

(\*\*\*) MAKROER BEGYNDER (\*\*\*)

$$[\text{ph}_1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_1 \ddot{=} a_{\text{Ph}}]])]$$

$$[\text{ph}_2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_2 \ddot{=} b_{\text{Ph}}]])]$$

$$[\text{ph}_3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_3 \ddot{=} c_{\text{Ph}}]])]$$

$$[\text{ph}_4 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_4 \ddot{=} d_{\text{Ph}}]])]$$

$$[\text{ph}_5 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_5 \ddot{=} e_{\text{Ph}}]])]$$

$$[\text{ph}_6 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{ph}_6 \ddot{=} f_{\text{Ph}}]])]$$

$$[x \wedge y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \wedge y \ddot{=} \dot{\lambda}((x \Rightarrow \dot{\lambda}(y)n))n]])]$$

$$[x \dot{\vee} y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \dot{\vee} y \ddot{=} \dot{\lambda}(x)n \Rightarrow y]])]$$

$$[x \Leftrightarrow y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \Leftrightarrow y \ddot{=} (x \Rightarrow y) \wedge (y \Rightarrow x)])]$$

$$[x \neq y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \neq y \ddot{=} \dot{\lambda}(x=y)n]])]$$

$$[x \notin y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \notin y \ddot{=} \dot{\lambda}(x \in y)n]])]$$

$$[x \subseteq y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \subseteq y \ddot{=} \forall(S1ob): ((S1ob) \in x \Rightarrow (S1ob) \in y)])]$$

$$[\{x\} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\{x\} \ddot{=} \{x, x\}]])]$$

$$[x \cup y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \cup y \ddot{=} \cup\{\{x\}, \{y\}\}]])]$$

$$[x \cap y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \cap y \ddot{=} \{\text{ph} \in x \cup y \mid \text{ph}_3 \in x \wedge \text{ph}_3 \in y\}]])]$$

$$[\langle x, y \rangle \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\langle x, y \rangle \ddot{=} \{\{x\}, \{x, y\}\}]])]$$

$$[r(x, y) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[r(x, y) \ddot{=} \langle x, y \rangle \in r]])]$$

$$[\text{RefRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{RefRel}(r, x) \ddot{=} \forall s: (s \in x \Rightarrow r(s, s))]])]$$

$$[\text{SymRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{SymRel}(r, x) \ddot{=} \forall s, t: (s \in x \Rightarrow t \in x \Rightarrow r(s, t) \Rightarrow r(t, s))]])]$$

$$[\text{TransRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{TransRel}(r, x) \ddot{=} \forall s, t, u: (s \in x \Rightarrow t \in x \Rightarrow u \in x \Rightarrow r(s, t) \Rightarrow r(t, u) \Rightarrow r(s, u))]])]$$

$$[\text{EqRel}(r, x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{EqRel}(r, x) \ddot{=} \text{RefRel}(r, x) \wedge \text{SymRel}(r, x) \wedge \text{TransRel}(r, x)]])]$$

$$[\mathcal{BS} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\mathcal{BS} \ddot{=} \text{bs}]])]$$

$$[\text{OBS} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{OBS} \ddot{=} \overline{\text{bs}}]])]$$

$$[[x \in \text{bs}]_r \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \in \text{bs}]_r \ddot{=} \{\text{ph} \in \text{bs} \mid r(\text{ph}_1, x)\}]])]$$

$$[\text{bs}/r \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{bs}/r \ddot{=} \{\text{ph} \in P(\text{bs}) \mid \text{Ex}_{20} \in \text{bs} \wedge [\text{Ex}_{20} \in \text{bs}]_r == \text{ph}_2\}]])]$$

$$[\text{Partition}(p, \text{bs}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Partition}(p, \text{bs}) \ddot{=} (\forall s: (s \in p \Rightarrow s \neq \emptyset)) \wedge (\forall s, t: (s \in p \Rightarrow t \in p \Rightarrow s \neq t \Rightarrow s \cap t == \emptyset)) \wedge \cup p == \text{bs}]])]$$

(\*\*\*) EKSISTENS-VARIABLE (\*\*\*)

$$[x^{\text{Ex}} \xrightarrow{\text{val}} x \stackrel{r}{=} [x_{\text{Ex}}]]$$

$$[\text{Ex}_1 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Ex}_1 \ddot{=} a_{\text{Ex}}]])]$$

$$\begin{aligned}
& [EX_2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[EX_2 \doteq b_{Ex}]])] \\
& [EX_{10} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[EX_{10} \doteq j_{Ex}]])] \\
& [EX_{20} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[EX_{20} \doteq t_{Ex}]])] \\
& [\langle a \equiv b | x ::= t \rangle_{Ex} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\langle a \equiv b | x ::= t \rangle_{Ex} \doteq \\
& \langle [a] \equiv^0 [b] | [x] ::= [t] \rangle_{Ex}]]])]
\end{aligned}$$

$$[\langle a \equiv^0 b | x ::= t \rangle_{Ex} \xrightarrow{\text{val}} \lambda c. x^{Ex} \wedge \langle a \equiv^1 b | x ::= t \rangle_{Ex}]$$

$$[\langle a \equiv^1 b | x ::= t \rangle_{Ex} \xrightarrow{\text{val}} a!x!t!]$$

$$\text{If}(b \stackrel{r}{=} \bigvee_{\text{Obj}} u : v, F,$$

$$\text{If}(b^{Ex} \wedge b \stackrel{t}{=} x, a \stackrel{t}{=} t, \text{If}(\$$

$$a \stackrel{r}{=} b, \langle a^t \equiv^* b^t | x ::= t \rangle_{Ex}, F)))]$$

$$[\langle a \equiv^* b | x ::= t \rangle_{Ex} \xrightarrow{\text{val}} b!x!t! \text{If}(a, T, \text{If}(\langle a^h \equiv^1 b^h | x ::= t \rangle_{Ex}, \langle a^t \equiv^* b^t | x ::= t \rangle_{Ex}, F)))]$$

(\*\*\* AKSIOMATISK SYSTEM \*\*\*)

$$\begin{aligned}
& [\text{SystemQ} \xrightarrow{\text{stmt}} \forall m: \text{UB}(\text{us}[m], \text{SetOfFxs}) \oplus \forall \text{fx}: \forall \text{fy}: \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \\
& \text{P}(\text{P}(\text{Union}(\{N, Q\}))) \mid \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \\
& \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow \dot{\neg} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \\
& \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{r1})}: \overline{(\text{r1})} \in \text{f}_{\text{Ph}} \Rightarrow \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \\
& \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow \dot{\neg} (\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{f1})}: \forall_{\text{Obj}} \overline{(\text{f2})}: \forall_{\text{Obj}} \overline{(\text{f3})}: \forall_{\text{Obj}} \overline{(\text{f4})}: \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \\
& \{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})} \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{s1})}: \overline{(\text{s1})} \in N \Rightarrow \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{s2})}: \dot{\neg} (\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in \\
& \text{f}_{\text{Ph}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \forall_{\text{Obj}} \overline{(\epsilon)}: \dot{\neg} (\forall_{\text{Obj}} \overline{\text{n}}: \dot{\neg} (\forall_{\text{Obj}} \overline{\text{m}}: \dot{\neg} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{(\epsilon)}) \text{n}) \text{n}) \text{n}) \Rightarrow \\
& \overline{\text{n}} \leq \overline{\text{m}} \Rightarrow \dot{\neg} (\{ \text{ph} \in \{ \text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))) \} \mid \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \dot{\neg} (\forall_{\text{Obj}} \overline{\text{m}}: \dot{\neg} (\text{d}_{\text{Ph}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, ((\text{fx})[\overline{\text{m}}] + (\text{fy})[\overline{\text{m}}])\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \leq \overline{(\epsilon)} \Rightarrow \\
& \dot{\neg} (\dot{\neg} (\{ \text{ph} \in \{ \text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))) \} \mid \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \dot{\neg} (\forall_{\text{Obj}} \overline{\text{m}}: \dot{\neg} (\text{d}_{\text{Ph}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, ((\text{fx})[\overline{\text{m}}] + (\text{fy})[\overline{\text{m}}])\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid = \overline{(\epsilon)} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) = \\
& \{ \text{ph} \in \text{P}(\{ \text{ph} \in \text{P}(\{ \text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))) \} \mid \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{r1})}: \overline{(\text{r1})} \in \\
& \text{f}_{\text{Ph}} \Rightarrow \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\} \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{f1})}: \forall_{\text{Obj}} \overline{(\text{f2})}: \forall_{\text{Obj}} \overline{(\text{f3})}: \forall_{\text{Obj}} \overline{(\text{f4})}: \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \\
& \{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})} \text{n}) \text{n}) \Rightarrow \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{s1})}: \overline{(\text{s1})} \in N \Rightarrow \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{s2})}: \dot{\neg} (\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in \\
& \text{f}_{\text{Ph}) \text{n}) \text{n}) \text{n}) \text{n}) \text{n}) \mid \forall_{\text{Obj}} \overline{(\epsilon)}: \dot{\neg} (\forall_{\text{Obj}} \overline{\text{n}}: \dot{\neg} (\forall_{\text{Obj}} \overline{\text{m}}: \dot{\neg} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{(\epsilon)}) \text{n}) \text{n}) \text{n}) \Rightarrow \\
& \overline{\text{n}} \leq \overline{\text{m}} \Rightarrow \dot{\neg} (\{ \text{ph} \in \{ \text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))) \} \mid \\
& \dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op1})}: \dot{\neg} (\dot{\neg} (\forall_{\text{Obj}} \overline{(\text{op2})}: \dot{\neg} (\dot{\neg} (\overline{(\text{op1})} \in N \Rightarrow \dot{\neg} (\overline{(\text{op2})} \in Q) \text{n}) \text{n}) \Rightarrow
\end{aligned}$$

























































$$\begin{aligned}
& \overline{n} \leq \overline{m} \Rightarrow \dot{\vdash} (|\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\})\}| \mid \overline{(\epsilon)} \Rightarrow \dot{\vdash} (0 = \overline{(\epsilon)})n)n \Rightarrow \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in N} \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \\
& \dot{\vdash} (\overline{\text{a}_{\text{Ph}}} = \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{\text{m}})}: \dot{\vdash} (\overline{\text{d}_{\text{Ph}}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, (\overline{\text{fx}}[\overline{\text{m}}])\} + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\})\}) \mid \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in N} \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \\
& \dot{\vdash} (\overline{\text{a}_{\text{Ph}}} = \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{\text{m}})}: \dot{\vdash} (\overline{\text{f}_{\text{Ph}}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, (-u(\overline{\text{fx}}[\overline{\text{m}}]))\}\}n)n)[\overline{\text{m}}] \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{\text{m}})}: \dot{\vdash} (\overline{\text{d}_{\text{Ph}}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, (\overline{\text{fx}}[\overline{\text{m}}])\} + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\})\}) \mid \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in N} \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \\
& \dot{\vdash} (\overline{\text{a}_{\text{Ph}}} = \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{\text{m}})}: \dot{\vdash} (\overline{\text{f}_{\text{Ph}}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, (-u(\overline{\text{fx}}[\overline{\text{m}}]))\}\}n)n)[\overline{\text{m}}] \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{\text{m}})}: \dot{\vdash} (\overline{\text{d}_{\text{Ph}}} = \\
& \{\{\overline{\text{m}}, \overline{\text{m}}\}, \{\overline{\text{m}}, (\overline{\text{fx}}[\overline{\text{m}}])\} + \{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\})\}) \mid \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in N} \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \\
& \dot{\vdash} (\overline{\text{a}_{\text{Ph}}} = \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{r1}}))}: \overline{(\text{r1})} \in \\
& \text{f}_{\text{Ph}} \Rightarrow \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in N} \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \\
& \dot{\vdash} (\overline{(\text{r1})} = \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n \Rightarrow \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{f1}}))}: \overline{(\forall_{\text{obj}}(\overline{\text{f2}}))}: \overline{(\forall_{\text{obj}}(\overline{\text{f3}}))}: \overline{(\forall_{\text{obj}}(\overline{\text{f4}}))}: \{\{\overline{(\text{f1}), (\overline{\text{f1}})}\}, \{\overline{(\text{f1}), (\overline{\text{f2}})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \\
& \{\{\overline{(\text{f3}), (\overline{\text{f3}})}\}, \{\overline{(\text{f3}), (\overline{\text{f4}})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})}n)n \Rightarrow \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{s1}}))}: \overline{(\text{s1})} \in N \Rightarrow \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{s2}}))}: \dot{\vdash} (\{\{\overline{(\text{s1}), (\overline{\text{s1}})}\}, \{\overline{(\text{s1}), (\overline{\text{s2}})}\}\} \in \\
& \text{f}_{\text{Ph}}n)n)n)n) \mid \overline{(\forall_{\text{obj}}(\overline{\epsilon}))}: \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{n})}: \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{m})}: \dot{\vdash} (0 \leq \overline{(\epsilon)}) \Rightarrow \dot{\vdash} (0 = \overline{(\epsilon)})n)n) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\vdash} (|\{\text{ph} \in \{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\}))\})\}| \mid \overline{(\epsilon)} \Rightarrow \dot{\vdash} (0 = \overline{(\epsilon)})n)n) \Rightarrow \\
& \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in N} \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \\
& \dot{\vdash} (\overline{\text{a}_{\text{Ph}}} = \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{crs1}}))}: \dot{\vdash} (\overline{\text{c}_{\text{Ph}}} = \\
& \{\{\{\overline{\text{crs1}}, \overline{\text{crs1}}\}, \{\overline{\text{crs1}}, 0\}\}\}n)n)[\overline{\text{m}}] + (-ud_{\text{Ph}}[\overline{\text{m}}])) \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op1}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{op2}}))}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1}}) \in \\
& N \Rightarrow \dot{\vdash} (\overline{(\text{op2}}) \in Q})n)n \Rightarrow \dot{\vdash} (\overline{\text{a}_{\text{Ph}}} = \\
& \{\{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}\}n)n)n)n) \mid \dot{\vdash} (\overline{(\forall_{\text{obj}}(\overline{\text{crs1}}))}: \dot{\vdash} (\overline{\text{c}_{\text{Ph}}} = \\
& \{\{\{\overline{\text{crs1}}, \overline{\text{crs1}}\}, \{\overline{\text{crs1}}, 0\}\}\}n)n)[\overline{\text{m}}] + (-ud_{\text{Ph}}[\overline{\text{m}}])) = \overline{(\epsilon)}n)n)n) \oplus \\
& \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{x} * \underline{y}) + (\underline{x} * \underline{z})) \oplus \forall \underline{m}: \forall \underline{n}: \text{Nat}(\underline{m}) \Vdash \text{Nat}(\underline{n}) \Vdash \\
& \dot{\vdash} (\underline{m} \leq (\underline{n} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{m} = (\underline{n} + 1))n)n) \Vdash \underline{m} \leq \underline{n} \oplus \\
& \forall (\underline{sx}): \forall (\underline{sy}): \forall (\underline{sz}): (\underline{sx}) \in \{(\underline{sy}), (\underline{sz})\} \Vdash \dot{\vdash} ((\underline{sx}) = (\underline{sy})n) \Rightarrow (\underline{sx}) = (\underline{sz}) \oplus \forall \underline{x}: \underline{x} \in \\
& Q \Rightarrow (-\underline{ux}) \in Q \oplus \forall \underline{x}: \forall \underline{t}: \forall \underline{a}: \forall \underline{b}: \langle [\underline{a}] =^0 [\underline{b}] \mid [\underline{x}] := [\underline{t}] \rangle_{\text{Ex}} \Vdash \underline{a} \Vdash \underline{b} \oplus \\
& \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\dot{\vdash} (\underline{y} \leq \underline{x} \Rightarrow \dot{\vdash} (\text{if}(\underline{y} \leq \underline{x}, \underline{x}, \underline{y}) = \underline{x})n)n) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{y} \leq \underline{x})n) \Rightarrow \\
& \dot{\vdash} (\text{if}(\underline{y} \leq \underline{x}, \underline{x}, \underline{y}) = \underline{y})n) \oplus \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{m})n)n) \Vdash \\
& \underline{x}(\text{exp})\underline{m} = (\underline{x} * \underline{x}(\text{exp})(\underline{m} + (-u1))) \oplus \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z})) \oplus \\
& \forall (\underline{sx}): \forall (\underline{sy}): (\underline{sx}) = (\underline{sy}) \Vdash \{(\underline{sx}), (\underline{sx})\} = \{(\underline{sy}), (\underline{sy})\} \oplus \\
& \forall (\underline{v1}): \forall (\underline{v2}): \forall \underline{n}: \forall (\underline{\epsilon}): \forall (\underline{fx}): \forall_{\text{obj}}(\underline{\epsilon}): \dot{\vdash} (\overline{(\forall_{\text{obj}}\overline{n})}: \dot{\vdash} (\overline{(\forall_{\text{obj}}(\underline{v1}))}: \overline{(\forall_{\text{obj}}(\underline{v2}))}: \dot{\vdash} (0 \leq \\
& \underline{(\epsilon)}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{(\epsilon)})n)n) \Rightarrow \underline{n} \leq \underline{(v1)} \Rightarrow \underline{n} \leq \underline{(v2)} \Rightarrow \\
& \dot{\vdash} (|\{(\overline{\text{fx}}[\underline{(v1)}]) + (-u(\overline{\text{fx}}[\underline{(v2)}]))\}| \leq \underline{(\epsilon)}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|\{(\overline{\text{fx}}[\underline{(v1)}]) + (-u(\overline{\text{fx}}[\underline{(v2)}]))\}| =
\end{aligned}$$























$[\text{leqTotality} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\underline{x} \leq \underline{y}) \text{n} \Rightarrow \underline{y} \leq \underline{x}] [\text{leqTotality} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{leqAdditionAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} \leq \underline{y} \Rightarrow (\underline{x} + \underline{z}) \leq (\underline{y} + \underline{z})] [\text{leqAdditionAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{leqMultiplicationAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: 0 \leq \underline{z} \Rightarrow \underline{x} \leq \underline{y} \Rightarrow (\underline{x} * \underline{z}) \leq (\underline{y} * \underline{z})] [\text{leqMultiplicationAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{plusAssociativity} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} + \underline{y}) + \underline{z}) = (\underline{x} + (\underline{y} + \underline{z}))] [\text{plusAssociativity} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{plusCommutativity} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: (\underline{x} + \underline{y}) = (\underline{y} + \underline{x})] [\text{plusCommutativity} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Negative} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: (\underline{x} + (-\underline{ux})) = 0] [\text{Negative} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{plus0} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: (\underline{x} + 0) = \underline{x}] [\text{plus0} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{timesAssociativity} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: ((\underline{x} * \underline{y}) * \underline{z}) = (\underline{x} * (\underline{y} * \underline{z}))] [\text{timesAssociativity} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{timesCommutativity} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: (\underline{x} * \underline{y}) = (\underline{y} * \underline{x})] [\text{timesCommutativity} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{ReciprocalAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (\underline{x} = 0) \text{n} \Rightarrow (\underline{x} * \text{recx}) = 1] [\text{ReciprocalAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{times1} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: (\underline{x} * 1) = \underline{x}] [\text{times1} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Distribution} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: (\underline{x} * (\underline{y} + \underline{z})) = ((\underline{x} * \underline{y}) + (\underline{x} * \underline{z}))] [\text{Distribution} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[0\text{not1} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \dot{\vdash} (0 = 1) \text{n}] [0\text{not1} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{EqualityAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \Rightarrow \underline{x} = \underline{z} \Rightarrow \underline{y} = \underline{z}] [\text{EqualityAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{EqLeqAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} = \underline{y} \Rightarrow \underline{x} \leq \underline{y}] [\text{EqLeqAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{EqAdditionAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \Rightarrow (\underline{x} + \underline{z}) = (\underline{y} + \underline{z})] [\text{EqAdditionAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{EqMultiplicationAxiom} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \forall \underline{z}: \underline{x} = \underline{y} \Rightarrow (\underline{x} * \underline{z}) = (\underline{y} * \underline{z})] [\text{EqMultiplicationAxiom} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{A4(Axiom)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall (\underline{v1}): \forall \underline{a}: \forall \underline{b}: \langle \underline{a} \equiv \underline{b} \rangle (\underline{v1}) ::= \underline{x} \rangle_{\text{Me}} \Vdash \forall \text{obj} (\underline{v1}): \underline{b} \Rightarrow \underline{a}] [\text{A4(Axiom)} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$(*** \text{XX snydeaksiomer} ***)$

$[=\text{Reflexivity} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{rx}): \underline{(rx)} = \underline{(rx)}] [=\text{Reflexivity} \xrightarrow{\text{proof}} \text{Rule tactic}]$









$[(F2) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F2) \doteq \underline{(f2)}]])]$   
 $[(F3) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F3) \doteq \underline{(f3)}]])]$   
 $[(F4) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F4) \doteq \underline{(f4)}]])]$   
 $[(OP1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(OP1) \doteq \underline{(op1)}]])]$   
 $[(OP2) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(OP2) \doteq \underline{(op2)}]])]$   
 $[(R1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(R1) \doteq \underline{(r1)}]])]$   
 $[(S1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S1) \doteq \underline{(s1)}]])]$   
 $[(S2) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S2) \doteq \underline{(s2)}]])]$   
 $[(EPob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(EPob) \doteq \underline{(\epsilon)}]])]$   
 $[(CRS1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(CRS1ob) \doteq \underline{(crs1)}]])]$   
 $[(F1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F1ob) \doteq \underline{(f1)}]])]$   
 $[(F2ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F2ob) \doteq \underline{(f2)}]])]$   
 $[(F3ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F3ob) \doteq \underline{(f3)}]])]$   
 $[(F4ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(F4ob) \doteq \underline{(f4)}]])]$   
 $[(N1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(N1ob) \doteq \underline{(n1)}]])]$   
 $[(N2ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(N2ob) \doteq \underline{(n2)}]])]$   
 $[(OP1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(OP1ob) \doteq \underline{(op1)}]])]$   
 $[(OP2ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(OP2ob) \doteq \underline{(op2)}]])]$   
 $[(R1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(R1ob) \doteq \underline{(r1)}]])]$   
 $[(S1ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S1ob) \doteq \underline{(s1)}]])]$   
 $[(S2ob) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(S2ob) \doteq \underline{(s2)}]])]$   
 $[(fx) \leq_f (fy) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(fx) \leq_f (fy) \doteq (fx) <_f (fy) \dot{\vee} \text{SF}((fx), (fy))]])]$   
 $[\text{Ex3} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{Ex3} \doteq \text{cEx}]])]$   
 $[\exists(v1): a \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\exists(v1): a \doteq \dot{\neg} (\forall(v1): \dot{\neg} (a)n)]])]$   
 $[x < < == y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x < < == y \doteq x < < y \dot{\vee} x == y]])]$   
 $[(-1) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(-1) \doteq \underline{(-u1)}]])]$   
 $[2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[2 \doteq \underline{(1 + 1)}]])]$   
 $[3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[3 \doteq \underline{(2 + 1)}]])]$   
 $[1/2 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[1/2 \doteq \underline{\text{rec2}}]])]$   
 $[1/3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[1/3 \doteq \underline{\text{rec3}}]])]$   
 $[2/3 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[2/3 \doteq \underline{(2 * 1/3)}]])]$   
 $[x < y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x < y \doteq x < y \wedge x \neq y]])]$   
 $[x \neq y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x \neq y \doteq \dot{\neg} (x = y)n]])]$   
 $[(x - y) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(x - y) \doteq \underline{(x + (-uy))}]])]$   
 $[00 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[00 \doteq \underline{\text{R}(0f)}]])]$   
 $[01 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[01 \doteq \underline{\text{R}(1f)}]])]$   
 $[x!! == y \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[x!! == y \doteq \dot{\neg} (x == y)n]])]$   
 (\*\*\*) REGELLEMMMAER (\*\*\*)  
 (\*\*\*) UDSAGNSLOGIK (\*\*\*)





$\dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n$ ; JoinConjuncts  $\triangleright \dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n \triangleright \dot{\vdash} (\underline{c})n \gg \dot{\vdash} (\dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{c})n)n$ ],  $p_0, c$ ]

[FromNegatedOr  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \underline{b})n \vdash \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{b})n)n)n$ ]

[FromNegatedOr  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \underline{b})n \vdash$   
Repetition  $\triangleright \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \underline{b})n \gg \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow$   
 $\underline{b})n$ ; FromNegatedImPLY  $\triangleright \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \underline{b})n \gg \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{b})n)n)n$ ],  $p_0, c$ )]

[InductionAxiom  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall (v1): \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \langle \underline{b} \equiv \underline{a} \rangle (v1) := 0 \rangle_{\text{Me}} \vdash$   
 $\langle \underline{c} \equiv \underline{a} \rangle (v1) := ((v1) + 1) \rangle_{\text{Me}} \vdash \underline{b} \Rightarrow \forall_{\text{obj}} \underline{(v1)}: \underline{a} \Rightarrow \underline{c} \Rightarrow$

$\forall_{\text{obj}} \underline{(v1)}: \underline{a}$ ][InductionAxiom  $\xrightarrow{\text{proof}}$  Rule tactic]

[LessMinus1(N)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \forall \underline{n}: \text{Nat}(\underline{m}) \vdash \text{Nat}(\underline{n}) \vdash \dot{\vdash} (\underline{m} <=$   
 $(\underline{n} + 1) \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{m} = (\underline{n} + 1))n)n \vdash \underline{m} <= \underline{n}$ ][LessMinus1(N)  $\xrightarrow{\text{proof}}$  Rule tactic]

[Nonnegative(N)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{m}: \text{Nat}(\underline{m}) \vdash 0 <= \underline{m}$ ][Nonnegative(N)  $\xrightarrow{\text{proof}}$   
Rule tactic]

[Cauchy  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash$

$\forall (v1): \forall (v2): \forall \underline{n}: \forall (\epsilon): \forall (\underline{fx}): \forall_{\text{obj}} \underline{(\epsilon)}: \dot{\vdash} (\forall_{\text{obj}} \underline{n}: \dot{\vdash} (\forall_{\text{obj}} \underline{(v1)}: \forall_{\text{obj}} \underline{(v2)}: \dot{\vdash} (0 <=$   
 $(\underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = (\underline{\epsilon})n)n) \Rightarrow \underline{n} <= \underline{(v1)} \Rightarrow \underline{n} <= \underline{(v2)}) \Rightarrow$   
 $\dot{\vdash} (|((\underline{fx})[(v1)] + (-\underline{u}(\underline{fx})[(v2)]))| <= \underline{(\epsilon)}) \Rightarrow$

$\dot{\vdash} (\dot{\vdash} (|((\underline{fx})[(v1)] + (-\underline{u}(\underline{fx})[(v2)]))| = \underline{(\epsilon)})n)n)n$ ][Cauchy  $\xrightarrow{\text{proof}}$  Rule tactic]

[JoinConjuncts(2conditions)  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow$   
 $\underline{b} \Rightarrow \underline{d} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\vdash} (\underline{c} \Rightarrow \dot{\vdash} (\underline{d})n)n$ ]

[JoinConjuncts(2conditions)  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \underline{a} \Rightarrow \underline{b} \Rightarrow$   
 $\underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \underline{a} \vdash \underline{b} \vdash \text{MP2} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \triangleright \underline{a} \triangleright \underline{b} \gg \underline{c}; \text{MP2} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow$   
 $\underline{d} \triangleright \underline{a} \triangleright \underline{b} \gg \underline{d}; \text{JoinConjuncts} \triangleright \underline{c} \triangleright \underline{d} \gg \dot{\vdash} (\underline{c} \Rightarrow$

$\dot{\vdash} (\underline{d})n)n$ ;  $\forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \text{Ded} \triangleright \forall \underline{a}: \forall \underline{b}: \forall \underline{c}: \forall \underline{d}: \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \underline{a} \vdash \underline{b} \vdash$   
 $\dot{\vdash} (\underline{c} \Rightarrow \dot{\vdash} (\underline{d})n)n \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\vdash} (\underline{c} \Rightarrow \dot{\vdash} (\underline{d})n)n$ ;  $\underline{a} \Rightarrow$   
 $\underline{b} \Rightarrow \underline{c} \vdash \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \vdash \text{MP2} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \Rightarrow \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\vdash} (\underline{c} \Rightarrow$   
 $\dot{\vdash} (\underline{d})n)n \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{c} \triangleright \underline{a} \Rightarrow \underline{b} \Rightarrow \underline{d} \gg \underline{a} \Rightarrow \underline{b} \Rightarrow \dot{\vdash} (\underline{c} \Rightarrow \dot{\vdash} (\underline{d})n)n$ ],  $p_0, c$ ]

[FromNegatedAnd  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n) \vdash \underline{a} \vdash \dot{\vdash} (\underline{b})n$ ]

[FromNegatedAnd  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n) \vdash \underline{a} \vdash$   
Repetition  $\triangleright \dot{\vdash} (\dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n) \gg \dot{\vdash} (\dot{\vdash} (\underline{a} \Rightarrow$   
 $\dot{\vdash} (\underline{b})n)n$ ; RemoveDoubleNeg  $\triangleright \dot{\vdash} (\dot{\vdash} (\underline{a} \Rightarrow \dot{\vdash} (\underline{b})n)n) \gg \underline{a} \Rightarrow$   
 $\dot{\vdash} (\underline{b})n$ ; MP  $\triangleright \underline{a} \Rightarrow \dot{\vdash} (\underline{b})n \triangleright \underline{a} \gg \dot{\vdash} (\underline{b})n$ ],  $p_0, c$ ]

[ToNegatedOr  $\xrightarrow{\text{stmt}}$  SystemQ  $\vdash \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{b})n)n) \vdash \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow$   
 $\underline{b})n$ ]

[ToNegatedOr  $\xrightarrow{\text{proof}}$   $\lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \dot{\vdash} (\dot{\vdash} (\underline{a})n \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{b})n)n) \vdash$































$$\{\overline{(\text{crsl})}, \overline{(\text{crsl})}\}, \{\overline{(\text{crsl})}, 1\}\}n)n\}[\underline{m}]\}n)n\}[\underline{m}] + (-\text{ud}_{\text{Ph}}[\underline{m}])) =$$

$$\overline{(\epsilon)}n)n)n)n\}[\text{US0} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{ExpZero} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \underline{m} = 0 \vdash \underline{x}(\text{exp})\underline{m} = 1][\text{ExpZero} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{ExpPositive} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{m})n)n)n \vdash \underline{x}(\text{exp})\underline{m} = (\underline{x} * \underline{x}(\text{exp})(\underline{m} + (-u1)))][\text{ExpPositive} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{BSzero} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \underline{n} = 0 \vdash \text{BS}(\underline{m}, \underline{n}) = \text{rec}(1 + 1)(\text{exp})\underline{m}][\text{BSzero} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{BSpositive} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (0 \leq \underline{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{n})n)n)n \vdash \text{BS}(\underline{m}, \underline{n}) = (\text{rec}(1 + 1)(\text{exp})(\underline{m} + \underline{n}) + \text{BS}(\underline{m}, (\underline{n} + (-u1))))][\text{BSpositive} \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{UStescope}(\text{Zero}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \underline{n} = 0 \vdash \text{UStescope}(\underline{m}, \underline{n}) = |(\text{us}[\underline{m}] + (-\text{uus}[\underline{m} + 1]))][\text{UStescope}(\text{Zero}) \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[\text{UStescope}(\text{Positive}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{n}: \dot{\vdash} (0 \leq \underline{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{n})n)n)n \vdash \text{UStescope}(\underline{m}, \underline{n}) = (|(\text{us}[\underline{m} + \underline{n}] + (-\text{uus}[\underline{m} + (\underline{n} + 1)]))| + \text{UStescope}(\underline{m}, (\underline{n} + (-u1))))][\text{UStescope}(\text{Positive}) \xrightarrow{\text{proof}} \text{Rule tactic}]$$

$$[(x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \mathcal{M}_4(t, s, c, [[(x) \doteq (x)])]$$

$$[\text{EqAddition}(\text{R}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{(\text{fx})}: \forall \underline{(\text{fy})}: \forall \underline{(\text{fz})}: \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\vdash} (\dot{\vdash} (\overline{(\text{op1})} \in \text{N} \Rightarrow \dot{\vdash} (\overline{(\text{op2})} \in \text{Q})n)n} \Rightarrow \dot{\vdash} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\}n)n)n)n)n) \mid \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}} \overline{(\text{r1})}: (\text{r1}) \in \text{f}_{\text{Ph}} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\vdash} (\dot{\vdash} (\dot{\vdash} (\overline{(\text{op1})} \in \text{N} \Rightarrow \dot{\vdash} (\overline{(\text{op2})} \in \text{Q})n)n} \Rightarrow \dot{\vdash} (\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\}n)n)n)n)n) \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{f1})}: \forall_{\text{obj}} \overline{(\text{f2})}: \forall_{\text{obj}} \overline{(\text{f3})}: \forall_{\text{obj}} \overline{(\text{f4})}: \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})}n)n} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{s1})}: (\text{s1}) \in \text{N} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{s2})}: \dot{\vdash} (\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in \text{f}_{\text{Ph}}n)n)n)n) \mid \forall_{\text{obj}} \overline{(\epsilon)}: \dot{\vdash} (\forall_{\text{obj}} \overline{\text{n}}: \dot{\vdash} (\forall_{\text{obj}} \overline{\text{m}}: \dot{\vdash} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{(\epsilon)})n)n)n) \Rightarrow \overline{\text{n}} \leq \overline{\text{m}} \Rightarrow \dot{\vdash} (|(\overline{(\text{fx})}[\underline{m}] + (-\text{ud}_{\text{Ph}}[\underline{m}]))| \leq \overline{(\epsilon)}) \Rightarrow$$

$$\dot{\vdash} (\dot{\vdash} (|(\overline{(\text{fx})}[\underline{m}] + (-\text{ud}_{\text{Ph}}[\underline{m}]))| = \overline{(\epsilon)})n)n)n)n) = \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\text{P}(\text{Union}(\{\text{N}, \text{Q}\}))) \mid \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\vdash} (\dot{\vdash} (\dot{\vdash} (\overline{(\text{op1})} \in \text{N} \Rightarrow \dot{\vdash} (\overline{(\text{op2})} \in \text{Q})n)n} \Rightarrow \dot{\vdash} (\text{a}_{\text{Ph}} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\}n)n)n)n)n) \mid \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}} \overline{(\text{r1})}: (\text{r1}) \in \text{f}_{\text{Ph}} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\vdash} (\dot{\vdash} (\dot{\vdash} (\overline{(\text{op1})} \in \text{N} \Rightarrow \dot{\vdash} (\overline{(\text{op2})} \in \text{Q})n)n} \Rightarrow \dot{\vdash} (\overline{(\text{r1})} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\}n)n)n)n)n) \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{f1})}: \forall_{\text{obj}} \overline{(\text{f2})}: \forall_{\text{obj}} \overline{(\text{f3})}: \forall_{\text{obj}} \overline{(\text{f4})}: \{\{\overline{(\text{f1})}, \overline{(\text{f1})}\}, \{\overline{(\text{f1})}, \overline{(\text{f2})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \{\{\overline{(\text{f3})}, \overline{(\text{f3})}\}, \{\overline{(\text{f3})}, \overline{(\text{f4})}\}\} \in \text{f}_{\text{Ph}} \Rightarrow \overline{(\text{f1})} = \overline{(\text{f3})} \Rightarrow \overline{(\text{f2})} = \overline{(\text{f4})}n)n} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{s1})}: (\text{s1}) \in \text{N} \Rightarrow \dot{\vdash} (\forall_{\text{obj}} \overline{(\text{s2})}: \dot{\vdash} (\{\{\overline{(\text{s1})}, \overline{(\text{s1})}\}, \{\overline{(\text{s1})}, \overline{(\text{s2})}\}\} \in \text{f}_{\text{Ph}}n)n)n)n) \mid \forall_{\text{obj}} \overline{(\epsilon)}: \dot{\vdash} (\forall_{\text{obj}} \overline{\text{n}}: \dot{\vdash} (\forall_{\text{obj}} \overline{\text{m}}: \dot{\vdash} (0 \leq \overline{(\epsilon)} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{(\epsilon)})n)n)n) \Rightarrow \overline{\text{n}} \leq \overline{\text{m}} \Rightarrow \dot{\vdash} (|(\overline{(\text{fy})}[\underline{m}] + (-\text{ud}_{\text{Ph}}[\underline{m}]))| \leq \overline{(\epsilon)}) \Rightarrow$$

$$\dot{\vdash} (\dot{\vdash} (|(\overline{(\text{fy})}[\underline{m}] + (-\text{ud}_{\text{Ph}}[\underline{m}]))| = \overline{(\epsilon)})n)n)n)n) \vdash \{\text{ph} \in \text{P}(\{\text{ph} \in \text{P}(\{\text{ph} \in$$





































$$\begin{aligned}
& \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathbf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} \Rightarrow \mathbf{n} \Rightarrow \\
& \dot{\vdash} (\forall_{\text{obj}}(s1): (s1) \in \mathbf{N} \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(s2): \dot{\vdash} (\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \\
& \mathbf{f}_{Ph} \Rightarrow \mathbf{n}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{\epsilon}): \dot{\vdash} (\forall_{\text{obj}}\overline{n}: \dot{\vdash} (\forall_{\text{obj}}\overline{m}: \dot{\vdash} (0 \leq \overline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{\epsilon}) \Rightarrow \mathbf{n}) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\vdash} (|\{\{\text{ph} \in \{\text{ph} \in \mathbf{P}(\mathbf{P}(\text{Union}(\{\mathbf{N}, \mathbf{Q}\}))\})\})| \\
& \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\overline{\text{op1}}) \in \mathbf{N} \Rightarrow \dot{\vdash} (\overline{\text{op2}}) \in \mathbf{Q}) \Rightarrow \\
& \dot{\vdash} (\mathbf{a}_{Ph} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{crs1}}): \dot{\vdash} (\mathbf{c}_{Ph} = \\
& \{\{\overline{\text{crs1}}, \overline{\text{crs1}}\}, \{\overline{\text{crs1}}, 1\}\}) \Rightarrow \dot{\vdash} (\overline{m} + (-\text{ud}_{Ph}(\overline{m})) | \leq \overline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|\{\{\text{ph} \in \\
& \{\text{ph} \in \mathbf{P}(\mathbf{P}(\text{Union}(\{\mathbf{N}, \mathbf{Q}\}))\})\}) | \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\overline{\text{op1}}) \in \\
& \mathbf{N} \Rightarrow \dot{\vdash} (\overline{\text{op2}}) \in \mathbf{Q}) \Rightarrow \dot{\vdash} (\mathbf{a}_{Ph} = \\
& \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{crs1}}): \dot{\vdash} (\mathbf{c}_{Ph} = \\
& \{\{\overline{\text{crs1}}, \overline{\text{crs1}}\}, \{\overline{\text{crs1}}, 1\}\}) \Rightarrow \dot{\vdash} (\overline{m} + (-\text{ud}_{Ph}(\overline{m})) | = \\
& \overline{\epsilon}) \Rightarrow \dot{\vdash} (\text{ExpZero}(\mathbf{R}) \xrightarrow{\text{proof}} \text{Rule tactic}) \\
& [\text{ExpPositive}(\mathbf{R}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall (\underline{fx}): \dot{\vdash} (0 \leq \underline{m} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{m}) \Rightarrow \mathbf{n}) \vdash \\
& (\underline{fx})(\underline{exp}) \underline{m} = \{\text{ph} \in \mathbf{P}(\{\text{ph} \in \mathbf{P}(\{\text{ph} \in \mathbf{P}(\mathbf{P}(\text{Union}(\{\mathbf{N}, \mathbf{Q}\}))\})\})\}) | \\
& \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\overline{\text{op1}}) \in \mathbf{N} \Rightarrow \dot{\vdash} (\overline{\text{op2}}) \in \mathbf{Q}) \Rightarrow \\
& \dot{\vdash} (\mathbf{a}_{Ph} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}) \Rightarrow \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{r1}): (\overline{r1}) \in \\
& \mathbf{f}_{Ph} \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\overline{\text{op1}}) \in \mathbf{N} \Rightarrow \dot{\vdash} (\overline{\text{op2}}) \in \mathbf{Q}) \Rightarrow \\
& \dot{\vdash} (\overline{r1}) = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}) \Rightarrow \\
& \dot{\vdash} (\forall_{\text{obj}}(\overline{f1}): \forall_{\text{obj}}(\overline{f2}): \forall_{\text{obj}}(\overline{f3}): \forall_{\text{obj}}(\overline{f4}): \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \mathbf{f}_{Ph} \Rightarrow \\
& \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \mathbf{f}_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)} \Rightarrow \mathbf{n} \Rightarrow \\
& \dot{\vdash} (\forall_{\text{obj}}(s1): (s1) \in \mathbf{N} \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(s2): \dot{\vdash} (\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \\
& \mathbf{f}_{Ph} \Rightarrow \mathbf{n}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{\epsilon}): \dot{\vdash} (\forall_{\text{obj}}\overline{n}: \dot{\vdash} (\forall_{\text{obj}}\overline{m}: \dot{\vdash} (0 \leq \overline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{\epsilon}) \Rightarrow \mathbf{n}) \Rightarrow \\
& \overline{n} \leq \overline{m} \Rightarrow \dot{\vdash} (|\{\{\text{ph} \in \{\text{ph} \in \mathbf{P}(\mathbf{P}(\text{Union}(\{\mathbf{N}, \mathbf{Q}\}))\})\}) | \\
& \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\overline{\text{op1}}) \in \mathbf{N} \Rightarrow \dot{\vdash} (\overline{\text{op2}}) \in \mathbf{Q}) \Rightarrow \\
& \dot{\vdash} (\mathbf{a}_{Ph} = \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}\underline{m}: \dot{\vdash} (\mathbf{e}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{x}[\underline{m}] * \underline{y}[\underline{m}])\}\}) \Rightarrow \dot{\vdash} (\overline{m} + (-\text{ud}_{Ph}(\overline{m})) | \leq \overline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|\{\{\text{ph} \in \\
& \{\text{ph} \in \mathbf{P}(\mathbf{P}(\text{Union}(\{\mathbf{N}, \mathbf{Q}\}))\})\}) | \dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\overline{\text{op1}}) \in \\
& \mathbf{N} \Rightarrow \dot{\vdash} (\overline{\text{op2}}) \in \mathbf{Q}) \Rightarrow \dot{\vdash} (\mathbf{a}_{Ph} = \\
& \{\{\overline{\text{op1}}, \overline{\text{op1}}\}, \{\overline{\text{op1}}, \overline{\text{op2}}\}\}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}\underline{m}: \dot{\vdash} (\mathbf{e}_{Ph} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, (\underline{x}[\underline{m}] * \underline{y}[\underline{m}])\}\}) \Rightarrow \dot{\vdash} (\overline{m} + (-\text{ud}_{Ph}(\overline{m})) | = \\
& \overline{\epsilon}) \Rightarrow \dot{\vdash} (\text{ExpPositive}(\mathbf{R}) \xrightarrow{\text{proof}} \text{Rule tactic}) \\
& \text{---(26.10.06)}
\end{aligned}$$

$$\begin{aligned}
& [02 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [02 \doteq 01 + +01] \rrbracket)] \\
& [01//02 \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [01//02 \doteq 01//\text{temp02}] \rrbracket)] \\
& \text{---(28.10.06)}
\end{aligned}$$

$$\begin{aligned}
& [\text{ExpUnbounded}(\mathbf{R}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \\
& \forall \underline{m}: \forall (\underline{fx}): \dot{\vdash} (\forall_{\text{obj}}\underline{m}: \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{\epsilon}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}\overline{n}: \dot{\vdash} (\forall_{\text{obj}}\overline{m}: \dot{\vdash} (\dot{\vdash} (0 \leq \overline{\epsilon}) \Rightarrow \\
& \dot{\vdash} (\dot{\vdash} (0 = \overline{\epsilon}) \Rightarrow \mathbf{n}) \Rightarrow \dot{\vdash} (\overline{n} \leq \overline{m} \Rightarrow \underline{x}[\overline{m}] \leq \\
& (\underline{y}[\overline{m}] + (-\underline{u}(\overline{\epsilon})))) \Rightarrow \dot{\vdash} (\text{ExpUnbounded}(\mathbf{R}) \xrightarrow{\text{proof}} \text{Rule tactic}) \\
& \text{---(30.10.06)}
\end{aligned}$$

$$[\text{FromLeq}(\text{Advanced})(\mathbf{N}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall (\underline{m1}): \forall \underline{n}: \underline{m} \leq \underline{n} \vdash$$

$\dot{\vdash} (\forall_{\text{obj}}(\underline{m1}): \dot{\vdash} ((\underline{m} + (\underline{m1})) = \underline{n})n)n[\text{FromLeq}(\text{Advanced})(N) \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $\text{---}(3.11.06)$   
 $[\text{usFoelge} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{usFoelge} \doteq \text{us}]])]$   
 $[\text{FromLeastUpperBound} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \text{LUB}((\underline{fx}), (\underline{fys})) \vdash$   
 $\dot{\vdash} (\text{UB}((\underline{fx}), (\underline{fys})) \Rightarrow \dot{\vdash} (\text{UB}((\underline{fz}), (\underline{fys})) \Rightarrow$   
 $\dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{\epsilon}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}\bar{n}: \dot{\vdash} (\forall_{\text{obj}}\bar{m}: \dot{\vdash} (\dot{\vdash} (0 \leq \overline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \overline{\epsilon}))n)n)n \Rightarrow$   
 $\dot{\vdash} (\bar{n} \leq \bar{m} \Rightarrow x[\bar{m}] \leq (y[\bar{m}] + (-u(\underline{\epsilon}))))n)n)n)n)n \Rightarrow \underline{fx} =$   
 $\underline{fz})n)n][\text{FromLeastUpperBound} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{ToLeastUpperBound} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall(\underline{fx}): \forall(\underline{fz}): \forall(\underline{fys}): \text{UB}((\underline{fx}), (\underline{fys})) \vdash$   
 $\text{UB}((\underline{fz}), (\underline{fys})) \Rightarrow \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}\bar{\epsilon}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}\bar{n}: \dot{\vdash} (\forall_{\text{obj}}\bar{m}: \dot{\vdash} (\dot{\vdash} (0 \leq \overline{\epsilon}) \Rightarrow$   
 $\dot{\vdash} (\dot{\vdash} (0 = \overline{\epsilon}))n)n)n \Rightarrow \dot{\vdash} (\bar{n} \leq \bar{m} \Rightarrow x[\bar{m}] \leq$   
 $(y[\bar{m}] + (-u(\underline{\epsilon}))))n)n)n)n)n \Rightarrow \underline{fx} = \underline{fz}) \vdash$   
 $\text{LUB}((\underline{fx}), (\underline{fys}))][\text{ToLeastUpperBound} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{XSisNotUpperBound} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash$   
 $\forall \underline{m}: \dot{\vdash} (\text{UB}(x_s[\underline{m}], \text{SetOfF}_x))n][\text{XSisNotUpperBound} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $\text{---}(4.11.06)$   
 $[\text{xaF} \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{xaF} \doteq x_s]])]$   
 $[\text{ysFGreater} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \dot{\vdash} (x_s[\underline{m}] \leq y_s[\underline{m}] \Rightarrow \dot{\vdash} (\dot{\vdash} (x_s[\underline{m}] =$   
 $y_s[\underline{m}])n)n)n][\text{ysFGreater} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{ysFLess} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \dot{\vdash} (y_s[\underline{m}] \leq (x_s[\underline{m}] + \text{recm}) \Rightarrow \dot{\vdash} (\dot{\vdash} (y_s[\underline{m}] =$   
 $(x_s[\underline{m}] + \text{recm}))n)n)n][\text{ysFLess} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{SmallInverse} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall \underline{x}: \dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{x})n)n) \vdash$   
 $\dot{\vdash} (\forall_{\text{obj}}\bar{m}: \dot{\vdash} (\dot{\vdash} (\text{recm} \leq \underline{x} \Rightarrow \dot{\vdash} (\dot{\vdash} (\text{recm} = \underline{x})n)n)n)n)[\text{SmallInverse} \xrightarrow{\text{proof}}$   
 $\text{Rule tactic}]$   
 $\text{---}(6.11.06)$   
 $[x = y \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[x = y \doteq x = y]])]$   
 $[\text{OrderedPair}(x, y) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{OrderedPair}(x, y) \doteq \langle x, y \rangle]])]$   
 $[\text{MemberOfSeries}(\text{ImPLY}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall(\underline{fx}): \forall(\underline{sy}): \underline{m} \in N \Rightarrow$   
 $\dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{r1}): \overline{(\underline{r1})} \in \underline{fx}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{op1}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{op2}): \dot{\vdash} (\dot{\vdash} (\overline{(\underline{op1})} \in N \Rightarrow$   
 $\dot{\vdash} (\overline{(\underline{op2})} \in \underline{sy}))n)n) \Rightarrow \dot{\vdash} (\overline{(\underline{r1})} =$   
 $\{\{\overline{(\underline{op1})}, \overline{(\underline{op1})}\}, \{\overline{(\underline{op1})}, \overline{(\underline{op2})}\}\})n)n)n)n) \Rightarrow$   
 $\dot{\vdash} (\forall_{\text{obj}}(\underline{f1}): \forall_{\text{obj}}(\underline{f2}): \forall_{\text{obj}}(\underline{f3}): \forall_{\text{obj}}(\underline{f4}): \{\{\overline{(\underline{f1})}, \overline{(\underline{f1})}\}, \{\overline{(\underline{f1})}, \overline{(\underline{f2})}\}\} \in \underline{fx}) \Rightarrow$   
 $\{\{\overline{(\underline{f3})}, \overline{(\underline{f3})}\}, \{\overline{(\underline{f3})}, \overline{(\underline{f4})}\}\} \in \underline{fx}) \Rightarrow \overline{(\underline{f1})} = \overline{(\underline{f3})} \Rightarrow \overline{(\underline{f2})} = \overline{(\underline{f4})})n)n) \Rightarrow$   
 $\dot{\vdash} (\forall_{\text{obj}}(\underline{s1}): \overline{(\underline{s1})} \in N \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{s2}): \dot{\vdash} (\{\{\overline{(\underline{s1})}, \overline{(\underline{s1})}\}, \{\overline{(\underline{s1})}, \overline{(\underline{s2})}\}\} \in$   
 $\underline{fx}))n)n)n) \Rightarrow \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{fx}[\underline{m}]\}\} \in \underline{fx}][\text{MemberOfSeries}(\text{ImPLY}) \xrightarrow{\text{proof}}$   
 $\text{Rule tactic}]$   
 $[\text{MemberOfSeries} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall(\underline{fx}): \forall(\underline{sy}): \underline{m} \in N \vdash$   
 $\dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{r1}): \overline{(\underline{r1})} \in \underline{fx}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{op1}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{op2}): \dot{\vdash} (\dot{\vdash} (\overline{(\underline{op1})} \in N \Rightarrow$



$\lambda c. \text{Typeseries0}(\overline{[(\underline{fx})]}, \overline{[(\underline{sy})]}) \gg \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\neg}(\dot{\neg}(\overline{(\text{op1})} \in \mathbb{N} \Rightarrow \dot{\neg}(\overline{(\text{op2})} \in \underline{(sy)})n)n} \Rightarrow$   
 $\dot{\neg}(\overline{(r1)} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})n)n)n)n) \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(f1)}: \forall_{\text{obj}} \overline{(f2)}: \forall_{\text{obj}} \overline{(f3)}: \forall_{\text{obj}} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$   
 $\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(s1)}: \overline{(s1)} \in \mathbb{N} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$   
 $\underline{(fx)})n)n)n)n; \text{MemberOfSeries} \triangleright \underline{m} \in \mathbb{N} \triangleright \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\neg}(\dot{\neg}(\overline{(\text{op1})} \in \mathbb{N} \Rightarrow \dot{\neg}(\overline{(\text{op2})} \in \underline{(sy)})n)n} \Rightarrow$   
 $\dot{\neg}(\overline{(r1)} = \{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})n)n)n)n) \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(f1)}: \forall_{\text{obj}} \overline{(f2)}: \forall_{\text{obj}} \overline{(f3)}: \forall_{\text{obj}} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$   
 $\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(s1)}: \overline{(s1)} \in \mathbb{N} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$   
 $\underline{(fx)})n)n)n) \gg \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{(fx)[m]}\} \in \underline{(fx)}], p_0, c)$   
 $[\text{NatType} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \lambda c. \text{TypeNat0}(\overline{[\underline{m}]}) \Vdash \underline{m} \in \mathbb{N}] [\text{NatType} \xrightarrow{\text{proof}}$   
 $\text{Rule tactic}]$   
 $[\text{RationalType} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \lambda c. \text{TypeRational0}(\overline{[\underline{x}]}) \Vdash \underline{x} \in$   
 $\mathbb{Q}] [\text{RationalType} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{SeriesType} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{(fx)}: \forall \underline{(sy)}: \lambda c. \text{Typeseries0}(\overline{[(\underline{fx})]}, \overline{[(\underline{sy})]}) \Vdash$   
 $\dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \overline{(r1)}: \overline{(r1)} \in \underline{(fx)} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \overline{(\text{op1})}: \dot{\neg}(\dot{\neg}(\forall_{\text{obj}} \overline{(\text{op2})}: \dot{\neg}(\dot{\neg}(\overline{(\text{op1})} \in \mathbb{N} \Rightarrow$   
 $\dot{\neg}(\overline{(\text{op2})} \in \underline{(sy)})n)n} \Rightarrow \dot{\neg}(\overline{(r1)} =$   
 $\{\{\overline{(\text{op1})}, \overline{(\text{op1})}\}, \{\overline{(\text{op1})}, \overline{(\text{op2})}\}\})n)n)n)n) \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(f1)}: \forall_{\text{obj}} \overline{(f2)}: \forall_{\text{obj}} \overline{(f3)}: \forall_{\text{obj}} \overline{(f4)}: \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in \underline{(fx)} \Rightarrow$   
 $\{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in \underline{(fx)} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)})n)n \Rightarrow$   
 $\dot{\neg}(\forall_{\text{obj}} \overline{(s1)}: \overline{(s1)} \in \mathbb{N} \Rightarrow \dot{\neg}(\forall_{\text{obj}} \overline{(s2)}: \dot{\neg}(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in$   
 $\underline{(fx)})n)n)n) [\text{SeriesType} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{IsOrderedPair}((\underline{sx}), (\underline{sy}), (\underline{sz})) \xrightarrow{\text{macro}}$   
 $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \overline{[\text{IsOrderedPair}((\underline{sx}), (\underline{sy}), (\underline{sz})) \ddot{=}$   
 $\exists(\text{OP1ob}): \exists(\text{OP2ob}): (\text{OP1ob}) \in (\underline{sy}) \wedge (\text{OP2ob}) \in (\underline{sz}) \wedge (\underline{sx}) =$   
 $\text{OrderedPair}((\text{OP1ob}), (\text{OP2ob}))]])]$   
 $[\text{IsRelation}((\underline{sx}), (\underline{sy}), (\underline{sz})) \xrightarrow{\text{macro}}$   
 $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \overline{[\text{IsRelation}((\underline{sx}), (\underline{sy}), (\underline{sz})) \ddot{=} \forall(\text{R1ob}): ((\text{R1ob}) \in (\underline{sx}) \Rightarrow$   
 $\text{IsOrderedPair}((\text{R1ob}), (\underline{sy}), (\underline{sz}))]])]$   
 $[\text{isFunction}((\underline{sx}), (\underline{sy}), (\underline{sz})) \xrightarrow{\text{macro}}$   
 $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \overline{[\text{isFunction}((\underline{sx}), (\underline{sy}), (\underline{sz})) \ddot{=} \text{IsRelation}((\underline{sx}), (\underline{sy}), (\underline{sz})) \wedge$   
 $\forall(\text{F1ob}), (\text{F2ob}), (\text{F3ob}),$   
 $(\text{F4ob}): (\text{OrderedPair}((\text{F1ob}), (\text{F2ob})) \in (\underline{sx}) \Rightarrow \text{OrderedPair}((\text{F3ob}), (\text{F4ob})) \in$   
 $(\underline{sx}) \Rightarrow (\text{F1ob}) = (\text{F3ob}) \Rightarrow (\text{F2ob}) = (\text{F4ob}))]])]$   
 $[\text{IsSeries}((\underline{fx}), (\underline{fy})) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \overline{[\text{IsSeries}((\underline{fx}), (\underline{fy})) \ddot{=}$   
 $\text{isFunction}((\underline{fx}), \mathbb{N}, (\underline{fy})) \wedge \forall(\text{S1ob}): ((\text{S1ob}) \in \mathbb{N} \Rightarrow$   
 $\exists(\text{S2ob}): \text{OrderedPair}((\text{S1ob}), (\text{S2ob})) \in (\underline{fx}))]])]$



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$$\begin{aligned}
& [\text{TypeNat}(x) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{TypeNat}(x) \doteq \lambda c.\text{TypeNat}0(\ulcorner x \urcorner)])]) \\
& [\text{TypeNat}0(x) \xrightarrow{\text{val}} x \in_t [0] :: [\ulcorner v2n \urcorner] :: [\underline{m}] :: [\underline{n}] :: [\ulcorner n+1 \urcorner] :: [\underline{m}+0] :: \\
& [\underline{m}+\underline{n}] :: [\underline{o}] :: [\underline{p}] :: [\ulcorner (\underline{m}+\underline{n})+1 \urcorner] :: [\ulcorner \underline{m}+(\underline{m}1) \urcorner] :: [\underline{m}+(\underline{n}+1)] :: \\
& [\ulcorner \underline{m}1 \urcorner] :: [\ulcorner \underline{m}2 \urcorner] :: [\ulcorner \underline{n}1 \urcorner] :: [\ulcorner \underline{n}2 \urcorner] :: [\underline{m}] :: [\underline{n}] :: \underline{\top}] \\
& [\text{TypeSeries}(x, y) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{TypeSeries}(x, y) \doteq \\
& \lambda c.\text{Typeseries}0(\ulcorner x \urcorner, \ulcorner y \urcorner)])]) \\
& [\text{Typeseries}0(x, y) \xrightarrow{\text{val}} y!x \in_t [\underline{fx}] :: [\underline{fy}] :: [\underline{fz}] :: [\underline{us}] :: [\{\text{ph} \in \{\text{ph} \in \\
& P(\text{P}(\text{Union}(\{N, Q\})) \mid \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \\
& \dot{\neg}(\forall_{\text{obj}}\underline{m}: \dot{\neg}(\mathbf{d}_{\text{Ph}} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\underline{fx})[\underline{m}] + (\underline{fy})[\underline{m}]))\})n)n}] :: [\{\text{ph} \in \{\text{ph} \in \\
& P(\text{P}(\text{Union}(\{N, Q\})) \mid \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \\
& \dot{\neg}(\forall_{\text{obj}}(\overline{\text{crs}1}): \dot{\neg}(\mathbf{c}_{\text{Ph}} = \{\{\overline{\text{crs}1}, \overline{\text{crs}1}\}, \{\overline{\text{crs}1}, 0\}\})n)n}] :: [\{\text{ph} \in \{\text{ph} \in \\
& P(\text{P}(\text{Union}(\{N, Q\})) \mid \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \\
& \dot{\neg}(\forall_{\text{obj}}(\overline{\text{crs}1}): \dot{\neg}(\mathbf{c}_{\text{Ph}} = \{\{\overline{\text{crs}1}, \overline{\text{crs}1}\}, \{\overline{\text{crs}1}, 1\}\})n)n}] :: [\{\text{ph} \in \{\text{ph} \in \\
& P(\text{P}(\text{Union}(\{N, Q\})) \mid \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \\
& \dot{\neg}(\forall_{\text{obj}}\underline{m}: \dot{\neg}(\mathbf{e}_{\text{Ph}} = \{\{\underline{m}, \underline{m}\}, \{\underline{m}, ((\underline{fx})[\underline{m}]) * \{\text{ph} \in \{\text{ph} \in P(\text{P}(\text{Union}(\{N, Q\})) \mid \\
& \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \dot{\neg}(\forall_{\text{obj}}(\overline{\text{crs}1}): \dot{\neg}(\mathbf{c}_{\text{Ph}} = \\
& \{\{\overline{\text{crs}1}, \overline{\text{crs}1}\}, \{\overline{\text{crs}1}, 0\}\})n)n}[\underline{m}])\})n)n}] :: [\{\text{ph} \in \{\text{ph} \in \\
& P(\text{P}(\text{Union}(\{N, Q\})) \mid \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \\
& \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \\
& \dot{\neg}(\forall_{\text{obj}}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{fx})[\underline{m}] = 0))n \Rightarrow \dot{\neg}(\mathbf{f}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \text{rec}(\underline{fx})[\underline{m}]\})n)n \Rightarrow \dot{\neg}((\underline{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathbf{f}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n}] :: \underline{\top}] \\
& [\text{TypeRational}(x) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{TypeRational}(x) \doteq \\
& \lambda c.\text{TypeRational}0(\ulcorner x \urcorner)])]) \\
& [\text{TypeRational}0(x) \xrightarrow{\text{val}} x \in_t [\underline{x}] :: [\underline{y}] :: [\underline{z}] :: [0] :: [1] :: \underline{\top}] \\
& [\text{Max}(x, y) \xrightarrow{\text{macro}} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, [[\text{Max}(x, y) \doteq \text{if}(y <= x, x, y)])]) \\
& \text{---(7.11.06)} \\
& [\text{ReciprocalF} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{m}: \forall (\underline{fx}): \{\text{ph} \in \{\text{ph} \in P(\text{P}(\text{Union}(\{N, Q\})) \mid \\
& \dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}1}): \dot{\neg}(\dot{\neg}(\forall_{\text{obj}}(\overline{\text{op}2}): \dot{\neg}(\dot{\neg}(\overline{\text{op}1}) \in N \Rightarrow \dot{\neg}(\overline{\text{op}2}) \in Q))n)n \Rightarrow \\
& \dot{\neg}(\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op}1}, \overline{\text{op}1}\}, \{\overline{\text{op}1}, \overline{\text{op}2}\}\})n)n)n)n \mid \\
& \dot{\neg}(\forall_{\text{obj}}\underline{m}: \dot{\neg}(\dot{\neg}(\dot{\neg}(\dot{\neg}((\underline{fx})[\underline{m}] = 0))n \Rightarrow \dot{\neg}(\mathbf{f}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \text{rec}(\underline{fx})[\underline{m}]\})n)n \Rightarrow \dot{\neg}((\underline{fx})[\underline{m}] = 0 \Rightarrow \dot{\neg}(\mathbf{f}_{\text{Ph}} = \\
& \{\{\underline{m}, \underline{m}\}, \{\underline{m}, 0\}\})n)n)n}[\underline{m}] = \text{if}((\underline{fx})[\underline{m}] = 0, 0, \text{rec}(\underline{fx})[\underline{m}])][\text{ReciprocalF} \xrightarrow{\text{proof}} \\
& \text{Rule tactic}] \\
& \text{---(11.11.06)}
\end{aligned}$$

$[0f \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [0f \doteq \text{constantRationalSeries}(0)] \rrbracket)]$   
 $[1f \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [1f \doteq \text{constantRationalSeries}(1)] \rrbracket)]$   
 $[\text{cartProd}((sx)) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\text{cartProd}((sx)) \doteq \{\text{ph} \in \text{Power}(\text{Power}(\text{binaryUnion}((sx), (sy))) \mid \text{IsOrderedPair}(\text{ph}_1, (sx), (sy))\}] \rrbracket)]$   
 $[\text{constantRationalSeries}(x) \xrightarrow{\text{macro}}$   
 $\lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\text{constantRationalSeries}(x) \doteq \{\text{ph} \in \text{cartProd}(N) \mid \exists (\text{CRS1ob}): \text{ph}_3 = \text{OrderedPair}((\text{CRS1ob}), x)\}] \rrbracket)]$   
 $[\text{Sep2Formula} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall a: \forall b: \forall x: \forall y: y \in \{\text{ph} \in x \mid a\} \vdash \dot{\vdash} (y \in x \Rightarrow \dot{\vdash} (b)n)n] [\text{Sep2Formula} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[\text{Power}(x) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\text{Power}(x) \doteq P(x)] \rrbracket)]$   
 $\text{---}(12.11.06)$   
 $[\text{IsSubset}(x, y) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, \llbracket [\text{IsSubset}(x, y) \doteq x \subseteq y] \rrbracket)]$   
 $\text{---}(12.11.06)$   
 $[\text{Formula2Sep} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall a: \forall b: \forall x: \forall y: y \in x \vdash b \vdash y \in \{\text{ph} \in x \mid a\}] [\text{Formula2Sep} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $\text{---}(13.11.06)$   
 $[\text{SameSeries} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall m: \forall n: \forall (fx): \forall (sy): \lambda c. \text{TypeNat0}(\llbracket m \rrbracket) \Vdash \lambda c. \text{TypeNat0}(\llbracket n \rrbracket) \Vdash \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sy} \rrbracket) \Vdash m = n \vdash \underline{fx}[m] = \underline{fx}[n]]$   
 $[\text{SameSeries} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall m: \forall n: \forall (fx): \forall (sy): \lambda c. \text{TypeNat0}(\llbracket m \rrbracket) \Vdash \lambda c. \text{TypeNat0}(\llbracket n \rrbracket) \Vdash \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sy} \rrbracket) \Vdash m = n \vdash \text{memberOfSeries}(\text{Type}) \triangleright \lambda c. \text{TypeNat0}(\llbracket m \rrbracket) \triangleright \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sy} \rrbracket) \gg \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{fx}[\underline{m}]\}\} \in \underline{fx}; \text{memberOfSeries}(\text{Type}) \triangleright \lambda c. \text{TypeNat0}(\llbracket n \rrbracket) \triangleright \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sy} \rrbracket) \gg \{\{\underline{n}, \underline{n}\}, \{\underline{n}, \underline{fx}[\underline{n}]\}\} \in \underline{fx}; \text{UniqueMember}(\text{Type}) \triangleright \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sy} \rrbracket) \triangleright \{\{\underline{m}, \underline{m}\}, \{\underline{m}, \underline{fx}[\underline{m}]\}\} \in \underline{fx} \triangleright \{\{\underline{n}, \underline{n}\}, \{\underline{n}, \underline{fx}[\underline{n}]\}\} \in \underline{fx} \triangleright m = n \gg \underline{fx}[m] = \underline{fx}[n], p_0, c)]$   
 $[\text{UniqueMember}(\text{Type}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (fx): \forall (sx): \forall (sx1): \forall (sy): \forall (sy1): \forall (sz): \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sz} \rrbracket) \Vdash \{\{\underline{sx}, \underline{sx}\}, \{\underline{sx}, \underline{sx1}\}\} \in \underline{fx} \vdash \{\{\underline{sy}, \underline{sy}\}, \{\underline{sy}, \underline{sy1}\}\} \in \underline{fx} \vdash \underline{sx} = \underline{sy} \vdash \underline{sx1} = \underline{sy1}]$   
 $[\text{UniqueMember}(\text{Type}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\llbracket \text{SystemQ} \vdash \forall (fx): \forall (sx): \forall (sx1): \forall (sy): \forall (sy1): \forall (sz): \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sz} \rrbracket) \Vdash \{\{\underline{sx}, \underline{sx}\}, \{\underline{sx}, \underline{sx1}\}\} \in \underline{fx} \vdash \{\{\underline{sy}, \underline{sy}\}, \{\underline{sy}, \underline{sy1}\}\} \in \underline{fx} \vdash \underline{sx} = \underline{sy} \vdash \text{SeriesType} \triangleright \lambda c. \text{Typeseries0}(\llbracket \underline{fx} \rrbracket, \llbracket \underline{sz} \rrbracket) \gg \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{r1}): \overline{r1}) \in \underline{fx}) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{op1}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{op2}): \dot{\vdash} (\dot{\vdash} ((\overline{op1}) \in N \Rightarrow \dot{\vdash} ((\overline{op2}) \in \underline{sz}))n)n \Rightarrow \dot{\vdash} (\overline{r1}) = \{\{\overline{op1}, \overline{op1}\}, \{\overline{op1}, \overline{op2}\}\})n)n)n)n \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{f1}): \forall_{\text{obj}}(\overline{f2}): \forall_{\text{obj}}(\overline{f3}): \forall_{\text{obj}}(\overline{f4}): \{\{\overline{f1}, \overline{f1}\}, \{\overline{f1}, \overline{f2}\}\} \in \underline{fx}) \Rightarrow \{\{\overline{f3}, \overline{f3}\}, \{\overline{f3}, \overline{f4}\}\} \in \underline{fx}) \Rightarrow \overline{f1} = \overline{f3} \Rightarrow \overline{f2} = \overline{f4})n)n \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{s1}): \overline{s1}) \in N \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\overline{s2}): \dot{\vdash} (\{\{\overline{s1}, \overline{s1}\}, \{\overline{s1}, \overline{s2}\}\} \in \underline{fx}))n)n)n; \text{UniqueMember} \triangleright \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\overline{r1}): \overline{r1}) \in \underline{fx}) \Rightarrow$





$\forall_{\text{obj}} \overline{f4}: \{ \{ \underline{sx}, \underline{sx} \}, \{ \underline{sx}, \underline{sx1} \} \} \in \underline{fx} \Rightarrow \{ \{ \underline{sy}, \underline{sy} \}, \{ \underline{sy}, \overline{f4} \} \} \in$   
 $\underline{fx} \Rightarrow \underline{sx} = \underline{sy} \Rightarrow \underline{sx1} = \overline{f4} \gg \{ \{ \underline{sx}, \underline{sx} \}, \{ \underline{sx}, \underline{sx1} \} \} \in \underline{fx} \Rightarrow$   
 $\{ \{ \underline{sy}, \underline{sy} \}, \{ \underline{sy}, \underline{sy1} \} \} \in \underline{fx} \Rightarrow \underline{sx} = \underline{sy} \Rightarrow \underline{sx1} =$   
 $\underline{sy1}; \text{MP3} \triangleright \{ \{ \underline{sx}, \underline{sx} \}, \{ \underline{sx}, \underline{sx1} \} \} \in \underline{fx} \Rightarrow \{ \{ \underline{sy}, \underline{sy} \}, \{ \underline{sy}, \underline{sy1} \} \} \in$   
 $\underline{fx} \Rightarrow \underline{sx} = \underline{sy} \Rightarrow \underline{sx1} = \underline{sy1} \triangleright \{ \{ \underline{sx}, \underline{sx} \}, \{ \underline{sx}, \underline{sx1} \} \} \in$   
 $\underline{fx} \triangleright \{ \{ \underline{sy}, \underline{sy} \}, \{ \underline{sy}, \underline{sy1} \} \} \in \underline{fx} \triangleright \underline{sx} = \underline{sy} \gg \underline{sx1} = \underline{sy1}], p_0, c]$

$[A4 \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall (\underline{v1}): \forall \underline{a}: \forall \underline{b}: \langle \underline{a} \equiv \underline{b} | (\underline{v1}) := \underline{x} \rangle_{\text{Me}} \Vdash \forall_{\text{obj}} (\underline{v1}): \underline{b} \vdash \underline{a}]$

$\text{KVANTI} [A4 \xrightarrow{\text{proof}} \lambda c. \lambda x. \lambda \mathcal{P}. ([\text{SystemQ} \vdash \forall \underline{x}: \forall (\underline{v1}): \forall \underline{a}: \forall \underline{b}: \langle \underline{a} \equiv \underline{b} | (\underline{v1}) := \underline{x} \rangle_{\text{Me}} \Vdash$   
 $\forall_{\text{obj}} (\underline{v1}): \underline{b} \vdash A4(\text{Axiom}) \triangleright \langle \underline{a} \equiv \underline{b} | (\underline{v1}) := \underline{x} \rangle_{\text{Me}} \gg \forall_{\text{obj}} (\underline{v1}): \underline{b} \Rightarrow$   
 $\underline{a}; \text{MP} \triangleright \forall_{\text{obj}} (\underline{v1}): \underline{b} \Rightarrow \underline{a} \triangleright \forall_{\text{obj}} (\underline{v1}): \underline{b} \gg \underline{a}], p_0, c)]$   
 —(16.11.06)

$[\text{SameMember} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{sx}): \forall (\underline{sy}): \forall (\underline{sz}): \underline{sx} = \underline{sy} \vdash \underline{sx} \in \underline{sz} \vdash$   
 $(\underline{sy}) \in \underline{sz}][\text{SameMember} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{ToSetEquality} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{sx}): \forall (\underline{fy}): \forall_{\text{obj}} (\underline{s1}): \overline{(\underline{s1})} \in \underline{fx} \Rightarrow \overline{(\underline{s1})} \in \underline{fy} \vdash$   
 $\forall_{\text{obj}} (\underline{s1}): \overline{(\underline{s1})} \in \underline{fy} \Rightarrow \overline{(\underline{s1})} \in \underline{fx} \vdash \underline{fx} = \underline{fy}][\text{ToSetEquality} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[[\underline{px}, \underline{y}] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\underline{px}, \underline{y}) \doteq \{x, y\}]])]$

$[\text{SamePair} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{sx}): \forall (\underline{sx1}): \forall (\underline{sy}): \forall (\underline{sy1}): \underline{sx} = \underline{sx1} \vdash \underline{sy} =$   
 $\underline{sy1} \vdash \{ \underline{sx}, \underline{sy} \} = \{ \underline{sx1}, \underline{sy1} \}][\text{SamePair} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $[[\underline{sx}] \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\underline{sx}) \doteq \{x\}]])]$

$[\text{SameSingleton} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{sx}): \forall (\underline{sy}): \underline{sx} = \underline{sy} \vdash \{ \underline{sx}, \underline{sx} \} =$   
 $\{ \underline{sy}, \underline{sy} \}][\text{SameSingleton} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 —(17.11.06)

$[[\underline{fx}] +_f (\underline{fy}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\underline{fx}) +_f (\underline{fy}) \doteq \{ \text{ph} \in \text{cartProd}(\mathbb{N}) \mid$   
 $\exists \mathcal{M}: \text{ph}_4 = \text{OrderedPair}(\mathcal{M}, ((\underline{fx})[\mathcal{M}] + (\underline{fy})[\mathcal{M}]) \}]])]$

$[\text{Qclosed(Addition)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \in \mathbb{Q} \vdash \underline{y} \in \mathbb{Q} \vdash \underline{x} + \underline{y} \in$   
 $\mathbb{Q}][\text{Qclosed(Addition)} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{FromCartProd(1)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash$   
 $\forall (\underline{sx}): \forall (\underline{sx1}): \forall (\underline{sy}): \forall (\underline{sy1}): \{ \{ \underline{sx}, \underline{sx} \}, \{ \underline{sx}, \underline{sy} \} \} \in \{ \text{ph} \in$   
 $\text{P}(\text{P}(\text{Union}(\{ \{ \underline{sx1}, \underline{sy1} \} \}))) \mid \dot{\neg} (\forall_{\text{obj}} (\overline{\text{op1}}): \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} (\overline{\text{op2}}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{\text{op1}} \in$   
 $\underline{sx1} \Rightarrow \dot{\neg} (\overline{\text{op2}} \in \underline{sy1})) \text{n})) \text{n})) \Rightarrow \dot{\neg} (\text{a}_{\text{ph}} =$   
 $\{ \{ \overline{\text{op1}}, \overline{\text{op1}} \}, \{ \overline{\text{op1}}, \overline{\text{op2}} \} \} \text{n})) \text{n})) \text{n})) \text{n})) \vdash \underline{sx} \in$   
 $\underline{sx1}][\text{FromCartProd(1)} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{1rule fromCartProd(2)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash$   
 $\forall (\underline{sx}): \forall (\underline{sx1}): \forall (\underline{sy}): \forall (\underline{sy1}): \{ \{ \underline{sx}, \underline{sx} \}, \{ \underline{sx}, \underline{sy} \} \} \in \{ \text{ph} \in$   
 $\text{P}(\text{P}(\text{Union}(\{ \{ \underline{sx1}, \underline{sy1} \} \}))) \mid \dot{\neg} (\forall_{\text{obj}} (\overline{\text{op1}}): \dot{\neg} (\dot{\neg} (\forall_{\text{obj}} (\overline{\text{op2}}): \dot{\neg} (\dot{\neg} (\dot{\neg} (\overline{\text{op1}} \in$   
 $\underline{sx1} \Rightarrow \dot{\neg} (\overline{\text{op2}} \in \underline{sy1})) \text{n})) \text{n})) \Rightarrow \dot{\neg} (\text{a}_{\text{ph}} =$   
 $\{ \{ \overline{\text{op1}}, \overline{\text{op1}} \}, \{ \overline{\text{op1}}, \overline{\text{op2}} \} \} \text{n})) \text{n})) \text{n})) \text{n})) \vdash \underline{sy} \in$

$\frac{(\text{sy1})[[1\text{rule from CartProd}(2) \xrightarrow{\text{proof}} \text{Rule tactic}]]}{(18.11.06)}$

$[[(\text{fx}) *_{\text{f}} (\text{fy}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\text{fx}) *_{\text{f}} (\text{fy}) \doteq \{\text{ph} \in \text{cartProd}(\mathbb{N}) \mid \exists \mathcal{M}: \text{ph}_5 = \text{OrderedPair}(\mathcal{M}, ((\text{fx})[\mathcal{M}] * (\text{fy})[\mathcal{M}]])]])]]]$

$[\text{Qclosed}(\text{Multiplication}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \underline{x} \in \mathbb{Q} \vdash \underline{y} \in \mathbb{Q} \vdash (\underline{x} * \underline{y}) \in \mathbb{Q}][\text{Qclosed}(\text{Multiplication}) \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $\frac{}{(19.11.06)}$

$[\text{Pair2Formula} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{\text{sx}}): \forall (\underline{\text{sy}}): \forall (\underline{\text{sz}}): (\underline{\text{sx}} \in \{(\underline{\text{sy}}), (\underline{\text{sz}})\}) \vdash \dot{\vdash} ((\underline{\text{sx}} = (\underline{\text{sy}})) \text{n} \Rightarrow (\underline{\text{sx}} = (\underline{\text{sz}})))] [\text{Pair2Formula} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Formula2Pair} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{\text{sx}}): \forall (\underline{\text{sy}}): \forall (\underline{\text{sz}}): \dot{\vdash} ((\underline{\text{sx}} = (\underline{\text{sy}})) \text{n} \Rightarrow (\underline{\text{sx}} = (\underline{\text{sz}}) \vdash (\underline{\text{sx}} \in \{(\underline{\text{sy}}), (\underline{\text{sz}})\})))] [\text{Formula2Pair} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $\frac{}{(23.11.06)}$

$[\text{binaryUnion}(x, y) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{binaryUnion}(x, y) \doteq \text{Union}((\text{px}, y)])]])]$

$[\text{Formula2Union} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{\text{sx}}): \forall (\underline{\text{sy}}): \forall (\underline{\text{sz}}): \dot{\vdash} (\forall \text{obj}(\underline{\text{sy}}): \dot{\vdash} (\dot{\vdash} ((\underline{\text{sx}} \in (\underline{\text{sy}}) \Rightarrow \dot{\vdash} ((\underline{\text{sy}} \in (\underline{\text{sz}}) \text{n}) \text{n}) \text{n}) \text{n} \vdash (\underline{\text{sx}} \in \text{Union}((\underline{\text{sz}})))])])]$   $[\text{Formula2Union} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Formula2Power} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall (\underline{\text{sx}}): \forall (\underline{\text{sy}}): \forall \text{obj}(\underline{\text{s1}}): (\underline{\text{s1}} \in (\underline{\text{sx}}) \Rightarrow (\underline{\text{s1}} \in (\underline{\text{sy}}) \vdash (\underline{\text{sx}} \in \text{P}((\underline{\text{sy}})))])]$   $[\text{Formula2Power} \xrightarrow{\text{proof}} \text{Rule tactic}]$   
 $\frac{}{(28.11.06)}$

$[\text{SetOfRationalSeries} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{SetOfRationalSeries} \doteq \{\text{ph} \in \text{Power}(\text{cartProd}(\mathbb{N})) \mid \text{IsSeries}(\text{ph}_2, \mathbb{Q})\}]])]$

$[[1\text{f}/(\text{fx}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[1\text{f}/(\text{fx}) \doteq \{\text{ph} \in \text{cartProd}(\mathbb{N}) \mid \exists \mathcal{M}: ((\text{fx})[\mathcal{M}] \neq 0 \wedge \text{ph}_6 = \text{OrderedPair}(\mathcal{M}, \text{rec}(\text{fx})[\mathcal{M}])) \vee ((\text{fx})[\mathcal{M}] = 0 \wedge \text{ph}_6 = \text{OrderedPair}(\mathcal{M}, 0))\}]]]]]$

$[\text{Max} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \forall \underline{y}: \dot{\vdash} (\dot{\vdash} (\underline{y} \leq \underline{x} \Rightarrow \dot{\vdash} (\text{if}(\underline{y} \leq \underline{x}, \underline{x}, \underline{y}) = \underline{x}) \text{n}) \text{n}) \text{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (\underline{y} \leq \underline{x}) \text{n} \Rightarrow \dot{\vdash} (\text{if}(\underline{y} \leq \underline{x}, \underline{x}, \underline{y}) = \underline{y}) \text{n}) \text{n}])]$   $[\text{Max} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Numerical} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (\dot{\vdash} (0 \leq \underline{x} \Rightarrow \dot{\vdash} (|\underline{x}| = \underline{x}) \text{n}) \text{n}) \text{n} \Rightarrow \dot{\vdash} (\dot{\vdash} (0 \leq \underline{x}) \text{n} \Rightarrow \dot{\vdash} (|\underline{x}| = (-\underline{u}\underline{x})) \text{n}) \text{n}])]$   $[\text{Numerical} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Separation2formula}(1) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{x}: \forall \underline{y}: \underline{y} \in \{\text{ph} \in \underline{x} \mid \underline{a}\} \vdash \underline{y} \in \underline{x}][\text{Separation2formula}(1) \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{Separation2formula}(2) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{a}: \forall \underline{b}: \forall \underline{x}: \forall \underline{y}: \underline{y} \in \{\text{ph} \in \underline{x} \mid \underline{a}\} \vdash \underline{b}][\text{Separation2formula}(2) \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{QisClosed}(\text{Reciprocal})(\text{Imply}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (\underline{x} = 0) \text{n} \Rightarrow \underline{x} \in \mathbb{Q} \Rightarrow \text{rec} \underline{x} \in \mathbb{Q}][\text{QisClosed}(\text{Reciprocal})(\text{Imply}) \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{QisClosed}(\text{Reciprocal}) \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (\underline{x} = 0) \text{n} \vdash \underline{x} \in \mathbb{Q} \vdash \text{rec} \underline{x} \in \mathbb{Q}]$

$[\text{QisClosed}(\text{Reciprocal}) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\lceil \text{SystemQ} \vdash \forall \underline{x}: \dot{\vdash} (\underline{x} = 0) \text{n} \vdash \underline{x} \in \mathbb{Q} \vdash \text{QisClosed}(\text{Reciprocal})(\text{Imply}) \gg \dot{\vdash} (\underline{x} = 0) \text{n} \Rightarrow \underline{x} \in \mathbb{Q} \Rightarrow \text{rec} \underline{x} \in \mathbb{Q}; \text{MP2} \triangleright \dot{\vdash} (\underline{x} =$

$(0)n \Rightarrow \underline{x} \in \mathbb{Q} \Rightarrow \text{rec}\underline{x} \in \mathbb{Q} \triangleright \dot{\vdash} (\underline{x} = 0)n \triangleright \underline{x} \in \mathbb{Q} \gg \text{rec}\underline{x} \in \mathbb{Q}], p_0, c)]$   
 —(1.12.06)

$[\text{QisClosed(Negative)(ImPLY)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \underline{x} \in \mathbb{Q} \Rightarrow (-\underline{ux}) \in \mathbb{Q}][\text{QisClosed(Negative)(ImPLY)} \xrightarrow{\text{proof}} \text{Rule tactic}]$

$[\text{QisClosed(Negative)} \xrightarrow{\text{stmt}} \text{SystemQ} \vdash \forall \underline{x}: \underline{x} \in \mathbb{Q} \vdash (-\underline{ux}) \in \mathbb{Q}]$

$[\text{QisClosed(Negative)} \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}([\text{SystemQ} \vdash \forall \underline{x}: \underline{x} \in \mathbb{Q} \vdash \text{QisClosed(Negative)(ImPLY)} \gg \underline{x} \in \mathbb{Q} \Rightarrow (-\underline{ux}) \in \mathbb{Q}; \text{MP} \triangleright \underline{x} \in \mathbb{Q} \Rightarrow (-\underline{ux}) \in \mathbb{Q} \triangleright \underline{x} \in \mathbb{Q} \gg (-\underline{ux}) \in \mathbb{Q}], p_0, c)]$

$[-_f(\underline{fx}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[-_f(\underline{fx}) \doteq \{\text{ph} \in \text{cartProd}(N) \mid \exists \mathcal{M}: \text{ph}_6 = \text{OrderedPair}(\mathcal{M}, (-u(\underline{fx})[\mathcal{M}])\})]])]$

$[\text{SF}((\underline{fx}), (\underline{fy})) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{SF}((\underline{fx}), (\underline{fy})) \doteq \forall (\text{EPob}): \exists n: \forall m: (0 < (\text{EPob}) \Rightarrow n <= m \Rightarrow |((\underline{fx})[m] - (\underline{fy})[m])| < (\text{EPob}))]])]$   
 —(2.12.06)

$[(\underline{fx}) <_f (\underline{fy}) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[(\underline{fx}) <_f (\underline{fy}) \doteq \exists (\text{EPob}): \exists n: \forall m: 0 < (\text{EPob}) \wedge (n <= m \Rightarrow (\underline{fx})[m] <= ((\underline{fy})[m] - (\text{EPob}))]])]$   
 —(2.12.06)

$[\text{extractSeries}(t) \xrightarrow{\text{val}} t^{22121222111111}]$

$[\text{SetOfSeries}((\underline{sx})) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{SetOfSeries}((\underline{sx})) \doteq \{\text{ph} \in \text{Power}(\text{cartProd}(N)) \mid \text{IsSeries}(\text{ph}_6, (\underline{sx}))\}]])]$

$[\text{R}((\underline{fx})) \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. \tilde{\mathcal{M}}_4(t, s, c, [[\text{R}((\underline{fx})) \doteq \{\text{ph} \in \text{Power}(\text{SetOfSeries}(\mathbb{Q})) \mid \text{SF}((\underline{fx}), \text{ph}_4)\}]])]$

$[\text{ExpandList}(t, s, c) \xrightarrow{\text{val}} \text{t!s!c!If}(t^a, T, \text{StateExpand}(t^h, s, c) :: \text{ExpandList}(t^t, s, c))]$   
 (\*\*\*)

$[\underline{x} * \underline{y} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. * \text{Macro}(t^h :: \text{ExpandList}(t^t, s, c))]$

$[* \text{Macro}(t) \xrightarrow{\text{val}} \tilde{\mathcal{Q}}(t, [\{\underline{\text{ph}} \in \text{P}(\{\underline{\text{ph}} \in \text{P}(\{\underline{\text{ph}} \in \text{P}(\text{Union}(\{N, Q\}))) \mid \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\dot{\vdash} ((\underline{\text{op1}}) \in N \Rightarrow \dot{\vdash} ((\underline{\text{op2}}) \in Q)n)n \Rightarrow \dot{\vdash} (\underline{\text{aPh}} = \{\{\{\underline{\text{op1}}, \underline{\text{op1}}\}, \{\{\underline{\text{op1}}, \underline{\text{op2}}\}\})n)n)n)n)n) \mid \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{\text{r1}}): (\underline{\text{r1}}) \in \mathbf{fPh} \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\dot{\vdash} ((\underline{\text{op1}}) \in N \Rightarrow \dot{\vdash} ((\underline{\text{op2}}) \in Q)n)n \Rightarrow \dot{\vdash} ((\underline{\text{r1}}) = \{\{\{\underline{\text{op1}}, \underline{\text{op1}}\}, \{\{\underline{\text{op1}}, \underline{\text{op2}}\}\})n)n)n)n)n) \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{f1}}): \forall_{\text{obj}}(\underline{\text{f2}}): \forall_{\text{obj}}(\underline{\text{f3}}): \forall_{\text{obj}}(\underline{\text{f4}}): \{\{\{\underline{\text{f1}}, \underline{\text{f1}}\}, \{\{\underline{\text{f1}}, \underline{\text{f2}}\}\} \in \mathbf{fPh} \Rightarrow \{\{\{\underline{\text{f3}}, \underline{\text{f3}}\}, \{\{\underline{\text{f3}}, \underline{\text{f4}}\}\} \in \mathbf{fPh} \Rightarrow \underline{\text{f1}} = \underline{\text{f3}} \Rightarrow \underline{\text{f2}} = \underline{\text{f4}})n)n \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{s1}}): (\underline{\text{s1}}) \in N \Rightarrow \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{s2}}): \dot{\vdash} (\{\{\{\underline{\text{s1}}, \underline{\text{s1}}\}, \{\{\underline{\text{s1}}, \underline{\text{s2}}\}\} \in \mathbf{fPh})n)n)n) \mid \forall_{\text{obj}}(\underline{\epsilon}): \dot{\vdash} (\forall_{\text{obj}}\underline{n}: \dot{\vdash} (\forall_{\text{obj}}\underline{m}: \dot{\vdash} (0 <= \underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (0 = \underline{\epsilon})n)n) \Rightarrow \underline{n} <= \underline{m} \Rightarrow \dot{\vdash} (|(\{\underline{\text{ph}} \in \{\underline{\text{ph}} \in \text{P}(\text{Union}(\{N, Q\}))) \mid \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\dot{\vdash} ((\underline{\text{op1}}) \in N \Rightarrow \dot{\vdash} ((\underline{\text{op2}}) \in Q)n)n \Rightarrow \dot{\vdash} (\underline{\text{aPh}} = \{\{\{\underline{\text{op1}}, \underline{\text{op1}}\}, \{\{\underline{\text{op1}}, \underline{\text{op2}}\}\})n)n)n)n)n) \mid \dot{\vdash} (\forall_{\text{obj}}\underline{m}: \dot{\vdash} (\underline{\text{ePh}} = \{\{\underline{\text{m}}, \underline{\text{m}}\}, \{\underline{\text{m}}, (\underline{x}[\underline{\text{m}}] * \underline{y}[\underline{\text{m}}])\})n)n\}[\underline{\text{m}}] + (-\text{udPh}[\underline{\text{m}}]))| <= \underline{\epsilon}) \Rightarrow \dot{\vdash} (\dot{\vdash} (|(\{\underline{\text{ph}} \in \text{P}(\text{Union}(\{N, Q\}))) \mid \dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op1}}): \dot{\vdash} (\dot{\vdash} (\forall_{\text{obj}}(\underline{\text{op2}}): \dot{\vdash} (\dot{\vdash} (\dot{\vdash} ((\underline{\text{op1}}) \in N \Rightarrow \dot{\vdash} ((\underline{\text{op2}}) \in Q)n)n \Rightarrow \dot{\vdash} (\underline{\text{aPh}} = \{\{\{\underline{\text{op1}}, \underline{\text{op1}}\}, \{\{\underline{\text{op1}}, \underline{\text{op2}}\}\})n)n)n)n)n) \mid \dot{\vdash} (\forall_{\text{obj}}\underline{m}: \dot{\vdash} (\underline{\text{ePh}} = \{\{\underline{\text{m}}, \underline{\text{m}}\}, \{\underline{\text{m}}, (\underline{x}[\underline{\text{m}}] * \underline{y}[\underline{\text{m}}])\})n)n\}[\underline{\text{m}}] + (-\text{udPh}[\underline{\text{m}}]))| = \underline{\epsilon})n)n)n) \}], [\underline{x}] ::$







$$\begin{aligned}
& \dot{\dot{\dot{((op2)} \in Q)n}n} \Rightarrow \dot{\dot{\dot{(a_{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}n)n)n)n} \mid \\
& \dot{\dot{\dot{(\dot{(\forall_{obj}(r1): (r1) \in f_{Ph}} \Rightarrow \dot{\dot{(\forall_{obj}(op1): \dot{\dot{(\dot{(\forall_{obj}(op2): \dot{\dot{(\dot{(\dot{((op1)} \in N \Rightarrow \\
& \dot{\dot{((op2)} \in Q)n}n} \Rightarrow \dot{\dot{((r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}n)n)n)n} \Rightarrow \\
& \dot{\dot{(\forall_{obj}(f1): \forall_{obj}(f2): \forall_{obj}(f3): \forall_{obj}(f4): \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph}} \Rightarrow \\
& \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)}n)n \Rightarrow \\
& \dot{\dot{(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\dot{(\forall_{obj}(s2): \dot{\dot{(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \\
& f_{Ph})n)n)n}) \mid \forall_{obj}(\epsilon): \dot{\dot{(\forall_{obj}\bar{n}: \dot{\dot{(\forall_{obj}\bar{m}: \dot{\dot{(0 \leq (\epsilon)} \Rightarrow \dot{\dot{(\dot{(0 = (\overline{(\epsilon)})n)n} \Rightarrow \\
& \bar{n} \leq \bar{m} \Rightarrow \dot{\dot{(\dot{(|(\underline{(fy)}[\bar{m}] + (-\text{ud}_{Ph}[\bar{m}]|) \leq (\overline{(\epsilon)}) \Rightarrow \\
& \dot{\dot{(\dot{(|(\underline{(fy)}[\bar{m}] + (-\text{ud}_{Ph}[\bar{m}]|) = (\overline{(\epsilon)})n)n)n}) \mid \\
& [\text{lemma eqLeq}(R) \xrightarrow{\text{proof}} \lambda c. \lambda x. \mathcal{P}(\ulcorner \text{SystemQ} \vdash \forall(\underline{fx}): \forall(\underline{fy}): \{\text{ph} \in P(\{\text{ph} \in \\
& P(\{\text{ph} \in P(P(\text{Union}(\{N, Q\})) \mid \dot{\dot{(\forall_{obj}(\overline{op1}): \dot{\dot{(\dot{(\forall_{obj}(\overline{op2}): \dot{\dot{(\dot{(\dot{((op1)} \in N \Rightarrow \\
& N \Rightarrow \dot{\dot{((op2)} \in Q)n}n} \Rightarrow \dot{\dot{(a_{Ph}} = \\
& \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}n)n)n)n} \mid \dot{\dot{(\dot{(\forall_{obj}(\overline{r1}): (\overline{r1}) \in f_{Ph}} \Rightarrow \\
& \dot{\dot{(\forall_{obj}(\overline{op1}): \dot{\dot{(\dot{(\forall_{obj}(\overline{op2}): \dot{\dot{(\dot{(\dot{((op1)} \in N \Rightarrow \dot{\dot{((op2)} \in Q)n}n} \Rightarrow \\
& \dot{\dot{((r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}n)n)n)n} \Rightarrow \\
& \dot{\dot{(\forall_{obj}(\overline{f1}): \forall_{obj}(\overline{f2}): \forall_{obj}(\overline{f3}): \forall_{obj}(\overline{f4}): \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph}} \Rightarrow \\
& \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)}n)n \Rightarrow \\
& \dot{\dot{(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\dot{(\forall_{obj}(s2): \dot{\dot{(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \\
& f_{Ph})n)n)n}) \mid \forall_{obj}(\epsilon): \dot{\dot{(\forall_{obj}\bar{n}: \dot{\dot{(\forall_{obj}\bar{m}: \dot{\dot{(0 \leq (\overline{(\epsilon)} \Rightarrow \dot{\dot{(\dot{(0 = (\overline{(\epsilon)})n)n} \Rightarrow \\
& \bar{n} \leq \bar{m} \Rightarrow \dot{\dot{(\dot{(|(\underline{(fx)}[\bar{m}] + (-\text{ud}_{Ph}[\bar{m}]|) \leq (\overline{(\epsilon)}) \Rightarrow \\
& \dot{\dot{(\dot{(|(\underline{(fx)}[\bar{m}] + (-\text{ud}_{Ph}[\bar{m}]|) = (\overline{(\epsilon)})n)n)n}) = \{\text{ph} \in P(\{\text{ph} \in P(\{\text{ph} \in \\
& P(P(\text{Union}(\{N, Q\})) \mid \dot{\dot{(\forall_{obj}(\overline{op1}): \dot{\dot{(\dot{(\forall_{obj}(\overline{op2}): \dot{\dot{(\dot{(\dot{((op1)} \in N \Rightarrow \\
& \dot{\dot{((op2)} \in Q)n}n} \Rightarrow \dot{\dot{(a_{Ph}} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}n)n)n)n} \mid \\
& \dot{\dot{(\dot{(\forall_{obj}(\overline{r1}): (\overline{r1}) \in f_{Ph}} \Rightarrow \dot{\dot{(\forall_{obj}(\overline{op1}): \dot{\dot{(\dot{(\forall_{obj}(\overline{op2}): \dot{\dot{(\dot{(\dot{((op1)} \in N \Rightarrow \\
& \dot{\dot{((op2)} \in Q)n}n} \Rightarrow \dot{\dot{((r1)} = \{\{\overline{(op1)}, \overline{(op1)}\}, \{\overline{(op1)}, \overline{(op2)}\}\}n)n)n)n} \Rightarrow \\
& \dot{\dot{(\forall_{obj}(\overline{f1}): \forall_{obj}(\overline{f2}): \forall_{obj}(\overline{f3}): \forall_{obj}(\overline{f4}): \{\{\overline{(f1)}, \overline{(f1)}\}, \{\overline{(f1)}, \overline{(f2)}\}\} \in f_{Ph}} \Rightarrow \\
& \{\{\overline{(f3)}, \overline{(f3)}\}, \{\overline{(f3)}, \overline{(f4)}\}\} \in f_{Ph} \Rightarrow \overline{(f1)} = \overline{(f3)} \Rightarrow \overline{(f2)} = \overline{(f4)}n)n \Rightarrow \\
& \dot{\dot{(\forall_{obj}(s1): (s1) \in N \Rightarrow \dot{\dot{(\forall_{obj}(s2): \dot{\dot{(\{\{\overline{(s1)}, \overline{(s1)}\}, \{\overline{(s1)}, \overline{(s2)}\}\} \in \\
& f_{Ph})n)n)n}) \mid \forall_{obj}(\epsilon): \dot{\dot{(\forall_{obj}\bar{n}: \dot{\dot{(\forall_{obj}\bar{m}: \dot{\dot{(0 \leq (\overline{(\epsilon)} \Rightarrow \dot{\dot{(\dot{(0 = (\overline{(\epsilon)})n)n} \Rightarrow \\
& \bar{n} \leq \bar{m} \Rightarrow \dot{\dot{(\dot{(|(\underline{(fx)}[\bar{m}] + (-\text{ud}_{Ph}[\bar{m}]|) \leq (\overline{(\epsilon)}) \Rightarrow \\
& \dot{\dot{(\dot{(|(\underline{(fx)}[\bar{m}] + (-\text{ud}_{Ph}[\bar{m}]|) = (\overline{(\epsilon)})n)n)n}) = \{\text{ph} \in P(\{\text{ph} \in P(\{\text{ph} \in \\
\end{aligned}$$





















$\{\{\overline{f3}\}, \overline{f3}\}, \{\overline{f3}\}, \overline{f4}\} \in \mathbf{f}_{\text{Ph}} \Rightarrow \overline{f1} = \overline{f3} \Rightarrow \overline{f2} = \overline{f4})n \Rightarrow$   
 $\dot{\neg} (\forall_{\text{obj}}(s1): (s1) \in N \Rightarrow \dot{\neg} (\forall_{\text{obj}}(s2): \dot{\neg} (\{\{s1\}, (s1)\}, \{\{s1\}, (s2)\}\} \in$   
 $\mathbf{f}_{\text{Ph}})n)n)n) \mid \forall_{\text{obj}}(\epsilon): \dot{\neg} (\forall_{\text{obj}}\bar{n}: \dot{\neg} (\forall_{\text{obj}}\bar{m}: \dot{\neg} (0 \leq \epsilon) \Rightarrow \dot{\neg} (\dot{\neg} (0 = \overline{\epsilon}))n)n)n) \Rightarrow$   
 $\bar{n} \leq \bar{m} \Rightarrow \dot{\neg} (\{ \{ \text{ph} \in \{ \text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\})) \} \} \} \mid$   
 $\dot{\neg} (\forall_{\text{obj}}(\text{op1}): \dot{\neg} (\dot{\neg} (\forall_{\text{obj}}(\text{op2}): \dot{\neg} (\dot{\neg} (\dot{\neg} ((\text{op1}) \in N \Rightarrow \dot{\neg} ((\text{op2}) \in Q))n)n) \Rightarrow$   
 $\dot{\neg} (\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, (\text{op1})\}, \{(\text{op1}), \overline{\text{op2}}\}\})n)n)n)n)n) \mid$   
 $\dot{\neg} (\forall_{\text{obj}}\bar{m}: \dot{\neg} (\dot{\neg} (\dot{\neg} (\dot{\neg} (x[\bar{m}] = 0)n \Rightarrow \dot{\neg} (\mathbf{f}_{\text{Ph}} = \{\{\bar{m}, \bar{m}\}, \{\bar{m}, \text{recx}[\bar{m}]\}\})n)n)n) \Rightarrow$   
 $\dot{\neg} (x[\bar{m}] = 0 \Rightarrow \dot{\neg} (\mathbf{f}_{\text{Ph}} = \{\{\bar{m}, \bar{m}\}, \{\bar{m}, 0\}\})n)n)n)n) \mid \bar{m} + (-\text{ud}_{\text{Ph}}[\bar{m}]) \mid < =$   
 $\overline{\epsilon}) \Rightarrow \dot{\neg} (\dot{\neg} (\{ \{ \text{ph} \in \{ \text{ph} \in \text{P}(\text{P}(\text{Union}(\{N, Q\})) \} \} \} \mid$   
 $\dot{\neg} (\forall_{\text{obj}}(\text{op1}): \dot{\neg} (\dot{\neg} (\forall_{\text{obj}}(\text{op2}): \dot{\neg} (\dot{\neg} (\dot{\neg} ((\text{op1}) \in N \Rightarrow \dot{\neg} ((\text{op2}) \in Q))n)n) \Rightarrow$   
 $\dot{\neg} (\mathbf{a}_{\text{Ph}} = \{\{\overline{\text{op1}}, (\text{op1})\}, \{(\text{op1}), \overline{\text{op2}}\}\})n)n)n)n)n) \mid$   
 $\dot{\neg} (\forall_{\text{obj}}\bar{m}: \dot{\neg} (\dot{\neg} (\dot{\neg} (\dot{\neg} (x[\bar{m}] = 0)n \Rightarrow \dot{\neg} (\mathbf{f}_{\text{Ph}} = \{\{\bar{m}, \bar{m}\}, \{\bar{m}, \text{recx}[\bar{m}]\}\})n)n)n) \Rightarrow$   
 $\dot{\neg} (x[\bar{m}] = 0 \Rightarrow \dot{\neg} (\mathbf{f}_{\text{Ph}} = \{\{\bar{m}, \bar{m}\}, \{\bar{m}, 0\}\})n)n)n)n) \mid \bar{m} + (-\text{ud}_{\text{Ph}}[\bar{m}]) \mid =$   
 $\overline{\epsilon})n)n)n)n) \mid, [x] :: \text{extractSeries}(t^1) :: T)$   
 venter—

[kvanti]  $\xrightarrow{\text{prio}}$

### Preassociative

[kvanti], [base], [bracket \* end bracket], [big bracket \* end bracket], [ \$ \* \$ ],  
 [flush left [\*], [x], [y], [z], [[\*  $\bowtie$  \*], [[\*  $\rightarrow$  \*], [pyk], [tex], [name], [prio], [\*, [T],  
 [if(\*, \*, \*), [[\*  $\Rightarrow$  \*], [val], [claim], [ $\perp$ ], [f(\*), [(\*)], [F], [0], [1], [2], [3], [4], [5], [6],  
 [7], [8], [9], [0], [1], [2], [3], [4], [5], [6], [7], [8], [9], [a], [b], [c], [d], [e], [f], [g], [h], [i], [j],  
 [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v], [w], [(\*)<sup>M</sup>], [If(\*, \*, \*)],  
 [array{\*} \* end array], [l], [c], [r], [empty], [( \* | \* := \*)], [ $\mathcal{M}(*), \tilde{\mathcal{U}}(*), \mathcal{U}(*),$   
 $\mathcal{U}^M(*), [\mathbf{apply}(*, *), [\mathbf{apply}_1(*, *), [\text{identifier}(*), [\text{identifier}_1(*, *), [\text{array-}$   
 $\text{plus}(*, *), [\text{array-remove}(*, *, *), [\text{array-put}(*, *, *, *), [\text{array-add}(*, *, *, *, *)],$   
 [bit(\*, \*)], [bit<sub>1</sub>(\*, \*)], [rack], ["vector"], ["bibliography"], ["dictionary"],  
 ["body"], ["codex"], ["expansion"], ["code"], ["cache"], ["diagnose"], ["pyk"],  
 ["tex"], ["texname"], ["value"], ["message"], ["macro"], ["definition"],  
 ["unpack"], ["claim"], ["priority"], ["lambda"], ["apply"], ["true"], ["if"],  
 ["quote"], ["proclaim"], ["define"], ["introduce"], ["hide"], ["pre"], ["post"],  
 $\mathcal{E}(*, *, *), [\mathcal{E}_2(*, *, *, *), [\mathcal{E}_3(*, *, *, *), [\mathcal{E}_4(*, *, *, *), [\mathbf{lookup}(*, *, *),$   
 $\mathbf{abstract}(*, *, *, *), [[*], [\mathcal{M}(*, *, *), [\mathcal{M}_2(*, *, *, *), [\mathcal{M}^*(*, *, *), [\text{macro},$   
 $[\text{s}_0], [\mathbf{zip}(*, *), [\mathbf{assoc}_1(*, *, *), [(*)^{\text{P}}], [\text{self}], [[*  $\doteq$  *], [[*  $\doteq$  *], [[*  $\doteq$  *],$   
 $[[* \stackrel{\text{pyk}}{=} *], [[* \stackrel{\text{tex}}{=} *], [[* \stackrel{\text{name}}{=} *], [\mathbf{Priority\ table}[*], [\tilde{\mathcal{M}}_1], [\tilde{\mathcal{M}}_2(*), [\tilde{\mathcal{M}}_3(*),$   
 $[\tilde{\mathcal{M}}_4(*, *, *, *), [\mathcal{M}(*, *, *), [\mathcal{Q}(*, *, *), [\tilde{\mathcal{Q}}_2(*, *, *), [\tilde{\mathcal{Q}}_3(*, *, *, *), [\tilde{\mathcal{Q}}^*(*, *, *),$   
 $[(*)], [(*)], [\text{display}(*), [\text{statement}(*), [(*)], [(*)], [\mathbf{aspect}(*, *),$   
 $\mathbf{aspect}(*, *, *), [(\langle * \rangle), [\mathbf{tuple}_1(*), [\mathbf{tuple}_2(*), [\text{let}_2(*, *), [\text{let}_1(*, *),$   
 $[[* \stackrel{\text{claim}}{=} *], [\text{checker}], [\mathbf{check}(*, *), [\mathbf{check}_2(*, *, *), [\mathbf{check}_3(*, *, *),$   
 $\mathbf{check}^*(*, *), [\mathbf{check}_2^*(*, *, *), [(*)], [(*)], [(*)], [msg], [[*  $\stackrel{\text{msg}}{=} *], [ < \text{stmt} > ],$   
 $[\text{stmt}], [[* \stackrel{\text{stmt}}{=} *], [\text{HeadNil}'], [\text{HeadPair}'], [\text{Transitivity}'], [ $\perp$ ], [\text{Contra}'], [T<sub>E</sub>],  
 $[L_1], [\underline{*}], [A], [B], [C], [D], [E], [F], [G], [H], [I], [J], [K], [L], [M], [N], [O], [P], [Q],$   
 $[R], [S], [T], [U], [V], [W], [X], [Y], [Z], [( * | * := *)], [( * * | * := *)], [\emptyset], [\text{Remainder}],$   
 $[(*)^{\vee}], [\text{intro}(*, *, *, *), [\text{intro}(*, *, *), [\text{error}(*, *), [\text{error}_2(*, *), [\text{proof}(*, *, *),$   
 $[\text{proof}_2(*, *), [S(*, *), [S<sup>I</sup>(*, *), [S<sup>D</sup>(*, *), [S<sup>D</sup>(*, *, *), [S<sup>E</sup>(*, *), [S<sup>E</sup>(*, *, *)],$$$

$[S^+(*, *)]$ ,  $[S_1^+(*, *, *)]$ ,  $[S^-(*, *)]$ ,  $[S_1^-(*, *, *)]$ ,  $[S^*(*, *)]$ ,  $[S_1^*(*, *, *)]$ ,  
 $[S_2^*(*, *, *, *)]$ ,  $[S^{\textcircled{a}}(*, *)]$ ,  $[S_1^{\textcircled{a}}(*, *, *, *)]$ ,  $[S^{\dagger}(*, *)]$ ,  $[S_1^{\dagger}(*, *, *, *)]$ ,  $[S^{\#}(*, *)]$ ,  
 $[S_1^{\#}(*, *, *, *)]$ ,  $[S^{\text{i.e.}}(*, *)]$ ,  $[S_1^{\text{i.e.}}(*, *, *, *, *)]$ ,  $[S_2^{\text{i.e.}}(*, *, *, *, *)]$ ,  $[S^{\vee}(*, *)]$ ,  
 $[S_1^{\vee}(*, *, *, *)]$ ,  $[S^{\text{i}}(*, *)]$ ,  $[S_1^{\text{i}}(*, *, *, *)]$ ,  $[S_2^{\text{i}}(*, *, *, *, *)]$ ,  $[T(*)]$ ,  $[\text{claims}(*, *, *)]$ ,  
 $[\text{claims}_2(*, *, *)]$ ,  $[<\text{proof}>]$ ,  $[\text{proof}]$ ,  $[[\text{Lemma } *: *]]$ ,  $[[\text{Proof of } *: *]]$ ,  
 $[[* \text{ lemma } *: *]]$ ,  $[[* \text{ antilemma } *: *]]$ ,  $[[* \text{ rule } *: *]]$ ,  $[[* \text{ antirule } *: *]]$ ,  
 $[\text{verifier}]$ ,  $[\mathcal{V}_1(*)]$ ,  $[\mathcal{V}_2(*, *)]$ ,  $[\mathcal{V}_3(*, *, *, *)]$ ,  $[\mathcal{V}_4(*, *)]$ ,  $[\mathcal{V}_5(*, *, *, *, *)]$ ,  $[\mathcal{V}_6(*, *, *, *, *)]$ ,  
 $[\mathcal{V}_7(*, *, *, *, *)]$ ,  $[\text{Cut}(*, *)]$ ,  $[\text{Head}_{\oplus}(*)]$ ,  $[\text{Tail}_{\oplus}(*)]$ ,  $[\text{rule}_1(*, *)]$ ,  $[\text{rule}(*, *)]$ ,  
 $[\text{Rule tactic}]$ ,  $[\text{Plus}(*, *)]$ ,  $[[\text{Theory } *]]$ ,  $[\text{theory}_2(*, *)]$ ,  $[\text{theory}_3(*, *)]$ ,  
 $[\text{theory}_4(*, *, *, *)]$ ,  $[\text{HeadNil}''']$ ,  $[\text{HeadPair}''']$ ,  $[\text{Transitivity}''']$ ,  $[\text{Contra}''']$ ,  $[\text{HeadNil}]$ ,  
 $[\text{HeadPair}]$ ,  $[\text{Transitivity}]$ ,  $[\text{Contra}]$ ,  $[\text{T}_{\text{E}}]$ ,  $[\text{ragged right}]$ ,  
 $[\text{ragged right expansion}]$ ,  $[\text{parm}(*, *, *)]$ ,  $[\text{parm}^*(*, *, *)]$ ,  $[\text{inst}(*, *)]$ ,  
 $[\text{inst}^*(*, *)]$ ,  $[\text{occur}(*, *, *)]$ ,  $[\text{occur}^*(*, *, *)]$ ,  $[\text{unify}(* = *, *)]$ ,  $[\text{unify}^*(* = *, *)]$ ,  
 $[\text{unify}_2(* = *, *)]$ ,  $[\text{L}_a]$ ,  $[\text{L}_b]$ ,  $[\text{L}_c]$ ,  $[\text{L}_d]$ ,  $[\text{L}_e]$ ,  $[\text{L}_f]$ ,  $[\text{L}_g]$ ,  $[\text{L}_h]$ ,  $[\text{L}_i]$ ,  $[\text{L}_j]$ ,  $[\text{L}_k]$ ,  $[\text{L}_l]$ ,  $[\text{L}_m]$ ,  
 $[\text{L}_n]$ ,  $[\text{L}_o]$ ,  $[\text{L}_p]$ ,  $[\text{L}_q]$ ,  $[\text{L}_r]$ ,  $[\text{L}_s]$ ,  $[\text{L}_t]$ ,  $[\text{L}_u]$ ,  $[\text{L}_v]$ ,  $[\text{L}_w]$ ,  $[\text{L}_x]$ ,  $[\text{L}_y]$ ,  $[\text{L}_z]$ ,  $[\text{L}_A]$ ,  $[\text{L}_B]$ ,  $[\text{L}_C]$ ,  
 $[\text{L}_D]$ ,  $[\text{L}_E]$ ,  $[\text{L}_F]$ ,  $[\text{L}_G]$ ,  $[\text{L}_H]$ ,  $[\text{L}_I]$ ,  $[\text{L}_J]$ ,  $[\text{L}_K]$ ,  $[\text{L}_L]$ ,  $[\text{L}_M]$ ,  $[\text{L}_N]$ ,  $[\text{L}_O]$ ,  $[\text{L}_P]$ ,  $[\text{L}_Q]$ ,  $[\text{L}_R]$ ,  
 $[\text{L}_S]$ ,  $[\text{L}_T]$ ,  $[\text{L}_U]$ ,  $[\text{L}_V]$ ,  $[\text{L}_W]$ ,  $[\text{L}_X]$ ,  $[\text{L}_Y]$ ,  $[\text{L}_Z]$ ,  $[\text{L}_?]$ ,  $[\text{Reflexivity}]$ ,  $[\text{Reflexivity}_1]$ ,  
 $[\text{Commutativity}]$ ,  $[\text{Commutativity}_1]$ ,  $[<\text{tactic}>]$ ,  $[\text{tactic}]$ ,  $[[* \text{ tactic}^* *]]$ ,  $[\mathcal{P}(*, *, *)]$ ,  
 $[\mathcal{P}^*(*, *, *)]$ ,  $[\text{p}_0]$ ,  $[\text{conclude}_1(*, *)]$ ,  $[\text{conclude}_2(*, *, *)]$ ,  $[\text{conclude}_3(*, *, *, *)]$ ,  
 $[\text{conclude}_4(*, *)]$ ,  $[\text{check}]$ ,  $[[* \overset{\circ}{=} *]]$ ,  $[\text{RootVisible}(*)]$ ,  $[\text{A}]$ ,  $[\text{R}]$ ,  $[\text{C}]$ ,  $[\text{T}]$ ,  $[\text{L}]$ ,  $[\{*\}]$ ,  $[\bar{*}]$ ,  
 $[a]$ ,  $[b]$ ,  $[c]$ ,  $[d]$ ,  $[e]$ ,  $[f]$ ,  $[g]$ ,  $[h]$ ,  $[i]$ ,  $[j]$ ,  $[k]$ ,  $[l]$ ,  $[m]$ ,  $[n]$ ,  $[o]$ ,  $[p]$ ,  $[q]$ ,  $[r]$ ,  $[s]$ ,  $[t]$ ,  $[u]$ ,  $[v]$ ,  
 $[w]$ ,  $[x]$ ,  $[y]$ ,  $[z]$ ,  $[(*) \equiv * \mid * := *)]$ ,  $[(*) \overset{0}{\equiv} * \mid * := *)]$ ,  $[(*) \overset{1}{\equiv} * \mid * := *)]$ ,  $[(*) \overset{*}{\equiv} * \mid * := *)]$ ,  
 $[\text{Ded}(*, *)]$ ,  $[\text{Ded}_0(*, *)]$ ,  $[\text{Ded}_1(*, *, *)]$ ,  $[\text{Ded}_2(*, *, *)]$ ,  $[\text{Ded}_3(*, *, *, *)]$ ,  
 $[\text{Ded}_4(*, *, *, *)]$ ,  $[\text{Ded}_4^*(*, *, *, *)]$ ,  $[\text{Ded}_5(*, *, *)]$ ,  $[\text{Ded}_6(*, *, *, *)]$ ,  
 $[\text{Ded}_6^*(*, *, *, *)]$ ,  $[\text{Ded}_7(*, *)]$ ,  $[\text{Ded}_8(*, *)]$ ,  $[\text{Ded}_8^*(*, *)]$ ,  $[\text{S}]$ ,  $[\text{Neg}]$ ,  $[\text{MP}]$ ,  $[\text{Gen}]$ ,  
 $[\text{Ded}]$ ,  $[\text{S1}]$ ,  $[\text{S2}]$ ,  $[\text{S3}]$ ,  $[\text{S4}]$ ,  $[\text{S5}]$ ,  $[\text{S6}]$ ,  $[\text{S7}]$ ,  $[\text{S8}]$ ,  $[\text{S9}]$ ,  $[\text{Repetition}]$ ,  $[\text{A1}']$ ,  $[\text{A2}']$ ,  $[\text{A4}']$ ,  
 $[\text{A5}']$ ,  $[\text{Prop 3.2a}]$ ,  $[\text{Prop 3.2b}]$ ,  $[\text{Prop 3.2c}]$ ,  $[\text{Prop 3.2d}]$ ,  $[\text{Prop 3.2e}_1]$ ,  $[\text{Prop 3.2e}_2]$ ,  
 $[\text{Prop 3.2e}]$ ,  $[\text{Prop 3.2f}_1]$ ,  $[\text{Prop 3.2f}_2]$ ,  $[\text{Prop 3.2f}]$ ,  $[\text{Prop 3.2g}_1]$ ,  $[\text{Prop 3.2g}_2]$ ,  
 $[\text{Prop 3.2g}]$ ,  $[\text{Prop 3.2h}_1]$ ,  $[\text{Prop 3.2h}_2]$ ,  $[\text{Prop 3.2h}]$ ,  $[\text{Block}_1(*, *, *)]$ ,  $[\text{Block}_2(*, *)]$ ,  
 $[\text{UniqueMember}]$ ,  $[\text{UniqueMember}(\text{Type})]$ ,  $[\text{SameSeries}]$ ,  $[\text{A4}]$ ,  $[\text{SameMember}]$ ,  
 $[\text{Qclosed}(\text{Addition})]$ ,  $[\text{Qclosed}(\text{Multiplication})]$ ,  $[\text{FromCartProd}(1)]$ ,  
 $[\text{Irule fromCartProd}(2)]$ ,  $[\text{constantRationalSeries}(*)]$ ,  $[\text{cartProd}(*)]$ ,  $[\text{Power}(*)]$ ,  
 $[\text{binaryUnion}(*, *)]$ ,  $[\text{SetOfRationalSeries}]$ ,  $[\text{IsSubset}(*, *)]$ ,  $[(p*, *)]$ ,  $[(s*)]$ ,  
 $[(\dots)]$ ,  $[\text{Objekt-var}]$ ,  $[\text{Ex-var}]$ ,  $[\text{Ph-var}]$ ,  $[\text{Værdi}]$ ,  $[\text{Variabel}]$ ,  $[\text{Op}(*)]$ ,  $[\text{Op}(*, *)]$ ,  
 $[* \overset{=}{=} *]$ ,  $[\text{ContainsEmpty}(*)]$ ,  $[\text{Nat}(*)]$ ,  $[\text{Dedu}(*, *)]$ ,  $[\text{Dedu}_0(*, *)]$ ,  
 $[\text{Dedu}_s(*, *, *)]$ ,  $[\text{Dedu}_1(*, *, *)]$ ,  $[\text{Dedu}_2(*, *, *)]$ ,  $[\text{Dedu}_3(*, *, *, *)]$ ,  
 $[\text{Dedu}_4(*, *, *, *)]$ ,  $[\text{Dedu}_4^*(*, *, *, *)]$ ,  $[\text{Dedu}_5(*, *, *)]$ ,  $[\text{Dedu}_6(*, *, *, *)]$ ,  
 $[\text{Dedu}_6^*(*, *, *, *)]$ ,  $[\text{Dedu}_7(*, *)]$ ,  $[\text{Dedu}_8(*, *)]$ ,  $[\text{Dedu}_8^*(*, *)]$ ,  $[\text{EX}_1]$ ,  $[\text{EX}_2]$ ,  $[\text{EX}_3]$ ,  
 $[\text{EX}_{10}]$ ,  $[\text{EX}_{20}]$ ,  $[*_{\text{EX}}]$ ,  $[*^{\text{EX}}]$ ,  $[(*) \equiv * \mid * := *)_{\text{EX}}]$ ,  $[(*) \overset{0}{\equiv} * \mid * := *)_{\text{EX}}]$ ,  
 $[(*) \overset{1}{\equiv} * \mid * := *)_{\text{EX}}]$ ,  $[(*) \overset{*}{\equiv} * \mid * := *)_{\text{EX}}]$ ,  $[\text{ph}_1]$ ,  $[\text{ph}_2]$ ,  $[\text{ph}_3]$ ,  $[*_{\text{Ph}}]$ ,  $[*^{\text{Ph}}]$ ,  
 $[(*) \equiv * \mid * := *)_{\text{Ph}}]$ ,  $[(*) \overset{0}{\equiv} * \mid * := *)_{\text{Ph}}]$ ,  $[(*) \overset{1}{\equiv} * \mid * := *)_{\text{Ph}}]$ ,  
 $[(*) \overset{*}{\equiv} * \mid * := *)_{\text{Ph}}]$ ,  $[(*) \equiv * \mid * := *)_{\text{Me}}]$ ,  $[(*) \overset{1}{\equiv} * \mid * := *)_{\text{Me}}]$ ,  
 $[(*) \overset{*}{\equiv} * \mid * := *)_{\text{Me}}]$ ,  $[\text{bs}]$ ,  $[\text{OBS}]$ ,  $[\mathcal{BS}]$ ,  $[\emptyset]$ ,  $[\text{SystemQ}]$ ,  $[\text{MP}]$ ,  $[\text{Gen}]$ ,  $[\text{Repetition}]$ ,  
 $[\text{Neg}]$ ,  $[\text{Ded}]$ ,  $[\text{ExistIntro}]$ ,  $[\text{Extensionality}]$ ,  $[\emptyset\text{def}]$ ,  $[\text{PairDef}]$ ,  $[\text{UnionDef}]$ ,  
 $[\text{PowerDef}]$ ,  $[\text{SeparationDef}]$ ,  $[\text{AddDoubleNeg}]$ ,  $[\text{RemoveDoubleNeg}]$ ,

[AndCommutativity], [AutoImPLY], [Contrapositive], [FirstConjunct],  
 [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity],  
 [IffFirst], [IffSecond], [ImPLYTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4],  
 [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1],  
 [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],  
 [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power],  
 [SubsetInPower], [HelperPowerIsSub], [PowerIsSub],  
 [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality],  
 [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality],  
 [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry],  
 [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive],  
 [ERisSymmetric], [ERisTransitive], [ $\emptyset$ isSubset], [HelperMemberNot $\emptyset$ ],  
 [MemberNot $\emptyset$ ], [HelperUnique $\emptyset$ ], [Unique $\emptyset$ ], [= Reflexivity], [= Symmetry],  
 [Helper == Transitivity], [= Transitivity], [HelperTransferNotEq],  
 [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset],  
 [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset],  
 [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection],  
 [AutoMember], [HelperEqSysNot $\emptyset$ ], [EqSysNot $\emptyset$ ], [HelperEqSubset],  
 [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary],  
 [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset],  
 [EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImPLY], [BSsubset],  
 [Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)],  
 [(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [( $\epsilon$ )],  
 [( $\epsilon$ )<sub>1</sub>], [( $\epsilon$ )<sub>2</sub>], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)],  
 [(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)],  
 [(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X<sub>1</sub>], [X<sub>2</sub>],  
 [Y<sub>1</sub>], [Y<sub>2</sub>], [V<sub>1</sub>], [V<sub>2</sub>], [V<sub>3</sub>], [V<sub>4</sub>], [V<sub>2n</sub>], [M<sub>1</sub>], [M<sub>2</sub>], [N<sub>1</sub>], [N<sub>2</sub>], [N<sub>3</sub>], [ $\epsilon$ ], [ $\epsilon$ ]<sub>1</sub>], [ $\epsilon$ ]<sub>2</sub>,  
 [FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)],  
 [(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)],  
 [(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)],  
 [(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)],  
 [(S1ob)], [(S2ob)], [ph<sub>4</sub>], [ph<sub>5</sub>], [ph<sub>6</sub>], [NAT], [RATIONALSERIES], [SERIES],  
 [SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1],  
 [(-1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(- - 01)], [02], [01/02],  
 [PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)],  
 [Times1(R)], [lessAddition(R)], [PlusCommutativity(R)],  
 [LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)],  
 [Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom],  
 [EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom],  
 [QisClosed(Reciprocal)(ImPLY)], [QisClosed(Reciprocal)],  
 [QisClosed(Negative)(ImPLY)], [QisClosed(Negative)], [leqReflexivity],  
 [leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality],  
 [leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity],  
 [plusCommutativity], [Negative], [plus0], [timesAssociativity],  
 [timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1],  
 [lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)],

[(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy], [PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)], [ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0], [NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)], [NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)], [ExpPositive(R)], [BSzero], [BSpositive], [UStelelescope(Zero)], [UStelelescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound], [FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)], [FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound], [XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [Nat Type], [RationalType], [SeriesType], [Max], [Numerical], [NumericalF], [MemberOfSeries(Implied)], [JoinConjuncts(2conditions)], [prop lemma imply negation], [TND], [FromNegatedImplied], [ToNegatedImplied], [FromNegated(2 \* Implied)], [FromNegatedAnd], [FromNegatedOr], [ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 \* 2Disjuncts], [NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2], [LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)];

### Preassociative

[\*\_{\*}], [\* /indexintro(\*, \*, \*, \*)], [\* /intro(\*, \*, \*)], [\* /bothintro(\*, \*, \*, \*, \*)], [\* /nameintro(\*, \*, \*, \*)], [\* '], [\* [\* ]], [\* [\* → \*]], [\* [\* ⇒ \*]], [\* 0], [\* 1], [0b], [\* -color(\*)], [\* -color \* (\*)], [\* <sup>H</sup>], [\* <sup>T</sup>], [\* <sup>U</sup>], [\* <sup>h</sup>], [\* <sup>t</sup>], [\* <sup>s</sup>], [\* <sup>c</sup>], [\* <sup>d</sup>], [\* <sup>a</sup>], [\* <sup>C</sup>], [\* <sup>M</sup>], [\* <sup>B</sup>], [\* <sup>t</sup>], [\* <sup>i</sup>], [\* <sup>d</sup>], [\* <sup>R</sup>], [\* <sup>0</sup>], [\* <sup>1</sup>], [\* <sup>2</sup>], [\* <sup>3</sup>], [\* <sup>4</sup>], [\* <sup>5</sup>], [\* <sup>6</sup>], [\* <sup>7</sup>], [\* <sup>8</sup>], [\* <sup>9</sup>], [\* <sup>E</sup>], [\* <sup>V</sup>], [\* <sup>C</sup>], [\* <sup>C\*</sup>], [\* <sub>hide</sub>];

### Preassociative

[“ \* ”], [], [(\*)<sup>t</sup>], [string(\*) + \*], [string(\*) ++ \*], [\*, [\*], [! \*], [! \*], [# \*], [\$ \*], [% \*], [& \*], [' \*], [(\*)], [()\*], [\*\*], [+ \*], [, \*], [- \*], [.\*], [/ \*], [0 \*], [1 \*], [2 \*], [3 \*], [4 \*], [5 \*], [6 \*], [7 \*], [8 \*], [9 \*], [: \*], [; \*], [< \*], [= \*], [> \*], [?\*], [@ \*], [A \*], [B \*], [C \*], [D \*], [E \*], [F \*], [G \*], [H \*], [I \*], [J \*], [K \*], [L \*], [M \*], [N \*], [O \*], [P \*], [Q \*], [R \*], [S \*], [T \*], [U \*], [V \*], [W \*], [X \*], [Y \*], [Z \*], [[ \*], [\ \*], [ \*], [ ^ \*], [ \_ \*], [ \*], [ a \*], [ b \*], [ c \*], [ d \*], [ e \*], [ f \*], [ g \*], [ h \*], [ i \*], [ j \*], [ k \*], [ l \*], [ m \*], [ n \*], [ o \*], [ p \*], [ q \*], [ r \*], [ s \*], [ t \*], [ u \*], [ v \*], [ w \*], [ x \*], [ y \*], [ z \*], [{ \*}, [ \*], { \*}], [ ~ \*], [Preassociative \*; \*], [Postassociative \*; \*], [[ \*], \*], [priority \* end], [newline \*], [macro newline \*], [MacroIndent(\*)];

### Preassociative

[\* ' \*], [\* ' \*];

### Preassociative

[\*(exp)\*];

### Preassociative

[\*'], [R(\*)], [— — R(\*)], [rec\*];

### Preassociative

[\*/ \*], [\* ∩ \*], [\* [\*]];

### Preassociative

[∪ \*], [\* ∪ \*], [P(\*)];

### Preassociative

[{ \*}], [StateExpand(\*, \*, \*)], [extractSeries(\*)], [SetOfSeries(\*)], [— — Macro(\*)], [ExpandList(\*, \*, \*)], [\* \* Macro(\*)], [+ + Macro(\*)], [<< Macro(\*)],

[[Macro(\*), [01//Macro(\*)], [UB(\*, \*)], [LUB(\*, \*)], [BS(\*, \*)],  
 [UStescope(\*, \*)], [(\*)], [f \* |], [r \* |], [Limit(\*, \*)], [Union(\*)],  
 [IsOrderedPair(\*, \*, \*)], [IsRelation(\*, \*, \*)], [isFunction(\*, \*, \*)], [IsSeries(\*, \*)],  
 [IsNatural(\*, \*)], [OrderedPair(\*, \*)], [TypeNat(\*)], [TypeNat0(\*)],  
 [TypeRational(\*)], [TypeRational0(\*)], [TypeSeries(\*, \*)], [Typeseries0(\*, \*)];

**Preassociative**

[{\* , \*}], [(\* , \*)], [(-u\*)], [-f\*], [(- - \*)], [1f/\*], [01//temp\*];

**Preassociative**

[\*( , \*)], [RefRel(\*, \*)], [SymRel(\*, \*)], [TransRel(\*, \*)], [EqRel(\*, \*)], [{"\* ∈ \*"}],  
 [Partition(\*, \*)];

**Preassociative**

[\* · \*], [\* · 0 \*], [{"\*\* \*\*}], [\* \*<sub>f</sub> \*], [\* \* \* \*];

**Preassociative**

[\* + \*], [\* + 0 \*], [\* + 1 \*], [\* - \*], [\* - 0 \*], [\* - 1 \*], [{"\* + \*}], [{"\* - \*}], [\* +<sub>f</sub> \*],  
 [\* -<sub>f</sub> \*], [\* + + \*], [R(\*) - R(\*)];

**Preassociative**

[\* ∈ \*];

**Preassociative**

[| \* |], [if(\*, \*, \*)], [Max(\*, \*)], [Max(\*, \*)];

**Preassociative**

[\* = \*], [\* ≠ \*], [\* <= \*], [\* < \*], [\* <<sub>f</sub> \*], [\* ≤<sub>f</sub> \*], [SF(\*, \*)], [\* == \*],  
 [\*!! == \*], [\* << \*], [\* <<== \*];

**Preassociative**

[\* ∪ {\*}], [\* ∪ \*], [\* \ {\*}];

**Postassociative**

[\* ∴ \*], [\* ∴̇ \*], [\* ∴ ∴ \*], [\* +2\* \*], [\* ∴ ∴ \*], [\* +2\* \*];

**Postassociative**

[\* , \*];

**Preassociative**

[\* <sup>B</sup> ≈ \*], [\* <sup>D</sup> ≈ \*], [\* <sup>C</sup> ≈ \*], [\* <sup>P</sup> ≈ \*], [\* ≈ \*], [\* = \*], [\* <sup>+</sup> → \*], [\* <sup>t</sup> ≐ \*], [\* <sup>t\*</sup> ≐ \*], [\* <sup>r</sup> ≐ \*],  
 [\* ∈<sub>t</sub> \*], [\* ⊆<sub>T</sub> \*], [\* <sup>T</sup> ≐ \*], [\* <sup>s</sup> ≐ \*], [\* free in \*], [\* free in\* \*], [\* free for \* in \*],  
 [\* free for\* \* in \*], [\* ∈<sub>c</sub> \*], [\* < \*], [\* <' \*], [\* ≤' \*], [\* = \*], [\* ≠ \*], [\*<sup>var</sup>],  
 [\* #<sup>0</sup> \*], [\* #<sup>1</sup> \*], [\* #\* \*], [\* == \*], [\* ⊆ \*];

**Preassociative**

[¬\*], [¬(\* )n], [\* ∉ \*], [\* ≠ \*];

**Preassociative**

[\* ∧ \*], [\* <sup>λ</sup> \*], [\* <sup>λ</sup> \*], [\* ∧<sub>c</sub> \*], [\* <sup>λ</sup> \*];

**Preassociative**

[\* ∨ \*], [\* || \*], [\* <sup>∨</sup> \*];

**Postassociative**

[\* <sup>∨</sup> \*];

**Preassociative**

[∃\* : \*], [∀\* : \*], [∀<sub>obj</sub>\* : \*], [∃\* : \*];

**Postassociative**

[\* <sup>⇒</sup> \*], [\* ⇒ \*], [\* ⇔ \*], [\* ⇔ \*];



**Preassociative**

[{ph ∈ \* | \*}];

**Postassociative**

[\* : \*], [\* spy \*], [\*!\*];

**Preassociative**

[\*  $\left\{ \begin{array}{c} * \\ * \end{array} \right.$ ];

**Preassociative**

[λ \* . \*], [Λ \* . \*], [Λ \*], [if \* then \* else \*], [let \* = \* in \*], [let \* ÷ \* in \*];

**Preassociative**

[\*#\*];

**Preassociative**

[\*<sup>1</sup>], [\*<sup>▷</sup>], [\*<sup>∇</sup>], [\*<sup>+</sup>], [\*<sup>-</sup>], [\*<sup>\*</sup>];

**Preassociative**

[\* @ \*], [\* ▷ \*], [\* ▹ \*], [\* ≫ \*], [\* ⊇ \*];

**Postassociative**

[\* ⊢ \*], [\* ⊣ \*], [\* i.e. \*];

**Preassociative**

[∀\* : \*], [Π\* : \*];

**Postassociative**

[\* ⊕ \*];

**Postassociative**

[\* ; \*];

**Preassociative**

[\* proves \*];

**Preassociative**

[\* **proof of** \* : \*], [Line \* : \* ≫ \*; \*], [Last line \* ≫ \* □],  
[Line \* : Premise ≫ \*; \*], [Line \* : Side-condition ≫ \*; \*], [Arbitrary ≫ \*; \*],  
[Local ≫ \* = \*; \*], [Begin \*; \* : End; \*], [Last block line \* ≫ \*; \*],  
[Arbitrary ≫ \*; \*];

**Postassociative**

[\* | \*];

**Postassociative**

[\* , \*], [\* [ \* ]\*];

**Preassociative**

[\*&\*];

**Preassociative**

[\* \\ \*], [\* linebreak[4] \*], [\* \\ \*];]

## A Pyk definitioner

[UniqueMember  $\xrightarrow{\text{pyk}}$  “lemma uniqueMember”]

[UniqueMember(Type)  $\xrightarrow{\text{pyk}}$  “lemma uniqueMember(Type)”]

[SameSeries  $\xrightarrow{\text{pyk}}$  “lemma sameSeries”]

[A4  $\xrightarrow{\text{pyk}}$  “lemma a4”]  
 [SameMember  $\xrightarrow{\text{pyk}}$  “lemma sameMember”]  
 [Qclosed(Addition)  $\xrightarrow{\text{pyk}}$  “1rule Qclosed(Addition)”]  
 [Qclosed(Multiplication)  $\xrightarrow{\text{pyk}}$  “1rule Qclosed(Multiplication)”]  
 [FromCartProd(1)  $\xrightarrow{\text{pyk}}$  “1rule fromCartProd(1)”]  
 [1rule fromCartProd(2)  $\xrightarrow{\text{pyk}}$  “1rule fromCartProd(2)”]  
 [constantRationalSeries(\*)  $\xrightarrow{\text{pyk}}$  “constantRationalSeries( " )”]  
 [cartProd(\*)  $\xrightarrow{\text{pyk}}$  “cartProd( " , " )”]  
 [Power(\*)  $\xrightarrow{\text{pyk}}$  “P( " )”]  
 [binaryUnion(\*, \*)  $\xrightarrow{\text{pyk}}$  “binaryUnion( " , " )”]  
 [SetOfRationalSeries  $\xrightarrow{\text{pyk}}$  “setOfRationalSeries”]  
 [IsSubset(\*, \*)  $\xrightarrow{\text{pyk}}$  “isSubset( " , " )”]  
 [(p\*, \*)  $\xrightarrow{\text{pyk}}$  “(p " , " )”]  
 [(s\*)  $\xrightarrow{\text{pyk}}$  “(s " )”]  
 [(...)  $\xrightarrow{\text{pyk}}$  “cdots”]  
 [Objekt-var  $\xrightarrow{\text{pyk}}$  “object-var”]  
 [Ex-var  $\xrightarrow{\text{pyk}}$  “ex-var”]  
 [Ph-var  $\xrightarrow{\text{pyk}}$  “ph-var”]  
 [Værdi  $\xrightarrow{\text{pyk}}$  “vaerdi”]  
 [Variabel  $\xrightarrow{\text{pyk}}$  “variabel”]  
 [Op(\*)  $\xrightarrow{\text{pyk}}$  “op " end op”]  
 [Op(\*, \*)  $\xrightarrow{\text{pyk}}$  “op2 " comma " end op2”]  
 [\* ::= \*  $\xrightarrow{\text{pyk}}$  “define-equal " comma " end equal”]  
 [ContainsEmpty(\*)  $\xrightarrow{\text{pyk}}$  “contains-empty " end empty”]  
 [Nat(\*)  $\xrightarrow{\text{pyk}}$  “Nat( " )”]  
 [Dedu(\*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction " conclude " end 1deduction”]  
 [Dedu<sub>0</sub>(\*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction zero " conclude " end 1deduction”]  
 [Dedu<sub>s</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction side " conclude " condition " end 1deduction”]  
 [Dedu<sub>1</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction one " conclude " condition " end 1deduction”]  
 [Dedu<sub>2</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction two " conclude " condition " end 1deduction”]  
 [Dedu<sub>3</sub>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction three " conclude " condition " bound " end 1deduction”]  
 [Dedu<sub>4</sub>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction four " conclude " condition " bound " end 1deduction”]  
 [Dedu<sub>4</sub>\*(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  “1deduction four star " conclude " condition " bound " end 1deduction”]

$[\text{Dedu}_5(*, *, *) \xrightarrow{\text{pyk}} \text{"1deduction five " condition " bound " end 1deduction"}]$   
 $[\text{Dedu}_6(*, *, *, *) \xrightarrow{\text{pyk}} \text{"1deduction six " conclude " exception " bound " end 1deduction"}]$   
 $[\text{Dedu}_6^*(*, *, *, *) \xrightarrow{\text{pyk}} \text{"1deduction six star " conclude " exception " bound " end 1deduction"}]$   
 $[\text{Dedu}_7(*) \xrightarrow{\text{pyk}} \text{"1deduction seven " end 1deduction"}]$   
 $[\text{Dedu}_8(*, *) \xrightarrow{\text{pyk}} \text{"1deduction eight " bound " end 1deduction"}]$   
 $[\text{Dedu}_8^*(*, *) \xrightarrow{\text{pyk}} \text{"1deduction eight star " bound " end 1deduction"}]$   
 $[\text{Ex}_1 \xrightarrow{\text{pyk}} \text{"ex1"}]$   
 $[\text{Ex}_2 \xrightarrow{\text{pyk}} \text{"ex2"}]$   
 $[\text{Ex}_3 \xrightarrow{\text{pyk}} \text{"ex3"}]$   
 $[\text{Ex}_{10} \xrightarrow{\text{pyk}} \text{"ex10"}]$   
 $[\text{Ex}_{20} \xrightarrow{\text{pyk}} \text{"ex20"}]$   
 $[\text{*Ex} \xrightarrow{\text{pyk}} \text{"existential var " end var"}]$   
 $[\text{*Ex} \xrightarrow{\text{pyk}} \text{" " is existential var"}]$   
 $[\langle * \equiv * \mid * : == * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub " is " where " is " end sub"}]$   
 $[\langle * \equiv^0 * \mid * : == * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub0 " is " where " is " end sub"}]$   
 $[\langle * \equiv^1 * \mid * : == * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub1 " is " where " is " end sub"}]$   
 $[\langle * \equiv * * \mid * : == * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub* " is " where " is " end sub"}]$   
 $[\text{ph}_1 \xrightarrow{\text{pyk}} \text{"ph1"}]$   
 $[\text{ph}_2 \xrightarrow{\text{pyk}} \text{"ph2"}]$   
 $[\text{ph}_3 \xrightarrow{\text{pyk}} \text{"ph3"}]$   
 $[\text{*Ph} \xrightarrow{\text{pyk}} \text{"placeholder-var " end var"}]$   
 $[\text{*Ph} \xrightarrow{\text{pyk}} \text{" " is placeholder-var"}]$   
 $[\langle * \equiv * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub " is " where " is " end sub"}]$   
 $[\langle * \equiv^0 * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub0 " is " where " is " end sub"}]$   
 $[\langle * \equiv^1 * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub1 " is " where " is " end sub"}]$   
 $[\langle * \equiv * * \mid * : == * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub* " is " where " is " end sub"}]$   
 $[\langle * \equiv * \mid * : == * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} \text{"meta-sub " is " where " is " end sub"}]$   
 $[\langle * \equiv^1 * \mid * : == * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} \text{"meta-sub1 " is " where " is " end sub"}]$   
 $[\langle * \equiv * * \mid * : == * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} \text{"meta-sub* " is " where " is " end sub"}]$   
 $[\text{bs} \xrightarrow{\text{pyk}} \text{"var big set"}]$   
 $[\text{OBS} \xrightarrow{\text{pyk}} \text{"object big set"}]$   
 $[\mathcal{BS} \xrightarrow{\text{pyk}} \text{"meta big set"}]$   
 $[\emptyset \xrightarrow{\text{pyk}} \text{"zermelo empty set"}]$   
 $[\text{SystemQ} \xrightarrow{\text{pyk}} \text{"system Q"}]$

[MP  $\xrightarrow{\text{pyk}}$  “1rule mp”]  
 [Gen  $\xrightarrow{\text{pyk}}$  “1rule gen”]  
 [Repetition  $\xrightarrow{\text{pyk}}$  “1rule repetition”]  
 [Neg  $\xrightarrow{\text{pyk}}$  “1rule ad absurdum”]  
 [Ded  $\xrightarrow{\text{pyk}}$  “1rule deduction”]  
 [ExistIntro  $\xrightarrow{\text{pyk}}$  “1rule exist intro”]  
 [Extensionality  $\xrightarrow{\text{pyk}}$  “axiom extensionality”]  
 [ $\emptyset$ def  $\xrightarrow{\text{pyk}}$  “axiom empty set”]  
 [PairDef  $\xrightarrow{\text{pyk}}$  “axiom pair definition”]  
 [UnionDef  $\xrightarrow{\text{pyk}}$  “axiom union definition”]  
 [PowerDef  $\xrightarrow{\text{pyk}}$  “axiom power definition”]  
 [SeparationDef  $\xrightarrow{\text{pyk}}$  “axiom separation definition”]  
 [AddDoubleNeg  $\xrightarrow{\text{pyk}}$  “prop lemma add double neg”]  
 [RemoveDoubleNeg  $\xrightarrow{\text{pyk}}$  “prop lemma remove double neg”]  
 [AndCommutativity  $\xrightarrow{\text{pyk}}$  “prop lemma and commutativity”]  
 [AutoImply  $\xrightarrow{\text{pyk}}$  “prop lemma auto imply”]  
 [Contrapositive  $\xrightarrow{\text{pyk}}$  “prop lemma contrapositive”]  
 [FirstConjunct  $\xrightarrow{\text{pyk}}$  “prop lemma first conjunct”]  
 [SecondConjunct  $\xrightarrow{\text{pyk}}$  “prop lemma second conjunct”]  
 [FromContradiction  $\xrightarrow{\text{pyk}}$  “prop lemma from contradiction”]  
 [FromDisjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma from disjuncts”]  
 [IffCommutativity  $\xrightarrow{\text{pyk}}$  “prop lemma iff commutativity”]  
 [IffFirst  $\xrightarrow{\text{pyk}}$  “prop lemma iff first”]  
 [IffSecond  $\xrightarrow{\text{pyk}}$  “prop lemma iff second”]  
 [ImplyTransitivity  $\xrightarrow{\text{pyk}}$  “prop lemma imply transitivity”]  
 [JoinConjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma join conjuncts”]  
 [MP2  $\xrightarrow{\text{pyk}}$  “prop lemma mp2”]  
 [MP3  $\xrightarrow{\text{pyk}}$  “prop lemma mp3”]  
 [MP4  $\xrightarrow{\text{pyk}}$  “prop lemma mp4”]  
 [MP5  $\xrightarrow{\text{pyk}}$  “prop lemma mp5”]  
 [MT  $\xrightarrow{\text{pyk}}$  “prop lemma mt”]  
 [NegativeMT  $\xrightarrow{\text{pyk}}$  “prop lemma negative mt”]  
 [Technicality  $\xrightarrow{\text{pyk}}$  “prop lemma technicality”]  
 [Weakening  $\xrightarrow{\text{pyk}}$  “prop lemma weakening”]  
 [WeakenOr1  $\xrightarrow{\text{pyk}}$  “prop lemma weaken or first”]

[WeakenOr2  $\xrightarrow{\text{pyk}}$  “prop lemma weaken or second”]  
 [Formula2Pair  $\xrightarrow{\text{pyk}}$  “lemma formula2pair”]  
 [Pair2Formula  $\xrightarrow{\text{pyk}}$  “lemma pair2formula”]  
 [Formula2Union  $\xrightarrow{\text{pyk}}$  “lemma formula2union”]  
 [Union2Formula  $\xrightarrow{\text{pyk}}$  “lemma union2formula”]  
 [Formula2Sep  $\xrightarrow{\text{pyk}}$  “lemma formula2separation”]  
 [Sep2Formula  $\xrightarrow{\text{pyk}}$  “lemma separation2formula”]  
 [Formula2Power  $\xrightarrow{\text{pyk}}$  “lemma formula2power”]  
 [SubsetInPower  $\xrightarrow{\text{pyk}}$  “lemma subset in power set”]  
 [HelperPowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset0”]  
 [PowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset”]  
 [(Switch)HelperPowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset0-switch”]  
 [(Switch)PowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset-switch”]  
 [ToSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition”]  
 [HelperToSetEquality(t)  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition(t)0”]  
 [ToSetEquality(t)  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition(t)”]  
 [HelperFromSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality skip quantifier”]  
 [FromSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition”]  
 [HelperReflexivity  $\xrightarrow{\text{pyk}}$  “lemma reflexivity0”]  
 [Reflexivity  $\xrightarrow{\text{pyk}}$  “lemma reflexivity”]  
 [HelperSymmetry  $\xrightarrow{\text{pyk}}$  “lemma symmetry0”]  
 [Symmetry  $\xrightarrow{\text{pyk}}$  “lemma symmetry”]  
 [HelperTransitivity  $\xrightarrow{\text{pyk}}$  “lemma transitivity0”]  
 [Transitivity  $\xrightarrow{\text{pyk}}$  “lemma transitivity”]  
 [ERisReflexive  $\xrightarrow{\text{pyk}}$  “lemma er is reflexive”]  
 [ERisSymmetric  $\xrightarrow{\text{pyk}}$  “lemma er is symmetric”]  
 [ERisTransitive  $\xrightarrow{\text{pyk}}$  “lemma er is transitive”]  
 [ØisSubset  $\xrightarrow{\text{pyk}}$  “lemma empty set is subset”]  
 [HelperMemberNotØ  $\xrightarrow{\text{pyk}}$  “lemma member not empty0”]  
 [MemberNotØ  $\xrightarrow{\text{pyk}}$  “lemma member not empty”]  
 [HelperUniqueØ  $\xrightarrow{\text{pyk}}$  “lemma unique empty set0”]  
 [UniqueØ  $\xrightarrow{\text{pyk}}$  “lemma unique empty set”]  
 [== Reflexivity  $\xrightarrow{\text{pyk}}$  “lemma ==Reflexivity”]  
 [== Symmetry  $\xrightarrow{\text{pyk}}$  “lemma ==Symmetry”]  
 [Helper== Transitivity  $\xrightarrow{\text{pyk}}$  “lemma ==Transitivity0”]

$[== \text{Transitivity} \xrightarrow{\text{pyk}} \text{“lemma } == \text{Transitivity”}]$   
 $[\text{HelperTransferNotEq} \xrightarrow{\text{pyk}} \text{“lemma transfer } \sim \text{is0”}]$   
 $[\text{TransferNotEq} \xrightarrow{\text{pyk}} \text{“lemma transfer } \sim \text{is”}]$   
 $[\text{HelperPairSubset} \xrightarrow{\text{pyk}} \text{“lemma pair subset0”}]$   
 $[\text{Helper(2)PairSubset} \xrightarrow{\text{pyk}} \text{“lemma pair subset1”}]$   
 $[\text{PairSubset} \xrightarrow{\text{pyk}} \text{“lemma pair subset”}]$   
 $[\text{SamePair} \xrightarrow{\text{pyk}} \text{“lemma same pair”}]$   
 $[\text{SameSingleton} \xrightarrow{\text{pyk}} \text{“lemma same singleton”}]$   
 $[\text{UnionSubset} \xrightarrow{\text{pyk}} \text{“lemma union subset”}]$   
 $[\text{SameUnion} \xrightarrow{\text{pyk}} \text{“lemma same union”}]$   
 $[\text{SeparationSubset} \xrightarrow{\text{pyk}} \text{“lemma separation subset”}]$   
 $[\text{SameSeparation} \xrightarrow{\text{pyk}} \text{“lemma same separation”}]$   
 $[\text{SameBinaryUnion} \xrightarrow{\text{pyk}} \text{“lemma same binary union”}]$   
 $[\text{IntersectionSubset} \xrightarrow{\text{pyk}} \text{“lemma intersection subset”}]$   
 $[\text{SameIntersection} \xrightarrow{\text{pyk}} \text{“lemma same intersection”}]$   
 $[\text{AutoMember} \xrightarrow{\text{pyk}} \text{“lemma auto member”}]$   
 $[\text{HelperEqSysNot}\emptyset \xrightarrow{\text{pyk}} \text{“lemma eq-system not empty0”}]$   
 $[\text{EqSysNot}\emptyset \xrightarrow{\text{pyk}} \text{“lemma eq-system not empty”}]$   
 $[\text{HelperEqSubset} \xrightarrow{\text{pyk}} \text{“lemma eq subset0”}]$   
 $[\text{EqSubset} \xrightarrow{\text{pyk}} \text{“lemma eq subset”}]$   
 $[\text{HelperEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma equivalence nec condition0”}]$   
 $[\text{EqNecessary} \xrightarrow{\text{pyk}} \text{“lemma equivalence nec condition”}]$   
 $[\text{HelperNoneEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma none-equivalence nec condition0”}]$   
 $[\text{Helper(2)NoneEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma none-equivalence nec condition1”}]$   
 $[\text{NoneEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma none-equivalence nec condition”}]$   
 $[\text{EqClassIsSubset} \xrightarrow{\text{pyk}} \text{“lemma equivalence class is subset”}]$   
 $[\text{EqClassesAreDisjoint} \xrightarrow{\text{pyk}} \text{“lemma equivalence classes are disjoint”}]$   
 $[\text{AllDisjoint} \xrightarrow{\text{pyk}} \text{“lemma all disjoint”}]$   
 $[\text{AllDisjointImPLY} \xrightarrow{\text{pyk}} \text{“lemma all disjoint-imply”}]$   
 $[\text{BSsubset} \xrightarrow{\text{pyk}} \text{“lemma bs subset union(bs/r)”}]$   
 $[\text{Union(BS/R)subset} \xrightarrow{\text{pyk}} \text{“lemma union(bs/r) subset bs”}]$   
 $[\text{UnionIdentity} \xrightarrow{\text{pyk}} \text{“lemma union(bs/r) is bs”}]$   
 $[\text{EqSysIsPartition} \xrightarrow{\text{pyk}} \text{“theorem eq-system is partition”}]$   
 $[(x1) \xrightarrow{\text{pyk}} \text{“var x1”}]$   
 $[(x2) \xrightarrow{\text{pyk}} \text{“var x2”}]$

$[(y1) \xrightarrow{\text{pyk}} \text{“var } y1\text{”}]$   
 $[(y2) \xrightarrow{\text{pyk}} \text{“var } y2\text{”}]$   
 $[(v1) \xrightarrow{\text{pyk}} \text{“var } v1\text{”}]$   
 $[(v2) \xrightarrow{\text{pyk}} \text{“var } v2\text{”}]$   
 $[(v3) \xrightarrow{\text{pyk}} \text{“var } v3\text{”}]$   
 $[(v4) \xrightarrow{\text{pyk}} \text{“var } v4\text{”}]$   
 $[(v2n) \xrightarrow{\text{pyk}} \text{“var } v2n\text{”}]$   
 $[(m1) \xrightarrow{\text{pyk}} \text{“var } m1\text{”}]$   
 $[(m2) \xrightarrow{\text{pyk}} \text{“var } m2\text{”}]$   
 $[(n1) \xrightarrow{\text{pyk}} \text{“var } n1\text{”}]$   
 $[(n2) \xrightarrow{\text{pyk}} \text{“var } n2\text{”}]$   
 $[(n3) \xrightarrow{\text{pyk}} \text{“var } n3\text{”}]$   
 $[(\epsilon) \xrightarrow{\text{pyk}} \text{“var } \epsilon\text{”}]$   
 $[(\epsilon)_1 \xrightarrow{\text{pyk}} \text{“var } \epsilon_1\text{”}]$   
 $[(\epsilon_2) \xrightarrow{\text{pyk}} \text{“var } \epsilon_2\text{”}]$   
 $[(fep) \xrightarrow{\text{pyk}} \text{“var } fep\text{”}]$   
 $[(fx) \xrightarrow{\text{pyk}} \text{“var } fx\text{”}]$   
 $[(fy) \xrightarrow{\text{pyk}} \text{“var } fy\text{”}]$   
 $[(fz) \xrightarrow{\text{pyk}} \text{“var } fz\text{”}]$   
 $[(fu) \xrightarrow{\text{pyk}} \text{“var } fu\text{”}]$   
 $[(fv) \xrightarrow{\text{pyk}} \text{“var } fv\text{”}]$   
 $[(fw) \xrightarrow{\text{pyk}} \text{“var } fw\text{”}]$   
 $[(rx) \xrightarrow{\text{pyk}} \text{“var } rx\text{”}]$   
 $[(ry) \xrightarrow{\text{pyk}} \text{“var } ry\text{”}]$   
 $[(rz) \xrightarrow{\text{pyk}} \text{“var } rz\text{”}]$   
 $[(ru) \xrightarrow{\text{pyk}} \text{“var } ru\text{”}]$   
 $[(sx) \xrightarrow{\text{pyk}} \text{“var } sx\text{”}]$   
 $[(sx1) \xrightarrow{\text{pyk}} \text{“var } sx1\text{”}]$   
 $[(sy) \xrightarrow{\text{pyk}} \text{“var } sy\text{”}]$   
 $[(sy1) \xrightarrow{\text{pyk}} \text{“var } sy1\text{”}]$   
 $[(sz) \xrightarrow{\text{pyk}} \text{“var } sz\text{”}]$   
 $[(sz1) \xrightarrow{\text{pyk}} \text{“var } sz1\text{”}]$   
 $[(su) \xrightarrow{\text{pyk}} \text{“var } su\text{”}]$   
 $[(su1) \xrightarrow{\text{pyk}} \text{“var } su1\text{”}]$   
 $[(fxs) \xrightarrow{\text{pyk}} \text{“var } fxs\text{”}]$

$[(\text{fys}) \xrightarrow{\text{pyk}} \text{“var fys”}]$   
 $[(\text{crs1}) \xrightarrow{\text{pyk}} \text{“var crs1”}]$   
 $[(\text{f1}) \xrightarrow{\text{pyk}} \text{“var f1”}]$   
 $[(\text{f2}) \xrightarrow{\text{pyk}} \text{“var f2”}]$   
 $[(\text{f3}) \xrightarrow{\text{pyk}} \text{“var f3”}]$   
 $[(\text{f4}) \xrightarrow{\text{pyk}} \text{“var f4”}]$   
 $[(\text{op1}) \xrightarrow{\text{pyk}} \text{“var op1”}]$   
 $[(\text{op2}) \xrightarrow{\text{pyk}} \text{“var op2”}]$   
 $[(\text{r1}) \xrightarrow{\text{pyk}} \text{“var r1”}]$   
 $[(\text{s1}) \xrightarrow{\text{pyk}} \text{“var s1”}]$   
 $[(\text{s2}) \xrightarrow{\text{pyk}} \text{“var s2”}]$   
 $[\text{X}_1 \xrightarrow{\text{pyk}} \text{“meta x1”}]$   
 $[\text{X}_2 \xrightarrow{\text{pyk}} \text{“meta x2”}]$   
 $[\text{Y}_1 \xrightarrow{\text{pyk}} \text{“meta y1”}]$   
 $[\text{Y}_2 \xrightarrow{\text{pyk}} \text{“meta y2”}]$   
 $[\text{V}_1 \xrightarrow{\text{pyk}} \text{“meta v1”}]$   
 $[\text{V}_2 \xrightarrow{\text{pyk}} \text{“meta v2”}]$   
 $[\text{V}_3 \xrightarrow{\text{pyk}} \text{“meta v3”}]$   
 $[\text{V}_4 \xrightarrow{\text{pyk}} \text{“meta v4”}]$   
 $[\text{V}_{2n} \xrightarrow{\text{pyk}} \text{“meta v2n”}]$   
 $[\text{M}_1 \xrightarrow{\text{pyk}} \text{“meta m1”}]$   
 $[\text{M}_2 \xrightarrow{\text{pyk}} \text{“meta m2”}]$   
 $[\text{N}_1 \xrightarrow{\text{pyk}} \text{“meta n1”}]$   
 $[\text{N}_2 \xrightarrow{\text{pyk}} \text{“meta n2”}]$   
 $[\text{N}_3 \xrightarrow{\text{pyk}} \text{“meta n3”}]$   
 $[\epsilon \xrightarrow{\text{pyk}} \text{“meta ep”}]$   
 $[\epsilon 1 \xrightarrow{\text{pyk}} \text{“meta ep1”}]$   
 $[\epsilon 2 \xrightarrow{\text{pyk}} \text{“meta ep2”}]$   
 $[\text{FX} \xrightarrow{\text{pyk}} \text{“meta fx”}]$   
 $[\text{FY} \xrightarrow{\text{pyk}} \text{“meta fy”}]$   
 $[\text{FZ} \xrightarrow{\text{pyk}} \text{“meta fz”}]$   
 $[\text{FU} \xrightarrow{\text{pyk}} \text{“meta fu”}]$   
 $[\text{FV} \xrightarrow{\text{pyk}} \text{“meta fv”}]$   
 $[\text{FW} \xrightarrow{\text{pyk}} \text{“meta fw”}]$   
 $[\text{FEP} \xrightarrow{\text{pyk}} \text{“meta fep”}]$



$[RX \xrightarrow{\text{pyk}} \text{“meta rx”}]$   
 $[RY \xrightarrow{\text{pyk}} \text{“meta ry”}]$   
 $[RZ \xrightarrow{\text{pyk}} \text{“meta rz”}]$   
 $[RU \xrightarrow{\text{pyk}} \text{“meta ru”}]$   
 $[(SX) \xrightarrow{\text{pyk}} \text{“meta sx”}]$   
 $[(SX1) \xrightarrow{\text{pyk}} \text{“meta sx1”}]$   
 $[(SY) \xrightarrow{\text{pyk}} \text{“meta sy”}]$   
 $[(SY1) \xrightarrow{\text{pyk}} \text{“meta sy1”}]$   
 $[(SZ) \xrightarrow{\text{pyk}} \text{“meta sz”}]$   
 $[(SZ1) \xrightarrow{\text{pyk}} \text{“meta sz1”}]$   
 $[(SU) \xrightarrow{\text{pyk}} \text{“meta su”}]$   
 $[(SU1) \xrightarrow{\text{pyk}} \text{“meta su1”}]$   
 $[FXS \xrightarrow{\text{pyk}} \text{“meta fxs”}]$   
 $[FYS \xrightarrow{\text{pyk}} \text{“meta fys”}]$   
 $[(F1) \xrightarrow{\text{pyk}} \text{“meta f1”}]$   
 $[(F2) \xrightarrow{\text{pyk}} \text{“meta f2”}]$   
 $[(F3) \xrightarrow{\text{pyk}} \text{“meta f3”}]$   
 $[(F4) \xrightarrow{\text{pyk}} \text{“meta f4”}]$   
 $[(OP1) \xrightarrow{\text{pyk}} \text{“meta op1”}]$   
 $[(OP2) \xrightarrow{\text{pyk}} \text{“meta op2”}]$   
 $[(R1) \xrightarrow{\text{pyk}} \text{“meta r1”}]$   
 $[(S1) \xrightarrow{\text{pyk}} \text{“meta s1”}]$   
 $[(S2) \xrightarrow{\text{pyk}} \text{“meta s2”}]$   
 $[(EPob) \xrightarrow{\text{pyk}} \text{“object ep”}]$   
 $[(CRS1ob) \xrightarrow{\text{pyk}} \text{“object crs1”}]$   
 $[(F1ob) \xrightarrow{\text{pyk}} \text{“object f1”}]$   
 $[(F2ob) \xrightarrow{\text{pyk}} \text{“object f2”}]$   
 $[(F3ob) \xrightarrow{\text{pyk}} \text{“object f3”}]$   
 $[(F4ob) \xrightarrow{\text{pyk}} \text{“object f4”}]$   
 $[(N1ob) \xrightarrow{\text{pyk}} \text{“object n1”}]$   
 $[(N2ob) \xrightarrow{\text{pyk}} \text{“object n2”}]$   
 $[(OP1ob) \xrightarrow{\text{pyk}} \text{“object op1”}]$   
 $[(OP2ob) \xrightarrow{\text{pyk}} \text{“object op2”}]$   
 $[(R1ob) \xrightarrow{\text{pyk}} \text{“object r1”}]$   
 $[(S1ob) \xrightarrow{\text{pyk}} \text{“object s1”}]$

$[(S2ob) \xrightarrow{pyk} \text{“object s2”}]$   
 $[ph_4 \xrightarrow{pyk} \text{“ph4”}]$   
 $[ph_5 \xrightarrow{pyk} \text{“ph5”}]$   
 $[ph_6 \xrightarrow{pyk} \text{“ph6”}]$   
 $[NAT \xrightarrow{pyk} \text{“NAT”}]$   
 $[RATIONAL_SERIES \xrightarrow{pyk} \text{“RATIONAL\_SERIES”}]$   
 $[SERIES \xrightarrow{pyk} \text{“SERIES”}]$   
 $[SetOfReals \xrightarrow{pyk} \text{“setOfReals”}]$   
 $[SetOfFxs \xrightarrow{pyk} \text{“setOfFxs”}]$   
 $[N \xrightarrow{pyk} \text{“N”}]$   
 $[Q \xrightarrow{pyk} \text{“Q”}]$   
 $[X \xrightarrow{pyk} \text{“X”}]$   
 $[xs \xrightarrow{pyk} \text{“xs”}]$   
 $[xaF \xrightarrow{pyk} \text{“xsF”}]$   
 $[ysF \xrightarrow{pyk} \text{“ysF”}]$   
 $[us \xrightarrow{pyk} \text{“us”}]$   
 $[usFoelge \xrightarrow{pyk} \text{“usF”}]$   
 $[0 \xrightarrow{pyk} \text{“0”}]$   
 $[1 \xrightarrow{pyk} \text{“1”}]$   
 $[(-1) \xrightarrow{pyk} \text{“(-1)”}]$   
 $[2 \xrightarrow{pyk} \text{“2”}]$   
 $[3 \xrightarrow{pyk} \text{“3”}]$   
 $[1/2 \xrightarrow{pyk} \text{“1/2”}]$   
 $[1/3 \xrightarrow{pyk} \text{“1/3”}]$   
 $[2/3 \xrightarrow{pyk} \text{“2/3”}]$   
 $[0f \xrightarrow{pyk} \text{“0f”}]$   
 $[1f \xrightarrow{pyk} \text{“1f”}]$   
 $[00 \xrightarrow{pyk} \text{“00”}]$   
 $[01 \xrightarrow{pyk} \text{“01”}]$   
 $[(- - 01) \xrightarrow{pyk} \text{“(--01)”}]$   
 $[02 \xrightarrow{pyk} \text{“02”}]$   
 $[01//02 \xrightarrow{pyk} \text{“01//02”}]$   
 $[PlusAssociativity(R) \xrightarrow{pyk} \text{“lemma plusAssociativity(R)”}]$   
 $[PlusAssociativity(R)XX \xrightarrow{pyk} \text{“lemma plusAssociativity(R)XX”}]$   
 $[Plus0(R) \xrightarrow{pyk} \text{“lemma plus0(R)”}]$

[Negative(R)  $\xrightarrow{\text{pyk}}$  “lemma negative(R)”]  
 [Times1(R)  $\xrightarrow{\text{pyk}}$  “lemma times1(R)”]  
 [lessAddition(R)  $\xrightarrow{\text{pyk}}$  “lemma lessAddition(R)”]  
 [PlusCommutativity(R)  $\xrightarrow{\text{pyk}}$  “lemma plusCommutativity(R)”]  
 [LeqAntisymmetry(R)  $\xrightarrow{\text{pyk}}$  “lemma leqAntisymmetry(R)”]  
 [LeqTransitivity(R)  $\xrightarrow{\text{pyk}}$  “lemma leqTransitivity(R)”]  
 [leqAddition(R)  $\xrightarrow{\text{pyk}}$  “lemma leqAddition(R)”]  
 [Distribution(R)  $\xrightarrow{\text{pyk}}$  “lemma distribution(R)”]  
 [A4(Axiom)  $\xrightarrow{\text{pyk}}$  “axiom a4”]  
 [InductionAxiom  $\xrightarrow{\text{pyk}}$  “axiom induction”]  
 [EqualityAxiom  $\xrightarrow{\text{pyk}}$  “axiom equality”]  
 [EqLeqAxiom  $\xrightarrow{\text{pyk}}$  “axiom eqLeq”]  
 [EqAdditionAxiom  $\xrightarrow{\text{pyk}}$  “axiom eqAddition”]  
 [EqMultiplicationAxiom  $\xrightarrow{\text{pyk}}$  “axiom eqMultiplication”]  
 [QisClosed(Reciprocal)(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom QisClosed(reciprocal)”]  
 [QisClosed(Reciprocal)  $\xrightarrow{\text{pyk}}$  “lemma QisClosed(reciprocal)”]  
 [QisClosed(Negative)(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom QisClosed(negative)”]  
 [QisClosed(Negative)  $\xrightarrow{\text{pyk}}$  “lemma QisClosed(negative)”]  
 [leqReflexivity  $\xrightarrow{\text{pyk}}$  “axiom leqReflexivity”]  
 [leqAntisymmetryAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqAntisymmetry”]  
 [leqTransitivityAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqTransitivity”]  
 [leqTotality  $\xrightarrow{\text{pyk}}$  “axiom leqTotality”]  
 [leqAdditionAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqAddition”]  
 [leqMultiplicationAxiom  $\xrightarrow{\text{pyk}}$  “axiom leqMultiplication”]  
 [plusAssociativity  $\xrightarrow{\text{pyk}}$  “axiom plusAssociativity”]  
 [plusCommutativity  $\xrightarrow{\text{pyk}}$  “axiom plusCommutativity”]  
 [Negative  $\xrightarrow{\text{pyk}}$  “axiom negative”]  
 [plus0  $\xrightarrow{\text{pyk}}$  “axiom plus0”]  
 [timesAssociativity  $\xrightarrow{\text{pyk}}$  “axiom timesAssociativity”]  
 [timesCommutativity  $\xrightarrow{\text{pyk}}$  “axiom timesCommutativity”]  
 [ReciprocalAxiom  $\xrightarrow{\text{pyk}}$  “axiom reciprocal”]  
 [times1  $\xrightarrow{\text{pyk}}$  “axiom times1”]  
 [Distribution  $\xrightarrow{\text{pyk}}$  “axiom distribution”]  
 [0not1  $\xrightarrow{\text{pyk}}$  “axiom 0not1”]  
 [lemma eqLeq(R)  $\xrightarrow{\text{pyk}}$  “lemma eqLeq(R)”]

[TimesAssociativity(R)  $\xrightarrow{\text{pyk}}$  “lemma timesAssociativity(R)”]  
 [TimesCommutativity(R)  $\xrightarrow{\text{pyk}}$  “lemma timesCommutativity(R)”]  
 [(Adgic)SameR  $\xrightarrow{\text{pyk}}$  “1rule adhoc sameR”]  
 [Separation2formula(1)  $\xrightarrow{\text{pyk}}$  “lemma separation2formula(1)”]  
 [Separation2formula(2)  $\xrightarrow{\text{pyk}}$  “lemma separation2formula(2)”]  
 [Cauchy  $\xrightarrow{\text{pyk}}$  “axiom cauchy”]  
 [PlusF  $\xrightarrow{\text{pyk}}$  “axiom plusF”]  
 [ReciprocalF  $\xrightarrow{\text{pyk}}$  “axiom reciprocalF”]  
 [From ==  $\xrightarrow{\text{pyk}}$  “1rule from==”]  
 [To ==  $\xrightarrow{\text{pyk}}$  “1rule to==”]  
 [FromInR  $\xrightarrow{\text{pyk}}$  “1rule fromInR”]  
 [PlusR(Sym)  $\xrightarrow{\text{pyk}}$  “lemma plusR(Sym)”]  
 [ReciprocalR(Axiom)  $\xrightarrow{\text{pyk}}$  “axiom reciprocalR”]  
 [LessMinus1(N)  $\xrightarrow{\text{pyk}}$  “1rule lessMinus1(N)”]  
 [Nonnegative(N)  $\xrightarrow{\text{pyk}}$  “axiom nonnegative(N)”]  
 [US0  $\xrightarrow{\text{pyk}}$  “axiom US0”]  
 [NextXS(UpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextXS(upperBound)”]  
 [NextXS(NoUpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextXS(noUpperBound)”]  
 [NextUS(UpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextUS(upperBound)”]  
 [NextUS(NoUpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextUS(noUpperBound)”]  
 [ExpZero  $\xrightarrow{\text{pyk}}$  “1rule expZero”]  
 [ExpPositive  $\xrightarrow{\text{pyk}}$  “1rule expPositive”]  
 [ExpZero(R)  $\xrightarrow{\text{pyk}}$  “1rule expZero(R)”]  
 [ExpPositive(R)  $\xrightarrow{\text{pyk}}$  “1rule expPositive(R)”]  
 [BSzero  $\xrightarrow{\text{pyk}}$  “1rule base(1/2)Sum zero”]  
 [BSpositive  $\xrightarrow{\text{pyk}}$  “1rule base(1/2)Sum positive”]  
 [UStescope(Zero)  $\xrightarrow{\text{pyk}}$  “1rule UStescope zero”]  
 [UStescope(Positive)  $\xrightarrow{\text{pyk}}$  “1rule UStescope positive”]  
 [EqAddition(R)  $\xrightarrow{\text{pyk}}$  “1rule adhoc eqAddition(R)”]  
 [FromLimit  $\xrightarrow{\text{pyk}}$  “1rule fromLimit”]  
 [ToUpperBound  $\xrightarrow{\text{pyk}}$  “1rule toUpperBound”]  
 [FromUpperBound  $\xrightarrow{\text{pyk}}$  “1rule fromUpperBound”]  
 [USisUpperBound  $\xrightarrow{\text{pyk}}$  “axiom USisUpperBound”]  
 [0not1(R)  $\xrightarrow{\text{pyk}}$  “axiom 0not1(R)”]  
 [ExpUnbounded(R)  $\xrightarrow{\text{pyk}}$  “1rule expUnbounded”]

[FromLeq(Advanced)(N)  $\xrightarrow{\text{pyk}}$  “1rule fromLeq(Advanced)(N)”]  
 [FromLeastUpperBound  $\xrightarrow{\text{pyk}}$  “1rule fromLeastUpperBound”]  
 [ToLeastUpperBound  $\xrightarrow{\text{pyk}}$  “1rule toLeastUpperBound”]  
 [XSisNotUpperBound  $\xrightarrow{\text{pyk}}$  “axiom XSisNotUpperBound”]  
 [ysFGreater  $\xrightarrow{\text{pyk}}$  “axiom ysFGreater”]  
 [ysFLess  $\xrightarrow{\text{pyk}}$  “axiom ysFLess”]  
 [SmallInverse  $\xrightarrow{\text{pyk}}$  “1rule smallInverse”]  
 [NatType  $\xrightarrow{\text{pyk}}$  “axiom natType”]  
 [RationalType  $\xrightarrow{\text{pyk}}$  “axiom rationalType”]  
 [SeriesType  $\xrightarrow{\text{pyk}}$  “axiom seriesType”]  
 [Max  $\xrightarrow{\text{pyk}}$  “axiom max”]  
 [Numerical  $\xrightarrow{\text{pyk}}$  “axiom numerical”]  
 [NumericalF  $\xrightarrow{\text{pyk}}$  “axiom numericalF”]  
 [MemberOfSeries(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom memberOfSeries”]  
 [JoinConjuncts(2conditions)  $\xrightarrow{\text{pyk}}$  “prop lemma doubly conditioned join  
 conjuncts”]  
 [prop lemma imply negation  $\xrightarrow{\text{pyk}}$  “prop lemma imply negation”]  
 [TND  $\xrightarrow{\text{pyk}}$  “prop lemma tertium non datur”]  
 [FromNegatedImPLY  $\xrightarrow{\text{pyk}}$  “prop lemma from negated imply”]  
 [ToNegatedImPLY  $\xrightarrow{\text{pyk}}$  “prop lemma to negated imply”]  
 [FromNegated(2 \* ImPLY)  $\xrightarrow{\text{pyk}}$  “prop lemma from negated double imply”]  
 [FromNegatedAnd  $\xrightarrow{\text{pyk}}$  “prop lemma from negated and”]  
 [FromNegatedOr  $\xrightarrow{\text{pyk}}$  “prop lemma from negated or”]  
 [ToNegatedOr  $\xrightarrow{\text{pyk}}$  “prop lemma to negated or”]  
 [FromNegations  $\xrightarrow{\text{pyk}}$  “prop lemma from negations”]  
 [From3Disjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma from three disjuncts”]  
 [From2 \* 2Disjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma from two times two disjuncts”]  
 [NegateDisjunct1  $\xrightarrow{\text{pyk}}$  “prop lemma negate first disjunct”]  
 [NegateDisjunct2  $\xrightarrow{\text{pyk}}$  “prop lemma negate second disjunct”]  
 [ExpandDisjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma expand disjuncts”]  
 [SENC1  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition(1)”]  
 [SENC2  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition(2)”]  
 [LessLeq(R)  $\xrightarrow{\text{pyk}}$  “lemma lessLeq(R)”]  
 [MemberOfSeries  $\xrightarrow{\text{pyk}}$  “lemma memberOfSeries”]  
 [memberOfSeries(Type)  $\xrightarrow{\text{pyk}}$  “lemma memberOfSeries(Type)”]

$[*(\text{exp}) * \xrightarrow{\text{pyk}} \text{" " \wedge \text{ "}}]$   
 $[\text{R}(* ) \xrightarrow{\text{pyk}} \text{"R( " )} ]]$   
 $[- - \text{R}(* ) \xrightarrow{\text{pyk}} \text{"--R( " )} ]]$   
 $[\text{rec} * \xrightarrow{\text{pyk}} \text{"1/ "}]$   
 $[*/ * \xrightarrow{\text{pyk}} \text{"eq-system of " modulo "}]$   
 $[* \cap * \xrightarrow{\text{pyk}} \text{"intersection " comma " end intersection"}]$   
 $[*[*] \xrightarrow{\text{pyk}} \text{"[ " ; " } ]]$   
 $[\cup * \xrightarrow{\text{pyk}} \text{"union " end union"}]$   
 $[* \cup * \xrightarrow{\text{pyk}} \text{"binary-union " comma " end union"}]$   
 $[\text{P}(* ) \xrightarrow{\text{pyk}} \text{"power " end power"}]$   
 $[\{ * \} \xrightarrow{\text{pyk}} \text{"zermelo singleton " end singleton"}]$   
 $[\text{StateExpand}(*, *, *) \xrightarrow{\text{pyk}} \text{"stateExpand( " , " , " )} ]]$   
 $[\text{extractSeries}(* ) \xrightarrow{\text{pyk}} \text{"extractSeries( " )} ]]$   
 $[\text{SetOfSeries}(* ) \xrightarrow{\text{pyk}} \text{"setOfSeries( " )} ]]$   
 $[- - \text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"--Macro( " )} ]]$   
 $[\text{ExpandList}(*, *, *) \xrightarrow{\text{pyk}} \text{"expandList( " , " , " )} ]]$   
 $[* * \text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"**Macro( " )} ]]$   
 $[+ + \text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"++Macro( " )} ]]$   
 $[<< \text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"<<Macro( " )} ]]$   
 $[||\text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"||Macro( " )} ]]$   
 $[01//\text{Macro}(* ) \xrightarrow{\text{pyk}} \text{"01//Macro( " )} ]]$   
 $[\text{UB}(*, *) \xrightarrow{\text{pyk}} \text{"upperBound( " , " )} ]]$   
 $[\text{LUB}(*, *) \xrightarrow{\text{pyk}} \text{"leastUpperBound( " , " )} ]]$   
 $[\text{BS}(*, *) \xrightarrow{\text{pyk}} \text{"base(1/2)Sum( " , " )} ]]$   
 $[\text{UStelescope}(*, *) \xrightarrow{\text{pyk}} \text{"UStelescope( " , " )} ]]$   
 $[(*) \xrightarrow{\text{pyk}} \text{"( " )} ]]$   
 $[|f * | \xrightarrow{\text{pyk}} \text{"|f " |} ]]$   
 $[|r * | \xrightarrow{\text{pyk}} \text{"|r " |} ]]$   
 $[\text{Limit}(*, *) \xrightarrow{\text{pyk}} \text{"limit( " , " )} ]]$   
 $[\text{Union}(* ) \xrightarrow{\text{pyk}} \text{"U( " )} ]]$   
 $[\text{IsOrderedPair}(*, *, *) \xrightarrow{\text{pyk}} \text{"isOrderedPair( " , " , " )} ]]$   
 $[\text{IsRelation}(*, *, *) \xrightarrow{\text{pyk}} \text{"isRelation( " , " , " )} ]]$   
 $[\text{isFunction}(*, *, *) \xrightarrow{\text{pyk}} \text{"isFunction( " , " , " )} ]]$   
 $[\text{IsSeries}(*, *) \xrightarrow{\text{pyk}} \text{"isSeries( " , " )} ]]$   
 $[\text{IsNatural}(*, *) \xrightarrow{\text{pyk}} \text{"isNatural( " )} ]]$

$[\text{OrderedPair}(*, *) \xrightarrow{\text{pyk}} \text{"(o " , " )"}]$   
 $[\text{TypeNat}(*, *) \xrightarrow{\text{pyk}} \text{"typeNat( " )"}]$   
 $[\text{TypeNat0}(*, *) \xrightarrow{\text{pyk}} \text{"typeNat0( " )"}]$   
 $[\text{TypeRational}(*, *) \xrightarrow{\text{pyk}} \text{"typeRational( " )"}]$   
 $[\text{TypeRational0}(*, *) \xrightarrow{\text{pyk}} \text{"typeRational0( " )"}]$   
 $[\text{TypeSeries}(*, *) \xrightarrow{\text{pyk}} \text{"typeSeries( " , " )"}]$   
 $[\text{Typeseries0}(*, *) \xrightarrow{\text{pyk}} \text{"typeSeries0( " , " )"}]$   
 $[\{*, *\} \xrightarrow{\text{pyk}} \text{"zermelo pair " comma " end pair"}]$   
 $[\langle *, * \rangle \xrightarrow{\text{pyk}} \text{"zermelo ordered pair " comma " end pair"}]$   
 $[-u* \xrightarrow{\text{pyk}} \text{"_ "}]$   
 $[-f* \xrightarrow{\text{pyk}} \text{"_f "}]$   
 $[- - * \xrightarrow{\text{pyk}} \text{"_ "}]$   
 $[1f/* \xrightarrow{\text{pyk}} \text{"1f/ "}]$   
 $[01//temp* \xrightarrow{\text{pyk}} \text{"01// "}]$   
 $[*(*, *) \xrightarrow{\text{pyk}} \text{" " is related to " under "}]$   
 $[\text{RefRel}(*, *) \xrightarrow{\text{pyk}} \text{" " is reflexive relation in "}]$   
 $[\text{SymRel}(*, *) \xrightarrow{\text{pyk}} \text{" " is symmetric relation in "}]$   
 $[\text{TransRel}(*, *) \xrightarrow{\text{pyk}} \text{" " is transitive relation in "}]$   
 $[\text{EqRel}(*, *) \xrightarrow{\text{pyk}} \text{" " is equivalence relation in "}]$   
 $[[* \in *]_* \xrightarrow{\text{pyk}} \text{"equivalence class of " in " modulo "}]$   
 $[\text{Partition}(*, *) \xrightarrow{\text{pyk}} \text{" " is partition of "}]$   
 $[(***) \xrightarrow{\text{pyk}} \text{" " * "}]$   
 $[* * f * \xrightarrow{\text{pyk}} \text{" " * f "}]$   
 $[* * * * \xrightarrow{\text{pyk}} \text{" " * * "}]$   
 $[(* + *) \xrightarrow{\text{pyk}} \text{" " + "}]$   
 $[(* - *) \xrightarrow{\text{pyk}} \text{" " - "}]$   
 $[* + f * \xrightarrow{\text{pyk}} \text{" " + f "}]$   
 $[* - f * \xrightarrow{\text{pyk}} \text{" " - f "}]$   
 $[* + + * \xrightarrow{\text{pyk}} \text{" " ++ "}]$   
 $[\text{R}(*, *) \xrightarrow{\text{pyk}} \text{"R( " ) -- R( " )"}]$   
 $[* \in * \xrightarrow{\text{pyk}} \text{" " in0 "}]$   
 $[| * | \xrightarrow{\text{pyk}} \text{" | " |"}]$   
 $[\text{if}(*, *, *) \xrightarrow{\text{pyk}} \text{"if( " , " , " )"}]$   
 $[\text{Max}(*, *) \xrightarrow{\text{pyk}} \text{"max( " , " )"}]$   
 $[\text{Max}(*, *) \xrightarrow{\text{pyk}} \text{"maxR( " , " )"}]$

[\* = \*  $\xrightarrow{\text{pyk}}$  " = " ]

[\*  $\neq$  \*  $\xrightarrow{\text{pyk}}$  "  $\neq$  " ]

[\* <= \*  $\xrightarrow{\text{pyk}}$  " <= " ]

[\* < \*  $\xrightarrow{\text{pyk}}$  " < " ]

[\* <\_f \*  $\xrightarrow{\text{pyk}}$  " <\_f " ]

[\*  $\leq_f$  \*  $\xrightarrow{\text{pyk}}$  "  $\leq_f$  " ]

[SF(\*,\*)  $\xrightarrow{\text{pyk}}$  " sameF " ]

[\* == \*  $\xrightarrow{\text{pyk}}$  " == " ]

[\*!! == \*  $\xrightarrow{\text{pyk}}$  " !! == " ]

[\* << \*  $\xrightarrow{\text{pyk}}$  " << " ]

[\* <<== \*  $\xrightarrow{\text{pyk}}$  " <<== " ]

[\* == \*  $\xrightarrow{\text{pyk}}$  " zermelo is " ]

[\*  $\subseteq$  \*  $\xrightarrow{\text{pyk}}$  " is subset of " ]

[ $\dot{\neg}$ (\* )n  $\xrightarrow{\text{pyk}}$  "not0 " ]

[\*  $\notin$  \*  $\xrightarrow{\text{pyk}}$  " zermelo  $\sim$ in " ]

[\*  $\neq$  \*  $\xrightarrow{\text{pyk}}$  " zermelo  $\sim$ is " ]

[\*  $\wedge$  \*  $\xrightarrow{\text{pyk}}$  " and0 " ]

[\*  $\dot{\vee}$  \*  $\xrightarrow{\text{pyk}}$  " or0 " ]

[ $\exists$ \*: \*  $\xrightarrow{\text{pyk}}$  "exist0 " indeed " ]

[\*  $\Leftrightarrow$  \*  $\xrightarrow{\text{pyk}}$  " iff " ]

[{ph  $\in$  \* | \*}  $\xrightarrow{\text{pyk}}$  "the set of ph in " such that " end set"]

[kvanti  $\xrightarrow{\text{pyk}}$  "kvanti"]



## B T<sub>E</sub>X definitioner

[kvanti  $\xrightarrow{\text{tex}}$  “kvanti”]

[ $(\dots)$   $\xrightarrow{\text{tex}}$  “ $(\cdots)$ ”]

[Objekt-var  $\xrightarrow{\text{tex}}$  “ $\text{\texttt{Objekt-var}}$ ”]

[Ex-var  $\xrightarrow{\text{tex}}$  “ $\text{\texttt{Ex-var}}$ ”]

[Ph-var  $\xrightarrow{\text{tex}}$  “ $\text{\texttt{Ph-var}}$ ”]

[Værdi  $\xrightarrow{\text{tex}}$  “ $\text{\texttt{V\ae\{rdi}}$ ”]

[Variabel  $\xrightarrow{\text{tex}}$  “ $\text{\texttt{Variabel}}$ ”]

[Op(x)  $\xrightarrow{\text{tex}}$  “Op(#1.  
)”]

[Op(x, y)  $\xrightarrow{\text{tex}}$  “Op(#1.  
, #2.  
)”]

[ $x \doteq y$   $\xrightarrow{\text{tex}}$  “#1.  
 $\mathrel{\{\ddot{=}\}}$  #2.”]

[ContainsEmpty(x)  $\xrightarrow{\text{tex}}$  “ContainsEmpty(#1.  
)”]

[Dedu(x, y)  $\xrightarrow{\text{tex}}$  “  
Dedu(#1.  
, #2.  
)”]

[Dedu<sub>0</sub>(x, y)  $\xrightarrow{\text{tex}}$  “  
Dedu\_0(#1.  
, #2.  
)”]

[Dedu<sub>s</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “Dedu\_{s}(#1.  
, #2.  
, #3.  
)”]

[Dedu<sub>1</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “  
Dedu\_1(#1.  
, #2.  
)”]

, #3.  
)”]

[Dedu<sub>2</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “  
Dedu\_2(#1.

, #2.  
, #3.  
)”]

[Dedu<sub>3</sub>(x, y, z, u)  $\xrightarrow{\text{tex}}$  “  
Dedu\_3(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>4</sub>(x, y, z, u)  $\xrightarrow{\text{tex}}$  “  
Dedu\_4(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>4</sub><sup>\*</sup>(x, y, z, u)  $\xrightarrow{\text{tex}}$  “  
Dedu\_4^\*(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>5</sub>(x, y, z)  $\xrightarrow{\text{tex}}$  “  
Dedu\_5(#1.

, #2.  
, #3.  
)”]

[Dedu<sub>6</sub>(p, c, e, b)  $\xrightarrow{\text{tex}}$  “  
Dedu\_6(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>6</sub><sup>\*</sup>(p, c, e, b)  $\xrightarrow{\text{tex}}$  “  
Dedu\_6^\*(#1.

, #2.  
, #3.

, #4.  
)”]

[Dedu<sub>7</sub>(p)  $\xrightarrow{\text{tex}}$  “  
Dedu\_7(#1.  
)”]

[Dedu<sub>8</sub>(p, b)  $\xrightarrow{\text{tex}}$  “  
Dedu\_8(#1.  
, #2.  
)”]

[Dedu<sub>8</sub><sup>\*</sup>(p, b)  $\xrightarrow{\text{tex}}$  “  
Dedu\_8^\*(#1.  
, #2.  
)”]

[Ex<sub>1</sub>  $\xrightarrow{\text{tex}}$  “Ex\_{1}”]

[Ex<sub>2</sub>  $\xrightarrow{\text{tex}}$  “Ex\_{2}”]

[Ex<sub>10</sub>  $\xrightarrow{\text{tex}}$  “Ex\_{10}”]

[Ex<sub>20</sub>  $\xrightarrow{\text{tex}}$  “Ex\_{20}”]

[x<sub>Ex</sub>  $\xrightarrow{\text{tex}}$  “#1.  
\_{Ex}”]

[x<sup>Ex</sup>  $\xrightarrow{\text{tex}}$  “#1.  
^\_{Ex}”]

[(x≡y|z:=u)<sub>Ex</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv} #2.  
| #3.  
{:=} #4.  
\rangle\_{Ex} ”]

[(x≡<sup>0</sup>y|z:=u)<sub>Ex</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv}^0 #2.  
| #3.  
{:=} #4.  
\rangle\_{Ex} ”]

[(x≡<sup>1</sup>y|z:=u)<sub>Ex</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.  
{\equiv}^1 #2.  
| #3.  
{:=} #4.  
\rangle\_{Ex} ”]

$\langle x \equiv *y | z := u \rangle_{\text{Ex}} \xrightarrow{\text{tex}} \text{"\langle \rangle \#1.}$   
 $\{\text{equiv}\}^* \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ex}}$

$\text{ph}_1 \xrightarrow{\text{tex}} \text{"ph_{1}}"$

$\text{ph}_2 \xrightarrow{\text{tex}} \text{"ph_{2}}"$

$\text{ph}_3 \xrightarrow{\text{tex}} \text{"ph_{3}}"$

$\text{ph}_4 \xrightarrow{\text{tex}} \text{"ph_{4}}"$

$\text{ph}_5 \xrightarrow{\text{tex}} \text{"ph_{5}}"$

$\text{ph}_6 \xrightarrow{\text{tex}} \text{"ph_{6}}"$

$[*_{\text{Ph}} \xrightarrow{\text{tex}} \text{"\#1.}$   
 $\text{-}\{\text{Ph}\}]$

$[x^{\text{Ph}} \xrightarrow{\text{tex}} \text{"\#1.}$   
 $\wedge\{\text{Ph}\}]$

$\langle x \equiv y | z := u \rangle_{\text{Ph}} \xrightarrow{\text{tex}} \text{"\langle \rangle \#1.}$   
 $\{\text{equiv}\} \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

$\langle x \equiv^0 y | z := u \rangle_{\text{Ph}} \xrightarrow{\text{tex}} \text{"\langle \rangle \#1.}$   
 $\{\text{equiv}\}^0 \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

$\langle x \equiv^1 y | z := u \rangle_{\text{Ph}} \xrightarrow{\text{tex}} \text{"\langle \rangle \#1.}$   
 $\{\text{equiv}\}^1 \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

$\langle x \equiv *y | z := u \rangle_{\text{Ph}} \xrightarrow{\text{tex}} \text{"\langle \rangle \#1.}$   
 $\{\text{equiv}\}^* \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

[bs  $\xrightarrow{\text{tex}}$  “\mathsf {bs}”]

[OBS  $\xrightarrow{\text{tex}}$  “ \mathsf {OBS}”]

[BS  $\xrightarrow{\text{tex}}$  “{\cal BS}”]

[ $\emptyset$   $\xrightarrow{\text{tex}}$  “\mathrm{\O}”]

[SystemQ  $\xrightarrow{\text{tex}}$  “SystemQ”]

[MP  $\xrightarrow{\text{tex}}$  “MP”]

[Gen  $\xrightarrow{\text{tex}}$  “Gen”]

[Repetition  $\xrightarrow{\text{tex}}$  “Repetition”]

[Neg  $\xrightarrow{\text{tex}}$  “Neg”]

[Ded  $\xrightarrow{\text{tex}}$  “Ded”]

[ExistIntro  $\xrightarrow{\text{tex}}$  “ExistIntro”]

[Extensionality  $\xrightarrow{\text{tex}}$  “Extensionality”]

[ $\emptyset$ def  $\xrightarrow{\text{tex}}$  “\O{}def”]

[PairDef  $\xrightarrow{\text{tex}}$  “PairDef”]

[UnionDef  $\xrightarrow{\text{tex}}$  “UnionDef”]

[PowerDef  $\xrightarrow{\text{tex}}$  “PowerDef”]

[SeparationDef  $\xrightarrow{\text{tex}}$  “SeparationDef”]

[AddDoubleNeg  $\xrightarrow{\text{tex}}$  “AddDoubleNeg”]

[RemoveDoubleNeg  $\xrightarrow{\text{tex}}$  “RemoveDoubleNeg”]

[AndCommutativity  $\xrightarrow{\text{tex}}$  “AndCommutativity”]

[AutoImply  $\xrightarrow{\text{tex}}$  “AutoImply”]

[Contrapositive  $\xrightarrow{\text{tex}}$  “Contrapositive”]

[FirstConjunct  $\xrightarrow{\text{tex}}$  “FirstConjunct”]

[SecondConjunct  $\xrightarrow{\text{tex}}$  “SecondConjunct”]

[FromContradiction  $\xrightarrow{\text{tex}}$  “FromContradiction”]

[FromDisjuncts  $\xrightarrow{\text{tex}}$  “FromDisjuncts”]  
[IffCommutativity  $\xrightarrow{\text{tex}}$  “IffCommutativity”]  
[IffFirst  $\xrightarrow{\text{tex}}$  “IffFirst”]  
[IffSecond  $\xrightarrow{\text{tex}}$  “IffSecond”]  
[ImplyTransitivity  $\xrightarrow{\text{tex}}$  “ImplyTransitivity”]  
[JoinConjuncts  $\xrightarrow{\text{tex}}$  “JoinConjuncts”]  
[MP2  $\xrightarrow{\text{tex}}$  “MP2”]  
[MP3  $\xrightarrow{\text{tex}}$  “MP3”]  
[MP4  $\xrightarrow{\text{tex}}$  “MP4”]  
[MP5  $\xrightarrow{\text{tex}}$  “MP5”]  
[MT  $\xrightarrow{\text{tex}}$  “MT”]  
[NegativeMT  $\xrightarrow{\text{tex}}$  “NegativeMT”]  
[Technicality  $\xrightarrow{\text{tex}}$  “Technicality”]  
[Weakening  $\xrightarrow{\text{tex}}$  “Weakening”]  
[WeakenOr1  $\xrightarrow{\text{tex}}$  “WeakenOr1”]  
[WeakenOr2  $\xrightarrow{\text{tex}}$  “WeakenOr2”]  
[Pair2Formula  $\xrightarrow{\text{tex}}$  “Pair2Formula”]  
[Formula2Pair  $\xrightarrow{\text{tex}}$  “Formula2Pair”]  
[Union2Formula  $\xrightarrow{\text{tex}}$  “Union2Formula”]  
[Formula2Union  $\xrightarrow{\text{tex}}$  “Formula2Union”]  
[Formula2Power  $\xrightarrow{\text{tex}}$  “Formula2Power”]  
[Sep2Formula  $\xrightarrow{\text{tex}}$  “Sep2Formula”]  
[Formula2Sep  $\xrightarrow{\text{tex}}$  “Formula2Sep”]  
[SubsetInPower  $\xrightarrow{\text{tex}}$  “SubsetInPower”]  
[HelperPowerIsSub  $\xrightarrow{\text{tex}}$  “HelperPowerIsSub”]

[PowerIsSub  $\xrightarrow{\text{tex}}$  “PowerIsSub”]

[(Switch)HelperPowerIsSub  $\xrightarrow{\text{tex}}$  “(Switch)HelperPowerIsSub”]

[(Switch)PowerIsSub  $\xrightarrow{\text{tex}}$  “(Switch)PowerIsSub”]

[ToSetEquality  $\xrightarrow{\text{tex}}$  “ToSetEquality”]

[HelperToSetEquality(t)  $\xrightarrow{\text{tex}}$  “HelperToSetEquality(t)”]

[ToSetEquality(t)  $\xrightarrow{\text{tex}}$  “ToSetEquality(t)”]

[HelperFromSetEquality  $\xrightarrow{\text{tex}}$  “HelperFromSetEquality”]

[FromSetEquality  $\xrightarrow{\text{tex}}$  “FromSetEquality”]

[HelperReflexivity  $\xrightarrow{\text{tex}}$  “HelperReflexivity”]

[Reflexivity  $\xrightarrow{\text{tex}}$  “Reflexivity”]

[HelperSymmetry  $\xrightarrow{\text{tex}}$  “HelperSymmetry”]

[Symmetry  $\xrightarrow{\text{tex}}$  “Symmetry”]

[HelperTransitivity  $\xrightarrow{\text{tex}}$  “HelperTransitivity”]

[Transitivity  $\xrightarrow{\text{tex}}$  “Transitivity”],

[ERisReflexive  $\xrightarrow{\text{tex}}$  “ERisReflexive”]

[ERisSymmetric  $\xrightarrow{\text{tex}}$  “ERisSymmetric”]

[ERisTransitive  $\xrightarrow{\text{tex}}$  “ERisTransitive”]

[ $\emptyset$ isSubset  $\xrightarrow{\text{tex}}$  “ $\emptyset$ isSubset”]

[HelperMemberNot $\emptyset$   $\xrightarrow{\text{tex}}$  “HelperMemberNot $\emptyset$ ”]

[MemberNot $\emptyset$   $\xrightarrow{\text{tex}}$  “MemberNot $\emptyset$ ”]

[HelperUnique $\emptyset$   $\xrightarrow{\text{tex}}$  “HelperUnique $\emptyset$ ”]

[Unique $\emptyset$   $\xrightarrow{\text{tex}}$  “Unique $\emptyset$ ”]

[== Reflexivity  $\xrightarrow{\text{tex}}$  “==\!{ }Reflexivity”]

[== Symmetry  $\xrightarrow{\text{tex}}$  “==\!{ }Symmetry”]

[Helper == Transitivity  $\xrightarrow{\text{tex}}$  “Helper\!{ } ==\!{ }Transitivity”]

[==Transitivity  $\xrightarrow{\text{tex}}$  “\!\{ }==\!\{ }Transitivity”]

[HelperTransferNotEq  $\xrightarrow{\text{tex}}$  “HelperTransferNotEq”]

[TransferNotEq  $\xrightarrow{\text{tex}}$  “TransferNotEq”]

[HelperPairSubset  $\xrightarrow{\text{tex}}$  “HelperPairSubset”]

[Helper(2)PairSubset  $\xrightarrow{\text{tex}}$  “Helper(2)PairSubset”]

[PairSubset  $\xrightarrow{\text{tex}}$  “PairSubset”]

[SamePair  $\xrightarrow{\text{tex}}$  “SamePair”]

[SameSingleton  $\xrightarrow{\text{tex}}$  “SameSingleton”]

[UnionSubset  $\xrightarrow{\text{tex}}$  “UnionSubset”]

[SameUnion  $\xrightarrow{\text{tex}}$  “SameUnion”]

[SeparationSubset  $\xrightarrow{\text{tex}}$  “SeparationSubset”]

[SameSeparation  $\xrightarrow{\text{tex}}$  “SameSeparation”]

[SameBinaryUnion  $\xrightarrow{\text{tex}}$  “SameBinaryUnion”]

[IntersectionSubset  $\xrightarrow{\text{tex}}$  “IntersectionSubset”]

[SameIntersection  $\xrightarrow{\text{tex}}$  “SameIntersection”]

[AutoMember  $\xrightarrow{\text{tex}}$  “AutoMember”]

[HelperEqSysNot $\emptyset$   $\xrightarrow{\text{tex}}$  “HelperEqSysNot\O{ }”]

[EqSysNot $\emptyset$   $\xrightarrow{\text{tex}}$  “EqSysNot\O{ }”]

[HelperEqSubset  $\xrightarrow{\text{tex}}$  “HelperEqSubset”]

[EqSubset  $\xrightarrow{\text{tex}}$  “EqSubset”]

[EqNecessary  $\xrightarrow{\text{tex}}$  “EqNecessary”]

[HelperEqNecessary  $\xrightarrow{\text{tex}}$  “HelperEqNecessary”]

[HelperNoneEqNecessary  $\xrightarrow{\text{tex}}$  “HelperNoneEqNecessary”]

[Helper(2)NoneEqNecessary  $\xrightarrow{\text{tex}}$  “Helper(2)NoneEqNecessary”]

[NoneEqNecessary  $\xrightarrow{\text{tex}}$  “NoneEqNecessary”]



[EqClassIsSubset  $\xrightarrow{\text{tex}}$  “EqClassIsSubset”]

[EqClassesAreDisjoint  $\xrightarrow{\text{tex}}$  “EqClassesAreDisjoint”]

[AllDisjoint  $\xrightarrow{\text{tex}}$  “AllDisjoint”]

[AllDisjointImply  $\xrightarrow{\text{tex}}$  “AllDisjointImply”]

[BSsubset  $\xrightarrow{\text{tex}}$  “BSsubset”]

[Union(BS/R)subset  $\xrightarrow{\text{tex}}$  “Union(BS/R)subset”]

[UnionIdentity  $\xrightarrow{\text{tex}}$  “UnionIdentity”]

[EqSysIsPartition  $\xrightarrow{\text{tex}}$  “EqSysIsPartition”]

[x/y  $\xrightarrow{\text{tex}}$  “#1.  
/ #2.”]

[x  $\cap$  y  $\xrightarrow{\text{tex}}$  “#1.  
\cap #2.”]

[ $\cup$ x  $\xrightarrow{\text{tex}}$  “\cup #1.”]

[x  $\cup$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\cup} #2.”]

[P(x)  $\xrightarrow{\text{tex}}$  “P(#1.  
)”]

[{x}  $\xrightarrow{\text{tex}}$  “\{#1.  
\}”]

[{x, y}  $\xrightarrow{\text{tex}}$  “\{#1.  
, #2.  
\}”]

[<x, y>  $\xrightarrow{\text{tex}}$  “\langle #1.  
, #2.  
\rangle”],

[x  $\in$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\in} #2.”]

[z(x, y)  $\xrightarrow{\text{tex}}$  “#3.  
(#1.  
, #2.  
)”]

[RefRel(r, x)  $\xrightarrow{\text{tex}}$  “RefRel(#1.  
, #2.  
)”]

[SymRel(r, x)  $\xrightarrow{\text{tex}}$  “SymRel(#1.  
, #2.  
)”]

[TransRel(r, x)  $\xrightarrow{\text{tex}}$  “TransRel(#1.  
, #2.  
)”]

[EqRel(r, x)  $\xrightarrow{\text{tex}}$  “EqRel(#1.  
, #2.  
)”]

[[x  $\in$  bs]<sub>r</sub>  $\xrightarrow{\text{tex}}$  “[#1.  
\mathrel{\in} #2.  
]-{#3.  
}”]

[Partition(x, y)  $\xrightarrow{\text{tex}}$  “Partition(#1.  
, #2.  
)”]

[x == y  $\xrightarrow{\text{tex}}$  “#1.  
\!\mathrel{=} #2.”]

[x  $\subseteq$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\subseteq} #2.”]

[ $\dot{\neg}(x)$ n  $\xrightarrow{\text{tex}}$  “\dot{\neg}\, (#1.  
n)”]

[x  $\notin$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\notin} #2.”]

[x  $\neq$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\neq} #2.”]

[x  $\dot{\wedge}$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\dot{\wedge}} #2.”]

[x  $\dot{\vee}$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\dot{\vee}} #2.”]

[x  $\dot{\leftrightarrow}$  y  $\xrightarrow{\text{tex}}$  “#1.  
\mathrel{\dot{\leftrightarrow}} #2.”]

[{ph ∈ x | a}  $\xrightarrow{\text{tex}}$  “ \{ ph \mathrel{\in} \} #1.  
\mid #2.  
\}”]

[x ⇒ y  $\xrightarrow{\text{tex}}$  “(i#1.  
\Rightarrow #2.  
i”]

[Nat(x)  $\xrightarrow{\text{tex}}$  “Nat(#1.  
)”]

[(x≡y|z:=u)<sub>Me</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.  
\equiv #2.  
| #3.  
\{:=\} #4.  
\rangle\_{\text{Me}}”]

[(x≡<sup>1</sup>y|z:=u)<sub>Me</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.  
\equiv<sup>1</sup> #2.  
| #3.  
\{:=\} #4.  
\rangle\_{\text{Me}} ”]

[(x≡\*y|z:=u)<sub>Me</sub>  $\xrightarrow{\text{tex}}$  “\langle #1.  
\equiv<sup>\*</sup> #2.  
| #3.  
\{:=\} #4.  
\rangle\_{\text{Me}} ”]

[∃x:y  $\xrightarrow{\text{tex}}$  “  
\exists #1.  
: #2.”]

[(x1)  $\xrightarrow{\text{tex}}$  “(x1)”]

[(x2)  $\xrightarrow{\text{tex}}$  “(x2)”]

[(y1)  $\xrightarrow{\text{tex}}$  “(y1)”]

[(y2)  $\xrightarrow{\text{tex}}$  “(y2)”]

[(v1)  $\xrightarrow{\text{tex}}$  “(v1)”]

[(v2)  $\xrightarrow{\text{tex}}$  “(v2)”]

[(v3)  $\xrightarrow{\text{tex}}$  “(v3)”]

[(v4)  $\xrightarrow{\text{tex}}$  “(v4)”]

$[(v2n) \xrightarrow{\text{tex}} "(v2n)"]$

$[(n1) \xrightarrow{\text{tex}} "(n1)"]$

$[(n2) \xrightarrow{\text{tex}} "(n2)"]$

$[(n3) \xrightarrow{\text{tex}} "(n3)"]$

$[(m1) \xrightarrow{\text{tex}} "(m1)"]$

$[(m2) \xrightarrow{\text{tex}} "(m2)"]$

$[(\epsilon) \xrightarrow{\text{tex}} "(\backslash\epsilonpsilon)"]$

$[(\epsilon)_1 \xrightarrow{\text{tex}} "(\backslash\epsilonpsilon)_{1}"]$

$[(\epsilon 2) \xrightarrow{\text{tex}} "(\backslash\epsilonpsilon 2)"]$

$[(fx) \xrightarrow{\text{tex}} "(fx)"]$

$[(fy) \xrightarrow{\text{tex}} "(fy)"]$

$[(fz) \xrightarrow{\text{tex}} "(fz)"]$

$[(fu) \xrightarrow{\text{tex}} "(fu)"]$

$[(fv) \xrightarrow{\text{tex}} "(fv)"]$

$[(fw) \xrightarrow{\text{tex}} "(fw)"]$

$[(fep) \xrightarrow{\text{tex}} "(fep)"]$

$[(rx) \xrightarrow{\text{tex}} "(rx)"]$

$[(ry) \xrightarrow{\text{tex}} "(ry)"]$

$[(rz) \xrightarrow{\text{tex}} "(rz)"]$

$[(ru) \xrightarrow{\text{tex}} "(ru)"]$

$[(sx) \xrightarrow{\text{tex}} "(sx)"]$

$[(sx1) \xrightarrow{\text{tex}} "(sx1)"]$

$[(sy) \xrightarrow{\text{tex}} "(sy)"]$

$[(sy1) \xrightarrow{\text{tex}} "(sy1)"]$

$[(sz) \xrightarrow{\text{tex}} "(sz)"]$

$[(sz1) \xrightarrow{\text{tex}} \text{"(sz1)"}]$

$[(su) \xrightarrow{\text{tex}} \text{"(su)"}]$

$[(su1) \xrightarrow{\text{tex}} \text{"(su1)"}]$

$[(fxs) \xrightarrow{\text{tex}} \text{"(fxs)"}]$

$[(fys) \xrightarrow{\text{tex}} \text{"(fys)"}]$

$[(crs1) \xrightarrow{\text{tex}} \text{"(crs1)"}]$

$[(f1) \xrightarrow{\text{tex}} \text{"(f1)"}]$

$[(f2) \xrightarrow{\text{tex}} \text{"(f2)"}]$

$[(f3) \xrightarrow{\text{tex}} \text{"(f3)"}]$

$[(f4) \xrightarrow{\text{tex}} \text{"(f4)"}]$

$[(op1) \xrightarrow{\text{tex}} \text{"(op1)"}]$

$[(op2) \xrightarrow{\text{tex}} \text{"(op2)"}]$

$[(r1) \xrightarrow{\text{tex}} \text{"(r1)"}]$

$[(s1) \xrightarrow{\text{tex}} \text{"(s1)"}]$

$[(s2) \xrightarrow{\text{tex}} \text{"(s2)"}]$

$[X_1 \xrightarrow{\text{tex}} \text{"X_{1}"}]$

$[X_2 \xrightarrow{\text{tex}} \text{"X_{2}"}]$

$[Y_1 \xrightarrow{\text{tex}} \text{"Y_{1}"}]$

$[Y_2 \xrightarrow{\text{tex}} \text{"Y_{2}"}]$

$[V_1 \xrightarrow{\text{tex}} \text{"V_{1}"}]$

$[V_2 \xrightarrow{\text{tex}} \text{"V_{2}"}]$

$[V_3 \xrightarrow{\text{tex}} \text{"V_{3}"}]$

$[V_4 \xrightarrow{\text{tex}} \text{"V_{4}"}]$

$[V_{2n} \xrightarrow{\text{tex}} \text{"V_{2n}"}]$

$[\epsilon \xrightarrow{\text{tex}} \text{"\epsilon"}]$

[M<sub>1</sub>  $\xrightarrow{\text{tex}}$  “M\_{1}”]

[M<sub>2</sub>  $\xrightarrow{\text{tex}}$  “M\_{2}”]

[N<sub>1</sub>  $\xrightarrow{\text{tex}}$  “N\_{1} ”]

[N<sub>2</sub>  $\xrightarrow{\text{tex}}$  “N\_{2} ”]

[N<sub>3</sub>  $\xrightarrow{\text{tex}}$  “N\_{3} ”]

[e1  $\xrightarrow{\text{tex}}$  “\epsilon 1”]

[e2  $\xrightarrow{\text{tex}}$  “\epsilon 2”]

[FX  $\xrightarrow{\text{tex}}$  “FX”]

[FY  $\xrightarrow{\text{tex}}$  “FY”]

[FZ  $\xrightarrow{\text{tex}}$  “FZ”]

[FU  $\xrightarrow{\text{tex}}$  “FU”]

[FV  $\xrightarrow{\text{tex}}$  “FV”]

[FW  $\xrightarrow{\text{tex}}$  “FW”]

[FEP  $\xrightarrow{\text{tex}}$  “FEP”]

[RX  $\xrightarrow{\text{tex}}$  “RX”]

[RY  $\xrightarrow{\text{tex}}$  “RY”]

[RZ  $\xrightarrow{\text{tex}}$  “RZ”]

[RU  $\xrightarrow{\text{tex}}$  “RU”]

[(SX)  $\xrightarrow{\text{tex}}$  “(SX)”]

[(SX1)  $\xrightarrow{\text{tex}}$  “(SX1)”]

[(SY)  $\xrightarrow{\text{tex}}$  “(SY)”]

[(SY1)  $\xrightarrow{\text{tex}}$  “(SY1)”]

[(SZ)  $\xrightarrow{\text{tex}}$  “(SZ)”]

[(SZ1)  $\xrightarrow{\text{tex}}$  “(SZ1)”]

[(SU)  $\xrightarrow{\text{tex}}$  “(SU)”]

[(SU1)  $\xrightarrow{\text{tex}}$  “(SU1)”]

[FXS  $\xrightarrow{\text{tex}}$  “FXS”]

[FYS  $\xrightarrow{\text{tex}}$  “FYS”]

[(F1)  $\xrightarrow{\text{tex}}$  “(F1)”]

[(F2)  $\xrightarrow{\text{tex}}$  “(F2)”]

[(F3)  $\xrightarrow{\text{tex}}$  “(F3)”]

[(F4)  $\xrightarrow{\text{tex}}$  “(F4)”]

[(OP1)  $\xrightarrow{\text{tex}}$  “(OP1)”]

[(OP2)  $\xrightarrow{\text{tex}}$  “(OP2)”]

[(R1)  $\xrightarrow{\text{tex}}$  “(R1)”]

[(S1)  $\xrightarrow{\text{tex}}$  “(S1)”]

[(S2)  $\xrightarrow{\text{tex}}$  “(S2)”]

[(EPob)  $\xrightarrow{\text{tex}}$  “(EPob)”]

[(CRS1ob)  $\xrightarrow{\text{tex}}$  “(CRS1ob)”]

[(F1ob)  $\xrightarrow{\text{tex}}$  “(F1ob)”]

[(F2ob)  $\xrightarrow{\text{tex}}$  “(F2ob)”]

[(F3ob)  $\xrightarrow{\text{tex}}$  “(F3ob)”]

[(F4ob)  $\xrightarrow{\text{tex}}$  “(F4ob)”]

[(N1ob)  $\xrightarrow{\text{tex}}$  “(N1ob)”]

[(N2ob)  $\xrightarrow{\text{tex}}$  “(N2ob)”]

[(OP1ob)  $\xrightarrow{\text{tex}}$  “(OP1ob)”]

[(OP2ob)  $\xrightarrow{\text{tex}}$  “(OP2ob)”]

[(R1ob)  $\xrightarrow{\text{tex}}$  “(R1ob)”]

[(S1ob)  $\xrightarrow{\text{tex}}$  “(S1ob)”]

[(S2ob)  $\xrightarrow{\text{tex}}$  “(S2ob)”]

[Ex3  $\xrightarrow{\text{tex}}$  “Ex3”]

[NAT  $\xrightarrow{\text{tex}}$  “NAT”]

[RATIONALSERIES  $\xrightarrow{\text{tex}}$  “RATIONAL\_SERIES”]

[SERIES  $\xrightarrow{\text{tex}}$  “SERIES”]

[SetOfReals  $\xrightarrow{\text{tex}}$  “SetOfReals”]

[SetOfFxs  $\xrightarrow{\text{tex}}$  “SetOfFxs”]

[N  $\xrightarrow{\text{tex}}$  “N”]

[Q  $\xrightarrow{\text{tex}}$  “Q”]

[X  $\xrightarrow{\text{tex}}$  “X”]

[xs  $\xrightarrow{\text{tex}}$  “xs”]

[xaF  $\xrightarrow{\text{tex}}$  “xaF”]

[ysF  $\xrightarrow{\text{tex}}$  “ysF”]

[us  $\xrightarrow{\text{tex}}$  “us”]

[usFoelge  $\xrightarrow{\text{tex}}$  “usFoelge”]

[0  $\xrightarrow{\text{tex}}$  “0”]

[1  $\xrightarrow{\text{tex}}$  “1”]

[(-1)  $\xrightarrow{\text{tex}}$  “(-1)”]

[2  $\xrightarrow{\text{tex}}$  “2”]

[3  $\xrightarrow{\text{tex}}$  “3”]

[1/2  $\xrightarrow{\text{tex}}$  “1/2”]

[1/3  $\xrightarrow{\text{tex}}$  “1/3”]

[2/3  $\xrightarrow{\text{tex}}$  “2/3”]

[0f  $\xrightarrow{\text{tex}}$  “0f”]

[00  $\xrightarrow{\text{tex}}$  “00”]

[(- - 01)  $\xrightarrow{\text{tex}}$  “(-01)”]



[02  $\xrightarrow{\text{tex}}$  “02”]

[01//02  $\xrightarrow{\text{tex}}$  “01//02”]

[x = y  $\xrightarrow{\text{tex}}$  “#1.  
= #2.”]

[x ≠ y  $\xrightarrow{\text{tex}}$  “#1.  
\neq #2.”]

[x < y  $\xrightarrow{\text{tex}}$  “#1.  
< #2.”]

[x <= y  $\xrightarrow{\text{tex}}$  “#1.  
<= #2.”]

[x <<sub>f</sub> y  $\xrightarrow{\text{tex}}$  “#1.  
<\_{f} #2.”]

[x ≤<sub>f</sub> y  $\xrightarrow{\text{tex}}$  “#1.  
\leq\_{f} #2.”]

[SF(x,y)  $\xrightarrow{\text{tex}}$  “SF(#1.  
, #2.  
)”]

[x == y  $\xrightarrow{\text{tex}}$  “#1.  
== #2.”]

[x!! == y  $\xrightarrow{\text{tex}}$  “#1.  
!!== #2.”]

[x << y  $\xrightarrow{\text{tex}}$  “#1.  
<< #2.”]

[x <<== y  $\xrightarrow{\text{tex}}$  “#1.  
<<== #2.”]

[x[y]  $\xrightarrow{\text{tex}}$  “#1.  
[#2.  
]”]

[(-ux)  $\xrightarrow{\text{tex}}$  “(-u#1.  
)”]

[-<sub>f</sub>x  $\xrightarrow{\text{tex}}$  “-\_{f} #1.”]

$[(- - x) \xrightarrow{\text{tex}} “(--\#1.$   
)]”]

$[1f/x \xrightarrow{\text{tex}} “1f/\#1.”]$

$[01//tempx \xrightarrow{\text{tex}} “01//temp\#1.”]$

$[(x + y) \xrightarrow{\text{tex}} “(\#1.$   
+ $\#2.$   
)]”]

$[(x - y) \xrightarrow{\text{tex}} “(\#1.$   
- $\#2.$   
)]”]

$[(fx) +_f (fy) \xrightarrow{\text{tex}} “\#1.$   
+ $_{-}\{f\}\#2.”]$

$[(fx) -_f (fy) \xrightarrow{\text{tex}} “\#1.$   
- $_{-}\{f\}\#2.”]$

$[(fx) *_f (fy) \xrightarrow{\text{tex}} “\#1.$   
\* $_{-}\{f\}\#2.”]$

$[x + +y \xrightarrow{\text{tex}} “\#1.$   
++ $\#2.”]$

$[R((fx)) - -R((fy)) \xrightarrow{\text{tex}} “R(\#1.$   
) -- R( $\#2.$   
)]”]

$[(x * y) \xrightarrow{\text{tex}} “(\#1.$   
\* $\#2.$   
)]”]

$[x * *y \xrightarrow{\text{tex}} “\#1.$   
\*\* $\#2.”]$

$[x(\text{exp})y \xrightarrow{\text{tex}} “ \#1.$   
( $\text{exp}$ )  $\#2.”]$

$[\text{leqReflexivity} \xrightarrow{\text{tex}} “\text{leqReflexivity}”]$

$[\text{recx} \xrightarrow{\text{tex}} “\text{rec}\#1.”]$

$[|x| \xrightarrow{\text{tex}} “|\#1.$   
|”]

[StateExpand(t, s, c)  $\xrightarrow{\text{tex}}$  “StateExpand(#1.  
, #2.  
, #3.  
)”]

[extractSeries(t)  $\xrightarrow{\text{tex}}$  “extractSeries(#1.  
)”]

[|f|x|  $\xrightarrow{\text{tex}}$  “|f#1.  
”]

[|r|x|  $\xrightarrow{\text{tex}}$  “|r#1.  
”]

[SetOfSeries(x)  $\xrightarrow{\text{tex}}$  “SetOfSeries(#1.  
)”]

[ExpandList(x, y, z)  $\xrightarrow{\text{tex}}$  “ExpandList(#1.  
, #2.  
, #3.  
)”]

[\* \* Macro(x)  $\xrightarrow{\text{tex}}$  “\*\*Macro(#1.  
)”]

[+ + Macro(x)  $\xrightarrow{\text{tex}}$  “++Macro(#1.  
)”]

[- - Macro(x)  $\xrightarrow{\text{tex}}$  “--Macro(#1.  
)”]

[<< Macro(x)  $\xrightarrow{\text{tex}}$  “<<Macro(#1.  
)”]

[|Macro(x)  $\xrightarrow{\text{tex}}$  “|Macro(#1.  
)”]

[01//Macro(x)  $\xrightarrow{\text{tex}}$  “01//Macro(#1.  
)”]

[Max(x, y)  $\xrightarrow{\text{tex}}$  “Max(#1.  
, #2.  
)”]

[Max(x, y)  $\xrightarrow{\text{tex}}$  “Max(#1.  
, #2.  
)”]

[Limit(x, y)  $\xrightarrow{\text{tex}}$  “Limit(#1.  
, #2.  
)”]

[Union(x)  $\xrightarrow{\text{tex}}$  “Union(#1.  
)”]

[if(x, y, z)  $\xrightarrow{\text{tex}}$  “if(#1.  
, #2.  
, #3.  
)”]

[IsOrderedPair(x, y, z)  $\xrightarrow{\text{tex}}$  “IsOrderedPair(#1.  
, #2.  
, #3.  
)”]

[IsRelation(x, y, z)  $\xrightarrow{\text{tex}}$  “IsRelation(#1.  
, #2.  
, #3.  
)”]

[isFunction(x, y, z)  $\xrightarrow{\text{tex}}$  “isFunction(#1.  
, #2.  
, #3.  
)”]

[TypeNat(x)  $\xrightarrow{\text{tex}}$  “TypeNat(#1.  
)”]

[TypeNat0(x)  $\xrightarrow{\text{tex}}$  “TypeNat0(#1.  
)”]

[TypeRational(x)  $\xrightarrow{\text{tex}}$  “TypeRational(#1.  
)”]

[TypeRational0(x)  $\xrightarrow{\text{tex}}$  “TypeRational0(#1.  
)”]

[TypeSeries(x, y)  $\xrightarrow{\text{tex}}$  “TypeSeries(#1.  
, #2.  
)”]

[Typeseries0(x, y)  $\xrightarrow{\text{tex}}$  “Typeseries0(#1.  
, #2.  
)”]

[UB(x, y)  $\xrightarrow{\text{tex}}$  “UB(#1.  
, #2.  
)”]

[LUB(x, y)  $\xrightarrow{\text{tex}}$  “LUB(#1.  
, #2.  
)”]

[BS(x, y)  $\xrightarrow{\text{tex}}$  “BS(#1.  
, #2.  
)”]

[UStelescope(x, y)  $\xrightarrow{\text{tex}}$  “UStelescope(#1.  
, #2.  
)”]

[(x)  $\xrightarrow{\text{tex}}$  “(#1.  
)”]

[R(x)  $\xrightarrow{\text{tex}}$  “R(#1.  
)”]

[- - R(x)  $\xrightarrow{\text{tex}}$  “--R(#1.  
)”]

[IsSeries(x, y)  $\xrightarrow{\text{tex}}$  “IsSeries(#1.  
, #2.  
)”]

[IsNatural(xy, \*)  $\xrightarrow{\text{tex}}$  “IsNatural(#1.  
, #2.  
)”]

[OrderedPair(x, y)  $\xrightarrow{\text{tex}}$  “OrderedPair(#1.  
, #2.  
)”]

[leqAntisymmetryAxiom  $\xrightarrow{\text{tex}}$  “leqAntisymmetryAxiom”]

[leqTransitivityAxiom  $\xrightarrow{\text{tex}}$  “leqTransitivityAxiom”]

[leqTotality  $\xrightarrow{\text{tex}}$  “leqTotality”]

[leqAdditionAxiom  $\xrightarrow{\text{tex}}$  “leqAdditionAxiom”]

[leqMultiplicationAxiom  $\xrightarrow{\text{tex}}$  “leqMultiplicationAxiom”]

[plusAssociativity  $\xrightarrow{\text{tex}}$  “plusAssociativity”]

$[\text{plusCommutativity} \xrightarrow{\text{tex}} \text{“plusCommutativity”}]$   
 $[\text{Negative} \xrightarrow{\text{tex}} \text{“Negative”}]$   
 $[\text{plus0} \xrightarrow{\text{tex}} \text{“plus0”}]$   
 $[\text{timesAssociativity} \xrightarrow{\text{tex}} \text{“timesAssociativity”}]$   
 $[\text{timesCommutativity} \xrightarrow{\text{tex}} \text{“timesCommutativity”}]$   
 $[\text{ReciprocalAxiom} \xrightarrow{\text{tex}} \text{“ReciprocalAxiom”}]$   
 $[\text{times1} \xrightarrow{\text{tex}} \text{“times1”}]$   
 $[\text{plusAssociativity} \xrightarrow{\text{tex}} \text{“plusAssociativity”}]$   
 $[\text{plusCommutativity} \xrightarrow{\text{tex}} \text{“plusCommutativity”}]$   
 $[\text{Negative} \xrightarrow{\text{tex}} \text{“Negative”}]$   
 $[\text{Distribution} \xrightarrow{\text{tex}} \text{“Distribution”}]$   
 $[\text{0not1} \xrightarrow{\text{tex}} \text{“0not1”}]$   
 $[\text{A4(Axiom)} \xrightarrow{\text{tex}} \text{“A4(Axiom)”}]$   
 $[\text{InductionAxiom} \xrightarrow{\text{tex}} \text{“InductionAxiom”}]$   
 $[\text{EqualityAxiom} \xrightarrow{\text{tex}} \text{“EqualityAxiom”}]$   
 $[\text{EqLeqAxiom} \xrightarrow{\text{tex}} \text{“EqLeqAxiom”}]$   
 $[\text{EqAdditionAxiom} \xrightarrow{\text{tex}} \text{“EqAdditionAxiom”}]$   
 $[\text{EqMultiplicationAxiom} \xrightarrow{\text{tex}} \text{“EqMultiplicationAxiom”}]$   
 $[\text{SENC1} \xrightarrow{\text{tex}} \text{“SENC1”}]$   
 $[\text{SENC2} \xrightarrow{\text{tex}} \text{“SENC2”}]$   
 $[\text{Cauchy} \xrightarrow{\text{tex}} \text{“Cauchy”}]$   
 $[\text{PlusF} \xrightarrow{\text{tex}} \text{“PlusF”}]$   
 $[\text{ReciprocalF} \xrightarrow{\text{tex}} \text{“ReciprocalF”}]$   
 $[\text{From} \xrightarrow{\text{tex}} \text{“From”}]$   
 $[\text{To} \xrightarrow{\text{tex}} \text{“To”}]$

[FromInR  $\xrightarrow{\text{tex}}$  “FromInR”]

[ReciprocalR(Axiom)  $\xrightarrow{\text{tex}}$  “ReciprocalR(Axiom)”]

[US0  $\xrightarrow{\text{tex}}$  “US0”]

[NextXS(UpperBound)  $\xrightarrow{\text{tex}}$  “NextXS(UpperBound)”]

[NextXS(NoUpperBound)  $\xrightarrow{\text{tex}}$  “NextXS(NoUpperBound)”]

[NextUS(UpperBound)  $\xrightarrow{\text{tex}}$  “NextUS(UpperBound)”]

[NextUS(NoUpperBound)  $\xrightarrow{\text{tex}}$  “NextUS(NoUpperBound)”]

[ExpZero  $\xrightarrow{\text{tex}}$  “ExpZero”]

[ExpPositive  $\xrightarrow{\text{tex}}$  “ExpPositive”]

[ExpZero(R)  $\xrightarrow{\text{tex}}$  “ExpZero(R)”]

[ExpPositive(R)  $\xrightarrow{\text{tex}}$  “ExpPositive(R)”]

[LessMinus1(N)  $\xrightarrow{\text{tex}}$  “LessMinus1(N)”]

[Nonnegative(N)  $\xrightarrow{\text{tex}}$  “Nonnegative(N)”]

[BSzero  $\xrightarrow{\text{tex}}$  “BSzero”]

[BSpositive  $\xrightarrow{\text{tex}}$  “BSpositive”]

[USTelescope(Zero)  $\xrightarrow{\text{tex}}$  “USTelescope(Zero)”]

[USTelescope(Positive)  $\xrightarrow{\text{tex}}$  “USTelescope(Positive)”]

[EqAddition(R)  $\xrightarrow{\text{tex}}$  “EqAddition(R)”]

[FromLimit  $\xrightarrow{\text{tex}}$  “FromLimit”]

[ToUpperBound  $\xrightarrow{\text{tex}}$  “ToUpperBound”]

[FromUpperBound  $\xrightarrow{\text{tex}}$  “FromUpperBound”]

[USisUpperBound  $\xrightarrow{\text{tex}}$  “USisUpperBound”]

[0not1(R)  $\xrightarrow{\text{tex}}$  “0not1(R)”]

[ExpUnbounded(R)  $\xrightarrow{\text{tex}}$  “ExpUnbounded(R)”]

[FromLeq(Advanced)(N)  $\xrightarrow{\text{tex}}$  “FromLeq(Advanced)(N)”]

$[FromLeastUpperBound \xrightarrow{\text{tex}} \text{“FromLeastUpperBound”}]$   
 $[ToLeastUpperBound \xrightarrow{\text{tex}} \text{“ToLeastUpperBound”}]$   
 $[XSisNotUpperBound \xrightarrow{\text{tex}} \text{“XSisNotUpperBound”}]$   
 $[ysFGreater \xrightarrow{\text{tex}} \text{“ysFGreater”}]$   
 $[ysFLess \xrightarrow{\text{tex}} \text{“ysFLess”}]$   
 $[SmallInverse \xrightarrow{\text{tex}} \text{“SmallInverse”}]$   
 $[MemberOfSeries(ImPLY) \xrightarrow{\text{tex}} \text{“MemberOfSeries(ImPLY)”}]$   
 $[NatType \xrightarrow{\text{tex}} \text{“NatType”}]$   
 $[RationalType \xrightarrow{\text{tex}} \text{“RationalType”}]$   
 $[SeriesType \xrightarrow{\text{tex}} \text{“SeriesType”}]$   
 $[JoinConjuncts(2conditions) \xrightarrow{\text{tex}} \text{“JoinConjuncts(2conditions)”}]$   
 $[TND \xrightarrow{\text{tex}} \text{“TND”}]$   
 $[FromNegatedImPLY \xrightarrow{\text{tex}} \text{“FromNegatedImPLY”}]$   
 $[ToNegatedImPLY \xrightarrow{\text{tex}} \text{“ToNegatedImPLY”}]$   
 $[FromNegated(2 * ImPLY) \xrightarrow{\text{tex}} \text{“FromNegated(2*ImPLY)”}]$   
 $[FromNegatedAnd \xrightarrow{\text{tex}} \text{“FromNegatedAnd”}]$   
 $[FromNegatedOr \xrightarrow{\text{tex}} \text{“FromNegatedOr”}]$   
 $[ToNegatedOr \xrightarrow{\text{tex}} \text{“ToNegatedOr”}]$   
 $[FromNegations \xrightarrow{\text{tex}} \text{“FromNegations”}]$   
 $[From3Disjuncts \xrightarrow{\text{tex}} \text{“From3Disjuncts”}]$   
 $[NegateDisjunct1 \xrightarrow{\text{tex}} \text{“NegateDisjunct1”}]$   
 $[NegateDisjunct2 \xrightarrow{\text{tex}} \text{“NegateDisjunct2”}]$   
 $[ExpandDisjuncts \xrightarrow{\text{tex}} \text{“ExpandDisjuncts”}]$   
 $[From2 * 2Disjuncts \xrightarrow{\text{tex}} \text{“From2*2Disjuncts”}]$   
 $[PlusR(Sym) \xrightarrow{\text{tex}} \text{“PlusR(Sym)”}]$



[LessLeq(R)  $\xrightarrow{\text{tex}}$  “LessLeq(R)”]

[LeqAntisymmetry(R)  $\xrightarrow{\text{tex}}$  “LeqAntisymmetry(R)”]

[LeqTransitivity(R)  $\xrightarrow{\text{tex}}$  “LeqTransitivity(R)”]

[Plus0(R)  $\xrightarrow{\text{tex}}$  “Plus0(R)”]

[lessAddition(R)  $\xrightarrow{\text{tex}}$  “lessAddition(R)”]

[leqAddition(R)  $\xrightarrow{\text{tex}}$  “leqAddition(R)”]

[PlusAssociativity(R)XX  $\xrightarrow{\text{tex}}$  “PlusAssociativity(R)XX”]

[PlusAssociativity(R)  $\xrightarrow{\text{tex}}$  “PlusAssociativity(R)”]

[Negative(R)  $\xrightarrow{\text{tex}}$  “Negative(R)”]

[PlusCommutativity(R)  $\xrightarrow{\text{tex}}$  “PlusCommutativity(R)”]

[Times1(R)  $\xrightarrow{\text{tex}}$  “Times1(R)”]

[TimesAssociativity(R)  $\xrightarrow{\text{tex}}$  “TimesAssociativity(R)”]

[TimesCommutativity(R)  $\xrightarrow{\text{tex}}$  “TimesCommutativity(R)”]

[Distribution(R)  $\xrightarrow{\text{tex}}$  “Distribution(R)”]

[ $\exists x: y \xrightarrow{\text{tex}}$  “(AARRGGHH!-exist-bug!”]

[constantRationalSeries(x)  $\xrightarrow{\text{tex}}$  “constantRationalSeries(#1.  
)”]

[Power(x)  $\xrightarrow{\text{tex}}$  “Power(#1.  
)”]

[cartProd(x)  $\xrightarrow{\text{tex}}$  “cartProd(#1.  
)”]

[binaryUnion(x, y)  $\xrightarrow{\text{tex}}$  “binaryUnion(#1.  
, #2.  
)”]

[SetOfRationalSeries  $\xrightarrow{\text{tex}}$  “SetOfRationalSeries”]

[MemberOfSeries  $\xrightarrow{\text{tex}}$  “MemberOfSeries”]

[IsSubset(x, y)  $\xrightarrow{\text{tex}}$  “IsSubset(#1.  
, #2.  
)”]

[memberOfSeries(Type)  $\xrightarrow{\text{tex}}$  “memberOfSeries(Type)”]

[UniqueMember  $\xrightarrow{\text{tex}}$  “UniqueMember”]

[UniqueMember(Type)  $\xrightarrow{\text{tex}}$  “UniqueMember(Type)”]

[SameSeries  $\xrightarrow{\text{tex}}$  “SameSeries”]

[A4  $\xrightarrow{\text{tex}}$  “A4”]

[((sx)  $\xrightarrow{\text{tex}}$  “(s#1.  
)”]

[((px, y)  $\xrightarrow{\text{tex}}$  “(p#1.  
, #2.  
)”]

[SameMember  $\xrightarrow{\text{tex}}$  “SameMember”]

[Qclosed(Addition)  $\xrightarrow{\text{tex}}$  “Qclosed(Addition)”]

[Qclosed(Multiplication)  $\xrightarrow{\text{tex}}$  “Qclosed(Multiplication)”]

[FromCartProd(1)  $\xrightarrow{\text{tex}}$  “FromCartProd(1)”]

[FromCartProd(1)  $\xrightarrow{\text{tex}}$  “FromCartProd(1)”]

[Max  $\xrightarrow{\text{tex}}$  “Max”]

[Numerical  $\xrightarrow{\text{tex}}$  “Numerical”]

[NumericalF  $\xrightarrow{\text{tex}}$  “NumericalF”]

[Separation2formula(1)  $\xrightarrow{\text{tex}}$  “Separation2formula(1)”]

[Separation2formula(2)  $\xrightarrow{\text{tex}}$  “Separation2formula(2)”]

[QisClosed(Reciprocal)(ImPLY)  $\xrightarrow{\text{tex}}$  “QisClosed(Reciprocal)(ImPLY)”]

[QisClosed(Reciprocal)  $\xrightarrow{\text{tex}}$  “QisClosed(Reciprocal)”]

[QisClosed(Negative)(ImPLY)  $\xrightarrow{\text{tex}}$  “QisClosed(Negative)(ImPLY)”]

[QisClosed(Negative)  $\xrightarrow{\text{tex}}$  “QisClosed(Negative)”]

$[(\text{Adgic})\text{SameR} \xrightarrow{\text{tex}} \text{“}(\text{Adgic})\text{SameR”}]$