

(\*\* MAKROER BEGYNDER \*\*)

[ph<sub>1</sub> ≐ a<sub>Ph</sub>]

[ph<sub>2</sub> ≐ b<sub>Ph</sub>]

[ph<sub>3</sub> ≐ c<sub>Ph</sub>]

[ph<sub>4</sub> ≐ d<sub>Ph</sub>]

[ph<sub>5</sub> ≐ e<sub>Ph</sub>]

[ph<sub>6</sub> ≐ f<sub>Ph</sub>]

[x ∧ y ≐ ≐ ((x ⇒ ≐ (y)n)n)]

[x ∨ y ≐ ≐ (x)n ⇒ y]

[x ⇔ y ≐ (x ⇒ y) ∧ (y ⇒ x)]

[x ≠ y ≐ ≐ (x==y)n]

[x ∉ y ≐ ≐ (x ∈ y)n]

[x ⊆ y ≐ ≐ (S1ob): ((S1ob) ∈ x ⇒ (S1ob) ∈ y)]

[{x} ≐ {x, x}]

[x ∪ y ≐ ∪ {{x}, {y}}]

[x ∩ y ≐ {ph ∈ x ∪ y | ph<sub>3</sub> ∈ x ∧ ph<sub>3</sub> ∈ y}]

[⟨x, y⟩ ≐ {{x}, {x, y}}]

[r(x, y) ≐ ⟨x, y⟩ ∈ r]

[ReflRel(r, x) ≐ ≐ (s ∈ x ⇒ r(s, s))]

[SymRel(r, x) ≐ ≐ (s ∈ x ⇒ t ∈ x ⇒ r(s, t) ⇒ r(t, s))]

[TransRel(r, x) ≐

≐ (s, t, u: (s ∈ x ⇒ t ∈ x ⇒ u ∈ x ⇒ r(s, t) ⇒ r(t, u) ⇒ r(s, u)))]

[EqRel(r, x) ≐ ReflRel(r, x) ∧ SymRel(r, x) ∧ TransRel(r, x)]

[BS ≐ bs]

[OBS ≐ bs]

[[x ∈ bs]<sub>r</sub> ≐ {ph ∈ bs | r(ph<sub>1</sub>, x)}]

[bs/r ≐ {ph ∈ P(bs) | Ex<sub>20</sub> ∈ bs ∧ [Ex<sub>20</sub> ∈ bs]<sub>r</sub> == ph<sub>2</sub>}]

[Partition(p, bs) ≐ (≐ (s ∈ p ⇒ s ≠ ∅)) ∧

(≐ (s, t: (s ∈ p ⇒ t ∈ p ⇒ s ≠ t ⇒ s ∩ t == ∅)) ∧

∪ p == bs]

(\*\* EKSISTENS-VARIABLE \*\*)

[x<sup>Ex</sup> ≐ x <sup>r</sup> [x<sub>Ex</sub>]]

[EX<sub>1</sub> ≐ a<sub>Ex</sub>]

[EX<sub>2</sub> ≐ b<sub>Ex</sub>]

[EX<sub>10</sub> ≐ j<sub>Ex</sub>]

[EX<sub>20</sub> ≐ t<sub>Ex</sub>]

[⟨a≐b|x:==t⟩<sub>Ex</sub> ≐ ⟨[a]≐<sup>0</sup>[b]||[x]:==[t]⟩<sub>Ex</sub>]

[⟨a≐<sup>0</sup>b|x:==t⟩<sub>Ex</sub> ≐ λc.x<sup>Ex</sup> ∧ ⟨a≐<sup>1</sup>b|x:==t⟩<sub>Ex</sub>]

$[(a \equiv^1 b | x := t)_{\text{Ex}} \doteq a!x!t!]$

**if**  $b \stackrel{r}{\equiv} [\forall u: v]$  **then**  $F$  **else**

**if**  $b^{\text{Ex}} \wedge b \stackrel{t}{\equiv} x$  **then**  $a \stackrel{t}{\equiv} t$  **else**

$a \stackrel{r}{\equiv} b \wedge (a^t \equiv^* b^t | x := t)_{\text{Ex}}$

$[(a \equiv^* b | x := t)_{\text{Ex}} \doteq b!x!t! \text{If}(a, T, (a^h \equiv^1 b^h | x := t)_{\text{Ex}} \wedge (a^t \equiv^* b^t | x := t)_{\text{Ex}})]$

(\*\*\* AKSIOMATISK SYSTEM \*\*\*)

[Theory SystemQ]

[SystemQ rule MP:  $\Pi A, B: A \Rightarrow B \vdash A \vdash B$ ]

[SystemQ rule Gen:  $\Pi \mathcal{X}, A: A \vdash \forall \mathcal{X}: A$ ]

[SystemQ rule Repetition:  $\Pi A: A \vdash A$ ]

[SystemQ rule Neg:  $\Pi A, B: \dot{\neg}(B)_n \Rightarrow A \vdash \dot{\neg}(B)_n \Rightarrow \dot{\neg}(A)_n \vdash B$ ]

[SystemQ rule Ded:  $\Pi A, B: A \vdash B$ ]

[SystemQ rule ExistIntro:  $\Pi \mathcal{X}, T, A, B: (A \equiv B | \mathcal{X} := T)_{\text{Ex}} \Vdash A \vdash B$ ]

[SystemQ rule Extensionality:  $\Pi \mathcal{X}, \mathcal{Y}: \mathcal{X} == \mathcal{Y} \Leftrightarrow \forall s: (s \in \mathcal{X} \Leftrightarrow s \in \mathcal{Y})$ ]

[SystemQ rule  $\emptyset$ def:  $\Pi S: \dot{\neg}(S \in \emptyset)_n$ ]

[SystemQ rule PairDef:  $\Pi S, \mathcal{X}, \mathcal{Y}: S \in \{\mathcal{X}, \mathcal{Y}\} \Leftrightarrow S == \mathcal{X} \dot{\vee} S == \mathcal{Y}$ ]

[SystemQ rule UnionDef:  $\Pi S, \mathcal{X}: S \in \cup \mathcal{X} \Leftrightarrow (S \in \text{Ex}_{10} \wedge \text{Ex}_{10} \in \mathcal{X})$ ]

[SystemQ rule PowerDef:  $\Pi S, \mathcal{X}: S \in P(\mathcal{X}) \Leftrightarrow \forall s: (s \in S \Rightarrow s \in \mathcal{X})$ ]

[SystemQ rule SeparationDef:  $\Pi A, B, \mathcal{P}, \mathcal{X}, \mathcal{Z}: \mathcal{P}^{\text{Ph}} \wedge (B \equiv A | \mathcal{P} := \mathcal{Z})_{\text{Ph}} \Vdash \mathcal{Z} \in \{\text{ph} \in \mathcal{X} \mid A\} \Leftrightarrow \mathcal{Z} \in \mathcal{X} \wedge B$ ]

————— RRRRRRRRRRRRRRRR —————

(\*\*\* import fra A.M. \*\*\*)

[SystemQ rule TimesCommutativity(R):  $\Pi FX, FY: R(FX)**R(FY) == R(FY)*R(FX)$ ]

(\*\*\* aksiomer \*\*\*)

[SystemQ rule leqReflexivity:  $\Pi \mathcal{X}: \mathcal{X} <= \mathcal{X}$ ]

[SystemQ rule leqAntisymmetryAxiom:  $\Pi \mathcal{X}, \mathcal{Y}: \mathcal{X} <= \mathcal{Y} \Rightarrow \mathcal{Y} <= \mathcal{X} \Rightarrow \mathcal{X} = \mathcal{Y}$ ]

[SystemQ rule leqTransitivityAxiom:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: \mathcal{X} <= \mathcal{Y} \Rightarrow \mathcal{Y} <= \mathcal{Z} \Rightarrow \mathcal{X} <= \mathcal{Z}$ ]

[SystemQ rule leqTotality:  $\Pi \mathcal{X}, \mathcal{Y}: \mathcal{X} <= \mathcal{Y} \dot{\vee} \mathcal{Y} <= \mathcal{X}$ ]

[SystemQ rule leqAdditionAxiom:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: \mathcal{X} <= \mathcal{Y} \Rightarrow (\mathcal{X} + \mathcal{Z}) <= (\mathcal{Y} + \mathcal{Z})$ ]

[SystemQ rule leqMultiplicationAxiom:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: 0 <= \mathcal{Z} \Rightarrow \mathcal{X} <= \mathcal{Y} \Rightarrow (\mathcal{X} * \mathcal{Z}) <= (\mathcal{Y} * \mathcal{Z})$ ]

[SystemQ **rule** plusAssociativity:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: (((\mathcal{X} + \mathcal{Y})) + \mathcal{Z}) = (\mathcal{X} + ((\mathcal{Y} + \mathcal{Z})))$ ]

[SystemQ **rule** plusCommutativity:  $\Pi \mathcal{X}, \mathcal{Y}: (\mathcal{X} + \mathcal{Y}) = (\mathcal{Y} + \mathcal{X})$ ]

[SystemQ **rule** Negative:  $\Pi \mathcal{X}: (\mathcal{X} + ((-\text{u}\mathcal{X}))) = 0$ ]

[SystemQ **rule** plus0:  $\Pi \mathcal{X}: (\mathcal{X} + 0) = \mathcal{X}$ ]

[SystemQ **rule** timesAssociativity:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: (((\mathcal{X} * \mathcal{Y})) * \mathcal{Z}) = (\mathcal{X} * ((\mathcal{Y} * \mathcal{Z})))$ ]

[SystemQ **rule** timesCommutativity:  $\Pi \mathcal{X}, \mathcal{Y}: (\mathcal{X} * \mathcal{Y}) = (\mathcal{Y} * \mathcal{X})$ ]

[SystemQ **rule** ReciprocalAxiom:  $\Pi \mathcal{X}: \mathcal{X} \neq 0 \Rightarrow (\mathcal{X} * \text{rec}\mathcal{X}) = 1$ ]

[SystemQ **rule** times1:  $\Pi \mathcal{X}: (\mathcal{X} * 1) = \mathcal{X}$ ]

[SystemQ **rule** Distribution:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: (\mathcal{X} * ((\mathcal{Y} + \mathcal{Z}))) = (((\mathcal{X} * \mathcal{Y})) + ((\mathcal{X} * \mathcal{Z})))$ ]

[SystemQ **rule** 0not1:  $0 \neq 1$ ]

[SystemQ **rule** EqualityAxiom:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: \mathcal{X} = \mathcal{Y} \Rightarrow \mathcal{X} = \mathcal{Z} \Rightarrow \mathcal{Y} = \mathcal{Z}$ ]

[SystemQ **rule** EqLeqAxiom:  $\Pi \mathcal{X}, \mathcal{Y}: \mathcal{X} = \mathcal{Y} \Rightarrow \mathcal{X} <= \mathcal{Y}$ ]

[SystemQ **rule** EqAdditionAxiom:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: \mathcal{X} = \mathcal{Y} \Rightarrow (\mathcal{X} + \mathcal{Z}) = (\mathcal{Y} + \mathcal{Z})$ ]

[SystemQ **rule** EqMultiplicationAxiom:  $\Pi \mathcal{X}, \mathcal{Y}, \mathcal{Z}: \mathcal{X} = \mathcal{Y} \Rightarrow (\mathcal{X} * \mathcal{Z}) = (\mathcal{Y} * \mathcal{Z})$ ]

[SystemQ **rule** A4(Axiom):  $\Pi \mathcal{X}, V_1, \mathcal{A}, \mathcal{B}: \langle \mathcal{A} = \mathcal{B} | V_1 := \mathcal{X} \rangle_{\text{Me}} \Vdash \forall V_1: \mathcal{B} \Rightarrow \mathcal{A}$ ]

(\*\* XX snydeaksiomer \*\*)

[SystemQ **rule** ==Reflexivity:  $\Pi \text{RX}: \text{RX} == \text{RX}$ ]

[SystemQ **rule** ==Symmetry:  $\Pi \text{RX}, \text{RY}: \text{RX} == \text{RY} \vdash \text{RY} == \text{RX}$ ]

[SystemQ **rule** ==Transitivity:  $\Pi \text{RX}, \text{RY}, \text{RZ}: \text{RX} == \text{RY} \vdash \text{RY} == \text{RZ} \vdash \text{RX} == \text{RZ}$ ]

XX ikke 100procent identisk med originalen fra equivalence-relations [SystemQ **rule** ==Transitivity:  $\Pi \text{RX}, \text{RY}: \text{RX} == \text{RY} \vdash \text{FX} \in \text{RX} \vdash \text{FX} \in \text{RY}$ ]

XX boer bevises ud fra nummer 1 [SystemQ **rule** SENC2:  $\Pi \text{FX}, \text{RX}, \text{RY}: \text{RX} == \text{RY} \vdash \text{FX} \in \text{RY} \vdash \text{FX} \in \text{RX}$ ]

[SystemQ **rule** PlusF:  $\Pi \mathcal{M}, \text{FX}, \text{FY}: \text{FX} +_{\text{f}} \text{FY}[\mathcal{M}] = (\text{FX}[\mathcal{M}] + \text{FY}[\mathcal{M}])$ ]

[SystemQ **rule** From ==:  $\Pi \text{FX}, \text{FY}: \text{R}(\text{FX}) == \text{R}(\text{FY}) \vdash \text{SF}(\text{FX}, \text{FY})$ ]

[SystemQ **rule** To ==:  $\Pi \text{FX}, \text{FY}: \text{SF}(\text{FX}, \text{FY}) \vdash \text{R}(\text{FX}) == \text{R}(\text{FY})$ ]

[SystemQ **rule** FromInR:  $\Pi \text{FX}, \text{FY}: \text{FX} \in \text{R}(\text{FY}) \vdash \text{SF}(\text{FX}, \text{FY})$ ]

(\*\* makroer \*\*)

KVANTI

$[\overline{M_1} \doteq \overline{(m1)}] [\overline{M_2} \doteq \overline{(m2)}] [\overline{N_1} \doteq \overline{(n1)}] [\overline{N_2} \doteq \overline{(n2)}] [\overline{N_3} \doteq \overline{(n3)}] [\overline{\epsilon} \doteq \overline{(\epsilon)}]$

$[\overline{\epsilon_1} \doteq \overline{(\epsilon_1)}] [\overline{\epsilon_2} \doteq \overline{(\epsilon_2)}] [\overline{X_1} \doteq \overline{(x1)}] [\overline{X_2} \doteq \overline{(x2)}] [\overline{Y_1} \doteq \overline{(y1)}] [\overline{Y_2} \doteq \overline{(y2)}] [\overline{V_1} \doteq \overline{(v1)}]$

$[\overline{V_2} \doteq \overline{(v2)}] [\overline{V_3} \doteq \overline{(v3)}] [\overline{V_4} \doteq \overline{(v4)}] [\overline{V_{2n}} \doteq \overline{(v2n)}] [\overline{FX} \doteq \overline{(fx)}] [\overline{FY} \doteq \overline{(fy)}]$

$[\overline{FZ} \doteq \overline{(fz)}] [\overline{FU} \doteq \overline{(fu)}] [\overline{FV} \doteq \overline{(fv)}] [\overline{FW} \doteq \overline{(fw)}] [\overline{FEP} \doteq \overline{(fep)}] [\overline{RX} \doteq \overline{(rx)}]$

$[\overline{RY} \doteq \overline{(ry)}] [\overline{RZ} \doteq \overline{(rz)}] [\overline{RU} \doteq \overline{(ru)}] [\overline{(SX)} \doteq \overline{(sx)}] [\overline{(SX1)} \doteq \overline{(sx1)}] [\overline{(SY)} \doteq \overline{(sy)}]$

$[\overline{(SY1)} \doteq \overline{(sy1)}] [\overline{(SZ)} \doteq \overline{(sz)}] [\overline{(SZ1)} \doteq \overline{(sz1)}] [\overline{(SU)} \doteq \overline{(su)}] [\overline{(SU1)} \doteq \overline{(su1)}]$

$[\overline{FXS} \doteq \overline{(fxs)}] [\overline{FYS} \doteq \overline{(fys)}] [\overline{(F1)} \doteq \overline{(f1)}] [\overline{(F2)} \doteq \overline{(f2)}] [\overline{(F3)} \doteq \overline{(f3)}] [\overline{(F4)} \doteq \overline{(f4)}]$

$[\overline{(OP1)} \doteq \overline{(op1)}] [\overline{(OP2)} \doteq \overline{(op2)}] [\overline{(R1)} \doteq \overline{(r1)}] [\overline{(S1)} \doteq \overline{(s1)}] [\overline{(S2)} \doteq \overline{(s2)}]$

$[\overline{(EPob)} \doteq \overline{(\epsilon)}] [\overline{(CRS1ob)} \doteq \overline{(crs1)}] [\overline{(F1ob)} \doteq \overline{(f1)}] [\overline{(F2ob)} \doteq \overline{(f2)}] [\overline{(F3ob)} \doteq \overline{(f3)}]$

$[\overline{(F4ob)} \doteq \overline{(f4)}] [\overline{(N1ob)} \doteq \overline{(n1)}] [\overline{(N2ob)} \doteq \overline{(n2)}] [\overline{(OP1ob)} \doteq \overline{(op1)}]$

$[\overline{(OP2ob)} \doteq \overline{(op2)}] [\overline{(R1ob)} \doteq \overline{(r1)}] [\overline{(S1ob)} \doteq \overline{(s1)}] [\overline{(S2ob)} \doteq \overline{(s2)}]$

$[(fx) \leq_f (fy) \doteq (fx) <_f (fy) \dot{\vee} SF((fx), (fy))]$

[Ex3  $\doteq c_{Ex}$ ]

$[\exists(v1): a \doteq \dot{\neg}(\forall(v1): \dot{\neg}(a)n)n]$

$[x <<== y \doteq x << y \dot{\vee} x == y]$

$[(-1) \doteq (-u1)]$

$[2 \doteq (1 + 1)]$

$[3 \doteq (2 + 1)]$

$[1/2 \doteq rec2]$

$[1/3 \doteq rec3]$

$[2/3 \doteq (2 * 1/3)]$

$[x < y \doteq x <= y \wedge x \neq y]$

$[x \neq y \doteq \dot{\neg}(x = y)n]$

$[(x - y) \doteq (x + (-uy))]$

$[00 \doteq R(0f)]$

$[01 \doteq R(1f)]$

$[x!! == y \doteq \dot{\neg}(x == y)n]$

(\*\*\* REGELLEMMER \*\*\*)

(\*\*\* UDSAGNSLOGIK \*\*\*)

[SystemQ **lemma** ToNegatedImPLY:  $\Pi A, B: A \vdash \dot{\neg}(B)n \vdash \dot{\neg}((A \Rightarrow B)n)n]$

SystemQ **proof** of ToNegatedImPLY:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$A, B$	;
L03:	Premise $\gg$	$A$	;
L04:	Premise $\gg$	$\dot{\neg}(B)n$	;
L05:	Premise $\gg$	$\dot{\neg}(\dot{\neg}((A \Rightarrow B)n)n)$	;
L06:	RemoveDoubleNeg $\triangleright$ L05 $\gg$	$A \Rightarrow B$	;
L07:	MP $\triangleright$ L06 $\triangleright$ L03 $\gg$	$B$	;
L08:	FromContradiction $\triangleright$ L07 $\triangleright$ L04 $\gg$	$\dot{\neg}((A \Rightarrow B)n)$	;
L09:	Block $\gg$	End	;
L10:	Arbitrary $\gg$	$A, B$	;
L11:	Ded $\triangleright$ L09 $\gg$	$A \Rightarrow \dot{\neg}(B)n \Rightarrow \dot{\neg}(\dot{\neg}((A \Rightarrow B)n)n)$	;
L12:	Premise $\gg$	$A$	;
L13:	Premise $\gg$	$\dot{\neg}(B)n$	;
L14:	MP2 $\triangleright$ L11 $\triangleright$ L12 $\triangleright$ L13 $\gg$	$\dot{\neg}(\dot{\neg}((A \Rightarrow B)n)n) \Rightarrow \dot{\neg}((A \Rightarrow B)n)$	;
L15:	AutoImPLY $\gg$	$\dot{\neg}(\dot{\neg}((A \Rightarrow B)n)n) \Rightarrow \dot{\neg}(\dot{\neg}((A \Rightarrow B)n)n)$	;
L16:	Neg $\triangleright$ L14 $\triangleright$ L15 $\gg$	$\dot{\neg}((A \Rightarrow B)n)$	□

[SystemQ **lemma** TND:  $\Pi A: A \dot{\vee} \dot{\neg}(A)n]$

SystemQ **proof** of TND:

L01:	Arbitrary $\gg$	$A$	;
L02:	AutoImPLY $\gg$	$\dot{\neg}(A)n \Rightarrow \dot{\neg}(A)n$	;
L03:	Repetition $\triangleright$ L02 $\gg$	$A \dot{\vee} \dot{\neg}(A)n$	□

[SystemQ **lemma** FromNegations:  $\Pi A, B: A \Rightarrow B \vdash \dot{\neg}(A)n \Rightarrow B \vdash B]$

SystemQ **proof of** FromNegations:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L03:	Premise $\gg$	$\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$	;
L04:	TND $\gg$	$\mathcal{A} \dot{\vee} \dot{\neg}(\mathcal{A})_n$	;
L05:	FromDisjuncts $\triangleright$ L04 $\triangleright$ L02 $\triangleright$ L03 $\gg$	$\mathcal{B}$	□

[SystemQ **lemma** prop lemma imply negation:  $\Pi \mathcal{A}: \mathcal{A} \Rightarrow \dot{\neg}(\mathcal{A})_n \vdash \dot{\neg}(\mathcal{A})_n$ ]

SystemQ **proof of** prop lemma imply negation:

L01:	Arbitrary $\gg$	$\mathcal{A}$	;
L02:	Premise $\gg$	$\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{A})_n$	;
L03:	AutoImPLY $\gg$	$\dot{\neg}(\mathcal{A})_n \Rightarrow \dot{\neg}(\mathcal{A})_n$	;
L04:	TND $\gg$	$\mathcal{A} \dot{\vee} \dot{\neg}(\mathcal{A})_n$	;
L05:	FromDisjuncts $\triangleright$ L04 $\triangleright$ L02 $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})_n$	□

[SystemQ **lemma** From3Disjuncts:  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}: \mathcal{A} \dot{\vee} \mathcal{B} \dot{\vee} \mathcal{C} \vdash \mathcal{A} \Rightarrow \mathcal{D} \vdash \mathcal{B} \Rightarrow \mathcal{D} \vdash \mathcal{C} \Rightarrow \mathcal{D}$ ]

SystemQ **proof of** From3Disjuncts:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$	;
L03:	Premise $\gg$	$\mathcal{A} \dot{\vee} \mathcal{B} \dot{\vee} \mathcal{C}$	;
L04:	Premise $\gg$	$\mathcal{B} \Rightarrow \mathcal{D}$	;
L05:	Premise $\gg$	$\mathcal{C} \Rightarrow \mathcal{D}$	;
L06:	Premise $\gg$	$\dot{\neg}(\mathcal{A})_n$	;
L07:	Repetition $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})_n \Rightarrow (\mathcal{B} \dot{\vee} \mathcal{C})$	;
L08:	MP $\triangleright$ L07 $\triangleright$ L06 $\gg$	$\mathcal{B} \dot{\vee} \mathcal{C}$	;
L09:	FromDisjuncts $\triangleright$ L08 $\triangleright$ L04 $\triangleright$ L05 $\gg$	$\mathcal{D}$	;
L10:	Block $\gg$	End	;
L11:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$	;
L12:	Ded $\triangleright$ L10 $\gg$	$\mathcal{A} \dot{\vee} \mathcal{B} \dot{\vee} \mathcal{C} \Rightarrow (\mathcal{B} \Rightarrow \mathcal{D}) \Rightarrow (\mathcal{C} \Rightarrow \mathcal{D}) \Rightarrow \dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{D}$	;
L13:	AutoImPLY $\gg$	$(\mathcal{A} \Rightarrow \mathcal{D}) \Rightarrow \mathcal{A} \Rightarrow \mathcal{D}$	;
L14:	Premise $\gg$	$\mathcal{A} \dot{\vee} \mathcal{B} \dot{\vee} \mathcal{C}$	;
L15:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{D}$	;
L16:	Premise $\gg$	$\mathcal{B} \Rightarrow \mathcal{D}$	;
L17:	Premise $\gg$	$\mathcal{C} \Rightarrow \mathcal{D}$	;
L18:	MP3 $\triangleright$ L12 $\triangleright$ L14 $\triangleright$ L16 $\triangleright$ L17 $\gg$	$\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{D}$	;
L19:	MP $\triangleright$ L13 $\triangleright$ L15 $\gg$	$\mathcal{A} \Rightarrow \mathcal{D}$	;
L20:	FromNegations $\triangleright$ L19 $\triangleright$ L18 $\gg$	$\mathcal{D}$	□

[SystemQ **lemma** NegateDisjunct1:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \dot{\vee} \mathcal{B} \vdash \dot{\neg}(\mathcal{A})_n \vdash \mathcal{B}$ ]

SystemQ **proof of** NegateDisjunct1:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{A} \dot{\vee} \mathcal{B}$	;
L03:	Premise $\gg$	$\dot{\neg}(\mathcal{A})_n$	;
L04:	Repetition $\triangleright$ L02 $\gg$	$\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$	;

L05: MP  $\triangleright$  L04  $\triangleright$  L03  $\gg$   $\mathcal{B}$  □

[SystemQ **lemma** NegateDisjunct2:  $\Pi A, \mathcal{B}: A \dot{\vee} \mathcal{B} \vdash \dot{\neg}(\mathcal{B})_n \vdash A$ ]

SystemQ **proof of** NegateDisjunct2:

L01: Arbitrary  $\gg$   $A, \mathcal{B}$  ;  
L02: Premise  $\gg$   $A \dot{\vee} \mathcal{B}$  ;  
L03: Premise  $\gg$   $\dot{\neg}(\mathcal{B})_n$  ;  
L04: Repetition  $\triangleright$  L02  $\gg$   $\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$  ;  
L05: NegativeMT  $\triangleright$  L04  $\triangleright$  L03  $\gg$   $A$  □

(\*\*\*)

[SystemQ **lemma** ExpandDisjuncts:  $\Pi A, \mathcal{B}, \mathcal{C}, \mathcal{D}: A \dot{\vee} \mathcal{B} \vdash \mathcal{C} \dot{\vee} \mathcal{D} \vdash \mathcal{B} \dot{\vee} \mathcal{D} \dot{\vee} (A \dot{\wedge} \mathcal{C})$ ]

SystemQ **proof of** ExpandDisjuncts:

L01: Block  $\gg$  Begin ;  
L02: Arbitrary  $\gg$   $A, \mathcal{B}, \mathcal{C}, \mathcal{D}$  ;  
L03: Premise  $\gg$   $A \dot{\vee} \mathcal{B}$  ;  
L04: Premise  $\gg$   $\mathcal{C} \dot{\vee} \mathcal{D}$  ;  
L05: Premise  $\gg$   $\dot{\neg}(\mathcal{B})_n$  ;  
L06: Premise  $\gg$   $\dot{\neg}(\mathcal{D})_n$  ;  
L07: NegateDisjunct2  $\triangleright$  L03  $\triangleright$  L05  $\gg$   $A$  ;  
L08: NegateDisjunct2  $\triangleright$  L04  $\triangleright$  L06  $\gg$   $\mathcal{C}$  ;  
L09: JoinConjuncts  $\triangleright$  L07  $\triangleright$  L08  $\gg$   $A \dot{\wedge} \mathcal{C}$  ;  
L10: Block  $\gg$  End ;  
L11: Arbitrary  $\gg$   $A, \mathcal{B}, \mathcal{C}, \mathcal{D}$  ;  
L12: Ded  $\triangleright$  L10  $\gg$   $A \dot{\vee} \mathcal{B} \Rightarrow \mathcal{C} \dot{\vee} \mathcal{D} \Rightarrow \dot{\neg}(\mathcal{B})_n \Rightarrow$  ;  
 $\dot{\neg}(\mathcal{D})_n \Rightarrow A \dot{\wedge} \mathcal{C}$  ;  
L13: Premise  $\gg$   $A \dot{\vee} \mathcal{B}$  ;  
L14: Premise  $\gg$   $\mathcal{C} \dot{\vee} \mathcal{D}$  ;  
L15: MP2  $\triangleright$  L12  $\triangleright$  L13  $\triangleright$  L14  $\gg$   $\dot{\neg}(\mathcal{B})_n \Rightarrow \dot{\neg}(\mathcal{D})_n \Rightarrow A \dot{\wedge} \mathcal{C}$  ;  
L16: Repetition  $\triangleright$  L15  $\gg$   $\mathcal{B} \dot{\vee} \mathcal{D} \dot{\vee} (A \dot{\wedge} \mathcal{C})$  □

[SystemQ **lemma** From2 \* 2Disjuncts:  $\Pi A, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}: A \dot{\vee} \mathcal{B} \vdash \mathcal{C} \dot{\vee} \mathcal{D} \vdash A \Rightarrow \mathcal{C} \Rightarrow \mathcal{E} \vdash A \Rightarrow \mathcal{D} \Rightarrow \mathcal{E} \vdash \mathcal{B} \Rightarrow \mathcal{C} \Rightarrow \mathcal{E} \vdash \mathcal{B} \Rightarrow \mathcal{D} \Rightarrow \mathcal{E} \vdash \mathcal{E}$ ]

SystemQ **proof of** From2 \* 2Disjuncts:

L01: Block  $\gg$  Begin ;  
L02: Arbitrary  $\gg$   $A, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$  ;  
L03: Premise  $\gg$   $\mathcal{C} \dot{\vee} \mathcal{D}$  ;  
L04: Premise  $\gg$   $A \Rightarrow \mathcal{C} \Rightarrow \mathcal{E}$  ;  
L05: Premise  $\gg$   $A \Rightarrow \mathcal{D} \Rightarrow \mathcal{E}$  ;  
L06: Premise  $\gg$   $A$  ;  
L07: MP  $\triangleright$  L04  $\triangleright$  L06  $\gg$   $\mathcal{C} \Rightarrow \mathcal{E}$  ;  
L08: MP  $\triangleright$  L05  $\triangleright$  L06  $\gg$   $\mathcal{D} \Rightarrow \mathcal{E}$  ;  
L09: FromDisjuncts  $\triangleright$  L03  $\triangleright$  L07  $\triangleright$  ;  
 $\mathcal{L08} \gg$   $\mathcal{E}$  ;  
L10: Block  $\gg$  End ;  
L11: Block  $\gg$  Begin ;  
L12: Arbitrary  $\gg$   $A, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$  ;  
L13: Premise  $\gg$   $A \dot{\vee} \mathcal{B}$  ;

L14:	Premise $\gg$	$C \dot{\vee} D$	;
L15:	Premise $\gg$	$B \Rightarrow C \Rightarrow \mathcal{E}$	;
L16:	Premise $\gg$	$B \Rightarrow D \Rightarrow \mathcal{E}$	;
L17:	Premise $\gg$	$\dot{\neg}(A)_n$	;
L18:	NegateDisjunct1 $\triangleright$ L13 $\triangleright$ L17 $\gg$	$B$	;
L19:	MP $\triangleright$ L15 $\triangleright$ L18 $\gg$	$C \Rightarrow \mathcal{E}$	;
L20:	MP $\triangleright$ L16 $\triangleright$ L18 $\gg$	$D \Rightarrow \mathcal{E}$	;
L21:	FromDisjuncts $\triangleright$ L14 $\triangleright$ L19 $\triangleright$ L20 $\gg$	$\mathcal{E}$	;
L22:	Block $\gg$	End	;
L23:	Arbitrary $\gg$	$A, B, C, D, \mathcal{E}$	;
L24:	Ded $\triangleright$ L10 $\gg$	$C \dot{\vee} D \Rightarrow (A \Rightarrow C \Rightarrow \mathcal{E}) \Rightarrow$ $(A \Rightarrow D \Rightarrow \mathcal{E}) \Rightarrow A \Rightarrow \mathcal{E}$	;
L25:	Ded $\triangleright$ L22 $\gg$	$A \dot{\vee} B \Rightarrow C \dot{\vee} D \Rightarrow (B \Rightarrow$ $C \Rightarrow \mathcal{E}) \Rightarrow (B \Rightarrow D \Rightarrow \mathcal{E}) \Rightarrow$ $\dot{\neg}(A)_n \Rightarrow \mathcal{E}$	;
L26:	Premise $\gg$	$A \dot{\vee} B$	;
L27:	Premise $\gg$	$C \dot{\vee} D$	;
L28:	Premise $\gg$	$A \Rightarrow C \Rightarrow \mathcal{E}$	;
L29:	Premise $\gg$	$A \Rightarrow D \Rightarrow \mathcal{E}$	;
L30:	Premise $\gg$	$B \Rightarrow C \Rightarrow \mathcal{E}$	;
L31:	Premise $\gg$	$B \Rightarrow D \Rightarrow \mathcal{E}$	;
L32:	MP3 $\triangleright$ L24 $\triangleright$ L27 $\triangleright$ L28 $\triangleright$ L29 $\gg$	$A \Rightarrow \mathcal{E}$	;
L33:	MP4 $\triangleright$ L25 $\triangleright$ L26 $\triangleright$ L27 $\triangleright$ L30 $\triangleright$ L31 $\gg$	$\dot{\neg}(A)_n \Rightarrow \mathcal{E}$	;
L34:	FromNegations $\triangleright$ L32 $\triangleright$ L33 $\gg$	$\mathcal{E}$	$\square$

(\*\*\*) SAME-F (\*\*\*) XX-am

(\*\*\*) R-AFDELINGEN (\*\*\*) XX-am

(\*\*\*\*\*)

[SystemQ lemma FromNegatedImPLY:  $\Pi A, B: \dot{\neg}((A \Rightarrow B))_n \vdash A \dot{\wedge} \dot{\neg}(B)_n]$

SystemQ proof of FromNegatedImPLY:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$A, B$	;
L03:	Premise $\gg$	$A \Rightarrow \dot{\neg}(\dot{\neg}(B)_n)_n$	;
L04:	Premise $\gg$	$A$	;
L05:	MP $\triangleright$ L03 $\triangleright$ L04 $\gg$	$\dot{\neg}(\dot{\neg}(B)_n)_n$	;
L06:	RemoveDoubleNeg $\triangleright$ L05 $\gg$	$B$	;
L07:	Block $\gg$	End	;
L08:	Arbitrary $\gg$	$A, B$	;
L03:	Ded $\triangleright$ L07 $\gg$	$(A \Rightarrow \dot{\neg}(\dot{\neg}(B)_n)_n) \Rightarrow (A \Rightarrow B)$	;
L04:	Premise $\gg$	$\dot{\neg}((A \Rightarrow B))_n$	;
L05:	MT $\triangleright$ L03 $\triangleright$ L04 $\gg$	$\dot{\neg}((A \Rightarrow \dot{\neg}(\dot{\neg}(B)_n)_n))_n$	;
L09:	Repetition $\triangleright$ L05 $\gg$	$A \dot{\wedge} \dot{\neg}(B)_n$	$\square$

(\*\*\*)

[SystemQ lemma FromNegated(2 \* ImPLY):  $\Pi A, B, C: \dot{\neg}((A \Rightarrow B \Rightarrow C))_n \vdash A \dot{\wedge} B \dot{\wedge} \dot{\neg}(C)_n]$

SystemQ **proof of** FromNegated(2 \* Imply):

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}, \mathcal{C}$	;
L02:	Premise $\gg$	$\dot{\neg}((\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}))_n$	;
L03:	FromNegatedImply $\triangleright$ L02 $\gg$	$\mathcal{A} \dot{\wedge} \dot{\neg}((\mathcal{B} \Rightarrow \mathcal{C}))_n$	;
L04:	FirstConjunct $\triangleright$ L03 $\gg$	$\mathcal{A}$	;
L05:	SecondConjunct $\triangleright$ L03 $\gg$	$\dot{\neg}((\mathcal{B} \Rightarrow \mathcal{C}))_n$	;
L06:	FromNegatedImply $\triangleright$ L05 $\gg$	$\mathcal{B} \dot{\wedge} \dot{\neg}(\mathcal{C})_n$	;
L07:	FirstConjunct $\triangleright$ L06 $\gg$	$\mathcal{B}$	;
L08:	SecondConjunct $\triangleright$ L06 $\gg$	$\dot{\neg}(\mathcal{C})_n$	;
L09:	JoinConjuncts $\triangleright$ L04 $\triangleright$ L07 $\gg$	$\mathcal{A} \dot{\wedge} \mathcal{B}$	;
L10:	JoinConjuncts $\triangleright$ L09 $\triangleright$ L08 $\gg$	$\mathcal{A} \dot{\wedge} \mathcal{B} \dot{\wedge} \dot{\neg}(\mathcal{C})_n$	□

[SystemQ **lemma** FromNegatedOr:  $\Pi \mathcal{A}, \mathcal{B}: \dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n \vdash \dot{\neg}(\mathcal{A})_n \dot{\wedge} \dot{\neg}(\mathcal{B})_n$ ]

SystemQ **proof of** FromNegatedOr:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n$	;
L03:	Repetition $\triangleright$ L02 $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B})_n$	;
L04:	FromNegatedImply $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})_n \dot{\wedge} \dot{\neg}(\mathcal{B})_n$	□

[SystemQ **rule** InductionAxiom:  $\Pi V_1, \mathcal{A}, \mathcal{B}, \mathcal{C}: \langle \mathcal{B} \equiv \mathcal{A} | V_1 := 0 \rangle_{\text{Me}} \vdash \langle \mathcal{C} \equiv \mathcal{A} | V_1 := 1 \rangle_{\text{Me}} \vdash \mathcal{B} \Rightarrow \forall V_1: (\mathcal{A} \Rightarrow \mathcal{C}) \Rightarrow \forall V_1: \mathcal{A}$ ]

[SystemQ **rule** LessMinus1(N):  $\Pi \mathcal{M}, \mathcal{N}: \text{Nat}(\mathcal{M}) \vdash \text{Nat}(\mathcal{N}) \vdash \mathcal{M} < (\mathcal{N} + 1) \vdash \mathcal{M} \leq \mathcal{N}$ ]

[SystemQ **rule** Nonnegative(N):  $\Pi \mathcal{M}: \text{Nat}(\mathcal{M}) \vdash 0 \leq \mathcal{M}$ ]

[SystemQ **rule** Cauchy:  $\Pi V_1, V_2, \mathcal{N}, \epsilon, \text{FX}: \forall \epsilon: \exists \mathcal{N}: \forall V_1, V_2: (0 < \epsilon \Rightarrow \mathcal{N} \leq V_1 \Rightarrow \mathcal{N} \leq V_2 \Rightarrow |(\text{FX}[V_1] - \text{FX}[V_2])| < \epsilon)$ ]

[SystemQ **lemma** JoinConjuncts(2conditions):  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}: \mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C} \vdash \mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{D} \vdash \mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C} \dot{\wedge} \mathcal{D}$ ]

SystemQ **proof of** JoinConjuncts(2conditions):

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$	;
L03:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}$	;
L04:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{D}$	;
L05:	Premise $\gg$	$\mathcal{A}$	;
L06:	Premise $\gg$	$\mathcal{B}$	;
L07:	MP2 $\triangleright$ L03 $\triangleright$ L05 $\triangleright$ L06 $\gg$	$\mathcal{C}$	;
L08:	MP2 $\triangleright$ L04 $\triangleright$ L05 $\triangleright$ L06 $\gg$	$\mathcal{D}$	;
L09:	JoinConjuncts $\triangleright$ L07 $\triangleright$ L08 $\gg$	$\mathcal{C} \dot{\wedge} \mathcal{D}$	;
L10:	Block $\gg$	End	;
L11:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$	;
L03:	Ded $\triangleright$ L10 $\gg$	$(\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}) \Rightarrow (\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{D}) \Rightarrow \mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C} \dot{\wedge} \mathcal{D}$	;
L04:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C}$	;
L05:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{D}$	;
L12:	MP2 $\triangleright$ L03 $\triangleright$ L04 $\triangleright$ L05 $\gg$	$\mathcal{A} \Rightarrow \mathcal{B} \Rightarrow \mathcal{C} \dot{\wedge} \mathcal{D}$	□



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[SystemQ **lemma** FromNegatedAnd:  $\Pi \mathcal{A}, \mathcal{B}: \dot{\neg}((\mathcal{A} \dot{\wedge} \mathcal{B}))_n \vdash \mathcal{A} \vdash \dot{\neg}(\mathcal{B})_n$ ]

SystemQ **proof of** FromNegatedAnd:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\dot{\neg}((\mathcal{A} \dot{\wedge} \mathcal{B}))_n$	;
L03:	Premise $\gg$	$\mathcal{A}$	;
L04:	Repetition $\triangleright$ L02 $\gg$	$\dot{\neg}(\dot{\neg}((\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n))_n)_n$	;
L05:	RemoveDoubleNeg $\triangleright$ L04 $\gg$	$\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L06:	MP $\triangleright$ L05 $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{B})_n$	$\square$

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[SystemQ **lemma** ToNegatedOr:  $\Pi \mathcal{A}, \mathcal{B}: \dot{\neg}(\mathcal{A})_n \dot{\wedge} \dot{\neg}(\mathcal{B})_n \vdash \dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n$ ]

SystemQ **proof of** ToNegatedOr:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Premise $\gg$	$\dot{\neg}(\mathcal{A})_n \dot{\wedge} \dot{\neg}(\mathcal{B})_n$	;
L04:	Premise $\gg$	$\mathcal{A} \dot{\vee} \mathcal{B}$	;
L05:	FirstConjunct $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})_n$	;
L06:	SecondConjunct $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{B})_n$	;
L07:	NegateDisjunct1 $\triangleright$ L04 $\triangleright$ L05 $\gg$	$\mathcal{B}$	;
L08:	FromContradiction $\triangleright$ L07 $\triangleright$ L06 $\gg$	$\dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n$	;
L09:	Block $\gg$	End	;
L10:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Ded $\triangleright$ L09 $\gg$	$\dot{\neg}(\mathcal{A})_n \dot{\wedge} \dot{\neg}(\mathcal{B})_n \Rightarrow \mathcal{A} \dot{\vee} \mathcal{B} \Rightarrow$ $\dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n$	;
L04:	Premise $\gg$	$\dot{\neg}(\mathcal{A})_n \dot{\wedge} \dot{\neg}(\mathcal{B})_n$	;
L05:	MP $\triangleright$ L03 $\triangleright$ L04 $\gg$	$\mathcal{A} \dot{\vee} \mathcal{B} \Rightarrow \dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n$	;
L11:	prop lemma imply negation $\triangleright$ L05 $\gg$	$\dot{\neg}((\mathcal{A} \dot{\vee} \mathcal{B}))_n$	$\square$

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[SystemQ **rule** NextXS(UpperBound):  $\Pi \mathcal{M}: \text{UB}(01//02**(\text{xs}[\mathcal{M}]++\text{us}[\mathcal{M}]), \text{Se}$   
 $\text{xs}[(\mathcal{M} + 1)] == \text{xs}[\mathcal{M}]$ ]

[SystemQ **rule** NextXS(NoUpperBound):  $\Pi \mathcal{M}: \dot{\neg}(\text{UB}(01//02**(\text{xs}[\mathcal{M}]++\text{us}[\mathcal{M}]), \text{Se}$   
 $\text{xs}[(\mathcal{M} + 1)] == 01//02**(\text{xs}[\mathcal{M}] + \text{us}[\mathcal{M}]))$ ]

[SystemQ **rule** NextUS(UpperBound):  $\Pi \mathcal{M}: \text{UB}(01//02**(\text{xs}[\mathcal{M}]++\text{us}[\mathcal{M}]), \text{Se}$   
 $\text{us}[(\mathcal{M} + 1)] == 01//02**(\text{xs}[\mathcal{M}] + \text{us}[\mathcal{M}]))$ ]

[SystemQ **rule** NextUS(NoUpperBound):  $\Pi \mathcal{M}: \dot{\neg}(\text{UB}(01//02**(\text{xs}[\mathcal{M}]++\text{us}[\mathcal{M}]), \text{Se}$   
 $\text{us}[(\mathcal{M} + 1)] == \text{us}[\mathcal{M}]))$ ]

[SystemQ **rule** US0:  $\text{us}[0] == \text{xs}[0] + 01$ ]

[SystemQ **rule** ExpZero:  $\Pi \mathcal{M}, \mathcal{X}: \mathcal{M} = 0 \vdash \mathcal{X}(\text{exp})\mathcal{M} = 1$ ]

[SystemQ **rule** ExpPositive:  $\Pi \mathcal{M}, \mathcal{X}: 0 < \mathcal{M} \vdash \mathcal{X}(\text{exp})\mathcal{M} = (\mathcal{X} * \mathcal{X}(\text{exp}))((\mathcal{M} - 1))$ ]

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[SystemQ **rule** BSzero:  $\Pi \mathcal{M}, \mathcal{N}: \mathcal{N} = 0 \vdash \text{BS}(\mathcal{M}, \mathcal{N}) = 1/2(\text{exp})\mathcal{M}$ ]

[SystemQ **rule** BSpositive:  $\Pi \mathcal{M}, \mathcal{N}: 0 < \mathcal{N} \vdash \text{BS}(\mathcal{M}, \mathcal{N}) = (1/2(\text{exp})((\mathcal{M} + \mathcal{N})) + \text{BS}(\mathcal{M}, (\mathcal{N} - 1)))$ ]

[SystemQ **rule** UStelescope(Zero):  $\Pi \mathcal{M}, \mathcal{N}: \mathcal{N} = 0 \vdash \text{UStelescope}(\mathcal{M}, \mathcal{N}) = |(\text{us}[\mathcal{M}] - \text{us}[(\mathcal{M} + 1)])|$ ]

[SystemQ **rule** UStelescope(Positive):  $\Pi \mathcal{M}, \mathcal{N}: 0 < \mathcal{N} \vdash \text{UStelescope}(\mathcal{M}, \mathcal{N}) = |(\text{us}[(\mathcal{M} + \mathcal{N})] - \text{us}[(\mathcal{M} + ((\mathcal{N} + 1))])| + \text{UStelescope}(\mathcal{M}, (\mathcal{N} - 1))$ ]

[ $(x) \doteq (x)$ ]

[SystemQ **rule** EqAddition(R):  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) = \text{R}(\text{FY}) \vdash \text{R}(\text{FX}) + \text{R}(\text{FZ}) = \text{R}(\text{FY}) + \text{R}(\text{FZ})$ ]

[SystemQ **rule** PlusCommutativity(R):  $\Pi \text{FX}, \text{FY}: \text{R}(\text{FX}) ++ \text{R}(\text{FY}) == \text{R}(\text{FY}) + \text{R}(\text{FX})$ ]

[SystemQ **rule** PlusAssociativity(R):  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) ++ \text{R}(\text{FY}) ++ \text{R}(\text{FZ}) \text{R}(\text{FX}) + + (\text{R}(\text{FY}) + + \text{R}(\text{FZ}))$ ]

[SystemQ **rule** PlusAssociativity(R)XX:  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX} +_f \text{FY} +_f \text{FZ}) == \text{R}(\text{FX} +_f (\text{FY} +_f \text{FZ}))$ ]

[SystemQ **rule** Plus0(R):  $\Pi \text{FX}: \text{R}(\text{FX}) + + 00 == \text{R}(\text{FX})$ ]

[SystemQ **rule** Negative(R):  $\Pi \mathcal{M}, \text{FX}: \text{R}(\text{FX}) + + (- - \text{R}(\text{FX})) == 00$ ]

[SystemQ **rule** TimesAssociativity(R):  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) ** \text{R}(\text{FY}) ** \text{R}(\text{FZ}) \text{R}(\text{FX}) ** (\text{R}(\text{FY}) ** \text{R}(\text{FZ}))$ ]

[SystemQ **rule** Times1(R):  $\Pi \text{FX}: \text{R}(\text{FX}) * * 01 == \text{R}(\text{FX})$ ]

—————(21.10.06)

[SystemQ **rule** lessAddition(R):  $\Pi \mathcal{M}, \epsilon, \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) << \text{R}(\text{FY}) \vdash \text{R}(\text{FX}) + \text{R}(\text{FZ}) << \text{R}(\text{FY}) + \text{R}(\text{FZ})$ ]

—————(23.10.06)

[SystemQ **rule** LeqAntisymmetry(R):  $\Pi \text{FX}, \text{FY}: \text{R}(\text{FX}) <<== \text{R}(\text{FY}) \vdash \text{R}(\text{FY}) \text{R}(\text{FX}) \vdash \text{R}(\text{FX}) = \text{R}(\text{FY})$ ]

[SystemQ **rule** LeqTransitivity(R):  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) <<== \text{R}(\text{FY}) \vdash \text{R}(\text{FY}) <<== \text{R}(\text{FZ}) \vdash \text{R}(\text{FX}) <<== \text{R}(\text{FZ})$ ]

—————(24.10.06)

[SystemQ **rule** leqAddition(R):  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) <<== \text{R}(\text{FY}) \vdash \text{R}(\text{FX}) + \text{R}(\text{FZ}) <<== \text{R}(\text{FY}) + \text{R}(\text{FZ})$ ]

[SystemQ **rule** Distribution(R):  $\Pi \text{FX}, \text{FY}, \text{FZ}: \text{R}(\text{FX}) ** (\text{R}(\text{FY}) ++ \text{R}(\text{FZ})) = \text{R}(\text{FX}) ** \text{R}(\text{FY}) + + \text{R}(\text{FX}) ** \text{R}(\text{FZ})$ ]

[ $(- - 01) \doteq (- - 01)$ ]

—————(24.10.06)

[SystemQ **rule** FromLimit:  $\Pi \mathcal{M}, \mathcal{N}, \text{FEP}, \text{FX}, \text{FYS}: \text{Limit}(\text{FX}, \text{FYS}) \vdash \forall \text{FEP}: \exists \text{FEP} \Rightarrow \mathcal{M} <= \mathcal{N} \Rightarrow |\text{rFX} + + (- - \text{FYS}[\mathcal{N}])| << \text{FEP}$ ]

[SystemQ **rule** ToUpperBound:  $\Pi \text{FX}, \text{FY}, \text{FXS}: \text{FX} \in \text{FXS} \Rightarrow \text{FX} <<== \text{FY} \vdash \text{UB}(\text{FY}, \text{FXS})$ ]

[SystemQ **rule** FromUpperBound:  $\Pi \text{FX}, \text{FY}, \text{FXS}: \text{UB}(\text{FY}, \text{FXS}) \vdash \text{FX} \in \text{FXS} \text{R}(\text{FX}) <<== \text{FY}$ ]

[SystemQ **rule** USisUpperBound:  $\Pi \mathcal{M}: \text{UB}(\text{usFoelge}[\mathcal{M}], \text{SetOfFxs})$ ]

—————(25.10.06)

[SystemQ **rule** 0not1(R): 00 ≠ 01]  
 ——(25.10.06)

[SystemQ **rule** ExpZero(R):  $\Pi \mathcal{M}, \text{FX}: \mathcal{M} = 0 \vdash \text{FX}(\text{exp})\mathcal{M} = 01$ ]  
[SystemQ **rule** ExpPositive(R):  $\Pi \mathcal{M}, \text{FX}: 0 < \mathcal{M} \vdash \text{FX}(\text{exp})\mathcal{M} == \text{FX} * \text{FX}(\text{exp})((\mathcal{M} - 1))$ ]  
 ——(26.10.06)

[02 ≐ 01 + +01] [01//02 ≐ 01//temp02]  
 ——(28.10.06)

[SystemQ **rule** ExpUnbounded(R):  $\Pi \mathcal{M}, \text{FX}: \exists \mathcal{M}: \text{FX} << 02(\text{exp})\mathcal{M}$ ]  
 ——(30.10.06)

[SystemQ **rule** FromLeq(Advanced)(N):  $\Pi \mathcal{M}, \mathcal{M}_1, \mathcal{N}: \mathcal{M} <= \mathcal{N} \vdash \exists \mathcal{M}_1: (\mathcal{M} + \mathcal{M}_1) = \mathcal{N}$ ]  
 ——(3.11.06)

[usFoelge ≐ us]  
[SystemQ **rule** FromLeastUpperBound:  $\text{LUB}(\text{FX}, \text{FYS}) \vdash \text{UB}(\text{FX}, \text{FYS}) \wedge (\text{UB}(\text{FZ}, \text{FYS}) \Rightarrow \text{FX} <<== \text{FZ})$ ]  
[SystemQ **rule** ToLeastUpperBound:  $\Pi \text{FX}, \text{FZ}, \text{FYS}: \text{UB}(\text{FX}, \text{FYS}) \vdash \text{UB}(\text{FZ}, \text{FYS}) \wedge \text{FX} <<== \text{FZ} \vdash \text{LUB}(\text{FX}, \text{FYS})$ ]  
[SystemQ **rule** XSisNotUpperBound:  $\Pi \mathcal{M}: \neg (\text{UB}(\text{xs}[\mathcal{M}], \text{SetOfXs}))\mathcal{n}$ ]  
 ——(4.11.06)

[xaF ≐ xs]  
[SystemQ **rule** ysFGreater:  $\Pi \mathcal{M}: \text{xaF}[\mathcal{M}] < \text{ysF}[\mathcal{M}]$ ]  
[SystemQ **rule** ysFLess:  $\Pi \mathcal{M}: \text{ysF}[\mathcal{M}] < (\text{xaF}[\mathcal{M}] + \text{rec}\mathcal{M})$ ]  
[SystemQ **rule** SmallInverse:  $\Pi \mathcal{M}, \mathcal{X}: 0 < \mathcal{X} \vdash \exists \mathcal{M}: \text{rec}\mathcal{M} < \mathcal{X}$ ]  
 ——(6.11.06)

[x == y ≐ x = y]  
[OrderedPair(x, y) ≐ ⟨x, y⟩]  
[SystemQ **rule** MemberOfSeries(ImPLY):  $\Pi \mathcal{M}, \text{FX}, (\text{SY}): \mathcal{M} \in \mathbb{N} \Rightarrow \text{IsSeries}(\text{FX}, \text{OrderedPair}(\mathcal{M}, \text{FX}[\mathcal{M}]) \in \text{FX})$ ]  
[SystemQ **lemma** MemberOfSeries:  $\Pi \mathcal{M}, \text{FX}, (\text{SY}): \mathcal{M} \in \mathbb{N} \vdash \text{IsSeries}(\text{FX}, (\text{SY}) \text{OrderedPair}(\mathcal{M}, \text{FX}[\mathcal{M}]) \in \text{FX})$ ]  
SystemQ **proof of** MemberOfSeries:

L01: Arbitrary ≫  $\mathcal{M}, \text{FX}, (\text{SY})$  ;  
L02: Premise ≫  $\mathcal{M} \in \mathbb{N}$  ;  
L03: Premise ≫  $\text{IsSeries}(\text{FX}, (\text{SY}))$  ;  
L04: MemberOfSeries(ImPLY) ≫  $\mathcal{M} \in \mathbb{N} \Rightarrow \text{IsSeries}(\text{FX}, (\text{SY})) \Rightarrow \text{OrderedPair}(\mathcal{M}, \text{FX}[\mathcal{M}]) \in \text{FX}$  ;  
L05: MP2 ▷ L04 ▷ L02 ▷ L03 ≫  $\text{OrderedPair}(\mathcal{M}, \text{FX}[\mathcal{M}]) \in \text{FX}$  □

[SystemQ **lemma** memberOfSeries(Type):  $\Pi \mathcal{M}, \text{FX}, (\text{SY}): \text{TypeNat}(\mathcal{M}) \vdash \text{TypeOrderedPair}(\mathcal{M}, \text{FX}[\mathcal{M}]) \in \text{FX}$ ]  
SystemQ **proof of** memberOfSeries(Type):

L01: Arbitrary ≫  $\mathcal{M}, \text{FX}, (\text{SY})$  ;  
L02: Side-condition ≫  $\text{TypeNat}(\mathcal{M})$  ;  
L03: Side-condition ≫  $\text{TypeSeries}(\text{FX}, (\text{SY}))$  ;  
L04: NatType ▷ L02 ≫  $\mathcal{M} \in \mathbb{N}$  ;

L05: SeriesType  $\triangleright$  L03  $\gg$  IsSeries(FX, (SY)) ;  
L06: MemberOfSeries  $\triangleright$  L04  $\triangleright$  L05  $\gg$  OrderedPair( $\mathcal{M}$ , FX[ $\mathcal{M}$ ])  $\in$  FX  $\square$   
[SystemQ **rule** NatType:  $\Pi \mathcal{M}$ : TypeNat( $\mathcal{M}$ )  $\#$   $\mathcal{M} \in \mathbb{N}$ ]  
[SystemQ **rule** RationalType:  $\Pi \mathcal{X}$ : TypeRational( $\mathcal{X}$ )  $\#$   $\mathcal{X} \in \mathbb{Q}$ ]  
[SystemQ **rule** SeriesType:  $\Pi$ FX, (SY): TypeSeries(FX, (SY))  $\#$  IsSeries(FX, (SY))  
[IsOrderedPair((sx), (sy), (sz))  $\doteq$   $\exists$ (OP1ob):  $\exists$ (OP2ob): (OP1ob)  $\in$  (sy)  $\wedge$   
(OP2ob)  $\in$  (sz)  $\wedge$  (sx) = OrderedPair((OP1ob), (OP2ob))]  
[IsRelation((sx), (sy), (sz))  $\doteq$   $\forall$ (R1ob): ((R1ob)  $\in$  (sx)  $\Rightarrow$  IsOrderedPair((R1ob), (sy)))  
[isFunction((sx), (sy), (sz))  $\doteq$  IsRelation((sx), (sy), (sz))  $\wedge$   $\forall$ (F1ob), (F2ob),  
(F3ob), (F4ob): (OrderedPair((F1ob), (F2ob))  $\in$  (sx)  $\Rightarrow$  OrderedPair((F3ob), (F4ob))  
(sx)  $\Rightarrow$  (F1ob) = (F3ob)  $\Rightarrow$  (F2ob) = (F4ob))]  
[IsSeries((fx), (fy))  $\doteq$  isFunction((fx),  $\mathbb{N}$ , (fy))  $\wedge$   $\forall$ (S1ob): ((S1ob)  $\in$   $\mathbb{N}$   $\Rightarrow$   
 $\exists$ (S2ob): OrderedPair((S1ob), (S2ob))  $\in$  (fx))]  
XXhertype [TypeNat(x)  $\doteq$   $\lambda c$ .TypeNat0( $[x]$ )]  
[TypeNat0(x)  $\doteq$   $x \in_t$  ( $[0] :: [\mathbb{V}_{2n}] :: [\mathcal{M}] :: [\mathcal{N}] :: [(\mathcal{N}+1)] :: [(\mathcal{M}+0)] ::$   
 $[(\mathcal{M} + \mathcal{N})] :: [\mathcal{O}] :: [\mathcal{P}] :: [((\mathcal{M} + \mathcal{N}) + 1)] :: [(\mathcal{M} + \mathcal{M}_1)] :: [(\mathcal{M} + ((\mathcal{N} +$   
 $1)))] :: [\mathcal{M}_1] :: [\mathcal{M}_2] :: [\mathcal{N}_1] :: [\mathcal{N}_2] :: [m] :: [n] :: \mathbb{T}$ )]  
[TypeSeries(x, y)  $\doteq$   $\lambda c$ .Typeseries0( $[x]$ ,  $[y]$ )]  
[Typeseries0(x, y)  $\doteq$   $y!x \in_t$  ( $[FX] :: [FY] :: [FZ] :: [us] :: [FX +_f FY] ::$   
 $[of] :: [1f] :: [FX *_f of] :: [1f/FX] :: \mathbb{T}$ )]  
[TypeRational(x)  $\doteq$   $\lambda c$ .TypeRational0( $[x]$ )]  
[TypeRational0(x)  $\doteq$   $x \in_t$  ( $[\mathcal{X}] :: [\mathcal{Y}] :: [\mathcal{Z}] :: [0] :: [1] :: \mathbb{T}$ )]  
[Max(x, y)  $\doteq$  if(y  $\leq$  x, x, y)]  
—(7.11.06)  
[SystemQ **rule** ReciprocalF:  $\Pi \mathcal{M}$ , FX: 1f/FX[ $\mathcal{M}$ ] = if(FX[ $\mathcal{M}$ ] = 0, 0, recFX[ $\mathcal{M}$ ])  
—(11.11.06)  
[of  $\doteq$  constantRationalSeries(0)] [1f  $\doteq$  constantRationalSeries(1)] [cartProd((sx),  
{ph  $\in$  Power(Power(binaryUnion((sx), (sy)))) | IsOrderedPair(ph<sub>1</sub>, (sx), (sy))}]  
[constantRationalSeries(x)  $\doteq$  {ph  $\in$  cartProd( $\mathbb{N}$ ) |  $\exists$ (CRS1ob): ph<sub>3</sub> = OrderedP  
[SystemQ **rule** Sep2Formula:  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}$ :  $\mathcal{Y} \in \{\text{ph} \in \mathcal{X} \mid \mathcal{A}\} \vdash \mathcal{Y} \in \mathcal{X} \wedge \mathcal{B}$ ]  
[Power(x)  $\doteq$  P(x)]  
—(12.11.06)  
[IsSubset(x, y)  $\doteq$   $x \subseteq y$ ]  
—(12.11.06)  
[SystemQ **rule** Formula2Sep:  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}$ :  $\mathcal{Y} \in \mathcal{X} \vdash \mathcal{B} \vdash \mathcal{Y} \in \{\text{ph} \in \mathcal{X} \mid \mathcal{A}\}$ ]  
—(13.11.06)  
[SystemQ **lemma** SameSeries:  $\Pi \mathcal{M}, \mathcal{N}$ , FX, (SY): TypeNat( $\mathcal{M}$ )  $\#$  TypeNat( $\mathcal{N}$ )  
TypeSeries(FX, (SY))  $\#$   $\mathcal{M} = \mathcal{N} \vdash \text{FX}[\mathcal{M}] = \text{FX}[\mathcal{N}]$ ]  
SystemQ **proof of** SameSeries:  
L01: Arbitrary  $\gg$   $\mathcal{M}, \mathcal{N}, \text{FX}, (\text{SY})$  ;  
L02: Side-condition  $\gg$  TypeNat( $\mathcal{M}$ ) ;  
L03: Side-condition  $\gg$  TypeNat( $\mathcal{N}$ ) ;  
L04: Side-condition  $\gg$  TypeSeries(FX, (SY)) ;  
L05: Premise  $\gg$   $\mathcal{M} = \mathcal{N}$  ;  
L06: memberOfSeries(Type)  $\triangleright$  OrderedPair( $\mathcal{M}$ , FX[ $\mathcal{M}$ ])  $\in$  FX ;  
L02  $\triangleright$  L04  $\gg$

L07:  $\text{memberOfSeries}(\text{Type}) \triangleright$   
L03  $\triangleright$  L04  $\gg$   $\text{OrderedPair}(\mathcal{N}, \text{FX}[\mathcal{N}]) \in \text{FX}$  ;

L08:  $\text{UniqueMember}(\text{Type}) \triangleright$   
L04  $\triangleright$  L06  $\triangleright$  L07  $\triangleright$  L05  $\gg$   $\text{FX}[\mathcal{M}] = \text{FX}[\mathcal{N}]$   $\square$

[SystemQ **lemma**  $\text{UniqueMember}(\text{Type})$ :  $\text{IFX}, (\text{SX}), (\text{SX1}), (\text{SY}), (\text{SY1}), (\text{SZ})$ :  
 $\text{OrderedPair}((\text{SX}), (\text{SX1})) \in \text{FX} \vdash \text{OrderedPair}((\text{SY}), (\text{SY1})) \in \text{FX} \vdash (\text{SX}) =$   
 $(\text{SY}) \vdash (\text{SX1}) = (\text{SY1})$ ]

SystemQ **proof of**  $\text{UniqueMember}(\text{Type})$ :

L01: Arbitrary  $\gg$   $\text{FX}, (\text{SX}), (\text{SX1}), (\text{SY}), (\text{SY1}),$   
 $(\text{SZ})$  ;

L02: Side-condition  $\gg$   $\text{TypeSeries}(\text{FX}, (\text{SZ}))$  ;

L03: Premise  $\gg$   $\text{OrderedPair}((\text{SX}), (\text{SX1})) \in$   
 $\text{FX}$  ;

L04: Premise  $\gg$   $\text{OrderedPair}((\text{SY}), (\text{SY1})) \in$   
 $\text{FX}$  ;

L05: Premise  $\gg$   $(\text{SX}) = (\text{SY})$  ;

L06: SeriesType  $\triangleright$  L02  $\gg$   $\text{IsSeries}(\text{FX}, (\text{SZ}))$  ;

L07: UniqueMember  $\triangleright$  L06  $\triangleright$  L03  $\triangleright$   
L04  $\triangleright$  L05  $\gg$   $(\text{SX1}) = (\text{SY1})$   $\square$

[SystemQ **lemma**  $\text{UniqueMember}$ :  $\text{IFX}, (\text{SX}), (\text{SX1}), (\text{SY}), (\text{SY1}), (\text{SZ})$ :  $\text{IsSeries}$   
 $\text{OrderedPair}((\text{SX}), (\text{SX1})) \in \text{FX} \vdash \text{OrderedPair}((\text{SY}), (\text{SY1})) \in \text{FX} \vdash (\text{SX}) =$   
 $(\text{SY}) \vdash (\text{SX1}) = (\text{SY1})$ ]

SystemQ **proof of**  $\text{UniqueMember}$ :

L01: Arbitrary  $\gg$   $\text{FX}, (\text{SX}), (\text{SX1}), (\text{SY}), (\text{SY1}),$   
 $(\text{SZ})$  ;

L02: Premise  $\gg$   $\text{IsSeries}(\text{FX}, (\text{SZ}))$  ;

L03: Premise  $\gg$   $\text{OrderedPair}((\text{SX}), (\text{SX1})) \in$   
 $\text{FX}$  ;

L04: Premise  $\gg$   $\text{OrderedPair}((\text{SY}), (\text{SY1})) \in$   
 $\text{FX}$  ;

L05: Premise  $\gg$   $(\text{SX}) = (\text{SY})$  ;

L06: Repetition  $\triangleright$  L02  $\gg$   $\text{isFunction}(\text{FX}, \text{N}, (\text{SZ})) \hat{\wedge}$   
 $\forall (\text{S1ob}): ((\text{S1ob}) \in \text{N} \Rightarrow$   
 $\exists (\text{S2ob}): \text{OrderedPair}((\text{S1ob}), (\text{S2ob})) \in$   
 $\text{FX})$  ;

L07: FirstConjunct  $\triangleright$  L06  $\gg$   $\text{isFunction}(\text{FX}, \text{N}, (\text{SZ}))$  ;

L08: Repetition  $\triangleright$  L07  $\gg$   $\text{IsRelation}(\text{FX}, \text{N}, (\text{SZ})) \hat{\wedge}$   
 $\forall (\text{F1ob}), (\text{F2ob}), (\text{F3ob}),$   
 $(\text{F4ob}): (\text{OrderedPair}((\text{F1ob}), (\text{F2ob})) \in$   
 $\text{FX} \Rightarrow$   
 $\text{OrderedPair}((\text{F3ob}), (\text{F4ob})) \in$   
 $\text{FX} \Rightarrow (\text{F1ob}) = (\text{F3ob}) \Rightarrow$   
 $(\text{F2ob}) = (\text{F4ob}))$  ;

L09:	SecondConjunct $\triangleright$ L08 $\gg$	$\forall (F1ob), (F2ob), (F3ob),$ $(F4ob): (\text{OrderedPair}((F1ob), (F2ob))) \in$ $\text{FX} \Rightarrow$ $\text{OrderedPair}((F3ob), (F4ob)) \in$ $\text{FX} \Rightarrow (F1ob) = (F3ob) \Rightarrow$ $(F2ob) = (F4ob)$ ;
L10:	A4@(SX) $\triangleright$ L09 $\gg$	$\forall (F2ob), (F3ob),$ $(F4ob): (\text{OrderedPair}((SX), (F2ob))) \in$ $\text{FX} \Rightarrow$ $\text{OrderedPair}((F3ob), (F4ob)) \in$ $\text{FX} \Rightarrow (SX) = (F3ob) \Rightarrow$ $(F2ob) = (F4ob)$ ;
L11:	A4@(SX1) $\triangleright$ L10 $\gg$	$\forall (F3ob),$ $(F4ob): (\text{OrderedPair}((SX), (SX1))) \in$ $\text{FX} \Rightarrow$ $\text{OrderedPair}((F3ob), (F4ob)) \in$ $\text{FX} \Rightarrow (SX) = (F3ob) \Rightarrow$ $(SX1) = (F4ob)$ ;
L12:	A4@(SY) $\triangleright$ L11 $\gg$	$\forall (F4ob): (\text{OrderedPair}((SX), (SX1))) \in$ $\text{FX} \Rightarrow$ $\text{OrderedPair}((SY), (F4ob)) \in$ $\text{FX} \Rightarrow (SX) = (SY) \Rightarrow (SX1) =$ $(F4ob)$ ;
L13:	A4@(SY1) $\triangleright$ L12 $\gg$	$\text{OrderedPair}((SX), (SX1)) \in$ $\text{FX} \Rightarrow$ $\text{OrderedPair}((SY), (SY1)) \in$ $\text{FX} \Rightarrow (SX) = (SY) \Rightarrow (SX1) =$ $(SY1)$ ;
L14:	MP3 $\triangleright$ L13 $\triangleright$ L03 $\triangleright$ L04 $\triangleright$ L05 $\gg$	$(SX1) = (SY1)$ $\square$

[SystemQ **lemma** A4:  $\Pi \mathcal{X}, V_1, \mathcal{A}, \mathcal{B}: \langle \mathcal{A} \equiv \mathcal{B} \mid V_1 := \mathcal{X} \rangle_{\text{Me}} \Vdash \forall V_1: \mathcal{B} \vdash \mathcal{A}$ ]  
KVANTI SystemQ **proof of A4:**

L01:	Arbitrary $\gg$	$\mathcal{X}, V_1, \mathcal{A}, \mathcal{B}$ ;
L02:	Side-condition $\gg$	$\langle \mathcal{A} \equiv \mathcal{B} \mid V_1 := \mathcal{X} \rangle_{\text{Me}}$ ;
L03:	Premise $\gg$	$\forall V_1: \mathcal{B}$ ;
L04:	A4(Axiom) $\triangleright$ L02 $\gg$	$\forall V_1: \mathcal{B} \Rightarrow \mathcal{A}$ ;
L05:	MP $\triangleright$ L04 $\triangleright$ L03 $\gg$	$\mathcal{A}$ $\square$

—(16.11.06)

[SystemQ **rule** SameMember:  $\Pi(SX), (SY), (SZ): (SX) = (SY) \vdash (SX) \in (SZ) \vdash (SY) \in (SZ)$ ]

[SystemQ **rule** ToSetEquality:  $\Pi(SX), FY: \text{IsSubset}(FX, FY) \vdash \text{IsSubset}(FY, FX) \vdash FX = FY$ ]

$[(px, y) \doteq \{x, y\}]$

[SystemQ **rule** SamePair:  $\Pi(SX), (SX1), (SY), (SY1): (SX) = (SX1) \vdash (SY) = (SY1) \vdash (p(SX), (SY)) = (p(SX1), (SY1))$ ]

$[(sx) \doteq \{x\}]$

[SystemQ **rule** SameSingleton:  $\Pi(SX), (SY): (SX) = (SY) \vdash (s(SX)) = (s(SY))$ ]

—(17.11.06)

$[(fx) +_f (fy) \doteq \{ph \in \text{cartProd}(N) \mid \exists \mathcal{M}: ph_4 = \text{OrderedPair}(\mathcal{M}, ((fx)[\mathcal{M}] + (fy)[\mathcal{M}]))\}]$

[SystemQ **rule** Qclosed(Addition):  $\Pi \mathcal{X}, \mathcal{Y}: \mathcal{X} \in Q \vdash \mathcal{Y} \in Q \vdash (\mathcal{X} + \mathcal{Y}) \in Q$ ]

[SystemQ **rule** FromCartProd(1):  $\Pi(SX), (SX1), (SY), (SY1): \text{OrderedPair}((SX, \text{cartProd}((SX1)) \vdash (SX) \in (SX1))$ ]

[SystemQ **rule** 1rule fromCartProd(2):  $\Pi(SX), (SX1), (SY), (SY1): \text{OrderedPair}(\text{cartProd}((SX1)) \vdash (SY) \in (SY1))$ ]

—(18.11.06)

$[(fx) *_f (fy) \doteq \{ph \in \text{cartProd}(N) \mid \exists \mathcal{M}: ph_5 = \text{OrderedPair}(\mathcal{M}, ((fx)[\mathcal{M}] * (fy)[\mathcal{M}]))\}]$

[SystemQ **rule** Qclosed(Multiplication):  $\Pi \mathcal{X}, \mathcal{Y}: \mathcal{X} \in Q \vdash \mathcal{Y} \in Q \vdash (\mathcal{X} * \mathcal{Y}) \in Q$ ]

—(19.11.06)

[SystemQ **rule** Pair2Formula:  $\Pi(SX), (SY), (SZ): (SX) \in (p(SY), (SZ)) \vdash (SX) = (SY) \dot{\vee} (SX) = (SZ)$ ]

[SystemQ **rule** Formula2Pair:  $\Pi(SX), (SY), (SZ): (SX) = (SY) \dot{\vee} (SX) = (SZ) \vdash (SX) \in (p(SY), (SZ))$ ]

—(23.11.06)

[binaryUnion(x, y)  $\doteq$  Union((px, y))]

[SystemQ **rule** Formula2Union:  $\Pi(SX), (SY), (SZ): \exists(SY): (SX) \in (SY) \wedge (SY) (SZ) \vdash (SX) \in \text{Union}((SZ))$ ]

[SystemQ **rule** Formula2Power:  $\Pi(SX), (SY): \text{IsSubset}((SX), (SY)) \vdash (SX) \in \text{Power}((SY))$ ]

—(28.11.06)

[SetOfRationalSeries  $\doteq$   $\{ph \in \text{Power}(\text{cartProd}(N)) \mid \text{IsSeries}(ph_2, Q)\}$ ]

[If/(fx)  $\doteq$   $\{ph \in \text{cartProd}(N) \mid \exists \mathcal{M}: ((fx)[\mathcal{M}] \neq 0 \wedge ph_6 = \text{OrderedPair}(\mathcal{M}, \text{rec}((fx)[\mathcal{M}] = 0 \wedge ph_6 = \text{OrderedPair}(\mathcal{M}, 0)))\}$ ]

[SystemQ **rule** Max:  $\Pi \mathcal{X}, \mathcal{Y}: (\mathcal{Y} <= \mathcal{X} \wedge \text{Max}(\mathcal{X}, \mathcal{Y}) = \mathcal{X}) \dot{\vee} (\dot{\neg}(\mathcal{Y} <= \mathcal{X})_n \wedge \text{Max}(\mathcal{X}, \mathcal{Y}) = \mathcal{Y})$ ]

[SystemQ **rule** Numerical:  $\Pi \mathcal{X}: (0 <= \mathcal{X} \wedge |\mathcal{X}| = \mathcal{X}) \dot{\vee} (\dot{\neg}(0 <= \mathcal{X})_n \wedge |\mathcal{X}| = (-u\mathcal{X}))$ ]

[SystemQ **rule** Separation2formula(1):  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}: \mathcal{Y} \in \{ph \in \mathcal{X} \mid \mathcal{A}\} \vdash \mathcal{Y} \in \mathcal{X}$ ]

[SystemQ **rule** Separation2formula(2):  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{X}, \mathcal{Y}: \mathcal{Y} \in \{ph \in \mathcal{X} \mid \mathcal{A}\} \vdash \mathcal{B}$ ]

[SystemQ **rule** QisClosed(Reciprocal)(Imply):  $\Pi \mathcal{X}: \mathcal{X} \neq 0 \Rightarrow \mathcal{X} \in Q \Rightarrow \text{rec} \mathcal{X} \in Q$ ]

[SystemQ **lemma** QisClosed(Reciprocal):  $\Pi \mathcal{X}: \mathcal{X} \neq 0 \vdash \mathcal{X} \in Q \vdash \text{rec} \mathcal{X} \in Q$ ]

SystemQ **proof of** QisClosed(Reciprocal):

L01: Arbitrary  $\gg \mathcal{X} ;$

L02: Premise  $\gg \mathcal{X} \neq 0 ;$

L03: Premise  $\gg \mathcal{X} \in Q ;$

L04: QisClosed(Reciprocal)(Imply)  $\gg \mathcal{X} \neq 0 \Rightarrow \mathcal{X} \in Q \Rightarrow \text{rec} \mathcal{X} \in Q ;$

L05: MP2  $\triangleright$  L04  $\triangleright$  L02  $\triangleright$  L03  $\gg \text{rec} \mathcal{X} \in Q \quad \square$

—(1.12.06)

[SystemQ **rule** QisClosed(Negative)(ImPLY):  $\Pi \mathcal{X}: \mathcal{X} \in \mathbb{Q} \Rightarrow (-u\mathcal{X}) \in \mathbb{Q}$ ]

[SystemQ **lemma** QisClosed(Negative):  $\Pi \mathcal{X}: \mathcal{X} \in \mathbb{Q} \vdash (-u\mathcal{X}) \in \mathbb{Q}$ ]

SystemQ **proof of** QisClosed(Negative):

- L01: Arbitrary  $\gg$   $\mathcal{X}$  ;
- L02: Premise  $\gg$   $\mathcal{X} \in \mathbb{Q}$  ;
- L03: QisClosed(Negative)(ImPLY)  $\gg$   $\mathcal{X} \in \mathbb{Q} \Rightarrow (-u\mathcal{X}) \in \mathbb{Q}$  ;
- L04: MP  $\triangleright$  L03  $\triangleright$  L02  $\gg$   $(-u\mathcal{X}) \in \mathbb{Q}$   $\square$
- [ $-_f(\text{fx}) \doteq \{\text{ph} \in \text{cartProd}(\mathbb{N}) \mid \exists \mathcal{M}: \text{ph}_6 = \text{OrderedPair}(\mathcal{M}, (-u(\text{fx})[\mathcal{M}]))\}$ ]  
[SF((fx), (fy))  $\doteq \forall (\text{EPob}): \exists n: \forall m: (0 < (\text{EPob}) \Rightarrow n <= m \Rightarrow |((\text{fx})[m] - (\text{fy})[m])| < (\text{EPob}))$ ]  
—(2.12.06)  
[(fx)  $<_f$  (fy)  $\doteq \exists (\text{EPob}): \exists n: \forall m: 0 < (\text{EPob}) \wedge (n <= m \Rightarrow (\text{fx})[m] <= ((\text{fy})[m] - (\text{EPob})))$ ]  
—(2.12.06)  
[extractSeries(t)  $\doteq \mathfrak{t}^{221212221111111}$ ]  
[SetOfSeries((sx))  $\doteq \{\text{ph} \in \text{Power}(\text{cartProd}(\mathbb{N})) \mid \text{IsSeries}(\text{ph}_6, (\text{sx}))\}$ ]  
[R((fx))  $\doteq \{\text{ph} \in \text{Power}(\text{SetOfSeries}(\mathbb{Q})) \mid \text{SF}((\text{fx}), \text{ph}_4)\}$ ]  
[ExpandList(t, s, c)  $\doteq \text{t!s!c!if } \mathfrak{t}^{\mathfrak{a}} \text{ then } \mathfrak{T} \text{ else StateExpand}(\mathfrak{t}^{\mathfrak{h}}, \text{s}, \text{c}) :: \text{ExpandList}(\text{***})$   
([x \* \*y  $\xrightarrow{\text{macro}} \lambda \text{t.} \lambda \text{s.} \lambda \text{c.} * * \text{Macro}(\mathfrak{t}^{\mathfrak{h}} :: \text{ExpandList}(\mathfrak{t}^{\mathfrak{t}}, \text{s}, \text{c}))$ )]<sup>P</sup>  
[\*\*Macro(t)  $\xrightarrow{\text{val}} \tilde{\mathcal{Q}}(\text{t}, [\text{R}(x*fy)], ([x] :: \text{extractSeries}(\mathfrak{t}^1)) :: ([y] :: \text{extractSeries}(\text{T}))$ ]  
(\*\*\*)  
([x + +y  $\xrightarrow{\text{macro}} \lambda \text{t.} \lambda \text{s.} \lambda \text{c.} + + \text{Macro}(\mathfrak{t}^{\mathfrak{h}} :: \text{ExpandList}(\mathfrak{t}^{\mathfrak{t}}, \text{s}, \text{c}))$ )]<sup>P</sup>  
[+ + Macro(t)  $\xrightarrow{\text{val}} \tilde{\mathcal{Q}}(\text{t}, [\text{R}(x+fy)], ([x] :: \text{extractSeries}(\mathfrak{t}^1)) :: ([y] :: \text{extractSeries}(\text{T}))$ ]  
(T)]  
(\*\*\*)  
([( - - x)  $\xrightarrow{\text{macro}} \lambda \text{t.} \lambda \text{s.} \lambda \text{c.} - - \text{Macro}(\mathfrak{t}^{\mathfrak{h}} :: \text{ExpandList}(\mathfrak{t}^{\mathfrak{t}}, \text{s}, \text{c}))$ )]<sup>P</sup>  
[- - Macro(t)  $\xrightarrow{\text{val}} \tilde{\mathcal{Q}}(\text{t}, [\text{R}(-fx)], ([x] :: \text{extractSeries}(\mathfrak{t}^1)) :: \text{T})$ ]  
(\*\*\*\*)  
([x << y  $\xrightarrow{\text{macro}} \lambda \text{t.} \lambda \text{s.} \lambda \text{c.} << \text{Macro}(\mathfrak{t}^{\mathfrak{h}} :: \text{ExpandList}(\mathfrak{t}^{\mathfrak{t}}, \text{s}, \text{c}))$ )]<sup>P</sup>  
[<< Macro(t)  $\xrightarrow{\text{val}} \tilde{\mathcal{Q}}(\text{t}, [x <_f y], ([x] :: \text{extractSeries}(\mathfrak{t}^1)) :: ([y] :: \text{extractSeries}(\text{T}))$ ]  
(T)]  
—(3.12.06)  
[StateExpand(t, s, c)  $\doteq \text{t!s!c!}\mathcal{U}^{\text{M}}(\text{s}^{\mathfrak{h}} \text{ ' } \text{t} \text{ ' } \text{s} \text{ ' } \text{c})$ ]  
—(5.12.06)  
[SystemQ **lemma** lemma eqLeq(R):  $\text{IIFX}, \text{FY}: \text{R}(\text{FX}) = \text{R}(\text{FY}) \vdash \text{R}(\text{FX}) <<= \text{R}(\text{FY})$ ]  
SystemQ **proof of** lemma eqLeq(R):
- L01: Arbitrary  $\gg$  FX, FY ;
- L02: Premise  $\gg$  R(FX) = R(FY) ;
- L03: WeakenOr1  $\triangleright$  L02  $\gg$  R(FX) << R(FY)  $\dot{\vee}$  R(FX) = R(FY) ;



L04: Repetition  $\triangleright$  L03  $\gg$   $R(FX) \ll == R(FY)$   $\square$   
 —(5.12.06)

[SystemQ **lemma** LessLeq(R):  $\Pi FX, FY: R(FX) \ll R(FY) \vdash R(FX) \ll == R(FY)$ ]

SystemQ **proof of** LessLeq(R):

L01: Arbitrary  $\gg$   $FX, FY$  ;  
 L02: Premise  $\gg$   $R(FX) \ll R(FY)$  ;  
 L03: WeakenOr2  $\triangleright$  L02  $\gg$   $R(FX) \ll R(FY) \dot{\vee} R(FX) = R(FY)$  ;  
 L04: Repetition  $\triangleright$  L03  $\gg$   $R(FX) \ll == R(FY)$   $\square$   
 —(7.12.06)

[SystemQ **lemma** MP2:  $\Pi A, B, C: A \Rightarrow B \Rightarrow C \vdash A \vdash B \vdash C$ ]

SystemQ **proof of** MP2:

L01: Arbitrary  $\gg$   $A, B, C$  ;  
 L02: Premise  $\gg$   $A \Rightarrow B \Rightarrow C$  ;  
 L03: Premise  $\gg$   $A$  ;  
 L04: Premise  $\gg$   $B$  ;  
 L05: MP  $\triangleright$  L02  $\triangleright$  L03  $\gg$   $B \Rightarrow C$  ;  
 L06: MP  $\triangleright$  L05  $\triangleright$  L04  $\gg$   $C$   $\square$

[SystemQ **lemma** MP3:  $\Pi A, B, C, D: A \Rightarrow B \Rightarrow C \Rightarrow D \vdash A \vdash B \vdash C \vdash D$ ]

SystemQ **proof of** MP3:

L01: Arbitrary  $\gg$   $A, B, C, D$  ;  
 L02: Premise  $\gg$   $A \Rightarrow B \Rightarrow C \Rightarrow D$  ;  
 L03: Premise  $\gg$   $A$  ;  
 L04: Premise  $\gg$   $B$  ;  
 L05: Premise  $\gg$   $C$  ;  
 L06: MP2  $\triangleright$  L02  $\triangleright$  L03  $\triangleright$  L04  $\gg$   $C \Rightarrow D$  ;  
 L07: MP  $\triangleright$  L06  $\triangleright$  L05  $\gg$   $D$   $\square$

[SystemQ **lemma** MP4:  $\Pi A, B, C, D, E: A \Rightarrow B \Rightarrow C \Rightarrow D \Rightarrow E \vdash A \vdash B \vdash C \vdash D \vdash E$ ]

SystemQ **proof of** MP4:

L01: Arbitrary  $\gg$   $A, B, C, D, E$  ;  
 L02: Premise  $\gg$   $A \Rightarrow B \Rightarrow C \Rightarrow D \Rightarrow E$  ;  
 L03: Premise  $\gg$   $A$  ;  
 L04: Premise  $\gg$   $B$  ;  
 L05: Premise  $\gg$   $C$  ;  
 L06: Premise  $\gg$   $D$  ;  
 L07: MP2  $\triangleright$  L02  $\triangleright$  L03  $\triangleright$  L04  $\gg$   $C \Rightarrow D \Rightarrow E$  ;  
 L08: MP2  $\triangleright$  L07  $\triangleright$  L05  $\triangleright$  L06  $\gg$   $E$   $\square$

[SystemQ **lemma** MP5:  $\Pi A, B, C, D, E, F: A \Rightarrow B \Rightarrow C \Rightarrow D \Rightarrow E \Rightarrow F \vdash A \vdash B \vdash C \vdash D \vdash E \vdash F$ ]

SystemQ **proof of** MP5:

L01: Arbitrary  $\gg$   $A, B, C, D, E, F$  ;  
 L02: Premise  $\gg$   $A \Rightarrow B \Rightarrow C \Rightarrow D \Rightarrow E \Rightarrow F$  ;  
 L03: Premise  $\gg$   $A$  ;  
 L04: Premise  $\gg$   $B$  ;

L05:	Premise $\gg$	$C$	;
L06:	Premise $\gg$	$D$	;
L07:	Premise $\gg$	$\mathcal{E}$	;
L08:	MP3 $\triangleright$ L02 $\triangleright$ L03 $\triangleright$ L04 $\triangleright$ L05 $\gg$	$D \Rightarrow \mathcal{E} \Rightarrow \mathcal{F}$	;
L09:	MP2 $\triangleright$ L08 $\triangleright$ L06 $\triangleright$ L07 $\gg$	$\mathcal{F}$	□

[SystemQ lemma AutoImPLY:  $\Pi A: \mathcal{A} \Rightarrow \mathcal{A}$ ]

SystemQ proof of AutoImPLY:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}$	;
L03:	Premise $\gg$	$\mathcal{A}$	;
L04:	Repetition $\triangleright$ L03 $\gg$	$\mathcal{A}$	;
L05:	Block $\gg$	End	;
L06:	Arbitrary $\gg$	$\mathcal{A}$	;
L07:	Ded $\triangleright$ L05 $\gg$	$\mathcal{A} \Rightarrow \mathcal{A}$	□

[SystemQ lemma ImPLYTransitivity:  $\Pi A, B, C: \mathcal{A} \Rightarrow B \vdash B \Rightarrow C \vdash \mathcal{A} \Rightarrow C$ ]

SystemQ proof of ImPLYTransitivity:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, B, C$	;
L03:	Premise $\gg$	$\mathcal{A} \Rightarrow B$	;
L04:	Premise $\gg$	$B \Rightarrow C$	;
L05:	Premise $\gg$	$\mathcal{A}$	;
L06:	MP $\triangleright$ L03 $\triangleright$ L05 $\gg$	$B$	;
L07:	MP $\triangleright$ L04 $\triangleright$ L06 $\gg$	$C$	;
L08:	Block $\gg$	End	;
L09:	Arbitrary $\gg$	$\mathcal{A}, B, C$	;
L10:	Premise $\gg$	$\mathcal{A} \Rightarrow B$	;
L11:	Premise $\gg$	$B \Rightarrow C$	;
L12:	Ded $\triangleright$ L08 $\gg$	$(\mathcal{A} \Rightarrow B) \Rightarrow (B \Rightarrow C) \Rightarrow \mathcal{A} \Rightarrow C$	;
L13:	MP2 $\triangleright$ L12 $\triangleright$ L10 $\triangleright$ L11 $\gg$	$\mathcal{A} \Rightarrow C$	□

[SystemQ lemma Weakening:  $\Pi A, B: B \vdash \mathcal{A} \Rightarrow B$ ]

SystemQ proof of Weakening:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, B$	;
L03:	Premise $\gg$	$B$	;
L04:	Premise $\gg$	$\mathcal{A}$	;
L05:	Repetition $\triangleright$ L03 $\gg$	$B$	;
L06:	Block $\gg$	End	;
L07:	Arbitrary $\gg$	$\mathcal{A}, B$	;
L08:	Ded $\triangleright$ L06 $\gg$	$B \Rightarrow \mathcal{A} \Rightarrow B$	;
L09:	Premise $\gg$	$B$	;
L10:	MP $\triangleright$ L08 $\triangleright$ L09 $\gg$	$\mathcal{A} \Rightarrow B$	□

[SystemQ lemma FromContradiction:  $\Pi A, B: \mathcal{A} \vdash \neg(\mathcal{A}) \vdash B$ ]

SystemQ proof of FromContradiction:

L01:	Arbitrary $\gg$	$\mathcal{A}, B$	;
L02:	Premise $\gg$	$\mathcal{A}$	;
L03:	Premise $\gg$	$\neg(\mathcal{A})$	;

L04:	Weakening $\triangleright$ L02 $\gg$	$\dot{\neg}(\mathcal{B})n \Rightarrow \mathcal{A}$	;
L05:	Weakening $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{B})n \Rightarrow \dot{\neg}(\mathcal{A})n$	;
L06:	Neg $\triangleright$ L04 $\triangleright$ L05 $\gg$	$\mathcal{B}$	□

[SystemQ **lemma** RemoveDoubleNeg:  $\Pi \mathcal{A}: \dot{\neg}(\dot{\neg}(\mathcal{A})n)n \vdash \mathcal{A}$ ]

SystemQ **proof of** RemoveDoubleNeg:

L01:	Arbitrary $\gg$	$\mathcal{A}$	;
L02:	Premise $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})n)n$	;
L03:	Weakening $\triangleright$ L02 $\gg$	$\dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\dot{\neg}(\mathcal{A})n)n$	;
L04:	AutoImPLY $\gg$	$\dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\mathcal{A})n$	;
L05:	Neg $\triangleright$ L04 $\triangleright$ L03 $\gg$	$\mathcal{A}$	□

[SystemQ **lemma** AddDoubleNeg:  $\Pi \mathcal{A}: \mathcal{A} \vdash \dot{\neg}(\dot{\neg}(\mathcal{A})n)n$ ]

SystemQ **proof of** AddDoubleNeg:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}$	;
L03:	Premise $\gg$	$\dot{\neg}(\dot{\neg}(\dot{\neg}(\mathcal{A})n)n)n$	;
L04:	RemoveDoubleNeg $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})n$	;
L05:	Block $\gg$	End	;
L06:	Arbitrary $\gg$	$\mathcal{A}$	;
L07:	Ded $\triangleright$ L05 $\gg$	$\dot{\neg}(\dot{\neg}(\dot{\neg}(\mathcal{A})n)n)n \Rightarrow \dot{\neg}(\mathcal{A})n$	;
L08:	Premise $\gg$	$\mathcal{A}$	;
L09:	Weakening $\triangleright$ L08 $\gg$	$\dot{\neg}(\dot{\neg}(\dot{\neg}(\mathcal{A})n)n)n \Rightarrow \mathcal{A}$	;
L10:	Neg $\triangleright$ L09 $\triangleright$ L07 $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})n)n$	□

—(10.12.06)

[SystemQ **lemma** Technicality:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \Rightarrow \mathcal{B} \vdash \dot{\neg}(\dot{\neg}(\mathcal{A})n)n \Rightarrow \mathcal{B}$ ]

SystemQ **proof of** Technicality:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L04:	Premise $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})n)n$	;
L05:	RemoveDoubleNeg $\triangleright$ L04 $\gg$	$\mathcal{A}$	;
L06:	MP $\triangleright$ L03 $\triangleright$ L05 $\gg$	$\mathcal{B}$	;
L07:	Block $\gg$	End	;
L08:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L09:	Ded $\triangleright$ L07 $\gg$	$(\mathcal{A} \Rightarrow \mathcal{B}) \Rightarrow \dot{\neg}(\dot{\neg}(\mathcal{A})n)n \Rightarrow \mathcal{B}$	;
L10:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L11:	MP $\triangleright$ L09 $\triangleright$ L10 $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})n)n \Rightarrow \mathcal{B}$	□

[SystemQ **lemma** NegativeMT:  $\Pi \mathcal{A}, \mathcal{B}: \dot{\neg}(\mathcal{A})n \Rightarrow \mathcal{B} \vdash \dot{\neg}(\mathcal{B})n \vdash \mathcal{A}$ ]

SystemQ **proof of** NegativeMT:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\dot{\neg}(\mathcal{A})n \Rightarrow \mathcal{B}$	;
L03:	Premise $\gg$	$\dot{\neg}(\mathcal{B})n$	;
L04:	Weakening $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})n \Rightarrow \dot{\neg}(\mathcal{B})n$	;
L05:	Neg $\triangleright$ L02 $\triangleright$ L04 $\gg$	$\mathcal{A}$	□

[SystemQ **lemma** MT:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \Rightarrow \mathcal{B} \vdash \dot{\neg}(\mathcal{B})n \vdash \dot{\neg}(\mathcal{A})n$ ]

SystemQ **proof of** MT:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
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L02:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L03:	Premise $\gg$	$\dot{\neg}(\mathcal{B})_n$	;
L04:	Technicality $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})_n)_n \Rightarrow \mathcal{B}$	;
L05:	NegativeMT $\triangleright$ L04 $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{A})_n$	□

[SystemQ **lemma** Contrapositive:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \Rightarrow \mathcal{B} \vdash \dot{\neg}(\mathcal{B})_n \Rightarrow \dot{\neg}(\mathcal{A})_n$ ]

SystemQ **proof of** Contrapositive:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L04:	Premise $\gg$	$\dot{\neg}(\mathcal{B})_n$	;
L05:	MT $\triangleright$ L03 $\triangleright$ L04 $\gg$	$\dot{\neg}(\mathcal{A})_n$	;
L06:	Block $\gg$	End	;
L07:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L08:	Premise $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L09:	Ded $\triangleright$ L06 $\gg$	$(\mathcal{A} \Rightarrow \mathcal{B}) \Rightarrow \dot{\neg}(\mathcal{B})_n \Rightarrow \dot{\neg}(\mathcal{A})_n$	;
L10:	MP $\triangleright$ L09 $\triangleright$ L08 $\gg$	$\dot{\neg}(\mathcal{B})_n \Rightarrow \dot{\neg}(\mathcal{A})_n$	□

[SystemQ **lemma** JoinConjuncts:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \vdash \mathcal{B} \vdash \mathcal{A} \wedge \mathcal{B}$ ]

SystemQ **proof of** JoinConjuncts:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{A}$	;
L04:	Premise $\gg$	$\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L05:	MP $\triangleright$ L04 $\triangleright$ L03 $\gg$	$\dot{\neg}(\mathcal{B})_n$	;
L06:	Block $\gg$	End	;
L07:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L08:	Ded $\triangleright$ L06 $\gg$	$\mathcal{A} \Rightarrow (\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n) \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L09:	Premise $\gg$	$\mathcal{A}$	;
L10:	Premise $\gg$	$\mathcal{B}$	;
L11:	MP $\triangleright$ L08 $\triangleright$ L09 $\gg$	$(\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n) \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L12:	AddDoubleNeg $\triangleright$ L10 $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{B})_n)_n$	;
L13:	MT $\triangleright$ L11 $\triangleright$ L12 $\gg$	$\dot{\neg}((\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n))_n$	;
L14:	Repetition $\triangleright$ L13 $\gg$	$\mathcal{A} \wedge \mathcal{B}$	□

[SystemQ **lemma** SecondConjunct:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \wedge \mathcal{B} \vdash \mathcal{B}$ ]

SystemQ **proof of** SecondConjunct:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Premise $\gg$	$\dot{\neg}(\mathcal{B})_n$	;
L04:	Weakening $\triangleright$ L03 $\gg$	$\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L05:	Block $\gg$	End	;
L06:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L07:	Ded $\triangleright$ L05 $\gg$	$\dot{\neg}(\mathcal{B})_n \Rightarrow \mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L08:	Premise $\gg$	$\mathcal{A} \wedge \mathcal{B}$	;
L09:	Repetition $\triangleright$ L08 $\gg$	$\dot{\neg}((\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n))_n$	;
L10:	NegativeMT $\triangleright$ L07 $\triangleright$ L09 $\gg$	$\mathcal{B}$	□

—(10.12.06)

[SystemQ **lemma** AndCommutativity:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \wedge \mathcal{B} \vdash \mathcal{B} \wedge \mathcal{A}$ ]

SystemQ **proof of** AndCommutativity:

L01:	Block $\gg$	Begin	;
L02:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{B} \Rightarrow \dot{\neg}(\mathcal{A})_n$	;
L04:	Premise $\gg$	$\mathcal{A}$	;
L05:	AddDoubleNeg $\triangleright$ L04 $\gg$	$\dot{\neg}(\dot{\neg}(\mathcal{A})_n)_n$	;
L06:	MT $\triangleright$ L03 $\triangleright$ L05 $\gg$	$\dot{\neg}(\mathcal{B})_n$	;
L07:	Block $\gg$	End	;
L08:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L09:	Ded $\triangleright$ L07 $\gg$	$(\mathcal{B} \Rightarrow \dot{\neg}(\mathcal{A})_n) \Rightarrow \mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n$	;
L10:	Premise $\gg$	$\mathcal{A} \wedge \mathcal{B}$	;
L11:	Repetition $\gg$	$\dot{\neg}((\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{B})_n))_n$	;
L12:	MT $\triangleright$ L09 $\triangleright$ L11 $\gg$	$\dot{\neg}((\mathcal{B} \Rightarrow \dot{\neg}(\mathcal{A})_n))_n$	;
L13:	Repetition $\triangleright$ L12 $\gg$	$\mathcal{B} \wedge \mathcal{A}$	□

[SystemQ **lemma** FirstConjunct:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \wedge \mathcal{B} \vdash \mathcal{A}$ ]

SystemQ **proof of** FirstConjunct:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{A} \wedge \mathcal{B}$	;
L03:	AndCommutativity $\triangleright$ L02 $\gg$	$\mathcal{B} \wedge \mathcal{A}$	;
L04:	SecondConjunct $\triangleright$ L03 $\gg$	$\mathcal{A}$	□

[SystemQ **lemma** IffFirst:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \Leftrightarrow \mathcal{B} \vdash \mathcal{B} \vdash \mathcal{A}$ ]

SystemQ **proof of** IffFirst:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{A} \Leftrightarrow \mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{B}$	;
L04:	SecondConjunct $\triangleright$ L02 $\gg$	$\mathcal{B} \Rightarrow \mathcal{A}$	;
L05:	MP $\triangleright$ L04 $\triangleright$ L03 $\gg$	$\mathcal{A}$	□

[SystemQ **lemma** IffSecond:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \Leftrightarrow \mathcal{B} \vdash \mathcal{A} \vdash \mathcal{B}$ ]

SystemQ **proof of** IffSecond:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{A} \Leftrightarrow \mathcal{B}$	;
L03:	Premise $\gg$	$\mathcal{A}$	;
L04:	FirstConjunct $\triangleright$ L02 $\gg$	$\mathcal{A} \Rightarrow \mathcal{B}$	;
L05:	MP $\triangleright$ L04 $\triangleright$ L03 $\gg$	$\mathcal{B}$	□

[SystemQ **lemma** IffCommutativity:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \Leftrightarrow \mathcal{B} \vdash \mathcal{B} \Leftrightarrow \mathcal{A}$ ]

SystemQ **proof of** IffCommutativity:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{A} \Leftrightarrow \mathcal{B}$	;
L03:	Repetition $\triangleright$ L02 $\gg$	$(\mathcal{A} \Rightarrow \mathcal{B}) \wedge (\mathcal{B} \Rightarrow \mathcal{A})$	;
L04:	AndCommutativity $\triangleright$ L03 $\gg$	$(\mathcal{B} \Rightarrow \mathcal{A}) \wedge (\mathcal{A} \Rightarrow \mathcal{B})$	;
L05:	Repetition $\triangleright$ L04 $\gg$	$\mathcal{B} \Leftrightarrow \mathcal{A}$	□

—(10.12.06)

[SystemQ **lemma** WeakenOr1:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{B} \vdash \mathcal{A} \vee \mathcal{B}$ ]

SystemQ **proof of** WeakenOr1:

L01:	Arbitrary $\gg$	$\mathcal{A}, \mathcal{B}$	;
L02:	Premise $\gg$	$\mathcal{B}$	;

L03: Weakening  $\triangleright$  L02  $\gg$   $\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$  ;  
L04: Repetition  $\triangleright$  L03  $\gg$   $\mathcal{A} \dot{\vee} \mathcal{B}$   $\square$

[SystemQ **lemma** WeakenOr2:  $\Pi \mathcal{A}, \mathcal{B}: \mathcal{A} \vdash \mathcal{A} \dot{\vee} \mathcal{B}$ ]

SystemQ **proof** of WeakenOr2:

L01: Block  $\gg$  Begin ;  
L02: Arbitrary  $\gg$   $\mathcal{A}, \mathcal{B}$  ;  
L03: Premise  $\gg$   $\mathcal{A}$  ;  
L04: Premise  $\gg$   $\dot{\neg}(\mathcal{A})_n$  ;  
L05: FromContradiction  $\triangleright$  L03  $\triangleright$   
L04  $\gg$   $\mathcal{B}$  ;  
L06: Block  $\gg$  End ;  
L07: Arbitrary  $\gg$   $\mathcal{A}, \mathcal{B}$  ;  
L08: Ded  $\triangleright$  L06  $\gg$   $\mathcal{A} \Rightarrow \dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$  ;  
L09: Premise  $\gg$   $\mathcal{A}$  ;  
L10: MP  $\triangleright$  L08  $\triangleright$  L09  $\gg$   $\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$  ;  
L11: Repetition  $\triangleright$  L10  $\gg$   $\mathcal{A} \dot{\vee} \mathcal{B}$   $\square$

[SystemQ **lemma** FromDisjuncts:  $\Pi \mathcal{A}, \mathcal{B}, \mathcal{C}: \mathcal{A} \dot{\vee} \mathcal{B} \vdash \mathcal{A} \Rightarrow \mathcal{C} \vdash \mathcal{B} \Rightarrow \mathcal{C} \vdash \mathcal{C}$ ]

SystemQ **proof** of FromDisjuncts:

L01: Arbitrary  $\gg$   $\mathcal{A}, \mathcal{B}, \mathcal{C}$  ;  
L02: Premise  $\gg$   $\mathcal{A} \dot{\vee} \mathcal{B}$  ;  
L03: Premise  $\gg$   $\mathcal{A} \Rightarrow \mathcal{C}$  ;  
L04: Premise  $\gg$   $\mathcal{B} \Rightarrow \mathcal{C}$  ;  
L05: Repetition  $\triangleright$  L02  $\gg$   $\dot{\neg}(\mathcal{A})_n \Rightarrow \mathcal{B}$  ;  
L06: Contrapositive  $\triangleright$  L05  $\gg$   $\dot{\neg}(\mathcal{B})_n \Rightarrow \dot{\neg}(\dot{\neg}(\mathcal{A})_n)_n$  ;  
L07: Technicality  $\triangleright$  L03  $\gg$   $\dot{\neg}(\dot{\neg}(\mathcal{A})_n)_n \Rightarrow \mathcal{C}$  ;  
L08: ImplyTransitivity  $\triangleright$  L06  $\triangleright$  L07  $\gg$   
 $\dot{\neg}(\mathcal{B})_n \Rightarrow \mathcal{C}$  ;  
L09: Contrapositive  $\triangleright$  L08  $\gg$   $\dot{\neg}(\mathcal{C})_n \Rightarrow \dot{\neg}(\dot{\neg}(\mathcal{B})_n)_n$  ;  
L10: Contrapositive  $\triangleright$  L04  $\gg$   $\dot{\neg}(\mathcal{C})_n \Rightarrow \dot{\neg}(\mathcal{B})_n$  ;  
L11: Neg  $\triangleright$  L10  $\triangleright$  L09  $\gg$   $\mathcal{C}$   $\square$

[SystemQ **rule** NumericalF:  $\text{IIFX}: (\text{of} \leq_f \text{FX} \Rightarrow |\text{fFX}| = \text{FX}) \wedge (\dot{\neg}(\text{of} \leq_f \text{FX})_n \Rightarrow |\text{fFX}| = -_f \text{FX})$ ]

$([|x| \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. |[ \text{Macro}(t^h :: \text{ExpandList}(t^t, s, c))] ]])^P$

$[|[ \text{Macro}(t) \xrightarrow{\text{val}} \tilde{Q}(t, [R(|f|x)], ([x] :: \text{extractSeries}(t^1)) :: T) ] ]$

—(11.12.06)

[SystemQ **rule** (Adgic)SameR:  $\text{IIFX}, \text{FY}: \text{FX} = \text{FY} \vdash R(\text{FX}) = R(\text{FY})$ ]

—(11.12.06)

$([01//\text{tempx} \xrightarrow{\text{macro}} \lambda t. \lambda s. \lambda c. 01//\text{Macro}(t^h :: \text{ExpandList}(t^t, s, c))] ])^P$

$[01//\text{Macro}(t) \xrightarrow{\text{val}} \tilde{Q}(t, [R(1f/x)], ([x] :: \text{extractSeries}(t^1)) :: T) ]$

venter—

## Priority table

### Preassociative

[kvanti], [base], [bracket \* end bracket], [big bracket \* end bracket], [ \$ \* \$ ],  
[flush left [\*]], [x], [y], [z], [[\*  $\bowtie$  \*]], [[\*  $\xrightarrow{*}$  \*]], [pyk], [tex], [name], [prio], [\*], [T],

$[f(*, *, *)]$ ,  $[[* \xrightarrow{*} *]]$ ,  $[val]$ ,  $[claim]$ ,  $[\perp]$ ,  $[f(*)]$ ,  $[(*)^1]$ ,  $[F]$ ,  $[0]$ ,  $[1]$ ,  $[2]$ ,  $[3]$ ,  $[4]$ ,  $[5]$ ,  $[6]$ ,  $[7]$ ,  $[8]$ ,  $[9]$ ,  $[0]$ ,  $[1]$ ,  $[2]$ ,  $[3]$ ,  $[4]$ ,  $[5]$ ,  $[6]$ ,  $[7]$ ,  $[8]$ ,  $[9]$ ,  $[a]$ ,  $[b]$ ,  $[c]$ ,  $[d]$ ,  $[e]$ ,  $[f]$ ,  $[g]$ ,  $[h]$ ,  $[i]$ ,  $[j]$ ,  $[k]$ ,  $[l]$ ,  $[m]$ ,  $[n]$ ,  $[o]$ ,  $[p]$ ,  $[q]$ ,  $[r]$ ,  $[s]$ ,  $[t]$ ,  $[u]$ ,  $[v]$ ,  $[w]$ ,  $[(*)^M]$ ,  $[If(*, *, *)]$ ,  $[array\{*\} * \text{end array}]$ ,  $[l]$ ,  $[c]$ ,  $[r]$ ,  $[empty]$ ,  $[\{* | * := *\}]$ ,  $[\mathcal{M}(*)]$ ,  $[\tilde{\mathcal{U}}(*)]$ ,  $[\mathcal{U}(*)]$ ,  $[\mathcal{U}^M(*)]$ ,  $[\mathbf{apply}(*, *)]$ ,  $[\mathbf{apply}_1(*, *)]$ ,  $[\text{identifier}(*)]$ ,  $[\text{identifier}_1(*, *)]$ ,  $[\text{array-plus}(*, *)]$ ,  $[\text{array-remove}(*, *, *)]$ ,  $[\text{array-put}(*, *, *, *)]$ ,  $[\text{array-add}(*, *, *, *, *)]$ ,  $[\text{bit}(*, *)]$ ,  $[\text{bit}_1(*, *)]$ ,  $[\text{rack}]$ ,  $["\text{vector}"]$ ,  $["\text{bibliography}"]$ ,  $["\text{dictionary}"]$ ,  $["\text{body}"]$ ,  $["\text{codex}"]$ ,  $["\text{expansion}"]$ ,  $["\text{code}"]$ ,  $["\text{cache}"]$ ,  $["\text{diagnose}"]$ ,  $["\text{pyk}"]$ ,  $["\text{tex}"]$ ,  $["\text{texname}"]$ ,  $["\text{value}"]$ ,  $["\text{message}"]$ ,  $["\text{macro}"]$ ,  $["\text{definition}"]$ ,  $["\text{unpack}"]$ ,  $["\text{claim}"]$ ,  $["\text{priority}"]$ ,  $["\text{lambda}"]$ ,  $["\text{apply}"]$ ,  $["\text{true}"]$ ,  $["\text{if}"]$ ,  $["\text{quote}"]$ ,  $["\text{proclaim}"]$ ,  $["\text{define}"]$ ,  $["\text{introduce}"]$ ,  $["\text{hide}"]$ ,  $["\text{pre}"]$ ,  $["\text{post}"]$ ,  $[\mathcal{E}(*, *, *)]$ ,  $[\mathcal{E}_2(*, *, *, *, *)]$ ,  $[\mathcal{E}_3(*, *, *, *, *)]$ ,  $[\mathcal{E}_4(*, *, *, *, *)]$ ,  $[\mathbf{lookup}(*, *, *)]$ ,  $[\mathbf{abstract}(*, *, *, *, *)]$ ,  $[[*]]$ ,  $[\mathcal{M}(*, *, *, *)]$ ,  $[\mathcal{M}_2(*, *, *, *, *)]$ ,  $[\mathcal{M}^*(*, *, *, *)]$ ,  $[\text{macro}]$ ,  $[s_0]$ ,  $[\mathbf{zip}(*, *)]$ ,  $[\mathbf{assoc}_1(*, *, *)]$ ,  $[(*)^P]$ ,  $[\text{self}]$ ,  $[[* \doteq *]]$ ,  $[[* \dot{=} *]]$ ,  $[[* \dot{=} *]]$ ,  $[[* \stackrel{\text{pyk}}{=} *]]$ ,  $[[* \stackrel{\text{tex}}{=} *]]$ ,  $[[* \stackrel{\text{name}}{=} *]]$ ,  $[\mathbf{Priority table}[*]]$ ,  $[\tilde{\mathcal{M}}_1]$ ,  $[\tilde{\mathcal{M}}_2(*)]$ ,  $[\tilde{\mathcal{M}}_3(*)]$ ,  $[\tilde{\mathcal{M}}_4(*, *, *, *, *)]$ ,  $[\tilde{\mathcal{M}}(*, *, *, *)]$ ,  $[\tilde{\mathcal{Q}}(*, *, *, *)]$ ,  $[\tilde{\mathcal{Q}}_2(*, *, *, *)]$ ,  $[\tilde{\mathcal{Q}}_3(*, *, *, *, *)]$ ,  $[\tilde{\mathcal{Q}}^*(*, *, *, *)]$ ,  $[(*)]$ ,  $[(*)]$ ,  $[\text{display}(*)]$ ,  $[\text{statement}(*)]$ ,  $[[*]]$ ,  $[[*^-]]$ ,  $[\mathbf{aspect}(*, *)]$ ,  $[\mathbf{aspect}(*, *, *)]$ ,  $[(*)]$ ,  $[\mathbf{tuple}_1(*)]$ ,  $[\mathbf{tuple}_2(*)]$ ,  $[\text{let}_2(*, *)]$ ,  $[\text{let}_1(*, *)]$ ,  $[[* \stackrel{\text{claim}}{=} *]]$ ,  $[\text{checker}]$ ,  $[\mathbf{check}(*, *)]$ ,  $[\mathbf{check}_2(*, *, *)]$ ,  $[\mathbf{check}_3(*, *, *, *)]$ ,  $[\mathbf{check}^*(*, *)]$ ,  $[\mathbf{check}_2^*(*, *, *, *)]$ ,  $[[*]]$ ,  $[[*^-]]$ ,  $[[*^\circ]]$ ,  $[\text{msg}]$ ,  $[[* \stackrel{\text{msg}}{=} *]]$ ,  $[\text{<stmt>}]$ ,  $[\text{stmt}]$ ,  $[[* \stackrel{\text{stmt}}{=} *]]$ ,  $[\text{HeadNil}']$ ,  $[\text{HeadPair}']$ ,  $[\text{Transitivity}']$ ,  $[\perp]$ ,  $[\text{Contra}']$ ,  $[\text{T}'_E]$ ,  $[\mathcal{L}_1]$ ,  $[\mathcal{A}]$ ,  $[\mathcal{B}]$ ,  $[\mathcal{C}]$ ,  $[\mathcal{D}]$ ,  $[\mathcal{E}]$ ,  $[\mathcal{F}]$ ,  $[\mathcal{G}]$ ,  $[\mathcal{H}]$ ,  $[\mathcal{I}]$ ,  $[\mathcal{J}]$ ,  $[\mathcal{K}]$ ,  $[\mathcal{L}]$ ,  $[\mathcal{M}]$ ,  $[\mathcal{N}]$ ,  $[\mathcal{O}]$ ,  $[\mathcal{P}]$ ,  $[\mathcal{Q}]$ ,  $[\mathcal{R}]$ ,  $[\mathcal{S}]$ ,  $[\mathcal{T}]$ ,  $[\mathcal{U}]$ ,  $[\mathcal{V}]$ ,  $[\mathcal{W}]$ ,  $[\mathcal{X}]$ ,  $[\mathcal{Y}]$ ,  $[\mathcal{Z}]$ ,  $[[* | * := *]]$ ,  $[[*^* | * := *]]$ ,  $[\emptyset]$ ,  $[\text{Remainder}]$ ,  $[(*)^\vee]$ ,  $[\text{intro}(*, *, *, *, *)]$ ,  $[\text{intro}(*, *, *, *)]$ ,  $[\text{error}(*, *)]$ ,  $[\text{error}_2(*, *)]$ ,  $[\text{proof}(*, *, *)]$ ,  $[\text{proof}_2(*, *)]$ ,  $[\mathcal{S}(*, *)]$ ,  $[\mathcal{S}^1(*, *)]$ ,  $[\mathcal{S}^\triangleright(*, *)]$ ,  $[\mathcal{S}_1^\triangleright(*, *, *, *)]$ ,  $[\mathcal{S}^E(*, *)]$ ,  $[\mathcal{S}_1^E(*, *, *, *)]$ ,  $[\mathcal{S}^+(*, *)]$ ,  $[\mathcal{S}_1^+(*, *, *, *)]$ ,  $[\mathcal{S}^-(*, *)]$ ,  $[\mathcal{S}_1^-(*, *, *, *)]$ ,  $[\mathcal{S}^*(*, *)]$ ,  $[\mathcal{S}_1^*(*, *, *, *)]$ ,  $[\mathcal{S}_2^*(*, *, *, *, *)]$ ,  $[\mathcal{S}^\oplus(*, *)]$ ,  $[\mathcal{S}_1^\oplus(*, *, *, *, *)]$ ,  $[\mathcal{S}^\ominus(*, *)]$ ,  $[\mathcal{S}_1^\ominus(*, *, *, *, *)]$ ,  $[\mathcal{S}^\#(*, *)]$ ,  $[\mathcal{S}_1^\#(*, *, *, *, *)]$ ,  $[\mathcal{S}^{i.e.}(*, *)]$ ,  $[\mathcal{S}_1^{i.e.}(*, *, *, *, *)]$ ,  $[\mathcal{S}_2^{e.}(*, *, *, *, *, *)]$ ,  $[\mathcal{S}^\vee(*, *)]$ ,  $[\mathcal{S}_1^\vee(*, *, *, *, *)]$ ,  $[\mathcal{S}^i(*, *)]$ ,  $[\mathcal{S}_1^i(*, *, *, *, *)]$ ,  $[\mathcal{S}_2^i(*, *, *, *, *, *)]$ ,  $[\mathcal{T}(*)]$ ,  $[\text{claims}(*, *, *)]$ ,  $[\text{claims}_2(*, *, *)]$ ,  $[\text{<proof>}]$ ,  $[\text{proof}]$ ,  $[[\mathbf{Lemma} * : *]]$ ,  $[[\mathbf{Proof of} * : *]]$ ,  $[[* \mathbf{lemma} * : *]]$ ,  $[[* \mathbf{antilemma} * : *]]$ ,  $[[* \mathbf{rule} * : *]]$ ,  $[[* \mathbf{antirule} * : *]]$ ,  $[\text{verifier}]$ ,  $[\mathcal{V}_1(*)]$ ,  $[\mathcal{V}_2(*, *)]$ ,  $[\mathcal{V}_3(*, *, *, *)]$ ,  $[\mathcal{V}_4(*, *)]$ ,  $[\mathcal{V}_5(*, *, *, *, *)]$ ,  $[\mathcal{V}_6(*, *, *, *, *)]$ ,  $[\mathcal{V}_7(*, *, *, *, *)]$ ,  $[\text{Cut}(*, *)]$ ,  $[\text{Head}_\oplus(*)]$ ,  $[\text{Tail}_\oplus(*)]$ ,  $[\text{rule}_1(*, *)]$ ,  $[\text{rule}(*, *)]$ ,  $[\text{Rule tactic}]$ ,  $[\text{Plus}(*, *)]$ ,  $[[\mathbf{Theory} *]]$ ,  $[\text{theory}_2(*, *)]$ ,  $[\text{theory}_3(*, *)]$ ,  $[\text{theory}_4(*, *, *)]$ ,  $[\text{HeadNil}''']$ ,  $[\text{HeadPair}''']$ ,  $[\text{Transitivity}''']$ ,  $[\text{Contra}''']$ ,  $[\text{HeadNil}]$ ,  $[\text{HeadPair}]$ ,  $[\text{Transitivity}]$ ,  $[\text{Contra}]$ ,  $[\text{T}'_E]$ ,  $[\text{ragged right}]$ ,  $[\text{ragged right expansion}]$ ,  $[\text{parm}(*, *, *)]$ ,  $[\text{parm}^*(*, *, *)]$ ,  $[\text{inst}(*, *)]$ ,  $[\text{inst}^*(*, *)]$ ,  $[\text{occur}(*, *, *)]$ ,  $[\text{occur}^*(*, *, *)]$ ,  $[\text{unify}(* = *, *)]$ ,  $[\text{unify}^*(* = *, *)]$ ,  $[\text{unify}_2(* = *, *)]$ ,  $[\text{L}_a]$ ,  $[\text{L}_b]$ ,  $[\text{L}_c]$ ,  $[\text{L}_d]$ ,  $[\text{L}_e]$ ,  $[\text{L}_f]$ ,  $[\text{L}_g]$ ,  $[\text{L}_h]$ ,  $[\text{L}_i]$ ,  $[\text{L}_j]$ ,  $[\text{L}_k]$ ,  $[\text{L}_l]$ ,  $[\text{L}_m]$ ,  $[\text{L}_n]$ ,  $[\text{L}_o]$ ,  $[\text{L}_p]$ ,  $[\text{L}_q]$ ,  $[\text{L}_r]$ ,  $[\text{L}_s]$ ,  $[\text{L}_t]$ ,  $[\text{L}_u]$ ,  $[\text{L}_v]$ ,  $[\text{L}_w]$ ,  $[\text{L}_x]$ ,  $[\text{L}_y]$ ,  $[\text{L}_z]$ ,  $[\text{L}_A]$ ,  $[\text{L}_B]$ ,  $[\text{L}_C]$ ,  $[\text{L}_D]$ ,  $[\text{L}_E]$ ,  $[\text{L}_F]$ ,  $[\text{L}_G]$ ,  $[\text{L}_H]$ ,  $[\text{L}_I]$ ,  $[\text{L}_J]$ ,  $[\text{L}_K]$ ,  $[\text{L}_L]$ ,  $[\text{L}_M]$ ,  $[\text{L}_N]$ ,  $[\text{L}_O]$ ,  $[\text{L}_P]$ ,  $[\text{L}_Q]$ ,  $[\text{L}_R]$ ,  $[\text{L}_S]$ ,  $[\text{L}_T]$ ,  $[\text{L}_U]$ ,  $[\text{L}_V]$ ,  $[\text{L}_W]$ ,  $[\text{L}_X]$ ,  $[\text{L}_Y]$ ,  $[\text{L}_Z]$ ,  $[\text{L}_?]$ ,  $[\text{Reflexivity}]$ ,  $[\text{Reflexivity}_1]$ ,  $[\text{Commutativity}]$ ,  $[\text{Commutativity}_1]$ ,  $[\text{<tactic>}]$ ,  $[\text{tactic}]$ ,  $[[* \stackrel{\text{tactic}}{=} *]]$ ,  $[\mathcal{P}(*, *, *)]$ ,  $[\mathcal{P}^*(*, *, *)]$ ,  $[\text{p}_0]$ ,  $[\text{conclude}_1(*, *)]$ ,  $[\text{conclude}_2(*, *, *)]$ ,  $[\text{conclude}_3(*, *, *, *)]$ ,

[conclude<sub>4</sub>(\* , \* )], [check], [ $[* \overset{\circ}{=} *$ ]], [RootVisible(\* )], [A], [R], [C], [T], [L], [ $\{*\}$ ], [ $\bar{*}$ ],  
 [a], [b], [c], [d], [e], [f], [g], [h], [i], [j], [k], [l], [m], [n], [o], [p], [q], [r], [s], [t], [u], [v],  
 [w], [x], [y], [z], [ $\langle * \equiv * \mid * := * \rangle$ ]], [ $\langle * \equiv^0 * \mid * := * \rangle$ ]], [ $\langle * \equiv^1 * \mid * := * \rangle$ ]], [ $\langle * \equiv^* * \mid * := * \rangle$ ]],  
 [Ded(\* , \* )], [Ded<sub>0</sub>(\* , \* )], [Ded<sub>1</sub>(\* , \* , \* )], [Ded<sub>2</sub>(\* , \* , \* )], [Ded<sub>3</sub>(\* , \* , \* , \* )],  
 [Ded<sub>4</sub>(\* , \* , \* , \* )], [Ded<sub>4</sub><sup>\*</sup>(\* , \* , \* , \* )], [Ded<sub>5</sub>(\* , \* , \* )], [Ded<sub>6</sub>(\* , \* , \* , \* )],  
 [Ded<sub>6</sub><sup>\*</sup>(\* , \* , \* , \* )], [Ded<sub>7</sub>(\* )], [Ded<sub>8</sub>(\* , \* )], [Ded<sub>8</sub><sup>\*</sup>(\* , \* )], [S], [Neg], [MP], [Gen],  
 [Ded], [S1], [S2], [S3], [S4], [S5], [S6], [S7], [S8], [S9], [Repetition], [A1'], [A2'], [A4'],  
 [A5'], [Prop 3.2a], [Prop 3.2b], [Prop 3.2c], [Prop 3.2d], [Prop 3.2e<sub>1</sub>], [Prop 3.2e<sub>2</sub>],  
 [Prop 3.2e], [Prop 3.2f<sub>1</sub>], [Prop 3.2f<sub>2</sub>], [Prop 3.2f], [Prop 3.2g<sub>1</sub>], [Prop 3.2g<sub>2</sub>],  
 [Prop 3.2g], [Prop 3.2h<sub>1</sub>], [Prop 3.2h<sub>2</sub>], [Prop 3.2h], [Block<sub>1</sub>(\* , \* , \* )], [Block<sub>2</sub>(\* )],  
 [UniqueMember], [UniqueMember(Type)], [SameSeries], [A4], [SameMember],  
 [Qclosed(Addition)], [Qclosed(Multiplication)], [FromCartProd(1)],  
 [1rule fromCartProd(2)], [constantRationalSeries(\* )], [cartProd(\* )], [Power(\* )],  
 [binaryUnion(\* , \* )], [SetOfRationalSeries], [IsSubset(\* , \* )], [(p\* , \* )], [(s\* )],  
 [( $\cdot \cdot \cdot$ )], [Objekt-var], [Ex-var], [Ph-var], [Værdi], [Variabel], [Op(\* )], [Op(\* , \* )],  
 [\* := \* ], [ContainsEmpty(\* )], [Nat(\* )], [Dedu(\* , \* )], [Dedu<sub>0</sub>(\* , \* )],  
 [Dedu<sub>s</sub>(\* , \* , \* )], [Dedu<sub>1</sub>(\* , \* , \* )], [Dedu<sub>2</sub>(\* , \* , \* )], [Dedu<sub>3</sub>(\* , \* , \* , \* )],  
 [Dedu<sub>4</sub>(\* , \* , \* , \* )], [Dedu<sub>4</sub><sup>\*</sup>(\* , \* , \* , \* )], [Dedu<sub>5</sub>(\* , \* , \* )], [Dedu<sub>6</sub>(\* , \* , \* , \* )],  
 [Dedu<sub>6</sub><sup>\*</sup>(\* , \* , \* , \* )], [Dedu<sub>7</sub>(\* )], [Dedu<sub>8</sub>(\* , \* )], [Dedu<sub>8</sub><sup>\*</sup>(\* , \* )], [EX<sub>1</sub>], [EX<sub>2</sub>], [EX<sub>3</sub>],  
 [EX<sub>10</sub>], [EX<sub>20</sub>], [ $*_{\text{Ex}}$ ], [ $*^{\text{Ex}}$ ]], [ $\langle * \equiv * \mid * := * \rangle_{\text{Ex}}$ ]], [ $\langle * \equiv^0 * \mid * := * \rangle_{\text{Ex}}$ ]],  
 [ $\langle * \equiv^1 * \mid * := * \rangle_{\text{Ex}}$ ]], [ $\langle * \equiv^* * \mid * := * \rangle_{\text{Ex}}$ ]], [ph<sub>1</sub>], [ph<sub>2</sub>], [ph<sub>3</sub>], [ $*_{\text{Ph}}$ ]], [ $*^{\text{Ph}}$ ]],  
 [ $\langle * \equiv * \mid * := * \rangle_{\text{Ph}}$ ]], [ $\langle * \equiv^0 * \mid * := * \rangle_{\text{Ph}}$ ]], [ $\langle * \equiv^1 * \mid * := * \rangle_{\text{Ph}}$ ]],  
 [ $\langle * \equiv^* * \mid * := * \rangle_{\text{Ph}}$ ]], [ $\langle * \equiv * \mid * := * \rangle_{\text{Me}}$ ]], [ $\langle * \equiv^1 * \mid * := * \rangle_{\text{Me}}$ ]],  
 [ $\langle * \equiv^* * \mid * := * \rangle_{\text{Me}}$ ]], [bs], [OBS], [BS], [∅], [SystemQ], [MP], [Gen], [Repetition],  
 [Neg], [Ded], [ExistIntro], [Extensionality], [∅def], [PairDef], [UnionDef],  
 [PowerDef], [SeparationDef], [AddDoubleNeg], [RemoveDoubleNeg],  
 [AndCommutativity], [AutoImPLY], [Contrapositive], [FirstConjunct],  
 [SecondConjunct], [FromContradiction], [FromDisjuncts], [IffCommutativity],  
 [IffFirst], [IffSecond], [ImPLYTransitivity], [JoinConjuncts], [MP2], [MP3], [MP4],  
 [MP5], [MT], [NegativeMT], [Technicality], [Weakening], [WeakenOr1],  
 [WeakenOr2], [Formula2Pair], [Pair2Formula], [Formula2Union],  
 [Union2Formula], [Formula2Sep], [Sep2Formula], [Formula2Power],  
 [SubsetInPower], [HelperPowerIsSub], [PowerIsSub],  
 [(Switch)HelperPowerIsSub], [(Switch)PowerIsSub], [ToSetEquality],  
 [HelperToSetEquality(t)], [ToSetEquality(t)], [HelperFromSetEquality],  
 [FromSetEquality], [HelperReflexivity], [Reflexivity], [HelperSymmetry],  
 [Symmetry], [HelperTransitivity], [Transitivity], [ERisReflexive],  
 [ERisSymmetric], [ERisTransitive], [∅isSubset], [HelperMemberNot∅],  
 [MemberNot∅], [HelperUnique∅], [Unique∅], [= Reflexivity], [= Symmetry],  
 [Helper == Transitivity], [= Transitivity], [HelperTransferNotEq],  
 [TransferNotEq], [HelperPairSubset], [Helper(2)PairSubset], [PairSubset],  
 [SamePair], [SameSingleton], [UnionSubset], [SameUnion], [SeparationSubset],  
 [SameSeparation], [SameBinaryUnion], [IntersectionSubset], [SameIntersection],  
 [AutoMember], [HelperEqSysNot∅], [EqSysNot∅], [HelperEqSubset],  
 [EqSubset], [HelperEqNecessary], [EqNecessary], [HelperNoneEqNecessary],  
 [Helper(2)NoneEqNecessary], [NoneEqNecessary], [EqClassIsSubset],



[EqClassesAreDisjoint], [AllDisjoint], [AllDisjointImPLY], [BSsubset],  
[Union(BS/R)subset], [UnionIdentity], [EqSysIsPartition], [(x1)], [(x2)], [(y1)],  
[(y2)], [(v1)], [(v2)], [(v3)], [(v4)], [(v2n)], [(m1)], [(m2)], [(n1)], [(n2)], [(n3)], [(ε)],  
[(ε<sub>1</sub>)], [(ε<sub>2</sub>)], [(fep)], [(fx)], [(fy)], [(fz)], [(fu)], [(fv)], [(fw)], [(rx)], [(ry)], [(rz)],  
[(ru)], [(sx)], [(sx1)], [(sy)], [(sy1)], [(sz)], [(sz1)], [(su)], [(su1)], [(fxs)], [(fys)],  
[(crs1)], [(f1)], [(f2)], [(f3)], [(f4)], [(op1)], [(op2)], [(r1)], [(s1)], [(s2)], [X<sub>1</sub>], [X<sub>2</sub>],  
[Y<sub>1</sub>], [Y<sub>2</sub>], [V<sub>1</sub>], [V<sub>2</sub>], [V<sub>3</sub>], [V<sub>4</sub>], [V<sub>2n</sub>], [M<sub>1</sub>], [M<sub>2</sub>], [N<sub>1</sub>], [N<sub>2</sub>], [N<sub>3</sub>], [ε], [ε<sub>1</sub>], [ε<sub>2</sub>],  
[FX], [FY], [FZ], [FU], [FV], [FW], [FEP], [RX], [RY], [RZ], [RU], [(SX)], [(SX1)],  
[(SY)], [(SY1)], [(SZ)], [(SZ1)], [(SU)], [(SU1)], [FXS], [FYS], [(F1)], [(F2)], [(F3)],  
[(F4)], [(OP1)], [(OP2)], [(R1)], [(S1)], [(S2)], [(EPob)], [(CRS1ob)], [(F1ob)],  
[(F2ob)], [(F3ob)], [(F4ob)], [(N1ob)], [(N2ob)], [(OP1ob)], [(OP2ob)], [(R1ob)],  
[(S1ob)], [(S2ob)], [ph<sub>4</sub>], [ph<sub>5</sub>], [ph<sub>6</sub>], [NAT], [RATIONALSERIES], [SERIES],  
[SetOfReals], [SetOfFxs], [N], [Q], [X], [xs], [xaF], [ysF], [us], [usFoelge], [0], [1],  
[(-1)], [2], [3], [1/2], [1/3], [2/3], [0f], [1f], [00], [01], [(- - 01)], [02], [01/02],  
[PlusAssociativity(R)], [PlusAssociativity(R)XX], [Plus0(R)], [Negative(R)],  
[Times1(R)], [lessAddition(R)], [PlusCommutativity(R)],  
[LeqAntisymmetry(R)], [LeqTransitivity(R)], [leqAddition(R)],  
[Distribution(R)], [A4(Axiom)], [InductionAxiom], [EqualityAxiom],  
[EqLeqAxiom], [EqAdditionAxiom], [EqMultiplicationAxiom],  
[QisClosed(Reciprocal)(ImPLY)], [QisClosed(Reciprocal)],  
[QisClosed(Negative)(ImPLY)], [QisClosed(Negative)], [leqReflexivity],  
[leqAntisymmetryAxiom], [leqTransitivityAxiom], [leqTotality],  
[leqAdditionAxiom], [leqMultiplicationAxiom], [plusAssociativity],  
[plusCommutativity], [Negative], [plus0], [timesAssociativity],  
[timesCommutativity], [ReciprocalAxiom], [times1], [Distribution], [0not1],  
[lemma eqLeq(R)], [TimesAssociativity(R)], [TimesCommutativity(R)],  
[(Adgic)SameR], [Separation2formula(1)], [Separation2formula(2)], [Cauchy],  
[PlusF], [ReciprocalF], [From ==], [To ==], [FromInR], [PlusR(Sym)],  
[ReciprocalR(Axiom)], [LessMinus1(N)], [Nonnegative(N)], [US0],  
[NextXS(UpperBound)], [NextXS(NoUpperBound)], [NextUS(UpperBound)],  
[NextUS(NoUpperBound)], [ExpZero], [ExpPositive], [ExpZero(R)],  
[ExpPositive(R)], [BSzero], [BSpositive], [UStelescope(Zero)],  
[UStelescope(Positive)], [EqAddition(R)], [FromLimit], [ToUpperBound],  
[FromUpperBound], [USisUpperBound], [0not1(R)], [ExpUnbounded(R)],  
[FromLeq(Advanced)(N)], [FromLeastUpperBound], [ToLeastUpperBound],  
[XSisNotUpperBound], [ysFGreater], [ysFLess], [SmallInverse], [Nat Type],  
[RationalType], [SeriesType], [Max], [Numerical], [NumericalF],  
[MemberOfSeries(ImPLY)], [JoinConjuncts(2conditions)],  
[prop lemma imply negation], [TND], [FromNegatedImPLY], [ToNegatedImPLY],  
[FromNegated(2 \* ImPLY)], [FromNegatedAnd], [FromNegatedOr],  
[ToNegatedOr], [FromNegations], [From3Disjuncts], [From2 \* 2Disjuncts],  
[NegateDisjunct1], [NegateDisjunct2], [ExpandDisjuncts], [SENC1], [SENC2],  
[LessLeq(R)], [MemberOfSeries], [memberOfSeries(Type)];

### Preassociative

[\*\_{\*}], [\*/indexintro(\*, \*, \*, \*)], [\*/intro(\*, \*, \*)], [\*/bothintro(\*, \*, \*, \*, \*)],  
[\*/nameintro(\*, \*, \*, \*)], [\*'], [\* \*], [\* \* → \*], [\* \* ⇒ \*], [\*0], [\*1], [0b], [\*-color(\*)],

[\*color\*(\*)], [\*H], [\*T], [\*U], [\*h], [\*t], [\*s], [\*c], [\*d], [\*a], [\*C], [\*M], [\*B], [\*F], [\*i], [\*d], [\*R], [\*0], [\*1], [\*2], [\*3], [\*4], [\*5], [\*6], [\*7], [\*8], [\*9], [\*E], [\*V], [\*C], [\*C<sup>n</sup>], [\*hide];

### Preassociative

[“ \* ”], [], [(\*)<sup>t</sup>], [string(\*) + \*], [string(\*) ++ \*], [\*], [\*], [! \*], [“ \* ”], [# \*], [\$ \*], [% \*], [& \*], [’ \*], [(\*)], [() \*], [\*\*], [+ \*], [ \* ], [- \*], [ . \* ], [ / \* ], [0 \*], [1 \*], [2 \*], [3 \*], [4 \*], [5 \*], [6 \*], [7 \*], [8 \*], [9 \*], [: \*], [; \*], [< \*], [= \*], [> \*], [? \*], [@ \*], [A \*], [B \*], [C \*], [D \*], [E \*], [F \*], [G \*], [H \*], [I \*], [J \*], [K \*], [L \*], [M \*], [N \*], [O \*], [P \*], [Q \*], [R \*], [S \*], [T \*], [U \*], [V \*], [W \*], [X \*], [Y \*], [Z \*], [[ \* ], [\ \* ], [ ] \*], [ ^ \* ], [ \_ \* ], [ ‘ \* ], [ a \* ], [ b \* ], [ c \* ], [ d \* ], [ e \* ], [ f \* ], [ g \* ], [ h \* ], [ i \* ], [ j \* ], [ k \* ], [ l \* ], [ m \* ], [ n \* ], [ o \* ], [ p \* ], [ q \* ], [ r \* ], [ s \* ], [ t \* ], [ u \* ], [ v \* ], [ w \* ], [ x \* ], [ y \* ], [ z \* ], [ { \* }, [ | \* }, [ } \* ], [ ~ \* ], [Preassociative \*; \*], [Postassociative \*; \*], [ \* ], [ \* ], [priority \* end], [newline \*], [macro newline \*], [MacroIndent(\*)];

### Preassociative

[ \* ’ \* ], [ \* ‘ \* ];

### Preassociative

[\*(exp)\*];

### Preassociative

[\*], [R(\*)], [ - - R(\*)], [rec\*];

### Preassociative

[\*/ \*], [ \* ∩ \* ], [ \* [ \* ] ];

### Preassociative

[∪ \*], [ \* ∪ \* ], [P(\*)];

### Preassociative

[{ \* }], [StateExpand(\*, \*, \*)], [extractSeries(\*)], [SetOfSeries(\*)], [ - - Macro(\*)], [ExpandList(\*, \*, \*)], [\*\* Macro(\*)], [+ + Macro(\*)], [<< Macro(\*)], [|Macro(\*)], [01//Macro(\*)], [UB(\*, \*)], [LUB(\*, \*)], [BS(\*, \*)], [USteelescope(\*, \*)], [(\*)], [|f \* |], [|r \* |], [Limit(\*, \*)], [Union(\*)], [IsOrderedPair(\*, \*, \*)], [IsRelation(\*, \*, \*)], [isFunction(\*, \*, \*)], [IsSeries(\*, \*)], [IsNatural(\*, \*)], [OrderedPair(\*, \*)], [TypeNat(\*)], [TypeNat0(\*)], [TypeRational(\*)], [TypeRational0(\*)], [TypeSeries(\*, \*)], [Typeseries0(\*, \*)];

### Preassociative

[{ \* , \* }], [( \* , \* )], [(-u\*)], [-f\*], [(- - \*)], [1f/\*], [01//temp\*];

### Preassociative

[\*( \* , \* )], [RefRel(\*, \*)], [SymRel(\*, \*)], [TransRel(\*, \*)], [EqRel(\*, \*)], [( \* ∈ \* ) \* ], [Partition(\*, \*)];

### Preassociative

[ \* · \* ], [ \* · 0 \* ], [( \* \* \* )], [ \* \*<sub>f</sub> \* ], [ \* \* \* \* ];

### Preassociative

[ \* + \* ], [ \* + 0 \* ], [ \* + 1 \* ], [ \* - \* ], [ \* - 0 \* ], [ \* - 1 \* ], [( \* + \* )], [( \* - \* )], [ \* +<sub>f</sub> \* ], [ \* -<sub>f</sub> \* ], [ \* + + \* ], [R(\*) - - R(\*)];

### Preassociative

[ \* ∈ \* ];

### Preassociative

[| \* |], [if(\*, \*, \*)], [Max(\*, \*)], [Max(\*, \*)];

### Preassociative

$[* = *], [* \neq *], [* \leq *], [* < *], [* <_f *], [* \leq_f *], [SF(*, *)], [* == *],$   
 $[*!! == *], [* << *], [* << == *];$

**Preassociative**

$[* \cup \{*\}], [* \cup *], [* \setminus \{*\}];$

**Postassociative**

$[* \dot{:} *], [* \dot{:} *], [* \dot{:} *], [* +2* *], [* :: *], [* +2* *];$

**Postassociative**

$[*, *];$

**Preassociative**

$[* \overset{B}{\approx} *], [* \overset{D}{\approx} *], [* \overset{C}{\approx} *], [* \overset{P}{\approx} *], [* \approx *], [* = *], [* \dashv *], [* \overset{t}{=} *], [* \overset{t^*}{=} *], [* \overset{r}{=} *],$   
 $[* \in_t *], [* \subseteq_T *], [* \overset{T}{=} *], [* \overset{s}{=} *], [* \text{free in } *], [* \text{free in}^* *], [* \text{free for } * \text{ in } *],$   
 $[* \text{free for}^* * \text{ in } *], [* \in_c *], [* < *], [* <' *], [* \leq' *], [* = *], [* \neq *], [*^{\text{var}}],$   
 $[* \#^0 *], [* \#^1 *], [* \#^* *], [* == *], [* \subseteq *];$

**Preassociative**

$[\neg *], [\dot{\neg} (*n)], [* \notin *], [* \neq *];$

**Preassociative**

$[* \wedge *], [* \ddot{\wedge} *], [* \tilde{\wedge} *], [* \wedge_c *], [* \dot{\wedge} *];$

**Preassociative**

$[* \vee *], [* \parallel *], [* \ddot{\vee} *];$

**Postassociative**

$[* \dot{\vee} *];$

**Preassociative**

$[\exists *: *], [\forall *: *], [\forall_{\text{obj}} *: *], [\exists *: *];$

**Postassociative**

$[* \dot{\Rightarrow} *], [* \Rightarrow *], [* \Leftrightarrow *], [* \dot{\Leftrightarrow} *];$

**Preassociative**

$[\{\text{ph} \in * \mid *\}];$

**Postassociative**

$[* : *], [* \text{spy } *], [* ! *];$

**Preassociative**

$[* \left\{ \begin{array}{l} * \\ * \end{array} \right.];$

**Preassociative**

$[\lambda *. *], [\Lambda *. *], [\Lambda *], [\text{if } * \text{ then } * \text{ else } *], [\text{let } * = * \text{ in } *], [\text{let } * \ddot{=} * \text{ in } *];$

**Preassociative**

$[* \# *];$

**Preassociative**

$[*^I], [*^\triangleright], [*^V], [*^+], [*^-], [*^*];$

**Preassociative**

$[* @ *], [* \triangleright *], [* \blacktriangleright *], [* \gg *], [* \triangleright *];$

**Postassociative**

$[* \vdash *], [* \vdash *], [* \text{i.e. } *];$

**Preassociative**

$[\forall *: *], [\Pi *: *];$

**Postassociative**

[\*  $\oplus$  \*];

**Postassociative**

[\*, \*];

**Preassociative**

[\* proves \*];

**Preassociative**

[\* **proof of** \* : \*], [Line \* : \*  $\gg$  \*; \*], [Last line \*  $\gg$  \*  $\square$ ],

[Line \* : Premise  $\gg$  \*; \*], [Line \* : Side-condition  $\gg$  \*; \*], [Arbitrary  $\gg$  \*; \*],

[Local  $\gg$  \* = \*; \*], [Begin \*; \* : End; \*], [Last block line \*  $\gg$  \*; \*],

[Arbitrary  $\gg$  \*; \*];

**Postassociative**

[\* | \*];

**Postassociative**

[\* , \*], [\* [\* ]\*];

**Preassociative**

[\*&\*];

**Preassociative**

[\* \\ \*], [\* linebreak[4] \*], [\* \\ \*]; **End table**

## A Pyk definitioner

[UniqueMember  $\xrightarrow{\text{pyk}}$  “lemma uniqueMember”]

[UniqueMember(Type)  $\xrightarrow{\text{pyk}}$  “lemma uniqueMember(Type)”]

[SameSeries  $\xrightarrow{\text{pyk}}$  “lemma sameSeries”]

[A4  $\xrightarrow{\text{pyk}}$  “lemma a4”]

[SameMember  $\xrightarrow{\text{pyk}}$  “lemma sameMember”]

[Qclosed(Addition)  $\xrightarrow{\text{pyk}}$  “1rule Qclosed(Addition)”]

[Qclosed(Multiplication)  $\xrightarrow{\text{pyk}}$  “1rule Qclosed(Multiplication)”]

[FromCartProd(1)  $\xrightarrow{\text{pyk}}$  “1rule fromCartProd(1)”]

[1rule fromCartProd(2)  $\xrightarrow{\text{pyk}}$  “1rule fromCartProd(2)”]

[constantRationalSeries(\*)  $\xrightarrow{\text{pyk}}$  “constantRationalSeries( " )”]

[cartProd(\*)  $\xrightarrow{\text{pyk}}$  “cartProd( " , " )”]

[Power(\*)  $\xrightarrow{\text{pyk}}$  “P( " )”]

[binaryUnion(\*, \*)  $\xrightarrow{\text{pyk}}$  “binaryUnion( " , " )”]

[SetOfRationalSeries  $\xrightarrow{\text{pyk}}$  “setOfRationalSeries”]

[IsSubset(\*, \*)  $\xrightarrow{\text{pyk}}$  “isSubset( " , " )”]

[(p\*, \*)  $\xrightarrow{\text{pyk}}$  “(p " , " )”]

[(s\*)  $\xrightarrow{\text{pyk}}$  “(s " )”]

[( $\dots$ )  $\xrightarrow{\text{pyk}}$  “cdots”]

[Objekt-var  $\xrightarrow{\text{pyk}}$  "object-var"]  
 [Ex-var  $\xrightarrow{\text{pyk}}$  "ex-var"]  
 [Ph-var  $\xrightarrow{\text{pyk}}$  "ph-var"]  
 [Værdi  $\xrightarrow{\text{pyk}}$  "vaerdi"]  
 [Variabel  $\xrightarrow{\text{pyk}}$  "variabel"]  
 [Op(\*)  $\xrightarrow{\text{pyk}}$  "op " end op"]  
 [Op(\*, \*)  $\xrightarrow{\text{pyk}}$  "op2 " comma " end op2"]  
 [\* ::= \*  $\xrightarrow{\text{pyk}}$  "define-equal " comma " end equal"]  
 [ContainsEmpty(\*)  $\xrightarrow{\text{pyk}}$  "contains-empty " end empty"]  
 [Nat(\*)  $\xrightarrow{\text{pyk}}$  "Nat( " )"]  
 [Dedu(\*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction " conclude " end 1deduction"]  
 [Dedu<sub>0</sub>(\*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction zero " conclude " end 1deduction"]  
 [Dedu<sub>s</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction side " conclude " condition " end 1deduction"]  
 [Dedu<sub>1</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction one " conclude " condition " end 1deduction"]  
 [Dedu<sub>2</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction two " conclude " condition " end 1deduction"]  
 [Dedu<sub>3</sub>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction three " conclude " condition " bound " end 1deduction"]  
 [Dedu<sub>4</sub>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction four " conclude " condition " bound " end 1deduction"]  
 [Dedu<sub>4</sub><sup>\*</sup>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction four star " conclude " condition " bound " end 1deduction"]  
 [Dedu<sub>5</sub>(\*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction five " condition " bound " end 1deduction"]  
 [Dedu<sub>6</sub>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction six " conclude " exception " bound " end 1deduction"]  
 [Dedu<sub>6</sub><sup>\*</sup>(\*, \*, \*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction six star " conclude " exception " bound " end 1deduction"]  
 [Dedu<sub>7</sub>(\*)  $\xrightarrow{\text{pyk}}$  "1deduction seven " end 1deduction"]  
 [Dedu<sub>8</sub>(\*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction eight " bound " end 1deduction"]  
 [Dedu<sub>8</sub><sup>\*</sup>(\*, \*)  $\xrightarrow{\text{pyk}}$  "1deduction eight star " bound " end 1deduction"]  
 [Ex<sub>1</sub>  $\xrightarrow{\text{pyk}}$  "ex1"]  
 [Ex<sub>2</sub>  $\xrightarrow{\text{pyk}}$  "ex2"]  
 [Ex<sub>3</sub>  $\xrightarrow{\text{pyk}}$  "ex3"]  
 [Ex<sub>10</sub>  $\xrightarrow{\text{pyk}}$  "ex10"]  
 [Ex<sub>20</sub>  $\xrightarrow{\text{pyk}}$  "ex20"]  
 [\*<sub>Ex</sub>  $\xrightarrow{\text{pyk}}$  "existential var " end var"]  
 [\*<sub>Ex</sub>  $\xrightarrow{\text{pyk}}$  " " is existential var"]

$[\langle * \equiv * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub " is " where " is " end sub"}]$   
 $[\langle * \equiv^0 * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub0 " is " where " is " end sub"}]$   
 $[\langle * \equiv^1 * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub1 " is " where " is " end sub"}]$   
 $[\langle * \equiv^* * \mid * := * \rangle_{\text{Ex}} \xrightarrow{\text{pyk}} \text{"exist-sub* " is " where " is " end sub"}]$   
 $[\text{ph}_1 \xrightarrow{\text{pyk}} \text{"ph1"}]$   
 $[\text{ph}_2 \xrightarrow{\text{pyk}} \text{"ph2"}]$   
 $[\text{ph}_3 \xrightarrow{\text{pyk}} \text{"ph3"}]$   
 $[\text{*Ph} \xrightarrow{\text{pyk}} \text{"placeholder-var " end var"}]$   
 $[\text{*Ph} \xrightarrow{\text{pyk}} \text{" " is placeholder-var"}]$   
 $[\langle * \equiv * \mid * := * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub " is " where " is " end sub"}]$   
 $[\langle * \equiv^0 * \mid * := * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub0 " is " where " is " end sub"}]$   
 $[\langle * \equiv^1 * \mid * := * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub1 " is " where " is " end sub"}]$   
 $[\langle * \equiv^* * \mid * := * \rangle_{\text{Ph}} \xrightarrow{\text{pyk}} \text{"ph-sub* " is " where " is " end sub"}]$   
 $[\langle * \equiv * \mid * := * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} \text{"meta-sub " is " where " is " end sub"}]$   
 $[\langle * \equiv^1 * \mid * := * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} \text{"meta-sub1 " is " where " is " end sub"}]$   
 $[\langle * \equiv^* * \mid * := * \rangle_{\text{Me}} \xrightarrow{\text{pyk}} \text{"meta-sub* " is " where " is " end sub"}]$   
 $[\text{bs} \xrightarrow{\text{pyk}} \text{"var big set"}]$   
 $[\text{OBS} \xrightarrow{\text{pyk}} \text{"object big set"}]$   
 $[\mathcal{BS} \xrightarrow{\text{pyk}} \text{"meta big set"}]$   
 $[\emptyset \xrightarrow{\text{pyk}} \text{"zermelo empty set"}]$   
 $[\text{SystemQ} \xrightarrow{\text{pyk}} \text{"system Q"}]$   
 $[\text{MP} \xrightarrow{\text{pyk}} \text{"1rule mp"}]$   
 $[\text{Gen} \xrightarrow{\text{pyk}} \text{"1rule gen"}]$   
 $[\text{Repetition} \xrightarrow{\text{pyk}} \text{"1rule repetition"}]$   
 $[\text{Neg} \xrightarrow{\text{pyk}} \text{"1rule ad absurdum"}]$   
 $[\text{Ded} \xrightarrow{\text{pyk}} \text{"1rule deduction"}]$   
 $[\text{ExistIntro} \xrightarrow{\text{pyk}} \text{"1rule exist intro"}]$   
 $[\text{Extensionality} \xrightarrow{\text{pyk}} \text{"axiom extensionality"}]$   
 $[\emptyset\text{def} \xrightarrow{\text{pyk}} \text{"axiom empty set"}]$   
 $[\text{PairDef} \xrightarrow{\text{pyk}} \text{"axiom pair definition"}]$   
 $[\text{UnionDef} \xrightarrow{\text{pyk}} \text{"axiom union definition"}]$   
 $[\text{PowerDef} \xrightarrow{\text{pyk}} \text{"axiom power definition"}]$   
 $[\text{SeparationDef} \xrightarrow{\text{pyk}} \text{"axiom separation definition"}]$   
 $[\text{AddDoubleNeg} \xrightarrow{\text{pyk}} \text{"prop lemma add double neg"}]$   
 $[\text{RemoveDoubleNeg} \xrightarrow{\text{pyk}} \text{"prop lemma remove double neg"}]$

[AndCommutativity  $\xrightarrow{\text{pyk}}$  “prop lemma and commutativity”]  
 [AutoImPLY  $\xrightarrow{\text{pyk}}$  “prop lemma auto imply”]  
 [Contrapositive  $\xrightarrow{\text{pyk}}$  “prop lemma contrapositive”]  
 [FirstConjunct  $\xrightarrow{\text{pyk}}$  “prop lemma first conjunct”]  
 [SecondConjunct  $\xrightarrow{\text{pyk}}$  “prop lemma second conjunct”]  
 [FromContradiction  $\xrightarrow{\text{pyk}}$  “prop lemma from contradiction”]  
 [FromDisjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma from disjuncts”]  
 [IffCommutativity  $\xrightarrow{\text{pyk}}$  “prop lemma iff commutativity”]  
 [IffFirst  $\xrightarrow{\text{pyk}}$  “prop lemma iff first”]  
 [IffSecond  $\xrightarrow{\text{pyk}}$  “prop lemma iff second”]  
 [ImPLYTransitivity  $\xrightarrow{\text{pyk}}$  “prop lemma imply transitivity”]  
 [JoinConjuncts  $\xrightarrow{\text{pyk}}$  “prop lemma join conjuncts”]  
 [MP2  $\xrightarrow{\text{pyk}}$  “prop lemma mp2”]  
 [MP3  $\xrightarrow{\text{pyk}}$  “prop lemma mp3”]  
 [MP4  $\xrightarrow{\text{pyk}}$  “prop lemma mp4”]  
 [MP5  $\xrightarrow{\text{pyk}}$  “prop lemma mp5”]  
 [MT  $\xrightarrow{\text{pyk}}$  “prop lemma mt”]  
 [NegativeMT  $\xrightarrow{\text{pyk}}$  “prop lemma negative mt”]  
 [Technicality  $\xrightarrow{\text{pyk}}$  “prop lemma technicality”]  
 [Weakening  $\xrightarrow{\text{pyk}}$  “prop lemma weakening”]  
 [WeakenOr1  $\xrightarrow{\text{pyk}}$  “prop lemma weaken or first”]  
 [WeakenOr2  $\xrightarrow{\text{pyk}}$  “prop lemma weaken or second”]  
 [Formula2Pair  $\xrightarrow{\text{pyk}}$  “lemma formula2pair”]  
 [Pair2Formula  $\xrightarrow{\text{pyk}}$  “lemma pair2formula”]  
 [Formula2Union  $\xrightarrow{\text{pyk}}$  “lemma formula2union”]  
 [Union2Formula  $\xrightarrow{\text{pyk}}$  “lemma union2formula”]  
 [Formula2Sep  $\xrightarrow{\text{pyk}}$  “lemma formula2separation”]  
 [Sep2Formula  $\xrightarrow{\text{pyk}}$  “lemma separation2formula”]  
 [Formula2Power  $\xrightarrow{\text{pyk}}$  “lemma formula2power”]  
 [SubsetInPower  $\xrightarrow{\text{pyk}}$  “lemma subset in power set”]  
 [HelperPowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset0”]  
 [PowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset”]  
 [(Switch)HelperPowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset0-switch”]  
 [(Switch)PowerIsSub  $\xrightarrow{\text{pyk}}$  “lemma power set is subset-switch”]  
 [ToSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition”]

[HelperToSetEquality(t)  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition(t)0”]  
 [ToSetEquality(t)  $\xrightarrow{\text{pyk}}$  “lemma set equality suff condition(t)”]  
 [HelperFromSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality skip quantifier”]  
 [FromSetEquality  $\xrightarrow{\text{pyk}}$  “lemma set equality nec condition”]  
 [HelperReflexivity  $\xrightarrow{\text{pyk}}$  “lemma reflexivity0”]  
 [Reflexivity  $\xrightarrow{\text{pyk}}$  “lemma reflexivity”]  
 [HelperSymmetry  $\xrightarrow{\text{pyk}}$  “lemma symmetry0”]  
 [Symmetry  $\xrightarrow{\text{pyk}}$  “lemma symmetry”]  
 [HelperTransitivity  $\xrightarrow{\text{pyk}}$  “lemma transitivity0”]  
 [Transitivity  $\xrightarrow{\text{pyk}}$  “lemma transitivity”]  
 [ERisReflexive  $\xrightarrow{\text{pyk}}$  “lemma er is reflexive”]  
 [ERisSymmetric  $\xrightarrow{\text{pyk}}$  “lemma er is symmetric”]  
 [ERisTransitive  $\xrightarrow{\text{pyk}}$  “lemma er is transitive”]  
 [ØisSubset  $\xrightarrow{\text{pyk}}$  “lemma empty set is subset”]  
 [HelperMemberNotØ  $\xrightarrow{\text{pyk}}$  “lemma member not empty0”]  
 [MemberNotØ  $\xrightarrow{\text{pyk}}$  “lemma member not empty”]  
 [HelperUniqueØ  $\xrightarrow{\text{pyk}}$  “lemma unique empty set0”]  
 [UniqueØ  $\xrightarrow{\text{pyk}}$  “lemma unique empty set”]  
 [== Reflexivity  $\xrightarrow{\text{pyk}}$  “lemma ==Reflexivity”]  
 [== Symmetry  $\xrightarrow{\text{pyk}}$  “lemma ==Symmetry”]  
 [Helper==Transitivity  $\xrightarrow{\text{pyk}}$  “lemma ==Transitivity0”]  
 [== Transitivity  $\xrightarrow{\text{pyk}}$  “lemma ==Transitivity”]  
 [HelperTransferNotEq  $\xrightarrow{\text{pyk}}$  “lemma transfer ~is0”]  
 [TransferNotEq  $\xrightarrow{\text{pyk}}$  “lemma transfer ~is”]  
 [HelperPairSubset  $\xrightarrow{\text{pyk}}$  “lemma pair subset0”]  
 [Helper(2)PairSubset  $\xrightarrow{\text{pyk}}$  “lemma pair subset1”]  
 [PairSubset  $\xrightarrow{\text{pyk}}$  “lemma pair subset”]  
 [SamePair  $\xrightarrow{\text{pyk}}$  “lemma same pair”]  
 [SameSingleton  $\xrightarrow{\text{pyk}}$  “lemma same singleton”]  
 [UnionSubset  $\xrightarrow{\text{pyk}}$  “lemma union subset”]  
 [SameUnion  $\xrightarrow{\text{pyk}}$  “lemma same union”]  
 [SeparationSubset  $\xrightarrow{\text{pyk}}$  “lemma separation subset”]  
 [SameSeparation  $\xrightarrow{\text{pyk}}$  “lemma same separation”]  
 [SameBinaryUnion  $\xrightarrow{\text{pyk}}$  “lemma same binary union”]  
 [IntersectionSubset  $\xrightarrow{\text{pyk}}$  “lemma intersection subset”]



$[\text{SameIntersection} \xrightarrow{\text{pyk}} \text{“lemma same intersection”}]$   
 $[\text{AutoMember} \xrightarrow{\text{pyk}} \text{“lemma auto member”}]$   
 $[\text{HelperEqSysNot}\emptyset \xrightarrow{\text{pyk}} \text{“lemma eq-system not empty0”}]$   
 $[\text{EqSysNot}\emptyset \xrightarrow{\text{pyk}} \text{“lemma eq-system not empty”}]$   
 $[\text{HelperEqSubset} \xrightarrow{\text{pyk}} \text{“lemma eq subset0”}]$   
 $[\text{EqSubset} \xrightarrow{\text{pyk}} \text{“lemma eq subset”}]$   
 $[\text{HelperEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma equivalence nec condition0”}]$   
 $[\text{EqNecessary} \xrightarrow{\text{pyk}} \text{“lemma equivalence nec condition”}]$   
 $[\text{HelperNoneEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma none-equivalence nec condition0”}]$   
 $[\text{Helper(2)NoneEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma none-equivalence nec condition1”}]$   
 $[\text{NoneEqNecessary} \xrightarrow{\text{pyk}} \text{“lemma none-equivalence nec condition”}]$   
 $[\text{EqClassIsSubset} \xrightarrow{\text{pyk}} \text{“lemma equivalence class is subset”}]$   
 $[\text{EqClassesAreDisjoint} \xrightarrow{\text{pyk}} \text{“lemma equivalence classes are disjoint”}]$   
 $[\text{AllDisjoint} \xrightarrow{\text{pyk}} \text{“lemma all disjoint”}]$   
 $[\text{AllDisjointImply} \xrightarrow{\text{pyk}} \text{“lemma all disjoint-imply”}]$   
 $[\text{BSsubset} \xrightarrow{\text{pyk}} \text{“lemma bs subset union(bs/r)”}]$   
 $[\text{Union(BS/R)subset} \xrightarrow{\text{pyk}} \text{“lemma union(bs/r) subset bs”}]$   
 $[\text{UnionIdentity} \xrightarrow{\text{pyk}} \text{“lemma union(bs/r) is bs”}]$   
 $[\text{EqSysIsPartition} \xrightarrow{\text{pyk}} \text{“theorem eq-system is partition”}]$   
 $[(x1) \xrightarrow{\text{pyk}} \text{“var x1”}]$   
 $[(x2) \xrightarrow{\text{pyk}} \text{“var x2”}]$   
 $[(y1) \xrightarrow{\text{pyk}} \text{“var y1”}]$   
 $[(y2) \xrightarrow{\text{pyk}} \text{“var y2”}]$   
 $[(v1) \xrightarrow{\text{pyk}} \text{“var v1”}]$   
 $[(v2) \xrightarrow{\text{pyk}} \text{“var v2”}]$   
 $[(v3) \xrightarrow{\text{pyk}} \text{“var v3”}]$   
 $[(v4) \xrightarrow{\text{pyk}} \text{“var v4”}]$   
 $[(v2n) \xrightarrow{\text{pyk}} \text{“var v2n”}]$   
 $[(m1) \xrightarrow{\text{pyk}} \text{“var m1”}]$   
 $[(m2) \xrightarrow{\text{pyk}} \text{“var m2”}]$   
 $[(n1) \xrightarrow{\text{pyk}} \text{“var n1”}]$   
 $[(n2) \xrightarrow{\text{pyk}} \text{“var n2”}]$   
 $[(n3) \xrightarrow{\text{pyk}} \text{“var n3”}]$   
 $[(\epsilon) \xrightarrow{\text{pyk}} \text{“var ep”}]$   
 $[(\epsilon)_1 \xrightarrow{\text{pyk}} \text{“var ep1”}]$

$[(\epsilon 2) \xrightarrow{\text{pyk}} \text{“var ep2”}]$   
 $[(\text{fep}) \xrightarrow{\text{pyk}} \text{“var fep”}]$   
 $[(\text{fx}) \xrightarrow{\text{pyk}} \text{“var fx”}]$   
 $[(\text{fy}) \xrightarrow{\text{pyk}} \text{“var fy”}]$   
 $[(\text{fz}) \xrightarrow{\text{pyk}} \text{“var fz”}]$   
 $[(\text{fu}) \xrightarrow{\text{pyk}} \text{“var fu”}]$   
 $[(\text{fv}) \xrightarrow{\text{pyk}} \text{“var fv”}]$   
 $[(\text{fw}) \xrightarrow{\text{pyk}} \text{“var fw”}]$   
 $[(\text{rx}) \xrightarrow{\text{pyk}} \text{“var rx”}]$   
 $[(\text{ry}) \xrightarrow{\text{pyk}} \text{“var ry”}]$   
 $[(\text{rz}) \xrightarrow{\text{pyk}} \text{“var rz”}]$   
 $[(\text{ru}) \xrightarrow{\text{pyk}} \text{“var ru”}]$   
 $[(\text{sx}) \xrightarrow{\text{pyk}} \text{“var sx”}]$   
 $[(\text{sx1}) \xrightarrow{\text{pyk}} \text{“var sx1”}]$   
 $[(\text{sy}) \xrightarrow{\text{pyk}} \text{“var sy”}]$   
 $[(\text{sy1}) \xrightarrow{\text{pyk}} \text{“var sy1”}]$   
 $[(\text{sz}) \xrightarrow{\text{pyk}} \text{“var sz”}]$   
 $[(\text{sz1}) \xrightarrow{\text{pyk}} \text{“var sz1”}]$   
 $[(\text{su}) \xrightarrow{\text{pyk}} \text{“var su”}]$   
 $[(\text{su1}) \xrightarrow{\text{pyk}} \text{“var su1”}]$   
 $[(\text{fxs}) \xrightarrow{\text{pyk}} \text{“var fxs”}]$   
 $[(\text{fys}) \xrightarrow{\text{pyk}} \text{“var fys”}]$   
 $[(\text{crs1}) \xrightarrow{\text{pyk}} \text{“var crs1”}]$   
 $[(\text{f1}) \xrightarrow{\text{pyk}} \text{“var f1”}]$   
 $[(\text{f2}) \xrightarrow{\text{pyk}} \text{“var f2”}]$   
 $[(\text{f3}) \xrightarrow{\text{pyk}} \text{“var f3”}]$   
 $[(\text{f4}) \xrightarrow{\text{pyk}} \text{“var f4”}]$   
 $[(\text{op1}) \xrightarrow{\text{pyk}} \text{“var op1”}]$   
 $[(\text{op2}) \xrightarrow{\text{pyk}} \text{“var op2”}]$   
 $[(\text{r1}) \xrightarrow{\text{pyk}} \text{“var r1”}]$   
 $[(\text{s1}) \xrightarrow{\text{pyk}} \text{“var s1”}]$   
 $[(\text{s2}) \xrightarrow{\text{pyk}} \text{“var s2”}]$   
 $[\text{X}_1 \xrightarrow{\text{pyk}} \text{“meta x1”}]$   
 $[\text{X}_2 \xrightarrow{\text{pyk}} \text{“meta x2”}]$   
 $[\text{Y}_1 \xrightarrow{\text{pyk}} \text{“meta y1”}]$

$[Y_2 \xrightarrow{\text{pyk}} \text{“meta y2”}]$   
 $[V_1 \xrightarrow{\text{pyk}} \text{“meta v1”}]$   
 $[V_2 \xrightarrow{\text{pyk}} \text{“meta v2”}]$   
 $[V_3 \xrightarrow{\text{pyk}} \text{“meta v3”}]$   
 $[V_4 \xrightarrow{\text{pyk}} \text{“meta v4”}]$   
 $[V_{2n} \xrightarrow{\text{pyk}} \text{“meta v2n”}]$   
 $[M_1 \xrightarrow{\text{pyk}} \text{“meta m1”}]$   
 $[M_2 \xrightarrow{\text{pyk}} \text{“meta m2”}]$   
 $[N_1 \xrightarrow{\text{pyk}} \text{“meta n1”}]$   
 $[N_2 \xrightarrow{\text{pyk}} \text{“meta n2”}]$   
 $[N_3 \xrightarrow{\text{pyk}} \text{“meta n3”}]$   
 $[\epsilon \xrightarrow{\text{pyk}} \text{“meta ep”}]$   
 $[\epsilon 1 \xrightarrow{\text{pyk}} \text{“meta ep1”}]$   
 $[\epsilon 2 \xrightarrow{\text{pyk}} \text{“meta ep2”}]$   
 $[FX \xrightarrow{\text{pyk}} \text{“meta fx”}]$   
 $[FY \xrightarrow{\text{pyk}} \text{“meta fy”}]$   
 $[FZ \xrightarrow{\text{pyk}} \text{“meta fz”}]$   
 $[FU \xrightarrow{\text{pyk}} \text{“meta fu”}]$   
 $[FV \xrightarrow{\text{pyk}} \text{“meta fv”}]$   
 $[FW \xrightarrow{\text{pyk}} \text{“meta fw”}]$   
 $[FEP \xrightarrow{\text{pyk}} \text{“meta fep”}]$   
 $[RX \xrightarrow{\text{pyk}} \text{“meta rx”}]$   
 $[RY \xrightarrow{\text{pyk}} \text{“meta ry”}]$   
 $[RZ \xrightarrow{\text{pyk}} \text{“meta rz”}]$   
 $[RU \xrightarrow{\text{pyk}} \text{“meta ru”}]$   
 $[(SX) \xrightarrow{\text{pyk}} \text{“meta sx”}]$   
 $[(SX1) \xrightarrow{\text{pyk}} \text{“meta sx1”}]$   
 $[(SY) \xrightarrow{\text{pyk}} \text{“meta sy”}]$   
 $[(SY1) \xrightarrow{\text{pyk}} \text{“meta sy1”}]$   
 $[(SZ) \xrightarrow{\text{pyk}} \text{“meta sz”}]$   
 $[(SZ1) \xrightarrow{\text{pyk}} \text{“meta sz1”}]$   
 $[(SU) \xrightarrow{\text{pyk}} \text{“meta su”}]$   
 $[(SU1) \xrightarrow{\text{pyk}} \text{“meta su1”}]$   
 $[FXS \xrightarrow{\text{pyk}} \text{“meta fxs”}]$   
 $[FYS \xrightarrow{\text{pyk}} \text{“meta fys”}]$

$[(F1) \xrightarrow{\text{pyk}} \text{“meta f1”}]$   
 $[(F2) \xrightarrow{\text{pyk}} \text{“meta f2”}]$   
 $[(F3) \xrightarrow{\text{pyk}} \text{“meta f3”}]$   
 $[(F4) \xrightarrow{\text{pyk}} \text{“meta f4”}]$   
 $[(OP1) \xrightarrow{\text{pyk}} \text{“meta op1”}]$   
 $[(OP2) \xrightarrow{\text{pyk}} \text{“meta op2”}]$   
 $[(R1) \xrightarrow{\text{pyk}} \text{“meta r1”}]$   
 $[(S1) \xrightarrow{\text{pyk}} \text{“meta s1”}]$   
 $[(S2) \xrightarrow{\text{pyk}} \text{“meta s2”}]$   
 $[(EPob) \xrightarrow{\text{pyk}} \text{“object ep”}]$   
 $[(CRS1ob) \xrightarrow{\text{pyk}} \text{“object crs1”}]$   
 $[(F1ob) \xrightarrow{\text{pyk}} \text{“object f1”}]$   
 $[(F2ob) \xrightarrow{\text{pyk}} \text{“object f2”}]$   
 $[(F3ob) \xrightarrow{\text{pyk}} \text{“object f3”}]$   
 $[(F4ob) \xrightarrow{\text{pyk}} \text{“object f4”}]$   
 $[(N1ob) \xrightarrow{\text{pyk}} \text{“object n1”}]$   
 $[(N2ob) \xrightarrow{\text{pyk}} \text{“object n2”}]$   
 $[(OP1ob) \xrightarrow{\text{pyk}} \text{“object op1”}]$   
 $[(OP2ob) \xrightarrow{\text{pyk}} \text{“object op2”}]$   
 $[(R1ob) \xrightarrow{\text{pyk}} \text{“object r1”}]$   
 $[(S1ob) \xrightarrow{\text{pyk}} \text{“object s1”}]$   
 $[(S2ob) \xrightarrow{\text{pyk}} \text{“object s2”}]$   
 $[\text{ph}_4 \xrightarrow{\text{pyk}} \text{“ph4”}]$   
 $[\text{ph}_5 \xrightarrow{\text{pyk}} \text{“ph5”}]$   
 $[\text{ph}_6 \xrightarrow{\text{pyk}} \text{“ph6”}]$   
 $[\text{NAT} \xrightarrow{\text{pyk}} \text{“NAT”}]$   
 $[\text{RATIONAL}_S\text{ERIES} \xrightarrow{\text{pyk}} \text{“RATIONAL\_SERIES”}]$   
 $[\text{SERIES} \xrightarrow{\text{pyk}} \text{“SERIES”}]$   
 $[\text{SetOfReals} \xrightarrow{\text{pyk}} \text{“setOfReals”}]$   
 $[\text{SetOfFxs} \xrightarrow{\text{pyk}} \text{“setOfFxs”}]$   
 $[\text{N} \xrightarrow{\text{pyk}} \text{“N”}]$   
 $[\text{Q} \xrightarrow{\text{pyk}} \text{“Q”}]$   
 $[\text{X} \xrightarrow{\text{pyk}} \text{“X”}]$   
 $[\text{xs} \xrightarrow{\text{pyk}} \text{“xs”}]$   
 $[\text{xaF} \xrightarrow{\text{pyk}} \text{“xsF”}]$

$[ysF \xrightarrow{pyk} \text{“ysF”}]$   
 $[us \xrightarrow{pyk} \text{“us”}]$   
 $[usFoelge \xrightarrow{pyk} \text{“usF”}]$   
 $[0 \xrightarrow{pyk} \text{“0”}]$   
 $[1 \xrightarrow{pyk} \text{“1”}]$   
 $[(-1) \xrightarrow{pyk} \text{“(-1)”}]$   
 $[2 \xrightarrow{pyk} \text{“2”}]$   
 $[3 \xrightarrow{pyk} \text{“3”}]$   
 $[1/2 \xrightarrow{pyk} \text{“1/2”}]$   
 $[1/3 \xrightarrow{pyk} \text{“1/3”}]$   
 $[2/3 \xrightarrow{pyk} \text{“2/3”}]$   
 $[0f \xrightarrow{pyk} \text{“0f”}]$   
 $[1f \xrightarrow{pyk} \text{“1f”}]$   
 $[00 \xrightarrow{pyk} \text{“00”}]$   
 $[01 \xrightarrow{pyk} \text{“01”}]$   
 $[(- - 01) \xrightarrow{pyk} \text{“(-01)”}]$   
 $[02 \xrightarrow{pyk} \text{“02”}]$   
 $[01//02 \xrightarrow{pyk} \text{“01//02”}]$   
 $[PlusAssociativity(R) \xrightarrow{pyk} \text{“lemma plusAssociativity(R)”}]$   
 $[PlusAssociativity(R)XX \xrightarrow{pyk} \text{“lemma plusAssociativity(R)XX”}]$   
 $[Plus0(R) \xrightarrow{pyk} \text{“lemma plus0(R)”}]$   
 $[Negative(R) \xrightarrow{pyk} \text{“lemma negative(R)”}]$   
 $[Times1(R) \xrightarrow{pyk} \text{“lemma times1(R)”}]$   
 $[lessAddition(R) \xrightarrow{pyk} \text{“lemma lessAddition(R)”}]$   
 $[PlusCommutativity(R) \xrightarrow{pyk} \text{“lemma plusCommutativity(R)”}]$   
 $[LeqAntisymmetry(R) \xrightarrow{pyk} \text{“lemma leqAntisymmetry(R)”}]$   
 $[LeqTransitivity(R) \xrightarrow{pyk} \text{“lemma leqTransitivity(R)”}]$   
 $[leqAddition(R) \xrightarrow{pyk} \text{“lemma leqAddition(R)”}]$   
 $[Distribution(R) \xrightarrow{pyk} \text{“lemma distribution(R)”}]$   
 $[A4(Axiom) \xrightarrow{pyk} \text{“axiom a4”}]$   
 $[InductionAxiom \xrightarrow{pyk} \text{“axiom induction”}]$   
 $[EqualityAxiom \xrightarrow{pyk} \text{“axiom equality”}]$   
 $[EqLeqAxiom \xrightarrow{pyk} \text{“axiom eqLeq”}]$   
 $[EqAdditionAxiom \xrightarrow{pyk} \text{“axiom eqAddition”}]$   
 $[EqMultiplicationAxiom \xrightarrow{pyk} \text{“axiom eqMultiplication”}]$

$[QisClosed(Reciprocal)(Imply) \xrightarrow{pyk} \text{“axiom QisClosed(reciprocal)”}]$   
 $[QisClosed(Reciprocal) \xrightarrow{pyk} \text{“lemma QisClosed(reciprocal)”}]$   
 $[QisClosed(Negative)(Imply) \xrightarrow{pyk} \text{“axiom QisClosed(negative)”}]$   
 $[QisClosed(Negative) \xrightarrow{pyk} \text{“lemma QisClosed(negative)”}]$   
 $[leqReflexivity \xrightarrow{pyk} \text{“axiom leqReflexivity”}]$   
 $[leqAntisymmetryAxiom \xrightarrow{pyk} \text{“axiom leqAntisymmetry”}]$   
 $[leqTransitivityAxiom \xrightarrow{pyk} \text{“axiom leqTransitivity”}]$   
 $[leqTotality \xrightarrow{pyk} \text{“axiom leqTotality”}]$   
 $[leqAdditionAxiom \xrightarrow{pyk} \text{“axiom leqAddition”}]$   
 $[leqMultiplicationAxiom \xrightarrow{pyk} \text{“axiom leqMultiplication”}]$   
 $[plusAssociativity \xrightarrow{pyk} \text{“axiom plusAssociativity”}]$   
 $[plusCommutativity \xrightarrow{pyk} \text{“axiom plusCommutativity”}]$   
 $[Negative \xrightarrow{pyk} \text{“axiom negative”}]$   
 $[plus0 \xrightarrow{pyk} \text{“axiom plus0”}]$   
 $[timesAssociativity \xrightarrow{pyk} \text{“axiom timesAssociativity”}]$   
 $[timesCommutativity \xrightarrow{pyk} \text{“axiom timesCommutativity”}]$   
 $[ReciprocalAxiom \xrightarrow{pyk} \text{“axiom reciprocal”}]$   
 $[times1 \xrightarrow{pyk} \text{“axiom times1”}]$   
 $[Distribution \xrightarrow{pyk} \text{“axiom distribution”}]$   
 $[0not1 \xrightarrow{pyk} \text{“axiom 0not1”}]$   
 $[lemma eqLeq(R) \xrightarrow{pyk} \text{“lemma eqLeq(R)”}]$   
 $[TimesAssociativity(R) \xrightarrow{pyk} \text{“lemma timesAssociativity(R)”}]$   
 $[TimesCommutativity(R) \xrightarrow{pyk} \text{“lemma timesCommutativity(R)”}]$   
 $[(Adgic)SameR \xrightarrow{pyk} \text{“1rule adhoc sameR”}]$   
 $[Separation2formula(1) \xrightarrow{pyk} \text{“lemma separation2formula(1)”}]$   
 $[Separation2formula(2) \xrightarrow{pyk} \text{“lemma separation2formula(2)”}]$   
 $[Cauchy \xrightarrow{pyk} \text{“axiom cauchy”}]$   
 $[PlusF \xrightarrow{pyk} \text{“axiom plusF”}]$   
 $[ReciprocalF \xrightarrow{pyk} \text{“axiom reciprocalF”}]$   
 $[From == \xrightarrow{pyk} \text{“1rule from==”}]$   
 $[To == \xrightarrow{pyk} \text{“1rule to==”}]$   
 $[FromInR \xrightarrow{pyk} \text{“1rule fromInR”}]$   
 $[PlusR(Sym) \xrightarrow{pyk} \text{“lemma plusR(Sym)”}]$   
 $[ReciprocalR(Axiom) \xrightarrow{pyk} \text{“axiom reciprocalR”}]$   
 $[LessMinus1(N) \xrightarrow{pyk} \text{“1rule lessMinus1(N)”}]$

[Nonnegative(N)  $\xrightarrow{\text{pyk}}$  “axiom nonnegative(N)”]  
 [US0  $\xrightarrow{\text{pyk}}$  “axiom US0”]  
 [NextXS(UpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextXS(upperBound)”]  
 [NextXS(NoUpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextXS(noUpperBound)”]  
 [NextUS(UpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextUS(upperBound)”]  
 [NextUS(NoUpperBound)  $\xrightarrow{\text{pyk}}$  “1rule nextUS(noUpperBound)”]  
 [ExpZero  $\xrightarrow{\text{pyk}}$  “1rule expZero”]  
 [ExpPositive  $\xrightarrow{\text{pyk}}$  “1rule expPositive”]  
 [ExpZero(R)  $\xrightarrow{\text{pyk}}$  “1rule expZero(R)”]  
 [ExpPositive(R)  $\xrightarrow{\text{pyk}}$  “1rule expPositive(R)”]  
 [BSzero  $\xrightarrow{\text{pyk}}$  “1rule base(1/2)Sum zero”]  
 [BSpositive  $\xrightarrow{\text{pyk}}$  “1rule base(1/2)Sum positive”]  
 [UStelescope(Zero)  $\xrightarrow{\text{pyk}}$  “1rule UStelescope zero”]  
 [UStelescope(Positive)  $\xrightarrow{\text{pyk}}$  “1rule UStelescope positive”]  
 [EqAddition(R)  $\xrightarrow{\text{pyk}}$  “1rule adhoc eqAddition(R)”]  
 [FromLimit  $\xrightarrow{\text{pyk}}$  “1rule fromLimit”]  
 [ToUpperBound  $\xrightarrow{\text{pyk}}$  “1rule toUpperBound”]  
 [FromUpperBound  $\xrightarrow{\text{pyk}}$  “1rule fromUpperBound”]  
 [USisUpperBound  $\xrightarrow{\text{pyk}}$  “axiom USisUpperBound”]  
 [0not1(R)  $\xrightarrow{\text{pyk}}$  “axiom 0not1(R)”]  
 [ExpUnbounded(R)  $\xrightarrow{\text{pyk}}$  “1rule expUnbounded”]  
 [FromLeq(Advanced)(N)  $\xrightarrow{\text{pyk}}$  “1rule fromLeq(Advanced)(N)”]  
 [FromLeastUpperBound  $\xrightarrow{\text{pyk}}$  “1rule fromLeastUpperBound”]  
 [ToLeastUpperBound  $\xrightarrow{\text{pyk}}$  “1rule toLeastUpperBound”]  
 [XSisNotUpperBound  $\xrightarrow{\text{pyk}}$  “axiom XSisNotUpperBound”]  
 [ysFGreater  $\xrightarrow{\text{pyk}}$  “axiom ysFGreater”]  
 [ysFLess  $\xrightarrow{\text{pyk}}$  “axiom ysFLess”]  
 [SmallInverse  $\xrightarrow{\text{pyk}}$  “1rule smallInverse”]  
 [NatType  $\xrightarrow{\text{pyk}}$  “axiom natType”]  
 [RationalType  $\xrightarrow{\text{pyk}}$  “axiom rationalType”]  
 [SeriesType  $\xrightarrow{\text{pyk}}$  “axiom seriesType”]  
 [Max  $\xrightarrow{\text{pyk}}$  “axiom max”]  
 [Numerical  $\xrightarrow{\text{pyk}}$  “axiom numerical”]  
 [NumericalF  $\xrightarrow{\text{pyk}}$  “axiom numericalF”]  
 [MemberOfSeries(ImPLY)  $\xrightarrow{\text{pyk}}$  “axiom memberOfSeries”]

$[\text{JoinConjuncts}(2\text{conditions}) \xrightarrow{\text{pyk}} \text{“prop lemma doubly conditioned join conjuncts”}]$   
 $[\text{prop lemma imply negation} \xrightarrow{\text{pyk}} \text{“prop lemma imply negation”}]$   
 $[\text{TND} \xrightarrow{\text{pyk}} \text{“prop lemma tertium non datur”}]$   
 $[\text{FromNegatedImPLY} \xrightarrow{\text{pyk}} \text{“prop lemma from negated imply”}]$   
 $[\text{ToNegatedImPLY} \xrightarrow{\text{pyk}} \text{“prop lemma to negated imply”}]$   
 $[\text{FromNegated}(2 * \text{ImPLY}) \xrightarrow{\text{pyk}} \text{“prop lemma from negated double imply”}]$   
 $[\text{FromNegatedAnd} \xrightarrow{\text{pyk}} \text{“prop lemma from negated and”}]$   
 $[\text{FromNegatedOr} \xrightarrow{\text{pyk}} \text{“prop lemma from negated or”}]$   
 $[\text{ToNegatedOr} \xrightarrow{\text{pyk}} \text{“prop lemma to negated or”}]$   
 $[\text{FromNegations} \xrightarrow{\text{pyk}} \text{“prop lemma from negations”}]$   
 $[\text{From3Disjuncts} \xrightarrow{\text{pyk}} \text{“prop lemma from three disjuncts”}]$   
 $[\text{From2} * 2\text{Disjuncts} \xrightarrow{\text{pyk}} \text{“prop lemma from two times two disjuncts”}]$   
 $[\text{NegateDisjunct1} \xrightarrow{\text{pyk}} \text{“prop lemma negate first disjunct”}]$   
 $[\text{NegateDisjunct2} \xrightarrow{\text{pyk}} \text{“prop lemma negate second disjunct”}]$   
 $[\text{ExpandDisjuncts} \xrightarrow{\text{pyk}} \text{“prop lemma expand disjuncts”}]$   
 $[\text{SENC1} \xrightarrow{\text{pyk}} \text{“lemma set equality nec condition(1)”}]$   
 $[\text{SENC2} \xrightarrow{\text{pyk}} \text{“lemma set equality nec condition(2)”}]$   
 $[\text{LessLeq}(R) \xrightarrow{\text{pyk}} \text{“lemma lessLeq(R)”}]$   
 $[\text{MemberOfSeries} \xrightarrow{\text{pyk}} \text{“lemma memberOfSeries”}]$   
 $[\text{memberOfSeries}(\text{Type}) \xrightarrow{\text{pyk}} \text{“lemma memberOfSeries(Type)”}]$   
 $[\text{*}(\text{exp})\text{*} \xrightarrow{\text{pyk}} \text{“} \wedge \text{”}]$   
 $[\text{R}(\text{*}) \xrightarrow{\text{pyk}} \text{“R( " )”}]$   
 $[\text{--} \text{R}(\text{*}) \xrightarrow{\text{pyk}} \text{“--R( " )”}]$   
 $[\text{rec}\text{*} \xrightarrow{\text{pyk}} \text{“1/ ”}]$   
 $[\text{*/}\text{*} \xrightarrow{\text{pyk}} \text{“eq-system of " modulo "”}]$   
 $[\text{*} \cap \text{*} \xrightarrow{\text{pyk}} \text{“intersection " comma " end intersection”}]$   
 $[\text{*}[\text{*}] \xrightarrow{\text{pyk}} \text{“[ " ; " ]”}]$   
 $[\text{U}\text{*} \xrightarrow{\text{pyk}} \text{“union " end union”}]$   
 $[\text{*} \cup \text{*} \xrightarrow{\text{pyk}} \text{“binary-union " comma " end union”}]$   
 $[\text{P}(\text{*}) \xrightarrow{\text{pyk}} \text{“power " end power”}]$   
 $[\{\text{*}\} \xrightarrow{\text{pyk}} \text{“zermelo singleton " end singleton”}]$   
 $[\text{StateExpand}(\text{*}, \text{*}, \text{*}) \xrightarrow{\text{pyk}} \text{“stateExpand( " , " , " )”}]$   
 $[\text{extractSeries}(\text{*}) \xrightarrow{\text{pyk}} \text{“extractSeries( " )”}]$   
 $[\text{SetOfSeries}(\text{*}) \xrightarrow{\text{pyk}} \text{“setOfSeries( " )”}]$



$[- - \text{Macro}(\ast) \xrightarrow{\text{pyk}} "--\text{Macro}(\ " )"]$   
 $[\text{ExpandList}(\ast, \ast, \ast) \xrightarrow{\text{pyk}} "expandList(\ " , \ " , \ ")"]$   
 $[\ast \ast \text{Macro}(\ast) \xrightarrow{\text{pyk}} "\ast\ast\text{Macro}(\ " )"]$   
 $[+ + \text{Macro}(\ast) \xrightarrow{\text{pyk}} "++\text{Macro}(\ " )"]$   
 $[<< \text{Macro}(\ast) \xrightarrow{\text{pyk}} "<<\text{Macro}(\ " )"]$   
 $[|\text{Macro}(\ast) \xrightarrow{\text{pyk}} "|\text{Macro}(\ " )"]$   
 $[01//\text{Macro}(\ast) \xrightarrow{\text{pyk}} "01//\text{Macro}(\ " )"]$   
 $[\text{UB}(\ast, \ast) \xrightarrow{\text{pyk}} "upperBound(\ " , \ ")"]$   
 $[\text{LUB}(\ast, \ast) \xrightarrow{\text{pyk}} "leastUpperBound(\ " , \ ")"]$   
 $[\text{BS}(\ast, \ast) \xrightarrow{\text{pyk}} "base(1/2)\text{Sum}(\ " , \ ")"]$   
 $[\text{UStelescope}(\ast, \ast) \xrightarrow{\text{pyk}} "UStelescope(\ " , \ ")"]$   
 $[(\ast) \xrightarrow{\text{pyk}} "(\ " )"]$   
 $[|f \ast | \xrightarrow{\text{pyk}} "|f \ " |"]$   
 $[|r \ast | \xrightarrow{\text{pyk}} "|r \ " |"]$   
 $[\text{Limit}(\ast, \ast) \xrightarrow{\text{pyk}} "limit(\ " , \ ")"]$   
 $[\text{Union}(\ast) \xrightarrow{\text{pyk}} "U(\ " )"]$   
 $[\text{IsOrderedPair}(\ast, \ast, \ast) \xrightarrow{\text{pyk}} "isOrderedPair(\ " , \ " , \ ")"]$   
 $[\text{IsRelation}(\ast, \ast, \ast) \xrightarrow{\text{pyk}} "isRelation(\ " , \ " , \ ")"]$   
 $[\text{isFunction}(\ast, \ast, \ast) \xrightarrow{\text{pyk}} "isFunction(\ " , \ " , \ ")"]$   
 $[\text{IsSeries}(\ast, \ast) \xrightarrow{\text{pyk}} "isSeries(\ " , \ ")"]$   
 $[\text{IsNatural}(\ast, \ast) \xrightarrow{\text{pyk}} "isNatural(\ " )"]$   
 $[\text{OrderedPair}(\ast, \ast) \xrightarrow{\text{pyk}} "(o \ " , \ ")"]$   
 $[\text{TypeNat}(\ast) \xrightarrow{\text{pyk}} "typeNat(\ " )"]$   
 $[\text{TypeNat0}(\ast) \xrightarrow{\text{pyk}} "typeNat0(\ " )"]$   
 $[\text{TypeRational}(\ast) \xrightarrow{\text{pyk}} "typeRational(\ " )"]$   
 $[\text{TypeRational0}(\ast) \xrightarrow{\text{pyk}} "typeRational0(\ " )"]$   
 $[\text{TypeSeries}(\ast, \ast) \xrightarrow{\text{pyk}} "typeSeries(\ " , \ ")"]$   
 $[\text{Typeseries0}(\ast, \ast) \xrightarrow{\text{pyk}} "typeSeries0(\ " , \ ")"]$   
 $[\{\ast, \ast\} \xrightarrow{\text{pyk}} "zermelo pair \ " comma \ " end pair"]$   
 $[\langle \ast, \ast \rangle \xrightarrow{\text{pyk}} "zermelo ordered pair \ " comma \ " end pair"]$   
 $[(-u\ast) \xrightarrow{\text{pyk}} "- \ " ]$   
 $[-f\ast \xrightarrow{\text{pyk}} "-f \ " ]$   
 $[(- - \ast) \xrightarrow{\text{pyk}} "-- \ " ]$   
 $[1f/\ast \xrightarrow{\text{pyk}} "1f/ \ " ]$   
 $[01//temp\ast \xrightarrow{\text{pyk}} "01// \ " ]$

$[*(*,*) \xrightarrow{\text{pyk}} \text{" is related to " under "}]$   
 $[\text{RefRel}(*,*) \xrightarrow{\text{pyk}} \text{" is reflexive relation in "}]$   
 $[\text{SymRel}(*,*) \xrightarrow{\text{pyk}} \text{" is symmetric relation in "}]$   
 $[\text{TransRel}(*,*) \xrightarrow{\text{pyk}} \text{" is transitive relation in "}]$   
 $[\text{EqRel}(*,*) \xrightarrow{\text{pyk}} \text{" is equivalence relation in "}]$   
 $[[* \in *]_* \xrightarrow{\text{pyk}} \text{"equivalence class of " in " modulo "}]$   
 $[\text{Partition}(*,*) \xrightarrow{\text{pyk}} \text{" is partition of "}]$   
 $[(***) \xrightarrow{\text{pyk}} \text{" * "}]$   
 $[* *_f * \xrightarrow{\text{pyk}} \text{" *_f "}]$   
 $[* ** * \xrightarrow{\text{pyk}} \text{" ** "}]$   
 $[(* + *) \xrightarrow{\text{pyk}} \text{" + "}]$   
 $[(* - *) \xrightarrow{\text{pyk}} \text{" - "}]$   
 $[* +_f * \xrightarrow{\text{pyk}} \text{" +_f "}]$   
 $[* -_f * \xrightarrow{\text{pyk}} \text{" -_f "}]$   
 $[* ++ * \xrightarrow{\text{pyk}} \text{" ++ "}]$   
 $[\text{R}(*,*) \xrightarrow{\text{pyk}} \text{"R( " ) -- R( " )"}]$   
 $[* \in * \xrightarrow{\text{pyk}} \text{" in0 "}]$   
 $[|*| \xrightarrow{\text{pyk}} \text{" | "}]$   
 $[\text{if}(*,*,*) \xrightarrow{\text{pyk}} \text{"if( " , " , " )"}]$   
 $[\text{Max}(*,*) \xrightarrow{\text{pyk}} \text{"max( " , " )"}]$   
 $[\text{Max}(*,*) \xrightarrow{\text{pyk}} \text{"maxR( " , " )"}]$   
 $[* = * \xrightarrow{\text{pyk}} \text{" = "}]$   
 $[* \neq * \xrightarrow{\text{pyk}} \text{" != "}]$   
 $[* <= * \xrightarrow{\text{pyk}} \text{" <= "}]$   
 $[* < * \xrightarrow{\text{pyk}} \text{" < "}]$   
 $[* <_f * \xrightarrow{\text{pyk}} \text{" <_f "}]$   
 $[* \leq_f * \xrightarrow{\text{pyk}} \text{" <=f "}]$   
 $[\text{SF}(*,*) \xrightarrow{\text{pyk}} \text{" sameF "}]$   
 $[* == * \xrightarrow{\text{pyk}} \text{" == "}]$   
 $[*!! == * \xrightarrow{\text{pyk}} \text{" !!== "}]$   
 $[* << * \xrightarrow{\text{pyk}} \text{" << "}]$   
 $[* <<== * \xrightarrow{\text{pyk}} \text{" <<== "}]$   
 $[* == * \xrightarrow{\text{pyk}} \text{" zermelo is "}]$   
 $[* \subseteq * \xrightarrow{\text{pyk}} \text{" is subset of "}]$   
 $[\dot{\neg}(*)_n \xrightarrow{\text{pyk}} \text{"not0 "}]$

[\*  $\notin$  \*  $\xrightarrow{\text{pyk}}$  “~in ”]

[\*  $\neq$  \*  $\xrightarrow{\text{pyk}}$  “~is ”]

[\*  $\wedge$  \*  $\xrightarrow{\text{pyk}}$  “and0 ”]

[\*  $\vee$  \*  $\xrightarrow{\text{pyk}}$  “or0 ”]

[ $\exists$ \*: \*  $\xrightarrow{\text{pyk}}$  “exist0 ” indeed ”]

[\*  $\Leftrightarrow$  \*  $\xrightarrow{\text{pyk}}$  “iff ”]

[{ph  $\in$  \* | \*}  $\xrightarrow{\text{pyk}}$  “the set of ph in ” such that ” end set”]

[kvanti  $\xrightarrow{\text{pyk}}$  “kvanti”]

)<sup>P</sup>

## B T<sub>E</sub>X definitioner

[kvanti <sup>tex</sup> ≡ “kvanti”]

[( $\cdots$ ) <sup>tex</sup> ≡ “(\cdots{ })”]

[Objekt-var <sup>tex</sup> ≡ “\texttt{Objekt-var}”]

[Ex-var <sup>tex</sup> ≡ “\texttt{Ex-var}”]

[Ph-var <sup>tex</sup> ≡ “\texttt{Ph-var}”]

[Værdi <sup>tex</sup> ≡ “\texttt{V\ae{ }rdi}”]

[Variabel <sup>tex</sup> ≡ “\texttt{Variabel}”]

[Op(x) <sup>tex</sup> ≡ “Op(#1.  
)”]

[Op(x, y) <sup>tex</sup> ≡ “Op(#1.  
, #2.  
)”]

[ $x \doteq y$  <sup>tex</sup> ≡ “#1.  
\mathrel {\ddot{=} } #2.”]

[ContainsEmpty(x) <sup>tex</sup> ≡ “ContainsEmpty(#1.  
)”]

[Dedu(x, y) <sup>tex</sup> ≡ “  
Dedu(#1.  
, #2.  
)”]

[Dedu<sub>0</sub>(x, y) <sup>tex</sup> ≡ “  
Dedu\_0(#1.  
, #2.  
)”]

[Dedu<sub>s</sub>(x, y, z) <sup>tex</sup> ≡ “Dedu\_{s}(#1.  
, #2.  
, #3.  
)”]

[Dedu<sub>1</sub>(x, y, z) <sup>tex</sup> ≡ “  
Dedu\_1(#1.  
, #2.  
)”]

, #3.  
)”]

[Dedu<sub>2</sub>(x, y, z)  $\stackrel{\text{tex}}{=} “$   
Dedu\_2(#1.

, #2.  
, #3.  
)”]

[Dedu<sub>3</sub>(x, y, z, u)  $\stackrel{\text{tex}}{=} “$   
Dedu\_3(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>4</sub>(x, y, z, u)  $\stackrel{\text{tex}}{=} “$   
Dedu\_4(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>4</sub><sup>\*</sup>(x, y, z, u)  $\stackrel{\text{tex}}{=} “$   
Dedu\_4<sup>\*</sup>(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>5</sub>(x, y, z)  $\stackrel{\text{tex}}{=} “$   
Dedu\_5(#1.

, #2.  
, #3.  
)”]

[Dedu<sub>6</sub>(p, c, e, b)  $\stackrel{\text{tex}}{=} “$   
Dedu\_6(#1.

, #2.  
, #3.  
, #4.  
)”]

[Dedu<sub>6</sub><sup>\*</sup>(p, c, e, b)  $\stackrel{\text{tex}}{=} “$   
Dedu\_6<sup>\*</sup>(#1.

, #2.  
, #3.

, #4.  
)”]

[Dedu<sub>7</sub>(p) <sup>tex</sup> ≡ “  
Dedu\_7(#1.  
)”]

[Dedu<sub>8</sub>(p, b) <sup>tex</sup> ≡ “  
Dedu\_8(#1.  
, #2.  
)”]

[Dedu<sub>8</sub><sup>\*</sup>(p, b) <sup>tex</sup> ≡ “  
Dedu\_8^\*(#1.  
, #2.  
)”]

[EX<sub>1</sub> <sup>tex</sup> ≡ “EX\_{1}”]

[EX<sub>2</sub> <sup>tex</sup> ≡ “EX\_{2}”]

[EX<sub>10</sub> <sup>tex</sup> ≡ “EX\_{10}”]

[EX<sub>20</sub> <sup>tex</sup> ≡ “EX\_{20}”]

[x<sub>EX</sub> <sup>tex</sup> ≡ “#1.  
\_{EX}”]

[x<sup>EX</sup> <sup>tex</sup> ≡ “#1.  
^{EX}”]

[(x≡y|z:=u)<sub>EX</sub> <sup>tex</sup> ≡ “\langle #1.  
{\equiv} #2.  
| #3.  
{:=} #4.  
\rangle\_{EX} ”]

[(x≡<sup>0</sup>y|z:=u)<sub>EX</sub> <sup>tex</sup> ≡ “\langle #1.  
{\equiv}^0 #2.  
| #3.  
{:=} #4.  
\rangle\_{EX} ”]

[(x≡<sup>1</sup>y|z:=u)<sub>EX</sub> <sup>tex</sup> ≡ “\langle #1.  
{\equiv}^1 #2.  
| #3.  
{:=} #4.  
\rangle\_{EX} ”]

$\langle x \equiv *y | z := u \rangle_{\text{Ex}} \stackrel{\text{tex}}{=} \backslash\langle \#1.$   
 $\{\equiv\}^* \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ex}}$

$[\text{ph}_1 \stackrel{\text{tex}}{=} \text{ph}_{\{1\}}]$

$[\text{ph}_2 \stackrel{\text{tex}}{=} \text{ph}_{\{2\}}]$

$[\text{ph}_3 \stackrel{\text{tex}}{=} \text{ph}_{\{3\}}]$

$[\text{ph}_4 \stackrel{\text{tex}}{=} \text{ph}_{\{4\}}]$

$[\text{ph}_5 \stackrel{\text{tex}}{=} \text{ph}_{\{5\}}]$

$[\text{ph}_6 \stackrel{\text{tex}}{=} \text{ph}_{\{6\}}]$

$[*_{\text{Ph}} \stackrel{\text{tex}}{=} \#1.$   
 $\text{-}\{\text{Ph}\}]$

$[x^{\text{Ph}} \stackrel{\text{tex}}{=} \#1.$   
 $\wedge\{\text{Ph}\}]$

$\langle x \equiv y | z := u \rangle_{\text{Ph}} \stackrel{\text{tex}}{=} \backslash\langle \#1.$   
 $\{\equiv\} \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

$\langle x \equiv^0 y | z := u \rangle_{\text{Ph}} \stackrel{\text{tex}}{=} \backslash\langle \#1.$   
 $\{\equiv\}^0 \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

$\langle x \equiv^1 y | z := u \rangle_{\text{Ph}} \stackrel{\text{tex}}{=} \backslash\langle \#1.$   
 $\{\equiv\}^1 \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

$\langle x \equiv *y | z := u \rangle_{\text{Ph}} \stackrel{\text{tex}}{=} \backslash\langle \#1.$   
 $\{\equiv\}^* \#2.$   
 $| \#3.$   
 $\{:=\} \#4.$   
 $\rangle_{\text{Ph}}$

[bs <sup>tex</sup> ≡ “\mathsf {bs}”]

[OBS <sup>tex</sup> ≡ “ \mathsf {OBS}”]

[BS <sup>tex</sup> ≡ “{\cal BS}”]

[∅ <sup>tex</sup> ≡ “\mathrm{\O}”]

[SystemQ <sup>tex</sup> ≡ “SystemQ”]

[MP <sup>tex</sup> ≡ “MP”]

[Gen <sup>tex</sup> ≡ “Gen”]

[Repetition <sup>tex</sup> ≡ “Repetition”]

[Neg <sup>tex</sup> ≡ “Neg”]

[Ded <sup>tex</sup> ≡ “Ded”]

[ExistIntro <sup>tex</sup> ≡ “ExistIntro”]

[Extensionality <sup>tex</sup> ≡ “Extensionality”]

[∅def <sup>tex</sup> ≡ “\O{}def”]

[PairDef <sup>tex</sup> ≡ “PairDef”]

[UnionDef <sup>tex</sup> ≡ “UnionDef”]

[PowerDef <sup>tex</sup> ≡ “PowerDef”]

[SeparationDef <sup>tex</sup> ≡ “SeparationDef”]

[AddDoubleNeg <sup>tex</sup> ≡ “AddDoubleNeg”]

[RemoveDoubleNeg <sup>tex</sup> ≡ “RemoveDoubleNeg”]

[AndCommutativity <sup>tex</sup> ≡ “AndCommutativity”]

[AutoImply <sup>tex</sup> ≡ “AutoImply”]

[Contrapositive <sup>tex</sup> ≡ “Contrapositive”]

[FirstConjunct <sup>tex</sup> ≡ “FirstConjunct”]

[SecondConjunct <sup>tex</sup> ≡ “SecondConjunct”]

[FromContradiction <sup>tex</sup> ≡ “FromContradiction”]



[FromDisjuncts  $\stackrel{\text{tex}}{=} \text{“FromDisjuncts”}$ ]  
 [IffCommutativity  $\stackrel{\text{tex}}{=} \text{“IffCommutativity”}$ ]  
 [IffFirst  $\stackrel{\text{tex}}{=} \text{“IffFirst”}$ ]  
 [IffSecond  $\stackrel{\text{tex}}{=} \text{“IffSecond”}$ ]  
 [ImplyTransitivity  $\stackrel{\text{tex}}{=} \text{“ImplyTransitivity”}$ ]  
 [JoinConjuncts  $\stackrel{\text{tex}}{=} \text{“JoinConjuncts”}$ ]  
 [MP2  $\stackrel{\text{tex}}{=} \text{“MP2”}$ ]  
 [MP3  $\stackrel{\text{tex}}{=} \text{“MP3”}$ ]  
 [MP4  $\stackrel{\text{tex}}{=} \text{“MP4”}$ ]  
 [MP5  $\stackrel{\text{tex}}{=} \text{“MP5”}$ ]  
 [MT  $\stackrel{\text{tex}}{=} \text{“MT”}$ ]  
 [NegativeMT  $\stackrel{\text{tex}}{=} \text{“NegativeMT”}$ ]  
 [Technicality  $\stackrel{\text{tex}}{=} \text{“Technicality”}$ ]  
 [Weakening  $\stackrel{\text{tex}}{=} \text{“Weakening”}$ ]  
 [WeakenOr1  $\stackrel{\text{tex}}{=} \text{“WeakenOr1”}$ ]  
 [WeakenOr2  $\stackrel{\text{tex}}{=} \text{“WeakenOr2”}$ ]  
 [Pair2Formula  $\stackrel{\text{tex}}{=} \text{“Pair2Formula”}$ ]  
 [Formula2Pair  $\stackrel{\text{tex}}{=} \text{“Formula2Pair”}$ ]  
 [Union2Formula  $\stackrel{\text{tex}}{=} \text{“Union2Formula”}$ ]  
 [Formula2Union  $\stackrel{\text{tex}}{=} \text{“Formula2Union”}$ ]  
 [Formula2Power  $\stackrel{\text{tex}}{=} \text{“Formula2Power”}$ ]  
 [Sep2Formula  $\stackrel{\text{tex}}{=} \text{“Sep2Formula”}$ ]  
 [Formula2Sep  $\stackrel{\text{tex}}{=} \text{“Formula2Sep”}$ ]  
 [SubsetInPower  $\stackrel{\text{tex}}{=} \text{“SubsetInPower”}$ ]  
 [HelperPowerIsSub  $\stackrel{\text{tex}}{=} \text{“HelperPowerIsSub”}$ ]

[PowerIsSub  $\stackrel{\text{tex}}{\equiv}$  “PowerIsSub”]

[(Switch)HelperPowerIsSub  $\stackrel{\text{tex}}{\equiv}$  “(Switch)HelperPowerIsSub”]

[(Switch)PowerIsSub  $\stackrel{\text{tex}}{\equiv}$  “(Switch)PowerIsSub”]

[ToSetEquality  $\stackrel{\text{tex}}{\equiv}$  “ToSetEquality”]

[HelperToSetEquality(t)  $\stackrel{\text{tex}}{\equiv}$  “HelperToSetEquality(t)”]

[ToSetEquality(t)  $\stackrel{\text{tex}}{\equiv}$  “ToSetEquality(t)”]

[HelperFromSetEquality  $\stackrel{\text{tex}}{\equiv}$  “HelperFromSetEquality”]

[FromSetEquality  $\stackrel{\text{tex}}{\equiv}$  “FromSetEquality”]

[HelperReflexivity  $\stackrel{\text{tex}}{\equiv}$  “HelperReflexivity”]

[Reflexivity  $\stackrel{\text{tex}}{\equiv}$  “Reflexivity”]

[HelperSymmetry  $\stackrel{\text{tex}}{\equiv}$  “HelperSymmetry”]

[Symmetry  $\stackrel{\text{tex}}{\equiv}$  “Symmetry”]

[HelperTransitivity  $\stackrel{\text{tex}}{\equiv}$  “HelperTransitivity”]

[Transitivity  $\stackrel{\text{tex}}{\equiv}$  “Transitivity”],

[ERisReflexive  $\stackrel{\text{tex}}{\equiv}$  “ERisReflexive”]

[ERisSymmetric  $\stackrel{\text{tex}}{\equiv}$  “ERisSymmetric”]

[ERisTransitive  $\stackrel{\text{tex}}{\equiv}$  “ERisTransitive”]

[ $\emptyset$ isSubset  $\stackrel{\text{tex}}{\equiv}$  “ $\emptyset$ isSubset”]

[HelperMemberNot $\emptyset$   $\stackrel{\text{tex}}{\equiv}$  “HelperMemberNot $\emptyset$ ”]

[MemberNot $\emptyset$   $\stackrel{\text{tex}}{\equiv}$  “MemberNot $\emptyset$ ”]

[HelperUnique $\emptyset$   $\stackrel{\text{tex}}{\equiv}$  “HelperUnique $\emptyset$ ”]

[Unique $\emptyset$   $\stackrel{\text{tex}}{\equiv}$  “Unique $\emptyset$ ”]

[== Reflexivity  $\stackrel{\text{tex}}{\equiv}$  “==\!{ }Reflexivity”]

[== Symmetry  $\stackrel{\text{tex}}{\equiv}$  “==\!{ }Symmetry”]

[Helper == Transitivity  $\stackrel{\text{tex}}{\equiv}$  “Helper\!{ }==\!{ }Transitivity”]

[ $\equiv$ Transitivity  $\stackrel{\text{tex}}{=} \text{"\!\{ }\equiv\!\{ }\text{Transitivity}"}$ ]

[HelperTransferNotEq  $\stackrel{\text{tex}}{=} \text{"HelperTransferNotEq}"}$ ]

[TransferNotEq  $\stackrel{\text{tex}}{=} \text{"TransferNotEq}"}$ ]

[HelperPairSubset  $\stackrel{\text{tex}}{=} \text{"HelperPairSubset}"}$ ]

[Helper(2)PairSubset  $\stackrel{\text{tex}}{=} \text{"Helper(2)PairSubset}"}$ ]

[PairSubset  $\stackrel{\text{tex}}{=} \text{"PairSubset}"}$ ]

[SamePair  $\stackrel{\text{tex}}{=} \text{"SamePair}"}$ ]

[SameSingleton  $\stackrel{\text{tex}}{=} \text{"SameSingleton}"}$ ]

[UnionSubset  $\stackrel{\text{tex}}{=} \text{"UnionSubset}"}$ ]

[SameUnion  $\stackrel{\text{tex}}{=} \text{"SameUnion}"}$ ]

[SeparationSubset  $\stackrel{\text{tex}}{=} \text{"SeparationSubset}"}$ ]

[SameSeparation  $\stackrel{\text{tex}}{=} \text{"SameSeparation}"}$ ]

[SameBinaryUnion  $\stackrel{\text{tex}}{=} \text{"SameBinaryUnion}"}$ ]

[IntersectionSubset  $\stackrel{\text{tex}}{=} \text{"IntersectionSubset}"}$ ]

[SameIntersection  $\stackrel{\text{tex}}{=} \text{"SameIntersection}"}$ ]

[AutoMember  $\stackrel{\text{tex}}{=} \text{"AutoMember}"}$ ]

[HelperEqSysNot $\emptyset$   $\stackrel{\text{tex}}{=} \text{"HelperEqSysNot\O{ }}"}$ ]

[EqSysNot $\emptyset$   $\stackrel{\text{tex}}{=} \text{"EqSysNot\O{ }}"}$ ]

[HelperEqSubset  $\stackrel{\text{tex}}{=} \text{"HelperEqSubset}"}$ ]

[EqSubset  $\stackrel{\text{tex}}{=} \text{"EqSubset}"}$ ]

[EqNecessary  $\stackrel{\text{tex}}{=} \text{"EqNecessary}"}$ ]

[HelperEqNecessary  $\stackrel{\text{tex}}{=} \text{"HelperEqNecessary}"}$ ]

[HelperNoneEqNecessary  $\stackrel{\text{tex}}{=} \text{"HelperNoneEqNecessary}"}$ ]

[Helper(2)NoneEqNecessary  $\stackrel{\text{tex}}{=} \text{"Helper(2)NoneEqNecessary}"}$ ]

[NoneEqNecessary  $\stackrel{\text{tex}}{=} \text{"NoneEqNecessary}"}$ ]

[EqClassIsSubset  $\stackrel{\text{tex}}{=} \text{“EqClassIsSubset”}$ ]

[EqClassesAreDisjoint  $\stackrel{\text{tex}}{=} \text{“EqClassesAreDisjoint”}$ ]

[AllDisjoint  $\stackrel{\text{tex}}{=} \text{“AllDisjoint”}$ ]

[AllDisjointImply  $\stackrel{\text{tex}}{=} \text{“AllDisjointImply”}$ ]

[BSsubset  $\stackrel{\text{tex}}{=} \text{“BSsubset”}$ ]

[Union(BS/R)subset  $\stackrel{\text{tex}}{=} \text{“Union(BS/R)subset”}$ ]

[UnionIdentity  $\stackrel{\text{tex}}{=} \text{“UnionIdentity”}$ ]

[EqSysIsPartition  $\stackrel{\text{tex}}{=} \text{“EqSysIsPartition”}$ ]

[x/y  $\stackrel{\text{tex}}{=} \text{“\#1.}$   
 $\text{/ \#2.”}$ ]

[x  $\cap$  y  $\stackrel{\text{tex}}{=} \text{“\#1.}$   
 $\backslash\text{cap \#2.”}$ ]

[ $\cup$ x  $\stackrel{\text{tex}}{=} \text{“\backslashcup \#1.”}$ ]

[x  $\cup$  y  $\stackrel{\text{tex}}{=} \text{“\#1.}$   
 $\backslash\text{mathrel{\backslashcup} \#2.”}$ ]

[P(x)  $\stackrel{\text{tex}}{=} \text{“P(\#1.}$   
 $\text{)”}$ ]

[{x}  $\stackrel{\text{tex}}{=} \text{“\{\#1.}$   
 $\backslash\text{”}$ ]

[{x, y}  $\stackrel{\text{tex}}{=} \text{“\{\#1.}$   
 $\text{\#2.}$   
 $\backslash\text{”}$ ]

[ $\langle$ x, y $\rangle$   $\stackrel{\text{tex}}{=} \text{“\langle \#1.}$   
 $\text{\#2.}$   
 $\backslash\text{rangle”}$ ],

[x  $\in$  y  $\stackrel{\text{tex}}{=} \text{“\#1.}$   
 $\backslash\text{mathrel{\backslashin} \#2.”}$ ]

[z(x, y)  $\stackrel{\text{tex}}{=} \text{“\#3.}$   
 $\text{(\#1.}$   
 $\text{\#2.}$   
 $\text{)”}$ ]

[RefRel(r, x)  $\stackrel{\text{tex}}{\equiv}$  “RefRel(#1.  
, #2.  
)”]

[SymRel(r, x)  $\stackrel{\text{tex}}{\equiv}$  “SymRel(#1.  
, #2.  
)”]

[TransRel(r, x)  $\stackrel{\text{tex}}{\equiv}$  “TransRel(#1.  
, #2.  
)”]

[EqRel(r, x)  $\stackrel{\text{tex}}{\equiv}$  “EqRel(#1.  
, #2.  
)”]

[[x  $\in$  bs]<sub>r</sub>  $\stackrel{\text{tex}}{\equiv}$  “[#1.  
\mathrel{\in} #2.  
]-{#3.  
}”]

[Partition(x, y)  $\stackrel{\text{tex}}{\equiv}$  “Partition(#1.  
, #2.  
)”]

[x == y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\!\mathrel{=} #2.”]

[x  $\subseteq$  y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\mathrel{\subseteq} #2.”]

[ $\dot{\neg}(x)$   $\stackrel{\text{tex}}{\equiv}$  “\dot{\neg}\, (#1.  
n”]

[x  $\notin$  y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\mathrel{\notin} #2.”]

[x  $\neq$  y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\mathrel{\neq} #2.”]

[x  $\dot{\wedge}$  y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\mathrel{\dot{\wedge}} #2.”]

[x  $\dot{\vee}$  y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\mathrel{\dot{\vee}} #2.”]

[x  $\dot{\leftrightarrow}$  y  $\stackrel{\text{tex}}{\equiv}$  “#1.  
\mathrel{\dot{\leftrightarrow}} #2.”]

[{ph ∈ x | a} <sup>tex</sup> “ \{ ph \mathrel{\in} #1.  
\mid #2.  
\}”]

[x ⇒ y <sup>tex</sup> “(i#1.  
\Rightarrow #2.  
i”]

[Nat(x) <sup>tex</sup> “Nat(#1.  
)”]

[(x≡y|z:=u)<sub>Me</sub> <sup>tex</sup> “\langle #1.  
\equiv #2.  
| #3.  
{:=} #4.  
\rangle\_{Me}”]

[(x≡<sup>1</sup>y|z:=u)<sub>Me</sub> <sup>tex</sup> “\langle #1.  
\equiv<sup>1</sup> #2.  
| #3.  
{:=} #4.  
\rangle\_{Me} ”]

[(x≡\*y|z:=u)<sub>Me</sub> <sup>tex</sup> “\langle #1.  
\equiv\* #2.  
| #3.  
{:=} #4.  
\rangle\_{Me} ”]

[∃x:y <sup>tex</sup> “  
\exists #1.  
: #2.”]

[(x1) <sup>tex</sup> “(x1)”]

[(x2) <sup>tex</sup> “(x2)”]

[(y1) <sup>tex</sup> “(y1)”]

[(y2) <sup>tex</sup> “(y2)”]

[(v1) <sup>tex</sup> “(v1)”]

[(v2) <sup>tex</sup> “(v2)”]

[(v3) <sup>tex</sup> “(v3)”]

[(v4) <sup>tex</sup> “(v4)”]

$[(v2n) \stackrel{\text{tex}}{=} "(v2n)"]$

$[(n1) \stackrel{\text{tex}}{=} "(n1)"]$

$[(n2) \stackrel{\text{tex}}{=} "(n2)"]$

$[(n3) \stackrel{\text{tex}}{=} "(n3)"]$

$[(m1) \stackrel{\text{tex}}{=} "(m1)"]$

$[(m2) \stackrel{\text{tex}}{=} "(m2)"]$

$[(\epsilon) \stackrel{\text{tex}}{=} "(\backslash\epsilonpsilon)"]$

$[(\epsilon)_1 \stackrel{\text{tex}}{=} "(\backslash\epsilonpsilon)_{-}\{1\}"]$

$[(\epsilon 2) \stackrel{\text{tex}}{=} "(\backslash\epsilonpsilon 2)"]$

$[(fx) \stackrel{\text{tex}}{=} "(fx)"]$

$[(fy) \stackrel{\text{tex}}{=} "(fy)"]$

$[(fz) \stackrel{\text{tex}}{=} "(fz)"]$

$[(fu) \stackrel{\text{tex}}{=} "(fu)"]$

$[(fv) \stackrel{\text{tex}}{=} "(fv)"]$

$[(fw) \stackrel{\text{tex}}{=} "(fw)"]$

$[(fep) \stackrel{\text{tex}}{=} "(fep)"]$

$[(rx) \stackrel{\text{tex}}{=} "(rx)"]$

$[(ry) \stackrel{\text{tex}}{=} "(ry)"]$

$[(rz) \stackrel{\text{tex}}{=} "(rz)"]$

$[(ru) \stackrel{\text{tex}}{=} "(ru)"]$

$[(sx) \stackrel{\text{tex}}{=} "(sx)"]$

$[(sx1) \stackrel{\text{tex}}{=} "(sx1)"]$

$[(sy) \stackrel{\text{tex}}{=} "(sy)"]$

$[(sy1) \stackrel{\text{tex}}{=} "(sy1)"]$

$[(sz) \stackrel{\text{tex}}{=} "(sz)"]$

$[(sz1) \stackrel{\text{tex}}{=} "(sz1)"]$

$[(su) \stackrel{\text{tex}}{=} "(su)"]$

$[(su1) \stackrel{\text{tex}}{=} "(su1)"]$

$[(fxs) \stackrel{\text{tex}}{=} "(fxs)"]$

$[(fys) \stackrel{\text{tex}}{=} "(fys)"]$

$[(crs1) \stackrel{\text{tex}}{=} "(crs1)"]$

$[(f1) \stackrel{\text{tex}}{=} "(f1)"]$

$[(f2) \stackrel{\text{tex}}{=} "(f2)"]$

$[(f3) \stackrel{\text{tex}}{=} "(f3)"]$

$[(f4) \stackrel{\text{tex}}{=} "(f4)"]$

$[(op1) \stackrel{\text{tex}}{=} "(op1)"]$

$[(op2) \stackrel{\text{tex}}{=} "(op2)"]$

$[(r1) \stackrel{\text{tex}}{=} "(r1)"]$

$[(s1) \stackrel{\text{tex}}{=} "(s1)"]$

$[(s2) \stackrel{\text{tex}}{=} "(s2)"]$

$[X_1 \stackrel{\text{tex}}{=} "X_{\{1\}}"]$

$[X_2 \stackrel{\text{tex}}{=} "X_{\{2\}}"]$

$[Y_1 \stackrel{\text{tex}}{=} "Y_{\{1\}}"]$

$[Y_2 \stackrel{\text{tex}}{=} "Y_{\{2\}}"]$

$[V_1 \stackrel{\text{tex}}{=} "V_{\{1\}}"]$

$[V_2 \stackrel{\text{tex}}{=} "V_{\{2\}}"]$

$[V_3 \stackrel{\text{tex}}{=} "V_{\{3\}}"]$

$[V_4 \stackrel{\text{tex}}{=} "V_{\{4\}}"]$

$[V_{2n} \stackrel{\text{tex}}{=} "V_{\{2n\}}"]$

$[\epsilon \stackrel{\text{tex}}{=} "\epsilon"]$



[M<sub>1</sub> <sup>tex</sup> ≡ “M\_{1}”]

[M<sub>2</sub> <sup>tex</sup> ≡ “M\_{2}”]

[N<sub>1</sub> <sup>tex</sup> ≡ “N\_{1} ”]

[N<sub>2</sub> <sup>tex</sup> ≡ “N\_{2} ”]

[N<sub>3</sub> <sup>tex</sup> ≡ “N\_{3} ”]

[ε<sub>1</sub> <sup>tex</sup> ≡ “\epsilon 1”]

[ε<sub>2</sub> <sup>tex</sup> ≡ “\epsilon 2”]

[FX <sup>tex</sup> ≡ “FX”]

[FY <sup>tex</sup> ≡ “FY”]

[FZ <sup>tex</sup> ≡ “FZ”]

[FU <sup>tex</sup> ≡ “FU”]

[FV <sup>tex</sup> ≡ “FV”]

[FW <sup>tex</sup> ≡ “FW”]

[FEP <sup>tex</sup> ≡ “FEP”]

[RX <sup>tex</sup> ≡ “RX”]

[RY <sup>tex</sup> ≡ “RY”]

[RZ <sup>tex</sup> ≡ “RZ”]

[RU <sup>tex</sup> ≡ “RU”]

[(SX) <sup>tex</sup> ≡ “(SX)”]

[(SX1) <sup>tex</sup> ≡ “(SX1)”]

[(SY) <sup>tex</sup> ≡ “(SY)”]

[(SY1) <sup>tex</sup> ≡ “(SY1)”]

[(SZ) <sup>tex</sup> ≡ “(SZ)”]

[(SZ1) <sup>tex</sup> ≡ “(SZ1)”]

[(SU) <sup>tex</sup> ≡ “(SU)”]

[(SU1)<sup>tex</sup> ≡ “(SU1)”]

[FXS<sup>tex</sup> ≡ “FXS”]

[FYS<sup>tex</sup> ≡ “FYS”]

[(F1)<sup>tex</sup> ≡ “(F1)”]

[(F2)<sup>tex</sup> ≡ “(F2)”]

[(F3)<sup>tex</sup> ≡ “(F3)”]

[(F4)<sup>tex</sup> ≡ “(F4)”]

[(OP1)<sup>tex</sup> ≡ “(OP1)”]

[(OP2)<sup>tex</sup> ≡ “(OP2)”]

[(R1)<sup>tex</sup> ≡ “(R1)”]

[(S1)<sup>tex</sup> ≡ “(S1)”]

[(S2)<sup>tex</sup> ≡ “(S2)”]

[(EPob)<sup>tex</sup> ≡ “(EPob)”]

[(CRS1ob)<sup>tex</sup> ≡ “(CRS1ob)”]

[(F1ob)<sup>tex</sup> ≡ “(F1ob)”]

[(F2ob)<sup>tex</sup> ≡ “(F2ob)”]

[(F3ob)<sup>tex</sup> ≡ “(F3ob)”]

[(F4ob)<sup>tex</sup> ≡ “(F4ob)”]

[(N1ob)<sup>tex</sup> ≡ “(N1ob)”]

[(N2ob)<sup>tex</sup> ≡ “(N2ob)”]

[(OP1ob)<sup>tex</sup> ≡ “(OP1ob)”]

[(OP2ob)<sup>tex</sup> ≡ “(OP2ob)”]

[(R1ob)<sup>tex</sup> ≡ “(R1ob)”]

[(S1ob)<sup>tex</sup> ≡ “(S1ob)”]

[(S2ob)<sup>tex</sup> ≡ “(S2ob)”]

[Ex3  $\stackrel{\text{tex}}{=} \text{"Ex3"}$ ]

[NAT  $\stackrel{\text{tex}}{=} \text{"NAT"}$ ]

[RATIONALSERIES  $\stackrel{\text{tex}}{=} \text{"RATIONAL_SERIES"}$ ]

[SERIES  $\stackrel{\text{tex}}{=} \text{"SERIES"}$ ]

[SetOfReals  $\stackrel{\text{tex}}{=} \text{"SetOfReals"}$ ]

[SetOfFxs  $\stackrel{\text{tex}}{=} \text{"SetOfFxs"}$ ]

[N  $\stackrel{\text{tex}}{=} \text{"N"}$ ]

[Q  $\stackrel{\text{tex}}{=} \text{"Q"}$ ]

[X  $\stackrel{\text{tex}}{=} \text{"X"}$ ]

[xs  $\stackrel{\text{tex}}{=} \text{"xs"}$ ]

[xaF  $\stackrel{\text{tex}}{=} \text{"xaF"}$ ]

[ysF  $\stackrel{\text{tex}}{=} \text{"ysF"}$ ]

[us  $\stackrel{\text{tex}}{=} \text{"us"}$ ]

[usFoelge  $\stackrel{\text{tex}}{=} \text{"usFoelge"}$ ]

[0  $\stackrel{\text{tex}}{=} \text{"0"}$ ]

[1  $\stackrel{\text{tex}}{=} \text{"1"}$ ]

[(-1)  $\stackrel{\text{tex}}{=} \text{"(-1)"}$ ]

[2  $\stackrel{\text{tex}}{=} \text{"2"}$ ]

[3  $\stackrel{\text{tex}}{=} \text{"3"}$ ]

[1/2  $\stackrel{\text{tex}}{=} \text{"1/2"}$ ]

[1/3  $\stackrel{\text{tex}}{=} \text{"1/3"}$ ]

[2/3  $\stackrel{\text{tex}}{=} \text{"2/3"}$ ]

[0f  $\stackrel{\text{tex}}{=} \text{"0f"}$ ]

[00  $\stackrel{\text{tex}}{=} \text{"00"}$ ]

[(- - 01)  $\stackrel{\text{tex}}{=} \text{"(-01)"}$ ]

[02 <sup>tex</sup> ≡ “02”]

[01//02 <sup>tex</sup> ≡ “01//02”]

[x = y <sup>tex</sup> ≡ “#1.  
= #2.”]

[x ≠ y <sup>tex</sup> ≡ “#1.  
\neq #2.”]

[x < y <sup>tex</sup> ≡ “#1.  
< #2.”]

[x <= y <sup>tex</sup> ≡ “#1.  
<= #2.”]

[x <<sub>f</sub> y <sup>tex</sup> ≡ “#1.  
<\_{f}#2.”]

[x ≤<sub>f</sub> y <sup>tex</sup> ≡ “#1.  
\leq\_{f}#2.”]

[SF(x,y) <sup>tex</sup> ≡ “SF(#1.  
, #2.  
)”]

[x == y <sup>tex</sup> ≡ “#1.  
== #2.”]

[x!! == y <sup>tex</sup> ≡ “#1.  
!!== #2.”]

[x << y <sup>tex</sup> ≡ “#1.  
<< #2.”]

[x <<== y <sup>tex</sup> ≡ “#1.  
<<== #2.”]

[x[y] <sup>tex</sup> ≡ “#1.  
[#2.  
]”]

[(-ux) <sup>tex</sup> ≡ “(-u#1.  
)”]

[-<sub>f</sub>x <sup>tex</sup> ≡ “-\_{f}#1.”]

$[(- - x)^{\text{tex}} \equiv (--\#1.$   
)]

$[1f/x^{\text{tex}} \equiv "1f/\#1." ]$

$[01//tempx^{\text{tex}} \equiv "01//temp\#1." ]$

$[(x + y)^{\text{tex}} \equiv "(#1.$   
+ $\#2.$   
)]

$[(x - y)^{\text{tex}} \equiv "(#1.$   
- $\#2.$   
)]

$[(fx) +_f (fy)^{\text{tex}} \equiv "\#1.$   
+\_{-}\{f\}\#2." ]

$[(fx) -_f (fy)^{\text{tex}} \equiv "\#1.$   
-\_{-}\{f\}\#2." ]

$[(fx) *_f (fy)^{\text{tex}} \equiv "\#1.$   
\*\_{-}\{f\}\#2." ]

$[x + +y^{\text{tex}} \equiv "\#1.$   
++ $\#2." ]$

$[R((fx)) - -R((fy))^{\text{tex}} \equiv "R(\#1.$   
) -- R( $\#2.$   
)]

$[(x * y)^{\text{tex}} \equiv "(#1.$   
\* $\#2.$   
)]

$[x * *y^{\text{tex}} \equiv "\#1.$   
\*\* $\#2." ]$

$[x(\text{exp})y^{\text{tex}} \equiv "\#1.$   
( $\text{exp}$ )  $\#2." ]$

$[\text{leqReflexivity}^{\text{tex}} \equiv "\text{leqReflexivity}"]$

$[\text{recx}^{\text{tex}} \equiv "\text{rec}\#1." ]$

$[|x|^{\text{tex}} \equiv "| \#1.$   
|"]

[StateExpand(t, s, c)  $\stackrel{\text{tex}}{=} \text{“StateExpand(\#1.}$   
#2.  
#3.  
)”]

[extractSeries(t)  $\stackrel{\text{tex}}{=} \text{“extractSeries(\#1.}$   
)”]

[|f|x|  $\stackrel{\text{tex}}{=} \text{“|f\#1.}$   
”]

[|r|x|  $\stackrel{\text{tex}}{=} \text{“|r\#1.}$   
”]

[SetOfSeries(x)  $\stackrel{\text{tex}}{=} \text{“SetOfSeries(\#1.}$   
)”]

[ExpandList(x, y, z)  $\stackrel{\text{tex}}{=} \text{“ExpandList(\#1.}$   
#2.  
#3.  
)”]

[\* \* Macro(x)  $\stackrel{\text{tex}}{=} \text{“**Macro(\#1.}$   
)”]

[+ + Macro(x)  $\stackrel{\text{tex}}{=} \text{“++Macro(\#1.}$   
)”]

[- - Macro(x)  $\stackrel{\text{tex}}{=} \text{“--Macro(\#1.}$   
)”]

[<< Macro(x)  $\stackrel{\text{tex}}{=} \text{“<<Macro(\#1.}$   
)”]

[|Macro(x)  $\stackrel{\text{tex}}{=} \text{“|Macro(\#1.}$   
)”]

[01//Macro(x)  $\stackrel{\text{tex}}{=} \text{“01//Macro(\#1.}$   
)”]

[Max(x, y)  $\stackrel{\text{tex}}{=} \text{“Max(\#1.}$   
#2.  
)”]

[Max(x, y)  $\stackrel{\text{tex}}{=} \text{“Max(\#1.}$   
#2.  
)”]

[Limit(x, y)  $\stackrel{\text{tex}}{=} \text{“Limit(\#1.}$   
 , #2.  
 )”]

[Union(x)  $\stackrel{\text{tex}}{=} \text{“Union(\#1.}$   
 )”]

[if(x, y, z)  $\stackrel{\text{tex}}{=} \text{“if(\#1.}$   
 , #2.  
 , #3.  
 )”]

[IsOrderedPair(x, y, z)  $\stackrel{\text{tex}}{=} \text{“IsOrderedPair(\#1.}$   
 , #2.  
 , #3.  
 )”]

[IsRelation(x, y, z)  $\stackrel{\text{tex}}{=} \text{“IsRelation(\#1.}$   
 , #2.  
 , #3.  
 )”]

[isFunction(x, y, z)  $\stackrel{\text{tex}}{=} \text{“isFunction(\#1.}$   
 , #2.  
 , #3.  
 )”]

[TypeNat(x)  $\stackrel{\text{tex}}{=} \text{“TypeNat(\#1.}$   
 )”]

[TypeNat0(x)  $\stackrel{\text{tex}}{=} \text{“TypeNat0(\#1.}$   
 )”]

[TypeRational(x)  $\stackrel{\text{tex}}{=} \text{“TypeRational(\#1.}$   
 )”]

[TypeRational0(x)  $\stackrel{\text{tex}}{=} \text{“TypeRational0(\#1.}$   
 )”]

[TypeSeries(x, y)  $\stackrel{\text{tex}}{=} \text{“TypeSeries(\#1.}$   
 , #2.  
 )”]

[Typeseries0(x, y)  $\stackrel{\text{tex}}{=} \text{“Typeseries0(\#1.}$   
 , #2.  
 )”]

[UB(x, y) <sup>tex</sup> ≡ “UB(#1.  
, #2.  
)”]

[LUB(x, y) <sup>tex</sup> ≡ “LUB(#1.  
, #2.  
)”]

[BS(x, y) <sup>tex</sup> ≡ “BS(#1.  
, #2.  
)”]

[UStelescope(x, y) <sup>tex</sup> ≡ “UStelescope(#1.  
, #2.  
)”]

[(x) <sup>tex</sup> ≡ “(#1.  
)”]

[R(x) <sup>tex</sup> ≡ “R(#1.  
)”]

[- - R(x) <sup>tex</sup> ≡ “--R(#1.  
)”]

[IsSeries(x, y) <sup>tex</sup> ≡ “IsSeries(#1.  
, #2.  
)”]

[IsNatural(xy, \*) <sup>tex</sup> ≡ “IsNatural(#1.  
, #2.  
)”]

[OrderedPair(x, y) <sup>tex</sup> ≡ “OrderedPair(#1.  
, #2.  
)”]

[leqAntisymmetryAxiom <sup>tex</sup> ≡ “leqAntisymmetryAxiom”]

[leqTransitivityAxiom <sup>tex</sup> ≡ “leqTransitivityAxiom”]

[leqTotality <sup>tex</sup> ≡ “leqTotality”]

[leqAdditionAxiom <sup>tex</sup> ≡ “leqAdditionAxiom”]

[leqMultiplicationAxiom <sup>tex</sup> ≡ “leqMultiplicationAxiom”]

[plusAssociativity <sup>tex</sup> ≡ “plusAssociativity”]



$\text{[plusCommutativity}^{\text{tex}} \text{ "plusCommutativity"}]$   
 $\text{[Negative}^{\text{tex}} \text{ "Negative"}]$   
 $\text{[plus0}^{\text{tex}} \text{ "plus0"}]$   
 $\text{[timesAssociativity}^{\text{tex}} \text{ "timesAssociativity"}]$   
 $\text{[timesCommutativity}^{\text{tex}} \text{ "timesCommutativity"}]$   
 $\text{[ReciprocalAxiom}^{\text{tex}} \text{ "ReciprocalAxiom"}]$   
 $\text{[times1}^{\text{tex}} \text{ "times1"}]$   
 $\text{[plusAssociativity}^{\text{tex}} \text{ "plusAssociativity"}]$   
 $\text{[plusCommutativity}^{\text{tex}} \text{ "plusCommutativity"}]$   
 $\text{[Negative}^{\text{tex}} \text{ "Negative"}]$   
 $\text{[Distribution}^{\text{tex}} \text{ "Distribution"}]$   
 $\text{[0not1}^{\text{tex}} \text{ "0not1"}]$   
 $\text{[A4(Axiom)}^{\text{tex}} \text{ "A4(Axiom)}]$   
 $\text{[InductionAxiom}^{\text{tex}} \text{ "InductionAxiom"}]$   
 $\text{[EqualityAxiom}^{\text{tex}} \text{ "EqualityAxiom"}]$   
 $\text{[EqLeqAxiom}^{\text{tex}} \text{ "EqLeqAxiom"}]$   
 $\text{[EqAdditionAxiom}^{\text{tex}} \text{ "EqAdditionAxiom"}]$   
 $\text{[EqMultiplicationAxiom}^{\text{tex}} \text{ "EqMultiplicationAxiom"}]$   
 $\text{[SENC1}^{\text{tex}} \text{ "SENC1"}]$   
 $\text{[SENC2}^{\text{tex}} \text{ "SENC2"}]$   
 $\text{[Cauchy}^{\text{tex}} \text{ "Cauchy"}]$   
 $\text{[PlusF}^{\text{tex}} \text{ "PlusF"}]$   
 $\text{[ReciprocalF}^{\text{tex}} \text{ "ReciprocalF"}]$   
 $\text{[From ==}^{\text{tex}} \text{ "From=="}]$   
 $\text{[To ==}^{\text{tex}} \text{ "To=="}]$

[FromInR  $\stackrel{\text{tex}}{\equiv}$  "FromInR"]

[ReciprocalR(Axiom)  $\stackrel{\text{tex}}{\equiv}$  "ReciprocalR(Axiom)"]

[US0  $\stackrel{\text{tex}}{\equiv}$  "US0"]

[NextXS(UpperBound)  $\stackrel{\text{tex}}{\equiv}$  "NextXS(UpperBound)"]

[NextXS(NoUpperBound)  $\stackrel{\text{tex}}{\equiv}$  "NextXS(NoUpperBound)"]

[NextUS(UpperBound)  $\stackrel{\text{tex}}{\equiv}$  "NextUS(UpperBound)"]

[NextUS(NoUpperBound)  $\stackrel{\text{tex}}{\equiv}$  "NextUS(NoUpperBound)"]

[ExpZero  $\stackrel{\text{tex}}{\equiv}$  "ExpZero"]

[ExpPositive  $\stackrel{\text{tex}}{\equiv}$  "ExpPositive"]

[ExpZero(R)  $\stackrel{\text{tex}}{\equiv}$  "ExpZero(R)"]

[ExpPositive(R)  $\stackrel{\text{tex}}{\equiv}$  "ExpPositive(R)"]

[LessMinus1(N)  $\stackrel{\text{tex}}{\equiv}$  "LessMinus1(N)"]

[Nonnegative(N)  $\stackrel{\text{tex}}{\equiv}$  "Nonnegative(N)"]

[BSzero  $\stackrel{\text{tex}}{\equiv}$  "BSzero"]

[BSpositive  $\stackrel{\text{tex}}{\equiv}$  "BSpositive"]

[USTelescope(Zero)  $\stackrel{\text{tex}}{\equiv}$  "USTelescope(Zero)"]

[USTelescope(Positive)  $\stackrel{\text{tex}}{\equiv}$  "USTelescope(Positive)"]

[EqAddition(R)  $\stackrel{\text{tex}}{\equiv}$  "EqAddition(R)"]

[FromLimit  $\stackrel{\text{tex}}{\equiv}$  "FromLimit"]

[ToUpperBound  $\stackrel{\text{tex}}{\equiv}$  "ToUpperBound"]

[FromUpperBound  $\stackrel{\text{tex}}{\equiv}$  "FromUpperBound"]

[USisUpperBound  $\stackrel{\text{tex}}{\equiv}$  "USisUpperBound"]

[0not1(R)  $\stackrel{\text{tex}}{\equiv}$  "0not1(R)"]

[ExpUnbounded(R)  $\stackrel{\text{tex}}{\equiv}$  "ExpUnbounded(R)"]

[FromLeq(Advanced)(N)  $\stackrel{\text{tex}}{\equiv}$  "FromLeq(Advanced)(N)"]

[FromLeastUpperBound  $\stackrel{\text{tex}}{=} \text{“FromLeastUpperBound”}$ ]

[ToLeastUpperBound  $\stackrel{\text{tex}}{=} \text{“ToLeastUpperBound”}$ ]

[XSisNotUpperBound  $\stackrel{\text{tex}}{=} \text{“XSisNotUpperBound”}$ ]

[ysFGreater  $\stackrel{\text{tex}}{=} \text{“ysFGreater”}$ ]

[ysFLess  $\stackrel{\text{tex}}{=} \text{“ysFLess”}$ ]

[SmallInverse  $\stackrel{\text{tex}}{=} \text{“SmallInverse”}$ ]

[MemberOfSeries(ImPLY)  $\stackrel{\text{tex}}{=} \text{“MemberOfSeries(ImPLY)”}$ ]

[NatType  $\stackrel{\text{tex}}{=} \text{“NatType”}$ ]

[RationalType  $\stackrel{\text{tex}}{=} \text{“RationalType”}$ ]

[SeriesType  $\stackrel{\text{tex}}{=} \text{“SeriesType”}$ ]

[JoinConjuncts(2conditions)  $\stackrel{\text{tex}}{=} \text{“JoinConjuncts(2conditions)”}$ ]

[TND  $\stackrel{\text{tex}}{=} \text{“TND”}$ ]

[FromNegatedImPLY  $\stackrel{\text{tex}}{=} \text{“FromNegatedImPLY”}$ ]

[ToNegatedImPLY  $\stackrel{\text{tex}}{=} \text{“ToNegatedImPLY”}$ ]

[FromNegated(2 \* ImPLY)  $\stackrel{\text{tex}}{=} \text{“FromNegated(2*ImPLY)”}$ ]

[FromNegatedAnd  $\stackrel{\text{tex}}{=} \text{“FromNegatedAnd”}$ ]

[FromNegatedOr  $\stackrel{\text{tex}}{=} \text{“FromNegatedOr”}$ ]

[ToNegatedOr  $\stackrel{\text{tex}}{=} \text{“ToNegatedOr”}$ ]

[FromNegations  $\stackrel{\text{tex}}{=} \text{“FromNegations”}$ ]

[From3Disjuncts  $\stackrel{\text{tex}}{=} \text{“From3Disjuncts”}$ ]

[NegateDisjunct1  $\stackrel{\text{tex}}{=} \text{“NegateDisjunct1”}$ ]

[NegateDisjunct2  $\stackrel{\text{tex}}{=} \text{“NegateDisjunct2”}$ ]

[ExpandDisjuncts  $\stackrel{\text{tex}}{=} \text{“ExpandDisjuncts”}$ ]

[From2 \* 2Disjuncts  $\stackrel{\text{tex}}{=} \text{“From2*2Disjuncts”}$ ]

[PlusR(Sym)  $\stackrel{\text{tex}}{=} \text{“PlusR(Sym)”}$ ]

[LessLeq(R)  $\stackrel{\text{tex}}{=} \text{“LessLeq(R)”}$ ]

[LeqAntisymmetry(R)  $\stackrel{\text{tex}}{=} \text{“LeqAntisymmetry(R)”}$ ]

[LeqTransitivity(R)  $\stackrel{\text{tex}}{=} \text{“LeqTransitivity(R)”}$ ]

[Plus0(R)  $\stackrel{\text{tex}}{=} \text{“Plus0(R)”}$ ]

[lessAddition(R)  $\stackrel{\text{tex}}{=} \text{“lessAddition(R)”}$ ]

[leqAddition(R)  $\stackrel{\text{tex}}{=} \text{“leqAddition(R)”}$ ]

[PlusAssociativity(R)XX  $\stackrel{\text{tex}}{=} \text{“PlusAssociativity(R)XX”}$ ]

[PlusAssociativity(R)  $\stackrel{\text{tex}}{=} \text{“PlusAssociativity(R)”}$ ]

[Negative(R)  $\stackrel{\text{tex}}{=} \text{“Negative(R)”}$ ]

[PlusCommutativity(R)  $\stackrel{\text{tex}}{=} \text{“PlusCommutativity(R)”}$ ]

[Times1(R)  $\stackrel{\text{tex}}{=} \text{“Times1(R)”}$ ]

[TimesAssociativity(R)  $\stackrel{\text{tex}}{=} \text{“TimesAssociativity(R)”}$ ]

[TimesCommutativity(R)  $\stackrel{\text{tex}}{=} \text{“TimesCommutativity(R)”}$ ]

[Distribution(R)  $\stackrel{\text{tex}}{=} \text{“Distribution(R)”}$ ]

[ $\exists x: y \stackrel{\text{tex}}{=} \text{“(AARRGGHH!-exist-bug!)”}$ ]

[constantRationalSeries(x)  $\stackrel{\text{tex}}{=} \text{“constantRationalSeries(\#1.)”}$ ]

[Power(x)  $\stackrel{\text{tex}}{=} \text{“Power(\#1.)”}$ ]

[cartProd(x)  $\stackrel{\text{tex}}{=} \text{“cartProd(\#1.)”}$ ]

[binaryUnion(x, y)  $\stackrel{\text{tex}}{=} \text{“binaryUnion(\#1., \#2.)”}$ ]

[SetOfRationalSeries  $\stackrel{\text{tex}}{=} \text{“SetOfRationalSeries”}$ ]

[MemberOfSeries  $\stackrel{\text{tex}}{=} \text{“MemberOfSeries”}$ ]

[IsSubset(x, y) <sup>tex</sup> ≡ “IsSubset(#1.  
, #2.  
)”]

[memberOfSeries(Type) <sup>tex</sup> ≡ “memberOfSeries(Type)”]

[UniqueMember <sup>tex</sup> ≡ “UniqueMember”]

[UniqueMember(Type) <sup>tex</sup> ≡ “UniqueMember(Type)”]

[SameSeries <sup>tex</sup> ≡ “SameSeries”]

[A4 <sup>tex</sup> ≡ “A4”]

[(sx) <sup>tex</sup> ≡ “(s#1.  
)”]

[(px, y) <sup>tex</sup> ≡ “(p#1.  
, #2.  
)”]

[SameMember <sup>tex</sup> ≡ “SameMember”]

[Qclosed(Addition) <sup>tex</sup> ≡ “Qclosed(Addition)”]

[Qclosed(Multiplication) <sup>tex</sup> ≡ “Qclosed(Multiplication)”]

[FromCartProd(1) <sup>tex</sup> ≡ “FromCartProd(1)”]

[FromCartProd(1) <sup>tex</sup> ≡ “FromCartProd(1)”]

[Max <sup>tex</sup> ≡ “Max”]

[Numerical <sup>tex</sup> ≡ “Numerical”]

[NumericalF <sup>tex</sup> ≡ “NumericalF”]

[Separation2formula(1) <sup>tex</sup> ≡ “Separation2formula(1)”]

[Separation2formula(2) <sup>tex</sup> ≡ “Separation2formula(2)”]

[QisClosed(Reciprocal)(ImPLY) <sup>tex</sup> ≡ “QisClosed(Reciprocal)(ImPLY)”]

[QisClosed(Reciprocal) <sup>tex</sup> ≡ “QisClosed(Reciprocal)”]

[QisClosed(Negative)(ImPLY) <sup>tex</sup> ≡ “QisClosed(Negative)(ImPLY)”]

[QisClosed(Negative) <sup>tex</sup> ≡ “QisClosed(Negative)”]

$[(\text{Adgic})\text{SameR} \stackrel{\text{tex}}{=} \text{"}(\text{Adgic})\text{SameR"}]$