Logiweb codex of peanorapport

peanorapport

[peanorapport $\stackrel{\text{pyk}}{\rightarrow}$ “peanorapport”]

$\dot{0}$

[$\dot{0}$ $\stackrel{\text{tex}}{\rightarrow}$ “
\dot{0}”]

[$\dot{0}$ $\stackrel{\text{pyk}}{\rightarrow}$ “peano zero”]

$\dot{1}$

[$\dot{1} \stackrel{\text{macro}}{\rightarrow} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\dot{1} \ddot{=} \dot{0}']\rceil)$]

[$\dot{1}$ $\stackrel{\text{tex}}{\rightarrow}$ “
\dot{1}”]

[$\dot{1}$ $\stackrel{\text{pyk}}{\rightarrow}$ “peano one”]

$\dot{2}$

[$\dot{2} \stackrel{\text{macro}}{\rightarrow} \lambda t.\lambda s.\lambda c.\tilde{\mathcal{M}}_4(t, s, c, \lceil[\dot{2} \ddot{=} \dot{1}']\rceil)$]

[$\dot{2}$ $\stackrel{\text{tex}}{\rightarrow}$ “
\dot{2}”]

[$\dot{2}$ $\stackrel{\text{pyk}}{\rightarrow}$ “peano two”]

$\dot{a}$

$[\dot{a} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{a} \ddot{=} \dot{\mathsf{a}}] \rceil)]$

$[\dot{a} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{a}}"]

$[\dot{a} \stackrel{\text{pyk}}{\to}$ "peano a"]

$\dot{b}$

$[\dot{b} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{b} \ddot{=} \dot{\mathsf{b}}] \rceil)]$

$[\dot{b} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{b}}"]

$[\dot{b} \stackrel{\text{pyk}}{\to}$ "peano b"]

$\dot{c}$

$[\dot{c} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{c} \ddot{=} \dot{\mathsf{c}}] \rceil)]$

$[\dot{c} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{c}}"]

$[\dot{c} \stackrel{\text{pyk}}{\to}$ "peano c"]

$\dot{d}$

$[\dot{d} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{d} \ddot{=} \dot{\mathsf{d}}] \rceil)]$

$[\dot{d} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{d}}"]

$[\dot{d} \stackrel{\text{pyk}}{\to}$ "peano d"]

$\dot{e}$

$[\dot{e} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{e} \ddot{=} \dot{\mathsf{e}}] \rceil)]$

$[\dot{e} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{e}}"]

$[\dot{e} \stackrel{\text{pyk}}{\to}$ "peano e"]

$\dot{f}$

$[\dot{f} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{f} \ddot{=} \dot{\mathsf{f}}]\rceil)]$

$[\dot{f} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{f}}"]

$[\dot{f} \stackrel{\text{pyk}}{\rightarrow}$ "peano f"]

$\dot{g}$

$[\dot{g} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{g} \ddot{=} \dot{\mathsf{g}}]\rceil)]$

$[\dot{g} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{g}}"]

$[\dot{g} \stackrel{\text{pyk}}{\rightarrow}$ "peano g"]

$\dot{h}$

$[\dot{h} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{h} \ddot{=} \dot{\mathsf{h}}]\rceil)]$

$[\dot{h} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{h}}"]

$[\dot{h} \stackrel{\text{pyk}}{\rightarrow}$ "peano h"]

$\dot{i}$

$[\dot{i} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{i} \ddot{=} \dot{\mathsf{i}}]\rceil)]$

$[\dot{i} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{i}}"]

$[\dot{i} \stackrel{\text{pyk}}{\rightarrow}$ "peano i"]

$\dot{j}$

$[\dot{j} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{j} \ddot{=} \dot{\mathsf{j}}]\rceil)]$

$[\dot{j} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{j}}"]

$[\dot{j} \stackrel{\text{pyk}}{\rightarrow}$ "peano j"]

$\dot{k}$

$[\dot{k} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{k} \ddot{=} \dot{\mathsf{k}}]\rceil)]$

$[\dot{k} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{k}}"]

$[\dot{k} \stackrel{\text{pyk}}{\to}$ "peano k"]

$\dot{l}$

$[\dot{l} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{l} \ddot{=} \dot{\mathsf{l}}]\rceil)]$

$[\dot{l} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{l}}"]

$[\dot{l} \stackrel{\text{pyk}}{\to}$ "peano l"]

$\dot{m}$

$[\dot{m} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{m} \ddot{=} \dot{\mathsf{m}}]\rceil)]$

$[\dot{m} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{m}}"]

$[\dot{m} \stackrel{\text{pyk}}{\to}$ "peano m"]

$\dot{n}$

$[\dot{n} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{n} \ddot{=} \dot{\mathsf{n}}]\rceil)]$

$[\dot{n} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{n}}"]

$[\dot{n} \stackrel{\text{pyk}}{\to}$ "peano n"]

$\dot{o}$

$[\dot{o} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil[\dot{o} \ddot{=} \dot{\mathsf{o}}]\rceil)]$

$[\dot{o} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{o}}"]

$[\dot{o} \stackrel{\text{pyk}}{\to}$ "peano o"]

$\dot{p}$

$[\dot{p} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{p} \ddot{=} \dot{\mathsf{p}}] \rceil)]$

$[\dot{p} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{p}}"]

$[\dot{p} \stackrel{\text{pyk}}{\rightarrow}$ "peano p"]

$\dot{q}$

$[\dot{q} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{q} \ddot{=} \dot{\mathsf{q}}] \rceil)]$

$[\dot{q} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{q}}"]

$[\dot{q} \stackrel{\text{pyk}}{\rightarrow}$ "peano q"]

$\dot{r}$

$[\dot{r} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{r} \ddot{=} \dot{\mathsf{r}}] \rceil)]$

$[\dot{r} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{r}}"]

$[\dot{r} \stackrel{\text{pyk}}{\rightarrow}$ "peano r"]

$\dot{s}$

$[\dot{s} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{s} \ddot{=} \dot{\mathsf{s}}] \rceil)]$

$[\dot{s} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{s}}"]

$[\dot{s} \stackrel{\text{pyk}}{\rightarrow}$ "peano s"]

$\dot{t}$

$[\dot{t} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{t} \ddot{=} \dot{\mathsf{t}}] \rceil)]$

$[\dot{t} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{t}}"]

$[\dot{t} \stackrel{\text{pyk}}{\rightarrow}$ "peano t"]

$\dot{u}$

$[\dot{u} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{u} \stackrel{..}{=} \dot{\mathsf{u}}] \rceil)]$

$[\dot{u} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{u}}"]

$[\dot{u} \stackrel{\text{pyk}}{\rightarrow}$ "peano u"]

$\dot{v}$

$[\dot{v} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{v} \stackrel{..}{=} \dot{\mathsf{v}}] \rceil)]$

$[\dot{v} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{v}}"]

$[\dot{v} \stackrel{\text{pyk}}{\rightarrow}$ "peano v"]

$\dot{w}$

$[\dot{w} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{w} \stackrel{..}{=} \dot{\mathsf{w}}] \rceil)]$

$[\dot{w} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{w}}"]

$[\dot{w} \stackrel{\text{pyk}}{\rightarrow}$ "peano w"]

$\dot{x}$

$[\dot{x} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{x} \stackrel{..}{=} \dot{\mathsf{x}}] \rceil)]$

$[\dot{x} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{x}}"]

$[\dot{x} \stackrel{\text{pyk}}{\rightarrow}$ "peano x"]

$\dot{y}$

$[\dot{y} \stackrel{\text{macro}}{\rightarrow} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t}, \mathsf{s}, \mathsf{c}, \lceil [\dot{y} \stackrel{..}{=} \dot{\mathsf{y}}] \rceil)]$

$[\dot{y} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{\mathit{y}}"]

$[\dot{y} \stackrel{\text{pyk}}{\rightarrow}$ "peano y"]

$\dot{z}$

$[\dot{z} \stackrel{\text{macro}}{\to} \lambda \mathsf{t}.\lambda \mathsf{s}.\lambda \mathsf{c}.\tilde{\mathcal{M}}_4(\mathsf{t},\mathsf{s},\mathsf{c},\lceil[\dot{z} \ddot{=} \dot{\mathsf{z}}]\rceil)]$

$[\dot{z} \stackrel{\text{tex}}{\to}$ "
\dot{\mathit{z}}"]

$[\dot{z} \stackrel{\text{pyk}}{\to}$ "peano z"]

$\dot{\text{nonfree}}(*,*)$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to}$
$\text{If}(\mathsf{y}^{\mathcal{P}}, \neg\, [\, \mathsf{x} \stackrel{\text{t}}{=} \mathsf{y}\,]\,,$
$\text{If}(\neg\, [\, \mathsf{y} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{x}\colon \mathsf{y} \rceil\,]\,, \dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}),$
$\text{If}(\mathsf{x} \stackrel{\text{t}}{=}\ [\, \mathsf{y}^1\,]\,, \mathsf{T}, \dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^2))))]$

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}(#1.
, #2.
)"]

$[\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree * in * end nonfree"]

$\dot{\text{nonfree}}^*(*,*)$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{val}}{\to} \mathsf{x}!\text{If}(\mathsf{y}, \mathsf{T}, \text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{y}^{\text{h}}), \dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}^{\text{t}}), \mathsf{F}))]$

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{tex}}{\to}$ "
\dot{nonfree}^*(#1.
, #2.
)"]

$[\dot{\text{nonfree}}^*(\mathsf{x},\mathsf{y}) \stackrel{\text{pyk}}{\to}$ "peano nonfree star * in * end nonfree"]

$\dot{\text{free}}\langle *|* := *\rangle$

$[\dot{\text{free}}\langle \mathsf{a}|\mathsf{x} := \mathsf{b}\rangle \stackrel{\text{val}}{\to} \mathsf{x}!\ [\ \mathsf{b}!$
$\text{If}(\mathsf{a}^{\mathcal{P}}, \mathsf{T},$
$\text{If}(\neg\, [\, \mathsf{a} \stackrel{\text{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil\,]\,, \dot{\text{free}}^*\langle \mathsf{a}^{\text{t}}|\mathsf{x} := \mathsf{b}\rangle,$
$\text{If}(\mathsf{a}^1 \stackrel{\text{t}}{=} \mathsf{x}, \mathsf{T},$
$\text{If}(\dot{\text{nonfree}}(\mathsf{x},\mathsf{a}^2), \mathsf{T},$

$\mathrm{If}(\neg \dot{\mathrm{nonfree}}(\mathsf{a}^1, \mathsf{b}), \mathsf{F},$
$\dot{\mathrm{free}}\langle \mathsf{a}^2 | \mathsf{x} := \mathsf{b} \rangle)))))]\ ]$

[$\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{tex}}{\to}$ "
\dot{free}\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\mathrm{free}}\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{pyk}}{\to}$ "peano free * set * to * end free"]

$\dot{\mathrm{free}}^*\langle * | * := * \rangle$

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{val}}{\to} \mathsf{x}!\ [\ \mathsf{b}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\dot{\mathrm{free}}\langle \mathsf{a}^{\mathrm{h}} | \mathsf{x} := \mathsf{b} \rangle, \dot{\mathrm{free}}^*\langle \mathsf{a}^{\mathrm{t}} | \mathsf{x} := \mathsf{b} \rangle, \mathsf{F}))\ ]\ ]$

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{tex}}{\to}$ "
\dot{free}{}^*\langle #1.
| #2.
:= #3.
\rangle"]

[$\dot{\mathrm{free}}^*\langle \mathsf{a} | \mathsf{x} := \mathsf{b} \rangle \stackrel{\mathrm{pyk}}{\to}$ "peano free star * set * to * end free"]

$* \equiv \langle * | * := * \rangle$

[$\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{val}}{\to} \mathsf{a}!\ [\ \mathsf{x}!\ [\ \mathsf{c}!$
$\mathrm{If}(\mathrm{If}(\mathsf{b} \stackrel{\mathrm{r}}{=} \lceil \dot{\forall} \mathsf{u}\colon \mathsf{v} \rceil, \mathsf{b}^1 \stackrel{\mathrm{t}}{=} \mathsf{x}, \mathsf{F}), \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{b},$
$\mathrm{If}(\mathsf{b}^{\mathcal{P}} \wedge\ [\ \mathsf{b} \stackrel{\mathrm{t}}{=} \mathsf{x}\ ]\ , \mathsf{a} \stackrel{\mathrm{t}}{=} \mathsf{c}, \mathrm{If}(\ [$
$\mathsf{a}\ ]\ \stackrel{\mathrm{r}}{=} \mathsf{b}, \mathsf{a}^{\mathrm{t}} \equiv \langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F})))\ ]\ \ ]\ ]$

[$\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{tex}}{\to}$ "#1.
{\equiv}\langle #2.
|#3.
:=#4.
\rangle"]

[$\mathsf{a} \equiv \langle \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{pyk}}{\to}$ "peano sub * is * where * is * end sub"]

$* \equiv \langle^* * | * := * \rangle$

[$\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{val}}{\to} \mathsf{b}!\ [\ \mathsf{x}!\ [\ \mathsf{c}!\mathrm{If}(\mathsf{a}, \mathsf{T}, \mathrm{If}(\mathsf{a}^{\mathrm{h}} \equiv \langle \mathsf{b}^{\mathrm{h}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{a}^{\mathrm{t}} \equiv \langle^* \mathsf{b}^{\mathrm{t}} | \mathsf{x} := \mathsf{c} \rangle, \mathsf{F}))\ ]\ \ ]\ ]$

[$\mathsf{a} \equiv \langle^* \mathsf{b} | \mathsf{x} := \mathsf{c} \rangle \stackrel{\mathrm{tex}}{\to}$ "#1.

{\equiv}\langle^* #2.
|#3.
:=#4.
\rangle"]

[$\mathsf{a}{\equiv}\langle^*\mathsf{b}|\mathsf{x} := \mathsf{c}\rangle \stackrel{\text{pyk}}{\rightarrow}$ "peano sub star * is * where * is * end sub"]

S′

[$\mathrm{S}' \stackrel{\text{stmt}}{\rightarrow}$ $[\,[\forall\underline{a}\colon\forall\underline{b}\colon [\,[\underline{a} \dot{+} [\underline{b}']\,] \stackrel{p}{=} [\,[\underline{a} \dot{+} \underline{b}]'\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon [\,[\,[\dot{\neg}\underline{b}] \dot{\Rightarrow} \dot{\neg}\underline{a}\,] \dot{\Rightarrow} [\,[\,[\dot{\neg}\underline{b}] \dot{\Rightarrow} \underline{a}\,] \dot{\Rightarrow} \underline{b}\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon [\,[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [\underline{a}' \stackrel{p}{=} [\underline{b}']\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon [\,[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [\underline{a} \vdash \underline{b}]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon [\,[\underline{a}' \stackrel{p}{=} [\underline{b}']\,] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{b}]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]\,]\,]\,] \oplus [\,[\forall\underline{x}\colon\forall\underline{a}\colon\forall\underline{b}\colon [\mathrm{nonfree}(\lceil\underline{x}\rceil, \lceil\underline{a}\rceil) \Vdash [\,[\dot{\forall}\underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{b}]\,] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{b}]\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon [\,[\underline{a} \dot{\cdot} [\underline{b}']\,] \stackrel{p}{=} [\,[\underline{a} \dot{\cdot} \underline{b}] \dot{+} \underline{a}\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon [\,[\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [\,[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]\,] \dot{\Rightarrow} [\,[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [\,[\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [\,[\underline{a} \stackrel{p}{=} \underline{c}] \dot{\Rightarrow} [\underline{b} \stackrel{p}{=} \underline{c}]\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon\forall\underline{x}\colon [\underline{b}{\equiv}\langle\underline{a}|\underline{x} := \dot{0}\rangle \Vdash [\underline{c}{\equiv}\langle\underline{a}|\underline{x} := \underline{x}'\rangle \Vdash [\underline{b} \dot{\Rightarrow} [\,[\dot{\forall}\underline{x}\colon [\underline{a} \dot{\Rightarrow} \underline{c}]\,] \dot{\Rightarrow} \dot{\forall}\underline{x}\colon \underline{a}\,]\,]\,]\,]\,]\,] \oplus [\,[\forall\underline{a}\colon \neg [\dot{0} \stackrel{p}{=} [\underline{a}']\,]\,]\,] \oplus [\,[\forall\underline{x}\colon\forall\underline{a}\colon [\underline{a} \vdash \dot{\forall}\underline{x}\colon \underline{a}]\,]\,] \oplus [\,[\forall\underline{c}\colon\forall\underline{a}\colon\forall\underline{x}\colon\forall\underline{b}\colon [\lceil\underline{a}\rceil{\equiv}\langle\lceil\underline{b}\rceil|\lceil\underline{x}\rceil := \lceil\underline{c}\rceil\rangle \Vdash [\,[\dot{\forall}\underline{x}\colon \underline{b}] \dot{\Rightarrow} \underline{a}\,]\,]\,]\,] \oplus \forall\underline{a}\colon [\,[\underline{a} \dot{\cdot} \dot{0}] \stackrel{p}{=} \dot{0}\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]\,]$]

[$\mathrm{S}' \stackrel{\text{tex}}{\rightarrow}$ "
S'"]

[$\mathrm{S}' \stackrel{\text{pyk}}{\rightarrow}$ "system prime s"]

A1′

[$\mathrm{A1}' \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{A1}' \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall\underline{a}\colon\forall\underline{b}\colon [\,\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{a}]\,]$]

[$\mathrm{A1}' \stackrel{\text{tex}}{\rightarrow}$ "
A1'"]

[$\mathrm{A1}' \stackrel{\text{pyk}}{\rightarrow}$ "axiom prime a one"]

A2′

[$\mathrm{A2}' \stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[$\mathrm{A2}' \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall\underline{a}\colon\forall\underline{b}\colon\forall\underline{c}\colon [\,[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]\,] \dot{\Rightarrow} [\,[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]\,]\,]$]

$[A2' \stackrel{tex}{\rightarrow}$ "
A2'"]

$[A2' \stackrel{pyk}{\rightarrow}$ "axiom prime a two"]

A3′

$[A3' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[A3' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ [\ \dot{\neg}\ \underline{b}\ ]\ \dot{\Rightarrow}\ \dot{\neg}\ \underline{a}\ ]\ \dot{\Rightarrow}\ [\ [\ [\ \dot{\neg}\ \underline{b}\ ]\ \dot{\Rightarrow}\ \underline{a}\ ]\ \dot{\Rightarrow}\ \underline{b}\ ]\ ]\ ]$

$[A3' \stackrel{tex}{\rightarrow}$ "
A3'"]

$[A3' \stackrel{pyk}{\rightarrow}$ "axiom prime a three"]

A4′

$[A4' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[A4' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{c}\colon \forall \underline{a}\colon \forall \underline{x}\colon \forall \underline{b}\colon [\ \lceil \underline{a} \rceil \equiv \langle \lceil \underline{b} \rceil | \lceil \underline{x} \rceil := \lceil \underline{c} \rceil \rangle \Vdash [\ [\ \dot{\forall} \underline{x}\colon \underline{b}\ ]\ \dot{\Rightarrow}\ \underline{a}\ ]\ ]\ ]$

$[A4' \stackrel{tex}{\rightarrow}$ "
A4'"]

$[A4' \stackrel{pyk}{\rightarrow}$ "axiom prime a four"]

A5′

$[A5' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[A5' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon \forall \underline{b}\colon [\ \dot{\text{nonfree}}(\lceil \underline{x} \rceil, \lceil \underline{a} \rceil) \Vdash [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a}\ \dot{\Rightarrow}\ \underline{b}\ ]\ ]\ \dot{\Rightarrow}\ [\ \underline{a}\ \dot{\Rightarrow}\ \dot{\forall} \underline{x}\colon \underline{b}\ ]\ ]\ ]\ ]$

$[A5' \stackrel{tex}{\rightarrow}$ "
A5'"]

$[A5' \stackrel{pyk}{\rightarrow}$ "axiom prime a five"]

S1′

$[S1' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S1' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon [\ [\ \underline{a} \stackrel{p}{=} \underline{b}\ ]\ \dot{\Rightarrow}\ [\ [\ \underline{a} \stackrel{p}{=} \underline{c}\ ]\ \dot{\Rightarrow}\ [\ \underline{b} \stackrel{p}{=} \underline{c}\ ]\ ]\ ]\ ]$

$[S1' \stackrel{tex}{\rightarrow}$ "
S1""]

$[S1' \stackrel{pyk}{\rightarrow}$ "axiom prime s one"]

S2′

$[S2' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S2' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\underline{a} \stackrel{p}{=} \underline{b}] \dot{\Rightarrow} [\underline{a}' \stackrel{p}{=} [\underline{b}']\ ]\ ]]$

$[S2' \stackrel{tex}{\rightarrow}$ "
S2""]

$[S2' \stackrel{pyk}{\rightarrow}$ "axiom prime s two"]

S3′

$[S3' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S3' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{a} \colon \neg [\dot{0} \stackrel{p}{=} [\underline{a}']\ ]]$

$[S3' \stackrel{tex}{\rightarrow}$ "
S3""]

$[S3' \stackrel{pyk}{\rightarrow}$ "axiom prime s three"]

S4′

$[S4' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S4' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\ [\underline{a}' \stackrel{p}{=} [\underline{b}']\ ] \dot{\Rightarrow} [\underline{a} \stackrel{p}{=} \underline{b}]\ ]]$

$[S4' \stackrel{tex}{\rightarrow}$ "
S4""]

$[S4' \stackrel{pyk}{\rightarrow}$ "axiom prime s four"]

S5′

$[S5' \stackrel{proof}{\rightarrow}$ Rule tactic]

$[S5' \stackrel{stmt}{\rightarrow} S' \vdash \forall \underline{a} \colon [\ [\underline{a} \dot{+} \dot{0}] \stackrel{p}{=} \underline{a}\ ]]$

[S5′ $\stackrel{\text{tex}}{\rightarrow}$ "
S5""]

[S5′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s five"]

S6′

[S6′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S6′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{+} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{+} \underline{b}\ ]\ '\ ]\ ]$]

[S6′ $\stackrel{\text{tex}}{\rightarrow}$ "
S6""]

[S6′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s six"]

S7′

[S7′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S7′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon [\ [\ \underline{a} \dot{\cdot} \dot{0}\ ] \stackrel{p}{=} \dot{0}\ ]$]

[S7′ $\stackrel{\text{tex}}{\rightarrow}$ "
S7""]

[S7′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s seven"]

S8′

[S8′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S8′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\cdot} [\ \underline{b}'\ ]\ ] \stackrel{p}{=} [\ [\ \underline{a} \dot{\cdot} \underline{b}\ ] \dot{+} \underline{a}\ ]\ ]$]

[S8′ $\stackrel{\text{tex}}{\rightarrow}$ "
S8""]

[S8′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s eight"]

S9′

[S9′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[S9′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon \forall \underline{c}\colon \forall \underline{x}\colon [\ \underline{b} \equiv \langle \underline{a} | \underline{x} := \dot{0} \rangle \Vdash [\ \underline{c} \equiv \langle \underline{a} | \underline{x} := \underline{x}' \rangle \Vdash [\ \underline{b} \dot{\Rightarrow} [\ [\ \dot{\forall} \underline{x}\colon [\ \underline{a} \dot{\Rightarrow} \underline{c}\ ]\ ] \dot{\Rightarrow} \dot{\forall} \underline{x}\colon \underline{a}\ ]\ ]\ ]\ ]$]

[S9′ $\stackrel{\text{tex}}{\rightarrow}$ "
S9′"]

[S9′ $\stackrel{\text{pyk}}{\rightarrow}$ "axiom prime s nine"]

MP′

[MP′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[MP′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{a}\colon \forall \underline{b}\colon [\ [\ \underline{a} \dot{\Rightarrow} \underline{b}\ ] \vdash [\ \underline{a} \vdash \underline{b}\ ]\ ]$]

[MP′ $\stackrel{\text{tex}}{\rightarrow}$ "
MP′"]

[MP′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime mp"]

Gen′

[Gen′ $\stackrel{\text{proof}}{\rightarrow}$ Rule tactic]

[Gen′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{x}\colon \forall \underline{a}\colon [\ \underline{a} \vdash \dot{\forall} \underline{x}\colon \underline{a}\ ]$]

[Gen′ $\stackrel{\text{tex}}{\rightarrow}$ "
Gen′"]

[Gen′ $\stackrel{\text{pyk}}{\rightarrow}$ "rule prime gen"]

L3.2(a)′

[L3.2(a)′ $\stackrel{\text{proof}}{\rightarrow}$ $\lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{t}\colon [\ [\ S5' \gg [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ ]\ ; [\ [\ S1' \gg [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ]\ ; [\ [\ [\ [\ \text{MPTwice} \rhd [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \dot{\Rightarrow} [\ [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ] \dot{\Rightarrow} [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ] \rhd [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ ] \rhd [\ [\ \underline{t} \dot{+} \dot{0}\ ] \stackrel{p}{=} \underline{t}\ ]\ ] \gg [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]\ ]\ ]\ ]\rceil, p_0, c)$]

[L3.2(a)′ $\stackrel{\text{stmt}}{\rightarrow}$ $S' \vdash \forall \underline{t}\colon [\ \underline{t} \stackrel{p}{=} \underline{t}\ ]$]

[L3.2(a)′ $\stackrel{\text{tex}}{\rightarrow}$ "L3.2 (a)′"]

[L3.2(a)′ $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime three two a"]

L3.2(b)′

$[\mathrm{L3.2(b)'} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S'} \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\ [\ \mathrm{S1'} \gg [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Taut1} \rhd [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]\ ] \gg [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(a)'} \gg [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ;\ [\ [\ [\ \mathrm{MP'} \rhd [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]\ ] \rhd [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ] \gg [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]\ ]\ ]\ ] \rceil, p_0, \mathsf{c})]$

$[\mathrm{L3.2(b)'} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]$

$[\mathrm{L3.2(b)'} \overset{\mathrm{tex}}{\rightarrow}$ “L3.2 (b)′”]

$[\mathrm{L3.2(b)'} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma prime three two b”]

L3.2(c)′

$[\mathrm{L3.2(c)'} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S'} \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \mathrm{S1'} \gg [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(b)'} \gg [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Taut2} \rhd [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}]\ ]\ ] \rhd [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ] \gg [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]\ ]\ ] \rceil, p_0, \mathsf{c})]$

$[\mathrm{L3.2(c)'} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{r}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]$

$[\mathrm{L3.2(c)'} \overset{\mathrm{tex}}{\rightarrow}$ “L3.2 (c)′”]

$[\mathrm{L3.2(c)'} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma prime three two c”]

L3.2(d)′

$[\mathrm{L3.2(d)'} \overset{\mathrm{proof}}{\rightarrow} \lambda \mathsf{c}.\lambda \mathsf{x}.\mathcal{P}(\lceil \mathrm{S'} \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\ \mathrm{L3.2(c)'} \gg [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Taut1} \rhd [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ] \gg [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{L3.2(b)'} \gg [\ [\underline{\mathsf{s}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{Taut2} \rhd [\ [\underline{\mathsf{s}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ] \rhd [\ [\underline{\mathsf{t}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ] \gg [\ [\underline{\mathsf{s}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Taut1} \rhd [\ [\underline{\mathsf{s}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ] \gg [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{s}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]\ ]\ ]\ ]\ ] \rceil, p_0, \mathsf{c})]$

$[\mathrm{L3.2(d)'} \overset{\mathrm{stmt}}{\rightarrow} \mathrm{S'} \vdash \forall \underline{\mathsf{t}}\colon \forall \underline{\mathsf{r}}\colon \forall \underline{\mathsf{s}}\colon [\ [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\ [\underline{\mathsf{s}} \overset{\mathrm{p}}{=} \underline{\mathsf{t}}] \mathrel{\dot{\Rightarrow}} [\underline{\mathsf{r}} \overset{\mathrm{p}}{=} \underline{\mathsf{s}}]\ ]\ ]\ ]$

$[\mathrm{L3.2(d)'} \overset{\mathrm{tex}}{\rightarrow}$ “L3.2 (d)′”]

$[\mathrm{L3.2(d)'} \overset{\mathrm{pyk}}{\rightarrow}$ “lemma prime three two d”]

L3.2(f)′

$$[\mathrm{L3.2(f)}' \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [[\mathrm{S5}' \gg [[\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}]] ; [[\mathrm{L3.2(b)}' \gg [[[\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}] \dot{\Rightarrow} [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]]]] ; [[[[\mathrm{MP}' \rhd [[[\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}] \dot{\Rightarrow} [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]]]] \rhd [[\dot{0} \dot{+} \dot{0}] \stackrel{p}{=} \dot{0}]] \gg [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]]] ; [[\mathrm{S6}' \gg [[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]] ; [[\mathrm{A1}' \gg [[[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]] ; [[[[\mathrm{MP}' \rhd [[[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]]] \rhd [[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]] \gg [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]] ; [[\mathrm{M1.7} \gg [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]] ; [[\mathrm{S2}' \gg [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]] ; [[[[\mathrm{Taut2} \rhd [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]] \rhd [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]] \gg [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]] ; [[\mathrm{L3.2(d)}' \gg [[\dot{t}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [[[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]]]] ; [[[[\mathrm{Taut2} \rhd [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]] \rhd [[\dot{t}' \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [[[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]]] \gg [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]]]] ; [[[[\mathrm{Taut3} \rhd [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']]]]] \rhd [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [[[\dot{0} \dot{+} [\dot{t}']] \stackrel{p}{=} [[\dot{0} \dot{+} [\dot{t}]] ']] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]]] \gg [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] ; [[[\mathrm{Gen}' \rhd [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] \gg \dot{\forall}\dot{t}\colon [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]] ; [[\mathrm{S9}' \gg [[\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]] \dot{\Rightarrow} [[\dot{\forall}\dot{t}\colon [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] \dot{\Rightarrow} \dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]]] ; [[[[\mathrm{MPTwice} \rhd [[\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]] \dot{\Rightarrow} [[\dot{\forall}\dot{t}\colon [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] \dot{\Rightarrow} \dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]]] \rhd [\dot{0} \stackrel{p}{=} [\dot{0} \dot{+} \dot{0}]]] \rhd \dot{\forall}\dot{t}\colon [[\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]] \dot{\Rightarrow} [\dot{t}' \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}']]]]]] \gg \dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]]]]]]]]]]]]]]]] \rceil, p_0, c)]$$

$$[\mathrm{L3.2(f)}' \stackrel{\text{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dot{+} [\dot{t}]]]]$$

[L3.2(f)′ $\stackrel{\text{tex}}{\rightarrow}$ “L3.2 (f)’”]

[L3.2(f)′ $\stackrel{\text{pyk}}{\rightarrow}$ “lemma prime three two f”]

L3.2(g)′

$$[\mathrm{L3.2(g)}' \stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [[\mathrm{S5}' \gg [[\dot{t}' \dot{+} \dot{0}] \stackrel{p}{=} [\dot{t}']]] ; [[\mathrm{S5}' \gg [[\dot{t} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{t}]]] ; [[\mathrm{S2}' \gg [[[\dot{t} \dot{+} \dot{0}] \stackrel{p}{=} [\dot{t}]] \dot{\Rightarrow} [[\dot{t} \dot{+} \dot{0}] ' \stackrel{p}{=} [\dot{t}']]$$

$] \,] \, ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{t} \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{t} \,] \,] \dot{\Rightarrow} [\, [\, \dot{t} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{t}' \,] \,] \,] \,] \rhd [\, [$
$\dot{t} \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{t} \,] \,] \,] \gg [\, [\, \dot{t} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{t}' \,] \,] \,] \, ; [\, [\, \mathrm{L3.2(d)}' \gg [\, [\, [\, \dot{t}' \dot{+} \dot{0} \,]$
$\stackrel{p}{=} [\, \dot{t}' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{t}' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} \dot{0} \,]' \,] \,] \,] \,] \,] \, ;$
$[\, [\, [\, [\, [\, \mathrm{MPTwice} \rhd [\, [\, [\, \dot{t}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{t}' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t} \dot{+} \dot{0} \,]' \stackrel{p}{=} [\, \dot{t}' \,] \,] \dot{\Rightarrow} [$
$[\, \dot{t}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} \dot{0} \,]' \,] \,] \,] \,] \,] \rhd [\, [\, \dot{t}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, \dot{t}' \,] \,] \,] \rhd [\, [\, \dot{t} \dot{+} \dot{0} \,]' \stackrel{p}{=} [$
$\dot{t}' \,] \,] \,] \gg [\, [\, \dot{t}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} \dot{0} \,]' \,] \,] \,] \, ; [\, [\, \mathrm{S6}' \gg [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [$
$\dot{t}' \dot{+} [\, \dot{r} \,] \,]' \,] \,] \,] \, ; [\, [\, [\, \mathrm{Weaken} \rhd [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \,] \,] \,]$
$\gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,]$
$]' \,] \,] \,] \,] \, ; [\, [\, \mathrm{S2}' \gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,]$
$]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \, ; [\, [\, \mathrm{L3.2(c)}' \gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,]$
$]' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [$
$\dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \,] \,] \, ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \,] \,]$
$\dot{\Rightarrow} [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,]$
$]'' \,] \,] \,] \,] \,] \rhd [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \,] \,] \,] \gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]$
$' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \, ; [\, [\, [\, [$
$\mathrm{Taut2} \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+}$
$[\, \dot{r} \,] \,]'' \,] \,] \,] \,] \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,]$
$\stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [$
$\dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \, ; [\, [\, \mathrm{S6}' \gg [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [$
$\dot{r} \,] \,]' \,] \,] \,] \, ; [\, [\, \mathrm{S2}' \gg [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t} \dot{+} [\, \dot{r}'$
$] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \, ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}$
$] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \,] \rhd [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [$
$[\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \,] \gg [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \, ; [\, [$
$\mathrm{L3.2(d)}' \gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [$
$[\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \,] \,] \, ; [\, [\, [\, [$
$\mathrm{Taut2} \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+}$
$[\, \dot{r} \,] \,]'' \,] \,] \,] \,] \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,]$
$]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \,] \,] \gg [$
$[\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]''$
$] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \,] \, ; [\, [\, [\, \mathrm{Taut1} \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r}$
$] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \dot{\Rightarrow} [\, [$
$\dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \,] \,] \gg [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,]$
$]'' \,] \,] \dot{\Rightarrow} [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [$
$\dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \,] \,] \, ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,]$
$\dot{\Rightarrow} [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,]$
$]' \,] \,] \,] \,] \,] \rhd [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]'' \,] \,] \,] \gg [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,]$
$\stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \, ; [\, [\, [$
$\mathrm{Gen}' \rhd [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [$
$\dot{r}' \,] \,]' \,] \,] \,] \,] \gg \dot{\forall} \dot{r} \colon [\, [\, [\, \dot{t}' \dot{+} [\, \dot{r} \,] \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r} \,] \,]' \,] \,] \dot{\Rightarrow} [\, [\, \dot{t}' \dot{+} [\, \dot{r}' \,]$
$] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} [\, \dot{r}' \,] \,]' \,] \,] \,] \,] \, ; [\, [\, \mathrm{S9}' \gg [\, [\, [\, \dot{t}' \dot{+} \dot{0} \,] \stackrel{p}{=} [\, [\, \dot{t} \dot{+} \dot{0} \,]' \,] \,] \dot{\Rightarrow} [$

$[\,\dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}' \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ [\ \mathrm{MPTwice} \rhd [\ [\ [\ \dot{t}' \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} \dot{0}\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}' \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ]\ \rhd [\ [\ \dot{t}' \dot{+} \dot{0}\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} \dot{0}\ ]\ '\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{r}\colon [\ [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{t}' \dot{+} [\ \dot{r}'\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}'\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \gg \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ;\ [\ [\ \mathrm{Gen}' \rhd \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ \gg \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ \rceil, p_0, c)]$

$[\mathrm{L3.2(g)}' \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\,]$

$[\mathrm{L3.2(g)}' \stackrel{\mathrm{tex}}{\rightarrow}$ "L3.2 (g)'"]

$[\mathrm{L3.2(g)}' \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma prime three two g"]

L3.2(g)'II

$[\mathrm{L3.2(g)}'\mathrm{II} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash [\ [\ \mathrm{L3.2(g)}' \gg \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ;\ [\ \lceil \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \rceil \equiv \langle \lceil \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \rceil | \lceil \dot{t} \rceil := \lceil \dot{s} \rceil \rangle \Vdash [\ [\ [\ \mathrm{A4}' \unrhd \lceil \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \rceil \equiv \langle \lceil \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \rceil | \lceil \dot{t} \rceil := \lceil \dot{s} \rceil \rangle\ ] \gg [\ [\ \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{t}\colon \dot{\forall}\dot{r}\colon [\ [\ \dot{t}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{t} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ \gg \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ ;\ [\ \lceil [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ] \rceil \equiv \langle \lceil [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ] \rceil | \lceil \dot{r} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [\ [\ [\ \mathrm{A4}' \unrhd \lceil [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ] \rceil \equiv \langle \lceil [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ] \rceil | \lceil \dot{r} \rceil := \lceil \dot{t} \rceil \rangle\ ] \gg [\ [\ \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ]\ ]\ \gg [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{Gen}' \rhd [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ \gg \dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ;\ [\ \lceil [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ] \rceil \equiv \langle \lceil [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ] \rceil | \lceil \dot{s} \rceil := \lceil \dot{r} \rceil \rangle \Vdash [\ [\ [\ \mathrm{A4}' \unrhd \lceil [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ] \rceil \equiv \langle \lceil [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ] \rceil | \lceil \dot{s} \rceil := \lceil \dot{r} \rceil \rangle\ ] \gg [\ [\ \dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ;\ [\ [\ [\ \mathrm{MP}' \rhd [\ [\ \dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ] \dot{\Rightarrow} [\ [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ \rhd \dot{\forall}\dot{s}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ \gg [\ [\ \dot{r}' \dot{+} [\ \dot{t}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{r} \dot{+} [\ \dot{t}\ ]\ ]\ '\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ ]\ \rceil, p_0, c)]$

$[\mathrm{L3.2(g)}'\mathrm{II} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash [\ \lceil \dot{\forall}\dot{r}\colon [\ [\ \dot{s}' \dot{+} [\ \dot{r}\ ]\ ] \stackrel{p}{=} [\ [\ \dot{s} \dot{+} [\ \dot{r}\ ]\ ]\ '\ ]\ ] \rceil \equiv \langle \lceil \dot{\forall}\dot{r}\colon [\ [$

$\dot{t}' \dotplus [\dot{r}]] \stackrel{p}{=} [[\dot{t} \dotplus [\dot{r}]]']] \rceil \| \lceil \dot{t} \rceil := \lceil \dot{s} \rceil \rangle \Vdash [\lceil [\dot{s}' \dotplus [\dot{t}]] \stackrel{p}{=} [[\dot{s} \dotplus [\dot{t}]]'] \rceil \equiv \langle \lceil [\dot{s}' \dotplus [\dot{r}]] \stackrel{p}{=} [[\dot{s} \dotplus [\dot{r}]]'] \rceil \| \lceil \dot{r} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [\lceil [\dot{r}' \dotplus [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dotplus [\dot{t}]]'] \rceil \equiv \langle \lceil [\dot{s}' \dotplus [\dot{t}]] \stackrel{p}{=} [[\dot{s} \dotplus [\dot{t}]]'] \rceil \| \lceil \dot{s} \rceil := \lceil \dot{r} \rceil \rangle \Vdash [[\dot{r}' \dotplus [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dotplus [\dot{t}]]']]]]]]]$

[L3.2(g)′II $\stackrel{\text{tex}}{\rightarrow}$ "L3.2 (g)' II"]

[L3.2(g)′II $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime three two g rev"]

L3.2(h)′basis

[L3.2(h)′basis $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [[S5' \gg [[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{t}]]] ; [[L3.2(f)' \gg \dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]] ; [\lceil \dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]] \rceil \equiv \langle \lceil \dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]] \rceil \| \lceil \dot{t} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [[[A4' \rhd\!\!\rhd \lceil \dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]] \rceil \equiv \langle \lceil \dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]] \rceil \| \lceil \dot{t} \rceil := \lceil \dot{t} \rceil \rangle] \gg [[\dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]] \dot{\Rightarrow} [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]]] ; [[[[MP' \rhd [[\dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]] \dot{\Rightarrow} [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]]] \rhd \dot{\forall}\dot{t}\colon [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]] \gg [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]] ; [[L3.2(c)' \gg [[[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{t}]] \dot{\Rightarrow} [[\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]]]]] ; [[[[[MPTwice \rhd [[[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{t}]] \dot{\Rightarrow} [[\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]]]] \rhd [[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{t}]]] \rhd [\dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]] \gg [[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]] ; [[S9' \gg [[[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{\forall}\dot{r}\colon [[[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dotplus [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dotplus [\dot{t}]]]]]] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]]]]]] ; [[[MP' \rhd [[[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{\forall}\dot{r}\colon [[[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dotplus [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dotplus [\dot{t}]]]]]] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]]]]]] \rhd [[\dot{t} \dotplus \dot{0}] \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]]]]] \gg [[\dot{\forall}\dot{r}\colon [[[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dotplus [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dotplus [\dot{t}]]]]]] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]]]]]]]]]]]]] \rceil, p_0, c)$]

[L3.2(h)′basis $\stackrel{\text{stmt}}{\rightarrow} S' \vdash [\lceil \dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]] \rceil \equiv \langle \lceil \dot{t} \stackrel{p}{=} [\dot{0} \dotplus [\dot{t}]] \rceil \| \lceil \dot{t} \rceil := \lceil \dot{t} \rceil \rangle \Vdash [[\dot{\forall}\dot{r}\colon [[[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]] \dot{\Rightarrow} [[\dot{t} \dotplus [\dot{r}']] \stackrel{p}{=} [\dot{r}' \dotplus [\dot{t}]]]]]] \dot{\Rightarrow} \dot{\forall}\dot{r}\colon [[\dot{t} \dotplus [\dot{r}]] \stackrel{p}{=} [\dot{r} \dotplus [\dot{t}]]]]]]]$]

[L3.2(h)′basis $\stackrel{\text{tex}}{\rightarrow}$ "L3.2 (h)' basis"]

[L3.2(h)′basis $\stackrel{\text{pyk}}{\rightarrow}$ "lemma prime three two h base"]

L3.2(h)′

[L3.2(h)′ $\stackrel{\text{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash [[S6' \gg [[\dot{t} \dotplus [\dot{r}']] \stackrel{p}{=} [[\dot{t} \dotplus [\dot{r}]]']]] ; [[L3.2(g)'II \gg [[\dot{r}' \dotplus [\dot{t}]] \stackrel{p}{=} [[\dot{r} \dotplus [\dot{t}]]']]] ; [[S2' \gg [[[$

ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [[ṫ ∔ [ṙ]]′ $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]] ; [[
L3.2(c)′ ≫ [[[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṫ ∔ [ṙ]]′]] ⇒̇ [[[ṫ ∔ [ṙ]]′ $\stackrel{p}{=}$ [[
ṙ ∔ [ṫ]]′]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]]] ; [[[[MP′ ▷
[[[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṫ ∔ [ṙ]]′]] ⇒̇ [[[ṫ ∔ [ṙ]]′ $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′
]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]]] ▷ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṫ ∔
[ṙ]]′]]] ≫ [[[ṫ ∔ [ṙ]]′ $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$
[[ṙ ∔ [ṫ]]′]]]] ; [[[[Taut2 ▷ [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇
[[ṫ ∔ [ṙ]]′ $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]] ▷ [[[ṫ ∔ [ṙ]]′ $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]
′]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]] ≫ [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔
[ṫ]]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]] ; [[L3.2(d)′ ≫ [[[
ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]] ⇒̇ [[[ṙ′ ∔ [ṫ]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]
⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]] ; [[[[Taut2 ▷ [[[ṫ ∔ [ṙ]]
$\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]]] ▷ [[[ṫ ∔ [ṙ′
]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]] ⇒̇ [[[ṙ′ ∔ [ṫ]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]] ⇒̇ [[
ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]]] ≫ [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [
[[ṙ′ ∔ [ṫ]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]]
] ; [[[[Taut4 ▷ [[ṙ′ ∔ [ṫ]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]]] ▷ [[[ṫ ∔ [ṙ]]
$\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [[[ṙ′ ∔ [ṫ]] $\stackrel{p}{=}$ [[ṙ ∔ [ṫ]]′]] ⇒̇ [[ṫ ∔ [ṙ′]]
$\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]]] ≫ [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [[ṫ ∔ [ṙ′]
] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]] ; [[[Gen′ ▷ [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [
[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]] ≫ ∀̇ṙ: [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇
[[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]] ; [[L3.2(h)′basis ≫ [[∀̇ṙ: [[[ṫ ∔ [ṙ
]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]] ⇒̇ ∀̇ṙ: [[ṫ ∔ [
ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]]]]] ; [[[[MP′ ▷ [[∀̇ṙ: [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ
]]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔ [ṫ]]]]]] ⇒̇ ∀̇ṙ: [[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [
ṫ]]]]] ▷ ∀̇ṙ: [[[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]] ⇒̇ [[ṫ ∔ [ṙ′]] $\stackrel{p}{=}$ [ṙ′ ∔
[ṫ]]]]] ≫ ∀̇ṙ: [[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]]] ; [[Gen′ ▷ ∀̇ṙ: [[ṫ ∔ [
ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]]] ≫ ∀̇ṫ: ∀̇ṙ: [[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]]]]]]]]]
]]]]]]] ⌉, p_0, c)]

[L3.2(h)′ $\stackrel{\text{stmt}}{\rightarrow}$ S′ ⊢ ∀̇ṫ: ∀̇ṙ: [[ṫ ∔ [ṙ]] $\stackrel{p}{=}$ [ṙ ∔ [ṫ]]]]]

[L3.2(h)′ $\stackrel{\text{tex}}{\rightarrow}$ “L3.2 (h)′”]

[L3.2(h)′ $\stackrel{\text{pyk}}{\rightarrow}$ “lemma prime three two h”]

Taut1

[Taut1 $\stackrel{\text{proof}}{\rightarrow}$ λc.λx.𝒫(⌈S′ ⊢ ∀a̲: ∀b̲: ∀c̲: [[a̲ ⇒̇ [b̲ ⇒̇ c̲]] ⊢ [[A2′ ≫ [[a̲ ⇒̇
[b̲ ⇒̇ c̲]] ⇒̇ [[a̲ ⇒̇ b̲] ⇒̇ [a̲ ⇒̇ c̲]]]] ; [[[[MP′ ▷ [[a̲ ⇒̇ [b̲ ⇒̇ c̲
]] ⇒̇ [[a̲ ⇒̇ b̲] ⇒̇ [a̲ ⇒̇ c̲]]]] ▷ [a̲ ⇒̇ [b̲ ⇒̇ c̲]]] ≫ [[a̲ ⇒̇ b̲]
⇒̇ [a̲ ⇒̇ c̲]]] ; [[[Weaken ▷ [[a̲ ⇒̇ b̲] ⇒̇ [a̲ ⇒̇ c̲]]] ≫ [b̲ ⇒̇ [[

$\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] ; [[A2' \gg [[\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] ; [[A1' \gg [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]]] ; [[[[\mathrm{MPTwice} \rhd [[\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] \dot{\Rightarrow} [[\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]] \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] \rhd [\underline{b} \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \rhd [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{b}]]] \gg [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]]]]]] \rceil, p_0, c)]$

$[\mathrm{Taut1} \stackrel{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \vdash [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]]$

$[\mathrm{Taut1} \stackrel{\mathrm{tex}}{\rightarrow}$ "Taut 1"$]$

$[\mathrm{Taut1} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma tautology one"$]$

Taut2

$[\mathrm{Taut2} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{d} \colon \forall \underline{e} \colon \forall \underline{f} \colon [[\underline{d} \dot{\Rightarrow} \underline{e}] \vdash [[\underline{e} \dot{\Rightarrow} \underline{f}] \vdash [[A1' \gg [[\underline{e} \dot{\Rightarrow} \underline{f}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]]]] ; [[[[MP' \rhd [[\underline{e} \dot{\Rightarrow} \underline{f}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]]]] \rhd [\underline{e} \dot{\Rightarrow} \underline{f}]] \gg [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]]] ; [[A2' \gg [[\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]] \dot{\Rightarrow} [[\underline{d} \dot{\Rightarrow} \underline{e}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} \underline{f}]]]] ; [[[[\mathrm{MPTwice} \rhd [[\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]] \dot{\Rightarrow} [[\underline{d} \dot{\Rightarrow} \underline{e}] \dot{\Rightarrow} [\underline{d} \dot{\Rightarrow} \underline{f}]]]] \rhd [\underline{d} \dot{\Rightarrow} [\underline{e} \dot{\Rightarrow} \underline{f}]]] \rhd [\underline{d} \dot{\Rightarrow} \underline{e}]] \gg [\underline{d} \dot{\Rightarrow} \underline{f}]]]]]]] \rceil, p_0, c)]$

$[\mathrm{Taut2} \stackrel{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{d} \colon \forall \underline{e} \colon \forall \underline{f} \colon [[\underline{d} \dot{\Rightarrow} \underline{e}] \vdash [[\underline{e} \dot{\Rightarrow} \underline{f}] \vdash [\underline{d} \dot{\Rightarrow} \underline{f}]]]]$

$[\mathrm{Taut2} \stackrel{\mathrm{tex}}{\rightarrow}$ "Taut 2"$]$

$[\mathrm{Taut2} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma tautology two"$]$

Taut3

$[\mathrm{Taut3} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \vdash [[A2' \gg [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] ; [[[[\mathrm{MPTwice} \rhd [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \dot{\Rightarrow} [[\underline{a} \dot{\Rightarrow} \underline{b}] \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]]] \rhd [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]]] \rhd [\underline{a} \dot{\Rightarrow} \underline{b}]] \gg [\underline{a} \dot{\Rightarrow} \underline{c}]]]]] \rceil, p_0, c)]$

$[\mathrm{Taut3} \stackrel{\mathrm{stmt}}{\rightarrow} S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [[\underline{a} \dot{\Rightarrow} \underline{b}] \vdash [[\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]] \vdash [\underline{a} \dot{\Rightarrow} \underline{c}]]]]$

$[\mathrm{Taut3} \stackrel{\mathrm{tex}}{\rightarrow}$ "Taut 3"$]$

$[\mathrm{Taut3} \stackrel{\mathrm{pyk}}{\rightarrow}$ "lemma tautology three"$]$

Taut4

$[\mathrm{Taut4} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil S' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [\underline{a} \vdash [[\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]] \vdash [[[\mathrm{Taut1} \rhd [\underline{b} \dot{\Rightarrow} [\underline{a} \dot{\Rightarrow} \underline{c}]]] \gg [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]]] ; [[[MP' \rhd [\underline{a} \dot{\Rightarrow} [\underline{b} \dot{\Rightarrow} \underline{c}]]] \rhd \underline{a}] \gg [\underline{b} \dot{\Rightarrow} \underline{c}]]]]] \rceil, p_0, c)]$

$[\mathrm{Taut4} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [\, \underline{a} \vdash [\, [\, \underline{b} \dot{\Rightarrow} [\, \underline{a} \dot{\Rightarrow} \underline{c} \,] \,] \vdash [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \,] \,]$

$[\mathrm{Taut4} \stackrel{\mathrm{tex}}{\rightarrow}$ “Taut 4”$]$

$[\mathrm{Taut4} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma tautology four”$]$

Weaken

$[\mathrm{Weaken} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\, \underline{a} \vdash [\, [\, \mathrm{A1}' \gg [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,] \,] ; [\, [\, [\, \mathrm{MP}' \rhd [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,] \,] \rhd \underline{a} \,] \gg [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,] \,] \,] \rceil, p_0, c)]$

$[\mathrm{Weaken} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon [\, \underline{a} \vdash [\, \underline{b} \dot{\Rightarrow} \underline{a} \,] \,] \,]$

$[\mathrm{Weaken} \stackrel{\mathrm{tex}}{\rightarrow}$ “Weaken”$]$

$[\mathrm{Weaken} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma weaken”$]$

M1.7

$[\mathrm{M1.7} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{b} \colon [\, [\, \mathrm{A1}' \gg [\, \underline{b} \dot{\Rightarrow} [\, [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} \underline{b} \,] \,] \,] ; [\, [\, \mathrm{A2}' \gg [\, [\, \underline{b} \dot{\Rightarrow} [\, [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} \underline{b} \,] \,] \dot{\Rightarrow} [\, [\, \underline{b} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] \,] ; [\, [\, [\, [\, \mathrm{MP}' \rhd [\, [\, \underline{b} \dot{\Rightarrow} [\, [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} \underline{b} \,] \,] \dot{\Rightarrow} [\, [\, \underline{b} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] \,] \rhd [\, \underline{b} \dot{\Rightarrow} [\, [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \dot{\Rightarrow} \underline{b} \,] \,] \,] \gg [\, [\, \underline{b} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] ; [\, [\, \mathrm{A1}' \gg [\, \underline{b} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] ; [\, [\, [\, \mathrm{MP}' \rhd [\, [\, \underline{b} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] \rhd [\, \underline{b} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] \gg [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,] \,] \,] \,] \,] \rceil, p_0, c)]$

$[\mathrm{M1.7} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{b} \colon [\, \underline{b} \dot{\Rightarrow} \underline{b} \,] \,]$

$[\mathrm{M1.7} \stackrel{\mathrm{tex}}{\rightarrow}$ “
M1.7”$]$

$[\mathrm{M1.7} \stackrel{\mathrm{pyk}}{\rightarrow}$ “mendelson one seven”$]$

MPTwice

$[\mathrm{MPTwice} \stackrel{\mathrm{proof}}{\rightarrow} \lambda c.\lambda x.\mathcal{P}(\lceil \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [\, [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \vdash [\, \underline{a} \vdash [\, \underline{b} \vdash [\, [\, [\, [\, \mathrm{MP}' \rhd [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \,] \rhd \underline{a} \,] \gg [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] ; [\, [\, [\, \mathrm{MP}' \rhd [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \rhd \underline{b} \,] \gg \underline{c} \,] \,] \,] \,] \,] \rceil, p_0, c)]$

$[\mathrm{MPTwice} \stackrel{\mathrm{stmt}}{\rightarrow} \mathrm{S}' \vdash \forall \underline{a} \colon \forall \underline{b} \colon \forall \underline{c} \colon [\, [\, \underline{a} \dot{\Rightarrow} [\, \underline{b} \dot{\Rightarrow} \underline{c} \,] \,] \vdash [\, \underline{a} \vdash [\, \underline{b} \vdash \underline{c} \,] \,] \,] \,]$

$[\mathrm{MPTwice} \stackrel{\mathrm{tex}}{\rightarrow}$ “MPTwice”$]$

$[\mathrm{MPTwice} \stackrel{\mathrm{pyk}}{\rightarrow}$ “lemma mp twice”$]$

$\dot{*}$

$[\dot{\mathsf{x}} \stackrel{\text{tex}}{\rightarrow}$ "
\dot{#1.
}"]

$[\dot{\mathsf{x}} \stackrel{\text{pyk}}{\rightarrow}$ "* peano var"]

$*'$

$[\mathsf{x}' \stackrel{\text{tex}}{\rightarrow}$ "#1.'"]

$[\mathsf{x}' \stackrel{\text{pyk}}{\rightarrow}$ "* peano succ"]

$* \mathop{\dot{\cdot}} *$

$[\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{\cdot}} #2."]

$[\mathsf{x} \mathop{\dot{\cdot}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano times *"]

$* \mathop{\dot{+}} *$

$[\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\mathop{\dot{+}} #2."]

$[\mathsf{x} \mathop{\dot{+}} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano plus *"]

$* \stackrel{p}{=} *$

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{tex}}{\rightarrow}$ "#1.
\stackrel{p}{=} #2."]

$[\mathsf{x} \stackrel{p}{=} \mathsf{y} \stackrel{\text{pyk}}{\rightarrow}$ "* peano is *"]

$*^{\cal P}$

$[\mathsf{x}^{\cal P} \stackrel{\text{val}}{\rightarrow} \mathsf{x} \stackrel{r}{=} \lceil \dot{\mathsf{x}} \rceil]$

$[\mathsf{x}^{\cal P} \stackrel{\text{tex}}{\rightarrow}$ "#1.
{} ^ {\cal P}"]

$[\mathsf{x}^{\mathcal{P}} \stackrel{\text{pyk}}{\to}$ "$*$ is peano var"$]$

$\dot{\neg}\, *$

$[\dot{\neg}\, \mathsf{x} \stackrel{\text{tex}}{\to}$ "
\dot{\neg}\, #1."]

$[\dot{\neg}\, \mathsf{x} \stackrel{\text{pyk}}{\to}$ "peano not $*$"$]$

$\dot{\forall} *\colon *$

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{tex}}{\to}$ "
\dot{\forall} #1.
\colon #2."]

$[\dot{\forall}\mathsf{x}\colon \mathsf{y} \stackrel{\text{pyk}}{\to}$ "peano all $*$ indeed $*$"$]$

$* \mathrel{\dot{\Rightarrow}} *$

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{tex}}{\to}$ "#1.
\mathrel{\dot{\Rightarrow}} #2."]

$[\mathsf{x} \mathrel{\dot{\Rightarrow}} \mathsf{y} \stackrel{\text{pyk}}{\to}$ "$*$ peano imply $*$"$]$

The pyk compiler, version 0.grue.20050603 by Klaus Grue
GRD-2005-06-30.UTC:07:17:36.765255 = MJD-53551.TAI:07:18:08.765255 = LGT-4626832688765255e-6