Logiweb codex of kvanti

Up Help

$\exists *\colon *$, $* \Rightarrow *$, kvanti, UniqueMember, UniqueMember(Type), SameSeries, A4, SameMember, Qclosed(Addition), Qclosed(Multiplication), FromCartProd(1), 1rule fromCartProd(2), constantRationalSeries($*$), cartProd($*$), Power($*$), binaryUnion($*, *$), SetOfRationalSeries, IsSubset($*, *$), (p$*, *$), (s$*$), $(\cdots)$, **Objekt-var**, **Ex-var**, **Ph-var**, **Værdi**, **Variabel**, Op($*$), Op($*, *$), $* \mathrel{\ddot{=}=} *$, ContainsEmpty($*$), Nat($*$), Dedu($*, *$), $\mathrm{Dedu}_0(*, *)$, $\mathrm{Dedu}_s(*, *, *)$, $\mathrm{Dedu}_1(*, *, *)$, $\mathrm{Dedu}_2(*, *, *)$, $\mathrm{Dedu}_3(*, *, *, *)$, $\mathrm{Dedu}_4(*, *, *, *)$, $\mathrm{Dedu}_4^*(*, *, *, *)$, $\mathrm{Dedu}_5(*, *, *)$, $\mathrm{Dedu}_6(*, *, *, *)$, $\mathrm{Dedu}_6^*(*, *, *, *)$, $\mathrm{Dedu}_7(*)$, $\mathrm{Dedu}_8(*, *)$, $\mathrm{Dedu}_8^*(*, *)$, Ex_1, Ex_2, Ex3, Ex_{10}, Ex_{20}, $*_{\mathrm{Ex}}$, $*^{\mathrm{Ex}}$, $\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ex}}$, $\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ex}}$, $\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ex}}$, $\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ex}}$, ph_1, ph_2, ph_3, $*_{\mathrm{Ph}}$, $*^{\mathrm{Ph}}$, $\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ph}}$, $\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ph}}$, $\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ph}}$, $\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ph}}$, $\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$, $\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}}$, $\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}}$, bs, OBS, $\mathcal{BS}$, Ø, SystemQ, MP, Gen, Repetition, Neg, Ded, ExistIntro, Extensionality, Ødef, PairDef, UnionDef, PowerDef, SeparationDef, AddDoubleNeg, RemoveDoubleNeg, AndCommutativity, AutoImply, Contrapositive, FirstConjunct, SecondConjunct, FromContradiction, FromDisjuncts, IffCommutativity, IffFirst, IffSecond, ImplyTransitivity, JoinConjuncts, MP2, MP3, MP4, MP5, MT, NegativeMT, Technicality, Weakening, WeakenOr1, WeakenOr2, Formula2Pair, Pair2Formula, Formula2Union, Union2Formula, Formula2Sep, Sep2Formula, Formula2Power, SubsetInPower, HelperPowerIsSub, PowerIsSub, (Switch)HelperPowerIsSub, (Switch)PowerIsSub, ToSetEquality, HelperToSetEquality(t), ToSetEquality(t), HelperFromSetEquality, FromSetEquality, HelperReflexivity, Reflexivity, HelperSymmetry, Symmetry, HelperTransitivity, Transitivity, ERisReflexive, ERisSymmetric, ERisTransitive, ØisSubset, HelperMemberNotØ, MemberNotØ, HelperUniqueØ, UniqueØ, ==Reflexivity, ==Symmetry, Helper==Transitivity, ==Transitivity, HelperTransferNotEq, TransferNotEq, HelperPairSubset, Helper(2)PairSubset, PairSubset, SamePair, SameSingleton, UnionSubset, SameUnion, SeparationSubset, SameSeparation, SameBinaryUnion, IntersectionSubset, SameIntersection, AutoMember, HelperEqSysNotØ, EqSysNotØ, HelperEqSubset, EqSubset, HelperEqNecessary, EqNecessary, HelperNoneEqNecessary, Helper(2)NoneEqNecessary, NoneEqNecessary, EqClassIsSubset, EqClassesAreDisjoint, AllDisjoint, AllDisjointImply, BSsubset, Union(BS/R)subset, UnionIdentity, EqSysIsPartition, (x1), (x2), (y1), (y2), (v1), (v2), (v3), (v4), (v2n), (m1), (m2), (n1), (n2), (n3), (ϵ), $(\epsilon)_1$, $(\epsilon 2)$, (fep), (fx), (fy), (fz), (fu), (fv), (fw), (rx), (ry), (rz), (ru), (sx), (sx1), (sy), (sy1), (sz), (sz1), (su), (su1), (fxs), (fys), (crs1), (f1), (f2), (f3), (f4), (op1), (op2), (r1), (s1), (s2), X_1, X_2, Y_1, Y_2, V_1, V_2, V_3, V_4, V_{2n}, M_1, M_2, N_1, N_2, N_3, ϵ, ϵ1, ϵ2, FX, FY, FZ, FU, FV, FW, FEP, RX, RY, RZ, RU, (SX), (SX1), (SY), (SY1), (SZ), (SZ1), (SU), (SU1), FXS, FYS, (F1), (F2), (F3), (F4), (OP1), (OP2), (R1), (S1), (S2), (EPob), (CRS1ob), (F1ob), (F2ob), (F3ob), (F4ob), (N1ob), (N2ob), (OP1ob), (OP2ob), (R1ob), (S1ob), (S2ob), ph_4, ph_5, ph_6, NAT, $RATIONAL_SERIES$, SERIES, SetOfReals, SetOfFxs, N, Q, X, xs, xaF, ysF, us, usFoelge, 0, 1, (−1), 2, 3, 1/2, 1/3, 2/3, 0f, 1f, 00, 01, (− − 01), 02, 01//02, PlusAssociativity(R), PlusAssociativity(R)XX, Plus0(R), Negative(R), Times1(R), lessAddition(R), PlusCommutativity(R), LeqAntisymmetry(R), LeqTransitivity(R), leqAddition(R), Distribution(R), A4(Axiom), InductionAxiom, EqualityAxiom, EqLeqAxiom, EqAdditionAxiom, EqMultiplicationAxiom, QisClosed(Reciprocal)(Imply), QisClosed(Reciprocal), QisClosed(Negative)(Imply), QisClosed(Negative), leqReflexivity, leqAntisymmetryAxiom, leqTransitivityAxiom, leqTotality, leqAdditionAxiom, leqMultiplicationAxiom, plusAssociativity, plusCommutativity, Negative, plus0, timesAssociativity, timesCommutativity, ReciprocalAxiom, times1, Distribution, 0not1, lemma eqLeq(R), TimesAssociativity(R), TimesCommutativity(R), (Adgic)SameR, Separation2formula(1), Separation2formula(2), Cauchy, PlusF, ReciprocalF, From ==, To ==, FromInR, PlusR(Sym), ReciprocalR(Axiom), LessMinus1(N), Nonnegative(N), US0, NextXS(UpperBound), NextXS(NoUpperBound), NextUS(UpperBound), NextUS(NoUpperBound), ExpZero, ExpPositive, ExpZero(R), ExpPositive(R), BSzero, BSpositive, UStelescope(Zero), UStelescope(Positive), EqAddition(R), FromLimit, ToUpperBound, FromUpperBound, USisUpperBound, 0not1(R), ExpUnbounded(R), FromLeq(Advanced)(N), FromLeastUpperBound, ToLeastUpperBound, XSisNotUpperBound, ysFGreater, ysFLess, SmallInverse, NatType, RationalType, SeriesType, Max, Numerical, NumericalF, MemberOfSeries(Imply), JoinConjuncts(2conditions), prop lemma imply negation, TND, FromNegatedImply, ToNegatedImply, FromNegated(2 ∗ Imply), FromNegatedAnd, FromNegatedOr, ToNegatedOr, FromNegations, From3Disjuncts, From2 ∗ 2Disjuncts, NegateDisjunct1, NegateDisjunct2, ExpandDisjuncts, SENC1, SENC2, LessLeq(R), MemberOfSeries, memberOfSeries(Type), ∗(exp)∗, R(∗), − − R(∗), rec∗, ∗/∗, ∗ ∩ ∗, ∗[∗], ∪∗, ∗ ∪ ∗, P(∗), {∗}, StateExpand(∗, ∗, ∗), extractSeries(∗), SetOfSeries(∗), − − Macro(∗), ExpandList(∗, ∗, ∗), ∗ ∗ Macro(∗), + + Macro(∗), << Macro(∗), ||Macro(∗), 01//Macro(∗), UB(∗, ∗), LUB(∗, ∗), BS(∗, ∗), UStelescope(∗, ∗), (∗), |f ∗ |, |r ∗ |, Limit(∗, ∗), Union(∗), IsOrderedPair(∗, ∗, ∗), IsRelation(∗, ∗, ∗), isFunction(∗, ∗, ∗), IsSeries(∗, ∗), IsNatural(∗, ∗), OrderedPair(∗, ∗), TypeNat(∗), TypeNat0(∗), TypeRational(∗), TypeRational0(∗), TypeSeries(∗, ∗), Typeseries0(∗, ∗), {∗, ∗}, ⟨∗, ∗⟩, (−u∗), $-_f$∗, (− − ∗), 1f/∗, 01//temp∗, ∗(∗, ∗), ReflRel(∗, ∗), SymRel(∗, ∗), TransRel(∗, ∗), EqRel(∗, ∗), $[* \in *]_*$, Partition(∗, ∗), (∗ ∗ ∗), $* *_f *$, ∗ ∗ ∗∗, (∗ + ∗), (∗ − ∗), $* +_f *$, $* -_f *$, ∗ + +∗, R(∗) − −R(∗), ∗ ∈ ∗, | ∗ |, if(∗, ∗, ∗), Max(∗, ∗), Max(∗, ∗), ∗ = ∗, ∗ ≠ ∗, ∗ <= ∗, ∗ < ∗, $* <_f *$, $* \leq_f *$, SF(∗, ∗), ∗ == ∗, ∗!! == ∗, ∗ << ∗, ∗ <<== ∗, ∗===∗, ∗ ⊆ ∗, $\dot{\neg}$ (∗)n, ∗ ∉ ∗, ∗ ≠ ∗,

$* \dot{\wedge} *$, $* \dot{\vee} *$, $\exists *\colon *$, $* \dot{\Leftrightarrow} *$, $\{\mathrm{ph} \in * \mid *\}$,

$\exists *\colon *$

[$\exists \mathsf{x}\colon \mathsf{y} \stackrel{\mathrm{tex}}{\to}$ "(AARRGGHH!-exist-bug!)"]

$* \Rightarrow *$

[$\mathsf{x} \Rightarrow \mathsf{y} \stackrel{\mathrm{tex}}{\to}$ "(i#1.
\Rightarrow #2.
)i"]

kvanti

[kvanti $\stackrel{\mathrm{prio}}{\to}$
Preassociative
[kvanti], [base], [bracket $*$ end bracket], [big bracket $*$ end bracket], [\$ $*$ \$], [**flush left** [$*$]], [x], [y], [z], [[$* \bowtie *$]], [[$* \stackrel{*}{\to} *$]], [pyk], [tex], [name], [prio], [$*$], [T], [if($*, *, *$)], [[$* \stackrel{*}{\Rightarrow} *$]], [val], [claim], [$\bot$], [f($*$)], [$(*)^{\mathrm{I}}$], [$\mathsf{F}$], [$\underline{0}$], [$\underline{1}$], [$\underline{2}$], [$\underline{3}$], [$\underline{4}$], [$\underline{5}$], [$\underline{6}$], [$\underline{7}$], [$\underline{8}$], [$\underline{9}$], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [$(*)^{\mathrm{M}}$], [If($*, *, *$)], [array{$*$} $*$ end array], [l], [c], [r], [empty], [$\langle * \mid * := * \rangle$], [$\mathcal{M}(*)$], [$\tilde{\mathcal{U}}(*)$], [$\mathcal{U}(*)$], [$\mathcal{U}^{\mathrm{M}}(*)$], [**apply**($*, *$)], [$\mathbf{apply}_1(*, *)$], [identifier($*$)], [$\mathrm{identifier}_1(*, *)$], [array-plus($*, *$)], [array-remove($*, *, *$)], [array-put($*, *, *, *$)], [array-add($*, *, *, *, *$)], [bit($*, *$)], [$\mathrm{bit}_1(*, *)$], [rack], ["vector"], ["bibliography"], ["dictionary"], ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"], ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"], ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"], ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"], [$\mathcal{E}(*, *, *)$], [$\mathcal{E}_2(*, *, *, *, *)$], [$\mathcal{E}_3(*, *, *, *)$], [$\mathcal{E}_4(*, *, *, *)$], [**lookup**($*, *, *$)], [**abstract**($*, *, *, *$)], [$\lceil * \rceil$], [$\mathcal{M}(*, *, *)$], [$\mathcal{M}_2(*, *, *, *)$], [$\mathcal{M}^*(*, *, *)$], [macro], [$\mathrm{s}_0$], [**zip**($*, *$)], [$\mathbf{assoc}_1(*, *, *)$], [$(*)^{\mathbf{P}}$], [self], [[$* \ddot{=} *$]], [[$* \doteq *$]], [[$* \acute{=} *$]], [[$* \stackrel{\mathrm{pyk}}{=} *$]], [[$* \stackrel{\mathrm{tex}}{=} *$]], [[$* \stackrel{\mathrm{name}}{=} *$]], [**Priority table**[$*$]], [$\tilde{\mathcal{M}}_1$], [$\tilde{\mathcal{M}}_2(*)$], [$\tilde{\mathcal{M}}_3(*)$], [$\tilde{\mathcal{M}}_4(*, *, *, *)$], [$\tilde{\mathcal{M}}(*, *, *)$], [$\tilde{\mathcal{Q}}(*, *, *)$], [$\tilde{\mathcal{Q}}_2(*, *, *)$], [$\tilde{\mathcal{Q}}_3(*, *, *, *)$], [$\tilde{\mathcal{Q}}^*(*, *, *)$], [($*$)], [($*$)], [display($*$)], [statement($*$)], [$[*]^{\cdot}$], [$[*]^{-}$], [**aspect**($*, *$)], [**aspect**($*, *, *$)], [$\langle * \rangle$], [$\mathbf{tuple}_1(*)$], [$\mathbf{tuple}_2(*)$], [$\mathrm{let}_2(*, *)$], [$\mathrm{let}_1(*, *)$], [[$* \stackrel{\mathrm{claim}}{=} *$]], [checker], [**check**($*, *$)], [$\mathbf{check}_2(*, *, *)$], [$\mathbf{check}_3(*, *, *)$], [$\mathbf{check}^*(*, *)$], [$\mathbf{check}^*_2(*, *, *)$], [$[*]^{\cdot}$], [$[*]^{-}$], [$[*]^{\circ}$], [msg], [[$* \stackrel{\mathrm{msg}}{=} *$]], [<stmt>], [stmt], [[$* \stackrel{\mathrm{stmt}}{=} *$]], [HeadNil′], [HeadPair′], [Transitivity′], [$\perp\!\!\!\perp$], [Contra′], [T'_{E}], [L_1], [$\underline{*}$], [$\mathcal{A}$], [$\mathcal{B}$], [$\mathcal{C}$], [$\mathcal{D}$], [$\mathcal{E}$], [$\mathcal{F}$], [$\mathcal{G}$], [$\mathcal{H}$], [$\mathcal{I}$], [$\mathcal{J}$], [$\mathcal{K}$], [$\mathcal{L}$], [$\mathcal{M}$], [$\mathcal{N}$], [$\mathcal{O}$], [$\mathcal{P}$], [$\mathcal{Q}$], [$\mathcal{R}$], [$\mathcal{S}$], [$\mathcal{T}$], [$\mathcal{U}$], [$\mathcal{V}$], [$\mathcal{W}$], [$\mathcal{X}$], [$\mathcal{Y}$], [$\mathcal{Z}$], [$\langle * \mid * := * \rangle$], [$\langle^* * \mid * := * \rangle$], [$\emptyset$], [Remainder],

$[(*)^{\mathbf{v}}]$, $[\mathrm{intro}(*,*,*,*)]$, $[\mathrm{intro}(*,*,*)]$, $[\mathrm{error}(*,*)]$, $[\mathrm{error}_2(*,*)]$, $[\mathrm{proof}(*,*,*)]$,
$[\mathrm{proof}_2(*,*)]$, $[\mathcal{S}(*,*)]$, $[\mathcal{S}^{\mathrm{I}}(*,*)]$, $[\mathcal{S}^{\rhd}(*,*)]$, $[\mathcal{S}_1^{\rhd}(*,*,*)]$, $[\mathcal{S}^{\mathrm{E}}(*,*)]$, $[\mathcal{S}_1^{\mathrm{E}}(*,*,*)]$,
$[\mathcal{S}^{+}(*,*)]$, $[\mathcal{S}_1^{+}(*,*,*)]$, $[\mathcal{S}^{-}(*,*)]$, $[\mathcal{S}_1^{-}(*,*,*)]$, $[\mathcal{S}^{*}(*,*)]$, $[\mathcal{S}_1^{*}(*,*,*)]$,
$[\mathcal{S}_2^{*}(*,*,*,*)]$, $[\mathcal{S}^{@}(*,*)]$, $[\mathcal{S}_1^{@}(*,*,*)]$, $[\mathcal{S}^{\vdash}(*,*)]$, $[\mathcal{S}_1^{\vdash}(*,*,*,*)]$, $[\mathcal{S}^{\Vdash}(*,*)]$,
$[\mathcal{S}_1^{\Vdash}(*,*,*,*)]$, $[\mathcal{S}^{\mathrm{i.e.}}(*,*)]$, $[\mathcal{S}_1^{\mathrm{i.e.}}(*,*,*,*)]$, $[\mathcal{S}_2^{\mathrm{i.e.}}(*,*,*,*,*)]$, $[\mathcal{S}^{\forall}(*,*)]$,
$[\mathcal{S}_1^{\forall}(*,*,*,*)]$, $[\mathcal{S}^{;}(*,*)]$, $[\mathcal{S}_1^{;}(*,*,*)]$, $[\mathcal{S}_2^{;}(*,*,*,*)]$, $[\mathcal{T}(*)]$, $[\mathrm{claims}(*,*,*)]$,
$[\mathrm{claims}_2(*,*,*)]$, [<proof>], [proof], [[**Lemma** $*$: $*$]], [[**Proof of** $*$: $*$]],
[[$*$ **lemma** $*$: $*$]], [[$*$ **antilemma** $*$: $*$]], [[$*$ **rule** $*$: $*$]], [[$*$ **antirule** $*$: $*$]],
[verifier], $[\mathcal{V}_1(*)]$, $[\mathcal{V}_2(*,*)]$, $[\mathcal{V}_3(*,*,*,*)]$, $[\mathcal{V}_4(*,*)]$, $[\mathcal{V}_5(*,*,*,*)]$, $[\mathcal{V}_6(*,*,*,*)]$,
$[\mathcal{V}_7(*,*,*,*)]$, $[\mathrm{Cut}(*,*)]$, $[\mathrm{Head}_{\oplus}(*)]$, $[\mathrm{Tail}_{\oplus}(*)]$, $[\mathrm{rule}_1(*,*)]$, $[\mathrm{rule}(*,*)]$,
[Rule tactic], $[\mathrm{Plus}(*,*)]$, [[**Theory** $*$]], $[\mathrm{theory}_2(*,*)]$, $[\mathrm{theory}_3(*,*)]$,
$[\mathrm{theory}_4(*,*,*)]$, [HeadNil$''$], [HeadPair$''$], [Transitivity$''$], [Contra$''$], [HeadNil],
[HeadPair], [Transitivity], [Contra], $[\mathrm{T_E}]$, [ragged right],
[ragged right expansion], $[\mathrm{parm}(*,*,*)]$, $[\mathrm{parm}^*(*,*,*)]$, $[\mathrm{inst}(*,*)]$,
$[\mathrm{inst}^*(*,*)]$, $[\mathrm{occur}(*,*,*)]$, $[\mathrm{occur}^*(*,*,*)]$, $[\mathrm{unify}(*=*,*)]$, $[\mathrm{unify}^*(*=*,*)]$,
$[\mathrm{unify}_2(*=*,*)]$, $[\mathrm{L_a}]$, $[\mathrm{L_b}]$, $[\mathrm{L_c}]$, $[\mathrm{L_d}]$, $[\mathrm{L_e}]$, $[\mathrm{L_f}]$, $[\mathrm{L_g}]$, $[\mathrm{L_h}]$, $[\mathrm{L_i}]$, $[\mathrm{L_j}]$, $[\mathrm{L_k}]$, $[\mathrm{L_l}]$, $[\mathrm{L_m}]$,
$[\mathrm{L_n}]$, $[\mathrm{L_o}]$, $[\mathrm{L_p}]$, $[\mathrm{L_q}]$, $[\mathrm{L_r}]$, $[\mathrm{L_s}]$, $[\mathrm{L_t}]$, $[\mathrm{L_u}]$, $[\mathrm{L_v}]$, $[\mathrm{L_w}]$, $[\mathrm{L_x}]$, $[\mathrm{L_y}]$, $[\mathrm{L_z}]$, $[\mathrm{L_A}]$, $[\mathrm{L_B}]$, $[\mathrm{L_C}]$,
$[\mathrm{L_D}]$, $[\mathrm{L_E}]$, $[\mathrm{L_F}]$, $[\mathrm{L_G}]$, $[\mathrm{L_H}]$, $[\mathrm{L_I}]$, $[\mathrm{L_J}]$, $[\mathrm{L_K}]$, $[\mathrm{L_L}]$, $[\mathrm{L_M}]$, $[\mathrm{L_N}]$, $[\mathrm{L_O}]$, $[\mathrm{L_P}]$, $[\mathrm{L_Q}]$, $[\mathrm{L_R}]$,
$[\mathrm{L_S}]$, $[\mathrm{L_T}]$, $[\mathrm{L_U}]$, $[\mathrm{L_V}]$, $[\mathrm{L_W}]$, $[\mathrm{L_X}]$, $[\mathrm{L_Y}]$, $[\mathrm{L_Z}]$, $[\mathrm{L_?}]$, [Reflexivity], $[\mathrm{Reflexivity}_1]$,
[Commutativity], $[\mathrm{Commutativity}_1]$, [<tactic>], [tactic], $[[* \stackrel{\mathrm{tactic}}{=} *]]$, $[\mathcal{P}(*,*,*)]$,
$[\mathcal{P}^*(*,*,*)]$, $[\mathrm{p_0}]$, $[\mathrm{conclude}_1(*,*)]$, $[\mathrm{conclude}_2(*,*,*)]$, $[\mathrm{conclude}_3(*,*,*,*)]$,
$[\mathrm{conclude}_4(*,*)]$, [check], $[[* \stackrel{\circ}{=} *]]$, $[\mathrm{RootVisible}(*)]$, [A], [R], [C], [T], [L], $[\{*\}]$, $[\overline{*}]$,
$[a]$, $[b]$, $[c]$, $[d]$, $[e]$, $[f]$, $[g]$, $[h]$, $[i]$, $[j]$, $[k]$, $[l]$, $[m]$, $[n]$, $[o]$, $[p]$, $[q]$, $[r]$, $[s]$, $[t]$, $[u]$, $[v]$,
$[w]$, $[x]$, $[y]$, $[z]$, $[\langle * \equiv * \mid * := * \rangle]$, $[\langle * \equiv^0 * \mid * := * \rangle]$, $[\langle * \equiv^1 * \mid * := * \rangle]$, $[\langle * \equiv^* * \mid * := * \rangle]$,
$[\mathrm{Ded}(*,*)]$, $[\mathrm{Ded}_0(*,*)]$, $[\mathrm{Ded}_1(*,*,*)]$, $[\mathrm{Ded}_2(*,*,*)]$, $[\mathrm{Ded}_3(*,*,*,*)]$,
$[\mathrm{Ded}_4(*,*,*,*)]$, $[\mathrm{Ded}_4^*(*,*,*,*)]$, $[\mathrm{Ded}_5(*,*,*)]$, $[\mathrm{Ded}_6(*,*,*,*)]$,
$[\mathrm{Ded}_6^*(*,*,*,*)]$, $[\mathrm{Ded}_7(*)]$, $[\mathrm{Ded}_8(*,*)]$, $[\mathrm{Ded}_8^*(*,*)]$, [S], [Neg], [MP], [Gen],
[Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1$'$], [A2$'$], [A4$'$],
[A5$'$], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop $3.2\mathrm{e}_1$], [Prop $3.2\mathrm{e}_2$],
[Prop 3.2e], [Prop $3.2\mathrm{f}_1$], [Prop $3.2\mathrm{f}_2$], [Prop 3.2f], [Prop $3.2\mathrm{g}_1$], [Prop $3.2\mathrm{g}_2$],
[Prop 3.2g], [Prop $3.2\mathrm{h}_1$], [Prop $3.2\mathrm{h}_2$], [Prop 3.2h], $[\mathrm{Block}_1(*,*,*)]$, $[\mathrm{Block}_2(*)]$,
[UniqueMember], [UniqueMember(Type)], [SameSeries], [A4], [SameMember],
[Qclosed(Addition)], [Qclosed(Multiplication)], [FromCartProd(1)],
[1rule fromCartProd(2)], [constantRationalSeries($*$)], [cartProd($*$)], [Power($*$)],
[binaryUnion($*,*$)], [SetOfRationalSeries], [IsSubset($*,*$)], [(p$*,*$)], [(s$*$)],
$[(\cdots)]$, [`Objekt-var`], [`Ex-var`], [`Ph-var`], [`Værdi`], [`Variabel`], $[\mathrm{Op}(*)]$, $[\mathrm{Op}(*,*)]$,
$[* \mathrel{\ddot{=}} *]$, [ContainsEmpty($*$)], $[\mathrm{Nat}(*)]$, $[\mathrm{Dedu}(*,*)]$, $[\mathrm{Dedu}_0(*,*)]$,
$[\mathrm{Dedu}_s(*,*,*)]$, $[\mathrm{Dedu}_1(*,*,*)]$, $[\mathrm{Dedu}_2(*,*,*)]$, $[\mathrm{Dedu}_3(*,*,*,*)]$,
$[\mathrm{Dedu}_4(*,*,*,*)]$, $[\mathrm{Dedu}_4^*(*,*,*,*)]$, $[\mathrm{Dedu}_5(*,*,*)]$, $[\mathrm{Dedu}_6(*,*,*,*)]$,
$[\mathrm{Dedu}_6^*(*,*,*,*)]$, $[\mathrm{Dedu}_7(*)]$, $[\mathrm{Dedu}_8(*,*)]$, $[\mathrm{Dedu}_8^*(*,*)]$, $[\mathrm{Ex}_1]$, $[\mathrm{Ex}_2]$, [Ex3],
$[\mathrm{Ex}_{10}]$, $[\mathrm{Ex}_{20}]$, $[*_{\mathrm{Ex}}]$, $[*^{\mathrm{Ex}}]$, $[\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ex}}]$, $[\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ex}}]$,
$[\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ex}}]$, $[\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ex}}]$, $[\mathrm{ph}_1]$, $[\mathrm{ph}_2]$, $[\mathrm{ph}_3]$, $[*_{\mathrm{Ph}}]$, $[*^{\mathrm{Ph}}]$,
$[\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ph}}]$, $[\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ph}}]$, $[\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ph}}]$,
$[\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ph}}]$, $[\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}]$, $[\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}}]$,
$[\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}}]$, [bs], [OBS], $[\mathcal{BS}]$, [Ø], [SystemQ], [MP], [Gen], [Repetition],

[Neg], [Ded], [ExistIntro], [Extensionality], [∅def], [PairDef], [UnionDef], [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg], [AndCommutativity], [AutoImply], [Contrapositive], [FirstConjunct], [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity], [IffFirst], [IffSecond], [ImplyTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4], [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1], [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union], [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power], [SubsetInPower], [HelperPowerIsSub], [PowerIsSub], [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality], [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality], [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry], [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive], [ERisSymmetric], [ERisTransitive], [∅isSubset], [HelperMemberNot∅], [MemberNot∅], [HelperUnique∅], [Unique∅], [==Reflexivity], [==Symmetry], [Helper==Transitivity], [==Transitivity], [HelperTransferNotEq], [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset], [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset], [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection], [AutoMember], [HelperEqSysNot∅], [EqSysNot∅], [HelperEqSubset], [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary], [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset], [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImply], [BSsubset], [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)], [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ϵ)], [$(\epsilon)_1$], [(ϵ2)], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)], [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)], [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X_1], [X_2], [Y_1], [Y_2], [V_1], [V_2], [V_3], [V_4], [V_{2n}], [M_1], [M_2], [N_1], [N_2], [N_3], [ϵ], [ϵ1], [ϵ2], [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)], [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)], [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)], [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)], [(S1ob)], [(S2ob)], [ph_4], [ph_5], [ph_6], [NAT], [$RATIONAL_S$ERIES], [SERIES], [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1], [(−1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(− − 01)], [02], [01//02], [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)], [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)], [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)], [Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom], [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom], [QisClosed(Reciprocal)(Imply)], [QisClosed(Reciprocal)], [QisClosed(Negative)(Imply)], [QisClosed(Negative)], [leqReflexivity], [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality], [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity], [plusCommutativity], [Negative], [plus0], [timesAssociativity], [timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1], [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)], [(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [UStelescope(Zero)], [UStelescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [NatType], [RationalType], [SeriesType], [Max], [Numerical], [NumericalF], [MemberOfSeries(Imply)], [JoinConjuncts(2conditions)], [prop lemma imply negation], [TND], [FromNegatedImply], [ToNegatedImply], [FromNegated(2 * Imply)], [FromNegatedAnd], [FromNegatedOr], [ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 * 2Disjuncts], [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2], [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)];

Preassociative

[*_{*}], [*/indexintro(*, *, *, *)], [*/intro(*, *, *)], [*/bothintro(*, *, *, *, *)], [*/nameintro(*, *, *, *)], [*′], [*[*]], [*[*→*]], [*[*⇒*]], [*0], [*1], [0b], [*-color(*)], [*-color*(*)], [$*^{\mathrm{H}}$], [$*^{\mathrm{T}}$], [$*^{\mathrm{U}}$], [$*^{\mathrm{h}}$], [$*^{\mathrm{t}}$], [$*^{\mathrm{s}}$], [$*^{\mathrm{c}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{a}}$], [$*^{\mathrm{C}}$], [$*^{\mathrm{M}}$], [$*^{\mathrm{B}}$], [$*^{\mathrm{r}}$], [$*^{\mathrm{i}}$], [$*^{\mathrm{d}}$], [$*^{\mathrm{R}}$], [$*^{0}$], [$*^{1}$], [$*^{2}$], [$*^{3}$], [$*^{4}$], [$*^{5}$], [$*^{6}$], [$*^{7}$], [$*^{8}$], [$*^{9}$], [$*^{\mathrm{E}}$], [$*^{\mathcal{V}}$], [$*^{\mathcal{C}}$], [$*^{\mathcal{C}^*}$], [$*^{\mathrm{hide}}$];

Preassociative

[“ * ”], [], [(*)$^{\mathbf{t}}$], [string(*) + *], [string(*) ++ *], [
*], [*], [!*], ["*], [#*], [$*], [%*], [&*], [’*], [(*], [)*], [**], [+*], [, *], [-*], [.*], [/*], [0*], [1*], [2*], [3*], [4*], [5*], [6*], [7*], [8*], [9*], [:*], [; *], [<*], [=*], [>*], [?*], [@*], [A*], [B*], [C*], [D*], [E*], [F*], [G*], [H*], [I*], [J*], [K*], [L*], [M*], [N*], [O*], [P*], [Q*], [R*], [S*], [T*], [U*], [V*], [W*], [X*], [Y*], [Z*], [[*], [*], []*], [ˆ*], [_*], [‘*], [a*], [b*], [c*], [d*], [e*], [f*], [g*], [h*], [i*], [j*], [k*], [l*], [m*], [n*], [o*], [p*], [q*], [r*], [s*], [t*], [u*], [v*], [w*], [x*], [y*], [z*], [{*], [|*], [}*], [˜*], [**Preassociative** *; *], [**Postassociative** *; *], [[*], *], [priority * end], [newline *], [macro newline *], [MacroIndent(*)];

Preassociative

[* ’ *], [* ‘ *];

Preassociative

[*(exp)*];

Preassociative

[*′], [R(*)], [− − R(*)], [rec*];

Preassociative

[*/*], [* ∩ *], [*[*]];

Preassociative

[∪*], [* ∪ *], [P(*)];

Preassociative

$[\{*\}]$, $[\mathrm{StateExpand}(*,*,*)]$, $[\mathrm{extractSeries}(*)]$, $[\mathrm{SetOfSeries}(*)]$, $[--\mathrm{Macro}(*)]$, $[\mathrm{ExpandList}(*,*,*)]$, $[**\mathrm{Macro}(*)]$, $[++\mathrm{Macro}(*)]$, $[<<\mathrm{Macro}(*)]$, $[||\mathrm{Macro}(*)]$, $[01//\mathrm{Macro}(*)]$, $[\mathrm{UB}(*,*)]$, $[\mathrm{LUB}(*,*)]$, $[\mathrm{BS}(*,*)]$, $[\mathrm{UStelescope}(*,*)]$, $[(*)]$, $[|\mathrm{f}*|]$, $[|\mathrm{r}*|]$, $[\mathrm{Limit}(*,*)]$, $[\mathrm{Union}(*)]$, $[\mathrm{IsOrderedPair}(*,*,*)]$, $[\mathrm{IsRelation}(*,*,*)]$, $[\mathrm{isFunction}(*,*,*)]$, $[\mathrm{IsSeries}(*,*)]$, $[\mathrm{IsNatural}(*,*)]$, $[\mathrm{OrderedPair}(*,*)]$, $[\mathrm{TypeNat}(*)]$, $[\mathrm{TypeNat0}(*)]$, $[\mathrm{TypeRational}(*)]$, $[\mathrm{TypeRational0}(*)]$, $[\mathrm{TypeSeries}(*,*)]$, $[\mathrm{Typeseries0}(*,*)]$;

Preassociative

$[\{*,*\}]$, $[\langle *,*\rangle]$, $[(-\mathrm{u}*)]$, $[-_{\mathrm{f}}*]$, $[(--*)]$, $[1\mathrm{f}/*]$, $[01//\mathrm{temp}*]$;

Preassociative

$[*(*,*)]$, $[\mathrm{ReflRel}(*,*)]$, $[\mathrm{SymRel}(*,*)]$, $[\mathrm{TransRel}(*,*)]$, $[\mathrm{EqRel}(*,*)]$, $[[* \in *]_*]$, $[\mathrm{Partition}(*,*)]$;

Preassociative

$[* \cdot *]$, $[* \cdot_0 *]$, $[(* * *)]$, $[* *_{\mathrm{f}} *]$, $[* * **]$;

Preassociative

$[* + *]$, $[* +_0 *]$, $[* +_1 *]$, $[* - *]$, $[* -_0 *]$, $[* -_1 *]$, $[(* + *)]$, $[(* - *)]$, $[* +_{\mathrm{f}} *]$, $[* -_{\mathrm{f}} *]$, $[* + + *]$, $[\mathrm{R}(*) - -\mathrm{R}(*)]$;

Preassociative

$[* \in *]$;

Preassociative

$[|*|]$, $[\mathrm{if}(*,*,*)]$, $[\mathrm{Max}(*,*)]$, $[\mathrm{Max}(*,*)]$;

Preassociative

$[* = *]$, $[* \neq *]$, $[* <= *]$, $[* < *]$, $[* <_{\mathrm{f}} *]$, $[* \leq_{\mathrm{f}} *]$, $[\mathrm{SF}(*,*)]$, $[* == *]$, $[*!! == *]$, $[* << *]$, $[* <<== *]$;

Preassociative

$[* \cup \{*\}]$, $[* \cup *]$, $[* \backslash \{*\}]$;

Postassociative

$[* \therefore *]$, $[* \underline{\therefore} *]$, $[* \underline{::} *]$, $[* \underline{+2*} *]$, $[* :: *]$, $[* +2* *]$;

Postassociative

$[*,*]$;

Preassociative

$[* \overset{\mathrm{B}}{\approx} *]$, $[* \overset{\mathrm{D}}{\approx} *]$, $[* \overset{\mathrm{C}}{\approx} *]$, $[* \overset{\mathrm{P}}{\approx} *]$, $[* \approx *]$, $[* = *]$, $[* \overset{+}{\rightarrow} *]$, $[* \overset{\mathrm{t}}{=} *]$, $[* \overset{\mathrm{t}^*}{=} *]$, $[* \overset{\mathrm{r}}{=} *]$, $[* \in_{\mathrm{t}} *]$, $[* \subseteq_{\mathrm{T}} *]$, $[* \overset{\mathrm{T}}{=} *]$, $[* \overset{\mathrm{s}}{=} *]$, $[* \text{ free in } *]$, $[* \text{ free in}^* *]$, $[* \text{ free for } * \text{ in } *]$, $[* \text{ free for}^* * \text{ in } *]$, $[* \in_{\mathrm{c}} *]$, $[* < *]$, $[* <' *]$, $[* \leq' *]$, $[* = *]$, $[* \neq *]$, $[*^{\mathrm{var}}]$, $[*\#^0*]$, $[*\#^1*]$, $[*\#^**]$, $[* == *]$, $[* \subseteq *]$;

Preassociative

$[\neg *]$, $[\dot{\neg}(*)\mathrm{n}]$, $[* \notin *]$, $[* \neq *]$;

Preassociative

$[* \wedge *]$, $[* \ddot{\wedge} *]$, $[* \tilde{\wedge} *]$, $[* \wedge_{\mathrm{c}} *]$, $[* \dot{\wedge} *]$;

Preassociative

$[* \vee *]$, $[* \parallel *]$, $[* \ddot{\vee} *]$;

Postassociative

$[* \dot{\vee} *]$;

Preassociative

$[\exists *: *]$, $[\forall *: *]$, $[\forall_{\mathrm{obj}} *: *]$, $[\exists *: *]$;

Postassociative
$[* \mathrel{\ddot{\Rightarrow}} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \mathrel{\dot{\Leftrightarrow}} *]$;
Preassociative
$[\{\mathrm{ph} \in * \mid *\}]$;
Postassociative
$[* : *], [* \text{ spy } *], [*!*]$;
Preassociative
$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.]$;
Preassociative
$[\lambda * .*], [\Lambda * .*], [\Lambda *], [\textbf{if } * \textbf{ then } * \textbf{ else } *], [\textbf{let } * = * \textbf{ in } *], [\textbf{let } * \mathrel{\ddot{=}} * \text{ in } *]$;
Preassociative
$[* \# *]$;
Preassociative
$[*^{\mathrm{I}}], [*^{\triangleright}], [*^{\mathrm{V}}], [*^{+}], [*^{-}], [*^{*}]$;
Preassociative
$[* @ *], [* \rhd *], [* \mathrel{\blacktriangleright} *], [* \gg *], [* \unrhd *]$;
Postassociative
$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *]$;
Preassociative
$[\forall * : *], [\Pi * : *]$;
Postassociative
$[* \oplus *]$;
Postassociative
$[*; *]$;
Preassociative
$[* \text{ proves } *]$;
Preassociative
$[* \textbf{ proof of } * : *], [\text{Line} * : * \gg *; *], [\text{Last line} * \gg * \square]$,
$[\text{Line} * : \text{Premise} \gg *; *], [\text{Line} * : \text{Side-condition} \gg *; *], [\text{Arbitrary} \gg *; *]$,
$[\text{Local} \gg * = *; *], [\text{Begin} *; * : \text{End}; *], [\text{Last block line} * \gg * ;]$,
$[\text{Arbitrary} \gg *; *]$;
Postassociative
$[* \mid *]$;
Postassociative
$[* , *], [* [*] *]$;
Preassociative
$[* \& *]$;
Preassociative
$[* \backslash\backslash *], [* \text{ linebreak[4] } *], [* \backslash\backslash *];]$

$[\text{kvanti} \stackrel{\text{tex}}{\to} \text{“kvanti”}]$

$[\text{kvanti} \stackrel{\text{pyk}}{\to} \text{“kvanti”}]$

UniqueMember

$[\mathrm{UniqueMember} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash$
$\forall \underline{(fx)}: \forall \underline{(sx)}: \forall \underline{(sx1)}: \forall \underline{(sy)}: \forall \underline{(sy1)}: \forall \underline{(sz)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \vdash$
$\{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \vdash \underline{(sx)} = \underline{(sy)} \vdash$
$\mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n; \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \gg \dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} =$
$\overline{(f4)})n)n; \mathrm{Repetition} \rhd \dot{\neg}(\forall_{obj} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj} \overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj} \overline{(f1)}: \forall_{obj} \overline{(f2)}: \forall_{obj} \overline{(f3)}: \forall_{obj} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \gg$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow \dot{\neg}((\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} =$
$\overline{(f4)})n)n; \mathrm{SecondConjunct} \rhd \dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \gg$
$\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} =$
$\overline{(f4)}; \mathrm{A4} \mathbin{@} \underline{(sx)} \rhd \forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in$
$\underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} \gg$
$\forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \underline{(sx)} = \overline{(f3)} \Rightarrow \overline{(f2)} =$
$\overline{(f4)}; \mathrm{A4} \mathbin{@} \underline{(sx1)} \rhd \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \underline{(sx)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} \gg$
$\forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \underline{(sx)} = \overline{(f3)} \Rightarrow \underline{(sx1)} =$
$\overline{(f4)}; \mathrm{A4} \mathbin{@} \underline{(sy)} \rhd \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \underline{(sx)} = \overline{(f3)} \Rightarrow \underline{(sx1)} = \overline{(f4)} \gg$
$\forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \Rightarrow \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \overline{(f4)}\}\} \in$
$\underline{(fx)} \Rightarrow \underline{(sx)} = \underline{(sy)} \Rightarrow \underline{(sx1)} = \overline{(f4)}; \mathrm{A4} \mathbin{@} \underline{(sy1)} \rhd$
$\forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \Rightarrow \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \overline{(f4)}\}\} \in$
$\underline{(fx)} \Rightarrow \underline{(sx)} = \underline{(sy)} \Rightarrow \underline{(sx1)} = \overline{(f4)} \gg \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \Rightarrow \underline{(sx)} = \underline{(sy)} \Rightarrow \underline{(sx1)} =$
$\underline{(sy1)}; \mathrm{MP3} \rhd \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \Rightarrow \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in$
$\underline{(fx)} \Rightarrow \underline{(sx)} = \underline{(sy)} \Rightarrow \underline{(sx1)} = \underline{(sy1)} \rhd \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in$
$\underline{(fx)} \rhd \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \rhd \underline{(sx)} = \underline{(sy)} \gg \underline{(sx1)} = \underline{(sy1)}], p_0, \mathsf{c})]$

$[\mathrm{UniqueMember} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash$
$\forall\underline{(fx)}\colon \forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \forall\underline{(sz)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(op2)} \in \underline{(sz)})n)n \Rightarrow$
$\dot{\neg}((\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \vdash \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \vdash$

$\{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \vdash \underline{(sx)} = \underline{(sy)} \vdash \underline{(sx1)} = \underline{(sy1)}]$

[UniqueMember $\stackrel{\text{tex}}{\to}$ "UniqueMember"]

[UniqueMember $\stackrel{\text{pyk}}{\to}$ "lemma uniqueMember"]

UniqueMember(Type)

[UniqueMember(Type) $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{(fx)} \colon \forall \underline{(sx)} \colon \forall \underline{(sx1)} \colon \forall \underline{(sy)} \colon \forall \underline{(sy1)} \colon \forall \underline{(sz)} \colon \lambda \mathsf{c}.\text{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sz)} \rceil) \Vdash \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \vdash \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \vdash \underline{(sx)} = \underline{(sy)} \vdash \text{SeriesType} \rhd\!\!\rhd \lambda \mathsf{c}.\text{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sz)} \rceil) \gg \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(r1)} \colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(op1)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(op2)} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \text{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \underline{(sz)})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(f1)} \colon \forall_{\text{obj}} \overline{(f2)} \colon \forall_{\text{obj}} \overline{(f3)} \colon \forall_{\text{obj}} \overline{(f4)} \colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(s1)} \colon \overline{(s1)} \in \text{N} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(s2)} \colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})\text{n})\text{n})\text{n})\text{n}; \text{UniqueMember} \rhd \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(r1)} \colon \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(op1)} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(op2)} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \text{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \underline{(sz)})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(f1)} \colon \forall_{\text{obj}} \overline{(f2)} \colon \forall_{\text{obj}} \overline{(f3)} \colon \forall_{\text{obj}} \overline{(f4)} \colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(s1)} \colon \overline{(s1)} \in \text{N} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} \overline{(s2)} \colon \dot{\neg}\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})\text{n})\text{n})\text{n})\text{n} \rhd \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \rhd \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \rhd \underline{(sx)} = \underline{(sy)} \gg \underline{(sx1)} = \underline{(sy1)} \rceil, \text{p}_0, \mathsf{c})]$

[UniqueMember(Type) $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{(fx)} \colon \forall \underline{(sx)} \colon \forall \underline{(sx1)} \colon \forall \underline{(sy)} \colon \forall \underline{(sy1)} \colon \forall \underline{(sz)} \colon \lambda \mathsf{c}.\text{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sz)} \rceil) \Vdash \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sx1)}\}\} \in \underline{(fx)} \vdash \{\{\underline{(sy)}, \underline{(sy)}\}, \{\underline{(sy)}, \underline{(sy1)}\}\} \in \underline{(fx)} \vdash \underline{(sx)} = \underline{(sy)} \vdash \underline{(sx1)} = \underline{(sy1)}]$

[UniqueMember(Type) $\stackrel{\text{tex}}{\to}$ "UniqueMember(Type)"]

[UniqueMember(Type) $\stackrel{\text{pyk}}{\to}$ "lemma uniqueMember(Type)"]

SameSeries

[SameSeries $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{m}} \colon \forall \underline{\mathsf{n}} \colon \forall \underline{(fx)} \colon \forall \underline{(sy)} \colon \lambda \mathsf{c}.\text{TypeNat0}(\lceil \underline{\mathsf{m}} \rceil) \Vdash \lambda \mathsf{c}.\text{TypeNat0}(\lceil \underline{\mathsf{n}} \rceil) \Vdash \lambda \mathsf{c}.\text{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sy)} \rceil) \Vdash \underline{\mathsf{m}} = \underline{\mathsf{n}} \vdash \text{memberOfSeries(Type)} \rhd\!\!\rhd \lambda \mathsf{c}.\text{TypeNat0}(\lceil \underline{\mathsf{m}} \rceil) \rhd\!\!\rhd \lambda \mathsf{c}.\text{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sy)} \rceil) \gg$

$\{\{\underline{m},\underline{m}\},\{\underline{m},\underline{(fx)}[\underline{m}]\}\} \in \underline{(fx)}; \mathrm{memberOfSeries}(\mathrm{Type}) \unrhd \lambda c.\mathrm{TypeNat0}(\lceil \underline{n} \rceil) \unrhd \lambda c.\mathrm{Typeseries0}(\lceil \underline{(fx)}\rceil, \lceil \underline{(sy)} \rceil) \gg \{\{\underline{n},\underline{n}\},\{\underline{n},\underline{(fx)}[\underline{n}]\}\} \in \underline{(fx)}; \mathrm{UniqueMember}(\mathrm{Type}) \unrhd \lambda c.\mathrm{Typeseries0}(\lceil\underline{(fx)}\rceil,\lceil\underline{(sy)}\rceil) \rhd \{\{\underline{m},\underline{m}\},\{\underline{m},\underline{(fx)}[\underline{m}]\}\} \in \underline{(fx)} \rhd \{\{\underline{n},\underline{n}\},\{\underline{n},\underline{(fx)}[\underline{n}]\}\} \in \underline{(fx)} \rhd \underline{m} = \underline{n} \gg \underline{(fx)}[\underline{m}] = \underline{(fx)}[\underline{n}]\rceil, p_0, c)]$

$[\mathrm{SameSeries} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{m}\colon \forall \underline{n}\colon \forall \underline{(fx)}\colon \forall \underline{(sy)}\colon \lambda c.\mathrm{TypeNat0}(\lceil \underline{m} \rceil) \Vdash \lambda c.\mathrm{TypeNat0}(\lceil \underline{n} \rceil) \Vdash \lambda c.\mathrm{Typeseries0}(\lceil \underline{(fx)} \rceil, \lceil \underline{(sy)} \rceil) \Vdash \underline{m} = \underline{n} \vdash \underline{(fx)}[\underline{m}] = \underline{(fx)}[\underline{n}]]$

[SameSeries $\overset{\mathrm{tex}}{\rightarrow}$ "SameSeries"]

[SameSeries $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma sameSeries"]

A4

$[\mathrm{A4} \overset{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{(v1)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \langle \underline{a} \equiv \underline{b} | \underline{(v1)} {:}{=}{=} \underline{x} \rangle_{\mathrm{Me}} \Vdash \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{b} \vdash \mathrm{A4}(\mathrm{Axiom}) \unrhd \langle \underline{a} \equiv \underline{b} | \underline{(v1)} {:}{=}{=} \underline{x} \rangle_{\mathrm{Me}} \gg \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{b} \Rightarrow \underline{a}; \mathrm{MP} \rhd \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{b} \Rightarrow \underline{a} \rhd \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{b} \gg \underline{a} \rceil, p_0, c)]$

$[\mathrm{A4} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{(v1)}\colon \forall \underline{a}\colon \forall \underline{b}\colon \langle \underline{a} \equiv \underline{b} | \underline{(v1)} {:}{=}{=} \underline{x} \rangle_{\mathrm{Me}} \Vdash \forall_{\mathrm{obj}} \underline{(v1)}\colon \underline{b} \vdash \underline{a}]$

[A4 $\overset{\mathrm{tex}}{\rightarrow}$ "A4"]

[A4 $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma a4"]

SameMember

[SameMember $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{SameMember} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{(sx)}\colon \forall \underline{(sy)}\colon \forall \underline{(sz)}\colon \underline{(sx)} = \underline{(sy)} \vdash \underline{(sx)} \in \underline{(sz)} \vdash \underline{(sy)} \in \underline{(sz)}]$

[SameMember $\overset{\mathrm{tex}}{\rightarrow}$ "SameMember"]

[SameMember $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma sameMember"]

Qclosed(Addition)

[Qclosed(Addition) $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

$[\mathrm{Qclosed}(\mathrm{Addition}) \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} \in \mathsf{Q} \vdash \underline{y} \in \mathsf{Q} \vdash (\underline{x} + \underline{y}) \in \mathsf{Q}]$

[Qclosed(Addition) $\overset{\mathrm{tex}}{\rightarrow}$ "Qclosed(Addition)"]

[Qclosed(Addition) $\overset{\mathrm{pyk}}{\rightarrow}$ "1rule Qclosed(Addition)"]

Qclosed(Multiplication)

[Qclosed(Multiplication) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Qclosed(Multiplication) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \underline{\mathsf{x}} \in \mathrm{Q} \vdash \underline{\mathsf{y}} \in \mathrm{Q} \vdash (\underline{\mathsf{x}} * \underline{\mathsf{y}}) \in \mathrm{Q}$]

[Qclosed(Multiplication) $\stackrel{\text{tex}}{\rightarrow}$ "Qclosed(Multiplication)"]

[Qclosed(Multiplication) $\stackrel{\text{pyk}}{\rightarrow}$ "1rule Qclosed(Multiplication)"]

FromCartProd(1)

[FromCartProd(1) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[FromCartProd(1) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sx1})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sy1})}\colon \{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})}, \underline{(\mathrm{sy1})}\}))) \mid \dot{\neg}\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\, (\dot{\neg}\, (\dot{\neg}\, (\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\, (\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \underline{(\mathrm{sx})} \in \underline{(\mathrm{sx1})}$]

[FromCartProd(1) $\stackrel{\text{tex}}{\rightarrow}$ "FromCartProd(1)"]

[FromCartProd(1) $\stackrel{\text{pyk}}{\rightarrow}$ "1rule fromCartProd(1)"]

1rule fromCartProd(2)

[1rule fromCartProd(2) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[1rule fromCartProd(2) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{(\mathrm{sx})}\colon \forall \underline{(\mathrm{sx1})}\colon \forall \underline{(\mathrm{sy})}\colon \forall \underline{(\mathrm{sy1})}\colon \{\{\underline{(\mathrm{sx})}, \underline{(\mathrm{sx})}\}, \{\underline{(\mathrm{sx})}, \underline{(\mathrm{sy})}\}\} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\underline{(\mathrm{sx1})}, \underline{(\mathrm{sy1})}\}))) \mid \dot{\neg}\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\, (\dot{\neg}\, (\dot{\neg}\, (\overline{(\mathrm{op1})} \in$
$\underline{(\mathrm{sx1})} \Rightarrow \dot{\neg}\, (\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy1})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \underline{(\mathrm{sy})} \in \underline{(\mathrm{sy1})}$]

[1rule fromCartProd(2) $\stackrel{\text{pyk}}{\rightarrow}$ "1rule fromCartProd(2)"]

constantRationalSeries(*)

[constantRationalSeries(x) $\stackrel{\text{macro}}{\rightarrow}$
$\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil$[constantRationalSeries($\mathsf{x}$) $\ddot{=} \{\mathrm{ph} \in \mathrm{cartProd}(\mathrm{N}) \mid$
$\exists (\mathrm{CRS1ob})\colon \mathrm{ph}_3 = \mathrm{OrderedPair}((\mathrm{CRS1ob}), \mathsf{x})\}]\rceil)$]

[constantRationalSeries(x) $\stackrel{\text{tex}}{\to}$ "constantRationalSeries(#1.
)"]

[constantRationalSeries(∗) $\stackrel{\text{pyk}}{\to}$ "constantRationalSeries(")"]

cartProd(∗)

[cartProd((sx)) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil$[cartProd((sx)) $\ddot{=}$ {ph ∈ Power(Power(binaryUnion((sx), (sy)))) | IsOrderedPair(ph_1, (sx), (sy))}]$\rceil$)]

[cartProd(x) $\stackrel{\text{tex}}{\to}$ "cartProd(#1.
)"]

[cartProd(∗) $\stackrel{\text{pyk}}{\to}$ "cartProd(" , ")"]

Power(∗)

[Power(x) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil$[Power(x) $\ddot{=}$ P(x)]$\rceil$)]

[Power(x) $\stackrel{\text{tex}}{\to}$ "Power(#1.
)"]

[Power(∗) $\stackrel{\text{pyk}}{\to}$ "P(")"]

binaryUnion(∗, ∗)

[binaryUnion(x, y) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil$[binaryUnion(x, y) $\ddot{=}$ Union((px, y))]$\rceil$)]

[binaryUnion(x, y) $\stackrel{\text{tex}}{\to}$ "binaryUnion(#1.
, #2.
)"]

[binaryUnion(∗, ∗) $\stackrel{\text{pyk}}{\to}$ "binaryUnion(" , ")"]

SetOfRationalSeries

[SetOfRationalSeries $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil$[SetOfRationalSeries $\ddot{=}$ {ph ∈ Power(cartProd(N)) | IsSeries(ph_2, Q)}]$\rceil$)]

[SetOfRationalSeries $\stackrel{\text{tex}}{\to}$ "SetOfRationalSeries"]

[SetOfRationalSeries $\stackrel{\text{pyk}}{\to}$ "setOfRationalSeries"]

IsSubset$(*, *)$

$[\mathrm{IsSubset}(\mathsf{x},\mathsf{y}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{IsSubset}(\mathsf{x},\mathsf{y}) \ddot{=} \mathsf{x} \subseteq \mathsf{y}]\rceil)]$

[IsSubset$(\mathsf{x},\mathsf{y}) \stackrel{\mathrm{tex}}{\to}$ "IsSubset(#1.
, #2.
)"]

[IsSubset$(*, *) \stackrel{\mathrm{pyk}}{\to}$ "isSubset(" , ")"]

$(\mathrm{p}*, *)$

$[(\mathrm{p}\mathsf{x},\mathsf{y}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{p}\mathsf{x},\mathsf{y}) \ddot{=} \{\mathsf{x},\mathsf{y}\}]\rceil)]$

[$(\mathrm{p}\mathsf{x},\mathsf{y}) \stackrel{\mathrm{tex}}{\to}$ "(p#1.
, #2.
)"]

[$(\mathrm{p}*, *) \stackrel{\mathrm{pyk}}{\to}$ "(p " , ")"]

$(\mathrm{s}*)$

$[(\mathrm{s}\mathsf{x}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{s}\mathsf{x}) \ddot{=} \{\mathsf{x}\}]\rceil)]$

[$(\mathrm{s}\mathsf{x}) \stackrel{\mathrm{tex}}{\to}$ "(s#1.
)"]

[$(\mathrm{s}*) \stackrel{\mathrm{pyk}}{\to}$ "(s ")"]

$(\cdots)$

[$(\cdots) \stackrel{\mathrm{tex}}{\to}$ "(\cdots{})"]

[$(\cdots) \stackrel{\mathrm{pyk}}{\to}$ "cdots"]

`Objekt-var`

[`Objekt-var` $\stackrel{\mathrm{tex}}{\to}$ "\texttt{Objekt-var}"]

[`Objekt-var` $\stackrel{\mathrm{pyk}}{\to}$ "object-var"]

Ex-var

[`Ex-var` $\stackrel{\text{tex}}{\to}$ "\texttt{Ex-var}"]

[`Ex-var` $\stackrel{\text{pyk}}{\to}$ "ex-var"]

Ph-var

[`Ph-var` $\stackrel{\text{tex}}{\to}$ "\texttt{Ph-var}"]

[`Ph-var` $\stackrel{\text{pyk}}{\to}$ "ph-var"]

Værdi

[`Værdi` $\stackrel{\text{tex}}{\to}$ "\texttt{V\ae{}rdi}"]

[`Værdi` $\stackrel{\text{pyk}}{\to}$ "vaerdi"]

Variabel

[`Variabel` $\stackrel{\text{tex}}{\to}$ "\texttt{Variabel}"]

[`Variabel` $\stackrel{\text{pyk}}{\to}$ "variabel"]

Op($*$)

[Op(x) $\stackrel{\text{tex}}{\to}$ "Op(#1.
)"]

[Op($*$) $\stackrel{\text{pyk}}{\to}$ "op " end op"]

Op($*, *$)

[Op(x, y) $\stackrel{\text{tex}}{\to}$ "Op(#1.
, #2.
)"]

[Op($*, *$) $\stackrel{\text{pyk}}{\to}$ "op2 " comma " end op2"]

$* \mathrel{\ddot{==}} *$

[$\mathsf{x} \mathrel{\ddot{==}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel {\ddot {==}} #2."]

[$* \mathrel{\ddot{==}} * \stackrel{\text{pyk}}{\rightarrow}$ "define-equal " comma " end equal"]

ContainsEmpty($*$)

[ContainsEmpty(x) $\stackrel{\text{tex}}{\rightarrow}$ "ContainsEmpty(#1.
)"]

[ContainsEmpty($*$) $\stackrel{\text{pyk}}{\rightarrow}$ "contains-empty " end empty"]

Nat($*$)

[Nat(x) $\stackrel{\text{tex}}{\rightarrow}$ "Nat(#1.
)"]

[Nat($*$) $\stackrel{\text{pyk}}{\rightarrow}$ "Nat(")"]

Dedu($*, *$)

[Dedu(x, y) $\stackrel{\text{tex}}{\rightarrow}$ "
Dedu(#1.
, #2.
)"]

[Dedu($*, *$) $\stackrel{\text{pyk}}{\rightarrow}$ "1deduction " conclude " end 1deduction"]

$\text{Dedu}_0(*, *)$

[$\text{Dedu}_0(\mathsf{x}, \mathsf{y})$ $\stackrel{\text{tex}}{\rightarrow}$ "
Dedu_0(#1.
, #2.
)"]

[$\text{Dedu}_0(*, *)$ $\stackrel{\text{pyk}}{\rightarrow}$ "1deduction zero " conclude " end 1deduction"]

$\mathrm{Dedu_s}(*, *, *)$

[$\mathrm{Dedu_s}(\mathsf{x}, \mathsf{y}, \mathsf{z}) \overset{\text{tex}}{\to}$ "Dedu_{s}(#1.
, #2.
, #3.
)"]

[$\mathrm{Dedu_s}(*, *, *) \overset{\text{pyk}}{\to}$ "1deduction side " conclude " condition " end 1deduction"]

$\mathrm{Dedu_1}(*, *, *)$

[$\mathrm{Dedu_1}(\mathsf{x}, \mathsf{y}, \mathsf{z}) \overset{\text{tex}}{\to}$ "
Dedu_1(#1.
, #2.
, #3.
)"]

[$\mathrm{Dedu_1}(*, *, *) \overset{\text{pyk}}{\to}$ "1deduction one " conclude " condition " end 1deduction"]

$\mathrm{Dedu_2}(*, *, *)$

[$\mathrm{Dedu_2}(\mathsf{x}, \mathsf{y}, \mathsf{z}) \overset{\text{tex}}{\to}$ "
Dedu_2(#1.
, #2.
, #3.
)"]

[$\mathrm{Dedu_2}(*, *, *) \overset{\text{pyk}}{\to}$ "1deduction two " conclude " condition " end 1deduction"]

$\mathrm{Dedu_3}(*, *, *, *)$

[$\mathrm{Dedu_3}(\mathsf{x}, \mathsf{y}, \mathsf{z}, \mathsf{u}) \overset{\text{tex}}{\to}$ "
Dedu_3(#1.
, #2.
, #3.
, #4.
)"]

[$\mathrm{Dedu_3}(*, *, *, *) \overset{\text{pyk}}{\to}$ "1deduction three " conclude " condition " bound " end 1deduction"]

$\mathrm{Dedu}_4(*,*,*,*)$

[$\mathrm{Dedu}_4(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \stackrel{\text{tex}}{\rightarrow}$ "
Dedu_4(#1.
, #2.
, #3.
, #4.
)"]

[$\mathrm{Dedu}_4(*,*,*,*) \stackrel{\text{pyk}}{\rightarrow}$ "1deduction four " conclude " condition " bound " end 1deduction"]

$\mathrm{Dedu}_4^*(*,*,*,*)$

[$\mathrm{Dedu}_4^*(\mathsf{x},\mathsf{y},\mathsf{z},\mathsf{u}) \stackrel{\text{tex}}{\rightarrow}$ "
Dedu_4^*(#1.
, #2.
, #3.
, #4.
)"]

[$\mathrm{Dedu}_4^*(*,*,*,*) \stackrel{\text{pyk}}{\rightarrow}$ "1deduction four star " conclude " condition " bound " end 1deduction"]

$\mathrm{Dedu}_5(*,*,*)$

[$\mathrm{Dedu}_5(\mathsf{x},\mathsf{y},\mathsf{z}) \stackrel{\text{tex}}{\rightarrow}$ "
Dedu_5(#1.
, #2.
, #3.
)"]

[$\mathrm{Dedu}_5(*,*,*) \stackrel{\text{pyk}}{\rightarrow}$ "1deduction five " condition " bound " end 1deduction"]

$\mathrm{Dedu}_6(*,*,*,*)$

[$\mathrm{Dedu}_6(\mathsf{p},\mathsf{c},\mathsf{e},\mathsf{b}) \stackrel{\text{tex}}{\rightarrow}$ "
Dedu_6(#1.
, #2.
, #3.
, #4.

)”]

[$\mathrm{Dedu}_6(*, *, *, *) \stackrel{\mathrm{pyk}}{\to}$ “1deduction six " conclude " exception " bound " end 1deduction”]

$\mathrm{Dedu}_6^*(*, *, *, *)$

[$\mathrm{Dedu}_6^*(\mathsf{p}, \mathsf{c}, \mathsf{e}, \mathsf{b}) \stackrel{\mathrm{tex}}{\to}$ “
Dedu_6^*(#1.
, #2.
, #3.
, #4.
)”]

[$\mathrm{Dedu}_6^*(*, *, *, *) \stackrel{\mathrm{pyk}}{\to}$ “1deduction six star " conclude " exception " bound " end 1deduction”]

$\mathrm{Dedu}_7(*)$

[$\mathrm{Dedu}_7(\mathsf{p}) \stackrel{\mathrm{tex}}{\to}$ “
Dedu_7(#1.
)”]

[$\mathrm{Dedu}_7(*) \stackrel{\mathrm{pyk}}{\to}$ “1deduction seven " end 1deduction”]

$\mathrm{Dedu}_8(*, *)$

[$\mathrm{Dedu}_8(\mathsf{p}, \mathsf{b}) \stackrel{\mathrm{tex}}{\to}$ “
Dedu_8(#1.
, #2.
)”]

[$\mathrm{Dedu}_8(*, *) \stackrel{\mathrm{pyk}}{\to}$ “1deduction eight " bound " end 1deduction”]

$\mathrm{Dedu}_8^*(*, *)$

[$\mathrm{Dedu}_8^*(\mathsf{p}, \mathsf{b}) \stackrel{\mathrm{tex}}{\to}$ “
Dedu_8^*(#1.
, #2.
)”]

$[\mathrm{Dedu}_8^*(*,*) \stackrel{\mathrm{pyk}}{\rightarrow}$ "1deduction eight star " bound " end 1deduction"$]$

Ex_1

$[\mathrm{Ex}_1 \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{Ex}_1 \ddot{=} \mathsf{a}_{\mathrm{Ex}}]\rceil)]$

$[\mathrm{Ex}_1 \stackrel{\mathrm{tex}}{\rightarrow}$ "Ex_{1}"$]$

$[\mathrm{Ex}_1 \stackrel{\mathrm{pyk}}{\rightarrow}$ "ex1"$]$

Ex_2

$[\mathrm{Ex}_2 \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{Ex}_2 \ddot{=} \mathsf{b}_{\mathrm{Ex}}]\rceil)]$

$[\mathrm{Ex}_2 \stackrel{\mathrm{tex}}{\rightarrow}$ "Ex_{2}"$]$

$[\mathrm{Ex}_2 \stackrel{\mathrm{pyk}}{\rightarrow}$ "ex2"$]$

Ex3

$[\mathrm{Ex3} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{Ex3} \ddot{=} \mathsf{c}_{\mathrm{Ex}}]\rceil)]$

$[\mathrm{Ex3} \stackrel{\mathrm{tex}}{\rightarrow}$ "Ex3"$]$

$[\mathrm{Ex3} \stackrel{\mathrm{pyk}}{\rightarrow}$ "ex3"$]$

Ex_{10}

$[\mathrm{Ex}_{10} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{Ex}_{10} \ddot{=} \mathsf{j}_{\mathrm{Ex}}]\rceil)]$

$[\mathrm{Ex}_{10} \stackrel{\mathrm{tex}}{\rightarrow}$ "Ex_{10}"$]$

$[\mathrm{Ex}_{10} \stackrel{\mathrm{pyk}}{\rightarrow}$ "ex10"$]$

Ex_{20}

$[\mathrm{Ex}_{20} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{Ex}_{20} \ddot{=} \mathsf{t}_{\mathrm{Ex}}]\rceil)]$

$[\mathrm{Ex}_{20} \stackrel{\mathrm{tex}}{\rightarrow}$ "Ex_{20}"$]$

$[\mathrm{Ex}_{20} \stackrel{\mathrm{pyk}}{\rightarrow}$ "ex20"$]$

$*_{\mathrm{Ex}}$

$[\mathsf{x}_{\mathrm{Ex}} \stackrel{\text{tex}}{\rightarrow}$ “#1.
_{Ex}”]

$[*_{\mathrm{Ex}} \stackrel{\text{pyk}}{\rightarrow}$ “existential var " end var”]

$*^{\mathrm{Ex}}$

$[\mathsf{x}^{\mathrm{Ex}} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{\text{r}}{=} \lceil \mathsf{x}_{\mathrm{Ex}} \rceil]$

$[\mathsf{x}^{\mathrm{Ex}} \stackrel{\text{tex}}{\rightarrow}$ “#1.
^{Ex}”]

$[*^{\mathrm{Ex}} \stackrel{\text{pyk}}{\rightarrow}$ “" is existential var”]

$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ex}}$

$[\langle \mathsf{a} \equiv \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\langle \mathsf{a} \equiv \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}} \ddot{=} \langle \lceil \mathsf{a} \rceil \equiv^0 \lceil \mathsf{b} \rceil | \lceil \mathsf{x} \rceil :== \lceil \mathsf{t} \rceil \rangle_{\mathrm{Ex}}] \rceil)]$

$[\langle \mathsf{x} \equiv \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ex}} \stackrel{\text{tex}}{\rightarrow}$ “\langle #1.
{\equiv} #2.
| #3.
{:==} #4.
\rangle_{Ex} ”]

$[\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ex}} \stackrel{\text{pyk}}{\rightarrow}$ “exist-sub " is " where " is " end sub”]

$\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ex}}$

$[\langle \mathsf{a} \equiv^0 \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}} \stackrel{\text{val}}{\rightarrow} \lambda \mathsf{c}.\mathsf{x}^{\mathrm{Ex}} \wedge \langle \mathsf{a} \equiv^1 \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}}]$

$[\langle \mathsf{x} \equiv^0 \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ex}} \stackrel{\text{tex}}{\rightarrow}$ “\langle #1.
{\equiv}^0 #2.
| #3.
{:==} #4.
\rangle_{Ex} ”]

$[\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ex}} \stackrel{\text{pyk}}{\rightarrow}$ “exist-sub0 " is " where " is " end sub”]

$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ex}}$

$[\langle \mathsf{a} \equiv^1 \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}} \stackrel{\mathrm{val}}{\to} \mathsf{a!x!t!}$
$\mathrm{If}(\mathsf{b} \stackrel{\mathrm{r}}{=} \lceil \forall_{\mathrm{obj}} \mathsf{u} \colon \mathsf{v} \rceil, \mathsf{F},$
$\mathrm{If}(\mathsf{b}^{\mathrm{Ex}} \wedge \mathsf{b} \stackrel{\mathrm{t}}{=} \mathsf{x}, \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{t}, \mathrm{If}($
$\mathsf{a} \stackrel{\mathrm{r}}{=} \mathsf{b}, \langle \mathsf{a}^{\mathrm{t}} \equiv^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}}, \mathsf{F})))]$

$[\langle \mathsf{x} \equiv^1 \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ex}} \stackrel{\mathrm{tex}}{\to}$ "\langle #1.
{\equiv}^1 #2.
| #3.
{:==} #4.
\rangle_{Ex} "]

$[\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ex}} \stackrel{\mathrm{pyk}}{\to}$ "exist-sub1 " is " where " is " end sub"]

$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ex}}$

$[\langle \mathsf{a} \equiv^* \mathsf{b} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}} \stackrel{\mathrm{val}}{\to} \mathsf{b!x!t!}\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\langle \mathsf{a}^{\mathrm{h}} \equiv^1 \mathsf{b}^{\mathrm{h}} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}}, \langle \mathsf{a}^{\mathrm{t}} \equiv^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} :== \mathsf{t} \rangle_{\mathrm{Ex}}, \mathsf{F}))]$

$[\langle \mathsf{x} \equiv^* \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ex}} \stackrel{\mathrm{tex}}{\to}$ "\langle #1.
{\equiv}^* #2.
| #3.
{:==} #4.
\rangle_{Ex} "]

$[\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ex}} \stackrel{\mathrm{pyk}}{\to}$ "exist-sub* " is " where " is " end sub"]

ph_1

$[\mathrm{ph}_1 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{ph}_1 \doteq \mathsf{a}_{\mathrm{Ph}}] \rceil)]$

$[\mathrm{ph}_1 \stackrel{\mathrm{tex}}{\to}$ "ph_{1}"]

$[\mathrm{ph}_1 \stackrel{\mathrm{pyk}}{\to}$ "ph1"]

ph_2

$[\mathrm{ph}_2 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{ph}_2 \doteq \mathsf{b}_{\mathrm{Ph}}] \rceil)]$

$[\mathrm{ph}_2 \stackrel{\mathrm{tex}}{\to}$ "ph_{2}"]

$[\mathrm{ph}_2 \stackrel{\mathrm{pyk}}{\to}$ "ph2"]

ph_3

$[\mathrm{ph}_3 \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{ph}_3 \ddot{=} \mathsf{c}_{\mathrm{Ph}}]\rceil)]$

$[\mathrm{ph}_3 \stackrel{\text{tex}}{\rightarrow}$ "ph_{3}"]

$[\mathrm{ph}_3 \stackrel{\text{pyk}}{\rightarrow}$ "ph3"]

$*_{\mathrm{Ph}}$

$[*_{\mathrm{Ph}} \stackrel{\text{tex}}{\rightarrow}$ "#1.
_{Ph} "]

$[*_{\mathrm{Ph}} \stackrel{\text{pyk}}{\rightarrow}$ "placeholder-var " end var"]

$*^{\mathrm{Ph}}$

$[\mathsf{x}^{\mathrm{Ph}} \stackrel{\text{tex}}{\rightarrow}$ "#1.
^{Ph}"]

$[*^{\mathrm{Ph}} \stackrel{\text{pyk}}{\rightarrow}$ "" is placeholder-var"]

$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ph}}$

$[\langle \mathsf{x} \equiv \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ph}} \stackrel{\text{tex}}{\rightarrow}$ "\langle #1.
{\equiv} #2.
| #3.
{:==} #4.
\rangle_{Ph} "]

$[\langle * \equiv * \mid * :== * \rangle_{\mathrm{Ph}} \stackrel{\text{pyk}}{\rightarrow}$ "ph-sub " is " where " is " end sub"]

$\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ph}}$

$[\langle \mathsf{x} \equiv^0 \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ph}} \stackrel{\text{tex}}{\rightarrow}$ "\langle #1.
{\equiv}^0 #2.
| #3.
{:==} #4.
\rangle_{Ph} "]

$[\langle * \equiv^0 * \mid * :== * \rangle_{\mathrm{Ph}} \stackrel{\text{pyk}}{\rightarrow}$ "ph-sub0 " is " where " is " end sub"]

$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ph}}$

[$\langle \mathsf{x} \equiv^1 \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ph}} \stackrel{\text{tex}}{\to}$ "\langle #1.
{\equiv}^1 #2.
| #3.
{:==} #4.
\rangle_{Ph} "]

[$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Ph}} \stackrel{\text{pyk}}{\to}$ "ph-sub1 " is " where " is " end sub"]

$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ph}}$

[$\langle \mathsf{x} \equiv^* \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Ph}} \stackrel{\text{tex}}{\to}$ "\langle #1.
{\equiv}^∗ #2.
| #3.
{:==} #4.
\rangle_{Ph} "]

[$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Ph}} \stackrel{\text{pyk}}{\to}$ "ph-sub∗ " is " where " is " end sub"]

$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}}$

[$\langle \mathsf{x} \equiv \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Me}} \stackrel{\text{tex}}{\to}$ "\langle #1.
{\equiv} #2.
| #3.
{:==} #4.
\rangle_{Me}"]

[$\langle * \equiv * \mid * :== * \rangle_{\mathrm{Me}} \stackrel{\text{pyk}}{\to}$ "meta-sub " is " where " is " end sub"]

$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}}$

[$\langle \mathsf{x} \equiv^1 \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Me}} \stackrel{\text{tex}}{\to}$ "\langle #1.
{\equiv}^1 #2.
| #3.
{:==} #4.
\rangle_{Me} "]

[$\langle * \equiv^1 * \mid * :== * \rangle_{\mathrm{Me}} \stackrel{\text{pyk}}{\to}$ "meta-sub1 " is " where " is " end sub"]

$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}}$

[$\langle \mathsf{x} \equiv^* \mathsf{y} | \mathsf{z} :== \mathsf{u} \rangle_{\mathrm{Me}} \overset{\mathrm{tex}}{\to}$ "\langle #1.
{\equiv}ˆ* #2.
| #3.
{:==} #4.
\rangle_{Me} "]

[$\langle * \equiv^* * \mid * :== * \rangle_{\mathrm{Me}} \overset{\mathrm{pyk}}{\to}$ "meta-sub* " is " where " is " end sub"]

bs

[$\mathsf{bs} \overset{\mathrm{tex}}{\to}$ "\mathsf {bs}"]

[$\mathsf{bs} \overset{\mathrm{pyk}}{\to}$ "var big set"]

OBS

[$\mathsf{OBS} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{OBS} \ddot{=} \overline{\mathsf{bs}}] \rceil)$]

[$\mathsf{OBS} \overset{\mathrm{tex}}{\to}$ " \mathsf {OBS}"]

[$\mathsf{OBS} \overset{\mathrm{pyk}}{\to}$ "object big set"]

$\mathcal{BS}$

[$\mathcal{BS} \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathcal{BS} \ddot{=} \underline{\mathsf{bs}}] \rceil)$]

[$\mathcal{BS} \overset{\mathrm{tex}}{\to}$ "{\cal BS}"]

[$\mathcal{BS} \overset{\mathrm{pyk}}{\to}$ "meta big set"]

Ø

[$\mathrm{\O} \overset{\mathrm{tex}}{\to}$ "\mathrm{\O}"]

[$\mathrm{\O} \overset{\mathrm{pyk}}{\to}$ "zermelo empty set"]

SystemQ

$[\text{SystemQ} \overset{\text{stmt}}{\rightarrow} \forall \underline{\mathsf{m}}\colon \text{UB}(\text{us}[\underline{\mathsf{m}}], \text{SetOfFxs}) \oplus \forall(\text{fx})\colon \forall(\text{fy})\colon \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in$
$\text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow$
$\dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{r1})\colon (\text{r1}) \in \mathsf{f}_{\text{Ph}} \Rightarrow \dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow$
$\dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot{-}\,((\text{r1}) = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\forall_{\text{obj}}(\text{f1})\colon \forall_{\text{obj}}(\text{f2})\colon \forall_{\text{obj}}(\text{f3})\colon \forall_{\text{obj}}(\text{f4})\colon \{\{(\text{f1}), (\text{f1})\}, \{(\text{f1}), (\text{f2})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\{\{(\text{f3}), (\text{f3})\}, \{(\text{f3}), (\text{f4})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow (\text{f1}) = (\text{f3}) \Rightarrow (\text{f2}) = (\text{f4}))\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\forall_{\text{obj}}(\text{s1})\colon (\text{s1}) \in \text{N} \Rightarrow \dot{-}\,(\forall_{\text{obj}}(\text{s2})\colon \dot{-}\,(\{\{(\text{s1}), (\text{s1})\}, \{(\text{s1}), (\text{s2})\}\} \in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}}(\epsilon)\colon \dot{-}\,(\forall_{\text{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\text{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= (\epsilon) \Rightarrow \dot{-}\,(\dot{-}\,(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{-}\,(\forall_{\text{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{d}_{\text{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\text{fx})[\underline{\mathsf{m}}] + (\text{fy})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}] + (-\text{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{-}\,(\forall_{\text{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{d}_{\text{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\text{fx})[\underline{\mathsf{m}}] + (\text{fy})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}] + (-\text{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\text{n})\text{n})\text{n})\text{n})\text{n}\} =$
$\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{r1})\colon (\text{r1}) \in$
$\mathsf{f}_{\text{Ph}} \Rightarrow \dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,((\text{r1}) = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\forall_{\text{obj}}(\text{f1})\colon \forall_{\text{obj}}(\text{f2})\colon \forall_{\text{obj}}(\text{f3})\colon \forall_{\text{obj}}(\text{f4})\colon \{\{(\text{f1}), (\text{f1})\}, \{(\text{f1}), (\text{f2})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\{\{(\text{f3}), (\text{f3})\}, \{(\text{f3}), (\text{f4})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow (\text{f1}) = (\text{f3}) \Rightarrow (\text{f2}) = (\text{f4}))\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\forall_{\text{obj}}(\text{s1})\colon (\text{s1}) \in \text{N} \Rightarrow \dot{-}\,(\forall_{\text{obj}}(\text{s2})\colon \dot{-}\,(\{\{(\text{s1}), (\text{s1})\}, \{(\text{s1}), (\text{s2})\}\} \in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}}(\epsilon)\colon \dot{-}\,(\forall_{\text{obj}}\overline{\mathsf{n}}\colon \dot{-}\,(\forall_{\text{obj}}\overline{\mathsf{m}}\colon \dot{-}\,(0 <= (\epsilon) \Rightarrow \dot{-}\,(\dot{-}\,(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{-}\,(\forall_{\text{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{d}_{\text{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\text{fy})[\underline{\mathsf{m}}] + (\text{fx})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}] + (-\text{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{-}\,(\forall_{\text{obj}}\underline{\mathsf{m}}\colon \dot{-}\,(\mathsf{d}_{\text{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\text{fy})[\underline{\mathsf{m}}] + (\text{fx})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}] + (-\text{ud}_{\text{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\text{n})\text{n})\text{n})\text{n})\text{n}\} \oplus$
$\forall(\text{fx})\colon \forall(\text{fy})\colon \forall(\text{fz})\colon \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{r1})\colon (\text{r1}) \in$
$\mathsf{f}_{\text{Ph}} \Rightarrow \dot{-}\,(\forall_{\text{obj}}(\text{op1})\colon \dot{-}\,(\dot{-}\,(\forall_{\text{obj}}(\text{op2})\colon \dot{-}\,(\dot{-}\,(\dot{-}\,((\text{op1}) \in \text{N} \Rightarrow \dot{-}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{-}\,((\text{r1}) = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot{-}\,(\forall_{\text{obj}}(\text{f1})\colon \forall_{\text{obj}}(\text{f2})\colon \forall_{\text{obj}}(\text{f3})\colon \forall_{\text{obj}}(\text{f4})\colon \{\{(\text{f1}), (\text{f1})\}, \{(\text{f1}), (\text{f2})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$

$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}]*(\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]*(\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus \forall(\mathrm{fx})\colon \forall(\mathrm{rx})\colon \forall(\mathrm{ry})\colon (\mathrm{rx}) = (\mathrm{ry}) \vdash (\mathrm{fx}) \in (\mathrm{rx}) \vdash (\mathrm{fx}) \in (\mathrm{ry}) \oplus$
$\forall\underline{\mathsf{m}}\colon \mathrm{UB}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-ud_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-ud_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}, SetOfReals) \vdash xs[(\underline{\mathsf{m}} + 1)] = xs[\underline{\mathsf{m}}] \oplus \forall\underline{\mathsf{a}}: \forall\underline{\mathsf{b}}: \forall\underline{\mathsf{x}}: \forall\underline{\mathsf{y}}: \underline{\mathsf{y}} \in \underline{\mathsf{x}} \vdash \underline{\mathsf{b}} \vdash$

$\underline{y} \in \{ph \in \underline{x} \mid \underline{a}\} \oplus \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \underline{x} = \underline{y} \oplus \forall \underline{s}\colon \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}(\underline{s} \in$
$\{\underline{x}, \underline{y}\} \Rightarrow \dot{\neg}(\underline{s}==\underline{x})n \Rightarrow \underline{s}==\underline{y} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{s}==\underline{x})n \Rightarrow \underline{s}==\underline{y} \Rightarrow \underline{s} \in \{\underline{x}, \underline{y}\})n)n \oplus$
$\forall \underline{m}\colon \forall \underline{(fx)}\colon \forall \underline{(sy)}\colon \underline{m} \in N \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \underline{(fx)} \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(fx)}[\underline{m}]\}\} \in \underline{(fx)} \oplus \forall \underline{m}\colon \forall \underline{n}\colon \underline{n} = 0 \vdash BS(\underline{m}, \underline{n}) =$
$rec(1+1)(exp)\underline{m} \oplus \forall \underline{(sx)}\colon \forall \underline{(fy)}\colon \forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)} \vdash$
$\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in \underline{(fy)} \Rightarrow \overline{(s1)} \in \underline{(fx)} \vdash \underline{(fx)} = \underline{(fy)} \oplus \forall \underline{x}\colon (\underline{x} + 0) = \underline{x} \oplus$
$\forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in$
$f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\underline{(fy)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fy)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{n}\colon \dot{\neg}(\forall_{obj}\overline{m}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow$
$\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{m}] + (-u\underline{(fy)}[\overline{m}]))| =$
$\overline{(\epsilon)})n)n)n)n)n \oplus \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \Rightarrow \underline{x} <= \underline{y} \oplus \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b} \oplus$
$\forall \underline{m}\colon \dot{\neg}(ysF[\underline{m}] <= (xs[\underline{m}] + rec\underline{m}) \Rightarrow \dot{\neg}(\dot{\neg}(ysF[\underline{m}] = (xs[\underline{m}] + rec\underline{m}))n)n)n \oplus$
$\forall \underline{x}\colon \forall \underline{y}\colon \underline{x} \in Q \vdash \underline{y} \in Q \vdash (\underline{x} * \underline{y}) \in Q \oplus \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \forall \underline{(fz)}\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\})))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in f_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] + (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] + (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{f1})\colon \forall_{\mathrm{obj}}(\mathrm{f2})\colon \forall_{\mathrm{obj}}(\mathrm{f3})\colon \forall_{\mathrm{obj}}(\mathrm{f4})\colon \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s1})\colon (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}(\mathrm{s2})\colon \dot{\neg}\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] + (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] + (\mathrm{fz})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus$
$\forall(\mathrm{fx}){:}\,\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\,(\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\,\forall_{\mathrm{obj}}(\mathrm{f2}){:}\,\forall_{\mathrm{obj}}(\mathrm{f3}){:}\,\forall_{\mathrm{obj}}(\mathrm{f4}){:}\,\{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\,(\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\,\dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon){:}\,\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot\neg\,(0 <= (\epsilon) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon))\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1}){:}\,\dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{crs1}){:}\,\dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{r1}){:}\,(\mathrm{r1}) \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op1}){:}\,\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{op2}){:}\,\dot\neg\,(\dot\neg\,(\dot\neg\,((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot\neg\,((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{f1}){:}\,\forall_{\mathrm{obj}}(\mathrm{f2}){:}\,\forall_{\mathrm{obj}}(\mathrm{f3}){:}\,\forall_{\mathrm{obj}}(\mathrm{f4}){:}\,\{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s1}){:}\,(\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}(\mathrm{s2}){:}\,\dot\neg\,(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon){:}\,\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot\neg\,(0 <= (\epsilon) \Rightarrow \dot\neg\,(\dot\neg\,(0 = (\epsilon))\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot\neg\,(\dot\neg\,(|((\mathrm{fx})[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus \forall\underline{\mathsf{x}}{:}\,(\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}} \oplus \forall(\mathrm{sx}){:}\,\forall(\mathrm{sy}){:}\,\forall(\mathrm{sz}){:}\,\dot\neg\,((\mathrm{sx}) =$

(sy))n ⇒ (sx) = (sz) ⊢ (sx) ∈ {(sy), (sz)} ⊕ ∀a: ∀b: a ⊢ b ⊕
∀(fx): ∀(sy): λc.Typeseries0(⌈(fx)⌉, ⌈(sy)⌉) ⊩ ∸¬(∸¬(∀_obj (r1): (r1) ∈ (fx) ⇒
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ (sy))n)n ⇒
∸¬((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸¬(∀_obj (f1): ∀_obj (f2): ∀_obj (f3): ∀_obj (f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ (fx) ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ (fx) ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸¬(∀_obj (s1): (s1) ∈ N ⇒ ∸¬(∀_obj (s2): ∸¬({{(s1), (s1)}, {(s1), (s2)}} ∈
(fx))n)n)n)n ⊕ ∀(rx): ∀(ry): (rx) = (ry) ⊢ (ry) = (rx) ⊕ ∀m: ∀x: m = 0 ⊢
x(exp)m = 1 ⊕ ∀x: ∀y: ∀z: 0 <= z ⇒ x <= y ⇒ (x ∗ z) <= (y ∗ z) ⊕
∀(sx): ∀(sx1): ∀(sy): ∀(sy1): (sx) = (sx1) ⊢ (sy) = (sy1) ⊢ {(sx), (sy)} =
{(sx1), (sy1)} ⊕ ∀a: ∀b: ∀x: ∀y: y ∈ {ph ∈ x | a} ⊢ b ⊕ ∀(fx): ∀(fy): ∀(fz): {ph ∈
P({ph ∈ P({ph ∈ P(P(Union({N, Q}))) |
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬(a_Ph = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) | ∸¬(∸¬(∀_obj (r1): (r1) ∈
f_Ph ⇒ ∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸¬(∀_obj (f1): ∀_obj (f2): ∀_obj (f3): ∀_obj (f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ f_Ph ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ f_Ph ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸¬(∀_obj (s1): (s1) ∈ N ⇒ ∸¬(∀_obj (s2): ∸¬({{(s1), (s1)}, {(s1), (s2)}} ∈
f_Ph)n)n)n)n}) | ∀_obj (ϵ): ∸¬(∀_obj n: ∸¬(∀_obj m: ∸¬(0 <= (ϵ) ⇒ ∸¬(∸¬(0 = (ϵ))n)n)n ⇒
n <= m ⇒ ∸¬(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬(a_Ph = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸¬(∀_obj m: ∸¬(e_Ph =
{{m, m}, {m, ((fx)[m] ∗ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬(a_Ph = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸¬(∀_obj m: ∸¬(d_Ph =
{{m, m}, {m, ((fy)[m] + (fz)[m])}})n)n}[m])}})n)n}[m] + (−ud_Ph[m]))| <=
(ϵ) ⇒ ∸¬(∸¬(|({ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬(a_Ph = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸¬(∀_obj m: ∸¬(e_Ph =
{{m, m}, {m, ((fx)[m] ∗ {ph ∈ {ph ∈ P(P(Union({N, Q}))) |
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬(a_Ph = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n} | ∸¬(∀_obj m: ∸¬(d_Ph =
{{m, m}, {m, ((fy)[m] + (fz)[m])}})n)n}[m])}})n)n}[m] + (−ud_Ph[m]))| =
(ϵ))n)n)n)n)n} = {ph ∈ P({ph ∈ P({ph ∈ P(P(Union({N, Q}))) |
∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬(a_Ph = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n}) | ∸¬(∸¬(∀_obj (r1): (r1) ∈
f_Ph ⇒ ∸¬(∀_obj (op1): ∸¬(∸¬(∀_obj (op2): ∸¬(∸¬(∸¬((op1) ∈ N ⇒ ∸¬((op2) ∈ Q)n)n ⇒
∸¬((r1) = {{(op1), (op1)}, {(op1), (op2)}})n)n)n)n)n)n ⇒
∸¬(∀_obj (f1): ∀_obj (f2): ∀_obj (f3): ∀_obj (f4): {{(f1), (f1)}, {(f1), (f2)}} ∈ f_Ph ⇒
{{(f3), (f3)}, {(f3), (f4)}} ∈ f_Ph ⇒ (f1) = (f3) ⇒ (f2) = (f4))n)n ⇒
∸¬(∀_obj (s1): (s1) ∈ N ⇒ ∸¬(∀_obj (s2): ∸¬({{(s1), (s1)}, {(s1), (s2)}} ∈

$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \oplus \mathrm{LUB}(\underline{(fx)}, \underline{(fys)}) \vdash \dot\neg(\mathrm{UB}(\underline{(fx)}, \underline{(fys)}) \Rightarrow \dot\neg(\mathrm{UB}(\underline{(fz)}, \underline{(fys)}) \Rightarrow$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \underline{(fx)} = \underline{(fz)})n)n \oplus$
$\forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(fxs)}\colon \underline{(fx)} \in \underline{(fxs)} \Rightarrow$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \underline{(fx)} = \underline{(fy)} \vdash$
$\mathrm{UB}(\underline{(fy)}, \underline{(fxs)}) \oplus \forall\underline{(fx)}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\underline{m}: \dot\neg(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\overline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) | \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$f_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{n}: \dot\neg(\forall_{obj}\overline{m}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot\neg(|(\overline{(fx)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\overline{(fx)}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} \oplus \dot\neg(0 = 1)n \oplus \forall\underline{m}: Nat(\underline{m}) \Vdash$
$0 <= \underline{m} \oplus \forall\overline{(sx)}: \forall\overline{(sy)}: \forall\overline{(sz)}: \dot\neg(\forall_{obj}\overline{(sy)}: \dot\neg(\dot\neg(\overline{(sx)} \in \overline{(sy)} \Rightarrow \dot\neg(\overline{(sy)} \in$
$\overline{(sz)})n)n)n)n \vdash \overline{(sx)} \in Union(\overline{(sz)}) \oplus \forall\underline{x}: \forall\underline{y}: \dot\neg(\underline{x} == \underline{y} \Rightarrow \forall_{obj}\overline{s}: \dot\neg(\overline{s} \in \underline{x} \Rightarrow \overline{s} \in$
$\underline{y} \Rightarrow \dot\neg(\overline{s} \in \underline{y} \Rightarrow \overline{s} \in \underline{x})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{s}: \dot\neg(\overline{s} \in \underline{x} \Rightarrow \overline{s} \in \underline{y} \Rightarrow \dot\neg(\overline{s} \in \underline{y} \Rightarrow \overline{s} \in$
$\underline{x})n)n \Rightarrow \underline{x} == \underline{y})n)n \oplus \forall\underline{x}: \dot\neg(\dot\neg(0 <= \underline{x} \Rightarrow \dot\neg(|\underline{x}| = \underline{x})n)n)n \Rightarrow \dot\neg(\dot\neg(0 <=$
$\underline{x})n \Rightarrow \dot\neg(|\underline{x}| = (-u\underline{x}))n)n \oplus \forall\overline{(rx)}: \forall\overline{(ry)}: \forall\overline{(rz)}: \overline{(rx)} = \overline{(ry)} \vdash \overline{(ry)} = \overline{(rz)} \vdash$
$\overline{(rx)} = \overline{(rz)} \oplus \forall\underline{m}: \forall\overline{(fx)}: \underline{m} = 0 \vdash \overline{(fx)}(exp)\underline{m} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) |$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in f_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$f_{Ph})n)n)n)n\}) | \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{n}: \dot\neg(\forall_{obj}\overline{m}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) | \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} | \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} \oplus$
$\forall\underline{x}: \forall\underline{y}: (\underline{x} + \underline{y}) = (\underline{y} + \underline{x}) \oplus \forall\underline{m}: \forall\overline{(fx)}: \forall\overline{(fy)}: \{ph \in \{ph \in P(P(Union(\{N, Q\}))) |$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]+\overline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]=(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]+\overline{(\mathrm{fy})}[\underline{\mathsf{m}}])\oplus$
$\forall\overline{(\mathrm{v1})}\colon\forall\underline{\mathsf{a}}\colon\forall\underline{\mathsf{b}}\colon\forall\underline{\mathsf{c}}\colon\langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\overline{(\mathrm{v1})}{:}{=}{=}0\rangle_{\mathrm{Me}}\Vdash\langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\overline{(\mathrm{v1})}{:}{=}{=}(\overline{(\mathrm{v1})}+1)\rangle_{\mathrm{Me}}\Vdash\underline{\mathsf{b}}\Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\mathrm{v1})}\colon\underline{\mathsf{a}}\Rightarrow\underline{\mathsf{c}}\Rightarrow\forall_{\mathrm{obj}}\overline{(\mathrm{v1})}\colon\underline{\mathsf{a}}\oplus\forall\underline{\mathsf{m}}\colon\dot\neg(\mathrm{UB}(\mathrm{xs}[\underline{\mathsf{m}}],\mathrm{SetOfFxs}))\mathrm{n}\oplus$
$\forall\overline{(\mathrm{sx})}\colon\forall\overline{(\mathrm{sy})}\colon\forall\overline{(\mathrm{sz})}\colon\overline{(\mathrm{sx})}=\overline{(\mathrm{sy})}\vdash\overline{(\mathrm{sx})}\in\overline{(\mathrm{sz})}\vdash\overline{(\mathrm{sy})}\in\overline{(\mathrm{sz})}\oplus\forall\underline{\mathsf{m}}\colon\forall\underline{\mathsf{n}}\colon\underline{\mathsf{n}}=0\vdash$
$\mathrm{UStelescope}(\underline{\mathsf{m}},\underline{\mathsf{n}})=|(\mathrm{us}[\underline{\mathsf{m}}]+(-\mathrm{uus}[(\underline{\mathsf{m}}+1)]))|\oplus\forall\overline{(\mathrm{fx})}\colon\forall\overline{(\mathrm{fy})}\colon\forall\overline{(\mathrm{fz})}\colon\{\mathrm{ph}\in$
$\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]+\overline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]+\overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]+\overline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]+\overline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}=\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(\mathrm{f1})}=\overline{(\mathrm{f3})}\Rightarrow\overline{(\mathrm{f2})}=\overline{(\mathrm{f4})})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})}\in\mathrm{N}\Rightarrow\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n}\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot\neg(|(\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\mid\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(\mathrm{fx})}[\underline{\mathsf{m}}]+\{\mathrm{ph}\in\{\mathrm{ph}\in\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))))\mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})}\in\mathrm{N}\Rightarrow\dot\neg(\overline{(\mathrm{op2})}\in\mathrm{Q})\mathrm{n})\mathrm{n}\Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon (\underline{\mathsf{x}} * \underline{\mathsf{y}}) = (\underline{\mathsf{y}} * \underline{\mathsf{x}}) \oplus \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \underline{(fx)} \in \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon \underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} * \underline{\mathsf{z}}) =$
$(\underline{\mathsf{y}} * \underline{\mathsf{z}}) \oplus \forall\underline{\mathsf{a}}\colon \underline{\mathsf{a}} \vdash \underline{\mathsf{a}} \oplus \forall\underline{\mathsf{m}}\colon \lambda\mathsf{c}.TypeNat0(\lceil\underline{\mathsf{m}}\rceil) \Vdash \underline{\mathsf{m}} \in N \oplus$
$\forall\underline{(sx)}\colon \forall\underline{(sx1)}\colon \forall\underline{(sy)}\colon \forall\underline{(sy1)}\colon \{\{\underline{(sx)}, \underline{(sx)}\}, \{\underline{(sx)}, \underline{(sy)}\}\} \in \{ph \in$
$P(P(Union(\{\underline{(sx1)}, \underline{(sy1)}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$\underline{(sx1)} \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \vdash \underline{(sy)} \in \underline{(sy1)} \oplus \forall\underline{\mathsf{m}}\colon UB(\{ph \in$
$P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}, \mathrm{SetOfReals}) \vdash \mathrm{us}[(\underline{\mathsf{m}} + 1)] = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus \forall\overline{(\mathrm{sx})}\colon \forall\overline{(\mathrm{sy})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \overline{(\mathrm{sx})} \Rightarrow \overline{(\mathrm{s1})} \in \overline{(\mathrm{sy})} \vdash \overline{(\mathrm{sx})} \in$
$\mathrm{P}(\overline{(\mathrm{sy})}) \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \dot{\neg}(\underline{\mathsf{x}} <= \underline{\mathsf{y}})\mathrm{n} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{x}} \oplus \forall\underline{\mathsf{s}}\colon \forall\underline{\mathsf{x}}\colon \dot{\neg}(\underline{\mathsf{s}} \in \mathrm{P}(\underline{\mathsf{x}}) \Rightarrow \forall_{\mathrm{obj}}\overline{\mathsf{s}}\colon \overline{\mathsf{s}} \in \underline{\mathsf{s}} \Rightarrow$
$\overline{\mathsf{s}} \in \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{s}}\colon \overline{\mathsf{s}} \in \underline{\mathsf{s}} \Rightarrow \overline{\mathsf{s}} \in \underline{\mathsf{x}} \Rightarrow \underline{\mathsf{s}} \in \mathrm{P}(\underline{\mathsf{x}}))\mathrm{n})\mathrm{n} \oplus \forall\overline{(\mathrm{fx})}\colon \forall\overline{(\mathrm{fy})}\colon \overline{(\mathrm{fx})} = \overline{(\mathrm{fy})} \vdash$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fz})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <= ((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus$

$\forall\underline{\mathsf{m}}\colon\forall\underline{(\mathrm{fx})}\colon\dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{-}\,(\dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <=$
$(\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \oplus \forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\forall\underline{(\mathrm{fxs})}\colon\mathrm{UB}(\underline{(\mathrm{fy})},\underline{(\mathrm{fxs})}) \vdash \underline{(\mathrm{fx})} \in$
$\underline{(\mathrm{fxs})} \vdash \dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{-}\,(\dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{fx})} =$
$\underline{(\mathrm{fy})} \oplus \forall\underline{\mathsf{m}}\colon\forall\underline{(\epsilon)}\colon\forall\underline{(\mathrm{fx})}\colon\forall\underline{(\mathrm{fy})}\colon\forall\underline{(\mathrm{fz})}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{-}\,(\dot{-}\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{-}\,(\dot{-}\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{-}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{-}\,(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \oplus \mathrm{us}[0] = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon\overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon\forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon\{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon\overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon\dot{-}\,(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{-}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\mathsf{x}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot{-}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}\,(\dot{-}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{-}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\mathsf{x}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon\dot{-}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})},\overline{(\mathrm{crs1})}\},\{\overline{(\mathrm{crs1})},1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus \forall\underline{\mathsf{x}}\colon\underline{\mathsf{x}} <= \underline{\mathsf{x}} \oplus \forall\underline{\mathsf{s}}\colon\dot{-}\,(\underline{\mathsf{s}} \in \emptyset)\mathrm{n} \oplus$
$\forall\underline{(\mathrm{fx})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{-}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{-}\,(\dot{-}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}\,(\dot{-}\,(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon\dot{-}\,(\dot{-}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon\dot{-}\,(\dot{-}\,(\dot{-}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\mathrm{fx})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\mathrm{fx})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$|\mathrm{f}\overline{(\mathrm{fx})}| = \overline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= ((\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\mathrm{fx})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\mathrm{fx})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$|\mathrm{f}\overline{(\mathrm{fx})}| = \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{u}\overline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\})\mathrm{n})\mathrm{n} \oplus \forall\underline{\mathsf{m}}\colon \forall\overline{(\mathrm{fx})}\colon \dot{\neg}(0 <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{\mathsf{m}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \overline{(\mathrm{fx})}(\mathrm{exp})\underline{\mathsf{m}} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$

$\{\{\underline{m},\underline{m}\},\{\underline{m},(x[\underline{m}]*y[\underline{m}])\}\})n)n\}[\overline{m}]+(-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in$
$\{ph \in P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1) \in$
$N \Rightarrow \dot\neg((op2) \in Q)n)n \Rightarrow \dot\neg(a_{Ph} =$
$\{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}{:}\,\dot\neg(e_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(x[\underline{m}]*y[\underline{m}])\}\})n)n\}[\overline{m}]+(-ud_{Ph}[\overline{m}]))| = \overline{(\epsilon)})n)n)n)n)n\} \oplus$
$\forall\underline{x}{:}\,(\underline{x}+(-u\underline{x})) = 0 \oplus \forall\underline{m}{:}\,\forall(fx){:}\,\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}{:}\,\dot\neg(\dot\neg(\dot\neg(\dot\neg(((fx)[\underline{m}] = 0)n \Rightarrow \dot\neg(f_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m}, rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot\neg(((fx)[\underline{m}] = 0 \Rightarrow \dot\neg(f_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}] = if((fx)[\underline{m}] = 0, 0, rec(fx)[\underline{m}]) \oplus \forall\underline{x}{:}\,\forall\underline{y}{:}\,\forall\underline{z}{:}\,\underline{x} =$
$\underline{y} \Rightarrow \underline{x} = \underline{z} \Rightarrow \underline{y} = \underline{z} \oplus \forall\underline{m}{:}\,\dot\neg(xs[\underline{m}] <= ysF[\underline{m}] \Rightarrow \dot\neg(\dot\neg(xs[\underline{m}] = ysF[\underline{m}])n)n)n \oplus$
$\forall\underline{x}{:}\,\forall\underline{y}{:}\,\underline{x} \in Q \vdash \underline{y} \in Q \vdash (\underline{x}+\underline{y})\in Q \oplus \forall\underline{m}{:}\,\forall\underline{n}{:}\,\dot\neg(0 <= \underline{n} \Rightarrow \dot\neg(\dot\neg(0 = \underline{n})n)n)n \vdash$
$UStelescope(\underline{m},\underline{n}) = (|(us[(\underline{m}+\underline{n})] + (-uus[(\underline{m} + (\underline{n}+1))]))| +$
$UStelescope(\underline{m},(\underline{n}+(-u1)))) \oplus \forall(fx){:}\,\forall(fy){:}\,\forall(fz){:}\,\{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow$
$\dot\neg((op2)\in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}(r1){:}\,(r1)\in f_{Ph} \Rightarrow \dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow$
$\dot\neg((op2)\in Q)n)n \Rightarrow \dot\neg((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}(f1){:}\,\forall_{obj}(f2){:}\,\forall_{obj}(f3){:}\,\forall_{obj}(f4){:}\,\{\{(f1),(f1)\},\{(f1),(f2)\}\}\in f_{Ph} \Rightarrow$
$\{\{(f3),(f3)\},\{(f3),(f4)\}\}\in f_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot\neg(\forall_{obj}(s1){:}\,(s1)\in N \Rightarrow \dot\neg(\forall_{obj}(s2){:}\,\dot\neg(\{\{(s1),(s1)\},\{(s1),(s2)\}\}\in$
$f_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}{:}\,\dot\neg(\forall_{obj}\overline{n}{:}\,\dot\neg(\forall_{obj}\overline{m}{:}\,\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{n} <= \overline{m} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}{:}\,\dot\neg(d_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}{:}\,\dot\neg(d_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(((fx)[\underline{m}]+(fy)[\underline{m}])\}\})n)n\}[\underline{m}]+(fz)[\underline{m}])\}\})n)n\}[\overline{m}] +$
$(-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}{:}\,\dot\neg(d_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{m}{:}\,\dot\neg(d_{Ph} =$
$\{\{\underline{m},\underline{m}\},\{\underline{m},(((fx)[\underline{m}]+(fy)[\underline{m}])\}\})n)n\}[\underline{m}]+(fz)[\underline{m}])\}\})n)n\}[\overline{m}]+(-ud_{Ph}[\overline{m}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}(r1){:}\,(r1)\in$
$f_{Ph} \Rightarrow \dot\neg(\forall_{obj}(op1){:}\,\dot\neg(\dot\neg(\forall_{obj}(op2){:}\,\dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}(f1){:}\,\forall_{obj}(f2){:}\,\forall_{obj}(f3){:}\,\forall_{obj}(f4){:}\,\{\{(f1),(f1)\},\{(f1),(f2)\}\}\in f_{Ph} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fx)}[\underline{\mathsf{m}}] + \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}] + \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fx)}[\underline{\mathsf{m}}] + \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\overline{(fy)}[\underline{\mathsf{m}}] + \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} \oplus \forall\underline{\mathsf{x}}\colon \dot{\neg}\,(\underline{\mathsf{x}} = 0)n \Rightarrow (\underline{\mathsf{x}} * \mathrm{rec}\underline{\mathsf{x}}) = 1 \oplus \forall\underline{\mathsf{x}}\colon \dot{\neg}\,(\underline{\mathsf{x}} = 0)n \Rightarrow \underline{\mathsf{x}} \in \mathrm{Q} \Rightarrow$
$\mathrm{rec}\underline{\mathsf{x}} \in \mathrm{Q} \oplus \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \dot{\neg}\,(\underline{\mathsf{b}})n \Rightarrow \underline{\mathsf{a}} \vdash \dot{\neg}\,(\underline{\mathsf{b}})n \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})n \vdash \underline{\mathsf{b}} \oplus$
$\forall\underline{\mathsf{x}}\colon \lambda\mathsf{c}.\mathrm{TypeRational0}(\lceil\underline{\mathsf{x}}\rceil) \Vdash \underline{\mathsf{x}} \in \mathrm{Q} \oplus \forall\underline{(rx)}\colon \underline{(rx)} = \underline{(rx)} \oplus \forall\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathrm{UB}(\{ph \in$
$\mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}, \mathrm{SetOfReals}))\mathrm{n} \vdash \mathrm{us}[(\underline{\mathsf{m}} + 1)] = \mathrm{us}[\underline{\mathsf{m}}] \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow$
$(\underline{\mathsf{x}} + \underline{\mathsf{z}}) <= (\underline{\mathsf{y}} + \underline{\mathsf{z}}) \oplus \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{p}}\colon \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{z}}\colon \underline{\mathsf{p}}^{\mathrm{Ph}} \wedge \langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{\mathsf{p}}{:=}{=}\underline{\mathsf{z}}\rangle_{\mathrm{Ph}} \Vdash \dot{\neg}(\underline{\mathsf{z}} \in \{\mathrm{ph} \in \underline{\mathsf{x}} \mid$
$\underline{\mathsf{a}}\} \Rightarrow \dot{\neg}(\underline{\mathsf{z}} \in \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{z}} \in \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{z}} \in \{\mathrm{ph} \in \underline{\mathsf{x}} \mid \underline{\mathsf{a}}\})\mathrm{n})\mathrm{n} \oplus$
$\forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \underline{\mathsf{y}} \in \{\mathrm{ph} \in \underline{\mathsf{x}} \mid \underline{\mathsf{a}}\} \vdash \underline{\mathsf{y}} \in \underline{\mathsf{x}} \oplus$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon\forall_{\mathrm{obj}}\overline{(f2)}\colon\forall_{\mathrm{obj}}\overline{(f3)}\colon\forall_{\mathrm{obj}}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon\dot{\neg}\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}\,(|((fy)[\overline{\mathsf{m}}]+(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|((fy)[\overline{\mathsf{m}}]+(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}\vdash$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(\dot{\neg}\,(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\dot{\neg}\,(\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}]+(fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]<=(\{ph\in\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow$
$\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}}=\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fy)[\underline{\mathsf{m}}]+(fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+$
$(-u\overline{(\epsilon)})))n)n)n)n)n)n)n\Rightarrow\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon\forall_{\mathrm{obj}}\overline{(f2)}\colon\forall_{\mathrm{obj}}\overline{(f3)}\colon\forall_{\mathrm{obj}}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon\dot{\neg}\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}\,(|(\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}]+(fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon\dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((fx)[\underline{\mathsf{m}}]+(fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}=$
$\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{\mathrm{Ph}}\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}\,(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon\forall_{\mathrm{obj}}\overline{(f2)}\colon\forall_{\mathrm{obj}}\overline{(f3)}\colon\forall_{\mathrm{obj}}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{\mathrm{Ph}}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon\dot{\neg}\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\})\mid\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon\dot{\neg}\,(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}\,(\dot{\neg}\,(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}\,(|(\{ph\in\{ph\in P(P(Union(\{N,Q\})))\mid$
$\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(op1)}\in N\Rightarrow\dot{\neg}\,(\overline{(op2)}\in Q)n)n\Rightarrow$

$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \oplus$
$\forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{m1})}\colon \forall\underline{\mathsf{n}}\colon \underline{\mathsf{m}} <= \underline{\mathsf{n}} \vdash \dot{-}(\forall_{\mathrm{obj}}\underline{(\mathrm{m1})}\colon \dot{-}((\underline{\mathsf{m}} + \underline{(\mathrm{m1})}) = \underline{\mathsf{n}})n)n \oplus$
$\forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\mathrm{fep})}\colon \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fys})}\colon \mathrm{Limit}(\underline{(\mathrm{fx})}, \underline{(\mathrm{fys})}) \vdash$
$\forall_{\mathrm{obj}}\underline{(\mathrm{fep})}\colon \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{-}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$
$\underline{\mathsf{m}} <= \underline{\mathsf{n}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow |\mathrm{f}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{u}\mathsf{x}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}|[\overline{\mathsf{m}}] <=$
$(\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n)n \oplus \forall\underline{\mathsf{m}}\colon \forall\underline{(\mathrm{fx})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{-}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{-}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{-}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{-}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-u\underline{(fx)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \oplus$
$\forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}})) = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}})) \oplus \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon Nat(\underline{\mathsf{m}}) \Vdash Nat(\underline{\mathsf{n}}) \Vdash$
$\dot{\neg}(\underline{\mathsf{m}} <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{m}} = (\underline{\mathsf{n}} + 1))n)n)n \vdash \underline{\mathsf{m}} <= \underline{\mathsf{n}} \oplus$
$\forall\underline{(sx)}\colon \forall\underline{(sy)}\colon \forall\underline{(sz)}\colon \underline{(sx)} \in \{\underline{(sy)}, \underline{(sz)}\} \vdash \dot{\neg}(\underline{(sx)} = \underline{(sy)})n \Rightarrow \underline{(sx)} = \underline{(sz)} \oplus \forall\underline{\mathsf{x}}\colon \underline{\mathsf{x}} \in$
$Q \Rightarrow (-u\underline{\mathsf{x}}) \in Q \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{t}}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \langle\lceil\underline{\mathsf{a}}\rceil\equiv^0\lceil\underline{\mathsf{b}}\rceil|\lceil\underline{\mathsf{x}}\rceil{:}{=}{=}\lceil\underline{\mathsf{t}}\rceil\rangle_{\mathrm{Ex}} \Vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \oplus$
$\forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \dot{\neg}(\dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(if(\underline{\mathsf{y}} <= \underline{\mathsf{x}}, \underline{\mathsf{x}}, \underline{\mathsf{y}}) = \underline{\mathsf{x}})n)n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{y}} <= \underline{\mathsf{x}})n \Rightarrow$
$\dot{\neg}(if(\underline{\mathsf{y}} <= \underline{\mathsf{x}}, \underline{\mathsf{x}}, \underline{\mathsf{y}}) = \underline{\mathsf{y}})n)n \oplus \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{x}}\colon \dot{\neg}(0 <= \underline{\mathsf{m}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{m}})n)n)n \vdash$
$\underline{\mathsf{x}}(exp)\underline{\mathsf{m}} = (\underline{\mathsf{x}} * \underline{\mathsf{x}}(exp)(\underline{\mathsf{m}} + (-u1))) \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon ((\underline{\mathsf{x}} + \underline{\mathsf{y}}) + \underline{\mathsf{z}}) = (\underline{\mathsf{x}} + (\underline{\mathsf{y}} + \underline{\mathsf{z}})) \oplus$
$\forall\underline{(sx)}\colon \forall\underline{(sy)}\colon \underline{(sx)} = \underline{(sy)} \vdash \{\underline{(sx)}, \underline{(sx)}\} = \{\underline{(sy)}, \underline{(sy)}\} \oplus$
$\forall\underline{(v1)}\colon \forall\underline{(v2)}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(\epsilon)}\colon \forall\underline{(fx)}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\underline{(v1)}\colon \forall_{\mathrm{obj}}\underline{(v2)}\colon \dot{\neg}(0 <=$
$\underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{(\epsilon)})n)n)n \Rightarrow \underline{\mathsf{n}} <= \underline{(v1)} \Rightarrow \underline{\mathsf{n}} <= \underline{(v2)} \Rightarrow$
$\dot{\neg}(|(\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\underline{(v1)}] + (-u\underline{(fx)}[\underline{(v2)}]))| =$
$\underline{(\epsilon)})n)n)n)n)n \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{(v1)}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \langle\underline{\mathsf{a}}\equiv\underline{\mathsf{b}}|\underline{(v1)}{:}{=}{=}\underline{\mathsf{x}}\rangle_{\mathrm{Me}} \Vdash \forall_{\mathrm{obj}}\underline{(v1)}\colon \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \oplus$
$\forall\underline{(fx)}\colon \forall\underline{(fz)}\colon \forall\underline{(fys)}\colon UB(\underline{(fx)}, \underline{(fys)}) \vdash UB(\underline{(fz)}, \underline{(fys)}) \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \underline{(fx)} = \underline{(fz)} \vdash$
$LUB(\underline{(fx)}, \underline{(fys)}) \oplus \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \dot{\neg}(0 <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{n}})n)n)n \vdash BS(\underline{\mathsf{m}}, \underline{\mathsf{n}}) =$
$(rec(1 + 1)(exp)(\underline{\mathsf{m}} + \underline{\mathsf{n}}) + BS(\underline{\mathsf{m}}, (\underline{\mathsf{n}} + (-u1)))) \oplus \forall\underline{\mathsf{x}}\colon \forall\underline{\mathsf{y}}\colon \forall\underline{\mathsf{z}}\colon ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) * \underline{\mathsf{z}}) =$
$(\underline{\mathsf{x}} * (\underline{\mathsf{y}} * \underline{\mathsf{z}})) \oplus \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n \vdash \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}{:}\,\forall_{\mathrm{obj}}\overline{(f2)}{:}\,\forall_{\mathrm{obj}}\overline{(f3)}{:}\,\forall_{\mathrm{obj}}\overline{(f4)}{:}\,\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}{:}\,\overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}{:}\,\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}{:}\,\overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}{:}\,\forall_{\mathrm{obj}}\overline{(f2)}{:}\,\forall_{\mathrm{obj}}\overline{(f3)}{:}\,\forall_{\mathrm{obj}}\overline{(f4)}{:}\,\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}{:}\,\overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}{:}\,\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \oplus \forall\underline{\mathsf{x}}{:}\,\forall\underline{\mathsf{y}}{:}\,\forall\underline{\mathsf{z}}{:}\,\underline{\mathsf{x}} = \underline{\mathsf{y}} \Rightarrow (\underline{\mathsf{x}} + \underline{\mathsf{z}}) =$
$(\underline{\mathsf{y}} + \underline{\mathsf{z}}) \oplus \forall\underline{\mathsf{x}}{:}\,\forall\underline{\mathsf{a}}{:}\,\underline{\mathsf{a}} \vdash \forall_{\mathrm{obj}}\underline{\mathsf{x}}{:}\,\underline{\mathsf{a}} \oplus \forall\underline{\mathsf{m}}{:}\,\forall\underline{\mathsf{x}}{:}\,\dot{\neg}(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{\mathsf{x}})n)n)n \vdash$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(\mathrm{rec}\underline{\mathsf{m}} <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}(\dot{\neg}(\mathrm{rec}\underline{\mathsf{m}} = \underline{\mathsf{x}})n)n)n)n)n \oplus$
$\forall\underline{(sx)}{:}\,\forall\underline{(sx1)}{:}\,\forall\underline{(sy)}{:}\,\forall\underline{(sy1)}{:}\,\{\{\underline{(sx)},\underline{(sx)}\},\{\underline{(sx)},\underline{(sy)}\}\} \in \{ph \in$
$P(P(\mathrm{Union}(\{\underline{(sx1)},\underline{(sy1)}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$\underline{(sx1)} \Rightarrow \dot{\neg}(\overline{(op2)} \in \underline{(sy1)})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \vdash \underline{(sx)} \in \underline{(sx1)} \oplus$
$\forall\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathrm{UB}(\{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(r1)}{:}\,\overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(f1)}{:}\,\forall_{\mathrm{obj}}\overline{(f2)}{:}\,\forall_{\mathrm{obj}}\overline{(f3)}{:}\,\forall_{\mathrm{obj}}\overline{(f4)}{:}\,\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(s1)}{:}\,\overline{(s1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(s2)}{:}\,\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}{:}\,\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}{:}\,\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph \in \{ph \in P(P(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}{:}\,\dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph \in \{ph \in P(P(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op1)}{:}\,\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(op2)}{:}\,\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = 0)n \Rightarrow \dot{\neg}(f_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(d_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}]\}\})n)n \Rightarrow \dot{\neg}(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(d_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = 0 \Rightarrow \dot{\neg}(f_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n)n\}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(d_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (x[\underline{m}] + y[\underline{m}])\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-ud_{Ph}[\overline{m}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(e_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(d_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(c_{Ph} =$

$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}\colon \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}, SetOfReals))n \vdash xs[(\underline{\mathsf{m}} + 1)] = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)}))n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus \forall\underline{\mathsf{a}}\colon\forall\underline{\mathsf{b}}\colon\forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon \underline{\mathsf{y}} \in \{\mathrm{ph} \in \underline{\mathsf{x}} \mid \underline{\mathsf{a}}\} \vdash \dot\neg(\underline{\mathsf{y}} \in \underline{\mathsf{x}} \Rightarrow \dot\neg(\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \oplus$
$\forall\underline{\mathsf{x}}\colon\forall\underline{\mathsf{y}}\colon\forall\underline{\mathsf{z}}\colon \underline{\mathsf{x}} <= \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{y}} <= \underline{\mathsf{z}} \Rightarrow \underline{\mathsf{x}} <= \underline{\mathsf{z}} \oplus \forall\underline{\mathsf{s}}\colon\forall\underline{\mathsf{x}}\colon \dot\neg(\underline{\mathsf{s}} \in \cup\underline{\mathsf{x}} \Rightarrow \dot\neg(\underline{\mathsf{s}} \in \mathsf{j}_{\mathrm{Ex}} \Rightarrow$
$\dot\neg(\mathsf{j}_{\mathrm{Ex}} \in \underline{\mathsf{x}})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\dot\neg(\underline{\mathsf{s}} \in \mathsf{j}_{\mathrm{Ex}} \Rightarrow \dot\neg(\mathsf{j}_{\mathrm{Ex}} \in \underline{\mathsf{x}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{s}} \in \cup\underline{\mathsf{x}})\mathrm{n})\mathrm{n} \oplus$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fx})[\underline{\mathsf{m}}] * (\mathrm{fy})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\mathrm{fy})[\underline{\mathsf{m}}] * (\mathrm{fx})[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{rx})}\colon \forall\underline{(\mathrm{ry})}\colon \underline{(\mathrm{rx})} = \underline{(\mathrm{ry})} \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{ry})} \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{rx})} \oplus$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|((fy)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|=\overline{(\epsilon)})n)n)n)n)n\}\vdash$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{n}\colon\dot\neg(\forall_{obj}\overline{m}\colon\dot\neg(\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))n)n)n\Rightarrow$
$\dot\neg(\overline{n}<=\overline{m}\Rightarrow(fy)[\overline{m}]<=((fx)[\overline{m}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n\Rightarrow\{ph\in P(\{ph\in$
$P(\{ph\in P(P(Union(\{N,Q\})))\mid\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in$
$N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\mathsf{a}_{Ph}=$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot\neg(\forall_{obj}\overline{n}\colon\dot\neg(\forall_{obj}\overline{m}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))n)n)n\Rightarrow$
$\overline{n}<=\overline{m}\Rightarrow\dot\neg(|((fy)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg(\dot\neg(|((fy)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|=\overline{(\epsilon)})n)n)n)n)n\}=\{ph\in P(\{ph\in P(\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow$
$\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow$
$\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot\neg(\forall_{obj}\overline{n}\colon\dot\neg(\forall_{obj}\overline{m}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))n)n)n\Rightarrow$
$\overline{n}<=\overline{m}\Rightarrow\dot\neg(|((fx)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg(\dot\neg(|((fx)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|=\overline{(\epsilon)})n)n)n)n)n\}\vdash\{ph\in P(\{ph\in P(\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow$
$\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow$
$\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot\neg(\forall_{obj}\overline{n}\colon\dot\neg(\forall_{obj}\overline{m}\colon\dot\neg(0<=\overline{(\epsilon)}\Rightarrow\dot\neg(\dot\neg(0=\overline{(\epsilon)}))n)n)n\Rightarrow$
$\overline{n}<=\overline{m}\Rightarrow\dot\neg(|((fx)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot\neg(\dot\neg(|((fx)[\overline{m}]+(-u\mathsf{d}_{Ph}[\overline{m}]))|=\overline{(\epsilon)})n)n)n)n)n\}=\{ph\in P(\{ph\in P(\{ph\in$
$P(P(Union(\{N,Q\})))\mid\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow$
$\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in\mathsf{f}_{Ph}\Rightarrow\dot\neg(\forall_{obj}\overline{(op1)}\colon\dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon\dot\neg(\dot\neg(\dot\neg(\overline{(op1)}\in N\Rightarrow$
$\dot\neg(\overline{(op2)}\in Q)n)n\Rightarrow\dot\neg(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot\neg(\forall_{obj}\overline{(s2)}\colon\dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \oplus \dot{\neg}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n}]$

[SystemQ $\stackrel{\mathrm{tex}}{\rightarrow}$ “SystemQ”]

[SystemQ $\stackrel{\mathrm{pyk}}{\rightarrow}$ “system Q”]

MP

[MP $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b}$]

[MP $\stackrel{\text{tex}}{\rightarrow}$ “MP”]

[MP $\stackrel{\text{pyk}}{\rightarrow}$ “1rule mp”]

Gen

[Gen $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{a}\colon \underline{a} \vdash \forall_{\text{obj}} \underline{x}\colon \underline{a}$]

[Gen $\stackrel{\text{tex}}{\rightarrow}$ “Gen”]

[Gen $\stackrel{\text{pyk}}{\rightarrow}$ “1rule gen”]

Repetition

[Repetition $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Repetition $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}\colon \underline{a} \vdash \underline{a}$]

[Repetition $\stackrel{\text{tex}}{\rightarrow}$ “Repetition”]

[Repetition $\stackrel{\text{pyk}}{\rightarrow}$ “1rule repetition”]

Neg

[Neg $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Neg $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}\,(\underline{b})\text{n} \Rightarrow \underline{a} \vdash \dot{\neg}\,(\underline{b})\text{n} \Rightarrow \dot{\neg}\,(\underline{a})\text{n} \vdash \underline{b}$]

[Neg $\stackrel{\text{tex}}{\rightarrow}$ “Neg”]

[Neg $\stackrel{\text{pyk}}{\rightarrow}$ “1rule ad absurdum”]

Ded

[Ded $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Ded $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \underline{b}$]

[Ded $\stackrel{\text{tex}}{\rightarrow}$ “Ded”]

[Ded $\stackrel{\text{pyk}}{\rightarrow}$ “1rule deduction”]

ExistIntro

[ExistIntro $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ExistIntro $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \langle \lceil \underline{\mathsf{a}} \rceil \equiv^{0} \lceil \underline{\mathsf{b}} \rceil | \lceil \underline{\mathsf{x}} \rceil {:}{=}{=} \lceil \underline{\mathsf{t}} \rceil \rangle_{\mathrm{Ex}} \Vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}$]

[ExistIntro $\stackrel{\text{tex}}{\rightarrow}$ “ExistIntro”]

[ExistIntro $\stackrel{\text{pyk}}{\rightarrow}$ “1rule exist intro”]

Extensionality

[Extensionality $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Extensionality $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\,(\underline{\mathsf{x}} {=}{=} \underline{\mathsf{y}} \Rightarrow \forall_{\mathrm{obj}} \bar{\mathsf{s}}\colon \dot{\neg}\,(\bar{\mathsf{s}} \in \underline{\mathsf{x}} \Rightarrow \bar{\mathsf{s}} \in \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\bar{\mathsf{s}} \in \underline{\mathsf{y}} \Rightarrow \bar{\mathsf{s}} \in \underline{\mathsf{x}})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \bar{\mathsf{s}}\colon \dot{\neg}\,(\bar{\mathsf{s}} \in \underline{\mathsf{x}} \Rightarrow \bar{\mathsf{s}} \in \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\bar{\mathsf{s}} \in \underline{\mathsf{y}} \Rightarrow \bar{\mathsf{s}} \in \underline{\mathsf{x}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{x}} {=}{=} \underline{\mathsf{y}})\mathrm{n})\mathrm{n}$]

[Extensionality $\stackrel{\text{tex}}{\rightarrow}$ “Extensionality”]

[Extensionality $\stackrel{\text{pyk}}{\rightarrow}$ “axiom extensionality”]

Ødef

[Ødef $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Ødef $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{s}}\colon \dot{\neg}\,(\underline{\mathsf{s}} \in \emptyset)\mathrm{n}$]

[Ødef $\stackrel{\text{tex}}{\rightarrow}$ “\O{}def”]

[Ødef $\stackrel{\text{pyk}}{\rightarrow}$ “axiom empty set”]

PairDef

[PairDef $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[PairDef $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{s}}\colon \forall \underline{\mathsf{x}}\colon \forall \underline{\mathsf{y}}\colon \dot{\neg}\,(\underline{\mathsf{s}} \in \{\underline{\mathsf{x}}, \underline{\mathsf{y}}\} \Rightarrow \dot{\neg}\,(\underline{\mathsf{s}} {=}{=} \underline{\mathsf{x}})\mathrm{n} \Rightarrow \underline{\mathsf{s}} {=}{=} \underline{\mathsf{y}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{s}} {=}{=} \underline{\mathsf{x}})\mathrm{n} \Rightarrow \underline{\mathsf{s}} {=}{=} \underline{\mathsf{y}} \Rightarrow \underline{\mathsf{s}} \in \{\underline{\mathsf{x}}, \underline{\mathsf{y}}\})\mathrm{n})\mathrm{n}$]

[PairDef $\stackrel{\text{tex}}{\rightarrow}$ “PairDef”]

[PairDef $\stackrel{\text{pyk}}{\rightarrow}$ "axiom pair definition"]

UnionDef

[UnionDef $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[UnionDef $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{s}\colon \forall \underline{x}\colon \dot{\neg}(\underline{s} \in \cup\underline{x} \Rightarrow \dot{\neg}(\underline{s} \in j_{\mathrm{Ex}} \Rightarrow \dot{\neg}(j_{\mathrm{Ex}} \in \underline{x})n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{s} \in j_{\mathrm{Ex}} \Rightarrow \dot{\neg}(j_{\mathrm{Ex}} \in \underline{x})n)n \Rightarrow \underline{s} \in \cup\underline{x})n)n$]

[UnionDef $\stackrel{\text{tex}}{\rightarrow}$ "UnionDef"]

[UnionDef $\stackrel{\text{pyk}}{\rightarrow}$ "axiom union definition"]

PowerDef

[PowerDef $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[PowerDef $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{s}\colon \forall \underline{x}\colon \dot{\neg}(\underline{s} \in \mathrm{P}(\underline{x}) \Rightarrow \forall_{\mathrm{obj}}\bar{s}\colon \bar{s} \in \underline{s} \Rightarrow \bar{s} \in \underline{x} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\bar{s}\colon \bar{s} \in \underline{s} \Rightarrow \bar{s} \in \underline{x} \Rightarrow \underline{s} \in \mathrm{P}(\underline{x}))n)n$]

[PowerDef $\stackrel{\text{tex}}{\rightarrow}$ "PowerDef"]

[PowerDef $\stackrel{\text{pyk}}{\rightarrow}$ "axiom power definition"]

SeparationDef

[SeparationDef $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[SeparationDef $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{p}\colon \forall \underline{x}\colon \forall \underline{z}\colon \underline{p}^{\mathrm{Ph}} \wedge \langle \underline{b} \equiv \underline{a} | \underline{p} := = \underline{z} \rangle_{\mathrm{Ph}} \Vdash \dot{\neg}(\underline{z} \in \{\mathrm{ph} \in \underline{x} \mid \underline{a}\} \Rightarrow \dot{\neg}(\underline{z} \in \underline{x} \Rightarrow \dot{\neg}(\underline{b})n)n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{z} \in \underline{x} \Rightarrow \dot{\neg}(\underline{b})n)n \Rightarrow \underline{z} \in \{\mathrm{ph} \in \underline{x} \mid \underline{a}\})n)n$]

[SeparationDef $\stackrel{\text{tex}}{\rightarrow}$ "SeparationDef"]

[SeparationDef $\stackrel{\text{pyk}}{\rightarrow}$ "axiom separation definition"]

AddDoubleNeg

[AddDoubleNeg $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil$SystemQ $\vdash \forall \underline{a}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \vdash$ RemoveDoubleNeg $\rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \gg \dot{\neg}(\underline{a})n; \forall \underline{a}\colon$ Ded $\rhd \forall \underline{a}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \vdash \dot{\neg}(\underline{a})n \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \Rightarrow \dot{\neg}(\underline{a})n; \underline{a} \vdash$ Weakening $\rhd \underline{a} \gg \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \Rightarrow \underline{a};$ Neg $\rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \Rightarrow \underline{a} \rhd \dot{\neg}(\dot{\neg}(\dot{\neg}(\underline{a})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n)n\rceil, p_0, c)$]

[AddDoubleNeg $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}\colon \underline{a} \vdash \dot{\neg}(\dot{\neg}(\underline{a})n)n$]

[AddDoubleNeg $\stackrel{\text{tex}}{\rightarrow}$ "AddDoubleNeg"]

[AddDoubleNeg $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma add double neg"]

RemoveDoubleNeg

[RemoveDoubleNeg $\stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \vdash$
$\text{Weakening} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n}; \text{AutoImply} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow$
$\dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \text{Neg} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \gg \underline{\mathsf{a}}\rceil, \text{p}_0, \mathsf{c})$]

[RemoveDoubleNeg $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \vdash \underline{\mathsf{a}}$]

[RemoveDoubleNeg $\stackrel{\text{tex}}{\rightarrow}$ "RemoveDoubleNeg"]

[RemoveDoubleNeg $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma remove double neg"]

AndCommutativity

[AndCommutativity $\stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \vdash \underline{\mathsf{a}} \vdash$
$\text{AddDoubleNeg} \rhd \underline{\mathsf{a}} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n}; \text{MT} \rhd \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \gg$
$\dot{\neg}\,(\underline{\mathsf{b}})\text{n}; \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \text{Ded} \rhd \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \vdash \underline{\mathsf{a}} \vdash \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \gg \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{a}} \Rightarrow$
$\dot{\neg}\,(\underline{\mathsf{b}})\text{n}; \dot{\neg}\,(\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \vdash \text{Repetition} \gg \dot{\neg}\,(\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n}; \text{MT} \rhd \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow$
$\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\text{n} \rhd \dot{\neg}\,(\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n}; \text{Repetition} \rhd \dot{\neg}\,(\underline{\mathsf{b}} \Rightarrow$
$\dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[AndCommutativity $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \dot{\neg}\,(\underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \vdash \dot{\neg}\,(\underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n})\text{n}$]

[AndCommutativity $\stackrel{\text{tex}}{\rightarrow}$ "AndCommutativity"]

[AndCommutativity $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma and commutativity"]

AutoImply

[AutoImply $\stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \underline{\mathsf{a}} \vdash \text{Repetition} \rhd \underline{\mathsf{a}} \gg$
$\underline{\mathsf{a}}; \forall\underline{\mathsf{a}}\colon \text{Ded} \rhd \forall\underline{\mathsf{a}}\colon \underline{\mathsf{a}} \vdash \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}\rceil, \text{p}_0, \mathsf{c})$]

[AutoImply $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{\mathsf{a}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}}$]

[AutoImply $\stackrel{\text{tex}}{\rightarrow}$ "AutoImply"]

[AutoImply $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma auto imply"]

Contrapositive

$[\text{Contrapositive} \stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \text{MT} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \gg \dot{\neg}(\underline{\mathsf{a}})\text{n}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \dot{\neg}(\underline{\mathsf{a}})\text{n} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}; \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}\rceil, \text{p}_0, \mathsf{c})]$

$[\text{Contrapositive} \stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}]$

$[\text{Contrapositive} \stackrel{\text{tex}}{\to}$ "Contrapositive"$]$

$[\text{Contrapositive} \stackrel{\text{pyk}}{\to}$ "prop lemma contrapositive"$]$

FirstConjunct

$[\text{FirstConjunct} \stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \vdash \text{AndCommutativity} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n}; \text{SecondConjunct} \rhd \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n})\text{n} \gg \underline{\mathsf{a}}\rceil, \text{p}_0, \mathsf{c})]$

$[\text{FirstConjunct} \stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \vdash \underline{\mathsf{a}}]$

$[\text{FirstConjunct} \stackrel{\text{tex}}{\to}$ "FirstConjunct"$]$

$[\text{FirstConjunct} \stackrel{\text{pyk}}{\to}$ "prop lemma first conjunct"$]$

SecondConjunct

$[\text{SecondConjunct} \stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \text{Weakening} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \gg \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{b}})\text{n} \vdash \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n} \gg \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n}; \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \vdash \text{Repetition} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n}; \text{NegativeMT} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \gg \underline{\mathsf{b}}\rceil, \text{p}_0, \mathsf{c})]$

$[\text{SecondConjunct} \stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\text{n})\text{n} \vdash \underline{\mathsf{b}}]$

$[\text{SecondConjunct} \stackrel{\text{tex}}{\to}$ "SecondConjunct"$]$

$[\text{SecondConjunct} \stackrel{\text{pyk}}{\to}$ "prop lemma second conjunct"$]$

FromContradiction

$[\text{FromContradiction} \stackrel{\text{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \dot{\neg}(\underline{\mathsf{a}})\text{n} \vdash \text{Weakening} \rhd \underline{\mathsf{a}} \gg \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \underline{\mathsf{a}}; \text{Weakening} \rhd \dot{\neg}(\underline{\mathsf{a}})\text{n} \gg \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n}; \text{Neg} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \underline{\mathsf{a}} \rhd \dot{\neg}(\underline{\mathsf{b}})\text{n} \Rightarrow \dot{\neg}(\underline{\mathsf{a}})\text{n} \gg \underline{\mathsf{b}}\rceil, \text{p}_0, \mathsf{c})]$

$[\text{FromContradiction} \stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \dot{\neg}(\underline{\mathsf{a}})\text{n} \vdash \underline{\mathsf{b}}]$

[FromContradiction $\overset{\text{tex}}{\to}$ "FromContradiction"]

[FromContradiction $\overset{\text{pyk}}{\to}$ "prop lemma from contradiction"]

FromDisjuncts

[FromDisjuncts $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{b}}; \mathrm{Contrapositive} \rhd \dot{\neg}(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n}; \mathrm{Technicality} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{c}}; \mathrm{ImplyTransitivity} \rhd \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{c}} \gg \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \underline{\mathsf{c}}; \mathrm{Contrapositive} \rhd \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \underline{\mathsf{c}} \gg \dot{\neg}(\underline{\mathsf{c}})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n}; \mathrm{Contrapositive} \rhd \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \gg \dot{\neg}(\underline{\mathsf{c}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n}; \mathrm{Neg} \rhd \dot{\neg}(\underline{\mathsf{c}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \rhd \dot{\neg}(\underline{\mathsf{c}})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{c}}\rceil, \mathrm{p}_0, \mathsf{c})$]

[FromDisjuncts $\overset{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \dot{\neg}(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{c}}$]

[FromDisjuncts $\overset{\text{tex}}{\to}$ "FromDisjuncts"]

[FromDisjuncts $\overset{\text{pyk}}{\to}$ "prop lemma from disjuncts"]

IffCommutativity

[IffCommutativity $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n}; \mathrm{AndCommutativity} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})$]

[IffCommutativity $\overset{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}})\mathrm{n})\mathrm{n}$]

[IffCommutativity $\overset{\text{tex}}{\to}$ "IffCommutativity"]

[IffCommutativity $\overset{\text{pyk}}{\to}$ "prop lemma iff commutativity"]

IffFirst

[IffFirst $\overset{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{b}} \vdash \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}}; \mathrm{MP} \rhd \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{a}}\rceil, \mathrm{p}_0, \mathsf{c})$]

[IffFirst $\overset{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{a}}$]

[IffFirst $\overset{\text{tex}}{\to}$ "IffFirst"]

[IffFirst $\overset{\text{pyk}}{\to}$ "prop lemma iff first"]

IffSecond

$[\text{IffSecond} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{a}} \vdash \text{FirstConjunct} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}}; \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \gg \underline{\mathsf{b}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{IffSecond} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}}]$

$[\text{IffSecond} \stackrel{\text{tex}}{\rightarrow}$ "IffSecond"$]$

$[\text{IffSecond} \stackrel{\text{pyk}}{\rightarrow}$ "prop lemma iff second"$]$

ImplyTransitivity

$[\text{ImplyTransitivity} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \vdash \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \gg \underline{\mathsf{b}}; \text{MP} \rhd \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{c}}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{c}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}; \text{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{ImplyTransitivity} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}]$

$[\text{ImplyTransitivity} \stackrel{\text{tex}}{\rightarrow}$ "ImplyTransitivity"$]$

$[\text{ImplyTransitivity} \stackrel{\text{pyk}}{\rightarrow}$ "prop lemma imply transitivity"$]$

JoinConjuncts

$[\text{JoinConjuncts} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \vdash \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \rhd \underline{\mathsf{a}} \gg \dot{\neg}(\underline{\mathsf{b}})\mathrm{n}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \vdash \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n}; \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \rhd \underline{\mathsf{a}} \gg \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n}; \text{AddDoubleNeg} \rhd \underline{\mathsf{b}} \gg \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n}; \text{MT} \rhd \underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n}; \text{Repetition} \rhd \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{JoinConjuncts} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \dot{\neg}(\underline{\mathsf{a}} \Rightarrow \dot{\neg}(\underline{\mathsf{b}})\mathrm{n})\mathrm{n}]$

$[\text{JoinConjuncts} \stackrel{\text{tex}}{\rightarrow}$ "JoinConjuncts"$]$

$[\text{JoinConjuncts} \stackrel{\text{pyk}}{\rightarrow}$ "prop lemma join conjuncts"$]$

MP2

$[\text{MP2} \stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \rhd \underline{\mathsf{a}} \gg \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}}; \text{MP} \rhd \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{c}}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\text{MP2} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}}]$

$[\mathrm{MP2} \stackrel{\mathrm{tex}}{\rightarrow} \text{“MP2”}]$

$[\mathrm{MP2} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“prop lemma mp2”}]$

MP3

$[\mathrm{MP3} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \vdash \mathrm{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \rhd \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}}; \mathrm{MP} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \rhd \underline{\mathsf{c}} \gg \underline{\mathsf{d}} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MP3} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \vdash \underline{\mathsf{d}}]$

$[\mathrm{MP3} \stackrel{\mathrm{tex}}{\rightarrow} \text{“MP3”}]$

$[\mathrm{MP3} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“prop lemma mp3”}]$

MP4

$[\mathrm{MP4} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \vdash \underline{\mathsf{d}} \vdash \mathrm{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \rhd \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}}; \mathrm{MP2} \rhd \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \rhd \underline{\mathsf{c}} \rhd \underline{\mathsf{d}} \gg \underline{\mathsf{e}} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MP4} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \vdash \underline{\mathsf{d}} \vdash \underline{\mathsf{e}}]$

$[\mathrm{MP4} \stackrel{\mathrm{tex}}{\rightarrow} \text{“MP4”}]$

$[\mathrm{MP4} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“prop lemma mp4”}]$

MP5

$[\mathrm{MP5} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \vdash \underline{\mathsf{d}} \vdash \underline{\mathsf{e}} \vdash \mathrm{MP3} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \rhd \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \rhd \underline{\mathsf{c}} \gg \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}}; \mathrm{MP2} \rhd \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \rhd \underline{\mathsf{d}} \rhd \underline{\mathsf{e}} \gg \underline{\mathsf{f}} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MP5} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \forall \underline{\mathsf{e}}\colon \forall \underline{\mathsf{f}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{e}} \Rightarrow \underline{\mathsf{f}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \underline{\mathsf{c}} \vdash \underline{\mathsf{d}} \vdash \underline{\mathsf{e}} \vdash \underline{\mathsf{f}}]$

$[\mathrm{MP5} \stackrel{\mathrm{tex}}{\rightarrow} \text{“MP5”}]$

$[\mathrm{MP5} \stackrel{\mathrm{pyk}}{\rightarrow} \text{“prop lemma mp5”}]$

MT

$[\mathrm{MT} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \vdash \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \vdash \mathrm{Technicality} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{b}}; \mathrm{NegativeMT} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \rhd \dot{\neg}\,(\underline{\mathsf{b}})\mathrm{n} \gg \dot{\neg}\,(\underline{\mathsf{a}})\mathrm{n} \rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{MT} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}\, (\underline{b})\mathrm{n} \vdash \dot{\neg}\, (\underline{a})\mathrm{n}]$

$[\mathrm{MT} \stackrel{\mathrm{tex}}{\rightarrow}$ "MT"$]$

$[\mathrm{MT} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma mt"$]$

NegativeMT

$[\mathrm{NegativeMT} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}\, (\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}\, (\underline{b})\mathrm{n} \vdash \mathrm{Weakening} \rhd \dot{\neg}\, (\underline{b})\mathrm{n} \gg \dot{\neg}\, (\underline{a})\mathrm{n} \Rightarrow \dot{\neg}\, (\underline{b})\mathrm{n}; \mathrm{Neg} \rhd \dot{\neg}\, (\underline{a})\mathrm{n} \Rightarrow \underline{b} \rhd \dot{\neg}\, (\underline{a})\mathrm{n} \Rightarrow \dot{\neg}\, (\underline{b})\mathrm{n} \gg \underline{a}\rceil, \mathrm{p}_0, c)]$

$[\mathrm{NegativeMT} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}\, (\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}\, (\underline{b})\mathrm{n} \vdash \underline{a}]$

$[\mathrm{NegativeMT} \stackrel{\mathrm{tex}}{\rightarrow}$ "NegativeMT"$]$

$[\mathrm{NegativeMT} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma negative mt"$]$

Technicality

$[\mathrm{Technicality} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \vdash \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \gg \underline{a}; \mathrm{MP} \rhd \underline{a} \Rightarrow \underline{b} \rhd \underline{a} \gg \underline{b}; \forall \underline{a}\colon \forall \underline{b}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \vdash \underline{b} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \Rightarrow \underline{b}; \underline{a} \Rightarrow \underline{b} \vdash \mathrm{MP} \rhd \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \Rightarrow \underline{b} \rhd \underline{a} \Rightarrow \underline{b} \gg \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \Rightarrow \underline{b}\rceil, \mathrm{p}_0, c)]$

$[\mathrm{Technicality} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}\, (\dot{\neg}\, (\underline{a})\mathrm{n})\mathrm{n} \Rightarrow \underline{b}]$

$[\mathrm{Technicality} \stackrel{\mathrm{tex}}{\rightarrow}$ "Technicality"$]$

$[\mathrm{Technicality} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma technicality"$]$

Weakening

$[\mathrm{Weakening} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{b} \vdash \underline{a} \vdash \mathrm{Repetition} \rhd \underline{b} \gg \underline{b}; \forall \underline{a}\colon \forall \underline{b}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \underline{b} \vdash \underline{a} \vdash \underline{b} \gg \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b}; \underline{b} \vdash \mathrm{MP} \rhd \underline{b} \Rightarrow \underline{a} \Rightarrow \underline{b} \rhd \underline{b} \gg \underline{a} \Rightarrow \underline{b}\rceil, \mathrm{p}_0, c)]$

$[\mathrm{Weakening} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{b} \vdash \underline{a} \Rightarrow \underline{b}]$

$[\mathrm{Weakening} \stackrel{\mathrm{tex}}{\rightarrow}$ "Weakening"$]$

$[\mathrm{Weakening} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma weakening"$]$

WeakenOr1

[WeakenOr1 $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{b}} \vdash \text{Weakening} \rhd \underline{\mathsf{b}} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}; \text{Repetition} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}\rceil, \text{p}_0, \mathsf{c})$]

[WeakenOr1 $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{b}} \vdash \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}$]

[WeakenOr1 $\stackrel{\text{tex}}{\to}$ "WeakenOr1"]

[WeakenOr1 $\stackrel{\text{pyk}}{\to}$ "prop lemma weaken or first"]

WeakenOr2

[WeakenOr2 $\stackrel{\text{proof}}{\to}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \vdash \text{FromContradiction} \rhd \underline{\mathsf{a}} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \gg \underline{\mathsf{b}}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \vdash \underline{\mathsf{b}} \gg \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}; \underline{\mathsf{a}} \vdash \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}} \rhd \underline{\mathsf{a}} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}; \text{Repetition} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}\rceil, \text{p}_0, \mathsf{c})$]

[WeakenOr2 $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \vdash \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \underline{\mathsf{b}}$]

[WeakenOr2 $\stackrel{\text{tex}}{\to}$ "WeakenOr2"]

[WeakenOr2 $\stackrel{\text{pyk}}{\to}$ "prop lemma weaken or second"]

Formula2Pair

[Formula2Pair $\stackrel{\text{proof}}{\to}$ Rule tactic]

[Formula2Pair $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sz})}\colon \dot{\neg}\,(\underline{(\text{sx})} = \underline{(\text{sy})})\text{n} \Rightarrow \underline{(\text{sx})} = \underline{(\text{sz})} \vdash \underline{(\text{sx})} \in \{\underline{(\text{sy})}, \underline{(\text{sz})}\}$]

[Formula2Pair $\stackrel{\text{tex}}{\to}$ "Formula2Pair"]

[Formula2Pair $\stackrel{\text{pyk}}{\to}$ "lemma formula2pair"]

Pair2Formula

[Pair2Formula $\stackrel{\text{proof}}{\to}$ Rule tactic]

[Pair2Formula $\stackrel{\text{stmt}}{\to}$ $\text{SystemQ} \vdash \forall \underline{(\text{sx})}\colon \forall \underline{(\text{sy})}\colon \forall \underline{(\text{sz})}\colon \underline{(\text{sx})} \in \{\underline{(\text{sy})}, \underline{(\text{sz})}\} \vdash \dot{\neg}\,(\underline{(\text{sx})} = \underline{(\text{sy})})\text{n} \Rightarrow \underline{(\text{sx})} = \underline{(\text{sz})}$]

[Pair2Formula $\stackrel{\text{tex}}{\to}$ "Pair2Formula"]

[Pair2Formula $\stackrel{\text{pyk}}{\to}$ "lemma pair2formula"]

Formula2Union

[Formula2Union $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Formula2Union $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}: \forall \underline{(sy)}: \forall \underline{(sz)}: \dot{\neg}\, (\forall_{\text{obj}} \underline{(sy)}: \dot{\neg}\, (\dot{\neg}\, (\underline{(sx)} \in \underline{(sy)} \Rightarrow \dot{\neg}\, (\underline{(sy)} \in \underline{(sz)})\text{n})\text{n})\text{n})\text{n} \vdash \underline{(sx)} \in \text{Union}(\underline{(sz)})$]

[Formula2Union $\stackrel{\text{tex}}{\rightarrow}$ "Formula2Union"]

[Formula2Union $\stackrel{\text{pyk}}{\rightarrow}$ "lemma formula2union"]

Union2Formula

[Union2Formula $\stackrel{\text{tex}}{\rightarrow}$ "Union2Formula"]

[Union2Formula $\stackrel{\text{pyk}}{\rightarrow}$ "lemma union2formula"]

Formula2Sep

[Formula2Sep $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Formula2Sep $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{x}: \forall \underline{y}: \underline{y} \in \underline{x} \vdash \underline{b} \vdash \underline{y} \in \{\text{ph} \in \underline{x} \mid \underline{a}\}$]

[Formula2Sep $\stackrel{\text{tex}}{\rightarrow}$ "Formula2Sep"]

[Formula2Sep $\stackrel{\text{pyk}}{\rightarrow}$ "lemma formula2separation"]

Sep2Formula

[Sep2Formula $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Sep2Formula $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{x}: \forall \underline{y}: \underline{y} \in \{\text{ph} \in \underline{x} \mid \underline{a}\} \vdash \dot{\neg}\, (\underline{y} \in \underline{x} \Rightarrow \dot{\neg}\, (\underline{b})\text{n})\text{n}$]

[Sep2Formula $\stackrel{\text{tex}}{\rightarrow}$ "Sep2Formula"]

[Sep2Formula $\stackrel{\text{pyk}}{\rightarrow}$ "lemma separation2formula"]

Formula2Power

[Formula2Power $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Formula2Power $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}: \forall \underline{(sy)}: \forall_{\text{obj}} \overline{(s1)}: \overline{(s1)} \in \underline{(sx)} \Rightarrow \overline{(s1)} \in \underline{(sy)} \vdash \underline{(sx)} \in \text{P}(\underline{(sy)})$]

[Formula2Power $\stackrel{\text{tex}}{\rightarrow}$ "Formula2Power"]

[Formula2Power $\stackrel{\text{pyk}}{\rightarrow}$ "lemma formula2power"]

SubsetInPower

[SubsetInPower $\stackrel{\text{tex}}{\rightarrow}$ "SubsetInPower"]

[SubsetInPower $\stackrel{\text{pyk}}{\rightarrow}$ "lemma subset in power set"]

HelperPowerIsSub

[HelperPowerIsSub $\stackrel{\text{tex}}{\rightarrow}$ "HelperPowerIsSub"]

[HelperPowerIsSub $\stackrel{\text{pyk}}{\rightarrow}$ "lemma power set is subset0"]

PowerIsSub

[PowerIsSub $\stackrel{\text{tex}}{\rightarrow}$ "PowerIsSub"]

[PowerIsSub $\stackrel{\text{pyk}}{\rightarrow}$ "lemma power set is subset"]

(Switch)HelperPowerIsSub

[(Switch)HelperPowerIsSub $\stackrel{\text{tex}}{\rightarrow}$ "(Switch)HelperPowerIsSub"]

[(Switch)HelperPowerIsSub $\stackrel{\text{pyk}}{\rightarrow}$ "lemma power set is subset0-switch"]

(Switch)PowerIsSub

[(Switch)PowerIsSub $\stackrel{\text{tex}}{\rightarrow}$ "(Switch)PowerIsSub"]

[(Switch)PowerIsSub $\stackrel{\text{pyk}}{\rightarrow}$ "lemma power set is subset-switch"]

ToSetEquality

[ToSetEquality $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ToSetEquality $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}\colon \forall \underline{(fy)}\colon \forall_{\text{obj}} \overline{(s1)}\colon \overline{(s1)} \in \underline{(fx)} \Rightarrow \overline{(s1)} \in \underline{(fy)} \vdash \forall_{\text{obj}} \overline{(s1)}\colon \overline{(s1)} \in \underline{(fy)} \Rightarrow \overline{(s1)} \in \underline{(fx)} \vdash \underline{(fx)} = \underline{(fy)}$]

[ToSetEquality $\overset{\text{tex}}{\rightarrow}$ "ToSetEquality"]

[ToSetEquality $\overset{\text{pyk}}{\rightarrow}$ "lemma set equality suff condition"]

HelperToSetEquality(t)

[HelperToSetEquality(t) $\overset{\text{tex}}{\rightarrow}$ "HelperToSetEquality(t)"]

[HelperToSetEquality(t) $\overset{\text{pyk}}{\rightarrow}$ "lemma set equality suff condition(t)0"]

ToSetEquality(t)

[ToSetEquality(t) $\overset{\text{tex}}{\rightarrow}$ "ToSetEquality(t)"]

[ToSetEquality(t) $\overset{\text{pyk}}{\rightarrow}$ "lemma set equality suff condition(t)"]

HelperFromSetEquality

[HelperFromSetEquality $\overset{\text{tex}}{\rightarrow}$ "HelperFromSetEquality"]

[HelperFromSetEquality $\overset{\text{pyk}}{\rightarrow}$ "lemma set equality skip quantifier"]

FromSetEquality

[FromSetEquality $\overset{\text{tex}}{\rightarrow}$ "FromSetEquality"]

[FromSetEquality $\overset{\text{pyk}}{\rightarrow}$ "lemma set equality nec condition"]

HelperReflexivity

[HelperReflexivity $\overset{\text{tex}}{\rightarrow}$ "HelperReflexivity"]

[HelperReflexivity $\overset{\text{pyk}}{\rightarrow}$ "lemma reflexivity0"]

Reflexivity

[Reflexivity $\stackrel{\text{tex}}{\to}$ "Reflexivity"]

[Reflexivity $\stackrel{\text{pyk}}{\to}$ "lemma reflexivity"]

HelperSymmetry

[HelperSymmetry $\stackrel{\text{tex}}{\to}$ "HelperSymmetry"]

[HelperSymmetry $\stackrel{\text{pyk}}{\to}$ "lemma symmetry0"]

Symmetry

[Symmetry $\stackrel{\text{tex}}{\to}$ "Symmetry"]

[Symmetry $\stackrel{\text{pyk}}{\to}$ "lemma symmetry"]

HelperTransitivity

[HelperTransitivity $\stackrel{\text{tex}}{\to}$ "HelperTransitivity"]

[HelperTransitivity $\stackrel{\text{pyk}}{\to}$ "lemma transitivity0"]

Transitivity

[Transitivity $\stackrel{\text{tex}}{\to}$ "Transitivity"]

[Transitivity $\stackrel{\text{pyk}}{\to}$ "lemma transitivity"]

ERisReflexive

[ERisReflexive $\stackrel{\text{tex}}{\to}$ "ERisReflexive"]

[ERisReflexive $\stackrel{\text{pyk}}{\to}$ "lemma er is reflexive"]

ERisSymmetric

[ERisSymmetric $\stackrel{\text{tex}}{\to}$ "ERisSymmetric"]

[ERisSymmetric $\stackrel{\text{pyk}}{\to}$ "lemma er is symmetric"]

ERisTransitive

[ERisTransitive $\stackrel{\text{tex}}{\to}$ "ERisTransitive"]

[ERisTransitive $\stackrel{\text{pyk}}{\to}$ "lemma er is transitive"]

ØisSubset

[ØisSubset $\stackrel{\text{tex}}{\to}$ "\O{}isSubset"]

[ØisSubset $\stackrel{\text{pyk}}{\to}$ "lemma empty set is subset"]

HelperMemberNotØ

[HelperMemberNotØ $\stackrel{\text{tex}}{\to}$ "HelperMemberNot\O{}"]

[HelperMemberNotØ $\stackrel{\text{pyk}}{\to}$ "lemma member not empty0"]

MemberNotØ

[MemberNotØ $\stackrel{\text{tex}}{\to}$ "MemberNot\O{}"]

[MemberNotØ $\stackrel{\text{pyk}}{\to}$ "lemma member not empty"]

HelperUniqueØ

[HelperUniqueØ $\stackrel{\text{tex}}{\to}$ "HelperUnique\O{}"]

[HelperUniqueØ $\stackrel{\text{pyk}}{\to}$ "lemma unique empty set0"]

UniqueØ

[UniqueØ $\stackrel{\text{tex}}{\to}$ "Unique\O{}"]

[UniqueØ $\stackrel{\text{pyk}}{\to}$ "lemma unique empty set"]

==Reflexivity

[==Reflexivity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[==Reflexivity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\text{rx})}\colon \underline{(\text{rx})} = \underline{(\text{rx})}$]

[==Reflexivity $\stackrel{\text{tex}}{\rightarrow}$ "==\!{}Reflexivity"]

[==Reflexivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma ==Reflexivity"]

==Symmetry

[==Symmetry $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[==Symmetry $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\text{rx})}\colon \forall \underline{(\text{ry})}\colon \underline{(\text{rx})} = \underline{(\text{ry})} \vdash \underline{(\text{ry})} = \underline{(\text{rx})}$]

[==Symmetry $\stackrel{\text{tex}}{\rightarrow}$ "==\!{}Symmetry"]

[==Symmetry $\stackrel{\text{pyk}}{\rightarrow}$ "lemma ==Symmetry"]

Helper==Transitivity

[Helper==Transitivity $\stackrel{\text{tex}}{\rightarrow}$ "Helper\!{}==\!{}Transitivity"]

[Helper==Transitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma ==Transitivity0"]

==Transitivity

[==Transitivity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[==Transitivity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\text{rx})}\colon \forall \underline{(\text{ry})}\colon \forall \underline{(\text{rz})}\colon \underline{(\text{rx})} = \underline{(\text{ry})} \vdash \underline{(\text{ry})} = \underline{(\text{rz})} \vdash \underline{(\text{rx})} = \underline{(\text{rz})}$]

[==Transitivity $\stackrel{\text{tex}}{\rightarrow}$ "\!{}==\!{}Transitivity"]

[==Transitivity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma ==Transitivity"]

HelperTransferNotEq

[HelperTransferNotEq $\stackrel{\text{tex}}{\rightarrow}$ "HelperTransferNotEq"]

[HelperTransferNotEq $\stackrel{\text{pyk}}{\rightarrow}$ "lemma transfer ~is0"]

TransferNotEq

[TransferNotEq $\overset{\text{tex}}{\rightarrow}$ "TransferNotEq"]

[TransferNotEq $\overset{\text{pyk}}{\rightarrow}$ "lemma transfer ˜is"]

HelperPairSubset

[HelperPairSubset $\overset{\text{tex}}{\rightarrow}$ "HelperPairSubset"]

[HelperPairSubset $\overset{\text{pyk}}{\rightarrow}$ "lemma pair subset0"]

Helper(2)PairSubset

[Helper(2)PairSubset $\overset{\text{tex}}{\rightarrow}$ "Helper(2)PairSubset"]

[Helper(2)PairSubset $\overset{\text{pyk}}{\rightarrow}$ "lemma pair subset1"]

PairSubset

[PairSubset $\overset{\text{tex}}{\rightarrow}$ "PairSubset"]

[PairSubset $\overset{\text{pyk}}{\rightarrow}$ "lemma pair subset"]

SamePair

[SamePair $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[SamePair $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}\colon \forall \underline{(sx1)}\colon \forall \underline{(sy)}\colon \forall \underline{(sy1)}\colon \underline{(sx)} = \underline{(sx1)} \vdash \underline{(sy)} = \underline{(sy1)} \vdash \{\underline{(sx)}, \underline{(sy)}\} = \{\underline{(sx1)}, \underline{(sy1)}\}$]

[SamePair $\overset{\text{tex}}{\rightarrow}$ "SamePair"]

[SamePair $\overset{\text{pyk}}{\rightarrow}$ "lemma same pair"]

SameSingleton

[SameSingleton $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[SameSingleton $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(sx)}\colon \forall \underline{(sy)}\colon \underline{(sx)} = \underline{(sy)} \vdash \{\underline{(sx)}, \underline{(sx)}\} = \{\underline{(sy)}, \underline{(sy)}\}$]

[SameSingleton $\stackrel{\text{tex}}{\rightarrow}$ "SameSingleton"]

[SameSingleton $\stackrel{\text{pyk}}{\rightarrow}$ "lemma same singleton"]

UnionSubset

[UnionSubset $\stackrel{\text{tex}}{\rightarrow}$ "UnionSubset"]

[UnionSubset $\stackrel{\text{pyk}}{\rightarrow}$ "lemma union subset"]

SameUnion

[SameUnion $\stackrel{\text{tex}}{\rightarrow}$ "SameUnion"]

[SameUnion $\stackrel{\text{pyk}}{\rightarrow}$ "lemma same union"]

SeparationSubset

[SeparationSubset $\stackrel{\text{tex}}{\rightarrow}$ "SeparationSubset"]

[SeparationSubset $\stackrel{\text{pyk}}{\rightarrow}$ "lemma separation subset"]

SameSeparation

[SameSeparation $\stackrel{\text{tex}}{\rightarrow}$ "SameSeparation"]

[SameSeparation $\stackrel{\text{pyk}}{\rightarrow}$ "lemma same separation"]

SameBinaryUnion

[SameBinaryUnion $\stackrel{\text{tex}}{\rightarrow}$ "SameBinaryUnion"]

[SameBinaryUnion $\stackrel{\text{pyk}}{\rightarrow}$ "lemma same binary union"]

IntersectionSubset

[IntersectionSubset $\stackrel{\text{tex}}{\rightarrow}$ "IntersectionSubset"]

[IntersectionSubset $\stackrel{\text{pyk}}{\rightarrow}$ "lemma intersection subset"]

SameIntersection

[SameIntersection $\stackrel{\text{tex}}{\to}$ "SameIntersection"]

[SameIntersection $\stackrel{\text{pyk}}{\to}$ "lemma same intersection"]

AutoMember

[AutoMember $\stackrel{\text{tex}}{\to}$ "AutoMember"]

[AutoMember $\stackrel{\text{pyk}}{\to}$ "lemma auto member"]

HelperEqSysNot∅

[HelperEqSysNot∅ $\stackrel{\text{tex}}{\to}$ "HelperEqSysNot\O{}"]

[HelperEqSysNot∅ $\stackrel{\text{pyk}}{\to}$ "lemma eq-system not empty0"]

EqSysNot∅

[EqSysNot∅ $\stackrel{\text{tex}}{\to}$ "EqSysNot\O{}"]

[EqSysNot∅ $\stackrel{\text{pyk}}{\to}$ "lemma eq-system not empty"]

HelperEqSubset

[HelperEqSubset $\stackrel{\text{tex}}{\to}$ "HelperEqSubset"]

[HelperEqSubset $\stackrel{\text{pyk}}{\to}$ "lemma eq subset0"]

EqSubset

[EqSubset $\stackrel{\text{tex}}{\to}$ "EqSubset"]

[EqSubset $\stackrel{\text{pyk}}{\to}$ "lemma eq subset"]

HelperEqNecessary

[HelperEqNecessary $\stackrel{\text{tex}}{\to}$ "HelperEqNecessary"]

[HelperEqNecessary $\stackrel{\text{pyk}}{\to}$ "lemma equivalence nec condition0"]

EqNecessary

[EqNecessary $\stackrel{\text{tex}}{\to}$ "EqNecessary"]

[EqNecessary $\stackrel{\text{pyk}}{\to}$ "lemma equivalence nec condition"]

HelperNoneEqNecessary

[HelperNoneEqNecessary $\stackrel{\text{tex}}{\to}$ "HelperNoneEqNecessary"]

[HelperNoneEqNecessary $\stackrel{\text{pyk}}{\to}$ "lemma none-equivalence nec condition0"]

Helper(2)NoneEqNecessary

[Helper(2)NoneEqNecessary $\stackrel{\text{tex}}{\to}$ "Helper(2)NoneEqNecessary"]

[Helper(2)NoneEqNecessary $\stackrel{\text{pyk}}{\to}$ "lemma none-equivalence nec condition1"]

NoneEqNecessary

[NoneEqNecessary $\stackrel{\text{tex}}{\to}$ "NoneEqNecessary"]

[NoneEqNecessary $\stackrel{\text{pyk}}{\to}$ "lemma none-equivalence nec condition"]

EqClassIsSubset

[EqClassIsSubset $\stackrel{\text{tex}}{\to}$ "EqClassIsSubset"]

[EqClassIsSubset $\stackrel{\text{pyk}}{\to}$ "lemma equivalence class is subset"]

EqClassesAreDisjoint

[EqClassesAreDisjoint $\stackrel{\text{tex}}{\to}$ "EqClassesAreDisjoint"]

[EqClassesAreDisjoint $\stackrel{\text{pyk}}{\to}$ "lemma equivalence classes are disjoint"]

AllDisjoint

[AllDisjoint $\stackrel{\text{tex}}{\rightarrow}$ "AllDisjoint"]

[AllDisjoint $\stackrel{\text{pyk}}{\rightarrow}$ "lemma all disjoint"]

AllDisjointImply

[AllDisjointImply $\stackrel{\text{tex}}{\rightarrow}$ "AllDisjointImply"]

[AllDisjointImply $\stackrel{\text{pyk}}{\rightarrow}$ "lemma all disjoint-imply"]

BSsubset

[BSsubset $\stackrel{\text{tex}}{\rightarrow}$ "BSsubset"]

[BSsubset $\stackrel{\text{pyk}}{\rightarrow}$ "lemma bs subset union(bs/r)"]

Union(BS/R)subset

[Union(BS/R)subset $\stackrel{\text{tex}}{\rightarrow}$ "Union(BS/R)subset"]

[Union(BS/R)subset $\stackrel{\text{pyk}}{\rightarrow}$ "lemma union(bs/r) subset bs"]

UnionIdentity

[UnionIdentity $\stackrel{\text{tex}}{\rightarrow}$ "UnionIdentity"]

[UnionIdentity $\stackrel{\text{pyk}}{\rightarrow}$ "lemma union(bs/r) is bs"]

EqSysIsPartition

[EqSysIsPartition $\stackrel{\text{tex}}{\rightarrow}$ "EqSysIsPartition"]

[EqSysIsPartition $\stackrel{\text{pyk}}{\rightarrow}$ "theorem eq-system is partition"]

(x1)

[(x1) $\stackrel{\text{tex}}{\rightarrow}$ "(x1)"]

$[(x1) \stackrel{\text{pyk}}{\rightarrow}$ "var x1"$]$

(x2)

$[(x2) \stackrel{\text{tex}}{\rightarrow}$ "(x2)"$]$

$[(x2) \stackrel{\text{pyk}}{\rightarrow}$ "var x2"$]$

(y1)

$[(y1) \stackrel{\text{tex}}{\rightarrow}$ "(y1)"$]$

$[(y1) \stackrel{\text{pyk}}{\rightarrow}$ "var y1"$]$

(y2)

$[(y2) \stackrel{\text{tex}}{\rightarrow}$ "(y2)"$]$

$[(y2) \stackrel{\text{pyk}}{\rightarrow}$ "var y2"$]$

(v1)

$[(v1) \stackrel{\text{tex}}{\rightarrow}$ "(v1)"$]$

$[(v1) \stackrel{\text{pyk}}{\rightarrow}$ "var v1"$]$

(v2)

$[(v2) \stackrel{\text{tex}}{\rightarrow}$ "(v2)"$]$

$[(v2) \stackrel{\text{pyk}}{\rightarrow}$ "var v2"$]$

(v3)

$[(v3) \stackrel{\text{tex}}{\rightarrow}$ "(v3)"$]$

$[(v3) \stackrel{\text{pyk}}{\rightarrow}$ "var v3"$]$

(v4)

[(v4) $\stackrel{\text{tex}}{\to}$ "(v4)"]

[(v4) $\stackrel{\text{pyk}}{\to}$ "var v4"]

(v2n)

[(v2n) $\stackrel{\text{tex}}{\to}$ "(v2n)"]

[(v2n) $\stackrel{\text{pyk}}{\to}$ "var v2n"]

(m1)

[(m1) $\stackrel{\text{tex}}{\to}$ "(m1)"]

[(m1) $\stackrel{\text{pyk}}{\to}$ "var m1"]

(m2)

[(m2) $\stackrel{\text{tex}}{\to}$ "(m2)"]

[(m2) $\stackrel{\text{pyk}}{\to}$ "var m2"]

(n1)

[(n1) $\stackrel{\text{tex}}{\to}$ "(n1)"]

[(n1) $\stackrel{\text{pyk}}{\to}$ "var n1"]

(n2)

[(n2) $\stackrel{\text{tex}}{\to}$ "(n2)"]

[(n2) $\stackrel{\text{pyk}}{\to}$ "var n2"]

(n3)

$[(n3) \stackrel{\text{tex}}{\to}$ "(n3)"]

$[(n3) \stackrel{\text{pyk}}{\to}$ "var n3"]

(ϵ)

$[(\epsilon) \stackrel{\text{tex}}{\to}$ "(\epsilon)"]

$[(\epsilon) \stackrel{\text{pyk}}{\to}$ "var ep"]

$(\epsilon)_1$

$[(\epsilon)_1 \stackrel{\text{tex}}{\to}$ "(\epsilon)_{1}"]

$[(\epsilon)_1 \stackrel{\text{pyk}}{\to}$ "var ep1"]

$(\epsilon 2)$

$[(\epsilon 2) \stackrel{\text{tex}}{\to}$ "(\epsilon 2)"]

$[(\epsilon 2) \stackrel{\text{pyk}}{\to}$ "var ep2"]

(fep)

$[(fep) \stackrel{\text{tex}}{\to}$ "(fep)"]

$[(fep) \stackrel{\text{pyk}}{\to}$ "var fep"]

(fx)

$[(fx) \stackrel{\text{tex}}{\to}$ "(fx)"]

$[(fx) \stackrel{\text{pyk}}{\to}$ "var fx"]

(fy)

[(fy) $\stackrel{\text{tex}}{\rightarrow}$ "(fy)"]

[(fy) $\stackrel{\text{pyk}}{\rightarrow}$ "var fy"]

(fz)

[(fz) $\stackrel{\text{tex}}{\rightarrow}$ "(fz)"]

[(fz) $\stackrel{\text{pyk}}{\rightarrow}$ "var fz"]

(fu)

[(fu) $\stackrel{\text{tex}}{\rightarrow}$ "(fu)"]

[(fu) $\stackrel{\text{pyk}}{\rightarrow}$ "var fu"]

(fv)

[(fv) $\stackrel{\text{tex}}{\rightarrow}$ "(fv)"]

[(fv) $\stackrel{\text{pyk}}{\rightarrow}$ "var fv"]

(fw)

[(fw) $\stackrel{\text{tex}}{\rightarrow}$ "(fw)"]

[(fw) $\stackrel{\text{pyk}}{\rightarrow}$ "var fw"]

(rx)

[(rx) $\stackrel{\text{tex}}{\rightarrow}$ "(rx)"]

[(rx) $\stackrel{\text{pyk}}{\rightarrow}$ "var rx"]

(ry)

[(ry) $\stackrel{\text{tex}}{\rightarrow}$ "(ry)"]

[(ry) $\stackrel{\text{pyk}}{\rightarrow}$ "var ry"]

(rz)

[(rz) $\stackrel{\text{tex}}{\rightarrow}$ "(rz)"]

[(rz) $\stackrel{\text{pyk}}{\rightarrow}$ "var rz"]

(ru)

[(ru) $\stackrel{\text{tex}}{\rightarrow}$ "(ru)"]

[(ru) $\stackrel{\text{pyk}}{\rightarrow}$ "var ru"]

(sx)

[(sx) $\stackrel{\text{tex}}{\rightarrow}$ "(sx)"]

[(sx) $\stackrel{\text{pyk}}{\rightarrow}$ "var sx"]

(sx1)

[(sx1) $\stackrel{\text{tex}}{\rightarrow}$ "(sx1)"]

[(sx1) $\stackrel{\text{pyk}}{\rightarrow}$ "var sx1"]

(sy)

[(sy) $\stackrel{\text{tex}}{\rightarrow}$ "(sy)"]

[(sy) $\stackrel{\text{pyk}}{\rightarrow}$ "var sy"]

(sy1)

[(sy1) $\stackrel{\text{tex}}{\to}$ "(sy1)"]

[(sy1) $\stackrel{\text{pyk}}{\to}$ "var sy1"]

(sz)

[(sz) $\stackrel{\text{tex}}{\to}$ "(sz)"]

[(sz) $\stackrel{\text{pyk}}{\to}$ "var sz"]

(sz1)

[(sz1) $\stackrel{\text{tex}}{\to}$ "(sz1)"]

[(sz1) $\stackrel{\text{pyk}}{\to}$ "var sz1"]

(su)

[(su) $\stackrel{\text{tex}}{\to}$ "(su)"]

[(su) $\stackrel{\text{pyk}}{\to}$ "var su"]

(su1)

[(su1) $\stackrel{\text{tex}}{\to}$ "(su1)"]

[(su1) $\stackrel{\text{pyk}}{\to}$ "var su1"]

(fxs)

[(fxs) $\stackrel{\text{tex}}{\to}$ "(fxs)"]

[(fxs) $\stackrel{\text{pyk}}{\to}$ "var fxs"]

(fys)

$[(\mathrm{fys}) \stackrel{\mathrm{tex}}{\rightarrow}$ "(fys)"$]$

$[(\mathrm{fys}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "var fys"$]$

(crs1)

$[(\mathrm{crs1}) \stackrel{\mathrm{tex}}{\rightarrow}$ "(crs1)"$]$

$[(\mathrm{crs1}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "var crs1"$]$

(f1)

$[(\mathrm{f1}) \stackrel{\mathrm{tex}}{\rightarrow}$ "(f1)"$]$

$[(\mathrm{f1}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "var f1"$]$

(f2)

$[(\mathrm{f2}) \stackrel{\mathrm{tex}}{\rightarrow}$ "(f2)"$]$

$[(\mathrm{f2}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "var f2"$]$

(f3)

$[(\mathrm{f3}) \stackrel{\mathrm{tex}}{\rightarrow}$ "(f3)"$]$

$[(\mathrm{f3}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "var f3"$]$

(f4)

$[(\mathrm{f4}) \stackrel{\mathrm{tex}}{\rightarrow}$ "(f4)"$]$

$[(\mathrm{f4}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "var f4"$]$

(op1)

[(op1) $\overset{\text{tex}}{\rightarrow}$ "(op1)"]

[(op1) $\overset{\text{pyk}}{\rightarrow}$ "var op1"]

(op2)

[(op2) $\overset{\text{tex}}{\rightarrow}$ "(op2)"]

[(op2) $\overset{\text{pyk}}{\rightarrow}$ "var op2"]

(r1)

[(r1) $\overset{\text{tex}}{\rightarrow}$ "(r1)"]

[(r1) $\overset{\text{pyk}}{\rightarrow}$ "var r1"]

(s1)

[(s1) $\overset{\text{tex}}{\rightarrow}$ "(s1)"]

[(s1) $\overset{\text{pyk}}{\rightarrow}$ "var s1"]

(s2)

[(s2) $\overset{\text{tex}}{\rightarrow}$ "(s2)"]

[(s2) $\overset{\text{pyk}}{\rightarrow}$ "var s2"]

X_1

$[X_1 \overset{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [X_1 \ddot{=} \underline{(x1)}] \rceil)]$

[X_1 $\overset{\text{tex}}{\rightarrow}$ "X_{1}"]

[X_1 $\overset{\text{pyk}}{\rightarrow}$ "meta x1"]

X_2

$[\mathrm{X}_2 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{X}_2 \ddot{=} \underline{(\mathrm{x2})}] \rceil)]$

$[\mathrm{X}_2 \stackrel{\mathrm{tex}}{\to}$ "X_{2}"]

$[\mathrm{X}_2 \stackrel{\mathrm{pyk}}{\to}$ "meta x2"]

Y_1

$[\mathrm{Y}_1 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{Y}_1 \ddot{=} \underline{(\mathrm{y1})}] \rceil)]$

$[\mathrm{Y}_1 \stackrel{\mathrm{tex}}{\to}$ "Y_{1}"]

$[\mathrm{Y}_1 \stackrel{\mathrm{pyk}}{\to}$ "meta y1"]

Y_2

$[\mathrm{Y}_2 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{Y}_2 \ddot{=} \underline{(\mathrm{y2})}] \rceil)]$

$[\mathrm{Y}_2 \stackrel{\mathrm{tex}}{\to}$ "Y_{2}"]

$[\mathrm{Y}_2 \stackrel{\mathrm{pyk}}{\to}$ "meta y2"]

V_1

$[\mathrm{V}_1 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{V}_1 \ddot{=} \underline{(\mathrm{v1})}] \rceil)]$

$[\mathrm{V}_1 \stackrel{\mathrm{tex}}{\to}$ "V_{1}"]

$[\mathrm{V}_1 \stackrel{\mathrm{pyk}}{\to}$ "meta v1"]

V_2

$[\mathrm{V}_2 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{V}_2 \ddot{=} \underline{(\mathrm{v2})}] \rceil)]$

$[\mathrm{V}_2 \stackrel{\mathrm{tex}}{\to}$ "V_{2}"]

$[\mathrm{V}_2 \stackrel{\mathrm{pyk}}{\to}$ "meta v2"]

V_3

$[\mathrm{V}_3 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{V}_3 \ddot{=} \underline{(\mathrm{v3})}] \rceil)]$

$[\mathrm{V}_3 \stackrel{\mathrm{tex}}{\to}$ "V_{3}"]

$[\mathrm{V}_3 \stackrel{\mathrm{pyk}}{\to}$ "meta v3"]

V_4

$[\mathrm{V}_4 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{V}_4 \ddot{=} \underline{(\mathrm{v4})}] \rceil)]$

$[\mathrm{V}_4 \stackrel{\mathrm{tex}}{\to}$ "V_{4}"]

$[\mathrm{V}_4 \stackrel{\mathrm{pyk}}{\to}$ "meta v4"]

$\mathrm{V}_{2\mathrm{n}}$

$[\mathrm{V}_{2\mathrm{n}} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{V}_{2\mathrm{n}} \ddot{=} \underline{(\mathrm{v2n})}] \rceil)]$

$[\mathrm{V}_{2\mathrm{n}} \stackrel{\mathrm{tex}}{\to}$ "V_{2n}"]

$[\mathrm{V}_{2\mathrm{n}} \stackrel{\mathrm{pyk}}{\to}$ "meta v2n"]

M_1

$[\mathrm{M}_1 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{M}_1 \ddot{=} \underline{(\mathrm{m1})}] \rceil)]$

$[\mathrm{M}_1 \stackrel{\mathrm{tex}}{\to}$ "M_{1}"]

$[\mathrm{M}_1 \stackrel{\mathrm{pyk}}{\to}$ "meta m1"]

M_2

$[\mathrm{M}_2 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{M}_2 \ddot{=} \underline{(\mathrm{m2})}] \rceil)]$

$[\mathrm{M}_2 \stackrel{\mathrm{tex}}{\to}$ "M_{2}"]

$[\mathrm{M}_2 \stackrel{\mathrm{pyk}}{\to}$ "meta m2"]

N_1

$[\mathrm{N}_1 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{N}_1 \ddot{=} \underline{(\mathrm{n1})}] \rceil)]$

$[\mathrm{N}_1 \stackrel{\text{tex}}{\to}$ "N_{1} "]

$[\mathrm{N}_1 \stackrel{\text{pyk}}{\to}$ "meta n1"]

N_2

$[\mathrm{N}_2 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{N}_2 \ddot{=} \underline{(\mathrm{n2})}] \rceil)]$

$[\mathrm{N}_2 \stackrel{\text{tex}}{\to}$ "N_{2} "]

$[\mathrm{N}_2 \stackrel{\text{pyk}}{\to}$ "meta n2"]

N_3

$[\mathrm{N}_3 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathrm{N}_3 \ddot{=} \underline{(\mathrm{n3})}] \rceil)]$

$[\mathrm{N}_3 \stackrel{\text{tex}}{\to}$ "N_{3} "]

$[\mathrm{N}_3 \stackrel{\text{pyk}}{\to}$ "meta n3"]

ϵ

$[\epsilon \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\epsilon \ddot{=} \underline{(\epsilon)}] \rceil)]$

$[\epsilon \stackrel{\text{tex}}{\to}$ "\epsilon"]

$[\epsilon \stackrel{\text{pyk}}{\to}$ "meta ep"]

$\epsilon 1$

$[\epsilon 1 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\epsilon 1 \ddot{=} \underline{(\epsilon)_1}] \rceil)]$

$[\epsilon 1 \stackrel{\text{tex}}{\to}$ "\epsilon 1"]

$[\epsilon 1 \stackrel{\text{pyk}}{\to}$ "meta ep1"]

$\epsilon 2$

$[\epsilon 2 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\epsilon 2 \doteq \underline{(\epsilon 2)}]\rceil)]$

$[\epsilon 2 \stackrel{\text{tex}}{\to}$ "\epsilon 2"]

$[\epsilon 2 \stackrel{\text{pyk}}{\to}$ "meta ep2"]

FX

$[\mathrm{FX} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{FX} \doteq \underline{(\mathrm{fx})}]\rceil)]$

$[\mathrm{FX} \stackrel{\text{tex}}{\to}$ "FX"]

$[\mathrm{FX} \stackrel{\text{pyk}}{\to}$ "meta fx"]

FY

$[\mathrm{FY} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{FY} \doteq \underline{(\mathrm{fy})}]\rceil)]$

$[\mathrm{FY} \stackrel{\text{tex}}{\to}$ "FY"]

$[\mathrm{FY} \stackrel{\text{pyk}}{\to}$ "meta fy"]

FZ

$[\mathrm{FZ} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{FZ} \doteq \underline{(\mathrm{fz})}]\rceil)]$

$[\mathrm{FZ} \stackrel{\text{tex}}{\to}$ "FZ"]

$[\mathrm{FZ} \stackrel{\text{pyk}}{\to}$ "meta fz"]

FU

$[\mathrm{FU} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{FU} \doteq \underline{(\mathrm{fu})}]\rceil)]$

$[\mathrm{FU} \stackrel{\text{tex}}{\to}$ "FU"]

$[\mathrm{FU} \stackrel{\text{pyk}}{\to}$ "meta fu"]

FV

$[\mathrm{FV} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{FV} \ddot{=} \underline{(\mathrm{fv})}]\rceil)]$

$[\mathrm{FV} \stackrel{\text{tex}}{\to} \text{“FV”}]$

$[\mathrm{FV} \stackrel{\text{pyk}}{\to} \text{“meta fv”}]$

FW

$[\mathrm{FW} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{FW} \ddot{=} \underline{(\mathrm{fw})}]\rceil)]$

$[\mathrm{FW} \stackrel{\text{tex}}{\to} \text{“FW”}]$

$[\mathrm{FW} \stackrel{\text{pyk}}{\to} \text{“meta fw”}]$

FEP

$[\mathrm{FEP} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{FEP} \ddot{=} \underline{(\mathrm{fep})}]\rceil)]$

$[\mathrm{FEP} \stackrel{\text{tex}}{\to} \text{“FEP”}]$

$[\mathrm{FEP} \stackrel{\text{pyk}}{\to} \text{“meta fep”}]$

RX

$[\mathrm{RX} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{RX} \ddot{=} \underline{(\mathrm{rx})}]\rceil)]$

$[\mathrm{RX} \stackrel{\text{tex}}{\to} \text{“RX”}]$

$[\mathrm{RX} \stackrel{\text{pyk}}{\to} \text{“meta rx”}]$

RY

$[\mathrm{RY} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{RY} \ddot{=} \underline{(\mathrm{ry})}]\rceil)]$

$[\mathrm{RY} \stackrel{\text{tex}}{\to} \text{“RY”}]$

$[\mathrm{RY} \stackrel{\text{pyk}}{\to} \text{“meta ry”}]$

RZ

$[\mathrm{RZ} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{RZ} \ddot{=} \underline{(\mathrm{rz})}]\rceil)]$

$[\mathrm{RZ} \stackrel{\mathrm{tex}}{\to}$ "RZ"$]$

$[\mathrm{RZ} \stackrel{\mathrm{pyk}}{\to}$ "meta rz"$]$

RU

$[\mathrm{RU} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{RU} \ddot{=} \underline{(\mathrm{ru})}]\rceil)]$

$[\mathrm{RU} \stackrel{\mathrm{tex}}{\to}$ "RU"$]$

$[\mathrm{RU} \stackrel{\mathrm{pyk}}{\to}$ "meta ru"$]$

(SX)

$[(\mathrm{SX}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SX}) \ddot{=} \underline{(\mathrm{sx})}]\rceil)]$

$[(\mathrm{SX}) \stackrel{\mathrm{tex}}{\to}$ "(SX)"$]$

$[(\mathrm{SX}) \stackrel{\mathrm{pyk}}{\to}$ "meta sx"$]$

(SX1)

$[(\mathrm{SX1}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SX1}) \ddot{=} \underline{(\mathrm{sx1})}]\rceil)]$

$[(\mathrm{SX1}) \stackrel{\mathrm{tex}}{\to}$ "(SX1)"$]$

$[(\mathrm{SX1}) \stackrel{\mathrm{pyk}}{\to}$ "meta sx1"$]$

(SY)

$[(\mathrm{SY}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SY}) \ddot{=} \underline{(\mathrm{sy})}]\rceil)]$

$[(\mathrm{SY}) \stackrel{\mathrm{tex}}{\to}$ "(SY)"$]$

$[(\mathrm{SY}) \stackrel{\mathrm{pyk}}{\to}$ "meta sy"$]$

(SY1)

$[(\mathrm{SY1}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SY1}) \ddot{=} \underline{(\mathrm{sy1})}]\rceil)]$

$[(\mathrm{SY1}) \stackrel{\mathrm{tex}}{\to}$ "(SY1)"]

$[(\mathrm{SY1}) \stackrel{\mathrm{pyk}}{\to}$ "meta sy1"]

(SZ)

$[(\mathrm{SZ}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SZ}) \ddot{=} \underline{(\mathrm{sz})}]\rceil)]$

$[(\mathrm{SZ}) \stackrel{\mathrm{tex}}{\to}$ "(SZ)"]

$[(\mathrm{SZ}) \stackrel{\mathrm{pyk}}{\to}$ "meta sz"]

(SZ1)

$[(\mathrm{SZ1}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SZ1}) \ddot{=} \underline{(\mathrm{sz1})}]\rceil)]$

$[(\mathrm{SZ1}) \stackrel{\mathrm{tex}}{\to}$ "(SZ1)"]

$[(\mathrm{SZ1}) \stackrel{\mathrm{pyk}}{\to}$ "meta sz1"]

(SU)

$[(\mathrm{SU}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SU}) \ddot{=} \underline{(\mathrm{su})}]\rceil)]$

$[(\mathrm{SU}) \stackrel{\mathrm{tex}}{\to}$ "(SU)"]

$[(\mathrm{SU}) \stackrel{\mathrm{pyk}}{\to}$ "meta su"]

(SU1)

$[(\mathrm{SU1}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{SU1}) \ddot{=} \underline{(\mathrm{su1})}]\rceil)]$

$[(\mathrm{SU1}) \stackrel{\mathrm{tex}}{\to}$ "(SU1)"]

$[(\mathrm{SU1}) \stackrel{\mathrm{pyk}}{\to}$ "meta su1"]

FXS

$[\mathrm{FXS} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{FXS} \ddot{=} \underline{(\mathrm{fxs})}]\rceil)]$

$[\mathrm{FXS} \stackrel{\mathrm{tex}}{\to} \text{“FXS”}]$

$[\mathrm{FXS} \stackrel{\mathrm{pyk}}{\to} \text{“meta fxs”}]$

FYS

$[\mathrm{FYS} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{FYS} \ddot{=} \underline{(\mathrm{fys})}]\rceil)]$

$[\mathrm{FYS} \stackrel{\mathrm{tex}}{\to} \text{“FYS”}]$

$[\mathrm{FYS} \stackrel{\mathrm{pyk}}{\to} \text{“meta fys”}]$

(F1)

$[(\mathrm{F1}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F1}) \ddot{=} \underline{(\mathrm{f1})}]\rceil)]$

$[(\mathrm{F1}) \stackrel{\mathrm{tex}}{\to} \text{“(F1)”}]$

$[(\mathrm{F1}) \stackrel{\mathrm{pyk}}{\to} \text{“meta f1”}]$

(F2)

$[(\mathrm{F2}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F2}) \ddot{=} \underline{(\mathrm{f2})}]\rceil)]$

$[(\mathrm{F2}) \stackrel{\mathrm{tex}}{\to} \text{“(F2)”}]$

$[(\mathrm{F2}) \stackrel{\mathrm{pyk}}{\to} \text{“meta f2”}]$

(F3)

$[(\mathrm{F3}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F3}) \ddot{=} \underline{(\mathrm{f3})}]\rceil)]$

$[(\mathrm{F3}) \stackrel{\mathrm{tex}}{\to} \text{“(F3)”}]$

$[(\mathrm{F3}) \stackrel{\mathrm{pyk}}{\to} \text{“meta f3”}]$

(F4)

$[(\mathrm{F4}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F4}) \ddot{=} \underline{(\mathrm{f4})}]\rceil)]$

$[(\mathrm{F4}) \stackrel{\text{tex}}{\to}$ "(F4)"$]$

$[(\mathrm{F4}) \stackrel{\text{pyk}}{\to}$ "meta f4"$]$

(OP1)

$[(\mathrm{OP1}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{OP1}) \ddot{=} \underline{(\mathrm{op1})}]\rceil)]$

$[(\mathrm{OP1}) \stackrel{\text{tex}}{\to}$ "(OP1)"$]$

$[(\mathrm{OP1}) \stackrel{\text{pyk}}{\to}$ "meta op1"$]$

(OP2)

$[(\mathrm{OP2}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{OP2}) \ddot{=} \underline{(\mathrm{op2})}]\rceil)]$

$[(\mathrm{OP2}) \stackrel{\text{tex}}{\to}$ "(OP2)"$]$

$[(\mathrm{OP2}) \stackrel{\text{pyk}}{\to}$ "meta op2"$]$

(R1)

$[(\mathrm{R1}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{R1}) \ddot{=} \underline{(\mathrm{r1})}]\rceil)]$

$[(\mathrm{R1}) \stackrel{\text{tex}}{\to}$ "(R1)"$]$

$[(\mathrm{R1}) \stackrel{\text{pyk}}{\to}$ "meta r1"$]$

(S1)

$[(\mathrm{S1}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{S1}) \ddot{=} \underline{(\mathrm{s1})}]\rceil)]$

$[(\mathrm{S1}) \stackrel{\text{tex}}{\to}$ "(S1)"$]$

$[(\mathrm{S1}) \stackrel{\text{pyk}}{\to}$ "meta s1"$]$

(S2)

$[(\mathrm{S2}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{S2}) \ddot{=} \underline{(\mathrm{s2})}]\rceil)]$

$[(\mathrm{S2}) \stackrel{\mathrm{tex}}{\to}$ "(S2)"$]$

$[(\mathrm{S2}) \stackrel{\mathrm{pyk}}{\to}$ "meta s2"$]$

(EPob)

$[(\mathrm{EPob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{EPob}) \ddot{=} \overline{(\epsilon)}]\rceil)]$

$[(\mathrm{EPob}) \stackrel{\mathrm{tex}}{\to}$ "(EPob)"$]$

$[(\mathrm{EPob}) \stackrel{\mathrm{pyk}}{\to}$ "object ep"$]$

(CRS1ob)

$[(\mathrm{CRS1ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{CRS1ob}) \ddot{=} \overline{(\mathrm{crs1})}]\rceil)]$

$[(\mathrm{CRS1ob}) \stackrel{\mathrm{tex}}{\to}$ "(CRS1ob)"$]$

$[(\mathrm{CRS1ob}) \stackrel{\mathrm{pyk}}{\to}$ "object crs1"$]$

(F1ob)

$[(\mathrm{F1ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F1ob}) \ddot{=} \overline{(\mathrm{f1})}]\rceil)]$

$[(\mathrm{F1ob}) \stackrel{\mathrm{tex}}{\to}$ "(F1ob)"$]$

$[(\mathrm{F1ob}) \stackrel{\mathrm{pyk}}{\to}$ "object f1"$]$

(F2ob)

$[(\mathrm{F2ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F2ob}) \ddot{=} \overline{(\mathrm{f2})}]\rceil)]$

$[(\mathrm{F2ob}) \stackrel{\mathrm{tex}}{\to}$ "(F2ob)"$]$

$[(\mathrm{F2ob}) \stackrel{\mathrm{pyk}}{\to}$ "object f2"$]$

(F3ob)

$[(\mathrm{F3ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F3ob}) \ddot{=} \overline{(\mathrm{f3})}]\rceil)]$

$[(\mathrm{F3ob}) \stackrel{\mathrm{tex}}{\to}$ "(F3ob)"$]$

$[(\mathrm{F3ob}) \stackrel{\mathrm{pyk}}{\to}$ "object f3"$]$

(F4ob)

$[(\mathrm{F4ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{F4ob}) \ddot{=} \overline{(\mathrm{f4})}]\rceil)]$

$[(\mathrm{F4ob}) \stackrel{\mathrm{tex}}{\to}$ "(F4ob)"$]$

$[(\mathrm{F4ob}) \stackrel{\mathrm{pyk}}{\to}$ "object f4"$]$

(N1ob)

$[(\mathrm{N1ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{N1ob}) \ddot{=} \overline{(\mathrm{n1})}]\rceil)]$

$[(\mathrm{N1ob}) \stackrel{\mathrm{tex}}{\to}$ "(N1ob)"$]$

$[(\mathrm{N1ob}) \stackrel{\mathrm{pyk}}{\to}$ "object n1"$]$

(N2ob)

$[(\mathrm{N2ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{N2ob}) \ddot{=} \overline{(\mathrm{n2})}]\rceil)]$

$[(\mathrm{N2ob}) \stackrel{\mathrm{tex}}{\to}$ "(N2ob)"$]$

$[(\mathrm{N2ob}) \stackrel{\mathrm{pyk}}{\to}$ "object n2"$]$

(OP1ob)

$[(\mathrm{OP1ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{OP1ob}) \ddot{=} \overline{(\mathrm{op1})}]\rceil)]$

$[(\mathrm{OP1ob}) \stackrel{\mathrm{tex}}{\to}$ "(OP1ob)"$]$

$[(\mathrm{OP1ob}) \stackrel{\mathrm{pyk}}{\to}$ "object op1"$]$

(OP2ob)

$[(\mathrm{OP2ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{OP2ob}) \ddot{=} \overline{(\mathrm{op2})}]\rceil)]$

$[(\mathrm{OP2ob}) \stackrel{\mathrm{tex}}{\to}$ "(OP2ob)"$]$

$[(\mathrm{OP2ob}) \stackrel{\mathrm{pyk}}{\to}$ "object op2"$]$

(R1ob)

$[(\mathrm{R1ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{R1ob}) \ddot{=} \overline{(\mathrm{r1})}]\rceil)]$

$[(\mathrm{R1ob}) \stackrel{\mathrm{tex}}{\to}$ "(R1ob)"$]$

$[(\mathrm{R1ob}) \stackrel{\mathrm{pyk}}{\to}$ "object r1"$]$

(S1ob)

$[(\mathrm{S1ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{S1ob}) \ddot{=} \overline{(\mathrm{s1})}]\rceil)]$

$[(\mathrm{S1ob}) \stackrel{\mathrm{tex}}{\to}$ "(S1ob)"$]$

$[(\mathrm{S1ob}) \stackrel{\mathrm{pyk}}{\to}$ "object s1"$]$

(S2ob)

$[(\mathrm{S2ob}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathrm{S2ob}) \ddot{=} \overline{(\mathrm{s2})}]\rceil)]$

$[(\mathrm{S2ob}) \stackrel{\mathrm{tex}}{\to}$ "(S2ob)"$]$

$[(\mathrm{S2ob}) \stackrel{\mathrm{pyk}}{\to}$ "object s2"$]$

ph_4

$[\mathrm{ph}_4 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{ph}_4 \ddot{=} \mathsf{d}_{\mathrm{Ph}}]\rceil)]$

$[\mathrm{ph}_4 \stackrel{\mathrm{tex}}{\to}$ "ph_{4}"$]$

$[\mathrm{ph}_4 \stackrel{\mathrm{pyk}}{\to}$ "ph4"$]$

ph_5

$[\mathrm{ph}_5 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{ph}_5 \ddot{=} \mathsf{e}_{\mathrm{Ph}}]\rceil)]$

$[\mathrm{ph}_5 \stackrel{\mathrm{tex}}{\to}$ "ph_{5}"]

$[\mathrm{ph}_5 \stackrel{\mathrm{pyk}}{\to}$ "ph5"]

ph_6

$[\mathrm{ph}_6 \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathrm{ph}_6 \ddot{=} \mathsf{f}_{\mathrm{Ph}}]\rceil)]$

$[\mathrm{ph}_6 \stackrel{\mathrm{tex}}{\to}$ "ph_{6}"]

$[\mathrm{ph}_6 \stackrel{\mathrm{pyk}}{\to}$ "ph6"]

NAT

[NAT $\stackrel{\mathrm{tex}}{\to}$ "NAT"]

[NAT $\stackrel{\mathrm{pyk}}{\to}$ "NAT"]

$\mathrm{RATIONAL}_{\mathrm{S}}\mathrm{ERIES}$

[$\mathrm{RATIONAL}_{\mathrm{S}}\mathrm{ERIES} \stackrel{\mathrm{tex}}{\to}$ "RATIONAL_SERIES"]

[$\mathrm{RATIONAL}_{\mathrm{S}}\mathrm{ERIES} \stackrel{\mathrm{pyk}}{\to}$ "RATIONAL_SERIES"]

SERIES

[SERIES $\stackrel{\mathrm{tex}}{\to}$ "SERIES"]

[SERIES $\stackrel{\mathrm{pyk}}{\to}$ "SERIES"]

SetOfReals

[SetOfReals $\stackrel{\mathrm{tex}}{\to}$ "SetOfReals"]

[SetOfReals $\stackrel{\mathrm{pyk}}{\to}$ "setOfReals"]

SetOfFxs

[SetOfFxs $\stackrel{\text{tex}}{\rightarrow}$ "SetOfFxs"]

[SetOfFxs $\stackrel{\text{pyk}}{\rightarrow}$ "setOfFxs"]

N

[N $\stackrel{\text{tex}}{\rightarrow}$ "N"]

[N $\stackrel{\text{pyk}}{\rightarrow}$ "N"]

Q

[Q $\stackrel{\text{tex}}{\rightarrow}$ "Q"]

[Q $\stackrel{\text{pyk}}{\rightarrow}$ "Q"]

X

[X $\stackrel{\text{tex}}{\rightarrow}$ "X"]

[X $\stackrel{\text{pyk}}{\rightarrow}$ "X"]

xs

[xs $\stackrel{\text{tex}}{\rightarrow}$ "xs"]

[xs $\stackrel{\text{pyk}}{\rightarrow}$ "xs"]

xaF

[xaF $\stackrel{\text{macro}}{\rightarrow}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\text{xaF} \ddot{=} \text{xs}]\rceil)$]

[xaF $\stackrel{\text{tex}}{\rightarrow}$ "xaF"]

[xaF $\stackrel{\text{pyk}}{\rightarrow}$ "xsF"]

ysF

$[\text{ysF} \overset{\text{tex}}{\to} \text{“ysF”}]$

$[\text{ysF} \overset{\text{pyk}}{\to} \text{“ysF”}]$

us

$[\text{us} \overset{\text{tex}}{\to} \text{“us”}]$

$[\text{us} \overset{\text{pyk}}{\to} \text{“us”}]$

usFoelge

$[\text{usFoelge} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\text{usFoelge} \ddot{=} \text{us}]\rceil)]$

$[\text{usFoelge} \overset{\text{tex}}{\to} \text{“usFoelge”}]$

$[\text{usFoelge} \overset{\text{pyk}}{\to} \text{“usF”}]$

0

$[0 \overset{\text{tex}}{\to} \text{“0”}]$

$[0 \overset{\text{pyk}}{\to} \text{“0”}]$

1

$[1 \overset{\text{tex}}{\to} \text{“1”}]$

$[1 \overset{\text{pyk}}{\to} \text{“1”}]$

(-1)

$[(-1) \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[(-1) \ddot{=} (-\text{u}1)]\rceil)]$

$[(-1) \overset{\text{tex}}{\to} \text{“(-1)”}]$

$[(-1) \overset{\text{pyk}}{\to} \text{“(-1)”}]$

2

$[2 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[2 \doteq (1+1)]\rceil)]$

$[2 \stackrel{\text{tex}}{\to} \text{“2”}]$

$[2 \stackrel{\text{pyk}}{\to} \text{“2”}]$

3

$[3 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[3 \doteq (2+1)]\rceil)]$

$[3 \stackrel{\text{tex}}{\to} \text{“3”}]$

$[3 \stackrel{\text{pyk}}{\to} \text{“3”}]$

1/2

$[1/2 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[1/2 \doteq \text{rec2}]\rceil)]$

$[1/2 \stackrel{\text{tex}}{\to} \text{“1/2”}]$

$[1/2 \stackrel{\text{pyk}}{\to} \text{“1/2”}]$

1/3

$[1/3 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[1/3 \doteq \text{rec3}]\rceil)]$

$[1/3 \stackrel{\text{tex}}{\to} \text{“1/3”}]$

$[1/3 \stackrel{\text{pyk}}{\to} \text{“1/3”}]$

2/3

$[2/3 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[2/3 \doteq (2 * 1/3)]\rceil)]$

$[2/3 \stackrel{\text{tex}}{\to} \text{“2/3”}]$

$[2/3 \stackrel{\text{pyk}}{\to} \text{“2/3”}]$

0f

$[0\text{f} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[0\text{f} \doteq \text{constantRationalSeries}(0)]\rceil)]$

$[0f \stackrel{\text{tex}}{\rightarrow}$ "0f"$]$

$[0f \stackrel{\text{pyk}}{\rightarrow}$ "0f"$]$

1f

$[1f \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[1f \ddot{=} \text{constantRationalSeries}(1)]\rceil)]$

$[1f \stackrel{\text{pyk}}{\rightarrow}$ "1f"$]$

00

$[00 \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[00 \ddot{=} \text{R}(0f)]\rceil)]$

$[00 \stackrel{\text{tex}}{\rightarrow}$ "00"$]$

$[00 \stackrel{\text{pyk}}{\rightarrow}$ "00"$]$

01

$[01 \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[01 \ddot{=} \text{R}(1f)]\rceil)]$

$[01 \stackrel{\text{pyk}}{\rightarrow}$ "01"$]$

$(- - 01)$

$[(- - 01) \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(- - 01) \ddot{=} (- - 01)]\rceil)]$

$[(- - 01) \stackrel{\text{tex}}{\rightarrow}$ "(--01)"$]$

$[(- - 01) \stackrel{\text{pyk}}{\rightarrow}$ "(--01)"$]$

02

$[02 \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[02 \ddot{=} 01 + +01]\rceil)]$

$[02 \stackrel{\text{tex}}{\rightarrow}$ "02"$]$

$[02 \stackrel{\text{pyk}}{\rightarrow}$ "02"$]$

01//02

$[01//02 \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[01//02 \ddot{=} 01//\text{temp02}]\rceil)]$

$[01//02 \stackrel{\text{tex}}{\to} \text{“01//02”}]$

$[01//02 \stackrel{\text{pyk}}{\to} \text{“01//02”}]$

PlusAssociativity(R)

$[\text{PlusAssociativity(R)} \stackrel{\text{proof}}{\to} \text{Rule tactic}]$

$[\text{PlusAssociativity(R)} \stackrel{\text{stmt}}{\to} \text{SystemQ} \vdash \forall(\text{fx}): \forall(\text{fy}): \forall(\text{fz}): \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{r1}): (\text{r1}) \in \mathsf{f}_{\text{Ph}} \Rightarrow \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg((\text{r1}) = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow \dot\neg(\forall_{\text{obj}}(\text{f1}): \forall_{\text{obj}}(\text{f2}): \forall_{\text{obj}}(\text{f3}): \forall_{\text{obj}}(\text{f4}): \{\{(\text{f1}),(\text{f1})\},\{(\text{f1}),(\text{f2})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow \{\{(\text{f3}),(\text{f3})\},\{(\text{f3}),(\text{f4})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow (\text{f1}) = (\text{f3}) \Rightarrow (\text{f2}) = (\text{f4}))\text{n})\text{n} \Rightarrow \dot\neg(\forall_{\text{obj}}(\text{s1}): (\text{s1}) \in \text{N} \Rightarrow \dot\neg(\forall_{\text{obj}}(\text{s2}): \dot\neg(\{\{(\text{s1}),(\text{s1})\},\{(\text{s1}),(\text{s2})\}\} \in \mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}}(\epsilon): \dot\neg(\forall_{\text{obj}}\overline{\mathsf{n}}: \dot\neg(\forall_{\text{obj}}\overline{\mathsf{m}}: \dot\neg(0 <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{\text{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{\text{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\text{fx})[\underline{\mathsf{m}}] + (\text{fy})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\underline{\mathsf{m}}] + (\text{fz})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}] + (-\text{u}\mathsf{d}_{\text{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow \dot\neg(\dot\neg(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{\text{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot\neg(\forall_{\text{obj}}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{\text{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},((\text{fx})[\underline{\mathsf{m}}]+(\text{fy})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\underline{\mathsf{m}}]+(\text{fz})[\underline{\mathsf{m}}])\}\})\text{n})\text{n}\}[\overline{\mathsf{m}}]+(-\text{u}\mathsf{d}_{\text{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\text{n})\text{n})\text{n})\text{n})\text{n}\} = \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N},\text{Q}\}))) \mid \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{r1}): (\text{r1}) \in \mathsf{f}_{\text{Ph}} \Rightarrow \dot\neg(\forall_{\text{obj}}(\text{op1}): \dot\neg(\dot\neg(\forall_{\text{obj}}(\text{op2}): \dot\neg(\dot\neg(\dot\neg((\text{op1}) \in \text{N} \Rightarrow \dot\neg((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot\neg((\text{r1}) = \{\{(\text{op1}),(\text{op1})\},\{(\text{op1}),(\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow \dot\neg(\forall_{\text{obj}}(\text{f1}): \forall_{\text{obj}}(\text{f2}): \forall_{\text{obj}}(\text{f3}): \forall_{\text{obj}}(\text{f4}): \{\{(\text{f1}),(\text{f1})\},\{(\text{f1}),(\text{f2})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$

$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] + \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] + \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}]$

[PlusAssociativity(R) $\stackrel{\text{tex}}{\to}$ "PlusAssociativity(R)"]

[PlusAssociativity(R) $\stackrel{\text{pyk}}{\to}$ "lemma plusAssociativity(R)"]

PlusAssociativity(R)XX

[PlusAssociativity(R)XX $\stackrel{\text{proof}}{\to}$ Rule tactic]

[PlusAssociativity(R)XX $\stackrel{\text{stmt}}{\to}$ SystemQ $\vdash \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \forall\underline{(fz)}\colon \{ph \in \mathrm{P}(\{ph \in$
$\mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in$
$\mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] + (fy)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] + (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}]+(fy)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]+(fz)[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(f1)}\colon \forall_{\mathrm{obj}} \overline{(f2)}\colon \forall_{\mathrm{obj}} \overline{(f3)}\colon \forall_{\mathrm{obj}} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] + (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fy)[\underline{\mathsf{m}}] + (fz)[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}]$

[PlusAssociativity(R)XX $\stackrel{\text{tex}}{\to}$ "PlusAssociativity(R)XX"]

[PlusAssociativity(R)XX $\stackrel{\text{pyk}}{\to}$ "lemma plusAssociativity(R)XX"]

Plus0(R)

[Plus0(R) $\stackrel{\text{proof}}{\to}$ Rule tactic]

[Plus0(R) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\mathrm{fx})}\colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}$]

[Plus0(R) $\stackrel{\text{tex}}{\rightarrow}$ "Plus0(R)"]

[Plus0(R) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma plus0(R)"]

Negative(R)

[Negative(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$$
\begin{aligned}
&[\mathrm{Negative(R)} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}{:}\, \forall \underline{(\mathrm{fx})}{:}\, \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \\
&\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \\
&\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ | \\
&\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}{:}\, \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \\
&\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}{:}\, \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}{:}\, \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}{:}\, \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}{:}\, \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \\
&\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}{:}\, \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}{:}\, \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \\
&\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}} \overline{(\epsilon)}{:}\, \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}}{:}\, \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}}{:}\, \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \\
&\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ | \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}{:}\, \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} = \\
&\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ | \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}{:}\, \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} = \\
&\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \\
&\dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ | \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}{:}\, \dot{\neg}\,(\mathsf{d}_{\mathrm{Ph}} = \\
&\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ | \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}{:}\, \dot{\neg}\,(\mathsf{f}_{\mathrm{Ph}} = \\
&\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{u}\underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \\
&\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ | \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}{:}\, \overline{(\mathrm{r1})} \in \\
&\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}{:}\, \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}{:}\, \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}{:}\, \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}{:}\, \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \\
&\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}{:}\, \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}{:}\, \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \\
&\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}} \overline{(\epsilon)}{:}\, \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}}{:}\, \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}}{:}\, \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \\
&\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ | \\
&\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \\
&\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}{:}\, \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \\
&\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \\
&\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}{:}\, \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \\
&\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \\
&\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}{:}\, \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} = \\
&\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]
\end{aligned}
$$

[Negative(R) $\overset{\mathrm{tex}}{\rightarrow}$ “Negative(R)”]

[Negative(R) $\overset{\text{pyk}}{\rightarrow}$ "lemma negative(R)"]

Times1(R)

[Times1(R) $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Times1(R) $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall (fx): \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\})))) \mid \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}(r1): (r1) \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow$
$\dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1): \forall_{obj}(f2): \forall_{obj}(f3): \forall_{obj}(f4): \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2): \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon): \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1): \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((fx)[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(\mathrm{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}(crs1): \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{(crs1), (crs1)\}, \{(crs1), 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(\mathrm{Union}(\{N, Q\})))) \mid$
$\dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}(r1): (r1) \in$
$\mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}(op1): \dot{\neg}(\dot{\neg}(\forall_{obj}(op2): \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow$
$\dot{\neg}((r1) = \{\{(op1), (op1)\}, \{(op1), (op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(f1): \forall_{obj}(f2): \forall_{obj}(f3): \forall_{obj}(f4): \{\{(f1), (f1)\}, \{(f1), (f2)\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{(f3), (f3)\}, \{(f3), (f4)\}\} \in \mathsf{f}_{Ph} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\neg}(\forall_{obj}(s2): \dot{\neg}(\{\{(s1), (s1)\}, \{(s1), (s2)\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon): \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = (\epsilon))n)n)n)n)n\}$]

[Times1(R) $\overset{\text{tex}}{\rightarrow}$ "Times1(R)"]

[Times1(R) $\overset{\text{pyk}}{\to}$ "lemma times1(R)"]

lessAddition(R)

[lessAddition(R) $\overset{\text{proof}}{\to}$ Rule tactic]

[lessAddition(R) $\overset{\text{stmt}}{\to}$ SystemQ $\vdash$
$\forall \underline{\mathsf{m}} \colon \forall \underline{(\epsilon)} \colon \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \forall \underline{(\mathrm{fz})} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}(\dot{\neg}(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$]

[lessAddition(R) $\overset{\text{tex}}{\to}$ "lessAddition(R)"]

[lessAddition(R) $\overset{\text{pyk}}{\to}$ "lemma lessAddition(R)"]

PlusCommutativity(R)

[PlusCommutativity(R) $\overset{\text{proof}}{\to}$ Rule tactic]

[PlusCommutativity(R) $\overset{\text{stmt}}{\to}$ SystemQ $\vdash \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, ((\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{d}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}]+\underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}=$
$\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{d}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fy)}[\underline{\mathsf{m}}]+\underline{(fx)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}\mid\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}\colon\dot{\neg}(\mathsf{d}_{Ph}=$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fy)}[\underline{\mathsf{m}}]+\underline{(fx)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}]$

[PlusCommutativity(R) $\overset{tex}{\rightarrow}$ "PlusCommutativity(R)"]

[PlusCommutativity(R) $\overset{pyk}{\rightarrow}$ "lemma plusCommutativity(R)"]

LeqAntisymmetry(R)

[LeqAntisymmetry(R) $\overset{proof}{\rightarrow}$ Rule tactic]

[LeqAntisymmetry(R) $\overset{stmt}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{(fx)}\colon\forall\underline{(fy)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=$
$\overline{(\epsilon)})n)n)n\Rightarrow\dot{\neg}(\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\underline{(fx)}[\overline{\mathsf{m}}]<=(\underline{(fy)}[\overline{\mathsf{m}}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n\Rightarrow$
$\{ph\in P(\{ph\in P(\{ph\in P(P(Union(\{N,Q\}))))\mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\})\mid\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon\overline{(r1)}\in$
$\mathsf{f}_{Ph}\Rightarrow\dot{\neg}(\forall_{obj}\overline{(op1)}\colon\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon\dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)}\in N\Rightarrow\dot{\neg}(\overline{(op2)}\in Q)n)n\Rightarrow$
$\dot{\neg}(\overline{(r1)}=\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon\{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\}\in\mathsf{f}_{Ph}\Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\}\in\mathsf{f}_{Ph}\Rightarrow\overline{(f1)}=\overline{(f3)}\Rightarrow\overline{(f2)}=\overline{(f4)})n)n\Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon\overline{(s1)}\in N\Rightarrow\dot{\neg}(\forall_{obj}\overline{(s2)}\colon\dot{\neg}(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\})\mid\forall_{obj}\overline{(\epsilon)}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon\dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon\dot{\neg}(0<=\overline{(\epsilon)}\Rightarrow\dot{\neg}(\dot{\neg}(0=\overline{(\epsilon)})n)n)n\Rightarrow$
$\overline{\mathsf{n}}<=\overline{\mathsf{m}}\Rightarrow\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}]+(-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))|<=\overline{(\epsilon)}\Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}]+(-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}=\{ph\in P(\{ph\in P(\{ph\in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \overline{(\mathrm{fy})}[\overline{\mathsf{m}}] <= (\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\overline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$

$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[LeqAntisymmetry(R) $\overset{\mathrm{tex}}{\rightarrow}$ "LeqAntisymmetry(R)"]

[LeqAntisymmetry(R) $\overset{\mathrm{pyk}}{\rightarrow}$ "lemma leqAntisymmetry(R)"]

LeqTransitivity(R)

[LeqTransitivity(R) $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[LeqTransitivity(R) $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fz})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <=$
$(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (fy)[\overline{\mathsf{m}}] <= ((fz)[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fy)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fy)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$
$\dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fz)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fz)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\dot{-}(\dot{-}(\forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(\dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot{-}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (fx)[\overline{\mathsf{m}}] <= ((fz)[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in$
$N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{-}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{-}(\dot{-}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow \dot{-}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{-}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{-}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{-}(\forall_{obj}\overline{(s2)}\colon \dot{-}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{-}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{-}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((fx)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((fx)[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{-}(\forall_{obj}\overline{(op1)}\colon \dot{-}(\dot{-}(\forall_{obj}\overline{(op2)}\colon \dot{-}(\dot{-}(\dot{-}(\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1}): (\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}): \forall_{\mathrm{obj}}(\mathrm{f2}): \forall_{\mathrm{obj}}(\mathrm{f3}): \forall_{\mathrm{obj}}(\mathrm{f4}): \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}): (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}): \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon): \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fz})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[LeqTransitivity(R) $\stackrel{\text{tex}}{\rightarrow}$ "LeqTransitivity(R)"]

[LeqTransitivity(R) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma leqTransitivity(R)"]

leqAddition(R)

[leqAddition(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[leqAddition(R) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall(\mathrm{fx}): \forall(\mathrm{fy}): \forall(\mathrm{fz}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\epsilon): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (\mathrm{fx})[\overline{\mathsf{m}}] <=$
$((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathrm{u}(\epsilon))))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1}): (\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}): \forall_{\mathrm{obj}}(\mathrm{f2}): \forall_{\mathrm{obj}}(\mathrm{f3}): \forall_{\mathrm{obj}}(\mathrm{f4}): \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}): (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}): \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon): \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fx})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1}): (\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1}): \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2}): \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\mathrm{r1}) = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{f1}): \forall_{\mathrm{obj}}(\mathrm{f2}): \forall_{\mathrm{obj}}(\mathrm{f3}): \forall_{\mathrm{obj}}(\mathrm{f4}): \{\{(\mathrm{f1}), (\mathrm{f1})\}, \{(\mathrm{f1}), (\mathrm{f2})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{(\mathrm{f3}), (\mathrm{f3})\}, \{(\mathrm{f3}), (\mathrm{f4})\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (\mathrm{f1}) = (\mathrm{f3}) \Rightarrow (\mathrm{f2}) = (\mathrm{f4}))\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s1}): (\mathrm{s1}) \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{s2}): \dot{\neg}(\{\{(\mathrm{s1}), (\mathrm{s1})\}, \{(\mathrm{s1}), (\mathrm{s2})\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}(\epsilon): \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}: \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}: \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= (\epsilon) \Rightarrow$
$\dot{\neg}(\dot{\neg}(|((\mathrm{fy})[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = (\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash$

$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |\ \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\})))\ |$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] + \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[leqAddition(R) $\stackrel{\mathrm{tex}}{\rightarrow}$ "leqAddition(R)"]

[leqAddition(R) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma leqAddition(R)"]

Distribution(R)

[Distribution(R) $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[Distribution(R) $\stackrel{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \overline{(fx)}\colon \forall \overline{(fy)}\colon \forall \overline{(fz)}\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj} \overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(f1)}\colon \forall_{obj} \overline{(f2)}\colon \forall_{obj} \overline{(f3)}\colon \forall_{obj} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj} \overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj} \overline{(\epsilon)}\colon \dot\neg(\forall_{obj} \overline{\mathsf{n}}\colon \dot\neg(\forall_{obj} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] + \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\overline{(fy)}[\underline{\mathsf{m}}] + \overline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)}))n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj} \overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(f1)}\colon \forall_{obj} \overline{(f2)}\colon \forall_{obj} \overline{(f3)}\colon \forall_{obj} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj} \overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj} \overline{(\epsilon)}\colon \dot\neg(\forall_{obj} \overline{\mathsf{n}}\colon \dot\neg(\forall_{obj} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fz})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[Distribution(R) $\overset{\mathrm{tex}}{\to}$ “Distribution(R)”]

[Distribution(R) $\overset{\mathrm{pyk}}{\to}$ “lemma distribution(R)”]

A4(Axiom)

[A4(Axiom) $\overset{\mathrm{proof}}{\to}$ Rule tactic]

[A4(Axiom) $\overset{\mathrm{stmt}}{\to}$ SystemQ $\vdash \forall\underline{\mathsf{x}}\colon \forall\underline{(\mathrm{v1})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \langle\underline{\mathsf{a}}{\equiv}\underline{\mathsf{b}}|\underline{(\mathrm{v1})}{:=}{=}\underline{\mathsf{x}}\rangle_{\mathrm{Me}} \Vdash$
$\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{a}}$]

[A4(Axiom) $\overset{\mathrm{tex}}{\to}$ “A4(Axiom)”]

[A4(Axiom) $\overset{\mathrm{pyk}}{\to}$ “axiom a4”]

InductionAxiom

[InductionAxiom $\overset{\mathrm{proof}}{\to}$ Rule tactic]

[InductionAxiom $\overset{\mathrm{stmt}}{\to}$ SystemQ $\vdash \forall\underline{(\mathrm{v1})}\colon \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \langle\underline{\mathsf{b}}{\equiv}\underline{\mathsf{a}}|\underline{(\mathrm{v1})}{:=}{=}0\rangle_{\mathrm{Me}} \Vdash$
$\langle\underline{\mathsf{c}}{\equiv}\underline{\mathsf{a}}|\underline{(\mathrm{v1})}{:=}{=}(\underline{(\mathrm{v1})} + 1)\rangle_{\mathrm{Me}} \Vdash \underline{\mathsf{b}} \Rightarrow \overline{\forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{c}}} \Rightarrow \forall_{\mathrm{obj}}\underline{(\mathrm{v1})}\colon \underline{\mathsf{a}}$]

[InductionAxiom $\overset{\mathrm{tex}}{\to}$ “InductionAxiom”]

[InductionAxiom $\stackrel{\text{pyk}}{\to}$ "axiom induction"]

EqualityAxiom

[EqualityAxiom $\stackrel{\text{proof}}{\to}$ Rule tactic]

[EqualityAxiom $\stackrel{\text{stmt}}{\to}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \Rightarrow \underline{x} = \underline{z} \Rightarrow \underline{y} = \underline{z}$]

[EqualityAxiom $\stackrel{\text{tex}}{\to}$ "EqualityAxiom"]

[EqualityAxiom $\stackrel{\text{pyk}}{\to}$ "axiom equality"]

EqLeqAxiom

[EqLeqAxiom $\stackrel{\text{proof}}{\to}$ Rule tactic]

[EqLeqAxiom $\stackrel{\text{stmt}}{\to}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} = \underline{y} \Rightarrow \underline{x} <= \underline{y}$]

[EqLeqAxiom $\stackrel{\text{tex}}{\to}$ "EqLeqAxiom"]

[EqLeqAxiom $\stackrel{\text{pyk}}{\to}$ "axiom eqLeq"]

EqAdditionAxiom

[EqAdditionAxiom $\stackrel{\text{proof}}{\to}$ Rule tactic]

[EqAdditionAxiom $\stackrel{\text{stmt}}{\to}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z})$]

[EqAdditionAxiom $\stackrel{\text{tex}}{\to}$ "EqAdditionAxiom"]

[EqAdditionAxiom $\stackrel{\text{pyk}}{\to}$ "axiom eqAddition"]

EqMultiplicationAxiom

[EqMultiplicationAxiom $\stackrel{\text{proof}}{\to}$ Rule tactic]

[EqMultiplicationAxiom $\stackrel{\text{stmt}}{\to}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} = \underline{y} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})$]

[EqMultiplicationAxiom $\stackrel{\text{tex}}{\to}$ "EqMultiplicationAxiom"]

[EqMultiplicationAxiom $\stackrel{\text{pyk}}{\to}$ "axiom eqMultiplication"]

QisClosed(Reciprocal)(Imply)

[QisClosed(Reciprocal)(Imply) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[QisClosed(Reciprocal)(Imply) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \dot{\neg}\, (\underline{x} = 0)\mathrm{n} \Rightarrow \underline{x} \in \mathrm{Q} \Rightarrow \mathrm{rec}\underline{x} \in \mathrm{Q}$]

[QisClosed(Reciprocal)(Imply) $\stackrel{\text{tex}}{\rightarrow}$ "QisClosed(Reciprocal)(Imply)"]

[QisClosed(Reciprocal)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ "axiom QisClosed(reciprocal)"]

QisClosed(Reciprocal)

[QisClosed(Reciprocal) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathrm{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \dot{\neg}\, (\underline{x} = 0)\mathrm{n} \vdash \underline{x} \in \mathrm{Q} \vdash$ QisClosed(Reciprocal)(Imply) $\gg \dot{\neg}\, (\underline{x} = 0)\mathrm{n} \Rightarrow \underline{x} \in \mathrm{Q} \Rightarrow \mathrm{rec}\underline{x} \in \mathrm{Q}; \mathrm{MP2} \rhd \dot{\neg}\, (\underline{x} = 0)\mathrm{n} \Rightarrow \underline{x} \in \mathrm{Q} \Rightarrow \mathrm{rec}\underline{x} \in \mathrm{Q} \rhd \dot{\neg}\, (\underline{x} = 0)\mathrm{n} \rhd \underline{x} \in \mathrm{Q} \gg \mathrm{rec}\underline{x} \in \mathrm{Q}\rceil, \mathrm{p}_0, \mathsf{c})$]

[QisClosed(Reciprocal) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \dot{\neg}\, (\underline{x} = 0)\mathrm{n} \vdash \underline{x} \in \mathrm{Q} \vdash \mathrm{rec}\underline{x} \in \mathrm{Q}$]

[QisClosed(Reciprocal) $\stackrel{\text{tex}}{\rightarrow}$ "QisClosed(Reciprocal)"]

[QisClosed(Reciprocal) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma QisClosed(reciprocal)"]

QisClosed(Negative)(Imply)

[QisClosed(Negative)(Imply) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[QisClosed(Negative)(Imply) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \underline{x} \in \mathrm{Q} \Rightarrow (-\mathrm{u}\underline{x}) \in \mathrm{Q}$]

[QisClosed(Negative)(Imply) $\stackrel{\text{tex}}{\rightarrow}$ "QisClosed(Negative)(Imply)"]

[QisClosed(Negative)(Imply) $\stackrel{\text{pyk}}{\rightarrow}$ "axiom QisClosed(negative)"]

QisClosed(Negative)

[QisClosed(Negative) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathrm{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{x}\colon \underline{x} \in \mathrm{Q} \vdash$ QisClosed(Negative)(Imply) $\gg \underline{x} \in \mathrm{Q} \Rightarrow (-\mathrm{u}\underline{x}) \in \mathrm{Q}; \mathrm{MP} \rhd \underline{x} \in \mathrm{Q} \Rightarrow (-\mathrm{u}\underline{x}) \in \mathrm{Q} \rhd \underline{x} \in \mathrm{Q} \gg (-\mathrm{u}\underline{x}) \in \mathrm{Q}\rceil, \mathrm{p}_0, \mathsf{c})$]

[QisClosed(Negative) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \underline{x} \in \mathrm{Q} \vdash (-\mathrm{u}\underline{x}) \in \mathrm{Q}$]

[QisClosed(Negative) $\stackrel{\text{tex}}{\rightarrow}$ "QisClosed(Negative)"]

[QisClosed(Negative) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma QisClosed(negative)"]

leqReflexivity

[leqReflexivity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[leqReflexivity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \underline{x} <= \underline{x}$]

[leqReflexivity $\stackrel{\text{tex}}{\rightarrow}$ “leqReflexivity”]

[leqReflexivity $\stackrel{\text{pyk}}{\rightarrow}$ “axiom leqReflexivity”]

leqAntisymmetryAxiom

[leqAntisymmetryAxiom $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[leqAntisymmetryAxiom $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \underline{x} = \underline{y}$]

[leqAntisymmetryAxiom $\stackrel{\text{tex}}{\rightarrow}$ “leqAntisymmetryAxiom”]

[leqAntisymmetryAxiom $\stackrel{\text{pyk}}{\rightarrow}$ “axiom leqAntisymmetry”]

leqTransitivityAxiom

[leqTransitivityAxiom $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[leqTransitivityAxiom $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{x} <= \underline{z}$]

[leqTransitivityAxiom $\stackrel{\text{tex}}{\rightarrow}$ “leqTransitivityAxiom”]

[leqTransitivityAxiom $\stackrel{\text{pyk}}{\rightarrow}$ “axiom leqTransitivity”]

leqTotality

[leqTotality $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[leqTotality $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\, (\underline{x} <= \underline{y})\text{n} \Rightarrow \underline{y} <= \underline{x}$]

[leqTotality $\stackrel{\text{tex}}{\rightarrow}$ “leqTotality”]

[leqTotality $\stackrel{\text{pyk}}{\rightarrow}$ “axiom leqTotality”]

leqAdditionAxiom

[leqAdditionAxiom $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic] [leqAdditionAxiom $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon \underline{x} <= \underline{y} \Rightarrow (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z})$]

[leqAdditionAxiom $\stackrel{\text{tex}}{\rightarrow}$ "leqAdditionAxiom"]

[leqAdditionAxiom $\stackrel{\text{pyk}}{\rightarrow}$ "axiom leqAddition"]

leqMultiplicationAxiom

[leqMultiplicationAxiom $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[leqMultiplicationAxiom $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon 0 <= \underline{z} \Rightarrow \underline{x} <= \underline{y} \Rightarrow (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z})$]

[leqMultiplicationAxiom $\stackrel{\text{tex}}{\rightarrow}$ "leqMultiplicationAxiom"]

[leqMultiplicationAxiom $\stackrel{\text{pyk}}{\rightarrow}$ "axiom leqMultiplication"]

plusAssociativity

[plusAssociativity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[plusAssociativity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z}))$]

[plusAssociativity $\stackrel{\text{tex}}{\rightarrow}$ "plusAssociativity"]

[plusAssociativity $\stackrel{\text{pyk}}{\rightarrow}$ "axiom plusAssociativity"]

plusCommutativity

[plusCommutativity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[plusCommutativity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon (\underline{x} + \underline{y}) = (\underline{y} + \underline{x})$]

[plusCommutativity $\stackrel{\text{tex}}{\rightarrow}$ "plusCommutativity"]

[plusCommutativity $\stackrel{\text{pyk}}{\rightarrow}$ "axiom plusCommutativity"]

Negative

[Negative $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Negative $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon (\underline{x} + (-\mathrm{u}\underline{x})) = 0$]

[Negative $\stackrel{\text{tex}}{\rightarrow}$ "Negative"]

[Negative $\stackrel{\text{pyk}}{\rightarrow}$ "axiom negative"]

plus0

[plus0 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[plus0 $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon (\underline{x} + 0) = \underline{x}$]

[plus0 $\stackrel{\text{tex}}{\rightarrow}$ "plus0"]

[plus0 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom plus0"]

timesAssociativity

[timesAssociativity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[timesAssociativity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \forall \underline{z}\colon ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{y} * \underline{z}))$]

[timesAssociativity $\stackrel{\text{tex}}{\rightarrow}$ "timesAssociativity"]

[timesAssociativity $\stackrel{\text{pyk}}{\rightarrow}$ "axiom timesAssociativity"]

timesCommutativity

[timesCommutativity $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[timesCommutativity $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon (\underline{x} * \underline{y}) = (\underline{y} * \underline{x})$]

[timesCommutativity $\stackrel{\text{tex}}{\rightarrow}$ "timesCommutativity"]

[timesCommutativity $\stackrel{\text{pyk}}{\rightarrow}$ "axiom timesCommutativity"]

ReciprocalAxiom

[ReciprocalAxiom $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ReciprocalAxiom $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \dot{\neg} (\underline{x} = 0)\text{n} \Rightarrow (\underline{x} * \text{rec}\underline{x}) = 1$]

[ReciprocalAxiom $\stackrel{\text{tex}}{\rightarrow}$ "ReciprocalAxiom"]

[ReciprocalAxiom $\stackrel{\text{pyk}}{\rightarrow}$ "axiom reciprocal"]

times1

[times1 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[times1 $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}} \colon (\underline{\mathsf{x}} * 1) = \underline{\mathsf{x}}$]

[times1 $\stackrel{\text{tex}}{\rightarrow}$ "times1"]

[times1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom times1"]

Distribution

[Distribution $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Distribution $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \forall \underline{\mathsf{z}} \colon (\underline{\mathsf{x}} * (\underline{\mathsf{y}} + \underline{\mathsf{z}})) = ((\underline{\mathsf{x}} * \underline{\mathsf{y}}) + (\underline{\mathsf{x}} * \underline{\mathsf{z}}))$]

[Distribution $\stackrel{\text{tex}}{\rightarrow}$ "Distribution"]

[Distribution $\stackrel{\text{pyk}}{\rightarrow}$ "axiom distribution"]

0not1

[0not1 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[0not1 $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \dot{\neg}\,(0 = 1)\mathrm{n}$]

[0not1 $\stackrel{\text{tex}}{\rightarrow}$ "0not1"]

[0not1 $\stackrel{\text{pyk}}{\rightarrow}$ "axiom 0not1"]

lemma eqLeq(R)

[lemma eqLeq(R) $\stackrel{\text{proof}}{\rightarrow}$ $\lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] +$
$(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \mathrm{WeakenOr1} \rhd \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})n)n \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$

$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}; \mathrm{Repetition} \triangleright \dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <=$
$(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}], p_0, \mathsf{c})]$

$[\text{lemma eqLeq}(R) \overset{stmt}{\rightarrow} SystemQ \vdash \forall\underline{(fx)}\colon \forall\underline{(fy)}\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= (\underline{(fy)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$

$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{-}(\dot{-}(|((\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[lemma eqLeq(R) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma eqLeq(R)”]

TimesAssociativity(R)

[TimesAssociativity(R) $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[TimesAssociativity(R) $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \forall \underline{(\mathrm{fz})} \colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{-}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{-}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{-}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{-}(0 <= \overline{(\epsilon)} \Rightarrow \dot{-}(\dot{-}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{-}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{-}(\dot{-}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{-}(\dot{-}(\dot{-}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{-}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{-}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{-}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{-}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$

$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}]*\underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]*\underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \underline{(fy)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in$
$\mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fy)}[\underline{\mathsf{m}}] * \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\}]$

[TimesAssociativity(R) $\stackrel{\text{tex}}{\rightarrow}$ "TimesAssociativity(R)"]

[TimesAssociativity(R) $\stackrel{\text{pyk}}{\rightarrow}$ "lemma timesAssociativity(R)"]

TimesCommutativity(R)

[TimesCommutativity(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

$$[\mathrm{TimesCommutativity(R)} \overset{\mathrm{stmt}}{\to} \mathrm{SystemQ} \vdash \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fx})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fy})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(\mathrm{fy})}[\underline{\mathsf{m}}] * \underline{(\mathrm{fx})}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$$

[TimesCommutativity(R) $\overset{\mathrm{tex}}{\to}$ "TimesCommutativity(R)"]

[TimesCommutativity(R) $\overset{\mathrm{pyk}}{\to}$ "lemma timesCommutativity(R)"]

(Adgic)SameR

[(Adgic)SameR $\overset{\mathrm{proof}}{\to}$ Rule tactic]

[(Adgic)SameR $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\mathrm{fx})} \colon \forall \underline{(\mathrm{fy})} \colon \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \vdash \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})} \colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})} \colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})} \colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})} \colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\,(\dot{\neg}\,(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}$]

[(Adgic)SameR $\overset{\text{tex}}{\rightarrow}$ "(Adgic)SameR"]

[(Adgic)SameR $\overset{\text{pyk}}{\rightarrow}$ "1rule adhoc sameR"]

Separation2formula(1)

[Separation2formula(1) $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Separation2formula(1) $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{y}} \in \{\mathrm{ph} \in \underline{\mathsf{x}} \mid \underline{\mathsf{a}}\} \vdash \underline{\mathsf{y}} \in \underline{\mathsf{x}}$]

[Separation2formula(1) $\overset{\text{tex}}{\rightarrow}$ "Separation2formula(1)"]

[Separation2formula(1) $\overset{\text{pyk}}{\rightarrow}$ "lemma separation2formula(1)"]

Separation2formula(2)

[Separation2formula(2) $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Separation2formula(2) $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{a}} \colon \forall \underline{\mathsf{b}} \colon \forall \underline{\mathsf{x}} \colon \forall \underline{\mathsf{y}} \colon \underline{\mathsf{y}} \in \{\mathrm{ph} \in \underline{\mathsf{x}} \mid \underline{\mathsf{a}}\} \vdash \underline{\mathsf{b}}$]

[Separation2formula(2) $\overset{\text{tex}}{\rightarrow}$ "Separation2formula(2)"]

[Separation2formula(2) $\overset{\text{pyk}}{\rightarrow}$ "lemma separation2formula(2)"]

Cauchy

[Cauchy $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Cauchy $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall \underline{(v1)}: \forall \underline{(v2)}: \forall \underline{\mathsf{n}}: \forall \underline{(\epsilon)}: \forall \underline{(fx)}: \forall_{obj} \underline{(\epsilon)}: \dot{\neg} (\forall_{obj} \underline{\mathsf{n}}: \dot{\neg} (\forall_{obj} \underline{(v1)}: \forall_{obj} \underline{(v2)}: \dot{\neg} (0 <= \underline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \underline{(\epsilon)}) n) n) n \Rightarrow \underline{\mathsf{n}} <= \underline{(v1)} \Rightarrow \underline{\mathsf{n}} <= \underline{(v2)} \Rightarrow \dot{\neg} (|(\underline{(fx)}[\underline{(v1)}] + (-u \underline{(fx)}[\underline{(v2)}]))| <= \underline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (|(\underline{(fx)}[\underline{(v1)}] + (-u \underline{(fx)}[\underline{(v2)}]))| = \underline{(\epsilon)}) n) n) n) n) n$]

[Cauchy $\overset{\text{tex}}{\rightarrow}$ "Cauchy"]

[Cauchy $\overset{\text{pyk}}{\rightarrow}$ "axiom cauchy"]

PlusF

[PlusF $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[PlusF $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall \underline{\mathsf{m}}: \forall \underline{(fx)}: \forall \underline{(fy)}: \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg} (\forall_{obj} \underline{(op1)}: \dot{\neg} (\dot{\neg} (\forall_{obj} \underline{(op2)}: \dot{\neg} (\dot{\neg} (\dot{\neg} (\underline{(op1)} \in N \Rightarrow \dot{\neg} (\underline{(op2)} \in Q) n) n \Rightarrow \dot{\neg} (\mathsf{a}_{Ph} = \{\{\underline{(op1)}, \underline{(op1)}\}, \{\underline{(op1)}, \underline{(op2)}\}\}) n) n) n) n) n) n\} \mid \dot{\neg} (\forall_{obj} \underline{\mathsf{m}}: \dot{\neg} (\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\underline{(fx)}[\underline{\mathsf{m}}] + \underline{(fy)}[\underline{\mathsf{m}}])\}\}) n) n\}[\underline{\mathsf{m}}] = (\underline{(fx)}[\underline{\mathsf{m}}] + \underline{(fy)}[\underline{\mathsf{m}}])$]

[PlusF $\overset{\text{tex}}{\rightarrow}$ "PlusF"]

[PlusF $\overset{\text{pyk}}{\rightarrow}$ "axiom plusF"]

ReciprocalF

[ReciprocalF $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[ReciprocalF $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$ $\forall \underline{\mathsf{m}}: \forall \underline{(fx)}: \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg} (\forall_{obj} \underline{(op1)}: \dot{\neg} (\dot{\neg} (\forall_{obj} \underline{(op2)}: \dot{\neg} (\dot{\neg} (\dot{\neg} (\underline{(op1)} \in N \Rightarrow \dot{\neg} (\underline{(op2)} \in Q) n) n \Rightarrow \dot{\neg} (\mathsf{a}_{Ph} = \{\{\underline{(op1)}, \underline{(op1)}\}, \{\underline{(op1)}, \underline{(op2)}\}\}) n) n) n) n) n) n\} \mid \dot{\neg} (\forall_{obj} \underline{\mathsf{m}}: \dot{\neg} (\dot{\neg} (\dot{\neg} (\dot{\neg} (\underline{(fx)}[\underline{\mathsf{m}}] = 0) n \Rightarrow \dot{\neg} (\mathsf{f}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec \underline{(fx)}[\underline{\mathsf{m}}]\}\}) n) n) n \Rightarrow \dot{\neg} (\underline{(fx)}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg} (\mathsf{f}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\}) n) n) n) n\}[\underline{\mathsf{m}}] = if(\underline{(fx)}[\underline{\mathsf{m}}] = 0, 0, rec \underline{(fx)}[\underline{\mathsf{m}}])$]

[ReciprocalF $\overset{\text{tex}}{\rightarrow}$ "ReciprocalF"]

[ReciprocalF $\overset{\text{pyk}}{\rightarrow}$ "axiom reciprocalF"]

From ==

[From $==\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[From $==\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj} \overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(f1)}\colon \forall_{obj} \overline{(f2)}\colon \forall_{obj} \overline{(f3)}\colon \forall_{obj} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj} \overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj} \overline{(\epsilon)}\colon \dot\neg(\forall_{obj} \overline{\mathsf{n}}\colon \dot\neg(\forall_{obj} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)}))n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj} \overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(f1)}\colon \forall_{obj} \overline{(f2)}\colon \forall_{obj} \overline{(f3)}\colon \forall_{obj} \overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj} \overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj} \overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj} \overline{(\epsilon)}\colon \dot\neg(\forall_{obj} \overline{\mathsf{n}}\colon \dot\neg(\forall_{obj} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\underline{(fy)}[\overline{\mathsf{m}}] +$
$(-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)}))n)n)n)n)n\} \vdash \forall_{obj} \overline{(\epsilon)}\colon \dot\neg(\forall_{obj} \overline{\mathsf{n}}\colon \dot\neg(\forall_{obj} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)}))n)n)n)n)n$]

[From $==\stackrel{\text{tex}}{\rightarrow}$ "From=="]

[From $==\stackrel{\text{pyk}}{\rightarrow}$ "1rule from=="]

To ==

[To $==\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[To $==\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \forall_{obj} \overline{(\epsilon)}\colon \dot\neg(\forall_{obj} \overline{\mathsf{n}}\colon \dot\neg(\forall_{obj} \overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-u\underline{(fy)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)}))n)n)n)n)n \vdash \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\mathrm{Union}(\{N, Q\}))) \mid \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj} \overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj} \overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj} \overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[To $\overset{\mathrm{tex}}{\rightarrow}$ “To==”]

[To $\overset{\mathrm{pyk}}{\rightarrow}$ “1rule to==”]

FromInR

[FromInR $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[FromInR $\overset{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \underline{(\mathrm{fx})} \in \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] +$
$(-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \vdash \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fy})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[FromInR $\overset{\mathrm{tex}}{\rightarrow}$ “FromInR”]

[FromInR $\overset{\mathrm{pyk}}{\rightarrow}$ “1rule fromInR”]

PlusR(Sym)

[PlusR(Sym) $\stackrel{\text{tex}}{\rightarrow}$ “PlusR(Sym)”]

[PlusR(Sym) $\stackrel{\text{pyk}}{\rightarrow}$ “lemma plusR(Sym)”]

ReciprocalR(Axiom)

[ReciprocalR(Axiom) $\stackrel{\text{tex}}{\rightarrow}$ “ReciprocalR(Axiom)”]

[ReciprocalR(Axiom) $\stackrel{\text{pyk}}{\rightarrow}$ “axiom reciprocalR”]

LessMinus1(N)

[LessMinus1(N) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[LessMinus1(N) $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}} \colon \forall \underline{\mathsf{n}} \colon \mathrm{Nat}(\underline{\mathsf{m}}) \Vdash \mathrm{Nat}(\underline{\mathsf{n}}) \Vdash \dot{\neg}\,(\underline{\mathsf{m}} <= (\underline{\mathsf{n}} + 1) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{m}} = (\underline{\mathsf{n}} + 1))\mathrm{n})\mathrm{n})\mathrm{n} \vdash \underline{\mathsf{m}} <= \underline{\mathsf{n}}$]

[LessMinus1(N) $\stackrel{\text{tex}}{\rightarrow}$ “LessMinus1(N)”]

[LessMinus1(N) $\stackrel{\text{pyk}}{\rightarrow}$ “1rule lessMinus1(N)”]

Nonnegative(N)

[Nonnegative(N) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Nonnegative(N) $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}} \colon \mathrm{Nat}(\underline{\mathsf{m}}) \Vdash 0 <= \underline{\mathsf{m}}$]

[Nonnegative(N) $\stackrel{\text{tex}}{\rightarrow}$ “Nonnegative(N)”]

[Nonnegative(N) $\stackrel{\text{pyk}}{\rightarrow}$ “axiom nonnegative(N)”]

US0

[US0 $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[US0 $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \mathrm{us}[0] = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})} \colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})} \colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})} \colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

[US0 $\overset{\text{tex}}{\to}$ "US0"]

[US0 $\overset{\text{pyk}}{\to}$ "axiom US0"]

NextXS(UpperBound)

[NextXS(UpperBound) $\overset{\text{proof}}{\to}$ Rule tactic]

[NextXS(UpperBound) $\overset{\text{stmt}}{\to}$ SystemQ $\vdash \forall\underline{\mathsf{m}}: UB(\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$

$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}, \mathrm{SetOfReals}) \vdash \mathrm{xs}[(\underline{\mathsf{m}} + 1)] = \mathrm{xs}[\underline{\mathsf{m}}]]$

[NextXS(UpperBound) $\stackrel{\mathrm{tex}}{\rightarrow}$ “NextXS(UpperBound)”]

[NextXS(UpperBound) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “1rule nextXS(upperBound)”]

NextXS(NoUpperBound)

[NextXS(NoUpperBound) $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[NextXS(NoUpperBound) $\stackrel{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall\underline{\mathsf{m}}\colon \dot{\neg}(\mathrm{UB}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}: \dot\neg(\forall_{obj}\overline{\mathsf{n}}: \dot\neg(\forall_{obj}\overline{\mathsf{m}}: \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)}))n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\dot\neg(\dot\neg(\dot\neg(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, rec\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{Ph} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}: \dot\neg(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}: \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{crs1})\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{(\mathrm{crs1}), (\mathrm{crs1})\}, \{(\mathrm{crs1}), 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow \dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}, \mathrm{SetOfReals}))\mathrm{n} \vdash \mathrm{xs}[(\underline{\mathsf{m}} + 1)] = \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$
$\dot{\neg}((\mathrm{op2}) \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(\mathrm{op1}), (\mathrm{op1})\}, \{(\mathrm{op1}), (\mathrm{op2})\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{r1})\colon (\mathrm{r1}) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op1})\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(\mathrm{op2})\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\mathrm{op1}) \in \mathrm{N} \Rightarrow$

$\dot\neg\,((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\dot\neg\,(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg\,(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$

$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}]$

$[\mathrm{NextXS(NoUpperBound)} \stackrel{\mathrm{tex}}{\to} \text{“NextXS(NoUpperBound)”}]$

[NextXS(NoUpperBound) $\stackrel{\text{pyk}}{\rightarrow}$ “1rule nextXS(noUpperBound)”]

NextUS(UpperBound)

[NextUS(UpperBound) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[NextUS(UpperBound) $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{m}\colon \text{UB}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in$
$\text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow$
$\dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\}) \mid$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{r1})\colon (\text{r1}) \in \mathsf{f}_{\text{Ph}} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow$
$\dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow \dot{\neg}\,((\text{r1}) = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{f1})\colon \forall_{\text{obj}} (\text{f2})\colon \forall_{\text{obj}} (\text{f3})\colon \forall_{\text{obj}} (\text{f4})\colon \{\{(\text{f1}), (\text{f1})\}, \{(\text{f1}), (\text{f2})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow$
$\{\{(\text{f3}), (\text{f3})\}, \{(\text{f3}), (\text{f4})\}\} \in \mathsf{f}_{\text{Ph}} \Rightarrow (\text{f1}) = (\text{f3}) \Rightarrow (\text{f2}) = (\text{f4}))\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{s1})\colon (\text{s1}) \in \text{N} \Rightarrow \dot{\neg}\,(\forall_{\text{obj}} (\text{s2})\colon \dot{\neg}\,(\{\{(\text{s1}), (\text{s1})\}, \{(\text{s1}), (\text{s2})\}\} \in$
$\mathsf{f}_{\text{Ph}})\text{n})\text{n})\text{n})\text{n}\}) \mid \forall_{\text{obj}} (\epsilon)\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= (\epsilon) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = (\epsilon))\text{n})\text{n})\text{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} \underline{m}\colon \dot{\neg}\,(\mathsf{e}_{\text{Ph}} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid$
$\dot{\neg}\,(\forall_{\text{obj}} \underline{m}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} \underline{m}\colon \dot{\neg}\,(\mathsf{d}_{\text{Ph}} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} (\text{crs1})\colon \dot{\neg}\,(\mathsf{c}_{\text{Ph}} =$
$\{\{(\text{crs1}), (\text{crs1})\}, \{(\text{crs1}), 1\}\})\text{n})\text{n}\}[\underline{m}] + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} (\text{crs1})\colon \dot{\neg}\,(\mathsf{c}_{\text{Ph}} =$
$\{\{(\text{crs1}), (\text{crs1})\}, \{(\text{crs1}), 1\}\})\text{n})\text{n}\}[\underline{m}])\}\})\text{n})\text{n}\}[\underline{m}] = 0)\text{n} \Rightarrow \dot{\neg}\,(\mathsf{f}_{\text{Ph}} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, \text{rec}\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} \underline{m}\colon \dot{\neg}\,(\mathsf{d}_{\text{Ph}} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} (\text{crs1})\colon \dot{\neg}\,(\mathsf{c}_{\text{Ph}} =$
$\{\{(\text{crs1}), (\text{crs1})\}, \{(\text{crs1}), 1\}\})\text{n})\text{n}\}[\underline{m}] + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid$
$\dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow \dot{\neg}\,((\text{op2}) \in \text{Q})\text{n})\text{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\text{Ph}} = \{\{(\text{op1}), (\text{op1})\}, \{(\text{op1}), (\text{op2})\}\})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n}\} \mid \dot{\neg}\,(\forall_{\text{obj}} (\text{crs1})\colon \dot{\neg}\,(\mathsf{c}_{\text{Ph}} =$
$\{\{(\text{crs1}), (\text{crs1})\}, \{(\text{crs1}), 1\}\})\text{n})\text{n}\}[\underline{m}])\}\})\text{n})\text{n}\}[\underline{m}]\}\})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\{\text{ph} \in \{\text{ph} \in$
$\text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot{\neg}\,(\forall_{\text{obj}} (\text{op1})\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} (\text{op2})\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,((\text{op1}) \in \text{N} \Rightarrow$

$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}(crs1)\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},0\}\})n)n)n)n\}[\underline{\mathsf{m}}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\mathsf{x}[\underline{\mathsf{m}}]+\mathsf{y}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}, SetOfReals) \vdash us[(\underline{\mathsf{m}} + 1)] = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow$
$\dot\neg((op2)\in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}(r1)\colon (r1)\in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N\Rightarrow$
$\dot\neg((op2)\in Q)n)n \Rightarrow \dot\neg((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}(f1)\colon\forall_{obj}(f2)\colon\forall_{obj}(f3)\colon\forall_{obj}(f4)\colon \{\{(f1),(f1)\},\{(f1),(f2)\}\}\in \mathsf{f}_{Ph}\Rightarrow$
$\{\{(f3),(f3)\},\{(f3),(f4)\}\}\in\mathsf{f}_{Ph}\Rightarrow (f1)=(f3)\Rightarrow(f2)=(f4))n)n\Rightarrow$
$\dot\neg(\forall_{obj}(s1)\colon (s1)\in N\Rightarrow\dot\neg(\forall_{obj}(s2)\colon\dot\neg(\{\{(s1),(s1)\},\{(s1),(s2)\}\}\in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}(\epsilon)\colon\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon\dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon\dot\neg(0 <= (\epsilon)\Rightarrow\dot\neg(\dot\neg(0=(\epsilon))n)n\Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph\in\{ph\in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon\dot\neg(\mathsf{e}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph\in\{ph\in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon\dot\neg(\dot\neg(\dot\neg(\dot\neg(\{ph\in\{ph\in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph\in\{ph\in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}(crs1)\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}(crs1)\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}] = 0)n \Rightarrow \dot\neg(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}}, rec\{ph\in\{ph\in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{Ph} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\{ph\in\{ph\in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}(crs1)\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{\mathsf{m}}] + \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid$
$\dot\neg(\forall_{obj}(op1)\colon \dot\neg(\dot\neg(\forall_{obj}(op2)\colon \dot\neg(\dot\neg(\dot\neg((op1)\in N \Rightarrow \dot\neg((op2)\in Q)n)n \Rightarrow$
$\dot\neg(\mathsf{a}_{Ph} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}(crs1)\colon \dot\neg(\mathsf{c}_{Ph} =$
$\{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{\mathsf{m}}])\}\})n)n\}[\underline{\mathsf{m}}]\}\})n)n)n \Rightarrow \dot\neg(\{ph\in\{ph\in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(crs1)\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(crs1)\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},0\}\})n)n)n)n\}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\mathsf{x}[\underline{m}] + \mathsf{y}[\underline{m}])\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{m}] + (-u\mathsf{d}_{\mathrm{Ph}}[\overline{m}]))| = (\epsilon))n)n)n)n)n\}]$

[NextUS(UpperBound) $\overset{\mathrm{tex}}{\to}$ "NextUS(UpperBound)"]

[NextUS(UpperBound) $\overset{\mathrm{pyk}}{\to}$ "1rule nextUS(upperBound)"]

NextUS(NoUpperBound)

[NextUS(NoUpperBound) $\overset{\mathrm{proof}}{\to}$ Rule tactic]

[NextUS(NoUpperBound) $\overset{\mathrm{stmt}}{\to}$ $SystemQ \vdash \forall \underline{m}\colon \dot{\neg}(UB(\{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(r1)\colon (r1) \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}((r1) = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(f1)\colon \forall_{\mathrm{obj}}(f2)\colon \forall_{\mathrm{obj}}(f3)\colon \forall_{\mathrm{obj}}(f4)\colon \{\{(f1),(f1)\},\{(f1),(f2)\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{(f3),(f3)\},\{(f3),(f4)\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow (f1) = (f3) \Rightarrow (f2) = (f4))n)n \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(s1)\colon (s1) \in N \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}(s2)\colon \dot{\neg}(\{\{(s1),(s1)\},\{(s1),(s2)\}\} \in \mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}(\epsilon)\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{n}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{m}\colon \dot{\neg}(0 <= (\epsilon) \Rightarrow \dot{\neg}(\dot{\neg}(0 = (\epsilon))n)n)n \Rightarrow \overline{n} <= \overline{m} \Rightarrow \dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{m}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{m},\underline{m}\},\{\underline{m},(\{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(crs1)\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} = \{\{(crs1),(crs1)\},\{(crs1),1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N,Q\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}(op1)\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}(op2)\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((op1) \in N \Rightarrow \dot{\neg}((op2) \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{(op1),(op1)\},\{(op1),(op2)\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{\mathrm{obj}}(crs1)\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] * \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot\neg(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot\neg(\mathsf{d}_{\mathrm{Ph}} =$

$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}]\}\})n)n)n \Rightarrow \dot{\neg}(\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\mathsf{d}_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}] + \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\underline{m}])\}\})n)n\}[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n)n\}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{m}: \dot{\neg}(\mathsf{d}_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\mathsf{x}[\underline{m}] + \mathsf{y}[\underline{m}])\}\})n)n\}[\underline{m}])\}\})n)n\}[\overline{\mathsf{m}}] + (-u\mathsf{d}_{Ph}[\overline{\mathsf{m}}]))| =$
$(\epsilon))n)n)n)n)n\}, SetOfReals))n \vdash us[(\underline{m} + 1)] = us[\underline{m}]]$

[NextUS(NoUpperBound) $\overset{tex}{\to}$ "NextUS(NoUpperBound)"]

[NextUS(NoUpperBound) $\overset{pyk}{\to}$ "1rule nextUS(noUpperBound)"]

ExpZero

[ExpZero $\overset{proof}{\to}$ Rule tactic]

[ExpZero $\overset{stmt}{\to}$ SystemQ $\vdash \forall\underline{m}: \forall\underline{x}: \underline{m} = 0 \vdash \underline{x}(exp)\underline{m} = 1$]

[ExpZero $\overset{tex}{\to}$ "ExpZero"]

[ExpZero $\overset{pyk}{\to}$ "1rule expZero"]

ExpPositive

[ExpPositive $\overset{proof}{\to}$ Rule tactic]

[ExpPositive $\overset{stmt}{\to}$ SystemQ $\vdash \forall\underline{m}: \forall\underline{x}: \dot{\neg}(0 <= \underline{m} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \underline{m})n)n)n \vdash$
$\underline{x}(exp)\underline{m} = (\underline{x} * \underline{x}(exp)(\underline{m} + (-u1)))$]

[ExpPositive $\overset{tex}{\to}$ "ExpPositive"]

[ExpPositive $\stackrel{\text{pyk}}{\rightarrow}$ "1rule expPositive"]

ExpZero(R)

[ExpZero(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ExpZero(R) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}} \colon \forall \underline{(fx)} \colon \underline{\mathsf{m}} = 0 \vdash \underline{(fx)}(exp)\underline{\mathsf{m}} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid \dot\neg(\forall_{obj}\overline{(op1)} \colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)} \colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)} \colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)} \colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)} \colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)} \colon \forall_{obj}\overline{(f2)} \colon \forall_{obj}\overline{(f3)} \colon \forall_{obj}\overline{(f4)} \colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)} \colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)} \colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)} \colon \dot\neg(\forall_{obj}\overline{\mathsf{n}} \colon \dot\neg(\forall_{obj}\overline{\mathsf{m}} \colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)} \colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)} \colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)} \colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)} \colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)} \colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)} \colon \dot\neg(\mathsf{c}_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}$]

[ExpZero(R) $\stackrel{\text{tex}}{\rightarrow}$ "ExpZero(R)"]

[ExpZero(R) $\stackrel{\text{pyk}}{\rightarrow}$ "1rule expZero(R)"]

ExpPositive(R)

[ExpPositive(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ExpPositive(R) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}} \colon \forall \underline{(fx)} \colon \dot\neg(0 <= \underline{\mathsf{m}} \Rightarrow \dot\neg(\dot\neg(0 = \underline{\mathsf{m}})n)n)n \vdash \underline{(fx)}(exp)\underline{\mathsf{m}} = \{ph \in P(\{ph \in P(\{ph \in P(P(Union(\{N, Q\})))) \mid \dot\neg(\forall_{obj}\overline{(op1)} \colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)} \colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)} \colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)} \colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)} \colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)} \colon \forall_{obj}\overline{(f2)} \colon \forall_{obj}\overline{(f3)} \colon \forall_{obj}\overline{(f4)} \colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)} \colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)} \colon \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$

$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] * \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}\,(\mathsf{e}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] * \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

[ExpPositive(R) $\overset{\text{tex}}{\to}$ “ExpPositive(R)”]

[ExpPositive(R) $\overset{\text{pyk}}{\to}$ “1rule expPositive(R)”]

BSzero

[BSzero $\overset{\text{proof}}{\to}$ Rule tactic]

[BSzero $\overset{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \underline{\mathsf{n}} = 0 \vdash \mathrm{BS}(\underline{\mathsf{m}}, \underline{\mathsf{n}}) = \mathrm{rec}(1+1)(\exp)\underline{\mathsf{m}}$]

[BSzero $\overset{\text{tex}}{\to}$ “BSzero”]

[BSzero $\overset{\text{pyk}}{\to}$ “1rule base(1/2)Sum zero”]

BSpositive

[BSpositive $\overset{\text{proof}}{\to}$ Rule tactic]

[BSpositive $\overset{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \dot{\neg}\,(0 <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathsf{n}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{BS}(\underline{\mathsf{m}}, \underline{\mathsf{n}}) = (\mathrm{rec}(1+1)(\exp)(\underline{\mathsf{m}} + \underline{\mathsf{n}}) + \mathrm{BS}(\underline{\mathsf{m}}, (\underline{\mathsf{n}} + (-\mathrm{u}1))))$]

[BSpositive $\overset{\text{tex}}{\to}$ “BSpositive”]

[BSpositive $\overset{\text{pyk}}{\to}$ “1rule base(1/2)Sum positive”]

UStelescope(Zero)

[UStelescope(Zero) $\overset{\text{proof}}{\to}$ Rule tactic]

[UStelescope(Zero) $\overset{\text{stmt}}{\to}$ $\mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{n}}\colon \underline{\mathsf{n}} = 0 \vdash \mathrm{UStelescope}(\underline{\mathsf{m}}, \underline{\mathsf{n}}) = |(\mathrm{us}[\underline{\mathsf{m}}] + (-\mathrm{uus}[(\underline{\mathsf{m}} + 1)]))|$]

[UStelescope(Zero) $\overset{\text{tex}}{\to}$ “UStelescope(Zero)”]

[UStelescope(Zero) $\overset{\text{pyk}}{\to}$ “1rule UStelescope zero”]

UStelescope(Positive)

[UStelescope(Positive) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[UStelescope(Positive) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{m}\colon \forall \underline{n}\colon \dot{\neg}(0 <= \underline{n} \Rightarrow \dot{\neg}(\dot{\neg}(0 =$
$\underline{n})n)n)n \vdash \text{UStelescope}(\underline{m}, \underline{n}) =$
$(|(\text{us}[(\underline{m} + \underline{n})] + (-\text{uus}[(\underline{m} + (\underline{n} + 1))]))| + \text{UStelescope}(\underline{m}, (\underline{n} + (-\text{u}1))))$]

[UStelescope(Positive) $\stackrel{\text{tex}}{\rightarrow}$ “UStelescope(Positive)”]

[UStelescope(Positive) $\stackrel{\text{pyk}}{\rightarrow}$ “1rule UStelescope positive”]

EqAddition(R)

[EqAddition(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[EqAddition(R) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(fx)}\colon \forall \underline{(fy)}\colon \forall \underline{(fz)}\colon \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fx)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}\colon \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(|(\underline{(fy)}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} \vdash \{ph \in P(\{ph \in P(\{ph \in$
$P(P(\text{Union}(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}\colon \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$

$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fx)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} =$
$\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(r1)}\colon \overline{(r1)} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(f1)}\colon \forall_{\mathrm{obj}}\overline{(f2)}\colon \forall_{\mathrm{obj}}\overline{(f3)}\colon \forall_{\mathrm{obj}}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(s1)}\colon \overline{(s1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\forall_{\mathrm{obj}}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fy)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|(\{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},(\underline{(fy)}[\underline{\mathsf{m}}] + \underline{(fz)}[\underline{\mathsf{m}}])\}\})n)n\}[\overline{\mathsf{m}}] + (-\mathrm{u}\mathsf{d}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\}]$

[EqAddition(R) $\stackrel{\mathrm{tex}}{\rightarrow}$ "EqAddition(R)"]

[EqAddition(R) $\stackrel{\mathrm{pyk}}{\rightarrow}$ "1rule adhoc eqAddition(R)"]

FromLimit

[FromLimit $\stackrel{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[FromLimit $\stackrel{\mathrm{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{m}}\colon \forall\underline{\mathsf{n}}\colon \forall\underline{(fep)}\colon \forall\underline{(fx)}\colon \forall\underline{(fys)}\colon \mathrm{Limit}(\underline{(fx)},\underline{(fys)}) \vdash$
$\forall_{\mathrm{obj}}\underline{(fep)}\colon \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\underline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid$
$\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(op2)} \in \mathrm{Q})n)n \Rightarrow$
$\dot\neg\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(crs1)}\colon \dot\neg\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(crs1)},\overline{(crs1)}\},\{\overline{(crs1)},0\}\})n)n\}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))n)n)n)n)n)n \Rightarrow$

$\underline{\mathsf{m}} <= \underline{\mathsf{n}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow |\mathrm{f}\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{u}\mathsf{x}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}|[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[FromLimit $\overset{\mathrm{tex}}{\rightarrow}$ "FromLimit"]

[FromLimit $\overset{\mathrm{pyk}}{\rightarrow}$ "1rule fromLimit"]

ToUpperBound

[ToUpperBound $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[ToUpperBound $\overset{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fxs})}\colon \underline{(\mathrm{fx})} \in \underline{(\mathrm{fxs})} \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})} \vdash \mathrm{UB}(\underline{(\mathrm{fy})}, \underline{(\mathrm{fxs})})$]

[ToUpperBound $\overset{\mathrm{tex}}{\rightarrow}$ "ToUpperBound"]

[ToUpperBound $\overset{\mathrm{pyk}}{\rightarrow}$ "1rule toUpperBound"]

FromUpperBound

[FromUpperBound $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[FromUpperBound $\overset{\mathrm{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \forall\underline{(\mathrm{fxs})}\colon \mathrm{UB}(\underline{(\mathrm{fy})}, \underline{(\mathrm{fxs})}) \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{fxs})} \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{fx})} = \underline{(\mathrm{fy})}$]

[FromUpperBound $\overset{\mathrm{tex}}{\rightarrow}$ "FromUpperBound"]

[FromUpperBound $\overset{\mathrm{pyk}}{\rightarrow}$ "1rule fromUpperBound"]

USisUpperBound

[USisUpperBound $\overset{\mathrm{proof}}{\rightarrow}$ Rule tactic]

[USisUpperBound $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}}\colon \mathrm{UB}(\mathrm{us}[\underline{\mathsf{m}}], \mathrm{SetOfFxs})$]

[USisUpperBound $\stackrel{\text{tex}}{\rightarrow}$ “USisUpperBound”]

[USisUpperBound $\stackrel{\text{pyk}}{\rightarrow}$ “axiom USisUpperBound”]

0not1(R)

[0not1(R) $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[0not1(R) $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \dot{\neg}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} =$
$\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\ |\ \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{crs1})}\colon \dot{\neg}(\mathsf{c}_{\mathrm{Ph}} =$

$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 1\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\})\mathrm{n}]$

[0not1(R) $\overset{\text{tex}}{\rightarrow}$ "0not1(R)"]

[0not1(R) $\overset{\text{pyk}}{\rightarrow}$ "axiom 0not1(R)"]

ExpUnbounded(R)

[ExpUnbounded(R) $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[ExpUnbounded(R) $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{\mathsf{m}} \colon \forall \underline{(\mathrm{fx})} \colon \dot{\neg}\, (\forall_{\mathrm{obj}} \underline{\mathsf{m}} \colon \dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\, (\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\, (\dot{\neg}\, (0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}\, (\dot{\neg}\, (0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\, (\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}]$

[ExpUnbounded(R) $\overset{\text{tex}}{\rightarrow}$ "ExpUnbounded(R)"]

[ExpUnbounded(R) $\overset{\text{pyk}}{\rightarrow}$ "1rule expUnbounded"]

FromLeq(Advanced)(N)

[FromLeq(Advanced)(N) $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[FromLeq(Advanced)(N) $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}} \colon \forall \underline{(\mathrm{m1})} \colon \forall \underline{\mathsf{n}} \colon \underline{\mathsf{m}} <= \underline{\mathsf{n}} \vdash$
$\dot{\neg}\, (\forall_{\mathrm{obj}} \underline{(\mathrm{m1})} \colon \dot{\neg}\, ((\underline{\mathsf{m}} + \underline{(\mathrm{m1})}) = \underline{\mathsf{n}})\mathrm{n})\mathrm{n}]$

[FromLeq(Advanced)(N) $\overset{\text{tex}}{\rightarrow}$ "FromLeq(Advanced)(N)"]

[FromLeq(Advanced)(N) $\overset{\text{pyk}}{\rightarrow}$ "1rule fromLeq(Advanced)(N)"]

FromLeastUpperBound

[FromLeastUpperBound $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[FromLeastUpperBound $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \mathrm{LUB}(\underline{(\mathrm{fx})}, \underline{(\mathrm{fys})}) \vdash$
$\dot{\neg}\, (\mathrm{UB}(\underline{(\mathrm{fx})}, \underline{(\mathrm{fys})}) \Rightarrow \dot{\neg}\, (\mathrm{UB}(\underline{(\mathrm{fz})}, \underline{(\mathrm{fys})}) \Rightarrow$
$\dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} \overline{(\epsilon)} \colon \dot{\neg}\, (\dot{\neg}\, (\forall_{\mathrm{obj}} \overline{\mathsf{n}} \colon \dot{\neg}\, (\forall_{\mathrm{obj}} \overline{\mathsf{m}} \colon \dot{\neg}\, (\dot{\neg}\, (0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\, (\dot{\neg}\, (0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\, (\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \underline{(\mathrm{fx})} = \underline{(\mathrm{fz})})\mathrm{n})\mathrm{n}]$

[FromLeastUpperBound $\overset{\text{tex}}{\rightarrow}$ "FromLeastUpperBound"]

[FromLeastUpperBound $\overset{\text{pyk}}{\rightarrow}$ "1rule fromLeastUpperBound"]

ToLeastUpperBound

[ToLeastUpperBound $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ToLeastUpperBound $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\text{fx})}\colon \forall \underline{(\text{fz})}\colon \forall \underline{(\text{fys})}\colon \text{UB}(\underline{(\text{fx})}, \underline{(\text{fys})}) \vdash \text{UB}(\underline{(\text{fz})}, \underline{(\text{fys})}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\text{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\text{n})\text{n})\text{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\text{u}\overline{(\epsilon)})))\text{n})\text{n})\text{n})\text{n})\text{n})\text{n})\text{n} \Rightarrow \underline{(\text{fx})} = \underline{(\text{fz})} \vdash \text{LUB}(\underline{(\text{fx})}, \underline{(\text{fys})})$]

[ToLeastUpperBound $\stackrel{\text{tex}}{\rightarrow}$ “ToLeastUpperBound”]

[ToLeastUpperBound $\stackrel{\text{pyk}}{\rightarrow}$ “1rule toLeastUpperBound”]

XSisNotUpperBound

[XSisNotUpperBound $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[XSisNotUpperBound $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}}\colon \dot{\neg}\,(\text{UB}(\text{xs}[\underline{\mathsf{m}}], \text{SetOfFxs}))\text{n}$]

[XSisNotUpperBound $\stackrel{\text{tex}}{\rightarrow}$ “XSisNotUpperBound”]

[XSisNotUpperBound $\stackrel{\text{pyk}}{\rightarrow}$ “axiom XSisNotUpperBound”]

ysFGreater

[ysFGreater $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ysFGreater $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}}\colon \dot{\neg}\,(\text{xs}[\underline{\mathsf{m}}] <= \text{ysF}[\underline{\mathsf{m}}] \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\text{xs}[\underline{\mathsf{m}}] = \text{ysF}[\underline{\mathsf{m}}])\text{n})\text{n})\text{n}$]

[ysFGreater $\stackrel{\text{tex}}{\rightarrow}$ “ysFGreater”]

[ysFGreater $\stackrel{\text{pyk}}{\rightarrow}$ “axiom ysFGreater”]

ysFLess

[ysFLess $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[ysFLess $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}}\colon \dot{\neg}\,(\text{ysF}[\underline{\mathsf{m}}] <= (\text{xs}[\underline{\mathsf{m}}] + \text{rec}\underline{\mathsf{m}}) \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\text{ysF}[\underline{\mathsf{m}}] = (\text{xs}[\underline{\mathsf{m}}] + \text{rec}\underline{\mathsf{m}}))\text{n})\text{n})\text{n}$]

[ysFLess $\stackrel{\text{tex}}{\rightarrow}$ “ysFLess”]

[ysFLess $\stackrel{\text{pyk}}{\rightarrow}$ “axiom ysFLess”]

SmallInverse

[SmallInverse $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[SmallInverse $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}}\colon \forall \underline{\mathsf{x}}\colon \dot{\neg}\,(0 <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n} \vdash$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \underline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(\mathrm{rec}\underline{\mathsf{m}} <= \underline{\mathsf{x}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\mathrm{rec}\underline{\mathsf{m}} = \underline{\mathsf{x}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$]

[SmallInverse $\stackrel{\text{tex}}{\rightarrow}$ "SmallInverse"]

[SmallInverse $\stackrel{\text{pyk}}{\rightarrow}$ "1rule smallInverse"]

NatType

[NatType $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[NatType $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{m}}\colon \lambda \mathsf{c}.\mathrm{TypeNat0}(\lceil \underline{\mathsf{m}} \rceil) \Vdash \underline{\mathsf{m}} \in \mathrm{N}$]

[NatType $\stackrel{\text{tex}}{\rightarrow}$ "NatType"]

[NatType $\stackrel{\text{pyk}}{\rightarrow}$ "axiom natType"]

RationalType

[RationalType $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[RationalType $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{\mathsf{x}}\colon \lambda \mathsf{c}.\mathrm{TypeRational0}(\lceil \underline{\mathsf{x}} \rceil) \Vdash \underline{\mathsf{x}} \in \mathrm{Q}$]

[RationalType $\stackrel{\text{tex}}{\rightarrow}$ "RationalType"]

[RationalType $\stackrel{\text{pyk}}{\rightarrow}$ "axiom rationalType"]

SeriesType

[SeriesType $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[SeriesType $\stackrel{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{sy})}\colon \lambda \mathsf{c}.\mathrm{Typeseries0}(\lceil \underline{(\mathrm{fx})} \rceil, \lceil \underline{(\mathrm{sy})} \rceil) \Vdash$
$\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot{\neg}\,(\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{r1})} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}} \overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{s2})}\colon \dot{\neg}\,(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}$]

[SeriesType $\stackrel{\text{tex}}{\rightarrow}$ "SeriesType"]

[SeriesType $\overset{\text{pyk}}{\rightarrow}$ "axiom seriesType"]

Max

[Max $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Max $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \forall \underline{y}\colon \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x} \Rightarrow \dot{\neg}\,(\mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{y} <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathrm{if}(\underline{y} <= \underline{x}, \underline{x}, \underline{y}) = \underline{y})\mathrm{n})\mathrm{n}$]

[Max $\overset{\text{tex}}{\rightarrow}$ "Max"]

[Max $\overset{\text{pyk}}{\rightarrow}$ "axiom max"]

Numerical

[Numerical $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[Numerical $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall \underline{x}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \underline{x} \Rightarrow \dot{\neg}\,(|\underline{x}| = \underline{x})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 <= \underline{x})\mathrm{n} \Rightarrow \dot{\neg}\,(|\underline{x}| = (-\mathrm{u}\underline{x}))\mathrm{n})\mathrm{n}$]

[Numerical $\overset{\text{tex}}{\rightarrow}$ "Numerical"]

[Numerical $\overset{\text{pyk}}{\rightarrow}$ "axiom numerical"]

NumericalF

[NumericalF $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[NumericalF $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash$
$\forall \underline{(\mathrm{fx})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(\dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fx})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\forall_{\mathrm{obj}} \overline{(\epsilon)}\colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{n}}\colon \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{\mathsf{m}}\colon \dot{\neg}\,(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}\,(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |$
$\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op1})}\colon \dot{\neg}\,(\dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{op2})}\colon \dot{\neg}\,(\dot{\neg}\,(\dot{\neg}\,(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}\,(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\ |\ \dot{\neg}\,(\forall_{\mathrm{obj}} \overline{(\mathrm{crs1})}\colon \dot{\neg}\,(\mathsf{c}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{crs1})}, \overline{(\mathrm{crs1})}\}, \{\overline{(\mathrm{crs1})}, 0\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{u}\underline{(\mathrm{fx})}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$|f\underline{(fx)}| = \underline{(fx)} \Rightarrow \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow$
$\dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] <= (\underline{(fx)}[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow$
$\forall_{obj}\overline{(\epsilon)}: \dot{\neg}(\forall_{obj}\overline{\mathsf{n}}: \dot{\neg}(\forall_{obj}\overline{\mathsf{m}}: \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})n)n)n \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow$
$\dot{\neg}(|(\{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-u\underline{(fx)}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{ph \in$
$\{ph \in P(P(Union(\{N, Q\}))) \mid \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in$
$N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow \dot{\neg}(\mathsf{a}_{Ph} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\overline{(crs1)}: \dot{\neg}(\mathsf{c}_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\overline{\mathsf{m}}] + (-u\underline{(fx)}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n)n \Rightarrow$
$|f\underline{(fx)}| = \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow \dot{\neg}(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot{\neg}(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot{\neg}(\forall_{obj}\underline{\mathsf{m}}: \dot{\neg}(\mathsf{f}_{Ph} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-u\underline{(fx)}[\underline{\mathsf{m}}])\}\})n)n\})n)n]$

[NumericalF $\overset{\text{tex}}{\rightarrow}$ "NumericalF"]

[NumericalF $\overset{\text{pyk}}{\rightarrow}$ "axiom numericalF"]

MemberOfSeries(Imply)

[MemberOfSeries(Imply) $\overset{\text{proof}}{\rightarrow}$ Rule tactic]

[MemberOfSeries(Imply) $\overset{\text{stmt}}{\rightarrow}$ SystemQ $\vdash \forall\underline{\mathsf{m}}: \forall\underline{(fx)}: \forall\underline{(sy)}: \underline{\mathsf{m}} \in N \Rightarrow$
$\dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{obj}\overline{(op1)}: \dot{\neg}(\dot{\neg}(\forall_{obj}\overline{(op2)}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(op1)} \in N \Rightarrow$
$\dot{\neg}(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot{\neg}(\overline{(r1)} =$
$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot{\neg}(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot{\neg}(\forall_{obj}\overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\underline{(fx)})n)n)n)n \Rightarrow \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)}$]

[MemberOfSeries(Imply) $\overset{\text{tex}}{\rightarrow}$ "MemberOfSeries(Imply)"]

[MemberOfSeries(Imply) $\overset{\text{pyk}}{\rightarrow}$ "axiom memberOfSeries"]

JoinConjuncts(2conditions)

[JoinConjuncts(2conditions) $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \text{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \rhd \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{c}}; \text{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \rhd \underline{\mathsf{a}} \rhd \underline{\mathsf{b}} \gg \underline{\mathsf{d}}; \text{JoinConjuncts} \rhd \underline{\mathsf{c}} \rhd \underline{\mathsf{d}} \gg \dot{\neg}\,(\underline{\mathsf{c}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{d}})\text{n})\text{n}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \vdash \dot{\neg}\,(\underline{\mathsf{c}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{d}})\text{n})\text{n} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{c}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{d}})\text{n})\text{n}; \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \vdash \text{MP2} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \Rightarrow \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{c}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{d}})\text{n})\text{n} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \rhd \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \gg \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{c}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{d}})\text{n})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[JoinConjuncts(2conditions) $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \forall \underline{\mathsf{c}}\colon \forall \underline{\mathsf{d}}\colon \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{c}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \underline{\mathsf{d}} \vdash \underline{\mathsf{a}} \Rightarrow \underline{\mathsf{b}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{c}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{d}})\text{n})\text{n}$]

[JoinConjuncts(2conditions) $\stackrel{\text{tex}}{\rightarrow}$ "JoinConjuncts(2conditions)"]

[JoinConjuncts(2conditions) $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma doubly conditioned join conjuncts"]

prop lemma imply negation

[prop lemma imply negation $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \vdash \text{AutoImply} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \text{TND} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \text{FromDisjuncts} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \rhd \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[prop lemma imply negation $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \vdash \dot{\neg}\,(\underline{\mathsf{a}})\text{n}$]

[prop lemma imply negation $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma imply negation"]

TND

[TND $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \text{AutoImply} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}; \text{Repetition} \rhd \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \gg \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}\rceil, \text{p}_0, \mathsf{c})$]

[TND $\stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \dot{\neg}\,(\underline{\mathsf{a}})\text{n} \Rightarrow \dot{\neg}\,(\underline{\mathsf{a}})\text{n}$]

[TND $\stackrel{\text{tex}}{\rightarrow}$ "TND"]

[TND $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma tertium non datur"]

FromNegatedImply

[FromNegatedImply $\stackrel{\text{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \vdash \underline{\mathsf{a}} \vdash \text{MP} \rhd \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \rhd \underline{\mathsf{a}} \gg \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n}; \text{RemoveDoubleNeg} \rhd \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \gg \underline{\mathsf{b}}; \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \text{Ded} \rhd \forall \underline{\mathsf{a}}\colon \forall \underline{\mathsf{b}}\colon \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \vdash \underline{\mathsf{a}} \vdash \underline{\mathsf{b}} \gg \underline{\mathsf{a}} \Rightarrow \dot{\neg}\,(\dot{\neg}\,(\underline{\mathsf{b}})\text{n})\text{n} \Rightarrow \underline{\mathsf{a}} \Rightarrow$

$\underline{b}; \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n} \vdash \mathrm{MT} \rhd \underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})\mathrm{n})\mathrm{n} \Rightarrow \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n} \gg \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNegatedImply} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n} \vdash \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNegatedImply} \stackrel{\mathrm{tex}}{\rightarrow}$ "FromNegatedImply"$]$

$[\mathrm{FromNegatedImply} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma from negated imply"$]$

ToNegatedImply

$[\mathrm{ToNegatedImply} \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \vdash \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \gg \underline{a} \Rightarrow \underline{b}; \mathrm{MP} \rhd \underline{a} \Rightarrow \underline{b} \rhd \underline{a} \gg \underline{b}; \mathrm{FromContradiction} \rhd \underline{b} \rhd \dot{\neg}(\underline{b})\mathrm{n} \gg \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n}; \forall \underline{a}\colon \forall \underline{b}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \vdash \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n} \gg \underline{a} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n}; \underline{a} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \mathrm{MP2} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n} \rhd \underline{a} \rhd \dot{\neg}(\underline{b})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n}; \mathrm{AutoImply} \gg \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n}; \mathrm{Neg} \rhd \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n} \rhd \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{ToNegatedImply} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \dot{\neg}(\underline{a} \Rightarrow \underline{b})\mathrm{n}]$

$[\mathrm{ToNegatedImply} \stackrel{\mathrm{tex}}{\rightarrow}$ "ToNegatedImply"$]$

$[\mathrm{ToNegatedImply} \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma to negated imply"$]$

FromNegated$(2 * \mathrm{Imply})$

$[\mathrm{FromNegated}(2 * \mathrm{Imply}) \stackrel{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \dot{\neg}(\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c})\mathrm{n} \vdash \mathrm{FromNegatedImply} \rhd \dot{\neg}(\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c})\mathrm{n} \gg \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b} \Rightarrow \underline{c})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b} \Rightarrow \underline{c})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{a}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b} \Rightarrow \underline{c})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{b} \Rightarrow \underline{c})\mathrm{n}; \mathrm{FromNegatedImply} \rhd \dot{\neg}(\underline{b} \Rightarrow \underline{c})\mathrm{n} \gg \dot{\neg}(\underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{c})\mathrm{n})\mathrm{n})\mathrm{n}; \mathrm{FirstConjunct} \rhd \dot{\neg}(\underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{c})\mathrm{n})\mathrm{n})\mathrm{n} \gg \underline{b}; \mathrm{SecondConjunct} \rhd \dot{\neg}(\underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{c})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{c})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \underline{a} \rhd \underline{b} \gg \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n})\mathrm{n}; \mathrm{JoinConjuncts} \rhd \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\underline{c})\mathrm{n} \gg \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{c})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{FromNegated}(2 * \mathrm{Imply}) \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \dot{\neg}(\underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c})\mathrm{n} \vdash \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{c})\mathrm{n})\mathrm{n})\mathrm{n}]$

$[\mathrm{FromNegated}(2 * \mathrm{Imply}) \stackrel{\mathrm{tex}}{\rightarrow}$ "FromNegated(2*Imply)"$]$

$[\mathrm{FromNegated}(2 * \mathrm{Imply}) \stackrel{\mathrm{pyk}}{\rightarrow}$ "prop lemma from negated double imply"$]$

FromNegatedAnd

[FromNegatedAnd $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \vdash \underline{a} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \gg \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n; \mathrm{RemoveDoubleNeg} \rhd \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \gg \underline{a} \Rightarrow \dot{\neg}(\underline{b})n; \mathrm{MP} \rhd \underline{a} \Rightarrow \dot{\neg}(\underline{b})n \rhd \underline{a} \gg \dot{\neg}(\underline{b})n\rceil, p_0, c)$]

[FromNegatedAnd $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n)n \vdash \underline{a} \vdash \dot{\neg}(\underline{b})n$]

[FromNegatedAnd $\stackrel{\text{tex}}{\rightarrow}$ "FromNegatedAnd"]

[FromNegatedAnd $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma from negated and"]

FromNegatedOr

[FromNegatedOr $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \vdash \mathrm{Repetition} \rhd \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \mathrm{FromNegatedImply} \rhd \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n\rceil, p_0, c)$]

[FromNegatedOr $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \vdash \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n$]

[FromNegatedOr $\stackrel{\text{tex}}{\rightarrow}$ "FromNegatedOr"]

[FromNegatedOr $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma from negated or"]

ToNegatedOr

[ToNegatedOr $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \vdash \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \mathrm{FirstConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \gg \dot{\neg}(\underline{a})n; \mathrm{SecondConjunct} \rhd \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \gg \dot{\neg}(\underline{b})n; \mathrm{NegateDisjunct1} \rhd \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{a})n \gg \underline{b}; \mathrm{FromContradiction} \rhd \underline{b} \rhd \dot{\neg}(\underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \forall \underline{a}\colon \forall \underline{b}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \vdash \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \vdash \mathrm{MP} \rhd \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \rhd \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \gg \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \text{prop lemma imply negation} \rhd \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n\rceil, p_0, c)$]

[ToNegatedOr $\stackrel{\text{stmt}}{\rightarrow}$ $\mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n)n \vdash \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n$]

[ToNegatedOr $\stackrel{\text{tex}}{\rightarrow}$ "ToNegatedOr"]

[ToNegatedOr $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma to negated or"]

FromNegations

[FromNegations $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \vdash$ $\text{TND} \gg \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{a})\text{n}; \text{FromDisjuncts} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{a})\text{n} \rhd \underline{a} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow$ $\underline{b} \gg \underline{b}\rceil, \text{p}_0, c)$]

[FromNegations $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \vdash \underline{b}$]

[FromNegations $\stackrel{\text{tex}}{\rightarrow}$ "FromNegations"]

[FromNegations $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma from negations"]

From3Disjuncts

[From3Disjuncts $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow$ $\underline{c} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \dot{\neg}(\underline{a})\text{n} \vdash \text{Repetition} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \gg \dot{\neg}(\underline{a})\text{n} \Rightarrow$ $\dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c}; \text{MP} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \rhd \dot{\neg}(\underline{a})\text{n} \gg \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c}; \text{FromDisjuncts} \rhd$ $\dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \rhd \underline{b} \Rightarrow \underline{d} \rhd \underline{c} \Rightarrow \underline{d} \gg \underline{d}; \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \dot{\neg}(\underline{a})\text{n} \Rightarrow$ $\dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \dot{\neg}(\underline{a})\text{n} \vdash \underline{d} \gg \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow$ $\underline{c} \Rightarrow \underline{d} \Rightarrow \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{d}; \text{AutoImply} \gg \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{d}; \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow$ $\underline{d} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \text{MP3} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{c} \Rightarrow \underline{d} \Rightarrow$ $\dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{d} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \rhd \underline{b} \Rightarrow \underline{d} \rhd \underline{c} \Rightarrow \underline{d} \gg \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{d}; \text{MP} \rhd \underline{a} \Rightarrow$ $\underline{d} \Rightarrow \underline{a} \Rightarrow \underline{d} \rhd \underline{a} \Rightarrow \underline{d} \gg \underline{a} \Rightarrow \underline{d}; \text{FromNegations} \rhd \underline{a} \Rightarrow \underline{d} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{d} \gg \underline{d}\rceil, \text{p}_0, c)$]

[From3Disjuncts $\stackrel{\text{stmt}}{\rightarrow}$ $\text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \dot{\neg}(\underline{a})\text{n} \Rightarrow \dot{\neg}(\underline{b})\text{n} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{d} \vdash$ $\underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \underline{d}$]

[From3Disjuncts $\stackrel{\text{tex}}{\rightarrow}$ "From3Disjuncts"]

[From3Disjuncts $\stackrel{\text{pyk}}{\rightarrow}$ "prop lemma from three disjuncts"]

From2 ∗ 2Disjuncts

[From2 ∗ 2Disjuncts $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \forall \underline{e}\colon \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \vdash$ $\underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash \underline{a} \vdash \text{MP} \rhd \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \rhd \underline{a} \gg \underline{c} \Rightarrow \underline{e}; \text{MP} \rhd \underline{a} \Rightarrow \underline{d} \Rightarrow$ $\underline{e} \rhd \underline{a} \gg \underline{d} \Rightarrow \underline{e}; \text{FromDisjuncts} \rhd \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \rhd \underline{c} \Rightarrow \underline{e} \rhd \underline{d} \Rightarrow \underline{e} \gg$ $\underline{e}; \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \forall \underline{e}\colon \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash$ $\dot{\neg}(\underline{a})\text{n} \vdash \text{NegateDisjunct1} \rhd \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{a})\text{n} \gg \underline{b}; \text{MP} \rhd \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \rhd \underline{b} \gg$ $\underline{c} \Rightarrow \underline{e}; \text{MP} \rhd \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \rhd \underline{b} \gg \underline{d} \Rightarrow \underline{e}; \text{FromDisjuncts} \rhd \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \rhd \underline{c} \Rightarrow$ $\underline{e} \rhd \underline{d} \Rightarrow \underline{e} \gg \underline{e}; \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \forall \underline{e}\colon \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \forall \underline{e}\colon \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{c} \Rightarrow$ $\underline{e} \vdash \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash \underline{a} \vdash \underline{e} \gg \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \Rightarrow \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \Rightarrow \underline{a} \Rightarrow$ $\underline{e}; \text{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \forall \underline{e}\colon \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash$ $\dot{\neg}(\underline{a})\text{n} \vdash \underline{e} \gg \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \Rightarrow \dot{\neg}(\underline{a})\text{n} \Rightarrow$ $\underline{e}; \dot{\neg}(\underline{a})\text{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\text{n} \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{d} \Rightarrow$

$\underline{e} \vdash \mathrm{MP3} \rhd \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \Rightarrow \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \Rightarrow \underline{a} \Rightarrow \underline{e} \rhd \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \rhd \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \rhd \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \gg \underline{a} \Rightarrow \underline{e}; \mathrm{MP4} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \Rightarrow \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{e} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \rhd \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \rhd \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \gg \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{e}; \mathrm{FromNegations} \rhd \underline{a} \Rightarrow \underline{e} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{e} \gg \underline{e}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{From2} * \mathrm{2Disjuncts} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \forall \underline{e}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash \underline{e}]$

$[\mathrm{From2} * \mathrm{2Disjuncts} \overset{\mathrm{tex}}{\rightarrow}$ “From2∗2Disjuncts”$]$

$[\mathrm{From2} * \mathrm{2Disjuncts} \overset{\mathrm{pyk}}{\rightarrow}$ “prop lemma from two times two disjuncts”$]$

NegateDisjunct1

$[\mathrm{NegateDisjunct1} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{a})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \gg \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b}; \mathrm{MP} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{a})\mathrm{n} \gg \underline{b}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NegateDisjunct1} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{a})\mathrm{n} \vdash \underline{b}]$

$[\mathrm{NegateDisjunct1} \overset{\mathrm{tex}}{\rightarrow}$ “NegateDisjunct1”$]$

$[\mathrm{NegateDisjunct1} \overset{\mathrm{pyk}}{\rightarrow}$ “prop lemma negate first disjunct”$]$

NegateDisjunct2

$[\mathrm{NegateDisjunct2} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \mathrm{Repetition} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \gg \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b}; \mathrm{NegativeMT} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{b})\mathrm{n} \gg \underline{a}\rceil, \mathrm{p}_0, \mathsf{c})]$

$[\mathrm{NegateDisjunct2} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \underline{a}]$

$[\mathrm{NegateDisjunct2} \overset{\mathrm{tex}}{\rightarrow}$ “NegateDisjunct2”$]$

$[\mathrm{NegateDisjunct2} \overset{\mathrm{pyk}}{\rightarrow}$ “prop lemma negate second disjunct”$]$

ExpandDisjuncts

$[\mathrm{ExpandDisjuncts} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \dot{\neg}(\underline{d})\mathrm{n} \vdash \mathrm{NegateDisjunct2} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{b})\mathrm{n} \gg \underline{a}; \mathrm{NegateDisjunct2} \rhd \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \rhd \dot{\neg}(\underline{d})\mathrm{n} \gg \underline{c}; \mathrm{JoinConjuncts} \rhd \underline{a} \rhd \underline{c} \gg \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n})\mathrm{n}; \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \mathrm{Ded} \rhd \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{d}\colon \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \vdash \dot{\neg}(\underline{b})\mathrm{n} \vdash \dot{\neg}(\underline{d})\mathrm{n} \vdash \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n})\mathrm{n} \gg \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n} \Rightarrow \dot{\neg}(\underline{d})\mathrm{n} \Rightarrow \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n})\mathrm{n}; \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \vdash \mathrm{MP2} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \Rightarrow \dot{\neg}(\underline{b})\mathrm{n} \Rightarrow \dot{\neg}(\underline{d})\mathrm{n} \Rightarrow \dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{c})\mathrm{n})\mathrm{n} \rhd \dot{\neg}(\underline{a})\mathrm{n} \Rightarrow \underline{b} \rhd \dot{\neg}(\underline{c})\mathrm{n} \Rightarrow \underline{d} \gg \dot{\neg}(\underline{b})\mathrm{n} \Rightarrow$

$\dot\neg\,(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{a}} \Rightarrow \dot\neg\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n}; \mathrm{Repetition} \rhd \dot\neg\,(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{a}} \Rightarrow \dot\neg\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n} \gg \dot\neg\,(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{a}} \Rightarrow \dot\neg\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n}\rceil, \mathrm{p}_0, \mathsf{c})]$

[ExpandDisjuncts $\overset{\text{stmt}}{\to}$ SystemQ $\vdash \forall\underline{\mathsf{a}}\colon \forall\underline{\mathsf{b}}\colon \forall\underline{\mathsf{c}}\colon \forall\underline{\mathsf{d}}\colon \dot\neg\,(\underline{\mathsf{a}})\mathrm{n} \Rightarrow \underline{\mathsf{b}} \vdash \dot\neg\,(\underline{\mathsf{c}})\mathrm{n} \Rightarrow \underline{\mathsf{d}} \vdash \dot\neg\,(\underline{\mathsf{b}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{d}})\mathrm{n} \Rightarrow \dot\neg\,(\underline{\mathsf{a}} \Rightarrow \dot\neg\,(\underline{\mathsf{c}})\mathrm{n})\mathrm{n}$]

[ExpandDisjuncts $\overset{\text{tex}}{\to}$ "ExpandDisjuncts"]

[ExpandDisjuncts $\overset{\text{pyk}}{\to}$ "prop lemma expand disjuncts"]

SENC1

[SENC1 $\overset{\text{proof}}{\to}$ Rule tactic]

[SENC1 $\overset{\text{stmt}}{\to}$ SystemQ $\vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{rx})}\colon \forall\underline{(\mathrm{ry})}\colon \underline{(\mathrm{rx})} = \underline{(\mathrm{ry})} \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{rx})} \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{ry})}$]

[SENC1 $\overset{\text{tex}}{\to}$ "SENC1"]

[SENC1 $\overset{\text{pyk}}{\to}$ "lemma set equality nec condition(1)"]

SENC2

[SENC2 $\overset{\text{proof}}{\to}$ Rule tactic]

[SENC2 $\overset{\text{stmt}}{\to}$ SystemQ $\vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{rx})}\colon \forall\underline{(\mathrm{ry})}\colon \underline{(\mathrm{rx})} = \underline{(\mathrm{ry})} \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{ry})} \vdash \underline{(\mathrm{fx})} \in \underline{(\mathrm{rx})}$]

[SENC2 $\overset{\text{tex}}{\to}$ "SENC2"]

[SENC2 $\overset{\text{pyk}}{\to}$ "lemma set equality nec condition(2)"]

LessLeq(R)

[LessLeq(R) $\overset{\text{proof}}{\to}$ $\lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall\underline{(\mathrm{fx})}\colon \forall\underline{(\mathrm{fy})}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{WeakenOr2} \rhd \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \gg \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(\mathrm{fx})}[\overline{\mathsf{m}}] <= (\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg\,(\dot\neg\,(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg\,(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg\,(\mathsf{a}_{\mathrm{Ph}} =$

$\{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow \dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg\,(\forall_{obj}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow$
$\dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow$
$\dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg\,(\forall_{obj}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})n)n)n)n)n\};$ Repetition $\rhd \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(\dot\neg\,(0 <=$
$\overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow \dot\neg\,(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow (fx)[\overline{\mathsf{m}}] <=$
$((fy)[\overline{\mathsf{m}}] + (-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow$
$\dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow$
$\dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg\,(\forall_{obj}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg\,(\dot\neg\,(|((fx)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow$
$\dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg\,(\mathsf{a}_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{Ph} \Rightarrow \dot\neg\,(\forall_{obj}\overline{(op1)}\colon \dot\neg\,(\dot\neg\,(\forall_{obj}\overline{(op2)}\colon \dot\neg\,(\dot\neg\,(\dot\neg\,(\overline{(op1)} \in N \Rightarrow$
$\dot\neg\,(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg\,(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(f1)}\colon \forall_{obj}\overline{(f2)}\colon \forall_{obj}\overline{(f3)}\colon \forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathsf{f}_{Ph} \Rightarrow$
$\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathsf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg\,(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg\,(\forall_{obj}\overline{(s2)}\colon \dot\neg\,(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$
$\mathsf{f}_{Ph})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg\,(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg\,(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg\,(\dot\neg\,(0 = \overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg\,(|((fy)[\overline{\mathsf{m}}] + (-\mathsf{ud}_{Ph}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$

$\dot\neg(\dot\neg(|((\underline{(fy)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\} \gg$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0=\overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{(fy)}[\overline{\mathsf{m}}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0=\overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\underline{(fx)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\underline{(fx)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$
$P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0=\overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\underline{(fy)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\underline{(fy)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\}], p_0, \mathsf{c})]$

$[LessLeq(R) \overset{stmt}{\to} SystemQ \vdash$
$\forall\underline{(fx)}\colon\forall\underline{(fy)}\colon \dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 =$
$\overline{(\epsilon)})n)n)n \Rightarrow \dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{(fy)}[\overline{\mathsf{m}}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \vdash$
$\dot\neg(\dot\neg(\forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(\dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0=\overline{(\epsilon)})n)n)n \Rightarrow$
$\dot\neg(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \underline{(fx)}[\overline{\mathsf{m}}] <= ((\underline{(fy)}[\overline{\mathsf{m}}]+(-u\overline{(\epsilon)})))n)n)n)n)n)n)n \Rightarrow \{ph \in P(\{ph \in$
$P(\{ph \in P(P(Union(\{N,Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in$
$N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n\}) \mid \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}\colon \overline{(r1)} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(\overline{(r1)} = \{\{\overline{(op1)},\overline{(op1)}\},\{\overline{(op1)},\overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(f1)}\colon\forall_{obj}\overline{(f2)}\colon\forall_{obj}\overline{(f3)}\colon\forall_{obj}\overline{(f4)}\colon \{\{\overline{(f1)},\overline{(f1)}\},\{\overline{(f1)},\overline{(f2)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(f3)},\overline{(f3)}\},\{\overline{(f3)},\overline{(f4)}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$
$\dot\neg(\forall_{obj}\overline{(s1)}\colon \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}\colon \dot\neg(\{\{\overline{(s1)},\overline{(s1)}\},\{\overline{(s1)},\overline{(s2)}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})n)n)n)n\}) \mid \forall_{obj}\overline{(\epsilon)}\colon \dot\neg(\forall_{obj}\overline{\mathsf{n}}\colon \dot\neg(\forall_{obj}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0=\overline{(\epsilon)})n)n)n \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|((\underline{(fx)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow$
$\dot\neg(\dot\neg(|((\underline{(fx)}[\overline{\mathsf{m}}]+(-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))|=\overline{(\epsilon)})n)n)n)n)n\} = \{ph \in P(\{ph \in P(\{ph \in$

$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N},\mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\underline{(\mathrm{fy})}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}]$

$[\mathrm{LessLeq(R)} \stackrel{\mathrm{tex}}{\to}$ "LessLeq(R)"$]$

$[\mathrm{LessLeq(R)} \stackrel{\mathrm{pyk}}{\to}$ "lemma lessLeq(R)"$]$

MemberOfSeries

$[\mathrm{MemberOfSeries} \stackrel{\mathrm{proof}}{\to} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{SystemQ} \vdash \forall \underline{\mathsf{m}}\colon \forall \underline{(\mathrm{fx})}\colon \forall \underline{(\mathrm{sy})}\colon \underline{\mathsf{m}} \in \mathrm{N} \vdash \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \vdash \mathrm{MemberOfSeries(Imply)} \gg \underline{\mathsf{m}} \in \mathrm{N} \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \in \underline{(\mathrm{fx})}; \mathrm{MP2} \rhd \underline{\mathsf{m}} \in \mathrm{N} \Rightarrow \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})},\overline{(\mathrm{f1})}\},\{\overline{(\mathrm{f1})},\overline{(\mathrm{f2})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \{\{\overline{(\mathrm{f3})},\overline{(\mathrm{f3})}\},\{\overline{(\mathrm{f3})},\overline{(\mathrm{f4})}\}\} \in \underline{(\mathrm{fx})} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})},\overline{(\mathrm{s1})}\},\{\overline{(\mathrm{s1})},\overline{(\mathrm{s2})}\}\} \in \underline{(\mathrm{fx})})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \{\{\underline{\mathsf{m}},\underline{\mathsf{m}}\},\{\underline{\mathsf{m}},\underline{(\mathrm{fx})}[\underline{\mathsf{m}}]\}\} \in \underline{(\mathrm{fx})} \rhd \underline{\mathsf{m}} \in \mathrm{N} \rhd \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \underline{(\mathrm{fx})} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}((\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}((\overline{(\mathrm{op2})} \in \underline{(\mathrm{sy})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}((\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})},\overline{(\mathrm{op1})}\},\{\overline{(\mathrm{op1})},\overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$

$\dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})n)n)n)n \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)}\rceil, p_0, \mathsf{c})]$

$[\text{MemberOfSeries} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{\mathsf{m}}: \forall\underline{(fx)}: \forall\underline{(sy)}: \underline{\mathsf{m}} \in N \vdash \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})n)n)n)n \vdash \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)}]$

$[\text{MemberOfSeries} \stackrel{\text{tex}}{\rightarrow} \text{“MemberOfSeries”}]$

$[\text{MemberOfSeries} \stackrel{\text{pyk}}{\rightarrow} \text{“lemma memberOfSeries”}]$

memberOfSeries(Type)

$[\text{memberOfSeries(Type)} \stackrel{\text{proof}}{\rightarrow} \lambda\mathsf{c}.\lambda\mathsf{x}.\mathcal{P}(\lceil \text{SystemQ} \vdash \forall\underline{\mathsf{m}}: \forall\underline{(fx)}: \forall\underline{(sy)}: \lambda\mathsf{c}.\text{TypeNat0}(\lceil\underline{\mathsf{m}}\rceil) \Vdash \lambda\mathsf{c}.\text{Typeseries0}(\lceil\underline{(fx)}\rceil, \lceil\underline{(sy)}\rceil) \Vdash \overline{\text{NatType} \rhd\!\!\rhd \lambda\mathsf{c}.\text{TypeNat0}(\lceil\underline{\mathsf{m}}\rceil) \gg \underline{\mathsf{m}} \in N}; \overline{\text{SeriesType}} \rhd\!\!\rhd \lambda\mathsf{c}.\text{Typeseries0}(\lceil\underline{(fx)}\rceil, \lceil\underline{(sy)}\rceil) \gg \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})n)n)n)n; \text{MemberOfSeries} \rhd \underline{\mathsf{m}} \in N \rhd \dot\neg(\dot\neg(\forall_{obj}\overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot\neg(\forall_{obj}\overline{(op1)}: \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}: \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in \underline{(sy)})n)n \Rightarrow \dot\neg(\overline{(r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(f1)}: \forall_{obj}\overline{(f2)}: \forall_{obj}\overline{(f3)}: \forall_{obj}\overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow \dot\neg(\forall_{obj}\overline{(s1)}: \overline{(s1)} \in N \Rightarrow \dot\neg(\forall_{obj}\overline{(s2)}: \dot\neg(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \underline{(fx)})n)n)n)n \gg \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)}\rceil, p_0, \mathsf{c})]$

$[\text{memberOfSeries(Type)} \stackrel{\text{stmt}}{\rightarrow} \text{SystemQ} \vdash \forall\underline{\mathsf{m}}: \forall\underline{(fx)}: \forall\underline{(sy)}: \lambda\mathsf{c}.\text{TypeNat0}(\lceil\underline{\mathsf{m}}\rceil) \Vdash \lambda\mathsf{c}.\text{Typeseries0}(\lceil\underline{(fx)}\rceil, \lceil\underline{(sy)}\rceil) \Vdash \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \underline{(fx)}[\underline{\mathsf{m}}]\}\} \in \underline{(fx)}]$

$[\text{memberOfSeries(Type)} \stackrel{\text{tex}}{\rightarrow} \text{“memberOfSeries(Type)”}]$

[memberOfSeries(Type) $\overset{\text{pyk}}{\to}$ "lemma memberOfSeries(Type)"]

$*(\exp)*$

[x(exp)y $\overset{\text{tex}}{\to}$ " #1.
(exp) #2."]

[$*(\exp)*$ $\overset{\text{pyk}}{\to}$ "" ^ ""]

R(*)

[R((fx)) $\overset{\text{macro}}{\to}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\mathrm{R}((\mathrm{fx})) \doteq \{\mathrm{ph} \in \mathrm{Power}(\mathrm{SetOfSeries}(\mathrm{Q})) \mid \mathrm{SF}((\mathrm{fx}), \mathrm{ph}_4)\}]\rceil)$]

[R(x) $\overset{\text{tex}}{\to}$ "R(#1.
)"]

[R(*) $\overset{\text{pyk}}{\to}$ "R(")"]

$- -$ R(*)

[$- -$ R(x) $\overset{\text{tex}}{\to}$ "--R(#1.
)"]

[$- -$ R(*) $\overset{\text{pyk}}{\to}$ "--R(")"]

rec*

[recx $\overset{\text{tex}}{\to}$ "rec#1."]

[rec* $\overset{\text{pyk}}{\to}$ "1/ ""]

/

[bs/r $\overset{\text{macro}}{\to}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\mathrm{bs/r} \doteq \{\mathrm{ph} \in \mathrm{P}(\mathrm{bs}) \mid \mathrm{Ex}_{20} \in \mathrm{bs} \dot{\wedge} [\mathrm{Ex}_{20} \in \mathrm{bs}]_{\mathrm{r}} {=}{=} \mathrm{ph}_2\}]\rceil)$]

[x/y $\overset{\text{tex}}{\to}$ "#1.
/ #2."]

[*/* $\overset{\text{pyk}}{\to}$ "eq-system of " modulo ""]

$* \cap *$

$[\mathsf{x} \cap \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \cap \mathsf{y} \ddot{=} \{\mathrm{ph} \in \mathsf{x} \cup \mathsf{y} \mid \mathrm{ph}_3 \in \mathsf{x} \dot{\wedge} \mathrm{ph}_3 \in \mathsf{y}\}]\rceil)]$

$[\mathsf{x} \cap \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\cap #2."]

$[* \cap * \stackrel{\text{pyk}}{\to}$ "intersection " comma " end intersection"]

$*[*]$

$[\mathsf{x}[\mathsf{y}] \stackrel{\text{tex}}{\to}$ "#1.
[#2.
]"]

$[*[*] \stackrel{\text{pyk}}{\to}$ "[" ; "]"]

$\cup *$

$[\cup \mathsf{x} \stackrel{\text{tex}}{\to}$ "\cup #1."]

$[\cup * \stackrel{\text{pyk}}{\to}$ "union " end union"]

$* \cup *$

$[\mathsf{x} \cup \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \cup \mathsf{y} \ddot{=} \cup\{\{\mathsf{x}\}, \{\mathsf{y}\}\}]\rceil)]$

$[\mathsf{x} \cup \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\cup} #2."]

$[* \cup * \stackrel{\text{pyk}}{\to}$ "binary-union " comma " end union"]

$\mathrm{P}(*)$

$[\mathrm{P}(\mathsf{x}) \stackrel{\text{tex}}{\to}$ "P(#1.
)"]

$[\mathrm{P}(*) \stackrel{\text{pyk}}{\to}$ "power " end power"]

$\{*\}$

$[\{\mathsf{x}\} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\{\mathsf{x}\} \ddot{=} \{\mathsf{x},\mathsf{x}\}]\rceil)]$

$[\{\mathsf{x}\} \overset{\text{tex}}{\to}$ "\{#1.
\}"]

$[\{*\} \overset{\text{pyk}}{\to}$ "zermelo singleton " end singleton"]

StateExpand$(*,*,*)$

$[\text{StateExpand}(\mathsf{t},\mathsf{s},\mathsf{c}) \overset{\text{val}}{\to} \mathsf{t}!\mathsf{s}!\mathsf{c}!\mathcal{U}^{\text{M}}(\mathsf{s}^{\text{h}} \text{`}\, \mathsf{t} \text{`}\, \mathsf{s} \text{`}\, \mathsf{c})]$

$[\text{StateExpand}(\mathsf{t},\mathsf{s},\mathsf{c}) \overset{\text{tex}}{\to}$ "StateExpand(#1.
, #2.
, #3.
)"]

$[\text{StateExpand}(*,*,*) \overset{\text{pyk}}{\to}$ "stateExpand(" , " , ")"]

extractSeries$(*)$

$[\text{extractSeries}(\mathsf{t}) \overset{\text{val}}{\to} \mathsf{t}^{22121222111111}]$

$[\text{extractSeries}(\mathsf{t}) \overset{\text{tex}}{\to}$ "extractSeries(#1.
)"]

$[\text{extractSeries}(*) \overset{\text{pyk}}{\to}$ "extractSeries(")"]

SetOfSeries$(*)$

$[\text{SetOfSeries}((\text{sx})) \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\text{SetOfSeries}((\text{sx})) \ddot{=} \{\text{ph} \in \text{Power}(\text{cartProd}(\text{N})) \mid \text{IsSeries}(\text{ph}_6,(\text{sx}))\}]\rceil)]$

$[\text{SetOfSeries}(\mathsf{x}) \overset{\text{tex}}{\to}$ "SetOfSeries(#1.
)"]

$[\text{SetOfSeries}(*) \overset{\text{pyk}}{\to}$ "setOfSeries(")"]

$--\mathrm{Macro}(*)$

$[--\mathrm{Macro}(\mathsf{t}) \stackrel{\mathrm{val}}{\to} \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot\neg(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot\neg(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot\neg(0 <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{f}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{ux}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot\neg(\dot\neg(|(\{\mathrm{ph} \in \{\mathrm{ph} \in$
$\mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow$
$\dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot\neg(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot\neg(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (-\mathrm{ux}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \lceil\mathsf{x}\rceil :: \mathrm{extractSeries}(\mathsf{t}^1) :: \mathsf{T})]$

$[--\mathrm{Macro}(\mathsf{x}) \stackrel{\mathrm{tex}}{\to}$ "--Macro(#1.
)"]

$[--\mathrm{Macro}(*) \stackrel{\mathrm{pyk}}{\to}$ "--Macro(")"]

ExpandList$(*, *, *)$

$[\mathrm{ExpandList}(\mathsf{t}, \mathsf{s}, \mathsf{c}) \stackrel{\mathrm{val}}{\to} \mathsf{t}!\mathsf{s}!\mathsf{c}!\mathrm{If}(\mathsf{t}^{\mathrm{a}}, \mathsf{T},$
$\mathrm{StateExpand}(\mathsf{t}^{\mathrm{h}}, \mathsf{s}, \mathsf{c}) :: \mathrm{ExpandList}(\mathsf{t}^{\mathrm{t}}, \mathsf{s}, \mathsf{c}))]$

$[\mathrm{ExpandList}(\mathsf{x}, \mathsf{y}, \mathsf{z}) \stackrel{\mathrm{tex}}{\to}$ "ExpandList(#1.
, #2.
, #3.
)"]

$[\mathrm{ExpandList}(*, *, *) \stackrel{\mathrm{pyk}}{\to}$ "expandList(" , " , ")"]

$**\mathrm{Macro}(*)$

$[**\mathrm{Macro}(\mathsf{t}) \stackrel{\mathrm{val}}{\to} \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot\neg(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot\neg(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot\neg(\dot\neg(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$

$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] * \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{e}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] * \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \lceil \mathsf{x} \rceil ::$
$\mathrm{extractSeries}(\mathsf{t}^1) :: \lceil \mathsf{y} \rceil :: \mathrm{extractSeries}(\mathsf{t}^2) :: \mathsf{T})]$

$[* * \mathrm{Macro}(\mathsf{x}) \stackrel{\mathrm{tex}}{\to}$ "**Macro(#1.
)"]

$[* * \mathrm{Macro}(*) \stackrel{\mathrm{pyk}}{\to}$ "**Macro(")"]

$+ + \mathrm{Macro}(*)$

$[+ + \mathrm{Macro}(\mathsf{t}) \stackrel{\mathrm{val}}{\to} \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in$
$\mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in$
$\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in$
$\mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} =$
$\{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid \dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\mathsf{d}_{\mathrm{Ph}} =$
$\{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, (\mathsf{x}[\underline{\mathsf{m}}] + \mathsf{y}[\underline{\mathsf{m}}])\}\})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \lceil \mathsf{x} \rceil ::$
$\mathrm{extractSeries}(\mathsf{t}^1) :: \lceil \mathsf{y} \rceil :: \mathrm{extractSeries}(\mathsf{t}^2) :: \mathsf{T})]$

$[+ + \mathrm{Macro}(\mathsf{x}) \stackrel{\mathrm{tex}}{\to}$ "++Macro(#1.

)”]

[+ + Macro(∗) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “++Macro(")”]

<< Macro(∗)

$$[<< \mathrm{Macro}(\mathsf{t}) \stackrel{\mathrm{val}}{\rightarrow} \tilde{\mathcal{Q}}(\mathsf{t}, \lceil \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \mathsf{x}[\overline{\mathsf{m}}] <= (\mathsf{y}[\overline{\mathsf{m}}] + (-\mathrm{u}\overline{(\epsilon)})))\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\rceil, \lceil\mathsf{x}\rceil :: \mathrm{extractSeries}(\mathsf{t}^1) :: \lceil\mathsf{y}\rceil :: \mathrm{extractSeries}(\mathsf{t}^2) :: \mathsf{T})]$$

[<< Macro(x) $\stackrel{\mathrm{tex}}{\rightarrow}$ “<<Macro(#1.
)”]

[<< Macro(∗) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “<<Macro(")”]

||Macro(∗)

$$[||\mathrm{Macro}(\mathsf{t}) \stackrel{\mathrm{val}}{\rightarrow} \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in \mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow \overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(|\mathsf{fx}|[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(|\mathsf{fx}|[\overline{\mathsf{m}}] + (-\mathrm{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \lceil\mathsf{x}\rceil :: \mathrm{extractSeries}(\mathsf{t}^1) :: \mathsf{T})]$$

[||Macro(x) $\stackrel{\mathrm{tex}}{\rightarrow}$ “||Macro(#1.
)”]

[||Macro(∗) $\stackrel{\mathrm{pyk}}{\rightarrow}$ “||Macro(")”]

01//Macro(∗)

$$[01//\mathrm{Macro}(\mathsf{t}) \stackrel{\mathrm{val}}{\rightarrow} \tilde{\mathcal{Q}}(\mathsf{t}, \lceil\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\}))) \mid \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{r1})}\colon \overline{(\mathrm{r1})} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow \dot{\neg}(\overline{(\mathrm{r1})} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$$

$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{f1})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f2})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f3})}\colon \forall_{\mathrm{obj}}\overline{(\mathrm{f4})}\colon \{\{\overline{(\mathrm{f1})}, \overline{(\mathrm{f1})}\}, \{\overline{(\mathrm{f1})}, \overline{(\mathrm{f2})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow$
$\{\{\overline{(\mathrm{f3})}, \overline{(\mathrm{f3})}\}, \{\overline{(\mathrm{f3})}, \overline{(\mathrm{f4})}\}\} \in \mathsf{f}_{\mathrm{Ph}} \Rightarrow \overline{(\mathrm{f1})} = \overline{(\mathrm{f3})} \Rightarrow \overline{(\mathrm{f2})} = \overline{(\mathrm{f4})})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s1})}\colon \overline{(\mathrm{s1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{s2})}\colon \dot{\neg}(\{\{\overline{(\mathrm{s1})}, \overline{(\mathrm{s1})}\}, \{\overline{(\mathrm{s1})}, \overline{(\mathrm{s2})}\}\} \in$
$\mathsf{f}_{\mathrm{Ph}})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}) \mid \forall_{\mathrm{obj}}\overline{(\epsilon)}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{n}}\colon \dot{\neg}(\forall_{\mathrm{obj}}\overline{\mathsf{m}}\colon \dot{\neg}(0 <= \overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(0 = \overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\overline{\mathsf{n}} <= \overline{\mathsf{m}} \Rightarrow \dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\mathsf{x}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\mathsf{x}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{x}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| <=$
$\overline{(\epsilon)} \Rightarrow \dot{\neg}(\dot{\neg}(|(\{\mathrm{ph} \in \{\mathrm{ph} \in \mathrm{P}(\mathrm{P}(\mathrm{Union}(\{\mathrm{N}, \mathrm{Q}\})))) \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op1})}\colon \dot{\neg}(\dot{\neg}(\forall_{\mathrm{obj}}\overline{(\mathrm{op2})}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\overline{(\mathrm{op1})} \in \mathrm{N} \Rightarrow \dot{\neg}(\overline{(\mathrm{op2})} \in \mathrm{Q})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{a}_{\mathrm{Ph}} = \{\{\overline{(\mathrm{op1})}, \overline{(\mathrm{op1})}\}, \{\overline{(\mathrm{op1})}, \overline{(\mathrm{op2})}\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\} \mid$
$\dot{\neg}(\forall_{\mathrm{obj}}\underline{\mathsf{m}}\colon \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}(\mathsf{x}[\underline{\mathsf{m}}] = 0)\mathrm{n} \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, \mathrm{rec}\mathsf{x}[\underline{\mathsf{m}}]\}\})\mathrm{n})\mathrm{n})\mathrm{n} \Rightarrow$
$\dot{\neg}(\mathsf{x}[\underline{\mathsf{m}}] = 0 \Rightarrow \dot{\neg}(\mathsf{f}_{\mathrm{Ph}} = \{\{\underline{\mathsf{m}}, \underline{\mathsf{m}}\}, \{\underline{\mathsf{m}}, 0\}\})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}[\overline{\mathsf{m}}] + (-\mathsf{ud}_{\mathrm{Ph}}[\overline{\mathsf{m}}]))| =$
$\overline{(\epsilon)})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n})\mathrm{n}\}\rceil, \lceil\mathsf{x}\rceil :: \mathrm{extractSeries}(\mathsf{t}^1) :: \mathsf{T})]$

[01//Macro(x) $\stackrel{\text{tex}}{\to}$ "01//Macro(#1.
)"]

[01//Macro($*$) $\stackrel{\text{pyk}}{\to}$ "01//Macro(")"]

UB($*,*$)

[UB(x,y) $\stackrel{\text{tex}}{\to}$ "UB(#1.
, #2.
)"]

[UB($*,*$) $\stackrel{\text{pyk}}{\to}$ "upperBound(" , ")"]

LUB($*,*$)

[LUB(x,y) $\stackrel{\text{tex}}{\to}$ "LUB(#1.
, #2.
)"]

[LUB($*,*$) $\stackrel{\text{pyk}}{\to}$ "leastUpperBound(" , ")"]

BS($*,*$)

[BS(x,y) $\stackrel{\text{tex}}{\to}$ "BS(#1.
, #2.
)"]

$[\mathrm{BS}(*,*) \stackrel{\mathrm{pyk}}{\to}$ "base(1/2)Sum(" , ")"]

UStelescope(∗, ∗)

$[\mathrm{UStelescope}(\mathsf{x},\mathsf{y}) \stackrel{\mathrm{tex}}{\to}$ "UStelescope(#1.
, #2.
)"]

$[\mathrm{UStelescope}(*,*) \stackrel{\mathrm{pyk}}{\to}$ "UStelescope(" , ")"]

(∗)

$[(\mathsf{x}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[(\mathsf{x}) \ddot{=} (\mathsf{x})]\rceil)]$

$[(\mathsf{x}) \stackrel{\mathrm{tex}}{\to}$ "(#1.
)"]

$[(*) \stackrel{\mathrm{pyk}}{\to}$ "(")"]

|f ∗ |

$[|\mathrm{f}\mathsf{x}| \stackrel{\mathrm{tex}}{\to}$ "|f#1.
|"]

$[|\mathrm{f} * | \stackrel{\mathrm{pyk}}{\to}$ "|f " |"]

|r ∗ |

$[|\mathrm{r}\mathsf{x}| \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.||\mathrm{Macro}(\mathsf{t}^{\mathrm{h}} :: \mathrm{ExpandList}(\mathsf{t}^{\mathrm{t}},\mathsf{s},\mathsf{c}))]$

$[|\mathrm{r}\mathsf{x}| \stackrel{\mathrm{tex}}{\to}$ "|r#1.
|"]

$[|\mathrm{r} * | \stackrel{\mathrm{pyk}}{\to}$ "|r " |"]

Limit(∗, ∗)

$[\mathrm{Limit}(\mathsf{x},\mathsf{y}) \stackrel{\mathrm{tex}}{\to}$ "Limit(#1.
, #2.
)"]

$[\text{Limit}(*,*) \stackrel{\text{pyk}}{\to}$ "limit(" , ")"]

Union(*)

$[\text{Union}(\mathsf{x}) \stackrel{\text{tex}}{\to}$ "Union(#1.
)"]

$[\text{Union}(*) \stackrel{\text{pyk}}{\to}$ "U(")"]

IsOrderedPair(*, *, *)

$[\text{IsOrderedPair}((\text{sx}),(\text{sy}),(\text{sz})) \stackrel{\text{macro}}{\to}$
$\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[\text{IsOrderedPair}((\text{sx}),(\text{sy}),(\text{sz})) \ddot{=}$
$\exists(\text{OP1ob})\colon \exists(\text{OP2ob})\colon (\text{OP1ob}) \in (\text{sy}) \dot{\wedge} (\text{OP2ob}) \in (\text{sz}) \dot{\wedge} (\text{sx}) =$
$\text{OrderedPair}((\text{OP1ob}),(\text{OP2ob}))]\rceil)]$

$[\text{IsOrderedPair}(\mathsf{x},\mathsf{y},\mathsf{z}) \stackrel{\text{tex}}{\to}$ "IsOrderedPair(#1.
, #2.
, #3.
)"]

$[\text{IsOrderedPair}(*,*,*) \stackrel{\text{pyk}}{\to}$ "isOrderedPair(" , " , ")"]

IsRelation(*, *, *)

$[\text{IsRelation}((\text{sx}),(\text{sy}),(\text{sz})) \stackrel{\text{macro}}{\to}$
$\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[\text{IsRelation}((\text{sx}),(\text{sy}),(\text{sz})) \ddot{=} \forall(\text{R1ob})\colon ((\text{R1ob}) \in (\text{sx}) \Rightarrow$
$\text{IsOrderedPair}((\text{R1ob}),(\text{sy}),(\text{sz})))]\rceil)]$

$[\text{IsRelation}(\mathsf{x},\mathsf{y},\mathsf{z}) \stackrel{\text{tex}}{\to}$ "IsRelation(#1.
, #2.
, #3.
)"]

$[\text{IsRelation}(*,*,*) \stackrel{\text{pyk}}{\to}$ "isRelation(" , " , ")"]

isFunction(*, *, *)

$[\text{isFunction}((\text{sx}),(\text{sy}),(\text{sz})) \stackrel{\text{macro}}{\to}$
$\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[\text{isFunction}((\text{sx}),(\text{sy}),(\text{sz})) \ddot{=} \text{IsRelation}((\text{sx}),(\text{sy}),(\text{sz})) \dot{\wedge}$
$\forall(\text{F1ob}),(\text{F2ob}),(\text{F3ob}),$

(F4ob): (OrderedPair((F1ob), (F2ob)) $\in$ (sx) $\Rightarrow$ OrderedPair((F3ob), (F4ob)) $\in$ (sx) $\Rightarrow$ (F1ob) = (F3ob) $\Rightarrow$ (F2ob) = (F4ob))]⌉)]

[isFunction($\mathsf{x}, \mathsf{y}, \mathsf{z}$) $\stackrel{\text{tex}}{\rightarrow}$ "isFunction(#1.
, #2.
, #3.
)"]

[isFunction($*, *, *$) $\stackrel{\text{pyk}}{\rightarrow}$ "isFunction(" , " , ")"]

IsSeries($*, *$)

[IsSeries((fx), (fy)) $\stackrel{\text{macro}}{\rightarrow}$ $\lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c},$ ⌈[IsSeries((fx), (fy)) $\ddot{=}$ isFunction((fx), N, (fy)) $\dot{\wedge}$ $\forall$(S1ob): ((S1ob) $\in$ N $\Rightarrow$ $\exists$(S2ob): OrderedPair((S1ob), (S2ob)) $\in$ (fx))]⌉)]

[IsSeries(x, y) $\stackrel{\text{tex}}{\rightarrow}$ "IsSeries(#1.
, #2.
)"]

[IsSeries($*, *$) $\stackrel{\text{pyk}}{\rightarrow}$ "isSeries(" , ")"]

IsNatural($*, *$)

[IsNatural($\mathsf{xy}, *$) $\stackrel{\text{tex}}{\rightarrow}$ "IsNatural(#1.
, #2.
)"]

[IsNatural($*, *$) $\stackrel{\text{pyk}}{\rightarrow}$ "isNatural(")"]

OrderedPair($*, *$)

[OrderedPair(x, y) $\stackrel{\text{macro}}{\rightarrow}$ $\lambda\mathsf{t}.\lambda\mathsf{s}.\lambda\mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c},$ ⌈[OrderedPair(x, y) $\ddot{=}$ $\langle \mathsf{x}, \mathsf{y} \rangle$]⌉)]

[OrderedPair(x, y) $\stackrel{\text{tex}}{\rightarrow}$ "OrderedPair(#1.
, #2.
)"]

[OrderedPair($*, *$) $\stackrel{\text{pyk}}{\rightarrow}$ "(o " , ")"]

TypeNat($*$)

$[\mathrm{TypeNat}(\mathsf{x}) \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{TypeNat}(\mathsf{x}) \ddot{=} \lambda \mathsf{c}.\mathrm{TypeNat0}(\lceil \mathsf{x} \rceil)]\rceil)]$

$[\mathrm{TypeNat}(\mathsf{x}) \overset{\mathrm{tex}}{\to}$ “TypeNat(#1.
)”]

$[\mathrm{TypeNat}(*) \overset{\mathrm{pyk}}{\to}$ “typeNat(")”]

TypeNat0($*$)

$[\mathrm{TypeNat0}(\mathsf{x}) \overset{\mathrm{val}}{\to} \mathsf{x} \in_{\mathrm{t}} \lceil 0 \rceil :: \lceil \underline{(\mathrm{v2n})} \rceil :: \lceil \underline{\mathsf{m}} \rceil :: \lceil \underline{\mathsf{n}} \rceil :: \lceil (\underline{\mathsf{n}} + 1) \rceil :: \lceil (\underline{\mathsf{m}} + 0) \rceil :: \lceil (\underline{\mathsf{m}} + \underline{\mathsf{n}}) \rceil :: \lceil \underline{\mathsf{o}} \rceil :: \lceil \underline{\mathsf{p}} \rceil :: \lceil ((\underline{\mathsf{m}} + \underline{\mathsf{n}}) + 1) \rceil :: \lceil (\underline{\mathsf{m}} + \underline{(\mathrm{m1})}) \rceil :: \lceil (\underline{\mathsf{m}} + (\underline{\mathsf{n}} + 1)) \rceil :: \lceil \underline{(\mathrm{m1})} \rceil :: \lceil \underline{(\mathrm{m2})} \rceil :: \lceil \underline{(\mathrm{n1})} \rceil :: \lceil \underline{(\mathrm{n2})} \rceil :: \lceil \overline{\mathsf{m}} \rceil :: \lceil \overline{\mathsf{n}} \rceil :: \mathsf{T}]$

$[\mathrm{TypeNat0}(\mathsf{x}) \overset{\mathrm{tex}}{\to}$ “TypeNat0(#1.
)”]

$[\mathrm{TypeNat0}(*) \overset{\mathrm{pyk}}{\to}$ “typeNat0(")”]

TypeRational($*$)

$[\mathrm{TypeRational}(\mathsf{x}) \overset{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{TypeRational}(\mathsf{x}) \ddot{=} \lambda \mathsf{c}.\mathrm{TypeRational0}(\lceil \mathsf{x} \rceil)]\rceil)]$

$[\mathrm{TypeRational}(\mathsf{x}) \overset{\mathrm{tex}}{\to}$ “TypeRational(#1.
)”]

$[\mathrm{TypeRational}(*) \overset{\mathrm{pyk}}{\to}$ “typeRational(")”]

TypeRational0($*$)

$[\mathrm{TypeRational0}(\mathsf{x}) \overset{\mathrm{val}}{\to} \mathsf{x} \in_{\mathrm{t}} \lceil \underline{\mathsf{x}} \rceil :: \lceil \underline{\mathsf{y}} \rceil :: \lceil \underline{\mathsf{z}} \rceil :: \lceil 0 \rceil :: \lceil 1 \rceil :: \mathsf{T}]$

$[\mathrm{TypeRational0}(\mathsf{x}) \overset{\mathrm{tex}}{\to}$ “TypeRational0(#1.
)”]

$[\mathrm{TypeRational0}(*) \overset{\mathrm{pyk}}{\to}$ “typeRational0(")”]

TypeSeries(∗, ∗)

[TypeSeries(x, y) $\stackrel{\text{macro}}{\to}$ $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\mathrm{TypeSeries}(x, y) \ddot{=}$ $\lambda c.\mathrm{Typeseries0}(\lceil x\rceil, \lceil y\rceil)]\rceil)]$

[TypeSeries(x, y) $\stackrel{\text{tex}}{\to}$ “TypeSeries(#1.
, #2.
)”]

[TypeSeries(∗, ∗) $\stackrel{\text{pyk}}{\to}$ “typeSeries(" , ")”]

Typeseries0(∗, ∗)

[Typeseries0(x, y) $\stackrel{\text{val}}{\to}$ $y!x \in_t \lceil\underline{(fx)}\rceil :: \lceil\underline{(fy)}\rceil :: \lceil\underline{(fz)}\rceil :: \lceil us\rceil :: \lceil\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(d_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] + \underline{(fy)}[\underline{m}])\}\})n)n\}\rceil :: \lceil\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}\rceil :: \lceil\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} = \{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 1\}\})n)n\}\rceil :: \lceil\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(e_{Ph} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{(fx)}[\underline{m}] * \{ph \in \{ph \in P(P(Union(\{N, Q\}))) \mid$
$\dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow \dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow$
$\dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid \dot\neg(\forall_{obj}\overline{(crs1)}\colon \dot\neg(c_{Ph} =$
$\{\{\overline{(crs1)}, \overline{(crs1)}\}, \{\overline{(crs1)}, 0\}\})n)n\}[\underline{m}])\}\})n)n\}\rceil :: \lceil\{ph \in \{ph \in$
$P(P(Union(\{N, Q\}))) \mid \dot\neg(\forall_{obj}\overline{(op1)}\colon \dot\neg(\dot\neg(\forall_{obj}\overline{(op2)}\colon \dot\neg(\dot\neg(\dot\neg(\overline{(op1)} \in N \Rightarrow$
$\dot\neg(\overline{(op2)} \in Q)n)n \Rightarrow \dot\neg(a_{Ph} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\})n)n)n)n)n)n\} \mid$
$\dot\neg(\forall_{obj}\underline{m}\colon \dot\neg(\dot\neg(\dot\neg(\dot\neg(\underline{(fx)}[\underline{m}] = 0)n \Rightarrow \dot\neg(f_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, rec(fx)[\underline{m}]\}\})n)n)n \Rightarrow \dot\neg(\underline{(fx)}[\underline{m}] = 0 \Rightarrow \dot\neg(f_{Ph} =$
$\{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n)n\}\rceil :: T]$

[Typeseries0(x, y) $\stackrel{\text{tex}}{\to}$ “Typeseries0(#1.
, #2.
)”]

[Typeseries0(∗, ∗) $\stackrel{\text{pyk}}{\to}$ “typeSeries0(" , ")”]

$\{*, *\}$

$[\{\mathsf{x}, \mathsf{y}\} \stackrel{\text{tex}}{\to}$ "\{#1.
, #2.
\}"]

$[\{*, *\} \stackrel{\text{pyk}}{\to}$ "zermelo pair " comma " end pair"]

$\langle *, * \rangle$

$[\langle \mathsf{x}, \mathsf{y} \rangle \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\langle \mathsf{x}, \mathsf{y} \rangle \ddot{=} \{\{\mathsf{x}\}, \{\mathsf{x}, \mathsf{y}\}\}] \rceil)]$

$[\langle \mathsf{x}, \mathsf{y} \rangle \stackrel{\text{tex}}{\to}$ "\langle #1.
, #2.
\rangle"]

$[\langle *, * \rangle \stackrel{\text{pyk}}{\to}$ "zermelo ordered pair " comma " end pair"]

$(-\mathrm{u}*)$

$[(-\mathrm{u}\mathsf{x}) \stackrel{\text{tex}}{\to}$ "(-u#1.
)"]

$[(-\mathrm{u}*) \stackrel{\text{pyk}}{\to}$ "- ""]

$-_{\mathrm{f}}*$

$[-_{\mathrm{f}}(\mathrm{fx}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [-_{\mathrm{f}}(\mathrm{fx}) \ddot{=} \{\mathrm{ph} \in \mathrm{cartProd(N)} \mid \exists \mathcal{M} \colon \mathrm{ph}_6 = \mathrm{OrderedPair}(\mathcal{M}, (-\mathrm{u}(\mathrm{fx})[\mathcal{M}]))\}] \rceil)]$

$[-_{\mathrm{f}}\mathsf{x} \stackrel{\text{tex}}{\to}$ "-_{f}#1."]

$[-_{\mathrm{f}}* \stackrel{\text{pyk}}{\to}$ "-f ""]

$(- - *)$

$[(- - \mathsf{x}) \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}. - -\mathrm{Macro}(\mathsf{t}^{\mathrm{h}} :: \mathrm{ExpandList}(\mathsf{t}^{\mathrm{t}}, \mathsf{s}, \mathsf{c}))]$

$[(- - \mathsf{x}) \stackrel{\text{tex}}{\to}$ "(--#1.
)"]

$[(- - *) \stackrel{\text{pyk}}{\to}$ "-- ""]

1f/*

[1f/(fx) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[$1f/(fx) $\ddot{=}$ {ph ∈ cartProd(N) | $\exists \mathcal{M}\colon ((\mathrm{fx})[\mathcal{M}] \neq 0 \mathrel{\dot{\wedge}} \mathrm{ph}_6 = \mathrm{OrderedPair}(\mathcal{M}, \mathrm{rec}(\mathrm{fx})[\mathcal{M}])) \mathrel{\dot{\vee}} ((\mathrm{fx})[\mathcal{M}] = 0 \mathrel{\dot{\wedge}} \mathrm{ph}_6 = \mathrm{OrderedPair}(\mathcal{M}, 0))$}$]\rceil)$]

[1f/x $\stackrel{\text{tex}}{\to}$ "1f/#1."]

[1f/* $\stackrel{\text{pyk}}{\to}$ "1f/ ""]

01//temp*

[01//tempx $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.$01//Macro($\mathsf{t}^{\mathrm{h}}$ :: ExpandList($\mathsf{t}^{\mathrm{t}}, \mathsf{s}, \mathsf{c}$))]

[01//tempx $\stackrel{\text{tex}}{\to}$ "01//temp#1."]

[01//temp* $\stackrel{\text{pyk}}{\to}$ "01// ""]

(, *)

[$\mathsf{r}(\mathsf{x},\mathsf{y})$ $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{r}(\mathsf{x},\mathsf{y}) \ddot{=} \langle \mathsf{x},\mathsf{y}\rangle \in \mathsf{r}]\rceil)$]

[$\mathsf{z}(\mathsf{x},\mathsf{y})$ $\stackrel{\text{tex}}{\to}$ "#3.
(#1.
, #2.
)"]

[*(*, *) $\stackrel{\text{pyk}}{\to}$ "" is related to " under ""]

ReflRel(*, *)

[ReflRel(r,x) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[$ReflRel($\mathsf{r},\mathsf{x}$) $\ddot{=} \forall s\colon (s \in \mathsf{x} \Rightarrow \mathsf{r}(s,s))]\rceil)$]

[ReflRel(r,x) $\stackrel{\text{tex}}{\to}$ "ReflRel(#1.
, #2.
)"]

[ReflRel(*, *) $\stackrel{\text{pyk}}{\to}$ "" is reflexive relation in ""]

SymRel(*, *)

[SymRel(r,x) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[$SymRel($\mathsf{r},\mathsf{x}$) $\ddot{=} \forall s, t\colon (s \in \mathsf{x} \Rightarrow t \in \mathsf{x} \Rightarrow \mathsf{r}(s,t) \Rightarrow \mathsf{r}(t,s))]\rceil)$]

[SymRel(r, x) $\stackrel{\text{tex}}{\to}$ “SymRel(#1.
, #2.
)”]

[SymRel(∗, ∗) $\stackrel{\text{pyk}}{\to}$ “" is symmetric relation in "”]

TransRel(∗, ∗)

[TransRel(r, x) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\text{TransRel}(\mathsf{r}, \mathsf{x}) \doteq \forall s, t, u\colon (s \in \mathsf{x} \Rightarrow t \in \mathsf{x} \Rightarrow u \in \mathsf{x} \Rightarrow \mathsf{r}(s, t) \Rightarrow \mathsf{r}(t, u) \Rightarrow \mathsf{r}(s, u))]\rceil)$]

[TransRel(r, x) $\stackrel{\text{tex}}{\to}$ “TransRel(#1.
, #2.
)”]

[TransRel(∗, ∗) $\stackrel{\text{pyk}}{\to}$ “" is transitive relation in "”]

EqRel(∗, ∗)

[EqRel(r, x) $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\text{EqRel}(\mathsf{r}, \mathsf{x}) \doteq \text{ReflRel}(\mathsf{r}, \mathsf{x}) \dot{\wedge} \text{SymRel}(\mathsf{r}, \mathsf{x}) \dot{\wedge} \text{TransRel}(\mathsf{r}, \mathsf{x})]\rceil)$]

[EqRel(r, x) $\stackrel{\text{tex}}{\to}$ “EqRel(#1.
, #2.
)”]

[EqRel(∗, ∗) $\stackrel{\text{pyk}}{\to}$ “" is equivalence relation in "”]

$[* \in *]_*$

[$[\mathsf{x} \in \mathsf{bs}]_\mathsf{r}$ $\stackrel{\text{macro}}{\to}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[[\mathsf{x} \in \mathsf{bs}]_\mathsf{r} \doteq \{\text{ph} \in \mathsf{bs} \mid \mathsf{r}(\text{ph}_1, \mathsf{x})\}]\rceil)$]

[$[\mathsf{x} \in \mathsf{bs}]_\mathsf{r}$ $\stackrel{\text{tex}}{\to}$ “[#1.
\mathrel{\in} #2.
]_{#3.
}”]

[$[* \in *]_*$ $\stackrel{\text{pyk}}{\to}$ “equivalence class of " in " modulo "”]

Partition(∗, ∗)

[Partition(p, bs) $\stackrel{\text{macro}}{\to}$ λt.λs.λc.$\tilde{\mathcal{M}}_4$(t, s, c, ⌈[Partition(p, bs) $\ddot{=}$ ($\forall s$: ($s \in$ p ⇒ $s \neq$ ∅)) $\dot{\wedge}$
($\forall s, t$: ($s \in$ p ⇒ $t \in$ p ⇒ $s \neq t$ ⇒ $s \cap t$==∅)) $\dot{\wedge}$
∪ p==bs]⌉)]

[Partition(x, y) $\stackrel{\text{tex}}{\to}$ "Partition(#1.
, #2.
)"]

[Partition(∗, ∗) $\stackrel{\text{pyk}}{\to}$ "" is partition of ""]

(∗ ∗ ∗)

[(x ∗ y) $\stackrel{\text{tex}}{\to}$ "(#1.
∗#2.
)"]

[(∗ ∗ ∗) $\stackrel{\text{pyk}}{\to}$ "" ∗ ""]

$* *_{\mathrm{f}} *$

[(fx) $*_{\mathrm{f}}$ (fy) $\stackrel{\text{macro}}{\to}$ λt.λs.λc.$\tilde{\mathcal{M}}_4$(t, s, c, ⌈[(fx) $*_{\mathrm{f}}$ (fy) $\ddot{=}$ {ph ∈ cartProd(N) | $\exists \mathcal{M}$: ph_5 = OrderedPair($\mathcal{M}$, ((fx)[$\mathcal{M}$] ∗ (fy)[$\mathcal{M}$]))}]⌉)]

[(fx) $*_{\mathrm{f}}$ (fy) $\stackrel{\text{tex}}{\to}$ "#1.
∗_{f}#2."]

[$* *_{\mathrm{f}} *$ $\stackrel{\text{pyk}}{\to}$ "" ∗f ""]

∗ ∗ ∗∗

[x ∗ ∗y $\stackrel{\text{macro}}{\to}$ λt.λs.λc. ∗ ∗Macro(t^{h} :: ExpandList(t^{t}, s, c))]

[x ∗ ∗y $\stackrel{\text{tex}}{\to}$ "#1.
∗∗#2."]

[∗ ∗ ∗∗ $\stackrel{\text{pyk}}{\to}$ "" ∗∗ ""]

$(* + *)$

$[(\mathsf{x} + \mathsf{y}) \overset{\text{tex}}{\to}$ "(#1.
+#2.
)"]

$[(* + *) \overset{\text{pyk}}{\to}$ "" + ""]

$(* - *)$

$[(\mathsf{x} - \mathsf{y}) \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[(\mathsf{x} - \mathsf{y}) \ddot{=} (\mathsf{x} + (-\mathrm{u}\mathsf{y}))]\rceil)]$

$[(\mathsf{x} - \mathsf{y}) \overset{\text{tex}}{\to}$ "(#1.
-#2.
)"]

$[(* - *) \overset{\text{pyk}}{\to}$ "" - ""]

$* +_{\mathrm{f}} *$

$[(\mathrm{fx}) +_{\mathrm{f}} (\mathrm{fy}) \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[(\mathrm{fx}) +_{\mathrm{f}} (\mathrm{fy}) \ddot{=} \{\mathrm{ph} \in \mathrm{cartProd}(\mathrm{N}) \mid \exists \mathcal{M} \colon \mathrm{ph}_4 = \mathrm{OrderedPair}(\mathcal{M}, ((\mathrm{fx})[\mathcal{M}] + (\mathrm{fy})[\mathcal{M}]))\}]\rceil)]$

$[(\mathrm{fx}) +_{\mathrm{f}} (\mathrm{fy}) \overset{\text{tex}}{\to}$ "#1.
+_{f}#2."]

$[* +_{\mathrm{f}} * \overset{\text{pyk}}{\to}$ "" +f ""]

$* -_{\mathrm{f}} *$

$[(\mathrm{fx}) -_{\mathrm{f}} (\mathrm{fy}) \overset{\text{tex}}{\to}$ "#1.
-_{f}#2."]

$[* -_{\mathrm{f}} * \overset{\text{pyk}}{\to}$ "" -f ""]

$* + +*$

$[\mathsf{x} + +\mathsf{y} \overset{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}. + +\mathrm{Macro}(\mathsf{t}^{\mathrm{h}} :: \mathrm{ExpandList}(\mathsf{t}^{\mathrm{t}}, \mathsf{s}, \mathsf{c}))]$

$[\mathsf{x} + +\mathsf{y} \overset{\text{tex}}{\to}$ "#1.
++#2."]

$[* + +* \overset{\text{pyk}}{\to}$ "" ++ ""]

R(*) − −R(*)

[R((fx)) − −R((fy)) $\stackrel{\text{tex}}{\rightarrow}$ "R(#1.
) -- R(#2.
)"]

[R(*) − −R(*) $\stackrel{\text{pyk}}{\rightarrow}$ "R(") -- R(")"]

* ∈ *

[x ∈ y $\stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\in} #2."]

[* ∈ * $\stackrel{\text{pyk}}{\rightarrow}$ "" in0 ""]

| * |

[|x| $\stackrel{\text{tex}}{\rightarrow}$ "|#1.
|"]

[| * | $\stackrel{\text{pyk}}{\rightarrow}$ "| " |"]

if(*, *, *)

[if(x, y, z) $\stackrel{\text{tex}}{\rightarrow}$ "if(#1.
, #2.
, #3.
)"]

[if(*, *, *) $\stackrel{\text{pyk}}{\rightarrow}$ "if(" , " , ")"]

Max(*, *)

[Max(x, y) $\stackrel{\text{macro}}{\rightarrow}$ $\lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathrm{Max}(\mathsf{x}, \mathsf{y}) \ddot{=} \mathrm{if}(\mathsf{y} <= \mathsf{x}, \mathsf{x}, \mathsf{y})]\rceil)$]

[Max(x, y) $\stackrel{\text{tex}}{\rightarrow}$ "Max(#1.
, #2.
)"]

[Max(*, *) $\stackrel{\text{pyk}}{\rightarrow}$ "max(" , ")"]

$\mathrm{Max}(*, *)$

$[\mathrm{Max}(\mathsf{x}, \mathsf{y}) \stackrel{\mathrm{tex}}{\rightarrow}$ "Max(#1.
, #2.
)"]

$[\mathrm{Max}(*, *) \stackrel{\mathrm{pyk}}{\rightarrow}$ "maxR(" , ")"]

$* = *$

$[\mathsf{x} = \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
= #2."]

$[* = * \stackrel{\mathrm{pyk}}{\rightarrow}$ "" = ""]

$* \neq *$

$[\mathsf{x} \neq \mathsf{y} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} \neq \mathsf{y} \ddot{=} \dot{\neg}(\mathsf{x} = \mathsf{y})\mathrm{n}] \rceil)]$

$[\mathsf{x} \neq \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
\neq #2."]

$[* \neq * \stackrel{\mathrm{pyk}}{\rightarrow}$ "" != ""]

$* <= *$

$[\mathsf{x} <= \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
<= #2."]

$[* <= * \stackrel{\mathrm{pyk}}{\rightarrow}$ "" <= ""]

$* < *$

$[\mathsf{x} < \mathsf{y} \stackrel{\mathrm{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\mathsf{x} < \mathsf{y} \ddot{=} \mathsf{x} <= \mathsf{y} \dot{\wedge} \mathsf{x} \neq \mathsf{y}] \rceil)]$

$[\mathsf{x} < \mathsf{y} \stackrel{\mathrm{tex}}{\rightarrow}$ "#1.
< #2."]

$[* < * \stackrel{\mathrm{pyk}}{\rightarrow}$ "" < ""]

$* <_{\mathrm{f}} *$

$[(\mathrm{fx}) <_{\mathrm{f}} (\mathrm{fy}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[(\mathrm{fx}) <_{\mathrm{f}} (\mathrm{fy}) \ddot{=} \exists(\mathrm{EPob})\colon \exists n\colon \forall m\colon 0 < (\mathrm{EPob}) \dot{\wedge} (n <= m \Rightarrow (\mathrm{fx})[m] <= ((\mathrm{fy})[m] - (\mathrm{EPob})))]\rceil)]$

$[\mathsf{x} <_{\mathrm{f}} \mathsf{y} \stackrel{\mathrm{tex}}{\to}$ "#1.
<_{f}#2."]

$[* <_{\mathrm{f}} * \stackrel{\mathrm{pyk}}{\to}$ "" <f ""]

$* \leq_{\mathrm{f}} *$

$[(\mathrm{fx}) \leq_{\mathrm{f}} (\mathrm{fy}) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[(\mathrm{fx}) \leq_{\mathrm{f}} (\mathrm{fy}) \ddot{=} (\mathrm{fx}) <_{\mathrm{f}} (\mathrm{fy}) \dot{\vee} \mathrm{SF}((\mathrm{fx}),(\mathrm{fy}))]\rceil)]$

$[\mathsf{x} \leq_{\mathrm{f}} \mathsf{y} \stackrel{\mathrm{tex}}{\to}$ "#1.
\leq_{f}#2."]

$[* \leq_{\mathrm{f}} * \stackrel{\mathrm{pyk}}{\to}$ "" <=f ""]

SF(∗, ∗)

$[\mathrm{SF}((\mathrm{fx}),(\mathrm{fy})) \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[\mathrm{SF}((\mathrm{fx}),(\mathrm{fy})) \ddot{=} \forall(\mathrm{EPob})\colon \exists n\colon \forall m\colon (0 < (\mathrm{EPob}) \Rightarrow n <= m \Rightarrow |((\mathrm{fx})[m] - (\mathrm{fy})[m])| < (\mathrm{EPob}))]\rceil)]$

$[\mathrm{SF}(\mathsf{x},\mathsf{y}) \stackrel{\mathrm{tex}}{\to}$ "SF(#1.
, #2.
)"]

$[\mathrm{SF}(*,*) \stackrel{\mathrm{pyk}}{\to}$ "" sameF ""]

$* == *$

$[\mathsf{x} == \mathsf{y} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[\mathsf{x} == \mathsf{y} \ddot{=} \mathsf{x} = \mathsf{y}]\rceil)]$

$[\mathsf{x} == \mathsf{y} \stackrel{\mathrm{tex}}{\to}$ "#1.
== #2."]

$[* == * \stackrel{\mathrm{pyk}}{\to}$ "" == ""]

$*!! == *$

$[\mathsf{x}!! == \mathsf{y} \stackrel{\mathrm{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c}, \lceil[\mathsf{x}!! == \mathsf{y} \ddot{=} \dot{\neg}(\mathsf{x} == \mathsf{y})\mathrm{n}]\rceil)]$

$[\mathsf{x}!! == \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
!!== #2."]

$[*!! == * \stackrel{\text{pyk}}{\to}$ "" !!== ""]

$* << *$

$[\mathsf{x} << \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}. << \text{Macro}(\mathsf{t}^{\text{h}} :: \text{ExpandList}(\mathsf{t}^{\text{t}}, \mathsf{s}, \mathsf{c}))]$

$[\mathsf{x} << \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
<< #2."]

$[* << * \stackrel{\text{pyk}}{\to}$ "" << ""]

$* <<== *$

$[\mathsf{x} <<== \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathsf{x} <<== \mathsf{y} \doteq \mathsf{x} << \mathsf{y} \dot{\vee} \mathsf{x} == \mathsf{y}]\rceil)]$

$[\mathsf{x} <<== \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
<<== #2."]

$[* <<== * \stackrel{\text{pyk}}{\to}$ "" <<== ""]

$* == *$

$[\mathsf{x}{==}\mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\!\mathrel{==}\! #2."]

$[*{==}* \stackrel{\text{pyk}}{\to}$ "" zermelo is ""]

$* \subseteq *$

$[\mathsf{x} \subseteq \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathsf{x} \subseteq \mathsf{y} \doteq \forall(\text{S1ob})\colon ((\text{S1ob}) \in \mathsf{x} \Rightarrow (\text{S1ob}) \in \mathsf{y})]\rceil)]$

$[\mathsf{x} \subseteq \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\subseteq} #2."]

$[* \subseteq * \stackrel{\text{pyk}}{\to}$ "" is subset of ""]

$\dot{\neg}\,(*)\mathrm{n}$

$[\dot{\neg}\,(\mathsf{x})\mathrm{n} \stackrel{\text{tex}}{\to}$ "\dot{\neg}\,(#1.
)n"]

$[\dot{\neg}\,(*)\mathrm{n} \stackrel{\text{pyk}}{\to}$ "not0 ""]

$* \notin *$

$[\mathsf{x} \notin \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \notin \mathsf{y} \ddot{=} \dot{\neg}\,(\mathsf{x} \in \mathsf{y})\mathrm{n}]\rceil)]$

$[\mathsf{x} \notin \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\notin} #2."]

$[* \notin * \stackrel{\text{pyk}}{\to}$ "" zermelo ~in ""]

$* \neq *$

$[\mathsf{x} \neq \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \neq \mathsf{y} \ddot{=} \dot{\neg}\,(\mathsf{x}{=}{=}\mathsf{y})\mathrm{n}]\rceil)]$

$[\mathsf{x} \neq \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\neq} #2."]

$[* \neq * \stackrel{\text{pyk}}{\to}$ "" zermelo ~is ""]

$* \dot{\wedge} *$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\wedge} \mathsf{y} \ddot{=} \dot{\neg}\,((\mathsf{x} \Rightarrow \dot{\neg}\,(\mathsf{y})\mathrm{n}))\mathrm{n}]\rceil)]$

$[\mathsf{x} \dot{\wedge} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\wedge}} #2."]

$[* \dot{\wedge} * \stackrel{\text{pyk}}{\to}$ "" and0 ""]

$* \dot{\vee} *$

$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\mathsf{x} \dot{\vee} \mathsf{y} \ddot{=} \dot{\neg}\,(\mathsf{x})\mathrm{n} \Rightarrow \mathsf{y}]\rceil)]$

$[\mathsf{x} \dot{\vee} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\vee}} #2."]

$[* \dot{\vee} * \stackrel{\text{pyk}}{\to}$ "" or0 ""]

$\exists *: *$

$[\exists(\text{v1})\colon \mathsf{a} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\exists(\text{v1})\colon \mathsf{a} \mathrel{\ddot{=}} \dot{\neg}\,(\forall(\text{v1})\colon \dot{\neg}\,(\mathsf{a})\text{n})\text{n}]\rceil)]$

[$\exists \mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "
\exists #1.
\colon #2."]

[$\exists *: * \stackrel{\text{pyk}}{\rightarrow}$ "exist0 " indeed ""]

$* \mathrel{\dot{\Leftrightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \mathrel{\ddot{=}} (\mathsf{x} \Rightarrow \mathsf{y}) \mathrel{\dot{\wedge}} (\mathsf{y} \Rightarrow \mathsf{x})]\rceil)]$

[$\mathsf{x} \mathrel{\dot{\Leftrightarrow}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathrel{\dot{\Leftrightarrow}} #2."]

[$* \mathrel{\dot{\Leftrightarrow}} * \stackrel{\text{pyk}}{\rightarrow}$ "" iff ""]

$\{\text{ph} \in * \mid *\}$

[$\{\text{ph} \in \mathsf{x} \mid \mathsf{a}\} \stackrel{\text{tex}}{\rightarrow}$ " \{ ph \mathrel{\in} #1.
\mid #2.
\}"]

[$\{\text{ph} \in * \mid *\} \stackrel{\text{pyk}}{\rightarrow}$ "the set of ph in " such that " end set"]

The pyk compiler, version 0.grue.20060417+ by Klaus Grue
GRD-2006-12-15.UTC:00:32:42.052453 = MJD-54084.TAI:00:33:15.052453 =
LGT-4672859595052453e-6