(R)”]
 [IntervalSize(Base) $\xrightarrow{\text{pyk}}$ “lemma intervalSize base”]
 [IntervalSize(Indu) $\xrightarrow{\text{pyk}}$ “lemma intervalSize indu”]
 [IntervalSize $\xrightarrow{\text{pyk}}$ “lemma intervalSize”]
 [XS < US $\xrightarrow{\text{pyk}}$ “lemma XSlessUS”]
 [lemma USdecreasing(+1) $\xrightarrow{\text{pyk}}$ “lemma USdecreasing(+1)”]
 [CloseUS $\xrightarrow{\text{pyk}}$ “lemma closeUS”]
 [CloseUS(n + 1) $\xrightarrow{\text{pyk}}$ “lemma closeUS(n+1)”]
 [AllNegated(ImPLY) $\xrightarrow{\text{pyk}}$ “pred lemma allNegated(ImPLY)”]
 [ExistNegated(ImPLY) $\xrightarrow{\text{pyk}}$ “pred lemma existNegated(ImPLY)”]

[IntroExist(Helper) $\xrightarrow{\text{pyk}}$ “pred lemma intro exist helper”]
 [IntroExist $\xrightarrow{\text{pyk}}$ “pred lemma intro exist”]
 [ExistMP $\xrightarrow{\text{pyk}}$ “pred lemma exist mp”]
 [ExistMP2 $\xrightarrow{\text{pyk}}$ “pred lemma exist mp2”]
 [TwiceExistMP $\xrightarrow{\text{pyk}}$ “pred lemma 2exist mp”]
 [TwiceExistMP2 $\xrightarrow{\text{pyk}}$ “pred lemma 2exist mp2”]
 [EAE – MP $\xrightarrow{\text{pyk}}$ “pred lemma EAE mp”]
 [AddAll $\xrightarrow{\text{pyk}}$ “pred lemma addAll”]
 [AddExist(Helper1) $\xrightarrow{\text{pyk}}$ “pred lemma addExist helper1”]
 [AddExist(Helper2) $\xrightarrow{\text{pyk}}$ “pred lemma addExist helper2”]
 [AddExist $\xrightarrow{\text{pyk}}$ “pred lemma addExist”]
 [AddExist(SimpleAnt) $\xrightarrow{\text{pyk}}$ “pred lemma addExist(SimpleAnt)”]
 [AddExist(Simple) $\xrightarrow{\text{pyk}}$ “pred lemma addExist(Simple)”]
 [AddEAE $\xrightarrow{\text{pyk}}$ “pred lemma addEAE”]
 [AEA – negated $\xrightarrow{\text{pyk}}$ “pred lemma AEAnegated”]
 [EEA – negated $\xrightarrow{\text{pyk}}$ “pred lemma EEAnegated”]
 [Induction $\xrightarrow{\text{pyk}}$ “lemma induction”]
 [leqAntisymmetry $\xrightarrow{\text{pyk}}$ “lemma leqAntisymmetry”]
 [leqTransitivity $\xrightarrow{\text{pyk}}$ “lemma leqTransitivity”]
 [leqAddition $\xrightarrow{\text{pyk}}$ “lemma leqAddition”]
 [leqMultiplication $\xrightarrow{\text{pyk}}$ “lemma leqMultiplication”]
 [Reciprocal $\xrightarrow{\text{pyk}}$ “lemma reciprocal”]
 [Equality $\xrightarrow{\text{pyk}}$ “lemma equality”]
 [eqLeq $\xrightarrow{\text{pyk}}$ “lemma eqLeq”]
 [eqAddition $\xrightarrow{\text{pyk}}$ “lemma eqAddition”]
 [eqMultiplication $\xrightarrow{\text{pyk}}$ “lemma eqMultiplication”]
 [LeqMultiplicationLeft $\xrightarrow{\text{pyk}}$ “lemma leqMultiplicationLeft”]
 [LeqLessEq $\xrightarrow{\text{pyk}}$ “lemma leqLessEq”]
 [LessLeq $\xrightarrow{\text{pyk}}$ “lemma lessLeq”]
 [FromLeqGeq $\xrightarrow{\text{pyk}}$ “lemma from leqGeq”]
 [subLeqRight $\xrightarrow{\text{pyk}}$ “lemma subLeqRight”]
 [subLeqLeft $\xrightarrow{\text{pyk}}$ “lemma subLeqLeft”]
 [Leq + 1 $\xrightarrow{\text{pyk}}$ “lemma leqPlus1”]
 [PositiveToRight(Leq) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Leq)”]
 [PositiveToRight(Leq)(1term) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Leq)(1 term)”]

$[\text{lemma negativeToRight}(\text{Leq}) \xrightarrow{\text{pyk}} \text{"lemma negativeToRight}(\text{Leq})"]$
 $[\text{PositiveToLeft}(\text{Leq}) \xrightarrow{\text{pyk}} \text{"lemma positiveToLeft}(\text{Leq})"]$
 $[\text{negativeToLeft}(\text{Leq}) \xrightarrow{\text{pyk}} \text{"lemma negativeToLeft}(\text{Leq})"]$
 $[\text{negativeToLeft}(\text{Leq})(1\text{term}) \xrightarrow{\text{pyk}} \text{"lemma negativeToLeft}(\text{Leq})(1\text{ term})"]$
 $[\text{LeqAdditionLeft} \xrightarrow{\text{pyk}} \text{"lemma leqAdditionLeft"}]$
 $[\text{leqSubtraction} \xrightarrow{\text{pyk}} \text{"lemma leqSubtraction"}]$
 $[\text{leqSubtractionLeft} \xrightarrow{\text{pyk}} \text{"lemma leqSubtractionLeft"}]$
 $[\text{thirdGeq} \xrightarrow{\text{pyk}} \text{"lemma thirdGeq"}]$
 $[\text{LeqNegated} \xrightarrow{\text{pyk}} \text{"lemma leqNegated"}]$
 $[\text{AddEquations}(\text{Leq}) \xrightarrow{\text{pyk}} \text{"lemma addEquations}(\text{Leq})"]$
 $[\text{MultiplyEquations}(\text{Leq}) \xrightarrow{\text{pyk}} \text{"lemma multiplyEquations}(\text{Leq})"]$
 $[\text{ThirdGeqSeries} \xrightarrow{\text{pyk}} \text{"lemma thirdGeqSeries"}]$
 $[\text{LeqNeqLess} \xrightarrow{\text{pyk}} \text{"lemma leqNeqLess"}]$
 $[\text{FromLess} \xrightarrow{\text{pyk}} \text{"lemma fromLess"}]$
 $[\text{ToLess} \xrightarrow{\text{pyk}} \text{"lemma toLess"}]$
 $[\text{fromNotLess} \xrightarrow{\text{pyk}} \text{"lemma fromNotLess"}]$
 $[\text{toNotLess} \xrightarrow{\text{pyk}} \text{"lemma toNotLess"}]$
 $[\text{NegativeLessPositive} \xrightarrow{\text{pyk}} \text{"lemma negativeLessPositive"}]$
 $[\text{leqLessTransitivity} \xrightarrow{\text{pyk}} \text{"lemma leqLessTransitivity"}]$
 $[\text{LessLeqTransitivity} \xrightarrow{\text{pyk}} \text{"lemma lessLeqTransitivity"}]$
 $[\text{LessTransitivity} \xrightarrow{\text{pyk}} \text{"lemma lessTransitivity"}]$
 $[\text{LessTotality} \xrightarrow{\text{pyk}} \text{"lemma lessTotality"}]$
 $[\text{SubLessRight} \xrightarrow{\text{pyk}} \text{"lemma subLessRight"}]$
 $[\text{SubLessLeft} \xrightarrow{\text{pyk}} \text{"lemma subLessLeft"}]$
 $[\text{SwitchTerms}(x < y - z) \xrightarrow{\text{pyk}} \text{"lemma switchTerms}(x < y - z)"]$
 $[\text{SwitchTerms}(x - y < z) \xrightarrow{\text{pyk}} \text{"lemma switchTerms}(x - y < z)"]$
 $[\text{LessAddition} \xrightarrow{\text{pyk}} \text{"lemma lessAddition"}]$
 $[\text{LessAdditionLeft} \xrightarrow{\text{pyk}} \text{"lemma lessAdditionLeft"}]$
 $[\text{LessMultiplication} \xrightarrow{\text{pyk}} \text{"lemma lessMultiplication"}]$
 $[\text{LessMultiplicationLeft} \xrightarrow{\text{pyk}} \text{"lemma lessMultiplicationLeft"}]$
 $[\text{LessDivision} \xrightarrow{\text{pyk}} \text{"lemma lessDivision"}]$
 $[\text{PositiveToRight}(\text{Less}) \xrightarrow{\text{pyk}} \text{"lemma positiveToRight}(\text{Less})"]$
 $[\text{PositiveToLeft}(\text{Less}) \xrightarrow{\text{pyk}} \text{"lemma positiveToLeft}(\text{Less})"]$
 $[\text{NegativeToLeft}(\text{Less}) \xrightarrow{\text{pyk}} \text{"lemma negativeToLeft}(\text{Less})"]$
 $[\text{NegativeToRight}(\text{Less}) \xrightarrow{\text{pyk}} \text{"lemma negativeToRight}(\text{Less})"]$

[AddEquations(Less) $\xrightarrow{\text{pyk}}$ “lemma addEquations(Less)”]
 [AddEquations(LeqLess) $\xrightarrow{\text{pyk}}$ “lemma addEquations(LeqLess)”]
 [reciprocalToLeft(Less) $\xrightarrow{\text{pyk}}$ “lemma reciprocalToLeft(Less)”]
 [LessNegated $\xrightarrow{\text{pyk}}$ “lemma lessNegated”]
 [PositiveNonzero $\xrightarrow{\text{pyk}}$ “lemma positiveNonzero”]
 [PositiveNegated $\xrightarrow{\text{pyk}}$ “lemma positiveNegated”]
 [NonpositiveNegated $\xrightarrow{\text{pyk}}$ “lemma nonpositiveNegated”]
 [NegativeNegated $\xrightarrow{\text{pyk}}$ “lemma negativeNegated”]
 [NonnegativeNegated $\xrightarrow{\text{pyk}}$ “lemma nonnegativeNegated”]
 [PositiveHalved $\xrightarrow{\text{pyk}}$ “lemma positiveHalved”]
 [PositiveInverted $\xrightarrow{\text{pyk}}$ “lemma positiveInverted”]
 [NonnegativeNumerical $\xrightarrow{\text{pyk}}$ “lemma nonnegativeNumerical”]
 [NegativeNumerical $\xrightarrow{\text{pyk}}$ “lemma negativeNumerical”]
 [PositiveNumerical $\xrightarrow{\text{pyk}}$ “lemma positiveNumerical”]
 [lemma nonpositiveNumerical $\xrightarrow{\text{pyk}}$ “lemma nonpositiveNumerical”]
 [|0| = 0 $\xrightarrow{\text{pyk}}$ “lemma |0|=0”]
 [0 <= |x| $\xrightarrow{\text{pyk}}$ “lemma 0<=|x|”]
 [x <= |x| $\xrightarrow{\text{pyk}}$ “lemma x<=|x|”]
 [FromPositiveNumerical $\xrightarrow{\text{pyk}}$ “lemma fromPositiveNumerical”]
 [SameNumerical $\xrightarrow{\text{pyk}}$ “lemma sameNumerical”]
 [SignNumerical(+) $\xrightarrow{\text{pyk}}$ “lemma signNumerical(+)”]
 [SignNumerical $\xrightarrow{\text{pyk}}$ “lemma signNumerical”]
 [ToNumericalLess $\xrightarrow{\text{pyk}}$ “lemma toNumericalLess”]
 [FromNumericalGreater $\xrightarrow{\text{pyk}}$ “lemma fromNumericalGreater”]
 [NumericalDifference $\xrightarrow{\text{pyk}}$ “lemma numericalDifference”]
 [NumericalDifferenceLess(Helper) $\xrightarrow{\text{pyk}}$ “lemma numericalDifferenceLess helper”]
 [NumericalDifferenceLess $\xrightarrow{\text{pyk}}$ “lemma numericalDifferenceLess”]
 [SplitNumericalSumHelper $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSumHelper”]
 [splitNumericalSum(++) $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSum(++)”]
 [splitNumericalSum(--) $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSum(--)”]
 [splitNumericalSum(+ - small) $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSum(+-, smallNegative)”]
 [splitNumericalSum(+ - big) $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSum(+-, bigNegative)”]
 [splitNumericalSum(+ -) $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSum(+ -)”]
 [splitNumericalSum(- +) $\xrightarrow{\text{pyk}}$ “lemma splitNumericalSum(- +)”]

$[\text{splitNumericalSum} \xrightarrow{\text{pyk}} \text{"lemma splitNumericalSum"}]$
 $[\text{SplitNumericalProduct}(++) \xrightarrow{\text{pyk}} \text{"lemma splitNumericalProduct}(++)"]$
 $[\text{SplitNumericalProduct}(+-) \xrightarrow{\text{pyk}} \text{"lemma splitNumericalProduct}(+-)"]$
 $[\text{SplitNumericalProduct} \xrightarrow{\text{pyk}} \text{"lemma splitNumericalProduct"}]$
 $[\text{insertMiddleTerm}(\text{Numerical}) \xrightarrow{\text{pyk}} \text{"lemma insertMiddleTerm}(\text{Numerical})"]$
 $[\text{insertTwoMiddleTerms}(\text{Numerical}) \xrightarrow{\text{pyk}} \text{"lemma insertTwoMiddleTerms}(\text{Numerical})"]$
 $[\text{Three2twoTerms} \xrightarrow{\text{pyk}} \text{"lemma three2twoTerms"}]$
 $[\text{Three2threeTerms} \xrightarrow{\text{pyk}} \text{"lemma three2threeTerms"}]$
 $[\text{Three2twoFactors} \xrightarrow{\text{pyk}} \text{"lemma three2twoFactors"}]$
 $[\text{Three2threeFactors} \xrightarrow{\text{pyk}} \text{"lemma three2threeFactors"}]$
 $[\text{Times}(-1) \xrightarrow{\text{pyk}} \text{"lemma times}(-1)"]$
 $[\text{Times}(-1)\text{Left} \xrightarrow{\text{pyk}} \text{"lemma times}(-1)\text{Left}"]$
 $[\text{MaxLeq}(1) \xrightarrow{\text{pyk}} \text{"lemma leqMax1"}]$
 $[\text{MaxLeq}(2) \xrightarrow{\text{pyk}} \text{"lemma leqMax2"}]$
 $[\text{LessThanMax} \xrightarrow{\text{pyk}} \text{"lemma lessThanMax"}]$
 $[x + y = z \text{Backwards} \xrightarrow{\text{pyk}} \text{"lemma } x+y=z \text{Backwards}"]$
 $[x * y = z \text{Backwards} \xrightarrow{\text{pyk}} \text{"lemma } x*y=z \text{Backwards}"]$
 $[x = x + (y - y) \xrightarrow{\text{pyk}} \text{"lemma } x=x+(y-y)"]$
 $[x = x + y - y \xrightarrow{\text{pyk}} \text{"lemma } x=x+y-y"]$
 $[x = x * y * (1/y) \xrightarrow{\text{pyk}} \text{"lemma } x=x*y*(1/y)"]$
 $[\text{insertMiddleTerm}(\text{Sum}) \xrightarrow{\text{pyk}} \text{"lemma insertMiddleTerm}(\text{Sum})"]$
 $[\text{insertTwoMiddleTerms}(\text{Sum}) \xrightarrow{\text{pyk}} \text{"lemma insertTwoMiddleTerms}(\text{Sum})"]$
 $[\text{insertMiddleTerm}(\text{Difference}) \xrightarrow{\text{pyk}} \text{"lemma insertMiddleTerm}(\text{Difference})"]$
 $[x * 0 + x = x \xrightarrow{\text{pyk}} \text{"lemma } x*0+x=x"]$
 $[x * 0 = 0 \xrightarrow{\text{pyk}} \text{"lemma } x*0=0"]$
 $[\text{NonnegativeFactors} \xrightarrow{\text{pyk}} \text{"lemma nonnegativeFactors"}]$
 $[\text{NonzeroFactors} \xrightarrow{\text{pyk}} \text{"lemma nonzeroFactors"}]$
 $[\text{PositiveFactors} \xrightarrow{\text{pyk}} \text{"lemma positiveFactors"}]$
 $[\text{PlusTimesMinus} \xrightarrow{\text{pyk}} \text{"lemma plusTimesMinus"}]$
 $[\text{MinusTimesMinus} \xrightarrow{\text{pyk}} \text{"lemma minusTimesMinus"}]$
 $[(-1) * (-1) + (-1) * 1 = 0 \xrightarrow{\text{pyk}} \text{"lemma } (-1)*(-1)+(-1)*1=0"]$
 $[(-1) * (-1) = 1 \xrightarrow{\text{pyk}} \text{"lemma } (-1)*(-1)=1"]$
 $[0 < 1 \text{Helper} \xrightarrow{\text{pyk}} \text{"lemma } 0<1 \text{Helper}"]$
 $[0 < 1 \xrightarrow{\text{pyk}} \text{"lemma } 0<1"]]$

$[0 < 2 \xrightarrow{\text{pyk}} \text{"lemma } 0 < 2\text{"}]$
 $[0 < 3 \xrightarrow{\text{pyk}} \text{"lemma } 0 < 3\text{"}]$
 $[0 < 1/2 \xrightarrow{\text{pyk}} \text{"lemma } 0 < 1/2\text{"}]$
 $[0 < 1/3 \xrightarrow{\text{pyk}} \text{"lemma } 0 < 1/3\text{"}]$
 $[\text{TwoWholes} \xrightarrow{\text{pyk}} \text{"lemma } x+x=2*x\text{"}]$
 $[\text{ThreeWholes} \xrightarrow{\text{pyk}} \text{"lemma } x+x+x=3*x\text{"}]$
 $[\text{TwoHalves} \xrightarrow{\text{pyk}} \text{"lemma } (1/2)x+(1/2)x=x\text{"}]$
 $[\text{ThreeThirds} \xrightarrow{\text{pyk}} \text{"lemma } (1/3)x+(1/3)x+(1/3)x=x\text{"}]$
 $[-x - y = -(x + y) \xrightarrow{\text{pyk}} \text{"lemma } -x-y=-(x+y)\text{"}]$
 $[-x * y = -(x * y) \xrightarrow{\text{pyk}} \text{"lemma } -x*y=-(x*y)\text{"}]$
 $[-0 = 0 \xrightarrow{\text{pyk}} \text{"lemma } -0=0\text{"}]$
 $[\text{SFsymmetry} \xrightarrow{\text{pyk}} \text{"lemma sameFSymmetry"}]$
 $[\text{SFtransitivity} \xrightarrow{\text{pyk}} \text{"lemma sameFtransitivity"}]$
 $[\text{f2R(Plus)} \xrightarrow{\text{pyk}} \text{"lemma f2R(Plus)"}]$
 $[\text{f2R(Times)} \xrightarrow{\text{pyk}} \text{"lemma f2R(Times)"}]$
 $[<< \text{TransitivityHelper(Q)} \xrightarrow{\text{pyk}} \text{"lemma } <<\text{TransitivityHelper(Q)}\text{"}]$
 $[<< \text{Transitivity} \xrightarrow{\text{pyk}} \text{"lemma } <<\text{Transitivity}"]$
 $[<<== \text{Reflexivity} \xrightarrow{\text{pyk}} \text{"lemma } <<==\text{Reflexivity}"]$
 $[<<== \text{AntisymmetryHelper(Q)} \xrightarrow{\text{pyk}} \text{"lemma } <<==\text{AntisymmetryHelper(Q)}"]$
 $[\text{FromNot } < \text{f(Weak)}(\text{Helper}) \xrightarrow{\text{pyk}} \text{"lemma fromNot}<\text{f(Weak) helper}"]$
 $[\text{FromNot } < \text{f(Weak)} \xrightarrow{\text{pyk}} \text{"lemma fromNot}<\text{f(Weak)}"]$
 $[\text{FromNot } < \text{f(Strong)}(\text{Helper2}) \xrightarrow{\text{pyk}} \text{"lemma fromNot}<\text{f(Strong) helper2}"]$
 $[\text{FromNot } < \text{f(Strong)}(\text{Helper}) \xrightarrow{\text{pyk}} \text{"lemma fromNot}<\text{f(Strong) helper}"]$
 $[\text{FromNot } < \text{f(Strong)} \xrightarrow{\text{pyk}} \text{"lemma fromNot}<\text{f(Strong)}"]$
 $[\text{fromNotSameF(Strongest)}(\text{Helper2}) \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Strongest) helper2"}]$
 $[\text{fromNotSameF(Strongest)}(\text{Helper}) \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Strongest) helper"}]$
 $[\text{fromNotSameF(Strongest)} \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Strongest)}"]$
 $[\text{ToLess(F)}(\text{Helper}) \xrightarrow{\text{pyk}} \text{"lemma toLess(F) helper"}]$
 $[\text{ToLess(F)} \xrightarrow{\text{pyk}} \text{"lemma toLess(F)}"]$
 $[\text{FromNot } << \xrightarrow{\text{pyk}} \text{"lemma fromNot}<<"}]$
 $[\text{ToLess(R)} \xrightarrow{\text{pyk}} \text{"lemma toLess(R)}"]$
 $[\text{LeqTotality(R)} \xrightarrow{\text{pyk}} \text{"lemma leqTotality(R)}"]$
 $[\text{FromNotSameF(Weak)}(\text{Helper}) \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Weak)(Helper)}"]$

$[\text{FromNotSameF(Weak)} \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Weak)"}]$
 $[\text{FromNotLess(F)} \xrightarrow{\text{pyk}} \text{"lemma fromNotLess(F)"}]$
 $[== \text{Addition} \xrightarrow{\text{pyk}} \text{"lemma ==Addition"}]$
 $[== \text{AdditionLeft} \xrightarrow{\text{pyk}} \text{"lemma ==AdditionLeft"}]$
 $[\text{Fpart} - \text{Bounded(Base)} \xrightarrow{\text{pyk}} \text{"lemma fpart-Bounded base"}]$
 $[\text{Fpart} - \text{Bounded(InduHelper)} \xrightarrow{\text{pyk}} \text{"lemma fpart-Bounded indu helper"}]$
 $[\text{Fpart} - \text{Bounded(Indu)} \xrightarrow{\text{pyk}} \text{"lemma fpart-Bounded indu"}]$
 $[\text{Fpart} - \text{Bounded} \xrightarrow{\text{pyk}} \text{"lemma fpart-Bounded"}]$
 $[\text{F} - \text{Bounded(Helper)} \xrightarrow{\text{pyk}} \text{"lemma f-Bounded helper"}]$
 $[\text{F} - \text{Bounded} \xrightarrow{\text{pyk}} \text{"lemma f-Bounded"}]$
 $[\text{SameFmultiplication(Helper)} \xrightarrow{\text{pyk}} \text{"lemma sameFmultiplication helper"}]$
 $[\text{SameFmultiplication} \xrightarrow{\text{pyk}} \text{"lemma sameFmultiplication"}]$
 $[\text{EqMultiplication(R)} \xrightarrow{\text{pyk}} \text{"lemma eqMultiplication(R)"}]$
 $[\text{EqMultiplicationLeft(R)} \xrightarrow{\text{pyk}} \text{"lemma eqMultiplicationLeft(R)"}]$
 $[\text{x} * 0 = 0(\text{F}) \xrightarrow{\text{pyk}} \text{"lemma x*0=0(F)"}]$
 $[\text{x} * 0 = 0(\text{R}) \xrightarrow{\text{pyk}} \text{"lemma x*0=0(R)"}]$
 $[\text{LessMultiplication(F)(Helper2)} \xrightarrow{\text{pyk}} \text{"lemma lessMultiplication(F) helper2"}]$
 $[\text{LessMultiplication(F)(Helper)} \xrightarrow{\text{pyk}} \text{"lemma lessMultiplication(F) helper"}]$
 $[\text{LessMultiplication(F)} \xrightarrow{\text{pyk}} \text{"lemma lessMultiplication(F)"}]$
 $[\text{LessMultiplication(R)} \xrightarrow{\text{pyk}} \text{"lemma lessMultiplication(R)"}]$
 $[\text{LeqMultiplication(R)} \xrightarrow{\text{pyk}} \text{"lemma leqMultiplication(R)"}]$
 $[\text{PlusAssociativity(F)} \xrightarrow{\text{pyk}} \text{"lemma plusAssociativity(F)"}]$
 $[\text{Plus0(F)} \xrightarrow{\text{pyk}} \text{"lemma plus0(F)"}]$
 $[\text{PlusCommutativity(F)} \xrightarrow{\text{pyk}} \text{"lemma plusCommutativity(F)"}]$
 $[\text{TimesAssociativity(F)} \xrightarrow{\text{pyk}} \text{"lemma timesAssociativity(F)"}]$
 $[\text{TimesIf} \xrightarrow{\text{pyk}} \text{"lemma timesIf"}]$
 $[\text{Cauchy(2)(Helper)} \xrightarrow{\text{pyk}} \text{"lemma 2cauchy helper"}]$
 $[\text{Cauchy(2)} \xrightarrow{\text{pyk}} \text{"lemma 2cauchy"}]$
 $[\text{ReciprocalFnonzero} \xrightarrow{\text{pyk}} \text{"lemma reciprocalF nonzero"}]$
 $[\text{ReciprocalFnynonzero} \xrightarrow{\text{pyk}} \text{"lemma reciprocalFnynonzero"}]$
 $[(\text{Eventually} = \text{f})2\text{sameF(Helper)} \xrightarrow{\text{pyk}} \text{"lemma eventually=f to sameF helper"}]$
 $[(\text{Eventually} = \text{f})2\text{sameF} \xrightarrow{\text{pyk}} \text{"lemma eventually=f to sameF"}]$
 $[\text{FromNotSameF(Strong)(Helper2)} \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Strong) helper2"}]$
 $[\text{FromNotSameF(Strong)(Helper)} \xrightarrow{\text{pyk}} \text{"lemma fromNotSameF(Strong) helper"}]$

[FromNotSameF(Strong) $\xrightarrow{\text{pyk}}$ “lemma fromNotSameF(Strong)”]
 [SameFreciprocal(Helper) $\xrightarrow{\text{pyk}}$ “lemma sameFreciprocal helper”]
 [SameFreciprocal $\xrightarrow{\text{pyk}}$ “lemma sameFreciprocal”]
 [From!! == $\xrightarrow{\text{pyk}}$ “lemma from!!==”]
 [Reciprocal(R) $\xrightarrow{\text{pyk}}$ “lemma reciprocal(R)”]
 [TimesCommutativity(F) $\xrightarrow{\text{pyk}}$ “lemma timesCommutativity(F)”]
 [Distribution(F) $\xrightarrow{\text{pyk}}$ “lemma distribution(F)”]
 [FromMax(1) $\xrightarrow{\text{pyk}}$ “lemma fromMax(1)”]
 [FromMax(2) $\xrightarrow{\text{pyk}}$ “lemma fromMax(2)”]
 [ToNegatedAnd $\xrightarrow{\text{pyk}}$ “prop lemma to negated and”]
 [PositiveToRight(Less)(1term) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Less)(1 term)”]
 [(A)to(E) $\xrightarrow{\text{pyk}}$ “pred lemma (A~)to(~E)”]
 [lemma ==Transitivity4 $\xrightarrow{\text{pyk}}$ “lemma ==Transitivity4”]
 [Plus0Left(R) $\xrightarrow{\text{pyk}}$ “lemma plus0Left(R)”]
 [x = x + (y - y)(R) $\xrightarrow{\text{pyk}}$ “lemma x=x+(y-y)(R)”]
 [x = x + y - y(R) $\xrightarrow{\text{pyk}}$ “lemma x=x+y-y(R)”]
 [PositiveToRight(Eq)(R) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Eq)(R)”]
 [SubtractEquations(R) $\xrightarrow{\text{pyk}}$ “lemma subtractEquations(R)”]
 [NeqAddition(R) $\xrightarrow{\text{pyk}}$ “lemma neqAddition(R)”]
 [EqAdditionLeft(R) $\xrightarrow{\text{pyk}}$ “lemma eqAdditionLeft(R)”]
 [Three2twoTerms(R) $\xrightarrow{\text{pyk}}$ “lemma three2twoTerms(R)”]
 [PositiveToRight(Less)(R) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Less)(R)”]
 [Three2threeTerms(R) $\xrightarrow{\text{pyk}}$ “lemma three2threeTerms(R)”]
 [PositiveToRight(Less)(1term)(R) $\xrightarrow{\text{pyk}}$ “lemma positiveToRight(Less)(1 term)(R)”]
 [ToLeq(Advanced)(R) $\xrightarrow{\text{pyk}}$ “lemma toLeq(Advanced)(R)”]
 [LeqNeqLess(R) $\xrightarrow{\text{pyk}}$ “lemma leqNeqLess(R)”]
 [SubLeqLeft(R) $\xrightarrow{\text{pyk}}$ “lemma subLeqLeft(R)”]
 [LeqLessTransitivity(R) $\xrightarrow{\text{pyk}}$ “lemma leqLessTransitivity(R)”]
 [NegativeToLeft(Eq)(R) $\xrightarrow{\text{pyk}}$ “lemma negativeToLeft(Eq)(R)”]
 [NegativeToRight(Less)(R) $\xrightarrow{\text{pyk}}$ “lemma negativeToRight(Less)(R)”]
 [!! == Symmetry $\xrightarrow{\text{pyk}}$ “lemma !!==Symmetry”]
 [NegativeToRight(Eq)(R) $\xrightarrow{\text{pyk}}$ “lemma negativeToRight(Eq)(R)”]
 [NegativeToRight(Eq)(1term)(R) $\xrightarrow{\text{pyk}}$ “lemma negativeToRight(Eq)(1 term)(R)”]
 [DoubleMinus(R) $\xrightarrow{\text{pyk}}$ “lemma doubleMinus(R)”]

$[\text{UniqueNegative}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma uniqueNegative}(\mathbb{R})"]$
 $[\text{SubtractEquationsLeft}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma subtractEquationsLeft}(\mathbb{R})"]$
 $[\text{EqNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma eqNegated}(\mathbb{R})"]$
 $[\text{NeqNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma neqNegated}(\mathbb{R})"]$
 $[\text{LeqNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma leqNegated}(\mathbb{R})"]$
 $[\text{LessNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma lessNegated}(\mathbb{R})"]$
 $[-0 = 0(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma -0=0}(\mathbb{R})"]$
 $[\text{NegativeNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma negativeNegated}(\mathbb{R})"]$
 $[\text{FromLeqGeq}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma from leqGeq}(\mathbb{R})"]$
 $[\text{SubLeqRight}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma subLeqRight}(\mathbb{R})"]$
 $[\text{FromLess}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma fromLess}(\mathbb{R})"]$
 $[\text{NonnegativeNumerical}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma nonnegativeNumerical}(\mathbb{R})"]$
 $[\text{NegativeNumerical}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma negativeNumerical}(\mathbb{R})"]$
 $[0 <= |x|(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma 0<=|x|}(\mathbb{R})"]$
 $[\text{PositiveNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma positiveNegated}(\mathbb{R})"]$
 $[\text{AddEquations}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma addEquations}(\mathbb{R})"]$
 $[\text{DistributionOut}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma distributionOut}(\mathbb{R})"]$
 $[== \text{Transitivity5} \xrightarrow{\text{pyk}} \text{"lemma ==Transitivity5"}]$
 $[x * 0 + x = x(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma x*0+x=x}(\mathbb{R})"]$
 $[x * 0 = 0(\mathbb{R})(\text{fff}) \xrightarrow{\text{pyk}} \text{"lemma x*0=0}(\mathbb{R})\text{fff}"]$
 $[\text{Times}(-1)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma times(-1)}(\mathbb{R})"]$
 $[\text{Times}(-1)\text{Left}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma times(-1)Left}(\mathbb{R})"]$
 $[-x - y = -(x + y)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma -x-y=-(x+y)}(\mathbb{R})"]$
 $[\text{LessTotality}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma lessTotality}(\mathbb{R})"]$
 $[\text{PositiveNumerical}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma positiveNumerical}(\mathbb{R})"]$
 $[\text{SignNumerical}(+)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma signNumerical}(+)(\mathbb{R})"]$
 $[\text{SameNumerical}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma sameNumerical}(\mathbb{R})"]$
 $[\text{MinusNegated}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma minusNegated}(\mathbb{R})"]$
 $[\text{SignNumerical}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma signNumerical}(\mathbb{R})"]$
 $[\text{NumericalDifference}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma numericalDifference}(\mathbb{R})"]$
 $[x <= |x|(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma x<=|x|}(\mathbb{R})"]$
 $[\text{USlimitIsUpperBound}(\text{Helper}) \xrightarrow{\text{pyk}} \text{"lemma USlimitIsUpperBound helper"}]$
 $[\text{USlimitIsUpperBound} \xrightarrow{\text{pyk}} \text{"lemma USlimitIsUpperBound"}]$
 $[(-1) * (-1) + (-1) * 1 = 0(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma (-1)*(-1)+(-1)*1=0}(\mathbb{R})"]$
 $[(-1) * (-1) = 1(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma (-1)*(-1)=1}(\mathbb{R})"]$

$[0 < 1\text{Helper}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } 0 < 1\text{Helper}(\mathbb{R})\text{"}]$
 $[0 < 1(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } 0 < 1(\mathbb{R})\text{"}]$
 $[\text{ExpZero}(\text{Exact})(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma expZero exact}(\mathbb{R})\text{"}]$
 $[\text{PositiveBase}(\mathbb{R})(\text{Base}) \xrightarrow{\text{pyk}} \text{"lemma positiveBase}(\mathbb{R}) \text{ base"}]$
 $[\text{Three2twoFactors}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma three2twoFactors}(\mathbb{R})\text{"}]$
 $[x = x * y * (1/y)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } x=x*y*(1/y)(\mathbb{R})\text{"}]$
 $[\text{NeqMultiplication}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma neqMultiplication}(\mathbb{R})\text{"}]$
 $[\text{LessTransitivity}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma lessTransitivity}(\mathbb{R})\text{"}]$
 $[0 < 2(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } 0 < 2(\mathbb{R})\text{"}]$
 $[\text{SameExp}(\mathbb{R})(\text{Base}) \xrightarrow{\text{pyk}} \text{"lemma sameExp}(\mathbb{R}) \text{ base"}]$
 $[\text{SameExp}(\mathbb{R})(\text{Indu}) \xrightarrow{\text{pyk}} \text{"lemma sameExp}(\mathbb{R}) \text{ indu"}]$
 $[\text{SameExp}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma sameExp}(\mathbb{R})\text{"}]$
 $[\text{SubNeqLeft}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma subNeqLeft}(\mathbb{R})\text{"}]$
 $[\text{SubNeqRight}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma subNeqRight}(\mathbb{R})\text{"}]$
 $[\text{NonzeroFactors}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma nonzeroFactors}(\mathbb{R})\text{"}]$
 $[\text{NonnegativeFactors}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma nonnegativeFactors}(\mathbb{R})\text{"}]$
 $[\text{PositiveFactors}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma positiveFactors}(\mathbb{R})\text{"}]$
 $[\text{LessDivision}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma lessDivision}(\mathbb{R})\text{"}]$
 $[0 < 1/2(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } 0 < 1/2(\mathbb{R})\text{"}]$
 $[\text{PositiveToRight}(\text{Eq})(1\text{term})(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma positiveToRight}(\text{Eq})(1 \text{ term})(\mathbb{R})\text{"}]$
 $[\text{Exp}(+1)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma exp}(+1)(\mathbb{R})\text{"}]$
 $[\text{PositiveBase}(\mathbb{R})(\text{Indu}) \xrightarrow{\text{pyk}} \text{"lemma positiveBase}(\mathbb{R}) \text{ indu"}]$
 $[\text{PositiveBase}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma positiveBase}(\mathbb{R})\text{"}]$
 $[-x * y = -(x * y)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } -x*y=-(x*y)(\mathbb{R})\text{"}]$
 $[\text{PositiveToLeft}(\text{Eq})(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma positiveToLeft}(\text{Eq})(\mathbb{R})\text{"}]$
 $[\text{Times1Left}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma times1Left}(\mathbb{R})\text{"}]$
 $[== \text{Transitivity6} \xrightarrow{\text{pyk}} \text{"lemma ==Transitivity6"}]$
 $[x + x = 2 * x(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } x+x=2*x(\mathbb{R})\text{"}]$
 $[(1/2)x + (1/2)x = x(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } (1/2)x+(1/2)x=x(\mathbb{R})\text{"}]$
 $[\text{DistributionOut}(\text{Minus})(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma distributionOut}(\text{Minus})(\mathbb{R})\text{"}]$
 $[(1/2)(x + y) - x = (1/2)(y - x)(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma } (1/2)(x+y)-x=(1/2)(y-x)(\mathbb{R})\text{"}]$
 $[\text{IntervalSize}(\mathbb{R})(\text{Base}) \xrightarrow{\text{pyk}} \text{"lemma intervalSize}(\mathbb{R}) \text{ base"}]$
 $[\text{LessMultiplicationLeft}(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma lessMultiplicationLeft}(\mathbb{R})\text{"}]$
 $[\text{NegativeToLeft}(\text{Less})(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma negativeToLeft}(\text{Less})(\mathbb{R})\text{"}]$
 $[\text{NegativeToLeft}(\text{Less})(1\text{term})(\mathbb{R}) \xrightarrow{\text{pyk}} \text{"lemma negativeToLeft}(\text{Less})(1$

term)(R)"]

[$y - (1/2)(x + y) = (1/2)(y - x)(R) \xrightarrow{\text{pyk}}$ "lemma $y - (1/2)(x + y) = (1/2)(y - x)(R)$ "]

[IntervalSize(R)(Indu) $\xrightarrow{\text{pyk}}$ "lemma intervalSize(R) indu"]

[IntervalSize(R) $\xrightarrow{\text{pyk}}$ "lemma intervalSize(R)"]

[XSlessUS(R) $\xrightarrow{\text{pyk}}$ "lemma XSlessUS(R)"]

[USdecreasing(+1)(R) $\xrightarrow{\text{pyk}}$ "lemma USdecreasing(+1)(R)"]

[ExpUnbounded(Base) $\xrightarrow{\text{pyk}}$ "lemma expUnbounded base"]

[ExpUnbounded(Indu) $\xrightarrow{\text{pyk}}$ "lemma expUnbounded indu"]

[ExpUnbounded $\xrightarrow{\text{pyk}}$ "lemma expUnbounded"]

[$1 \leq x + 1(N) \xrightarrow{\text{pyk}}$ "lemma $1 \leq x + 1(N)$ "]

[NonzeroProduct(2)(R) $\xrightarrow{\text{pyk}}$ "lemma nonzeroProduct(2)(R)"]

[PositiveNonzero(R) $\xrightarrow{\text{pyk}}$ "lemma positiveNonzero(R)"]

[NonreciprocalToRight(Eq)(1term)(R) $\xrightarrow{\text{pyk}}$ "lemma
nonreciprocalToRight(Eq)(1 term)(R)"]

[ExpNonzero(Base) $\xrightarrow{\text{pyk}}$ "lemma expNonzero base"]

[ExpNonzero(Indu) $\xrightarrow{\text{pyk}}$ "lemma expNonzero indu"]

[ExpNonzero $\xrightarrow{\text{pyk}}$ "lemma expNonzero"]

[ExpNonzero(2) $\xrightarrow{\text{pyk}}$ "lemma expNonzero(2)"]

[MultiplyEquations(R) $\xrightarrow{\text{pyk}}$ "lemma multiplyEquations(R)"]

[HalfBase(Base) $\xrightarrow{\text{pyk}}$ "lemma halfBase base"]

[HalfBase(Indu) $\xrightarrow{\text{pyk}}$ "lemma halfBase indu"]

[HalfBase $\xrightarrow{\text{pyk}}$ "lemma halfBase"]

[Three2threeFactors(R) $\xrightarrow{\text{pyk}}$ "lemma three2threeFactors(R)"]

[$x * y = z$ Backwards(R) $\xrightarrow{\text{pyk}}$ "lemma $x * y = z$ Backwards(R)"]

[PositiveInverted(R) $\xrightarrow{\text{pyk}}$ "lemma positiveInverted(R)"]

[ReciprocalToRight(Less)(R) $\xrightarrow{\text{pyk}}$ "lemma reciprocalToRight(Less)(R)"]

[ReciprocalToRight(Less)(1term)(R) $\xrightarrow{\text{pyk}}$ "lemma reciprocalToRight(Less)(1
term)(R)"]

[NonreciprocalToLeft(Less)(R) $\xrightarrow{\text{pyk}}$ "lemma nonreciprocalToLeft(Less)(R)"]

[$1 < x * y(R) \xrightarrow{\text{pyk}}$ "lemma $1 < x * y(R)$ "]

[SwitchFactors($1/x < y$)(R) $\xrightarrow{\text{pyk}}$ "lemma switchFactors($1/x < y$)(R)"]

[SmallHalving $\xrightarrow{\text{pyk}}$ "lemma smallHalving"]

[IntervalSize(anyPositive) $\xrightarrow{\text{pyk}}$ "lemma intervalSize(anyPositive)"]

[USdecreasing(+n)(Base) $\xrightarrow{\text{pyk}}$ "lemma USdecreasing(+n) base"]

[USdecreasing(+n)(Indu) $\xrightarrow{\text{pyk}}$ "lemma USdecreasing(+n) indu"]

[USdecreasing(+n) $\xrightarrow{\text{pyk}}$ "lemma USdecreasing(+n)"]

[USdecreasing $\xrightarrow{\text{pyk}}$ “lemma USdecreasing”]
 [LeqAdditionLeft(R) $\xrightarrow{\text{pyk}}$ “lemma leqAdditionLeft(R)”]
 [ToNotLess(R) $\xrightarrow{\text{pyk}}$ “lemma toNotLess(R)”]
 [LimitOfUSIsLeq $\xrightarrow{\text{pyk}}$ “lemma limitOfUSIsLeq”]
 [SubtractEquations(Less)(R) $\xrightarrow{\text{pyk}}$ “lemma subtractEquations(Less)(R)”]
 [SubtractEquationsLeft(Less)(R) $\xrightarrow{\text{pyk}}$ “lemma
 subtractEquationsLeft(Less)(R)”]
 [LessNegated(Negative)(R) $\xrightarrow{\text{pyk}}$ “lemma lessNegated(Negative)(R)”]
 [FromNegatedAnd(ImPLY) $\xrightarrow{\text{pyk}}$ “prop lemma from negated and (imply)”]
 [RemoveDoubleNeg(Consequent) $\xrightarrow{\text{pyk}}$ “prop lemma remove double neg
 (consequent)”]
 [FromNotUpperBound $\xrightarrow{\text{pyk}}$ “lemma fromNotUpperBound”]
 [DistributionOut $\xrightarrow{\text{pyk}}$ “lemma distributionOut”]
 [DistributionOutLeft $\xrightarrow{\text{pyk}}$ “lemma distributionOutLeft”]
 [DistributionLeft $\xrightarrow{\text{pyk}}$ “lemma distributionLeft”]
 [LeqNUB $\xrightarrow{\text{pyk}}$ “lemma leqNUB”]
 [USlimitIsLeastUpperBound(Helper) $\xrightarrow{\text{pyk}}$ “lemma USlimitIsLeastUpperBound
 helper”]
 [USlimitIsLeastUpperBound $\xrightarrow{\text{pyk}}$ “lemma USlimitIsLeastUpperBound”]
 [FromNotLess(R) $\xrightarrow{\text{pyk}}$ “lemma fromNotLess(R)”]
 [ExistMP3 $\xrightarrow{\text{pyk}}$ “pred lemma exist mp3”]
 [GreaterPositive(N) $\xrightarrow{\text{pyk}}$ “lemma greaterPositive(N)”]
 [ysFClose(Helper) $\xrightarrow{\text{pyk}}$ “lemma ysFClose helper”]
 [ysFClose $\xrightarrow{\text{pyk}}$ “lemma ysFClose”]
 [ysFCAuchy(Helper) $\xrightarrow{\text{pyk}}$ “lemma ysFCAuchy helper”]
 [ysFCAuchy $\xrightarrow{\text{pyk}}$ “lemma ysFCAuchy”]
 [CartProdIsRelation $\xrightarrow{\text{pyk}}$ “lemma cartProdIsRelation”]
 [FromSubset $\xrightarrow{\text{pyk}}$ “lemma fromSubset”]
 [SubsetIsRelation $\xrightarrow{\text{pyk}}$ “lemma subsetIsRelation”]
 [ToSeries $\xrightarrow{\text{pyk}}$ “lemma toSeries”]
 [FromSeries $\xrightarrow{\text{pyk}}$ “lemma fromSeries”]
 [SeriesSubsetCP $\xrightarrow{\text{pyk}}$ “lemma seriesSubsetCP”]
 [ValueType $\xrightarrow{\text{pyk}}$ “lemma valueType”]
 [RemoveOr $\xrightarrow{\text{pyk}}$ “prop lemma remove or”]
 [FromSingleton $\xrightarrow{\text{pyk}}$ “lemma fromSingleton”]
 [InPair(1) $\xrightarrow{\text{pyk}}$ “lemma inPair(1)”]

[InPair(2) $\xrightarrow{\text{pyk}}$ “lemma inPair(2)”]
 [SameMember(2) $\xrightarrow{\text{pyk}}$ “lemma sameMember(2)”]
 [ToBinaryUnion(1) $\xrightarrow{\text{pyk}}$ “lemma toBinaryUnion(1)”]
 [ToBinaryUnion(2) $\xrightarrow{\text{pyk}}$ “lemma toBinaryUnion(2)”]
 [FromOrderedPair(TwoLevels) $\xrightarrow{\text{pyk}}$ “lemma fromOrderedPair(twoLevels)”]
 [ToCartProd(Helper) $\xrightarrow{\text{pyk}}$ “lemma toCartProd helper”]
 [ToCartProd $\xrightarrow{\text{pyk}}$ “lemma toCartProd”]
 [NonreciprocalToRight(Eq) $\xrightarrow{\text{pyk}}$ “lemma nonreciprocalToRight(Eq)”]
 [NonreciprocalToLeft(Eq)(1term) $\xrightarrow{\text{pyk}}$ “lemma nonreciprocalToLeft(Eq)(1 term)”]
 [SameReciprocal $\xrightarrow{\text{pyk}}$ “lemma sameReciprocal”]
 [CPseparationIsRelation $\xrightarrow{\text{pyk}}$ “lemma CPseparationIsRelation”]
 [OrderedPairEquality $\xrightarrow{\text{pyk}}$ “lemma orderedPairEquality”]
 [ReciprocalIsFunction $\xrightarrow{\text{pyk}}$ “lemma reciprocalIsFunction”]
 [ReciprocalIsTotal $\xrightarrow{\text{pyk}}$ “lemma reciprocalIsTotal”]
 [ReciprocalIsRationalSeries $\xrightarrow{\text{pyk}}$ “lemma reciprocalIsRationalSeries”]
 [CrsIsRelation $\xrightarrow{\text{pyk}}$ “lemma crsIsRelation”]
 [CrsIsFunction $\xrightarrow{\text{pyk}}$ “lemma crsIsFunction”]
 [CrsIsTotal $\xrightarrow{\text{pyk}}$ “lemma crsIsTotal”]
 [CrsIsSeries $\xrightarrow{\text{pyk}}$ “lemma crsIsSeries”]
 [CrsLookup $\xrightarrow{\text{pyk}}$ “lemma crsLookup”]
 [0f $\xrightarrow{\text{pyk}}$ “lemma 0f”]
 [1f $\xrightarrow{\text{pyk}}$ “lemma 1f”]
 [ToSingleton $\xrightarrow{\text{pyk}}$ “lemma toSingleton”]
 [FromSameSingleton $\xrightarrow{\text{pyk}}$ “lemma fromSameSingleton”]
 [SingletonmembersEqual $\xrightarrow{\text{pyk}}$ “lemma singletonmembersEqual”]
 [UnequalsNotInSingleton $\xrightarrow{\text{pyk}}$ “lemma unequalsNotInSingleton”]
 [NonsingletonmembersUnequal $\xrightarrow{\text{pyk}}$ “lemma nonsingletonmembersUnequal”]
 [FromOrderedPair $\xrightarrow{\text{pyk}}$ “lemma fromOrderedPair”]
 [FromOrderedPair(1) $\xrightarrow{\text{pyk}}$ “lemma fromOrderedPair(1)”]
 [FromOrderedPair(2) $\xrightarrow{\text{pyk}}$ “lemma fromOrderedPair(2)”]
 [FromCartProd $\xrightarrow{\text{pyk}}$ “lemma fromCartProd”]
 [FromCartProd(1) $\xrightarrow{\text{pyk}}$ “lemma fromCartProd(1)”]
 [FromCartProd(2) $\xrightarrow{\text{pyk}}$ “lemma fromCartProd(2)”]
 [sameOrderedPair $\xrightarrow{\text{pyk}}$ “lemma sameOrderedPair”]

[InSeriesHelper $\xrightarrow{\text{pyk}}$ "lemma inSeries helper"]
 [InSeries $\xrightarrow{\text{pyk}}$ "lemma inSeries"]
 [To = f(Subset)(Helper) $\xrightarrow{\text{pyk}}$ "lemma to=f subset helper"]
 [To = f(Subset) $\xrightarrow{\text{pyk}}$ "lemma to=f subset"]
 [To = f $\xrightarrow{\text{pyk}}$ "lemma to=f"]
 [productIsFunction $\xrightarrow{\text{pyk}}$ "lemma productIsFunction"]
 [productIsTotal $\xrightarrow{\text{pyk}}$ "lemma productIsTotal"]
 [ProductIsRationalSeries $\xrightarrow{\text{pyk}}$ "lemma productIsRationalSeries"]
 [TimesF $\xrightarrow{\text{pyk}}$ "lemma timesF"]
 [-x + (1/2)x = -(1/2)x $\xrightarrow{\text{pyk}}$ "lemma -x+(1/2)x=-(1/2)x"]
 [PositiveTripled $\xrightarrow{\text{pyk}}$ "lemma positiveTripled"]
 [PositiveDividedBy3 $\xrightarrow{\text{pyk}}$ "lemma positiveDividedBy3"]
 [|x - x| = 0 $\xrightarrow{\text{pyk}}$ "lemma |x-x|=0"]
 [1 < 2 $\xrightarrow{\text{pyk}}$ "lemma 1<2"]
 [1/3 < 2/3 $\xrightarrow{\text{pyk}}$ "lemma 1/3<2/3"]
 [(1/3)x + (1/3)x = (2/3)x $\xrightarrow{\text{pyk}}$ "lemma (1/3)x+(1/3)x=(2/3)x"]
 [(2/3)x + (1/3)x = x $\xrightarrow{\text{pyk}}$ "lemma (2/3)x+(1/3)x=x"]
 [-x + (2/3)x = -(1/3)x $\xrightarrow{\text{pyk}}$ "lemma -x+(2/3)x=-(1/3)x"]
 [-(1/3)x - (1/3)x = -(2/3)x $\xrightarrow{\text{pyk}}$ "lemma -(1/3)x-(1/3)x=-(2/3)x"]
 [-x + (1/3)x = -(2/3)x $\xrightarrow{\text{pyk}}$ "lemma -x+(1/3)x=-(2/3)x"]
 [PreserveLessGreater $\xrightarrow{\text{pyk}}$ "lemma preserveLessGreater"]
 [ClosetolessIsLess $\xrightarrow{\text{pyk}}$ "lemma closetolessIsLess"]
 [SubLessLeft(F) $\xrightarrow{\text{pyk}}$ "lemma subLessLeft(F)"]
 [SubLessLeft(R) $\xrightarrow{\text{pyk}}$ "lemma subLessLeft(R)"]
 [ClosetogreaterIsGreater $\xrightarrow{\text{pyk}}$ "lemma closetogreaterIsGreater"]
 [SubLessRight(F) $\xrightarrow{\text{pyk}}$ "lemma subLessRight(F)"]
 [SubLessRight(R) $\xrightarrow{\text{pyk}}$ "lemma subLessRight(R)"]
 [Tester1 $\xrightarrow{\text{pyk}}$ "tester1"]
 [Tester2 $\xrightarrow{\text{pyk}}$ "tester2"]
 [Tester3 $\xrightarrow{\text{pyk}}$ "tester3"]
 [Tester4 $\xrightarrow{\text{pyk}}$ "tester4"]
 [Tester5 $\xrightarrow{\text{pyk}}$ "tester5"]
 [Tester6 $\xrightarrow{\text{pyk}}$ "tester6"]
 [sup $\xrightarrow{\text{pyk}}$ "sup"]

[sup $\xrightarrow{\text{tex}}$ “sup”]

[x(exp)y $\xrightarrow{\text{tex}}$ “(#1.
(expARGH!) #2.
)”]

[To!! == $\xrightarrow{\text{tex}}$ “To!!==”]

[ToNegatedDoubleImPLY $\xrightarrow{\text{tex}}$ “ToNegatedDoubleImPLY”]

[AddNegatedAll $\xrightarrow{\text{tex}}$ “AddNegatedAll”]

[(A)to(E)(ImPLY) $\xrightarrow{\text{tex}}$ “(A)to(\sim E \sim)(ImPLY)”]

[(E)to(A)(ImPLY) $\xrightarrow{\text{tex}}$ “(E)to(\sim A \sim)(ImPLY)”]

[(E)to(A)(ImPLY) $\xrightarrow{\text{tex}}$ “(E \sim)to(\sim A)(ImPLY)”]

[ToNegatedAEA $\xrightarrow{\text{tex}}$ “ToNegatedAEA ”]

[NonreciprocalToRight(Eq)(1term) $\xrightarrow{\text{tex}}$ “NonreciprocalToRight(Eq)(1 term)”]

[DistributionOut(Minus) $\xrightarrow{\text{tex}}$ “DistributionOut(Minus)”]

[PositiveToRight(Eq)(1term) $\xrightarrow{\text{tex}}$ “PositiveToRight(Eq)(1 term)”]

[SameSeries(NumDiff) $\xrightarrow{\text{tex}}$ “SameSeries(NumDiff)”]

[PlusAssociativity(4terms) $\xrightarrow{\text{tex}}$ “PlusAssociativity(4 terms)”]

[NonzeroProduct(2) $\xrightarrow{\text{tex}}$ “NonzeroProduct(2)”]

[SwitchTerms(x <= y - z) $\xrightarrow{\text{tex}}$ “SwitchTerms(x<=y-z)”]

[lemma eqLeq(R) $\xrightarrow{\text{tex}}$ “eqLeq(R)”]

[ThirdGeqSeries $\xrightarrow{\text{tex}}$ “ThirdGeqSeries”]

[negativeToLeft(Leq) $\xrightarrow{\text{tex}}$ “negativeToLeft(Leq)”]

[negativeToLeft(Leq)(1term) $\xrightarrow{\text{tex}}$ “negativeToLeft(Leq)(1 term)”]

[NegativeToLeft(Less)(1term) $\xrightarrow{\text{tex}}$ “NegativeToLeft(Less)(1 term)”]

[(1/2)(x + y) - x = (1/2)(y - x) $\xrightarrow{\text{tex}}$ “(1/2)(x+y)-x=(1/2)(y-x)”]

[y - (1/2)(x + y) = (1/2)(y - x) $\xrightarrow{\text{tex}}$ “y-(1/2)(x+y)=(1/2)(y-x)”]

$[(+1)\text{IsPositive}(\mathbb{N}) \xrightarrow{\text{tex}} \text{"(+1)\text{IsPositive}(\mathbb{N})"}]$
 $[\text{ExpZero}(\text{Exact}) \xrightarrow{\text{tex}} \text{"ExpZero(Exact)"}]$
 $[\text{SameExp}(\text{Base}) \xrightarrow{\text{tex}} \text{"SameExp(Base)"}]$
 $[\text{SameExp}(\text{Indu}) \xrightarrow{\text{tex}} \text{"SameExp(Indu)"}]$
 $[\text{SameExp} \xrightarrow{\text{tex}} \text{"SameExp"}]$
 $[\text{Exp}(+1) \xrightarrow{\text{tex}} \text{"Exp(+1)"}]$
 $[\text{BSzero}(\text{Exact}) \xrightarrow{\text{tex}} \text{"BSzero(Exact)"}]$
 $[\text{SameBS}(2)(\text{Base}) \xrightarrow{\text{tex}} \text{"SameBS(2)(Base)"}]$
 $[\text{SameBS}(2)(\text{Indu}) \xrightarrow{\text{tex}} \text{"SameBS(2)(Indu)"}]$
 $[\text{SameBS}(2) \xrightarrow{\text{tex}} \text{"SameBS(2)"}]$
 $[\text{BS}(+1) \xrightarrow{\text{tex}} \text{"BS(+1)"}]$
 $[\text{BSbound}(\text{Exact})(\text{Base}) \xrightarrow{\text{tex}} \text{"BSbound(Exact)(Base)"}]$
 $[\text{BSbound}(\text{Exact})(\text{Indu}) \xrightarrow{\text{tex}} \text{"BSbound(Exact)(Indu)"}]$
 $[\text{BSbound}(\text{Exact}) \xrightarrow{\text{tex}} \text{"BSbound(Exact)"}]$
 $[\text{BSbound} \xrightarrow{\text{tex}} \text{"BSbound"}]$
 $[\text{UStelescope}(\text{Zero})(\text{Exact}) \xrightarrow{\text{tex}} \text{"UStelescope(Zero)(Exact)"}]$
 $[\text{SameTelescope}(2)(\text{Base}) \xrightarrow{\text{tex}} \text{"SameTelescope(2)(Base)"}]$
 $[\text{SameTelescope}(2)(\text{Indu}) \xrightarrow{\text{tex}} \text{"SameTelescope(2)(Indu)"}]$
 $[\text{SameTelescope}(2) \xrightarrow{\text{tex}} \text{"SameTelescope(2)"}]$
 $[\text{UStelescope}(+1) \xrightarrow{\text{tex}} \text{"UStelescope(+1)"}]$
 $[\text{TelescopeNumerical}(\text{Base}) \xrightarrow{\text{tex}} \text{"TelescopeNumerical(Base)"}]$
 $[\text{TelescopeNumerical}(\text{Indu}) \xrightarrow{\text{tex}} \text{"TelescopeNumerical(Indu)"}]$
 $[\text{TelescopeNumerical} \xrightarrow{\text{tex}} \text{"TelescopeNumerical"}]$
 $[\text{TelescopeBound}(\text{Base}) \xrightarrow{\text{tex}} \text{"TelescopeBound(Base)"}]$
 $[\text{TelescopeBound}(\text{Indu}) \xrightarrow{\text{tex}} \text{"TelescopeBound(Indu)"}]$

[TelescopeBound $\xrightarrow{\text{tex}}$ “TelescopeBound”]

[LessNeq(F)(Helper) $\xrightarrow{\text{tex}}$ “LessNeq(F)(Helper)”]

[LessNeq(F) $\xrightarrow{\text{tex}}$ “LessNeq(F)”]

[LessNeq(R) $\xrightarrow{\text{tex}}$ “LessNeq(R)”]

[IntervalSize(Base) $\xrightarrow{\text{tex}}$ “IntervalSize(Base)”]

[IntervalSize(Indu) $\xrightarrow{\text{tex}}$ “IntervalSize(Indu)”]

[IntervalSize $\xrightarrow{\text{tex}}$ “IntervalSize”]

[XS < US $\xrightarrow{\text{tex}}$ “XS<US”]

[PositiveBase(Base) $\xrightarrow{\text{tex}}$ “PositiveBase(Base)”]

[PositiveBase(Indu) $\xrightarrow{\text{tex}}$ “PositiveBase(Indu)”]

[PositiveBase $\xrightarrow{\text{tex}}$ “PositiveBase”]

[CloseUS $\xrightarrow{\text{tex}}$ “CloseUS”]

[CloseUS(n + 1) $\xrightarrow{\text{tex}}$ “CloseUS(n+1)”]

[Induction $\xrightarrow{\text{tex}}$ “Induction”]

[leqAntisymmetry $\xrightarrow{\text{tex}}$ “leqAntisymmetry”]

[leqTransitivity $\xrightarrow{\text{tex}}$ “leqTransitivity”]

[leqAddition $\xrightarrow{\text{tex}}$ “leqAddition”]

[Reciprocal $\xrightarrow{\text{tex}}$ “Reciprocal”]

[Equality $\xrightarrow{\text{tex}}$ “Equality”]

[eqLeq $\xrightarrow{\text{tex}}$ “eqLeq”]

[eqAddition $\xrightarrow{\text{tex}}$ “eqAddition”]

[eqMultiplication $\xrightarrow{\text{tex}}$ “eqMultiplication”]

[eqReflexivity $\xrightarrow{\text{tex}}$ “eqReflexivity”]

[eqSymmetry $\xrightarrow{\text{tex}}$ “eqSymmetry”]

[eqTransitivity $\xrightarrow{\text{tex}}$ “eqTransitivity”]

$[\text{eqTransitivity4} \xrightarrow{\text{tex}} \text{“eqTransitivity4”}]$
 $[\text{eqTransitivity5} \xrightarrow{\text{tex}} \text{“eqTransitivity5”}]$
 $[\text{eqTransitivity6} \xrightarrow{\text{tex}} \text{“eqTransitivity6”}]$
 $[\text{lemma plus0Left} \xrightarrow{\text{tex}} \text{“plus0Left”}]$
 $[\text{lemma times1Left} \xrightarrow{\text{tex}} \text{“times1Left”}]$
 $[\text{lemma eqMultiplicationLeft} \xrightarrow{\text{tex}} \text{“EqMultiplicationLeft”}]$
 $[\text{DistributionLeft} \xrightarrow{\text{tex}} \text{“DistributionLeft”}]$
 $[\text{DistributionOut} \xrightarrow{\text{tex}} \text{“DistributionOut”}]$
 $[\text{DistributionOutLeft} \xrightarrow{\text{tex}} \text{“DistributionOutLeft”}]$
 $[\text{Three2twoTerms} \xrightarrow{\text{tex}} \text{“Three2twoTerms”}]$
 $[\text{Three2threeTerms} \xrightarrow{\text{tex}} \text{“Three2threeTerms”}]$
 $[\text{Three2twoFactors} \xrightarrow{\text{tex}} \text{“Three2twoFactors”}]$
 $[\text{Three2threeFactors} \xrightarrow{\text{tex}} \text{“Three2threeFactors”}]$
 $[\text{AddEquations} \xrightarrow{\text{tex}} \text{“AddEquations”}]$
 $[\text{SubtractEquations} \xrightarrow{\text{tex}} \text{“SubtractEquations”}]$
 $[\text{SubtractEquationsLeft} \xrightarrow{\text{tex}} \text{“SubtractEquationsLeft”}]$
 $[\text{MultiplyEquations} \xrightarrow{\text{tex}} \text{“MultiplyEquations”}]$
 $[\text{EqNegated} \xrightarrow{\text{tex}} \text{“EqNegated”}]$
 $[\text{PositiveToRight(Eq)} \xrightarrow{\text{tex}} \text{“PositiveToRight(Eq)”}]$
 $[\text{PositiveToLeft(Eq)} \xrightarrow{\text{tex}} \text{“PositiveToLeft(Eq)”}]$
 $[\text{PositiveToLeft(Eq)(1term)} \xrightarrow{\text{tex}} \text{“PositiveToLeft(Eq)(1 term)”}]$
 $[\text{NegativeToLeft(Eq)} \xrightarrow{\text{tex}} \text{“NegativeToLeft(Eq)”}]$
 $[\text{reciprocalToLeft(Less)} \xrightarrow{\text{tex}} \text{“reciprocalToLeft(Less)”}]$
 $[\text{LessNeq} \xrightarrow{\text{tex}} \text{“LessNeq”}]$
 $[\text{NeqSymmetry} \xrightarrow{\text{tex}} \text{“NeqSymmetry”}]$

$[\text{NeqNegated} \xrightarrow{\text{tex}} \text{“NeqNegated”}]$
 $[\text{SubNeqRight} \xrightarrow{\text{tex}} \text{“SubNeqRight”}]$
 $[\text{SubNeqLeft} \xrightarrow{\text{tex}} \text{“SubNeqLeft”}]$
 $[\text{NegativeToRight}(\text{Neq})(1\text{term}) \xrightarrow{\text{tex}} \text{“NegativeToRight}(\text{Neq})(1 \text{ term)”}]$
 $[\text{NeqAddition} \xrightarrow{\text{tex}} \text{“NeqAddition”}]$
 $[\text{NeqMultiplication} \xrightarrow{\text{tex}} \text{“NeqMultiplication”}]$
 $[\text{UniqueNegative} \xrightarrow{\text{tex}} \text{“UniqueNegative”}]$
 $[\text{DoubleMinus} \xrightarrow{\text{tex}} \text{“DoubleMinus”}]$
 $[\text{LeqMultiplicationLeft} \xrightarrow{\text{tex}} \text{“LeqMultiplicationLeft”}]$
 $[\text{LeqLessEq} \xrightarrow{\text{tex}} \text{“LeqLessEq”}]$
 $[\text{LessLeq} \xrightarrow{\text{tex}} \text{“LessLeq”}]$
 $[\text{FromLeqGeq} \xrightarrow{\text{tex}} \text{“FromLeqGeq”}]$
 $[\text{subLeqRight} \xrightarrow{\text{tex}} \text{“subLeqRight”}]$
 $[\text{subLeqLeft} \xrightarrow{\text{tex}} \text{“subLeqLeft”}]$
 $[\text{Leq} + 1 \xrightarrow{\text{tex}} \text{“Leq+1”}]$
 $[\text{PositiveToRight}(\text{Leq}) \xrightarrow{\text{tex}} \text{“PositiveToRight}(\text{Leq)”}]$
 $[\text{PositiveToRight}(\text{Leq})(1\text{term}) \xrightarrow{\text{tex}} \text{“PositiveToRight}(\text{Leq})(1 \text{ term)”}]$
 $[\text{PositiveToLeft}(\text{Leq}) \xrightarrow{\text{tex}} \text{“PositiveToLeft}(\text{Leq)”}]$
 $[\text{LeqAdditionLeft} \xrightarrow{\text{tex}} \text{“LeqAdditionLeft”}]$
 $[\text{leqSubtraction} \xrightarrow{\text{tex}} \text{“leqSubtraction”}]$
 $[\text{leqSubtractionLeft} \xrightarrow{\text{tex}} \text{“leqSubtractionLeft”}]$
 $[\text{leqMultiplication} \xrightarrow{\text{tex}} \text{“leqMultiplication”}]$
 $[\text{thirdGeq} \xrightarrow{\text{tex}} \text{“thirdGeq”}]$
 $[\text{LeqNegated} \xrightarrow{\text{tex}} \text{“LeqNegated”}]$
 $[\text{AddEquations}(\text{Leq}) \xrightarrow{\text{tex}} \text{“AddEquations}(\text{Leq)”}]$

[MultiplyEquations(Leq) $\xrightarrow{\text{tex}}$ “MultiplyEquations(Leq)”]

[LeqNeqLess $\xrightarrow{\text{tex}}$ “LeqNeqLess”]

[FromLess $\xrightarrow{\text{tex}}$ “FromLess”]

[ToLess $\xrightarrow{\text{tex}}$ “ToLess”]

[fromNotLess $\xrightarrow{\text{tex}}$ “fromNotLess”]

[toNotLess $\xrightarrow{\text{tex}}$ “toNotLess”]

[LessAddition $\xrightarrow{\text{tex}}$ “LessAddition”]

[LessAdditionLeft $\xrightarrow{\text{tex}}$ “LessAdditionLeft”]

[LessMultiplication $\xrightarrow{\text{tex}}$ “LessMultiplication”]

[LessMultiplicationLeft $\xrightarrow{\text{tex}}$ “LessMultiplicationLeft”]

[LessDivision $\xrightarrow{\text{tex}}$ “LessDivision”]

[PositiveToRight(Less) $\xrightarrow{\text{tex}}$ “PositiveToRight(Less)”]

[PositiveToLeft(Less) $\xrightarrow{\text{tex}}$ “PositiveToLeft(Less)”]

[NegativeToLeft(Less) $\xrightarrow{\text{tex}}$ “NegativeToLeft(Less)”]

[NegativeToRight(Less) $\xrightarrow{\text{tex}}$ “NegativeToRight(Less)”]

[AddEquations(Less) $\xrightarrow{\text{tex}}$ “AddEquations(Less)”]

[AddEquations(LeqLess) $\xrightarrow{\text{tex}}$ “AddEquations(LeqLess)”]

[NegativeLessPositive $\xrightarrow{\text{tex}}$ “NegativeLessPositive”]

[leqLessTransitivity $\xrightarrow{\text{tex}}$ “leqLessTransitivity”]

[LessLeqTransitivity $\xrightarrow{\text{tex}}$ “LessLeqTransitivity”]

[LessTransitivity $\xrightarrow{\text{tex}}$ “LessTransitivity”]

[LessTotality $\xrightarrow{\text{tex}}$ “LessTotality”]

[SubLessRight $\xrightarrow{\text{tex}}$ “SubLessRight”]

[SubLessLeft $\xrightarrow{\text{tex}}$ “SubLessLeft”]

[SwitchTerms($x < y - z$) $\xrightarrow{\text{tex}}$ “SwitchTerms($x < y - z$)”]

[SwitchTerms(x - y < z) $\xrightarrow{\text{tex}}$ "SwitchTerms(x-y<z)"]

[LessNegated $\xrightarrow{\text{tex}}$ "LessNegated"]

[PositiveNonzero $\xrightarrow{\text{tex}}$ "PositiveNonzero"]

[PositiveNegated $\xrightarrow{\text{tex}}$ "PositiveNegated"]

[NonpositiveNegated $\xrightarrow{\text{tex}}$ "NonpositiveNegated"]

[NegativeNegated $\xrightarrow{\text{tex}}$ "NegativeNegated"]

[NonnegativeNegated $\xrightarrow{\text{tex}}$ "NonnegativeNegated"]

[PositiveInverted $\xrightarrow{\text{tex}}$ "PositiveInverted"]

[PositiveHalved $\xrightarrow{\text{tex}}$ "PositiveHalved"]

[NonnegativeNumerical $\xrightarrow{\text{tex}}$ "NonnegativeNumerical"]

[NegativeNumerical $\xrightarrow{\text{tex}}$ "NegativeNumerical"]

[PositiveNumerical $\xrightarrow{\text{tex}}$ "PositiveNumerical"]

[|0| = 0 $\xrightarrow{\text{tex}}$ "|0|=0"]

[0 <= |x| $\xrightarrow{\text{tex}}$ "0<=|x|"]

[x <= |x| $\xrightarrow{\text{tex}}$ "x<=|x|"]

[FromPositiveNumerical $\xrightarrow{\text{tex}}$ "FromPositiveNumerical"]

[SameNumerical $\xrightarrow{\text{tex}}$ "SameNumerical"]

[SignNumerical(+) $\xrightarrow{\text{tex}}$ "SignNumerical(+)]

[SignNumerical $\xrightarrow{\text{tex}}$ "SignNumerical"]

[ToNumericalLess $\xrightarrow{\text{tex}}$ "ToNumericalLess"]

[FromNumericalGreater $\xrightarrow{\text{tex}}$ "FromNumericalGreater"]

[NumericalDifference $\xrightarrow{\text{tex}}$ "NumericalDifference"]

[NumericalDifferenceLess(Helper) $\xrightarrow{\text{tex}}$ "NumericalDifferenceLess(Helper)"]

[NumericalDifferenceLess $\xrightarrow{\text{tex}}$ "NumericalDifferenceLess"]

[SplitNumericalSumHelper $\xrightarrow{\text{tex}}$ "SplitNumericalSumHelper"]

$\text{[splitNumericalSum}(++) \xrightarrow{\text{tex}} \text{"splitNumericalSum}(++)\text{"}$
 $\text{[splitNumericalSum}(--)\xrightarrow{\text{tex}} \text{"splitNumericalSum}(--)\text{"}$
 $\text{[splitNumericalSum}(+ - \text{small}) \xrightarrow{\text{tex}} \text{"splitNumericalSum}(+ - \text{small})\text{"}$
 $\text{[splitNumericalSum}(+ - \text{big}) \xrightarrow{\text{tex}} \text{"splitNumericalSum}(+ - \text{big})\text{"}$
 $\text{[splitNumericalSum}(+-)\xrightarrow{\text{tex}} \text{"splitNumericalSum}(+-)\text{"}$
 $\text{[splitNumericalSum}(-+)\xrightarrow{\text{tex}} \text{"splitNumericalSum}(-+)\text{"}$
 $\text{[splitNumericalSum} \xrightarrow{\text{tex}} \text{"splitNumericalSum"}\text{"}$
 $\text{[SplitNumericalProduct}(++) \xrightarrow{\text{tex}} \text{"SplitNumericalProduct}(++)\text{"}$
 $\text{[SplitNumericalProduct}(+-)\xrightarrow{\text{tex}} \text{"SplitNumericalProduct}(+-)\text{"}$
 $\text{[SplitNumericalProduct} \xrightarrow{\text{tex}} \text{"SplitNumericalProduct"}\text{"}$
 $\text{[insertMiddleTerm}(\text{Numerical}) \xrightarrow{\text{tex}} \text{"insertMiddleTerm}(\text{Numerical})\text{"}$
 $\text{[insertTwoMiddleTerms}(\text{Numerical}) \xrightarrow{\text{tex}} \text{"insertTwoMiddleTerms}(\text{Numerical})\text{"}$
 $\text{[MaxLeq}(1) \xrightarrow{\text{tex}} \text{"MaxLeq}(1)\text{"}$
 $\text{[MaxLeq}(2) \xrightarrow{\text{tex}} \text{"MaxLeq}(2)\text{"}$
 $\text{[LessThanMax} \xrightarrow{\text{tex}} \text{"LessThanMax"}\text{"}$
 $\text{[x + y = zBackwards} \xrightarrow{\text{tex}} \text{"x+y=zBackwards"}\text{"}$
 $\text{[x * y = zBackwards} \xrightarrow{\text{tex}} \text{"x*y=zBackwards"}\text{"}$
 $\text{[x = x + (y - y)} \xrightarrow{\text{tex}} \text{"x=x+(y-y)}\text{"}$
 $\text{[x = x + y - y} \xrightarrow{\text{tex}} \text{"x=x+y-y"}\text{"}$
 $\text{[x = x * y * (1/y)} \xrightarrow{\text{tex}} \text{"x=x*y*(1/y)}\text{"}$
 $\text{[insertMiddleTerm}(\text{Sum}) \xrightarrow{\text{tex}} \text{"insertMiddleTerm}(\text{Sum})\text{"}$
 $\text{[insertTwoMiddleTerms}(\text{Sum}) \xrightarrow{\text{tex}} \text{"insertTwoMiddleTerms}(\text{Sum})\text{"}$
 $\text{[insertMiddleTerm}(\text{Difference}) \xrightarrow{\text{tex}} \text{"insertMiddleTerm}(\text{Difference})\text{"}$
 $\text{[x * 0 + x = x} \xrightarrow{\text{tex}} \text{"x*0+x=x"}\text{"}$
 $\text{[NonnegativeFactors} \xrightarrow{\text{tex}} \text{"NonnegativeFactors"}\text{"}$

[NonzeroFactors $\xrightarrow{\text{tex}}$ “NonzeroFactors”]

[PositiveFactors $\xrightarrow{\text{tex}}$ “PositiveFactors”]

[PlusTimesMinus $\xrightarrow{\text{tex}}$ “PlusTimesMinus”]

[MinusTimesMinus $\xrightarrow{\text{tex}}$ “MinusTimesMinus”]

[$x * 0 = 0 \xrightarrow{\text{tex}}$ “ $x*0=0$ ”]

[$(-1) * (-1) + (-1) * 1 = 0 \xrightarrow{\text{tex}}$ “ $(-1)*(-1)+(-1)*1=0$ ”]

[$(-1) * (-1) = 1 \xrightarrow{\text{tex}}$ “ $(-1)*(-1)=1$ ”]

[$0 < 1$ Helper $\xrightarrow{\text{tex}}$ “ $0<1$ Helper”]

[$0 < 1 \xrightarrow{\text{tex}}$ “ $0<1$ ”]

[$0 < 2 \xrightarrow{\text{tex}}$ “ $0<2$ ”]

[$0 < 3 \xrightarrow{\text{tex}}$ “ $0<3$ ”]

[$0 < 1/2 \xrightarrow{\text{tex}}$ “ $0<1/2$ ”]

[$0 < 1/3 \xrightarrow{\text{tex}}$ “ $0<1/3$ ”]

[TwoWholes $\xrightarrow{\text{tex}}$ “TwoWholes”]

[ThreeWholes $\xrightarrow{\text{tex}}$ “ThreeWholes”]

[TwoHalves $\xrightarrow{\text{tex}}$ “TwoHalves”]

[ThreeThirds $\xrightarrow{\text{tex}}$ “ThreeThirds”]

[$-x - y = -(x + y) \xrightarrow{\text{tex}}$ “ $-x-y=-(x+y)$ ”]

[$-x * y = -(x * y) \xrightarrow{\text{tex}}$ “ $-x*y=-(x*y)$ ”]

[MinusNegated $\xrightarrow{\text{tex}}$ “MinusNegated”]

[Times(-1) $\xrightarrow{\text{tex}}$ “Times(-1)”]

[Times(-1)Left $\xrightarrow{\text{tex}}$ “Times(-1)Left”]

[$-0 = 0 \xrightarrow{\text{tex}}$ “ $-0=0$ ”]

[AllNegated(ImPLY) $\xrightarrow{\text{tex}}$ “AllNegated(ImPLY)”]

[ExistNegated(ImPLY) $\xrightarrow{\text{tex}}$ “ExistNegated(ImPLY)”]

[IntroExist(Helper) $\xrightarrow{\text{tex}}$ “IntroExist(Helper)”]

[IntroExist $\xrightarrow{\text{tex}}$ “IntroExist”]

[ExistMP $\xrightarrow{\text{tex}}$ “ExistMP”]

[ExistMP2 $\xrightarrow{\text{tex}}$ “ExistMP2”]

[TwiceExistMP $\xrightarrow{\text{tex}}$ “TwiceExistMP”]

[TwiceExistMP2 $\xrightarrow{\text{tex}}$ “TwiceExistMP2”]

[EAE – MP $\xrightarrow{\text{tex}}$ “EAE-MP”]

[AddAll $\xrightarrow{\text{tex}}$ “AddAll ”]

[AddExist(Helper1) $\xrightarrow{\text{tex}}$ “AddExist(Helper1)”]

[AddExist(Helper2) $\xrightarrow{\text{tex}}$ “AddExist(Helper2)”]

[AddExist $\xrightarrow{\text{tex}}$ “AddExist”]

[AddExist(SimpleAnt) $\xrightarrow{\text{tex}}$ “AddExist(SimpleAnt)”]

[AddExist(Simple) $\xrightarrow{\text{tex}}$ “AddExist(Simple)”]

[AddEAE $\xrightarrow{\text{tex}}$ “AddEAE”]

[AEA – negated $\xrightarrow{\text{tex}}$ “AEA-negated”]

[EEA – negated $\xrightarrow{\text{tex}}$ “EEA-negated”]

[ToNegatedAnd $\xrightarrow{\text{tex}}$ “ToNegatedAnd”]

[PositiveToRight(Less)(1term) $\xrightarrow{\text{tex}}$ “PositiveToRight(Less)(1 term)”]

[(A)to(E) $\xrightarrow{\text{tex}}$ “(A~)to(~E)”]

[eqTransitivity4 $\xrightarrow{\text{tex}}$ “eqTransitivity4”]

[Plus0Left(R) $\xrightarrow{\text{tex}}$ “Plus0Left(R)”]

[x = x + (y – y)(R) $\xrightarrow{\text{tex}}$ “x=x+(y-y)(R)”]

[x = x + y – y(R) $\xrightarrow{\text{tex}}$ “x=x+y-y(R)”]

[PositiveToRight(Eq)(R) $\xrightarrow{\text{tex}}$ “PositiveToRight(Eq)(R)”]

[SubtractEquations(R) $\xrightarrow{\text{tex}}$ “SubtractEquations(R)”]

$[\text{NeqAddition}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NeqAddition}(\mathbb{R})"]$
 $[\text{EqAdditionLeft}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"EqAdditionLeft}(\mathbb{R})"]$
 $[\text{Three2twoTerms}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"Three2twoTerms}(\mathbb{R})"]$
 $[\text{PositiveToRight(Less)}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"PositiveToRight(Less)}(\mathbb{R})"]$
 $[\text{Three2threeTerms}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"Three2threeTerms}(\mathbb{R})"]$
 $[\text{PositiveToRight(Less)}(1\text{term})(\mathbb{R}) \xrightarrow{\text{tex}} \text{"PositiveToRight(Less)}(1\text{ term})(\mathbb{R})"]$
 $[\text{ToLeq(Advanced)}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"ToLeq(Advanced)}(\mathbb{R})"]$
 $[\text{LeqNeqLess}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"LeqNeqLess}(\mathbb{R})"]$
 $[\text{SubLeqLeft}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"SubLeqLeft}(\mathbb{R})"]$
 $[\text{LeqLessTransitivity}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"LeqLessTransitivity}(\mathbb{R})"]$
 $[\text{NegativeToLeft(Eq)}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NegativeToLeft(Eq)}(\mathbb{R})"]$
 $[\text{NegativeToRight(Less)}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NegativeToRight(Less)}(\mathbb{R})"]$
 $[!! == \text{Symmetry} \xrightarrow{\text{tex}} \text{"!!==Symmetry"}]$
 $[\text{NegativeToRight(Eq)}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NegativeToRight(Eq)}(\mathbb{R})"]$
 $[\text{NegativeToRight(Eq)}(1\text{term})(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NegativeToRight(Eq)}(1\text{ term})(\mathbb{R})"]$
 $[\text{SubLeqRight}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"SubLeqRight}(\mathbb{R})"]$
 $[\text{DoubleMinus}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"DoubleMinus}(\mathbb{R})"]$
 $[\text{UniqueNegative}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"UniqueNegative}(\mathbb{R})"]$
 $[\text{SubtractEquationsLeft}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"SubtractEquationsLeft}(\mathbb{R})"]$
 $[\text{EqNegated}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"EqNegated}(\mathbb{R})"]$
 $[\text{NeqNegated}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NeqNegated}(\mathbb{R})"]$
 $[\text{LeqNegated}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"LeqNegated}(\mathbb{R})"]$
 $[\text{LessNegated}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"LessNegated}(\mathbb{R})"]$
 $[-0 = 0(\mathbb{R}) \xrightarrow{\text{tex}} \text{"-0=0}(\mathbb{R})"]$
 $[\text{NegativeNegated}(\mathbb{R}) \xrightarrow{\text{tex}} \text{"NegativeNegated}(\mathbb{R})"]$

[FromLeqGeq(R) $\xrightarrow{\text{tex}}$ "FromLeqGeq(R)"]

[FromLess(R) $\xrightarrow{\text{tex}}$ "FromLess(R)"]

[NonnegativeNumerical(R) $\xrightarrow{\text{tex}}$ "NonnegativeNumerical(R)"]

[NegativeNumerical(R) $\xrightarrow{\text{tex}}$ "NegativeNumerical(R)"]

[0 <= |x|(R) $\xrightarrow{\text{tex}}$ "0<=|x|(R)"]

[PositiveNegated(R) $\xrightarrow{\text{tex}}$ "PositiveNegated(R)"]

[AddEquations(R) $\xrightarrow{\text{tex}}$ "AddEquations(R)"]

[DistributionOut(R) $\xrightarrow{\text{tex}}$ "DistributionOut(R)"]

[== Transitivity5 $\xrightarrow{\text{tex}}$ "==Transitivity5"]

[x * 0 = 0(R)(fff) $\xrightarrow{\text{tex}}$ "x*0=0(R)(fff)"]

[x * 0 + x = x(R) $\xrightarrow{\text{tex}}$ "x*0+x=x(R)"]

[Times(-1)(R) $\xrightarrow{\text{tex}}$ "Times(-1)(R)"]

[Times(-1)Left(R) $\xrightarrow{\text{tex}}$ "Times(-1)Left(R)"]

[-x - y = -(x + y)(R) $\xrightarrow{\text{tex}}$ "-x-y=-(x+y)(R)"]

[LessTotality(R) $\xrightarrow{\text{tex}}$ "LessTotality(R)"]

[SignNumerical(+)(R) $\xrightarrow{\text{tex}}$ "SignNumerical(+)(R)"]

[PositiveNumerical(R) $\xrightarrow{\text{tex}}$ "PositiveNumerical(R)"]

[SameNumerical(R) $\xrightarrow{\text{tex}}$ "SameNumerical(R)"]

[MinusNegated(R) $\xrightarrow{\text{tex}}$ "MinusNegated(R)"]

[SignNumerical(R) $\xrightarrow{\text{tex}}$ "SignNumerical(R)"]

[NumericalDifference(R) $\xrightarrow{\text{tex}}$ "NumericalDifference(R)"]

[USlimitIsUpperBound(Helper) $\xrightarrow{\text{tex}}$ "USlimitIsUpperBound(Helper)"]

[USlimitIsUpperBound $\xrightarrow{\text{tex}}$ "USlimitIsUpperBound"]

[x <= |x|(R) $\xrightarrow{\text{tex}}$ "x<=|x|(R)"]

[(-1) * (-1) + (-1) * 1 = 0(R) $\xrightarrow{\text{tex}}$ "(-1)*(-1)+(-1)*1=0(R)"]

$[(-1) * (-1) = 1(\mathbb{R}) \xrightarrow{\text{tex}} "(-1)*(-1)=1(\mathbb{R})"]$
 $[0 < 1\text{Helper}(\mathbb{R}) \xrightarrow{\text{tex}} "0<1\text{Helper}(\mathbb{R})"]$
 $[0 < 1(\mathbb{R}) \xrightarrow{\text{tex}} "0<1(\mathbb{R})"]$
 $[\text{ExpZero}(\text{Exact})(\mathbb{R}) \xrightarrow{\text{tex}} "\text{ExpZero}(\text{Exact})(\mathbb{R})"]$
 $[\text{PositiveBase}(\mathbb{R})(\text{Base}) \xrightarrow{\text{tex}} "\text{PositiveBase}(\mathbb{R})(\text{Base})"]$
 $[\text{Three2twoFactors}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{Three2twoFactors}(\mathbb{R})"]$
 $[x = x * y * (1/y)(\mathbb{R}) \xrightarrow{\text{tex}} "x=x*y*(1/y)(\mathbb{R})"]$
 $[\text{NeqMultiplication}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{NeqMultiplication}(\mathbb{R})"]$
 $[\text{LessTransitivity}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{LessTransitivity}(\mathbb{R})"]$
 $[0 < 2(\mathbb{R}) \xrightarrow{\text{tex}} "0<2(\mathbb{R})"]$
 $[\text{SameExp}(\mathbb{R})(\text{Base}) \xrightarrow{\text{tex}} "\text{SameExp}(\mathbb{R})(\text{Base})"]$
 $[\text{SameExp}(\mathbb{R})(\text{Indu}) \xrightarrow{\text{tex}} "\text{SameExp}(\mathbb{R})(\text{Indu})"]$
 $[\text{SameExp}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{SameExp}(\mathbb{R})"]$
 $[\text{SubNeqLeft}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{SubNeqLeft}(\mathbb{R})"]$
 $[\text{SubNeqRight}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{SubNeqRight}(\mathbb{R})"]$
 $[\text{NonzeroFactors}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{NonzeroFactors}(\mathbb{R})"]$
 $[\text{NonnegativeFactors}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{NonnegativeFactors}(\mathbb{R})"]$
 $[\text{PositiveFactors}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{PositiveFactors}(\mathbb{R})"]$
 $[\text{LessDivision}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{LessDivision}(\mathbb{R})"]$
 $[0 < 1/2(\mathbb{R}) \xrightarrow{\text{tex}} "0<1/2(\mathbb{R})"]$
 $[\text{PositiveToRight}(\text{Eq})(1\text{term})(\mathbb{R}) \xrightarrow{\text{tex}} "\text{PositiveToRight}(\text{Eq})(1\text{ term})(\mathbb{R})"]$
 $[\text{Exp}(+1)(\mathbb{R}) \xrightarrow{\text{tex}} "\text{Exp}(+1)(\mathbb{R})"]$
 $[\text{PositiveBase}(\mathbb{R})(\text{Indu}) \xrightarrow{\text{tex}} "\text{PositiveBase}(\mathbb{R})(\text{Indu})"]$
 $[\text{PositiveBase}(\mathbb{R}) \xrightarrow{\text{tex}} "\text{PositiveBase}(\mathbb{R})"]$
 $[-x * y = -(x * y)(\mathbb{R}) \xrightarrow{\text{tex}} "-x*y=-(x*y)(\mathbb{R})"]$

[PositiveToLeft(Eq)(R) $\xrightarrow{\text{tex}}$ “PositiveToLeft(Eq)(R)”]

[Times1Left(R) $\xrightarrow{\text{tex}}$ “Times1Left(R)”]

[==Transitivity6 $\xrightarrow{\text{tex}}$ “==Transitivity6”]

[$x + x = 2 * x$ (R) $\xrightarrow{\text{tex}}$ “ $x+x=2*x$ (R)”]

[(1/2)x + (1/2)x = x(R) $\xrightarrow{\text{tex}}$ “(1/2)x+(1/2)x=x(R)”]

[DistributionOut(Minus)(R) $\xrightarrow{\text{tex}}$ “DistributionOut(Minus)(R)”]

[(1/2)(x + y) - x = (1/2)(y - x)(R) $\xrightarrow{\text{tex}}$ “(1/2)(x+y)-x=(1/2)(y-x)(R)”]

[IntervalSize(R)(Base) $\xrightarrow{\text{tex}}$ “IntervalSize(R)(Base)”]

[LessMultiplicationLeft(R) $\xrightarrow{\text{tex}}$ “LessMultiplicationLeft(R)”]

[NegativeToLeft(Less)(R) $\xrightarrow{\text{tex}}$ “NegativeToLeft(Less)(R)”]

[NegativeToLeft(Less)(1term)(R) $\xrightarrow{\text{tex}}$ “NegativeToLeft(Less)(1 term)(R)”]

[$y - (1/2)(x + y) = (1/2)(y - x)$ (R) $\xrightarrow{\text{tex}}$ “ $y-(1/2)(x+y)=(1/2)(y-x)$ (R)”]

[IntervalSize(R)(Indu) $\xrightarrow{\text{tex}}$ “IntervalSize(R)(Indu)”]

[IntervalSize(R) $\xrightarrow{\text{tex}}$ “IntervalSize(R)”]

[XSlessUS(R) $\xrightarrow{\text{tex}}$ “XSlessUS(R)”]

[USdecreasing(+1)(R) $\xrightarrow{\text{tex}}$ “USdecreasing(+1)(R)”]

[ExpUnbounded(Base) $\xrightarrow{\text{tex}}$ “ExpUnbounded(Base)”]

[ExpUnbounded(Indu) $\xrightarrow{\text{tex}}$ “ExpUnbounded(Indu)”]

[ExpUnbounded $\xrightarrow{\text{tex}}$ “ExpUnbounded”]

[$1 \leq x + 1$ (N) $\xrightarrow{\text{tex}}$ “ $1 \leq x+1$ (N)”]

[NonzeroProduct(2)(R) $\xrightarrow{\text{tex}}$ “NonzeroProduct(2)(R)”]

[PositiveNonzero(R) $\xrightarrow{\text{tex}}$ “PositiveNonzero(R)”]

[NonreciprocalToRight(Eq)(1term)(R) $\xrightarrow{\text{tex}}$ “NonreciprocalToRight(Eq)(1 term)(R)”]

[ExpNonzero(Base) $\xrightarrow{\text{tex}}$ “ExpNonzero(Base)”]

[ExpNonzero(Indu) $\xrightarrow{\text{tex}}$ “ExpNonzero(Indu)”]

[ExpNonzero $\xrightarrow{\text{tex}}$ “ExpNonzero”]

[ExpNonzero(2) $\xrightarrow{\text{tex}}$ “ExpNonzero(2)”]

[MultiplyEquations(R) $\xrightarrow{\text{tex}}$ “MultiplyEquations(R)”]

[HalfBase(Base) $\xrightarrow{\text{tex}}$ “HalfBase(Base)”]

[HalfBase(Indu) $\xrightarrow{\text{tex}}$ “HalfBase(Indu)”]

[HalfBase $\xrightarrow{\text{tex}}$ “HalfBase”]

[Three2threeFactors(R) $\xrightarrow{\text{tex}}$ “Three2threeFactors(R)”]

[$x * y = z$ Backwards(R) $\xrightarrow{\text{tex}}$ “ $x*y=z$ Backwards(R)”]

[PositiveInverted(R) $\xrightarrow{\text{tex}}$ “PositiveInverted(R)”]

[ReciprocalToRight(Less)(R) $\xrightarrow{\text{tex}}$ “ReciprocalToRight(Less)(R)”]

[ReciprocalToRight(Less)(1term)(R) $\xrightarrow{\text{tex}}$ “ReciprocalToRight(Less)(1term)(R)”]

[NonreciprocalToLeft(Less)(R) $\xrightarrow{\text{tex}}$ “NonreciprocalToLeft(Less)(R)”]

[$1 < x * y$ (R) $\xrightarrow{\text{tex}}$ “ $1 < x*y$ (R)”]

[SwitchFactors($1/x < y$)(R) $\xrightarrow{\text{tex}}$ “SwitchFactors($1/x < y$)(R)”]

[SmallHalving $\xrightarrow{\text{tex}}$ “SmallHalving”]

[IntervalSize(anyPositive) $\xrightarrow{\text{tex}}$ “IntervalSize(anyPositive)”]

[USdecreasing(+n)(Base) $\xrightarrow{\text{tex}}$ “USdecreasing(+n)(Base)”]

[USdecreasing(+n)(Indu) $\xrightarrow{\text{tex}}$ “USdecreasing(+n)(Indu)”]

[USdecreasing(+n) $\xrightarrow{\text{tex}}$ “USdecreasing(+n)”]

[USdecreasing $\xrightarrow{\text{tex}}$ “USdecreasing”]

[LeqAdditionLeft(R) $\xrightarrow{\text{tex}}$ “LeqAdditionLeft(R)”]

[ToNotLess(R) $\xrightarrow{\text{tex}}$ “ToNotLess(R)”]

[LimitOfUSIsLeq $\xrightarrow{\text{tex}}$ “LimitOfUSIsLeq”]

$[SubtractEquations(Less)(R) \xrightarrow{tex} "SubtractEquations(Less)(R)"]$
 $[SubtractEquationsLeft(Less)(R) \xrightarrow{tex} "SubtractEquationsLeft(Less)(R)"]$
 $[LessNegated(Negative)(R) \xrightarrow{tex} "LessNegated(Negative)(R)"]$
 $[FromNegatedAnd(Implied) \xrightarrow{tex} "FromNegatedAnd(Implied)"]$
 $[RemoveDoubleNeg(Consequent) \xrightarrow{tex} "RemoveDoubleNeg(Consequent)"]$
 $[FromNotUpperBound \xrightarrow{tex} "FromNotUpperBound"]$
 $[LeqNUB \xrightarrow{tex} "LeqNUB"]$
 $[USlimitIsLeastUpperBound(Helper) \xrightarrow{tex} "USlimitIsLeastUpperBound(Helper)"]$
 $[USlimitIsLeastUpperBound \xrightarrow{tex} "USlimitIsLeastUpperBound"]$
 $[ExistMP3 \xrightarrow{tex} "ExistMP3"]$
 $[GreaterPositive(N) \xrightarrow{tex} "GreaterPositive(N)"]$
 $[ysFClose(Helper) \xrightarrow{tex} "ysFClose(Helper)"]$
 $[ysFClose \xrightarrow{tex} "ysFClose"]$
 $[ysFCAuchy(Helper) \xrightarrow{tex} "ysFCAuchy(Helper)"]$
 $[ysFCAuchy \xrightarrow{tex} "ysFCAuchy"]$
 $[SFsymmetry \xrightarrow{tex} "SFsymmetry"]$
 $[SFtransitivity \xrightarrow{tex} "SFtransitivity"]$
 $[lemma =f to sameF \xrightarrow{tex} "=fToSameF "]$
 $[lemma plusF(Sym) \xrightarrow{tex} "PlusF(Sym)"]$
 $[lemma timesF(Sym) \xrightarrow{tex} "TimesF(Sym)"]$
 $[f2R(Plus) \xrightarrow{tex} "f2R(Plus)"]$
 $[f2R(Times) \xrightarrow{tex} "f2R(Times)"]$
 $[<< TransitivityHelper(Q) \xrightarrow{tex} "<<TransitivityHelper(Q)"]$
 $[<< Transitivity \xrightarrow{tex} "<<Transitivity"]$

[<<== Reflexivity $\xrightarrow{\text{tex}}$ "<<==Reflexivity"]

[<<== AntisymmetryHelper(Q) $\xrightarrow{\text{tex}}$ "<<==AntisymmetryHelper(Q)"]

[FromNotSameF(Weak)(Helper) $\xrightarrow{\text{tex}}$ "FromNotSameF(Weak)(Helper)"]

[FromNotSameF(Weak) $\xrightarrow{\text{tex}}$ "FromNotSameF(Weak)"]

[FromNotLess(F) $\xrightarrow{\text{tex}}$ "FromNotLess(F)"]

[Plus0(F) $\xrightarrow{\text{tex}}$ "Plus0(F)"]

[== Addition $\xrightarrow{\text{tex}}$ "==Addition"]

[== AdditionLeft $\xrightarrow{\text{tex}}$ "==AdditionLeft"]

[Fpart – Bounded(Base) $\xrightarrow{\text{tex}}$ "Fpart-Bounded(Base)"]

[Fpart – Bounded(InduHelper) $\xrightarrow{\text{tex}}$ "Fpart-Bounded(InduHelper)"]

[Fpart – Bounded(Indu) $\xrightarrow{\text{tex}}$ "Fpart-Bounded(Indu)"]

[Fpart – Bounded $\xrightarrow{\text{tex}}$ "Fpart-Bounded"]

[F – Bounded $\xrightarrow{\text{tex}}$ "F-Bounded"]

[F – Bounded(Helper) $\xrightarrow{\text{tex}}$ "F-Bounded(Helper)"]

[SameFmultiplication(Helper) $\xrightarrow{\text{tex}}$ "SameFmultiplication(Helper)"]

[SameFmultiplication $\xrightarrow{\text{tex}}$ "SameFmultiplication"]

[FromNot < f(Weak)(Helper) $\xrightarrow{\text{tex}}$ "FromNot<f(Weak)(Helper)"]

[FromNot < f(Weak) $\xrightarrow{\text{tex}}$ "FromNot<f(Weak)"]

[FromNot < f(Strong)(Helper2) $\xrightarrow{\text{tex}}$ "FromNot<f(Strong)(Helper2)"]

[FromNot < f(Strong)(Helper) $\xrightarrow{\text{tex}}$ "FromNot<f(Strong)(Helper)"]

[FromNot < f(Strong) $\xrightarrow{\text{tex}}$ "FromNot<f(Strong)"]

[fromNotSameF(Strongest)(Helper2) $\xrightarrow{\text{tex}}$
"fromNotSameF(Strongest)(Helper2)"]

[fromNotSameF(Strongest)(Helper) $\xrightarrow{\text{tex}}$ "fromNotSameF(Strongest)(Helper)"]

[fromNotSameF(Strongest) $\xrightarrow{\text{tex}}$ "fromNotSameF(Strongest)"]

[ToLess(F)(Helper) $\xrightarrow{\text{tex}}$ “ToLess(F)(Helper)”]

[ToLess(F) $\xrightarrow{\text{tex}}$ “ToLess(F)”]

[LessMultiplication(F)(Helper2) $\xrightarrow{\text{tex}}$ “LessMultiplication(F)(Helper2)”]

[LessMultiplication(F)(Helper) $\xrightarrow{\text{tex}}$ “LessMultiplication(F)(Helper)”]

[LessMultiplication(F) $\xrightarrow{\text{tex}}$ “LessMultiplication(F)”]

[EqMultiplication(R) $\xrightarrow{\text{tex}}$ “EqMultiplication(R)”]

[EqMultiplicationLeft(R) $\xrightarrow{\text{tex}}$ “EqMultiplicationLeft(R)”]

[PlusAssociativity(F) $\xrightarrow{\text{tex}}$ “PlusAssociativity(F)”]

[FromNot << $\xrightarrow{\text{tex}}$ “FromNot<<”]

[ToLess(R) $\xrightarrow{\text{tex}}$ “ToLess(R)”]

[LeqTotality(R) $\xrightarrow{\text{tex}}$ “LeqTotality(R)”]

[x * 0 = 0(F) $\xrightarrow{\text{tex}}$ “x*0=0(F)”]

[x * 0 = 0(R) $\xrightarrow{\text{tex}}$ “x*0=0(R)”]

[PlusCommutativity(F) $\xrightarrow{\text{tex}}$ “PlusCommutativity(F)”]

[Cauchy(2)(Helper) $\xrightarrow{\text{tex}}$ “Cauchy(2)(Helper)”]

[Cauchy(2) $\xrightarrow{\text{tex}}$ “Cauchy(2)”]

[TimesAssociativity(F) $\xrightarrow{\text{tex}}$ “TimesAssociativity(F)”]

[LessMultiplication(R) $\xrightarrow{\text{tex}}$ “LessMultiplication(R)”]

[LeqMultiplication(R) $\xrightarrow{\text{tex}}$ “LeqMultiplication(R)”]

[Times1f $\xrightarrow{\text{tex}}$ “Times1f”]

[ReciprocalFnonzero $\xrightarrow{\text{tex}}$ “ReciprocalFnonzero”]

[ReciprocalFnynonzero $\xrightarrow{\text{tex}}$ “ReciprocalFnynonzero”]

[(Eventually = f)2sameF(Helper) $\xrightarrow{\text{tex}}$ “(Eventually=f)2sameF(Helper)”]

[(Eventually = f)2sameF $\xrightarrow{\text{tex}}$ “(Eventually=f)2sameF”]

[FromNotSameF(Strong)(Helper2) $\xrightarrow{\text{tex}}$ “FromNotSameF(Strong)(Helper2)”]

[FromNotSameF(Strong)(Helper) $\xrightarrow{\text{tex}}$ “FromNotSameF(Strong)(Helper)”]

[FromNotSameF(Strong) $\xrightarrow{\text{tex}}$ “FromNotSameF(Strong)”]

[SameFreciprocal(Helper) $\xrightarrow{\text{tex}}$ “SameFreciprocal(Helper)”]

[SameFreciprocal $\xrightarrow{\text{tex}}$ “SameFreciprocal”]

[From!! == $\xrightarrow{\text{tex}}$ “From!!==”]

[Reciprocal(R) $\xrightarrow{\text{tex}}$ “Reciprocal(R)”]

[TimesCommutativity(F) $\xrightarrow{\text{tex}}$ “TimesCommutativity(F)”]

[Distribution(F) $\xrightarrow{\text{tex}}$ “Distribution(F)”]

[FromNotLess(R) $\xrightarrow{\text{tex}}$ “FromNotLess(R)”]

[ToNegatedAnd(1) $\xrightarrow{\text{tex}}$ “ToNegatedAnd(1)”]

[FromMax(1) $\xrightarrow{\text{tex}}$ “FromMax(1)”]

[FromMax(2) $\xrightarrow{\text{tex}}$ “FromMax(2)”]

[CartProdIsRelation $\xrightarrow{\text{tex}}$ “CartProdIsRelation”]

[FromSubset $\xrightarrow{\text{tex}}$ “FromSubset”]

[SubsetIsRelation $\xrightarrow{\text{tex}}$ “SubsetIsRelation”]

[SeriesSubsetCP $\xrightarrow{\text{tex}}$ “SeriesSubsetCP”]

[ValueType $\xrightarrow{\text{tex}}$ “ValueType”]

[ToSeries $\xrightarrow{\text{tex}}$ “ToSeries”]

[FromSeries $\xrightarrow{\text{tex}}$ “FromSeries”]

[RemoveOr $\xrightarrow{\text{tex}}$ “RemoveOr”]

[FromSingleton $\xrightarrow{\text{tex}}$ “FromSingleton”]

[InPair(1) $\xrightarrow{\text{tex}}$ “InPair(1)”]

[InPair(2) $\xrightarrow{\text{tex}}$ “InPair(2)”]

[SameMember(2) $\xrightarrow{\text{tex}}$ “SameMember(2)”]

[ToBinaryUnion(1) $\xrightarrow{\text{tex}}$ “ToBinaryUnion(1)”]

[ToBinaryUnion(2) $\xrightarrow{\text{tex}}$ “ToBinaryUnion(2)”]
 [FromOrderedPair(TwoLevels) $\xrightarrow{\text{tex}}$ “FromOrderedPair(TwoLevels)”]
 [ToCartProd(Helper) $\xrightarrow{\text{tex}}$ “ToCartProd(Helper)”]
 [ToCartProd $\xrightarrow{\text{tex}}$ “ToCartProd”]
 [NonreciprocalToRight(Eq) $\xrightarrow{\text{tex}}$ “NonreciprocalToRight(Eq)”]
 [NonreciprocalToLeft(Eq)(1 term) $\xrightarrow{\text{tex}}$ “NonreciprocalToLeft(Eq)(1 term)”]
 [SameReciprocal $\xrightarrow{\text{tex}}$ “SameReciprocal”]
 [CPseparationIsRelation $\xrightarrow{\text{tex}}$ “CPseparationIsRelation”]
 [OrderedPairEquality $\xrightarrow{\text{tex}}$ “OrderedPairEquality”]
 [ReciprocalIsFunction $\xrightarrow{\text{tex}}$ “ReciprocalIsFunction”]
 [ReciprocalIsTotal $\xrightarrow{\text{tex}}$ “ReciprocalIsTotal”]
 [ReciprocalIsRationalSeries $\xrightarrow{\text{tex}}$ “ReciprocalIsRationalSeries”]
 [CrsIsRelation $\xrightarrow{\text{tex}}$ “CrsIsRelation”]
 [CrsIsFunction $\xrightarrow{\text{tex}}$ “CrsIsFunction ”]
 [CrsIsTotal $\xrightarrow{\text{tex}}$ “CrsIsTotal”]
 [CrsIsSeries $\xrightarrow{\text{tex}}$ “CrsIsSeries”]
 [CrsLookup $\xrightarrow{\text{tex}}$ “CrsLookup”]
 [0f $\xrightarrow{\text{tex}}$ “0f”]
 [1f $\xrightarrow{\text{tex}}$ “1f”]
 [ToSingleton $\xrightarrow{\text{tex}}$ “ToSingleton”]
 [FromSameSingleton $\xrightarrow{\text{tex}}$ “FromSameSingleton”]
 [SingletonmembersEqual $\xrightarrow{\text{tex}}$ “SingletonmembersEqual”]
 [UnequalsNotInSingleton $\xrightarrow{\text{tex}}$ “UnequalsNotInSingleton”]
 [NonsingletonmembersUnequal $\xrightarrow{\text{tex}}$ “NonsingletonmembersUnequal”]
 [FromOrderedPair $\xrightarrow{\text{tex}}$ “FromOrderedPair”]

[FromOrderedPair(1) $\xrightarrow{\text{tex}}$ “FromOrderedPair(1)”]

[FromOrderedPair(2) $\xrightarrow{\text{tex}}$ “FromOrderedPair(2)”]

[FromCartProd $\xrightarrow{\text{tex}}$ “FromCartProd”]

[FromCartProd(1) $\xrightarrow{\text{tex}}$ “FromCartProd(1)”]

[FromCartProd(2) $\xrightarrow{\text{tex}}$ “FromCartProd(2)”]

[sameOrderedPair $\xrightarrow{\text{tex}}$ “sameOrderedPair”]

[InSeriesHelper $\xrightarrow{\text{tex}}$ “InSeriesHelper”]

[InSeries $\xrightarrow{\text{tex}}$ “InSeries”]

[To = f(Subset)(Helper) $\xrightarrow{\text{tex}}$ “To=f(Subset)(Helper)”]

[To = f(Subset) $\xrightarrow{\text{tex}}$ “To=f(Subset)”]

[To = f $\xrightarrow{\text{tex}}$ “To=f”]

[Tester1 $\xrightarrow{\text{tex}}$ “Tester1”]

[Tester2 $\xrightarrow{\text{tex}}$ “Tester2”]

[Tester3 $\xrightarrow{\text{tex}}$ “Tester3”]

[Tester4 $\xrightarrow{\text{tex}}$ “Tester4”]

[Tester5 $\xrightarrow{\text{tex}}$ “Tester5”]

[Tester6 $\xrightarrow{\text{tex}}$ “Tester6”]

[productIsFunction $\xrightarrow{\text{tex}}$ “productIsFunction”]

[productIsTotal $\xrightarrow{\text{tex}}$ “productIsTotal”]

[ProductIsRationalSeries $\xrightarrow{\text{tex}}$ “ProductIsRationalSeries”]

[TimesF $\xrightarrow{\text{tex}}$ “TimesF”]

[-x + (1/2)x = -(1/2)x $\xrightarrow{\text{tex}}$ “-x+(1/2)x=-(1/2)x”]

[ClosetolessIsLess $\xrightarrow{\text{tex}}$ “ClosetolessIsLess”]

[SubLessLeft(F) $\xrightarrow{\text{tex}}$ “SubLessLeft(F)”]

[SubLessLeft(R) $\xrightarrow{\text{tex}}$ “SubLessLeft(R)”]

[ClosetogreaterIsGreater $\xrightarrow{\text{tex}}$ “ClosetogreaterIsGreater”]

[SubLessRight(F) $\xrightarrow{\text{tex}}$ “SubLessRight(F)”]

[SubLessRight(R) $\xrightarrow{\text{tex}}$ “SubLessRight(R)”]

[PositiveTripled $\xrightarrow{\text{tex}}$ “PositiveTripled”]

[PositiveDividedBy3 $\xrightarrow{\text{tex}}$ “PositiveDividedBy3”]

[$|x - x| = 0$ $\xrightarrow{\text{tex}}$ “ $|x-x|=0$ ”]

[$1 < 2$ $\xrightarrow{\text{tex}}$ “ $1<2$ ”]

[$1/3 < 2/3$ $\xrightarrow{\text{tex}}$ “ $1/3<2/3$ ”]

[$(1/3)x + (1/3)x = (2/3)x$ $\xrightarrow{\text{tex}}$ “ $(1/3)x+(1/3)x=(2/3)x$ ”]

[$(2/3)x + (1/3)x = x$ $\xrightarrow{\text{tex}}$ “ $(2/3)x+(1/3)x=x$ ”]

[$-x + (2/3)x = -(1/3)x$ $\xrightarrow{\text{tex}}$ “ $-x+(2/3)x=-(1/3)x$ ”]

[PreserveLessGreater $\xrightarrow{\text{tex}}$ “PreserveLessGreater”]

[$-(1/3)x - (1/3)x = -(2/3)x$ $\xrightarrow{\text{tex}}$ “ $-(1/3)x-(1/3)x=-(2/3)x$ ”]

[$-x + (1/3)x = -(2/3)x$ $\xrightarrow{\text{tex}}$ “ $-x+(1/3)x=-(2/3)x$ ”]