Logiweb codex of sup

Up Help

Nat(∗), $\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$, $\langle * \equiv^{1} * \mid * :== * \rangle_{\mathrm{Me}}$, $\langle * \equiv^{*} * \mid * :== * \rangle_{\mathrm{Me}}$, lemma plus0Left, lemma times1Left, lemma eqMultiplicationLeft, lemma eqLeq(R), lemma =f to sameF, lemma plusF(Sym), lemma timesF(Sym), ∗(exp)∗, sup, To!! ==, ToNegatedDoubleImply, ToNegatedAnd(1), AddNegatedAll, (A)to(E)(Imply), (E)to(A)(Imply), (E)to(A)(Imply), ToNegatedAEA, UniqueNegative, DoubleMinus, MinusNegated, eqReflexivity, eqSymmetry, eqTransitivity, eqTransitivity4, eqTransitivity5, eqTransitivity6, AddEquations, SubtractEquations, SubtractEquationsLeft, MultiplyEquations, EqNegated, PositiveToRight(Eq), PositiveToLeft(Eq), PositiveToLeft(Eq)(1term), NegativeToLeft(Eq), NonreciprocalToRight(Eq)(1term), DistributionOut(Minus), PositiveToRight(Eq)(1term), SameSeries(NumDiff), PlusAssociativity(4terms), LessNeq, NeqSymmetry, NeqNegated, SubNeqRight, SubNeqLeft, NegativeToRight(Neq)(1term), NeqAddition, NeqMultiplication, NonzeroProduct(2), SwitchTerms($x <= y - z$), NegativeToLeft(Less)(1term), (+1)IsPositive(N), $(1/2)(x + y) - x = (1/2)(y - x)$, $y - (1/2)(x + y) = (1/2)(y - x)$, ExpZero(Exact), SameExp(Base), SameExp(Indu), SameExp, Exp(+1), PositiveBase(Base), PositiveBase(Indu), PositiveBase, BSzero(Exact), SameBS(2)(Base), SameBS(2)(Indu), SameBS(2), BS(+1), BSbound(Exact)(Base), BSbound(Exact)(Indu), BSbound(Exact), BSbound, UStelescope(Zero)(Exact), SameTelescope(2)(Base), SameTelescope(2)(Indu), SameTelescope(2), UStelescope(+1), TelescopeNumerical(Base), TelescopeNumerical(Indu), TelescopeNumerical, TelescopeBound(Base), TelescopeBound(Indu), TelescopeBound, LessNeq(F)(Helper), LessNeq(F), LessNeq(R), IntervalSize(Base), IntervalSize(Indu), IntervalSize, $XS < US$, lemma USdecreasing(+1), CloseUS, CloseUS($n + 1$), AllNegated(Imply), ExistNegated(Imply), IntroExist(Helper), IntroExist, ExistMP, ExistMP2, TwiceExistMP, TwiceExistMP2, $EAE - MP$, AddAll, AddExist(Helper1), AddExist(Helper2), AddExist, AddExist(SimpleAnt), AddExist(Simple), AddEAE, $AEA -$ negated, $EEA -$ negated, Induction, leqAntisymmetry, leqTransitivity, leqAddition, leqMultiplication, Reciprocal, Equality, eqLeq, eqAddition, eqMultiplication, LeqMultiplicationLeft, LeqLessEq, LessLeq, FromLeqGeq, subLeqRight, subLeqLeft, $Leq + 1$, PositiveToRight(Leq), PositiveToRight(Leq)(1term), lemma negativeToRight(Leq), PositiveToLeft(Leq), negativeToLeft(Leq), negativeToLeft(Leq)(1term), LeqAdditionLeft, leqSubtraction, leqSubtractionLeft, thirdGeq, LeqNegated, AddEquations(Leq), MultiplyEquations(Leq), ThirdGeqSeries, LeqNeqLess, FromLess, ToLess, fromNotLess, toNotLess, NegativeLessPositive, leqLessTransitivity, LessLeqTransitivity, LessTransitivity, LessTotality, SubLessRight, SubLessLeft, SwitchTerms(x < y − z), SwitchTerms(x − y < z), LessAddition, LessAdditionLeft, LessMultiplication, LessMultiplicationLeft, LessDivision, PositiveToRight(Less), PositiveToLeft(Less), NegativeToLeft(Less), NegativeToRight(Less), AddEquations(Less), AddEquations(LeqLess), reciprocalToLeft(Less), LessNegated, PositiveNonzero, PositiveNegated, NonpositiveNegated, NegativeNegated, NonnegativeNegated, PositiveHalved, PositiveInverted, NonnegativeNumerical, NegativeNumerical, PositiveNumerical, lemma nonpositiveNumerical, |0| = 0, 0 <= |x|, x <= |x|, FromPositiveNumerical, SameNumerical, SignNumerical(+), SignNumerical, ToNumericalLess, FromNumericalGreater, NumericalDifference, NumericalDifferenceLess(Helper), NumericalDifferenceLess, SplitNumericalSumHelper, splitNumericalSum(++), splitNumericalSum(−−), splitNumericalSum(+ − small), splitNumericalSum(+ − big), splitNumericalSum(+−), splitNumericalSum(−+), splitNumericalSum, SplitNumericalProduct(++), SplitNumericalProduct(+−), SplitNumericalProduct, insertMiddleTerm(Numerical), insertTwoMiddleTerms(Numerical), Three2twoTerms, Three2threeTerms, Three2twoFactors, Three2threeFactors, Times(−1), Times(−1)Left, MaxLeq(1), MaxLeq(2), LessThanMax, x + y = zBackwards, x ∗ y = zBackwards, x = x + (y − y), x = x + y − y, x = x ∗ y ∗ (1/y), insertMiddleTerm(Sum), insertTwoMiddleTerms(Sum), insertMiddleTerm(Difference), x ∗ 0 + x = x, x ∗ 0 = 0, NonnegativeFactors, NonzeroFactors, PositiveFactors, PlusTimesMinus, MinusTimesMinus, (−1) ∗ (−1) + (−1) ∗ 1 = 0, (−1) ∗ (−1) = 1, 0 < 1Helper, 0 < 1, 0 < 2, 0 < 3, 0 < 1/2, 0 < 1/3, TwoWholes, ThreeWholes, TwoHalves, ThreeThirds, −x − y = −(x + y), −x ∗ y = −(x ∗ y), −0 = 0, SFsymmetry, SFtransitivity, f2R(Plus), f2R(Times), << TransitivityHelper(Q), << Transitivity, <<== Reflexivity, <<== AntisymmetryHelper(Q), FromNot < f(Weak)(Helper), FromNot < f(Weak), FromNot < f(Strong)(Helper2), FromNot < f(Strong)(Helper), FromNot < f(Strong), fromNotSameF(Strongest)(Helper2), fromNotSameF(Strongest)(Helper), fromNotSameF(Strongest), ToLess(F)(Helper), ToLess(F), FromNot <<, ToLess(R), LeqTotality(R), FromNotSameF(Weak)(Helper), FromNotSameF(Weak), FromNotLess(F), == Addition, == AdditionLeft, Fpart − Bounded(Base), Fpart − Bounded(InduHelper), Fpart − Bounded(Indu), Fpart − Bounded, F − Bounded(Helper), F − Bounded, SameFmultiplication(Helper), SameFmultiplication, EqMultiplication(R), EqMultiplicationLeft(R), x ∗ 0 = 0(F), x ∗ 0 = 0(R), LessMultiplication(F)(Helper2), LessMultiplication(F)(Helper), LessMultiplication(F), LessMultiplication(R), LeqMultiplication(R), PlusAssociativity(F), Plus0(F), PlusCommutativity(F), TimesAssociativity(F), Times1f, Cauchy(2)(Helper), Cauchy(2), ReciprocalFnonzero, ReciprocalFnynonzero, (Eventually = f)2sameF(Helper), (Eventually = f)2sameF, FromNotSameF(Strong)(Helper2), FromNotSameF(Strong)(Helper), FromNotSameF(Strong), SameFreciprocal(Helper), SameFreciprocal, From!! ==, Reciprocal(R), TimesCommutativity(F), Distribution(F), FromMax(1), FromMax(2), ToNegatedAnd, PositiveToRight(Less)(1term), (A)to(E), lemma ==Transitivity4, Plus0Left(R), x = x + (y − y)(R), x = x + y − y(R), PositiveToRight(Eq)(R), SubtractEquations(R), NeqAddition(R), EqAdditionLeft(R), Three2twoTerms(R), PositiveToRight(Less)(R), Three2threeTerms(R), PositiveToRight(Less)(1term)(R), ToLeq(Advanced)(R), LeqNeqLess(R), SubLeqLeft(R), LeqLessTransitivity(R), NegativeToLeft(Eq)(R), NegativeToRight(Less)(R), !! == Symmetry, NegativeToRight(Eq)(R), NegativeToRight(Eq)(1term)(R), DoubleMinus(R), UniqueNegative(R), SubtractEquationsLeft(R), EqNegated(R), NeqNegated(R), LeqNegated(R), LessNegated(R), −0 = 0(R), NegativeNegated(R), FromLeqGeq(R), SubLeqRight(R), FromLess(R), NonnegativeNumerical(R), NegativeNumerical(R), 0 <= |x|(R), PositiveNegated(R), AddEquations(R), DistributionOut(R), == Transitivity5, x ∗ 0 + x = x(R), x ∗ 0 = 0(R)(fff), Times(−1)(R), Times(−1)Left(R), −x − y = −(x + y)(R), LessTotality(R), PositiveNumerical(R), SignNumerical(+)(R), SameNumerical(R), MinusNegated(R), SignNumerical(R), NumericalDifference(R), x <= |x|(R), USlimitIsUpperBound(Helper), USlimitIsUpperBound, (−1) ∗ (−1) + (−1) ∗ 1 = 0(R), (−1) ∗ (−1) = 1(R), 0 < 1Helper(R), 0 < 1(R), ExpZero(Exact)(R), PositiveBase(R)(Base), Three2twoFactors(R), x = x ∗ y ∗ (1/y)(R), NeqMultiplication(R), LessTransitivity(R), 0 < 2(R), SameExp(R)(Base), SameExp(R)(Indu), SameExp(R), SubNeqLeft(R), SubNeqRight(R), NonzeroFactors(R), NonnegativeFactors(R), PositiveFactors(R), LessDivision(R), 0 < 1/2(R), PositiveToRight(Eq)(1term)(R), Exp(+1)(R), PositiveBase(R)(Indu), PositiveBase(R), −x ∗ y = −(x ∗ y)(R), PositiveToLeft(Eq)(R), Times1Left(R), == Transitivity6, x + x = 2 ∗ x(R), (1/2)x + (1/2)x = x(R), DistributionOut(Minus)(R), (1/2)(x + y) − x = (1/2)(y − x)(R), IntervalSize(R)(Base), LessMultiplicationLeft(R), NegativeToLeft(Less)(R), NegativeToLeft(Less)(1term)(R), y − (1/2)(x + y) = (1/2)(y − x)(R), IntervalSize(R)(Indu), IntervalSize(R), XSlessUS(R), USdecreasing(+1)(R), ExpUnbounded(Base), ExpUnbounded(Indu), ExpUnbounded, 1 <= x + 1(N), NonzeroProduct(2)(R), PositiveNonzero(R), NonreciprocalToRight(Eq)(1term)(R), ExpNonzero(Base), ExpNonzero(Indu), ExpNonzero, ExpNonzero(2), MultiplyEquations(R), HalfBase(Base), HalfBase(Indu), HalfBase, Three2threeFactors(R), x ∗ y = zBackwards(R), PositiveInverted(R), ReciprocalToRight(Less)(R), ReciprocalToRight(Less)(1term)(R), NonreciprocalToLeft(Less)(R), 1 < x ∗ y(R), SwitchFactors(1/x < y)(R), SmallHalving, IntervalSize(anyPositive), USdecreasing(+n)(Base), USdecreasing(+n)(Indu), USdecreasing(+n), USdecreasing, LeqAdditionLeft(R), ToNotLess(R), LimitOfUSIsLeq, SubtractEquations(Less)(R), SubtractEquationsLeft(Less)(R), LessNegated(Negative)(R), FromNegatedAnd(Imply), RemoveDoubleNeg(Consequent), FromNotUpperBound, DistributionOut, DistributionOutLeft, DistributionLeft, LeqNUB, USlimitIsLeastUpperBound(Helper), USlimitIsLeastUpperBound, FromNotLess(R), ExistMP3, GreaterPositive(N), ysFClose(Helper), ysFClose, ysFCauchy(Helper), ysFCauchy, CartProdIsRelation, FromSubset, SubsetIsRelation, ToSeries, FromSeries, SeriesSubsetCP, ValueType, RemoveOr, FromSingleton, InPair(1), InPair(2), SameMember(2), ToBinaryUnion(1), ToBinaryUnion(2), FromOrderedPair(TwoLevels), ToCartProd(Helper), ToCartProd, NonreciprocalToRight(Eq), NonreciprocalToLeft(Eq)(1term), SameReciprocal, CPseparationIsRelation, OrderedPairEquality, ReciprocalIsFunction, ReciprocalIsTotal, ReciprocalIsRationalSeries, CrsIsRelation, CrsIsFunction, CrsIsTotal, CrsIsSeries, CrsLookup, 0f, 1f, ToSingleton, FromSameSingleton, SingletonmembersEqual, UnequalsNotInSingleton, NonsingletonmembersUnequal, FromOrderedPair, FromOrderedPair(1), FromOrderedPair(2), FromCartProd, FromCartProd(1), FromCartProd(2), sameOrderedPair, InSeriesHelper, InSeries, To = f(Subset)(Helper), To = f(Subset), To = f, productIsFunction, productIsTotal, ProductIsRationalSeries, TimesF, $-x + (1/2)x = -(1/2)x$, PositiveTripled, PositiveDividedBy3, $|x - x| = 0$, $1 < 2$, $1/3 < 2/3$, $(1/3)x + (1/3)x = (2/3)x$, $(2/3)x + (1/3)x = x$, $-x + (2/3)x = -(1/3)x$, $-(1/3)x - (1/3)x = -(2/3)x$, $-x + (1/3)x = -(2/3)x$, PreserveLessGreater, ClosetolessIsLess, SubLessLeft(F), SubLessLeft(R), ClosetogreaterIsGreater, SubLessRight(F), SubLessRight(R), Tester1, Tester2, Tester3, Tester4, Tester5, Tester6,

Nat($*$)

$[\mathrm{Nat}(x) \stackrel{\text{macro}}{\to} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\mathrm{Nat}(x) \ddot{=} \lambda c.\lceil x\rceil \in_t (\lceil V_{2n}\rceil :: \lceil\mathcal{M}\rceil :: \lceil\mathcal{N}\rceil :: T)]\rceil)]$

$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$

$[\langle a \equiv b | x {:==} t\rangle_{\mathrm{Me}} \stackrel{\text{macro}}{\to} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\langle a \equiv b | x {:==} t\rangle_{\mathrm{Me}} \ddot{=} \langle\lceil a\rceil \equiv^1 \lceil b\rceil | \lceil x\rceil {:==} \lceil t\rceil\rangle_{\mathrm{Me}}]\rceil)]$

$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}}$

$[\langle a \equiv^1 b | x {:==} t\rangle_{\mathrm{Me}} \stackrel{\text{val}}{\to} a!x!t!$
$\mathrm{If}(\mathrm{If}(b \stackrel{r}{=} \lceil\forall_{\mathrm{obj}} u \colon v\rceil, b^1 \stackrel{t}{=} x, F), a \stackrel{t}{=} b,$
$\mathrm{If}(b \stackrel{t}{=} x, a \stackrel{t}{=} t, \mathrm{If}($
$a \stackrel{r}{=} b, \langle a^t \equiv^* b^t | x {:==} t\rangle_{\mathrm{Me}}, F)))]$

$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}}$

$[\langle \mathsf{a} \equiv^* \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Me}} \stackrel{\mathrm{val}}{\to} \mathsf{b}!\mathsf{x}!\mathsf{t}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\langle \mathsf{a}^{\mathrm{h}} \equiv^1 \mathsf{b}^{\mathrm{h}} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Me}}, \langle \mathsf{a}^{\mathrm{t}} \equiv^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Me}}, \mathsf{F}))]$

lemma plus0Left

[lemma plus0Left $\stackrel{\mathrm{tex}}{\to}$ “plus0Left”]

lemma times1Left

[lemma times1Left $\stackrel{\mathrm{tex}}{\to}$ “times1Left”]

lemma eqMultiplicationLeft

[lemma eqMultiplicationLeft $\stackrel{\mathrm{tex}}{\to}$ “EqMultiplicationLeft”]

lemma eqLeq(R)

[lemma eqLeq(R) $\stackrel{\mathrm{tex}}{\to}$ “eqLeq(R)”]

lemma =f to sameF

[lemma =f to sameF $\stackrel{\mathrm{tex}}{\to}$ “=fToSameF ”]

lemma plusF(Sym)

[lemma plusF(Sym) $\stackrel{\mathrm{tex}}{\to}$ “PlusF(Sym)”]

lemma timesF(Sym)

[lemma timesF(Sym) $\stackrel{\mathrm{tex}}{\to}$ “TimesF(Sym)”]

$*(\mathrm{exp})*$

[x(exp)y $\stackrel{\text{tex}}{\to}$ "(#1.
(expARGH!) #2.
)"]

sup

[sup $\stackrel{\text{prio}}{\to}$
Preassociative
[sup], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [\$ $*$ \$],
[**flush left** $[*]$], [x], [y], [z], $[[* \bowtie *]]$, $[[* \stackrel{*}{\to} *]]$, [pyk], [tex], [name], [prio], $[*]$, [T],
$[\mathrm{if}(*,*,*)]$, $[[* \stackrel{*}{\Rightarrow} *]]$, [val], [claim], $[\bot]$, $[\mathrm{f}(*)]$, $[(*)^{\mathrm{I}}]$, [F], $[\underline{0}]$, $[\underline{1}]$, $[\underline{2}]$, $[\underline{3}]$, $[\underline{4}]$, $[\underline{5}]$, $[\underline{6}]$,
$[\underline{7}]$, $[\underline{8}]$, $[\underline{9}]$, [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],
[k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], $[(*)^{\mathrm{M}}]$, $[\mathrm{If}(*,*,*)]$,
$[\mathrm{array}\{*\} * \mathrm{end\ array}]$, [l], [c], [r], [empty], $[\langle * \mid * := * \rangle]$, $[\mathcal{M}(*)]$, $[\tilde{\mathcal{U}}(*)]$, $[\mathcal{U}(*)]$,
$[\mathcal{U}^{\mathrm{M}}(*)]$, [**apply**$(*,*)$], [$\mathbf{apply}_1(*,*)$], $[\mathrm{identifier}(*)]$, $[\mathrm{identifier}_1(*,*)]$, [array-plus$(*,*)$], [array-remove$(*,*,*)$], [array-put$(*,*,*,*)$], [array-add$(*,*,*,*,*)$],
$[\mathrm{bit}(*,*)]$, $[\mathrm{bit}_1(*,*)]$, [rack], ["vector"], ["bibliography"], ["dictionary"],
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],
$[\mathcal{E}(*,*,*)]$, $[\mathcal{E}_2(*,*,*,*,*)]$, $[\mathcal{E}_3(*,*,*,*)]$, $[\mathcal{E}_4(*,*,*,*)]$, [**lookup**$(*,*,*)$],
[**abstract**$(*,*,*,*)$], $[\lceil * \rceil]$, $[\mathcal{M}(*,*,*)]$, $[\mathcal{M}_2(*,*,*,*)]$, $[\mathcal{M}^*(*,*,*)]$, [macro],
$[\mathrm{s}_0]$, [**zip**$(*,*)$], [$\mathbf{assoc}_1(*,*,*)$], $[(*)^{\mathsf{P}}]$, [self], $[[* \ddot{=} *]]$, $[[* \dot{=} *]]$, $[[* \acute{=} *]]$,
$[[* \stackrel{\text{pyk}}{=} *]]$, $[[* \stackrel{\text{tex}}{=} *]]$, $[[* \stackrel{\text{name}}{=} *]]$, [**Priority table**$[*]$], $[\tilde{\mathcal{M}}_1]$, $[\tilde{\mathcal{M}}_2(*)]$, $[\tilde{\mathcal{M}}_3(*)]$,
$[\tilde{\mathcal{M}}_4(*,*,*,*)]$, $[\tilde{\mathcal{M}}(*,*,*)]$, $[\tilde{\mathcal{Q}}(*,*,*)]$, $[\tilde{\mathcal{Q}}_2(*,*,*)]$, $[\tilde{\mathcal{Q}}_3(*,*,*,*)]$, $[\tilde{\mathcal{Q}}^*(*,*,*)]$,
$[(*)]$, $[(*)]$, $[\mathrm{display}(*)]$, $[\mathrm{statement}(*)]$, $[[*]^{\cdot}]$, $[[*]^{-}]$, [**aspect**$(*,*)$],
[**aspect**$(*,*,*)$], $[\langle * \rangle]$, [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], $[\mathrm{let}_2(*,*)]$, $[\mathrm{let}_1(*,*)]$,
$[[* \stackrel{\text{claim}}{=} *]]$, [checker], [**check**$(*,*)$], [$\mathbf{check}_2(*,*,*)$], [$\mathbf{check}_3(*,*,*)$],
[$\mathbf{check}^*(*,*)$], [$\mathbf{check}^*_2(*,*,*)$], $[[*]^{\cdot}]$, $[[*]^{-}]$, $[[*]^{\circ}]$, [msg], $[[* \stackrel{\text{msg}}{=} *]]$, [<stmt>],
[stmt], $[[* \stackrel{\text{stmt}}{=} *]]$, [HeadNil′], [HeadPair′], [Transitivity′], $[\perp\!\!\!\perp]$, [Contra′], $[\mathrm{T}'_{\mathrm{E}}]$,
$[\mathrm{L}_1]$, $[\underline{*}]$, $[\mathcal{A}]$, $[\mathcal{B}]$, $[\mathcal{C}]$, $[\mathcal{D}]$, $[\mathcal{E}]$, $[\mathcal{F}]$, $[\mathcal{G}]$, $[\mathcal{H}]$, $[\mathcal{I}]$, $[\mathcal{J}]$, $[\mathcal{K}]$, $[\mathcal{L}]$, $[\mathcal{M}]$, $[\mathcal{N}]$, $[\mathcal{O}]$, $[\mathcal{P}]$, $[\mathcal{Q}]$,
$[\mathcal{R}]$, $[\mathcal{S}]$, $[\mathcal{T}]$, $[\mathcal{U}]$, $[\mathcal{V}]$, $[\mathcal{W}]$, $[\mathcal{X}]$, $[\mathcal{Y}]$, $[\mathcal{Z}]$, $[\langle * \mid * := * \rangle]$, $[\langle^* * \mid * := * \rangle]$, $[\emptyset]$, [Remainder],
$[(*)^{\mathsf{v}}]$, $[\mathrm{intro}(*,*,*,*)]$, $[\mathrm{intro}(*,*,*)]$, $[\mathrm{error}(*,*)]$, $[\mathrm{error}_2(*,*)]$, $[\mathrm{proof}(*,*,*)]$,
$[\mathrm{proof}_2(*,*)]$, $[\mathcal{S}(*,*)]$, $[\mathcal{S}^{\mathrm{I}}(*,*)]$, $[\mathcal{S}^{\rhd}(*,*)]$, $[\mathcal{S}^{\rhd}_1(*,*,*)]$, $[\mathcal{S}^{\mathrm{E}}(*,*)]$, $[\mathcal{S}^{\mathrm{E}}_1(*,*,*)]$,
$[\mathcal{S}^{+}(*,*)]$, $[\mathcal{S}^{+}_1(*,*,*)]$, $[\mathcal{S}^{-}(*,*)]$, $[\mathcal{S}^{-}_1(*,*,*)]$, $[\mathcal{S}^{*}(*,*)]$, $[\mathcal{S}^{*}_1(*,*,*)]$,
$[\mathcal{S}^{*}_2(*,*,*,*)]$, $[\mathcal{S}^{@}(*,*)]$, $[\mathcal{S}^{@}_1(*,*,*)]$, $[\mathcal{S}^{\vdash}(*,*)]$, $[\mathcal{S}^{\vdash}_1(*,*,*,*)]$, $[\mathcal{S}^{\Vdash}(*,*)]$,
$[\mathcal{S}^{\Vdash}_1(*,*,*,*)]$, $[\mathcal{S}^{\mathrm{i.e.}}(*,*)]$, $[\mathcal{S}^{\mathrm{i.e.}}_1(*,*,*,*)]$, $[\mathcal{S}^{\mathrm{i.e.}}_2(*,*,*,*,*)]$, $[\mathcal{S}^{\forall}(*,*)]$,
$[\mathcal{S}^{\forall}_1(*,*,*,*)]$, $[\mathcal{S}^{;}(*,*)]$, $[\mathcal{S}^{;}_1(*,*,*)]$, $[\mathcal{S}^{;}_2(*,*,*,*)]$, $[\mathcal{T}(*)]$, $[\mathrm{claims}(*,*,*)]$,
$[\mathrm{claims}_2(*,*,*)]$, [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]],

[[$\ast$ **lemma** $\ast{:}\,\ast$]], [[$\ast$ **antilemma** $\ast{:}\,\ast$]], [[$\ast$ **rule** $\ast{:}\,\ast$]], [[$\ast$ **antirule** $\ast{:}\,\ast$]], [verifier], [$\mathcal{V}_1(\ast)$], [$\mathcal{V}_2(\ast,\ast)$], [$\mathcal{V}_3(\ast,\ast,\ast,\ast)$], [$\mathcal{V}_4(\ast,\ast)$], [$\mathcal{V}_5(\ast,\ast,\ast,\ast)$], [$\mathcal{V}_6(\ast,\ast,\ast,\ast)$], [$\mathcal{V}_7(\ast,\ast,\ast,\ast)$], [Cut$(\ast,\ast)$], [$\mathrm{Head}_\oplus(\ast)$], [$\mathrm{Tail}_\oplus(\ast)$], [$\mathrm{rule}_1(\ast,\ast)$], [rule$(\ast,\ast)$], [Rule tactic], [Plus$(\ast,\ast)$], [[**Theory** $\ast$]], [$\mathrm{theory}_2(\ast,\ast)$], [$\mathrm{theory}_3(\ast,\ast)$], [$\mathrm{theory}_4(\ast,\ast,\ast)$], [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil], [HeadPair], [Transitivity], [Contra], [$\mathrm{T_E}$], [ragged right], [ragged right expansion], [parm$(\ast,\ast,\ast)$], [parm$^\ast(\ast,\ast,\ast)$], [inst$(\ast,\ast)$], [inst$^\ast(\ast,\ast)$], [occur$(\ast,\ast,\ast)$], [occur$^\ast(\ast,\ast,\ast)$], [unify$(\ast=\ast,\ast)$], [unify$^\ast(\ast=\ast,\ast)$], [$\mathrm{unify}_2(\ast=\ast,\ast)$], [$\mathrm{L_a}$], [$\mathrm{L_b}$], [$\mathrm{L_c}$], [$\mathrm{L_d}$], [$\mathrm{L_e}$], [$\mathrm{L_f}$], [$\mathrm{L_g}$], [$\mathrm{L_h}$], [$\mathrm{L_i}$], [$\mathrm{L_j}$], [$\mathrm{L_k}$], [$\mathrm{L_l}$], [$\mathrm{L_m}$], [$\mathrm{L_n}$], [$\mathrm{L_o}$], [$\mathrm{L_p}$], [$\mathrm{L_q}$], [$\mathrm{L_r}$], [$\mathrm{L_s}$], [$\mathrm{L_t}$], [$\mathrm{L_u}$], [$\mathrm{L_v}$], [$\mathrm{L_w}$], [$\mathrm{L_x}$], [$\mathrm{L_y}$], [$\mathrm{L_z}$], [$\mathrm{L_A}$], [$\mathrm{L_B}$], [$\mathrm{L_C}$], [$\mathrm{L_D}$], [$\mathrm{L_E}$], [$\mathrm{L_F}$], [$\mathrm{L_G}$], [$\mathrm{L_H}$], [$\mathrm{L_I}$], [$\mathrm{L_J}$], [$\mathrm{L_K}$], [$\mathrm{L_L}$], [$\mathrm{L_M}$], [$\mathrm{L_N}$], [$\mathrm{L_O}$], [$\mathrm{L_P}$], [$\mathrm{L_Q}$], [$\mathrm{L_R}$], [$\mathrm{L_S}$], [$\mathrm{L_T}$], [$\mathrm{L_U}$], [$\mathrm{L_V}$], [$\mathrm{L_W}$], [$\mathrm{L_X}$], [$\mathrm{L_Y}$], [$\mathrm{L_Z}$], [$\mathrm{L_?}$], [Reflexivity], [$\mathrm{Reflexivity}_1$], [Commutativity], [$\mathrm{Commutativity}_1$], [<tactic>], [tactic], [[$\ast \stackrel{\text{tactic}}{=} \ast$]], [$\mathcal{P}(\ast,\ast,\ast)$], [$\mathcal{P}^\ast(\ast,\ast,\ast)$], [$\mathrm{p_0}$], [$\mathrm{conclude}_1(\ast,\ast)$], [$\mathrm{conclude}_2(\ast,\ast,\ast)$], [$\mathrm{conclude}_3(\ast,\ast,\ast,\ast)$], [$\mathrm{conclude}_4(\ast,\ast)$], [check], [[$\ast \stackrel{\circ}{=} \ast$]], [RootVisible$(\ast)$], [A], [R], [C], [T], [L], [$\{\ast\}$], [$\overline{\ast}$], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [x], [y], [z], [$\langle \ast\equiv\ast \mid \ast:=\ast\rangle$], [$\langle \ast\equiv^0\ast \mid \ast:=\ast\rangle$], [$\langle \ast\equiv^1\ast \mid \ast:=\ast\rangle$], [$\langle \ast\equiv^\ast\ast \mid \ast:=\ast\rangle$], [Ded$(\ast,\ast)$], [$\mathrm{Ded}_0(\ast,\ast)$], [$\mathrm{Ded}_1(\ast,\ast,\ast)$], [$\mathrm{Ded}_2(\ast,\ast,\ast)$], [$\mathrm{Ded}_3(\ast,\ast,\ast,\ast)$], [$\mathrm{Ded}_4(\ast,\ast,\ast,\ast)$], [$\mathrm{Ded}^\ast_4(\ast,\ast,\ast,\ast)$], [$\mathrm{Ded}_5(\ast,\ast,\ast)$], [$\mathrm{Ded}_6(\ast,\ast,\ast,\ast)$], [$\mathrm{Ded}^\ast_6(\ast,\ast,\ast,\ast)$], [$\mathrm{Ded}_7(\ast)$], [$\mathrm{Ded}_8(\ast,\ast)$], [$\mathrm{Ded}^\ast_8(\ast,\ast)$], [S], [Neg], [MP], [Gen], [Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1$'$], [A2$'$], [A4$'$], [A5$'$], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop $3.2\mathrm{e}_1$], [Prop $3.2\mathrm{e}_2$], [Prop 3.2e], [Prop $3.2\mathrm{f}_1$], [Prop $3.2\mathrm{f}_2$], [Prop 3.2f], [Prop $3.2\mathrm{g}_1$], [Prop $3.2\mathrm{g}_2$], [Prop 3.2g], [Prop $3.2\mathrm{h}_1$], [Prop $3.2\mathrm{h}_2$], [Prop 3.2h], [$\mathrm{Block}_1(\ast,\ast,\ast)$], [$\mathrm{Block}_2(\ast)$], [kvanti], [UniqueMember], [UniqueMember(Type)], [SameSeries], [A4], [SameMember], [Qclosed(Addition)], [Qclosed(Multiplication)], [FromCartProd(1)], [1rule fromCartProd(2)], [constantRationalSeries$(\ast)$], [cartProd$(\ast)$], [Power$(\ast)$], [binaryUnion$(\ast,\ast)$], [SetOfRationalSeries], [IsSubset$(\ast,\ast)$], [(p$\ast,\ast$)], [(s$\ast$)], [$(\cdots)$], [`Objekt-var`], [`Ex-var`], [`Ph-var`], [`Værdi`], [`Variabel`], [Op$(\ast)$], [Op$(\ast,\ast)$], [$\ast \stackrel{..}{==} \ast$], [ContainsEmpty$(\ast)$], [Nat$(\ast)$], [Dedu$(\ast,\ast)$], [$\mathrm{Dedu}_0(\ast,\ast)$], [$\mathrm{Dedu}_\mathrm{s}(\ast,\ast,\ast)$], [$\mathrm{Dedu}_1(\ast,\ast,\ast)$], [$\mathrm{Dedu}_2(\ast,\ast,\ast)$], [$\mathrm{Dedu}_3(\ast,\ast,\ast,\ast)$], [$\mathrm{Dedu}_4(\ast,\ast,\ast,\ast)$], [$\mathrm{Dedu}^\ast_4(\ast,\ast,\ast,\ast)$], [$\mathrm{Dedu}_5(\ast,\ast,\ast)$], [$\mathrm{Dedu}_6(\ast,\ast,\ast,\ast)$], [$\mathrm{Dedu}^\ast_6(\ast,\ast,\ast,\ast)$], [$\mathrm{Dedu}_7(\ast)$], [$\mathrm{Dedu}_8(\ast,\ast)$], [$\mathrm{Dedu}^\ast_8(\ast,\ast)$], [$\mathrm{Ex}_1$], [$\mathrm{Ex}_2$], [Ex3], [$\mathrm{Ex}_{10}$], [$\mathrm{Ex}_{20}$], [$\ast_{\mathrm{Ex}}$], [$\ast^{\mathrm{Ex}}$], [$\langle \ast\equiv\ast \mid \ast:==\ast\rangle_{\mathrm{Ex}}$], [$\langle \ast\equiv^0\ast \mid \ast:==\ast\rangle_{\mathrm{Ex}}$], [$\langle \ast\equiv^1\ast \mid \ast:==\ast\rangle_{\mathrm{Ex}}$], [$\langle \ast\equiv^\ast\ast \mid \ast:==\ast\rangle_{\mathrm{Ex}}$], [$\mathrm{ph}_1$], [$\mathrm{ph}_2$], [$\mathrm{ph}_3$], [$\ast_{\mathrm{Ph}}$], [$\ast^{\mathrm{Ph}}$], [$\langle \ast\equiv\ast \mid \ast:==\ast\rangle_{\mathrm{Ph}}$], [$\langle \ast\equiv^0\ast \mid \ast:==\ast\rangle_{\mathrm{Ph}}$], [$\langle \ast\equiv^1\ast \mid \ast:==\ast\rangle_{\mathrm{Ph}}$], [$\langle \ast\equiv^\ast\ast \mid \ast:==\ast\rangle_{\mathrm{Ph}}$], [$\langle \ast\equiv\ast \mid \ast:==\ast\rangle_{\mathrm{Me}}$], [$\langle \ast\equiv^1\ast \mid \ast:==\ast\rangle_{\mathrm{Me}}$], [$\langle \ast\equiv^\ast\ast \mid \ast:==\ast\rangle_{\mathrm{Me}}$], [bs], [OBS], [$\mathcal{BS}$], [Ø], [SystemQ], [MP], [Gen], [Repetition], [Neg], [Ded], [ExistIntro], [Extensionality], [Ødef], [PairDef], [UnionDef], [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg], [AndCommutativity], [AutoImply], [Contrapositive], [FirstConjunct], [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity], [IffFirst], [IffSecond], [ImplyTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4], [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1], [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],

[Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power], [SubsetInPower], [HelperPowerIsSub], [PowerIsSub], [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality], [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality], [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry], [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive], [ERisSymmetric], [ERisTransitive], [∅isSubset], [HelperMemberNot∅], [MemberNot∅], [HelperUnique∅], [Unique∅], [==Reflexivity], [==Symmetry], [Helper==Transitivity], [==Transitivity], [HelperTransferNotEq], [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset], [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset], [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection], [AutoMember], [HelperEqSysNot∅], [EqSysNot∅], [HelperEqSubset], [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary], [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset], [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImply], [BSsubset], [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)], [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ϵ)], [(ϵ)$_1$], [(ϵ2)], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)], [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)], [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X_1], [X_2], [Y_1], [Y_2], [V_1], [V_2], [V_3], [V_4], [V_{2n}], [M_1], [M_2], [N_1], [N_2], [N_3], [ϵ], [ϵ1], [ϵ2], [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)], [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)], [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)], [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)], [(S1ob)], [(S2ob)], [ph_4], [ph_5], [ph_6], [NAT], [RATIONAL$_S$ERIES], [SERIES], [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1], [(−1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(− − 01)], [02], [01//02], [lemma plus0Left], [lemma times1Left], [lemma eqAdditionLeft], [lemma eqMultiplicationLeft], [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)], [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)], [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)], [Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom], [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom], [QisClosed(Reciprocal)(Imply)], [QisClosed(Reciprocal)], [QisClosed(Negative)(Imply)], [QisClosed(Negative)], [leqReflexivity], [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality], [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity], [plusCommutativity], [Negative], [plus0], [timesAssociativity], [timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1], [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)], [lemma =f to sameF], [lemma plusF(Sym)], [lemma timesF(Sym)], [Separation2formula(1)], [Separation2formula(2)], [IfThenElse(T)], [IfThenElse(F)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==], [From <<], [to <<], [FromInR], [PlusR], [PlusR(Sym)], [TimesR],

[TimesR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [UStelescope(Zero)], [UStelescope(Positive)], [EqAddition(R)], [Unminus(R)], [FromLimit], [ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [NatType], [RationalType], [SeriesType], [Max], [Numerical], [MemberOfSeries(Imply)], [JoinConjuncts(2conditions)], [prop lemma imply negation], [TND], [FromNegatedImply], [ToNegatedImply], [FromNegated(2 ∗ Imply)], [FromNegatedAnd], [FromNegatedOr], [ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 ∗ 2Disjuncts], [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2], [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)], [To!! ==], [ToNegatedDoubleImply], [ToNegatedAnd(1)], [AddNegatedAll], [(A)to(E)(Imply)], [(E)to(A)(Imply)], [(E)to(A)(Imply)], [ToNegatedAEA], [UniqueNegative], [DoubleMinus], [MinusNegated], [eqReflexivity], [eqSymmetry], [eqTransitivity], [eqTransitivity4], [eqTransitivity5], [eqTransitivity6], [AddEquations], [SubtractEquations], [SubtractEquationsLeft], [MultiplyEquations], [EqNegated], [PositiveToRight(Eq)], [PositiveToLeft(Eq)], [PositiveToLeft(Eq)(1term)], [NegativeToLeft(Eq)], [NonreciprocalToRight(Eq)(1term)], [DistributionOut(Minus)], [PositiveToRight(Eq)(1term)], [SameSeries(NumDiff)], [PlusAssociativity(4terms)], [LessNeq], [NeqSymmetry], [NeqNegated], [SubNeqRight], [SubNeqLeft], [NegativeToRight(Neq)(1term)], [NeqAddition], [NeqMultiplication], [NonzeroProduct(2)], [SwitchTerms(x <= y − z)], [NegativeToLeft(Less)(1term)], [(+1)IsPositive(N)], [(1/2)(x + y) − x = (1/2)(y − x)], [y − (1/2)(x + y) = (1/2)(y − x)], [ExpZero(Exact)], [SameExp(Base)], [SameExp(Indu)], [SameExp], [Exp(+1)], [PositiveBase(Base)], [PositiveBase(Indu)], [PositiveBase], [BSzero(Exact)], [SameBS(2)(Base)], [SameBS(2)(Indu)], [SameBS(2)], [BS(+1)], [BSbound(Exact)(Base)], [BSbound(Exact)(Indu)], [BSbound(Exact)], [BSbound], [UStelescope(Zero)(Exact)], [SameTelescope(2)(Base)], [SameTelescope(2)(Indu)], [SameTelescope(2)], [UStelescope(+1)], [TelescopeNumerical(Base)], [TelescopeNumerical(Indu)], [TelescopeNumerical], [TelescopeBound(Base)], [TelescopeBound(Indu)], [TelescopeBound], [LessNeq(F)(Helper)], [LessNeq(F)], [LessNeq(R)], [IntervalSize(Base)], [IntervalSize(Indu)], [IntervalSize], [XS < US], [lemma USdecreasing(+1)], [CloseUS], [CloseUS(n + 1)], [AllNegated(Imply)], [ExistNegated(Imply)], [IntroExist(Helper)], [IntroExist], [ExistMP], [ExistMP2], [TwiceExistMP], [TwiceExistMP2], [EAE − MP], [AddAll], [AddExist(Helper1)], [AddExist(Helper2)], [AddExist], [AddExist(SimpleAnt)], [AddExist(Simple)], [AddEAE], [AEA − negated], [EEA − negated], [Induction], [leqAntisymmetry], [leqTransitivity], [leqAddition], [leqMultiplication], [Reciprocal], [Equality], [eqLeq], [eqAddition], [eqMultiplication], [LeqMultiplicationLeft], [LeqLessEq], [LessLeq], [FromLeqGeq], [subLeqRight], [subLeqLeft], [Leq + 1], [PositiveToRight(Leq)], [PositiveToRight(Leq)(1term)], [lemma negativeToRight(Leq)], [PositiveToLeft(Leq)], [negativeToLeft(Leq)], [negativeToLeft(Leq)(1term)], [LeqAdditionLeft], [leqSubtraction], [leqSubtractionLeft], [thirdGeq], [LeqNegated], [AddEquations(Leq)], [MultiplyEquations(Leq)], [ThirdGeqSeries], [LeqNeqLess], [FromLess], [ToLess], [fromNotLess], [toNotLess], [NegativeLessPositive], [leqLessTransitivity], [LessLeqTransitivity], [LessTransitivity], [LessTotality], [SubLessRight], [SubLessLeft], [SwitchTerms(x < y − z)], [SwitchTerms(x − y < z)], [LessAddition], [LessAdditionLeft], [LessMultiplication], [LessMultiplicationLeft], [LessDivision], [PositiveToRight(Less)], [PositiveToLeft(Less)], [NegativeToLeft(Less)], [NegativeToRight(Less)], [AddEquations(Less)], [AddEquations(LeqLess)], [reciprocalToLeft(Less)], [LessNegated], [PositiveNonzero], [PositiveNegated], [NonpositiveNegated], [NegativeNegated], [NonnegativeNegated], [PositiveHalved], [PositiveInverted], [NonnegativeNumerical], [NegativeNumerical], [PositiveNumerical], [lemma nonpositiveNumerical], [|0| = 0], [0 <= |x|], [x <= |x|], [FromPositiveNumerical], [SameNumerical], [SignNumerical(+)], [SignNumerical], [ToNumericalLess], [FromNumericalGreater], [NumericalDifference], [NumericalDifferenceLess(Helper)], [NumericalDifferenceLess], [SplitNumericalSumHelper], [splitNumericalSum(++)], [splitNumericalSum(−−)], [splitNumericalSum(+ − small)], [splitNumericalSum(+ − big)], [splitNumericalSum(+−)], [splitNumericalSum(−+)], [splitNumericalSum], [SplitNumericalProduct(++)], [SplitNumericalProduct(+−)], [SplitNumericalProduct], [insertMiddleTerm(Numerical)], [insertTwoMiddleTerms(Numerical)], [Three2twoTerms], [Three2threeTerms], [Three2twoFactors], [Three2threeFactors], [Times(−1)], [Times(−1)Left], [MaxLeq(1)], [MaxLeq(2)], [LessThanMax], [x + y = zBackwards], [x ∗ y = zBackwards], [x = x + (y − y)], [x = x + y − y], [x = x ∗ y ∗ (1/y)], [insertMiddleTerm(Sum)], [insertTwoMiddleTerms(Sum)], [insertMiddleTerm(Difference)], [x ∗ 0 + x = x], [x ∗ 0 = 0], [NonnegativeFactors], [NonzeroFactors], [PositiveFactors], [PlusTimesMinus], [MinusTimesMinus], [(−1) ∗ (−1) + (−1) ∗ 1 = 0], [(−1) ∗ (−1) = 1], [0 < 1Helper], [0 < 1], [0 < 2], [0 < 3], [0 < 1/2], [0 < 1/3], [TwoWholes], [ThreeWholes], [TwoHalves], [ThreeThirds], [−x − y = −(x + y)], [−x ∗ y = −(x ∗ y)], [−0 = 0], [SFsymmetry], [SFtransitivity], [f2R(Plus)], [f2R(Times)], [<< TransitivityHelper(Q)], [<< Transitivity], [<<== Reflexivity], [<<== AntisymmetryHelper(Q)], [FromNot < f(Weak)(Helper)], [FromNot < f(Weak)], [FromNot < f(Strong)(Helper2)], [FromNot < f(Strong)(Helper)], [FromNot < f(Strong)], [fromNotSameF(Strongest)(Helper2)], [fromNotSameF(Strongest)(Helper)], [fromNotSameF(Strongest)], [ToLess(F)(Helper)], [ToLess(F)], [FromNot <<], [ToLess(R)], [LeqTotality(R)], [FromNotSameF(Weak)(Helper)], [FromNotSameF(Weak)], [FromNotLess(F)], [== Addition], [== AdditionLeft], [Fpart − Bounded(Base)], [Fpart − Bounded(InduHelper)],

[Fpart − Bounded(Indu)], [Fpart − Bounded], [F − Bounded(Helper)],
[F − Bounded], [SameFmultiplication(Helper)], [SameFmultiplication],
[EqMultiplication(R)], [EqMultiplicationLeft(R)], [x ∗ 0 = 0(F)], [x ∗ 0 = 0(R)],
[LessMultiplication(F)(Helper2)], [LessMultiplication(F)(Helper)],
[LessMultiplication(F)], [LessMultiplication(R)], [LeqMultiplication(R)],
[PlusAssociativity(F)], [Plus0(F)], [PlusCommutativity(F)],
[TimesAssociativity(F)], [Times1f], [Cauchy(2)(Helper)], [Cauchy(2)],
[ReciprocalFnonzero], [ReciprocalFnynonzero],
[(Eventually = f)2sameF(Helper)], [(Eventually = f)2sameF],
[FromNotSameF(Strong)(Helper2)], [FromNotSameF(Strong)(Helper)],
[FromNotSameF(Strong)], [SameFreciprocal(Helper)], [SameFreciprocal],
[From!! ==], [Reciprocal(R)], [TimesCommutativity(F)], [Distribution(F)],
[FromMax(1)], [FromMax(2)], [ToNegatedAnd], [PositiveToRight(Less)(1term)],
[(A)to(E)], [lemma ==Transitivity4], [Plus0Left(R)], [x = x + (y − y)(R)],
[x = x + y − y(R)], [PositiveToRight(Eq)(R)], [SubtractEquations(R)],
[NeqAddition(R)], [EqAdditionLeft(R)], [Three2twoTerms(R)],
[PositiveToRight(Less)(R)], [Three2threeTerms(R)],
[PositiveToRight(Less)(1term)(R)], [ToLeq(Advanced)(R)], [LeqNeqLess(R)],
[SubLeqLeft(R)], [LeqLessTransitivity(R)], [NegativeToLeft(Eq)(R)],
[NegativeToRight(Less)(R)], [!! == Symmetry], [NegativeToRight(Eq)(R)],
[NegativeToRight(Eq)(1term)(R)], [DoubleMinus(R)], [UniqueNegative(R)],
[SubtractEquationsLeft(R)], [EqNegated(R)], [NeqNegated(R)],
[LeqNegated(R)], [LessNegated(R)], [−0 = 0(R)], [NegativeNegated(R)],
[FromLeqGeq(R)], [SubLeqRight(R)], [FromLess(R)],
[NonnegativeNumerical(R)], [NegativeNumerical(R)], [0 <= |x|(R)],
[PositiveNegated(R)], [AddEquations(R)], [DistributionOut(R)],
[== Transitivity5], [x ∗ 0 + x = x(R)], [x ∗ 0 = 0(R)(fff)], [Times(−1)(R)],
[Times(−1)Left(R)], [−x − y = −(x + y)(R)], [LessTotality(R)],
[PositiveNumerical(R)], [SignNumerical(+)(R)], [SameNumerical(R)],
[MinusNegated(R)], [SignNumerical(R)], [NumericalDifference(R)],
[x <= |x|(R)], [USlimitIsUpperBound(Helper)], [USlimitIsUpperBound],
[(−1) ∗ (−1) + (−1) ∗ 1 = 0(R)], [(−1) ∗ (−1) = 1(R)], [0 < 1Helper(R)],
[0 < 1(R)], [ExpZero(Exact)(R)], [PositiveBase(R)(Base)],
[Three2twoFactors(R)], [x = x ∗ y ∗ (1/y)(R)], [NeqMultiplication(R)],
[LessTransitivity(R)], [0 < 2(R)], [SameExp(R)(Base)], [SameExp(R)(Indu)],
[SameExp(R)], [SubNeqLeft(R)], [SubNeqRight(R)], [NonzeroFactors(R)],
[NonnegativeFactors(R)], [PositiveFactors(R)], [LessDivision(R)], [0 < 1/2(R)],
[PositiveToRight(Eq)(1term)(R)], [Exp(+1)(R)], [PositiveBase(R)(Indu)],
[PositiveBase(R)], [−x ∗ y = −(x ∗ y)(R)], [PositiveToLeft(Eq)(R)],
[Times1Left(R)], [== Transitivity6], [x + x = 2 ∗ x(R)],
[(1/2)x + (1/2)x = x(R)], [DistributionOut(Minus)(R)],
[(1/2)(x + y) − x = (1/2)(y − x)(R)], [IntervalSize(R)(Base)],
[LessMultiplicationLeft(R)], [NegativeToLeft(Less)(R)],
[NegativeToLeft(Less)(1term)(R)], [y − (1/2)(x + y) = (1/2)(y − x)(R)],
[IntervalSize(R)(Indu)], [IntervalSize(R)], [XSlessUS(R)],
[USdecreasing(+1)(R)], [ExpUnbounded(Base)], [ExpUnbounded(Indu)],

[ExpUnbounded], [1 <= x + 1(N)], [NonzeroProduct(2)(R)], [PositiveNonzero(R)], [NonreciprocalToRight(Eq)(1term)(R)], [ExpNonzero(Base)], [ExpNonzero(Indu)], [ExpNonzero], [ExpNonzero(2)], [MultiplyEquations(R)], [HalfBase(Base)], [HalfBase(Indu)], [HalfBase], [Three2threeFactors(R)], [x * y = zBackwards(R)], [PositiveInverted(R)], [ReciprocalToRight(Less)(R)], [ReciprocalToRight(Less)(1term)(R)], [NonreciprocalToLeft(Less)(R)], [1 < x * y(R)], [SwitchFactors(1/x < y)(R)], [SmallHalving], [IntervalSize(anyPositive)], [USdecreasing(+n)(Base)], [USdecreasing(+n)(Indu)], [USdecreasing(+n)], [USdecreasing], [LeqAdditionLeft(R)], [ToNotLess(R)], [LimitOfUSIsLeq], [SubtractEquations(Less)(R)], [SubtractEquationsLeft(Less)(R)], [LessNegated(Negative)(R)], [FromNegatedAnd(Imply)], [RemoveDoubleNeg(Consequent)], [FromNotUpperBound], [DistributionOut], [DistributionOutLeft], [DistributionLeft], [LeqNUB], [USlimitIsLeastUpperBound(Helper)], [USlimitIsLeastUpperBound], [FromNotLess(R)], [ExistMP3], [GreaterPositive(N)], [ysFClose(Helper)], [ysFClose], [ysFCauchy(Helper)], [ysFCauchy], [CartProdIsRelation], [FromSubset], [SubsetIsRelation], [ToSeries], [FromSeries], [SeriesSubsetCP], [ValueType], [RemoveOr], [FromSingleton], [InPair(1)], [InPair(2)], [SameMember(2)], [ToBinaryUnion(1)], [ToBinaryUnion(2)], [FromOrderedPair(TwoLevels)], [ToCartProd(Helper)], [ToCartProd], [NonreciprocalToRight(Eq)], [NonreciprocalToLeft(Eq)(1term)], [SameReciprocal], [CPseparationIsRelation], [OrderedPairEquality], [ReciprocalIsFunction], [ReciprocalIsTotal], [ReciprocalIsRationalSeries], [CrsIsRelation], [CrsIsFunction], [CrsIsTotal], [CrsIsSeries], [CrsLookup], [0f], [1f], [ToSingleton], [FromSameSingleton], [SingletonmembersEqual], [UnequalsNotInSingleton], [NonsingletonmembersUnequal], [FromOrderedPair], [FromOrderedPair(1)], [FromOrderedPair(2)], [FromCartProd], [FromCartProd(1)], [FromCartProd(2)], [sameOrderedPair], [InSeriesHelper], [InSeries], [To = f(Subset)(Helper)], [To = f(Subset)], [To = f], [productIsFunction], [productIsTotal], [ProductIsRationalSeries], [TimesF], [−x + (1/2)x = −(1/2)x], [PositiveTripled], [PositiveDividedBy3], [|x − x| = 0], [1 < 2], [1/3 < 2/3], [(1/3)x + (1/3)x = (2/3)x], [(2/3)x + (1/3)x = x], [−x + (2/3)x = −(1/3)x], [−(1/3)x − (1/3)x = −(2/3)x], [−x + (1/3)x = −(2/3)x], [PreserveLessGreater], [ClosetolessIsLess], [SubLessLeft(F)], [SubLessLeft(R)], [ClosetogreaterIsGreater], [SubLessRight(F)], [SubLessRight(R)];

Preassociative

[Tester1], [Tester2], [Tester3], [Tester4], [Tester5], [Tester6];

Preassociative

[$*_\{*\}$], [$*$/indexintro($*, *, *, *$)], [$*$/intro($*, *, *$)], [$*$/bothintro($*, *, *, *, *$)], [$*$/nameintro($*, *, *, *$)], [$*'$], [$*[\,*\,]$], [$*[*\longrightarrow*]$], [$*[*\Rightarrow*]$], [$*0$], [$*1$], [0b], [$*$-color($*$)], [$*$-color*($*$)], [$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^0$], [$*^1$], [$*^2$], [$*^3$], [$*^4$], [$*^5$], [$*^6$], [$*^7$], [$*^8$], [$*^9$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*^{\mathrm{hide}}$];

Preassociative

[“ ∗ ”], [], [(∗)$^{\mathbf{t}}$], [string(∗) + ∗], [string(∗) ++ ∗], [
∗], [∗], [!∗], ["∗], [#∗], [$∗], [%∗], [&∗], [’∗], [(∗], [)∗], [∗∗], [+∗], [, ∗], [-∗], [.∗], [/∗],
[0∗], [1∗], [2∗], [3∗], [4∗], [5∗], [6∗], [7∗], [8∗], [9∗], [:∗], [; ∗], [<∗], [=∗], [>∗], [?∗],
[@∗], [A∗], [B∗], [C∗], [D∗], [E∗], [F∗], [G∗], [H∗], [I∗], [J∗], [K∗], [L∗], [M∗], [N∗],
[O∗], [P∗], [Q∗], [R∗], [S∗], [T∗], [U∗], [V∗], [W∗], [X∗], [Y∗], [Z∗], [[∗], [\∗], []∗], [ˆ∗],
[_∗], [‘∗], [a∗], [b∗], [c∗], [d∗], [e∗], [f∗], [g∗], [h∗], [i∗], [j∗], [k∗], [l∗], [m∗], [n∗], [o∗],
[p∗], [q∗], [r∗], [s∗], [t∗], [u∗], [v∗], [w∗], [x∗], [y∗], [z∗], [{∗], [|∗], [}∗], [˜∗],
[**Preassociative** ∗; ∗], [**Postassociative** ∗; ∗], [[∗], ∗], [priority ∗ end],
[newline ∗], [macro newline ∗], [MacroIndent(∗)];
Preassociative
[∗ ’ ∗], [∗ ‘ ∗];
Preassociative
[∗(exp)∗];
Preassociative
[∗′], [R(∗)], [− − R(∗)], [rec∗];
Preassociative
[∗/∗], [∗ ∩ ∗], [∗[∗]];
Preassociative
[∪∗], [∗ ∪ ∗], [P(∗)];
Preassociative
[{∗}], [StateExpand(∗, ∗, ∗)], [extractSeries(∗)], [SetOfSeries(∗)], [− − Macro(∗)],
[ExpandList(∗, ∗, ∗)], [∗ ∗ Macro(∗)], [+ + Macro(∗)], [<< Macro(∗)], [UB(∗, ∗)],
[LUB(∗, ∗)], [BS(∗, ∗)], [UStelescope(∗, ∗)], [(∗)], [|r ∗ |], [Limit(∗, ∗)], [Union(∗)],
[IsOrderedPair(∗, ∗, ∗)], [IsRelation(∗, ∗, ∗)], [isFunction(∗, ∗, ∗)], [IsSeries(∗, ∗)],
[IsNatural(∗, ∗)], [OrderedPair(∗, ∗)], [TypeNat(∗)], [TypeNat0(∗)],
[TypeRational(∗)], [TypeRational0(∗)], [TypeSeries(∗, ∗)], [Typeseries0(∗, ∗)];
Preassociative
[{∗, ∗}], [⟨∗, ∗⟩], [(−u∗)], [$-_{\mathrm{f}}$∗], [(− − ∗)], [1f/∗], [1fny/∗], [01//temp∗];
Preassociative
[∗(∗, ∗)], [ReflRel(∗, ∗)], [SymRel(∗, ∗)], [TransRel(∗, ∗)], [EqRel(∗, ∗)], [[∗ ∈ ∗]$_{∗}$],
[Partition(∗, ∗)];
Preassociative
[∗ · ∗], [∗ $\cdot_0$ ∗], [(∗ ∗ ∗)], [∗ $*_{\mathrm{f}}$ ∗], [∗ ∗ ∗∗];
Preassociative
[∗ + ∗], [∗ $+_0$ ∗], [∗ $+_1$ ∗], [∗ − ∗], [∗ $-_0$ ∗], [∗ $-_1$ ∗], [(∗ + ∗)], [(∗ − ∗)], [∗ $+_{\mathrm{f}}$ ∗],
[∗ $-_{\mathrm{f}}$ ∗], [∗ + +∗], [R(∗) − −R(∗)];
Preassociative
[∗ ∈ ∗];
Preassociative
[| ∗ |], [if(∗, ∗, ∗)], [Max(∗, ∗)], [Max(∗, ∗)];
Preassociative
[∗ = ∗], [∗ ≠ ∗], [∗ <= ∗], [∗ < ∗], [∗ $<_{\mathrm{f}}$ ∗], [∗ $\leq_{\mathrm{f}}$ ∗], [SF(∗, ∗)], [∗ == ∗],
[∗!! == ∗], [∗ << ∗], [∗ <<== ∗];
Preassociative
[∗ ∪ {∗}], [∗ ∪ ∗], [∗\{∗}];
Postassociative

$[* \therefore *], [* \underline{\therefore} *], [* \underline{::} *], [* \underline{+2*} *], [* :: *], [* +2* *];$
Postassociative
$[*, *];$
Preassociative
$[* \overset{B}{\approx} *], [* \overset{D}{\approx} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \overset{+}{\rightarrow} *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$
$[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{ free in } *], [* \text{ free in}^* *], [* \text{ free for } * \text{ in } *],$
$[* \text{ free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\text{var}}],$
$[*\#^0*], [*\#^1*], [*\#^**], [*{==}*], [* \subseteq *];$
Preassociative
$[\neg *], [\dot{\neg}\,(*)\text{n}], [* \notin *], [* \neq *];$
Preassociative
$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$
Preassociative
$[* \vee *], [* \parallel *], [* \ddot{\vee} *];$
Postassociative
$[* \dot{\vee} *];$
Preassociative
$[\exists *\colon *], [\forall *\colon *], [\forall_{\text{obj}} *\colon *], [\exists *\colon *];$
Postassociative
$[* \ddot{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \dot{\Leftrightarrow} *];$
Preassociative
$[\{\text{ph} \in * \mid *\}];$
Postassociative
$[* : *], [* \text{ spy } *], [*!*];$
Preassociative
$[* \begin{cases} * \\ * \end{cases}];$
Preassociative
$[\lambda * .*], [\Lambda * .*], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \ddot{=} * \text{ in } *];$
Preassociative
$[*\#*];$
Preassociative
$[*^{\text{I}}], [*^{\triangleright}], [*^{\text{V}}], [*^{+}], [*^{-}], [*^{*}];$
Preassociative
$[* @ *], [* \triangleright *], [* \Rrightarrow *], [* \gg *], [* \trianglerighteq *];$
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$
Preassociative
$[\forall *\colon *], [\Pi *\colon *];$
Postassociative
$[* \oplus *];$
Postassociative
$[*; *];$
Preassociative
$[* \text{ proves } *];$

Preassociative
[$*$ **proof of** $*:*$], [Line $*:*\gg *;*$], [Last line $*\gg *\,\Box$],
[Line $*$: Premise $\gg *;*$], [Line $*$: Side-condition $\gg *;*$], [Arbitrary $\gg *;*$],
[Local $\gg * = *;*$], [Begin $*;*$: End; $*$], [Last block line $*\gg *$;],
[Arbitrary $\gg *;*$];
Postassociative
[$* \mid *$];
Postassociative
[$*\ ,\ *$], [$*[\ *\ ]*$];
Preassociative
[$*\&*$];
Preassociative
[$*\backslash\backslash*$], [$*$ linebreak[4] $*$], [$*\backslash\backslash*$];]

[sup $\overset{\text{tex}}{\rightarrow}$ "sup"]

[sup $\overset{\text{pyk}}{\rightarrow}$ "sup"]

To!! ==

[To!! ==$\overset{\text{tex}}{\rightarrow}$ "To!!=="]

[To!! ==$\overset{\text{pyk}}{\rightarrow}$ "lemma to!!=="]

ToNegatedDoubleImply

[ToNegatedDoubleImply $\overset{\text{tex}}{\rightarrow}$ "ToNegatedDoubleImply"]

[ToNegatedDoubleImply $\overset{\text{pyk}}{\rightarrow}$ "prop lemma to negated double imply"]

ToNegatedAnd(1)

[ToNegatedAnd(1) $\overset{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}\,(\underline{a})\text{n} \vdash \underline{a} \vdash \text{FromContradiction} \rhd \underline{a} \rhd \dot{\neg}\,(\underline{a})\text{n} \gg \dot{\neg}\,(\underline{b})\text{n}; \forall \underline{a}\colon \forall \underline{b}\colon \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}\,(\underline{a})\text{n} \vdash \underline{a} \vdash \dot{\neg}\,(\underline{b})\text{n} \gg \dot{\neg}\,(\underline{a})\text{n} \Rightarrow \underline{a} \Rightarrow \dot{\neg}\,(\underline{b})\text{n}; \dot{\neg}\,(\underline{a})\text{n} \vdash \text{MP} \rhd \dot{\neg}\,(\underline{a})\text{n} \Rightarrow \underline{a} \Rightarrow \dot{\neg}\,(\underline{b})\text{n} \rhd \dot{\neg}\,(\underline{a})\text{n} \gg \underline{a} \Rightarrow \dot{\neg}\,(\underline{b})\text{n}; \text{ToNegatedAnd} \rhd \underline{a} \Rightarrow \dot{\neg}\,(\underline{b})\text{n} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{a} \Rightarrow \dot{\neg}\,(\underline{b})\text{n})\text{n})\text{n}\rceil, \text{p}_0, c)$]

[ToNegatedAnd(1) $\overset{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}\,(\underline{a})\text{n} \vdash \dot{\neg}\,(\dot{\neg}\,(\underline{a} \Rightarrow \dot{\neg}\,(\underline{b})\text{n})\text{n})\text{n}$]

[ToNegatedAnd(1) $\overset{\text{tex}}{\rightarrow}$ "ToNegatedAnd(1)"]

[ToNegatedAnd(1) $\overset{\text{pyk}}{\rightarrow}$ "prop lemma to negated and(1)"]

AddNegatedAll

[AddNegatedAll $\stackrel{\text{tex}}{\rightarrow}$ “AddNegatedAll”]

[AddNegatedAll $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma addNegatedAll”]

(A)to(E)(Imply)

[(A)to(E)(Imply) $\stackrel{\text{tex}}{\rightarrow}$ “(A)to(˜E˜)(Imply)”]

[(A)to(E)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma (A)to(˜E˜)(Imply)”]

(E)to(A)(Imply)

[(E)to(A)(Imply) $\stackrel{\text{tex}}{\rightarrow}$ “(E)to(˜A˜)(Imply)”]

[(E)to(A)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma (E)to(˜A˜)(Imply)”]

(E)to(A)(Imply)

[(E)to(A)(Imply) $\stackrel{\text{tex}}{\rightarrow}$ “(E˜)to(˜A)(Imply)”]

[(E)to(A)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma (E˜)to(˜A)(Imply)”]

ToNegatedAEA

[ToNegatedAEA $\stackrel{\text{tex}}{\rightarrow}$ “ToNegatedAEA ”]

[ToNegatedAEA $\stackrel{\text{pyk}}{\rightarrow}$ “pred lemma toNegatedAEA”]

UniqueNegative

[UniqueNegative $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = 0 \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = 0 \vdash \mathrm{plusCommutativity} \gg (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{y}}); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{y}}) \rhd (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = 0 \gg (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = 0; \mathrm{PositiveToRight(Eq)} \rhd (\underline{\mathsf{y}} + \underline{\mathsf{x}}) = 0 \gg \underline{\mathsf{y}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})); \mathrm{plusCommutativity} \gg (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{z}}); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = (\underline{\mathsf{x}} + \underline{\mathsf{z}}) \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = 0 \gg (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = 0; \mathrm{PositiveToRight(Eq)} \rhd (\underline{\mathsf{z}} + \underline{\mathsf{x}}) = 0 \gg \underline{\mathsf{z}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})); \mathrm{eqSymmetry} \rhd \underline{\mathsf{z}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})) \gg (0 + (-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{z}}; \mathrm{eqTransitivity} \rhd \underline{\mathsf{y}} = (0 + (-\mathrm{u}\underline{\mathsf{x}})) \rhd (0 + (-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{z}} \gg \underline{\mathsf{y}} = \underline{\mathsf{z}}\rceil, \mathrm{p}_0, \mathsf{c})$]

$[\text{UniqueNegative} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = 0 \vdash (\underline{x} + \underline{z}) = 0 \vdash \underline{y} = \underline{z}]$

$[\text{UniqueNegative} \stackrel{\text{tex}}{\rightarrow}$ "UniqueNegative"$]$

$[\text{UniqueNegative} \stackrel{\text{pyk}}{\rightarrow}$ "lemma uniqueNegative"$]$

DoubleMinus

$[\text{DoubleMinus} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \text{Negative} \gg ((-\text{u}\underline{x}) + (-\text{u}(-\text{u}\underline{x}))) = 0; \text{x} + \text{y} = \text{zBackwards} \rhd ((-\text{u}\underline{x}) + (-\text{u}(-\text{u}\underline{x}))) = 0 \gg 0 = ((-\text{u}(-\text{u}\underline{x})) + (-\text{u}\underline{x})); \text{NegativeToLeft(Eq)} \rhd 0 = ((-\text{u}(-\text{u}\underline{x})) + (-\text{u}\underline{x})) \gg (0 + \underline{x}) = (-\text{u}(-\text{u}\underline{x})); \text{lemma plus0Left} \gg (0 + \underline{x}) = \underline{x}; \text{Equality} \rhd (0 + \underline{x}) = (-\text{u}(-\text{u}\underline{x})) \rhd (0 + \underline{x}) = \underline{x} \gg (-\text{u}(-\text{u}\underline{x})) = \underline{x} \rceil, \text{p}_0, c)]$

$[\text{DoubleMinus} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon (-\text{u}(-\text{u}\underline{x})) = \underline{x}]$

$[\text{DoubleMinus} \stackrel{\text{tex}}{\rightarrow}$ "DoubleMinus"$]$

$[\text{DoubleMinus} \stackrel{\text{pyk}}{\rightarrow}$ "lemma doubleMinus"$]$

MinusNegated

$[\text{MinusNegated} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \text{DoubleMinus} \gg (-\text{u}(-\text{u}\underline{y})) = \underline{y}; \text{eqAddition} \rhd (-\text{u}(-\text{u}\underline{y})) = \underline{y} \gg ((-\text{u}(-\text{u}\underline{y})) + (-\text{u}\underline{x})) = (\underline{y} + (-\text{u}\underline{x})); \text{eqSymmetry} \rhd ((-\text{u}(-\text{u}\underline{y})) + (-\text{u}\underline{x})) = (\underline{y} + (-\text{u}\underline{x})) \gg (\underline{y} + (-\text{u}\underline{x})) = ((-\text{u}(-\text{u}\underline{y})) + (-\text{u}\underline{x})); -\text{x} - \text{y} = -(\text{x} + \text{y}) \gg ((-\text{u}(-\text{u}\underline{y})) + (-\text{u}\underline{x})) = (-\text{u}((-\text{u}\underline{y}) + \underline{x})); \text{plusCommutativity} \gg ((-\text{u}\underline{y}) + \underline{x}) = (\underline{x} + (-\text{u}\underline{y})); \text{EqNegated} \rhd ((-\text{u}\underline{y}) + \underline{x}) = (\underline{x} + (-\text{u}\underline{y})) \gg (-\text{u}((-\text{u}\underline{y}) + \underline{x})) = (-\text{u}(\underline{x} + (-\text{u}\underline{y}))); \text{eqTransitivity4} \rhd (\underline{y} + (-\text{u}\underline{x})) = ((-\text{u}(-\text{u}\underline{y})) + (-\text{u}\underline{x})) \rhd ((-\text{u}(-\text{u}\underline{y})) + (-\text{u}\underline{x})) = (-\text{u}((-\text{u}\underline{y}) + \underline{x})) \rhd (-\text{u}((-\text{u}\underline{y}) + \underline{x})) = (-\text{u}(\underline{x} + (-\text{u}\underline{y}))) \gg (\underline{y} + (-\text{u}\underline{x})) = (-\text{u}(\underline{x} + (-\text{u}\underline{y}))); \text{eqSymmetry} \rhd (\underline{y} + (-\text{u}\underline{x})) = (-\text{u}(\underline{x} + (-\text{u}\underline{y}))) \gg (-\text{u}(\underline{x} + (-\text{u}\underline{y}))) = (\underline{y} + (-\text{u}\underline{x})) \rceil, \text{p}_0, c)]$

$[\text{MinusNegated} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon (-\text{u}(\underline{x} + (-\text{u}\underline{y}))) = (\underline{y} + (-\text{u}\underline{x}))]$

$[\text{MinusNegated} \stackrel{\text{tex}}{\rightarrow}$ "MinusNegated"$]$

$[\text{MinusNegated} \stackrel{\text{pyk}}{\rightarrow}$ "lemma minusNegated"$]$

eqReflexivity

$[\text{eqReflexivity} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \text{leqReflexivity} \gg \underline{x} <= \underline{x}; \text{leqAntisymmetry} \rhd \underline{x} <= \underline{x} \rhd \underline{x} <= \underline{x} \gg \underline{x} = \underline{x} \rceil, \text{p}_0, c)]$

$[\text{eqReflexivity} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \underline{x} = \underline{x}]$

$[\text{eqReflexivity} \stackrel{\text{tex}}{\rightarrow} \text{"eqReflexivity"}]$

$[\text{eqReflexivity} \stackrel{\text{pyk}}{\rightarrow} \text{"lemma eqReflexivity"}]$

eqSymmetry

$[\text{eqSymmetry} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash \text{eqReflexivity} \gg \underline{x} = \underline{x}; \text{Equality} \rhd \underline{x} = \underline{y} \rhd \underline{x} = \underline{x} \gg \underline{y} = \underline{x}\rceil, \text{p}_0, c)]$

$[\text{eqSymmetry} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash \underline{y} = \underline{x}]$

$[\text{eqSymmetry} \stackrel{\text{tex}}{\rightarrow} \text{"eqSymmetry"}]$

$[\text{eqSymmetry} \stackrel{\text{pyk}}{\rightarrow} \text{"lemma eqSymmetry"}]$

eqTransitivity

$[\text{eqTransitivity} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \underline{y} = \underline{z} \vdash \text{eqSymmetry} \rhd \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \text{Equality} \rhd \underline{y} = \underline{x} \rhd \underline{y} = \underline{z} \gg \underline{x} = \underline{z}\rceil, \text{p}_0, c)]$

$[\text{eqTransitivity} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \underline{y} = \underline{z} \vdash \underline{x} = \underline{z}]$

$[\text{eqTransitivity} \stackrel{\text{tex}}{\rightarrow} \text{"eqTransitivity"}]$

$[\text{eqTransitivity} \stackrel{\text{pyk}}{\rightarrow} \text{"lemma eqTransitivity"}]$

eqTransitivity4

$[\text{eqTransitivity4} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} = \underline{y} \vdash \underline{y} = \underline{z} \vdash \underline{z} = \underline{u} \vdash \text{eqTransitivity} \rhd \underline{x} = \underline{y} \rhd \underline{y} = \underline{z} \gg \underline{x} = \underline{z}; \text{eqTransitivity} \rhd \underline{x} = \underline{z} \rhd \underline{z} = \underline{u} \gg \underline{x} = \underline{u}\rceil, \text{p}_0, c)]$

$[\text{eqTransitivity4} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} = \underline{y} \vdash \underline{y} = \underline{z} \vdash \underline{z} = \underline{u} \vdash \underline{x} = \underline{u}]$

$[\text{eqTransitivity4} \stackrel{\text{tex}}{\rightarrow} \text{"eqTransitivity4"}]$

$[\text{eqTransitivity4} \stackrel{\text{pyk}}{\rightarrow} \text{"lemma eqTransitivity4"}]$

eqTransitivity5

$[\mathrm{eqTransitivity5} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \mathrm{eqTransitivity4} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{z}} = \underline{\mathsf{u}} \gg \underline{\mathsf{x}} = \underline{\mathsf{u}}; \mathrm{eqTransitivity} \rhd \underline{\mathsf{x}} = \underline{\mathsf{u}} \rhd \underline{\mathsf{u}} = \underline{\mathsf{v}} \gg \underline{\mathsf{x}} = \underline{\mathsf{v}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{eqTransitivity5} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{v}}]$

$[\mathrm{eqTransitivity5} \stackrel{\mathrm{tex}}{\rightarrow}$ “eqTransitivity5”$]$

$[\mathrm{eqTransitivity5} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma eqTransitivity5”$]$

eqTransitivity6

$[\mathrm{eqTransitivity6} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \forall \underline{\mathsf{w}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \underline{\mathsf{v}} = \underline{\mathsf{w}} \vdash \mathrm{eqTransitivity5} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{z}} = \underline{\mathsf{u}} \rhd \underline{\mathsf{u}} = \underline{\mathsf{v}} \gg \underline{\mathsf{x}} = \underline{\mathsf{v}}; \mathrm{eqTransitivity} \rhd \underline{\mathsf{x}} = \underline{\mathsf{v}} \rhd \underline{\mathsf{v}} = \underline{\mathsf{w}} \gg \underline{\mathsf{x}} = \underline{\mathsf{w}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{eqTransitivity6} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \forall \underline{\mathsf{v}}\colon \forall \underline{\mathsf{w}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \underline{\mathsf{u}} = \underline{\mathsf{v}} \vdash \underline{\mathsf{v}} = \underline{\mathsf{w}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{w}}]$

$[\mathrm{eqTransitivity6} \stackrel{\mathrm{tex}}{\rightarrow}$ “eqTransitivity6”$]$

$[\mathrm{eqTransitivity6} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma eqTransitivity6”$]$

AddEquations

$[\mathrm{AddEquations} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \mathrm{eqAddition} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}); \mathrm{lemma\ eqAdditionLeft} \rhd \underline{\mathsf{z}} = \underline{\mathsf{u}} \gg (\underline{\mathsf{y}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rhd (\underline{\mathsf{y}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{AddEquations} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}})]$

$[\mathrm{AddEquations} \stackrel{\mathrm{tex}}{\rightarrow}$ “AddEquations”$]$

$[\mathrm{AddEquations} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma addEquations”$]$

SubtractEquations

$[\mathrm{SubtractEquations} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \vdash \underline{\mathsf{z}} = \underline{\mathsf{u}} \vdash \mathrm{eqAddition} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + (-\mathrm{u}\underline{\mathsf{z}})) = ((\underline{\mathsf{y}} + \underline{\mathsf{u}}) + (-\mathrm{u}\underline{\mathsf{z}})); \mathrm{lemma\ plus0Left} \gg (0 + \underline{\mathsf{z}}) = \underline{\mathsf{z}}; \mathrm{eqTransitivity} \rhd (0 + \underline{\mathsf{z}}) =$

$\underline{z} \rhd \underline{z} = \underline{u} \gg (0 + \underline{z}) = \underline{u}$; PositiveToRight(Eq) $\rhd (0 + \underline{z}) = \underline{u} \gg 0 = (\underline{u} + (-u\underline{z}))$; eqSymmetry $\rhd 0 = (\underline{u} + (-u\underline{z})) \gg (\underline{u} + (-u\underline{z})) = 0$; lemma eqAdditionLeft $\rhd (\underline{u} + (-u\underline{z})) = 0 \gg (\underline{y} + (\underline{u} + (-u\underline{z}))) = (\underline{y}+0)$; plusAssociativity $\gg ((\underline{y}+\underline{u})+(-u\underline{z})) = (\underline{y}+(\underline{u}+(-u\underline{z})))$; plus0 $\gg (\underline{y}+0) = \underline{y}$; eqTransitivity4 $\rhd ((\underline{y} + \underline{u}) + (-u\underline{z})) = (\underline{y} + (\underline{u} + (-u\underline{z}))) \rhd (\underline{y} + (\underline{u} + (-u\underline{z}))) = (\underline{y} + 0) \rhd (\underline{y} + 0) = \underline{y} \gg ((\underline{y} + \underline{u}) + (-u\underline{z})) = \underline{y}$; $\mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{x} = ((\underline{x} + \underline{z}) + (-u\underline{z}))$; eqTransitivity4 $\rhd \underline{x} = ((\underline{x} + \underline{z}) + (-u\underline{z})) \rhd ((\underline{x} + \underline{z}) + (-u\underline{z})) = ((\underline{y} + \underline{u}) + (-u\underline{z})) \rhd ((\underline{y} + \underline{u}) + (-u\underline{z})) = \underline{y} \gg \underline{x} = \underline{y}\rceil, p_0, \mathsf{c})]$

[SubtractEquations $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{z} = \underline{u} \vdash \underline{x} = \underline{y}$]

[SubtractEquations $\stackrel{\text{tex}}{\rightarrow}$ "SubtractEquations"]

[SubtractEquations $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subtractEquations"]

SubtractEquationsLeft

[SubtractEquationsLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{x} = \underline{y} \vdash$ plusCommutativity $\gg (\underline{z} + \underline{x}) = (\underline{x} + \underline{z})$; plusCommutativity $\gg (\underline{y} + \underline{u}) = (\underline{u} + \underline{y})$; eqTransitivity4 $\rhd (\underline{z} + \underline{x}) = (\underline{x} + \underline{z}) \rhd (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \rhd (\underline{y} + \underline{u}) = (\underline{u} + \underline{y}) \gg (\underline{z} + \underline{x}) = (\underline{u} + \underline{y})$; SubtractEquations $\rhd (\underline{z} + \underline{x}) = (\underline{u} + \underline{y}) \rhd \underline{x} = \underline{y} \gg \underline{z} = \underline{u}\rceil, p_0, \mathsf{c})]$

[SubtractEquationsLeft $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon (\underline{x} + \underline{z}) = (\underline{y} + \underline{u}) \vdash \underline{x} = \underline{y} \vdash \underline{z} = \underline{u}$]

[SubtractEquationsLeft $\stackrel{\text{tex}}{\rightarrow}$ "SubtractEquationsLeft"]

[SubtractEquationsLeft $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subtractEquationsLeft"]

MultiplyEquations

[MultiplyEquations $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash$ eqMultiplication $\rhd \underline{x} = \underline{y} \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})$; lemma eqMultiplicationLeft $\rhd \underline{z} = \underline{u} \gg (\underline{y} * \underline{z}) = (\underline{y} * \underline{u})$; eqTransitivity $\rhd (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \rhd (\underline{y} * \underline{z}) = (\underline{y} * \underline{u}) \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{u})\rceil, p_0, \mathsf{c})]$

[MultiplyEquations $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon \underline{x} = \underline{y} \vdash \underline{z} = \underline{u} \vdash (\underline{x} * \underline{z}) = (\underline{y} * \underline{u})$]

[MultiplyEquations $\stackrel{\text{tex}}{\rightarrow}$ "MultiplyEquations"]

[MultiplyEquations $\stackrel{\text{pyk}}{\rightarrow}$ "lemma multiplyEquations"]

EqNegated

$[\text{EqNegated} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \text{Negative} \gg (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{x}})) = 0; \text{Negative} \gg (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) = 0; \text{eqSymmetry} \rhd (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) = 0 \gg 0 = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})); \text{eqTransitivity} \rhd (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{x}})) = 0 \rhd 0 = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) \gg (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{x}})) = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})); \text{SubtractEquationsLeft} \rhd (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{x}})) = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg (-\text{u}\underline{\mathsf{x}}) = (-\text{u}\underline{\mathsf{y}})\rceil, \text{p}_0, \mathsf{c})]$

$[\text{EqNegated} \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash (-\text{u}\underline{\mathsf{x}}) = (-\text{u}\underline{\mathsf{y}})]$

$[\text{EqNegated} \overset{\text{tex}}{\to}$ “EqNegated”$]$

$[\text{EqNegated} \overset{\text{pyk}}{\to}$ “lemma eqNegated”$]$

PositiveToRight(Eq)

$[\text{PositiveToRight(Eq)} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = \underline{\mathsf{z}} \vdash \text{eqAddition} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = \underline{\mathsf{z}} \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})) = (\underline{\mathsf{z}} + (-\text{u}\underline{\mathsf{y}})); \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})); \text{eqTransitivity} \rhd \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})) \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-\text{u}\underline{\mathsf{y}})) = (\underline{\mathsf{z}} + (-\text{u}\underline{\mathsf{y}})) \gg \underline{\mathsf{x}} = (\underline{\mathsf{z}} + (-\text{u}\underline{\mathsf{y}}))\rceil, \text{p}_0, \mathsf{c})]$

$[\text{PositiveToRight(Eq)} \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} = (\underline{\mathsf{z}} + (-\text{u}\underline{\mathsf{y}}))]$

$[\text{PositiveToRight(Eq)} \overset{\text{tex}}{\to}$ “PositiveToRight(Eq)”$]$

$[\text{PositiveToRight(Eq)} \overset{\text{pyk}}{\to}$ “lemma positiveToRight(Eq)”$]$

PositiveToLeft(Eq)

$[\text{PositiveToLeft(Eq)} \overset{\text{tex}}{\to}$ “PositiveToLeft(Eq)”$]$

$[\text{PositiveToLeft(Eq)} \overset{\text{pyk}}{\to}$ “lemma positiveToLeft(Eq)”$]$

PositiveToLeft(Eq)(1term)

$[\text{PositiveToLeft(Eq)(1term)} \overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \text{eqAddition} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{y}})) = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})); \text{Negative} \gg (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) = 0; \text{eqTransitivity} \rhd (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{y}})) = (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) \rhd (\underline{\mathsf{y}} + (-\text{u}\underline{\mathsf{y}})) = 0 \gg (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{y}})) = 0\rceil, \text{p}_0, \mathsf{c})]$

$[\text{PositiveToLeft(Eq)(1term)} \overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash (\underline{\mathsf{x}} + (-\text{u}\underline{\mathsf{y}})) = 0]$

$[\text{PositiveToLeft(Eq)(1term)} \overset{\text{tex}}{\to}$ “PositiveToLeft(Eq)(1 term)”$]$

[PositiveToLeft(Eq)(1term) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma positiveToLeft(Eq)(1 term)"]

NegativeToLeft(Eq)

[NegativeToLeft(Eq) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = (\underline{y} + (-u\underline{z})) \vdash \mathrm{eqAddition} \rhd \underline{x} = (\underline{y} + (-u\underline{z})) \gg (\underline{x} + \underline{z}) = ((\underline{y} + (-u\underline{z})) + \underline{z}); \mathrm{Three2threeTerms} \gg ((\underline{y} + (-u\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-u\underline{z})); \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})); \mathrm{eqSymmetry} \rhd \underline{y} = ((\underline{y} + \underline{z}) + (-u\underline{z})) \gg ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y}; \mathrm{eqTransitivity4} \rhd (\underline{x} + \underline{z}) = ((\underline{y} + (-u\underline{z})) + \underline{z}) \rhd ((\underline{y} + (-u\underline{z})) + \underline{z}) = ((\underline{y} + \underline{z}) + (-u\underline{z})) \rhd ((\underline{y} + \underline{z}) + (-u\underline{z})) = \underline{y} \gg (\underline{x} + \underline{z}) = \underline{y}\rceil, p_0, c)$]

[NegativeToLeft(Eq) $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = (\underline{y} + (-u\underline{z})) \vdash (\underline{x} + \underline{z}) = \underline{y}$]

[NegativeToLeft(Eq) $\stackrel{\text{tex}}{\rightarrow}$ "NegativeToLeft(Eq)"]

[NegativeToLeft(Eq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeToLeft(Eq)"]

NonreciprocalToRight(Eq)(1term)

[NonreciprocalToRight(Eq)(1term) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon (\underline{x} * \underline{y}) = 1 \vdash \mathrm{eqMultiplication} \rhd (\underline{x} * \underline{y}) = 1 \gg ((\underline{x} * \underline{y}) * \mathrm{rec}\underline{y}) = (1 * \mathrm{rec}\underline{y}); 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n; \mathrm{PositiveNonzero} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \gg \dot{\neg}(1 = 0)n; \mathrm{eqSymmetry} \rhd (\underline{x} * \underline{y}) = 1 \gg 1 = (\underline{x} * \underline{y}); \mathrm{SubNeqLeft} \rhd 1 = (\underline{x} * \underline{y}) \rhd \dot{\neg}(1 = 0)n \gg \dot{\neg}((\underline{x} * \underline{y}) = 0)n; \mathrm{NonzeroProduct(2)} \rhd \dot{\neg}((\underline{x} * \underline{y}) = 0)n \gg \dot{\neg}(\underline{y} = 0)n; \mathrm{x} = \mathrm{x} * \mathrm{y} * (1/\mathrm{y}) \rhd \dot{\neg}(\underline{y} = 0)n \gg \underline{x} = ((\underline{x} * \underline{y}) * \mathrm{rec}\underline{y}); \text{lemma times1Left} \gg (1 * \mathrm{rec}\underline{y}) = \mathrm{rec}\underline{y}; \mathrm{eqTransitivity4} \rhd \underline{x} = ((\underline{x} * \underline{y}) * \mathrm{rec}\underline{y}) \rhd ((\underline{x} * \underline{y}) * \mathrm{rec}\underline{y}) = (1 * \mathrm{rec}\underline{y}) \rhd (1 * \mathrm{rec}\underline{y}) = \mathrm{rec}\underline{y} \gg \underline{x} = \mathrm{rec}\underline{y}\rceil, p_0, c)$]

[NonreciprocalToRight(Eq)(1term) $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon (\underline{x} * \underline{y}) = 1 \vdash \underline{x} = \mathrm{rec}\underline{y}$]

[NonreciprocalToRight(Eq)(1term) $\stackrel{\text{tex}}{\rightarrow}$ "NonreciprocalToRight(Eq)(1 term)"]

[NonreciprocalToRight(Eq)(1term) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma nonreciprocalToRight(Eq)(1 term)"]

DistributionOut(Minus)

[DistributionOut(Minus) $\stackrel{\text{tex}}{\rightarrow}$ "DistributionOut(Minus)"]

[DistributionOut(Minus) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma distributionOut(Minus)"]

PositiveToRight(Eq)(1term)

[PositiveToRight(Eq)(1term) $\overset{\text{tex}}{\rightarrow}$ "PositiveToRight(Eq)(1 term)"]

[PositiveToRight(Eq)(1term) $\overset{\text{pyk}}{\rightarrow}$ "lemma positiveToRight(Eq)(1 term)"]

SameSeries(NumDiff)

[SameSeries(NumDiff) $\overset{\text{tex}}{\rightarrow}$ "SameSeries(NumDiff)"]

[SameSeries(NumDiff) $\overset{\text{pyk}}{\rightarrow}$ "lemma sameSeries(NumDiff)"]

PlusAssociativity(4terms)

[PlusAssociativity(4terms) $\overset{\text{tex}}{\rightarrow}$ "PlusAssociativity(4 terms)"]

[PlusAssociativity(4terms) $\overset{\text{pyk}}{\rightarrow}$ "lemma plusAssociativity(4 terms)"]

LessNeq

[LessNeq $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n})\text{n})\text{n} \vdash \text{Repetition} \rhd \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n})\text{n})\text{n}; \text{SecondConjunct} \rhd \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[LessNeq $\overset{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\,(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n})\text{n})\text{n} \vdash \dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n}$]

[LessNeq $\overset{\text{tex}}{\rightarrow}$ "LessNeq"]

[LessNeq $\overset{\text{pyk}}{\rightarrow}$ "lemma lessNeq"]

NeqSymmetry

[NeqSymmetry $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{y}} = \underline{\mathsf{x}} \vdash \text{eqSymmetry} \rhd \underline{\mathsf{y}} = \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}; \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{y}} = \underline{\mathsf{x}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{y}} = \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}}; \dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n} \vdash \text{MT} \rhd \underline{\mathsf{y}} = \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{y}} = \underline{\mathsf{x}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[NeqSymmetry $\overset{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\,(\underline{\mathsf{x}} = \underline{\mathsf{y}})\text{n} \vdash \dot{\neg}\,(\underline{\mathsf{y}} = \underline{\mathsf{x}})\text{n}$]

[NeqSymmetry $\overset{\text{tex}}{\rightarrow}$ "NeqSymmetry"]

[NeqSymmetry $\overset{\text{pyk}}{\rightarrow}$ "lemma neqSymmetry"]

NeqNegated

$[\text{NeqNegated} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \vdash \text{EqNegated} \rhd (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})); \text{DoubleMinus} \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{x}}; \text{eqSymmetry} \rhd (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) = \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})); \text{DoubleMinus} \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}}; \text{eqTransitivity4} \rhd \underline{\mathsf{x}} = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) \rhd (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{x}})) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) \rhd (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}; \text{FromContradiction} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}; \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \vdash \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \Rightarrow (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \Rightarrow \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}; \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash \text{MP} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \Rightarrow (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \Rightarrow \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \Rightarrow \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}; \text{prop lemma imply negation} \rhd (-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}) \Rightarrow \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{NeqNegated} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}]$

$[\text{NeqNegated} \stackrel{\text{tex}}{\rightarrow}$ "NeqNegated"$]$

$[\text{NeqNegated} \stackrel{\text{pyk}}{\rightarrow}$ "lemma neqNegated"$]$

SubNeqRight

$[\text{SubNeqRight} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{x}})\mathrm{n} \vdash \text{NeqSymmetry} \rhd \dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{x}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n}; \text{SubNeqLeft} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n}; \text{NeqSymmetry} \rhd \dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{y}})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{SubNeqRight} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{x}})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{y}})\mathrm{n}]$

$[\text{SubNeqRight} \stackrel{\text{tex}}{\rightarrow}$ "SubNeqRight"$]$

$[\text{SubNeqRight} \stackrel{\text{pyk}}{\rightarrow}$ "lemma subNeqRight"$]$

SubNeqLeft

$[\text{SubNeqLeft} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n} \vdash \text{EqualityAxiom} \gg \underline{\mathsf{y}} = \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{y}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}}; \text{eqSymmetry} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{y}} = \underline{\mathsf{x}}; \text{MP} \rhd \underline{\mathsf{y}} = \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{y}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{y}} = \underline{\mathsf{x}} \gg \underline{\mathsf{y}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}}; \text{Contrapositive} \rhd \underline{\mathsf{y}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}} \gg \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n}; \text{MP} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{SubNeqLeft} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n}]$

$[\text{SubNeqLeft} \stackrel{\text{tex}}{\rightarrow}$ "SubNeqLeft"$]$

$[\text{SubNeqLeft} \stackrel{\text{pyk}}{\rightarrow}$ "lemma subNeqLeft"$]$

NegativeToRight(Neq)(1term)

[NegativeToRight(Neq)(1term) $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \text{PositiveToLeft(Eq)(1term)} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0; \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0 \gg \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0; \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0)\mathrm{n} \vdash \text{MT} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0 \rhd \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0)\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[NegativeToRight(Neq)(1term) $\overset{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = 0)\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n}$]

[NegativeToRight(Neq)(1term) $\overset{\text{tex}}{\rightarrow}$ "NegativeToRight(Neq)(1 term)"]

[NegativeToRight(Neq)(1term) $\overset{\text{pyk}}{\rightarrow}$ "lemma negativeToRight(Neq)(1 term)"]

NeqAddition

[NeqAddition $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \vdash \text{eqReflexivity} \gg \underline{\mathsf{z}} = \underline{\mathsf{z}}; \text{SubtractEquations} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rhd \underline{\mathsf{z}} = \underline{\mathsf{z}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}; \text{FromContradiction} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n}; \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \vdash \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n}; \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash \text{MP} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n}; \text{prop lemma imply negation} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[NeqAddition $\overset{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n}$]

[NeqAddition $\overset{\text{tex}}{\rightarrow}$ "NeqAddition"]

[NeqAddition $\overset{\text{pyk}}{\rightarrow}$ "lemma neqAddition"]

NeqMultiplication

[NeqMultiplication $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \vdash \mathrm{x} = \mathrm{x} * \mathrm{y} * (1/\mathrm{y}) \rhd \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}); \text{eqMultiplication} \rhd (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \gg ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}); \mathrm{x} = \mathrm{x} * \mathrm{y} * (1/\mathrm{y}) \rhd \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \gg \underline{\mathsf{y}} = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}); \text{eqSymmetry} \rhd \underline{\mathsf{y}} = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) \gg ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) = \underline{\mathsf{y}}; \text{eqTransitivity4} \rhd \underline{\mathsf{x}} = ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) \rhd ((\underline{\mathsf{x}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) \rhd ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) = \underline{\mathsf{y}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}; \text{FromContradiction} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n}; \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \vdash \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n}; \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \vdash \text{MP2} \rhd \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}))\mathrm{n}; \text{prop lemma imply negation} \rhd (\underline{\mathsf{x}} * \underline{\mathsf{z}}) =$

$(\underline{y} * \underline{z}) \Rightarrow \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n} \gg \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NeqMultiplication} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{z} = 0)\mathrm{n} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n}]$

[NeqMultiplication $\overset{\mathrm{tex}}{\to}$ "NeqMultiplication"]

[NeqMultiplication $\overset{\mathrm{pyk}}{\to}$ "lemma neqMultiplication"]

NonzeroProduct(2)

$[\mathrm{NonzeroProduct(2)} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} = 0 \vdash \text{lemma eqMultiplicationLeft} \rhd \underline{y} = 0 \gg (\underline{x} * \underline{y}) = (\underline{x} * 0); \mathrm{x} * 0 = 0 \gg (\underline{x} * 0) = 0; \mathrm{eqTransitivity} \rhd (\underline{x} * \underline{y}) = (\underline{x} * 0) \rhd (\underline{x} * 0) = 0 \gg (\underline{x} * \underline{y}) = 0; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} = 0 \vdash (\underline{x} * \underline{y}) = 0 \gg \underline{y} = 0 \Rightarrow (\underline{x} * \underline{y}) = 0; \dot{\neg}((\underline{x} * \underline{y}) = 0)\mathrm{n} \vdash \mathrm{MT} \rhd \underline{y} = 0 \Rightarrow (\underline{x} * \underline{y}) = 0 \rhd \dot{\neg}((\underline{x} * \underline{y}) = 0)\mathrm{n} \gg \dot{\neg}(\underline{y} = 0)\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NonzeroProduct(2)} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}((\underline{x} * \underline{y}) = 0)\mathrm{n} \vdash \dot{\neg}(\underline{y} = 0)\mathrm{n}]$

[NonzeroProduct(2) $\overset{\mathrm{tex}}{\to}$ "NonzeroProduct(2)"]

[NonzeroProduct(2) $\overset{\mathrm{pyk}}{\to}$ "lemma nonzeroProduct(2)"]

SwitchTerms(x <= y − z)

[SwitchTerms(x <= y − z) $\overset{\mathrm{tex}}{\to}$ "SwitchTerms(x<=y-z)"]

[SwitchTerms(x <= y − z) $\overset{\mathrm{pyk}}{\to}$ "lemma switchTerms(x<=y-z)"]

NegativeToLeft(Less)(1term)

[NegativeToLeft(Less)(1term) $\overset{\mathrm{tex}}{\to}$ "NegativeToLeft(Less)(1 term)"]

[NegativeToLeft(Less)(1term) $\overset{\mathrm{pyk}}{\to}$ "lemma negativeToLeft(Less)(1 term)"]

(+1)IsPositive(N)

[(+1)IsPositive(N) $\overset{\mathrm{tex}}{\to}$ "(+1)IsPositive(N)"]

[(+1)IsPositive(N) $\overset{\mathrm{pyk}}{\to}$ "lemma +1IsPositive(N)"]

$(1/2)(x + y) - x = (1/2)(y - x)$

[$(1/2)(x + y) - x = (1/2)(y - x) \stackrel{tex}{\rightarrow}$ "(1/2)(x+y)-x=(1/2)(y-x)"]

[$(1/2)(x + y) - x = (1/2)(y - x) \stackrel{pyk}{\rightarrow}$ "lemma (1/2)(x+y)-x=(1/2)(y-x)"]

$y - (1/2)(x + y) = (1/2)(y - x)$

[$y - (1/2)(x + y) = (1/2)(y - x) \stackrel{tex}{\rightarrow}$ "y-(1/2)(x+y)=(1/2)(y-x)"]

[$y - (1/2)(x + y) = (1/2)(y - x) \stackrel{pyk}{\rightarrow}$ "lemma y-(1/2)(x+y)=(1/2)(y-x)"]

ExpZero(Exact)

[ExpZero(Exact) $\stackrel{tex}{\rightarrow}$ "ExpZero(Exact) "]

[ExpZero(Exact) $\stackrel{pyk}{\rightarrow}$ "lemma expZero exact"]

SameExp(Base)

[SameExp(Base) $\stackrel{tex}{\rightarrow}$ "SameExp(Base)"]

[SameExp(Base) $\stackrel{pyk}{\rightarrow}$ "lemma sameExp base"]

SameExp(Indu)

[SameExp(Indu) $\stackrel{tex}{\rightarrow}$ "SameExp(Indu)"]

[SameExp(Indu) $\stackrel{pyk}{\rightarrow}$ "lemma sameExp indu"]

SameExp

[SameExp $\stackrel{tex}{\rightarrow}$ "SameExp"]

[SameExp $\stackrel{pyk}{\rightarrow}$ "lemma sameExp"]

Exp(+1)

[Exp(+1) $\stackrel{\text{tex}}{\to}$ "Exp(+1)"]

[Exp(+1) $\stackrel{\text{pyk}}{\to}$ "lemma exp(+1)"]

PositiveBase(Base)

[PositiveBase(Base) $\stackrel{\text{tex}}{\to}$ "PositiveBase(Base)"]

[PositiveBase(Base) $\stackrel{\text{pyk}}{\to}$ "lemma positiveBase base"]

PositiveBase(Indu)

[PositiveBase(Indu) $\stackrel{\text{tex}}{\to}$ "PositiveBase(Indu)"]

[PositiveBase(Indu) $\stackrel{\text{pyk}}{\to}$ "lemma positiveBase indu"]

PositiveBase

[PositiveBase $\stackrel{\text{tex}}{\to}$ "PositiveBase"]

[PositiveBase $\stackrel{\text{pyk}}{\to}$ "lemma positiveBase"]

BSzero(Exact)

[BSzero(Exact) $\stackrel{\text{tex}}{\to}$ "BSzero(Exact)"]

[BSzero(Exact) $\stackrel{\text{pyk}}{\to}$ "lemma base(1/2)Sum zero exact"]

SameBS(2)(Base)

[SameBS(2)(Base) $\stackrel{\text{tex}}{\to}$ "SameBS(2)(Base)"]

[SameBS(2)(Base) $\stackrel{\text{pyk}}{\to}$ "lemma sameBase(1/2)Sum second base"]

SameBS(2)(Indu)

[SameBS(2)(Indu) $\stackrel{\text{tex}}{\rightarrow}$ “SameBS(2)(Indu)”]

[SameBS(2)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameBase(1/2)Sum second indu”]

SameBS(2)

[SameBS(2) $\stackrel{\text{tex}}{\rightarrow}$ “SameBS(2)”]

[SameBS(2) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameBase(1/2)Sum second”]

BS(+1)

[BS(+1) $\stackrel{\text{tex}}{\rightarrow}$ “BS(+1)”]

[BS(+1) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum(+1)”]

BSbound(Exact)(Base)

[BSbound(Exact)(Base) $\stackrel{\text{tex}}{\rightarrow}$ “BSbound(Exact)(Base)”]

[BSbound(Exact)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum exact bound base”]

BSbound(Exact)(Indu)

[BSbound(Exact)(Indu) $\stackrel{\text{tex}}{\rightarrow}$ “BSbound(Exact)(Indu)”]

[BSbound(Exact)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum exact bound indu”]

BSbound(Exact)

[BSbound(Exact) $\stackrel{\text{tex}}{\rightarrow}$ “BSbound(Exact)”]

[BSbound(Exact) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum exact bound”]

BSbound

[BSbound $\stackrel{\text{tex}}{\rightarrow}$ “BSbound”]

[BSbound $\stackrel{\text{pyk}}{\rightarrow}$ “lemma base(1/2)Sum bound”]

UStelescope(Zero)(Exact)

[UStelescope(Zero)(Exact) $\stackrel{\text{tex}}{\rightarrow}$ “UStelescope(Zero)(Exact)”]

[UStelescope(Zero)(Exact) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma UStelescope zero exact”]

SameTelescope(2)(Base)

[SameTelescope(2)(Base) $\stackrel{\text{tex}}{\rightarrow}$ “SameTelescope(2)(Base)”]

[SameTelescope(2)(Base) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameTelescope second base”]

SameTelescope(2)(Indu)

[SameTelescope(2)(Indu) $\stackrel{\text{tex}}{\rightarrow}$ “SameTelescope(2)(Indu)”]

[SameTelescope(2)(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameTelescope second indu”]

SameTelescope(2)

[SameTelescope(2) $\stackrel{\text{tex}}{\rightarrow}$ “SameTelescope(2)”]

[SameTelescope(2) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma sameTelescope second”]

UStelescope(+1)

[UStelescope(+1) $\stackrel{\text{tex}}{\rightarrow}$ “UStelescope(+1)”]

[UStelescope(+1) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma UStelescope(+1)”]

TelescopeNumerical(Base)

[TelescopeNumerical(Base) $\stackrel{\text{tex}}{\rightarrow}$ "TelescopeNumerical(Base)"]

[TelescopeNumerical(Base) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma telescopeNumerical base"]

TelescopeNumerical(Indu)

[TelescopeNumerical(Indu) $\stackrel{\text{tex}}{\rightarrow}$ "TelescopeNumerical(Indu)"]

[TelescopeNumerical(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma telescopeNumerical indu"]

TelescopeNumerical

[TelescopeNumerical $\stackrel{\text{tex}}{\rightarrow}$ "TelescopeNumerical"]

[TelescopeNumerical $\stackrel{\text{pyk}}{\rightarrow}$ "lemma telescopeNumerical"]

TelescopeBound(Base)

[TelescopeBound(Base) $\stackrel{\text{tex}}{\rightarrow}$ "TelescopeBound(Base)"]

[TelescopeBound(Base) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma telescopeBound base"]

TelescopeBound(Indu)

[TelescopeBound(Indu) $\stackrel{\text{tex}}{\rightarrow}$ "TelescopeBound(Indu)"]

[TelescopeBound(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma telescopeBound indu"]

TelescopeBound

[TelescopeBound $\stackrel{\text{tex}}{\rightarrow}$ "TelescopeBound"]

[TelescopeBound $\stackrel{\text{pyk}}{\rightarrow}$ "lemma telescopeBound"]

LessNeq(F)(Helper)

[LessNeq(F)(Helper) $\stackrel{\text{tex}}{\rightarrow}$ "LessNeq(F)(Helper)"]

[LessNeq(F)(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessNeq(F) helper"]

LessNeq(F)

[LessNeq(F) $\stackrel{\text{tex}}{\rightarrow}$ "LessNeq(F)"]

[LessNeq(F) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessNeq(F)"]

LessNeq(R)

[LessNeq(R) $\stackrel{\text{tex}}{\rightarrow}$ "LessNeq(R)"]

[LessNeq(R) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessNeq(R)"]

IntervalSize(Base)

[IntervalSize(Base) $\stackrel{\text{tex}}{\rightarrow}$ "IntervalSize(Base)"]

[IntervalSize(Base) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma intervalSize base"]

IntervalSize(Indu)

[IntervalSize(Indu) $\stackrel{\text{tex}}{\rightarrow}$ "IntervalSize(Indu)"]

[IntervalSize(Indu) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma intervalSize indu"]

IntervalSize

[IntervalSize $\stackrel{\text{tex}}{\rightarrow}$ "IntervalSize"]

[IntervalSize $\stackrel{\text{pyk}}{\rightarrow}$ "lemma intervalSize"]

XS < US

[XS < US $\stackrel{\text{tex}}{\rightarrow}$ "XS<US"]

[XS < US $\stackrel{\text{pyk}}{\rightarrow}$ "lemma XSlessUS"]

lemma USdecreasing(+1)

[lemma USdecreasing(+1) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma USdecreasing(+1)"]

CloseUS

[CloseUS $\stackrel{\text{tex}}{\rightarrow}$ "CloseUS"]

[CloseUS $\stackrel{\text{pyk}}{\rightarrow}$ "lemma closeUS"]

CloseUS(n + 1)

[CloseUS(n + 1) $\stackrel{\text{tex}}{\rightarrow}$ "CloseUS(n+1)"]

[CloseUS(n + 1) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma closeUS(n+1)"]

AllNegated(Imply)

[AllNegated(Imply) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n \vdash$ $\text{A4} @ \underline{x} \rhd \forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n \gg \dot{\neg}(\dot{\neg}(\underline{a})n)n; \text{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{a})n)n \gg$ $\underline{a}; \text{Gen} \rhd \underline{a} \gg \forall_{obj} \underline{(v1)}: \underline{a}; \forall \underline{(v1)}: \forall \underline{a}: \text{Ded} \rhd \forall \underline{(v1)}: \forall \underline{a}: \forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n \vdash$ $\forall_{obj} \underline{(v1)}: \underline{a} \gg \forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n \Rightarrow$ $\forall_{obj} \underline{(v1)}: \underline{a}; \text{Contrapositive} \rhd \forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n \Rightarrow \forall_{obj} \underline{(v1)}: \underline{a} \gg$ $\dot{\neg}(\forall_{obj} \underline{(v1)}: \underline{a})n \Rightarrow \dot{\neg}(\forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n; \text{Repetition} \rhd \dot{\neg}(\forall_{obj} \underline{(v1)}: \underline{a})n \Rightarrow$ $\dot{\neg}(\forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n \gg \dot{\neg}(\forall_{obj} \underline{(v1)}: \underline{a})n \Rightarrow$ $\dot{\neg}(\forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n \rceil, p_0, c)$]

[AllNegated(Imply) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{(v1)}: \forall \underline{a}: \dot{\neg}(\forall_{obj} \underline{(v1)}: \underline{a})n \Rightarrow$ $\dot{\neg}(\forall_{obj} \underline{(v1)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n$]

[AllNegated(Imply) $\stackrel{\text{tex}}{\rightarrow}$ "AllNegated(Imply)"]

[AllNegated(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma allNegated(Imply)"]

ExistNegated(Imply)

[ExistNegated(Imply) $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n} \vdash \text{Repetition} \rhd \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n} \gg \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n}; \text{RemoveDoubleNeg} \rhd \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n} \gg \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \text{Ded} \rhd \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n} \vdash \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \gg \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[ExistNegated(Imply) $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n}$]

[ExistNegated(Imply) $\overset{\text{tex}}{\to}$ "ExistNegated(Imply)"]

[ExistNegated(Imply) $\overset{\text{pyk}}{\to}$ "pred lemma existNegated(Imply)"]

IntroExist(Helper)

[IntroExist(Helper) $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \vdash \text{A4} @ \underline{\mathsf{x}} \unrhd \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \rhd \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \vdash \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \gg \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[IntroExist(Helper) $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}$]

[IntroExist(Helper) $\overset{\text{tex}}{\to}$ "IntroExist(Helper)"]

[IntroExist(Helper) $\overset{\text{pyk}}{\to}$ "pred lemma intro exist helper"]

IntroExist

[IntroExist $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \text{IntroExist(Helper)} @ \underline{\mathsf{x}} \unrhd \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \gg \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \underline{\mathsf{a}} \vdash \text{AddDoubleNeg} \rhd \underline{\mathsf{a}} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n}; \text{MT} \rhd \forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n}; \text{Repetition} \rhd \dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \gg \dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[IntroExist $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{(\text{v1})}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \equiv \dot{\neg}\,(\underline{\mathsf{b}})\text{n} | \underline{(\text{v1})} {:=}{=} \underline{\mathsf{x}}\rangle_{\text{Me}} \Vdash \underline{\mathsf{a}} \vdash \dot{\neg}\,(\forall_{\text{obj}} \underline{(\text{v1})}\colon \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n}$]

[IntroExist $\overset{\text{tex}}{\to}$ "IntroExist"]

[IntroExist $\overset{\text{pyk}}{\to}$ "pred lemma intro exist"]

ExistMP

$[\text{ExistMP} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \vdash \dot{\neg}(\underline{b})n \vdash \text{MT} \rhd \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{b})n \gg \dot{\neg}(\underline{a})n; \text{Gen} \rhd \dot{\neg}(\underline{a})n \gg \forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n; \text{Repetition} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \gg \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n; \text{FromContradiction} \rhd \forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \gg \dot{\neg}(\dot{\neg}(\underline{b})n)n; \forall \underline{(v1)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \text{Ded} \rhd \forall \underline{(v1)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \vdash \dot{\neg}(\underline{b})n \vdash \dot{\neg}(\dot{\neg}(\underline{b})n)n \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n; \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \vdash \text{MP2} \rhd \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \rhd \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \gg \dot{\neg}(\underline{b})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n; \text{prop lemma imply negation} \rhd \dot{\neg}(\underline{b})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n \gg \dot{\neg}(\dot{\neg}(\underline{b})n)n; \text{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{b})n)n \gg \underline{b}\rceil, p_0, c)]$

$[\text{ExistMP} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \vdash \underline{b}]$

$[\text{ExistMP} \stackrel{\text{tex}}{\rightarrow}$ "ExistMP"$]$

$[\text{ExistMP} \stackrel{\text{pyk}}{\rightarrow}$ "pred lemma exist mp"$]$

ExistMP2

$[\text{ExistMP2} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{b})n)n \vdash \text{ExistMP} \rhd \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \gg \underline{b} \Rightarrow \underline{c}; \text{ExistMP} \rhd \underline{b} \Rightarrow \underline{c} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{b})n)n \gg \underline{c}\rceil, p_0, c)]$

$[\text{ExistMP2} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\underline{a})n)n \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{b})n)n \vdash \underline{c}]$

$[\text{ExistMP2} \stackrel{\text{tex}}{\rightarrow}$ "ExistMP2"$]$

$[\text{ExistMP2} \stackrel{\text{pyk}}{\rightarrow}$ "pred lemma exist mp2"$]$

TwiceExistMP

$[\text{TwiceExistMP} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(v2)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \vdash \text{ExistMP} \rhd \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \gg \underline{b}; \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \text{Ded} \rhd \forall \underline{(v2)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \vdash \underline{b} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b}; \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n)n)n \vdash \text{MP} \rhd \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b} \rhd \underline{a} \Rightarrow \underline{b} \gg \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b}; \text{ExistMP} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n \Rightarrow \underline{b} \rhd \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n)n)n \gg \underline{b}\rceil, p_0, c)]$

$[\text{TwiceExistMP} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\forall_{\text{obj}} \underline{(v1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{a})n)n)n)n \vdash \underline{b}]$

$[\mathrm{TwiceExistMP} \stackrel{\mathrm{tex}}{\to} \text{“TwiceExistMP”}]$

$[\mathrm{TwiceExistMP} \stackrel{\mathrm{pyk}}{\to} \text{“pred lemma 2exist mp”}]$

TwiceExistMP2

$[\mathrm{TwiceExistMP2} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{(v3)}\colon \forall \underline{(v4)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v4)}\colon \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{TwiceExistMP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow$
$\underline{\mathsf{c}} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}; \mathrm{TwiceExistMP} \rhd \underline{\mathsf{b}} \Rightarrow$
$\underline{\mathsf{c}} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v4)}\colon \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{c}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{TwiceExistMP2} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{(v3)}\colon \forall \underline{(v4)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow$
$\underline{\mathsf{c}} \vdash \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v4)}\colon \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{c}}]$

$[\mathrm{TwiceExistMP2} \stackrel{\mathrm{tex}}{\to} \text{“TwiceExistMP2”}]$

$[\mathrm{TwiceExistMP2} \stackrel{\mathrm{pyk}}{\to} \text{“pred lemma 2exist mp2”}]$

EAE – MP

$[\mathrm{EAE}-\mathrm{MP} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(v2)}\colon \forall \underline{(v3)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \mathrm{A4} @ \underline{(v2)} \rhd \forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n}; \mathrm{ExistMP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg$
$\underline{\mathsf{b}}; \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{(v3)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \mathrm{Ded} \rhd \forall \underline{(v2)}\colon \forall \underline{(v3)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{b}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{\mathsf{b}}; \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v1)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow$
$\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \gg \forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{\mathsf{b}}; \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{\mathsf{b}} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v1)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{b}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{EAE}-\mathrm{MP} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{(v3)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{(v1)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v2)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(v3)}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{b}}]$

$[\mathrm{EAE}-\mathrm{MP} \stackrel{\mathrm{tex}}{\to} \text{“EAE-MP”}]$

$[\mathrm{EAE}-\mathrm{MP} \stackrel{\mathrm{pyk}}{\to} \text{“pred lemma EAE mp”}]$

AddAll

$[\mathrm{AddAll} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{\mathsf{a}} \vdash$
$\mathrm{A4} \rhd \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{\mathsf{a}} \gg \underline{\mathsf{a}}; \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \gg \underline{\mathsf{b}}; \mathrm{Gen} \rhd \underline{\mathsf{b}} \gg$

$\forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}}; \forall\underline{(v1)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \mathrm{Ded} \rhd \forall\underline{(v1)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \vdash \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}}; \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \mathrm{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \gg \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{AddAll} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{(v1)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}}]$

$[\mathrm{AddAll} \overset{\mathrm{tex}}{\rightarrow}$ "AddAll "]

$[\mathrm{AddAll} \overset{\mathrm{pyk}}{\rightarrow}$ "pred lemma addAll"]

AddExist(Helper1)

$[\mathrm{AddExist(Helper1)} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{y}}\colon \forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} | \underline{(v2)} {:=}{=} \underline{\mathsf{y}}\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \vdash \mathrm{A4} @ \underline{\mathsf{y}} \rhd \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{b}})\mathrm{n}; \mathrm{MT} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{a}})\mathrm{n}; \mathrm{MT} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \rhd \dot{\neg}(\underline{\mathsf{a}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{c}})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(\underline{\mathsf{c}})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n}; \mathrm{FromContradiction} \rhd \forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \forall\underline{\mathsf{y}}\colon \forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \mathrm{Ded} \rhd \forall\underline{\mathsf{y}}\colon \forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} | \underline{(v2)} {:=}{=} \underline{\mathsf{y}}\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \gg \langle \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} | \underline{(v2)} {:=}{=} \underline{\mathsf{y}}\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{AddExist(Helper1)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{y}}\colon \forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} | \underline{(v2)} {:=}{=} \underline{\mathsf{y}}\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}]$

$[\mathrm{AddExist(Helper1)} \overset{\mathrm{tex}}{\rightarrow}$ "AddExist(Helper1)"]

$[\mathrm{AddExist(Helper1)} \overset{\mathrm{pyk}}{\rightarrow}$ "pred lemma addExist helper1"]

AddExist(Helper2)

$[\mathrm{AddExist(Helper2)} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{y}}\colon \forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} | \underline{(v2)} {:=}{=} \underline{\mathsf{y}}\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \vdash \mathrm{AddExist(Helper1)} \unrhd \langle \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} | \underline{(v2)} {:=}{=} \underline{\mathsf{y}}\rangle_{\mathrm{Me}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \mathrm{MP3} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \text{prop lemma imply negation} \rhd \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \gg$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \mathrm{Ded} \rhd$
$\forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \gg \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash$
$\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[AddExist(Helper2) $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow$
$\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}]$

[AddExist(Helper2) $\stackrel{\mathrm{tex}}{\rightarrow}$ "AddExist(Helper2)"]

[AddExist(Helper2) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "pred lemma addExist helper2"]

AddExist

[AddExist $\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash$
$\mathrm{AddExist(Helper2)} \unrhd \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}; \mathrm{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \gg$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[AddExist $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}]$

[AddExist $\stackrel{\mathrm{tex}}{\rightarrow}$ "AddExist"]

[AddExist $\stackrel{\mathrm{pyk}}{\rightarrow}$ "pred lemma addExist"]

AddExist(SimpleAnt)

[AddExist(SimpleAnt) $\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\mathrm{AutoImply} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}; \mathrm{AddExist} @ \underline{\mathsf{y}} \unrhd \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \rhd \underline{\mathsf{a}} \Rightarrow$
$\underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[AddExist(SimpleAnt) $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{\mathsf{y}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{d}}\colon \langle \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \equiv \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n} | \underline{(\mathrm{v2})} := = \underline{\mathsf{y}} \rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(\mathrm{v2})}\colon \dot{\neg}\,(\underline{\mathsf{d}})\mathrm{n})\mathrm{n}]$

[AddExist(SimpleAnt) $\stackrel{\mathrm{tex}}{\rightarrow}$ "AddExist(SimpleAnt)"]

[AddExist(SimpleAnt) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "pred lemma addExist(SimpleAnt)"]

AddExist(Simple)

[AddExist(Simple) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall (v1){:}\, \forall (v2){:}\, \forall \underline{\mathsf{a}}{:}\, \forall \underline{\mathsf{b}}{:}\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\text{AutoImply} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}};\, \text{AddExist} \mathbin{@} (v2) \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \gg$
$\dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n} \rceil, \text{p}_0, \mathsf{c})$]

[AddExist(Simple) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall (v1){:}\, \forall (v2){:}\, \forall \underline{\mathsf{a}}{:}\, \forall \underline{\mathsf{b}}{:}\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n}$]

[AddExist(Simple) $\stackrel{\text{tex}}{\rightarrow}$ "AddExist(Simple)"]

[AddExist(Simple) $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma addExist(Simple)"]

AddEAE

[AddEAE $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall (v1){:}\, \forall (v2){:}\, \forall (v3){:}\, \forall \underline{\mathsf{a}}{:}\, \forall \underline{\mathsf{b}}{:}\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\text{AddExist(Simple)} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n};\, \text{AddAll} \rhd \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n} \gg$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n};\, \text{AddExist(Simple)} \rhd$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n} \Rightarrow \forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n} \gg$
$\dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n})\text{n})\text{n} \rceil, \text{p}_0, \mathsf{c})$]

[AddEAE $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall (v1){:}\, \forall (v2){:}\, \forall (v3){:}\, \forall \underline{\mathsf{a}}{:}\, \forall \underline{\mathsf{b}}{:}\, \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash$
$\dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\underline{\mathsf{b}})\text{n})\text{n})\text{n})\text{n}$]

[AddEAE $\stackrel{\text{tex}}{\rightarrow}$ "AddEAE"]

[AddEAE $\stackrel{\text{pyk}}{\rightarrow}$ "pred lemma addEAE"]

AEA − negated

[AEA − negated $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash$
$\forall (v1){:}\, \forall (v2){:}\, \forall (v3){:}\, \forall \underline{\mathsf{a}}{:}\, \dot{\neg}\, (\forall_{\text{obj}} (v1){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n})\text{n})\text{n} \vdash$
$\text{AllNegated(Imply)} \gg \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n})\text{n};\, \text{AddAll} \rhd$
$\dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n})\text{n} \gg \forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n} \Rightarrow$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n})\text{n};\, \text{ExistNegated(Imply)} \gg$
$\dot{\neg}\, (\dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n})\text{n})\text{n} \Rightarrow$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n};\, \text{ImplyTransitivity} \rhd$
$\dot{\neg}\, (\dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n})\text{n})\text{n} \Rightarrow$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n} \rhd \forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n} \Rightarrow$
$\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \dot{\neg}\, (\dot{\neg}\, (\underline{\mathsf{a}})\text{n})\text{n})\text{n} \gg \dot{\neg}\, (\dot{\neg}\, (\forall_{\text{obj}} (v2){:}\, \dot{\neg}\, (\forall_{\text{obj}} (v3){:}\, \underline{\mathsf{a}})\text{n})\text{n})\text{n} \Rightarrow$

$\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n; \mathrm{AddExist(Simple)} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n \gg$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n)n)n; \mathrm{AllNegated(Imply)} \gg$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n \rhd$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n)n)n; \mathrm{MP} \rhd$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n)n)n \rhd$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \gg$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[AEA − negated $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{a}: \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \vdash$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n)n)n]$

[AEA − negated $\overset{\mathrm{tex}}{\rightarrow}$ "AEA-negated"]

[AEA − negated $\overset{\mathrm{pyk}}{\rightarrow}$ "pred lemma AEAnegated"]

EEA − negated

[EEA − negated $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{(v3)}: \forall\underline{a}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n \vdash$
$\mathrm{AllNegated(Imply)} \gg \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n \Rightarrow \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n; \mathrm{AddAll} \rhd$
$\dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n \Rightarrow \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n \gg \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n; \mathrm{ExistNegated(Imply)} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n \rhd \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n; \mathrm{AddAll} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n \gg$
$\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \Rightarrow$
$\forall_{obj}\underline{(v1)}: \forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \dot{\neg}(\dot{\neg}(\underline{a})n)n)n; \mathrm{ExistNegated(Imply)} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n \Rightarrow$
$\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n; \mathrm{ImplyTransitivity} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n)n)n \Rightarrow$
$\forall_{obj}\underline{(v1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(\forall_{obj}\underline{(v3)}: \underline{a})n)n)n \rhd$

$\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \underline{\mathsf{a}})n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})n)n)n \gg$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \underline{\mathsf{a}})n)n)n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})n)n)n; \mathrm{MP} \rhd$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \underline{\mathsf{a}})n)n)n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})n)n)n \rhd$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \underline{\mathsf{a}})n)n)n)n)n \gg$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{EEA-negated} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{(v3)}\colon \forall\underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \underline{\mathsf{a}})n)n)n)n)n \vdash$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{(v3)}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})n)n)n]$

[EEA − negated $\overset{\mathrm{tex}}{\rightarrow}$ "EEA-negated"]

[EEA − negated $\overset{\mathrm{pyk}}{\rightarrow}$ "pred lemma EEAnegated"]

Induction

$[\mathrm{Induction} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(v1)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}0\rangle_{\mathrm{Me}} \Vdash$
$\langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}(\underline{(v1)}+1)\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \mathrm{Gen} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \gg \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow$
$\underline{\mathsf{c}}; \mathrm{InductionAxiom} \unrhd \langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}0\rangle_{\mathrm{Me}} \unrhd \langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}(\underline{(v1)}+1)\rangle_{\mathrm{Me}} \gg \underline{\mathsf{b}} \Rightarrow$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}}; \mathrm{MP2} \rhd \underline{\mathsf{b}} \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \Rightarrow$
$\forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \rhd \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \gg \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}}; \mathrm{A4} @ \underline{(v1)} \rhd \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{a}} \gg \underline{\mathsf{a}}\rceil, p_0, \mathsf{c})]$

$[\mathrm{Induction} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{(v1)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}0\rangle_{\mathrm{Me}} \Vdash$
$\langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(v1)}{:=}{=}(\underline{(v1)}+1)\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}}]$

[Induction $\overset{\mathrm{tex}}{\rightarrow}$ "Induction"]

[Induction $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma induction"]

leqAntisymmetry

$[\mathrm{leqAntisymmetry} \overset{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{x}} \vdash$
$\mathrm{leqAntisymmetryAxiom} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}}; \mathrm{MP2} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <=$
$\underline{\mathsf{x}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \underline{\mathsf{y}} <= \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}}\rceil, p_0, \mathsf{c})]$

$[\mathrm{leqAntisymmetry} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \underline{\mathsf{y}} <= \underline{\mathsf{x}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{y}}]$

[leqAntisymmetry $\overset{\mathrm{tex}}{\rightarrow}$ "leqAntisymmetry"]

[leqAntisymmetry $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqAntisymmetry"]

leqTransitivity

[leqTransitivity $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{z} \vdash \text{leqTransitivityAxiom} \gg \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{x} <= \underline{z}; \text{MP2} \rhd \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{x} <= \underline{z} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{z} \gg \underline{x} <= \underline{z}\rceil, p_0, c)$]

[leqTransitivity $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{z} \vdash \underline{x} <= \underline{z}$]

[leqTransitivity $\stackrel{\text{tex}}{\rightarrow}$ "leqTransitivity"]

[leqTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqTransitivity"]

leqAddition

[leqAddition $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash \text{leqAdditionAxiom} \gg \underline{x} <= \underline{y} \Rightarrow (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}); \text{MP} \rhd \underline{x} <= \underline{y} \Rightarrow (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \rhd \underline{x} <= \underline{y} \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z})\rceil, p_0, c)$]

[leqAddition $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z})$]

[leqAddition $\stackrel{\text{tex}}{\rightarrow}$ "leqAddition"]

[leqAddition $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqAddition"]

leqMultiplication

[leqMultiplication $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash \text{leqMultiplicationAxiom} \gg 0 <= \underline{z} \Rightarrow \underline{x} <= \underline{y} \Rightarrow (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}); \text{MP2} \rhd 0 <= \underline{z} \Rightarrow \underline{x} <= \underline{y} \Rightarrow (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \rhd 0 <= \underline{z} \rhd \underline{x} <= \underline{y} \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z})\rceil, p_0, c)$]

[leqMultiplication $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z})$]

[leqMultiplication $\stackrel{\text{tex}}{\rightarrow}$ "leqMultiplication"]

[leqMultiplication $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqMultiplication"]

Reciprocal

[Reciprocal $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot\neg (\underline{x} = 0)\text{n} \vdash \text{ReciprocalAxiom} \gg \dot\neg (\underline{x} = 0)\text{n} \Rightarrow (\underline{x} * \text{rec}\underline{x}) = 1; \text{MP} \rhd \dot\neg (\underline{x} = 0)\text{n} \Rightarrow (\underline{x} * \text{rec}\underline{x}) = 1 \rhd \dot\neg (\underline{x} = 0)\text{n} \gg (\underline{x} * \text{rec}\underline{x}) = 1\rceil, p_0, c)$]

[Reciprocal $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \dot\neg (\underline{x} = 0)\text{n} \vdash (\underline{x} * \text{rec}\underline{x}) = 1$]

[Reciprocal $\stackrel{\text{tex}}{\to}$ "Reciprocal"]

[Reciprocal $\stackrel{\text{pyk}}{\to}$ "lemma reciprocal"]

Equality

[Equality $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{z}} \vdash \text{EqualityAxiom} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{y}} = \underline{\mathsf{z}}; \text{MP2} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{x}} = \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{y}} = \underline{\mathsf{z}} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \rhd \underline{\mathsf{x}} = \underline{\mathsf{z}} \gg \underline{\mathsf{y}} = \underline{\mathsf{z}} \rceil, \mathrm{p}_0, \mathsf{c})$]

[Equality $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{x}} = \underline{\mathsf{z}} \vdash \underline{\mathsf{y}} = \underline{\mathsf{z}}$]

[Equality $\stackrel{\text{tex}}{\to}$ "Equality"]

[Equality $\stackrel{\text{pyk}}{\to}$ "lemma equality"]

eqLeq

[eqLeq $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \text{EqLeqAxiom} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{y}}; \text{MP} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rceil, \mathrm{p}_0, \mathsf{c})$]

[eqLeq $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \underline{\mathsf{x}} <= \underline{\mathsf{y}}$]

[eqLeq $\stackrel{\text{tex}}{\to}$ "eqLeq"]

[eqLeq $\stackrel{\text{pyk}}{\to}$ "lemma eqLeq"]

eqAddition

[eqAddition $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \text{EqAdditionAxiom} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}); \text{MP} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rceil, \mathrm{p}_0, \mathsf{c})$]

[eqAddition $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}})$]

[eqAddition $\stackrel{\text{tex}}{\to}$ "eqAddition"]

[eqAddition $\stackrel{\text{pyk}}{\to}$ "lemma eqAddition"]

eqMultiplication

[eqMultiplication $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \text{EqMultiplicationAxiom} \gg \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) = (\underline{\mathsf{y}} * \underline{\mathsf{z}}); \text{MP} \rhd \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) =$

$(\underline{y} * \underline{z}) \rhd \underline{x} = \underline{y} \gg (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{eqMultiplication} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} = \underline{y} \vdash (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})]$

$[\mathrm{eqMultiplication} \overset{\mathrm{tex}}{\rightarrow}$ "eqMultiplication"$]$

$[\mathrm{eqMultiplication} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma eqMultiplication"$]$

LeqMultiplicationLeft

$[\mathrm{LeqMultiplicationLeft} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash \mathrm{leqMultiplication} \rhd 0 <= \underline{z} \rhd \underline{x} <= \underline{y} \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}); \mathrm{timesCommutativity} \gg (\underline{x} * \underline{z}) = (\underline{z} * \underline{x}); \mathrm{subLeqLeft} \rhd (\underline{x} * \underline{z}) = (\underline{z} * \underline{x}) \rhd (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \gg (\underline{z} * \underline{x}) <= (\underline{y} * \underline{z}); \mathrm{timesCommutativity} \gg (\underline{y} * \underline{z}) = (\underline{z} * \underline{y}); \mathrm{subLeqRight} \rhd (\underline{y} * \underline{z}) = (\underline{z} * \underline{y}) \rhd (\underline{z} * \underline{x}) <= (\underline{y} * \underline{z}) \gg (\underline{z} * \underline{x}) <= (\underline{z} * \underline{y})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LeqMultiplicationLeft} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash (\underline{z} * \underline{x}) <= (\underline{z} * \underline{y})]$

$[\mathrm{LeqMultiplicationLeft} \overset{\mathrm{tex}}{\rightarrow}$ "LeqMultiplicationLeft "$]$

$[\mathrm{LeqMultiplicationLeft} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqMultiplicationLeft"$]$

LeqLessEq

$[\mathrm{LeqLessEq} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{fromNotLess} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{y} <= \underline{x}; \mathrm{leqAntisymmetry} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{x} \gg \underline{x} = \underline{y}; \forall \underline{x} \colon \forall \underline{y} \colon \mathrm{Ded} \rhd \forall \underline{x} \colon \forall \underline{y} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{x} = \underline{y} \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = \underline{y}; \underline{x} <= \underline{y} \vdash \mathrm{MP} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = \underline{y} \rhd \underline{x} <= \underline{y} \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = \underline{y}; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = \underline{y} \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = \underline{y}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LeqLessEq} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = \underline{y}]$

$[\mathrm{LeqLessEq} \overset{\mathrm{tex}}{\rightarrow}$ "LeqLessEq"$]$

$[\mathrm{LeqLessEq} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqLessEq"$]$

LessLeq

[LessLeq $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})n)n)n \vdash \text{Repetition} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})n)n)n \gg \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})n)n)n; \text{FirstConjunct} \rhd \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})n)n)n \gg \underline{x} <= \underline{y}\rceil, p_0, c)$]

[LessLeq $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})n)n)n \vdash \underline{x} <= \underline{y}$]

[LessLeq $\stackrel{\text{tex}}{\rightarrow}$ "LessLeq"]

[LessLeq $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessLeq"]

FromLeqGeq

[FromLeqGeq $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \Rightarrow \underline{a} \vdash \underline{y} <= \underline{x} \Rightarrow \underline{a} \vdash \text{leqTotality} \gg \dot\neg(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x}; \text{FromDisjuncts} \rhd \dot\neg(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \Rightarrow \underline{a} \rhd \underline{y} <= \underline{x} \Rightarrow \underline{a} \gg \underline{a}\rceil, p_0, c)$]

[FromLeqGeq $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \Rightarrow \underline{a} \vdash \underline{y} <= \underline{x} \Rightarrow \underline{a} \vdash \underline{a}$]

[FromLeqGeq $\stackrel{\text{tex}}{\rightarrow}$ "FromLeqGeq"]

[FromLeqGeq $\stackrel{\text{pyk}}{\rightarrow}$ "lemma from leqGeq"]

subLeqRight

[subLeqRight $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \underline{z} <= \underline{x} \vdash \text{eqLeq} \rhd \underline{x} = \underline{y} \gg \underline{x} <= \underline{y}; \text{leqTransitivity} \rhd \underline{z} <= \underline{x} \rhd \underline{x} <= \underline{y} \gg \underline{z} <= \underline{y}\rceil, p_0, c)$]

[subLeqRight $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \underline{z} <= \underline{x} \vdash \underline{z} <= \underline{y}$]

[subLeqRight $\stackrel{\text{tex}}{\rightarrow}$ "subLeqRight"]

[subLeqRight $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subLeqRight"]

subLeqLeft

[subLeqLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \underline{x} <= \underline{z} \vdash \text{eqSymmetry} \rhd \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \text{eqLeq} \rhd \underline{y} = \underline{x} \gg \underline{y} <= \underline{x}; \text{leqTransitivity} \rhd \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{z} \gg \underline{y} <= \underline{z}\rceil, p_0, c)$]

[subLeqLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \underline{x} <= \underline{z} \vdash \underline{y} <= \underline{z}$]

[subLeqLeft $\stackrel{\text{tex}}{\rightarrow}$ "subLeqLeft"]

[subLeqLeft $\overset{\text{pyk}}{\to}$ "lemma subLeqLeft"]

Leq + 1

[Leq + 1 $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash 0 < 1 \gg \dot\neg(0 <= 1 \Rightarrow \dot\neg(\dot\neg(0 = 1)n)n)n; \text{LessAdditionLeft} \rhd \dot\neg(0 <= 1 \Rightarrow \dot\neg(\dot\neg(0 = 1)n)n)n \gg \dot\neg((\underline{\mathsf{y}} + 0) <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot\neg(\dot\neg((\underline{\mathsf{y}} + 0) = (\underline{\mathsf{y}} + 1))n)n)n; \text{plus0} \gg (\underline{\mathsf{y}} + 0) = \underline{\mathsf{y}}; \text{SubLessLeft} \rhd (\underline{\mathsf{y}} + 0) = \underline{\mathsf{y}} \rhd \dot\neg((\underline{\mathsf{y}} + 0) <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot\neg(\dot\neg((\underline{\mathsf{y}} + 0) = (\underline{\mathsf{y}} + 1))n)n)n \gg \dot\neg(\underline{\mathsf{y}} <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot\neg(\dot\neg(\underline{\mathsf{y}} = (\underline{\mathsf{y}} + 1))n)n)n; \text{leqLessTransitivity} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \rhd \dot\neg(\underline{\mathsf{y}} <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot\neg(\dot\neg(\underline{\mathsf{y}} = (\underline{\mathsf{y}} + 1))n)n)n \gg \dot\neg(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot\neg(\dot\neg(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + 1))n)n)n\rceil, p_0, \mathsf{c})$]

[Leq + 1 $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \dot\neg(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + 1) \Rightarrow \dot\neg(\dot\neg(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + 1))n)n)n$]

[Leq + 1 $\overset{\text{tex}}{\to}$ "Leq+1"]

[Leq + 1 $\overset{\text{pyk}}{\to}$ "lemma leqPlus1"]

PositiveToRight(Leq)

[PositiveToRight(Leq) $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) <= \underline{\mathsf{z}} \vdash \text{leqAddition} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{y}}) <= \underline{\mathsf{z}} \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-u\underline{\mathsf{y}})) <= (\underline{\mathsf{z}} + (-u\underline{\mathsf{y}})); x = x + y - y \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-u\underline{\mathsf{y}})); \text{eqSymmetry} \rhd \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-u\underline{\mathsf{y}})) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-u\underline{\mathsf{y}})) = \underline{\mathsf{x}}; \text{subLeqLeft} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-u\underline{\mathsf{y}})) = \underline{\mathsf{x}} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + (-u\underline{\mathsf{y}})) <= (\underline{\mathsf{z}} + (-u\underline{\mathsf{y}})) \gg \underline{\mathsf{x}} <= (\underline{\mathsf{z}} + (-u\underline{\mathsf{y}}))\rceil, p_0, \mathsf{c})$]

[PositiveToRight(Leq) $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) <= \underline{\mathsf{z}} \vdash \underline{\mathsf{x}} <= (\underline{\mathsf{z}} + (-u\underline{\mathsf{y}}))$]

[PositiveToRight(Leq) $\overset{\text{tex}}{\to}$ "PositiveToRight(Leq)"]

[PositiveToRight(Leq) $\overset{\text{pyk}}{\to}$ "lemma positiveToRight(Leq)"]

PositiveToRight(Leq)(1term)

[PositiveToRight(Leq)(1term) $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{y}} <= \underline{\mathsf{z}} \vdash \text{lemma plus0Left} \gg (0 + \underline{\mathsf{y}}) = \underline{\mathsf{y}}; \text{eqSymmetry} \rhd (0 + \underline{\mathsf{y}}) = \underline{\mathsf{y}} \gg \underline{\mathsf{y}} = (0 + \underline{\mathsf{y}}); \text{subLeqLeft} \rhd \underline{\mathsf{y}} = (0 + \underline{\mathsf{y}}) \rhd \underline{\mathsf{y}} <= \underline{\mathsf{z}} \gg (0 + \underline{\mathsf{y}}) <= \underline{\mathsf{z}}; \text{PositiveToRight(Leq)} \rhd (0 + \underline{\mathsf{y}}) <= \underline{\mathsf{z}} \gg 0 <= (\underline{\mathsf{z}} + (-u\underline{\mathsf{y}}))\rceil, p_0, \mathsf{c})$]

[PositiveToRight(Leq)(1term) $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{y}} <= \underline{\mathsf{z}} \vdash 0 <= (\underline{\mathsf{z}} + (-u\underline{\mathsf{y}}))$]

[PositiveToRight(Leq)(1term) $\stackrel{\text{tex}}{\to}$ "PositiveToRight(Leq)(1 term)"]

[PositiveToRight(Leq)(1term) $\stackrel{\text{pyk}}{\to}$ "lemma positiveToRight(Leq)(1 term)"]

lemma negativeToRight(Leq)

[lemma negativeToRight(Leq) $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + (-\text{u}\underline{y})) <= \underline{z} \vdash \text{leqAddition} \rhd (\underline{x} + (-\text{u}\underline{y})) <= \underline{z} \gg ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}) <= (\underline{z} + \underline{y}); \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\text{u}\underline{y})); \text{Three2threeTerms} \gg ((\underline{x} + \underline{y}) + (-\text{u}\underline{y})) = ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}); \text{eqTransitivity} \rhd \underline{x} = ((\underline{x} + \underline{y}) + (-\text{u}\underline{y})) \rhd ((\underline{x} + \underline{y}) + (-\text{u}\underline{y})) = ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}) \gg \underline{x} = ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}); \text{eqSymmetry} \rhd \underline{x} = ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}) \gg ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}) = \underline{x}; \text{subLeqLeft} \rhd ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}) = \underline{x} \rhd ((\underline{x} + (-\text{u}\underline{y})) + \underline{y}) <= (\underline{z} + \underline{y}) \gg \underline{x} <= (\underline{z} + \underline{y})\rceil, \text{p}_0, \mathsf{c})$]

[lemma negativeToRight(Leq) $\stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + (-\text{u}\underline{y})) <= \underline{z} \vdash \underline{x} <= (\underline{z} + \underline{y})$]

[lemma negativeToRight(Leq) $\stackrel{\text{pyk}}{\to}$ "lemma negativeToRight(Leq)"]

PositiveToLeft(Leq)

[PositiveToLeft(Leq) $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= (\underline{y} + \underline{z}) \vdash \text{leqAddition} \rhd \underline{x} <= (\underline{y} + \underline{z}) \gg (\underline{x} + (-\text{u}\underline{z})) <= ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})); \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})); \text{eqSymmetry} \rhd \underline{y} = ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) \gg ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) = \underline{y}; \text{subLeqRight} \rhd ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) = \underline{y} \rhd (\underline{x} + (-\text{u}\underline{z})) <= ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) \gg (\underline{x} + (-\text{u}\underline{z})) <= \underline{y}\rceil, \text{p}_0, \mathsf{c})$]

[PositiveToLeft(Leq) $\stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= (\underline{y} + \underline{z}) \vdash (\underline{x} + (-\text{u}\underline{z})) <= \underline{y}$]

[PositiveToLeft(Leq) $\stackrel{\text{tex}}{\to}$ "PositiveToLeft(Leq)"]

[PositiveToLeft(Leq) $\stackrel{\text{pyk}}{\to}$ "lemma positiveToLeft(Leq)"]

negativeToLeft(Leq)

[negativeToLeft(Leq) $\stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= (\underline{y} + (-\text{u}\underline{z})) \vdash \text{leqAddition} \rhd \underline{x} <= (\underline{y} + (-\text{u}\underline{z})) \gg (\underline{x} + \underline{z}) <= ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}); \text{x} = \text{x} + \text{y} - \text{y} \gg \underline{y} = ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})); \text{Three2threeTerms} \gg ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) = ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}); \text{eqTransitivity} \rhd \underline{y} = ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) \rhd ((\underline{y} + \underline{z}) + (-\text{u}\underline{z})) = ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}) \gg \underline{y} = ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}); \text{eqSymmetry} \rhd \underline{y} = ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}) \gg ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}) = \underline{y}; \text{subLeqRight} \rhd ((\underline{y} + (-\text{u}\underline{z})) + \underline{z}) =$

$\underline{y} \rhd (\underline{x}+\underline{z}) <= ((\underline{y}+(-u\underline{z}))+\underline{z}) \gg (\underline{x}+\underline{z}) <= \underline{y}\rceil, p_0, c)]$

[negativeToLeft(Leq) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= (\underline{y}+(-u\underline{z})) \vdash (\underline{x}+\underline{z}) <= \underline{y}$]

[negativeToLeft(Leq) $\stackrel{\text{tex}}{\rightarrow}$ "negativeToLeft(Leq)"]

[negativeToLeft(Leq) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeToLeft(Leq)"]

negativeToLeft(Leq)(1term)

[negativeToLeft(Leq)(1term) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{y}+(-u\underline{z})) \vdash$ negativeToLeft(Leq) $\rhd 0 <= (\underline{y}+(-u\underline{z})) \gg (0+\underline{z}) <= \underline{y};$ lemma plus0Left $\gg (0+\underline{z}) = \underline{z};$ subLeqLeft $\rhd (0+\underline{z}) = \underline{z} \gg \underline{z} <= \underline{y}\rceil, p_0, c)]$

[negativeToLeft(Leq)(1term) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{y}+(-u\underline{z})) \vdash \underline{z} <= \underline{y}$]

[negativeToLeft(Leq)(1term) $\stackrel{\text{tex}}{\rightarrow}$ "negativeToLeft(Leq)(1 term)"]

[negativeToLeft(Leq)(1term) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma negativeToLeft(Leq)(1 term)"]

LeqAdditionLeft

[LeqAdditionLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash$ leqAddition $\rhd \underline{x} <= \underline{y} \gg (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z});$ plusCommutativity $\gg (\underline{x}+\underline{z}) = (\underline{z}+\underline{x});$ plusCommutativity $\gg (\underline{y}+\underline{z}) = (\underline{z}+\underline{y});$ subLeqLeft $\rhd (\underline{x}+\underline{z}) = (\underline{z}+\underline{x}) \rhd (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \gg (\underline{z}+\underline{x}) <= (\underline{y}+\underline{z});$ subLeqRight $\rhd (\underline{y}+\underline{z}) = (\underline{z}+\underline{y}) \rhd (\underline{z}+\underline{x}) <= (\underline{y}+\underline{z}) \gg (\underline{z}+\underline{x}) <= (\underline{z}+\underline{y})\rceil, p_0, c)]$

[LeqAdditionLeft $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \vdash (\underline{z}+\underline{x}) <= (\underline{z}+\underline{y})$]

[LeqAdditionLeft $\stackrel{\text{tex}}{\rightarrow}$ "LeqAdditionLeft"]

[LeqAdditionLeft $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqAdditionLeft"]

leqSubtraction

[leqSubtraction $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \vdash$ leqAddition $\rhd (\underline{x}+\underline{z}) <= (\underline{y}+\underline{z}) \gg ((\underline{x}+\underline{z})+(-u\underline{z})) <= ((\underline{y}+\underline{z})+(-u\underline{z}));$ $x = x+y-y \gg \underline{x} = ((\underline{x}+\underline{z})+(-u\underline{z}));$ eqSymmetry $\rhd \underline{x} = ((\underline{x}+\underline{z})+(-u\underline{z})) \gg ((\underline{x}+\underline{z})+(-u\underline{z})) = \underline{x};$ $x = x+y-y \gg \underline{y} = ((\underline{y}+\underline{z})+(-u\underline{z}));$ eqSymmetry $\rhd \underline{y} = ((\underline{y}+\underline{z})+(-u\underline{z})) \gg ((\underline{y}+\underline{z})+(-u\underline{z})) = \underline{y};$ subLeqLeft $\rhd ((\underline{x}+\underline{z})+(-u\underline{z})) = \underline{x} \rhd ((\underline{x}+\underline{z})+(-u\underline{z})) <= ((\underline{y}+\underline{z})+(-u\underline{z})) \gg \underline{x} <= ((\underline{y}+\underline{z})+(-u\underline{z}));$ subLeqRight $\rhd ((\underline{y}+\underline{z})+(-u\underline{z})) = \underline{y} \rhd \underline{x} <= ((\underline{y}+\underline{z})+(-u\underline{z})) \gg \underline{x} <= \underline{y}\rceil, p_0, c)]$

[leqSubtraction $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \vdash \underline{x} <= \underline{y}$]

[leqSubtraction $\stackrel{\text{tex}}{\rightarrow}$ “leqSubtraction”]

[leqSubtraction $\stackrel{\text{pyk}}{\rightarrow}$ “lemma leqSubtraction”]

leqSubtractionLeft

[leqSubtractionLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \vdash$ plusCommutativity $\gg (\underline{z} + \underline{x}) = (\underline{x} + \underline{z});$ plusCommutativity $\gg (\underline{z} + \underline{y}) = (\underline{y} + \underline{z});$ subLeqLeft $\rhd (\underline{z} + \underline{x}) = (\underline{x} + \underline{z}) \rhd (\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \gg (\underline{x} + \underline{z}) <= (\underline{z} + \underline{y});$ subLeqRight $\rhd (\underline{z} + \underline{y}) = (\underline{y} + \underline{z}) \rhd (\underline{x} + \underline{z}) <= (\underline{z} + \underline{y}) \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z});$ leqSubtraction $\rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \gg \underline{x} <= \underline{y}\rceil, p_0, c)$]

[leqSubtractionLeft $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \vdash \underline{x} <= \underline{y}$]

[leqSubtractionLeft $\stackrel{\text{tex}}{\rightarrow}$ “leqSubtractionLeft”]

[leqSubtractionLeft $\stackrel{\text{pyk}}{\rightarrow}$ “lemma leqSubtractionLeft”]

thirdGeq

[thirdGeq $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash$ leqReflexivity $\gg \underline{y} <= \underline{y};$ JoinConjuncts $\rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{y} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{y})n)n;$ ExistIntro @ c_{Ex} @ $\underline{y} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{y})n)n \gg \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n; \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash$ leqReflexivity $\gg \underline{x} <= \underline{x};$ JoinConjuncts $\rhd \underline{x} <= \underline{x} \rhd \underline{y} <= \underline{x} \gg \dot{\neg}(\underline{x} <= \underline{x} \Rightarrow \dot{\neg}(\underline{y} <= \underline{x})n)n;$ ExistIntro @ c_{Ex} @ $\underline{x} \rhd \dot{\neg}(\underline{x} <= \underline{x} \Rightarrow \dot{\neg}(\underline{y} <= \underline{x})n)n \gg \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n; \forall \underline{x}\colon \forall \underline{y}\colon$ Ded $\rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n;$ Ded $\rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n \gg \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n;$ leqTotality $\gg \dot{\neg}(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x};$ FromDisjuncts $\rhd \dot{\neg}(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n \rhd \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n \gg \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n\rceil, p_0, c)$]

[thirdGeq $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= c_{Ex} \Rightarrow \dot{\neg}(\underline{y} <= c_{Ex})n)n$]

[thirdGeq $\stackrel{\text{tex}}{\rightarrow}$ “thirdGeq”]

[thirdGeq $\stackrel{\text{pyk}}{\rightarrow}$ “lemma thirdGeq”]

LeqNegated

$$[\mathrm{LeqNegated} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \mathrm{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x} + (-\mathrm{u}\underline{x})) <= (\underline{y} + (-\mathrm{u}\underline{x})); \mathrm{Negative} \gg (\underline{x} + (-\mathrm{u}\underline{x})) = 0; \mathrm{subLeqLeft} \rhd (\underline{x} + (-\mathrm{u}\underline{x})) = 0 \rhd (\underline{x} + (-\mathrm{u}\underline{x})) <= (\underline{y} + (-\mathrm{u}\underline{x})) \gg 0 <= (\underline{y} + (-\mathrm{u}\underline{x})); \mathrm{plusCommutativity} \gg (\underline{y} + (-\mathrm{u}\underline{x})) = ((-\mathrm{u}\underline{x}) + \underline{y}); \mathrm{subLeqRight} \rhd (\underline{y} + (-\mathrm{u}\underline{x})) = ((-\mathrm{u}\underline{x}) + \underline{y}) \rhd 0 <= (\underline{y} + (-\mathrm{u}\underline{x})) \gg 0 <= ((-\mathrm{u}\underline{x}) + \underline{y}); \mathrm{leqAddition} \rhd 0 <= ((-\mathrm{u}\underline{x}) + \underline{y}) \gg (0 + (-\mathrm{u}\underline{y})) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})); \mathrm{lemma\ plus0Left} \gg (0 + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{y}); \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg (-\mathrm{u}\underline{x}) = (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})); \mathrm{eqSymmetry} \rhd (-\mathrm{u}\underline{x}) = (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) \gg (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{x}); \mathrm{subLeqLeft} \rhd (0 + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{y}) \rhd (0 + (-\mathrm{u}\underline{y})) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) \gg (-\mathrm{u}\underline{y}) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})); \mathrm{subLeqRight} \rhd (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) = (-\mathrm{u}\underline{x}) \rhd (-\mathrm{u}\underline{y}) <= (((-\mathrm{u}\underline{x}) + \underline{y}) + (-\mathrm{u}\underline{y})) \gg (-\mathrm{u}\underline{y}) <= (-\mathrm{u}\underline{x}) \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{LeqNegated} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash (-\mathrm{u}\underline{y}) <= (-\mathrm{u}\underline{x})]$$

[LeqNegated $\overset{\mathrm{tex}}{\rightarrow}$ "LeqNegated"]

[LeqNegated $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqNegated"]

AddEquations(Leq)

$$[\mathrm{AddEquations(Leq)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash \mathrm{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}); \mathrm{LeqAdditionLeft} \rhd \underline{z} <= \underline{u} \gg (\underline{y} + \underline{z}) <= (\underline{y} + \underline{u}); \mathrm{leqTransitivity} \rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \rhd (\underline{y} + \underline{z}) <= (\underline{y} + \underline{u}) \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{u}) \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{AddEquations(Leq)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash (\underline{x} + \underline{z}) <= (\underline{y} + \underline{u})]$$

[AddEquations(Leq) $\overset{\mathrm{tex}}{\rightarrow}$ "AddEquations(Leq)"]

[AddEquations(Leq) $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma addEquations(Leq)"]

MultiplyEquations(Leq)

$$[\mathrm{MultiplyEquations(Leq)} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon 0 <= \underline{x} \vdash 0 <= \underline{z} \vdash \underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash \mathrm{leqMultiplication} \rhd 0 <= \underline{z} \rhd \underline{x} <= \underline{y} \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}); \mathrm{leqTransitivity} \rhd 0 <= \underline{x} \rhd \underline{x} <= \underline{y} \gg 0 <= \underline{y}; \mathrm{LeqMultiplicationLeft} \rhd 0 <= \underline{y} \rhd \underline{z} <= \underline{u} \gg (\underline{y} * \underline{z}) <= (\underline{y} * \underline{u}); \mathrm{leqTransitivity} \rhd (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \rhd (\underline{y} * \underline{z}) <= (\underline{y} * \underline{u}) \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{u}) \rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{MultiplyEquations(Leq)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon 0 <= \underline{x} \vdash 0 <= \underline{z} \vdash$$

$\underline{x} <= \underline{y} \vdash \underline{z} <= \underline{u} \vdash (\underline{x} * \underline{z}) <= (\underline{y} * \underline{u})]$

$[\mathrm{MultiplyEquations(Leq)} \overset{\mathrm{tex}}{\rightarrow}$ "MultiplyEquations(Leq)"]

$[\mathrm{MultiplyEquations(Leq)} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma multiplyEquations(Leq)"]

ThirdGeqSeries

$[\mathrm{ThirdGeqSeries} \overset{\mathrm{tex}}{\rightarrow}$ "ThirdGeqSeries"]

$[\mathrm{ThirdGeqSeries} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma thirdGeqSeries"]

LeqNeqLess

$[\mathrm{LeqNeqLess} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash \mathrm{JoinConjuncts} \rhd \underline{x} <= \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LeqNeqLess} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{x} = \underline{y})\mathrm{n} \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{LeqNeqLess} \overset{\mathrm{tex}}{\rightarrow}$ "LeqNeqLess"]

$[\mathrm{LeqNeqLess} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqNeqLess"]

FromLess

$[\mathrm{FromLess} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \mathrm{toNotLess} \rhd \underline{y} <= \underline{x} \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{AddDoubleNeg} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MT} \rhd \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromLess} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{y} <= \underline{x})\mathrm{n}]$

$[\mathrm{FromLess} \overset{\mathrm{tex}}{\rightarrow}$ "FromLess"]

$[\mathrm{FromLess} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma fromLess"]

ToLess

[ToLess $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \vdash \mathrm{fromNotLess} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \gg \underline{x} <= \underline{y}; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \vdash \underline{x} <= \underline{y} \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \Rightarrow \underline{x} <= \underline{y}; \dot{\neg}(\underline{x} <= \underline{y})n \vdash \mathrm{NegativeMT} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \Rightarrow \underline{x} <= \underline{y} \rhd \dot{\neg}(\underline{x} <= \underline{y})n \gg \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n\rceil, p_0, c)$]

[ToLess $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= \underline{y})n \vdash \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n$]

[ToLess $\stackrel{\text{tex}}{\rightarrow}$ "ToLess"]

[ToLess $\stackrel{\text{pyk}}{\rightarrow}$ "lemma toLess"]

fromNotLess

[fromNotLess $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \underline{x} <= \underline{y} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n; \mathrm{MP} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n \rhd \underline{x} <= \underline{y} \gg \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n \gg \underline{x} = \underline{y}; \mathrm{eqSymmetry} \rhd \underline{x} = \underline{y} \gg \underline{y} = \underline{x}; \mathrm{eqLeq} \rhd \underline{y} = \underline{x} \gg \underline{y} <= \underline{x}; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{x} \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x}; \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x}; \mathrm{AutoImply} \gg \underline{y} <= \underline{x} \Rightarrow \underline{y} <= \underline{x}; \mathrm{leqTotality} \gg \dot{\neg}(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x}; \mathrm{FromDisjuncts} \rhd \dot{\neg}(\underline{x} <= \underline{y})n \Rightarrow \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \rhd \underline{y} <= \underline{x} \Rightarrow \underline{y} <= \underline{x} \gg \underline{y} <= \underline{x}\rceil, p_0, c)$]

[fromNotLess $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \underline{y} <= \underline{x}$]

[fromNotLess $\stackrel{\text{tex}}{\rightarrow}$ "fromNotLess"]

[fromNotLess $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromNotLess"]

toNotLess

[toNotLess $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{x} \vdash \mathrm{leqAntisymmetry} \rhd \underline{y} <= \underline{x} \rhd \underline{x} <= \underline{y} \gg \underline{y} = \underline{x}; \mathrm{AddDoubleNeg} \rhd \underline{y} = \underline{x} \gg \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \vdash \underline{y} <= \underline{x} \vdash \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n \gg \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n; \underline{x} <= \underline{y} \vdash \mathrm{MP} \rhd \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n \rhd \underline{x} <= \underline{y} \gg \underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n; \mathrm{AddDoubleNeg} \rhd \underline{y} <=$

$\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n\rceil, p_0, \mathsf{c})]$

[toNotLess $\stackrel{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n)n$]

[toNotLess $\stackrel{\mathrm{tex}}{\rightarrow}$ “toNotLess”]

[toNotLess $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma toNotLess”]

NegativeLessPositive

[NegativeLessPositive $\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg 0 <= \underline{x}; \mathrm{leqAddition} \rhd 0 <= \underline{x} \gg (0 + (-u\underline{x})) <= (\underline{x} + (-u\underline{x})); \mathrm{lemma\ plus0Left} \gg (0 + (-u\underline{x})) = (-u\underline{x}); \mathrm{Negative} \gg (\underline{x} + (-u\underline{x})) = 0; \mathrm{subLeqLeft} \rhd (0 + (-u\underline{x})) = (-u\underline{x}) \rhd (0 + (-u\underline{x})) <= (\underline{x} + (-u\underline{x})) \gg (-u\underline{x}) <= (\underline{x} + (-u\underline{x})); \mathrm{subLeqRight} \rhd (\underline{x} + (-u\underline{x})) = 0 \rhd (-u\underline{x}) <= (\underline{x} + (-u\underline{x})) \gg (-u\underline{x}) <= 0; \mathrm{leqLessTransitivity} \rhd (-u\underline{x}) <= 0 \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg \dot{\neg}((-u\underline{x}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = \underline{x})n)n)n\rceil, p_0, \mathsf{c})$]

[NegativeLessPositive $\stackrel{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x} \colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \dot{\neg}((-u\underline{x}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = \underline{x})n)n)n$]

[NegativeLessPositive $\stackrel{\mathrm{tex}}{\rightarrow}$ “NegativeLessPositive”]

[NegativeLessPositive $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma negativeLessPositive”]

leqLessTransitivity

[leqLessTransitivity $\stackrel{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \underline{x} = \underline{z} \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \underline{y} <= \underline{z}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \dot{\neg}(\underline{y} = \underline{z})n; \mathrm{subLeqLeft} \rhd \underline{x} = \underline{z} \rhd \underline{x} <= \underline{y} \gg \underline{z} <= \underline{y}; \mathrm{leqAntisymmetry} \rhd \underline{y} <= \underline{z} \rhd \underline{z} <= \underline{y} \gg \underline{y} = \underline{z}; \mathrm{FromContradiction} \rhd \underline{y} = \underline{z} \rhd \dot{\neg}(\underline{y} = \underline{z})n \gg \dot{\neg}(\underline{x} = \underline{z})n; \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \mathrm{Ded} \rhd \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \underline{x} = \underline{z} \vdash \dot{\neg}(\underline{x} = \underline{z})n \gg \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \Rightarrow \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n; \underline{x} <= \underline{y} \vdash \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \vdash \mathrm{MP2} \rhd \underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \Rightarrow \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n \rhd \underline{x} <= \underline{y} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n; \mathrm{prop\ lemma\ imply\ negation} \rhd \underline{x} = \underline{z} \Rightarrow \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{x} = \underline{z})n; \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{z})n)n)n \gg \underline{y} <= \underline{z}; \mathrm{leqTransitivity} \rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{z} \gg \underline{x} <= \underline{z}; \mathrm{JoinConjuncts} \rhd \underline{x} <= \underline{z} \rhd \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n\rceil, p_0, \mathsf{c})$]

[leqLessTransitivity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \underline{x} <= \underline{y} \vdash \dot{\neg}\, (\underline{y} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{y} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\, (\underline{x} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}$]

[leqLessTransitivity $\stackrel{\text{tex}}{\rightarrow}$ "leqLessTransitivity"]

[leqLessTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqLessTransitivity"]

LessLeqTransitivity

[LessLeqTransitivity $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{y} <= \underline{z} \vdash \underline{z} = \underline{x} \vdash$ FirstConjunct $\rhd \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{x} <= \underline{y};$ SecondConjunct $\rhd \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n};$ subLeqRight $\rhd \underline{z} = \underline{x} \rhd \underline{y} <= \underline{z} \gg \underline{y} <= \underline{x};$ leqAntisymmetry $\rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{x} \gg \underline{x} = \underline{y};$ FromContradiction $\rhd \underline{x} = \underline{y} \rhd \dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n} \gg \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n}; \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon$ Ded $\rhd \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{y} <= \underline{z} \vdash \underline{z} = \underline{x} \vdash \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n} \gg \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{z} = \underline{x} \Rightarrow \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n}; \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{y} <= \underline{z} \vdash$ MP2 $\rhd \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{z} = \underline{x} \Rightarrow \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n} \rhd \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{y} <= \underline{z} \gg \underline{z} = \underline{x} \Rightarrow \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n};$ prop lemma imply negation $\rhd \underline{z} = \underline{x} \Rightarrow \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n} \gg \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n};$ NeqSymmetry $\rhd \dot{\neg}\, (\underline{z} = \underline{x})\mathrm{n} \gg \dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n};$ FirstConjunct $\rhd \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{x} <= \underline{y};$ leqTransitivity $\rhd \underline{x} <= \underline{y} \rhd \underline{y} <= \underline{z} \gg \underline{x} <= \underline{z};$ JoinConjuncts $\rhd \underline{x} <= \underline{z} \rhd \dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n} \gg \dot{\neg}\, (\underline{x} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[LessLeqTransitivity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{y} <= \underline{z} \vdash \dot{\neg}\, (\underline{x} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}$]

[LessLeqTransitivity $\stackrel{\text{tex}}{\rightarrow}$ "LessLeqTransitivity"]

[LessLeqTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessLeqTransitivity"]

LessTransitivity

[LessTransitivity $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\, (\underline{y} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{y} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ FirstConjunct $\rhd \dot{\neg}\, (\underline{y} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{y} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{y} <= \underline{z};$ LessLeqTransitivity $\rhd \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{y} <= \underline{z} \gg \dot{\neg}\, (\underline{x} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[LessTransitivity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \dot{\neg}\, (\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\, (\underline{y} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{y} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\, (\underline{x} <= \underline{z} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{x} = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}$]

[LessTransitivity $\stackrel{\text{tex}}{\rightarrow}$ "LessTransitivity"]

[LessTransitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessTransitivity"]

LessTotality

$[\text{LessTotality} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \dot{\neg}(\underline{x} = \underline{y})n \vdash \text{fromNotLess} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \gg \underline{y} <= \underline{x}; \text{NeqSymmetry} \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}(\underline{y} = \underline{x})n; \text{LeqNeqLess} \rhd \underline{y} <= \underline{x} \rhd \dot{\neg}(\underline{y} = \underline{x})n \gg \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \vdash \dot{\neg}(\underline{x} = \underline{y})n \vdash \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n; \text{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n\rceil, p_0, \mathsf{c})]$

$[\text{LessTotality} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n)n \Rightarrow \dot{\neg}(\underline{x} = \underline{y})n \Rightarrow \dot{\neg}(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \underline{x})n)n)n]$

[LessTotality $\stackrel{\text{tex}}{\rightarrow}$ "LessTotality"]

[LessTotality $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessTotality"]

SubLessRight

$[\text{SubLessRight} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \vdash \text{Repetition} \rhd \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \gg \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n; \text{FirstConjunct} \rhd \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \gg \underline{z} <= \underline{x}; \text{subLeqRight} \rhd \underline{x} = \underline{y} \rhd \underline{z} <= \underline{x} \gg \underline{z} <= \underline{y}; \text{SecondConjunct} \rhd \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \gg \dot{\neg}(\underline{z} = \underline{x})n; \text{SubNeqRight} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{z} = \underline{x})n \gg \dot{\neg}(\underline{z} = \underline{y})n; \text{JoinConjuncts} \rhd \underline{z} <= \underline{y} \rhd \dot{\neg}(\underline{z} = \underline{y})n \gg \dot{\neg}(\underline{z} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{y})n)n)n\rceil, p_0, \mathsf{c})]$

$[\text{SubLessRight} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{z} <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{x})n)n)n \vdash \dot{\neg}(\underline{z} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{y})n)n)n]$

[SubLessRight $\stackrel{\text{tex}}{\rightarrow}$ "SubLessRight"]

[SubLessRight $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subLessRight"]

SubLessLeft

$[\text{SubLessLeft} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \vdash \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \text{Repetition} \rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n; \text{FirstConjunct} \rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \underline{x} <= \underline{z}; \text{subLeqLeft} \rhd \underline{x} = \underline{y} \rhd \underline{x} <= \underline{z} \gg \underline{y} <= \underline{z}; \text{SecondConjunct} \rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \dot{\neg}(\underline{x} = \underline{z})n; \text{SubNeqLeft} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} = \underline{z})n \gg \dot{\neg}(\underline{y} = \underline{z})n; \text{JoinConjuncts} \rhd \underline{y} <= \underline{z} \rhd \dot{\neg}(\underline{y} = \underline{z})n \gg \dot{\neg}(\underline{y} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} =$

$\underline{z})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SubLessLeft} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{y}} = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{SubLessLeft} \stackrel{\mathrm{tex}}{\rightarrow}$ "SubLessLeft"]

$[\mathrm{SubLessLeft} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma subLessLeft"]

SwitchTerms(x < y − z)

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{z}})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{NegativeToLeft}(\mathrm{Less}) \rhd \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{z}})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plusCommutativity} \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{z}} + \underline{\mathsf{x}}); \mathrm{SubLessLeft} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{z}} + \underline{\mathsf{x}}) \rhd \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{z}} + \underline{\mathsf{x}}) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{z}} + \underline{\mathsf{x}}) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveToRight}(\mathrm{Less}) \rhd \dot{\neg}((\underline{\mathsf{z}} + \underline{\mathsf{x}}) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{z}} + \underline{\mathsf{x}}) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{z}} <= (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{z}} = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{z}})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{z}} <= (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{z}} = (\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}})))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{tex}}{\rightarrow}$ "SwitchTerms(x<y-z)"]

$[\mathrm{SwitchTerms}(x < y - z) \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma switchTerms(x<y-z)"]

SwitchTerms(x − y < z)

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{NegativeToRight}(\mathrm{Less}) \rhd \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{z}} + \underline{\mathsf{y}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{z}} + \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plusCommutativity} \gg (\underline{\mathsf{z}} + \underline{\mathsf{y}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}); \mathrm{SubLessRight} \rhd (\underline{\mathsf{z}} + \underline{\mathsf{y}}) = (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rhd \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{z}} + \underline{\mathsf{y}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{z}} + \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveToLeft}(\mathrm{Less}) \rhd \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} + \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{tex}}{\rightarrow}$ "SwitchTerms(x-y<z)"]

$[\mathrm{SwitchTerms}(x - y < z) \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma switchTerms(x-y<z)"]

LessAddition

[LessAddition $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \text{LessLeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \underline{x} <= \underline{y}; \text{leqAddition} \rhd \underline{x} <= \underline{y} \gg (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}); \text{LessNeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} = \underline{y})n; \text{NeqAddition} \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n; \text{JoinConjuncts} \rhd (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \rhd \dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n \gg \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n)n)n\rceil, p_0, c)$]

[LessAddition $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n)n)n$]

[LessAddition $\stackrel{\text{tex}}{\rightarrow}$ "LessAddition"]

[LessAddition $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessAddition"]

LessAdditionLeft

[LessAdditionLeft $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \text{LessAddition} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n)n)n; \text{plusCommutativity} \gg (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}); \text{SubLessLeft} \rhd (\underline{x} + \underline{z}) = (\underline{z} + \underline{x}) \rhd \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))n)n)n \gg \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{y} + \underline{z}))n)n)n; \text{plusCommutativity} \gg (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}); \text{SubLessRight} \rhd (\underline{y} + \underline{z}) = (\underline{z} + \underline{y}) \rhd \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{y} + \underline{z}))n)n)n \gg \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{z} + \underline{y}))n)n)n\rceil, p_0, c)$]

[LessAdditionLeft $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}((\underline{z} + \underline{x}) <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{z} + \underline{x}) = (\underline{z} + \underline{y}))n)n)n$]

[LessAdditionLeft $\stackrel{\text{tex}}{\rightarrow}$ "LessAdditionLeft"]

[LessAdditionLeft $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessAdditionLeft"]

LessMultiplication

[LessMultiplication $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \text{LessLeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \underline{x} <= \underline{y}; \text{LessLeq} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \gg 0 <= \underline{z}; \text{leqMultiplication} \rhd 0 <= \underline{z} \rhd \underline{x} <= \underline{y} \gg (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}); \text{LessNeq} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} = \underline{y})n; \text{LessNeq} \rhd \dot{\neg}(0 <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{z})n)n)n \gg \dot{\neg}(0 = \underline{z})n; \text{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{z})n \gg \dot{\neg}(\underline{z} = 0)n; \text{NeqMultiplication} \rhd \dot{\neg}(\underline{z} = 0)n \rhd \dot{\neg}(\underline{x} = \underline{y})n \gg \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))n; \text{LeqNeqLess} \rhd (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \rhd \dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))n \gg \dot{\neg}((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))n)n)n\rceil, p_0, c)$]

[LessMultiplication $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(0 <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}$]

[LessMultiplication $\stackrel{\text{tex}}{\rightarrow}$ "LessMultiplication"]

[LessMultiplication $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessMultiplication"]

LessMultiplicationLeft

[LessMultiplicationLeft $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(0 <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessMultiplication} \rhd \dot{\neg}\,(0 <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{timesCommutativity} \gg (\underline{x} * \underline{z}) = (\underline{z} * \underline{x}); \mathrm{timesCommutativity} \gg (\underline{y} * \underline{z}) = (\underline{z} * \underline{y}); \mathrm{SubLessLeft} \rhd (\underline{x} * \underline{z}) = (\underline{z} * \underline{x}) \rhd \dot{\neg}\,((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{z} * \underline{x}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{z} * \underline{x}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubLessRight} \rhd (\underline{y} * \underline{z}) = (\underline{z} * \underline{y}) \rhd \dot{\neg}\,((\underline{z} * \underline{x}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{z} * \underline{x}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{z} * \underline{x}) <= (\underline{z} * \underline{y}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{z} * \underline{x}) = (\underline{z} * \underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[LessMultiplicationLeft $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}\,(0 <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,((\underline{z} * \underline{x}) <= (\underline{z} * \underline{y}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{z} * \underline{x}) = (\underline{z} * \underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}$]

[LessMultiplicationLeft $\stackrel{\text{tex}}{\rightarrow}$ "LessMultiplicationLeft"]

[LessMultiplicationLeft $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessMultiplicationLeft"]

LessDivision

[LessDivision $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= \underline{z} \vdash \dot{\neg}\,((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{FromLess} \rhd \dot{\neg}\,((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}))\mathrm{n}; \mathrm{leqMultiplicationAxiom} \gg 0 <= \underline{z} \Rightarrow \underline{y} <= \underline{x} \Rightarrow (\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}); \mathrm{MP} \rhd 0 <= \underline{z} \Rightarrow \underline{y} <= \underline{x} \Rightarrow (\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}) \rhd 0 <= \underline{z} \gg \underline{y} <= \underline{x} \Rightarrow (\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}); \mathrm{Contrapositive} \rhd \underline{y} <= \underline{x} \Rightarrow (\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}) \gg \dot{\neg}\,((\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}))\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{y} <= \underline{x})\mathrm{n}; \mathrm{MP} \rhd \dot{\neg}\,((\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}))\mathrm{n} \Rightarrow \dot{\neg}\,(\underline{y} <= \underline{x})\mathrm{n} \rhd \dot{\neg}\,((\underline{y} * \underline{z}) <= (\underline{x} * \underline{z}))\mathrm{n} \gg \dot{\neg}\,(\underline{y} <= \underline{x})\mathrm{n}; \mathrm{ToLess} \rhd \dot{\neg}\,(\underline{y} <= \underline{x})\mathrm{n} \gg \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[LessDivision $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= \underline{z} \vdash \dot{\neg}\,((\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x} * \underline{z}) = (\underline{y} * \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}$]

[LessDivision $\stackrel{\text{tex}}{\rightarrow}$ "LessDivision"]

[LessDivision $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessDivision"]

PositiveToRight(Less)

[PositiveToRight(Less) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x}+\underline{y}) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{y}) = \underline{z})n)n)n \vdash \text{LessAddition} \rhd \dot{\neg}((\underline{x}+\underline{y}) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{y}) = \underline{z})n)n)n \gg \dot{\neg}(((\underline{x}+\underline{y})+(-u\underline{y})) <= (\underline{z}+(-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+\underline{y})+(-u\underline{y})) = (\underline{z}+(-u\underline{y})))n)n)n; x = x+y-y \gg \underline{x} = ((\underline{x}+\underline{y})+(-u\underline{y})); \text{eqSymmetry} \rhd \underline{x} = ((\underline{x}+\underline{y})+(-u\underline{y})) \gg ((\underline{x}+\underline{y})+(-u\underline{y})) = \underline{x}; \text{SubLessLeft} \rhd ((\underline{x}+\underline{y})+(-u\underline{y})) = \underline{x} \rhd \dot{\neg}(((\underline{x}+\underline{y})+(-u\underline{y})) <= (\underline{z}+(-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+\underline{y})+(-u\underline{y})) = (\underline{z}+(-u\underline{y})))n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{z}+(-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z}+(-u\underline{y})))n)n)n\rceil, p_0, c)$]

[PositiveToRight(Less) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x}+\underline{y}) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{y}) = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= (\underline{z}+(-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z}+(-u\underline{y})))n)n)n$]

[PositiveToRight(Less) $\stackrel{\text{tex}}{\rightarrow}$ "PositiveToRight(Less)"]

[PositiveToRight(Less) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma positiveToRight(Less)"]

PositiveToLeft(Less)

[PositiveToLeft(Less) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n \vdash \text{LessAddition} \rhd \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n \gg \dot{\neg}((\underline{x}+(-u\underline{z})) <= ((\underline{y}+\underline{z})+(-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = ((\underline{y}+\underline{z})+(-u\underline{z})))n)n)n; x = x+y-y \gg \underline{y} = ((\underline{y}+\underline{z})+(-u\underline{z})); \text{eqSymmetry} \rhd \underline{y} = ((\underline{y}+\underline{z})+(-u\underline{z})) \gg ((\underline{y}+\underline{z})+(-u\underline{z})) = \underline{y}; \text{SubLessRight} \rhd ((\underline{y}+\underline{z})+(-u\underline{z})) = \underline{y} \rhd \dot{\neg}((\underline{x}+(-u\underline{z})) <= ((\underline{y}+\underline{z})+(-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = ((\underline{y}+\underline{z})+(-u\underline{z})))n)n)n \gg \dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n\rceil, p_0, c)$]

[PositiveToLeft(Less) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n \vdash \dot{\neg}((\underline{x}+(-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-u\underline{z})) = \underline{y})n)n)n$]

[PositiveToLeft(Less) $\stackrel{\text{tex}}{\rightarrow}$ "PositiveToLeft(Less)"]

[PositiveToLeft(Less) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma positiveToLeft(Less)"]

NegativeToLeft(Less)

[NegativeToLeft(Less) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+(-u\underline{z})))n)n)n \vdash \text{LessAddition} \rhd \dot{\neg}(\underline{x} <= (\underline{y}+(-u\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+(-u\underline{z})))n)n)n \gg \dot{\neg}((\underline{x}+\underline{z}) <= ((\underline{y}+(-u\underline{z}))+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+\underline{z}) = ((\underline{y}+(-u\underline{z}))+\underline{z}))n)n)n; \text{Three2threeTerms} \gg ((\underline{y}+(-u\underline{z}))+\underline{z}) = ((\underline{y}+\underline{z})+(-u\underline{z})); x = x+y-y \gg \underline{y} = ((\underline{y}+\underline{z})+(-u\underline{z})); \text{eqSymmetry} \rhd \underline{y} = ((\underline{y}+\underline{z})+(-u\underline{z})) \gg ((\underline{y}+\underline{z})+(-u\underline{z})) = \underline{y}; \text{eqTransitivity} \rhd ((\underline{y}+(-u\underline{z}))+\underline{z}) = ((\underline{y}+\underline{z})+(-u\underline{z})) \rhd ((\underline{y}+\underline{z})+(-u\underline{z})) = \underline{y} \gg ((\underline{y}+(-u\underline{z}))+\underline{z}) = \underline{y}; \text{SubLessRight} \rhd$

$((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) = \underline{y} \rhd \dot{\neg}((\underline{x} + \underline{z}) <= ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = ((\underline{y} + (-\mathrm{u}\underline{z})) + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x} + \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{NegativeToLeft(Less)} \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{x} <= (\underline{y} + (-\mathrm{u}\underline{z})) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + (-\mathrm{u}\underline{z})))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{x} + \underline{z}) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\text{NegativeToLeft(Less)} \overset{\text{tex}}{\rightarrow}$ "NegativeToLeft(Less)"$]$

$[\text{NegativeToLeft(Less)} \overset{\text{pyk}}{\rightarrow}$ "lemma negativeToLeft(Less)"$]$

NegativeToRight(Less)

$[\text{NegativeToRight(Less)} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \text{LessAddition} \rhd \dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(((\underline{x} + (-\mathrm{u}\underline{y})) + \underline{y}) <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+\underline{y}) = (\underline{z}+\underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}; \text{Three2threeTerms} \gg ((\underline{x}+(-\mathrm{u}\underline{y}))+\underline{y}) = ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})); \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{x} = ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})); \text{eqSymmetry} \rhd \underline{x} = ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) \gg ((\underline{x} + \underline{y}) + (-\mathrm{u}\underline{y})) = \underline{x}; \text{eqTransitivity} \rhd ((\underline{x} + (-\mathrm{u}\underline{y})) + \underline{y}) = ((\underline{x}+\underline{y})+(-\mathrm{u}\underline{y})) \rhd ((\underline{x}+\underline{y})+(-\mathrm{u}\underline{y})) = \underline{x} \gg ((\underline{x}+(-\mathrm{u}\underline{y}))+\underline{y}) = \underline{x}; \text{SubLessLeft} \rhd ((\underline{x}+(-\mathrm{u}\underline{y}))+\underline{y}) = \underline{x} \rhd \dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+\underline{y}) <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+\underline{y}) = (\underline{z}+\underline{y}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{x} <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z} + \underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{NegativeToRight(Less)} \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{x} <= (\underline{z} + \underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{z} + \underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\text{NegativeToRight(Less)} \overset{\text{tex}}{\rightarrow}$ "NegativeToRight(Less)"$]$

$[\text{NegativeToRight(Less)} \overset{\text{pyk}}{\rightarrow}$ "lemma negativeToRight(Less)"$]$

AddEquations(Less)

$[\text{AddEquations(Less)} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \text{LessAddition} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y}+\underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \text{LessAdditionLeft} \rhd \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{y}+\underline{z}) <= (\underline{y} + \underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + \underline{z}) = (\underline{y} + \underline{u}))\mathrm{n})\mathrm{n})\mathrm{n}; \text{LessTransitivity} \rhd \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}((\underline{y} + \underline{z}) <= (\underline{y} + \underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + \underline{z}) = (\underline{y} + \underline{u}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{u}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{AddEquations(Less)} \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \forall \underline{u}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{z} <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} = \underline{u})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{x} + \underline{z}) <= (\underline{y} + \underline{u}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + \underline{z}) = (\underline{y} + \underline{u}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\text{AddEquations(Less)} \overset{\text{tex}}{\rightarrow}$ "AddEquations(Less)"$]$

[AddEquations(Less) $\stackrel{\mathrm{pyk}}{\to}$ "lemma addEquations(Less)"]

AddEquations(LeqLess)

[AddEquations(LeqLess) $\stackrel{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{z}} <= \underline{\mathsf{u}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{u}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{leqAddition} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}); \mathrm{LessAdditionLeft} \rhd \dot{\neg}(\underline{\mathsf{z}} <= \underline{\mathsf{u}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{u}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{y}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{y}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{leqLessTransitivity} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \rhd \dot{\neg}((\underline{\mathsf{y}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{y}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[AddEquations(LeqLess) $\stackrel{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \vdash \dot{\neg}(\underline{\mathsf{z}} <= \underline{\mathsf{u}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{z}} = \underline{\mathsf{u}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{u}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} + \underline{\mathsf{z}}) = (\underline{\mathsf{y}} + \underline{\mathsf{u}}))\mathrm{n})\mathrm{n})\mathrm{n}$]

[AddEquations(LeqLess) $\stackrel{\mathrm{tex}}{\to}$ "AddEquations(LeqLess)"]

[AddEquations(LeqLess) $\stackrel{\mathrm{pyk}}{\to}$ "lemma addEquations(LeqLess)"]

reciprocalToLeft(Less)

[reciprocalToLeft(Less) $\stackrel{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessMultiplication} \rhd \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Three2threeFactors} \gg ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}) = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}); \mathrm{PositiveNonzero} \rhd \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n}; \mathrm{x} = \mathrm{x} * \mathrm{y} * (1/\mathrm{y}) \rhd \dot{\neg}(\underline{\mathsf{z}} = 0)\mathrm{n} \gg \underline{\mathsf{y}} = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}); \mathrm{eqSymmetry} \rhd \underline{\mathsf{y}} = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) \gg ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) = \underline{\mathsf{y}}; \mathrm{eqTransitivity} \rhd ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}) = ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) \rhd ((\underline{\mathsf{y}} * \underline{\mathsf{z}}) * \mathrm{rec}\underline{\mathsf{z}}) = \underline{\mathsf{y}} \gg ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}) = \underline{\mathsf{y}}; \mathrm{SubLessRight} \rhd ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}) = \underline{\mathsf{y}} \rhd \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = ((\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) * \underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[reciprocalToLeft(Less) $\stackrel{\mathrm{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \dot{\neg}(0 <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{z}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}} * \underline{\mathsf{z}}) = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}$]

[reciprocalToLeft(Less) $\stackrel{\mathrm{tex}}{\to}$ "reciprocalToLeft(Less)"]

[reciprocalToLeft(Less) $\stackrel{\mathrm{pyk}}{\to}$ "lemma reciprocalToLeft(Less)"]

LessNegated

$$[\mathrm{LessNegated} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessLeq} \rhd \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{x}} <= \underline{\mathsf{y}}; \mathrm{LeqNegated} \rhd \underline{\mathsf{x}} <= \underline{\mathsf{y}} \gg (-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}); \mathrm{LessNeq} \rhd \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n}; \mathrm{NeqNegated} \rhd \dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}\underline{\mathsf{y}}))\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n}; \mathrm{LeqNeqLess} \rhd (-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}) \rhd \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{LessNegated} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) <= (-\mathrm{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{x}}))\mathrm{n})\mathrm{n})\mathrm{n}]$$

$$[\mathrm{LessNegated} \overset{\mathrm{tex}}{\to} \text{“LessNegated”}]$$

$$[\mathrm{LessNegated} \overset{\mathrm{pyk}}{\to} \text{“lemma lessNegated”}]$$

PositiveNonzero

$$[\mathrm{PositiveNonzero} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{PositiveNonzero} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} = 0)\mathrm{n}]$$

$$[\mathrm{PositiveNonzero} \overset{\mathrm{tex}}{\to} \text{“PositiveNonzero”}]$$

$$[\mathrm{PositiveNonzero} \overset{\mathrm{pyk}}{\to} \text{“lemma positiveNonzero”}]$$

PositiveNegated

$$[\mathrm{PositiveNegated} \overset{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{LessNegated} \rhd \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= (-\mathrm{u}0) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}0))\mathrm{n})\mathrm{n})\mathrm{n}; -0 = 0 \gg (-\mathrm{u}0) = 0; \mathrm{SubLessRight} \rhd (-\mathrm{u}0) = 0 \rhd \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= (-\mathrm{u}0) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = (-\mathrm{u}0))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = 0)\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$$

$$[\mathrm{PositiveNegated} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}\underline{\mathsf{x}}) = 0)\mathrm{n})\mathrm{n})\mathrm{n}]$$

$$[\mathrm{PositiveNegated} \overset{\mathrm{tex}}{\to} \text{“PositiveNegated”}]$$

$$[\mathrm{PositiveNegated} \overset{\mathrm{pyk}}{\to} \text{“lemma positiveNegated”}]$$

NonpositiveNegated

[NonpositiveNegated $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \underline{\mathsf{x}} <= 0 \vdash \text{LeqNegated} \rhd \underline{\mathsf{x}} <= 0 \gg (-\text{u}0) <= (-\text{u}\underline{\mathsf{x}}); -0 = 0 \gg (-\text{u}0) = 0; \text{subLeqLeft} \rhd (-\text{u}0) = 0 \rhd (-\text{u}0) <= (-\text{u}\underline{\mathsf{x}}) \gg 0 <= (-\text{u}\underline{\mathsf{x}})\rceil, \text{p}_0, \mathsf{c})$]

[NonpositiveNegated $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \underline{\mathsf{x}} <= 0 \vdash 0 <= (-\text{u}\underline{\mathsf{x}})$]

[NonpositiveNegated $\stackrel{\text{tex}}{\rightarrow}$ “NonpositiveNegated”]

[NonpositiveNegated $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonpositiveNegated”]

NegativeNegated

[NegativeNegated $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\text{n})\text{n})\text{n} \vdash \text{LessNegated} \rhd \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\text{n})\text{n})\text{n} \gg \dot{\neg}((-\text{u}0) <= (-\text{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\text{u}0) = (-\text{u}\underline{\mathsf{x}}))\text{n})\text{n})\text{n}; -0 = 0 \gg (-\text{u}0) = 0; \text{SubLessLeft} \rhd (-\text{u}0) = 0 \rhd \dot{\neg}((-\text{u}0) <= (-\text{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}((-\text{u}0) = (-\text{u}\underline{\mathsf{x}}))\text{n})\text{n})\text{n} \gg \dot{\neg}(0 <= (-\text{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\text{u}\underline{\mathsf{x}}))\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[NegativeNegated $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{x}} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{x}} = 0)\text{n})\text{n})\text{n} \vdash \dot{\neg}(0 <= (-\text{u}\underline{\mathsf{x}}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\text{u}\underline{\mathsf{x}}))\text{n})\text{n})\text{n}$]

[NegativeNegated $\stackrel{\text{tex}}{\rightarrow}$ “NegativeNegated”]

[NegativeNegated $\stackrel{\text{pyk}}{\rightarrow}$ “lemma negativeNegated”]

NonnegativeNegated

[NonnegativeNegated $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon 0 <= \underline{\mathsf{x}} \vdash \text{LeqNegated} \rhd 0 <= \underline{\mathsf{x}} \gg (-\text{u}\underline{\mathsf{x}}) <= (-\text{u}0); -0 = 0 \gg (-\text{u}0) = 0; \text{subLeqRight} \rhd (-\text{u}0) = 0 \rhd (-\text{u}\underline{\mathsf{x}}) <= (-\text{u}0) \gg (-\text{u}\underline{\mathsf{x}}) <= 0\rceil, \text{p}_0, \mathsf{c})$]

[NonnegativeNegated $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon 0 <= \underline{\mathsf{x}} \vdash (-\text{u}\underline{\mathsf{x}}) <= 0$]

[NonnegativeNegated $\stackrel{\text{tex}}{\rightarrow}$ “NonnegativeNegated”]

[NonnegativeNegated $\stackrel{\text{pyk}}{\rightarrow}$ “lemma nonnegativeNegated”]

PositiveHalved

[PositiveHalved $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})\text{n})\text{n})\text{n} \vdash 0 < 1/2 \gg \dot{\neg}(0 <= \text{rec}(1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \text{rec}(1+1))\text{n})\text{n})\text{n}; \text{LessMultiplicationLeft} \rhd \dot{\neg}(0 <= \text{rec}(1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$

$\mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\mathrm{rec}(1+1) * 0) <= (\mathrm{rec}(1+1) * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}(1+1) * 0) = (\mathrm{rec}(1+1) * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} * 0 = 0 \gg (\mathrm{rec}(1+1) * 0) = 0; \mathrm{SubLessLeft} \rhd (\mathrm{rec}(1+1) * 0) = 0 \rhd \dot{\neg}((\mathrm{rec}(1+1) * 0) <= (\mathrm{rec}(1+1) * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}(1+1) * 0) = (\mathrm{rec}(1+1) * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\mathrm{rec}(1+1) * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1) * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveHalved} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (\mathrm{rec}(1+1) * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}(1+1) * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{PositiveHalved} \stackrel{\mathrm{tex}}{\rightarrow}$ "PositiveHalved"$]$

$[\mathrm{PositiveHalved} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma positiveHalved"$]$

PositiveInverted

$[\mathrm{PositiveInverted} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{x}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = \underline{x})\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{x})\mathrm{n} \gg \dot{\neg}(\underline{x} = 0)\mathrm{n}; 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} * 0 = 0 \gg (\underline{x} * 0) = 0; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd (\underline{x} * 0) = 0 \gg 0 = (0 * \underline{x}); \mathrm{SubLessLeft} \rhd 0 = (0 * \underline{x}) \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * \underline{x}) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Reciprocal} \rhd \dot{\neg}(\underline{x} = 0)\mathrm{n} \gg (\underline{x} * \mathrm{rec}\underline{x}) = 1; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd (\underline{x} * \mathrm{rec}\underline{x}) = 1 \gg 1 = (\mathrm{rec}\underline{x} * \underline{x}); \mathrm{SubLessRight} \rhd 1 = (\mathrm{rec}\underline{x} * \underline{x}) \rhd \dot{\neg}((0 * \underline{x}) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * \underline{x}) <= (\mathrm{rec}\underline{x} * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = (\mathrm{rec}\underline{x} * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessDivision} \rhd 0 <= \underline{x} \rhd \dot{\neg}((0 * \underline{x}) <= (\mathrm{rec}\underline{x} * \underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((0 * \underline{x}) = (\mathrm{rec}\underline{x} * \underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \mathrm{rec}\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}\underline{x})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{PositiveInverted} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= \mathrm{rec}\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}\underline{x})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{PositiveInverted} \stackrel{\mathrm{tex}}{\rightarrow}$ "PositiveInverted"$]$

$[\mathrm{PositiveInverted} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma positiveInverted"$]$

NonnegativeNumerical

$[\mathrm{NonnegativeNumerical} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \mathrm{Numerical} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n}; \mathrm{AddDoubleNeg} \rhd 0 <= \underline{x} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n})\mathrm{n}; \mathrm{ToNegatedAnd}(1) \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegateDisjunct2} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n}; \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n} \gg |\underline{x}| = \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NonnegativeNumerical} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash |\underline{x}| = \underline{x}]$

[NonnegativeNumerical $\overset{\text{tex}}{\to}$ "NonnegativeNumerical"]

[NonnegativeNumerical $\overset{\text{pyk}}{\to}$ "lemma nonnegativeNumerical"]

NegativeNumerical

[NegativeNumerical $\overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \vdash \text{Numerical} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{x})\text{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\text{u}\underline{x}))\text{n})\text{n}; \text{FromLess} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \gg \dot{\neg}(0 <= \underline{x})\text{n}; \text{ToNegatedAnd}(1) \rhd \dot{\neg}(0 <= \underline{x})\text{n} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\text{n})\text{n})\text{n}; \text{NegateDisjunct1} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= \underline{x})\text{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\text{u}\underline{x}))\text{n})\text{n} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| = \underline{x})\text{n})\text{n})\text{n} \gg \dot{\neg}(\dot{\neg}(0 <= \underline{x})\text{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\text{u}\underline{x}))\text{n})\text{n}; \text{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{x})\text{n} \Rightarrow \dot{\neg}(|\underline{x}| = (-\text{u}\underline{x}))\text{n})\text{n} \gg |\underline{x}| = (-\text{u}\underline{x})\rceil, \text{p}_0, \mathsf{c})$]

[NegativeNumerical $\overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \vdash |\underline{x}| = (-\text{u}\underline{x})$]

[NegativeNumerical $\overset{\text{tex}}{\to}$ "NegativeNumerical"]

[NegativeNumerical $\overset{\text{pyk}}{\to}$ "lemma negativeNumerical"]

PositiveNumerical

[PositiveNumerical $\overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \vdash \text{LessLeq} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \gg 0 <= \underline{x}; \text{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}\rceil, \text{p}_0, \mathsf{c})$]

[PositiveNumerical $\overset{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \vdash |\underline{x}| = \underline{x}$]

[PositiveNumerical $\overset{\text{tex}}{\to}$ "PositiveNumerical"]

[PositiveNumerical $\overset{\text{pyk}}{\to}$ "lemma positiveNumerical"]

lemma nonpositiveNumerical

[lemma nonpositiveNumerical $\overset{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \vdash \text{NegativeNumerical} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \gg |\underline{x}| = (-\text{u}\underline{x}); \forall \underline{x}\colon \underline{x} = 0 \vdash \text{eqSymmetry} \rhd \underline{x} = 0 \gg 0 = \underline{x}; \text{eqLeq} \rhd 0 = \underline{x} \gg 0 <= \underline{x}; \text{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; -0 = 0 \gg (-\text{u}0) = 0; \text{eqSymmetry} \rhd (-\text{u}0) = 0 \gg 0 = (-\text{u}0); \text{EqNegated} \rhd 0 = \underline{x} \gg (-\text{u}0) = (-\text{u}\underline{x}); \text{eqTransitivity5} \rhd |\underline{x}| = \underline{x} \rhd \underline{x} = 0 \rhd 0 = (-\text{u}0) \rhd (-\text{u}0) = (-\text{u}\underline{x}) \gg |\underline{x}| = (-\text{u}\underline{x}); \forall \underline{x}\colon \text{Ded} \rhd \forall \underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \vdash |\underline{x}| = (-\text{u}\underline{x}) \gg \dot{\neg}(\underline{x} <=$

$0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow |\underline{x}| = (-\mathrm{u}\underline{x}); \mathrm{Ded} \rhd \forall \underline{x}\colon \underline{x} = 0 \vdash |\underline{x}| = (-\mathrm{u}\underline{x}) \gg \underline{x} = 0 \Rightarrow |\underline{x}| = (-\mathrm{u}\underline{x}); \underline{x} <= 0 \vdash \mathrm{LeqLessEq} \rhd \underline{x} <= 0 \gg \dot{\neg}(\dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = 0; \mathrm{FromDisjuncts} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{x} = 0 \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow |\underline{x}| = (-\mathrm{u}\underline{x}) \rhd \underline{x} = 0 \Rightarrow |\underline{x}| = (-\mathrm{u}\underline{x}) \gg |\underline{x}| = (-\mathrm{u}\underline{x})\rceil, \mathrm{p}_0, \mathsf{c})]$

[lemma nonpositiveNumerical $\stackrel{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \underline{x} <= 0 \vdash |\underline{x}| = (-\mathrm{u}\underline{x})$]

[lemma nonpositiveNumerical $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma nonpositiveNumerical”]

$|0| = 0$

$[|0| = 0 \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{leqReflexivity} \gg 0 <= 0; \mathrm{NonnegativeNumerical} \rhd 0 <= 0 \gg |0| = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[|0| = 0 \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash |0| = 0]$

[$|0| = 0$ $\stackrel{\mathrm{tex}}{\rightarrow}$ “|0|=0”]

[$|0| = 0$ $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma |0|=0”]

$0 <= |x|$

$[0 <= |x| \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{eqSymmetry} \rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \mathrm{subLeqRight} \rhd \underline{x} = |\underline{x}| \rhd 0 <= \underline{x} \gg 0 <= |\underline{x}|; \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x})\mathrm{n} \vdash \mathrm{ToLess} \rhd \dot{\neg}(0 <= \underline{x})\mathrm{n} \gg \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegativeNumerical} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg |\underline{x}| = (-\mathrm{u}\underline{x}); \mathrm{eqSymmetry} \rhd |\underline{x}| = (-\mathrm{u}\underline{x}) \gg (-\mathrm{u}\underline{x}) = |\underline{x}|; \mathrm{NegativeNegated} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (-\mathrm{u}\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessLeq} \rhd \dot{\neg}(0 <= (-\mathrm{u}\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= (-\mathrm{u}\underline{x}); \mathrm{subLeqRight} \rhd (-\mathrm{u}\underline{x}) = |\underline{x}| \rhd 0 <= (-\mathrm{u}\underline{x}) \gg 0 <= |\underline{x}|; \forall \underline{x}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon 0 <= \underline{x} \vdash 0 <= |\underline{x}| \gg 0 <= \underline{x} \Rightarrow 0 <= |\underline{x}|; \mathrm{Ded} \rhd \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x})\mathrm{n} \vdash 0 <= |\underline{x}| \gg \dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow 0 <= |\underline{x}|; \mathrm{FromNegations} \rhd 0 <= \underline{x} \Rightarrow 0 <= |\underline{x}| \rhd \dot{\neg}(0 <= \underline{x})\mathrm{n} \Rightarrow 0 <= |\underline{x}| \gg 0 <= |\underline{x}|\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 <= |x| \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 <= |\underline{x}|]$

[$0 <= |x|$ $\stackrel{\mathrm{tex}}{\rightarrow}$ “0<=|x|”]

[$0 <= |x|$ $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma 0<=|x|”]

x <= |x|

$[x <= |x| \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \text{NonnegativeNumerical} \gg |\underline{x}| = \underline{x}; \text{eqSymmetry} \rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \text{eqLeq} \rhd \underline{x} = |\underline{x}| \gg \underline{x} <= |\underline{x}|; \forall \underline{x}\colon \underline{x} <= 0 \vdash 0 <= |x| \gg 0 <= |\underline{x}|; \text{leqTransitivity} \rhd \underline{x} <= 0 \rhd 0 <= |\underline{x}| \gg \underline{x} <= |\underline{x}|; \forall \underline{x}\colon \text{Ded} \rhd \forall \underline{x}\colon 0 <= \underline{x} \vdash \underline{x} <= |\underline{x}| \gg 0 <= \underline{x} \Rightarrow \underline{x} <= |\underline{x}|; \text{Ded} \rhd \forall \underline{x}\colon \underline{x} <= 0 \vdash \underline{x} <= |\underline{x}| \gg \underline{x} <= 0 \Rightarrow \underline{x} <= |\underline{x}|; \text{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \underline{x} <= |\underline{x}| \rhd \underline{x} <= 0 \Rightarrow \underline{x} <= |\underline{x}| \gg \underline{x} <= |\underline{x}| \rceil, \mathrm{p}_0, \mathsf{c})]$

$[x <= |x| \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \underline{x} <= |\underline{x}|]$

$[x <= |x| \stackrel{\text{tex}}{\rightarrow}$ "x<=|x|"]

$[x <= |x| \stackrel{\text{pyk}}{\rightarrow}$ "lemma x<=|x|"]

FromPositiveNumerical

$[\text{FromPositiveNumerical} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon 0 <= \underline{x} \vdash \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \vdash \text{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \text{SubLessRight} \rhd |\underline{x}| = \underline{x} \rhd \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \gg \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n; \text{LessNeq} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg \dot{\neg}(0 = \underline{x})n; \text{NeqSymmetry} \rhd \dot{\neg}(0 = \underline{x})n \gg \dot{\neg}(\underline{x} = 0)n; \forall \underline{x}\colon \underline{x} <= 0 \vdash \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \vdash \text{lemma nonpositiveNumerical} \rhd \underline{x} <= 0 \gg |\underline{x}| = (-u\underline{x}); \text{SubLessRight} \rhd |\underline{x}| = (-u\underline{x}) \rhd \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \gg \dot{\neg}(0 <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-u\underline{x}))n)n)n; \text{PositiveNegated} \rhd \dot{\neg}(0 <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-u\underline{x}))n)n)n \gg \dot{\neg}((-u(-u\underline{x})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u(-u\underline{x})) = 0)n)n)n; \text{DoubleMinus} \gg (-u(-u\underline{x})) = \underline{x}; \text{SubLessLeft} \rhd (-u(-u\underline{x})) = \underline{x} \rhd \dot{\neg}((-u(-u\underline{x})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u(-u\underline{x})) = 0)n)n)n \gg \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n; \text{LessNeq} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n \gg \dot{\neg}(\underline{x} = 0)n; \forall \underline{x}\colon \text{Ded} \rhd \forall \underline{x}\colon 0 <= \underline{x} \vdash \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \vdash \dot{\neg}(\underline{x} = 0)n \gg 0 <= \underline{x} \Rightarrow \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \Rightarrow \dot{\neg}(\underline{x} = 0)n; \text{Ded} \rhd \forall \underline{x}\colon \underline{x} <= 0 \vdash \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \vdash \dot{\neg}(\underline{x} = 0)n \gg \underline{x} <= 0 \Rightarrow \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \Rightarrow \dot{\neg}(\underline{x} = 0)n; \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \vdash \text{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \Rightarrow \dot{\neg}(\underline{x} = 0)n \rhd \underline{x} <= 0 \Rightarrow \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \Rightarrow \dot{\neg}(\underline{x} = 0)n \gg \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \Rightarrow \dot{\neg}(\underline{x} = 0)n; \text{MP} \rhd \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \Rightarrow \dot{\neg}(\underline{x} = 0)n \rhd \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \gg \dot{\neg}(\underline{x} = 0)n \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{FromPositiveNumerical} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= |\underline{x}| \Rightarrow \dot{\neg}(\dot{\neg}(0 = |\underline{x}|)n)n)n \vdash \dot{\neg}(\underline{x} = 0)n]$

[FromPositiveNumerical $\stackrel{\text{tex}}{\rightarrow}$ "FromPositiveNumerical"]

[FromPositiveNumerical $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromPositiveNumerical"]

SameNumerical

[SameNumerical $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{x} = \underline{y} \vdash \text{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \text{subLeqRight} \rhd \underline{x} = \underline{y} \rhd 0 <= \underline{x} \gg 0 <= \underline{y}; \text{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \text{eqSymmetry} \rhd |\underline{y}| = \underline{y} \gg \underline{y} = |\underline{y}|; \text{eqTransitivity4} \rhd |\underline{x}| = \underline{x} \rhd \underline{x} = \underline{y} \rhd \underline{y} = |\underline{y}| \gg |\underline{x}| = |\underline{y}|; \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(0 <= \underline{x})n \vdash \underline{x} = \underline{y} \vdash \text{ToLess} \rhd \dot{\neg}(0 <= \underline{x})n \gg \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n; \text{NegativeNumerical} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n \gg |\underline{x}| = (-u\underline{x}); \text{SubLessLeft} \rhd \underline{x} = \underline{y} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)n)n)n \gg \dot{\neg}(\underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = 0)n)n)n; \text{NegativeNumerical} \rhd \dot{\neg}(\underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = 0)n)n)n \gg |\underline{y}| = (-u\underline{y}); \text{eqSymmetry} \rhd |\underline{y}| = (-u\underline{y}) \gg (-u\underline{y}) = |\underline{y}|; \text{EqNegated} \rhd \underline{x} = \underline{y} \gg (-u\underline{x}) = (-u\underline{y}); \text{eqTransitivity4} \rhd |\underline{x}| = (-u\underline{x}) \rhd (-u\underline{x}) = (-u\underline{y}) \rhd (-u\underline{y}) = |\underline{y}| \gg |\underline{x}| = |\underline{y}|; \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}| \gg 0 <= \underline{x} \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(0 <= \underline{x})n \vdash \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}| \gg \dot{\neg}(0 <= \underline{x})n \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \text{FromNegations} \rhd 0 <= \underline{x} \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \rhd \dot{\neg}(0 <= \underline{x})n \Rightarrow \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \gg \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}|; \text{MP} \rhd \underline{x} = \underline{y} \Rightarrow |\underline{x}| = |\underline{y}| \rhd \underline{x} = \underline{y} \gg |\underline{x}| = |\underline{y}| \rceil, p_0, c)$]

[SameNumerical $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \vdash |\underline{x}| = |\underline{y}|$]

[SameNumerical $\overset{\text{tex}}{\to}$ "SameNumerical"]

[SameNumerical $\overset{\text{pyk}}{\to}$ "lemma sameNumerical"]

SignNumerical(+)

[SignNumerical(+) $\overset{\text{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash \text{PositiveNumerical} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg |\underline{x}| = \underline{x}; \text{PositiveNegated} \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \gg \dot{\neg}((-u\underline{x}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = 0)n)n)n; \text{NegativeNumerical} \rhd \dot{\neg}((-u\underline{x}) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{x}) = 0)n)n)n \gg |(-u\underline{x})| = (-u(-u\underline{x})); \text{DoubleMinus} \gg (-u(-u\underline{x})) = \underline{x}; \text{eqTransitivity} \rhd |(-u\underline{x})| = (-u(-u\underline{x})) \rhd (-u(-u\underline{x})) = \underline{x} \gg |(-u\underline{x})| = \underline{x}; \text{eqSymmetry} \rhd |(-u\underline{x})| = \underline{x} \gg \underline{x} = |(-u\underline{x})|; \text{eqTransitivity} \rhd |\underline{x}| = \underline{x} \rhd \underline{x} = |(-u\underline{x})| \gg |\underline{x}| = |(-u\underline{x})| \rceil, p_0, c)$]

[SignNumerical(+) $\overset{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})n)n)n \vdash |\underline{x}| = |(-u\underline{x})|$]

[SignNumerical(+) $\overset{\text{tex}}{\to}$ "SignNumerical(+)"]

[SignNumerical(+) $\overset{\text{pyk}}{\to}$ "lemma signNumerical(+)"]

SignNumerical

[SignNumerical $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \vdash \text{NegativeNegated} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \gg \dot{\neg}(0 <= (-\text{u}\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\text{u}\underline{x}))\text{n})\text{n})\text{n}; \text{SignNumerical}(+) \rhd \dot{\neg}(0 <= (-\text{u}\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (-\text{u}\underline{x}))\text{n})\text{n})\text{n} \gg |(-\text{u}\underline{x})| = |(-\text{u}(-\text{u}\underline{x}))|; \text{DoubleMinus} \gg (-\text{u}(-\text{u}\underline{x})) = \underline{x}; \text{SameNumerical} \rhd (-\text{u}(-\text{u}\underline{x})) = \underline{x} \gg |(-\text{u}(-\text{u}\underline{x}))| = |\underline{x}|; \text{eqTransitivity} \rhd |(-\text{u}\underline{x})| = |(-\text{u}(-\text{u}\underline{x}))| \rhd |(-\text{u}(-\text{u}\underline{x}))| = |\underline{x}| \gg |(-\text{u}\underline{x})| = |\underline{x}|; \text{eqSymmetry} \rhd |(-\text{u}\underline{x})| = |\underline{x}| \gg |\underline{x}| = |(-\text{u}\underline{x})|; \forall \underline{x}\colon \underline{x} = 0 \vdash \text{EqNegated} \rhd \underline{x} = 0 \gg (-\text{u}\underline{x}) = (-\text{u}0); -0 = 0 \gg (-\text{u}0) = 0; \text{eqSymmetry} \rhd \underline{x} = 0 \gg 0 = \underline{x}; \text{eqTransitivity4} \rhd (-\text{u}\underline{x}) = (-\text{u}0) \rhd (-\text{u}0) = 0 \rhd 0 = \underline{x} \gg (-\text{u}\underline{x}) = \underline{x}; \text{eqSymmetry} \rhd (-\text{u}\underline{x}) = \underline{x} \gg \underline{x} = (-\text{u}\underline{x}); \text{SameNumerical} \rhd \underline{x} = (-\text{u}\underline{x}) \gg |\underline{x}| = |(-\text{u}\underline{x})|; \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \vdash \text{SignNumerical}(+) \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \gg |\underline{x}| = |(-\text{u}\underline{x})|; \forall \underline{x}\colon \text{Ded} \rhd \forall \underline{x}\colon \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \vdash |\underline{x}| = |(-\text{u}\underline{x})| \gg \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \Rightarrow |\underline{x}| = |(-\text{u}\underline{x})|; \text{Ded} \rhd \forall \underline{x}\colon \underline{x} = 0 \vdash |\underline{x}| = |(-\text{u}\underline{x})| \gg \underline{x} = 0 \Rightarrow |\underline{x}| = |(-\text{u}\underline{x})|; \text{Ded} \rhd \forall \underline{x}\colon \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \vdash |\underline{x}| = |(-\text{u}\underline{x})| \gg \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \Rightarrow |\underline{x}| = |(-\text{u}\underline{x})|; \text{LessTotality} \gg \dot{\neg}(\dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{x} = 0)\text{n} \Rightarrow \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n}; \text{From3Disjuncts} \rhd \dot{\neg}(\dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{x} = 0)\text{n} \Rightarrow \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \rhd \dot{\neg}(\underline{x} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = 0)\text{n})\text{n})\text{n} \Rightarrow |\underline{x}| = |(-\text{u}\underline{x})| \rhd \underline{x} = 0 \Rightarrow |\underline{x}| = |(-\text{u}\underline{x})| \rhd \dot{\neg}(0 <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{x})\text{n})\text{n})\text{n} \Rightarrow |\underline{x}| = |(-\text{u}\underline{x})| \gg |\underline{x}| = |(-\text{u}\underline{x})|\rceil, \text{p}_0, \mathsf{c})$]

[SignNumerical $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon |\underline{x}| = |(-\text{u}\underline{x})|$]

[SignNumerical $\stackrel{\text{tex}}{\rightarrow}$ “SignNumerical”]

[SignNumerical $\stackrel{\text{pyk}}{\rightarrow}$ “lemma signNumerical”]

ToNumericalLess

[ToNumericalLess $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash 0 <= \underline{x} \vdash \text{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \text{eqSymmetry} \rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|; \text{SubLessLeft} \rhd \underline{x} = |\underline{x}| \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \gg \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})\text{n})\text{n})\text{n}; \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}((-\text{u}\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-\text{u}\underline{y}) = \underline{x})\text{n})\text{n})\text{n} \vdash \underline{x} <= 0 \vdash \text{LessNegated} \rhd \dot{\neg}((-\text{u}\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-\text{u}\underline{y}) = \underline{x})\text{n})\text{n})\text{n} \gg \dot{\neg}((-\text{u}\underline{x}) <= (-\text{u}(-\text{u}\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}((-\text{u}\underline{x}) = (-\text{u}(-\text{u}\underline{y})))\text{n})\text{n})\text{n}; \text{lemma nonpositiveNumerical} \rhd \underline{x} <= 0 \gg |\underline{x}| = (-\text{u}\underline{x}); \text{eqSymmetry} \rhd |\underline{x}| = (-\text{u}\underline{x}) \gg (-\text{u}\underline{x}) = |\underline{x}|; \text{SubLessLeft} \rhd (-\text{u}\underline{x}) = |\underline{x}| \rhd \dot{\neg}((-\text{u}\underline{x}) <= (-\text{u}(-\text{u}\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}((-\text{u}\underline{x}) = (-\text{u}(-\text{u}\underline{y})))\text{n})\text{n})\text{n} \gg \dot{\neg}(|\underline{x}| <= (-\text{u}(-\text{u}\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = (-\text{u}(-\text{u}\underline{y})))\text{n})\text{n})\text{n}; \text{DoubleMinus} \gg (-\text{u}(-\text{u}\underline{y})) = \underline{y}; \text{SubLessRight} \rhd (-\text{u}(-\text{u}\underline{y})) = \underline{y} \rhd \dot{\neg}(|\underline{x}| <= (-\text{u}(-\text{u}\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = (-\text{u}(-\text{u}\underline{y})))\text{n})\text{n})\text{n} \gg \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})\text{n})\text{n})\text{n}; \forall \underline{x}\colon \forall \underline{y}\colon \text{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})\text{n})\text{n})\text{n} \vdash 0 <= \underline{x} \vdash$

$\dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \Rightarrow 0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n; \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{y}) = \underline{x})n)n)n \vdash \underline{x} <= 0 \vdash \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n \gg \dot{\neg}((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{y}) = \underline{x})n)n)n \Rightarrow \underline{x} <= 0 \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n; \dot{\neg}((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{y}) = \underline{x})n)n)n \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \Rightarrow 0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg 0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n; \mathrm{MP} \rhd \dot{\neg}((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{y}) = \underline{x})n)n)n \Rightarrow \underline{x} <= 0 \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n \rhd \dot{\neg}((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{y}) = \underline{x})n)n)n \gg \underline{x} <= 0 \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n; \mathrm{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n \rhd \underline{x} <= 0 \Rightarrow \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n \gg \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{ToNumericalLess} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}((-u\underline{y}) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((-u\underline{y}) = \underline{x})n)n)n \vdash \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(|\underline{x}| <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = \underline{y})n)n)n]$

$[\mathrm{ToNumericalLess} \stackrel{\mathrm{tex}}{\rightarrow}$ “ToNumericalLess”]

$[\mathrm{ToNumericalLess} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma toNumericalLess”]

FromNumericalGreater

$[\mathrm{FromNumericalGreater} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash 0 <= \underline{y} \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \mathrm{SubLessRight} \rhd |\underline{y}| = \underline{y} \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{WeakenOr1} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \underline{y} <= 0 \vdash \mathrm{lemma\ nonpositiveNumerical} \rhd \underline{y} <= 0 \gg |\underline{y}| = (-u\underline{y}); \mathrm{SubLessRight} \rhd |\underline{y}| = (-u\underline{y}) \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \gg \dot{\neg}(\underline{x} <= (-u\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (-u\underline{y}))n)n)n; \mathrm{LessNegated} \rhd \dot{\neg}(\underline{x} <= (-u\underline{y}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (-u\underline{y}))n)n)n \gg \dot{\neg}((-u(-u\underline{y})) <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((-u(-u\underline{y})) = (-u\underline{x}))n)n)n; \mathrm{DoubleMinus} \gg (-u(-u\underline{y})) = \underline{y}; \mathrm{SubLessLeft} \rhd (-u(-u\underline{y})) = \underline{y} \rhd \dot{\neg}((-u(-u\underline{y})) <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}((-u(-u\underline{y})) = (-u\underline{x}))n)n)n \gg \dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n; \mathrm{WeakenOr2} \rhd \dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash 0 <= \underline{y} \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow 0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \underline{y} <= 0 \vdash \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow \underline{y} <= 0 \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (-u\underline{x}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (-u\underline{x}))n)n)n)n \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(\underline{x} <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = |\underline{y}|)n)n)n \Rightarrow 0 <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <=$

$(-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(\underline{x} <= |\underline{y}| \Rightarrow \dot\neg(\dot\neg(\underline{x} = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd \dot\neg(\underline{x} <= |\underline{y}| \Rightarrow \dot\neg(\dot\neg(\underline{x} = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{y} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(\underline{x} <= |\underline{y}| \Rightarrow \dot\neg(\dot\neg(\underline{x} = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{y} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromLeqGeq} \rhd 0 <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{y} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNumericalGreater} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot\neg(\underline{x} <= |\underline{y}| \Rightarrow \dot\neg(\dot\neg(\underline{x} = |\underline{y}|)\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot\neg(\dot\neg(\underline{y} <= (-\mathrm{u}\underline{x}) \Rightarrow \dot\neg(\dot\neg(\underline{y} = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\underline{x} <= \underline{y} \Rightarrow \dot\neg(\dot\neg(\underline{x} = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNumericalGreater} \stackrel{\mathrm{tex}}{\rightarrow}$ "FromNumericalGreater"$]$

$[\mathrm{FromNumericalGreater} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromNumericalGreater"$]$

NumericalDifference

$[\mathrm{NumericalDifference} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{SignNumerical} \gg |(\underline{x} + (-\mathrm{u}\underline{y}))| = |(-\mathrm{u}(\underline{x} + (-\mathrm{u}\underline{y})))|; \mathrm{MinusNegated} \gg (-\mathrm{u}(\underline{x} + (-\mathrm{u}\underline{y}))) = (\underline{y} + (-\mathrm{u}\underline{x})); \mathrm{SameNumerical} \rhd (-\mathrm{u}(\underline{x} + (-\mathrm{u}\underline{y}))) = (\underline{y} + (-\mathrm{u}\underline{x})) \gg |(-\mathrm{u}(\underline{x} + (-\mathrm{u}\underline{y})))| = |(\underline{y} + (-\mathrm{u}\underline{x}))|; \mathrm{eqTransitivity} \rhd |(\underline{x} + (-\mathrm{u}\underline{y}))| = |(-\mathrm{u}(\underline{x} + (-\mathrm{u}\underline{y})))| \rhd |(-\mathrm{u}(\underline{x} + (-\mathrm{u}\underline{y})))| = |(\underline{y} + (-\mathrm{u}\underline{x}))| \gg |(\underline{x} + (-\mathrm{u}\underline{y}))| = |(\underline{y} + (-\mathrm{u}\underline{x}))|\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NumericalDifference} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |(\underline{x} + (-\mathrm{u}\underline{y}))| = |(\underline{y} + (-\mathrm{u}\underline{x}))|]$

$[\mathrm{NumericalDifference} \stackrel{\mathrm{tex}}{\rightarrow}$ "NumericalDifference"$]$

$[\mathrm{NumericalDifference} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma numericalDifference"$]$

NumericalDifferenceLess(Helper)

$[\mathrm{NumericalDifferenceLess(Helper)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= (\underline{x} + (-\mathrm{u}\underline{y})) \vdash \dot\neg((\underline{x} + (-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot\neg(\dot\neg((\underline{x} + (-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{leqLessTransitivity} \rhd 0 <= (\underline{x} + (-\mathrm{u}\underline{y})) \rhd \dot\neg((\underline{x} + (-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot\neg(\dot\neg((\underline{x} + (-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= \underline{z} \Rightarrow \dot\neg(\dot\neg(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveNegated} \rhd \dot\neg(0 <= \underline{z} \Rightarrow \dot\neg(\dot\neg(0 = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg((-\mathrm{u}\underline{z}) <= 0 \Rightarrow \dot\neg(\dot\neg((-\mathrm{u}\underline{z}) = 0)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessAdditionLeft} \rhd \dot\neg((-\mathrm{u}\underline{z}) <= 0 \Rightarrow \dot\neg(\dot\neg((-\mathrm{u}\underline{z}) = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg((\underline{y} + (-\mathrm{u}\underline{z})) <= (\underline{y} + 0) \Rightarrow \dot\neg(\dot\neg((\underline{y} + (-\mathrm{u}\underline{z})) = (\underline{y} + 0))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plus0} \gg (\underline{y}+0) = \underline{y}; \mathrm{SubLessRight} \rhd (\underline{y}+0) = \underline{y} \rhd \dot\neg((\underline{y}+(-\mathrm{u}\underline{z})) <=$

$(\underline{y}+0) \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = (\underline{y}+0))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{negativeToLeft(Leq)(1term)} \rhd 0 <= (\underline{x}+(-\mathrm{u}\underline{y})) \gg \underline{y} <= \underline{x}; \mathrm{LessLeqTransitivity} \rhd \dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{y})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \underline{y} <= \underline{x} \gg \dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegativeToRight(Less)} \rhd \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\underline{x} <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{z}+\underline{y}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{plusCommutativity} \gg (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}); \mathrm{SubLessRight} \rhd (\underline{z}+\underline{y}) = (\underline{y}+\underline{z}) \rhd \dot{\neg}\,(\underline{x} <= (\underline{z}+\underline{y}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{z}+\underline{y}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NumericalDifferenceLess(Helper)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon 0 <= (\underline{x}+(-\mathrm{u}\underline{y})) \vdash \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{NumericalDifferenceLess(Helper)} \stackrel{\mathrm{tex}}{\rightarrow}$ "NumericalDifferenceLess(Helper)"$]$

$[\mathrm{NumericalDifferenceLess(Helper)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma numericalDifferenceLess helper"$]$

NumericalDifferenceLess

$[\mathrm{NumericalDifferenceLess} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash 0 <= (\underline{x}+(-\mathrm{u}\underline{y})) \vdash \mathrm{NonnegativeNumerical} \rhd 0 <= (\underline{x}+(-\mathrm{u}\underline{y})) \gg |(\underline{x}+(-\mathrm{u}\underline{y}))| = (\underline{x}+(-\mathrm{u}\underline{y})); \mathrm{SubLessLeft} \rhd |(\underline{x}+(-\mathrm{u}\underline{y}))| = (\underline{x}+(-\mathrm{u}\underline{y})) \rhd \dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NumericalDifferenceLess(Helper)} \rhd 0 <= (\underline{x}+(-\mathrm{u}\underline{y})) \rhd \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{z})) = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{x} = (\underline{y}+\underline{z}))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}\,(0 <= (\underline{x}+(-\mathrm{u}\underline{y})))\mathrm{n} \vdash \mathrm{ToLess} \rhd \dot{\neg}\,(0 <= (\underline{x}+(-\mathrm{u}\underline{y})))\mathrm{n} \gg \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = 0)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegativeNumerical} \rhd \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg |(\underline{x}+(-\mathrm{u}\underline{y}))| = (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))); \mathrm{MinusNegated} \gg (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))) = (\underline{y}+(-\mathrm{u}\underline{x})); \mathrm{eqTransitivity} \rhd |(\underline{x}+(-\mathrm{u}\underline{y}))| = (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))) \rhd (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))) = (\underline{y}+(-\mathrm{u}\underline{x})) \gg |(\underline{x}+(-\mathrm{u}\underline{y}))| = (\underline{y}+(-\mathrm{u}\underline{x})); \mathrm{SubLessLeft} \rhd |(\underline{x}+(-\mathrm{u}\underline{y}))| = (\underline{y}+(-\mathrm{u}\underline{x})) \rhd \dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{x}+(-\mathrm{u}\underline{y}))| = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{x})) <= \underline{z} \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{y}+(-\mathrm{u}\underline{x})) = \underline{z})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NegativeNegated} \rhd \dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) <= 0 \Rightarrow \dot{\neg}\,(\dot{\neg}\,((\underline{x}+(-\mathrm{u}\underline{y})) = 0)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{SubLessRight} \rhd (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))) = (\underline{y}+(-\mathrm{u}\underline{x})) \rhd \dot{\neg}\,(0 <= (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (-\mathrm{u}(\underline{x}+(-\mathrm{u}\underline{y}))))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(0 <= (\underline{y}+(-\mathrm{u}\underline{x})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\underline{y}+(-\mathrm{u}\underline{x})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessLeq} \rhd \dot{\neg}\,(0 <= (\underline{y}+(-\mathrm{u}\underline{x})) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\underline{y}+(-\mathrm{u}\underline{x})))\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= (\underline{y}+(-\mathrm{u}\underline{x})); \mathrm{NumericalDifferenceLess(Helper)} \rhd 0 <=$

$(\underline{y} + (-u\underline{x})) \rhd \dot{\neg}((\underline{y} + (-u\underline{x})) <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{x})) = \underline{z})n)n)n \gg$
$\dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) = \underline{y})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (\underline{x} + \underline{z}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x} + \underline{z}))n)n)n)n)n; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow$
$\dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) = \underline{y})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (\underline{x} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} =$
$(\underline{x} + \underline{z}))n)n)n)n)n \gg \dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) =$
$\underline{y})n)n)n; \mathrm{NegativeToRight}(\mathrm{Less}) \rhd \dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) =$
$\underline{y})n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) =$
$\underline{y})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} <= (\underline{x} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x} + \underline{z}))n)n)n)n)n \gg \dot{\neg}(\underline{y} <=$
$(\underline{x} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x} + \underline{z}))n)n)n; \mathrm{PositiveToLeft}(\mathrm{Less}) \rhd \dot{\neg}(\underline{y} <= (\underline{x} + \underline{z}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x} + \underline{z}))n)n)n \gg \dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n; \mathrm{JoinConjuncts} \rhd \dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \rhd \dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n; \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \mathrm{Ded} \rhd \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash 0 <= (\underline{x} + (-u\underline{y})) \vdash \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow$
$\dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \Rightarrow$
$0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n; \mathrm{Ded} \rhd \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \vdash \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \Rightarrow$
$\dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n; \dot{\neg}(|(\underline{x}+(-u\underline{y}))| <=$
$\underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \Rightarrow 0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \rhd \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \gg$
$0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y}+\underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y}+\underline{z}))n)n)n)n)n; \mathrm{MP} \rhd \dot{\neg}(|(\underline{x}+(-u\underline{y}))| <= \underline{z} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{x}+(-u\underline{y}))| = \underline{z})n)n)n \Rightarrow \dot{\neg}(0 <= (\underline{x}+(-u\underline{y})))n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-u\underline{z})) <=$
$\underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$
$(\underline{y} + \underline{z}))n)n)n)n)n \rhd \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \gg$
$\dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n; \mathrm{FromNegations} \rhd$
$0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n \rhd \dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow$
$\dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) =$
$\underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n\rceil, p_0, c)]$

$[\mathrm{NumericalDifferenceLess} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n]$

[NumericalDifferenceLess $\stackrel{\text{tex}}{\rightarrow}$ “NumericalDifferenceLess”]

[NumericalDifferenceLess $\stackrel{\text{pyk}}{\rightarrow}$ “lemma numericalDifferenceLess”]

SplitNumericalSumHelper

[SplitNumericalSumHelper $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \vdash \mathrm{SignNumerical} \gg |\underline{x}| = |(-u\underline{x})|; \mathrm{SignNumerical} \gg |\underline{y}| = |(-u\underline{y})|; \mathrm{AddEquations} \rhd |\underline{x}| = |(-u\underline{x})| \rhd |\underline{y}| = |(-u\underline{y})| \gg (|\underline{x}| + |\underline{y}|) = (|(-u\underline{x})| + |(-u\underline{y})|); \mathrm{eqSymmetry} \rhd (|\underline{x}| + |\underline{y}|) = (|(-u\underline{x})| + |(-u\underline{y})|) \gg (|(-u\underline{x})| + |(-u\underline{y})|) = (|\underline{x}| + |\underline{y}|); -x - y = -(x + y) \gg ((-u\underline{x}) + (-u\underline{y})) = (-u(\underline{x} + \underline{y})); \mathrm{SameNumerical} \rhd ((-u\underline{x}) + (-u\underline{y})) = (-u(\underline{x} + \underline{y})) \gg |((-u\underline{x}) + (-u\underline{y}))| = |(-u(\underline{x} + \underline{y}))|; \mathrm{SignNumerical} \gg |(\underline{x} + \underline{y})| = |(-u(\underline{x} + \underline{y}))|; \mathrm{eqSymmetry} \rhd |(\underline{x} + \underline{y})| = |(-u(\underline{x} + \underline{y}))| \gg |(-u(\underline{x} + \underline{y}))| = |(\underline{x} + \underline{y})|; \mathrm{eqTransitivity} \rhd |((-u\underline{x}) + (-uy))| = |(-u(\underline{x} + \underline{y}))| \rhd |(-u(\underline{x} + \underline{y}))| = |(\underline{x} + \underline{y})| \gg |((-u\underline{x}) + (-uy))| = |(\underline{x} + \underline{y})|; \mathrm{subLeqRight} \rhd (|(-u\underline{x})| + |(-u\underline{y})|) = (|\underline{x}| + |\underline{y}|) \rhd |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \gg |((-u\underline{x}) + (-u\underline{y}))| <= (|\underline{x}| + |\underline{y}|); \mathrm{subLeqLeft} \rhd |((-u\underline{x}) + (-u\underline{y}))| = |(\underline{x} + \underline{y})| \rhd |((-u\underline{x}) + (-u\underline{y}))| <= (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)\rceil, p_0, c)$]

[SplitNumericalSumHelper $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)$]

[SplitNumericalSumHelper $\stackrel{\text{tex}}{\rightarrow}$ “SplitNumericalSumHelper”]

[SplitNumericalSumHelper $\stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSumHelper”]

splitNumericalSum(++)

[splitNumericalSum(++) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash \mathrm{AddEquations(Leq)} \rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg (0 + 0) <= (\underline{x} + \underline{y}); \mathrm{plus0} \gg (0 + 0) = 0; \mathrm{subLeqLeft} \rhd (0 + 0) = 0 \rhd (0 + 0) <= (\underline{x} + \underline{y}) \gg 0 <= (\underline{x} + \underline{y}); \mathrm{NonnegativeNumerical} \rhd 0 <= (\underline{x} + \underline{y}) \gg |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y}); \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \mathrm{AddEquations} \rhd |\underline{x}| = \underline{x} \rhd |\underline{y}| = \underline{y} \gg (|\underline{x}| + |\underline{y}|) = (\underline{x} + \underline{y}); \mathrm{eqSymmetry} \rhd (|\underline{x}| + |\underline{y}|) = (\underline{x} + \underline{y}) \gg (\underline{x} + \underline{y}) = (|\underline{x}| + |\underline{y}|); \mathrm{eqTransitivity} \rhd |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y}) \rhd (\underline{x} + \underline{y}) = (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| = (|\underline{x}| + |\underline{y}|); \mathrm{eqLeq} \rhd |(\underline{x} + \underline{y})| = (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)\rceil, p_0, c)$]

[splitNumericalSum(++) $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)$]

[splitNumericalSum(++) $\stackrel{\text{tex}}{\rightarrow}$ “splitNumericalSum(++)”]

[splitNumericalSum(++) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSum(++)”]

splitNumericalSum$(--)$

[splitNumericalSum$(--) \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash$ NonpositiveNegated $\rhd \underline{x} <= 0 \gg 0 <= (-u\underline{x});$ NonpositiveNegated $\rhd \underline{y} <= 0 \gg 0 <= (-u\underline{y});$ splitNumericalSum$(++) \rhd 0 <= (-u\underline{x}) \rhd 0 <= (-u\underline{y}) \gg |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|);$ SplitNumericalSumHelper $\rhd |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)\rceil, p_0, c)$]

[splitNumericalSum$(--) \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)$]

[splitNumericalSum$(--) \stackrel{\text{tex}}{\rightarrow}$ "splitNumericalSum(--)"]

[splitNumericalSum$(--) \stackrel{\text{pyk}}{\rightarrow}$ "lemma splitNumericalSum(--)"]

splitNumericalSum$(+-\text{small})$

[splitNumericalSum$(+-\text{small}) \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |\underline{y}| <= |\underline{x}| \vdash$ LeqAdditionLeft $\rhd \underline{y} <= 0 \gg (\underline{x} + \underline{y}) <= (\underline{x} + 0);$ plus0 $\gg (\underline{x} + 0) = \underline{x};$ subLeqRight $\rhd (\underline{x} + 0) = \underline{x} \rhd (\underline{x} + \underline{y}) <= (\underline{x} + 0) \gg (\underline{x} + \underline{y}) <= \underline{x};$ PositiveToRight(Leq)(1term) $\rhd |\underline{y}| <= |\underline{x}| \gg 0 <= (|\underline{x}| + (-u|\underline{y}|));$ lemma nonpositiveNumerical $\rhd \underline{y} <= 0 \gg |\underline{y}| = (-u\underline{y});$ EqNegated $\rhd |\underline{y}| = (-u\underline{y}) \gg (-u|\underline{y}|) = (-u(-u\underline{y}));$ DoubleMinus $\gg (-u(-u\underline{y})) = \underline{y};$ eqTransitivity $\rhd (-u|\underline{y}|) = (-u(-u\underline{y})) \rhd (-u(-u\underline{y})) = \underline{y} \gg (-u|\underline{y}|) = \underline{y};$ NonnegativeNumerical $\rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x};$ AddEquations $\rhd |\underline{x}| = \underline{x} \rhd (-u|\underline{y}|) = \underline{y} \gg (|\underline{x}| + (-u|\underline{y}|)) = (\underline{x} + \underline{y});$ subLeqRight $\rhd (|\underline{x}| + (-u|\underline{y}|)) = (\underline{x} + \underline{y}) \rhd 0 <= (|\underline{x}| + (-u|\underline{y}|)) \gg 0 <= (\underline{x} + \underline{y});$ NonnegativeNumerical $\rhd 0 <= (\underline{x} + \underline{y}) \gg |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y});$ eqSymmetry $\rhd |(\underline{x} + \underline{y})| = (\underline{x} + \underline{y}) \gg (\underline{x} + \underline{y}) = |(\underline{x} + \underline{y})|;$ eqSymmetry $\rhd |\underline{x}| = \underline{x} \gg \underline{x} = |\underline{x}|;$ subLeqLeft $\rhd (\underline{x} + \underline{y}) = |(\underline{x} + \underline{y})| \rhd (\underline{x} + \underline{y}) <= \underline{x} \gg |(\underline{x} + \underline{y})| <= \underline{x};$ subLeqRight $\rhd \underline{x} = |\underline{x}| \rhd |(\underline{x} + \underline{y})| <= \underline{x} \gg |(\underline{x} + \underline{y})| <= |\underline{x}|\rceil, p_0, c)$]

[splitNumericalSum$(+-\text{small}) \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |\underline{y}| <= |\underline{x}| \vdash |(\underline{x} + \underline{y})| <= |\underline{x}|$]

[splitNumericalSum$(+-\text{small}) \stackrel{\text{tex}}{\rightarrow}$ "splitNumericalSum(+-small)"]

[splitNumericalSum$(+-\text{small}) \stackrel{\text{pyk}}{\rightarrow}$ "lemma splitNumericalSum(+-, smallNegative)"]

splitNumericalSum$(+-\text{big})$

[splitNumericalSum$(+-\text{big}) \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \dot{\neg}\,(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{x}| = |\underline{y}|)n)n)n \vdash$ NonnegativeNegated $\rhd 0 <= \underline{x} \gg$

$(-u\underline{x}) <= 0$; NonpositiveNegated $\rhd \underline{y} <= 0 \gg 0 <= (-u\underline{y})$; SignNumerical $\gg |\underline{x}| = |(-u\underline{x})|$; SubLessLeft $\rhd |\underline{x}| = |(-u\underline{x})| \rhd \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)n)n)n \gg \dot{\neg}(|(-u\underline{x})| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|(-u\underline{x})| = |\underline{y}|)n)n)n$; SignNumerical $\gg |\underline{y}| = |(-u\underline{y})|$; SubLessRight $\rhd |\underline{y}| = |(-u\underline{y})| \rhd \dot{\neg}(|(-u\underline{x})| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|(-u\underline{x})| = |\underline{y}|)n)n)n \gg \dot{\neg}(|(-u\underline{x})| <= |(-u\underline{y})| \Rightarrow \dot{\neg}(\dot{\neg}(|(-u\underline{x})| = |(-u\underline{y})|)n)n)n$; LessLeq $\rhd \dot{\neg}(|(-u\underline{x})| <= |(-u\underline{y})| \Rightarrow \dot{\neg}(\dot{\neg}(|(-u\underline{x})| = |(-u\underline{y})|)n)n)n \gg |(-u\underline{x})| <= |(-u\underline{y})|$; splitNumericalSum(+ − small) $\rhd 0 <= (-u\underline{y}) \rhd (-u\underline{x}) <= 0 \rhd |(-u\underline{x})| <= |(-u\underline{y})| \gg |((-u\underline{y}) + (-u\underline{x}))| <= |(-u\underline{y})|$; SignNumerical $\gg |(\underline{x} + \underline{y})| = |(-u(\underline{x} + \underline{y}))|$; $-x - y = -(x + y) \gg ((-u\underline{x}) + (-u\underline{y})) = (-u(\underline{x} + \underline{y}))$; plusCommutativity $\gg ((-u\underline{x}) + (-u\underline{y})) = ((-u\underline{y}) + (-u\underline{x}))$; Equality $\rhd ((-u\underline{x}) + (-u\underline{y})) = (-u(\underline{x} + \underline{y})) \rhd ((-u\underline{x}) + (-u\underline{y})) = ((-u\underline{y}) + (-u\underline{x})) \gg (-u(\underline{x} + \underline{y})) = ((-u\underline{y}) + (-u\underline{x}))$; SameNumerical $\rhd (-u(\underline{x} + \underline{y})) = ((-u\underline{y}) + (-u\underline{x})) \gg |(-u(\underline{x} + \underline{y}))| = |((-u\underline{y}) + (-u\underline{x}))|$; eqTransitivity $\rhd |(\underline{x} + \underline{y})| = |(-u(\underline{x} + \underline{y}))| \rhd |(-u(\underline{x} + \underline{y}))| = |((-u\underline{y}) + (-u\underline{x}))| \gg |(\underline{x} + \underline{y})| = |((-u\underline{y}) + (-u\underline{x}))|$; eqSymmetry $\rhd |(\underline{x} + \underline{y})| = |((-u\underline{y})+(-u\underline{x}))| \gg |((-u\underline{y})+(-u\underline{x}))| = |(\underline{x}+\underline{y})|$; eqSymmetry $\rhd |\underline{y}| = |(-u\underline{y})| \gg |(-u\underline{y})| = |\underline{y}|$; subLeqLeft $\rhd |((-u\underline{y}) + (-u\underline{x}))| = |(\underline{x}+\underline{y})| \rhd |((-u\underline{y}) + (-u\underline{x}))| <= |(-u\underline{y})| \gg |(\underline{x} + \underline{y})| <= |(-u\underline{y})|$; subLeqRight $\rhd |(-u\underline{y})| = |\underline{y}| \rhd |(\underline{x} + \underline{y})| <= |(-u\underline{y})| \gg |(\underline{x} + \underline{y})| <= |\underline{y}|\rceil, p_0, c)]$

[splitNumericalSum(+ − big) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)n)n)n \vdash |(\underline{x} + \underline{y})| <= |\underline{y}|$]

[splitNumericalSum(+ − big) $\stackrel{\text{tex}}{\rightarrow}$ “splitNumericalSum(+-big)”]

[splitNumericalSum(+ − big) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma splitNumericalSum(+-, bigNegative)”]

splitNumericalSum(+−)

[splitNumericalSum(+−) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon |\underline{y}| <= |\underline{x}| \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash$ splitNumericalSum(+ − small) $\rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \rhd |\underline{y}| <= |\underline{x}| \gg |(\underline{x} + \underline{y})| <= |\underline{x}|$; $0 <= |x| \gg 0 <= |\underline{y}|$; LeqAdditionLeft $\rhd 0 <= |\underline{y}| \gg (|\underline{x}| + 0) <= (|\underline{x}| + |\underline{y}|)$; plus0 $\gg (|\underline{x}| + 0) = |\underline{x}|$; subLeqLeft $\rhd (|\underline{x}| + 0) = |\underline{x}| \rhd (|\underline{x}| + 0) <= (|\underline{x}| + |\underline{y}|) \gg |\underline{x}| <= (|\underline{x}| + |\underline{y}|)$; leqTransitivity $\rhd |(\underline{x} + \underline{y})| <= |\underline{x}| \rhd |\underline{x}| <= (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)$; $\forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(|\underline{y}| <= |\underline{x}|)n \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash$ ToLess $\rhd \dot{\neg}(|\underline{y}| <= |\underline{x}|)n \gg \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)n)n)n$; splitNumericalSum(+ − big) $\rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \rhd \dot{\neg}(|\underline{x}| <= |\underline{y}| \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{x}| = |\underline{y}|)n)n)n \gg |(\underline{x} + \underline{y})| <= |\underline{y}|$; $0 <= |x| \gg 0 <= |\underline{x}|$; leqAddition $\rhd 0 <= |\underline{x}| \gg (0 + |\underline{y}|) <= (|\underline{x}| + |\underline{y}|)$; lemma plus0Left $\gg (0 + |\underline{y}|) = |\underline{y}|$; subLeqLeft $\rhd (0 + |\underline{y}|) = |\underline{y}| \rhd (0 + |\underline{y}|) <= (|\underline{x}| + |\underline{y}|) \gg |\underline{y}| <= (|\underline{x}| + |\underline{y}|)$; leqTransitivity $\rhd |(\underline{x} + \underline{y})| <= |\underline{y}| \rhd |\underline{y}| <= (|\underline{x}| + |\underline{y}|) \gg |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)$; $\forall\underline{x}\colon\forall\underline{y}\colon$ Ded $\rhd \forall\underline{x}\colon\forall\underline{y}\colon |\underline{y}| <= |\underline{x}| \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|) \gg |\underline{y}| <= |\underline{x}| \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x} + \underline{y})| <= (|\underline{x}| + |\underline{y}|)$; Ded $\rhd \forall\underline{x}\colon\forall\underline{y}\colon \dot{\neg}(|\underline{y}| <= |\underline{x}|)n \vdash 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x} + \underline{y})| <=$

$(|\underline{x}|+|\underline{y}|) \gg \dot{\neg}(|\underline{y}| <= |\underline{x}|)n \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \mathrm{FromNegations} \rhd |\underline{y}| <= |\underline{x}| \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd \dot{\neg}(|\underline{y}| <= |\underline{x}|)n \Rightarrow 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{MP2} \rhd 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, p_0, \mathsf{c})]$

$[\mathrm{splitNumericalSum}(+-) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

[splitNumericalSum(+−) $\stackrel{\mathrm{tex}}{\rightarrow}$ “splitNumericalSum(+-)”]

[splitNumericalSum(+−) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma splitNumericalSum(+-)”]

splitNumericalSum(−+)

$[\mathrm{splitNumericalSum}(-+) \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash \mathrm{NonpositiveNegated} \rhd \underline{x} <= 0 \gg 0 <= (-u\underline{x}); \mathrm{NonnegativeNegated} \rhd 0 <= \underline{y} \gg (-u\underline{y}) <= 0; \mathrm{splitNumericalSum}(+-) \rhd 0 <= (-u\underline{x}) \rhd (-u\underline{y}) <= 0 \gg |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|); \mathrm{SplitNumericalSumHelper} \rhd |((-u\underline{x}) + (-u\underline{y}))| <= (|(-u\underline{x})| + |(-u\underline{y})|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, p_0, \mathsf{c})]$

$[\mathrm{splitNumericalSum}(-+) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

[splitNumericalSum(−+) $\stackrel{\mathrm{tex}}{\rightarrow}$ “splitNumericalSum(-+)”]

[splitNumericalSum(−+) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma splitNumericalSum(-+)”]

splitNumericalSum

$[\mathrm{splitNumericalSum} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash \mathrm{splitNumericalSum}(++) \rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \mathrm{splitNumericalSum}(+-) \rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash \mathrm{splitNumericalSum}(-+) \rhd \underline{x} <= 0 \rhd 0 <= \underline{y} \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash \mathrm{splitNumericalSum}(--) \rhd \underline{x} <= 0 \rhd \underline{y} <= 0 \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{Ded} \rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg 0 <= \underline{y} \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{FromLeqGeq} \rhd 0 <= \underline{x} \Rightarrow \underline{y} <=$

$0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|); \mathrm{FromLeqGeq} \rhd 0 <= \underline{y} \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \rhd \underline{y} <= 0 \Rightarrow |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|) \gg |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{splitNumericalSum} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon |(\underline{x}+\underline{y})| <= (|\underline{x}|+|\underline{y}|)]$

$[\mathrm{splitNumericalSum} \overset{\mathrm{tex}}{\rightarrow}$ "splitNumericalSum"$]$

$[\mathrm{splitNumericalSum} \overset{\mathrm{pyk}}{\rightarrow}$ "lemma splitNumericalSum"$]$

SplitNumericalProduct(++)

$[\mathrm{SplitNumericalProduct}(++) \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash \mathrm{NonnegativeFactors} \rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg 0 <= (\underline{x} * \underline{y}); \mathrm{NonnegativeNumerical} \rhd 0 <= (\underline{x} * \underline{y}) \gg |(\underline{x} * \underline{y})| = (\underline{x} * \underline{y}); \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{x} \gg |\underline{x}| = \underline{x}; \mathrm{NonnegativeNumerical} \rhd 0 <= \underline{y} \gg |\underline{y}| = \underline{y}; \mathrm{MultiplyEquations} \rhd |\underline{x}| = \underline{x} \rhd |\underline{y}| = \underline{y} \gg (|\underline{x}| * |\underline{y}|) = (\underline{x} * \underline{y}); \mathrm{eqSymmetry} \rhd (|\underline{x}| * |\underline{y}|) = (\underline{x} * \underline{y}) \gg (\underline{x} * \underline{y}) = (|\underline{x}| * |\underline{y}|); \mathrm{eqTransitivity} \rhd |(\underline{x} * \underline{y})| = (\underline{x} * \underline{y}) \rhd (\underline{x} * \underline{y}) = (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SplitNumericalProduct}(++) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)]$

$[\mathrm{SplitNumericalProduct}(++) \overset{\mathrm{tex}}{\rightarrow}$ "SplitNumericalProduct(++)"$]$

$[\mathrm{SplitNumericalProduct}(++) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma splitNumericalProduct(++)"$]$

SplitNumericalProduct(+−)

$[\mathrm{SplitNumericalProduct}(+-) \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash \mathrm{SignNumerical} \gg |(\underline{x} * \underline{y})| = |(-\mathrm{u}(\underline{x} * \underline{y}))|; \mathrm{eqSymmetry} \rhd |(\underline{x} * \underline{y})| = |(-\mathrm{u}(\underline{x} * \underline{y}))| \gg |(-\mathrm{u}(\underline{x} * \underline{y}))| = |(\underline{x} * \underline{y})|; \mathrm{PlusTimesMinus} \gg (\underline{x} * (-\mathrm{u}\underline{y})) = (-\mathrm{u}(\underline{x} * \underline{y})); \mathrm{SameNumerical} \rhd (\underline{x} * (-\mathrm{u}\underline{y})) = (-\mathrm{u}(\underline{x} * \underline{y})) \gg |(\underline{x} * (-\mathrm{u}\underline{y}))| = |(-\mathrm{u}(\underline{x} * \underline{y}))|; \mathrm{eqTransitivity} \rhd |(\underline{x} * (-\mathrm{u}\underline{y}))| = |(-\mathrm{u}(\underline{x} * \underline{y}))| \rhd |(-\mathrm{u}(\underline{x} * \underline{y}))| = |(\underline{x} * \underline{y})| \gg |(\underline{x} * (-\mathrm{u}\underline{y}))| = |(\underline{x} * \underline{y})|; \mathrm{SignNumerical} \gg |\underline{y}| = |(-\mathrm{u}\underline{y})|; \mathrm{eqSymmetry} \rhd |\underline{y}| = |(-\mathrm{u}\underline{y})| \gg |(-\mathrm{u}\underline{y})| = |\underline{y}|; \mathrm{lemma\ eqMultiplicationLeft} \rhd |(-\mathrm{u}\underline{y})| = |\underline{y}| \gg (|\underline{x}| * |(-\mathrm{u}\underline{y})|) = (|\underline{x}| * |\underline{y}|); \mathrm{NonpositiveNegated} \rhd \underline{y} <= 0 \gg 0 <= (-\mathrm{u}\underline{y}); \mathrm{SplitNumericalProduct}(++) \rhd 0 <= \underline{x} \rhd 0 <= (-\mathrm{u}\underline{y}) \gg |(\underline{x} * (-\mathrm{u}\underline{y}))| = (|\underline{x}| * |(-\mathrm{u}\underline{y})|); \mathrm{eqTransitivity} \rhd |(\underline{x} * (-\mathrm{u}\underline{y}))| = (|\underline{x}| * |(-\mathrm{u}\underline{y})|) \rhd (|\underline{x}| * |(-\mathrm{u}\underline{y})|) = (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * (-\mathrm{u}\underline{y}))| = (|\underline{x}| * |\underline{y}|); \mathrm{Equality} \rhd |(\underline{x} * (-\mathrm{u}\underline{y}))| = |(\underline{x} * \underline{y})| \rhd |(\underline{x} * (-\mathrm{u}\underline{y}))| = (|\underline{x}| * |\underline{y}|) \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{SplitNumericalProduct}(+-) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|)]$

[SplitNumericalProduct(+−) $\overset{\mathrm{tex}}{\to}$ "SplitNumericalProduct(+-)"]

[SplitNumericalProduct(+−) $\overset{\mathrm{pyk}}{\to}$ "lemma splitNumericalProduct(+-)"]

SplitNumericalProduct

[SplitNumericalProduct $\overset{\mathrm{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash$ SplitNumericalProduct(++) $\rhd 0 <= \underline{x} \rhd 0 <= \underline{y} \gg |(\underline{x} * \underline{y})| = (|\underline{x}| * |\underline{y}|); \forall\underline{x}\colon\forall\underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash$ SplitNumericalProduct(+−) $\rhd 0 <= \underline{x} \rhd \underline{y} <= 0 \gg |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|); \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash$ SplitNumericalProduct(+−) $\rhd 0 <= \underline{y} \rhd \underline{x} <= 0 \gg |(\underline{y}*\underline{x})| = (|\underline{y}|*|\underline{x}|)$; timesCommutativity $\gg (\underline{x}*\underline{y}) = (\underline{y}*\underline{x})$; SameNumerical $\rhd (\underline{x}*\underline{y}) = (\underline{y}*\underline{x}) \gg |(\underline{x}*\underline{y})| = |(\underline{y}*\underline{x})|$; timesCommutativity $\gg (|\underline{y}|*|\underline{x}|) = (|\underline{x}|*|\underline{y}|)$; eqTransitivity4 $\rhd |(\underline{x}*\underline{y})| = |(\underline{y}*\underline{x})| \rhd |(\underline{y}*\underline{x})| = (|\underline{y}|*|\underline{x}|) \rhd (|\underline{y}|*|\underline{x}|) = (|\underline{x}|*|\underline{y}|) \gg |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|); \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash$ NonpositiveNegated $\rhd \underline{x} <= 0 \gg 0 <= (-u\underline{x})$; NonpositiveNegated $\rhd \underline{y} <= 0 \gg 0 <= (-u\underline{y})$; SplitNumericalProduct(++) $\rhd 0 <= (-u\underline{x}) \rhd 0 <= (-u\underline{y}) \gg |((-u\underline{x})*(-u\underline{y}))| = (|(-u\underline{x})|*|(-u\underline{y})|)$; MinusTimesMinus $\gg ((-u\underline{x})*(-u\underline{y})) = (\underline{x}*\underline{y})$; SameNumerical $\rhd ((-u\underline{x})*(-u\underline{y})) = (\underline{x}*\underline{y}) \gg |((-u\underline{x})*(-u\underline{y}))| = |(\underline{x}*\underline{y})|$; eqSymmetry $\rhd |((-u\underline{x})*(-u\underline{y}))| = |(\underline{x}*\underline{y})| \gg |(\underline{x}*\underline{y})| = |((-u\underline{x})*(-u\underline{y}))|$; SignNumerical $\gg |\underline{x}| = |(-u\underline{x})|$; SignNumerical $\gg |\underline{y}| = |(-u\underline{y})|$; MultiplyEquations $\rhd |\underline{x}| = |(-u\underline{x})| \rhd |\underline{y}| = |(-u\underline{y})| \gg (|\underline{x}|*|\underline{y}|) = (|(-u\underline{x})|*|(-u\underline{y})|)$; eqSymmetry $\rhd (|\underline{x}|*|\underline{y}|) = (|(-u\underline{x})|*|(-u\underline{y})|) \gg (|(-u\underline{x})|*|(-u\underline{y})|) = (|\underline{x}|*|\underline{y}|)$; eqTransitivity4 $\rhd |(\underline{x}*\underline{y})| = |((-u\underline{x})*(-u\underline{y}))| \rhd |((-u\underline{x})*(-u\underline{y}))| = (|(-u\underline{x})|*|(-u\underline{y})|) \rhd (|(-u\underline{x})|*|(-u\underline{y})|) = (|\underline{x}|*|\underline{y}|) \gg |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|); \forall\underline{x}\colon\forall\underline{y}\colon$ Ded $\rhd \forall\underline{x}\colon\forall\underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$; Ded $\rhd \forall\underline{x}\colon\forall\underline{y}\colon 0 <= \underline{x} \vdash \underline{y} <= 0 \vdash |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$; Ded $\rhd \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} <= 0 \vdash 0 <= \underline{y} \vdash |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$; Ded $\rhd \forall\underline{x}\colon\forall\underline{y}\colon \underline{x} <= 0 \vdash \underline{y} <= 0 \vdash |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$; FromLeqGeq $\rhd 0 <= \underline{x} \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow 0 <= \underline{y} \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg 0 <= \underline{y} \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$; FromLeqGeq $\rhd 0 <= \underline{x} \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \rhd \underline{x} <= 0 \Rightarrow \underline{y} <= 0 \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg \underline{y} <= 0 \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$; FromLeqGeq $\rhd 0 <= \underline{y} \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \rhd \underline{y} <= 0 \Rightarrow |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|) \gg |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)\rceil, p_0, c)$]

[SplitNumericalProduct $\overset{\mathrm{stmt}}{\to}$ SystemQ $\vdash \forall\underline{x}\colon\forall\underline{y}\colon |(\underline{x}*\underline{y})| = (|\underline{x}|*|\underline{y}|)$]

[SplitNumericalProduct $\overset{\mathrm{tex}}{\to}$ "SplitNumericalProduct"]

[SplitNumericalProduct $\overset{\mathrm{pyk}}{\to}$ "lemma splitNumericalProduct"]

insertMiddleTerm(Numerical)

[insertMiddleTerm(Numerical) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \text{splitNumericalSum} \gg |((\underline{x} + (-u\underline{z})) + (\underline{z} + \underline{y}))| <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|); \text{insertMiddleTerm(Sum)} \gg (\underline{x} + \underline{y}) = ((\underline{x} + (-u\underline{z})) + (\underline{z} + \underline{y})); \text{SameNumerical} \rhd (\underline{x} + \underline{y}) = ((\underline{x} + (-u\underline{z})) + (\underline{z} + \underline{y})) \gg |(\underline{x} + \underline{y})| = |((\underline{x} + (-u\underline{z})) + (\underline{z} + \underline{y}))|; \text{eqSymmetry} \rhd |(\underline{x} + \underline{y})| = |((\underline{x} + (-u\underline{z})) + (\underline{z} + \underline{y}))| \gg |((\underline{x} + (-u\underline{z})) + (\underline{z} + \underline{y}))| = |(\underline{x}+\underline{y})|; \text{subLeqLeft} \rhd |((\underline{x}+(-u\underline{z}))+(\underline{z}+\underline{y}))| = |(\underline{x}+\underline{y})| \rhd |((\underline{x}+(-u\underline{z}))+(\underline{z}+\underline{y}))| <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|) \gg |(\underline{x} + \underline{y})| <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|)\rceil, p_0, c)$]

[insertMiddleTerm(Numerical) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon |(\underline{x} + \underline{y})| <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|)$]

[insertMiddleTerm(Numerical) $\stackrel{\text{tex}}{\rightarrow}$ “insertMiddleTerm(Numerical)”]

[insertMiddleTerm(Numerical) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma insertMiddleTerm(Numerical)”]

insertTwoMiddleTerms(Numerical)

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon \text{insertMiddleTerm(Numerical)} \gg |(\underline{x} + \underline{y})| <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|); \text{insertMiddleTerm(Numerical)} \gg |(\underline{z} + \underline{y})| <= (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|); \text{LeqAdditionLeft} \rhd |(\underline{z} + \underline{y})| <= (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|) \gg (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|) <= (|(\underline{x} + (-u\underline{z}))| + (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|)); \text{leqTransitivity} \rhd |(\underline{x} + \underline{y})| <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|) \rhd (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + \underline{y})|) <= (|(\underline{x} + (-u\underline{z}))| + (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|)) \gg |(\underline{x} + \underline{y})| <= (|(\underline{x} + (-u\underline{z}))| + (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|)); \text{plusAssociativity} \gg ((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{u}+\underline{y})|) = (|(\underline{x}+(-u\underline{z}))|+(|(\underline{z}+(-u\underline{u}))|+|(\underline{u}+\underline{y})|)); \text{eqSymmetry} \rhd ((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{u}+\underline{y})|) = (|(\underline{x}+(-u\underline{z}))|+(|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|)) \gg (|(\underline{x} + (-u\underline{z}))| + (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|)) = ((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{u}+\underline{y})|); \text{subLeqRight} \rhd (|(\underline{x}+(-u\underline{z}))|+(|(\underline{z}+(-u\underline{u}))|+|(\underline{u}+\underline{y})|)) = ((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+|(\underline{u}+\underline{y})|) \rhd |(\underline{x}+\underline{y})| <= (|(\underline{x} + (-u\underline{z}))| + (|(\underline{z} + (-u\underline{u}))| + |(\underline{u} + \underline{y})|)) \gg |(\underline{x} + \underline{y})| <= ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{u} + \underline{y})|)\rceil, p_0, c)$]

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{x} \colon \forall \underline{y} \colon \forall \underline{z} \colon \forall \underline{u} \colon |(\underline{x} + \underline{y})| <= ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{u} + \underline{y})|)$]

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{tex}}{\rightarrow}$ “insertTwoMiddleTerms(Numerical)”]

[insertTwoMiddleTerms(Numerical) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma insertTwoMiddleTerms(Numerical)”]

Three2twoTerms

$[\mathrm{Three2twoTerms} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon (\underline{\mathsf{y}} + \underline{\mathsf{z}}) = \underline{\mathsf{u}} \vdash \mathrm{lemma\ eqAdditionLeft} \rhd (\underline{\mathsf{y}}+\underline{\mathsf{z}}) = \underline{\mathsf{u}} \gg (\underline{\mathsf{x}}+(\underline{\mathsf{y}}+\underline{\mathsf{z}})) = (\underline{\mathsf{x}}+\underline{\mathsf{u}}); \mathrm{plusAssociativity} \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + \underline{\mathsf{z}})); \mathrm{eqTransitivity} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + \underline{\mathsf{z}})) \rhd (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + \underline{\mathsf{z}})) = (\underline{\mathsf{x}} + \underline{\mathsf{u}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + \underline{\mathsf{u}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Three2twoTerms} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon (\underline{\mathsf{y}} + \underline{\mathsf{z}}) = \underline{\mathsf{u}} \vdash ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + \underline{\mathsf{u}})]$

[Three2twoTerms $\stackrel{\mathrm{tex}}{\to}$ “Three2twoTerms”]

[Three2twoTerms $\stackrel{\mathrm{pyk}}{\to}$ “lemma three2twoTerms”]

Three2threeTerms

$[\mathrm{Three2threeTerms} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \mathrm{plusCommutativity} \gg (\underline{\mathsf{y}} + \underline{\mathsf{z}}) = (\underline{\mathsf{z}} + \underline{\mathsf{y}}); \mathrm{Three2twoTerms} \rhd (\underline{\mathsf{y}} + \underline{\mathsf{z}}) = (\underline{\mathsf{z}} + \underline{\mathsf{y}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + (\underline{\mathsf{z}} + \underline{\mathsf{y}})); \mathrm{plusAssociativity} \gg ((\underline{\mathsf{x}}+\underline{\mathsf{z}})+\underline{\mathsf{y}}) = (\underline{\mathsf{x}}+(\underline{\mathsf{z}}+\underline{\mathsf{y}})); \mathrm{eqSymmetry} \rhd ((\underline{\mathsf{x}}+\underline{\mathsf{z}})+\underline{\mathsf{y}}) = (\underline{\mathsf{x}} + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) \gg (\underline{\mathsf{x}} + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) = ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + \underline{\mathsf{y}}); \mathrm{eqTransitivity} \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) \rhd (\underline{\mathsf{x}} + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) = ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + \underline{\mathsf{y}}) \gg ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + \underline{\mathsf{y}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Three2threeTerms} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + \underline{\mathsf{y}})]$

[Three2threeTerms $\stackrel{\mathrm{tex}}{\to}$ “Three2threeTerms”]

[Three2threeTerms $\stackrel{\mathrm{pyk}}{\to}$ “lemma three2threeTerms”]

Three2twoFactors

$[\mathrm{Three2twoFactors} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon (\underline{\mathsf{y}} * \underline{\mathsf{z}}) = \underline{\mathsf{u}} \vdash \mathrm{lemma\ eqMultiplicationLeft} \rhd (\underline{\mathsf{y}} * \underline{\mathsf{z}}) = \underline{\mathsf{u}} \gg (\underline{\mathsf{x}} * (\underline{\mathsf{y}} * \underline{\mathsf{z}})) = (\underline{\mathsf{x}}*\underline{\mathsf{u}}); \mathrm{timesAssociativity} \gg ((\underline{\mathsf{x}}*\underline{\mathsf{y}})*\underline{\mathsf{z}}) = (\underline{\mathsf{x}}*(\underline{\mathsf{y}}*\underline{\mathsf{z}})); \mathrm{eqTransitivity} \rhd ((\underline{\mathsf{x}}*\underline{\mathsf{y}})*\underline{\mathsf{z}}) = (\underline{\mathsf{x}} * (\underline{\mathsf{y}} * \underline{\mathsf{z}})) \rhd (\underline{\mathsf{x}} * (\underline{\mathsf{y}} * \underline{\mathsf{z}})) = (\underline{\mathsf{x}} * \underline{\mathsf{u}}) \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \underline{\mathsf{z}}) = (\underline{\mathsf{x}} * \underline{\mathsf{u}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Three2twoFactors} \stackrel{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon (\underline{\mathsf{y}}*\underline{\mathsf{z}}) = \underline{\mathsf{u}} \vdash ((\underline{\mathsf{x}}*\underline{\mathsf{y}})*\underline{\mathsf{z}}) = (\underline{\mathsf{x}}*\underline{\mathsf{u}})]$

[Three2twoFactors $\stackrel{\mathrm{tex}}{\to}$ “Three2twoFactors”]

[Three2twoFactors $\stackrel{\mathrm{pyk}}{\to}$ “lemma three2twoFactors”]

Three2threeFactors

[Three2threeFactors $\stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \text{timesCommutativity} \gg (\underline{y} * \underline{z}) = (\underline{z} * \underline{y}); \text{Three2twoFactors} \rhd (\underline{y} * \underline{z}) = (\underline{z} * \underline{y}) \gg ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{z} * \underline{y})); \text{timesAssociativity} \gg ((\underline{x} * \underline{z}) * \underline{y}) = (\underline{x} * (\underline{z} * \underline{y})); \text{eqSymmetry} \rhd ((\underline{x} * \underline{z}) * \underline{y}) = (\underline{x} * (\underline{z} * \underline{y})) \gg (\underline{x} * (\underline{z} * \underline{y})) = ((\underline{x} * \underline{z}) * \underline{y}); \text{eqTransitivity} \rhd ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{z} * \underline{y})) \rhd (\underline{x} * (\underline{z} * \underline{y})) = ((\underline{x} * \underline{z}) * \underline{y}) \gg ((\underline{x} * \underline{y}) * \underline{z}) = ((\underline{x} * \underline{z}) * \underline{y})\rceil, p_0, c)$]

[Three2threeFactors $\stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon ((\underline{x} * \underline{y}) * \underline{z}) = ((\underline{x} * \underline{z}) * \underline{y})$]

[Three2threeFactors $\stackrel{\text{tex}}{\to}$ “Three2threeFactors”]

[Three2threeFactors $\stackrel{\text{pyk}}{\to}$ “lemma three2threeFactors”]

Times(−1)

[Times(−1) $\stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \text{Negative} \gg (1 + (-u1)) = 0; \text{plusCommutativity} \gg ((-u1) + 1) = (1 + (-u1)); \text{eqTransitivity} \rhd ((-u1) + 1) = (1 + (-u1)) \rhd (1 + (-u1)) = 0 \gg ((-u1) + 1) = 0; \text{lemma eqMultiplicationLeft} \rhd ((-u1) + 1) = 0 \gg (\underline{x} * ((-u1) + 1)) = (\underline{x} * 0); x * 0 = 0 \gg (\underline{x} * 0) = 0; \text{eqTransitivity} \rhd (\underline{x} * ((-u1) + 1)) = (\underline{x} * 0) \rhd (\underline{x} * 0) = 0 \gg (\underline{x} * ((-u1) + 1)) = 0; \text{Distribution} \gg (\underline{x} * ((-u1) + 1)) = ((\underline{x} * (-u1)) + (\underline{x} * 1)); \text{eqSymmetry} \rhd (\underline{x} * ((-u1) + 1)) = ((\underline{x} * (-u1)) + (\underline{x} * 1)) \gg ((\underline{x} * (-u1)) + (\underline{x} * 1)) = (\underline{x} * ((-u1) + 1)); \text{eqTransitivity} \rhd ((\underline{x} * (-u1)) + (\underline{x} * 1)) = (\underline{x} * ((-u1) + 1)) \rhd (\underline{x} * ((-u1) + 1)) = 0 \gg ((\underline{x} * (-u1)) + (\underline{x} * 1)) = 0; \text{PositiveToRight(Eq)} \rhd ((\underline{x} * (-u1)) + (\underline{x} * 1)) = 0 \gg (\underline{x} * (-u1)) = (0 + (-u(\underline{x} * 1))); \text{lemma plus0Left} \gg (0 + (-u(\underline{x} * 1))) = (-u(\underline{x} * 1)); \text{eqTransitivity} \rhd (\underline{x} * (-u1)) = (0 + (-u(\underline{x} * 1))) \rhd (0 + (-u(\underline{x} * 1))) = (-u(\underline{x} * 1)) \gg (\underline{x} * (-u1)) = (-u(\underline{x} * 1)); \text{times1} \gg (\underline{x} * 1) = \underline{x}; \text{EqNegated} \rhd (\underline{x} * 1) = \underline{x} \gg (-u(\underline{x} * 1)) = (-u\underline{x}); \text{eqTransitivity} \rhd (\underline{x} * (-u1)) = (-u(\underline{x} * 1)) \rhd (-u(\underline{x} * 1)) = (-u\underline{x}) \gg (\underline{x} * (-u1)) = (-u\underline{x})\rceil, p_0, c)$]

[Times(−1) $\stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{x}\colon (\underline{x} * (-u1)) = (-u\underline{x})$]

[Times(−1) $\stackrel{\text{tex}}{\to}$ “Times(-1)”]

[Times(−1) $\stackrel{\text{pyk}}{\to}$ “lemma times(-1)”]

Times(−1)Left

[Times(−1)Left $\stackrel{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \text{Times}(-1) \gg (\underline{x} * (-u1)) = (-u\underline{x}); \text{timesCommutativity} \gg ((-u1) * \underline{x}) = (\underline{x} * (-u1)); \text{eqTransitivity} \rhd ((-u1) * \underline{x}) = (\underline{x} * (-u1)) \rhd (\underline{x} * (-u1)) = (-u\underline{x}) \gg ((-u1) * \underline{x}) = (-u\underline{x})\rceil, p_0, c)$]

[Times(−1)Left $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon ((-u1) * \underline{x}) = (-u\underline{x})$]

[Times(−1)Left $\stackrel{\text{tex}}{\rightarrow}$ "Times(-1)Left"]

[Times(−1)Left $\stackrel{\text{pyk}}{\rightarrow}$ "lemma times(-1)Left"]

MaxLeq(1)

[MaxLeq(1) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash$ FromMax(1) $\rhd \underline{y} <= \underline{x} \gg$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x}$; eqSymmetry $\rhd$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x} \gg \underline{x} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; eqLeq $\rhd \underline{x} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; $\forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})$n $\vdash$ FromMax(2) $\rhd \dot{\neg}(\underline{y} <= \underline{x})$n $\gg$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y}$; eqSymmetry $\rhd$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y} \gg \underline{y} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; ToLess $\rhd \dot{\neg}(\underline{y} <= \underline{x})$n $\gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)$n; LessLeq $\rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)$n $\gg \underline{x} <= \underline{y}$; subLeqRight $\rhd \underline{y} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \underline{x} <= \underline{y} \gg \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; $\forall \underline{x}\colon \forall \underline{y}\colon$ Ded $\rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <= \underline{x} \Rightarrow \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; Ded $\rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})$n $\vdash \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \dot{\neg}(\underline{y} <= \underline{x})$n $\Rightarrow \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; FromNegations $\rhd \underline{y} <= \underline{x} \Rightarrow \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \dot{\neg}(\underline{y} <= \underline{x})$n $\Rightarrow \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})\rceil, p_0, c)$]

[MaxLeq(1) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$]

[MaxLeq(1) $\stackrel{\text{tex}}{\rightarrow}$ "MaxLeq(1)"]

[MaxLeq(1) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqMax1"]

MaxLeq(2)

[MaxLeq(2) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash$ FromMax(1) $\rhd \underline{y} <= \underline{x} \gg$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x}$; eqSymmetry $\rhd$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x} \gg \underline{x} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; subLeqRight $\rhd \underline{x} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \underline{y} <= \underline{x} \gg \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; $\forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})$n $\vdash$ FromMax(2) $\rhd \dot{\neg}(\underline{y} <= \underline{x})$n $\gg$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y}$; eqSymmetry $\rhd$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y} \gg \underline{y} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; eqLeq $\rhd \underline{y} =$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; $\forall \underline{x}\colon \forall \underline{y}\colon$ Ded $\rhd \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <= \underline{x} \vdash \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <= \underline{x} \Rightarrow \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; Ded $\rhd \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{y} <= \underline{x})$n $\vdash \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \dot{\neg}(\underline{y} <= \underline{x})$n $\Rightarrow \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$; FromNegations $\rhd \underline{y} <= \underline{x} \Rightarrow \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \rhd \dot{\neg}(\underline{y} <= \underline{x})$n $\Rightarrow \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) \gg \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})\rceil, p_0, c)$]

[MaxLeq(2) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{y} <=$ if$(\underline{y} <= \underline{x}, \underline{x}, \underline{y})$]

[MaxLeq(2) $\stackrel{\text{tex}}{\rightarrow}$ "MaxLeq(2)"]

[MaxLeq(2) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqMax2"]

LessThanMax

[LessThanMax $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \underline{a} \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \rhd \underline{a} \gg \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n; \mathrm{MaxLeq}(1) \gg \underline{y} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}); \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \rhd \underline{y} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \gg \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{a})n \vdash \mathrm{MP} \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \rhd \dot{\neg}(\underline{a})n \gg \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n; \mathrm{MaxLeq}(2) \gg \underline{z} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}); \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \rhd \underline{z} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \gg \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \underline{a} \vdash \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \gg \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \Rightarrow \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{a})n \vdash \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \Rightarrow \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \gg \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \mathrm{MP} \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n; \mathrm{FromNegations} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \rhd \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n \gg \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n\rceil, p_0, c)$]

[LessThanMax $\stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{a} \Rightarrow \dot{\neg}(\underline{x} <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{y})n)n)n \vdash \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{x} <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \underline{z})n)n)n \vdash \dot{\neg}(\underline{x} <= \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = \mathrm{if}(\underline{z} <= \underline{y}, \underline{y}, \underline{z}))n)n)n$]

[LessThanMax $\stackrel{\text{tex}}{\rightarrow}$ "LessThanMax"]

[LessThanMax $\stackrel{\text{pyk}}{\rightarrow}$ "lemma lessThanMax"]

x + y = zBackwards

[x + y = zBackwards $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = \underline{z} \vdash \mathrm{plusCommutativity} \gg (\underline{x} + \underline{y}) = (\underline{y} + \underline{x}); \mathrm{Equality} \rhd (\underline{x} + \underline{y}) = \underline{z} \gg \underline{z} = (\underline{y} + \underline{x})\rceil, p_0, c)$]

[x + y = zBackwards $\stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} + \underline{y}) = \underline{z} \vdash \underline{z} = (\underline{y} + \underline{x})$]

$[x + y = zBackwards \stackrel{tex}{\rightarrow}$ "x+y=zBackwards"]

$[x + y = zBackwards \stackrel{pyk}{\rightarrow}$ "lemma x+y=zBackwards"]

x ∗ y = zBackwards

$[x * y = zBackwards \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} * \underline{y}) = \underline{z} \vdash timesCommutativity \gg (\underline{x}*\underline{y}) = (\underline{y}*\underline{x}); Equality \rhd (\underline{x}*\underline{y}) = \underline{z} \gg \underline{z} = (\underline{y}*\underline{x})\rceil, p_0, c)]$

$[x * y = zBackwards \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon (\underline{x} * \underline{y}) = \underline{z} \vdash \underline{z} = (\underline{y} * \underline{x})]$

$[x * y = zBackwards \stackrel{tex}{\rightarrow}$ "x∗y=zBackwards"]

$[x * y = zBackwards \stackrel{pyk}{\rightarrow}$ "lemma x∗y=zBackwards"]

x = x + (y − y)

$[x = x + (y - y) \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon plus0 \gg (\underline{x} + 0) = \underline{x}; Negative \gg (\underline{y} + (-u\underline{y})) = 0; eqSymmetry \rhd (\underline{y} + (-u\underline{y})) = 0 \gg 0 = (\underline{y} + (-u\underline{y})); lemma\ eqAdditionLeft \rhd 0 = (\underline{y} + (-u\underline{y})) \gg (\underline{x} + 0) = (\underline{x} + (\underline{y} + (-u\underline{y}))); Equality \rhd (\underline{x} + 0) = \underline{x} \rhd (\underline{x} + 0) = (\underline{x} + (\underline{y} + (-u\underline{y}))) \gg \underline{x} = (\underline{x} + (\underline{y} + (-u\underline{y})))\rceil, p_0, c)]$

$[x = x + (y - y) \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = (\underline{x} + (\underline{y} + (-u\underline{y})))]$

$[x = x + (y - y) \stackrel{tex}{\rightarrow}$ "x=x+(y-y)"]

$[x = x + (y - y) \stackrel{pyk}{\rightarrow}$ "lemma x=x+(y-y)"]

x = x + y − y

$[x = x + y - y \stackrel{proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon x = x + (y - y) \gg \underline{x} = (\underline{x} + (\underline{y} + (-u\underline{y}))); plusAssociativity \gg ((\underline{x} + \underline{y}) + (-u\underline{y})) = (\underline{x} + (\underline{y} + (-u\underline{y}))); eqSymmetry \rhd ((\underline{x} + \underline{y}) + (-u\underline{y})) = (\underline{x} + (\underline{y} + (-u\underline{y}))) \gg (\underline{x} + (\underline{y} + (-u\underline{y}))) = ((\underline{x} + \underline{y}) + (-u\underline{y})); eqTransitivity \rhd \underline{x} = (\underline{x} + (\underline{y} + (-u\underline{y}))) \rhd (\underline{x} + (\underline{y} + (-u\underline{y}))) = ((\underline{x} + \underline{y}) + (-u\underline{y})) \gg \underline{x} = ((\underline{x} + \underline{y}) + (-u\underline{y}))\rceil, p_0, c)]$

$[x = x + y - y \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = ((\underline{x} + \underline{y}) + (-u\underline{y}))]$

$[x = x + y - y \stackrel{tex}{\rightarrow}$ "x=x+y-y"]

$[x = x + y - y \stackrel{pyk}{\rightarrow}$ "lemma x=x+y-y"]

$x = x * y * (1/y)$

[$x = x * y * (1/y) \stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\, (\underline{\mathsf{y}} = 0)\mathrm{n} \vdash \mathrm{times1} \gg (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}}; \mathrm{Reciprocal} \rhd \dot{\neg}\, (\underline{\mathsf{y}} = 0)\mathrm{n} \gg (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{y}}) = 1; \mathrm{Three2twoFactors} \rhd (\underline{\mathsf{y}} * \mathrm{rec}\underline{\mathsf{y}}) = 1 \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}}) = (\underline{\mathsf{x}} * 1); \mathrm{eqTransitivity} \rhd ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}}) = (\underline{\mathsf{x}} * 1) \rhd (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}} \gg ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}}) = \underline{\mathsf{x}}; \mathrm{eqSymmetry} \rhd ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}}) = \underline{\mathsf{x}} \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}})\rceil, \mathrm{p}_0, \mathsf{c})$]

[$x = x * y * (1/y) \stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\, (\underline{\mathsf{y}} = 0)\mathrm{n} \vdash \underline{\mathsf{x}} = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \mathrm{rec}\underline{\mathsf{y}})$]

[$x = x * y * (1/y) \stackrel{\text{tex}}{\rightarrow}$ "x=x*y*(1/y)"]

[$x = x * y * (1/y) \stackrel{\text{pyk}}{\rightarrow}$ "lemma x=x*y*(1/y)"]

insertMiddleTerm(Sum)

[insertMiddleTerm(Sum) $\stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \mathrm{x} = \mathrm{x} + \mathrm{y} - \mathrm{y} \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + (-\mathrm{u}\underline{\mathsf{z}})); \mathrm{Three2threeTerms} \gg ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + (-\mathrm{u}\underline{\mathsf{z}})) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}); \mathrm{eqTransitivity} \rhd \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + (-\mathrm{u}\underline{\mathsf{z}})) \rhd ((\underline{\mathsf{x}} + \underline{\mathsf{z}}) + (-\mathrm{u}\underline{\mathsf{z}})) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}) \gg \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}); \mathrm{eqAddition} \rhd \underline{\mathsf{x}} = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}) \gg (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}) + \underline{\mathsf{y}}); \mathrm{plusAssociativity} \gg (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}) + \underline{\mathsf{y}}) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}})); \mathrm{eqTransitivity} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}) + \underline{\mathsf{y}}) \rhd (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + \underline{\mathsf{z}}) + \underline{\mathsf{y}}) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) \gg (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}}))\rceil, \mathrm{p}_0, \mathsf{c})$]

[insertMiddleTerm(Sum) $\stackrel{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}}))$]

[insertMiddleTerm(Sum) $\stackrel{\text{tex}}{\rightarrow}$ "insertMiddleTerm(Sum)"]

[insertMiddleTerm(Sum) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma insertMiddleTerm(Sum)"]

insertTwoMiddleTerms(Sum)

[insertTwoMiddleTerms(Sum) $\stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \forall \underline{\mathsf{u}}\colon \mathrm{insertMiddleTerm(Sum)} \gg (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}})); \mathrm{insertMiddleTerm(Sum)} \gg (\underline{\mathsf{z}} + \underline{\mathsf{y}}) = ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}})); \mathrm{lemma\ eqAdditionLeft} \rhd (\underline{\mathsf{z}} + \underline{\mathsf{y}}) = ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}})) \gg ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}}))); \mathrm{plusAssociativity} \gg (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))) + (\underline{\mathsf{u}} + \underline{\mathsf{y}})) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}}))); \mathrm{eqSymmetry} \rhd (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))) + (\underline{\mathsf{u}} + \underline{\mathsf{y}})) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}}))) \gg ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}}))) = (((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))) + (\underline{\mathsf{u}} + \underline{\mathsf{y}})); \mathrm{eqTransitivity4} \rhd (\underline{\mathsf{x}} + \underline{\mathsf{y}}) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) \rhd ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + (\underline{\mathsf{z}} + \underline{\mathsf{y}})) = ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}}))) \rhd ((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{z}})) + ((\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}})) + (\underline{\mathsf{u}} + \underline{\mathsf{y}}))) =$

$(((\underline{x} + (-u\underline{z})) + (\underline{z} + (-u\underline{u}))) + (\underline{u} + \underline{y})) \gg (\underline{x} + \underline{y}) = (((\underline{x} + (-u\underline{z})) + (\underline{z} + (-u\underline{u}))) + (\underline{u} + \underline{y}))\rceil, p_0, c)]$

$[\text{insertTwoMiddleTerms(Sum)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \forall\underline{u}\colon (\underline{x} + \underline{y}) = (((\underline{x} + (-u\underline{z})) + (\underline{z} + (-u\underline{u}))) + (\underline{u} + \underline{y}))]$

$[\text{insertTwoMiddleTerms(Sum)} \xrightarrow{\text{tex}} \text{“insertTwoMiddleTerms(Sum)”}]$

$[\text{insertTwoMiddleTerms(Sum)} \xrightarrow{\text{pyk}} \text{“lemma insertTwoMiddleTerms(Sum)”}]$

insertMiddleTerm(Difference)

$[\text{insertMiddleTerm(Difference)} \xrightarrow{\text{proof}} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon \text{insertMiddleTerm(Sum)} \gg (\underline{x} + (-u\underline{y})) = ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))); \text{DoubleMinus} \gg (-u(-u\underline{z})) = \underline{z}; \text{lemma eqAdditionLeft} \rhd (-u(-u\underline{z})) = \underline{z} \gg (\underline{x} + (-u(-u\underline{z}))) = (\underline{x} + \underline{z}); \text{plusCommutativity} \gg ((-u\underline{z}) + (-u\underline{y})) = ((-u\underline{y}) + (-u\underline{z})); -x - y = -(x + y) \gg ((-u\underline{y}) + (-u\underline{z})) = (-u(\underline{y} + \underline{z})); \text{eqTransitivity} \rhd ((-u\underline{z}) + (-u\underline{y})) = ((-u\underline{y}) + (-u\underline{z})) \rhd ((-u\underline{y}) + (-u\underline{z})) = (-u(\underline{y} + \underline{z})) \gg ((-u\underline{z}) + (-u\underline{y})) = (-u(\underline{y} + \underline{z})); \text{AddEquations} \rhd (\underline{x} + (-u(-u\underline{z}))) = (\underline{x} + \underline{z}) \rhd ((-u\underline{z}) + (-u\underline{y})) = (-u(\underline{y} + \underline{z})) \gg ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z}))); \text{eqTransitivity} \rhd (\underline{x} + (-u\underline{y})) = ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))) \rhd ((\underline{x} + (-u(-u\underline{z}))) + ((-u\underline{z}) + (-u\underline{y}))) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z}))) \gg (\underline{x} + (-u\underline{y})) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z})))\rceil, p_0, c)]$

$[\text{insertMiddleTerm(Difference)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall\underline{x}\colon \forall\underline{y}\colon \forall\underline{z}\colon (\underline{x} + (-u\underline{y})) = ((\underline{x} + \underline{z}) + (-u(\underline{y} + \underline{z})))]$

$[\text{insertMiddleTerm(Difference)} \xrightarrow{\text{tex}} \text{“insertMiddleTerm(Difference)”}]$

$[\text{insertMiddleTerm(Difference)} \xrightarrow{\text{pyk}} \text{“lemma insertMiddleTerm(Difference)”}]$

$x * 0 + x = x$

$[x * 0 + x = x \xrightarrow{\text{proof}} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{x}\colon \text{times1} \gg (\underline{x} * 1) = \underline{x}; \text{eqSymmetry} \rhd (\underline{x} * 1) = \underline{x} \gg \underline{x} = (\underline{x} * 1); \text{lemma eqAdditionLeft} \rhd \underline{x} = (\underline{x} * 1) \gg ((\underline{x} * 0) + \underline{x}) = ((\underline{x} * 0) + (\underline{x} * 1)); \text{Distribution} \gg (\underline{x} * (0 + 1)) = ((\underline{x} * 0) + (\underline{x} * 1)); \text{eqSymmetry} \rhd (\underline{x} * (0 + 1)) = ((\underline{x} * 0) + (\underline{x} * 1)) \gg ((\underline{x} * 0) + (\underline{x} * 1)) = (\underline{x} * (0 + 1)); \text{lemma plus0Left} \gg (0 + 1) = 1; \text{lemma eqMultiplicationLeft} \rhd (0 + 1) = 1 \gg (\underline{x} * (0 + 1)) = (\underline{x} * 1); \text{eqTransitivity5} \rhd ((\underline{x} * 0) + \underline{x}) = ((\underline{x} * 0) + (\underline{x} * 1)) \rhd ((\underline{x} * 0) + (\underline{x} * 1)) = (\underline{x} * (0 + 1)) \rhd (\underline{x} * (0 + 1)) = (\underline{x} * 1) \rhd (\underline{x} * 1) = \underline{x} \gg ((\underline{x} * 0) + \underline{x}) = \underline{x}\rceil, p_0, c)]$

$[x * 0 + x = x \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall\underline{x}\colon ((\underline{x} * 0) + \underline{x}) = \underline{x}]$

[$x * 0 + x = x \stackrel{\text{tex}}{\rightarrow}$ “x*0+x=x”]

[$x * 0 + x = x \stackrel{\text{pyk}}{\rightarrow}$ “lemma x*0+x=x”]

$x * 0 = 0$

[$x * 0 = 0 \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon x = x + (y - y) \gg (\underline{x} * 0) = ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))); \text{plusAssociativity} \gg (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))); \text{eqSymmetry} \rhd (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = ((\underline{x}*0)+(\underline{x}+(-u\underline{x}))) \gg ((\underline{x}*0)+(\underline{x}+(-u\underline{x}))) = (((\underline{x}*0)+\underline{x})+(-u\underline{x})); x*0+x = x \gg ((\underline{x} * 0) + \underline{x}) = \underline{x}; \text{eqAddition} \rhd ((\underline{x} * 0) + \underline{x}) = \underline{x} \gg (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = (\underline{x} + (-u\underline{x})); \text{Negative} \gg (\underline{x} + (-u\underline{x})) = 0; \text{eqTransitivity5} \rhd (\underline{x} * 0) = ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))) \rhd ((\underline{x} * 0) + (\underline{x} + (-u\underline{x}))) = (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) \rhd (((\underline{x} * 0) + \underline{x}) + (-u\underline{x})) = (\underline{x} + (-u\underline{x})) \rhd (\underline{x} + (-u\underline{x})) = 0 \gg (\underline{x} * 0) = 0\rceil, p_0, c)$]

[$x * 0 = 0 \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon (\underline{x} * 0) = 0$]

[$x * 0 = 0 \stackrel{\text{tex}}{\rightarrow}$ “x*0=0”]

[$x * 0 = 0 \stackrel{\text{pyk}}{\rightarrow}$ “lemma x*0=0”]

NonnegativeFactors

[$\text{NonnegativeFactors} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash \text{leqMultiplication} \rhd 0 <= \underline{y} \rhd 0 <= \underline{x} \gg (0*\underline{y}) <= (\underline{x}*\underline{y}); \text{timesCommutativity} \gg (0*\underline{y}) = (\underline{y}*0); x*0 = 0 \gg (\underline{y}*0) = 0; \text{eqTransitivity} \rhd (0*\underline{y}) = (\underline{y}*0) \rhd (\underline{y}*0) = 0 \gg (0*\underline{y}) = 0; \text{subLeqLeft} \rhd (0*\underline{y}) = 0 \rhd (0*\underline{y}) <= (\underline{x}*\underline{y}) \gg 0 <= (\underline{x}*\underline{y})\rceil, p_0, c)$]

[$\text{NonnegativeFactors} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon 0 <= \underline{x} \vdash 0 <= \underline{y} \vdash 0 <= (\underline{x} * \underline{y})$]

[$\text{NonnegativeFactors} \stackrel{\text{tex}}{\rightarrow}$ “NonnegativeFactors”]

[$\text{NonnegativeFactors} \stackrel{\text{pyk}}{\rightarrow}$ “lemma nonnegativeFactors”]

NonzeroFactors

[$\text{NonzeroFactors} \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} = 0)n \vdash \dot{\neg}(\underline{y} = 0)n \vdash \text{NeqMultiplication} \rhd \dot{\neg}(\underline{y} = 0)n \rhd \dot{\neg}(\underline{x} = 0)n \gg \dot{\neg}((\underline{x} * \underline{y}) = (0 * \underline{y}))n; \text{timesCommutativity} \gg (0 * \underline{y}) = (\underline{y} * 0); x * 0 = 0 \gg (\underline{y} * 0) = 0; \text{eqTransitivity} \rhd (0 * \underline{y}) = (\underline{y} * 0) \rhd (\underline{y} * 0) = 0 \gg (0 * \underline{y}) = 0; \text{SubNeqRight} \rhd (0 * \underline{y}) = 0 \rhd \dot{\neg}((\underline{x} * \underline{y}) = (0 * \underline{y}))n \gg \dot{\neg}((\underline{x} * \underline{y}) = 0)n\rceil, p_0, c)$]

[$\text{NonzeroFactors} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{x} = 0)n \vdash \dot{\neg}(\underline{y} = 0)n \vdash \dot{\neg}((\underline{x} * \underline{y}) = 0)n$]

[NonzeroFactors $\stackrel{\text{tex}}{\to}$ "NonzeroFactors"]

[NonzeroFactors $\stackrel{\text{pyk}}{\to}$ "lemma nonzeroFactors"]

PositiveFactors

[PositiveFactors $\stackrel{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon \dot\neg(0 <= \underline{\mathsf{x}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot\neg(0 <= \underline{\mathsf{y}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \text{Repetition} \rhd \dot\neg(0 <= \underline{\mathsf{x}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= \underline{\mathsf{x}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n}; \text{FirstConjunct} \rhd \dot\neg(0 <= \underline{\mathsf{x}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{\mathsf{x}}; \text{SecondConjunct} \rhd \dot\neg(0 <= \underline{\mathsf{x}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 = \underline{\mathsf{x}})\mathrm{n}; \text{NeqSymmetry} \rhd \dot\neg(0 = \underline{\mathsf{x}})\mathrm{n} \gg \dot\neg(\underline{\mathsf{x}} = 0)\mathrm{n}; \text{Repetition} \rhd \dot\neg(0 <= \underline{\mathsf{y}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= \underline{\mathsf{y}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n}; \text{FirstConjunct} \rhd \dot\neg(0 <= \underline{\mathsf{y}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= \underline{\mathsf{y}}; \text{SecondConjunct} \rhd \dot\neg(0 <= \underline{\mathsf{y}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 = \underline{\mathsf{y}})\mathrm{n}; \text{NeqSymmetry} \rhd \dot\neg(0 = \underline{\mathsf{y}})\mathrm{n} \gg \dot\neg(\underline{\mathsf{y}} = 0)\mathrm{n}; \text{NonnegativeFactors} \rhd 0 <= \underline{\mathsf{x}} \rhd 0 <= \underline{\mathsf{y}} \gg 0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \text{NonzeroFactors} \rhd \dot\neg(\underline{\mathsf{x}} = 0)\mathrm{n} \rhd \dot\neg(\underline{\mathsf{y}} = 0)\mathrm{n} \gg \dot\neg((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n}; \text{NeqSymmetry} \rhd \dot\neg((\underline{\mathsf{x}} * \underline{\mathsf{y}}) = 0)\mathrm{n} \gg \dot\neg(0 = (\underline{\mathsf{x}} * \underline{\mathsf{y}}))\mathrm{n}; \text{JoinConjuncts} \rhd 0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \rhd \dot\neg(0 = (\underline{\mathsf{x}} * \underline{\mathsf{y}}))\mathrm{n} \gg \dot\neg(0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \Rightarrow \dot\neg(\dot\neg(0 = (\underline{\mathsf{x}} * \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n}; \text{Repetition} \rhd \dot\neg(0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \Rightarrow \dot\neg(\dot\neg(0 = (\underline{\mathsf{x}} * \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg(0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \Rightarrow \dot\neg(\dot\neg(0 = (\underline{\mathsf{x}} * \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[PositiveFactors $\stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon \dot\neg(0 <= \underline{\mathsf{x}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot\neg(0 <= \underline{\mathsf{y}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{y}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot\neg(0 <= (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \Rightarrow \dot\neg(\dot\neg(0 = (\underline{\mathsf{x}} * \underline{\mathsf{y}}))\mathrm{n})\mathrm{n})\mathrm{n}$]

[PositiveFactors $\stackrel{\text{tex}}{\to}$ "PositiveFactors"]

[PositiveFactors $\stackrel{\text{pyk}}{\to}$ "lemma positiveFactors"]

PlusTimesMinus

[PlusTimesMinus $\stackrel{\text{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon \text{Times}(-1)\text{Left} \gg ((-\mathrm{u}1) * \underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{y}}); \text{lemma eqMultiplicationLeft} \rhd ((-\mathrm{u}1) * \underline{\mathsf{y}}) = (-\mathrm{u}\underline{\mathsf{y}}) \gg (\underline{\mathsf{x}} * ((-\mathrm{u}1) * \underline{\mathsf{y}})) = (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})); \text{timesAssociativity} \gg ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * \underline{\mathsf{y}}) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * \underline{\mathsf{y}})); \text{timesCommutativity} \gg (\underline{\mathsf{x}} * (-\mathrm{u}1)) = ((-\mathrm{u}1) * \underline{\mathsf{x}}); \text{eqMultiplication} \rhd (\underline{\mathsf{x}} * (-\mathrm{u}1)) = ((-\mathrm{u}1) * \underline{\mathsf{x}}) \gg ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * \underline{\mathsf{y}}) = (((-\mathrm{u}1) * \underline{\mathsf{x}}) * \underline{\mathsf{y}}); \text{timesAssociativity} \gg (((-\mathrm{u}1) * \underline{\mathsf{x}}) * \underline{\mathsf{y}}) = ((-\mathrm{u}1) * (\underline{\mathsf{x}} * \underline{\mathsf{y}})); \text{Times}(-1)\text{Left} \gg ((-\mathrm{u}1) * (\underline{\mathsf{x}} * \underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})); \text{eqTransitivity4} \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * \underline{\mathsf{y}}) = (((-\mathrm{u}1) * \underline{\mathsf{x}}) * \underline{\mathsf{y}}) \rhd (((-\mathrm{u}1) * \underline{\mathsf{x}}) * \underline{\mathsf{y}}) = ((-\mathrm{u}1) * (\underline{\mathsf{x}} * \underline{\mathsf{y}})) \rhd ((-\mathrm{u}1) * (\underline{\mathsf{x}} * \underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) \gg ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * \underline{\mathsf{y}}) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})); \text{Equality} \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * \underline{\mathsf{y}}) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * \underline{\mathsf{y}}) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * \underline{\mathsf{y}})) \gg (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * \underline{\mathsf{y}})); \text{eqTransitivity} \rhd (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * \underline{\mathsf{y}})) \rhd (\underline{\mathsf{x}} * ((-\mathrm{u}1) * \underline{\mathsf{y}})) = (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})) \gg (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) = (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})); \text{eqSymmetry} \rhd (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}})) = (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})) \gg (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))\rceil, \mathrm{p}_0, \mathsf{c})$]

$[\mathrm{PlusTimesMinus} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon (\underline{\mathsf{x}} * (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(\underline{\mathsf{x}} * \underline{\mathsf{y}}))]$

$[\mathrm{PlusTimesMinus} \stackrel{\mathrm{tex}}{\rightarrow}$ "PlusTimesMinus"$]$

$[\mathrm{PlusTimesMinus} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma plusTimesMinus"$]$

MinusTimesMinus

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \mathrm{DoubleMinus} \gg (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}}; \mathrm{Times}(-1)\mathrm{Left} \gg ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}})) = (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) \rhd (-\mathrm{u}(-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}} \gg ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}}; \mathrm{lemma\ eqMultiplicationLeft} \rhd ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}})) = \underline{\mathsf{y}} \gg (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}); \mathrm{Times}(-1) \gg (\underline{\mathsf{x}} * (-\mathrm{u}1)) = (-\mathrm{u}\underline{\mathsf{x}}); \mathrm{eqMultiplication} \rhd (\underline{\mathsf{x}} * (-\mathrm{u}1)) = (-\mathrm{u}\underline{\mathsf{x}}) \gg ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})); \mathrm{timesAssociativity} \gg ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))); \mathrm{Equality} \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) \rhd ((\underline{\mathsf{x}} * (-\mathrm{u}1)) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) \gg ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) \rhd (\underline{\mathsf{x}} * ((-\mathrm{u}1) * (-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \gg ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * \underline{\mathsf{y}})\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon ((-\mathrm{u}\underline{\mathsf{x}}) * (-\mathrm{u}\underline{\mathsf{y}})) = (\underline{\mathsf{x}} * \underline{\mathsf{y}})]$

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{tex}}{\rightarrow}$ "MinusTimesMinus"$]$

$[\mathrm{MinusTimesMinus} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma minusTimesMinus"$]$

$(-1) * (-1) + (-1) * 1 = 0$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{DistributionOut} \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)); \mathrm{Negative} \gg (1 + (-\mathrm{u}1)) = 0; \mathrm{plusCommutativity} \gg ((-\mathrm{u}1) + 1) = (1 + (-\mathrm{u}1)); \mathrm{eqTransitivity} \rhd ((-\mathrm{u}1) + 1) = (1 + (-\mathrm{u}1)) \rhd (1 + (-\mathrm{u}1)) = 0 \gg ((-\mathrm{u}1) + 1) = 0; \mathrm{lemma\ eqMultiplicationLeft} \rhd ((-\mathrm{u}1) + 1) = 0 \gg ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)) = ((-\mathrm{u}1) * 0); \mathrm{x} * 0 = 0 \gg ((-\mathrm{u}1) * 0) = 0; \mathrm{eqTransitivity4} \rhd (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)) \rhd ((-\mathrm{u}1) * ((-\mathrm{u}1) + 1)) = ((-\mathrm{u}1) * 0) \rhd ((-\mathrm{u}1) * 0) = 0 \gg (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = 0\rceil, \mathrm{p}_0, \mathsf{c})]$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash (((-\mathrm{u}1) * (-\mathrm{u}1)) + ((-\mathrm{u}1) * 1)) = 0]$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{tex}}{\rightarrow}$ "(-1)*(-1)+(-1)*1=0"$]$

$[(-1) * (-1) + (-1) * 1 = 0 \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma (-1)*(-1)+(-1)*1=0"$]$

$(-1)*(-1)=1$

$[(-1)*(-1)=1 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \text{x}=\text{x}+(\text{y}-\text{y}) \gg ((-\text{u1})*(-\text{u1}))=(((-\text{u1})*(-\text{u1}))+(1+(-\text{u1}))); \text{times1} \gg ((-\text{u1})*1)=(-\text{u1}); \text{eqSymmetry} \rhd ((-\text{u1})*1)=(-\text{u1}) \gg (-\text{u1})=((-\text{u1})*1); \text{lemma eqAdditionLeft} \rhd (-\text{u1})=((-\text{u1})*1) \gg (1+(-\text{u1}))=(1+((-\text{u1})*1)); \text{lemma eqAdditionLeft} \rhd (1+(-\text{u1}))=(1+((-\text{u1})*1)) \gg (((-\text{u1})*(-\text{u1}))+(1+(-\text{u1})))=(((-\text{u1})*(-\text{u1}))+(1+((-\text{u1})*1))); \text{plusCommutativity} \gg (1+((-\text{u1})*1))=(((-\text{u1})*1)+1); \text{lemma eqAdditionLeft} \rhd (1+((-\text{u1})*1))=(((-\text{u1})*1)+1) \gg (((-\text{u1})*(-\text{u1}))+(1+((-\text{u1})*1)))=(((-\text{u1})*(-\text{u1}))+(((-\text{u1})*1)+1)); \text{plusAssociativity} \gg ((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1)=(((-\text{u1})*(-\text{u1}))+(((-\text{u1})*1)+1)); \text{eqSymmetry} \rhd ((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1)=(((-\text{u1})*(-\text{u1}))+(((-\text{u1})*1)+1)) \gg (((-\text{u1})*(-\text{u1}))+(((-\text{u1})*1)+1))=((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1); (-1)*(-1)+(-1)*1=0 \gg (((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))=0; \text{eqAddition} \rhd (((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))=0 \gg ((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1)=(0+1); \text{lemma plus0Left} \gg (0+1)=1; \text{eqTransitivity5} \rhd ((-\text{u1})*(-\text{u1}))=(((-\text{u1})*(-\text{u1}))+(1+(-\text{u1}))) \rhd (((-\text{u1})*(-\text{u1}))+(1+(-\text{u1})))=(((-\text{u1})*(-\text{u1}))+(1+((-\text{u1})*1))) \rhd (((-\text{u1})*(-\text{u1}))+(1+((-\text{u1})*1)))=(((-\text{u1})*(-\text{u1}))+(((-\text{u1})*1)+1)) \rhd (((-\text{u1})*(-\text{u1}))+(((-\text{u1})*1)+1))=((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1) \gg ((-\text{u1})*(-\text{u1}))=((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1); \text{eqTransitivity4} \rhd ((-\text{u1})*(-\text{u1}))=((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1) \rhd ((((-\text{u1})*(-\text{u1}))+((-\text{u1})*1))+1)=(0+1) \rhd (0+1)=1 \gg ((-\text{u1})*(-\text{u1}))=1\rceil, \text{p}_0, \mathsf{c})]$

$[(-1)*(-1)=1 \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash ((-\text{u1})*(-\text{u1}))=1]$

$[(-1)*(-1)=1 \stackrel{\text{tex}}{\rightarrow}$ "(-1)*(-1)=1"$]$

$[(-1)*(-1)=1 \stackrel{\text{pyk}}{\rightarrow}$ "lemma (-1)*(-1)=1"$]$

$0<1$Helper

$[0<1\text{Helper} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash 1<=0 \vdash \text{leqAddition} \rhd 1<=0 \gg (1+(-\text{u1}))<=(0+(-\text{u1})); \text{Negative} \gg (1+(-\text{u1}))=0; \text{subLeqLeft} \rhd (1+(-\text{u1}))=0 \rhd (1+(-\text{u1}))<=(0+(-\text{u1})) \gg 0<=(0+(-\text{u1})); \text{lemma plus0Left} \gg (0+(-\text{u1}))=(-\text{u1}); \text{subLeqRight} \rhd (0+(-\text{u1}))=(-\text{u1}) \rhd 0<=(0+(-\text{u1})) \gg 0<=(-\text{u1}); \text{leqMultiplication} \rhd 0<=(-\text{u1}) \rhd 0<=(-\text{u1}) \gg (0*(-\text{u1}))<=((-\text{u1})*(-\text{u1})); \text{x}*0=0 \gg ((-\text{u1})*0)=0; \text{timesCommutativity} \gg (0*(-\text{u1}))=((-\text{u1})*0); \text{eqTransitivity} \rhd (0*(-\text{u1}))=((-\text{u1})*0) \rhd ((-\text{u1})*0)=0 \gg (0*(-\text{u1}))=0; \text{subLeqLeft} \rhd (0*(-\text{u1}))=0 \rhd (0*(-\text{u1}))<=((-\text{u1})*(-\text{u1})) \gg 0<=((-\text{u1})*(-\text{u1})); (-1)*(-1)=1 \gg ((-\text{u1})*(-\text{u1}))=1; \text{subLeqRight} \rhd ((-\text{u1})*(-\text{u1}))=1 \rhd 0<=((-\text{u1})*(-\text{u1})) \gg 0<=1; \text{Ded} \rhd 1<=0 \vdash 0<=1 \gg 1<=0 \Rightarrow 0<=1\rceil, \text{p}_0, \mathsf{c})]$

$[0 < 1\text{Helper} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash 1 <= 0 \Rightarrow 0 <= 1]$

$[0 < 1\text{Helper} \stackrel{\text{tex}}{\rightarrow}$ "0<1Helper"$]$

$[0 < 1\text{Helper} \stackrel{\text{pyk}}{\rightarrow}$ "lemma 0<1Helper"$]$

$0 < 1$

$[0 < 1 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \text{leqTotality} \gg \dot{\neg}(0 <= 1)\text{n} \Rightarrow 1 <= 0; \text{AutoImply} \gg 0 <= 1 \Rightarrow 0 <= 1; 0 < 1\text{Helper} \gg 1 <= 0 \Rightarrow 0 <= 1; \text{FromDisjuncts} \rhd \dot{\neg}(0 <= 1)\text{n} \Rightarrow 1 <= 0 \rhd 0 <= 1 \Rightarrow 0 <= 1 \rhd 1 <= 0 \Rightarrow 0 <= 1 \gg 0 <= 1; 0\text{not}1 \gg \dot{\neg}(0 = 1)\text{n}; \text{JoinConjuncts} \rhd 0 <= 1 \rhd \dot{\neg}(0 = 1)\text{n} \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})]$

$[0 < 1 \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\text{n})\text{n})\text{n}]$

$[0 < 1 \stackrel{\text{tex}}{\rightarrow}$ "0<1"$]$

$[0 < 1 \stackrel{\text{pyk}}{\rightarrow}$ "lemma 0<1"$]$

$0 < 2$

$[0 < 2 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\text{n})\text{n})\text{n}; \text{LessAddition} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\text{n})\text{n})\text{n} \gg \dot{\neg}((0+1) <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}((0+1) = (1+1))\text{n})\text{n})\text{n}; \text{lemma plus0Left} \gg (0+1) = 1; \text{SubLessLeft} \rhd (0+1) = 1 \rhd \dot{\neg}((0+1) <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}((0+1) = (1+1))\text{n})\text{n})\text{n} \gg \dot{\neg}(1 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(1 = (1+1))\text{n})\text{n})\text{n}; \text{LessTransitivity} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\text{n})\text{n})\text{n} \rhd \dot{\neg}(1 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(1 = (1+1))\text{n})\text{n})\text{n} \gg \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})]$

$[0 < 2 \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\text{n})\text{n})\text{n}]$

$[0 < 2 \stackrel{\text{tex}}{\rightarrow}$ "0<2"$]$

$[0 < 2 \stackrel{\text{pyk}}{\rightarrow}$ "lemma 0<2"$]$

$0 < 3$

$[0 < 3 \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash 0 < 2 \gg \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\text{n})\text{n})\text{n}; \text{LessLeq} \rhd \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\text{n})\text{n})\text{n} \gg 0 <= (1+1); \text{Leq}+1 \rhd 0 <= (1+1) \gg \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\text{n})\text{n})\text{n}; \text{Repetition} \rhd \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\text{n})\text{n})\text{n} \gg \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})]$

$[0 < 3 \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\text{n})\text{n})\text{n}]$

$[0 < 3 \overset{\text{tex}}{\rightarrow}$ “0<3”$]$

$[0 < 3 \overset{\text{pyk}}{\rightarrow}$ “lemma 0<3”$]$

$0 < 1/2$

$[0 < 1/2 \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0 < 2 \gg \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg 0 <= (1+1); \mathrm{SecondConjunct} \rhd \dot{\neg}(0 <= (1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (1+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 = (1+1))\mathrm{n}; \mathrm{NeqSymmetry} \rhd \dot{\neg}(0 = (1+1))\mathrm{n} \gg \dot{\neg}((1+1) = 0)\mathrm{n}; 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} * 0 = 0 \gg ((1+1) * 0) = 0; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd ((1+1) * 0) = 0 \gg 0 = (0 * (1+1)); \mathrm{SubLessLeft} \rhd 0 = (0 * (1+1)) \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * (1+1)) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * (1+1)) = 1)\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Reciprocal} \rhd \dot{\neg}((1+1) = 0)\mathrm{n} \gg ((1+1) * \mathrm{rec}(1+1)) = 1; \mathrm{x} * \mathrm{y} = \mathrm{zBackwards} \rhd ((1+1) * \mathrm{rec}(1+1)) = 1 \gg 1 = (\mathrm{rec}(1+1) * (1+1)); \mathrm{SubLessRight} \rhd 1 = (\mathrm{rec}(1+1) * (1+1)) \rhd \dot{\neg}((0 * (1+1)) <= 1 \Rightarrow \dot{\neg}(\dot{\neg}((0 * (1+1)) = 1)\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((0 * (1+1)) <= (\mathrm{rec}(1+1) * (1+1)) \Rightarrow \dot{\neg}(\dot{\neg}((0 * (1+1)) = (\mathrm{rec}(1+1) * (1+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessDivision} \rhd 0 <= (1+1) \rhd \dot{\neg}((0 * (1+1)) <= (\mathrm{rec}(1+1) * (1+1)) \Rightarrow \dot{\neg}(\dot{\neg}((0 * (1+1)) = (\mathrm{rec}(1+1) * (1+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \mathrm{rec}(1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 < 1/2 \overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \dot{\neg}(0 <= \mathrm{rec}(1+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}(1+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0 < 1/2 \overset{\text{tex}}{\rightarrow}$ “0<1/2”$]$

$[0 < 1/2 \overset{\text{pyk}}{\rightarrow}$ “lemma 0<1/2”$]$

$0 < 1/3$

$[0 < 1/3 \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash 0 < 3 \gg \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveInverted} \rhd \dot{\neg}(0 <= ((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[0 < 1/3 \overset{\text{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \dot{\neg}(0 <= \mathrm{rec}((1+1)+1) \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[0 < 1/3 \overset{\text{tex}}{\rightarrow}$ “0<1/3”$]$

$[0 < 1/3 \overset{\text{pyk}}{\rightarrow}$ “lemma 0<1/3”$]$

TwoWholes

$[\mathrm{TwoWholes} \overset{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathrm{x}}\colon \mathrm{times1} \gg (\underline{\mathrm{x}} * 1) = \underline{\mathrm{x}}; \mathrm{eqSymmetry} \gg \underline{\mathrm{x}} = (\underline{\mathrm{x}} * 1); \mathrm{lemma\ eqAdditionLeft} \rhd \underline{\mathrm{x}} = (\underline{\mathrm{x}} * 1) \gg (\underline{\mathrm{x}} + \underline{\mathrm{x}}) =$

$(\underline{x} + (\underline{x} * 1)); \mathrm{eqAddition} \rhd \underline{x} = (\underline{x} * 1) \gg (\underline{x} + (\underline{x} * 1)) = ((\underline{x} * 1) + (\underline{x} * 1)); \mathrm{eqTransitivity} \rhd (\underline{x} + \underline{x}) = (\underline{x} + (\underline{x} * 1)) \rhd (\underline{x} + (\underline{x} * 1)) = ((\underline{x} * 1) + (\underline{x} * 1)) \gg (\underline{x} + \underline{x}) = ((\underline{x} * 1) + (\underline{x} * 1)); \mathrm{DistributionOut} \gg ((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)); \mathrm{Repetition} \rhd ((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)) \gg ((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)); \mathrm{timesCommutativity} \gg (\underline{x} * (1 + 1)) = ((1 + 1) * \underline{x}); \mathrm{eqTransitivity4} \rhd (\underline{x} + \underline{x}) = ((\underline{x} * 1) + (\underline{x} * 1)) \rhd ((\underline{x} * 1) + (\underline{x} * 1)) = (\underline{x} * (1 + 1)) \rhd (\underline{x} * (1 + 1)) = ((1 + 1) * \underline{x}) \gg (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x})\rceil, p_0, \mathsf{c})]$

$[\mathrm{TwoWholes} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x})]$

[TwoWholes $\stackrel{\mathrm{tex}}{\rightarrow}$ “TwoWholes”]

[TwoWholes $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma x+x=2∗x”]

ThreeWholes

$[\mathrm{ThreeWholes} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \mathrm{TwoWholes} \gg (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x}); \mathrm{lemma\ times1Left} \gg (1 * \underline{x}) = \underline{x}; \mathrm{eqSymmetry} \rhd (1 * \underline{x}) = \underline{x} \gg \underline{x} = (1 * \underline{x}); \mathrm{AddEquations} \rhd (\underline{x} + \underline{x}) = ((1 + 1) * \underline{x}) \rhd \underline{x} = (1 * \underline{x}) \gg ((\underline{x} + \underline{x}) + \underline{x}) = (((1 + 1) * \underline{x}) + (1 * \underline{x})); \mathrm{DistributionOutLeft} \gg (((1 + 1) * \underline{x}) + (1 * \underline{x})) = (\underline{x} * ((1 + 1) + 1)); \mathrm{timesCommutativity} \gg (\underline{x} * ((1 + 1) + 1)) = (((1 + 1) + 1) * \underline{x}); \mathrm{eqTransitivity4} \rhd ((\underline{x} + \underline{x}) + \underline{x}) = (((1+1)*\underline{x})+(1*\underline{x})) \rhd (((1+1)*\underline{x})+(1*\underline{x})) = (\underline{x}*((1+1)+1)) \rhd (\underline{x}*((1+1)+1)) = (((1+1)+1)*\underline{x}) \gg ((\underline{x}+\underline{x})+\underline{x}) = (((1+1)+1)*\underline{x}); \mathrm{Repetition} \rhd ((\underline{x}+\underline{x})+\underline{x}) = (((1 + 1) + 1) * \underline{x}) \gg ((\underline{x} + \underline{x}) + \underline{x}) = (((1 + 1) + 1) * \underline{x})\rceil, p_0, \mathsf{c})]$

$[\mathrm{ThreeWholes} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon ((\underline{x} + \underline{x}) + \underline{x}) = (((1 + 1) + 1) * \underline{x})]$

[ThreeWholes $\stackrel{\mathrm{tex}}{\rightarrow}$ “ThreeWholes”]

[ThreeWholes $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma x+x+x=3∗x”]

TwoHalves

$[\mathrm{TwoHalves} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 < 2 \gg \dot\neg(0 <= (1 + 1) \Rightarrow \dot\neg(\dot\neg(0 = (1 + 1))n)n)n; \mathrm{LessNeq} \rhd \dot\neg(0 <= (1 + 1) \Rightarrow \dot\neg(\dot\neg(0 = (1 + 1))n)n)n \gg \dot\neg(0 = (1 + 1))n; \mathrm{NeqSymmetry} \rhd \dot\neg(0 = (1 + 1))n \gg \dot\neg((1 + 1) = 0)n; \mathrm{TwoWholes} \gg ((\mathrm{rec}(1{+}1)*\underline{x})+(\mathrm{rec}(1{+}1)*\underline{x})) = ((1{+}1)*(\mathrm{rec}(1{+}1)*\underline{x})); \mathrm{timesAssociativity} \gg (((1 + 1) * \mathrm{rec}(1 + 1)) * \underline{x}) = ((1 + 1) * (\mathrm{rec}(1 + 1) * \underline{x})); \mathrm{eqSymmetry} \rhd (((1 + 1) * \mathrm{rec}(1 + 1)) * \underline{x}) = ((1 + 1) * (\mathrm{rec}(1 + 1) * \underline{x})) \gg ((1 + 1) * (\mathrm{rec}(1 + 1) * \underline{x})) = (((1{+}1)*\mathrm{rec}(1{+}1))*\underline{x}); \mathrm{Reciprocal} \rhd \dot\neg((1{+}1) = 0)n \gg ((1{+}1)*\mathrm{rec}(1{+}1)) = 1; \mathrm{eqMultiplication} \rhd ((1 + 1) * \mathrm{rec}(1 + 1)) = 1 \gg (((1 + 1) * \mathrm{rec}(1 + 1)) * \underline{x}) = (1 * \underline{x}); \mathrm{lemma\ times1Left} \gg (1 * \underline{x}) = \underline{x}; \mathrm{eqTransitivity5} \rhd ((\mathrm{rec}(1 + 1) * \underline{x}) + (\mathrm{rec}(1 + 1) * \underline{x})) = ((1 + 1) * (\mathrm{rec}(1 + 1) * \underline{x})) \rhd ((1 + 1) * (\mathrm{rec}(1 + 1) * \underline{x})) = (((1 + 1) * \mathrm{rec}(1 + 1)) * \underline{x}) \rhd (((1 + 1) * \mathrm{rec}(1 + 1)) * \underline{x}) = (1 * \underline{x}) \rhd (1 * \underline{x}) = \underline{x} \gg$

$((\mathrm{rec}(1+1) * \underline{x}) + (\mathrm{rec}(1+1) * \underline{x})) = \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{TwoHalves} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon ((\mathrm{rec}(1+1) * \underline{x}) + (\mathrm{rec}(1+1) * \underline{x})) = \underline{x}]$

$[\mathrm{TwoHalves} \stackrel{\mathrm{tex}}{\rightarrow}$ "TwoHalves"$]$

$[\mathrm{TwoHalves} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma (1/2)x+(1/2)x=x"$]$

ThreeThirds

$[\mathrm{ThreeThirds} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon 0 < 3 \gg \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{PositiveNonzero} \rhd \dot{\neg}\,(0 <= ((1+1)+1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = ((1+1)+1))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}\,(((1+1)+1) = 0)\mathrm{n}; \mathrm{ThreeWholes} \gg (((\mathrm{rec}((1+1)+1) * \underline{x}) + (\mathrm{rec}((1+1)+1) * \underline{x})) + (\mathrm{rec}((1+1)+1) * \underline{x})) = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{x})); \mathrm{timesAssociativity} \gg ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{x}) = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{x})); \mathrm{eqSymmetry} \rhd ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{x}) = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{x})) \gg (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{x})) = ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{x}); \mathrm{Reciprocal} \rhd \dot{\neg}\,(((1+1)+1) = 0)\mathrm{n} \gg (((1+1)+1) * \mathrm{rec}((1+1)+1)) = 1; \mathrm{eqMultiplication} \rhd (((1+1)+1) * \mathrm{rec}((1+1)+1)) = 1 \gg ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{x}) = (1 * \underline{x}); \mathrm{lemma\ times1Left} \gg (1 * \underline{x}) = \underline{x}; \mathrm{eqTransitivity5} \rhd (((\mathrm{rec}((1+1)+1) * \underline{x}) + (\mathrm{rec}((1+1)+1) * \underline{x})) + (\mathrm{rec}((1+1)+1) * \underline{x})) = (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{x})) \rhd (((1+1)+1) * (\mathrm{rec}((1+1)+1) * \underline{x})) = ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{x}) \rhd ((((1+1)+1) * \mathrm{rec}((1+1)+1)) * \underline{x}) = (1 * \underline{x}) \rhd (1 * \underline{x}) = \underline{x} \gg$
$(((\mathrm{rec}((1+1)+1) * \underline{x}) + (\mathrm{rec}((1+1)+1) * \underline{x})) + (\mathrm{rec}((1+1)+1) * \underline{x})) = \underline{x}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ThreeThirds} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall \underline{x}\colon (((\mathrm{rec}((1+1)+1) * \underline{x}) + (\mathrm{rec}((1+1)+1) * \underline{x})) + (\mathrm{rec}((1+1)+1) * \underline{x})) = \underline{x}]$

$[\mathrm{ThreeThirds} \stackrel{\mathrm{tex}}{\rightarrow}$ "ThreeThirds"$]$

$[\mathrm{ThreeThirds} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma (1/3)x+(1/3)x+(1/3)x=x"$]$

$-\mathrm{x} - \mathrm{y} = -(\mathrm{x} + \mathrm{y})$

$[-\mathrm{x} - \mathrm{y} = -(\mathrm{x} + \mathrm{y}) \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Times}(-1)\mathrm{Left} \gg ((-_{\mathrm{u}}1) * \underline{x}) = (-_{\mathrm{u}}\underline{x}); \mathrm{Times}(-1)\mathrm{Left} \gg ((-_{\mathrm{u}}1) * \underline{y}) = (-_{\mathrm{u}}\underline{y}); \mathrm{AddEquations} \rhd ((-_{\mathrm{u}}1) * \underline{x}) = (-_{\mathrm{u}}\underline{x}) \rhd ((-_{\mathrm{u}}1) * \underline{y}) = (-_{\mathrm{u}}\underline{y}) \gg (((-_{\mathrm{u}}1) * \underline{x}) + ((-_{\mathrm{u}}1) * \underline{y})) = ((-_{\mathrm{u}}\underline{x}) + (-_{\mathrm{u}}\underline{y})); \mathrm{eqSymmetry} \rhd (((-_{\mathrm{u}}1) * \underline{x}) + ((-_{\mathrm{u}}1) * \underline{y})) = ((-_{\mathrm{u}}\underline{x}) + (-_{\mathrm{u}}\underline{y})) \gg ((-_{\mathrm{u}}\underline{x}) + (-_{\mathrm{u}}\underline{y})) = (((-_{\mathrm{u}}1) * \underline{x}) + ((-_{\mathrm{u}}1) * \underline{y})); \mathrm{DistributionOut} \gg (((-_{\mathrm{u}}1) * \underline{x}) + ((-_{\mathrm{u}}1) * \underline{y})) = ((-_{\mathrm{u}}1) * (\underline{x} + \underline{y})); \mathrm{Times}(-1)\mathrm{Left} \gg ((-_{\mathrm{u}}1) * (\underline{x} + \underline{y})) = (-_{\mathrm{u}}(\underline{x} + \underline{y})); \mathrm{eqTransitivity4} \rhd ((-_{\mathrm{u}}\underline{x}) + (-_{\mathrm{u}}\underline{y})) = (((-_{\mathrm{u}}1) * \underline{x}) + ((-_{\mathrm{u}}1) * \underline{y})) \rhd (((-_{\mathrm{u}}1) * \underline{x}) + ((-_{\mathrm{u}}1) * \underline{y})) = ((-_{\mathrm{u}}1) * (\underline{x} + \underline{y})) \rhd ((-_{\mathrm{u}}1) * (\underline{x} + \underline{y})) = (-_{\mathrm{u}}(\underline{x} + \underline{y})) \gg ((-_{\mathrm{u}}\underline{x}) + (-_{\mathrm{u}}\underline{y})) = (-_{\mathrm{u}}(\underline{x} + \underline{y}))\rceil, \mathrm{p}_0, \mathsf{c})]$

$[-x-y=-(x+y) \stackrel{\rm stmt}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon ((-u\underline{x})+(-u\underline{y})) = (-u(\underline{x}+\underline{y}))]$

$[-x-y=-(x+y) \stackrel{\rm tex}{\rightarrow}$ "-x-y=-(x+y)"$]$

$[-x-y=-(x+y) \stackrel{\rm pyk}{\rightarrow}$ "lemma -x-y=-(x+y)"$]$

$-x*y=-(x*y)$

$[-x*y=-(x*y) \stackrel{\rm proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \mathrm{Times}(-1)\mathrm{Left} \gg ((-u1)*\underline{x}) = (-u\underline{x}); \mathrm{eqMultiplication} \rhd ((-u1)*\underline{x}) = (-u\underline{x}) \gg (((-u1)*\underline{x})*\underline{y}) = ((-u\underline{x})*\underline{y}); \mathrm{eqSymmetry} \rhd (((-u1)*\underline{x})*\underline{y}) = ((-u\underline{x})*\underline{y}) \gg ((-u\underline{x})*\underline{y}) = (((-u1)*\underline{x})*\underline{y}); \mathrm{timesAssociativity} \gg (((-u1)*\underline{x})*\underline{y}) = ((-u1)*(\underline{x}*\underline{y})); \mathrm{Times}(-1)\mathrm{Left} \gg ((-u1)*(\underline{x}*\underline{y})) = (-u(\underline{x}*\underline{y})); \mathrm{eqTransitivity4} \rhd ((-u\underline{x})*\underline{y}) = (((-u1)*\underline{x})*\underline{y}) \rhd (((-u1)*\underline{x})*\underline{y}) = ((-u1)*(\underline{x}*\underline{y})) \rhd ((-u1)*(\underline{x}*\underline{y})) = (-u(\underline{x}*\underline{y})) \gg ((-u\underline{x})*\underline{y}) = (-u(\underline{x}*\underline{y}))\rceil, p_0, c)]$

$[-x*y=-(x*y) \stackrel{\rm stmt}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon ((-u\underline{x})*\underline{y}) = (-u(\underline{x}*\underline{y}))]$

$[-x*y=-(x*y) \stackrel{\rm tex}{\rightarrow}$ "-x*y=-(x*y)"$]$

$[-x*y=-(x*y) \stackrel{\rm pyk}{\rightarrow}$ "lemma -x*y=-(x*y)"$]$

$-0=0$

$[-0=0 \stackrel{\rm proof}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \mathrm{Negative} \gg (0+(-u0)) = 0; \mathrm{plus0} \gg (0+0) = 0; \mathrm{UniqueNegative} \rhd (0+(-u0)) = 0 \rhd (0+0) = 0 \gg (-u0) = 0\rceil, p_0, c)]$

$[-0=0 \stackrel{\rm stmt}{\rightarrow} \mathrm{SystemQ} \vdash (-u0) = 0]$

$[-0=0 \stackrel{\rm tex}{\rightarrow}$ "-0=0"$]$

$[-0=0 \stackrel{\rm pyk}{\rightarrow}$ "lemma -0=0"$]$

SFsymmetry

[SFsymmetry $\stackrel{\rm tex}{\rightarrow}$ "SFsymmetry"]

[SFsymmetry $\stackrel{\rm pyk}{\rightarrow}$ "lemma sameFsymmetry"]

SFtransitivity

[SFtransitivity $\stackrel{\rm tex}{\rightarrow}$ "SFtransitivity"]

[SFtransitivity $\stackrel{\text{pyk}}{\to}$ “lemma sameFtransitivity”]

f2R(Plus)

[f2R(Plus) $\stackrel{\text{tex}}{\to}$ “f2R(Plus)”]

[f2R(Plus) $\stackrel{\text{pyk}}{\to}$ “lemma f2R(Plus)”]

f2R(Times)

[f2R(Times) $\stackrel{\text{tex}}{\to}$ “f2R(Times)”]

[f2R(Times) $\stackrel{\text{pyk}}{\to}$ “lemma f2R(Times)”]

<< TransitivityHelper(Q)

[<< TransitivityHelper(Q) $\stackrel{\text{tex}}{\to}$ “<<TransitivityHelper(Q)”]

[<< TransitivityHelper(Q) $\stackrel{\text{pyk}}{\to}$ “lemma <<TransitivityHelper(Q)”]

<< Transitivity

[<< Transitivity $\stackrel{\text{tex}}{\to}$ “<<Transitivity”]

[<< Transitivity $\stackrel{\text{pyk}}{\to}$ “lemma <<Transitivity”]

<<== Reflexivity

[<<== Reflexivity $\stackrel{\text{tex}}{\to}$ “<<==Reflexivity”]

[<<== Reflexivity $\stackrel{\text{pyk}}{\to}$ “lemma <<==Reflexivity”]

<<== AntisymmetryHelper(Q)

[<<== AntisymmetryHelper(Q) $\stackrel{\text{tex}}{\to}$ “<<==AntisymmetryHelper(Q)”]

[<<== AntisymmetryHelper(Q) $\stackrel{\text{pyk}}{\to}$ “lemma <<==AntisymmetryHelper(Q)”]

FromNot < f(Weak)(Helper)

[FromNot < f(Weak)(Helper) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\text{fx})\colon \forall (\text{fy})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \vdash \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \vdash \text{FromNegatedAnd} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \rhd \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \gg \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n}; \text{FromNegatedImply} \rhd \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n} \gg \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}((\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n}; \text{FirstConjunct} \rhd \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}((\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \gg \underline{\mathsf{n}} <= \underline{\mathsf{m}}; \text{SecondConjunct} \rhd \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}((\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \gg \dot{\neg}((\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n}; \text{ToLess} \rhd \dot{\neg}((\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n} \gg \dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) <= (\text{fx})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) = (\text{fx})[\underline{\mathsf{m}}])\text{n})\text{n})\text{n}; \text{JoinConjuncts} \rhd \underline{\mathsf{n}} <= \underline{\mathsf{m}} \rhd \dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) <= (\text{fx})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) = (\text{fx})[\underline{\mathsf{m}}])\text{n})\text{n})\text{n} \gg \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) <= (\text{fx})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) = (\text{fx})[\underline{\mathsf{m}}])\text{n})\text{n})\text{n})\text{n})\text{n}; \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\text{fx})\colon \forall (\text{fy})\colon \text{Ded} \rhd \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\text{fx})\colon \forall (\text{fy})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \vdash \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \vdash \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) <= (\text{fx})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) = (\text{fx})[\underline{\mathsf{m}}])\text{n})\text{n})\text{n})\text{n})\text{n} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) <= (\text{fx})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) = (\text{fx})[\underline{\mathsf{m}}])\text{n})\text{n})\text{n})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[FromNot < f(Weak)(Helper) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\text{fx})\colon \forall (\text{fy})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) <= (\text{fx})[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))) = (\text{fx})[\underline{\mathsf{m}}])\text{n})\text{n})\text{n})\text{n})\text{n}$]

[FromNot < f(Weak)(Helper) $\stackrel{\text{tex}}{\rightarrow}$ "FromNot<f(Weak)(Helper)"]

[FromNot < f(Weak)(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromNot<f(Weak) helper"]

FromNot < f(Weak)

[FromNot < f(Weak) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall (\epsilon)\colon \forall (\text{fx})\colon \forall (\text{fy})\colon \dot{\neg}(\forall_{\text{obj}}(\epsilon)\colon \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\text{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <= ((\text{fy})[\underline{\mathsf{m}}] + (-\text{u}(\epsilon))))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \vdash \text{Ded} \rhd \dot{\neg}(\forall_{\text{obj}}(\epsilon)\colon \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\text{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\text{fx})[\underline{\mathsf{m}}] <=$

$((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \mathrm{Ded} \rhd$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{MT} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{EEA} - \mathrm{negated} \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ \underline{(\epsilon)} \rhd$
$\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{A4} @ \underline{\mathsf{n}} \rhd$
$\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromNot} < \mathrm{f(Weak)(Helper)} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <=$
$((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)})) =$

$\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ExistMP} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{IntroExist} \mathbin{@} \underline{\mathsf{m}} \rhd \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNot} < \mathrm{f(Weak)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) <= \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))) = \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[FromNot < f(Weak) $\stackrel{\mathrm{tex}}{\rightarrow}$ “FromNot<f(Weak)”]

[FromNot < f(Weak) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma fromNot<f(Weak)”]

FromNot < f(Strong)(Helper2)

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon \forall\underline{\mathsf{u}}\colon \forall\underline{\mathsf{v}}\colon \dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}})) = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{x} <= |\mathrm{x}| \gg (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))|; \mathrm{leqLessTransitivity} \rhd (\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| \rhd \dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}})) <= (\mathrm{rec}((1+1)+1) * \underline{\mathsf{v}}) \Rightarrow$

$\dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{y})) = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{NumericalDifference} \gg$ $|(\underline{z}+(-\mathrm{u}\underline{u}))| = |(\underline{u}+(-\mathrm{u}\underline{z}))|; \mathrm{SubLessLeft} \rhd |(\underline{z}+(-\mathrm{u}\underline{u}))| =$ $|(\underline{u}+(-\mathrm{u}\underline{z}))| \rhd \dot{\neg}(|(\underline{z}+(-\mathrm{u}\underline{u}))| <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{z}+(-\mathrm{u}\underline{u}))| =$ $(\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|(\underline{u}+(-\mathrm{u}\underline{z}))| <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow$ $\dot{\neg}(\dot{\neg}(|(\underline{u}+(-\mathrm{u}\underline{z}))| = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{x} <= |\mathrm{x}| \gg (\underline{u}+(-\mathrm{u}\underline{z})) <= |(\underline{u}+$ $(-\mathrm{u}\underline{z}))|; \mathrm{leqLessTransitivity} \rhd (\underline{u}+(-\mathrm{u}\underline{z})) <= |(\underline{u}+(-\mathrm{u}\underline{z}))| \rhd \dot{\neg}(|(\underline{u}+(-\mathrm{u}\underline{z}))| <=$ $(\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{u}+(-\mathrm{u}\underline{z}))| = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}((\underline{u}+(-\mathrm{u}\underline{z})) <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{u}+(-\mathrm{u}\underline{z})) = (\mathrm{rec}((1+1)+$ $1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddEquations(Less)} \rhd \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{y})) <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow$ $\dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{y})) = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}((\underline{y}+(-\mathrm{u}\underline{u})) <=$ $(\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-\mathrm{u}\underline{u})) = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u}))) <= ((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow$ $\dot{\neg}(\dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u}))) = ((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*$ $\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{AddEquations(Less)} \rhd \dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u}))) <=$ $((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u}))) =$ $((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}((\underline{u}+(-\mathrm{u}\underline{z})) <=$ $(\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{u}+(-\mathrm{u}\underline{z})) = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) <=$ $(((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}((((\underline{x}+$ $(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) = (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*$ $\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{insertTwoMiddleTerms(Sum)} \gg (\underline{x}+(-\mathrm{u}\underline{z})) =$ $(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))); \mathrm{eqSymmetry} \rhd (\underline{x}+(-\mathrm{u}\underline{z})) =$ $(((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) \gg (((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+$ $(-\mathrm{u}\underline{z}))) = (\underline{x}+(-\mathrm{u}\underline{z})); \mathrm{SubLessLeft} \rhd (((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) =$ $(\underline{x}+(-\mathrm{u}\underline{z})) \rhd \dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) <=$ $(((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow$ $\dot{\neg}(\dot{\neg}((((\underline{x}+(-\mathrm{u}\underline{y}))+(\underline{y}+(-\mathrm{u}\underline{u})))+(\underline{u}+(-\mathrm{u}\underline{z}))) = (((\mathrm{rec}((1+1)+1)*\underline{v})+$ $(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <=$ $(((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow$ $\dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+$ $1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{ThreeThirds} \gg (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*$ $\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) = \underline{v}; \mathrm{SubLessRight} \rhd (((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+$ $1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) = \underline{v} \rhd \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= (((\mathrm{rec}((1+1)+$ $1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) =$ $(((\mathrm{rec}((1+1)+1)*\underline{v})+(\mathrm{rec}((1+1)+1)*\underline{v}))+(\mathrm{rec}((1+1)+1)*\underline{v})))\mathrm{n})\mathrm{n})\mathrm{n} \gg$ $\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$ $\forall\underline{x}\colon\forall\underline{y}\colon\forall\underline{z}\colon\forall\underline{u}\colon\forall\underline{v}\colon \dot{\neg}(|(\underline{x}+(-\mathrm{u}\underline{y}))| <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow$ $\dot{\neg}(\dot{\neg}(|(\underline{x}+(-\mathrm{u}\underline{y}))| = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(|(\underline{z}+(-\mathrm{u}\underline{u}))| <=$ $(\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{z}+(-\mathrm{u}\underline{u}))| = (\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ $\dot{\neg}((\underline{y}+(-\mathrm{u}\underline{u})) <= (\mathrm{rec}((1+1)+1)*\underline{v}) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y}+(-\mathrm{u}\underline{u})) =$ $(\mathrm{rec}((1+1)+1)*\underline{v}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x}+(-\mathrm{u}\underline{z})) = \underline{v})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{tex}}{\rightarrow}$ "FromNot<f(Strong)(Helper2)"]

$[\mathrm{FromNot} < \mathrm{f(Strong)(Helper2)} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fromNot<f(Strong) helper2"]

FromNot < f(Strong)(Helper)

[FromNot < f(Strong)(Helper) $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash$
$\forall(\text{v1}){:}\,\forall(\text{v2}){:}\,\forall \mathsf{m}{:}\,\forall(\text{n1}){:}\,\forall(\text{n2}){:}\,\forall(\epsilon){:}\,\forall(\text{fx}){:}\,\forall(\text{fy}){:}\,\dot\neg\,(0 <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow \dot\neg\,((\text{n1}) <= (\text{n2}) \Rightarrow$
$\dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) <= (\text{fx})[(\text{n2})] \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] +$
$(-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) = (\text{fx})[(\text{n2})])\text{n})\text{n})\text{n})\text{n})\text{n} \vdash \forall_{\text{obj}}(\text{v1}){:}\,\forall_{\text{obj}}(\text{v2}){:}\,\dot\neg\,(0 <=$
$(\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow (\text{n1}) <= (\text{v1}) \Rightarrow$
$(\text{n1}) <= (\text{v2}) \Rightarrow \dot\neg\,(\dot\neg\,(|((\text{fx})[(\text{v1})] + (-\text{u}(\text{fx})[(\text{v2})]))| <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((\text{fx})[(\text{v1})] + (-\text{u}(\text{fx})[(\text{v2})]))| = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((\text{fy})[(\text{v1})] + (-\text{u}(\text{fy})[(\text{v2})]))| <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((\text{fy})[(\text{v1})] + (-\text{u}(\text{fy})[(\text{v2})]))| = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n})\text{n})\text{n} \vdash$
$\dot\neg\,(0 <= (\epsilon) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon))\text{n})\text{n})\text{n} \vdash (\text{n2}) <= \mathsf{m} \vdash 0 < 3 \gg \dot\neg\,(0 <=$
$((1+1)+1) \Rightarrow \dot\neg\,(\dot\neg\,(0 = ((1+1)+1))\text{n})\text{n})\text{n};$ PositiveInverted $\rhd \dot\neg\,(0 <=$
$((1+1)+1) \Rightarrow \dot\neg\,(\dot\neg\,(0 = ((1+1)+1))\text{n})\text{n})\text{n} \gg \dot\neg\,(0 <= \text{rec}((1+1)+1) \Rightarrow$
$\dot\neg\,(\dot\neg\,(0 = \text{rec}((1+1)+1))\text{n})\text{n})\text{n};$ PositiveFactors $\rhd \dot\neg\,(0 <= \text{rec}((1+1)+1) \Rightarrow$
$\dot\neg\,(\dot\neg\,(0 = \text{rec}((1+1)+1))\text{n})\text{n})\text{n} \rhd \dot\neg\,(0 <= (\epsilon) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon))\text{n})\text{n})\text{n} \gg \dot\neg\,(0 <=$
$(\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n};$ MP $\rhd \dot\neg\,(0 <=$
$(\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow \dot\neg\,((\text{n1}) <=$
$(\text{n2}) \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) <= (\text{fx})[(\text{n2})] \Rightarrow$
$\dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) = (\text{fx})[(\text{n2})])\text{n})\text{n})\text{n})\text{n})\text{n} \rhd \dot\neg\,(0 <=$
$(\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \gg \dot\neg\,((\text{n1}) <=$
$(\text{n2}) \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) <= (\text{fx})[(\text{n2})] \Rightarrow$
$\dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) =$
$(\text{fx})[(\text{n2})])\text{n})\text{n})\text{n})\text{n})\text{n};$ FirstConjunct $\rhd \dot\neg\,((\text{n1}) <= (\text{n2}) \Rightarrow$
$\dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) <= (\text{fx})[(\text{n2})] \Rightarrow$
$\dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) = (\text{fx})[(\text{n2})])\text{n})\text{n})\text{n})\text{n})\text{n} \gg (\text{n1}) <=$
$(\text{n2});$ SecondConjunct $\rhd \dot\neg\,((\text{n1}) <= (\text{n2}) \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+$
$1)+1)*(\epsilon)))) <= (\text{fx})[(\text{n2})] \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) =$
$(\text{fx})[(\text{n2})])\text{n})\text{n})\text{n})\text{n})\text{n} \gg \dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) <=$
$(\text{fx})[(\text{n2})] \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) =$
$(\text{fx})[(\text{n2})])\text{n})\text{n})\text{n};$ SwitchTerms$(\text{x} - \text{y} < \text{z}) \rhd \dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+$
$1)*(\epsilon)))) <= (\text{fx})[(\text{n2})] \Rightarrow \dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{rec}((1+1)+1)*(\epsilon)))) =$
$(\text{fx})[(\text{n2})])\text{n})\text{n})\text{n} \gg \dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{fx})[(\text{n2})])) <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg\,(\dot\neg\,(((\text{fy})[(\text{n2})] + (-\text{u}(\text{fx})[(\text{n2})])) = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n};$ leqTransitivity $\rhd$
$(\text{n1}) <= (\text{n2}) \rhd (\text{n2}) <= \mathsf{m} \gg (\text{n1}) <= \mathsf{m};$ A4 @ $\mathsf{m} \rhd \forall_{\text{obj}}(\text{v1}){:}\,\forall_{\text{obj}}(\text{v2}){:}\,\dot\neg\,(0 <=$
$(\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow (\text{n1}) <= (\text{v1}) \Rightarrow$
$(\text{n1}) <= (\text{v2}) \Rightarrow \dot\neg\,(\dot\neg\,(|((\text{fx})[(\text{v1})] + (-\text{u}(\text{fx})[(\text{v2})]))| <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((\text{fx})[(\text{v1})] + (-\text{u}(\text{fx})[(\text{v2})]))| = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((\text{fy})[(\text{v1})] + (-\text{u}(\text{fy})[(\text{v2})]))| <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((\text{fy})[(\text{v1})] + (-\text{u}(\text{fy})[(\text{v2})]))| = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n})\text{n})\text{n} \gg$
$\forall_{\text{obj}}(\text{v2}){:}\,\dot\neg\,(0 <= (\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow$
$(\text{n1}) <= \mathsf{m} \Rightarrow (\text{n1}) <= (\text{v2}) \Rightarrow \dot\neg\,(\dot\neg\,(|((\text{fx})[\mathsf{m}] + (-\text{u}(\text{fx})[(\text{v2})]))| <=$
$(\text{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot\neg\,(\dot\neg\,(|((\text{fx})[\mathsf{m}] + (-\text{u}(\text{fx})[(\text{v2})]))| =$
$(\text{rec}((1+1)+1)*(\epsilon)))\text{n})\text{n})\text{n} \Rightarrow \dot\neg\,(\dot\neg\,(|((\text{fy})[\mathsf{m}] + (-\text{u}(\text{fy})[(\text{v2})]))| <=$

(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n; A4 @ (n2) ▷ $\forall_{obj}$(v2): ∸ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= m ⇒
(n1) <= (v2) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fx)[m] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫ ∸ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= m ⇒
(n1) <= (n2) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] +
(−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n; MP3 ▷ ∸ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= m ⇒
(n1) <= (n2) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ▷ ∸ (0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ▷ (n1) <=
m ▷ (n1) <= (n2) ≫ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n)n)n; FirstConjunct ▷ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫ ∸ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n; SecondConjunct ▷ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫
∸ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n; FromNot <
f(Strong)(Helper2) ▷ ∸ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ▷ ∸ (|((fx)[m] +
(−u(fx)[(n2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (|((fx)[m]+(−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ▷ ∸ (((fy)[(n2)] + (−u(fx)[(n2)])) <= (rec((1 + 1) +
1) ∗ (ϵ)) ⇒ ∸ (∸ (((fy)[(n2)] + (−u(fx)[(n2)])) = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ≫
∸ (((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸ (∸ (((fy)[m] + (−u(fx)[m])) =
(ϵ))n)n)n; ∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): Ded ▷
∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∸ (0 <= (rec((1+1)+1)∗(ϵ)) ⇒
∸ (∸ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸ ((n1) <= (n2) ⇒
∸ (∸ (((fy)[(n2)]+(−u(rec((1+1)+1)∗(ϵ)))) <= (fx)[(n2)] ⇒ ∸ (∸ (((fy)[(n2)]+
(−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n ⊢ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ∸ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒

$(n1) <= (v2) \Rightarrow \neg(\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \neg(\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n)n)n \vdash \neg(0 <= (\epsilon) \Rightarrow \neg(\neg(0 = (\epsilon))n)n)n \vdash (n2) <= \mathsf{m} \vdash \neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) <= (\epsilon) \Rightarrow \neg(\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) = (\epsilon))n)n)n \gg \neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \neg((n1) <= (n2) \Rightarrow \neg(\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \neg(\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n \Rightarrow \forall_{obj}(v1){:}\, \forall_{obj}(v2){:}\, \neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \neg(\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \neg(\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n)n)n \Rightarrow \neg(0 <= (\epsilon) \Rightarrow \neg(\neg(0 = (\epsilon))n)n)n \Rightarrow (n2) <= \mathsf{m} \Rightarrow \neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) <= (\epsilon) \Rightarrow \neg(\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) = (\epsilon))n)n)n\rceil, p_0, \mathsf{c})]$

$[FromNot < f(Strong)(Helper) \stackrel{stmt}{\rightarrow} SystemQ \vdash \forall(v1){:}\, \forall(v2){:}\, \forall\mathsf{m}{:}\, \forall(n1){:}\, \forall(n2){:}\, \forall(\epsilon){:}\, \forall(fx){:}\, \forall(fy){:}\, \neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \neg((n1) <= (n2) \Rightarrow \neg(\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) <= (fx)[(n2)] \Rightarrow \neg(\neg(((fy)[(n2)] + (-u(rec((1+1)+1) * (\epsilon)))) = (fx)[(n2)])n)n)n)n)n \Rightarrow \forall_{obj}(v1){:}\, \forall_{obj}(v2){:}\, \neg(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \neg(\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \neg(\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \neg(\neg(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n)n)n \Rightarrow \neg(0 <= (\epsilon) \Rightarrow \neg(\neg(0 = (\epsilon))n)n)n \Rightarrow (n2) <= \mathsf{m} \Rightarrow \neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) <= (\epsilon) \Rightarrow \neg(\neg(((fy)[\mathsf{m}] + (-u(fx)[\mathsf{m}])) = (\epsilon))n)n)n]$

$[FromNot < f(Strong)(Helper) \stackrel{tex}{\rightarrow}$ "FromNot<f(Strong)(Helper)"]

$[FromNot < f(Strong)(Helper) \stackrel{pyk}{\rightarrow}$ "lemma fromNot<f(Strong) helper"]

FromNot < f(Strong)

$[FromNot < f(Strong) \stackrel{proof}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall(v1){:}\, \forall(v2){:}\, \forall\mathsf{m}{:}\, \forall(n2){:}\, \forall(\epsilon){:}\, \forall(fx){:}\, \forall(fy){:}\, \neg(\neg(\forall_{obj}(\epsilon){:}\, \neg(\neg(\forall_{obj}\mathsf{n}{:}\, \neg(\forall_{obj}\mathsf{m}{:}\, \neg(\neg(0 <= (\epsilon) \Rightarrow \neg(\neg(0 = (\epsilon))n)n)n \Rightarrow \neg(\mathsf{n} <= \mathsf{m} \Rightarrow (fx)[\mathsf{m}] <= ((fy)[\mathsf{m}] + (-u(\epsilon))))n)n)n)n)n)n)n \vdash FromNot < f(Weak) \rhd \neg(\neg(\forall_{obj}(\epsilon){:}\, \neg(\neg(\forall_{obj}\mathsf{n}{:}\, \neg(\forall_{obj}\mathsf{m}{:}\, \neg(\neg(0 <= (\epsilon) \Rightarrow \neg(\neg(0 = (\epsilon))n)n)n \Rightarrow \neg(\mathsf{n} <= \mathsf{m} \Rightarrow (fx)[\mathsf{m}] <= ((fy)[\mathsf{m}] + (-u(\epsilon))))n)n)n)n)n)n)n \gg \forall_{obj}(\epsilon){:}\, \forall_{obj}\mathsf{n}{:}\, \neg(\forall_{obj}(n2){:}\, \neg(\neg(0 <= (\epsilon) \Rightarrow \neg(\neg(0 = (\epsilon))n)n)n \Rightarrow \neg(\mathsf{n} <= (n2) \Rightarrow \neg(\neg(((fy)[(n2)] + (-u(\epsilon))) <= (fx)[(n2)] \Rightarrow$

$\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(ϵ))) = (fx)[(n2)])n)n)n)n)n)n)n; A4 @(rec((1 + 1) + 1) ∗ (ϵ)) ▷ $\forall_{obj}$(ϵ): $\forall_{obj}$n: $\dot{\neg}$ ($\forall_{obj}$(n2): $\dot{\neg}$ ($\dot{\neg}$ (0 <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (ϵ))n)n)n ⇒ $\dot{\neg}$ (n <= (n2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(ϵ))) <= (fx)[(n2)] ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(ϵ))) = (fx)[(n2)])n)n)n)n)n)n)n ≫ $\forall_{obj}$n: $\dot{\neg}$ ($\forall_{obj}$(n2): $\dot{\neg}$ ($\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ (n <= (n2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n)n)n; A4 @ (n1) ▷ $\forall_{obj}$n: $\dot{\neg}$ ($\forall_{obj}$(n2): $\dot{\neg}$ ($\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ (n <= (n2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n)n)n ≫ $\dot{\neg}$ ($\forall_{obj}$(n2): $\dot{\neg}$ ($\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ ((n1) <= (n2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)]+(−u(rec((1+1)+1)∗(ϵ)))) <= (fx)[(n2)] ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)]+ (−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n)n)n; Cauchy(2) ≫ $\forall_{obj}$(ϵ): $\dot{\neg}$ ($\forall_{obj}$(n1): $\dot{\neg}$ ($\forall_{obj}$(v1): $\forall_{obj}$(v2): $\dot{\neg}$ (0 <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (ϵ))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n)n)n; A4 @(rec((1 + 1) + 1) ∗ (ϵ)) ▷ $\forall_{obj}$(ϵ): $\dot{\neg}$ ($\forall_{obj}$(n1): $\dot{\neg}$ ($\forall_{obj}$(v1): $\forall_{obj}$(v2): $\dot{\neg}$ (0 <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (ϵ))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (ϵ))n)n)n ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (ϵ))n)n)n)n)n)n)n ≫ $\dot{\neg}$ ($\forall_{obj}$(n1): $\dot{\neg}$ ($\forall_{obj}$(v1): $\forall_{obj}$(v2): $\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n)n)n; FromNot < f(Strong)(Helper) ≫ $\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ ((n1) <= (n2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n ⇒ $\forall_{obj}$(v1): $\forall_{obj}$(v2): $\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (|((fy)[(v1)]+(−u(fy)[(v2)]))| = (rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⇒ $\dot{\neg}$ (0 <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ $\dot{\neg}$ (((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n; ExistMP2 ▷ $\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ $\dot{\neg}$ ((n1) <= (n2) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) <= (fx)[(n2)] ⇒ $\dot{\neg}$ ($\dot{\neg}$ (((fy)[(n2)] + (−u(rec((1 + 1) + 1) ∗ (ϵ)))) = (fx)[(n2)])n)n)n)n)n ⇒ $\forall_{obj}$(v1): $\forall_{obj}$(v2): $\dot{\neg}$ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ $\dot{\neg}$ ($\dot{\neg}$ (0 =

$(\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})]+(-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n2})\colon \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{n1}) <= (\mathrm{n2}) \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*(\epsilon)))) <= (\mathrm{fx})[(\mathrm{n2})] \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[(\mathrm{n2})] + (-\mathrm{u}(\mathrm{rec}((1+1)+1)*(\epsilon)))) = (\mathrm{fx})[(\mathrm{n2})])\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{n1})\colon \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1})\colon \forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(0 <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n1}) <= (\mathrm{v1}) \Rightarrow (\mathrm{n1}) <= (\mathrm{v2}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})]+(-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fx})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| <= (\mathrm{rec}((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fy})[(\mathrm{v1})] + (-\mathrm{u}(\mathrm{fy})[(\mathrm{v2})]))| = (\mathrm{rec}((1+1)+1)*(\epsilon)))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNot} < \mathrm{f}(\mathrm{Strong}) \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall(\mathrm{v1})\colon \forall(\mathrm{v2})\colon \forall\underline{\mathsf{m}}\colon \forall(\mathrm{n2})\colon \forall(\epsilon)\colon \forall(\mathrm{fx})\colon \forall(\mathrm{fy})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{n2}) <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fx})[\underline{\mathsf{m}}])) = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNot} < \mathrm{f}(\mathrm{Strong}) \overset{\mathrm{tex}}{\to}$ "FromNot<f(Strong)"]

$[\mathrm{FromNot} < \mathrm{f}(\mathrm{Strong}) \overset{\mathrm{pyk}}{\to}$ "lemma fromNot<f(Strong)"]

fromNotSameF(Strongest)(Helper2)

$[\mathrm{fromNotSameF}(\mathrm{Strongest})(\mathrm{Helper2}) \overset{\mathrm{proof}}{\to} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon \forall\underline{\mathsf{u}}\colon \forall\underline{\mathsf{v}}\colon \dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| <= (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| <= (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{z}} + (-\mathrm{u}\underline{\mathsf{u}}))| = (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{v}} <= |(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{u}}))| \vdash \mathrm{NumericalDifference} \gg |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = |(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))|; \mathrm{SubLessLeft} \rhd |(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = |(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))| \rhd \dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| <= (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}} + (-\mathrm{u}\underline{\mathsf{y}}))| = (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))| <= (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))| = (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessNegated} \rhd \dot{\neg}(|(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))| <= (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}}))| = (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}(\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}})) <= (-\mathrm{u}|(\underline{\mathsf{y}} + (-\mathrm{u}\underline{\mathsf{x}}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-\mathrm{u}(\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}})) = (-\mathrm{u}|(\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}}))|))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{LessNegated} \rhd \dot{\neg}(|(\underline{\mathsf{z}}+(-\mathrm{u}\underline{\mathsf{u}}))| <= (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{z}}+(-\mathrm{u}\underline{\mathsf{u}}))| = (\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((-\mathrm{u}(\mathrm{rec}((1+1)+1)*\underline{\mathsf{v}})) <=$

$(-u|(\underline{z} + (-u\underline{u}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v})) =$
$(-u|(\underline{z} + (-u\underline{u}))|))n)n)n; \mathrm{AddEquations}(\mathrm{LeqLess}) \rhd \underline{v} <=$
$|(\underline{y} + (-u\underline{u}))| \rhd \dot{\neg}((-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v})) <= (-u|(\underline{y} + (-u\underline{x}))|) \Rightarrow$
$\dot{\neg}(\dot{\neg}((-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v})) = (-u|(\underline{y} + (-u\underline{x}))|))n)n)n \gg$
$\dot{\neg}((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) <= (|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) \Rightarrow$
$\dot{\neg}(\dot{\neg}((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) = (|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} +$
$(-u\underline{x}))|)))n)n)n; \mathrm{AddEquations}(\mathrm{Less}) \rhd \dot{\neg}((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) <=$
$(|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) \Rightarrow \dot{\neg}(\dot{\neg}((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) =$
$(|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)))n)n)n \rhd \dot{\neg}((-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v})) <=$
$(-u|(\underline{z}+(-u\underline{u}))|) \Rightarrow \dot{\neg}(\dot{\neg}((-u(\mathrm{rec}((1+1)+1)\ast\underline{v})) = (-u|(\underline{z}+(-u\underline{u}))|))n)n)n \gg$
$\dot{\neg}(((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) <=$
$(((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) =$
$(((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} +$
$(-u\underline{u}))|)))n)n)n; \mathrm{insertTwoMiddleTerms}(\mathrm{Numerical}) \gg |(\underline{y} + (-u\underline{u}))| <=$
$((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|); \mathrm{plusAssociativity} \gg$
$((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) =$
$(|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)); \mathrm{plusCommutativity} \gg$
$(|(\underline{y}+(-u\underline{x}))|+(|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)) = ((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+$
$|(\underline{y} + (-u\underline{x}))|); \mathrm{eqTransitivity} \rhd ((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) =$
$(|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)) \rhd (|(\underline{y} + (-u\underline{x}))| + (|(\underline{x} +$
$(-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|)) = ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{y} + (-u\underline{x}))|) \gg$
$((|(\underline{y}+(-u\underline{x}))|+|(\underline{x}+(-u\underline{z}))|)+|(\underline{z}+(-u\underline{u}))|) = ((|(\underline{x}+(-u\underline{z}))|+|(\underline{z}+(-u\underline{u}))|)+$
$|(\underline{y} + (-u\underline{x}))|); \mathrm{subLeqRight} \rhd ((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) =$
$((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{y} + (-u\underline{x}))|) \rhd |(\underline{y} + (-u\underline{u}))| <=$
$((|(\underline{y} + (-u\underline{x}))| + |(\underline{x} + (-u\underline{z}))|) + |(\underline{z} + (-u\underline{u}))|) \gg |(\underline{y} + (-u\underline{u}))| <=$
$((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{y} + (-u\underline{x}))|); \mathrm{PositiveToLeft}(\mathrm{Leq}) \rhd |(\underline{y} +$
$(-u\underline{u}))| <= ((|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) + |(\underline{y} + (-u\underline{x}))|) \gg (|(\underline{y} + (-u\underline{u}))| +$
$(-u|(\underline{y} + (-u\underline{x}))|)) <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|); \mathrm{PositiveToLeft}(\mathrm{Leq}) \rhd$
$(|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) <= (|(\underline{x} + (-u\underline{z}))| + |(\underline{z} + (-u\underline{u}))|) \gg$
$((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|)) <= |(\underline{x} +$
$(-u\underline{z}))|; \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 +$
$1) + 1) \ast \underline{v}))) <= ((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) =$
$((|(\underline{y} + (-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|)))n)n)n \rhd ((|(\underline{y} +$
$(-u\underline{u}))| + (-u|(\underline{y} + (-u\underline{x}))|)) + (-u|(\underline{z} + (-u\underline{u}))|)) <= |(\underline{x} + (-u\underline{z}))| \gg$
$\dot{\neg}(((\underline{v}+(-u(\mathrm{rec}((1+1)+1)\ast\underline{v})))+(-u(\mathrm{rec}((1+1)+1)\ast\underline{v}))) <= |(\underline{x}+(-u\underline{z}))| \Rightarrow$
$\dot{\neg}(\dot{\neg}(((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) =$
$|(\underline{x} + (-u\underline{z}))|)n)n)n; \mathrm{ThreeThirds} \gg$
$(((\mathrm{rec}((1 + 1) + 1) \ast \underline{v}) + (\mathrm{rec}((1 + 1) + 1) \ast \underline{v})) + (\mathrm{rec}((1 + 1) + 1) \ast \underline{v})) =$
$\underline{v}; \mathrm{PositiveToRight}(\mathrm{Eq}) \rhd (((\mathrm{rec}((1+1)+1)\ast\underline{v})+(\mathrm{rec}((1+1)+1)\ast\underline{v}))+(\mathrm{rec}((1+$
$1)+1)\ast\underline{v})) = \underline{v} \gg ((\mathrm{rec}((1+1)+1)\ast\underline{v})+(\mathrm{rec}((1+1)+1)\ast\underline{v})) = (\underline{v}+(-u(\mathrm{rec}((1+$
$1)+1)\ast\underline{v}))); \mathrm{PositiveToRight}(\mathrm{Eq}) \rhd ((\mathrm{rec}((1+1)+1)\ast\underline{v})+(\mathrm{rec}((1+1)+1)\ast\underline{v})) =$
$(\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) \gg (\mathrm{rec}((1 + 1) + 1) \ast \underline{v}) = ((\underline{v} + (-u(\mathrm{rec}((1 +$
$1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))); \mathrm{eqSymmetry} \rhd (\mathrm{rec}((1 + 1) + 1) \ast \underline{v}) =$
$((\underline{v} + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) + (-u(\mathrm{rec}((1 + 1) + 1) \ast \underline{v}))) \gg$

((v + (−u(rec((1 + 1) + 1) ∗ v))) + (−u(rec((1 + 1) + 1) ∗ v))) = (rec((1 + 1) + 1) ∗ v); SubLessLeft ⊳ ((v + (−u(rec((1 + 1) + 1) ∗ v))) + (−u(rec((1 + 1) + 1) ∗ v))) = (rec((1 + 1) + 1) ∗ v) ⊳ ¬̇(((v + (−u(rec((1 + 1) + 1) ∗ v))) + (−u(rec((1 + 1) + 1) ∗ v))) <= |(x + (−uz))| ⇒ ¬̇(¬̇(((v + (−u(rec((1 + 1) + 1) ∗ v))) + (−u(rec((1 + 1) + 1) ∗ v))) = |(x + (−uz))|)n)n)n ≫ ¬̇((rec((1 + 1) + 1) ∗ v) <= |(x + (−uz))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ v) = |(x + (−uz))|)n)n)n⌉, p_0, c)]

[fromNotSameF(Strongest)(Helper2) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ ⊢ ∀x: ∀y: ∀z: ∀u: ∀v: ¬̇(|(x + (−uy))| <= (rec((1 + 1) + 1) ∗ v) ⇒ ¬̇(¬̇(|(x + (−uy))| = (rec((1 + 1) + 1) ∗ v))n)n)n ⊢ ¬̇(|(z + (−uu))| <= (rec((1 + 1) + 1) ∗ v) ⇒ ¬̇(¬̇(|(z + (−uu))| = (rec((1 + 1) + 1) ∗ v))n)n)n ⊢ v <= |(y + (−uu))| ⊢ ¬̇((rec((1 + 1) + 1) ∗ v) <= |(x + (−uz))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ v) = |(x + (−uz))|)n)n)n]

[fromNotSameF(Strongest)(Helper2) $\stackrel{\text{tex}}{\rightarrow}$ "fromNotSameF(Strongest)(Helper2)"]

[fromNotSameF(Strongest)(Helper2) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma fromNotSameF(Strongest) helper2"]

fromNotSameF(Strongest)(Helper)

[fromNotSameF(Strongest)(Helper) $\stackrel{\text{proof}}{\rightarrow}$ λc.λx.$\mathcal{P}$(⌈SystemQ ⊢ ∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\text{obj}}$(v1): $\forall_{\text{obj}}$(v2): ¬̇(0 <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)]+(−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⊢ (n2) <= m ⊢ FromNegated(2 ∗ Imply) ⊳ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ≫ ¬̇(¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇((n1) <= (n2))n)n ⇒ ¬̇(¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n; FirstConjunct ⊳ ¬̇(¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇((n1) <= (n2))n)n ⇒ ¬̇(¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n ≫ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇((n1) <= (n2))n)n; FirstConjunct ⊳ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇((n1) <= (n2))n)n ≫ ¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n; SecondConjunct ⊳ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇((n1) <= (n2))n)n ≫ (n1) <= (n2); SecondConjunct ⊳ ¬̇(¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒ ¬̇((n1) <= (n2))n)n ⇒ ¬̇(¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| =

(ϵ))n)n)n)n)n)n ≫ ¬̇ (¬̇ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇ (¬̇ (|((fx)[(n2)] + (−u(fy)[(n2)]))| =
(ϵ))n)n)n)n; fromNotLess ⊳ ¬̇ (¬̇ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇ (¬̇ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ≫ (ϵ) <=
|((fx)[(n2)] + (−u(fy)[(n2)]))|; A4 @ m ⊳ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇ (¬̇ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ ¬̇ (¬̇ (|((fx)[(v1)]+(−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒
¬̇ (¬̇ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇ (¬̇ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇ (¬̇ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫
$\forall_{obj}$(v2): ¬̇ (0 <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇ (¬̇ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒
(n1) <= m ⇒ (n1) <= (v2) ⇒ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n; A4 @ (n2) ⊳ $\forall_{obj}$(v2): ¬̇ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇ (¬̇ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= m ⇒
(n1) <= (v2) ⇒ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫ ¬̇ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇ (¬̇ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= m ⇒
(n1) <= (n2) ⇒ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fy)[m] +
(−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n; 0 < 1/3 ≫ ¬̇ (0 <=
rec((1 + 1) + 1) ⇒ ¬̇ (¬̇ (0 = rec((1 + 1) + 1))n)n)n; PositiveFactors ⊳ ¬̇ (0 <=
rec((1 + 1) + 1) ⇒ ¬̇ (¬̇ (0 = rec((1 + 1) + 1))n)n)n ⊳ ¬̇ (0 <= (ϵ) ⇒ ¬̇ (¬̇ (0 =
(ϵ))n)n)n ≫ ¬̇ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n; leqTransitivity ⊳ (n1) <= (n2) ⊳ (n2) <= m ≫
(n1) <= m; MP3 ⊳ ¬̇ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= m ⇒ (n1) <= (n2) ⇒
¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⊳ ¬̇ (0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⊳ (n1) <=
m ⊳ (n1) <= (n2) ≫ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n)n)n; FirstConjunct ⊳ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇ (¬̇ (|((fy)[m] + (−u(fy)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫ ¬̇ (|((fx)[m] + (−u(fx)[(n2)]))| <=

(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n; SecondConjunct ⊳ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fx)[m] + (−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ≫ ∸ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| =
(rec((1+1)+1)∗(ϵ)))n)n)n; fromNotSameF(Strongest)(Helper2) ⊳ ∸ (|((fx)[m]+
(−u(fx)[(n2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (|((fx)[m]+(−u(fx)[(n2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⊳ ∸ (|((fy)[m] + (−u(fy)[(n2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (|((fy)[m] + (−u(fy)[(n2)]))| = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n ⊳ (ϵ) <= |((fx)[(n2)]+(−u(fy)[(n2)]))| ≫ ∸ ((rec((1+1)+1)∗(ϵ)) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ∸ (∸ ((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] +
(−u(fy)[m]))|)n)n)n; ∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∸ (∸ (0 <=
(ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒
∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| =
(ϵ))n)n)n)n ⊢ $\forall_{\text{obj}}$(v1): $\forall_{\text{obj}}$(v2): ∸ (0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒ (n1) <= (v2) ⇒
∸ (∸ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸ (∸ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⊢
FromNegatedImply ⊳ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ (n1) <= (n2) ⇒
∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| =
(ϵ))n)n)n)n ≫ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒ ∸ (∸ ((n1) <= (n2) ⇒
∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒ ∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| =
(ϵ))n)n)n)n)n)n; FirstConjunct ⊳ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ⇒
∸ (∸ ((n1) <= (n2) ⇒ ∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n ≫ ∸ (0 <= (ϵ) ⇒
∸ (∸ (0 = (ϵ))n)n)n; 0 < 1/3 ≫ ∸ (0 <= rec((1 + 1) + 1) ⇒ ∸ (∸ (0 =
rec((1 + 1) + 1))n)n)n; PositiveFactors ⊳ ∸ (0 <= rec((1 + 1) + 1) ⇒ ∸ (∸ (0 =
rec((1 + 1) + 1))n)n)n ⊳ ∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 = (ϵ))n)n)n ≫ ∸ (0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ∸ (∸ (0 = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n; ∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): Ded ⊳
∀(v1): ∀(v2): ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 =
(ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\text{obj}}$(v1): $\forall_{\text{obj}}$(v2): ∸ (0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ∸ (∸ (0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ ∸ (∸ (|((fx)[(v1)] + (−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒
∸ (∸ (|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
∸ (∸ (|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
∸ (∸ (|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⊢ (n2) <=
m ⊢ ∸ ((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸ (∸ ((rec((1 +
1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ≫ ∸ (∸ (0 <= (ϵ) ⇒ ∸ (∸ (0 =
(ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
∸ (∸ (|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⇒ $\forall_{\text{obj}}$(v1): $\forall_{\text{obj}}$(v2): ∸ (0 <=

(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)]+(−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⇒
(n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒
¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n; Ded ⊳
∀(v1): ∀(v2): ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⊢ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <=
(v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⊢ ¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ≫ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒
(n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⇒ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)]+(−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⇒
¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗
(ϵ)))n)n)n; JoinConjuncts(2conditions) ⊳ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⇒ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <=
(v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n ⇒ ¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⊳ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⇒ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)]+(−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇(¬̇(|((fy)[(v1)]+(−u(fy)[(v2)]))| = (rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⇒ (n2) <=
m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 +
1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ≫ ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 =
(ϵ))n)n)n ⇒ (n1) <= (n2) ⇒ ¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| <= (ϵ) ⇒
¬̇(¬̇(|((fx)[(n2)] + (−u(fy)[(n2)]))| = (ϵ))n)n)n)n ⇒ $\forall_{\mathrm{obj}}$(v1): $\forall_{\mathrm{obj}}$(v2): ¬̇(0 <=

$(\mathrm{rec}((1{+}1){+}1){*}\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1{+}1){+}1){*}\underline{(\epsilon)}))n)n)n \Rightarrow \underline{(n1)} <= \underline{(v1)} \Rightarrow \underline{(n1)} <= \underline{(v2)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\underline{(v1)}]{+}({-}u\underline{(fx)}[\underline{(v2)}]))| <= (\mathrm{rec}((1{+}1){+}1){*}\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\underline{(v1)}] + ({-}u\underline{(fx)}[\underline{(v2)}]))| = (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fy)}[\underline{(v1)}] + ({-}u\underline{(fy)}[\underline{(v2)}]))| <= (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fy)}[\underline{(v1)}] + ({-}u\underline{(fy)}[\underline{(v2)}]))| = (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}))n)n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}))n)n)n \Rightarrow \dot{\neg}(\underline{(n2)} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) <= |((\underline{(fx)}[\underline{\mathsf{m}}] + ({-}u\underline{(fy)}[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) = |((\underline{(fx)}[\underline{\mathsf{m}}] + ({-}u\underline{(fy)}[\underline{\mathsf{m}}]))|)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

[fromNotSameF(Strongest)(Helper) $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall\underline{(v1)}\colon\forall\underline{(v2)}\colon\forall\underline{\mathsf{m}}\colon\forall\underline{(n1)}\colon\forall\underline{(n2)}\colon\forall\underline{(\epsilon)}\colon\forall\underline{(fx)}\colon\forall\underline{(fy)}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{(n1)} <= \underline{(n2)} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\underline{(n2)}] + ({-}u\underline{(fy)}[\underline{(n2)}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\underline{(n2)}] + ({-}u\underline{(fy)}[\underline{(n2)}]))| = \underline{(\epsilon)})n)n)n)n \Rightarrow \forall_{\mathrm{obj}}\underline{(v1)}\colon\forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(0 <= (\mathrm{rec}((1{+}1){+}1){*}\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1{+}1){+}1){*}\underline{(\epsilon)}))n)n)n \Rightarrow \underline{(n1)} <= \underline{(v1)} \Rightarrow \underline{(n1)} <= \underline{(v2)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\underline{(v1)}]{+}({-}u\underline{(fx)}[\underline{(v2)}]))| <= (\mathrm{rec}((1{+}1){+}1){*}\underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\underline{(v1)}] + ({-}u\underline{(fx)}[\underline{(v2)}]))| = (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fy)}[\underline{(v1)}] + ({-}u\underline{(fy)}[\underline{(v2)}]))| <= (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fy)}[\underline{(v1)}] + ({-}u\underline{(fy)}[\underline{(v2)}]))| = (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}))n)n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}))n)n)n \Rightarrow \dot{\neg}(\underline{(n2)} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}((\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) <= |((\underline{(fx)}[\underline{\mathsf{m}}] + ({-}u\underline{(fy)}[\underline{\mathsf{m}}]))| \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{rec}((1+1)+1) * \underline{(\epsilon)}) = |((\underline{(fx)}[\underline{\mathsf{m}}] + ({-}u\underline{(fy)}[\underline{\mathsf{m}}]))|)n)n)n)n)n$]

[fromNotSameF(Strongest)(Helper) $\overset{\mathrm{tex}}{\rightarrow}$ "fromNotSameF(Strongest)(Helper)"]

[fromNotSameF(Strongest)(Helper) $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma fromNotSameF(Strongest) helper"]

fromNotSameF(Strongest)

[fromNotSameF(Strongest) $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall\underline{(v1)}\colon\forall\underline{(v2)}\colon\forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\forall\underline{(n1)}\colon\forall\underline{(n2)}\colon\forall\underline{(\epsilon)}\colon\forall\underline{(fx)}\colon\forall\underline{(fy)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}($ $\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$ Repetition $\rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n;$ Ded $\rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + ({-}u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(n1)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(n2)}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{(n1)} <= \underline{(n2)} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\underline{(n2)}] + ({-}u\underline{(fy)}[\underline{(n2)}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\underline{(n2)}] + ({-}u\underline{(fy)}[\underline{(n2)}]))| =$

$(\epsilon))n)n)n)n)n)n; \mathrm{AEA-negated} \rhd \dot{\neg}(\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(n2){:}\,\dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow$
$\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| =$
$(\epsilon))n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(n2){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n)n)n)n)n; \mathrm{Cauchy}(2) \gg$
$\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$(n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] +$
$(-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(\epsilon))n)n)n)n)n)n)n; \mathrm{A4}\,@(rec((1+1)+1) * (\epsilon)) \rhd$
$\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$(n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (\epsilon))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] +$
$(-u(fy)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(\epsilon))n)n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <=$
$(rec((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow (n1) <=$
$(v1) \Rightarrow (n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <=$
$(rec((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <=$
$(rec((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| =$
$(rec((1+1)+1) * (\epsilon)))n)n)n)n)n)n)n; \mathrm{fromNotSameF(Strongest)(Helper)} \gg$
$\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow$
$\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| =$
$(\epsilon))n)n)n)n \Rightarrow \forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow (n1) <= (v2) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}((n2) <= \mathsf{m} \Rightarrow \dot{\neg}((rec((1+1)+1) * (\epsilon)) <= |((fx)[\mathsf{m}] + (-u(fy)[\mathsf{m}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((rec((1+1)+1) * (\epsilon)) =$
$|((fx)[\mathsf{m}] + (-u(fy)[\mathsf{m}]))|)n)n)n)n)n; \mathrm{EAE-MP} \rhd \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon))n)n)n \Rightarrow (n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| = (\epsilon))n)n)n)n \Rightarrow \forall_{obj}(v1){:}\,\forall_{obj}(v2){:}\,\dot{\neg}(0 <=$
$(rec((1+1)+1)*(\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1)*(\epsilon)))n)n)n \Rightarrow (n1) <= (v1) \Rightarrow$
$(n1) <= (v2) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (rec((1+1)+1)*(\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| <= (rec((1+1)+1) * (\epsilon)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[(v1)] + (-u(fy)[(v2)]))| = (rec((1+1)+1) * (\epsilon)))n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (rec((1+1)+1) * (\epsilon)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (rec((1+1)+1) * (\epsilon)))n)n)n \Rightarrow$
$\dot{\neg}((n2) <= \mathsf{m} \Rightarrow \dot{\neg}((rec((1+1)+1) * (\epsilon)) <= |((fx)[\mathsf{m}] + (-u(fy)[\mathsf{m}]))| \Rightarrow$
$\dot{\neg}(\dot{\neg}((rec((1+1)+1) * (\epsilon)) = |((fx)[\mathsf{m}] + (-u(fy)[\mathsf{m}]))|)n)n)n)n)n \rhd$
$\dot{\neg}(\forall_{obj}(\epsilon){:}\,\dot{\neg}(\forall_{obj}(n1){:}\,\dot{\neg}(\forall_{obj}(n2){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$(n1) <= (n2) \Rightarrow \dot{\neg}(|((fx)[(n2)] + (-u(fy)[(n2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(n2)] +$

(−u(fy)[(n2)]))| = (ϵ))n)n)n)n)n)n)n)n ≫ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <=
(v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⇒ ¬̇(¬̇(0 <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇((n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n; ExistMP ⊳ $\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ (n1) <=
(v1) ⇒ (n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| =
(rec((1+1)+1)∗(ϵ)))n)n)n)n)n ⇒ ¬̇(¬̇(0 <= (rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇((n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊳ ¬̇($\forall_{obj}$(n1): ¬̇($\forall_{obj}$(v1): $\forall_{obj}$(v2): ¬̇(0 <=
(rec((1+1)+1)∗(ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1+1)+1)∗(ϵ)))n)n)n ⇒ (n1) <= (v1) ⇒
(n1) <= (v2) ⇒ ¬̇(¬̇(|((fx)[(v1)]+(−u(fx)[(v2)]))| <= (rec((1+1)+1)∗(ϵ)) ⇒
¬̇(¬̇(|((fx)[(v1)] + (−u(fx)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒
¬̇(¬̇(|((fy)[(v1)] + (−u(fy)[(v2)]))| = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n)n)n)n)n ≫
¬̇(¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇((n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒
¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n; Gen ⊳
¬̇(¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒
¬̇((n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒
¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫
$\forall_{obj}$m: ¬̇(¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇((n2) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n; IntroExist @ (n2) ⊳ $\forall_{obj}$m: ¬̇(¬̇(0 <=
(rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 = (rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇((n2) <=
m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <= |((fx)[m] + (−u(fy)[m]))| ⇒
¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫
¬̇($\forall_{obj}$n: ¬̇($\forall_{obj}$m: ¬̇(¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇((n1) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n)n)n; IntroExist @(rec((1 + 1) + 1) ∗ (ϵ)) ⊳
¬̇($\forall_{obj}$n: ¬̇($\forall_{obj}$m: ¬̇(¬̇(0 <= (rec((1 + 1) + 1) ∗ (ϵ)) ⇒ ¬̇(¬̇(0 =
(rec((1 + 1) + 1) ∗ (ϵ)))n)n)n ⇒ ¬̇((n1) <= m ⇒ ¬̇((rec((1 + 1) + 1) ∗ (ϵ)) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ¬̇(¬̇((rec((1 + 1) + 1) ∗ (ϵ)) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n)n)n ≫
¬̇($\forall_{obj}$(ϵ): ¬̇(¬̇($\forall_{obj}$(n1): ¬̇($\forall_{obj}$m: ¬̇(¬̇(0 <= (ϵ) ⇒ ¬̇(¬̇(0 = (ϵ))n)n)n ⇒

∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n)n)n)n)n⌉, p_0, c)]

[fromNotSameF(Strongest) $\overset{\text{stmt}}{\rightarrow}$ SystemQ ⊢ ∀(v1): ∀(v2): ∀m: ∀n: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∸($\forall_{\text{obj}}$(ϵ): ∸($\forall_{\text{obj}}$n: ∸($\forall_{\text{obj}}$m: ∸((ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ n <= m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ϵ) ⇒ ∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ϵ))n)n)n)n)n)n ⊢ ∸($\forall_{\text{obj}}$(ϵ): ∸(∸($\forall_{\text{obj}}$(n1): ∸($\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n)n)n)n)n]

[fromNotSameF(Strongest) $\overset{\text{tex}}{\rightarrow}$ "fromNotSameF(Strongest)"]

[fromNotSameF(Strongest) $\overset{\text{pyk}}{\rightarrow}$ "lemma fromNotSameF(Strongest)"]

ToLess(F)(Helper)

[ToLess(F)(Helper) $\overset{\text{proof}}{\rightarrow}$ λc.λx.$\mathcal{P}$(⌈SystemQ ⊢ ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ if((n2) <= (n1), (n1), (n2)) <= m ⊢ MaxLeq(1) ≫ (n1) <= if((n2) <= (n1), (n1), (n2)); leqTransitivity ⊳ (n1) <= if((n2) <= (n1), (n1), (n2)) ⊳ if((n2) <= (n1), (n1), (n2)) <= m ≫ (n1) <= m; MaxLeq(2) ≫ (n2) <= if((n2) <= (n1), (n1), (n2)); leqTransitivity ⊳ (n2) <= if((n2) <= (n1), (n1), (n2)) ⊳ if((n2) <= (n1), (n1), (n2)) <= m ≫ (n2) <= m; A4 @ m ⊳ $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n; FirstConjunct ⊳ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n; SecondConjunct ⊳ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ (n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n; MP ⊳ (n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ⊳ (n1) <= m ≫ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n; FromNumericalGreater ⊳ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n ≫ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n ⇒ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] +

(−u(fy)[m])))n)n)n; MP2 ⊳ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊳ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⊳ (n2) <= m ≫ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n; LessNegated ⊳ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ≫ ∸((−u(ϵ)) <= (−u((fy)[m] + (−u(fx)[m]))) ⇒ ∸(∸((−u(ϵ)) = (−u((fy)[m] + (−u(fx)[m]))))n)n)n; MinusNegated ≫ (−u((fy)[m] + (−u(fx)[m]))) = ((fx)[m] + (−u(fy)[m])); SubLessRight ⊳ (−u((fy)[m] + (−u(fx)[m]))) = ((fx)[m] + (−u(fy)[m])) ⊳ ∸((−u(ϵ)) <= (−u((fy)[m] + (−u(fx)[m]))) ⇒ ∸(∸((−u(ϵ)) = (−u((fy)[m] + (−u(fx)[m]))))n)n)n ≫ ∸((−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((−u(ϵ)) = ((fx)[m] + (−u(fy)[m])))n)n)n; LessLeq ⊳ ∸((−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((−u(ϵ)) = ((fx)[m] + (−u(fy)[m])))n)n)n ≫ (−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])); toNotLess ⊳ (−u(ϵ)) <= ((fx)[m] + (−u(fy)[m])) ≫ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n)n; NegateDisjunct1 ⊳ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n)n ⇒ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] + (−u(fy)[m])))n)n)n ⊳ ∸(∸(((fx)[m] + (−u(fy)[m])) <= (−u(ϵ)) ⇒ ∸(∸(((fx)[m] + (−u(fy)[m])) = (−u(ϵ)))n)n)n)n ≫ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] + (−u(fy)[m])))n)n)n; SwitchTerms(x < y − z) ⊳ ∸((ϵ) <= ((fx)[m] + (−u(fy)[m])) ⇒ ∸(∸((ϵ) = ((fx)[m] + (−u(fy)[m])))n)n)n ≫ ∸((fy)[m] <= ((fx)[m] + (−u(ϵ))) ⇒ ∸(∸((fy)[m] = ((fx)[m] + (−u(ϵ))))n)n)n; LessLeq ⊳ ∸((fy)[m] <= ((fx)[m] + (−u(ϵ))) ⇒ ∸(∸((fy)[m] = ((fx)[m] + (−u(ϵ))))n)n)n ≫ (fy)[m] <= ((fx)[m] + (−u(ϵ))); ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ A4 @ m ⊳ $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ≫ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ (ϵ) <= |((fx)[m] + (−u(fy)[m]))|)n)n; FirstConjunct ⊳ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ (ϵ) <= |((fx)[m] + (−u(fy)[m]))|)n)n ≫ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n; ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): Ded ⊳ ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ if((n2) <= (n1), (n1), (n2)) <= m ⊢ (fy)[m] <= ((fx)[m] + (−u(ϵ))) ≫ $\forall_{\text{obj}}$m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <=

((fx)[m] + (−u(ϵ))); Ded ▷ ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⊢ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ≫ ∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n; JoinConjuncts(2conditions) ▷ ∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ▷ ∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m]+(−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m]+(−u(fx)[m])) = (ϵ))n)n)n ⇒ if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ϵ))) ≫ ∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ϵ))))n)n⌉, p₀, c)]

[ToLess(F)(Helper) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ ⊢ ∀m: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ϵ) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ϵ) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ϵ) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ϵ))n)n)n ⇒ ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ϵ))))n)n]

[ToLess(F)(Helper) $\stackrel{\text{tex}}{\rightarrow}$ “ToLess(F)(Helper)”]

[ToLess(F)(Helper) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma toLess(F) helper”]

ToLess(F)

[ToLess(F) $\stackrel{\text{proof}}{\rightarrow}$ λc.λx.𝒫(⌈SystemQ ⊢ ∀m: ∀n: ∀(n1): ∀(n2): ∀(ϵ): ∀(fx): ∀(fy): ∸(∸(∸(∀_obj (ϵ): ∸(∸(∀_obj n: ∸(∀_obj m: ∸(∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <=

$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$
Repetition $\triangleright \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n$; FromNegatedOr $\triangleright$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n)n)n$; FirstConjunct $\triangleright$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n$; SecondConjunct $\triangleright$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n$; FromNot <
f(Strong) $\triangleright \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \gg$
$\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{(n2)} <= \underline{\mathsf{m}} \Rightarrow$

∸(((fy)[m] + (−u(fx)[m])) <= (ε) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) =
(ε))n)n)n; fromNotSameF(Strongest) ⊳ ∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(0 <=
(ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ n <= m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒
∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n)n)n ≫
∸(∀$_{\mathrm{obj}}$(ε): ∸(∸(∀$_{\mathrm{obj}}$(n1): ∸(∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
∸((n1) <= m ⇒ ∸((ε) <= |((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ε) =
|((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n)n)n)n)n; ToLess(F)(Helper) ≫
∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ε) <=
|((fx)[m] + (−u(fy)[m]))| ⇒ ∸(∸((ε) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒
∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] +
(−u(fx)[m])) <= (ε) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) = (ε))n)n)n ⇒ ∸(∸(0 <=
(ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒
(fy)[m] <= ((fx)[m] + (−u(ε))))n)n; TwiceExistMP ⊳ ∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ε) <= |((fx)[m] + (−u(fy)[m]))| ⇒
∸(∸((ε) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n ⇒ ∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ε) ⇒
∸(∸(((fy)[m] + (−u(fx)[m])) = (ε))n)n)n ⇒ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <=
((fx)[m] + (−u(ε))))n)n ⊳ ∸(∀$_{\mathrm{obj}}$(ε): ∸(∸(∀$_{\mathrm{obj}}$(n1): ∸(∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸((n1) <= m ⇒ ∸((ε) <= |((fx)[m] + (−u(fy)[m]))| ⇒
∸(∸((ε) = |((fx)[m] + (−u(fy)[m]))|)n)n)n)n)n)n)n)n)n ≫ ∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ε) ⇒
∸(∸(((fy)[m] + (−u(fx)[m])) = (ε))n)n)n ⇒ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <=
((fx)[m] + (−u(ε))))n)n; MP ⊳ ∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ (n2) <=
m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ε) ⇒ ∸(∸(((fy)[m] + (−u(fx)[m])) =
(ε))n)n)n ⇒ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ ∸(if((n2) <=
(n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n ⊳ ∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ (n2) <= m ⇒ ∸(((fy)[m] + (−u(fx)[m])) <= (ε) ⇒
∸(∸(((fy)[m] + (−u(fx)[m])) = (ε))n)n)n ≫ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <=
((fx)[m] + (−u(ε))))n)n; Gen ⊳ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n ≫
∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ ∸(if((n2) <=
(n1), (n1), (n2)) <= m ⇒ (fy)[m] <= ((fx)[m] +
(−u(ε))))n)n; IntroExist @ if((n2) <= (n1), (n1), (n2)) ⊳ ∀$_{\mathrm{obj}}$m: ∸(∸(0 <=
(ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ ∸(if((n2) <= (n1), (n1), (n2)) <= m ⇒
(fy)[m] <= ((fx)[m] + (−u(ε))))n)n ≫ ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸(n <= m ⇒ (fy)[m] <= ((fx)[m] +
(−u(ε))))n)n)n)n; IntroExist @ (ε) ⊳ ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸(n <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n)n)n ≫
∸(∀$_{\mathrm{obj}}$(ε): ∸(∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
∸(n <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n)n)n)n)n; Ded ⊳
∸(∀$_{\mathrm{obj}}$(ε): ∸(∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
∸(n <= m ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n)n)n)n)n ≫

$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{ToLess(F)} \stackrel{stmt}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(n1)}\colon \forall\underline{(n2)}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n]$

$[\mathrm{ToLess(F)} \stackrel{tex}{\rightarrow}$ "ToLess(F)"$]$

$[\mathrm{ToLess(F)} \stackrel{pyk}{\rightarrow}$ "lemma toLess(F)"$]$

FromNot <<

$[\mathrm{FromNot} <<\stackrel{proof}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \mathrm{AutoImply} \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n; \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \vdash$
$\mathrm{MT} \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \rhd$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{FromNot} <<\stackrel{stmt}{\rightarrow} \mathrm{SystemQ} \vdash$

$\forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[FromNot $<<\stackrel{\mathrm{tex}}{\to}$ "FromNot<<"]

[FromNot $<<\stackrel{\mathrm{pyk}}{\to}$ "lemma fromNot<<"]

ToLess(R)

[ToLess(R) $\stackrel{\mathrm{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\underline{(}\lceil$SystemQ $\vdash$
$\forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \vdash$
Repetition $\rhd\ \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}: \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}: \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}: \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}: \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \gg$
$\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(fx)}[\overline{\mathsf{m}}] <= (\overline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}: \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(op1)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}: \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow \dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}: \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}\,(\forall_{obj}\overline{(op1)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}: \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(op1)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(op2)}: \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}\,(\forall_{obj}\overline{(s2)}: \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\overline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n;$ FromNegatedOr $\triangleright$
$\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}\,(\dot{\neg}\,(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}\,(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$

$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \gg$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n)n)n; \mathrm{FirstConjunct} \rhd$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (fx)[\overline{\mathsf{m}}] <= ((fy)[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)}))))n)n)n)n)n)n \Rightarrow \dot{-}(\dot{-}(\{ph \in$
$P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}((\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}((\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}((\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{-}((\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}((\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{-}((\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{-}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n)n)n \gg$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (fx)[\overline{\mathsf{m}}] <= ((fy)[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)}))))n)n)n)n)n)n; \mathrm{SecondConjunct} \rhd$
$\dot{-}(\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (fx)[\overline{\mathsf{m}}] <= ((fy)[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)}))))n)n)n)n)n)n \Rightarrow \dot{-}(\dot{-}(\{ph \in$
$P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}((\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}((\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}=\{ph\in P(\{ph\in P(\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|((fy)[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(|((fy)[\overline{\mathsf{m}}]+$
$(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\})n)n)n\gg\dot{\neg}(\{ph\in P(\{ph\in P(\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|((fx)[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}=\{ph\in P(\{ph\in P(\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|((fy)[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fy)[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\})n;\mathrm{FromNot}<<$
$\rhd\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow(fx)[\overline{\mathsf{m}}]<=((fy)[\overline{\mathsf{m}}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n\gg$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow(fx)[\overline{\mathsf{m}}]<=((fy)[\overline{\mathsf{m}}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n;\mathrm{From!!}==$
$\rhd\dot{\neg}(\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\})n \gg$
$\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n; \mathrm{JoinConjuncts} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n)n)n; \mathrm{ToNegatedOr} \rhd$
$\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <=$
$\overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n)n \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg$
$\dot\neg(\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n)n; \mathrm{ToLess(F)} \rhd \dot\neg(\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg$
$\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n; \mathrm{Repetition} \rhd$
$\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fy)}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{ToLess(R)} \stackrel{stmt}{\to} \mathrm{SystemQ} \vdash$
$\forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \dot\neg(\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))\ |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |\ \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\})))\ |\ \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\})\ |$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\})\ |\ \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|((\underline{\mathrm{fy}})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\underline{\mathrm{fy}})[\overline{\mathsf{m}}] <= ((\underline{\mathrm{fx}})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[ToLess(R) $\overset{\mathrm{tex}}{\to}$ "ToLess(R)"]

[ToLess(R) $\overset{\mathrm{pyk}}{\to}$ "lemma toLess(R)"]

LeqTotality(R)

[LeqTotality(R) $\overset{\mathrm{tex}}{\to}$ "LeqTotality(R)"]

[LeqTotality(R) $\overset{\mathrm{pyk}}{\to}$ "lemma leqTotality(R)"]

FromNotSameF(Weak)(Helper)

[FromNotSameF(Weak)(Helper) $\overset{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \vdash 0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \vdash$ NonnegativeNumerical $\rhd 0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \gg |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))$; subLeqRight $\rhd |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \rhd \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \gg \underline{\mathsf{z}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))$; negativeToLeft(Leq) $\rhd \underline{\mathsf{z}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \gg (\underline{\mathsf{z}}+\underline{\mathsf{y}}) <= \underline{\mathsf{x}}$; plusCommutativity $\gg (\underline{\mathsf{z}}+\underline{\mathsf{y}}) = (\underline{\mathsf{y}}+\underline{\mathsf{z}})$; subLeqLeft $\rhd (\underline{\mathsf{z}}+\underline{\mathsf{y}}) = (\underline{\mathsf{y}}+\underline{\mathsf{z}}) \rhd (\underline{\mathsf{z}}+\underline{\mathsf{y}}) <= \underline{\mathsf{x}} \gg (\underline{\mathsf{y}}+\underline{\mathsf{z}}) <= \underline{\mathsf{x}}$; PositiveToRight(Leq) $\rhd (\underline{\mathsf{y}}+\underline{\mathsf{z}}) <= \underline{\mathsf{x}} \gg \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}}))$; WeakenOr1 $\rhd \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}})) \gg \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}}))$; $\forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \vdash \dot{\neg}(0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})))\mathrm{n} \vdash$ ToLess $\rhd \dot{\neg}(0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})))\mathrm{n} \gg \dot{\neg}((\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) <= 0 \Rightarrow \dot{\neg}(\dot{\neg}((\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) = 0)\mathrm{n})\mathrm{n})\mathrm{n}$; NegativeNumerical $\gg |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = (-\mathrm{u}(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})))$; MinusNegated $\gg (-\mathrm{u}(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}}))$; eqTransitivity $\rhd |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = (-\mathrm{u}(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))) \rhd (-\mathrm{u}(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))) = (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}})) \gg |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}}))$; subLeqRight $\rhd |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}})) \rhd \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \gg \underline{\mathsf{z}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}}))$; negativeToLeft(Leq) $\rhd \underline{\mathsf{z}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{x}})) \gg (\underline{\mathsf{z}}+\underline{\mathsf{x}}) <= \underline{\mathsf{y}}$; plusCommutativity $\gg (\underline{\mathsf{z}}+\underline{\mathsf{x}}) = (\underline{\mathsf{x}}+\underline{\mathsf{z}})$; subLeqLeft $\rhd (\underline{\mathsf{z}}+\underline{\mathsf{x}}) = (\underline{\mathsf{x}}+\underline{\mathsf{z}}) \rhd (\underline{\mathsf{z}}+\underline{\mathsf{x}}) <= \underline{\mathsf{y}} \gg (\underline{\mathsf{x}}+\underline{\mathsf{z}}) <= \underline{\mathsf{y}}$; PositiveToRight(Leq) $\rhd (\underline{\mathsf{x}}+\underline{\mathsf{z}}) <= \underline{\mathsf{y}} \gg \underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}}))$; WeakenOr2 $\rhd \underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})) \gg \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}}))$; $\forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon$ Ded $\rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \vdash 0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \vdash \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}})) \gg \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \Rightarrow 0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \Rightarrow \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}}))$; Ded $\rhd \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \forall \underline{\mathsf{z}}\colon \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \vdash \dot{\neg}(0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})))\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}})) \gg \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \Rightarrow \dot{\neg}(0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})))\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}}))$; $\dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$ fromNotLess $\rhd \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| <= \underline{\mathsf{z}} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| = \underline{\mathsf{z}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))|$; MP $\rhd \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \Rightarrow 0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \Rightarrow \dot{\neg}(\underline{\mathsf{x}} <= (\underline{\mathsf{y}}+(-\mathrm{u}\underline{\mathsf{z}})))\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{z}})) \rhd \underline{\mathsf{z}} <= |(\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}}))| \gg 0 <= (\underline{\mathsf{x}}+(-\mathrm{u}\underline{\mathsf{y}})) \Rightarrow$

$\dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z})); \mathrm{MP} \rhd \underline{z} <= |(\underline{x} + (-u\underline{y}))| \Rightarrow \dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z})) \rhd \underline{z} <= |(\underline{x} + (-u\underline{y}))| \gg \dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z})); \mathrm{FromNegations} \rhd 0 <= (\underline{x} + (-u\underline{y})) \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z})) \rhd \dot{\neg}(0 <= (\underline{x} + (-u\underline{y})))n \Rightarrow \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z})) \gg \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z}))\rceil, p_0, c)]$

[FromNotSameF(Weak)(Helper) $\overset{\text{stmt}}{\to}$ SystemQ $\vdash$ $\forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n)n \vdash \dot{\neg}(\underline{x} <= (\underline{y} + (-u\underline{z})))n \Rightarrow \underline{y} <= (\underline{x} + (-u\underline{z}))$]

[FromNotSameF(Weak)(Helper) $\overset{\text{tex}}{\to}$ "FromNotSameF(Weak)(Helper)"]

[FromNotSameF(Weak)(Helper) $\overset{\text{pyk}}{\to}$ "lemma fromNotSameF(Weak)(Helper)"]

FromNotSameF(Weak)

[FromNotSameF(Weak) $\overset{\text{proof}}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash$ $\forall \underline{m}: \forall \underline{n}: \forall \underline{(\epsilon)}: \forall \underline{(fx)}: \forall \underline{(fy)}: \dot{\neg}(\forall_{obj} \overline{(\epsilon)}: \dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$ Repetition $\rhd \dot{\neg}(\forall_{obj} \overline{(\epsilon)}: \dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj} \overline{(\epsilon)}: \dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n)n; \mathrm{Ded} \rhd \dot{\neg}(\forall_{obj} \overline{(\epsilon)}: \dot{\neg}(\forall_{obj} \overline{n}: \dot{\neg}(\forall_{obj} \overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj} \underline{(\epsilon)}: \dot{\neg}(\forall_{obj} \underline{n}: \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = \underline{(\epsilon)})n)n)n)n)n)n; \mathrm{AEA} - \mathrm{negated} \rhd \dot{\neg}(\forall_{obj} \underline{(\epsilon)}: \dot{\neg}(\forall_{obj} \underline{n}: \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = \underline{(\epsilon)})n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj} \underline{(\epsilon)}: \dot{\neg}(\forall_{obj} \underline{n}: \dot{\neg}(\forall_{obj} \underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = \underline{(\epsilon)})n)n)n)n)n)n)n)n)n; \forall \underline{m}: \forall \underline{n}: \forall \underline{(\epsilon)}: \forall \underline{(fx)}: \forall \underline{(fy)}: \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = \underline{(\epsilon)})n)n)n)n \vdash$ FromNegated$(2 * \mathrm{Imply}) \rhd \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = \underline{(\epsilon)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m})n)n \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| =$

(ε))n)n)n)n)n)n; FirstConjunct ▷ ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
∸(n <= m)n)n ⇒ ∸(∸(∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒
∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n)n)n ≫ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n ⇒ ∸(n <= m)n)n; SecondConjunct ▷ ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸(∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒
∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n)n)n ≫
∸(∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒ ∸(∸(|((fx)[m] + (−u(fy)[m]))| =
(ε))n)n)n)n; FromNotSameF(Weak)(Helper) ▷ ∸(∸(|((fx)[m]+(−u(fy)[m]))| <=
(ε) ⇒ ∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n ≫ ∸((fx)[m] <= ((fy)[m] +
(−u(ε))))n ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))); JoinConjuncts ▷ ∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸(n <= m)n)n ▷ ∸((fx)[m] <= ((fy)[m] + (−u(ε))))n ⇒
(fy)[m] <= ((fx)[m] + (−u(ε))) ≫ ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <= ((fy)[m] + (−u(ε))))n ⇒ (fy)[m] <=
((fx)[m] + (−u(ε))))n)n; Ded ▷ ∀m: ∀n: ∀(ε): ∀(fx): ∀(fy): ∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ n <= m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒
∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n ⊢ ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <= ((fy)[m] + (−u(ε))))n ⇒
(fy)[m] <= ((fx)[m] + (−u(ε))))n)n ≫ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒
n <= m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒
∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n ⇒ ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <= ((fy)[m] + (−u(ε))))n ⇒
(fy)[m] <= ((fx)[m] + (−u(ε))))n)n; AddEAE ▷ ∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ n <= m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒
∸(∸(|((fx)[m] + (−u(fy)[m]))| = (ε))n)n)n)n ⇒ ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 =
(ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <= ((fy)[m] + (−u(ε))))n ⇒
(fy)[m] <= ((fx)[m] + (−u(ε))))n)n ≫
∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ n <=
m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒ ∸(∸(|((fx)[m] + (−u(fy)[m]))| =
(ε))n)n)n)n)n)n)n)n ⇒ ∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <=
((fy)[m] + (−u(ε))))n ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n)n)n)n)n; MP ▷
∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ n <=
m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒ ∸(∸(|((fx)[m] + (−u(fy)[m]))| =
(ε))n)n)n)n)n)n)n)n ⇒ ∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <=
((fy)[m] + (−u(ε))))n ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n)n)n)n)n ▷
∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(∸(0 <= (ε) ⇒ ∸(∸(0 = (ε))n)n)n ⇒ n <=
m ⇒ ∸(|((fx)[m] + (−u(fy)[m]))| <= (ε) ⇒ ∸(∸(|((fx)[m] + (−u(fy)[m]))| =
(ε))n)n)n)n)n)n)n)n ≫ ∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$n: ∸(∀$_{\mathrm{obj}}$m: ∸(∸(∸(∸(0 <= (ε) ⇒
∸(∸(0 = (ε))n)n)n ⇒ ∸(n <= m)n)n ⇒ ∸(∸((fx)[m] <=
((fy)[m] + (−u(ε))))n ⇒ (fy)[m] <= ((fx)[m] + (−u(ε))))n)n)n)n)n)n⌉, p_0, c)]

[FromNotSameF(Weak) $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ ⊢
∀m: ∀n: ∀(ε): ∀(fx): ∀(fy): ∸(∀$_{\mathrm{obj}}$(ε): ∸(∀$_{\mathrm{obj}}$$\overline{\mathrm{n}}$: ∸(∀$_{\mathrm{obj}}$$\overline{\mathrm{m}}$: ∸(0 <= $\overline{(\epsilon)}$ ⇒ ∸(∸(0 =
$\overline{(\epsilon)}$)n)n)n ⇒ $\overline{\mathrm{n}}$ <= $\overline{\mathrm{m}}$ ⇒ ∸(|((fx)[$\overline{\mathrm{m}}$] + (−u(fy)[$\overline{\mathrm{m}}$]))| <= $\overline{(\epsilon)}$ ⇒

$\dot\neg(\dot\neg(|((fx)[\overline{m}] + (-u(fy)[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n)n \vdash$
$\dot\neg(\forall_{obj}\underline{(\epsilon)}: \dot\neg(\forall_{obj}\underline{n}: \dot\neg(\forall_{obj}\underline{m}: \dot\neg(\dot\neg(\dot\neg(\dot\neg(0 <= \underline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \underline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\underline{n} <= \underline{m})n)n \Rightarrow \dot\neg(\dot\neg(((fx)[\underline{m}] <= ((fy)[\underline{m}] + (-u(\epsilon))))n \Rightarrow (fy)[\underline{m}] <=$
$((fx)[\underline{m}] + (-u(\epsilon))))n)n)n)n)n)n]$

[FromNotSameF(Weak) $\overset{tex}{\to}$ “FromNotSameF(Weak)”]

[FromNotSameF(Weak) $\overset{pyk}{\to}$ “lemma fromNotSameF(Weak)”]

FromNotLess(F)

[FromNotLess(F) $\overset{tex}{\to}$ “FromNotLess(F)”]

[FromNotLess(F) $\overset{pyk}{\to}$ “lemma fromNotLess(F)”]

== Addition

[== Addition $\overset{tex}{\to}$ “==Addition”]

[== Addition $\overset{pyk}{\to}$ “lemma ==Addition”]

== AdditionLeft

[== AdditionLeft $\overset{tex}{\to}$ “==AdditionLeft”]

[== AdditionLeft $\overset{pyk}{\to}$ “lemma ==AdditionLeft”]

Fpart − Bounded(Base)

[Fpart − Bounded(Base) $\overset{proof}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{(v1)}: \forall\underline{(v2n)}: \forall\underline{(fx)}: \underline{(v2n)} <= 0 \vdash$ LeqLessEq $\rhd$ $\underline{(v2n)} <= 0 \gg \dot\neg(\dot\neg(\underline{(v2n)} <=$
$0 \Rightarrow \dot\neg(\dot\neg(\underline{(v2n)} = 0)n)n)n)n \Rightarrow \underline{(v2n)} = 0$; Nonnegative(N) $\gg 0 <=$
$\underline{(v2n)}$; toNotLess $\rhd 0 <= \underline{(v2n)} \gg \dot\neg(\dot\neg(\underline{(v2n)} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{(v2n)} =$
$0)n)n)n)n$; NegateDisjunct1 $\rhd \dot\neg(\dot\neg(\underline{(v2n)} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{(v2n)} = 0)n)n)n)n \Rightarrow$
$\underline{(v2n)} = 0 \rhd \dot\neg(\dot\neg(\underline{(v2n)} <= 0 \Rightarrow \dot\neg(\dot\neg(\underline{(v2n)} = 0)n)n)n)n \gg \underline{(v2n)} =$
0; SameSeries$\rhd \underline{(v2n)} = 0 \gg \underline{(fx)}[\underline{(v2n)}] = \underline{(fx)}[0]$; SameNumerical$\rhd \underline{(fx)}[\underline{(v2n)}] =$
$\underline{(fx)}[0] \gg |\underline{(fx)}[\underline{(v2n)}]| = |\underline{(fx)}[0]|$; eqAddition $\rhd |\underline{(fx)}[\underline{(v2n)}]| = |\underline{(fx)}[0]| \gg$
$(|\underline{(fx)}[\underline{(v2n)}]| + 1) = (|\underline{(fx)}[0]| + 1)$; leqReflexivity $\gg |\underline{(fx)}[\underline{(v2n)}]| <=$
$|\underline{(fx)}[\underline{(v2n)}]|$; Leq + 1 $\rhd |\underline{(fx)}[\underline{(v2n)}]| <= |\underline{(fx)}[\underline{(v2n)}]| \gg \dot\neg(|\underline{(fx)}[\underline{(v2n)}]| <=$
$(|\underline{(fx)}[\underline{(v2n)}]| + 1) \Rightarrow \dot\neg(\dot\neg(|\underline{(fx)}[\underline{(v2n)}]| =$
$(|\underline{(fx)}[\underline{(v2n)}]| + 1))n)n)n$; SubLessRight $\rhd (|\underline{(fx)}[\underline{(v2n)}]| + 1) =$

$(|(fx)[0]|+1) \rhd \dot{\neg}(|(fx)[(v2n)]| <= (|(fx)[(v2n)]|+1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$(|(fx)[(v2n)]|+1))n)n)n \gg \dot{\neg}(|(fx)[(v2n)]| <= (|(fx)[0]|+1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[0]|+1))n)n)n; \forall(v1)\colon\forall(v2n)\colon\forall(fx)\colon \mathrm{Ded} \rhd$
$\forall(v1)\colon\forall(v2n)\colon\forall(fx)\colon (v2n) <= 0 \vdash \dot{\neg}(|(fx)[(v2n)]| <= (|(fx)[0]|+1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[0]|+1))n)n)n \gg (v2n) <= 0 \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <=$
$(|(fx)[0]|+1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[0]|+1))n)n)n; \mathrm{Gen} \rhd (v2n) <= 0 \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (|(fx)[0]|+1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[0]|+1))n)n)n \gg$
$\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= 0 \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (|(fx)[0]|+1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[0]|+1))n)n)n; \mathrm{IntroExist} \mathbin{@} (|(fx)[0]|+1) \rhd$
$\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= 0 \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (|(fx)[0]|+1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[0]|+1))n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n)\colon (v2n) <=$
$0 \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{Base}) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(v1)\colon\forall(v2n)\colon\forall(fx)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= 0 \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <=$
$(v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n]$

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{Base}) \stackrel{\mathrm{tex}}{\rightarrow}$ "Fpart-Bounded(Base)"]

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{Base}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma fpart-Bounded base"]

Fpart − Bounded(InduHelper)

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{InduHelper}) \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ} \vdash$
$\forall(v1)\colon\forall(v2n)\colon\forall\underline{\mathsf{n}}\colon\forall(fx)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash (v2n) <= (\underline{\mathsf{n}}+1) \vdash \mathrm{LeqLessEq} \rhd (v2n) <=$
$(\underline{\mathsf{n}}+1) \gg \dot{\neg}(\dot{\neg}((v2n) <= (\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((v2n) = (\underline{\mathsf{n}}+1))n)n)n)n \Rightarrow (v2n) =$
$(\underline{\mathsf{n}}+1); \forall(v1)\colon\forall(v2n)\colon\forall\underline{\mathsf{n}}\colon\forall(fx)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash \dot{\neg}((v2n) <= (\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((v2n) =$
$(\underline{\mathsf{n}}+1))n)n)n \vdash \mathrm{LessMinus1(N)} \rhd \dot{\neg}((v2n) <= (\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((v2n) =$
$(\underline{\mathsf{n}}+1))n)n)n \gg (v2n) <= \underline{\mathsf{n}}; \mathrm{MP} \rhd (v2n) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \rhd (v2n) <= \underline{\mathsf{n}} \gg \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n; \mathrm{MaxLeq}(1) \gg (v1) <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}}+1)]|+1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}}+1)]|+1)); \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(|(fx)[(v2n)]| <=$
$(v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \rhd (v1) <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}}+1)]|+1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}}+1)]|+1)) \gg \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}}+1)]|+1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{\mathsf{n}}+1)]|+1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}}+1)]|+1)))n)n)n; \forall(v1)\colon\forall(v2n)\colon\forall\underline{\mathsf{n}}\colon\forall(fx)\colon (v2n) = (\underline{\mathsf{n}}+1) \vdash$
$\mathrm{SameSeries} \rhd (v2n) = (\underline{\mathsf{n}}+1) \gg (fx)[(v2n)] =$
$(fx)[(\underline{\mathsf{n}}+1)]; \mathrm{SameNumerical} \rhd (fx)[(v2n)] = (fx)[(\underline{\mathsf{n}}+1)] \gg |(fx)[(v2n)]| =$
$|(fx)[(\underline{\mathsf{n}}+1)]|; \mathrm{eqLeq} \rhd |(fx)[(v2n)]| = |(fx)[(\underline{\mathsf{n}}+1)]| \gg |(fx)[(v2n)]| <=$
$|(fx)[(\underline{\mathsf{n}}+1)]|; \mathrm{Leq}+1 \rhd |(fx)[(v2n)]| <= |(fx)[(\underline{\mathsf{n}}+1)]| \gg \dot{\neg}(|(fx)[(v2n)]| <=$
$(|(fx)[(\underline{\mathsf{n}}+1)]|+1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (|(fx)[(\underline{\mathsf{n}}+1)]|+1))n)n)n; \mathrm{MaxLeq}(2) \gg$
$(|(fx)[(\underline{\mathsf{n}}+1)]|+1) <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}}+1)]|+1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}}+1)]|+1)); \mathrm{LessLeqTransitivity} \rhd \dot{\neg}(|(fx)[(v2n)]| <=$

$(|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$(|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \gg \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd \forall(\mathrm{v1})\colon \forall(\mathrm{v2n})\colon \forall\underline{\mathsf{n}}\colon \forall(\mathrm{fx})\colon (\mathrm{v2n}) <=$
$\underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}((\mathrm{v2n}) <=$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{v2n}) = (\underline{\mathsf{n}}+1))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg (\mathrm{v2n}) <=$
$\underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{v2n}) <=$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{v2n}) = (\underline{\mathsf{n}}+1))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{MP} \rhd (\mathrm{v2n}) <=$
$\underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{v2n}) <=$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{v2n}) = (\underline{\mathsf{n}}+1))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\mathrm{v2n}) <= \underline{\mathsf{n}} \Rightarrow$
$\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}((\mathrm{v2n}) <=$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{v2n}) = (\underline{\mathsf{n}}+1))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd$
$\forall(\mathrm{v1})\colon \forall(\mathrm{v2n})\colon \forall\underline{\mathsf{n}}\colon \forall(\mathrm{fx})\colon (\mathrm{v2n}) = (\underline{\mathsf{n}}+1) \vdash \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg (\mathrm{v2n}) =$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FromDisjuncts} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{v2n}) <=$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{v2n}) = (\underline{\mathsf{n}}+1))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow (\mathrm{v2n}) = (\underline{\mathsf{n}}+1) \rhd \dot{\neg}((\mathrm{v2n}) <=$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(\dot{\neg}((\mathrm{v2n}) = (\underline{\mathsf{n}}+1))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\mathrm{v2n}) =$
$(\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = \mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <=$
$(\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{InduHelper}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall(\mathrm{v1})\colon \forall(\mathrm{v2n})\colon \forall\underline{\mathsf{n}}\colon \forall(\mathrm{fx})\colon (\mathrm{v2n}) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <= (\mathrm{v1}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| = (\mathrm{v1}))\mathrm{n})\mathrm{n})\mathrm{n} \vdash (\mathrm{v2n}) <= (\underline{\mathsf{n}}+1) \vdash \dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| <=$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fx})[(\mathrm{v2n})]| =$
$\mathrm{if}((|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1) <= (\mathrm{v1}), (\mathrm{v1}), (|(\mathrm{fx})[(\underline{\mathsf{n}}+1)]|+1)))\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{InduHelper}) \overset{\mathrm{tex}}{\rightarrow}$ "Fpart-Bounded(InduHelper)"$]$

$[\mathrm{Fpart}-\mathrm{Bounded}(\mathrm{InduHelper}) \overset{\mathrm{pyk}}{\rightarrow}$ "lemma fpart-Bounded indu helper"$]$

Fpart − Bounded(Indu)

[Fpart − Bounded(Indu) $\stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall (v1)\colon \forall (v2n)\colon \forall \underline{\mathsf{n}}\colon \forall (fx)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash (v2n) <= (\underline{\mathsf{n}} + 1) \vdash$
$\mathrm{Fpart - Bounded(InduHelper)} \rhd (v2n) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \rhd (v2n) <= (\underline{\mathsf{n}} + 1) \gg \dot{\neg}(|(fx)[(v2n)]| <=$
$\mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$\mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| +$
$1)))n)n)n; \forall (v1)\colon \forall (v2n)\colon \forall \underline{\mathsf{n}}\colon \forall (fx)\colon \mathrm{Ded} \rhd \forall (v1)\colon \forall (v2n)\colon \forall \underline{\mathsf{n}}\colon \forall (fx)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \vdash (v2n) <= (\underline{\mathsf{n}}+1) \vdash$
$\dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| +$
$1)))n)n)n \gg (v2n) <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| =$
$(v1))n)n)n \Rightarrow (v2n) <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)))n)n)n; \mathrm{AddAll} \rhd (v2n) <= \underline{\mathsf{n}} \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \Rightarrow (v2n) <=$
$(\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)))n)n)n \gg \forall_{\mathrm{obj}}(v2n)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)))n)n)n; \mathrm{AddExist(SimpleAnt)} @ \mathrm{if}((|(fx)[(\underline{\mathsf{n}} +$
$1)]| + 1) <= (v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \rhd \forall_{\mathrm{obj}}(v2n)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = \mathrm{if}((|(fx)[(\underline{\mathsf{n}} + 1)]| + 1) <=$
$(v1), (v1), (|(fx)[(\underline{\mathsf{n}} + 1)]| + 1)))n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n)\colon (v2n) <=$
$\underline{\mathsf{n}} \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n \rceil, p_0, \mathsf{c})]$

[Fpart − Bounded(Indu) $\stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall (v1)\colon \forall (v2n)\colon \forall \underline{\mathsf{n}}\colon \forall (fx)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= \underline{\mathsf{n}} \Rightarrow$
$\dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(v1)\colon \dot{\neg}(\forall_{\mathrm{obj}}(v2n)\colon (v2n) <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(|(fx)[(v2n)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2n)]| = (v1))n)n)n)n)n]$

[Fpart − Bounded(Indu) $\stackrel{\mathrm{tex}}{\rightarrow}$ “Fpart-Bounded(Indu)”]

[Fpart − Bounded(Indu) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma fpart-Bounded indu”]

Fpart − Bounded

[Fpart − Bounded $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{\mathsf{n}}\colon \forall \underline{(fx)}\colon$ Fpart − Bounded(Base) $\gg \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= 0 \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n$; Fpart − Bounded(Indu) $\gg \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n \Rightarrow \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= (\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n$; Induction $\rhd \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= 0 \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n \rhd \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n \Rightarrow \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= (\underline{\mathsf{n}}+1) \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n\rceil, p_0, \mathsf{c})$]

[Fpart − Bounded $\overset{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{(v1)}\colon \forall \underline{(v2)}\colon \forall \underline{\mathsf{n}}\colon \forall \underline{(fx)}\colon \dot{\neg}(\forall_{obj}\underline{(v1)}\colon \dot{\neg}(\forall_{obj}\underline{(v2)}\colon \underline{(v2)} <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n$]

[Fpart − Bounded $\overset{\text{tex}}{\rightarrow}$ “Fpart-Bounded”]

[Fpart − Bounded $\overset{\text{pyk}}{\rightarrow}$ “lemma fpart-Bounded”]

F − Bounded(Helper)

[F − Bounded(Helper) $\overset{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash$ NumericalDifferenceLess $\rhd \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \gg \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n$; FirstConjunct $\rhd \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n$; NegativeToRight(Less) $\rhd \dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \gg \dot{\neg}(\underline{y} <= (\underline{x} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x} + \underline{z}))n)n)n$; $x <= |x| \gg (\underline{x} + \underline{z}) <= |(\underline{x} + \underline{z})|$; LessLeqTransitivity $\rhd \dot{\neg}(\underline{y} <= (\underline{x} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = (\underline{x} + \underline{z}))n)n)n \rhd (\underline{x} + \underline{z}) <= |(\underline{x} + \underline{z})| \gg \dot{\neg}(\underline{y} <= |(\underline{x} + \underline{z})| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = |(\underline{x} + \underline{z})|)n)n)n$; MaxLeq(1) $\gg |(\underline{x} + \underline{z})| <= \mathrm{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|)$; LessLeqTransitivity $\rhd \dot{\neg}(\underline{y} <= |(\underline{x} + \underline{z})| \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = |(\underline{x} + \underline{z})|)n)n)n \rhd |(\underline{x} + \underline{z})| <= \mathrm{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|) \gg \dot{\neg}(\underline{y} <= \mathrm{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \mathrm{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|))n)n)n$; SecondConjunct $\rhd \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) <= \underline{x} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{y} + (-u\underline{z})) = \underline{x})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} =$

$(\underline{y} + \underline{z}))n)n)n)n)n \gg \dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n; \text{PositiveToLeft}(\text{Less}) \rhd \dot{\neg}(\underline{x} <= (\underline{y} + \underline{z}) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{x} = (\underline{y} + \underline{z}))n)n)n \gg \dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) = \underline{y})n)n)n; x <= |x| \gg (\underline{z} + (-u\underline{x})) <= |(\underline{z} + (-u\underline{x}))|; \text{NumericalDifference} \gg |(\underline{z} + (-u\underline{x}))| = |(\underline{x} + (-u\underline{z}))|; \text{subLeqRight} \rhd |(\underline{z} + (-u\underline{x}))| = |(\underline{x} + (-u\underline{z}))| \rhd (\underline{z} + (-u\underline{x})) <= |(\underline{z} + (-u\underline{x}))| \gg (\underline{z} + (-u\underline{x})) <= |(\underline{x} + (-u\underline{z}))|; \text{LeqNegated} \rhd (\underline{z} + (-u\underline{x})) <= |(\underline{x} + (-u\underline{z}))| \gg (-u|(\underline{x} + (-u\underline{z}))|) <= (-u(\underline{z} + (-u\underline{x}))); \text{MinusNegated} \gg (-u(\underline{z} + (-u\underline{x}))) = (\underline{x} + (-u\underline{z})); \text{subLeqRight} \rhd (-u(\underline{z} + (-u\underline{x}))) = (\underline{x} + (-u\underline{z})) \rhd (-u|(\underline{x} + (-u\underline{z}))|) <= (-u(\underline{z} + (-u\underline{x}))) \gg (-u|(\underline{x} + (-u\underline{z}))|) <= (\underline{x} + (-u\underline{z})); \text{leqLessTransitivity} \rhd (-u|(\underline{x} + (-u\underline{z}))|) <= (\underline{x} + (-u\underline{z})) \rhd \dot{\neg}((\underline{x} + (-u\underline{z})) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((\underline{x} + (-u\underline{z})) = \underline{y})n)n)n \gg \dot{\neg}((-u|(\underline{x} + (-u\underline{z}))|) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-u|(\underline{x} + (-u\underline{z}))|) = \underline{y})n)n)n; \text{MaxLeq}(2) \gg |(\underline{x} + (-u\underline{z}))| <= \text{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|); \text{LeqNegated} \rhd |(\underline{x}+(-u\underline{z}))| <= \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|) \gg (-u\text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|)) <= (-u|(\underline{x} + (-u\underline{z}))|); \text{leqLessTransitivity} \rhd (-u\text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|)) <= (-u|(\underline{x} + (-u\underline{z}))|) \rhd \dot{\neg}((-u|(\underline{x} + (-u\underline{z}))|) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-u|(\underline{x} + (-u\underline{z}))|) = \underline{y})n)n)n \gg \dot{\neg}((-u\text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|)) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-u\text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|)) = \underline{y})n)n)n; \text{ToNumericalLess} \rhd \dot{\neg}((-u\text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|)) <= \underline{y} \Rightarrow \dot{\neg}(\dot{\neg}((-u\text{if}(|(\underline{x}+(-u\underline{z}))| <= |(\underline{x}+\underline{z})|, |(\underline{x}+\underline{z})|, |(\underline{x}+(-u\underline{z}))|)) = \underline{y})n)n)n \rhd \dot{\neg}(\underline{y} <= \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{y} = \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|))n)n)n \gg \dot{\neg}(|\underline{y}| <= \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{y}| = \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|))n)n)n\rceil, p_0, \mathsf{c})]$

$[\text{F} - \text{Bounded}(\text{Helper}) \overset{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{x}: \forall\underline{y}: \forall\underline{z}: \dot{\neg}(|(\underline{x} + (-u\underline{y}))| <= \underline{z} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{x} + (-u\underline{y}))| = \underline{z})n)n)n \vdash \dot{\neg}(|\underline{y}| <= \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|) \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{y}| = \text{if}(|(\underline{x} + (-u\underline{z}))| <= |(\underline{x} + \underline{z})|, |(\underline{x} + \underline{z})|, |(\underline{x} + (-u\underline{z}))|))n)n)n]$

$[\text{F} - \text{Bounded}(\text{Helper}) \overset{\text{tex}}{\rightarrow}$ "F-Bounded(Helper)"]

$[\text{F} - \text{Bounded}(\text{Helper}) \overset{\text{pyk}}{\rightarrow}$ "lemma f-Bounded helper"]

F − Bounded

$[\text{F} - \text{Bounded} \overset{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{n}: \forall\underline{(fx)}: \forall_{obj}\underline{(v1)}: \forall_{obj}\underline{(v2)}: \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{(v1)} \Rightarrow \underline{n} <= \underline{(v2)} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| = 1)n)n)n \vdash \underline{n} <= \underline{(v2)} \vdash A4 @ \underline{n} \rhd \forall_{obj}\underline{(v1)}: \forall_{obj}\underline{(v2)}: \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{(v1)} \Rightarrow \underline{n} <= \underline{(v2)} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| = 1)n)n)n \gg \forall_{obj}\underline{(v2)}: \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= \underline{n} \Rightarrow \underline{n} <= \underline{(v2)} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\underline{n}] + (-u\underline{(fx)}[\underline{(v2)}]))| <=$

$1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| = 1)n)n)n; \mathrm{A4} @ (v2) \rhd \forall_{obj}(v2){:}\ \dot{\neg}(0 <=$
$1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \mathsf{n} <= \mathsf{n} \Rightarrow \mathsf{n} <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| =$
$1)n)n)n \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \mathsf{n} <= \mathsf{n} \Rightarrow \mathsf{n} <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| =$
$1)n)n)n; 0 < 1 \gg \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n; \mathrm{leqReflexivity} \gg \mathsf{n} <=$
$\mathsf{n}; \mathrm{MP3} \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \mathsf{n} <= \mathsf{n} \Rightarrow \mathsf{n} <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| =$
$1)n)n)n \rhd \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \rhd \mathsf{n} <= \mathsf{n} \rhd \mathsf{n} <= (v2) \gg$
$\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| =$
$1)n)n)n; \mathrm{F} - \mathrm{Bounded(Helper)} \rhd \dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| <= 1 \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\mathsf{n}] + (-u(fx)[(v2)]))| = 1)n)n)n \gg \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] +$
$1)|, |((fx)[\mathsf{n}] + (-u1))|))n)n)n; \forall(v1){:}\ \forall(v2){:}\ \forall\mathsf{n}{:}\ \forall(fx){:}\ \forall_{obj}(v2){:}\ (v2) <= \mathsf{n} \Rightarrow$
$\dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \vdash \dot{\neg}(\mathsf{n} <= (v2))n \vdash$
$\mathrm{A4} @ (v2) \rhd \forall_{obj}(v2){:}\ (v2) <= \mathsf{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \gg (v2) <= \mathsf{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n; \mathrm{ToLess} \rhd \dot{\neg}(\mathsf{n} <= (v2))n \gg \dot{\neg}((v2) <= \mathsf{n} \Rightarrow$
$\dot{\neg}(\dot{\neg}((v2) = \mathsf{n})n)n)n; \mathrm{LessLeq} \rhd \dot{\neg}((v2) <= \mathsf{n} \Rightarrow \dot{\neg}(\dot{\neg}((v2) = \mathsf{n})n)n)n \gg$
$(v2) <= \mathsf{n}; \mathrm{MP} \rhd (v2) <= \mathsf{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$(v1))n)n)n \rhd (v2) <= \mathsf{n} \gg \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$(v1))n)n)n; \forall(v1){:}\ \forall(v2){:}\ \forall\mathsf{n}{:}\ \forall(\epsilon){:}\ \forall(fx){:}\ \mathrm{Ded} \rhd$
$\forall(v1){:}\ \forall(v2){:}\ \forall\mathsf{n}{:}\ \forall(fx){:}\ \forall_{obj}(v1){:}\ \forall_{obj}(v2){:}\ \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow$
$\mathsf{n} <= (v1) \Rightarrow \mathsf{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= 1 \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = 1)n)n)n \vdash \mathsf{n} <= (v2) \vdash \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] +$
$1)|, |((fx)[\mathsf{n}] + (-u1))|))n)n)n \gg \forall_{obj}(v1){:}\ \forall_{obj}(v2){:}\ \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$1)n)n)n \Rightarrow \mathsf{n} <= (v1) \Rightarrow \mathsf{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= 1 \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = 1)n)n)n \Rightarrow \mathsf{n} <= (v2) \Rightarrow \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] + (-u1))|) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\mathsf{n}] + (-u1))| <= |((fx)[\mathsf{n}] + 1)|, |((fx)[\mathsf{n}] +$
$1)|, |((fx)[\mathsf{n}] + (-u1))|))n)n)n; \mathrm{Ded} \rhd \forall(v1){:}\ \forall(v2){:}\ \forall\mathsf{n}{:}\ \forall(fx){:}\ \forall_{obj}(v2){:}\ (v2) <=$
$\mathsf{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \vdash \dot{\neg}(\mathsf{n} <=$
$(v2))n \vdash \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \gg$
$\forall_{obj}(v2){:}\ (v2) <= \mathsf{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$(v1))n)n)n \Rightarrow \dot{\neg}(\mathsf{n} <= (v2))n \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$(v1))n)n)n; \mathrm{Cauchy} \gg \forall_{obj}(\epsilon){:}\ \dot{\neg}(\forall_{obj}\mathsf{n}{:}\ \dot{\neg}(\forall_{obj}(v1){:}\ \forall_{obj}(v2){:}\ \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \mathsf{n} <= (v1) \Rightarrow \mathsf{n} <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n)n)n; \mathrm{A4} @ 1 \rhd \forall_{obj}(\epsilon){:}\ \dot{\neg}(\forall_{obj}\mathsf{n}{:}\ \dot{\neg}(\forall_{obj}(v1){:}\ \forall_{obj}(v2){:}\ \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \mathsf{n} <= (v1) \Rightarrow \mathsf{n} <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$(\epsilon))n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\mathsf{n}{:}\ \dot{\neg}(\forall_{obj}(v1){:}\ \forall_{obj}(v2){:}\ \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$1)n)n)n \Rightarrow \mathsf{n} <= (v1) \Rightarrow \mathsf{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <=$

$1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| = 1)n)n)n)n)n; \mathrm{Fpart - Bounded} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n; \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}}(v1) \colon \forall_{\mathrm{obj}}(v2) \colon \dot{\neg}(0 <= 1 \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= (v1) \Rightarrow \underline{n} <= (v2) \Rightarrow$
$\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| =$
$1)n)n)n \Rightarrow \underline{n} <= (v2) \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|))n)n)n \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{n} \colon \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \forall_{\mathrm{obj}}(v2) \colon \dot{\neg}(0 <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = 1)n)n)n \Rightarrow \underline{n} <= (v1) \Rightarrow$
$\underline{n} <= (v2) \Rightarrow \dot{\neg}(|((fx)[(v1)] + (-u(fx)[(v2)]))| <= 1 \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[(v1)] +$
$(-u(fx)[(v2)]))| = 1)n)n)n)n)n \gg \underline{n} <= (v2) \Rightarrow \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\underline{n}]+(-u1))| <= |((fx)[\underline{n}]+1)|, |((fx)[\underline{n}]+1)|, |((fx)[\underline{n}]+$
$(-u1))|))n)n)n; \mathrm{ExistMP} \rhd \forall_{\mathrm{obj}}(v2) \colon (v2) <= \underline{n} \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= (v2))n \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \rhd \dot{\neg}(\forall_{\mathrm{obj}}(v1) \colon \dot{\neg}(\forall_{\mathrm{obj}}(v2) \colon (v2) <= \underline{n} \Rightarrow$
$\dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n)n)n \gg \dot{\neg}(\underline{n} <=$
$(v2))n \Rightarrow \dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$(v1))n)n)n; \mathrm{LessThanMax} \rhd \underline{n} <= (v2) \Rightarrow \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|))n)n)n \rhd \dot{\neg}(\underline{n} <= (v2))n \Rightarrow$
$\dot{\neg}(|(fx)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| = (v1))n)n)n \gg \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), (v1)))n)n)n; \mathrm{Gen} \rhd$
$\dot{\neg}(|(fx)[(v2)]| <= \mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), (v1)))n)n)n \gg$
$\forall_{\mathrm{obj}}(v2) \colon \dot{\neg}(|(fx)[(v2)]| <= \mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] +$
$(-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <= |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] +$
$(-u1))|), (v1)))n)n)n; \mathrm{IntroExist} \; @ \; \mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}]+1)|, |((fx)[\underline{n}]+1)|, |((fx)[\underline{n}]+(-u1))|), (v1)) \rhd \forall_{\mathrm{obj}}(v2) \colon \dot{\neg}(|(fx)[(v2)]| <=$
$\mathrm{if}((v1) <= \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), \mathrm{if}(|((fx)[\underline{n}] + (-u1))| <=$
$|((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + 1)|, |((fx)[\underline{n}] + (-u1))|), (v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|(fx)[(v2)]| =$

$\mathrm{if}(\overline{(v1)} <= \mathrm{if}(|((\overline{fx})[\underline{n}] + (-u1))| <=$
$|((\overline{fx})[\underline{n}] + 1)|, |((\overline{fx})[\underline{n}] + 1)|, |((\overline{fx})[\underline{n}] + (-u1))|), \mathrm{if}(|((\overline{fx})[\underline{n}] + (-u1))| <=$
$|((\overline{fx})[\underline{n}] + 1)|, |((\overline{fx})[\underline{n}] + 1)|, |((\overline{fx})[\underline{n}] + (-u1))|), \overline{(v1)}))n)n)n \gg$
$\dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| =$
$\underline{(v1)})n)n)n)n)n\rceil, p_0, c)]$

[F − Bounded $\overset{\mathrm{stmt}}{\to}$ SystemQ $\vdash$
$\forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{n}: \forall\underline{(\epsilon)}: \forall\underline{(fx)}: \dot{\neg}(\forall_{obj}\underline{(v1)}: \dot{\neg}(\forall_{obj}\underline{(v2)}: \dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|\underline{(fx)}[\underline{(v2)}]| = \underline{(v1)})n)n)n)n)n]$

[F − Bounded $\overset{\mathrm{tex}}{\to}$ “F-Bounded”]

[F − Bounded $\overset{\mathrm{pyk}}{\to}$ “lemma f-Bounded”]

SameFmultiplication(Helper)

[SameFmultiplication(Helper) $\overset{\mathrm{proof}}{\to}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{(v1)}: \forall\underline{(v2)}: \forall\underline{m}: \forall\underline{n}: \forall\underline{(\epsilon)}: \forall\underline{(fx)}: \forall\underline{(fy)}: \forall\underline{(fz)}: \forall_{obj}\underline{m}: \dot{\neg}(0 <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <=$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \vdash$
$\forall_{obj}\underline{(v2)}: \dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| = \underline{(v1)})n)n)n \vdash \dot{\neg}(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \vdash \underline{n} <= \underline{m} \vdash \mathrm{A4} @ \underline{(v2)} \rhd \forall_{obj}\underline{(v2)}: \dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| <=$
$\underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| = \underline{(v1)})n)n)n \gg \dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| = \underline{(v1)})n)n)n; 0 <= |x| \gg 0 <=$
$|\underline{(fz)}[\underline{(v2)}]|; \mathrm{leqLessTransitivity} \rhd 0 <= |\underline{(fz)}[\underline{(v2)}]| \rhd \dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| <= \underline{(v1)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|\underline{(fz)}[\underline{(v2)}]| = \underline{(v1)})n)n)n \gg \dot{\neg}(0 <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(v1)})n)n)n; \mathrm{PositiveInverted} \rhd \dot{\neg}(0 <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(v1)})n)n)n \gg$
$\dot{\neg}(0 <= \mathrm{rec}\underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}\underline{(v1)})n)n)n; \mathrm{PositiveFactors} \rhd \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \rhd \dot{\neg}(0 <= \mathrm{rec}\underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \mathrm{rec}\underline{(v1)})n)n)n \gg \dot{\neg}(0 <=$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n; \mathrm{A4} @ \underline{m} \rhd \forall_{obj}\underline{m}: \dot{\neg}(0 <=$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow$
$\dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \gg \dot{\neg}(0 <=$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow$
$\dot{\neg}(|((\underline{fx})[\underline{m}]+(-u\underline{(fy)}[\underline{m}]))| <= (\underline{(\epsilon)}*\mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{m}]+(-u\underline{(fy)}[\underline{m}]))| =$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n; \mathrm{MP2} \rhd \dot{\neg}(0 <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <=$
$(\underline{(\epsilon)}*\mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{m}]+(-u\underline{(fy)}[\underline{m}]))| = (\underline{(\epsilon)}*\mathrm{rec}\underline{(v1)}))n)n)n \rhd \dot{\neg}(0 <=$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \rhd \underline{n} <= \underline{m} \gg$
$\dot{\neg}(|((\underline{fx})[\underline{m}]+(-u\underline{(fy)}[\underline{m}]))| <= (\underline{(\epsilon)}*\mathrm{rec}\underline{(v1)}) \Rightarrow \dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{m}]+(-u\underline{(fy)}[\underline{m}]))| =$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n; \mathrm{reciprocalToLeft(Less)} \rhd \dot{\neg}(0 <= \underline{(v1)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(v1)})n)n)n \rhd \dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| = (\underline{(\epsilon)} * \mathrm{rec}\underline{(v1)}))n)n)n \gg$
$\dot{\neg}((|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| * \underline{(v1)}) <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}((|((\underline{fx})[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))| * \underline{(v1)}) = \underline{(\epsilon)})n)n)n; \mathrm{TimesF} \gg \{ph \in \{ph \in$

$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] =$
$(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}]); TimesF \gg \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] =$
$(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}]); EqNegated \triangleright \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] = (\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}]) \gg (-u\{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]) = (-u(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])); -x * y =$
$-(x * y) \gg ((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}]) = (-u(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])); eqSymmetry \triangleright$
$((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}]) = (-u(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])) \gg (-u(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])) =$
$((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}]); eqTransitivity \triangleright (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]) = (-u(\underline{(fy)}[\underline{m}] *$
$\underline{(fz)}[\underline{m}])) \triangleright (-u(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])) = ((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}]) \gg (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]) =$
$((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}]); AddEquations \triangleright \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] = (\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}]) \triangleright (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]) =$
$((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}]) \gg (\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}])) =$
$(((\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}]) + ((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}])); DistributionOutLeft \gg$
$(((\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}]) + ((-u\underline{(fy)}[\underline{m}]) * \underline{(fz)}[\underline{m}])) =$
$(\underline{(fz)}[\underline{m}] * (\underline{(fx)}[\underline{m}] + (-u\underline{(fy)}[\underline{m}]))); eqTransitivity \triangleright (\{ph \in \{ph \in$

$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}])) =$
$(((fx)[\underline{m}] * (fz)[\underline{m}]) + ((-u(fy)[\underline{m}]) * (fz)[\underline{m}])) \rhd (((fx)[\underline{m}] * (fz)[\underline{m}]) +$
$((-u(fy)[\underline{m}]) * (fz)[\underline{m}])) = ((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}]))) \gg (\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}])) =$
$((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}]))); SameNumerical \rhd (\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}])) =$
$((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}]))) \gg |(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$|((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}])))|; SplitNumericalProduct \gg$
$|((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}])))| = (|(fz)[\underline{m}]| * |((fx)[\underline{m}] +$
$(-u(fy)[\underline{m}]))|); timesCommutativity \gg (|(fz)[\underline{m}]| * |((fx)[\underline{m}] + (-u(fy)[\underline{m}]))|) =$
$(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| * |(fz)[\underline{m}]|); eqTransitivity4 \rhd |(\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$|((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}])))| \rhd |((fz)[\underline{m}] * ((fx)[\underline{m}] + (-u(fy)[\underline{m}])))| =$
$(|(fz)[\underline{m}]| * |((fx)[\underline{m}] + (-u(fy)[\underline{m}]))|) \rhd (|(fz)[\underline{m}]| * |((fx)[\underline{m}] + (-u(fy)[\underline{m}]))|) =$
$(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| * |(fz)[\underline{m}]|) \gg |(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$(|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|); \mathrm{eqSymmetry} \rhd |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| = (|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] +$
$(-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) \gg (|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) = |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))|; \mathrm{A4} @ \overline{(\mathrm{fz})}[\underline{\mathsf{m}}] \rhd$
$\forall_{\mathrm{obj}}\overline{(\mathrm{v2})}\colon \dot\neg(|\overline{(\mathrm{fz})}[\overline{(\mathrm{v2})}]| <= \overline{(\mathrm{v1})} \Rightarrow \dot\neg(\dot\neg(|\overline{(\mathrm{fz})}[\overline{(\mathrm{v2})}]| = \overline{(\mathrm{v1})})n)n)n \gg$
$|\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]| <= \overline{(\mathrm{v1})}; 0 <= |\mathrm{x}| \gg 0 <=$
$|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))|; \mathrm{LeqMultiplicationLeft} \rhd 0 <=$
$|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| \rhd |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]| <= \overline{(\mathrm{v1})} \gg$
$(|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) <= (|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| *$
$\overline{(\mathrm{v1})}); \mathrm{leqLessTransitivity} \rhd (|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) <=$
$(|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * \overline{(\mathrm{v1})}) \rhd \dot\neg(((|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * \overline{(\mathrm{v1})}) <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(((|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * \overline{(\mathrm{v1})}) = \overline{(\epsilon)})n)n)n \gg \dot\neg(((|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] +$
$(-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(((|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) =$
$\overline{(\epsilon)})n)n)n; \mathrm{SubLessLeft} \rhd (|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) = |(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| \rhd$
$\dot\neg(((|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(((|(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-u\overline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| * |\overline{(\mathrm{fz})}[\underline{\mathsf{m}}]|) = \overline{(\epsilon)})n)n)n \gg \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n\} \mid$

$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$(\epsilon))n)n)n; \forall(v1): \forall(v2): \forall\underline{m}: \forall\underline{n}: \forall(\epsilon): \forall(fx): \forall(fy): \forall(fz): \mathrm{Ded} \triangleright$
$\forall(v1): \forall(v2): \forall\underline{m}: \forall\underline{n}: \forall(\epsilon): \forall(fx): \forall(fy): \forall(fz): \forall_{obj}\underline{m}: \dot{\neg}(0 <= ((\epsilon) * rec(v1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = ((\epsilon) * rec(v1)))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| <=$
$((\epsilon) * rec(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| = ((\epsilon) * rec(v1)))n)n)n \vdash$
$\forall_{obj}(v2): \dot{\neg}(|(fz)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fz)[(v2)]| = (v1))n)n)n \vdash \dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \vdash \underline{n} <= \underline{m} \vdash \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| = (\epsilon))n)n)n \gg$
$\forall_{obj}\underline{m}: \dot{\neg}(0 <= ((\epsilon) * rec(v1)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * rec(v1)))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow$
$\dot{\neg}(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| <= ((\epsilon) * rec(v1)) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\underline{m}] + (-u(fy)[\underline{m}]))| = ((\epsilon) * rec(v1)))n)n)n \Rightarrow$
$\forall_{obj}(v2): \dot{\neg}(|(fz)[(v2)]| <= (v1) \Rightarrow \dot{\neg}(\dot{\neg}(|(fz)[(v2)]| = (v1))n)n)n \Rightarrow \dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$

$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[SameFmultiplication(Helper) $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(0 = (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <=$
$(\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| = (\underline{(\epsilon)} * \mathrm{rec}\underline{(\mathrm{v1})}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\mathrm{v2})}\colon \dot{\neg}\,(|\underline{(\mathrm{fz})}[\overline{(\mathrm{v2})}]| <= \underline{(\mathrm{v1})} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|\underline{(\mathrm{fz})}[\overline{(\mathrm{v2})}]| = \underline{(\mathrm{v1})})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}]$

[SameFmultiplication(Helper) $\overset{\mathrm{tex}}{\rightarrow}$ "SameFmultiplication(Helper)"]

[SameFmultiplication(Helper) $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma sameFmultiplication helper"]

SameFmultiplication

[SameFmultiplication $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil$SystemQ $\vdash$
$\forall\underline{(\mathrm{v1})}\colon \forall\underline{(\mathrm{v2})}\colon \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
Repetition $\rhd$ $\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};$ Ded $\rhd$ $\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$

$\forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n; \mathrm{A4}\ @((\epsilon) * \mathrm{rec}(\mathrm{v1})) \rhd \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n \gg \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= ((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))n)n)n)n)n; \mathrm{F} - \mathrm{Bounded} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v1})\colon \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(|(\mathrm{fz})[(\mathrm{v2})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fz})[(\mathrm{v2})]| =$
$(\mathrm{v1}))n)n)n)n)n; \mathrm{SameFmultiplication(Helper)} \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <=$
$((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <= ((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(|(\mathrm{fz})[(\mathrm{v2})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fz})[(\mathrm{v2})]| = (\mathrm{v1}))n)n)n \Rightarrow \dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n; \mathrm{ExistMP2} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= ((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon) * \mathrm{rec}(\mathrm{v1})))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| <=$
$((\epsilon) * \mathrm{rec}(\mathrm{v1})) \Rightarrow \dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\underline{\mathsf{m}}] + (-\mathrm{u}(\mathrm{fy})[\underline{\mathsf{m}}]))| = ((\epsilon) * \mathrm{rec}(\mathrm{v1})))n)n)n \Rightarrow$
$\forall_{\mathrm{obj}}(\mathrm{v2})\colon \dot{\neg}(|(\mathrm{fz})[(\mathrm{v2})]| <= (\mathrm{v1}) \Rightarrow \dot{\neg}(\dot{\neg}(|(\mathrm{fz})[(\mathrm{v2})]| = (\mathrm{v1}))n)n)n \Rightarrow \dot{\neg}(0 <=$
$(\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= (\overline{(\epsilon)} * \mathrm{rec}(\overline{\mathrm{v1}})) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\overline{(\epsilon)} * \mathrm{rec}(\overline{\mathrm{v1}})))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| <=$
$(\overline{(\epsilon)} * \mathrm{rec}(\overline{\mathrm{v1}})) \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]))| =$
$(\overline{(\epsilon)} * \mathrm{rec}(\overline{\mathrm{v1}})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}} \underline{(\mathrm{v1})}\colon \dot{\neg}(\forall_{\mathrm{obj}} \underline{(\mathrm{v2})}\colon \dot{\neg}(|\underline{(\mathrm{fz})}[\underline{(\mathrm{v2})}]| <= \underline{(\mathrm{v1})} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|\underline{(\mathrm{fz})}[\underline{(\mathrm{v2})}]| = \underline{(\mathrm{v1})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Gen} \rhd \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]))| = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$\underline{(\epsilon)})n)n)n; \mathrm{IntroExist} @ \underline{n} \rhd \forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <=$
$\underline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| = \underline{(\epsilon)})n)n)n \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{n}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\{ph \in$
$\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fy)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}]))| =$
$\underline{(\epsilon)})n)n)n)n)n; \mathrm{Gen} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\underline{n}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow$
$\underline{n} <= \underline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(e_{\mathrm{Ph}} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\underline{(fx)}[\underline{m}] * \underline{(fz)}[\underline{m}])\}\})n)n\}[\underline{m}] + (-u\{ph \in \{ph \in$

$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{op1}) \in N \Rightarrow \dot{\neg}(((\mathrm{op2}) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})n)n)n)n)n \gg \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{op1}) \in N \Rightarrow \dot{\neg}(((\mathrm{op2}) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{op1}) \in N \Rightarrow \dot{\neg}(((\mathrm{op2}) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})n)n)n)n)n; \mathrm{Ded} \triangleright \forall_{\mathrm{obj}}\underline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{op1}) \in N \Rightarrow \dot{\neg}(((\mathrm{op2}) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(((\mathrm{op1}) \in N \Rightarrow \dot{\neg}(((\mathrm{op2}) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u\{\mathrm{ph} \in \{\mathrm{ph} \in$
$P(P(\mathrm{Union}(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]))| =$
$\underline{(\epsilon)})n)n)n)n)n \gg \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in P(P(\mathrm{Union}(\{N,Q\}))) \mid$

$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n; \mathrm{Repetition} \rhd \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)})n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n \gg \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)})n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$

$$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}}=\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]*\underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))|= \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil,\mathrm{p}_0,\mathsf{c})]$$

$$[\mathrm{SameFmultiplication}\overset{\mathrm{stmt}}{\to}\mathrm{SystemQ}\vdash \forall\underline{(\mathrm{v1})}\colon\forall\underline{(\mathrm{v2})}\colon\forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\forall\underline{(\epsilon)}\colon\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\forall\underline{(\mathrm{fz})}\colon\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(0<= \overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]+(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]+(-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\vdash \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow \dot{\neg}\,(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot{\neg}\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}}= \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]*\underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathrm{u}\{\mathrm{ph}\in\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}}=\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]*\underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot{\neg}\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}}= \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]*\underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathrm{u}\{\mathrm{ph}\in\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}}=\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}]*\underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$$

[SameFmultiplication $\overset{\mathrm{tex}}{\to}$ “SameFmultiplication”]

[SameFmultiplication $\overset{\mathrm{pyk}}{\to}$ “lemma sameFmultiplication”]

EqMultiplication(R)

$$[\mathrm{EqMultiplication}(\mathrm{R})\overset{\mathrm{proof}}{\to}\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil\mathrm{SystemQ}\vdash\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\forall\underline{(\mathrm{fz})}\colon\{\mathrm{ph}\in \mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot{\neg}\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in \mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot{\neg}\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{\neg}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow \overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]+(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]+(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow$$

$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \text{From} == \rhd\{ph \in P(\{ph \in P(\{ph \in$
$P(P(\text{Union}(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\text{Union}(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n; \text{SameFmultiplication} \rhd \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-u(fy)[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \gg$
$\forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{-}(|(\{ph \in \{ph \in P(P(\text{Union}(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(\text{Union}(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$

$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n; To == \triangleright\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n \gg \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqReflexivity} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqReflexivity} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}; \mathrm{eqTransitivity4} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$

$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\ (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}((\mathrm{r1}) = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\ \forall_{\mathrm{obj}}(\mathrm{f2}){:}\ \forall_{\mathrm{obj}}(\mathrm{f3}){:}\ \forall_{\mathrm{obj}}(\mathrm{f4}){:}\ \{\{(\mathrm{f1}),(\mathrm{f1})\},\{(\mathrm{f1}),(\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}),(\mathrm{f3})\},\{(\mathrm{f3}),(\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\ (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\ \dot{-}(\{\{(\mathrm{s1}),(\mathrm{s1})\},\{(\mathrm{s1}),(\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\ \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\ \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\ \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\ (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}((\mathrm{r1}) = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\ \forall_{\mathrm{obj}}(\mathrm{f2}){:}\ \forall_{\mathrm{obj}}(\mathrm{f3}){:}\ \forall_{\mathrm{obj}}(\mathrm{f4}){:}\ \{\{(\mathrm{f1}),(\mathrm{f1})\},\{(\mathrm{f1}),(\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}),(\mathrm{f3})\},\{(\mathrm{f3}),(\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\ (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\ \dot{-}(\{\{(\mathrm{s1}),(\mathrm{s1})\},\{(\mathrm{s1}),(\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\ \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\ \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\ \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\ \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\ (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\ \dot{-}(\dot{-}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\ \dot{-}(\dot{-}(\dot{-}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{-}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}], p_0, \mathsf{c})]$

$[EqMultiplication(R) \stackrel{stmt}{\to} SystemQ \vdash \forall\underline{(fx)}: \forall\underline{(fy)}: \forall\underline{(fz)}: \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in$
$N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow$
$\dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow$
$\dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow$
$\dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow$
$\dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg((\overline{(op1)} \in N \Rightarrow \dot\neg((\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[EqMultiplication(R) $\overset{\mathrm{tex}}{\rightarrow}$ “EqMultiplication(R)”]

[EqMultiplication(R) $\overset{\mathrm{pyk}}{\rightarrow}$ “lemma eqMultiplication(R)”]

EqMultiplicationLeft(R)

[EqMultiplicationLeft(R) $\overset{\mathrm{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \mathrm{EqMultiplication(R)} \rhd \{ph \in$
$\mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg \{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=$
$\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg\,(0<=\overline{(\epsilon)}\Rightarrow\dot\neg\,(\dot\neg\,(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg\,(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg\,(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg\,(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\};\mathrm{TimesCommutativity}(\mathrm{R})\gg\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |$
$\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow\dot\neg\,(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg\,(0<=\overline{(\epsilon)}\Rightarrow\dot\neg\,(\dot\neg\,(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg\,(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg\,(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fz})[\underline{\mathsf{m}}]*(\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg\,(\mathsf{e}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fz})[\underline{\mathsf{m}}]*(\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=$
$\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$

$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; TimesCommutativity(R) \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}(r1)\colon (r1) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow$
$\dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}(f1)\colon \forall_{\mathrm{obj}}(f2)\colon \forall_{\mathrm{obj}}(f3)\colon \forall_{\mathrm{obj}}(f4)\colon \{\{(f1),(f1)\},\{(f1),(f2)\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(f3),(f3)\},\{(f3),(f4)\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}(s1)\colon (s1) \in N \Rightarrow \dot\neg(\forall_{\mathrm{obj}}(s2)\colon \dot\neg(\{\{(s1),(s1)\},\{(s1),(s2)\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(r1)\colon (r1) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}(f1)\colon \forall_{\mathrm{obj}}(f2)\colon \forall_{\mathrm{obj}}(f3)\colon \forall_{\mathrm{obj}}(f4)\colon \{\{(f1),(f1)\},\{(f1),(f2)\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(f3),(f3)\},\{(f3),(f4)\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}(s1)\colon (s1) \in N \Rightarrow \dot\neg(\forall_{\mathrm{obj}}(s2)\colon \dot\neg(\{\{(s1),(s1)\},\{(s1),(s2)\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}(op1)\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1) \in N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$

(ϵ))n)n)n)n)n}; ==Transitivity ▷ {ph ∈ P({ph ∈ P({ph ∈
P(P(Union({N, Q}))) | ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) |
∸(∸($\forall_{obj}$(r1): (r1) ∈ f_{Ph} ⇒ ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸($\forall_{obj}$(f1): $\forall_{obj}$(f2): $\forall_{obj}$(f3): $\forall_{obj}$(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ f_{Ph} ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ f_{Ph} ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸($\forall_{obj}$(s1): (s1) ∈ N ⇒ ∸($\forall_{obj}$(s2): ∸({{(s1), (s1)}, {(s1), (s2)}} ∈
f_{Ph})n)n)n)n}) | $\forall_{obj}$(ϵ): ∸($\forall_{obj}$n: ∸($\forall_{obj}$m: ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
n <= m ⇒ ∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$m: ∸(e_{Ph} =
{{m, m}, {m, ((fz)[m] ∗ (fx)[m])}})n)n}[m] + (−ud_{Ph}[m]))| <= (ϵ) ⇒
∸(∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$m: ∸(e_{Ph} =
{{m, m}, {m, ((fz)[m] ∗ (fx)[m])}})n)n}[m] + (−ud_{Ph}[m]))| = (ϵ))n)n)n)n)n} =
{ph ∈ P({ph ∈ P({ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) | ∸(∸($\forall_{obj}$(r1): (r1) ∈
f_{Ph} ⇒ ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸($\forall_{obj}$(f1): $\forall_{obj}$(f2): $\forall_{obj}$(f3): $\forall_{obj}$(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ f_{Ph} ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ f_{Ph} ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸($\forall_{obj}$(s1): (s1) ∈ N ⇒ ∸($\forall_{obj}$(s2): ∸({{(s1), (s1)}, {(s1), (s2)}} ∈
f_{Ph})n)n)n)n}) | $\forall_{obj}$(ϵ): ∸($\forall_{obj}$n: ∸($\forall_{obj}$m: ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒
n <= m ⇒ ∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$m: ∸(e_{Ph} =
{{m, m}, {m, ((fx)[m] ∗ (fz)[m])}})n)n}[m] + (−ud_{Ph}[m]))| <= (ϵ) ⇒
∸(∸(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸($\forall_{obj}$m: ∸(e_{Ph} =
{{m, m}, {m, ((fx)[m] ∗ (fz)[m])}})n)n}[m] + (−ud_{Ph}[m]))| =
(ϵ))n)n)n)n)n} ▷ {ph ∈ P({ph ∈ P({ph ∈ P(P(Union({N, Q}))) |
∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) | ∸(∸($\forall_{obj}$(r1): (r1) ∈
f_{Ph} ⇒ ∸($\forall_{obj}$(op1): ∸(∸($\forall_{obj}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸($\forall_{obj}$(f1): $\forall_{obj}$(f2): $\forall_{obj}$(f3): $\forall_{obj}$(f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ f_{Ph} ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ f_{Ph} ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸($\forall_{obj}$(s1): (s1) ∈ N ⇒ ∸($\forall_{obj}$(s2): ∸({{(s1), (s1)}, {(s1), (s2)}} ∈
f_{Ph})n)n)n)n}) | $\forall_{obj}$(ϵ): ∸($\forall_{obj}$n: ∸($\forall_{obj}$m: ∸(0 <= (ϵ) ⇒ ∸(∸(0 = (ϵ))n)n)n ⇒

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$

$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; ==Transitivity \triangleright \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fx)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1),(s1)\},\{(s1),(s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1),(s1)\},\{(s1),(s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fz})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{EqMultiplicationLeft}(\mathrm{R}) \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fz)[\underline{\mathsf{m}}] * (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

[EqMultiplicationLeft(R) $\stackrel{tex}{\rightarrow}$ "EqMultiplicationLeft(R)"]

[EqMultiplicationLeft(R) $\stackrel{pyk}{\rightarrow}$ "lemma eqMultiplicationLeft(R)"]

$x * 0 = 0(F)$

[$x * 0 = 0(F) \stackrel{proof}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil SystemQ \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(fx)}\colon TimesF \gg \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}]); NatType \gg \underline{\mathsf{m}} \in N; 0f \rhd \underline{\mathsf{m}} \in N \gg \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] = 0;$ lemma eqMultiplicationLeft $\rhd \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] = 0 \gg (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}]) = (\underline{(fx)}[\underline{\mathsf{m}}] * 0); x * 0 = 0 \gg (\underline{(fx)}[\underline{\mathsf{m}}] * 0) = 0;$ eqSymmetry $\rhd \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] = 0 \gg 0 = \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}];$ eqTransitivity5 $\rhd \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) \rhd (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]) = (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * 0) \rhd (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * 0) = 0 \rhd 0 =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]; \mathrm{Gen}\rhd\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$

$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}]; To =$
$f \rhd \forall_{obj}\underline{m}\colon \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}] \gg \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}\rceil, p_0, c)]$

$[x * 0 = 0(F) \stackrel{stmt}{\to} SystemQ \vdash \forall\underline{m}\colon \forall\overline{(fx)}\colon \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}]$

$[x * 0 = 0(F) \stackrel{tex}{\to}$ "x*0=0(F)"]

$[x * 0 = 0(F) \stackrel{pyk}{\to}$ "lemma x*0=0(F)"]

x * 0 = 0(R)

$[x * 0 = 0(R) \stackrel{proof}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall\overline{(fx)}\colon x * 0 = 0(F) \gg \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}; \mathrm{lemma} =\mathrm{f\ to\ sameF} \rhd \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\} = \{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\} \gg$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n; \mathrm{To} == \rhd\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{m},\underline{m}\},\{\underline{m},((fx)[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},((fx)[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-u\{ph \in \{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{m}]))| =$
$\overline{(\epsilon)})n)n)n)n)n \gg \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$f_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{n}\colon \dot\neg(\forall_{obj}\overline{m}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},((fx)[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},((fx)[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; \mathrm{eqReflexivity} \gg \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})n)n)n)n)n \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}; == Transitivity \triangleright \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\overline{(crs1)}\colon \dot{-}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\})))\ \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\})))\ \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$

$(\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in$
$N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}; \mathrm{Repetition} \rhd \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1)\colon \forall_{obj}(f2)\colon \forall_{obj}(f3)\colon \forall_{obj}(f4)\colon \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2)\colon \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1)\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{obj}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1)\colon (r1) \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}: \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}: \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}: \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \gg$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}: \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}: \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}: \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}: \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}: \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}: \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}: \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)}))n)n)n)n)n\}], p_0, \mathsf{c})]$

$[x * 0 = 0(R) \overset{stmt}{\to} SystemQ \vdash \forall\underline{(fx)}\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)}))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$

$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{n}: \dot{\neg}(\forall_{obj}\overline{m}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

$[x * 0 = 0(R) \stackrel{tex}{\to}$ "x*0=0(R)"]

$[x * 0 = 0(R) \stackrel{pyk}{\to}$ "lemma x*0=0(R)"]

LessMultiplication(F)(Helper2)

$[LessMultiplication(F)(Helper2) \stackrel{proof}{\to} \lambda c.\lambda x.\mathcal{P}(\lceil SystemQ \vdash \forall\underline{x}: \forall\underline{y}: \forall\underline{z}: \forall\underline{u}: \forall\underline{v}: \dot{\neg}(0 <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{u})n)n)n \vdash \dot{\neg}(0 <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{v})n)n)n \vdash \underline{x} <= (\underline{y} + (-u\underline{u})) \vdash 0 <= (\underline{z} + (-u\underline{v})) \vdash negativeToLeft(Leq) \rhd \underline{x} <= (\underline{y} + (-u\underline{u})) \gg (\underline{x} + \underline{u}) <= \underline{y}; plusCommutativity \gg (\underline{x} + \underline{u}) = (\underline{u} + \underline{x}); subLeqLeft \rhd (\underline{x} + \underline{u}) = (\underline{u} + \underline{x}) \rhd (\underline{x} + \underline{u}) <= \underline{y} \gg (\underline{u} + \underline{x}) <= \underline{y}; PositiveToRight(Leq) \rhd (\underline{u} + \underline{x}) <= \underline{y} \gg \underline{u} <= (\underline{y} + (-u\underline{x})); negativeToLeft(Leq)(1term) \rhd 0 <= (\underline{z} + (-u\underline{v})) \gg \underline{v} <= \underline{z}; LessLeq \rhd \dot{\neg}(0 <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{u})n)n)n \gg 0 <= \underline{u}; LessLeq \rhd \dot{\neg}(0 <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{v})n)n)n \gg 0 <= \underline{v}; MultiplyEquations(Leq) \rhd 0 <= \underline{u} \rhd 0 <= \underline{v} \rhd \underline{u} <= (\underline{y} + (-u\underline{x})) \rhd \underline{v} <= \underline{z} \gg (\underline{u} * \underline{v}) <= ((\underline{y} + (-u\underline{x})) * \underline{z}); timesCommutativity \gg ((\underline{y} + (-u\underline{x})) * \underline{z}) = (\underline{z} * (\underline{y} + (-u\underline{x}))); DistributionLeft \gg (\underline{z} * (\underline{y} + (-u\underline{x}))) = ((\underline{y} * \underline{z}) + ((-u\underline{x}) * \underline{z})); -x * y = -(x * y) \gg ((-u\underline{x}) * \underline{z}) = (-u(\underline{x} * \underline{z})); lemma\ eqAdditionLeft \rhd ((-u\underline{x}) * \underline{z}) = (-u(\underline{x} * \underline{z})) \gg ((\underline{y} * \underline{z}) + ((-u\underline{x}) * \underline{z})) = ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))); eqTransitivity4 \rhd ((\underline{y} + (-u\underline{x})) * \underline{z}) = (\underline{z} * (\underline{y} + (-u\underline{x}))) \rhd (\underline{z} * (\underline{y} + (-u\underline{x}))) = ((\underline{y} * \underline{z}) + ((-u\underline{x}) * \underline{z})) \rhd ((\underline{y} * \underline{z}) + ((-u\underline{x}) * \underline{z})) = ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))) \gg ((\underline{y} + (-u\underline{x})) * \underline{z}) = ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))); subLeqRight \rhd ((\underline{y} + (-u\underline{x})) * \underline{z}) = ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))) \rhd (\underline{u} * \underline{v}) <= ((\underline{y} + (-u\underline{x})) * \underline{z}) \gg (\underline{u} * \underline{v}) <= ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))); negativeToLeft(Leq) \rhd (\underline{u} * \underline{v}) <= ((\underline{y} * \underline{z}) + (-u(\underline{x} * \underline{z}))) \gg ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) <= (\underline{y} * \underline{z}); plusCommutativity \gg ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) = ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})); subLeqLeft \rhd ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) = ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})) \rhd ((\underline{u} * \underline{v}) + (\underline{x} * \underline{z})) <= (\underline{y} * \underline{z}) \gg ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})) <= (\underline{y} * \underline{z}); PositiveToRight(Leq) \rhd ((\underline{x} * \underline{z}) + (\underline{u} * \underline{v})) <= (\underline{y} * \underline{z}) \gg (\underline{x} * \underline{z}) <= ((\underline{y} * \underline{z}) + (-u(\underline{u} * \underline{v})))\rceil, p_0, c)]$

$[LessMultiplication(F)(Helper2) \stackrel{stmt}{\to} SystemQ \vdash \forall\underline{x}: \forall\underline{y}: \forall\underline{z}: \forall\underline{u}: \forall\underline{v}: \dot{\neg}(0 <= \underline{u} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{u})n)n)n \vdash \dot{\neg}(0 <= \underline{v} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{v})n)n)n \vdash \underline{x} <= (\underline{y} + (-u\underline{u})) \vdash 0 <= (\underline{z} + (-u\underline{v})) \vdash (\underline{x} * \underline{z}) <= ((\underline{y} * \underline{z}) + (-u(\underline{u} * \underline{v})))]$

$[LessMultiplication(F)(Helper2) \stackrel{tex}{\to}$ "LessMultiplication(F)(Helper2)"]

[LessMultiplication(F)(Helper2) $\stackrel{\text{pyk}}{\to}$ "lemma lessMultiplication(F) helper2"]

LessMultiplication(F)(Helper)

[LessMultiplication(F)(Helper) $\stackrel{\text{proof}}{\to}$ λc.λx.$\mathcal{P}$(⌈SystemQ ⊢
∀m: ∀n: ∀$(\epsilon)_1$: ∀(ϵ2): ∀(fx): ∀(fy): ∀(fz): $\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= $(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 =
$(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u$(\epsilon)_1$)))n)n ⊢
$\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= (ϵ2) ⇒ ¬̇ (¬̇ (0 = (ϵ2))n)n)n ⇒ ¬̇ (n <= m ⇒ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\text{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\text{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\text{obj}}$(crs1): ¬̇ (c_{Ph} = {{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <=
((fz)[m] + (−u(ϵ2)))n)n ⊢ A4 @ m ▷ $\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= $(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 =
$(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u$(\epsilon)_1$)))n)n ≫ ¬̇ (¬̇ (0 <=
$(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 = $(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <=
((fy)[m] + (−u$(\epsilon)_1$)))n)n; A4 @ m ▷ $\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= (ϵ2) ⇒ ¬̇ (¬̇ (0 =
(ϵ2))n)n)n ⇒ ¬̇ (n <= m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
¬̇ ($\forall_{\text{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\text{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒ ¬̇ ((op2) ∈ Q)n)n ⇒
¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ¬̇ ($\forall_{\text{obj}}$(crs1): ¬̇ (c_{Ph} =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2)))n)n ≫
¬̇ (¬̇ (0 <= (ϵ2) ⇒ ¬̇ (¬̇ (0 = (ϵ2))n)n)n ⇒ ¬̇ (n <= m ⇒ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\text{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\text{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\text{obj}}$(crs1): ¬̇ (c_{Ph} = {{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <=
((fz)[m] + (−u(ϵ2)))n)n; FirstConjunct ▷ ¬̇ (¬̇ (0 <= $(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 =
$(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u$(\epsilon)_1$)))n)n ≫ ¬̇ (0 <=
$(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 = $(\epsilon)_1$)n)n)n; FirstConjunct ▷ ¬̇ (¬̇ (0 <= (ϵ2) ⇒ ¬̇ (¬̇ (0 =
(ϵ2))n)n)n ⇒ ¬̇ (n <= m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
¬̇ ($\forall_{\text{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\text{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒ ¬̇ ((op2) ∈ Q)n)n ⇒
¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ¬̇ ($\forall_{\text{obj}}$(crs1): ¬̇ (c_{Ph} =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2)))n)n ≫ ¬̇ (0 <=
(ϵ2) ⇒ ¬̇ (¬̇ (0 = (ϵ2))n)n)n; PositiveFactors ▷ ¬̇ (0 <= $(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 =
$(\epsilon)_1$)n)n)n ▷ ¬̇ (0 <= (ϵ2) ⇒ ¬̇ (¬̇ (0 = (ϵ2))n)n)n ≫ ¬̇ (0 <= ($(\epsilon)_1$ ∗ (ϵ2)) ⇒
¬̇ (¬̇ (0 = ($(\epsilon)_1$ ∗
(ϵ2)))n)n)n; ∀m: ∀n: ∀$(\epsilon)_1$: ∀(ϵ2): ∀(fx): ∀(fy): ∀(fz): $\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= $(\epsilon)_1$ ⇒
¬̇ (¬̇ (0 = $(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u$(\epsilon)_1$)))n)n ⊢
$\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= (ϵ2) ⇒ ¬̇ (¬̇ (0 = (ϵ2))n)n)n ⇒ ¬̇ (n <= m ⇒ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ¬̇ ($\forall_{\text{obj}}$(op1): ¬̇ (¬̇ ($\forall_{\text{obj}}$(op2): ¬̇ (¬̇ (¬̇ ((op1) ∈ N ⇒
¬̇ ((op2) ∈ Q)n)n ⇒ ¬̇ (a_{Ph} = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
¬̇ ($\forall_{\text{obj}}$(crs1): ¬̇ (c_{Ph} = {{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <=
((fz)[m] + (−u(ϵ2)))n)n ⊢ n <= m ⊢ A4 @ m ▷ $\forall_{\text{obj}}$m: ¬̇ (¬̇ (0 <= $(\epsilon)_1$ ⇒
¬̇ (¬̇ (0 = $(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u$(\epsilon)_1$)))n)n ≫
¬̇ (¬̇ (0 <= $(\epsilon)_1$ ⇒ ¬̇ (¬̇ (0 = $(\epsilon)_1$)n)n)n ⇒ ¬̇ (n <= m ⇒ (fx)[m] <=

((fy)[m] + (−u(ϵ)$_1$)))n)n; A4 @ m ▷ ∀$_{\mathrm{obj}}$m: ∸(∸(0 <= (ϵ2) ⇒ ∸(∸(0 =
(ϵ2))n)n)n ⇒ ∸(n <= m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2))))n)n ≫
∸(∸(0 <= (ϵ2) ⇒ ∸(∸(0 = (ϵ2))n)n)n ⇒ ∸(n <= m ⇒ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |
∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ = {{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <=
((fz)[m] + (−u(ϵ2))))n)n; FirstConjunct ▷ ∸(∸(0 <= (ϵ)$_1$ ⇒ ∸(∸(0 =
(ϵ)$_1$)n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ)$_1$)))n)n ≫ ∸(0 <=
(ϵ)$_1$ ⇒ ∸(∸(0 = (ϵ)$_1$)n)n)n; FirstConjunct ▷ ∸(∸(0 <= (ϵ2) ⇒ ∸(∸(0 =
(ϵ2))n)n)n ⇒ ∸(n <= m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2))))n)n ≫ ∸(0 <=
(ϵ2) ⇒ ∸(∸(0 = (ϵ2))n)n)n; SecondConjunct ▷ ∸(∸(0 <= (ϵ)$_1$ ⇒ ∸(∸(0 =
(ϵ)$_1$)n)n)n ⇒ ∸(n <= m ⇒ (fx)[m] <= ((fy)[m] + (−u(ϵ)$_1$)))n)n ≫ n <= m ⇒
(fx)[m] <= ((fy)[m] + (−u(ϵ)$_1$)); MP ▷ n <= m ⇒ (fx)[m] <=
((fy)[m] + (−u(ϵ)$_1$)) ▷ n <= m ≫ (fx)[m] <=
((fy)[m] + (−u(ϵ)$_1$)); SecondConjunct ▷ ∸(∸(0 <= (ϵ2) ⇒ ∸(∸(0 =
(ϵ2))n)n)n ⇒ ∸(n <= m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2))))n)n ≫ n <=
m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2))); MP ▷ n <=
m ⇒ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2))) ▷ n <= m ≫
{ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] <= ((fz)[m] + (−u(ϵ2))); NatType ≫ m ∈
N; 0f ▷ m ∈ N ≫ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒ ∸((op2) ∈ Q)n)n ⇒
∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸(∀$_{\mathrm{obj}}$(crs1): ∸(c$_{\mathrm{Ph}}$ =
{{(crs1), (crs1)}, {(crs1), 0}})n)n}[m] = 0; subLeqLeft ▷ {ph ∈ {ph ∈
P(P(Union({N, Q}))) | ∸(∀$_{\mathrm{obj}}$(op1): ∸(∸(∀$_{\mathrm{obj}}$(op2): ∸(∸(∸((op1) ∈ N ⇒
∸((op2) ∈ Q)n)n ⇒ ∸(a$_{\mathrm{Ph}}$ = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} |

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}{:}\,\dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}{:}\,\dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))) \gg 0 <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2)));\mathrm{LessMultiplication}(\mathrm{F})(\mathrm{Helper2}) \rhd \dot\neg(0 <= (\epsilon)_1 \Rightarrow$
$\dot\neg(\dot\neg(0 = (\epsilon)_1)\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot\neg(0 <= (\epsilon 2) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon 2))\mathrm{n})\mathrm{n})\mathrm{n} \rhd (\mathrm{fx})[\underline{\mathsf{m}}] <=$
$((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)) \rhd 0 <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2))) \gg ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) <=$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2))));\mathrm{lemma\ timesF}(\mathrm{Sym}) \gg$
$((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}];\mathrm{subLeqLeft} \rhd ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \rhd ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) <=$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))) \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2))));\mathrm{lemma\ timesF}(\mathrm{Sym}) \gg$
$((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}];\mathrm{eqAddition} \rhd ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) =$
$\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] \gg$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))) = (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2))));\mathrm{subLeqRight} \rhd$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))) = (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))) \rhd \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$(((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}]) + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))) \gg \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}{:}\,\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}{:}\,\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u((\epsilon)_1 *$
$(\epsilon 2)))); \forall\underline{m}\colon \forall\underline{n}\colon \forall(\epsilon)_1\colon \forall(\epsilon 2)\colon \forall(fx)\colon \forall(fy)\colon \forall(fz)\colon Ded \rhd$
$\forall\underline{m}\colon \forall\underline{n}\colon \forall(\epsilon)_1\colon \forall(\epsilon 2)\colon \forall(fx)\colon \forall(fy)\colon \forall(fz)\colon \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow (fx)[\underline{m}] <= ((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n \vdash$
$\forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}] <=$
$((fz)[\underline{m}] + (-u(\epsilon 2))))n)n \vdash \dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon)_1 * (\epsilon 2)))n)n)n \gg \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <=$
$\underline{m} \Rightarrow (fx)[\underline{m}] <= ((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n \Rightarrow \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}] <= ((fz)[\underline{m}] + (-u(\epsilon 2))))n)n \Rightarrow \dot{\neg}(0 <=$
$((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon)_1 * (\epsilon 2)))n)n)n; Ded \rhd$
$\forall\underline{m}\colon \forall\underline{n}\colon \forall(\epsilon)_1\colon \forall(\epsilon 2)\colon \forall(fx)\colon \forall(fy)\colon \forall(fz)\colon \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow (fx)[\underline{m}] <= ((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n \vdash$
$\forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}] <=$
$((fz)[\underline{m}] + (-u(\epsilon 2))))n)n \vdash \underline{n} <= \underline{m} \vdash \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fy)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] + (-u((\epsilon)_1 *$
$(\epsilon 2)))) \gg \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow$
$(fx)[\underline{m}] <= ((fy)[\underline{m}] + (-u(\epsilon)_1)))n)n \Rightarrow \forall_{obj}\underline{m}\colon \dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{n} <= \underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}] <= ((fz)[\underline{m}] + (-u(\epsilon 2))))n)n \Rightarrow \underline{n} <=$
$\underline{m} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}\colon \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((fx)[\underline{m}] * (fz)[\underline{m}])\}\})n)n\}[\underline{m}] <= (\{ph \in \{ph \in$

$\mathrm{P(P(Union(\{N,Q\})))} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\epsilon)_1 *$
$(\epsilon 2)))); \mathrm{JoinConjuncts(2conditions)} \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))n)n \Rightarrow$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P(P(Union(\{N,Q\})))} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}){:}\,\dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\underline{\mathsf{m}}] <=$
$((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2)))n)n \Rightarrow \dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$((\epsilon)_1 * (\epsilon 2)))n)n)n \rhd \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))n)n \Rightarrow \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P(P(Union(\{N,Q\})))} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}){:}\,\dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2)))n)n \Rightarrow \underline{\mathsf{n}} <=$
$\underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P(P(Union(\{N,Q\})))} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P(P(Union(\{N,Q\})))} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\epsilon)_1 *$
$(\epsilon 2)))) \gg \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow$
$(\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))n)n \Rightarrow \forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P(P(Union(\{N,Q\})))} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1}){:}\,\dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}),(\mathrm{crs1})\},\{(\mathrm{crs1}),0\}\})n)n\}[\underline{\mathsf{m}}] <= ((\mathrm{fz})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon 2)))n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= ((\epsilon)_1 * (\epsilon 2)) \Rightarrow \dot{\neg}(\dot{\neg}(0 = ((\epsilon)_1 * (\epsilon 2)))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P(P(Union(\{N,Q\})))} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in$
$\mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P(P(Union(\{N,Q\})))} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}),(\mathrm{op1})\},\{(\mathrm{op1}),(\mathrm{op2})\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\epsilon)_1 * (\epsilon 2)))))n)n], \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{LessMultiplication(F)(Helper)} \xrightarrow{\mathrm{stmt}} \mathrm{SystemQ} \vdash$
$\forall\underline{\mathsf{m}}{:}\,\forall\underline{\mathsf{n}}{:}\,\forall(\epsilon)_1{:}\,\forall(\epsilon 2){:}\,\forall(\mathrm{fx}){:}\,\forall(\mathrm{fy}){:}\,\forall(\mathrm{fz}){:}\,\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon)_1 \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\epsilon)_1)n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\underline{\mathsf{m}}] <= ((\mathrm{fy})[\underline{\mathsf{m}}] + (-\mathrm{u}(\epsilon)_1)))n)n \Rightarrow$
$\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(0 <= (\epsilon 2) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon 2))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fz})}[\underline{\mathsf{m}}] + (-\mathrm{u}(\underline{(\epsilon 2)})))\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + (-\mathrm{u}((\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))))\mathrm{n})\mathrm{n}]$

[LessMultiplication(F)(Helper) $\overset{\mathrm{tex}}{\to}$ “LessMultiplication(F)(Helper)”]

[LessMultiplication(F)(Helper) $\overset{\mathrm{pyk}}{\to}$ “lemma lessMultiplication(F) helper”]

LessMultiplication(F)

[LessMultiplication(F) $\overset{\mathrm{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \forall \underline{(\epsilon)_1}\colon \forall \underline{(\epsilon 2)}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{fy})}\colon \forall \underline{(\mathrm{fz})}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\mathrm{Repetition} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Ded} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{(\epsilon)_1}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)_1})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\underline{\mathsf{m}}] <= ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + (-\mathrm{u}\underline{(\epsilon)_1})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}(\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] +$
$(-u(\overline{(\epsilon)})))n)n)n)n)n)n; Ded \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u(\overline{(\epsilon)})))n)n)n)n)n)n \gg$
$\dot{\neg}(\forall_{obj}\underline{(\epsilon 2)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon 2)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon 2)})n)n)n \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <=$
$(\underline{(fz)}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon 2)})))n)n)n)n)n)n; LessMultiplication(F)(Helper) \gg$
$\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)_1})n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(fx)}[\underline{\mathsf{m}}] <=$
$(\underline{(fy)}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon)_1})))n)n \Rightarrow \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon 2)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon 2)})n)n)n \Rightarrow$
$\dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <= (\underline{(fz)}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon 2)})))n)n \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)})))))n)n; TwiceExistMP2 \rhd \forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)_1} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)_1})n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \underline{(fx)}[\underline{\mathsf{m}}] <= (\underline{(fy)}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon)_1})))n)n \Rightarrow$
$\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon 2)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon 2)})n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{\mathsf{m}}] <=$
$(\underline{(fz)}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon 2)})))n)n \Rightarrow \dot{\neg}(\dot{\neg}(0 <= (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$

$\dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fy)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] + (-u(\overline{(\epsilon)_1} *$
$\overline{(\epsilon 2)})))))\rm n)n \rhd \dot\neg(\forall_{\rm obj}\overline{(\epsilon)_1}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)_1} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)_1})\rm n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \overline{(\rm fx)}[\underline{\mathsf{m}}] <=$
$(\overline{(\rm fy)}[\underline{\mathsf{m}}] + (-u\overline{(\epsilon)_1})))\rm n)n)n)n)n)n \rhd \dot\neg(\forall_{\rm obj}\overline{(\epsilon 2)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon 2)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon 2)})\rm n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$\rm P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow$
$\dot\neg(\overline{(\rm op2)} \in \rm Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\rm obj}\overline{(\rm crs1)}\colon \dot\neg(\mathsf{c}_{\rm Ph} = \{\{\overline{(\rm crs1)},\overline{(\rm crs1)}\},\{\overline{(\rm crs1)},0\}\})\rm n)n\}[\underline{\mathsf{m}}] <=$
$(\overline{(\rm fz)}[\underline{\mathsf{m}}] + (-u\overline{(\epsilon 2)})))\rm n)n)n)n)n)n \gg \dot\neg(\dot\neg(0 <= (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\overline{(\epsilon)_1} * \overline{(\epsilon 2)}))\rm n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow \dot\neg(\overline{(\rm op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fx)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\rm P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow$
$\dot\neg(\overline{(\rm op2)} \in \rm Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fy)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] + (-u(\overline{(\epsilon)_1} *$
$\overline{(\epsilon 2)})))))\rm n)n; Gen \rhd \dot\neg(\dot\neg(0 <= (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}) \Rightarrow \dot\neg(\dot\neg(0 = (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}))\rm n)n)n \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow \dot\neg(\overline{(\rm op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fx)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\rm P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow$
$\dot\neg(\overline{(\rm op2)} \in \rm Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fy)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] + (-u(\overline{(\epsilon)_1} *$
$\overline{(\epsilon 2)})))))\rm n)n \gg \forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}) \Rightarrow \dot\neg(\dot\neg(0 = (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}))\rm n)n)n \Rightarrow$
$\dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow \dot\neg(\overline{(\rm op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fx)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\rm P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow$
$\dot\neg(\overline{(\rm op2)} \in \rm Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fy)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] + (-u(\overline{(\epsilon)_1} *$
$\overline{(\epsilon 2)})))))\rm n)n; IntroExist\ @\ \underline{\mathsf{n}} \rhd \forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\overline{(\epsilon)_1} * \overline{(\epsilon 2)}))\rm n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow \dot\neg(\overline{(\rm op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fx)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$\rm P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{\rm obj}\overline{(\rm op1)}\colon \dot\neg(\dot\neg(\forall_{\rm obj}\overline{(\rm op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\rm op1)} \in N \Rightarrow$
$\dot\neg(\overline{(\rm op2)} \in \rm Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\rm Ph} = \{\{\overline{(\rm op1)},\overline{(\rm op1)}\},\{\overline{(\rm op1)},\overline{(\rm op2)}\}\})\rm n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\rm Ph} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\rm fy)}[\underline{\mathsf{m}}] * \overline{(\rm fz)}[\underline{\mathsf{m}}])\}\})\rm n)n\}[\underline{\mathsf{m}}] + (-u(\overline{(\epsilon)_1} *$
$\overline{(\epsilon 2)})))))\rm n)n \gg \dot\neg(\forall_{\rm obj}\underline{\mathsf{n}}\colon \dot\neg(\forall_{\rm obj}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= (\overline{(\epsilon)_1} * \overline{(\epsilon 2)}) \Rightarrow \dot\neg(\dot\neg(0 =$
$(\overline{(\epsilon)_1} * \overline{(\epsilon 2)}))\rm n)n)n \Rightarrow \dot\neg(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$

$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)}))))n)n)n)n; \mathrm{IntroExist} @(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \rhd \dot{\neg}(\forall_{obj}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$(\underline{(\epsilon)_1} * \underline{(\epsilon 2)}) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\underline{(\epsilon)_1} * \underline{(\epsilon 2)}))n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (-u(\underline{(\epsilon)_1} *$
$\underline{(\epsilon 2)}))))n)n)n)n \gg \dot{\neg}(\forall_{obj}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] +$
$(-u(\epsilon))))n)n)n)n)n)n; \mathrm{Ded} \rhd \dot{\neg}(\forall_{obj}\underline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \underline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\underline{\mathsf{n}} <= \underline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] +$
$(-u(\underline{(\epsilon)}))))n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u(\overline{(\epsilon)}))))n)n)n)n)n)n; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u(\overline{\epsilon})))n)n)n)n)n)n \gg \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u(\overline{\epsilon})))n)n)n)n)n)n\rceil, p_0, \mathsf{c})]$

$[\mathrm{LessMultiplication(F)} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\forall\underline{(\epsilon)_1}\colon\forall\underline{(\epsilon 2)}\colon\forall\underline{(fx)}\colon\forall\underline{(fy)}\colon\forall\underline{(fz)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fz)}[\overline{\mathsf{m}}] + (-u(\overline{\epsilon})))n)n)n)n)n)n \vdash \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u(\overline{\epsilon})))n)n)n)n)n)n \vdash \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u(\overline{\epsilon})))n)n)n)n)n)n]$

[LessMultiplication(F) $\stackrel{\mathrm{tex}}{\rightarrow}$ “LessMultiplication(F)”]

[LessMultiplication(F) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma lessMultiplication(F)”]

LessMultiplication(R)

$[\mathrm{LessMultiplication(R)} \stackrel{\mathrm{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(fx)}\colon\forall\underline{(fy)}\colon\forall\underline{(fz)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
Repetition $\rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};$ Repetition $\rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <=$
$((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <=$
$((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};$ LessMultiplication(F) $\rhd$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <=$
$((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <=$
$((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n};$ Repetition $\rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$

$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)}))))n)n)n)n)n)n \gg \dot{-}(\forall_{obj}\overline{(\epsilon)}: \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)}))))n)n)n)n)n)n; eqReflexivity \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}: \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{-}(\forall_{obj}\overline{\mathsf{n}}: \dot{-}(\forall_{obj}\overline{\mathsf{m}}: \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}: \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}: \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}: \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; \mathrm{SubLessLeft}(R) \rhd \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in N \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in N \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in N \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$

$\dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n \gg \dot{-}(\forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n; eqReflexivity \gg \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$

$\overline{(\epsilon)})n)n)n)n)n\}; \mathrm{SubLessRight(R)} \rhd \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}{:}\ \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}{:}\ \forall_{obj}\overline{(f2)}{:}\ \forall_{obj}\overline{(f3)}{:}\ \forall_{obj}\overline{(f4)}{:}\ \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}{:}\ \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}{:}\ \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}{:}\ \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\ \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}{:}\ \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}{:}\ \forall_{obj}\overline{(f2)}{:}\ \forall_{obj}\overline{(f3)}{:}\ \forall_{obj}\overline{(f4)}{:}\ \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}{:}\ \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}{:}\ \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}{:}\ \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\ \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \rhd \dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\ \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow \dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] <= (\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}{:}\ \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}{:}\ \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] * (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-u\overline{(\epsilon)}))))n)n)n)n)n)n \gg \dot{\neg}(\forall_{obj}\overline{(\epsilon)}{:}\ \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}{:}\ \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}{:}\ \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$