

## Up Help

$\exists * : *$ ,  $* \Rightarrow *$ , kvanti, UniqueMember, UniqueMember(Type), SameSeries, A4, SameMember, Qclosed(Addition), Qclosed(Multiplication), FromCartProd(1), 1rule fromCartProd(2), constantRationalSeries(\*), cartProd(\*), Power(\*), binaryUnion(\*, \*), SetOfRationalSeries, IsSubset(\*, \*), (p\*, \*), (s\*), ( $\dots$ ), Objekt-var, Ex-var, Ph-var, Værdi, Variabel, Op(\*), Op(\*, \*),  $* \equiv *$ , ContainsEmpty(\*), Nat(\*), Dedu(\*, \*), Dedu<sub>0</sub>(\*, \*), Dedu<sub>s</sub>(\*, \*, \*), Dedu<sub>1</sub>(\*, \*, \*), Dedu<sub>2</sub>(\*, \*, \*), Dedu<sub>3</sub>(\*, \*, \*, \*), Dedu<sub>4</sub>(\*, \*, \*, \*), Dedu<sub>4</sub><sup>\*</sup>(\*, \*, \*, \*), Dedu<sub>5</sub>(\*, \*, \*), Dedu<sub>6</sub>(\*, \*, \*, \*), Dedu<sub>6</sub><sup>\*</sup>(\*, \*, \*, \*), Dedu<sub>7</sub>(\*), Dedu<sub>8</sub>(\*, \*), Dedu<sub>8</sub><sup>\*</sup>(\*, \*), EX<sub>1</sub>, EX<sub>2</sub>, EX<sub>3</sub>, EX<sub>10</sub>, EX<sub>20</sub>, \*EX, \*EX<sup>Ex</sup>,  $\langle * \equiv * \mid * : \equiv * \rangle_{EX}$ ,  $\langle * \equiv^0 * \mid * : \equiv * \rangle_{EX}$ ,  $\langle * \equiv^1 * \mid * : \equiv * \rangle_{EX}$ ,  $\langle * \equiv^* * \mid * : \equiv * \rangle_{EX}$ , ph<sub>1</sub>, ph<sub>2</sub>, ph<sub>3</sub>, \*Ph, \*Ph<sup>Ph</sup>,  $\langle * \equiv * \mid * : \equiv * \rangle_{Ph}$ ,  $\langle * \equiv^0 * \mid * : \equiv * \rangle_{Ph}$ ,  $\langle * \equiv^1 * \mid * : \equiv * \rangle_{Ph}$ ,  $\langle * \equiv^* * \mid * : \equiv * \rangle_{Ph}$ ,  $\langle * \equiv * \mid * : \equiv * \rangle_{Me}$ ,  $\langle * \equiv^1 * \mid * : \equiv * \rangle_{Me}$ ,  $\langle * \equiv^* * \mid * : \equiv * \rangle_{Me}$ , bs, OBS, BS,  $\emptyset$ , SystemQ, MP, Gen, Repetition, Neg, Ded, ExistIntro, Extensionality,  $\emptyset$ def, PairDef, UnionDef, PowerDef, SeparationDef, AddDoubleNeg, RemoveDoubleNeg, AndCommutativity, AutoImPLY, Contrapositive, FirstConjunct, SecondConjunct, FromContradiction, FromDisjuncts, IffCommutativity, IffFirst, IffSecond, ImplyTransitivity, JoinConjuncts, MP2, MP3, MP4, MP5, MT, NegativeMT, Technicality, Weakening, WeakenOr1, WeakenOr2, Formula2Pair, Pair2Formula, Formula2Union, Union2Formula, Formula2Sep, Sep2Formula, Formula2Power, SubsetInPower, HelperPowerIsSub, PowerIsSub, (Switch)HelperPowerIsSub, (Switch)PowerIsSub, ToSetEquality, HelperToSetEquality(t), ToSetEquality(t), HelperFromSetEquality, FromSetEquality, HelperReflexivity, Reflexivity, HelperSymmetry, Symmetry, HelperTransitivity, Transitivity, ERisReflexive, ERisSymmetric, ERisTransitive,  $\emptyset$ isSubset, HelperMemberNot $\emptyset$ , MemberNot $\emptyset$ , HelperUnique $\emptyset$ , Unique $\emptyset$ , == Reflexivity, == Symmetry, Helper == Transitivity, == Transitivity, HelperTransferNotEq, TransferNotEq, HelperPairSubset, Helper(2)PairSubset, PairSubset, SamePair, SameSingleton, UnionSubset, SameUnion, SeparationSubset, SameSeparation, SameBinaryUnion, IntersectionSubset, SameIntersection, AutoMember, HelperEqSysNot $\emptyset$ , EqSysNot $\emptyset$ , HelperEqSubset, EqSubset, HelperEqNecessary, EqNecessary, HelperNoneEqNecessary, Helper(2)NoneEqNecessary, NoneEqNecessary, EqClassIsSubset, EqClassesAreDisjoint, AllDisjoint, AllDisjointImPLY, BSsubset, Union(BS/R)subset, UnionIdentity, EqSysIsPartition, (x1), (x2), (y1), (y2), (v1), (v2), (v3), (v4), (v2n), (m1), (m2), (n1), (n2), (n3), ( $\epsilon$ ), ( $\epsilon$ )<sub>1</sub>, ( $\epsilon$ )<sub>2</sub>, (fep), (fx), (fy), (fz), (fu), (fv), (fw), (rx), (ry), (rz), (ru), (sx), (sx1), (sy), (sy1), (sz), (sz1), (su), (su1), (fxs), (fys), (crs1), (f1), (f2), (f3), (f4), (op1), (op2),

(r1), (s1), (s2), X<sub>1</sub>, X<sub>2</sub>, Y<sub>1</sub>, Y<sub>2</sub>, V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub>, V<sub>4</sub>, V<sub>2n</sub>, M<sub>1</sub>, M<sub>2</sub>, N<sub>1</sub>, N<sub>2</sub>, N<sub>3</sub>,  $\epsilon$ ,  $\epsilon_1$ ,  $\epsilon_2$ , FX, FY, FZ, FU, FV, FW, FEP, RX, RY, RZ, RU, (SX), (SX1), (SY), (SY1), (SZ), (SZ1), (SU), (SU1), FXS, FYS, (F1), (F2), (F3), (F4), (OP1), (OP2), (R1), (S1), (S2), (EPob), (CRS1ob), (F1ob), (F2ob), (F3ob), (F4ob), (N1ob), (N2ob), (OP1ob), (OP2ob), (R1ob), (S1ob), (S2ob), ph<sub>4</sub>, ph<sub>5</sub>, ph<sub>6</sub>, NAT, RATIONAL<sub>S</sub>ERIES, SERIES, SetOfReals, SetOfFxs, N, Q, X, xs, xaF, ysF, us, usFoelge, 0, 1, (-1), 2, 3, 1/2, 1/3, 2/3, 0f, 1f, 00, 01, (- - 01), 02, 01//02, PlusAssociativity(R), PlusAssociativity(R)XX, Plus0(R), Negative(R), Times1(R), lessAddition(R), PlusCommutativity(R), LeqAntisymmetry(R), LeqTransitivity(R), leqAddition(R), Distribution(R), A4(Axiom), InductionAxiom, EqualityAxiom, EqLeqAxiom, EqAdditionAxiom, EqMultiplicationAxiom, QisClosed(Reciprocal)(ImPLY), QisClosed(Reciprocal), QisClosed(Negative)(ImPLY), QisClosed(Negative), leqReflexivity, leqAntisymmetryAxiom, leqTransitivityAxiom, leqTotality, leqAdditionAxiom, leqMultiplicationAxiom, plusAssociativity, plusCommutativity, Negative, plus0, timesAssociativity, timesCommutativity, ReciprocalAxiom, times1, Distribution, 0not1, lemma eqLeq(R), TimesAssociativity(R), TimesCommutativity(R), (Adgic)SameR, Separation2formula(1), Separation2formula(2), Cauchy, PlusF, ReciprocalF, From ==, To ==, FromInR, PlusR(Sym), ReciprocalR(Axiom), LessMinus1(N), Nonnegative(N), US0, NextXS(UpperBound), NextXS(NoUpperBound), NextUS(UpperBound), NextUS(NoUpperBound), ExpZero, ExpPositive, ExpZero(R), ExpPositive(R), BSzero, BSpositive, USteleScope(Zero), USteleScope(Positive), EqAddition(R), FromLimit, ToUpperBound, FromUpperBound, USisUpperBound, 0not1(R), ExpUnbounded(R), FromLeq(Advanced)(N), FromLeastUpperBound, ToLeastUpperBound, XSisNotUpperBound, ysFGreater, ysFLess, SmallInverse, NatType, RationalType, SeriesType, Max, Numerical, NumericalF, MemberOfSeries(ImPLY), JoinConjuncts(2conditions), prop lemma imply negation, TND, FromNegatedImPLY, ToNegatedImPLY, FromNegated(2 \* ImPLY), FromNegatedAnd, FromNegatedOr, ToNegatedOr, FromNegations, From3Disjuncts, From2 \* 2Disjuncts, NegateDisjunct1, NegateDisjunct2, ExpandDisjuncts, SENC1, SENC2, LessLeq(R), MemberOfSeries, memberOfSeries(Type), \*(exp)\*, R(\*), - - R(\*), rec\*, \*/\*, \*  $\cap$  \*, \*[ \* ],  $\cup$ \*, \*  $\cup$  \*, P(\*), { \* }, StateExpand(\*, \*, \*), extractSeries(\*), SetOfSeries(\*), - - Macro(\*), ExpandList(\*, \*, \*), \*\* Macro(\*), ++ Macro(\*), << Macro(\*), ||Macro(\*), 01//Macro(\*), UB(\*, \*), LUB(\*, \*), BS(\*, \*), USteleScope(\*, \*), (\*), |f \* |, |r \* |, Limit(\*, \*), Union(\*), IsOrderedPair(\*, \*, \*), IsRelation(\*, \*, \*), isFunction(\*, \*, \*), IsSeries(\*, \*), IsNatural(\*, \*), OrderedPair(\*, \*), TypeNat(\*), TypeNat0(\*), TypeRational(\*), TypeRational0(\*), TypeSeries(\*, \*), Typeseries0(\*, \*), { \* , \* }, < \* , \* >, (-u\*), -f\*, (- - \*), 1f/\*, 01//temp\*, \*( \* , \* ), ReflRel(\*, \*), SymRel(\*, \*), TransRel(\*, \*), EqRel(\*, \*), [  $\in$  ]\*, Partition(\*, \*), (\* \* \*), \* \* f \*, \* \* \* \* , (\* + \*), (\* - \*), \* +f \*, \* -f \*, \* + + \*, R(\*) - -R(\*), \*  $\in$  \*, | \* |, if(\*, \*, \*), Max(\*, \*), Max(\*, \*), \* = \*, \*  $\neq$  \*, \* < = \*, \* < \*, \* < f \*, \*  $\leq$  f \*, SF(\*, \*), \* == \*, \* !! == \*, \* << \*, \* << = \*, \* == \*, \*  $\subseteq$  \*,  $\dot{\cup}$  (\* )n, \*  $\notin$  \*, \*  $\neq$  \*,

\*  $\wedge$  \*, \*  $\dot{\vee}$  \*,  $\exists$  \*: \*, \*  $\Leftrightarrow$  \*, {ph  $\in$  \* | \*},

$\exists$  \*: \*

[ $\exists$ x: y  $\xrightarrow{\text{tex}}$  "(AARRGGHH!-exist-bug!)"]

\*  $\Rightarrow$  \*

[x  $\Rightarrow$  y  $\xrightarrow{\text{tex}}$  "(i#1.  
\ $\rightarrow$  #2.  
)i"]

kvanti

[kvanti  $\xrightarrow{\text{prio}}$

**Preassociative**

[kvanti], [base], [bracket \* end bracket], [big bracket \* end bracket], [ \$ \* \$ ],  
[flush left [\*]], [x], [y], [z], [[\*  $\bowtie$  \*]], [[\*  $\rightarrow$  \*]], [pyk], [tex], [name], [prio], [\*], [T],  
[if(\*, \*, \*)], [[\*  $\Rightarrow$  \*]], [val], [claim], [ $\perp$ ], [f(\*)], [(\*)<sup>I</sup>], [F], [0], [1], [2], [3], [4], [5], [6],  
[7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],  
[k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(\*)<sup>M</sup>], [If(\*, \*, \*)],  
[array{\*} \* end array], [l], [c], [r], [empty], [( \* | \* := \* )], [ $\mathcal{M}$ (\*)], [ $\tilde{\mathcal{U}}$ (\*)], [ $\mathcal{U}$ (\*)],  
[ $\mathcal{U}^M$ (\*)], [apply(\*, \*)], [apply<sub>1</sub>(\*, \*)], [identifier(\*)], [identifier<sub>1</sub>(\*, \*)], [array-  
plus(\*, \*)], [array-remove(\*, \*, \*)], [array-put(\*, \*, \*, \*)], [array-add(\*, \*, \*, \*, \*)],  
[bit(\*, \*)], [bit<sub>1</sub>(\*, \*)], [rack], ["vector"], ["bibliography"], ["dictionary"],  
["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],  
["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],  
["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],  
["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],  
[ $\mathcal{E}$ (\*, \*, \*)], [ $\mathcal{E}_2$ (\*, \*, \*, \*, \*)], [ $\mathcal{E}_3$ (\*, \*, \*, \*, \*)], [ $\mathcal{E}_4$ (\*, \*, \*, \*, \*)], [lookup(\*, \*, \*)],  
[abstract(\*, \*, \*, \*)], [[\*]], [ $\mathcal{M}$ (\*, \*, \*)], [ $\mathcal{M}_2$ (\*, \*, \*, \*)], [ $\mathcal{M}^*$ (\*, \*, \*)], [macro],  
[s<sub>0</sub>], [zip(\*, \*)], [assoc<sub>1</sub>(\*, \*, \*)], [(\*)<sup>P</sup>], [self], [[\*  $\doteq$  \*]], [[\*  $\doteq$  \*]], [[\*  $\doteq$  \*]],  
[[\*  $\xrightarrow{\text{pyk}}$  \*]], [[\*  $\xrightarrow{\text{tex}}$  \*]], [[\*  $\xrightarrow{\text{name}}$  \*]], [Priority table[\*]], [ $\tilde{\mathcal{M}}_1$ ], [ $\tilde{\mathcal{M}}_2$ (\*)], [ $\tilde{\mathcal{M}}_3$ (\*)],  
[ $\tilde{\mathcal{M}}_4$ (\*, \*, \*, \*, \*)], [ $\mathcal{M}$ (\*, \*, \*)], [ $\tilde{\mathcal{Q}}$ (\*, \*, \*)], [ $\tilde{\mathcal{Q}}_2$ (\*, \*, \*)], [ $\tilde{\mathcal{Q}}_3$ (\*, \*, \*, \*)], [ $\tilde{\mathcal{Q}}^*$ (\*, \*, \*)],  
[(\*)], [(\*)], [display(\*)], [statement(\*)], [[\*<sup>·</sup>]], [[\*<sup>-</sup>]], [aspect(\*, \*)],  
[aspect(\*, \*, \*)], [(\*)], [tuple<sub>1</sub>(\*)], [tuple<sub>2</sub>(\*)], [let<sub>2</sub>(\*, \*)], [let<sub>1</sub>(\*, \*)],  
[[\*  $\xrightarrow{\text{claim}}$  \*]], [checker], [check(\*, \*)], [check<sub>2</sub>(\*, \*, \*)], [check<sub>3</sub>(\*, \*, \*)],  
[check<sup>\*</sup>(\*, \*)], [check<sub>2</sub><sup>\*</sup>(\*, \*, \*)], [[\*<sup>·</sup>]], [[\*<sup>-</sup>]], [[\*<sup>°</sup>]], [msg], [[\*  $\xrightarrow{\text{msg}}$  \*]], [<stmt>],  
[stmt], [[\*  $\xrightarrow{\text{stmt}}$  \*]], [HeadNil'], [HeadPair'], [Transitivity'], [ $\perp$ ], [Contra'], [T<sub>E</sub>],  
[L<sub>1</sub>], [ $\underline{\ast}$ ], [A], [B], [C], [D], [E], [F], [G], [H], [I], [J], [K], [L], [M], [N], [O], [P], [Q],  
[R], [S], [T], [U], [V], [W], [X], [Y], [Z], [( \* | \* := \* )], [( \* \* | \* := \* )], [ $\emptyset$ ], [Remainder],

$(\ast)^{\vee}$ ,  $[\text{intro}(\ast, \ast, \ast, \ast)]$ ,  $[\text{intro}(\ast, \ast, \ast)]$ ,  $[\text{error}(\ast, \ast)]$ ,  $[\text{error}_2(\ast, \ast)]$ ,  $[\text{proof}(\ast, \ast, \ast)]$ ,  
 $[\text{proof}_2(\ast, \ast)]$ ,  $[\mathcal{S}(\ast, \ast)]$ ,  $[\mathcal{S}^I(\ast, \ast)]$ ,  $[\mathcal{S}^{\triangleright}(\ast, \ast)]$ ,  $[\mathcal{S}^{\triangleright}(\ast, \ast, \ast)]$ ,  $[\mathcal{S}^E(\ast, \ast)]$ ,  $[\mathcal{S}_1^E(\ast, \ast, \ast)]$ ,  
 $[\mathcal{S}^+(\ast, \ast)]$ ,  $[\mathcal{S}_1^+(\ast, \ast, \ast)]$ ,  $[\mathcal{S}^-(\ast, \ast)]$ ,  $[\mathcal{S}_1^-(\ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\ast}(\ast, \ast)]$ ,  $[\mathcal{S}_1^{\ast}(\ast, \ast, \ast)]$ ,  
 $[\mathcal{S}_2^{\ast}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\textcircled{\ast}}(\ast, \ast)]$ ,  $[\mathcal{S}_1^{\textcircled{\ast}}(\ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\perp}(\ast, \ast)]$ ,  $[\mathcal{S}_1^{\perp}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\#}(\ast, \ast)]$ ,  
 $[\mathcal{S}_1^{\#}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\text{i.e.}}(\ast, \ast)]$ ,  $[\mathcal{S}_1^{\text{i.e.}}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}_2^{\text{i.e.}}(\ast, \ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\vee}(\ast, \ast)]$ ,  
 $[\mathcal{S}_1^{\vee}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}^{\text{i}}(\ast, \ast)]$ ,  $[\mathcal{S}_1^{\text{i}}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{S}_2^{\text{i}}(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{T}(\ast)]$ ,  $[\text{claims}(\ast, \ast, \ast)]$ ,  
 $[\text{claims}_2(\ast, \ast, \ast)]$ ,  $[\text{<proof>}]$ ,  $[\text{proof}]$ ,  $[[\text{Lemma } \ast : \ast]]$ ,  $[[\text{Proof of } \ast : \ast]]$ ,  
 $[[\ast \text{ lemma } \ast : \ast]]$ ,  $[[\ast \text{ antilemma } \ast : \ast]]$ ,  $[[\ast \text{ rule } \ast : \ast]]$ ,  $[[\ast \text{ antirule } \ast : \ast]]$ ,  
 $[\text{verifier}]$ ,  $[\mathcal{V}_1(\ast)]$ ,  $[\mathcal{V}_2(\ast, \ast)]$ ,  $[\mathcal{V}_3(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{V}_4(\ast, \ast)]$ ,  $[\mathcal{V}_5(\ast, \ast, \ast, \ast)]$ ,  $[\mathcal{V}_6(\ast, \ast, \ast, \ast)]$ ,  
 $[\mathcal{V}_7(\ast, \ast, \ast, \ast)]$ ,  $[\text{Cut}(\ast, \ast)]$ ,  $[\text{Head}_{\oplus}(\ast)]$ ,  $[\text{Tail}_{\oplus}(\ast)]$ ,  $[\text{rule}_1(\ast, \ast)]$ ,  $[\text{rule}(\ast, \ast)]$ ,  
 $[\text{Rule tactic}]$ ,  $[\text{Plus}(\ast, \ast)]$ ,  $[[\text{Theory } \ast]]$ ,  $[\text{theory}_2(\ast, \ast)]$ ,  $[\text{theory}_3(\ast, \ast)]$ ,  
 $[\text{theory}_4(\ast, \ast, \ast)]$ ,  $[\text{HeadNil}''']$ ,  $[\text{HeadPair}''']$ ,  $[\text{Transitivity}''']$ ,  $[\text{Contra}''']$ ,  $[\text{HeadNil}]$ ,  
 $[\text{HeadPair}]$ ,  $[\text{Transitivity}]$ ,  $[\text{Contra}]$ ,  $[\text{T}_E]$ ,  $[\text{ragged right}]$ ,  
 $[\text{ragged right expansion}]$ ,  $[\text{parm}(\ast, \ast, \ast)]$ ,  $[\text{parm}^{\ast}(\ast, \ast, \ast)]$ ,  $[\text{inst}(\ast, \ast)]$ ,  
 $[\text{inst}^{\ast}(\ast, \ast)]$ ,  $[\text{occur}(\ast, \ast, \ast)]$ ,  $[\text{occur}^{\ast}(\ast, \ast, \ast)]$ ,  $[\text{unify}(\ast = \ast, \ast)]$ ,  $[\text{unify}^{\ast}(\ast = \ast, \ast)]$ ,  
 $[\text{unify}_2(\ast = \ast, \ast)]$ ,  $[\text{L}_a]$ ,  $[\text{L}_b]$ ,  $[\text{L}_c]$ ,  $[\text{L}_d]$ ,  $[\text{L}_e]$ ,  $[\text{L}_f]$ ,  $[\text{L}_g]$ ,  $[\text{L}_h]$ ,  $[\text{L}_i]$ ,  $[\text{L}_j]$ ,  $[\text{L}_k]$ ,  $[\text{L}_l]$ ,  $[\text{L}_m]$ ,  
 $[\text{L}_n]$ ,  $[\text{L}_o]$ ,  $[\text{L}_p]$ ,  $[\text{L}_q]$ ,  $[\text{L}_r]$ ,  $[\text{L}_s]$ ,  $[\text{L}_t]$ ,  $[\text{L}_u]$ ,  $[\text{L}_v]$ ,  $[\text{L}_w]$ ,  $[\text{L}_x]$ ,  $[\text{L}_y]$ ,  $[\text{L}_z]$ ,  $[\text{L}_A]$ ,  $[\text{L}_B]$ ,  $[\text{L}_C]$ ,  
 $[\text{L}_D]$ ,  $[\text{L}_E]$ ,  $[\text{L}_F]$ ,  $[\text{L}_G]$ ,  $[\text{L}_H]$ ,  $[\text{L}_I]$ ,  $[\text{L}_J]$ ,  $[\text{L}_K]$ ,  $[\text{L}_L]$ ,  $[\text{L}_M]$ ,  $[\text{L}_N]$ ,  $[\text{L}_O]$ ,  $[\text{L}_P]$ ,  $[\text{L}_Q]$ ,  $[\text{L}_R]$ ,  
 $[\text{L}_S]$ ,  $[\text{L}_T]$ ,  $[\text{L}_U]$ ,  $[\text{L}_V]$ ,  $[\text{L}_W]$ ,  $[\text{L}_X]$ ,  $[\text{L}_Y]$ ,  $[\text{L}_Z]$ ,  $[\text{L}_?]$ ,  $[\text{Reflexivity}]$ ,  $[\text{Reflexivity}_1]$ ,  
 $[\text{Commutativity}]$ ,  $[\text{Commutativity}_1]$ ,  $[\text{<tactic>}]$ ,  $[\text{tactic}]$ ,  $[[\ast^{\text{tactic}} \ast]]$ ,  $[\mathcal{P}(\ast, \ast, \ast)]$ ,  
 $[\mathcal{P}^{\ast}(\ast, \ast, \ast)]$ ,  $[\text{p}_0]$ ,  $[\text{conclude}_1(\ast, \ast)]$ ,  $[\text{conclude}_2(\ast, \ast, \ast)]$ ,  $[\text{conclude}_3(\ast, \ast, \ast, \ast)]$ ,  
 $[\text{conclude}_4(\ast, \ast)]$ ,  $[\text{check}]$ ,  $[[\ast \stackrel{\circ}{=} \ast]]$ ,  $[\text{RootVisible}(\ast)]$ ,  $[\text{A}]$ ,  $[\text{R}]$ ,  $[\text{C}]$ ,  $[\text{T}]$ ,  $[\text{L}]$ ,  $[\{\ast\}]$ ,  $[\bar{\ast}]$ ,  
 $[a]$ ,  $[b]$ ,  $[c]$ ,  $[d]$ ,  $[e]$ ,  $[f]$ ,  $[g]$ ,  $[h]$ ,  $[i]$ ,  $[j]$ ,  $[k]$ ,  $[l]$ ,  $[m]$ ,  $[n]$ ,  $[o]$ ,  $[p]$ ,  $[q]$ ,  $[r]$ ,  $[s]$ ,  $[t]$ ,  $[u]$ ,  $[v]$ ,  
 $[w]$ ,  $[x]$ ,  $[y]$ ,  $[z]$ ,  $[(\ast \equiv \ast \mid \ast := \ast)]$ ,  $[(\ast \equiv^0 \ast \mid \ast := \ast)]$ ,  $[(\ast \equiv^1 \ast \mid \ast := \ast)]$ ,  $[(\ast \equiv^{\ast} \ast \mid \ast := \ast)]$ ,  
 $[\text{Ded}(\ast, \ast)]$ ,  $[\text{Ded}_0(\ast, \ast)]$ ,  $[\text{Ded}_1(\ast, \ast, \ast)]$ ,  $[\text{Ded}_2(\ast, \ast, \ast)]$ ,  $[\text{Ded}_3(\ast, \ast, \ast, \ast)]$ ,  
 $[\text{Ded}_4(\ast, \ast, \ast, \ast)]$ ,  $[\text{Ded}_4^{\ast}(\ast, \ast, \ast, \ast)]$ ,  $[\text{Ded}_5(\ast, \ast, \ast)]$ ,  $[\text{Ded}_6(\ast, \ast, \ast, \ast)]$ ,  
 $[\text{Ded}_6^{\ast}(\ast, \ast, \ast, \ast)]$ ,  $[\text{Ded}_7(\ast)]$ ,  $[\text{Ded}_8(\ast, \ast)]$ ,  $[\text{Ded}_8^{\ast}(\ast, \ast)]$ ,  $[\text{S}]$ ,  $[\text{Neg}]$ ,  $[\text{MP}]$ ,  $[\text{Gen}]$ ,  
 $[\text{Ded}]$ ,  $[\text{S1}]$ ,  $[\text{S2}]$ ,  $[\text{S3}]$ ,  $[\text{S4}]$ ,  $[\text{S5}]$ ,  $[\text{S6}]$ ,  $[\text{S7}]$ ,  $[\text{S8}]$ ,  $[\text{S9}]$ ,  $[\text{Repetition}]$ ,  $[\text{A1}']$ ,  $[\text{A2}']$ ,  $[\text{A4}']$ ,  
 $[\text{A5}']$ ,  $[\text{Prop 3.2a}]$ ,  $[\text{Prop 3.2b}]$ ,  $[\text{Prop 3.2c}]$ ,  $[\text{Prop 3.2d}]$ ,  $[\text{Prop 3.2e}_1]$ ,  $[\text{Prop 3.2e}_2]$ ,  
 $[\text{Prop 3.2e}]$ ,  $[\text{Prop 3.2f}_1]$ ,  $[\text{Prop 3.2f}_2]$ ,  $[\text{Prop 3.2f}]$ ,  $[\text{Prop 3.2g}_1]$ ,  $[\text{Prop 3.2g}_2]$ ,  
 $[\text{Prop 3.2g}]$ ,  $[\text{Prop 3.2h}_1]$ ,  $[\text{Prop 3.2h}_2]$ ,  $[\text{Prop 3.2h}]$ ,  $[\text{Block}_1(\ast, \ast, \ast)]$ ,  $[\text{Block}_2(\ast)]$ ,  
 $[\text{UniqueMember}]$ ,  $[\text{UniqueMember}(\text{Type})]$ ,  $[\text{SameSeries}]$ ,  $[\text{A4}]$ ,  $[\text{SameMember}]$ ,  
 $[\text{Qclosed}(\text{Addition})]$ ,  $[\text{Qclosed}(\text{Multiplication})]$ ,  $[\text{FromCartProd}(1)]$ ,  
 $[\text{Irule fromCartProd}(2)]$ ,  $[\text{constantRationalSeries}(\ast)]$ ,  $[\text{cartProd}(\ast)]$ ,  $[\text{Power}(\ast)]$ ,  
 $[\text{binaryUnion}(\ast, \ast)]$ ,  $[\text{SetOfRationalSeries}]$ ,  $[\text{IsSubset}(\ast, \ast)]$ ,  $[(\ast, \ast)]$ ,  $[(\ast, \ast)]$ ,  
 $[(\cdot \cdot \cdot)]$ ,  $[\text{Objekt-var}]$ ,  $[\text{Ex-var}]$ ,  $[\text{Ph-var}]$ ,  $[\text{Værdi}]$ ,  $[\text{Variabel}]$ ,  $[\text{Op}(\ast)]$ ,  $[\text{Op}(\ast, \ast)]$ ,  
 $[\ast \equiv \ast]$ ,  $[\text{ContainsEmpty}(\ast)]$ ,  $[\text{Nat}(\ast)]$ ,  $[\text{Dedu}(\ast, \ast)]$ ,  $[\text{Dedu}_0(\ast, \ast)]$ ,  
 $[\text{Dedu}_s(\ast, \ast, \ast)]$ ,  $[\text{Dedu}_1(\ast, \ast, \ast)]$ ,  $[\text{Dedu}_2(\ast, \ast, \ast)]$ ,  $[\text{Dedu}_3(\ast, \ast, \ast, \ast)]$ ,  
 $[\text{Dedu}_4(\ast, \ast, \ast, \ast)]$ ,  $[\text{Dedu}_4^{\ast}(\ast, \ast, \ast, \ast)]$ ,  $[\text{Dedu}_5(\ast, \ast, \ast)]$ ,  $[\text{Dedu}_6(\ast, \ast, \ast, \ast)]$ ,  
 $[\text{Dedu}_6^{\ast}(\ast, \ast, \ast, \ast)]$ ,  $[\text{Dedu}_7(\ast)]$ ,  $[\text{Dedu}_8(\ast, \ast)]$ ,  $[\text{Dedu}_8^{\ast}(\ast, \ast)]$ ,  $[\text{EX}_1]$ ,  $[\text{EX}_2]$ ,  $[\text{EX}_3]$ ,  
 $[\text{EX}_{10}]$ ,  $[\text{EX}_{20}]$ ,  $[\ast_{\text{Ex}}]$ ,  $[\ast^{\text{Ex}}]$ ,  $[(\ast \equiv \ast \mid \ast := \ast)_{\text{Ex}}]$ ,  $[(\ast \equiv^0 \ast \mid \ast := \ast)_{\text{Ex}}]$ ,  
 $[(\ast \equiv^1 \ast \mid \ast := \ast)_{\text{Ex}}]$ ,  $[(\ast \equiv^{\ast} \ast \mid \ast := \ast)_{\text{Ex}}]$ ,  $[\text{ph}_1]$ ,  $[\text{ph}_2]$ ,  $[\text{ph}_3]$ ,  $[\ast_{\text{Ph}}]$ ,  $[\ast^{\text{Ph}}]$ ,  
 $[(\ast \equiv \ast \mid \ast := \ast)_{\text{Ph}}]$ ,  $[(\ast \equiv^0 \ast \mid \ast := \ast)_{\text{Ph}}]$ ,  $[(\ast \equiv^1 \ast \mid \ast := \ast)_{\text{Ph}}]$ ,  
 $[(\ast \equiv^{\ast} \ast \mid \ast := \ast)_{\text{Ph}}]$ ,  $[(\ast \equiv \ast \mid \ast := \ast)_{\text{Me}}]$ ,  $[(\ast \equiv^1 \ast \mid \ast := \ast)_{\text{Me}}]$ ,  
 $[(\ast \equiv^{\ast} \ast \mid \ast := \ast)_{\text{Me}}]$ ,  $[\text{bs}]$ ,  $[\text{OBS}]$ ,  $[\mathcal{BS}]$ ,  $[\emptyset]$ ,  $[\text{SystemQ}]$ ,  $[\text{MP}]$ ,  $[\text{Gen}]$ ,  $[\text{Repetition}]$ ,

[Neg], [Ded], [ExistIntro], [Extensionality], [Ødef], [PairDef], [UnionDef],  
 [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg],  
 [AndCommutativity], [AutoImPLY], [Contrapositive], [FirstConjunct],  
 [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity],  
 [IffFirst], [IffSecond], [ImPLYTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4],  
 [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1],  
 [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],  
 [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power],  
 [SubsetInPower], [HelperPowerIsSub], [PowerIsSub],  
 [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality],  
 [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality],  
 [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry],  
 [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive],  
 [ERisSymmetric], [ERisTransitive], [ØisSubset], [HelperMemberNotØ],  
 [MemberNotØ], [HelperUniqueØ], [UniqueØ], [= Reflexivity], [= Symmetry],  
 [Helper == Transitivity], [= Transitivity], [HelperTransferNotEq],  
 [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset],  
 [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset],  
 [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection],  
 [AutoMember], [HelperEqSysNotØ], [EqSysNotØ], [HelperEqSubset],  
 [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary],  
 [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset],  
 [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImPLY], [BSsubset],  
 [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)],  
 [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ε)],  
 [(ε<sub>1</sub>)], [(ε<sub>2</sub>)], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)],  
 [(ru)], [(sx)], [(sx<sub>1</sub>)], [(sy)], [(sy<sub>1</sub>)], [(sz)], [(sz<sub>1</sub>)], [(su)], [(su<sub>1</sub>)], [(fxs)], [(fys)],  
 [(crs<sub>1</sub>)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X<sub>1</sub>], [X<sub>2</sub>],  
 [Y<sub>1</sub>], [Y<sub>2</sub>], [V<sub>1</sub>], [V<sub>2</sub>], [V<sub>3</sub>], [V<sub>4</sub>], [V<sub>2n</sub>], [M<sub>1</sub>], [M<sub>2</sub>], [N<sub>1</sub>], [N<sub>2</sub>], [N<sub>3</sub>], [ε], [ε<sub>1</sub>], [ε<sub>2</sub>],  
 [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX<sub>1</sub>)],  
 [(SY)], [(SY<sub>1</sub>)], [(SZ)], [(SZ<sub>1</sub>)], [(SU)], [(SU<sub>1</sub>)], [FXS], [FYS], [(F1)], [(F2)], [(F3)],  
 [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)],  
 [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)],  
 [(S1ob)], [(S2ob)], [ph<sub>4</sub>], [ph<sub>5</sub>], [ph<sub>6</sub>], [NAT], [RATIONALSERIES], [SERIES],  
 [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1],  
 [(-1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(- - 01)], [02], [01/02],  
 [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)],  
 [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)],  
 [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)],  
 [Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom],  
 [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom],  
 [QisClosed(Reciprocal)(ImPLY)], [QisClosed(Reciprocal)],  
 [QisClosed(Negative)(ImPLY)], [QisClosed(Negative)], [leqReflexivity],  
 [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality],  
 [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity],  
 [plusCommutativity], [Negative], [plus0], [timesAssociativity],

[timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1], [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)], [(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [UStelescope(Zero)], [UStelescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [NatType], [RationalType], [SeriesType], [Max], [Numerical], [NumericalF], [MemberOfSeries(ImPLY)], [JoinConjuncts(2conditions)], [prop lemma imply negation], [TND], [FromNegatedImPLY], [ToNegatedImPLY], [FromNegated(2 \* ImPLY)], [FromNegatedAnd], [FromNegatedOr], [ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 \* 2Disjuncts], [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2], [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)];

### Preassociative

[\*\_{\*}], [\* /indexintro(\*, \*, \*, \*)], [\* /intro(\*, \*, \*)], [\* /bothintro(\*, \*, \*, \*, \*)], [\* /nameintro(\*, \*, \*, \*)], [\*'], [\* [\* ]], [\* [\* → \*]], [\* [\* ⇒ \*]], [\* 0], [\* 1], [0b], [\* -color(\*)], [\* -color \* (\*)], [\*<sup>H</sup>], [\*<sup>T</sup>], [\*<sup>U</sup>], [\*<sup>h</sup>], [\*<sup>t</sup>], [\*<sup>s</sup>], [\*<sup>c</sup>], [\*<sup>d</sup>], [\*<sup>a</sup>], [\*<sup>C</sup>], [\*<sup>M</sup>], [\*<sup>B</sup>], [\*<sup>f</sup>], [\*<sup>i</sup>], [\*<sup>d</sup>], [\*<sup>R</sup>], [\*<sup>0</sup>], [\*<sup>1</sup>], [\*<sup>2</sup>], [\*<sup>3</sup>], [\*<sup>4</sup>], [\*<sup>5</sup>], [\*<sup>6</sup>], [\*<sup>7</sup>], [\*<sup>8</sup>], [\*<sup>9</sup>], [\*<sup>E</sup>], [\*<sup>V</sup>], [\*<sup>C</sup>], [\*<sup>C\*</sup>], [\*hide];

### Preassociative

[“ \* ”], [], [(\*)<sup>t</sup>], [string(\*) + \*], [string(\*) ++ \*], [\*, [ \*], [! \*], [\" \*], [# \*], [\$ \*], [% \*], [& \*], [\' \*], [(\*)], [D \*], [\*\*], [+ \*], [ , \*], [- \*], [ . \*], [/ \*], [0 \*], [1 \*], [2 \*], [3 \*], [4 \*], [5 \*], [6 \*], [7 \*], [8 \*], [9 \*], [: \*], [; \*], [< \*], [= \*], [> \*], [? \*], [@ \*], [A \*], [B \*], [C \*], [D \*], [E \*], [F \*], [G \*], [H \*], [I \*], [J \*], [K \*], [L \*], [M \*], [N \*], [O \*], [P \*], [Q \*], [R \*], [S \*], [T \*], [U \*], [V \*], [W \*], [X \*], [Y \*], [Z \*], [[ \*], [\ \*], [ ] \*], [^ \*], [ \_ \*], [ ` \*], [ a \*], [ b \*], [ c \*], [ d \*], [ e \*], [ f \*], [ g \*], [ h \*], [ i \*], [ j \*], [ k \*], [ l \*], [ m \*], [ n \*], [ o \*], [ p \*], [ q \*], [ r \*], [ s \*], [ t \*], [ u \*], [ v \*], [ w \*], [ x \*], [ y \*], [ z \*], [{ \*}, [ | \*], [ } \*], [ ~ \*], [Preassociative \*; \*], [Postassociative \*; \*], [ [ \*], \* ], [priority \* end], [newline \*], [macro newline \*], [MacroIndent(\*)];

### Preassociative

[\* ' \*], [\* ' \*];

### Preassociative

[\*(exp)\*];

### Preassociative

[\*'], [R(\*)], [- - R(\*)], [rec\*];

### Preassociative

[\*/ \*], [\* ∩ \*], [\* \*];

### Preassociative

[∪ \*], [\* ∪ \*], [P(\*)];

### Preassociative

[{\*}], [StateExpand(\*, \*, \*)], [extractSeries(\*)], [SetOfSeries(\*)], [— — Macro(\*)],  
[ExpandList(\*, \*, \*)], [\* \* Macro(\*)], [+ + Macro(\*)], [<< Macro(\*)],  
[|Macro(\*)], [01//Macro(\*)], [UB(\*, \*)], [LUB(\*, \*)], [BS(\*, \*)],  
[UStescope(\*, \*)], [(\*)], [|f \* |], [|r \* |], [Limit(\*, \*)], [Union(\*)],  
[IsOrderedPair(\*, \*, \*)], [IsRelation(\*, \*, \*)], [isFunction(\*, \*, \*)], [IsSeries(\*, \*)],  
[IsNatural(\*, \*)], [OrderedPair(\*, \*)], [TypeNat(\*)], [TypeNat0(\*)],  
[TypeRational(\*)], [TypeRational0(\*)], [TypeSeries(\*, \*)], [Typeseries0(\*, \*)];

**Preassociative**

[{\* , \*}], [(<\*, \*)], [(-u\*)], [-\_f\*], [(- - \*)], [1f/\*], [01//temp\*];

**Preassociative**

[\*(\*, \*)], [RefRel(\*, \*)], [SymRel(\*, \*)], [TransRel(\*, \*)], [EqRel(\*, \*)], [[\* ∈ \*]\_\*],  
[Partition(\*, \*)];

**Preassociative**

[\* · \*], [\* · 0 \*], [(\*\* \*\*)], [\* \*\_f \*], [\* \*\* \*\*];

**Preassociative**

[\* + \*], [\* + 0 \*], [\* + 1 \*], [\* - \*], [\* - 0 \*], [\* - 1 \*], [(\* + \*)], [(\* - \*)], [\* +\_f \*],  
[\* -\_f \*], [\* + +\*], [R(\*) - R(\*)];

**Preassociative**

[\* ∈ \*];

**Preassociative**

[| \* |], [if(\*, \*, \*)], [Max(\*, \*)], [Max(\*, \*)];

**Preassociative**

[\* = \*], [\* ≠ \*], [\* <= \*], [\* < \*], [\* <\_f \*], [\* ≤\_f \*], [SF(\*, \*)], [\* == \*],  
[\* !! == \*], [\* << \*], [\* << == \*];

**Preassociative**

[\* ∪ {\*}], [\* ∪ \*], [\* \ {\*}];

**Postassociative**

[\* ∴ \*], [\* ∴\_\*], [\* ∴\_\*], [\* +2\* \*], [\* ∴\_\*], [\* +2\* \*];

**Postassociative**

[\*, \*];

**Preassociative**

[\*  $\overset{B}{\approx}$  \*], [\*  $\overset{D}{\approx}$  \*], [\*  $\overset{C}{\approx}$  \*], [\*  $\overset{P}{\approx}$  \*], [\*  $\approx$  \*], [\* = \*], [\*  $\overset{+}{\vdash}$  \*], [\*  $\overset{t}{=}$  \*], [\*  $\overset{t^*}{=}$  \*], [\*  $\overset{r}{=}$  \*],  
[\* ∈\_t \*], [\* ⊆\_T \*], [\*  $\overset{T}{=}$  \*], [\*  $\overset{s}{=}$  \*], [\* free in \*], [\* free in\* \*], [\* free for \* in \*],  
[\* free for\* \* in \*], [\* ∈\_c \*], [\* < \*], [\* <' \*], [\* ≤' \*], [\* = \*], [\* ≠ \*], [\*<sup>var</sup>],  
[\* #<sup>0</sup> \*], [\* #<sup>1</sup> \*], [\* #\* \*], [\* == \*], [\* ⊆ \*];

**Preassociative**

[¬\*], [¬(\*n)], [\* ∉ \*], [\* ≠ \*];

**Preassociative**

[\* ∧ \*], [\*  $\overset{\sim}{\wedge}$  \*], [\*  $\overset{\sim}{\wedge}$  \*], [\* ∧\_c \*], [\*  $\overset{\sim}{\wedge}$  \*];

**Preassociative**

[\* ∨ \*], [\* || \*], [\*  $\overset{\sim}{\vee}$  \*];

**Postassociative**

[\*  $\overset{\sim}{\vee}$  \*];

**Preassociative**

[∃\*: \*], [∀\*: \*], [∀<sub>obj</sub>\*: \*], [∃\*: \*];

**Postassociative**

$[* \overset{\Rightarrow}{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \Leftrightarrow *];$

**Preassociative**

$[\{\text{ph} \in * | *\}];$

**Postassociative**

$[* : *], [* \text{ spy } *], [*! *];$

**Preassociative**

$[* \left\{ \begin{array}{c} * \\ * \end{array} \right.];$

**Preassociative**

$[\lambda * . *], [\Lambda * . *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \overset{=}{=} * \text{ in } *];$

**Preassociative**

$[* \# *];$

**Preassociative**

$[* \uparrow], [* \triangleright], [* \vee], [* +], [* -], [* *];$

**Preassociative**

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *], [* \triangleleft *];$

**Postassociative**

$[* \vdash *], [* \Vdash *], [* \text{ i.e. } *];$

**Preassociative**

$[\forall * : *], [\Pi * : *];$

**Postassociative**

$[* \oplus *];$

**Postassociative**

$[* ; *];$

**Preassociative**

$[* \text{ proves } *];$

**Preassociative**

$[* \text{ proof of } * : *], [\text{Line } * : * \gg * ; *], [\text{Last line } * \gg * \square],$   
 $[\text{Line } * : \text{Premise } \gg * ; *], [\text{Line } * : \text{Side-condition } \gg * ; *], [\text{Arbitrary } \gg * ; *],$   
 $[\text{Local } \gg * = * ; *], [\text{Begin } * ; * : \text{End}; *], [\text{Last block line } * \gg * ; *],$   
 $[\text{Arbitrary } \gg * ; *];$

**Postassociative**

$[* | *];$

**Postassociative**

$[* , *], [* [ * ] *];$

**Preassociative**

$[* \& *];$

**Preassociative**

$[* \\ *], [* \text{ linebreak}[4] *], [* \\ *];$

$[\text{kvarianti} \xrightarrow{\text{tex}} \text{"kvarianti"}]$

$[\text{kvarianti} \xrightarrow{\text{pyk}} \text{"kvarianti"}]$



## UniqueMember

[UniqueMember  $\xrightarrow{\text{tex}}$  “UniqueMember”]

[UniqueMember  $\xrightarrow{\text{pyk}}$  “lemma uniqueMember”]

## UniqueMember(Type)

[UniqueMember(Type)  $\xrightarrow{\text{tex}}$  “UniqueMember(Type)”]

[UniqueMember(Type)  $\xrightarrow{\text{pyk}}$  “lemma uniqueMember(Type)”]

## SameSeries

[SameSeries  $\xrightarrow{\text{tex}}$  “SameSeries”]

[SameSeries  $\xrightarrow{\text{pyk}}$  “lemma sameSeries”]

## A4

[A4  $\xrightarrow{\text{tex}}$  “A4”]

[A4  $\xrightarrow{\text{pyk}}$  “lemma a4”]

## SameMember

[SameMember  $\xrightarrow{\text{tex}}$  “SameMember”]

[SameMember  $\xrightarrow{\text{pyk}}$  “lemma sameMember”]

## Qclosed(Addition)

[Qclosed(Addition)  $\xrightarrow{\text{tex}}$  “Qclosed(Addition)”]

[Qclosed(Addition)  $\xrightarrow{\text{pyk}}$  “1rule Qclosed(Addition)”]

## Qclosed(Multiplication)

[Qclosed(Multiplication)  $\xrightarrow{\text{tex}}$  “Qclosed(Multiplication)”]

[Qclosed(Multiplication)  $\xrightarrow{\text{pyk}}$  “1rule Qclosed(Multiplication)”]

FromCartProd(1)

[FromCartProd(1)  $\xrightarrow{\text{tex}}$  “FromCartProd(1)”]

[FromCartProd(1)  $\xrightarrow{\text{pyk}}$  “1rule fromCartProd(1)”]

**1rule fromCartProd(2)**

[1rule fromCartProd(2)  $\xrightarrow{\text{pyk}}$  “1rule fromCartProd(2)”]

constantRationalSeries(\*)

[constantRationalSeries(x)  $\xrightarrow{\text{tex}}$  “constantRationalSeries(#1.  
)”]

[constantRationalSeries(\*)  $\xrightarrow{\text{pyk}}$  “constantRationalSeries( " )”]

cartProd(\*)

[cartProd(x)  $\xrightarrow{\text{tex}}$  “cartProd(#1.  
)”]

[cartProd(\*)  $\xrightarrow{\text{pyk}}$  “cartProd( " , " )”]

Power(\*)

[Power(x)  $\xrightarrow{\text{tex}}$  “Power(#1.  
)”]

[Power(\*)  $\xrightarrow{\text{pyk}}$  “P( " )”]

binaryUnion(\*, \*)

[binaryUnion(x, y)  $\xrightarrow{\text{tex}}$  “binaryUnion(#1.  
, #2.”]

)”]

[binaryUnion(\*, \*)  $\xrightarrow{\text{pyk}}$  “binaryUnion( " , " )”]

## SetOfRationalSeries

[SetOfRationalSeries  $\xrightarrow{\text{tex}}$  “SetOfRationalSeries”]

[SetOfRationalSeries  $\xrightarrow{\text{pyk}}$  “setOfRationalSeries”]

## IsSubset(\*, \*)

[IsSubset(x, y)  $\xrightarrow{\text{tex}}$  “IsSubset(#1.  
, #2.  
)”]

[IsSubset(\*, \*)  $\xrightarrow{\text{pyk}}$  “isSubset( " , " )”]

## (p\*, \*)

[(px, y)  $\xrightarrow{\text{tex}}$  “(p#1.  
, #2.  
)”]

[(p\*, \*)  $\xrightarrow{\text{pyk}}$  “(p " , " )”]

## (s\*)

[(sx)  $\xrightarrow{\text{tex}}$  “(s#1.  
)”]

[(s\*)  $\xrightarrow{\text{pyk}}$  “(s " )”]

## (...)

[(...)  $\xrightarrow{\text{tex}}$  “(\cdots{ })”]

[(...)  $\xrightarrow{\text{pyk}}$  “cdots”]

## Objekt-var

[Objekt-var  $\xrightarrow{\text{tex}}$  “\texttt{Objekt-var}”]

[Objekt-var  $\xrightarrow{\text{pyk}}$  “object-var”]

## Ex-var

[Ex-var  $\xrightarrow{\text{tex}}$  “\texttt{Ex-var}”]

[Ex-var  $\xrightarrow{\text{pyk}}$  “ex-var”]

## Ph-var

[Ph-var  $\xrightarrow{\text{tex}}$  “\texttt{Ph-var}”]

[Ph-var  $\xrightarrow{\text{pyk}}$  “ph-var”]

## Værdi

[Værdi  $\xrightarrow{\text{tex}}$  “\texttt{V\ae{}rdi}”]

[Værdi  $\xrightarrow{\text{pyk}}$  “vaerdi”]

## Variabel

[Variabel  $\xrightarrow{\text{tex}}$  “\texttt{Variabel}”]

[Variabel  $\xrightarrow{\text{pyk}}$  “variabel”]

## Op(\*)

[Op(x)  $\xrightarrow{\text{tex}}$  “Op(#1.  
)”]

[Op(\*)  $\xrightarrow{\text{pyk}}$  “op " end op”]

Op(\*, \*)

[Op(x, y)  $\xrightarrow{\text{tex}}$  “Op(#1.  
, #2.  
)”]

[Op(\*, \*)  $\xrightarrow{\text{pyk}}$  “op2 " comma " end op2”]

\*  $\stackrel{..}{=}$  \*

[x  $\stackrel{..}{=} y$   $\xrightarrow{\text{tex}}$  “#1.  
\mathrel {\ddot{=}} #2.”]

[\*  $\stackrel{..}{=} *$   $\xrightarrow{\text{pyk}}$  “define-equal " comma " end equal”]

ContainsEmpty(\*)

[ContainsEmpty(x)  $\xrightarrow{\text{tex}}$  “ContainsEmpty(#1.  
)”]

[ContainsEmpty(\*)  $\xrightarrow{\text{pyk}}$  “contains-empty " end empty”]

Nat(\*)

[Nat(x)  $\xrightarrow{\text{tex}}$  “Nat(#1.  
)”]

[Nat(\*)  $\xrightarrow{\text{pyk}}$  “Nat( " )”]

Dedu(\*, \*)

[Dedu(x, y)  $\xrightarrow{\text{tex}}$  “  
Dedu(#1.  
, #2.  
)”]

[Dedu(\*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction " conclude " end 1deduction”]

Dedu<sub>0</sub>(\* , \*)

[Dedu<sub>0</sub>(x, y)  $\xrightarrow{\text{tex}}$  “  
Dedu\_0(#1.  
, #2.  
)”]

[Dedu<sub>0</sub>(\* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction zero " conclude " end 1deduction”]

Dedu<sub>s</sub>(\* , \* , \*)

[Dedu<sub>s</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “Dedu\_{s}(#1.  
, #2.  
, #3.  
)”]

[Dedu<sub>s</sub>(\* , \* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction side " conclude " condition " end 1deduction”]

Dedu<sub>1</sub>(\* , \* , \*)

[Dedu<sub>1</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “  
Dedu\_1(#1.  
, #2.  
, #3.  
)”]

[Dedu<sub>1</sub>(\* , \* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction one " conclude " condition " end 1deduction”]

Dedu<sub>2</sub>(\* , \* , \*)

[Dedu<sub>2</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “  
Dedu\_2(#1.  
, #2.  
, #3.  
)”]

[Dedu<sub>2</sub>(\* , \* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction two " conclude " condition " end 1deduction”]

Dedu<sub>3</sub>(\* , \* , \* , \*)

[Dedu<sub>3</sub>(x, y, z, u)  $\xrightarrow{\text{tex}}$  “  
Dedu\_3(#1.  
, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>3</sub>(\* , \* , \* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction three " conclude " condition " bound " end  
1deduction”]

Dedu<sub>4</sub>(\* , \* , \* , \*)

[Dedu<sub>4</sub>(x, y, z, u)  $\xrightarrow{\text{tex}}$  “  
Dedu\_4(#1.  
, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>4</sub>(\* , \* , \* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction four " conclude " condition " bound " end  
1deduction”]

Dedu<sub>4</sub><sup>\*</sup>(\* , \* , \* , \*)

[Dedu<sub>4</sub><sup>\*</sup>(x, y, z, u)  $\xrightarrow{\text{tex}}$  “  
Dedu\_4^\*(#1.  
, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>4</sub><sup>\*</sup>(\* , \* , \* , \*)  $\xrightarrow{\text{pyk}}$  “1deduction four star " conclude " condition " bound "  
end 1deduction”]

Dedu<sub>5</sub>(\* , \* , \*)

[Dedu<sub>5</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “  
Dedu\_5(#1.  
, #2.

, #3.  
 )”]  
 $[\text{Dedu}_5(*, *, *) \xrightarrow{\text{pyk}} \text{“1deduction five " condition " bound " end 1deduction”}]$

$\text{Dedu}_6(*, *, *, *)$

$[\text{Dedu}_6(\mathbf{p}, \mathbf{c}, \mathbf{e}, \mathbf{b}) \xrightarrow{\text{tex}} \text{“}$   
 $\text{Dedu}_6(\#1.$   
 , #2.  
 , #3.  
 , #4.  
 )”]  
 $[\text{Dedu}_6(*, *, *, *) \xrightarrow{\text{pyk}} \text{“1deduction six " conclude " exception " bound " end 1deduction”}]$

$\text{Dedu}_6^*(*, *, *, *)$

$[\text{Dedu}_6^*(\mathbf{p}, \mathbf{c}, \mathbf{e}, \mathbf{b}) \xrightarrow{\text{tex}} \text{“}$   
 $\text{Dedu}_6^*(\#1.$   
 , #2.  
 , #3.  
 , #4.  
 )”]  
 $[\text{Dedu}_6^*(*, *, *, *) \xrightarrow{\text{pyk}} \text{“1deduction six star " conclude " exception " bound " end 1deduction”}]$

$\text{Dedu}_7(*)$

$[\text{Dedu}_7(\mathbf{p}) \xrightarrow{\text{tex}} \text{“}$   
 $\text{Dedu}_7(\#1.$   
 )”]  
 $[\text{Dedu}_7(*) \xrightarrow{\text{pyk}} \text{“1deduction seven " end 1deduction”}]$

$\text{Dedu}_8(*, *)$

$[\text{Dedu}_8(\mathbf{p}, \mathbf{b}) \xrightarrow{\text{tex}} \text{“}$   
 $\text{Dedu}_8(\#1.$



, #2.  
)]

[Dedu<sub>8</sub>(\* , \*)  $\xrightarrow{\text{pyk}}$  "1deduction eight " bound " end 1deduction"]

Dedu<sub>8</sub><sup>\*</sup>(\* , \*)

[Dedu<sub>8</sub><sup>\*</sup>(p, b)  $\xrightarrow{\text{tex}}$  "  
Dedu\_8^\*(#1.  
, #2.  
)"]

[Dedu<sub>8</sub><sup>\*</sup>(\* , \*)  $\xrightarrow{\text{pyk}}$  "1deduction eight star " bound " end 1deduction"]

EX<sub>1</sub>

[EX<sub>1</sub>  $\xrightarrow{\text{macro}}$  λt.λs.λc.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[EX_1 \ddot{=} a_{EX}]])$ ]]

[EX<sub>1</sub>  $\xrightarrow{\text{tex}}$  "EX\_{1}"]

[EX<sub>1</sub>  $\xrightarrow{\text{pyk}}$  "ex1"]

EX<sub>2</sub>

[EX<sub>2</sub>  $\xrightarrow{\text{macro}}$  λt.λs.λc.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[EX_2 \ddot{=} b_{EX}]])$ ]]

[EX<sub>2</sub>  $\xrightarrow{\text{tex}}$  "EX\_{2}"]

[EX<sub>2</sub>  $\xrightarrow{\text{pyk}}$  "ex2"]

EX<sub>3</sub>

[EX<sub>3</sub>  $\xrightarrow{\text{macro}}$  λt.λs.λc.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[EX_3 \ddot{=} c_{EX}]])$ ]]

[EX<sub>3</sub>  $\xrightarrow{\text{tex}}$  "EX3"]

[EX<sub>3</sub>  $\xrightarrow{\text{pyk}}$  "ex3"]

EX<sub>10</sub>

[EX<sub>10</sub>  $\xrightarrow{\text{macro}}$  λt.λs.λc.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[EX_{10} \ddot{=} j_{EX}]])$ ]]

[EX<sub>10</sub> <sup>tex</sup> → “EX\_{10}”]

[EX<sub>10</sub> <sup>pyk</sup> → “ex10”]

EX<sub>20</sub>

[EX<sub>20</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[EX_{20} \doteq t_{EX}]])$ ]

[EX<sub>20</sub> <sup>tex</sup> → “EX\_{20}”]

[EX<sub>20</sub> <sup>pyk</sup> → “ex20”]

\*EX

[x<sub>EX</sub> <sup>tex</sup> → “#1.  
\_{EX}”]

[\*EX <sup>pyk</sup> → “existential var " end var”]

\*<sup>Ex</sup>

[x<sup>Ex</sup> <sup>val</sup> → x <sup>r</sup> = [x<sub>EX</sub>]]

[x<sup>Ex</sup> <sup>tex</sup> → “#1.  
^ {EX}”]

[\*<sup>Ex</sup> <sup>pyk</sup> → “" is existential var”]

⟨\*≡\* | \* ::=\*⟩<sub>EX</sub>

[⟨a≡b|x::=t⟩<sub>EX</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[⟨a≡b|x::=t⟩_{EX} \doteq$   
⟨[a]≡<sup>0</sup>[b]|[x]::=[t]⟩<sub>EX</sub>]])]

[⟨x≡y|z::=u⟩<sub>EX</sub> <sup>tex</sup> → “\langle #1.

{\equiv} #2.

| #3.

{::=} #4.

\rangle\_{EX} ”]

[⟨\*≡\* | \* ::=\*⟩<sub>EX</sub> <sup>pyk</sup> → “exist-sub " is " where " is " end sub”]

$\langle * \equiv^0 * \mid * := * \rangle_{\text{Ex}}$

$[\langle a \equiv^0 b \mid x := t \rangle_{\text{Ex}} \xrightarrow{\text{val}} \lambda c. x^{\text{Ex}} \wedge \langle a \equiv^1 b \mid x := t \rangle_{\text{Ex}}]$

$[\langle x \equiv^0 y \mid z := u \rangle_{\text{Ex}} \xrightarrow{\text{tex}} \text{"\langle \#1.}$

$\{\equiv\}^0 \#2.$

$\mid \#3.$

$\{:=\} \#4.$

$\rangle_{\text{Ex}} \text{"}$

$[\langle * \equiv^0 * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub0 " is " where " is " end sub"}]$

$\langle * \equiv^1 * \mid * := * \rangle_{\text{Ex}}$

$[\langle a \equiv^1 b \mid x := t \rangle_{\text{Ex}} \xrightarrow{\text{val}} a!x!t!$

$\text{If}(b \stackrel{r}{=} [\forall_{\text{obj}} u; v], F,$

$\text{If}(b^{\text{Ex}} \wedge b \stackrel{t}{=} x, a \stackrel{t}{=} t, \text{If}(\$

$a \stackrel{r}{=} b, \langle a^t \equiv^* b^t \mid x := t \rangle_{\text{Ex}}, F)))]$

$[\langle x \equiv^1 y \mid z := u \rangle_{\text{Ex}} \xrightarrow{\text{tex}} \text{"\langle \#1.}$

$\{\equiv\}^1 \#2.$

$\mid \#3.$

$\{:=\} \#4.$

$\rangle_{\text{Ex}} \text{"}$

$[\langle * \equiv^1 * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub1 " is " where " is " end sub"}]$

$\langle * \equiv^* * \mid * := * \rangle_{\text{Ex}}$

$[\langle a \equiv^* b \mid x := t \rangle_{\text{Ex}} \xrightarrow{\text{val}} b!x!t! \text{If}(a, T, \text{If}(\langle a^h \equiv^1 b^h \mid x := t \rangle_{\text{Ex}}, \langle a^t \equiv^* b^t \mid x := t \rangle_{\text{Ex}}, F)))]$

$[\langle x \equiv^* y \mid z := u \rangle_{\text{Ex}} \xrightarrow{\text{tex}} \text{"\langle \#1.}$

$\{\equiv\}^* \#2.$

$\mid \#3.$

$\{:=\} \#4.$

$\rangle_{\text{Ex}} \text{"}$

$[\langle * \equiv^* * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub* " is " where " is " end sub"}]$

$\text{ph}_1$

$[\text{ph}_1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_1 \doteq a_{\text{Ph}}]])]$

[ph<sub>1</sub>  $\xrightarrow{\text{tex}}$  “ph\_{1}”]

[ph<sub>1</sub>  $\xrightarrow{\text{pyk}}$  “ph1”]

ph<sub>2</sub>

[ph<sub>2</sub>  $\xrightarrow{\text{macro}}$  λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_2 \doteq \mathbf{b}_{\text{Ph}}]])$ ]

[ph<sub>2</sub>  $\xrightarrow{\text{tex}}$  “ph\_{2}”]

[ph<sub>2</sub>  $\xrightarrow{\text{pyk}}$  “ph2”]

ph<sub>3</sub>

[ph<sub>3</sub>  $\xrightarrow{\text{macro}}$  λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_3 \doteq \mathbf{c}_{\text{Ph}}]])$ ]

[ph<sub>3</sub>  $\xrightarrow{\text{tex}}$  “ph\_{3}”]

[ph<sub>3</sub>  $\xrightarrow{\text{pyk}}$  “ph3”]

\*Ph

[\*Ph  $\xrightarrow{\text{tex}}$  “#1.  
\_{Ph} ”]

[\*Ph  $\xrightarrow{\text{pyk}}$  “placeholder-var " end var”]

\*Ph

[x<sup>Ph</sup>  $\xrightarrow{\text{tex}}$  “#1.  
^\_{Ph} ”]

[\*Ph  $\xrightarrow{\text{pyk}}$  “" is placeholder-var”]

⟨\*≡\* | \* ::=\*⟩<sub>Ph</sub>

[⟨x≡y|z::=u⟩<sub>Ph</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.

{\equiv} #2.

| #3.

{::=} #4.

\rangle\_{\text{Ph}} ”]

$\langle * \equiv * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}}$  “ph-sub " is " where " is " end sub”]

$\langle * \equiv^0 * \mid * : == * \rangle_{\text{Ph}}$

$[\langle x \equiv^0 y \mid z : == u \rangle_{\text{Ph}} \xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv}^0 #2.  
| #3.  
{: == } #4.  
\rangle\_{\text{Ph}} ”]

$\langle * \equiv^0 * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}}$  “ph-sub0 " is " where " is " end sub”]

$\langle * \equiv^1 * \mid * : == * \rangle_{\text{Ph}}$

$[\langle x \equiv^1 y \mid z : == u \rangle_{\text{Ph}} \xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv}^1 #2.  
| #3.  
{: == } #4.  
\rangle\_{\text{Ph}} ”]

$\langle * \equiv^1 * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}}$  “ph-sub1 " is " where " is " end sub”]

$\langle * \equiv^* * \mid * : == * \rangle_{\text{Ph}}$

$[\langle x \equiv^* y \mid z : == u \rangle_{\text{Ph}} \xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv}^\* #2.  
| #3.  
{: == } #4.  
\rangle\_{\text{Ph}} ”]

$\langle * \equiv^* * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}}$  “ph-sub\* " is " where " is " end sub”]

$\langle * \equiv * \mid * : == * \rangle_{\text{Me}}$

$[\langle x \equiv y \mid z : == u \rangle_{\text{Me}} \xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv} #2.  
| #3.  
{: == } #4.  
\rangle\_{\text{Me}} ”]

$\langle * \equiv * \mid * : == * \rangle_{\text{Me}} \xrightarrow{\text{pyk}}$  “meta-sub " is " where " is " end sub”]

$\langle * \equiv^1 * \mid * := * \rangle_{\text{Me}}$

$[\langle x \equiv^1 y \mid z := u \rangle_{\text{Me}} \xrightarrow{\text{tex}} “\langle \text{equiv} \rangle^1 \#1.$   
 $\{\equiv\} \#2.$   
 $\mid \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Me}} ”]$

$[\langle * \equiv^1 * \mid * := * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} “\text{meta-sub1 } " \text{ is } " \text{ where } " \text{ is } " \text{ end sub}”]$

$\langle * \equiv^* * \mid * := * \rangle_{\text{Me}}$

$[\langle x \equiv^* y \mid z := u \rangle_{\text{Me}} \xrightarrow{\text{tex}} “\langle \text{equiv} \rangle^* \#1.$   
 $\{\equiv\} \#2.$   
 $\mid \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Me}} ”]$

$[\langle * \equiv^* * \mid * := * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} “\text{meta-sub* } " \text{ is } " \text{ where } " \text{ is } " \text{ end sub}”]$

**bs**

$[\text{bs} \xrightarrow{\text{tex}} “\text{mathsf } \{\text{bs}\}”]$

$[\text{bs} \xrightarrow{\text{pyk}} “\text{var big set}”]$

**OBS**

$[\text{OBS} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{OBS} \doteq \overline{\text{bs}}]])]$

$[\text{OBS} \xrightarrow{\text{tex}} “\text{mathsf } \{\text{OBS}\}”]$

$[\text{OBS} \xrightarrow{\text{pyk}} “\text{object big set}”]$

**$\mathcal{BS}$**

$[\mathcal{BS} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\mathcal{BS} \doteq \underline{\text{bs}}]])]$

$[\mathcal{BS} \xrightarrow{\text{tex}} “\{\text{cal BS}\}”]$

$[\mathcal{BS} \xrightarrow{\text{pyk}} “\text{meta big set}”]$

$\emptyset$  $[\emptyset \xrightarrow{\text{tex}} \text{"\mathrm{\{ \emptyset \}}"]$  $[\emptyset \xrightarrow{\text{pyk}} \text{"zermelo empty set"}]$ 

## SystemQ

$$\begin{aligned}
& [\text{SystemQ} \xrightarrow{\text{stmt}} \forall(\underline{fx}): \forall(\underline{fy}): \underline{R}(\underline{fx}) + +\underline{R}(\underline{fy}) == \underline{R}(\underline{fy}) + +\underline{R}(\underline{fx}) \oplus \\
& \forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \underline{R}(\underline{fx}) ** \underline{R}(\underline{fy}) ** \underline{R}(\underline{fz}) == \underline{R}(\underline{fx}) ** \underline{R}(\underline{fy}) ** \underline{R}(\underline{fz}) \oplus \\
& \forall(\underline{fx}): \forall(\underline{rx}): \forall(\underline{ry}): (\underline{rx}) == (\underline{ry}) \vdash (\underline{fx}) \in (\underline{rx}) \vdash (\underline{fx}) \in (\underline{ry}) \oplus \forall \underline{m}: \text{UB}(01//02 * \\
& ** \underline{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals}) \vdash \underline{xs}[\underline{m} + 1] == \underline{xs}[\underline{m}] \oplus \forall \underline{x}: \forall \underline{y}: \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \\
& \underline{x} \Rightarrow \underline{x} = \underline{y} \oplus \forall \underline{s}: \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{s} \in \{\underline{x}, \underline{y}\}) \Rightarrow \dot{\vdash} (\underline{s} == \underline{x}) \text{n} \Rightarrow \underline{s} == \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{s} == \\
& \underline{x}) \text{n} \Rightarrow \underline{s} == \underline{y} \Rightarrow \underline{s} \in \{\underline{x}, \underline{y}\}) \text{n} \oplus \forall \underline{m}: \forall \underline{n}: \underline{n} = 0 \vdash \text{BS}(\underline{m}, \underline{n}) = \text{rec}(1 + 1)(\text{exp})\underline{m} \oplus \\
& \forall \underline{x}: (\underline{x} + 0) = \underline{x} \oplus \forall(\underline{fx}): \forall(\underline{fy}): \underline{R}(\underline{fx}) == \underline{R}(\underline{fy}) \vdash \text{SF}(\underline{fx}, \underline{fy}) \oplus \forall \underline{x}: \forall \underline{y}: \underline{x} = \\
& \underline{y} \Rightarrow \underline{x} <= \underline{y} \oplus \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b} \oplus \forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \underline{R}(\underline{fx}) = \underline{R}(\underline{fy}) \vdash \\
& \underline{R}(\underline{fx}) + +\underline{R}(\underline{fz}) = \underline{R}(\underline{fy}) + +\underline{R}(\underline{fz}) \oplus \forall(\underline{fx}): \underline{R}(\underline{fx}) + +\underline{R}(0f) == \\
& \underline{R}(\underline{fx}) \oplus \forall \underline{x}: (\underline{x} * 1) = \underline{x} \oplus \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b} \oplus \forall(\underline{rx}): \forall(\underline{ry}): (\underline{rx}) == (\underline{ry}) \vdash (\underline{rx}) == \\
& (\underline{rx}) \oplus \forall \underline{m}: \forall \underline{x}: \underline{m} = 0 \vdash \underline{x}(\text{exp})\underline{m} = 1 \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: 0 <= \underline{z} \Rightarrow \underline{x} <= \underline{y} \Rightarrow \\
& (\underline{x} * \underline{z}) <= (\underline{y} * \underline{z}) \oplus \forall(\underline{fx}): \underline{R}(\underline{fx}) ** \underline{R}(1f) == \underline{R}(\underline{fx}) \oplus \dot{\vdash} (0 = 1) \text{n} \oplus \\
& \forall \underline{m}: \text{Nat}(\underline{m}) \Vdash 0 <= \underline{m} \oplus \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} == \underline{y}) \Rightarrow \forall_{\text{obj}} \underline{s}: \dot{\vdash} (\underline{s} \in \underline{x} \Rightarrow \underline{s} \in \underline{y}) \Rightarrow \dot{\vdash} (\underline{s} \in \\
& \underline{y} \Rightarrow \underline{s} \in \underline{x}) \text{n} \text{n} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \underline{s}: \dot{\vdash} (\underline{s} \in \underline{x} \Rightarrow \underline{s} \in \underline{y}) \Rightarrow \dot{\vdash} (\underline{s} \in \underline{y} \Rightarrow \underline{s} \in \underline{x}) \text{n} \text{n} \Rightarrow \underline{x} == \\
& \underline{y}) \text{n} \text{n} \oplus \forall(\underline{rx}): \forall(\underline{ry}): \forall(\underline{rz}): (\underline{rx}) == (\underline{ry}) \vdash (\underline{ry}) == (\underline{rz}) \vdash (\underline{rx}) == (\underline{rz}) \oplus \\
& \forall \underline{x}: \forall \underline{y}: (\underline{x} + \underline{y}) = (\underline{y} + \underline{x}) \oplus \forall \underline{m}: \forall(\underline{fx}): \forall(\underline{fy}): (\underline{fx}) +_f (\underline{fy})[\underline{m}] = ((\underline{fx})[\underline{m}] + (\underline{fy})[\underline{m}]) \oplus \\
& \forall(\underline{v1}): \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: (\underline{b} \equiv \underline{a})(\underline{v1}) == 0 \text{Me} \Vdash \langle \underline{c} \equiv \underline{a} \rangle (\underline{v1}) == ((\underline{v1} + 1)) \text{Me} \Vdash \underline{b} \Rightarrow \\
& \forall_{\text{obj}} (\underline{v1}): \underline{a} \Rightarrow \underline{c} \Rightarrow \forall_{\text{obj}} (\underline{v1}): \underline{a} \oplus \forall \underline{m}: \forall \underline{n}: \underline{n} = 0 \vdash \text{UStescope}(\underline{m}, \underline{n}) = \\
& |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + 1]))| \oplus \forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \underline{R}(\underline{fx}) + +\underline{R}(\underline{fy}) + +\underline{R}(\underline{fz}) = \\
& \underline{R}(\underline{fx}) + +\underline{R}(\underline{fy}) + +\underline{R}(\underline{fz}) \oplus \forall \underline{x}: \forall \underline{y}: (\underline{x} * \underline{y}) = (\underline{y} * \underline{x}) \oplus \forall(\underline{fx}): \forall(\underline{fy}): (\underline{fx}) \in \\
& \underline{R}(\underline{fy}) \vdash \text{SF}(\underline{fx}, \underline{fy}) \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z}) \oplus \forall \underline{a}: \underline{a} \vdash \underline{a} \oplus \\
& \forall \underline{m}: \text{UB}(01//02 * ** \underline{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals}) \vdash \text{us}[\underline{m} + 1] == \\
& 01//02 * ** \underline{xs}[\underline{m}] + + \text{us}[\underline{m}] \oplus \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} <= \underline{y}) \text{n} \Rightarrow \underline{y} <= \underline{x} \oplus \forall \underline{s}: \forall \underline{x}: \dot{\vdash} (\underline{s} \in \\
& \text{P}(\underline{x}) \Rightarrow \forall_{\text{obj}} \underline{s}: \underline{s} \in \underline{s} \Rightarrow \underline{s} \in \underline{x} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \underline{s}: \underline{s} \in \underline{s} \Rightarrow \underline{s} \in \underline{x} \Rightarrow \underline{s} \in \text{P}(\underline{x})) \text{n} \text{n} \oplus \\
& \text{us}[0] == \underline{xs}[0] + +\underline{R}(1f) \oplus \forall \underline{x}: \underline{x} <= \underline{x} \oplus \forall \underline{s}: \dot{\vdash} (\underline{s} \in \emptyset) \text{n} \oplus \forall \underline{x}: (\underline{x} + (-\underline{ux})) = 0 \oplus \\
& \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \Rightarrow \underline{x} = \underline{z} \Rightarrow \underline{y} = \underline{z} \oplus \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (0 <= \underline{n}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{n}) \text{n}) \text{n} \vdash \\
& \text{UStescope}(\underline{m}, \underline{n}) = (|(\text{us}[\underline{m} + \underline{n}] + (-\text{uus}[\underline{m} + (\underline{n} + 1)])|) + \text{UStescope}(\underline{m}, (\underline{n} + \\
& (-\underline{u1}))) \oplus \forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \underline{R}(\underline{fx}) +_f (\underline{fy}) +_f (\underline{fz}) == \underline{R}(\underline{fx}) +_f (\underline{fy}) +_f (\underline{fz}) \oplus \\
& \forall \underline{x}: \dot{\vdash} (\underline{x} = 0) \text{n} \Rightarrow (\underline{x} * \text{rec}\underline{x}) = 1 \oplus \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\underline{b}) \text{n} \Rightarrow \underline{a} \vdash \dot{\vdash} (\underline{b}) \text{n} \Rightarrow \dot{\vdash} (\underline{a}) \text{n} \vdash \underline{b} \oplus \\
& \forall(\underline{rx}): (\underline{rx}) == (\underline{rx}) \oplus \forall \underline{m}: \dot{\vdash} (\text{UB}(01//02 * ** \underline{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals})) \text{n} \vdash \\
& \text{us}[\underline{m} + 1] == \text{us}[\underline{m}] \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} <= \underline{y} \Rightarrow (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z}) \oplus \\
& \forall \underline{a}: \forall \underline{b}: \forall \underline{p}: \forall \underline{z}: \underline{p}^{\text{Ph}} \wedge (\underline{b} \equiv \underline{a}) \underline{p}: == \underline{z} \text{Ph} \Vdash \dot{\vdash} (\underline{z} \in \{\text{ph} \in \underline{x} \mid \underline{a}\}) \Rightarrow \dot{\vdash} (\underline{z} \in \underline{x} \Rightarrow \\
& \dot{\vdash} (\underline{b}) \text{n}) \text{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{z} \in \underline{x} \Rightarrow \dot{\vdash} (\underline{b}) \text{n}) \Rightarrow \underline{z} \in \{\text{ph} \in \underline{x} \mid \underline{a}\}) \text{n} \text{n} \oplus \\
& \forall \underline{m}: \forall(\underline{fx}): \underline{R}(\underline{fx}) + +(-\underline{R}(\underline{fx})) == \underline{R}(0f) \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * (\underline{y} + \underline{z})) = \\
& ((\underline{x} * \underline{y}) + (\underline{x} * \underline{z})) \oplus \forall \underline{m}: \forall \underline{n}: \text{Nat}(\underline{m}) \Vdash \text{Nat}(\underline{n}) \Vdash \dot{\vdash} (\underline{m} <= (\underline{n} + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{m} = \\
& (\underline{n} + 1)) \text{n}) \text{n} \vdash \underline{m} <= \underline{n} \oplus \forall \underline{x}: \forall \underline{t}: \forall \underline{a}: \forall \underline{b}: \langle [\underline{a}] \equiv^0 [\underline{b}] \mid [\underline{x}] == [\underline{t}] \rangle_{\text{Ex}} \Vdash \underline{a} \vdash \underline{b} \oplus
\end{aligned}$$

$$\begin{aligned}
& \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{m})n)n) \vdash \underline{x}(\text{exp})\underline{m} = \\
& (\underline{x} * \underline{x}(\text{exp})(\underline{m} + (-u1))) \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z})) \oplus \\
& \forall (\underline{v1}): \forall (\underline{v2}): \forall \underline{n}: \forall (\underline{\epsilon}): \forall (\underline{fx}): \forall_{\text{obj}}(\underline{\epsilon}): \dot{\vdash} (\forall_{\text{obj}}\underline{n}: \dot{\vdash} (\forall_{\text{obj}}(\underline{v1}): \forall_{\text{obj}}(\underline{v2}): \dot{\vdash} (0 \leq \\
& (\underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{\epsilon})n)n)n \Rightarrow \underline{n} \leq (\underline{v1}) \Rightarrow \underline{n} \leq (\underline{v2}) \Rightarrow \\
& \dot{\vdash} (|((\underline{fx})[(\underline{v1})] + (-u(\underline{fx})[(\underline{v2})]))| \leq (\underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|((\underline{fx})[(\underline{v1})] + (-u(\underline{fx})[(\underline{v2})]))| = \\
& (\underline{\epsilon})n)n)n)n) \oplus \forall \underline{x}: \forall (\underline{v1}): \forall \underline{a}: \forall \underline{b}: \langle \underline{a} \equiv \underline{b} \rangle (\underline{v1}) := \underline{x} \vdash_{\text{Me}} \vdash_{\text{obj}} \underline{v1}: \underline{b} \Rightarrow \underline{a} \oplus \\
& \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (0 \leq \underline{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{n})n)n) \vdash \text{BS}(\underline{m}, \underline{n}) = \\
& (\text{rec}(1 + 1)(\text{exp})(\underline{m} + \underline{n}) + \text{BS}(\underline{m}, (\underline{n} + (-u1)))) \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} * \underline{y}) * \underline{z}) = \\
& (\underline{x} * (\underline{y} * \underline{z})) \oplus \forall (\underline{fx}): \forall (\underline{fy}): \text{SF}((\underline{fx}), (\underline{fy})) \vdash \text{R}((\underline{fx})) == \text{R}((\underline{fy})) \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \\
& \underline{y} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z}) \oplus \forall \underline{x}: \forall \underline{a}: \underline{a} \vdash_{\text{obj}} \underline{x}: \underline{a} \oplus \\
& \forall \underline{m}: \dot{\vdash} (\text{UB}(01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals}))n \vdash \text{xs}[(\underline{m} + 1)] == \\
& 01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}] \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} \leq \underline{y} \Rightarrow \underline{y} \leq \underline{z} \Rightarrow \underline{x} \leq \underline{z} \oplus \\
& \forall \underline{s}: \forall \underline{x}: \dot{\vdash} (\underline{s} \in \cup \underline{x} \Rightarrow \dot{\vdash} (\underline{s} \in \text{j}_{\text{Ex}} \Rightarrow \dot{\vdash} (\text{j}_{\text{Ex}} \in \underline{x})n)n \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{s} \in \text{j}_{\text{Ex}} \Rightarrow \dot{\vdash} (\text{j}_{\text{Ex}} \in \\
& \underline{x})n)n \Rightarrow \underline{s} \in \cup \underline{x})n) \oplus \forall (\underline{fx}): \forall (\underline{fy}): \text{R}((\underline{fx})) * * \text{R}((\underline{fy})) == \text{R}((\underline{fy})) * * \text{R}((\underline{fx})) \oplus \\
& \forall (\underline{fx}): \forall (\underline{rx}): \forall (\underline{ry}): (\underline{rx}) == (\underline{ry}) \vdash (\underline{fx}) \in (\underline{ry}) \vdash (\underline{fx}) \in (\underline{rx})]
\end{aligned}$$

$$[\text{SystemQ} \xrightarrow{\text{tex}} \text{"SystemQ"}]$$

$$[\text{SystemQ} \xrightarrow{\text{pyk}} \text{"system Q"}]$$

## MP

$$[\text{MP} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{MP} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \underline{a} \vdash \underline{b}]$$

$$[\text{MP} \xrightarrow{\text{tex}} \text{"MP"}]$$

$$[\text{MP} \xrightarrow{\text{pyk}} \text{"1rule mp"}]$$

## Gen

$$[\text{Gen} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{Gen} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{a}: \underline{a} \vdash \forall_{\text{obj}} \underline{x}: \underline{a}]$$

$$[\text{Gen} \xrightarrow{\text{tex}} \text{"Gen"}]$$

$$[\text{Gen} \xrightarrow{\text{pyk}} \text{"1rule gen"}]$$

## Repetition

$$[\text{Repetition} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{Repetition} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{a}: \underline{a} \vdash \underline{a}]$$



[Repetition  $\xrightarrow{\text{tex}}$  “Repetition”]

[Repetition  $\xrightarrow{\text{pyk}}$  “1rule repetition”]

## Neg

[Neg  $\xrightarrow{\text{proof}}$  Rule tactic]

[Neg  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg} (\underline{b})n \Rightarrow \underline{a} \vdash \dot{\neg} (\underline{b})n \Rightarrow \dot{\neg} (\underline{a})n \vdash \underline{b}$ ]

[Neg  $\xrightarrow{\text{tex}}$  “Neg”]

[Neg  $\xrightarrow{\text{pyk}}$  “1rule ad absurdum”]

## Ded

[Ded  $\xrightarrow{\text{proof}}$  Rule tactic]

[Ded  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \vdash \underline{b}$ ]

[Ded  $\xrightarrow{\text{tex}}$  “Ded”]

[Ded  $\xrightarrow{\text{pyk}}$  “1rule deduction”]

## ExistIntro

[ExistIntro  $\xrightarrow{\text{proof}}$  Rule tactic]

[ExistIntro  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{t}: \forall \underline{a}: \forall \underline{b}: \langle [\underline{a}] \equiv^0 [\underline{b}] \mid [\underline{x}] ::= [\underline{t}] \rangle_{\text{Ex}} \Vdash \underline{a} \vdash \underline{b}$ ]

[ExistIntro  $\xrightarrow{\text{tex}}$  “ExistIntro”]

[ExistIntro  $\xrightarrow{\text{pyk}}$  “1rule exist intro”]

## Extensionality

[Extensionality  $\xrightarrow{\text{proof}}$  Rule tactic]

[Extensionality  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \dot{\neg} (\underline{x} == \underline{y}) \Rightarrow \forall_{\text{obj}} \bar{s}: \dot{\neg} (\bar{s} \in \underline{x} \Rightarrow \bar{s} \in \underline{y}) \Rightarrow \dot{\neg} (\bar{s} \in \underline{y} \Rightarrow \bar{s} \in \underline{x})n \Rightarrow \dot{\neg} (\forall_{\text{obj}} \bar{s}: \dot{\neg} (\bar{s} \in \underline{x} \Rightarrow \bar{s} \in \underline{y}) \Rightarrow \dot{\neg} (\bar{s} \in \underline{y} \Rightarrow \bar{s} \in \underline{x})n) \Rightarrow \underline{x} == \underline{y})n$ ]

[Extensionality  $\xrightarrow{\text{tex}}$  “Extensionality”]

[Extensionality  $\xrightarrow{\text{pyk}}$  “axiom extensionality”]

## Ødef

[Ødef  $\xrightarrow{\text{proof}}$  Rule tactic]

[Ødef  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{s}: \dot{\vdash} (\underline{s} \in \emptyset)n$ ]

[Ødef  $\xrightarrow{\text{tex}}$  “\O{}def”]

[Ødef  $\xrightarrow{\text{pyk}}$  “axiom empty set”]

## PairDef

[PairDef  $\xrightarrow{\text{proof}}$  Rule tactic]

[PairDef  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{s}: \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{s} \in \{\underline{x}, \underline{y}\}) \Rightarrow \dot{\vdash} (\underline{s} == \underline{x})n \Rightarrow \underline{s} == \underline{y} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{s} == \underline{x})n \Rightarrow \underline{s} == \underline{y} \Rightarrow \underline{s} \in \{\underline{x}, \underline{y}\})n)n$ ]

[PairDef  $\xrightarrow{\text{tex}}$  “PairDef”]

[PairDef  $\xrightarrow{\text{pyk}}$  “axiom pair definition”]

## UnionDef

[UnionDef  $\xrightarrow{\text{proof}}$  Rule tactic]

[UnionDef  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{s}: \forall \underline{x}: \dot{\vdash} (\underline{s} \in \bigcup \underline{x}) \Rightarrow \dot{\vdash} (\underline{s} \in \text{jEx} \Rightarrow \dot{\vdash} (\text{jEx} \in \underline{x})n)n \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{s} \in \text{jEx} \Rightarrow \dot{\vdash} (\text{jEx} \in \underline{x})n)n \Rightarrow \underline{s} \in \bigcup \underline{x})n)n$ ]

[UnionDef  $\xrightarrow{\text{tex}}$  “UnionDef”]

[UnionDef  $\xrightarrow{\text{pyk}}$  “axiom union definition”]

## PowerDef

[PowerDef  $\xrightarrow{\text{proof}}$  Rule tactic]

[PowerDef  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{s}: \forall \underline{x}: \dot{\vdash} (\underline{s} \in \text{P}(\underline{x})) \Rightarrow \forall \text{obj} \bar{s}: \bar{s} \in \underline{s} \Rightarrow \bar{s} \in \underline{x} \Rightarrow \dot{\vdash} (\forall \text{obj} \bar{s}: \bar{s} \in \underline{s} \Rightarrow \bar{s} \in \underline{x} \Rightarrow \underline{s} \in \text{P}(\underline{x}))n)n$ ]

[PowerDef  $\xrightarrow{\text{tex}}$  “PowerDef”]

[PowerDef  $\xrightarrow{\text{pyk}}$  “axiom power definition”]

# SeparationDef

[SeparationDef  $\xrightarrow{\text{proof}}$  Rule tactic]

[SeparationDef  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{p}: \forall \mathbf{x}: \forall \mathbf{z}: \mathbf{p}^{\text{Ph}} \wedge \langle \mathbf{b} \equiv \mathbf{a} \mid \mathbf{p} ::= \mathbf{z} \rangle_{\text{Ph}} \Vdash \dot{\neg} (\mathbf{z} \in \{\mathbf{ph} \in \mathbf{x} \mid \mathbf{a}\}) \Rightarrow \dot{\neg} (\mathbf{z} \in \mathbf{x} \Rightarrow \dot{\neg} (\mathbf{b})\mathbf{n})\mathbf{n} \Rightarrow \dot{\neg} (\dot{\neg} (\mathbf{z} \in \mathbf{x} \Rightarrow \dot{\neg} (\mathbf{b})\mathbf{n})\mathbf{n} \Rightarrow \mathbf{z} \in \{\mathbf{ph} \in \mathbf{x} \mid \mathbf{a}\})\mathbf{n})\mathbf{n}$ ]

[SeparationDef  $\xrightarrow{\text{tex}}$  “SeparationDef”]

[SeparationDef  $\xrightarrow{\text{pyk}}$  “axiom separation definition”]

# AddDoubleNeg

[AddDoubleNeg  $\xrightarrow{\text{tex}}$  “AddDoubleNeg”]

[AddDoubleNeg  $\xrightarrow{\text{pyk}}$  “prop lemma add double neg”]

# RemoveDoubleNeg

[RemoveDoubleNeg  $\xrightarrow{\text{tex}}$  “RemoveDoubleNeg”]

[RemoveDoubleNeg  $\xrightarrow{\text{pyk}}$  “prop lemma remove double neg”]

# AndCommutativity

[AndCommutativity  $\xrightarrow{\text{tex}}$  “AndCommutativity”]

[AndCommutativity  $\xrightarrow{\text{pyk}}$  “prop lemma and commutativity”]

# AutoImply

[AutoImply  $\xrightarrow{\text{tex}}$  “AutoImply”]

[AutoImply  $\xrightarrow{\text{pyk}}$  “prop lemma auto imply”]

# Contrapositive

[Contrapositive  $\xrightarrow{\text{tex}}$  “Contrapositive”]

[Contrapositive  $\xrightarrow{\text{pyk}}$  “prop lemma contrapositive”]

## FirstConjunct

[FirstConjunct  $\xrightarrow{\text{tex}}$  “FirstConjunct”]

[FirstConjunct  $\xrightarrow{\text{pyk}}$  “prop lemma first conjunct”]

## SecondConjunct

[SecondConjunct  $\xrightarrow{\text{tex}}$  “SecondConjunct”]

[SecondConjunct  $\xrightarrow{\text{pyk}}$  “prop lemma second conjunct”]

## FromContradiction

[FromContradiction  $\xrightarrow{\text{tex}}$  “FromContradiction”]

[FromContradiction  $\xrightarrow{\text{pyk}}$  “prop lemma from contradiction”]

## FromDisjuncts

[FromDisjuncts  $\xrightarrow{\text{tex}}$  “FromDisjuncts”]

[FromDisjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma from disjuncts”]

## IffCommutativity

[IffCommutativity  $\xrightarrow{\text{tex}}$  “IffCommutativity”]

[IffCommutativity  $\xrightarrow{\text{pyk}}$  “prop lemma iff commutativity”]

## IffFirst

[IffFirst  $\xrightarrow{\text{tex}}$  “IffFirst”]

[IffFirst  $\xrightarrow{\text{pyk}}$  “prop lemma iff first”]

## IffSecond

[IffSecond  $\xrightarrow{\text{tex}}$  “IffSecond”]

[IffSecond  $\xrightarrow{\text{pyk}}$  “prop lemma iff second”]

## ImPLYTransitivity

[ImPLYTransitivity  $\xrightarrow{\text{tex}}$  “ImPLYTransitivity”]

[ImPLYTransitivity  $\xrightarrow{\text{pyk}}$  “prop lemma imply transitivity”]

## JoinConjuncts

[JoinConjuncts  $\xrightarrow{\text{tex}}$  “JoinConjuncts”]

[JoinConjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma join conjuncts”]

## MP2

[MP2  $\xrightarrow{\text{tex}}$  “MP2”]

[MP2  $\xrightarrow{\text{pyk}}$  “prop lemma mp2”]

## MP3

[MP3  $\xrightarrow{\text{tex}}$  “MP3”]

[MP3  $\xrightarrow{\text{pyk}}$  “prop lemma mp3”]

## MP4

[MP4  $\xrightarrow{\text{tex}}$  “MP4”]

[MP4  $\xrightarrow{\text{pyk}}$  “prop lemma mp4”]

## MP5

[MP5  $\xrightarrow{\text{tex}}$  “MP5”]

[MP5  $\xrightarrow{\text{pyk}}$  “prop lemma mp5”]

# MT

[MT  $\xrightarrow{\text{tex}}$  “MT”]

[MT  $\xrightarrow{\text{pyk}}$  “prop lemma mt”]

# NegativeMT

[NegativeMT  $\xrightarrow{\text{tex}}$  “NegativeMT”]

[NegativeMT  $\xrightarrow{\text{pyk}}$  “prop lemma negative mt”]

# Technicality

[Technicality  $\xrightarrow{\text{tex}}$  “Technicality”]

[Technicality  $\xrightarrow{\text{pyk}}$  “prop lemma technicality”]

# Weakening

[Weakening  $\xrightarrow{\text{tex}}$  “Weakening”]

[Weakening  $\xrightarrow{\text{pyk}}$  “prop lemma weakening”]

# WeakenOr1

[WeakenOr1  $\xrightarrow{\text{tex}}$  “WeakenOr1”]

[WeakenOr1  $\xrightarrow{\text{pyk}}$  “prop lemma weaken or first”]

# WeakenOr2

[WeakenOr2  $\xrightarrow{\text{tex}}$  “WeakenOr2”]

[WeakenOr2  $\xrightarrow{\text{pyk}}$  “prop lemma weaken or second”]

# Formula2Pair

[Formula2Pair  $\xrightarrow{\text{tex}}$  “Formula2Pair”]

[Formula2Pair  $\xrightarrow{\text{pyk}}$  “lemma formula2pair”]

## Pair2Formula

[Pair2Formula  $\xrightarrow{\text{tex}}$  “Pair2Formula”]

[Pair2Formula  $\xrightarrow{\text{pyk}}$  “lemma pair2formula”]

## Formula2Union

[Formula2Union  $\xrightarrow{\text{tex}}$  “Formula2Union”]

[Formula2Union  $\xrightarrow{\text{pyk}}$  “lemma formula2union”]

## Union2Formula

[Union2Formula  $\xrightarrow{\text{tex}}$  “Union2Formula”]

[Union2Formula  $\xrightarrow{\text{pyk}}$  “lemma union2formula”]

## Formula2Sep

[Formula2Sep  $\xrightarrow{\text{tex}}$  “Formula2Sep”]

[Formula2Sep  $\xrightarrow{\text{pyk}}$  “lemma formula2separation”]

## Sep2Formula

[Sep2Formula  $\xrightarrow{\text{tex}}$  “Sep2Formula”]

[Sep2Formula  $\xrightarrow{\text{pyk}}$  “lemma separation2formula”]

## Formula2Power

[Formula2Power  $\xrightarrow{\text{tex}}$  “Formula2Power”]

[Formula2Power  $\xrightarrow{\text{pyk}}$  “lemma formula2power”]

## SubsetInPower

[SubsetInPower  $\xrightarrow{\text{tex}}$  “SubsetInPower”]

[SubsetInPower  $\xrightarrow{\text{pyk}}$  “lemma subset in power set”]

## HelperPowerIsSub

[HelperPowerIsSub  $\xrightarrow{\text{tex}}$  “HelperPowerIsSub”]

[HelperPowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset0”]

## PowerIsSub

[PowerIsSub  $\xrightarrow{\text{tex}}$  “PowerIsSub”]

[PowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset”]

## (Switch)HelperPowerIsSub

[(Switch)HelperPowerIsSub  $\xrightarrow{\text{tex}}$  “(Switch)HelperPowerIsSub”]

[(Switch)HelperPowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset0-switch”]

## (Switch)PowerIsSub

[(Switch)PowerIsSub  $\xrightarrow{\text{tex}}$  “(Switch)PowerIsSub”]

[(Switch)PowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset-switch”]

## ToSetEquality

[ToSetEquality  $\xrightarrow{\text{tex}}$  “ToSetEquality”]

[ToSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition”]

## HelperToSetEquality(t)

[HelperToSetEquality(t)  $\xrightarrow{\text{tex}}$  “HelperToSetEquality(t)”]



[HelperToSetEquality(t)  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition(t)0”]

## ToSetEquality(t)

[ToSetEquality(t)  $\xrightarrow{\text{tex}}$  “ToSetEquality(t)”]

[ToSetEquality(t)  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition(t)”]

## HelperFromSetEquality

[HelperFromSetEquality  $\xrightarrow{\text{tex}}$  “HelperFromSetEquality”]

[HelperFromSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality skip quantifier”]

## FromSetEquality

[FromSetEquality  $\xrightarrow{\text{tex}}$  “FromSetEquality”]

[FromSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition”]

## HelperReflexivity

[HelperReflexivity  $\xrightarrow{\text{tex}}$  “HelperReflexivity”]

[HelperReflexivity  $\xrightarrow{\text{pyk}}$  “lemma reflexivity0”]

## Reflexivity

[Reflexivity  $\xrightarrow{\text{tex}}$  “Reflexivity”]

[Reflexivity  $\xrightarrow{\text{pyk}}$  “lemma reflexivity”]

## HelperSymmetry

[HelperSymmetry  $\xrightarrow{\text{tex}}$  “HelperSymmetry”]

[HelperSymmetry  $\xrightarrow{\text{pyk}}$  “lemma symmetry0”]

# Symmetry

[Symmetry  $\xrightarrow{\text{tex}}$  “Symmetry”]

[Symmetry  $\xrightarrow{\text{pyk}}$  “lemma symmetry”]

# HelperTransitivity

[HelperTransitivity  $\xrightarrow{\text{tex}}$  “HelperTransitivity”]

[HelperTransitivity  $\xrightarrow{\text{pyk}}$  “lemma transitivity0”]

# Transitivity

[Transitivity  $\xrightarrow{\text{tex}}$  “Transitivity”]

[Transitivity  $\xrightarrow{\text{pyk}}$  “lemma transitivity”]

# ERisReflexive

[ERisReflexive  $\xrightarrow{\text{tex}}$  “ERisReflexive”]

[ERisReflexive  $\xrightarrow{\text{pyk}}$  “lemma er is reflexive”]

# ERisSymmetric

[ERisSymmetric  $\xrightarrow{\text{tex}}$  “ERisSymmetric”]

[ERisSymmetric  $\xrightarrow{\text{pyk}}$  “lemma er is symmetric”]

# ERisTransitive

[ERisTransitive  $\xrightarrow{\text{tex}}$  “ERisTransitive”]

[ERisTransitive  $\xrightarrow{\text{pyk}}$  “lemma er is transitive”]

# ØisSubset

[ØisSubset  $\xrightarrow{\text{tex}}$  “\O{}isSubset”]

[ $\emptyset$ isSubst  $\xrightarrow{\text{pyk}}$  “lemma empty set is subst”]

## HelperMemberNot $\emptyset$

[HelperMemberNot $\emptyset$   $\xrightarrow{\text{tex}}$  “HelperMemberNot\O{”}]

[HelperMemberNot $\emptyset$   $\xrightarrow{\text{pyk}}$  “lemma member not empty0”]

## MemberNot $\emptyset$

[MemberNot $\emptyset$   $\xrightarrow{\text{tex}}$  “MemberNot\O{”}]

[MemberNot $\emptyset$   $\xrightarrow{\text{pyk}}$  “lemma member not empty”]

## HelperUnique $\emptyset$

[HelperUnique $\emptyset$   $\xrightarrow{\text{tex}}$  “HelperUnique\O{”}]

[HelperUnique $\emptyset$   $\xrightarrow{\text{pyk}}$  “lemma unique empty set0”]

## Unique $\emptyset$

[Unique $\emptyset$   $\xrightarrow{\text{tex}}$  “Unique\O{”}]

[Unique $\emptyset$   $\xrightarrow{\text{pyk}}$  “lemma unique empty set”]

## == Reflexivity

[== Reflexivity  $\xrightarrow{\text{proof}}$  Rule tactic]

[== Reflexivity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall(\underline{\text{rx}}): (\underline{\text{rx}}) == (\underline{\text{rx}})$ ]

[== Reflexivity  $\xrightarrow{\text{tex}}$  “==\!\{Reflexivity”]

[== Reflexivity  $\xrightarrow{\text{pyk}}$  “lemma ==Reflexivity”]

## == Symmetry

[== Symmetry  $\xrightarrow{\text{proof}}$  Rule tactic]

[==Symmetry  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall(\underline{\text{rx}}): \forall(\underline{\text{ry}}): (\underline{\text{rx}}) == (\underline{\text{ry}}) \vdash (\underline{\text{ry}}) == (\underline{\text{rx}})$ ]

[==Symmetry  $\xrightarrow{\text{tex}}$  “==\!\{Symmetry”]

[==Symmetry  $\xrightarrow{\text{pyk}}$  “lemma ==Symmetry”]

## Helper == Transitivity

[Helper == Transitivity  $\xrightarrow{\text{tex}}$  “Helper\!\{==\!\{Transitivity”]

[Helper == Transitivity  $\xrightarrow{\text{pyk}}$  “lemma ==Transitivity0”]

## == Transitivity

[==Transitivity  $\xrightarrow{\text{proof}}$  Rule tactic]

[==Transitivity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall(\underline{\text{rx}}): \forall(\underline{\text{ry}}): \forall(\underline{\text{rz}}): (\underline{\text{rx}}) == (\underline{\text{ry}}) \vdash (\underline{\text{ry}}) == (\underline{\text{rz}}) \vdash (\underline{\text{rx}}) == (\underline{\text{rz}})$ ]

[==Transitivity  $\xrightarrow{\text{tex}}$  “\!\{==\!\{Transitivity”]

[==Transitivity  $\xrightarrow{\text{pyk}}$  “lemma ==Transitivity”]

## HelperTransferNotEq

[HelperTransferNotEq  $\xrightarrow{\text{tex}}$  “HelperTransferNotEq”]

[HelperTransferNotEq  $\xrightarrow{\text{pyk}}$  “lemma transfer ~is0”]

## TransferNotEq

[TransferNotEq  $\xrightarrow{\text{tex}}$  “TransferNotEq”]

[TransferNotEq  $\xrightarrow{\text{pyk}}$  “lemma transfer ~is”]

## HelperPairSubset

[HelperPairSubset  $\xrightarrow{\text{tex}}$  “HelperPairSubset”]

[HelperPairSubset  $\xrightarrow{\text{pyk}}$  “lemma pair subset0”]

## Helper(2)PairSubset

[Helper(2)PairSubset  $\xrightarrow{\text{tex}}$  "Helper(2)PairSubset"]

[Helper(2)PairSubset  $\xrightarrow{\text{pyk}}$  "lemma pair subset1"]

## PairSubset

[PairSubset  $\xrightarrow{\text{tex}}$  "PairSubset"]

[PairSubset  $\xrightarrow{\text{pyk}}$  "lemma pair subset"]

## SamePair

[SamePair  $\xrightarrow{\text{tex}}$  "SamePair"]

[SamePair  $\xrightarrow{\text{pyk}}$  "lemma same pair"]

## SameSingleton

[SameSingleton  $\xrightarrow{\text{tex}}$  "SameSingleton"]

[SameSingleton  $\xrightarrow{\text{pyk}}$  "lemma same singleton"]

## UnionSubset

[UnionSubset  $\xrightarrow{\text{tex}}$  "UnionSubset"]

[UnionSubset  $\xrightarrow{\text{pyk}}$  "lemma union subset"]

## SameUnion

[SameUnion  $\xrightarrow{\text{tex}}$  "SameUnion"]

[SameUnion  $\xrightarrow{\text{pyk}}$  "lemma same union"]

## SeparationSubset

[SeparationSubset  $\xrightarrow{\text{tex}}$  "SeparationSubset"]

[SeparationSubset  $\xrightarrow{\text{pyk}}$  “lemma separation subset”]

## SameSeparation

[SameSeparation  $\xrightarrow{\text{tex}}$  “SameSeparation”]

[SameSeparation  $\xrightarrow{\text{pyk}}$  “lemma same separation”]

## SameBinaryUnion

[SameBinaryUnion  $\xrightarrow{\text{tex}}$  “SameBinaryUnion”]

[SameBinaryUnion  $\xrightarrow{\text{pyk}}$  “lemma same binary union”]

## IntersectionSubset

[IntersectionSubset  $\xrightarrow{\text{tex}}$  “IntersectionSubset”]

[IntersectionSubset  $\xrightarrow{\text{pyk}}$  “lemma intersection subset”]

## SameIntersection

[SameIntersection  $\xrightarrow{\text{tex}}$  “SameIntersection”]

[SameIntersection  $\xrightarrow{\text{pyk}}$  “lemma same intersection”]

## AutoMember

[AutoMember  $\xrightarrow{\text{tex}}$  “AutoMember”]

[AutoMember  $\xrightarrow{\text{pyk}}$  “lemma auto member”]

## HelperEqSysNot $\emptyset$

[HelperEqSysNot $\emptyset$   $\xrightarrow{\text{tex}}$  “HelperEqSysNot\O{”}]

[HelperEqSysNot $\emptyset$   $\xrightarrow{\text{pyk}}$  “lemma eq-system not empty0”]

## EqSysNot $\emptyset$

[EqSysNot $\emptyset \xrightarrow{\text{tex}}$  “EqSysNot\O{ }”]

[EqSysNot $\emptyset \xrightarrow{\text{pyk}}$  “lemma eq-system not empty”]

## HelperEqSubset

[HelperEqSubset  $\xrightarrow{\text{tex}}$  “HelperEqSubset”]

[HelperEqSubset  $\xrightarrow{\text{pyk}}$  “lemma eq subset0”]

## EqSubset

[EqSubset  $\xrightarrow{\text{tex}}$  “EqSubset”]

[EqSubset  $\xrightarrow{\text{pyk}}$  “lemma eq subset”]

## HelperEqNecessary

[HelperEqNecessary  $\xrightarrow{\text{tex}}$  “HelperEqNecessary”]

[HelperEqNecessary  $\xrightarrow{\text{pyk}}$  “lemma equivalence nec condition0”]

## EqNecessary

[EqNecessary  $\xrightarrow{\text{tex}}$  “EqNecessary”]

[EqNecessary  $\xrightarrow{\text{pyk}}$  “lemma equivalence nec condition”]

## HelperNoneEqNecessary

[HelperNoneEqNecessary  $\xrightarrow{\text{tex}}$  “HelperNoneEqNecessary”]

[HelperNoneEqNecessary  $\xrightarrow{\text{pyk}}$  “lemma none-equivalence nec condition0”]

## Helper(2)NoneEqNecessary

[Helper(2)NoneEqNecessary  $\xrightarrow{\text{tex}}$  “Helper(2)NoneEqNecessary”]

[Helper(2)NoneEqNecessary  $\xrightarrow{\text{pyk}}$  “lemma none-equivalence nec condition1”]

## NoneEqNecessary

[NoneEqNecessary  $\xrightarrow{\text{tex}}$  “NoneEqNecessary”]

[NoneEqNecessary  $\xrightarrow{\text{pyk}}$  “lemma none-equivalence nec condition”]

## EqClassIsSubset

[EqClassIsSubset  $\xrightarrow{\text{tex}}$  “EqClassIsSubset”]

[EqClassIsSubset  $\xrightarrow{\text{pyk}}$  “lemma equivalence class is subset”]

## EqClassesAreDisjoint

[EqClassesAreDisjoint  $\xrightarrow{\text{tex}}$  “EqClassesAreDisjoint”]

[EqClassesAreDisjoint  $\xrightarrow{\text{pyk}}$  “lemma equivalence classes are disjoint”]

## AllDisjoint

[AllDisjoint  $\xrightarrow{\text{tex}}$  “AllDisjoint”]

[AllDisjoint  $\xrightarrow{\text{pyk}}$  “lemma all disjoint”]

## AllDisjointImply

[AllDisjointImply  $\xrightarrow{\text{tex}}$  “AllDisjointImply”]

[AllDisjointImply  $\xrightarrow{\text{pyk}}$  “lemma all disjoint-imply”]

## BSsubset

[BSsubset  $\xrightarrow{\text{tex}}$  “BSsubset”]



[BSsubset  $\xrightarrow{\text{pyk}}$  “lemma bs subset union(bs/r)”]

## Union(BS/R)subset

[Union(BS/R)subset  $\xrightarrow{\text{tex}}$  “Union(BS/R)subset”]

[Union(BS/R)subset  $\xrightarrow{\text{pyk}}$  “lemma union(bs/r) subset bs”]

## UnionIdentity

[UnionIdentity  $\xrightarrow{\text{tex}}$  “UnionIdentity”]

[UnionIdentity  $\xrightarrow{\text{pyk}}$  “lemma union(bs/r) is bs”]

## EqSysIsPartition

[EqSysIsPartition  $\xrightarrow{\text{tex}}$  “EqSysIsPartition”]

[EqSysIsPartition  $\xrightarrow{\text{pyk}}$  “theorem eq-system is partition”]

### (x1)

[(x1)  $\xrightarrow{\text{tex}}$  “(x1)”]

[(x1)  $\xrightarrow{\text{pyk}}$  “var x1”]

### (x2)

[(x2)  $\xrightarrow{\text{tex}}$  “(x2)”]

[(x2)  $\xrightarrow{\text{pyk}}$  “var x2”]

### (y1)

[(y1)  $\xrightarrow{\text{tex}}$  “(y1)”]

[(y1)  $\xrightarrow{\text{pyk}}$  “var y1”]

(y2)

$[(y2) \xrightarrow{\text{tex}} \text{"(y2)"}]$

$[(y2) \xrightarrow{\text{pyk}} \text{"var y2"}]$

(v1)

$[(v1) \xrightarrow{\text{tex}} \text{"(v1)"}]$

$[(v1) \xrightarrow{\text{pyk}} \text{"var v1"}]$

(v2)

$[(v2) \xrightarrow{\text{tex}} \text{"(v2)"}]$

$[(v2) \xrightarrow{\text{pyk}} \text{"var v2"}]$

(v3)

$[(v3) \xrightarrow{\text{tex}} \text{"(v3)"}]$

$[(v3) \xrightarrow{\text{pyk}} \text{"var v3"}]$

(v4)

$[(v4) \xrightarrow{\text{tex}} \text{"(v4)"}]$

$[(v4) \xrightarrow{\text{pyk}} \text{"var v4"}]$

(v2n)

$[(v2n) \xrightarrow{\text{tex}} \text{"(v2n)"}]$

$[(v2n) \xrightarrow{\text{pyk}} \text{"var v2n"}]$

(m1)

[(m1)  $\xrightarrow{\text{tex}}$  "(m1)"]

[(m1)  $\xrightarrow{\text{pyk}}$  "var m1"]

(m2)

[(m2)  $\xrightarrow{\text{tex}}$  "(m2)"]

[(m2)  $\xrightarrow{\text{pyk}}$  "var m2"]

(n1)

[(n1)  $\xrightarrow{\text{tex}}$  "(n1)"]

[(n1)  $\xrightarrow{\text{pyk}}$  "var n1"]

(n2)

[(n2)  $\xrightarrow{\text{tex}}$  "(n2)"]

[(n2)  $\xrightarrow{\text{pyk}}$  "var n2"]

(n3)

[(n3)  $\xrightarrow{\text{tex}}$  "(n3)"]

[(n3)  $\xrightarrow{\text{pyk}}$  "var n3"]

( $\epsilon$ )

[( $\epsilon$ )  $\xrightarrow{\text{tex}}$  "(\\epsilon)"]

[( $\epsilon$ )  $\xrightarrow{\text{pyk}}$  "var ep"]

( $\epsilon$ )<sub>1</sub>

[( $\epsilon$ )<sub>1</sub>  $\xrightarrow{\text{tex}}$  “(\epsilon)\_{1}”]

[( $\epsilon$ )<sub>1</sub>  $\xrightarrow{\text{pyk}}$  “var ep1”]

( $\epsilon$ )<sub>2</sub>

[( $\epsilon$ )<sub>2</sub>  $\xrightarrow{\text{tex}}$  “(\epsilon) 2”]

[( $\epsilon$ )<sub>2</sub>  $\xrightarrow{\text{pyk}}$  “var ep2”]

(fep)

[(fep)  $\xrightarrow{\text{tex}}$  “(fep)”]

[(fep)  $\xrightarrow{\text{pyk}}$  “var fep”]

(fx)

[(fx)  $\xrightarrow{\text{tex}}$  “(fx)”]

[(fx)  $\xrightarrow{\text{pyk}}$  “var fx”]

(fy)

[(fy)  $\xrightarrow{\text{tex}}$  “(fy)”]

[(fy)  $\xrightarrow{\text{pyk}}$  “var fy”]

(fz)

[(fz)  $\xrightarrow{\text{tex}}$  “(fz)”]

[(fz)  $\xrightarrow{\text{pyk}}$  “var fz”]

(fu)

[(fu)  $\xrightarrow{\text{tex}}$  "(fu)"]

[(fu)  $\xrightarrow{\text{pyk}}$  "var fu"]

(fv)

[(fv)  $\xrightarrow{\text{tex}}$  "(fv)"]

[(fv)  $\xrightarrow{\text{pyk}}$  "var fv"]

(fw)

[(fw)  $\xrightarrow{\text{tex}}$  "(fw)"]

[(fw)  $\xrightarrow{\text{pyk}}$  "var fw"]

(rx)

[(rx)  $\xrightarrow{\text{tex}}$  "(rx)"]

[(rx)  $\xrightarrow{\text{pyk}}$  "var rx"]

(ry)

[(ry)  $\xrightarrow{\text{tex}}$  "(ry)"]

[(ry)  $\xrightarrow{\text{pyk}}$  "var ry"]

(rz)

[(rz)  $\xrightarrow{\text{tex}}$  "(rz)"]

[(rz)  $\xrightarrow{\text{pyk}}$  "var rz"]

(ru)

[(ru)  $\xrightarrow{\text{tex}}$  "(ru)"]

[(ru)  $\xrightarrow{\text{pyk}}$  "var ru"]

(sx)

[(sx)  $\xrightarrow{\text{tex}}$  "(sx)"]

[(sx)  $\xrightarrow{\text{pyk}}$  "var sx"]

(sx1)

[(sx1)  $\xrightarrow{\text{tex}}$  "(sx1)"]

[(sx1)  $\xrightarrow{\text{pyk}}$  "var sx1"]

(sy)

[(sy)  $\xrightarrow{\text{tex}}$  "(sy)"]

[(sy)  $\xrightarrow{\text{pyk}}$  "var sy"]

(sy1)

[(sy1)  $\xrightarrow{\text{tex}}$  "(sy1)"]

[(sy1)  $\xrightarrow{\text{pyk}}$  "var sy1"]

(sz)

[(sz)  $\xrightarrow{\text{tex}}$  "(sz)"]

[(sz)  $\xrightarrow{\text{pyk}}$  "var sz"]

(sz1)

[(sz1)  $\xrightarrow{\text{tex}}$  "(sz1)"]

[(sz1)  $\xrightarrow{\text{pyk}}$  "var sz1"]

(su)

[(su)  $\xrightarrow{\text{tex}}$  "(su)"]

[(su)  $\xrightarrow{\text{pyk}}$  "var su"]

(su1)

[(su1)  $\xrightarrow{\text{tex}}$  "(su1)"]

[(su1)  $\xrightarrow{\text{pyk}}$  "var su1"]

(fxs)

[(fxs)  $\xrightarrow{\text{tex}}$  "(fxs)"]

[(fxs)  $\xrightarrow{\text{pyk}}$  "var fxs"]

(fys)

[(fys)  $\xrightarrow{\text{tex}}$  "(fys)"]

[(fys)  $\xrightarrow{\text{pyk}}$  "var fys"]

(crs1)

[(crs1)  $\xrightarrow{\text{tex}}$  "(crs1)"]

[(crs1)  $\xrightarrow{\text{pyk}}$  "var crs1"]

(f1)

[(f1)  $\xrightarrow{\text{tex}}$  "(f1)"]

[(f1)  $\xrightarrow{\text{pyk}}$  "var f1"]

(f2)

[(f2)  $\xrightarrow{\text{tex}}$  "(f2)"]

[(f2)  $\xrightarrow{\text{pyk}}$  "var f2"]

(f3)

[(f3)  $\xrightarrow{\text{tex}}$  "(f3)"]

[(f3)  $\xrightarrow{\text{pyk}}$  "var f3"]

(f4)

[(f4)  $\xrightarrow{\text{tex}}$  "(f4)"]

[(f4)  $\xrightarrow{\text{pyk}}$  "var f4"]

(op1)

[(op1)  $\xrightarrow{\text{tex}}$  "(op1)"]

[(op1)  $\xrightarrow{\text{pyk}}$  "var op1"]

(op2)

[(op2)  $\xrightarrow{\text{tex}}$  "(op2)"]

[(op2)  $\xrightarrow{\text{pyk}}$  "var op2"]



(r1)

[(r1)  $\xrightarrow{\text{tex}}$  “(r1)”]

[(r1)  $\xrightarrow{\text{pyk}}$  “var r1”]

(s1)

[(s1)  $\xrightarrow{\text{tex}}$  “(s1)”]

[(s1)  $\xrightarrow{\text{pyk}}$  “var s1”]

(s2)

[(s2)  $\xrightarrow{\text{tex}}$  “(s2)”]

[(s2)  $\xrightarrow{\text{pyk}}$  “var s2”]

$X_1$

[ $X_1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[X_1 \doteq \underline{(x1)}]])$ ]

[ $X_1 \xrightarrow{\text{tex}}$  “X\_{1}”]

[ $X_1 \xrightarrow{\text{pyk}}$  “meta x1”]

$X_2$

[ $X_2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[X_2 \doteq \underline{(x2)}]])$ ]

[ $X_2 \xrightarrow{\text{tex}}$  “X\_{2}”]

[ $X_2 \xrightarrow{\text{pyk}}$  “meta x2”]

$Y_1$

[ $Y_1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[Y_1 \doteq \underline{(y1)}]])$ ]

[ $Y_1 \xrightarrow{\text{tex}}$  “Y\_{1}”]

[ $Y_1 \xrightarrow{\text{pyk}}$  “meta y1”]

$Y_2$

$[Y_2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[Y_2 \doteq \underline{(y2)}]])]$

$[Y_2 \xrightarrow{\text{tex}} \text{“}Y_{-}\{2\}\text{”}]$

$[Y_2 \xrightarrow{\text{pyk}} \text{“meta } y2\text{”}]$

$V_1$

$[V_1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[V_1 \doteq \underline{(v1)}]])]$

$[V_1 \xrightarrow{\text{tex}} \text{“}V_{-}\{1\}\text{”}]$

$[V_1 \xrightarrow{\text{pyk}} \text{“meta } v1\text{”}]$

$V_2$

$[V_2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[V_2 \doteq \underline{(v2)}]])]$

$[V_2 \xrightarrow{\text{tex}} \text{“}V_{-}\{2\}\text{”}]$

$[V_2 \xrightarrow{\text{pyk}} \text{“meta } v2\text{”}]$

$V_3$

$[V_3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[V_3 \doteq \underline{(v3)}]])]$

$[V_3 \xrightarrow{\text{tex}} \text{“}V_{-}\{3\}\text{”}]$

$[V_3 \xrightarrow{\text{pyk}} \text{“meta } v3\text{”}]$

$V_4$

$[V_4 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[V_4 \doteq \underline{(v4)}]])]$

$[V_4 \xrightarrow{\text{tex}} \text{“}V_{-}\{4\}\text{”}]$

$[V_4 \xrightarrow{\text{pyk}} \text{“meta } v4\text{”}]$

V<sub>2n</sub>

[V<sub>2n</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[V_{2n} \ddot{=} \underline{(v2n)}]])$ ]]

[V<sub>2n</sub> <sup>tex</sup> → “V\_{2n}”]

[V<sub>2n</sub> <sup>pyk</sup> → “meta v2n”]

M<sub>1</sub>

[M<sub>1</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[M_1 \ddot{=} \underline{(m1)}]])$ ]]

[M<sub>1</sub> <sup>tex</sup> → “M\_{1}”]

[M<sub>1</sub> <sup>pyk</sup> → “meta m1”]

M<sub>2</sub>

[M<sub>2</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[M_2 \ddot{=} \underline{(m2)}]])$ ]]

[M<sub>2</sub> <sup>tex</sup> → “M\_{2}”]

[M<sub>2</sub> <sup>pyk</sup> → “meta m2”]

N<sub>1</sub>

[N<sub>1</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[N_1 \ddot{=} \underline{(n1)}]])$ ]]

[N<sub>1</sub> <sup>tex</sup> → “N\_{1} ”]

[N<sub>1</sub> <sup>pyk</sup> → “meta n1”]

N<sub>2</sub>

[N<sub>2</sub> <sup>macro</sup> → λt.λs.λc.  $\tilde{\mathcal{M}}_4(t, s, c, [[N_2 \ddot{=} \underline{(n2)}]])$ ]]

[N<sub>2</sub> <sup>tex</sup> → “N\_{2} ”]

[N<sub>2</sub> <sup>pyk</sup> → “meta n2”]

$N_3$

$[N_3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[N_3 \doteq \underline{(n3)}]])]$

$[N_3 \xrightarrow{\text{tex}} \text{"N_{3}"}]$

$[N_3 \xrightarrow{\text{pyk}} \text{"meta n3"}]$

$\epsilon$

$[\epsilon \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\epsilon \doteq \underline{(\epsilon)}]])]$

$[\epsilon \xrightarrow{\text{tex}} \text{"\epsilon"}]$

$[\epsilon \xrightarrow{\text{pyk}} \text{"meta ep"}]$

$\epsilon 1$

$[\epsilon 1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\epsilon 1 \doteq \underline{(\epsilon)_1}]])]$

$[\epsilon 1 \xrightarrow{\text{tex}} \text{"\epsilon 1"}]$

$[\epsilon 1 \xrightarrow{\text{pyk}} \text{"meta ep1"}]$

$\epsilon 2$

$[\epsilon 2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\epsilon 2 \doteq \underline{(\epsilon 2)}]])]$

$[\epsilon 2 \xrightarrow{\text{tex}} \text{"\epsilon 2"}]$

$[\epsilon 2 \xrightarrow{\text{pyk}} \text{"meta ep2"}]$

$FX$

$[FX \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[FX \doteq \underline{(fx)}]])]$

$[FX \xrightarrow{\text{tex}} \text{"FX"}]$

$[FX \xrightarrow{\text{pyk}} \text{"meta fx"}]$

FY

[FY  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{FY} \doteq \underline{(\text{fy})}]])$ ]

[FY  $\xrightarrow{\text{tex}}$  “FY”]

[FY  $\xrightarrow{\text{pyk}}$  “meta fy”]

FZ

[FZ  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{FZ} \doteq \underline{(\text{fz})}]])$ ]

[FZ  $\xrightarrow{\text{tex}}$  “FZ”]

[FZ  $\xrightarrow{\text{pyk}}$  “meta fz”]

FU

[FU  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{FU} \doteq \underline{(\text{fu})}]])$ ]

[FU  $\xrightarrow{\text{tex}}$  “FU”]

[FU  $\xrightarrow{\text{pyk}}$  “meta fu”]

FV

[FV  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{FV} \doteq \underline{(\text{fv})}]])$ ]

[FV  $\xrightarrow{\text{tex}}$  “FV”]

[FV  $\xrightarrow{\text{pyk}}$  “meta fv”]

FW

[FW  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{FW} \doteq \underline{(\text{fw})}]])$ ]

[FW  $\xrightarrow{\text{tex}}$  “FW”]

[FW  $\xrightarrow{\text{pyk}}$  “meta fw”]

## FEP

$[\text{FEP} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{FEP} \doteq \underline{(\text{fep})}]])]$

$[\text{FEP} \xrightarrow{\text{tex}} \text{“FEP”}]$

$[\text{FEP} \xrightarrow{\text{pyk}} \text{“meta fep”}]$

## RX

$[\text{RX} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{RX} \doteq \underline{(\text{rx})}]])]$

$[\text{RX} \xrightarrow{\text{tex}} \text{“RX”}]$

$[\text{RX} \xrightarrow{\text{pyk}} \text{“meta rx”}]$

## RY

$[\text{RY} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{RY} \doteq \underline{(\text{ry})}]])]$

$[\text{RY} \xrightarrow{\text{tex}} \text{“RY”}]$

$[\text{RY} \xrightarrow{\text{pyk}} \text{“meta ry”}]$

## RZ

$[\text{RZ} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{RZ} \doteq \underline{(\text{rz})}]])]$

$[\text{RZ} \xrightarrow{\text{tex}} \text{“RZ”}]$

$[\text{RZ} \xrightarrow{\text{pyk}} \text{“meta rz”}]$

## RU

$[\text{RU} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{RU} \doteq \underline{(\text{ru})}]])]$

$[\text{RU} \xrightarrow{\text{tex}} \text{“RU”}]$

$[\text{RU} \xrightarrow{\text{pyk}} \text{“meta ru”}]$

(SX)

$[(SX) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[(SX) \doteq (\underline{sx})]])]$

$[(SX) \xrightarrow{\text{tex}} \text{“(SX)”}]$

$[(SX) \xrightarrow{\text{pyk}} \text{“meta sx”}]$

(SX1)

$[(SX1) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[(SX1) \doteq (\underline{sx1})]])]$

$[(SX1) \xrightarrow{\text{tex}} \text{“(SX1)”}]$

$[(SX1) \xrightarrow{\text{pyk}} \text{“meta sx1”}]$

(SY)

$[(SY) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[(SY) \doteq (\underline{sy})]])]$

$[(SY) \xrightarrow{\text{tex}} \text{“(SY)”}]$

$[(SY) \xrightarrow{\text{pyk}} \text{“meta sy”}]$

(SY1)

$[(SY1) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[(SY1) \doteq (\underline{sy1})]])]$

$[(SY1) \xrightarrow{\text{tex}} \text{“(SY1)”}]$

$[(SY1) \xrightarrow{\text{pyk}} \text{“meta sy1”}]$

(SZ)

$[(SZ) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[(SZ) \doteq (\underline{sz})]])]$

$[(SZ) \xrightarrow{\text{tex}} \text{“(SZ)”}]$

$[(SZ) \xrightarrow{\text{pyk}} \text{“meta sz”}]$

(SZ1)

$[(SZ1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(SZ1) \ddot{=} (\underline{sz1})]])]$

$[(SZ1) \xrightarrow{\text{tex}} \text{“(SZ1)”}]$

$[(SZ1) \xrightarrow{\text{pyk}} \text{“meta sz1”}]$

(SU)

$[(SU) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(SU) \ddot{=} (\underline{su})]])]$

$[(SU) \xrightarrow{\text{tex}} \text{“(SU)”}]$

$[(SU) \xrightarrow{\text{pyk}} \text{“meta su”}]$

(SU1)

$[(SU1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(SU1) \ddot{=} (\underline{su1})]])]$

$[(SU1) \xrightarrow{\text{tex}} \text{“(SU1)”}]$

$[(SU1) \xrightarrow{\text{pyk}} \text{“meta su1”}]$

FXS

$[(FXS) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[FXS \ddot{=} (\underline{fxs})]])]$

$[(FXS) \xrightarrow{\text{tex}} \text{“FXS”}]$

$[(FXS) \xrightarrow{\text{pyk}} \text{“meta fxs”}]$

FYS

$[(FYS) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[FYS \ddot{=} (\underline{fys})]])]$

$[(FYS) \xrightarrow{\text{tex}} \text{“FYS”}]$

$[(FYS) \xrightarrow{\text{pyk}} \text{“meta fys”}]$



(F1)

$[(F1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F1) \doteq \underline{(f1)}])])]$

$[(F1) \xrightarrow{\text{tex}} \text{“(F1)”}]$

$[(F1) \xrightarrow{\text{pyk}} \text{“meta f1”}]$

(F2)

$[(F2) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F2) \doteq \underline{(f2)}])])]$

$[(F2) \xrightarrow{\text{tex}} \text{“(F2)”}]$

$[(F2) \xrightarrow{\text{pyk}} \text{“meta f2”}]$

(F3)

$[(F3) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F3) \doteq \underline{(f3)}])])]$

$[(F3) \xrightarrow{\text{tex}} \text{“(F3)”}]$

$[(F3) \xrightarrow{\text{pyk}} \text{“meta f3”}]$

(F4)

$[(F4) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F4) \doteq \underline{(f4)}])])]$

$[(F4) \xrightarrow{\text{tex}} \text{“(F4)”}]$

$[(F4) \xrightarrow{\text{pyk}} \text{“meta f4”}]$

(OP1)

$[(OP1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(OP1) \doteq \underline{(op1)}])])]$

$[(OP1) \xrightarrow{\text{tex}} \text{“(OP1)”}]$

$[(OP1) \xrightarrow{\text{pyk}} \text{“meta op1”}]$

(OP2)

$[(OP2) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(OP2) \doteq \underline{(op2)}])])]$

$[(OP2) \xrightarrow{\text{tex}} \text{“(OP2)”}]$

$[(OP2) \xrightarrow{\text{pyk}} \text{“meta op2”}]$

(R1)

$[(R1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(R1) \doteq \underline{(r1)}])])]$

$[(R1) \xrightarrow{\text{tex}} \text{“(R1)”}]$

$[(R1) \xrightarrow{\text{pyk}} \text{“meta r1”}]$

(S1)

$[(S1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S1) \doteq \underline{(s1)}])])]$

$[(S1) \xrightarrow{\text{tex}} \text{“(S1)”}]$

$[(S1) \xrightarrow{\text{pyk}} \text{“meta s1”}]$

(S2)

$[(S2) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S2) \doteq \underline{(s2)}])])]$

$[(S2) \xrightarrow{\text{tex}} \text{“(S2)”}]$

$[(S2) \xrightarrow{\text{pyk}} \text{“meta s2”}]$

(EPob)

$[(EPob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(EPob) \doteq \overline{(\epsilon)}])])]$

$[(EPob) \xrightarrow{\text{tex}} \text{“(EPob)”}]$

$[(EPob) \xrightarrow{\text{pyk}} \text{“object ep”}]$

(CRS1ob)

$[(\text{CRS1ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{CRS1ob}) \doteq \overline{(\text{crs1})}]])]$   
 $[(\text{CRS1ob}) \xrightarrow{\text{tex}} \text{“}(\text{CRS1ob})\text{”}]$   
 $[(\text{CRS1ob}) \xrightarrow{\text{pyk}} \text{“object crs1”}]$

(F1ob)

$[(\text{F1ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{F1ob}) \doteq \overline{(\text{f1})}]])]$   
 $[(\text{F1ob}) \xrightarrow{\text{tex}} \text{“}(\text{F1ob})\text{”}]$   
 $[(\text{F1ob}) \xrightarrow{\text{pyk}} \text{“object f1”}]$

(F2ob)

$[(\text{F2ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{F2ob}) \doteq \overline{(\text{f2})}]])]$   
 $[(\text{F2ob}) \xrightarrow{\text{tex}} \text{“}(\text{F2ob})\text{”}]$   
 $[(\text{F2ob}) \xrightarrow{\text{pyk}} \text{“object f2”}]$

(F3ob)

$[(\text{F3ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{F3ob}) \doteq \overline{(\text{f3})}]])]$   
 $[(\text{F3ob}) \xrightarrow{\text{tex}} \text{“}(\text{F3ob})\text{”}]$   
 $[(\text{F3ob}) \xrightarrow{\text{pyk}} \text{“object f3”}]$

(F4ob)

$[(\text{F4ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{F4ob}) \doteq \overline{(\text{f4})}]])]$   
 $[(\text{F4ob}) \xrightarrow{\text{tex}} \text{“}(\text{F4ob})\text{”}]$   
 $[(\text{F4ob}) \xrightarrow{\text{pyk}} \text{“object f4”}]$

(N1ob)

$[(\text{N1ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{N1ob}) \doteq \overline{(\text{n1})}]])]$

$[(\text{N1ob}) \xrightarrow{\text{tex}} \text{“}(\text{N1ob})\text{”}]$

$[(\text{N1ob}) \xrightarrow{\text{pyk}} \text{“object n1”}]$

(N2ob)

$[(\text{N2ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{N2ob}) \doteq \overline{(\text{n2})}]])]$

$[(\text{N2ob}) \xrightarrow{\text{tex}} \text{“}(\text{N2ob})\text{”}]$

$[(\text{N2ob}) \xrightarrow{\text{pyk}} \text{“object n2”}]$

(OP1ob)

$[(\text{OP1ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{OP1ob}) \doteq \overline{(\text{op1})}]])]$

$[(\text{OP1ob}) \xrightarrow{\text{tex}} \text{“}(\text{OP1ob})\text{”}]$

$[(\text{OP1ob}) \xrightarrow{\text{pyk}} \text{“object op1”}]$

(OP2ob)

$[(\text{OP2ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{OP2ob}) \doteq \overline{(\text{op2})}]])]$

$[(\text{OP2ob}) \xrightarrow{\text{tex}} \text{“}(\text{OP2ob})\text{”}]$

$[(\text{OP2ob}) \xrightarrow{\text{pyk}} \text{“object op2”}]$

(R1ob)

$[(\text{R1ob}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{R1ob}) \doteq \overline{(\text{r1})}]])]$

$[(\text{R1ob}) \xrightarrow{\text{tex}} \text{“}(\text{R1ob})\text{”}]$

$[(\text{R1ob}) \xrightarrow{\text{pyk}} \text{“object r1”}]$

(S1ob)

$[(S1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S1ob) \doteq \overline{(s1)}]])]$

$[(S1ob) \xrightarrow{\text{tex}} \text{“(S1ob)”}]$

$[(S1ob) \xrightarrow{\text{pyk}} \text{“object s1”}]$

(S2ob)

$[(S2ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S2ob) \doteq \overline{(s2)}]])]$

$[(S2ob) \xrightarrow{\text{tex}} \text{“(S2ob)”}]$

$[(S2ob) \xrightarrow{\text{pyk}} \text{“object s2”}]$

ph<sub>4</sub>

$[\text{ph}_4 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_4 \doteq \text{d}_{Ph}]])]$

$[\text{ph}_4 \xrightarrow{\text{tex}} \text{“ph_{4}”}]$

$[\text{ph}_4 \xrightarrow{\text{pyk}} \text{“ph4”}]$

ph<sub>5</sub>

$[\text{ph}_5 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_5 \doteq \text{e}_{Ph}]])]$

$[\text{ph}_5 \xrightarrow{\text{tex}} \text{“ph_{5}”}]$

$[\text{ph}_5 \xrightarrow{\text{pyk}} \text{“ph5”}]$

ph<sub>6</sub>

$[\text{ph}_6 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_6 \doteq \text{f}_{Ph}]])]$

$[\text{ph}_6 \xrightarrow{\text{tex}} \text{“ph_{6}”}]$

$[\text{ph}_6 \xrightarrow{\text{pyk}} \text{“ph6”}]$

## NAT

[NAT  $\xrightarrow{\text{tex}}$  "NAT"]

[NAT  $\xrightarrow{\text{pyk}}$  "NAT"]

## RATIONAL<sub>S</sub>SERIES

[RATIONAL<sub>S</sub>SERIES  $\xrightarrow{\text{tex}}$  "RATIONAL\_SERIES"]

[RATIONAL<sub>S</sub>SERIES  $\xrightarrow{\text{pyk}}$  "RATIONAL\_SERIES"]

## SERIES

[SERIES  $\xrightarrow{\text{tex}}$  "SERIES"]

[SERIES  $\xrightarrow{\text{pyk}}$  "SERIES"]

## SetOfReals

[SetOfReals  $\xrightarrow{\text{tex}}$  "SetOfReals"]

[SetOfReals  $\xrightarrow{\text{pyk}}$  "setOfReals"]

## SetOfFxs

[SetOfFxs  $\xrightarrow{\text{tex}}$  "SetOfFxs"]

[SetOfFxs  $\xrightarrow{\text{pyk}}$  "setOfFxs"]

## N

[N  $\xrightarrow{\text{tex}}$  "N"]

[N  $\xrightarrow{\text{pyk}}$  "N"]

## Q

[Q  $\xrightarrow{\text{tex}}$  "Q"]

$[Q \xrightarrow{\text{pyk}} \text{“Q”}]$

X

$[X \xrightarrow{\text{tex}} \text{“X”}]$

$[X \xrightarrow{\text{pyk}} \text{“X”}]$

XS

$[xs \xrightarrow{\text{tex}} \text{“xs”}]$

$[xs \xrightarrow{\text{pyk}} \text{“xs”}]$

xaF

$[xaF \xrightarrow{\text{tex}} \text{“xaF”}]$

$[xaF \xrightarrow{\text{pyk}} \text{“xsF”}]$

ysF

$[ysF \xrightarrow{\text{tex}} \text{“ysF”}]$

$[ysF \xrightarrow{\text{pyk}} \text{“ysF”}]$

us

$[us \xrightarrow{\text{tex}} \text{“us”}]$

$[us \xrightarrow{\text{pyk}} \text{“us”}]$

usFoelge

$[usFoelge \xrightarrow{\text{tex}} \text{“usFoelge”}]$

$[usFoelge \xrightarrow{\text{pyk}} \text{“usF”}]$

0

$[0 \xrightarrow{\text{tex}} \text{“0”}]$

$[0 \xrightarrow{\text{pyk}} \text{“0”}]$

1

$[1 \xrightarrow{\text{tex}} \text{“1”}]$

$[1 \xrightarrow{\text{pyk}} \text{“1”}]$

(-1)

$[(-1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(-1) \ddot{=} (-u1)])]]$

$[(-1) \xrightarrow{\text{tex}} \text{“(-1)”}]$

$[(-1) \xrightarrow{\text{pyk}} \text{“(-1)”}]$

2

$[2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[2 \ddot{=} (1 + 1)])]]$

$[2 \xrightarrow{\text{tex}} \text{“2”}]$

$[2 \xrightarrow{\text{pyk}} \text{“2”}]$

3

$[3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[3 \ddot{=} (2 + 1)])]]$

$[3 \xrightarrow{\text{tex}} \text{“3”}]$

$[3 \xrightarrow{\text{pyk}} \text{“3”}]$

1/2

$[1/2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[1/2 \ddot{=} \text{rec2}]])]$

$[1/2 \xrightarrow{\text{tex}} \text{“1/2”}]$

$[1/2 \xrightarrow{\text{pyk}} \text{“1/2”}]$



**1/3**

[1/3  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[1/3 \doteq \text{rec3}]])$ ]

[1/3  $\xrightarrow{\text{tex}}$  “1/3”]

[1/3  $\xrightarrow{\text{pyk}}$  “1/3”]

**2/3**

[2/3  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[2/3 \doteq (2 * 1/3)])])$ ]

[2/3  $\xrightarrow{\text{tex}}$  “2/3”]

[2/3  $\xrightarrow{\text{pyk}}$  “2/3”]

**0f**

[0f  $\xrightarrow{\text{tex}}$  “0f”]

[0f  $\xrightarrow{\text{pyk}}$  “0f”]

**1f**

[1f  $\xrightarrow{\text{pyk}}$  “1f”]

**00**

[00  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[00 \doteq \text{R}(0f)])])$ ]

[00  $\xrightarrow{\text{tex}}$  “00”]

[00  $\xrightarrow{\text{pyk}}$  “00”]

**01**

[01  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[01 \doteq \text{R}(1f)])])$ ]

[01  $\xrightarrow{\text{pyk}}$  “01”]

(- - 01)

[(- - 01)  $\xrightarrow{\text{tex}}$  “(-01)”]

[(- - 01)  $\xrightarrow{\text{pyk}}$  “(-01)”]

02

[02  $\xrightarrow{\text{tex}}$  “02”]

[02  $\xrightarrow{\text{pyk}}$  “02”]

01//02

[01//02  $\xrightarrow{\text{tex}}$  “01//02”]

[01//02  $\xrightarrow{\text{pyk}}$  “01//02”]

PlusAssociativity(R)

[PlusAssociativity(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[PlusAssociativity(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$

$\forall(\underline{\text{fx}}): \forall(\underline{\text{fy}}): \forall(\underline{\text{fz}}): \text{R}(\underline{\text{fx}}) ++ \text{R}(\underline{\text{fy}}) ++ \text{R}(\underline{\text{fz}}) = \text{R}(\underline{\text{fx}}) ++ \text{R}(\underline{\text{fy}}) ++ \text{R}(\underline{\text{fz}})]$

[PlusAssociativity(R)  $\xrightarrow{\text{tex}}$  “PlusAssociativity(R)”]

[PlusAssociativity(R)  $\xrightarrow{\text{pyk}}$  “lemma plusAssociativity(R)”]

PlusAssociativity(R)XX

[PlusAssociativity(R)XX  $\xrightarrow{\text{proof}}$  Rule tactic]

[PlusAssociativity(R)XX  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$

$\forall(\underline{\text{fx}}): \forall(\underline{\text{fy}}): \forall(\underline{\text{fz}}): \text{R}(\underline{\text{fx}}) +_f \underline{\text{fy}} +_f \underline{\text{fz}}) == \text{R}(\underline{\text{fx}} +_f \underline{\text{fy}} +_f \underline{\text{fz}})]$

[PlusAssociativity(R)XX  $\xrightarrow{\text{tex}}$  “PlusAssociativity(R)XX”]

[PlusAssociativity(R)XX  $\xrightarrow{\text{pyk}}$  “lemma plusAssociativity(R)XX”]

## Plus0(R)

[Plus0(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[Plus0(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{\text{fx}}: \text{R}(\underline{\text{fx}}) + +\text{R}(0\text{f}) == \text{R}(\underline{\text{fx}})$ ]

[Plus0(R)  $\xrightarrow{\text{tex}}$  “Plus0(R)”]

[Plus0(R)  $\xrightarrow{\text{pyk}}$  “lemma plus0(R)”]

## Negative(R)

[Negative(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[Negative(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{\text{m}}: \forall \underline{\text{fx}}: \text{R}(\underline{\text{fx}}) + +(- - \text{R}(\underline{\text{fx}})) == \text{R}(0\text{f})$ ]

[Negative(R)  $\xrightarrow{\text{tex}}$  “Negative(R)”]

[Negative(R)  $\xrightarrow{\text{pyk}}$  “lemma negative(R)”]

## Times1(R)

[Times1(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[Times1(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{\text{fx}}: \text{R}(\underline{\text{fx}}) * * \text{R}(1\text{f}) == \text{R}(\underline{\text{fx}})$ ]

[Times1(R)  $\xrightarrow{\text{tex}}$  “Times1(R)”]

[Times1(R)  $\xrightarrow{\text{pyk}}$  “lemma times1(R)”]

## lessAddition(R)

[lessAddition(R)  $\xrightarrow{\text{tex}}$  “lessAddition(R)”]

[lessAddition(R)  $\xrightarrow{\text{pyk}}$  “lemma lessAddition(R)”]

## PlusCommutativity(R)

[PlusCommutativity(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[PlusCommutativity(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{\text{fx}}: \forall \underline{\text{fy}}: \text{R}(\underline{\text{fx}}) + +\text{R}(\underline{\text{fy}}) == \text{R}(\underline{\text{fy}}) + +\text{R}(\underline{\text{fx}})$ ]

[PlusCommutativity(R)  $\xrightarrow{\text{tex}}$  “PlusCommutativity(R)”]

[PlusCommutativity(R)  $\xrightarrow{\text{pyk}}$  “lemma plusCommutativity(R)”]

## LeqAntisymmetry(R)

[LeqAntisymmetry(R)  $\xrightarrow{\text{tex}}$  “LeqAntisymmetry(R)”]

[LeqAntisymmetry(R)  $\xrightarrow{\text{pyk}}$  “lemma leqAntisymmetry(R)”]

## LeqTransitivity(R)

[LeqTransitivity(R)  $\xrightarrow{\text{tex}}$  “LeqTransitivity(R)”]

[LeqTransitivity(R)  $\xrightarrow{\text{pyk}}$  “lemma leqTransitivity(R)”]

## leqAddition(R)

[leqAddition(R)  $\xrightarrow{\text{tex}}$  “leqAddition(R)”]

[leqAddition(R)  $\xrightarrow{\text{pyk}}$  “lemma leqAddition(R)”]

## Distribution(R)

[Distribution(R)  $\xrightarrow{\text{tex}}$  “Distribution(R)”]

[Distribution(R)  $\xrightarrow{\text{pyk}}$  “lemma distribution(R)”]

## A4(Axiom)

[A4(Axiom)  $\xrightarrow{\text{proof}}$  Rule tactic]

[A4(Axiom)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall (\underline{v1}): \forall \underline{a}: \forall \underline{b}: \langle \underline{a} \equiv \underline{b} \mid (\underline{v1}): == \underline{x} \rangle_{\text{Me}} \Vdash$   
 $\forall_{\text{obj}} (\underline{v1}): \underline{b} \Rightarrow \underline{a}$ ]

[A4(Axiom)  $\xrightarrow{\text{tex}}$  “A4(Axiom)”]

[A4(Axiom)  $\xrightarrow{\text{pyk}}$  “axiom a4”]

## InductionAxiom

[InductionAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[InductionAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall(v1): \forall a: \forall b: \forall c: (\underline{b} \equiv \underline{a} | (v1) ::= 0)_{\text{Me}} \Vdash$   
 $\langle \underline{c} \equiv \underline{a} | (v1) ::= ((v1) + 1) \rangle_{\text{Me}} \Vdash \underline{b} \Rightarrow \forall_{\text{obj}}(v1): \underline{a} \Rightarrow \underline{c} \Rightarrow \forall_{\text{obj}}(v1): \underline{a}$ ]

[InductionAxiom  $\xrightarrow{\text{tex}}$  “InductionAxiom”]

[InductionAxiom  $\xrightarrow{\text{pyk}}$  “axiom induction”]

## EqualityAxiom

[EqualityAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[EqualityAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall x: \forall y: \forall z: \underline{x} = \underline{y} \Rightarrow \underline{x} = \underline{z} \Rightarrow \underline{y} = \underline{z}$ ]

[EqualityAxiom  $\xrightarrow{\text{tex}}$  “EqualityAxiom”]

[EqualityAxiom  $\xrightarrow{\text{pyk}}$  “axiom equality”]

## EqLeqAxiom

[EqLeqAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[EqLeqAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall x: \forall y: \underline{x} = \underline{y} \Rightarrow \underline{x} <= \underline{y}$ ]

[EqLeqAxiom  $\xrightarrow{\text{tex}}$  “EqLeqAxiom”]

[EqLeqAxiom  $\xrightarrow{\text{pyk}}$  “axiom eqLeq”]

## EqAdditionAxiom

[EqAdditionAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[EqAdditionAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall x: \forall y: \forall z: \underline{x} = \underline{y} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z})$ ]

[EqAdditionAxiom  $\xrightarrow{\text{tex}}$  “EqAdditionAxiom”]

[EqAdditionAxiom  $\xrightarrow{\text{pyk}}$  “axiom eqAddition”]

## EqMultiplicationAxiom

[EqMultiplicationAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[EqMultiplicationAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})$ ]

[EqMultiplicationAxiom  $\xrightarrow{\text{tex}}$  “EqMultiplicationAxiom”]

[EqMultiplicationAxiom  $\xrightarrow{\text{pyk}}$  “axiom eqMultiplication”]

## QisClosed(Reciprocal)(ImPLY)

[QisClosed(Reciprocal)(ImPLY)  $\xrightarrow{\text{tex}}$  “QisClosed(Reciprocal)(ImPLY)”]

[QisClosed(Reciprocal)(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom QisClosed(reciprocal)”]

## QisClosed(Reciprocal)

[QisClosed(Reciprocal)  $\xrightarrow{\text{tex}}$  “QisClosed(Reciprocal)”]

[QisClosed(Reciprocal)  $\xrightarrow{\text{pyk}}$  “lemma QisClosed(reciprocal)”]

## QisClosed(Negative)(ImPLY)

[QisClosed(Negative)(ImPLY)  $\xrightarrow{\text{tex}}$  “QisClosed(Negative)(ImPLY)”]

[QisClosed(Negative)(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom QisClosed(negative)”]

## QisClosed(Negative)

[QisClosed(Negative)  $\xrightarrow{\text{tex}}$  “QisClosed(Negative)”]

[QisClosed(Negative)  $\xrightarrow{\text{pyk}}$  “lemma QisClosed(negative)”]

## leqReflexivity

[leqReflexivity  $\xrightarrow{\text{proof}}$  Rule tactic]

[leqReflexivity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \underline{x} \leq \underline{x}$ ]

[leqReflexivity  $\xrightarrow{\text{tex}}$  “leqReflexivity”]

[leqReflexivity  $\xrightarrow{\text{pyk}}$  “axiom leqReflexivity”]

## leqAntisymmetryAxiom

[leqAntisymmetryAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[leqAntisymmetryAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{x} \Rightarrow \underline{x} = \underline{y}$ ]

[leqAntisymmetryAxiom  $\xrightarrow{\text{tex}}$  “leqAntisymmetryAxiom”]

[leqAntisymmetryAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqAntisymmetry”]

## leqTransitivityAxiom

[leqTransitivityAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[leqTransitivityAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} <= \underline{y} \Rightarrow \underline{y} <= \underline{z} \Rightarrow \underline{x} <= \underline{z}$ ]

[leqTransitivityAxiom  $\xrightarrow{\text{tex}}$  “leqTransitivityAxiom”]

[leqTransitivityAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqTransitivity”]

## leqTotality

[leqTotality  $\xrightarrow{\text{proof}}$  Rule tactic]

[leqTotality  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \neg (\underline{x} <= \underline{y}) \Rightarrow \underline{y} <= \underline{x}$ ]

[leqTotality  $\xrightarrow{\text{tex}}$  “leqTotality”]

[leqTotality  $\xrightarrow{\text{pyk}}$  “axiom leqTotality”]

## leqAdditionAxiom

[leqAdditionAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[leqAdditionAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} <= \underline{y} \Rightarrow (\underline{x} + \underline{z}) <= (\underline{y} + \underline{z})$ ]

[leqAdditionAxiom  $\xrightarrow{\text{tex}}$  “leqAdditionAxiom”]

[leqAdditionAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqAddition”]

## leqMultiplicationAxiom

[leqMultiplicationAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[leqMultiplicationAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: 0 \leq \underline{z} \Rightarrow \underline{x} \leq \underline{y} \Rightarrow (\underline{x} * \underline{z}) \leq (\underline{y} * \underline{z})$ ]

[leqMultiplicationAxiom  $\xrightarrow{\text{tex}}$  “leqMultiplicationAxiom”]

[leqMultiplicationAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqMultiplication”]

## plusAssociativity

[plusAssociativity  $\xrightarrow{\text{proof}}$  Rule tactic]

[plusAssociativity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z}))$ ]

[plusAssociativity  $\xrightarrow{\text{tex}}$  “plusAssociativity”]

[plusAssociativity  $\xrightarrow{\text{pyk}}$  “axiom plusAssociativity”]

## plusCommutativity

[plusCommutativity  $\xrightarrow{\text{proof}}$  Rule tactic]

[plusCommutativity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: (\underline{x} + \underline{y}) = (\underline{y} + \underline{x})$ ]

[plusCommutativity  $\xrightarrow{\text{tex}}$  “plusCommutativity”]

[plusCommutativity  $\xrightarrow{\text{pyk}}$  “axiom plusCommutativity”]

## Negative

[Negative  $\xrightarrow{\text{proof}}$  Rule tactic]

[Negative  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: (\underline{x} + (-\underline{u}\underline{x})) = 0$ ]

[Negative  $\xrightarrow{\text{tex}}$  “Negative”]

[Negative  $\xrightarrow{\text{pyk}}$  “axiom negative”]



## plus0

[plus0  $\xrightarrow{\text{proof}}$  Rule tactic]

[plus0  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: (\underline{x} + 0) = \underline{x}$ ]

[plus0  $\xrightarrow{\text{tex}}$  “plus0”]

[plus0  $\xrightarrow{\text{pyk}}$  “axiom plus0”]

## timesAssociativity

[timesAssociativity  $\xrightarrow{\text{proof}}$  Rule tactic]

[timesAssociativity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{y} * \underline{z}))$ ]

[timesAssociativity  $\xrightarrow{\text{tex}}$  “timesAssociativity”]

[timesAssociativity  $\xrightarrow{\text{pyk}}$  “axiom timesAssociativity”]

## timesCommutativity

[timesCommutativity  $\xrightarrow{\text{proof}}$  Rule tactic]

[timesCommutativity  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: (\underline{x} * \underline{y}) = (\underline{y} * \underline{x})$ ]

[timesCommutativity  $\xrightarrow{\text{tex}}$  “timesCommutativity”]

[timesCommutativity  $\xrightarrow{\text{pyk}}$  “axiom timesCommutativity”]

## ReciprocalAxiom

[ReciprocalAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[ReciprocalAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \dot{\vdash} (\underline{x} = 0) \Rightarrow (\underline{x} * \text{rec}\underline{x}) = 1$ ]

[ReciprocalAxiom  $\xrightarrow{\text{tex}}$  “ReciprocalAxiom”]

[ReciprocalAxiom  $\xrightarrow{\text{pyk}}$  “axiom reciprocal”]

## times1

[times1  $\xrightarrow{\text{proof}}$  Rule tactic]

[times1  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: (\underline{x} * 1) = \underline{x}$ ]

[times1  $\xrightarrow{\text{tex}}$  “times1”]

[times1  $\xrightarrow{\text{pyk}}$  “axiom times1”]

## Distribution

[Distribution  $\xrightarrow{\text{proof}}$  Rule tactic]

[Distribution  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{x} * \underline{y}) + (\underline{x} * \underline{z}))$ ]

[Distribution  $\xrightarrow{\text{tex}}$  “Distribution”]

[Distribution  $\xrightarrow{\text{pyk}}$  “axiom distribution”]

## 0not1

[0not1  $\xrightarrow{\text{proof}}$  Rule tactic]

[0not1  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \dot{\vdash} (0 = 1)_n$ ]

[0not1  $\xrightarrow{\text{tex}}$  “0not1”]

[0not1  $\xrightarrow{\text{pyk}}$  “axiom 0not1”]

## lemma eqLeq(**R**)

[lemma eqLeq(**R**)  $\xrightarrow{\text{pyk}}$  “lemma eqLeq(**R**)”]

## TimesAssociativity(**R**)

[TimesAssociativity(**R**)  $\xrightarrow{\text{proof}}$  Rule tactic]

[TimesAssociativity(**R**)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$   
 $\forall(\underline{fx}): \forall(\underline{fy}): \forall(\underline{fz}): \underline{R}(\underline{fx}) * * \underline{R}(\underline{fy}) * * \underline{R}(\underline{fz}) == \underline{R}(\underline{fx}) * * \underline{R}(\underline{fy}) * * \underline{R}(\underline{fz})$ ]

[TimesAssociativity(**R**)  $\xrightarrow{\text{tex}}$  “TimesAssociativity(**R**)”]

[TimesAssociativity(**R**)  $\xrightarrow{\text{pyk}}$  “lemma timesAssociativity(**R**)”]

## TimesCommutativity(R)

[TimesCommutativity(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[TimesCommutativity(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall(\underline{fx}): \forall(\underline{fy}): R(\underline{fx}) * *R(\underline{fy}) == R(\underline{fy}) * *R(\underline{fx})$ ]

[TimesCommutativity(R)  $\xrightarrow{\text{tex}}$  “TimesCommutativity(R)”]

[TimesCommutativity(R)  $\xrightarrow{\text{pyk}}$  “lemma timesCommutativity(R)”]

## (Adgic)SameR

[(Adgic)SameR  $\xrightarrow{\text{tex}}$  “(Adgic)SameR”]

[(Adgic)SameR  $\xrightarrow{\text{pyk}}$  “1rule adhoc sameR”]

## Separation2formula(1)

[Separation2formula(1)  $\xrightarrow{\text{tex}}$  “Separation2formula(1)”]

[Separation2formula(1)  $\xrightarrow{\text{pyk}}$  “lemma separation2formula(1)”]

## Separation2formula(2)

[Separation2formula(2)  $\xrightarrow{\text{tex}}$  “Separation2formula(2)”]

[Separation2formula(2)  $\xrightarrow{\text{pyk}}$  “lemma separation2formula(2)”]

## Cauchy

[Cauchy  $\xrightarrow{\text{proof}}$  Rule tactic]

[Cauchy  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$   
 $\forall(\underline{v1}): \forall(\underline{v2}): \forall \underline{n}: \forall(\underline{\epsilon}): \forall(\underline{fx}): \forall_{\text{obj}}(\underline{\epsilon}): \dot{\vdash} (\forall_{\text{obj}} \underline{n}: \dot{\vdash} (\forall_{\text{obj}}(\underline{v1}): \forall_{\text{obj}}(\underline{v2}): \dot{\vdash} (0 <=$   
 $(\underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{\epsilon}) \underline{n}) \underline{n}) \underline{n} \Rightarrow \underline{n} <= (\underline{v1}) \Rightarrow \underline{n} <= (\underline{v2}) \Rightarrow \dot{\vdash} (|((\underline{fx})[(\underline{v1})] +$   
 $(-\underline{u}(\underline{fx})[(\underline{v2})])| <= (\underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|((\underline{fx})[(\underline{v1})] + (-\underline{u}(\underline{fx})[(\underline{v2})])| = (\underline{\epsilon}) \underline{n}) \underline{n}) \underline{n}) \underline{n})$ ]

[Cauchy  $\xrightarrow{\text{tex}}$  “Cauchy”]

[Cauchy  $\xrightarrow{\text{pyk}}$  “axiom cauchy”]

## PlusF

[PlusF  $\xrightarrow{\text{proof}}$  Rule tactic]

[PlusF  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{fx}: \forall \underline{fy}: \underline{fx} +_f \underline{fy} [\underline{m}] = ((\underline{fx})[\underline{m}] + (\underline{fy})[\underline{m}])$ ]

[PlusF  $\xrightarrow{\text{tex}}$  “PlusF”]

[PlusF  $\xrightarrow{\text{pyk}}$  “axiom plusF”]

## ReciprocalF

[ReciprocalF  $\xrightarrow{\text{tex}}$  “ReciprocalF”]

[ReciprocalF  $\xrightarrow{\text{pyk}}$  “axiom reciprocalF”]

## From ==

[From ==  $\xrightarrow{\text{proof}}$  Rule tactic]

[From ==  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{fx}: \forall \underline{fy}: \text{R}(\underline{fx}) == \text{R}(\underline{fy}) \vdash \text{SF}(\underline{fx}, \underline{fy})$ ]

[From ==  $\xrightarrow{\text{tex}}$  “From==”]

[From ==  $\xrightarrow{\text{pyk}}$  “1rule from==”]

## To ==

[To ==  $\xrightarrow{\text{proof}}$  Rule tactic]

[To ==  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{fx}: \forall \underline{fy}: \text{SF}(\underline{fx}, \underline{fy}) \vdash \text{R}(\underline{fx}) == \text{R}(\underline{fy})$ ]

[To ==  $\xrightarrow{\text{tex}}$  “To==”]

[To ==  $\xrightarrow{\text{pyk}}$  “1rule to==”]

## FromInR

[FromInR  $\xrightarrow{\text{proof}}$  Rule tactic]

[FromInR  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{fx}: \forall \underline{fy}: \underline{fx} \in \text{R}(\underline{fy}) \vdash \text{SF}(\underline{fx}, \underline{fy})$ ]

[FromInR  $\xrightarrow{\text{tex}}$  “FromInR”]

[FromInR  $\xrightarrow{\text{pyk}}$  “1rule fromInR”]

## PlusR(Sym)

[PlusR(Sym)  $\xrightarrow{\text{tex}}$  “PlusR(Sym)”]

[PlusR(Sym)  $\xrightarrow{\text{pyk}}$  “lemma plusR(Sym)”]

## ReciprocalR(Axiom)

[ReciprocalR(Axiom)  $\xrightarrow{\text{tex}}$  “ReciprocalR(Axiom)”]

[ReciprocalR(Axiom)  $\xrightarrow{\text{pyk}}$  “axiom reciprocalR”]

## LessMinus1(N)

[LessMinus1(N)  $\xrightarrow{\text{proof}}$  Rule tactic]

[LessMinus1(N)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{n}: \text{Nat}(\underline{m}) \Vdash \text{Nat}(\underline{n}) \Vdash \dot{\vdash} (\underline{m} \leq (\underline{n} + 1)) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{m} = (\underline{n} + 1)) \underline{n}) \underline{n} \vdash \underline{m} \leq \underline{n}$ ]

[LessMinus1(N)  $\xrightarrow{\text{tex}}$  “LessMinus1(N)”]

[LessMinus1(N)  $\xrightarrow{\text{pyk}}$  “1rule lessMinus1(N)”]

## Nonnegative(N)

[Nonnegative(N)  $\xrightarrow{\text{proof}}$  Rule tactic]

[Nonnegative(N)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \text{Nat}(\underline{m}) \Vdash 0 \leq \underline{m}$ ]

[Nonnegative(N)  $\xrightarrow{\text{tex}}$  “Nonnegative(N)”]

[Nonnegative(N)  $\xrightarrow{\text{pyk}}$  “axiom nonnegative(N)”]

## US0

[US0  $\xrightarrow{\text{proof}}$  Rule tactic]

[US0  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \text{us}[0] == \text{xs}[0] + +\text{R}(1\text{f})$ ]

[US0  $\xrightarrow{\text{tex}}$  “US0”]

[US0  $\xrightarrow{\text{pyk}}$  “axiom US0”]

## NextXS(UpperBound)

[NextXS(UpperBound)  $\xrightarrow{\text{proof}}$  Rule tactic]

[NextXS(UpperBound)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$   
 $\forall \underline{m}: \text{UB}(01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals}) \vdash \text{xs}[(\underline{m} + 1)] == \text{xs}[\underline{m}]$ ]

[NextXS(UpperBound)  $\xrightarrow{\text{tex}}$  “NextXS(UpperBound)”]

[NextXS(UpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextXS(upperBound)”]

## NextXS(NoUpperBound)

[NextXS(NoUpperBound)  $\xrightarrow{\text{proof}}$  Rule tactic]

[NextXS(NoUpperBound)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \neg (\text{UB}(01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals})) \vdash \text{xs}[(\underline{m} + 1)] == 01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}]$ ]

[NextXS(NoUpperBound)  $\xrightarrow{\text{tex}}$  “NextXS(NoUpperBound)”]

[NextXS(NoUpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextXS(noUpperBound)”]

## NextUS(UpperBound)

[NextUS(UpperBound)  $\xrightarrow{\text{proof}}$  Rule tactic]

[NextUS(UpperBound)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \text{UB}(01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals}) \vdash \text{us}[(\underline{m} + 1)] == 01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}]$ ]

[NextUS(UpperBound)  $\xrightarrow{\text{tex}}$  “NextUS(UpperBound)”]

[NextUS(UpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextUS(upperBound)”]

## NextUS(NoUpperBound)

[NextUS(NoUpperBound)  $\xrightarrow{\text{proof}}$  Rule tactic]

[NextUS(NoUpperBound)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$   
 $\forall \underline{m}: \neg (\text{UB}(01//02 * * \text{xs}[\underline{m}] + + \text{us}[\underline{m}], \text{SetOfReals})) \vdash \text{us}[(\underline{m} + 1)] == \text{us}[\underline{m}]$ ]

[NextUS(NoUpperBound)  $\xrightarrow{\text{tex}}$  “NextUS(NoUpperBound)”]

[NextUS(NoUpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextUS(noUpperBound)”]

## ExpZero

[ExpZero  $\xrightarrow{\text{proof}}$  Rule tactic]

[ExpZero  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{x}: \underline{m} = 0 \vdash \underline{x}(\text{exp})\underline{m} = 1$ ]

[ExpZero  $\xrightarrow{\text{tex}}$  “ExpZero”]

[ExpZero  $\xrightarrow{\text{pyk}}$  “1rule expZero”]

## ExpPositive

[ExpPositive  $\xrightarrow{\text{proof}}$  Rule tactic]

[ExpPositive  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{m})n)n) \vdash \underline{x}(\text{exp})\underline{m} = (\underline{x} * \underline{x}(\text{exp})(\underline{m} + (-u1)))$ ]

[ExpPositive  $\xrightarrow{\text{tex}}$  “ExpPositive”]

[ExpPositive  $\xrightarrow{\text{pyk}}$  “1rule expPositive”]

## ExpZero(R)

[ExpZero(R)  $\xrightarrow{\text{tex}}$  “ExpZero(R)”]

[ExpZero(R)  $\xrightarrow{\text{pyk}}$  “1rule expZero(R)”]

## ExpPositive(R)

[ExpPositive(R)  $\xrightarrow{\text{tex}}$  “ExpPositive(R)”]

[ExpPositive(R)  $\xrightarrow{\text{pyk}}$  “1rule expPositive(R)”]

## BSzero

[BSzero  $\xrightarrow{\text{proof}}$  Rule tactic]

[BSzero  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{n}: \underline{n} = 0 \vdash \text{BS}(\underline{m}, \underline{n}) = \text{rec}(1 + 1)(\text{exp})\underline{m}$ ]

[BSzero  $\xrightarrow{\text{tex}}$  “BSzero”]

[BSzero  $\xrightarrow{\text{pyk}}$  “1rule base(1/2)Sum zero”]

## BSpositive

[BSpositive  $\xrightarrow{\text{proof}}$  Rule tactic]

[BSpositive  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (0 \leq \underline{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{n})\underline{n})\underline{n})\underline{n} \vdash$   
BS( $\underline{m}, \underline{n}$ ) = (rec(1 + 1)(exp)( $\underline{m} + \underline{n}$ ) + BS( $\underline{m}, (\underline{n} + (-u1))$ )))]

[BSpositive  $\xrightarrow{\text{tex}}$  “BSpositive”]

[BSpositive  $\xrightarrow{\text{pyk}}$  “1rule base(1/2)Sum positive”]

## UStelescope(Zero)

[UStelescope(Zero)  $\xrightarrow{\text{proof}}$  Rule tactic]

[UStelescope(Zero)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{n}: \underline{n} = 0 \vdash$  UStelescope( $\underline{m}, \underline{n}$ ) =  
|(us[ $\underline{m}$ ] + (-uus[ $\underline{m} + 1$ ]))|]

[UStelescope(Zero)  $\xrightarrow{\text{tex}}$  “UStelescope(Zero)”]

[UStelescope(Zero)  $\xrightarrow{\text{pyk}}$  “1rule UStelescope zero”]

## UStelescope(Positive)

[UStelescope(Positive)  $\xrightarrow{\text{proof}}$  Rule tactic]

[UStelescope(Positive)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (0 \leq \underline{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 =$   
 $\underline{n})\underline{n})\underline{n})\underline{n} \vdash$  UStelescope( $\underline{m}, \underline{n}$ ) =  
|(us[ $\underline{m} + \underline{n}$ ] + (-uus[ $\underline{m} + (\underline{n} + 1)$ ]))| + UStelescope( $\underline{m}, (\underline{n} + (-u1))$ )]

[UStelescope(Positive)  $\xrightarrow{\text{tex}}$  “UStelescope(Positive)”]

[UStelescope(Positive)  $\xrightarrow{\text{pyk}}$  “1rule UStelescope positive”]

## EqAddition(R)

[EqAddition(R)  $\xrightarrow{\text{proof}}$  Rule tactic]

[EqAddition(R)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall (\underline{fx}): \forall (\underline{fy}): \forall (\underline{fz}): \underline{R}(\underline{fx}) = \underline{R}(\underline{fy}) \vdash$   
 $\underline{R}(\underline{fx}) + \underline{R}(\underline{fz}) = \underline{R}(\underline{fy}) + \underline{R}(\underline{fz})$ ]



[EqAddition(R)  $\xrightarrow{\text{tex}}$  “EqAddition(R)”]

[EqAddition(R)  $\xrightarrow{\text{pyk}}$  “1rule adhoc eqAddition(R)”]

## FromLimit

[FromLimit  $\xrightarrow{\text{tex}}$  “FromLimit”]

[FromLimit  $\xrightarrow{\text{pyk}}$  “1rule fromLimit”]

## ToUpperBound

[ToUpperBound  $\xrightarrow{\text{tex}}$  “ToUpperBound”]

[ToUpperBound  $\xrightarrow{\text{pyk}}$  “1rule toUpperBound”]

## FromUpperBound

[FromUpperBound  $\xrightarrow{\text{tex}}$  “FromUpperBound”]

[FromUpperBound  $\xrightarrow{\text{pyk}}$  “1rule fromUpperBound”]

## USisUpperBound

[USisUpperBound  $\xrightarrow{\text{tex}}$  “USisUpperBound”]

[USisUpperBound  $\xrightarrow{\text{pyk}}$  “axiom USisUpperBound”]

## 0not1(R)

[0not1(R)  $\xrightarrow{\text{tex}}$  “0not1(R)”]

[0not1(R)  $\xrightarrow{\text{pyk}}$  “axiom 0not1(R)”]

## ExpUnbounded(R)

[ExpUnbounded(R)  $\xrightarrow{\text{tex}}$  “ExpUnbounded(R)”]

[ExpUnbounded(R)  $\xrightarrow{\text{pyk}}$  “1rule expUnbounded”]

## FromLeq(Advanced)(N)

[FromLeq(Advanced)(N)  $\xrightarrow{\text{tex}}$  “FromLeq(Advanced)(N)”]

[FromLeq(Advanced)(N)  $\xrightarrow{\text{pyk}}$  “1rule fromLeq(Advanced)(N)”]

## FromLeastUpperBound

[FromLeastUpperBound  $\xrightarrow{\text{tex}}$  “FromLeastUpperBound”]

[FromLeastUpperBound  $\xrightarrow{\text{pyk}}$  “1rule fromLeastUpperBound”]

## ToLeastUpperBound

[ToLeastUpperBound  $\xrightarrow{\text{tex}}$  “ToLeastUpperBound”]

[ToLeastUpperBound  $\xrightarrow{\text{pyk}}$  “1rule toLeastUpperBound”]

## XSisNotUpperBound

[XSisNotUpperBound  $\xrightarrow{\text{tex}}$  “XSisNotUpperBound”]

[XSisNotUpperBound  $\xrightarrow{\text{pyk}}$  “axiom XSisNotUpperBound”]

## ysFGreater

[ysFGreater  $\xrightarrow{\text{tex}}$  “ysFGreater”]

[ysFGreater  $\xrightarrow{\text{pyk}}$  “axiom ysFGreater”]

## ysFLess

[ysFLess  $\xrightarrow{\text{tex}}$  “ysFLess”]

[ysFLess  $\xrightarrow{\text{pyk}}$  “axiom ysFLess”]

## SmallInverse

[SmallInverse  $\xrightarrow{\text{tex}}$  “SmallInverse”]

[SmallInverse  $\xrightarrow{\text{pyk}}$  “1rule smallInverse”]

## NatType

[NatType  $\xrightarrow{\text{tex}}$  “NatType”]

[NatType  $\xrightarrow{\text{pyk}}$  “axiom natType”]

## RationalType

[RationalType  $\xrightarrow{\text{tex}}$  “RationalType”]

[RationalType  $\xrightarrow{\text{pyk}}$  “axiom rationalType”]

## SeriesType

[SeriesType  $\xrightarrow{\text{tex}}$  “SeriesType”]

[SeriesType  $\xrightarrow{\text{pyk}}$  “axiom seriesType”]

## Max

[Max  $\xrightarrow{\text{tex}}$  “Max”]

[Max  $\xrightarrow{\text{pyk}}$  “axiom max”]

## Numerical

[Numerical  $\xrightarrow{\text{tex}}$  “Numerical”]

[Numerical  $\xrightarrow{\text{pyk}}$  “axiom numerical”]

## NumericalF

[NumericalF  $\xrightarrow{\text{tex}}$  “NumericalF”]

[NumericalF  $\xrightarrow{\text{pyk}}$  “axiom numericalF”]

## MemberOfSeries(ImPLY)

[MemberOfSeries(ImPLY)  $\xrightarrow{\text{tex}}$  “MemberOfSeries(ImPLY)”]

[MemberOfSeries(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom memberOfSeries”]

## JoinConjuncts(2conditions)

[JoinConjuncts(2conditions)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \underline{a} \vdash \underline{b} \vdash \text{MP2} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \triangleright \underline{a} \triangleright \underline{b} \ggg \underline{c}; \text{MP2} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \triangleright \underline{a} \triangleright \underline{b} \ggg \underline{d}; \text{JoinConjuncts} \triangleright \underline{c} \triangleright \underline{d} \ggg \dot{\neg}(\underline{c} \Rightarrow$

$\dot{\neg}(\underline{d})n; \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \underline{a} \vdash \underline{b} \vdash \dot{\neg}(\underline{c} \Rightarrow \dot{\neg}(\underline{d})n) \ggg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c} \Rightarrow \dot{\neg}(\underline{d})n); \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c} \Rightarrow \dot{\neg}(\underline{d})n) \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \ggg \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c} \Rightarrow \dot{\neg}(\underline{d})n) \rrbracket, p_0, c)$

[JoinConjuncts(2conditions)  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\underline{c} \Rightarrow \dot{\neg}(\underline{d})n)$ ]

[JoinConjuncts(2conditions)  $\xrightarrow{\text{tex}}$  “JoinConjuncts(2conditions)”]

[JoinConjuncts(2conditions)  $\xrightarrow{\text{pyk}}$  “prop lemma doubly conditioned join conjuncts”]

## prop lemma imply negation

[prop lemma imply negation  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{a}: \underline{a} \Rightarrow \dot{\neg}(\underline{a})n \vdash \text{AutoImPLY} \ggg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n; \text{TND} \ggg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n; \text{FromDisjuncts} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n \triangleright \underline{a} \Rightarrow \dot{\neg}(\underline{a})n \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n \ggg \dot{\neg}(\underline{a})n \rrbracket, p_0, c)$

[prop lemma imply negation  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \underline{a} \Rightarrow \dot{\neg}(\underline{a})n \vdash \dot{\neg}(\underline{a})n$ ]

[prop lemma imply negation  $\xrightarrow{\text{pyk}}$  “prop lemma imply negation”]

## TND

[TND  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \underline{a}: \text{AutoImPLY} \ggg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n; \text{Repetition} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n \ggg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n \rrbracket, p_0, c)$

[TND  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n$ ]

[TND  $\xrightarrow{\text{tex}}$  “TND”]

[TND  $\xrightarrow{\text{pyk}}$  “prop lemma tertium non datur”]

## FromNegatedImply

[FromNegatedImply  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n) \vdash \mathbf{a} \vdash$   
 MP  $\triangleright \mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n) \triangleright \mathbf{a} \gg \neg(\neg(\mathbf{b})n)n$ ; RemoveDoubleNeg  $\triangleright \neg(\neg(\mathbf{b})n) \gg$   
 $\mathbf{b}; \forall \mathbf{a}: \forall \mathbf{b}: \text{Ded} \triangleright \forall \mathbf{a}: \forall \mathbf{b}: \mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n) \vdash \mathbf{a} \vdash \mathbf{b} \gg \mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n) \Rightarrow \mathbf{a} \Rightarrow$   
 $\mathbf{b}; \neg(\mathbf{a} \Rightarrow \mathbf{b})n \vdash \text{MT} \triangleright \mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n) \Rightarrow \mathbf{a} \Rightarrow \mathbf{b} \triangleright \neg(\mathbf{a} \Rightarrow \mathbf{b})n \gg \neg(\mathbf{a} \Rightarrow$   
 $\neg(\neg(\mathbf{b})n)n)n$ ; Repetition  $\triangleright \neg(\mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n)n) \gg \neg(\mathbf{a} \Rightarrow$   
 $\neg(\neg(\mathbf{b})n)n \rceil, p_0, c)$ ]

[FromNegatedImply  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \neg(\mathbf{a} \Rightarrow \mathbf{b})n \vdash \neg(\mathbf{a} \Rightarrow \neg(\neg(\mathbf{b})n)n)$ ]

[FromNegatedImply  $\xrightarrow{\text{tex}}$  “FromNegatedImply”]

[FromNegatedImply  $\xrightarrow{\text{pyk}}$  “prop lemma from negated imply”]

## ToNegatedImply

[ToNegatedImply  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \mathbf{a} \vdash \neg(\mathbf{b})n \vdash \neg(\mathbf{a} \Rightarrow$   
 $\mathbf{b})n) \vdash \text{RemoveDoubleNeg} \triangleright \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n) \gg \mathbf{a} \Rightarrow \mathbf{b}; \text{MP} \triangleright \mathbf{a} \Rightarrow \mathbf{b} \triangleright \mathbf{a} \gg$   
 $\mathbf{b}; \text{FromContradiction} \triangleright \mathbf{b} \triangleright \neg(\mathbf{b})n \gg \neg(\mathbf{a} \Rightarrow \mathbf{b})n$ ;  $\forall \mathbf{a}: \forall \mathbf{b}: \text{Ded} \triangleright \forall \mathbf{a}: \forall \mathbf{b}: \mathbf{a} \vdash$   
 $\neg(\mathbf{b})n \vdash \neg(\mathbf{a} \Rightarrow \mathbf{b})n \vdash \neg(\mathbf{a} \Rightarrow \mathbf{b})n \gg \mathbf{a} \Rightarrow \neg(\mathbf{b})n \Rightarrow \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n) \Rightarrow$   
 $\neg(\mathbf{a} \Rightarrow \mathbf{b})n$ ;  $\mathbf{a} \vdash \neg(\mathbf{b})n \vdash \text{MP2} \triangleright \mathbf{a} \Rightarrow \neg(\mathbf{b})n \Rightarrow \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n) \Rightarrow \neg(\mathbf{a} \Rightarrow$   
 $\mathbf{b})n \triangleright \mathbf{a} \triangleright \neg(\mathbf{b})n \gg \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n) \Rightarrow \neg(\mathbf{a} \Rightarrow \mathbf{b})n$ ; AutoImPLY  $\gg \neg(\neg(\mathbf{a} \Rightarrow$   
 $\mathbf{b})n) \Rightarrow \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n)$ ; Neg  $\triangleright \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n) \Rightarrow \neg(\mathbf{a} \Rightarrow \mathbf{b})n \triangleright \neg(\neg(\mathbf{a} \Rightarrow$   
 $\mathbf{b})n) \Rightarrow \neg(\neg(\mathbf{a} \Rightarrow \mathbf{b})n) \gg \neg(\mathbf{a} \Rightarrow \mathbf{b})n \rceil, p_0, c)$ ]

[ToNegatedImply  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \mathbf{a} \vdash \neg(\mathbf{b})n \vdash \neg(\mathbf{a} \Rightarrow \mathbf{b})n$ ]

[ToNegatedImply  $\xrightarrow{\text{tex}}$  “ToNegatedImply”]

[ToNegatedImply  $\xrightarrow{\text{pyk}}$  “prop lemma to negated imply”]

## FromNegated(2 \* ImPLY)

[FromNegated(2 \* ImPLY)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{c}: \neg(\mathbf{a} \Rightarrow \mathbf{b} \Rightarrow$   
 $\mathbf{c})n \vdash \text{FromNegatedImPLY} \triangleright \neg(\mathbf{a} \Rightarrow \mathbf{b} \Rightarrow \mathbf{c})n \gg \neg(\mathbf{a} \Rightarrow \neg(\neg(\mathbf{b} \Rightarrow$   
 $\mathbf{c})n)n)n$ ; FirstConjunct  $\triangleright \neg(\mathbf{a} \Rightarrow \neg(\neg(\mathbf{b} \Rightarrow \mathbf{c})n)n) \gg \mathbf{a}$ ; SecondConjunct  $\triangleright$   
 $\neg(\mathbf{a} \Rightarrow \neg(\neg(\mathbf{b} \Rightarrow \mathbf{c})n)n) \gg \neg(\mathbf{b} \Rightarrow \mathbf{c})n$ ; FromNegatedImPLY  $\triangleright \neg(\mathbf{b} \Rightarrow \mathbf{c})n \gg$   
 $\neg(\mathbf{b} \Rightarrow \neg(\neg(\mathbf{c})n)n)n$ ; FirstConjunct  $\triangleright \neg(\mathbf{b} \Rightarrow \neg(\neg(\mathbf{c})n)n) \gg$   
 $\mathbf{b}$ ; SecondConjunct  $\triangleright \neg(\mathbf{b} \Rightarrow \neg(\neg(\mathbf{c})n)n) \gg \neg(\mathbf{c})n$ ; JoinConjuncts  $\triangleright \mathbf{a} \triangleright \mathbf{b} \gg$   
 $\neg(\mathbf{a} \Rightarrow \neg(\mathbf{b})n)n$ ; JoinConjuncts  $\triangleright \neg(\mathbf{a} \Rightarrow \neg(\mathbf{b})n) \triangleright \neg(\mathbf{c})n \gg \neg(\neg(\mathbf{a} \Rightarrow$   
 $\neg(\mathbf{b})n) \Rightarrow \neg(\neg(\mathbf{c})n)n \rceil, p_0, c)$ ]

[FromNegated(2 \* ImPLY)  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{c}: \neg(\mathbf{a} \Rightarrow \mathbf{b} \Rightarrow \mathbf{c})n \vdash$   
 $\neg(\neg(\mathbf{a} \Rightarrow \neg(\mathbf{b})n) \Rightarrow \neg(\neg(\mathbf{c})n)n)$ ]

[FromNegated(2 \* Imply)  $\xrightarrow{\text{tex}}$  “FromNegated(2\*Imply)”]

[FromNegated(2 \* Imply)  $\xrightarrow{\text{pyk}}$  “prop lemma from negated double imply”]

## FromNegatedAnd

[FromNegatedAnd  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n) \vdash \underline{a} \vdash$   
Repetition  $\triangleright \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n) \gg \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow$   
 $\dot{\neg}(\underline{b})n)n); \text{RemoveDoubleNeg} \triangleright \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n) \gg \underline{a} \Rightarrow$   
 $\dot{\neg}(\underline{b})n; \text{MP} \triangleright \underline{a} \Rightarrow \dot{\neg}(\underline{b})n \triangleright \underline{a} \gg \dot{\neg}(\underline{b})n], p_0, c)$

[FromNegatedAnd  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a} \Rightarrow \dot{\neg}(\underline{b})n)n) \vdash \underline{a} \vdash \dot{\neg}(\underline{b})n]$

[FromNegatedAnd  $\xrightarrow{\text{tex}}$  “FromNegatedAnd”]

[FromNegatedAnd  $\xrightarrow{\text{pyk}}$  “prop lemma from negated and”]

## FromNegatedOr

[FromNegatedOr  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \vdash$   
Repetition  $\triangleright \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow$   
 $\underline{b})n; \text{FromNegatedImply} \triangleright \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n), p_0, c)]$

[FromNegatedOr  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \vdash \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow$   
 $\dot{\neg}(\dot{\neg}(\underline{b})n)n)$

[FromNegatedOr  $\xrightarrow{\text{tex}}$  “FromNegatedOr”]

[FromNegatedOr  $\xrightarrow{\text{pyk}}$  “prop lemma from negated or”]

## ToNegatedOr

[ToNegatedOr  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \vdash$   
 $\dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \text{FirstConjunct} \triangleright \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \gg$   
 $\dot{\neg}(\underline{a})n; \text{SecondConjunct} \triangleright \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \gg$   
 $\dot{\neg}(\underline{b})n; \text{NegateDisjunct1} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \triangleright \dot{\neg}(\underline{a})n \gg \underline{b}; \text{FromContradiction} \triangleright \underline{b} \triangleright$   
 $\dot{\neg}(\underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \forall \underline{a}: \forall \underline{b}: \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \vdash$   
 $\dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow$   
 $\dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \vdash \text{MP} \triangleright \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow$   
 $\dot{\neg}(\dot{\neg}(\underline{b})n)n) \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \triangleright \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \gg$   
 $\dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n; \text{prop lemma imply negation} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \Rightarrow$   
 $\dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n \gg \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n], p_0, c)$

[ToNegatedOr  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\dot{\neg}(\underline{b})n)n) \vdash \dot{\neg}(\dot{\neg}(\underline{a})n \Rightarrow \underline{b})n$ ]

[ToNegatedOr  $\xrightarrow{\text{tex}}$  “ToNegatedOr”]

[ToNegatedOr  $\xrightarrow{\text{pyk}}$  “prop lemma to negated or”]

## FromNegations

[FromNegations  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\text{[SystemQ } \vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \text{TND} \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n; \text{FromDisjuncts} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{a})n \triangleright \underline{a} \Rightarrow \underline{b} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \gg \underline{b}]$ , p0, c)]

[FromNegations  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \underline{a} \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \underline{b}$ ]

[FromNegations  $\xrightarrow{\text{tex}}$  “FromNegations”]

[FromNegations  $\xrightarrow{\text{pyk}}$  “prop lemma from negations”]

## From3Disjuncts

[From3Disjuncts  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\text{[SystemQ } \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \dot{\neg}(\underline{a})n \vdash \text{Repetition} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c}; \text{MP} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \triangleright \dot{\neg}(\underline{a})n \gg \dot{\neg}(\underline{b})n \Rightarrow \underline{c}; \text{FromDisjuncts} \triangleright \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \triangleright \underline{b} \Rightarrow \underline{d} \triangleright \underline{c} \Rightarrow \underline{d} \gg \underline{d}; \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \dot{\neg}(\underline{a})n \vdash \underline{d} \gg \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{c} \Rightarrow \underline{d} \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \underline{d}; \text{AutoImPLY} \gg \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{d}; \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{d} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \text{MP3} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{c} \Rightarrow \underline{d} \Rightarrow \dot{\neg}(\underline{a})n \Rightarrow \underline{d} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \triangleright \underline{b} \Rightarrow \underline{d} \triangleright \underline{c} \Rightarrow \underline{d} \gg \dot{\neg}(\underline{a})n \Rightarrow \underline{d}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{d} \triangleright \underline{a} \Rightarrow \underline{d} \gg \underline{a} \Rightarrow \underline{d}; \text{FromNegations} \triangleright \underline{a} \Rightarrow \underline{d} \triangleright \dot{\neg}(\underline{a})n \Rightarrow \underline{d} \gg \underline{d}]$ , p0, c)]

[From3Disjuncts  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \dot{\neg}(\underline{a})n \Rightarrow \dot{\neg}(\underline{b})n \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{d} \vdash \underline{b} \Rightarrow \underline{d} \vdash \underline{c} \Rightarrow \underline{d} \vdash \underline{d}$ ]

[From3Disjuncts  $\xrightarrow{\text{tex}}$  “From3Disjuncts”]

[From3Disjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma from three disjuncts”]

## From2 \* 2Disjuncts

[From2 \* 2Disjuncts  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\text{[SystemQ } \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \forall \underline{e}: \dot{\neg}(\underline{c})n \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash \underline{a} \vdash \text{MP} \triangleright \underline{a} \Rightarrow \underline{c} \Rightarrow \underline{e} \triangleright \underline{a} \gg \underline{c} \Rightarrow \underline{e}; \text{MP} \triangleright \underline{a} \Rightarrow \underline{d} \Rightarrow \underline{e} \triangleright \underline{a} \gg \underline{d} \Rightarrow \underline{e}; \text{FromDisjuncts} \triangleright \dot{\neg}(\underline{c})n \Rightarrow \underline{d} \triangleright \underline{c} \Rightarrow \underline{e} \triangleright \underline{d} \Rightarrow \underline{e} \gg \underline{e}; \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \forall \underline{e}: \dot{\neg}(\underline{a})n \Rightarrow \underline{b} \vdash \dot{\neg}(\underline{c})n \Rightarrow \underline{d} \vdash \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{e} \vdash \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{e} \vdash$ ]





# ExpandDisjuncts

[ExpandDisjuncts  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{c}: \forall \mathbf{d}: \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \vdash \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \vdash \dot{\neg}(\mathbf{b})n \vdash \dot{\neg}(\mathbf{d})n \vdash \text{NegateDisjunct2} \triangleright \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \triangleright \dot{\neg}(\mathbf{b})n \gg \mathbf{a}; \text{NegateDisjunct2} \triangleright \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \triangleright \dot{\neg}(\mathbf{d})n \gg \mathbf{c}; \text{JoinConjuncts} \triangleright \mathbf{a} \triangleright \mathbf{c} \gg \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n; \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{c}: \forall \mathbf{d}: \text{Ded} \triangleright \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{c}: \forall \mathbf{d}: \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \vdash \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \vdash \dot{\neg}(\mathbf{b})n \vdash \dot{\neg}(\mathbf{d})n \vdash \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n \gg \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \Rightarrow \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \Rightarrow \dot{\neg}(\mathbf{b})n \Rightarrow \dot{\neg}(\mathbf{d})n \Rightarrow \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n; \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \vdash \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \vdash \text{MP2} \triangleright \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \Rightarrow \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \Rightarrow \dot{\neg}(\mathbf{b})n \Rightarrow \dot{\neg}(\mathbf{d})n \Rightarrow \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n \triangleright \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \triangleright \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \gg \dot{\neg}(\mathbf{b})n \Rightarrow \dot{\neg}(\mathbf{d})n \Rightarrow \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n; \text{Repetition} \triangleright \dot{\neg}(\mathbf{b})n \Rightarrow \dot{\neg}(\mathbf{d})n \Rightarrow \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n \gg \dot{\neg}(\mathbf{b})n \Rightarrow \dot{\neg}(\mathbf{d})n \Rightarrow \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n \rrbracket, p_0, c)$ ]

[ExpandDisjuncts  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall \mathbf{a}: \forall \mathbf{b}: \forall \mathbf{c}: \forall \mathbf{d}: \dot{\neg}(\mathbf{a})n \Rightarrow \mathbf{b} \vdash \dot{\neg}(\mathbf{c})n \Rightarrow \mathbf{d} \vdash \dot{\neg}(\mathbf{b})n \Rightarrow \dot{\neg}(\mathbf{d})n \Rightarrow \dot{\neg}(\mathbf{a} \Rightarrow \dot{\neg}(\mathbf{c})n)n$ ]

[ExpandDisjuncts  $\xrightarrow{\text{tex}}$  “ExpandDisjuncts”]

[ExpandDisjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma expand disjuncts”]

## SENC1

[SENC1  $\xrightarrow{\text{proof}}$  Rule tactic]

[SENC1  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall(\underline{\mathbf{f}\mathbf{x}}): \forall(\underline{\mathbf{r}\mathbf{x}}): \forall(\underline{\mathbf{r}\mathbf{y}}): \underline{\mathbf{r}\mathbf{x}} == \underline{\mathbf{r}\mathbf{y}} \vdash \underline{\mathbf{f}\mathbf{x}} \in \underline{\mathbf{r}\mathbf{x}} \vdash \underline{\mathbf{f}\mathbf{x}} \in \underline{\mathbf{r}\mathbf{y}}$ ]

[SENC1  $\xrightarrow{\text{tex}}$  “SENC1”]

[SENC1  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition(1)”]

## SENC2

[SENC2  $\xrightarrow{\text{proof}}$  Rule tactic]

[SENC2  $\xrightarrow{\text{stmt}}$   $\text{SystemQ} \vdash \forall(\underline{\mathbf{f}\mathbf{x}}): \forall(\underline{\mathbf{r}\mathbf{x}}): \forall(\underline{\mathbf{r}\mathbf{y}}): \underline{\mathbf{r}\mathbf{x}} == \underline{\mathbf{r}\mathbf{y}} \vdash \underline{\mathbf{f}\mathbf{x}} \in \underline{\mathbf{r}\mathbf{y}} \vdash \underline{\mathbf{f}\mathbf{x}} \in \underline{\mathbf{r}\mathbf{x}}$ ]

[SENC2  $\xrightarrow{\text{tex}}$  “SENC2”]

[SENC2  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition(2)”]

## LessLeq(R)

[LessLeq(R)  $\xrightarrow{\text{tex}}$  “LessLeq(R)”]

[LessLeq(R)  $\xrightarrow{\text{pyk}}$  “lemma lessLeq(R)”]

## MemberOfSeries

[MemberOfSeries  $\xrightarrow{\text{tex}}$  “MemberOfSeries”]

[MemberOfSeries  $\xrightarrow{\text{pyk}}$  “lemma memberOfSeries”]

## memberOfSeries(Type)

[memberOfSeries(Type)  $\xrightarrow{\text{tex}}$  “memberOfSeries(Type)”]

[memberOfSeries(Type)  $\xrightarrow{\text{pyk}}$  “lemma memberOfSeries(Type)”]

## \*(exp)\*

[x(exp)y  $\xrightarrow{\text{tex}}$  “ #1.  
(exp) #2.”]

[\*(exp)\*  $\xrightarrow{\text{pyk}}$  “#1 ^ #2”]

## R(\*)

[R(x)  $\xrightarrow{\text{tex}}$  “R(#1.  
)”]

[R(\*)  $\xrightarrow{\text{pyk}}$  “R( #1 )”]

## --R(\*)

[--R(x)  $\xrightarrow{\text{tex}}$  “--R(#1.  
)”]

[--R(\*)  $\xrightarrow{\text{pyk}}$  “--R( #1 )”]

## rec\*

[recx  $\xrightarrow{\text{tex}}$  “rec#1.”]

[rec\*  $\xrightarrow{\text{pyk}}$  "1/ ""]

\*/\*

[bs/r  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{bs}/r \doteq \{\text{ph} \in P(\text{bs}) \mid \text{Ex}_{20} \in \text{bs} \wedge [\text{Ex}_{20} \in \text{bs}]_r = \text{ph}_2\}]]])]$

[x/y  $\xrightarrow{\text{tex}}$  "#1.  
/ #2."]

[\*/\*  $\xrightarrow{\text{pyk}}$  "eq-system of " modulo ""]

\*  $\cap$  \*

[x  $\cap$  y  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \cap y \doteq \{\text{ph} \in x \cup y \mid \text{ph}_3 \in x \wedge \text{ph}_3 \in y\}]]])]$

[x  $\cap$  y  $\xrightarrow{\text{tex}}$  "#1.  
\cap #2."]

[\*  $\cap$  \*  $\xrightarrow{\text{pyk}}$  "intersection " comma " end intersection"]

\*[\*]

[x[y]  $\xrightarrow{\text{tex}}$  "#1.  
[#2.  
]"]

[\*[\*]  $\xrightarrow{\text{pyk}}$  "[ " ; " ]"]

$\cup$ \*

[ $\cup$ x  $\xrightarrow{\text{tex}}$  "\cup #1."]

[ $\cup$ \*  $\xrightarrow{\text{pyk}}$  "union " end union"]

\*  $\cup$  \*

[x  $\cup$  y  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \cup y \doteq \cup\{\{x\}, \{y\}\}]]])]$

[x  $\cup$  y  $\xrightarrow{\text{tex}}$  "#1.  
\mathrel{\cup} #2."]

$[* \cup * \xrightarrow{\text{pyk}} \text{"binary-union " comma " end union"}]$

$P(*)$

$[P(x) \xrightarrow{\text{tex}} \text{"P(\#1.} \\ \text{)"}]$

$[P(*) \xrightarrow{\text{pyk}} \text{"power " end power"}]$

$\{*\}$

$[\{x\} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\{x\} \doteq \{x, x\}]])]$

$[\{x\} \xrightarrow{\text{tex}} \text{"\{\#1.} \\ \backslash\}"]]$

$[\{*\} \xrightarrow{\text{pyk}} \text{"zermelo singleton " end singleton"}]$

$\text{StateExpand}(*, *, *)$

$[\text{StateExpand}(t, s, c) \xrightarrow{\text{tex}} \text{"StateExpand(\#1.} \\ \text{, \#2.} \\ \text{, \#3.} \\ \text{)"}]$

$[\text{StateExpand}(*, *, *) \xrightarrow{\text{pyk}} \text{"stateExpand( " , " , " )"}]$

$\text{extractSeries}(*)$

$[\text{extractSeries}(t) \xrightarrow{\text{tex}} \text{"extractSeries(\#1.} \\ \text{)"}]$

$[\text{extractSeries}(*) \xrightarrow{\text{pyk}} \text{"extractSeries( " )"}]$

$\text{SetOfSeries}(*)$

$[\text{SetOfSeries}(x) \xrightarrow{\text{tex}} \text{"SetOfSeries(\#1.} \\ \text{)"}]$

$[\text{SetOfSeries}(*) \xrightarrow{\text{pyk}} \text{"setOfSeries( " )"}]$

-- Macro(\*)

[-- Macro(x)  $\xrightarrow{\text{tex}}$  "--Macro(#1.  
)"]

[-- Macro(\*)  $\xrightarrow{\text{pyk}}$  "--Macro( " )"]

ExpandList(\*, \*, \*)

[ExpandList(x, y, z)  $\xrightarrow{\text{tex}}$  "ExpandList(#1.  
, #2.  
, #3.  
)"]

[ExpandList(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "expandList( " , " , " )"]

\*\* Macro(\*)

[\*\* Macro(x)  $\xrightarrow{\text{tex}}$  "\*\*Macro(#1.  
)"]

[\*\* Macro(\*)  $\xrightarrow{\text{pyk}}$  "\*\*Macro( " )"]

++ Macro(\*)

[++ Macro(x)  $\xrightarrow{\text{tex}}$  "++Macro(#1.  
)"]

[++ Macro(\*)  $\xrightarrow{\text{pyk}}$  "++Macro( " )"]

<< Macro(\*)

[<< Macro(x)  $\xrightarrow{\text{tex}}$  "<<Macro(#1.  
)"]

[<< Macro(\*)  $\xrightarrow{\text{pyk}}$  "<<Macro( " )"]

$\|\text{Macro}(*)$

$[\|\text{Macro}(x) \xrightarrow{\text{tex}} \text{"}\|\text{Macro}(\#1.$   
)]

$[\|\text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"}\|\text{Macro}( " )"]$

$01//\text{Macro}(*)$

$[01//\text{Macro}(x) \xrightarrow{\text{tex}} \text{"}01//\text{Macro}(\#1.$   
)]

$[01//\text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"}01//\text{Macro}( " )"]$

$\text{UB}(*, *)$

$[\text{UB}(x, y) \xrightarrow{\text{tex}} \text{"}\text{UB}(\#1.$   
, #2.  
)]

$[\text{UB}(*, * ) \xrightarrow{\text{pyk}} \text{"}\text{upperBound}( " , " )"]$

$\text{LUB}(*, *)$

$[\text{LUB}(x, y) \xrightarrow{\text{tex}} \text{"}\text{LUB}(\#1.$   
, #2.  
)]

$[\text{LUB}(*, * ) \xrightarrow{\text{pyk}} \text{"}\text{leastUpperBound}( " , " )"]$

$\text{BS}(*, *)$

$[\text{BS}(x, y) \xrightarrow{\text{tex}} \text{"}\text{BS}(\#1.$   
, #2.  
)]

$[\text{BS}(*, * ) \xrightarrow{\text{pyk}} \text{"}\text{base}(1/2)\text{Sum}( " , " )"]$

## UStelescope(\*, \*)

[UStelescope(x, y)  $\xrightarrow{\text{tex}}$  “UStelescope(#1.  
, #2.  
)”]

[UStelescope(\*, \*)  $\xrightarrow{\text{pyk}}$  “UStelescope( " , " )”]

(\*)

[(x)  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(x) \doteq (x)])$ )]

[(x)  $\xrightarrow{\text{tex}}$  “(#1.  
)”]

[(\*)  $\xrightarrow{\text{pyk}}$  “( " )”]

|f \* |

[|fx|  $\xrightarrow{\text{tex}}$  “|f#1.  
|”]

[|f \* |  $\xrightarrow{\text{pyk}}$  “|f " |”]

|r \* |

[|rx|  $\xrightarrow{\text{tex}}$  “|r#1.  
|”]

[|r \* |  $\xrightarrow{\text{pyk}}$  “|r " |”]

## Limit(\*, \*)

[Limit(x, y)  $\xrightarrow{\text{tex}}$  “Limit(#1.  
, #2.  
)”]

[Limit(\*, \*)  $\xrightarrow{\text{pyk}}$  “limit( " , " )”]

## Union(\*)

[Union(x)  $\xrightarrow{\text{tex}}$  "Union(#1.  
)"]

[Union(\*)  $\xrightarrow{\text{pyk}}$  "U( " )"]

## IsOrderedPair(\*, \*, \*)

[IsOrderedPair(x, y, z)  $\xrightarrow{\text{tex}}$  "IsOrderedPair(#1.  
, #2.  
, #3.  
)"]

[IsOrderedPair(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "isOrderedPair( " , " , " )"]

## IsRelation(\*, \*, \*)

[IsRelation(x, y, z)  $\xrightarrow{\text{tex}}$  "IsRelation(#1.  
, #2.  
, #3.  
)"]

[IsRelation(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "isRelation( " , " , " )"]

## isFunction(\*, \*, \*)

[isFunction(x, y, z)  $\xrightarrow{\text{tex}}$  "isFunction(#1.  
, #2.  
, #3.  
)"]

[isFunction(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "isFunction( " , " , " )"]

## IsSeries(\*, \*)

[IsSeries(x, y)  $\xrightarrow{\text{tex}}$  "IsSeries(#1.  
, #2.  
)"]

[IsSeries(\*, \*)  $\xrightarrow{\text{pyk}}$  "isSeries( " , " )"]



IsNatural(\*, \*)

[IsNatural(xy, \*)  $\xrightarrow{\text{tex}}$  "IsNatural(#1.  
, #2.  
)"]

[IsNatural(\*, \*)  $\xrightarrow{\text{pyk}}$  "isNatural( " )"]

OrderedPair(\*, \*)

[OrderedPair(x, y)  $\xrightarrow{\text{tex}}$  "OrderedPair(#1.  
, #2.  
)"]

[OrderedPair(\*, \*)  $\xrightarrow{\text{pyk}}$  "(o " , " )"]

TypeNat(\*)

[TypeNat(x)  $\xrightarrow{\text{tex}}$  "TypeNat(#1.  
)"]

[TypeNat(\*)  $\xrightarrow{\text{pyk}}$  "typeNat( " )"]

TypeNat0(\*)

[TypeNat0(x)  $\xrightarrow{\text{tex}}$  "TypeNat0(#1.  
)"]

[TypeNat0(\*)  $\xrightarrow{\text{pyk}}$  "typeNat0( " )"]

TypeRational(\*)

[TypeRational(x)  $\xrightarrow{\text{tex}}$  "TypeRational(#1.  
)"]

[TypeRational(\*)  $\xrightarrow{\text{pyk}}$  "typeRational( " )"]

## TypeRational0(\*)

[TypeRational0(x)  $\xrightarrow{\text{tex}}$  “TypeRational0(#1.  
)”]

[TypeRational0(\*)  $\xrightarrow{\text{pyk}}$  “typeRational0( ” )”]

## TypeSeries(\*, \*)

[TypeSeries(x, y)  $\xrightarrow{\text{tex}}$  “TypeSeries(#1.  
, #2.  
)”]

[TypeSeries(\*, \*)  $\xrightarrow{\text{pyk}}$  “typeSeries( ” , ” )”]

## Typeseries0(\*, \*)

[Typeseries0(x, y)  $\xrightarrow{\text{tex}}$  “Typeseries0(#1.  
, #2.  
)”]

[Typeseries0(\*, \*)  $\xrightarrow{\text{pyk}}$  “typeSeries0( ” , ” )”]

## {\*, \*}

[{x, y}  $\xrightarrow{\text{tex}}$  “\{#1.  
, #2.  
\}”]

[{\*, \*}  $\xrightarrow{\text{pyk}}$  “zermelo pair ” comma ” end pair”]

## ⟨\*, \*⟩

[⟨x, y⟩  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\langle x, y \rangle \doteq \{\{x\}, \{x, y\}\]])]$

[⟨x, y⟩  $\xrightarrow{\text{tex}}$  “\langle #1.  
, #2.  
\rangle”]

[⟨\*, \*⟩  $\xrightarrow{\text{pyk}}$  “zermelo ordered pair ” comma ” end pair”]

$(-u*)$

$[(-ux) \xrightarrow{\text{tex}} "(-u\#1.)"]$

$[(-u*) \xrightarrow{\text{pyk}} "- u"]$

$-f*$

$[-fx \xrightarrow{\text{tex}} "-\{f\}\#1. "]$

$[-f* \xrightarrow{\text{pyk}} "-f u"]$

$(- - *)$

$[(- - x) \xrightarrow{\text{tex}} "(--\#1.)"]$

$[(- - *) \xrightarrow{\text{pyk}} "-- u"]$

$1f/*$

$[1f/x \xrightarrow{\text{tex}} "1f/\#1. "]$

$[1f/* \xrightarrow{\text{pyk}} "1f/ u"]$

$01//temp*$

$[01//tempx \xrightarrow{\text{tex}} "01//temp\#1. "]$

$[01//temp* \xrightarrow{\text{pyk}} "01// u"]$

$*(*, *)$

$[r(x, y) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[r(x, y) \doteq \langle x, y \rangle \in r]])]$

$[z(x, y) \xrightarrow{\text{tex}} "\#3.$

$(\#1.$

$, \#2.$

$)"]$

$[*(*, *) \xrightarrow{\text{pyk}} \text{"* is related to * under *"}]$

## RefRel(\*, \*)

$[\text{RefRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{RefRel}(r, x) \doteq \forall s: (s \in x \Rightarrow r(s, s))]])]$

$[\text{RefRel}(r, x) \xrightarrow{\text{tex}} \text{"RefRel(\#1.}, \#2. \text{)"}]$

$[\text{RefRel}(*, *) \xrightarrow{\text{pyk}} \text{"* is reflexive relation in *"}]$

## SymRel(\*, \*)

$[\text{SymRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{SymRel}(r, x) \doteq \forall s, t: (s \in x \Rightarrow t \in x \Rightarrow r(s, t) \Rightarrow r(t, s))]])]$

$[\text{SymRel}(r, x) \xrightarrow{\text{tex}} \text{"SymRel(\#1.}, \#2. \text{)"}]$

$[\text{SymRel}(*, *) \xrightarrow{\text{pyk}} \text{"* is symmetric relation in *"}]$

## TransRel(\*, \*)

$[\text{TransRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{TransRel}(r, x) \doteq \forall s, t, u: (s \in x \Rightarrow t \in x \Rightarrow u \in x \Rightarrow r(s, t) \Rightarrow r(t, u) \Rightarrow r(s, u))]])]$

$[\text{TransRel}(r, x) \xrightarrow{\text{tex}} \text{"TransRel(\#1.}, \#2. \text{)"}]$

$[\text{TransRel}(*, *) \xrightarrow{\text{pyk}} \text{"* is transitive relation in *"}]$

## EqRel(\*, \*)

$[\text{EqRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{EqRel}(r, x) \doteq \text{RefRel}(r, x) \wedge \text{SymRel}(r, x) \wedge \text{TransRel}(r, x)])]]]$

$[\text{EqRel}(r, x) \xrightarrow{\text{tex}} \text{"EqRel(\#1.}, \#2. \text{)"}]$

[EqRel(\*, \*)  $\xrightarrow{\text{pyk}}$  “ $\equiv$  is equivalence relation in  $\mathcal{M}$ ”]

[\*  $\in$  \*]<sub>\*</sub>

[[x  $\in$  bs]<sub>r</sub>  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \in \text{bs}]_r \doteq \{\text{ph} \in \text{bs} \mid r(\text{ph}_1, x)\}]]]$

[[x  $\in$  bs]<sub>r</sub>  $\xrightarrow{\text{tex}}$  “[#1.  
\mathrel{\in} #2.  
]\_{\#3}.”]

[[\*  $\in$  \*]<sub>\*</sub>  $\xrightarrow{\text{pyk}}$  “equivalence class of  $\ast$  in  $\mathcal{M}$  modulo  $\equiv$ ”]

Partition(\*, \*)

[Partition(p, bs)  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{Partition}(p, \text{bs}) \doteq (\forall s: (s \in p \Rightarrow s \neq \emptyset)) \wedge$   
( $\forall s, t: (s \in p \Rightarrow t \in p \Rightarrow s \neq t \Rightarrow s \cap t = \emptyset$ ))  $\wedge$   
 $\cup p = \text{bs}]]]$

[Partition(x, y)  $\xrightarrow{\text{tex}}$  “Partition(#1.  
, #2.  
)”]

[Partition(\*, \*)  $\xrightarrow{\text{pyk}}$  “ $\equiv$  is partition of  $\mathcal{M}$ ”]

(\* \* \*)

[(x \* y)  $\xrightarrow{\text{tex}}$  “(#1.  
\* #2.  
)”]

[(\* \* \*)  $\xrightarrow{\text{pyk}}$  “ $\ast \ast \ast$ ”]

\* \*<sub>f</sub> \*

[(fx) \*<sub>f</sub> (fy)  $\xrightarrow{\text{tex}}$  “#1.  
\*\_{\#2}.”]

[\* \*<sub>f</sub> \*  $\xrightarrow{\text{pyk}}$  “ $\ast \ast_f \ast$ ”]

\* \* \*

$[x * y \xrightarrow{\text{tex}} \text{"\#1.} \\ **\#2." ]$

$[* * * \xrightarrow{\text{pyk}} \text{" * * "}]$

$( * + * )$

$[(x + y) \xrightarrow{\text{tex}} \text{"(\#1.} \\ +\#2. \\ )" ]$

$[( * + * ) \xrightarrow{\text{pyk}} \text{" + "}]$

$( * - * )$

$[(x - y) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(x - y) \ddot{=} (x + (-uy))]])]$

$[(x - y) \xrightarrow{\text{tex}} \text{"(\#1.} \\ -\#2. \\ )" ]$

$[( * - * ) \xrightarrow{\text{pyk}} \text{" - "}]$

$* +_f *$

$[(fx) +_f (fy) \xrightarrow{\text{tex}} \text{"\#1.} \\ +_{-}\{f\}\#2." ]$

$[* +_f * \xrightarrow{\text{pyk}} \text{" +_f "}]$

$* -_f *$

$[(fx) -_f (fy) \xrightarrow{\text{tex}} \text{"\#1.} \\ -_{-}\{f\}\#2." ]$

$[* -_f * \xrightarrow{\text{pyk}} \text{" -_f "}]$

\* + +\*

[x + +y  $\xrightarrow{\text{tex}}$  “#1.  
++#2.”]

[\* + +\*  $\xrightarrow{\text{pyk}}$  “ ” ++ ”]

R(\*) -- R(\*)

[R((fx)) -- R((fy))  $\xrightarrow{\text{tex}}$  “R(#1.  
)-- R(#2.  
)”]

[R(\*) -- R(\*)  $\xrightarrow{\text{pyk}}$  “R( ” ) -- R( ” )”]

\* ∈ \*

[x ∈ y  $\xrightarrow{\text{tex}}$  “#1.  
 $\backslash\mathrel{\{ \in \}}$  #2.”]

[\* ∈ \*  $\xrightarrow{\text{pyk}}$  “ ” in0 ”]

| \* |

[|x|  $\xrightarrow{\text{tex}}$  “|#1.  
|”]

[| \* |  $\xrightarrow{\text{pyk}}$  “| ” |”]

if(\*, \*, \*)

[if(x, y, z)  $\xrightarrow{\text{tex}}$  “if(#1.  
, #2.  
, #3.  
)”]

[if(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  “if( ” , ” , ” )”]

Max(\*, \*)

[Max(x, y)  $\xrightarrow{\text{tex}}$  “Max(#1.  
, #2.  
)”]

[Max(\*, \*)  $\xrightarrow{\text{pyk}}$  “max( " , " )”]

Max(\*, \*)

[Max(x, y)  $\xrightarrow{\text{tex}}$  “Max(#1.  
, #2.  
)”]

[Max(\*, \*)  $\xrightarrow{\text{pyk}}$  “maxR( " , " )”]

\* = \*

[x = y  $\xrightarrow{\text{tex}}$  “#1.  
= #2.”]

[\* = \*  $\xrightarrow{\text{pyk}}$  “" = ””]

\*  $\neq$  \*

[x  $\neq$  y  $\xrightarrow{\text{macro}}$   $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \neq y \ddot{=} \dot{=} (x = y)n]])$ ]

[x  $\neq$  y  $\xrightarrow{\text{tex}}$  “#1.  
\neq #2.”]

[\*  $\neq$  \*  $\xrightarrow{\text{pyk}}$  “" != ””]

\*  $\leq$  \*

[x  $\leq$  y  $\xrightarrow{\text{tex}}$  “#1.  
 $\leq$  #2.”]

[\*  $\leq$  \*  $\xrightarrow{\text{pyk}}$  “"  $\leq$  ””]



\* < \*

$[x < y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x < y \doteq x <= y \wedge x \neq y]])]$

$[x < y \xrightarrow{\text{tex}} \text{"\#1. < \#2."}]$

$[* < * \xrightarrow{\text{pyk}} \text{"< "}]$

\* <\_f \*

$[x <_f y \xrightarrow{\text{tex}} \text{"\#1. <_{-}\{f\}\#2."}]$

$[* <_f * \xrightarrow{\text{pyk}} \text{"<_f "}]$

\* ≤\_f \*

$[(fx) \leq_f (fy) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(fx) \leq_f (fy) \doteq (fx) <_f (fy) \vee \text{SF}((fx), (fy))]])]$

$[x \leq_f y \xrightarrow{\text{tex}} \text{"\#1. \leq_{-}\{f\}\#2."}]$

$[* \leq_f * \xrightarrow{\text{pyk}} \text{"<=f "}]$

SF(\*, \*)

$[\text{SF}(x, y) \xrightarrow{\text{tex}} \text{"SF(\#1. , \#2.)"}]$

$[\text{SF}(*, *) \xrightarrow{\text{pyk}} \text{"sameF "}]$

\* == \*

$[x == y \xrightarrow{\text{tex}} \text{"\#1. == \#2."}]$

$[* == * \xrightarrow{\text{pyk}} \text{"== "}]$

\*!! == \*

[x!! == y  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x!! == y \ddot{=} \dot{\wedge} (x == y)n]])$ )]

[x!! == y  $\xrightarrow{\text{tex}}$  “#1.  
!!== #2.”]

[\*!! == \*  $\xrightarrow{\text{pyk}}$  “! !!== ”]

\* << \*

[x << y  $\xrightarrow{\text{tex}}$  “#1.  
<< #2.”]

[\* << \*  $\xrightarrow{\text{pyk}}$  “! << ”]

\* <<== \*

[x <<== y  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x <<== y \ddot{=} x << y \dot{\vee} x == y]])$ )]

[x <<== y  $\xrightarrow{\text{tex}}$  “#1.  
<<== #2.”]

[\* <<== \*  $\xrightarrow{\text{pyk}}$  “! <<== ”]

\* == \*

[x == y  $\xrightarrow{\text{tex}}$  “#1.  
\!\mathrel{==}\! #2.”]

[\* == \*  $\xrightarrow{\text{pyk}}$  “! zermelo is ”]

\*  $\subseteq$  \*

[x  $\subseteq$  y  $\xrightarrow{\text{macro}}$   $\lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \subseteq y \ddot{=} \forall (S1ob): ((S1ob) \in x \Rightarrow (S1ob) \in y)])$ )]

[x  $\subseteq$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\subseteq} #2.”]

[\*  $\subseteq$  \*  $\xrightarrow{\text{pyk}}$  “! is subset of ”]

$\dot{\neg} (*)n$

$[\dot{\neg}(x)n \xrightarrow{\text{tex}} "\dot{\neg}\{\backslash neg\}\,(\#1. n)"]$

$[\dot{\neg}(*)n \xrightarrow{\text{pyk}} "\text{not0 }"]$

$* \notin *$

$[x \notin y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \notin y \dot{\equiv} \dot{\neg}(x \in y)n]])]$

$[x \notin y \xrightarrow{\text{tex}} "\#1. \mathrel{\notin} \#2."]$

$[* \notin * \xrightarrow{\text{pyk}} "\" zermelo \sim in \"]$

$* \neq *$

$[x \neq y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \neq y \dot{\equiv} \dot{\neg}(x = y)n]])]$

$[x \neq y \xrightarrow{\text{tex}} "\#1. \mathrel{\neq} \#2."]$

$[* \neq * \xrightarrow{\text{pyk}} "\" zermelo \sim is \"]$

$* \dot{\wedge} *$

$[x \dot{\wedge} y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\wedge} y \dot{\equiv} \dot{\neg}((x \Rightarrow \dot{\neg}(y)n))n]])]$

$[x \dot{\wedge} y \xrightarrow{\text{tex}} "\#1. \mathrel{\dot{\wedge}} \#2."]$

$[* \dot{\wedge} * \xrightarrow{\text{pyk}} "\" and0 \"]$

$* \dot{\vee} *$

$[x \dot{\vee} y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\vee} y \dot{\equiv} \dot{\neg}(x)n \Rightarrow y]])]$

$[x \dot{\vee} y \xrightarrow{\text{tex}} "\#1. \mathrel{\dot{\vee}} \#2."]$

$[* \dot{\vee} * \xrightarrow{\text{pyk}} "\" or0 \"]$

$\exists * : *$

$[\exists (v1) : a \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\exists (v1) : a \dot{=} \dot{=} (\forall (v1) : \dot{=} (a)n)n]])]]$

$[\exists x : y \xrightarrow{\text{tex}} “$   
 $\backslash \text{exists} \#1.$   
 $\backslash \text{colon} \#2.”]$

$[\exists * : * \xrightarrow{\text{pyk}} “\text{exist0} \text{ " indeed} \text{ "}”]$

$* \dot{\leftrightarrow} *$

$[x \dot{\leftrightarrow} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\leftrightarrow} y \dot{=} (x \Rightarrow y) \wedge (y \Rightarrow x)])]]$

$[x \dot{\leftrightarrow} y \xrightarrow{\text{tex}} “\#1.$   
 $\backslash \text{mathrel}\{\dot{\Leftrightarrow}\} \#2.”]$

$[* \dot{\leftrightarrow} * \xrightarrow{\text{pyk}} “\text{" iff} \text{ "}”]$

$\{\text{ph} \in * \mid *\}$

$[\{\text{ph} \in x \mid a\} \xrightarrow{\text{tex}} “\{ \text{ph} \backslash \text{mathrel}\{\in\} \#1.$   
 $\backslash \text{mid} \#2.$   
 $\}”]$

$[\{\text{ph} \in * \mid *\} \xrightarrow{\text{pyk}} “\text{the set of ph in " such that " end set}”]$

*The pyk compiler, version 0.grue.20060417+ by Klaus Grue  
GRD-2006-12-15.UTC:00:19:10.164930 = MJD-54084.TAI:00:19:43.164930 =  
LGT-4672858783164930e-6*